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Mathematics

6

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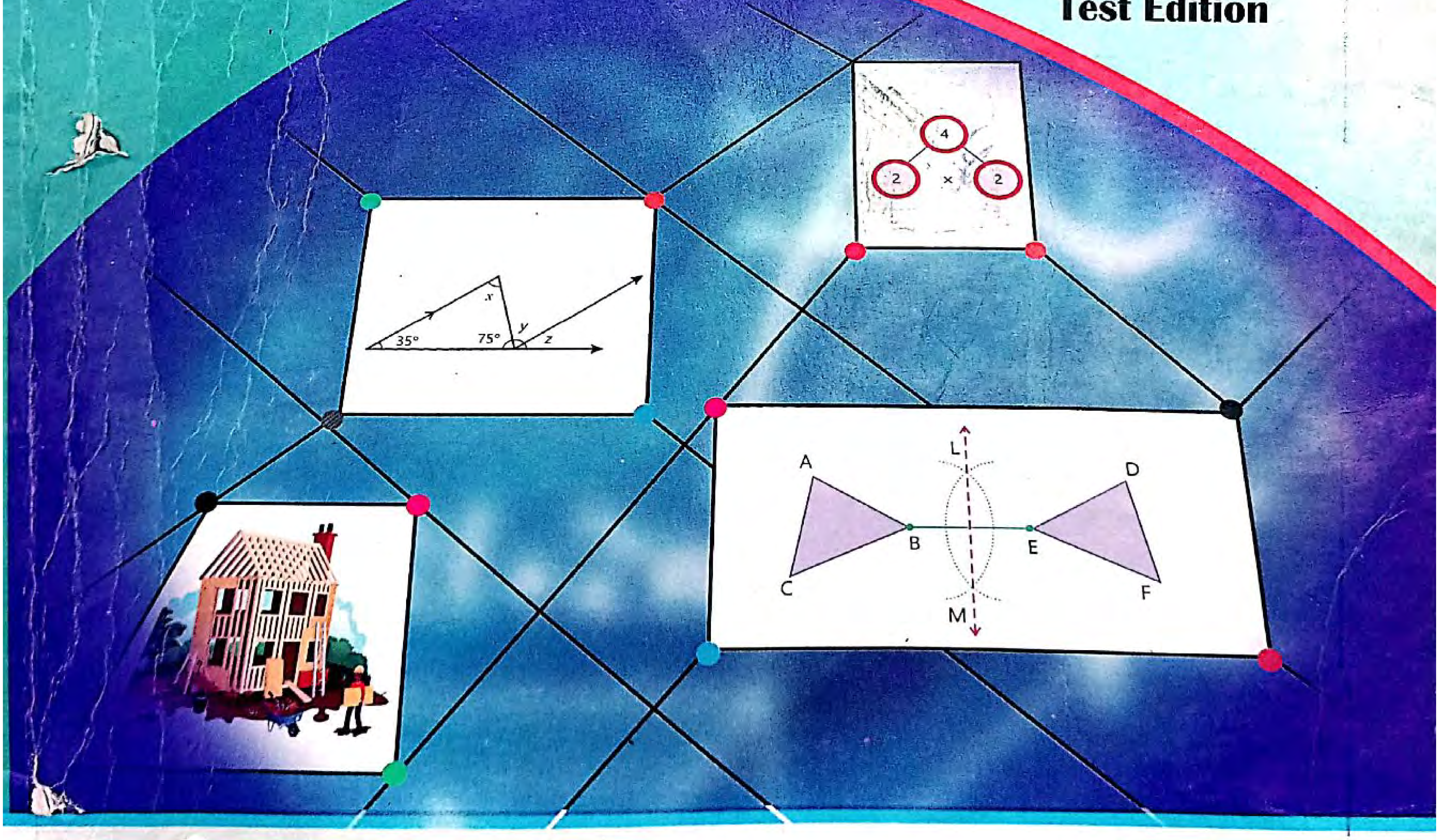

KHYBER PAKHTUNKHWA TEXTBOOK BOARD, PESHAWAR

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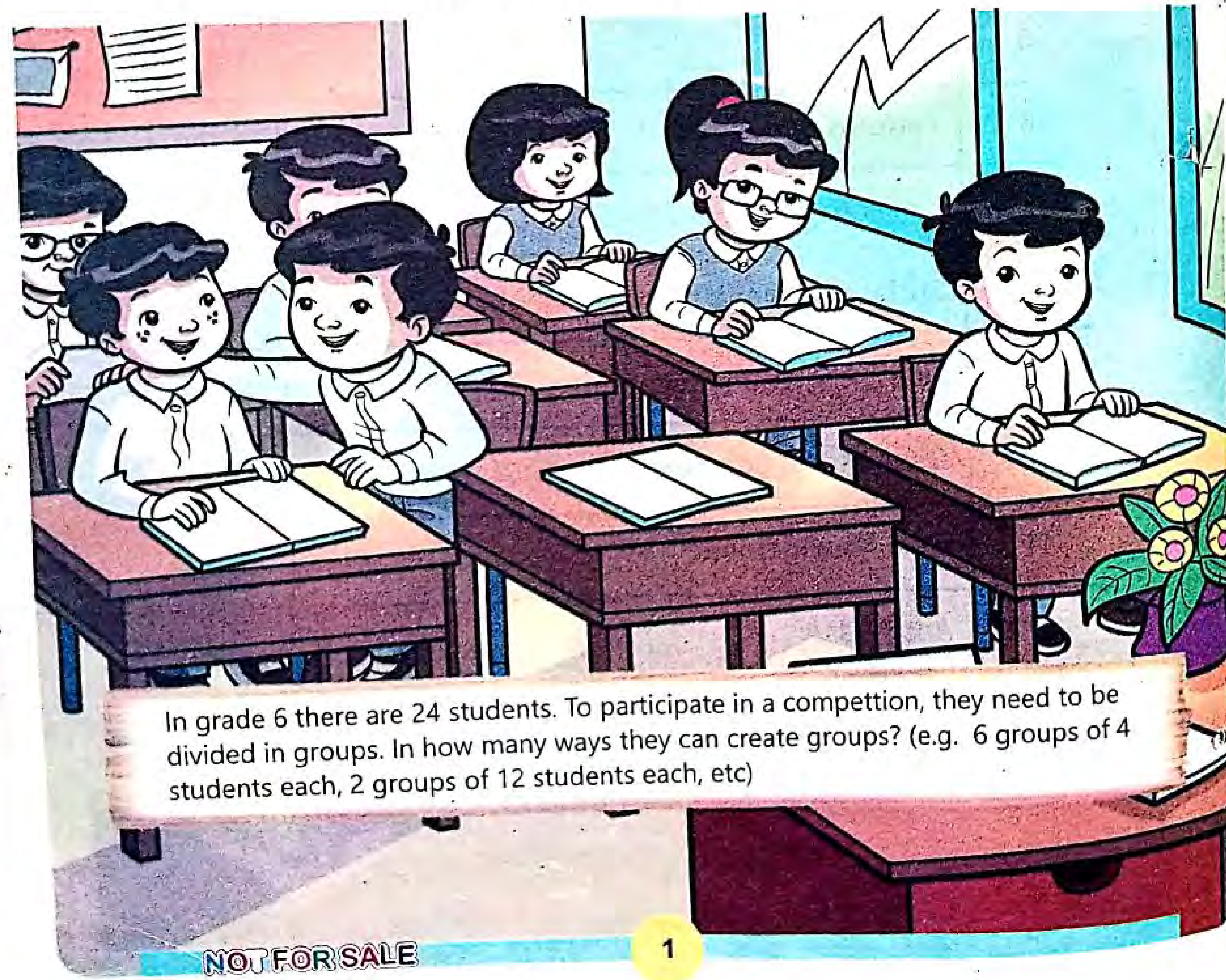
Content

Unit 1 Multiples and Factors

Student Learning Outcomes

After completing this unit, students will be able to:

- Identify:
  - Factors of up to 3-digit numbers
  - Multiples of up to 2-digit numbers
  - Prime factors of up to 4-digit numbers and express in index notation
- Identify base and exponent and express numbers given in expanded form in index notation and vice versa.
- Find H.C.F and L.C.M of two or three numbers (up to 3-digits) using various methods (for instance prime factorization and division method).
- Solve real-world word problems involving H.C.F and L.C.M.
- Recognise and calculate squares of up to 2-digit numbers.



In grade 6 there are 24 students. To participate in a competition, they need to be divided in groups. In how many ways they can create groups? (e.g. 6 groups of 4 students each, 2 groups of 12 students each, etc)

Introduction

We come across various routine problems where directly or indirectly the use of factors, multiples, HCF and LCM is included. Any situation where we divide the objects or things into groups and more number of items into small groups.

1.1 Factors and Multiples

1.1.1 Factors

A **factor** is a number that divides another number withou t leaving any remainder.

Ahmar has 15 pencils. He wants to make 5 equal groups of pencils. How can we tell if he can divide the pencils into 5 equal groups?

To find if he can divide the pencils into 5 equal groups or not, we need to check if 5 is a factor of 15 or not.

To find it out, we need to divide 15 by 5.

$$15 \div 5 = 3 \text{ r } 0$$

As 5 divides 15 exactly with zero remainder, so 5 is a factor of 15.

3 is also a factor of 15 as 3 divides 15 exactly.

$$15 \div 3 = 5 \text{ r } 0$$

Divisor or Factor → 5) 15 ← Dividend; 3 ← Quotient or factor; −15; 0 ← Remainder

Previous Knowledge Check

- What is meant by prime numbers?
- What is meant by factors of a number?
- Can you find the prime factors of the following numbers?
  a) 4 b) 12 c) 21

Quick Check

Can Ahmar divide 15 pencils in:
a) 2 equal groups?
b) 3 equal groups?

We can find factors of any number by dividing the number by 1, 2, 3, ... and so on. Start from 1 and continue until the factors start repeating.

Example 1: Find the factors of 16.

Solution: 16 = 1 × 16

16 = 2 × 8

16 = 4 × 4 (factors are repeating so we will stop here).

Factors of 16 are 1, 2, 4, 8 and 16.

Note it down

The smallest factor of any number is 1 and the greatest factor of any number is the number itself.

TEACHING POINT: Write different numbers on the board. Instruct the students to find the factors of the numbers and stop finding factors if factors start repeating.

Example 2: Find the factors of 56.

Solution: We can find the factors of 56 as:

56 = 1 × 56
56 = 2 × 28
56 = 4 × 14
56 = 7 × 8 (factors are repeating so, we will stop here)

Factors of 56 are 1, 2, 4, 7, 8, 14, 28 and 56.

All these factors divide the number 56 completely leaving no remainder.

From the above examples, we can see that:

- 1 is a factor of every number and it is the smallest factor of the numbers.
- Every number is the factor of itself and it is the greatest factor.

Note it down

The number greater than 1 has at least two factors.

Example 3: Is 15 a factor of 250 ?

Solution: To find if 15 is a factor of 250 or not, divide 250 by 15.

```
      16
15 ) 250
    -15
     100
   -  90
      10
```

As 15 does not divide 250 completely, it is not a factor of 250.

Quick Check

a) Find all the factors of 220.
b) Is 7 the factor of 52?

1.1.2 Multiples

Multiples are the numbers obtained by multiplying a certain number by another number/integer.

Example 1:

Find the first 10 multiples of 18.

Solution:

To find the multiples of 18 we recall the table of 18.

1 × 18 = 18, 2 × 18 = 36, 3 × 18 = 54, 4 × 18 = 72,
5× 18 = 90, 6 × 18 = 108 7 × 18 = 126, 8 × 18 = 144,
9 × 18 = 162, 10 × 18 = 180

Multiples of 18 are 18, 36, 54, 72, 90, 108, 126, 144, 162, 180.

Previous Knowledge Check

- How we can find the multiples of numbers?
- Find the first ten multiples of two1-digit numbers?

Teaching Point: Write different numbers on the board and explain to the students that a multiple is a dividend into which a factor can divide. Instruct them to find the first 5 multiples of the numbers.

Example 2:

Find the multiples of 24 less than 100.

Solution:

1 × 24= 24, 2 × 24= 48, 3 × 24 = 72, 4 × 24 = 96

Multiples of 24 less than 100 are 24, 48, 72 and 96.

Note it down

Every number is a multiple of all its factors.

Quick Check

Find all the multiples of 44 less than 200.

Example 3:

Find the multiples of 6 less than 40.

Solution:

1 × 6 = 6, 2 × 6 = 12, 3 × 6 = 18, 4 × 6 = 24,
5 × 6 = 30, 6 × 6 = 36

The multiples of 6 less than 40 are 6, 12, 18, 24, 30 and 36.

Quick Check

1. Find the factors of the following:
a) 48 b) 23 c) 45
2. Find the first 2 multiples of:
a) 13 b) 18 c) 35

Example 4:

Find the multiples of 9 between 30 and 80.

Solution:

4 × 9 = 36, 5 × 9 = 45, 6 × 9 = 54, 7 × 9 = 63,
8 × 9 = 72, 9 × 9 = 81 (stop here as this multiple of 9 is greater than 80)

The multiples of 9 between 30 and 80 are 36, 45, 54, 63 and 72.

Exercise 1.1

1 Find all the factors of the following numbers.

a) 26	b) 112	c) 150	d) 175	e) 68	f) 145
g) 80	h) 100	i) 23	j) 47	k) 58	l) 64
m) 42	n) 36	o) 70	p) 90		

2 Find the multiples of the given number less than 250.

a) 12	b) 25	c) 37	d) 48	e) 60	f) 75
g) 45	h) 57				

3 Find the multiples of 14 less than 500.

4 Is 18 a factor of 256?

1.2 Prime factorization

Prime factorization is a process in which a number is written as a product of its prime factors.

Previous Knowledge Check

- What is meant by prime factors?

Example 1:

Find the prime factors of 60.

Solution:

2	60
2	30
3	15
	5

Step I: First we break 60 into its factors as: 2×30

Step II: Now we break 30 into its factors as: 2×15

Step III: Now we break 15 into its factors as: 3×5

As 3 and 5 are prime numbers, so we stop here.

$60 = 2 \times 30$

$= 2 \times 2 \times 15$

$= 2 \times 2 \times 3 \times 5$

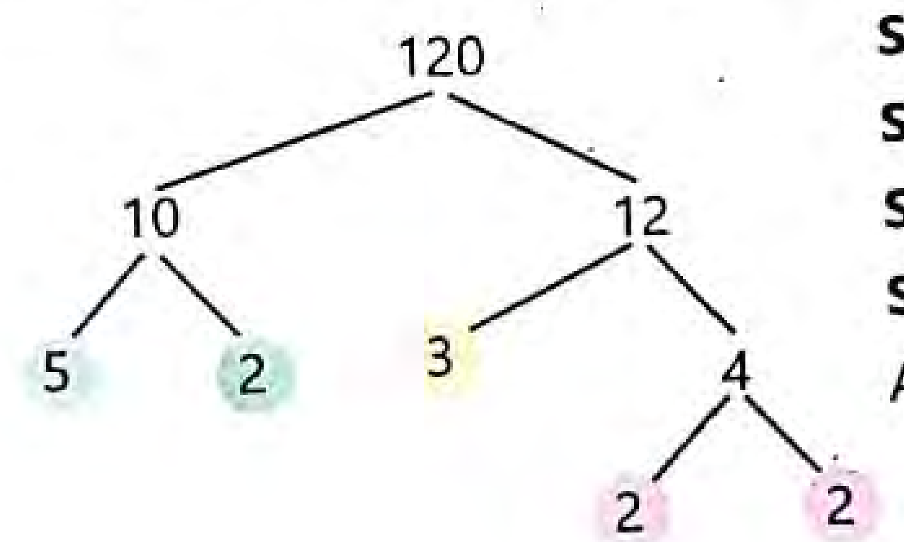
The prime factorization of $60 = 2 \times 2 \times 3 \times 5$.

Note it down

The order of factorization does not affect prime factorization.

Example 2:

Find prime factors of 120 by factor tree method.



Step I: First we break 120 into its factors as: 10 and 12

Step II: Now we break 10 into its factors as: 2×5

Step III: Now we break 12 into its factors as: 3×4

Step IV: Now we break 4 into its factors as: 2×2

As the end factors are all primes. So, we stop here.

So, the prime factors of 120 are $2 \times 2 \times 2 \times 3 \times 5$

We can also write the prime factors of 120 as: $2^3 \times 3 \times 5$

Here 2^3 is the representation of number in index notation.

Teaching Point: Make two groups of the students. Write some numbers on the board and instruct them to find its prime factors by using prime factorization and factor tree method.

1.2.1 Index Notation

Do you know what index notation means?

The **index** of a number represents how many times a number is multiplied by itself. It is a small raised number written next to the number which is being multiplied by itself. Index is also called power/exponent.

Look at $2 \times 2 \times 2 \times 2 \times 2 \times 2$.

It can be written in index form or in power/exponent as:

$2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

Here 2 is called the base and 6 is called the index or power. 2^6 represents that number 2 is 6 times multiplied by itself. We can read 2^6 as 2 to the power of 6.

In 2^6, 2 is called base and 6 is called exponent.

Quick Check

Write the following in index notation:

a) $3 \times 3 \times 5 \times 5$

b) $2 \times 2 \times 7 \times 11 \times 11 \times 13$

Example 3:

Write prime factors of 3408. Express the 'factors' in index notation.

2	3408
2	1704
2	852
2	426
3	213
	71

Step I: First we break 3408 into its factors as: 2×1704

Step II: Now we break 1704 into its factors as: 2×852

Step III: Now we break 852 into its factors as: 2×426

Step IV: Now we break 426 into its factors as: 2×213

Step V: Now we break 213 into its factors as: 3×71

Now 3 and 71 are prime factors so, we stop here.

$3408 = 2 \times 2 \times 2 \times 2 \times 3 \times 71$

Here 2 is repeated 4 times. So we can write it as:

$3408 = 2^4 \times 3 \times 71$

Here 3408 is the standard form, $2 \times 2 \times 2 \times 2 \times 3 \times 71$ is the expanded form and $2^4 \times 3 \times 71$ is the index form.

Quick Check

Find the prime factors and write the following in index notation:

a) 5662 b) 45981

Teaching Point: Share following online game with students to practice prime factorization https://www.mathplayground.com/factortrees.html

Example 4:

Write the prime factors of 2058 in the form of index notation.

2	2058
3	1029
7	343
7	49
	7

Standard form = 2058

Expanded form = $2 \times 3 \times 7 \times 7 \times 7$

Index form = $2 \times 3 \times 7^3$

Example 5: Express the 6^7 in expanded form.

As 6 is the base and 7 is the exponent.

6 raised to the power 7 means 6 is repeated 7 times or 6 is multiplied 7 times. We can write it in **expanded form** as:

$6^7 = 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

Exercise 1.2

1 **Find the prime factors of the following numbers and write it in the form of index notation.**

a) 144 b) 314 c) 5624 d) 3084 e) 7880
f) 6024 g) 8460 h) 2986

2 **Find the prime factors of the following numbers by using factor tree method and write it in the form of index notation.**

a) 2415 b) 208 c) 4256 d) 4834 e) 6745
f) 1268 g) 4580 h) 9046

3 **Write the following in expanded form.**

a) 2^6 b) 4×7^5 c) $3 \times 5^2 \times 11^4$ d) $2^2 \times 3^3 \times 5^6$ e) $13^6 \times 17^3$
f) 4×7^5 g) $3^2 \times 5^6 \times 17^2$ h) $7^2 \times 11^3 \times 29$

4 **Express the following in index notation.**

a) $5 \times 5 \times 5$ b) $2 \times 2 \times 5 \times 5 \times 5 \times 5$ c) $1 \times 2 \times 2 \times 7 \times 7 \times 7$
d) $11 \times 11 \times 13 \times 13 \times 13 \times 19 \times 19$ e) $7 \times 7 \times 7 \times 11 \times 11$

1.3 Highest Common Factor (HCF) and Least Common Multiple (LCM)

1.3.1 Highest Common Factor

I have two pieces of wires of length 180 cm and 222 cm. How can I know the length of equal smaller pieces in which I can cut the wires into equal pieces, without any leftover wire?

We can cut the wires into equal pieces by finding greatest possible length of wire that exactly divides 180 cm and 222 cm.

The Highest Common Factor (HCF) of two or more numbers is the greatest factor that divides the given numbers exactly without any remainder.

Let's learn how to find the HCF of two or more numbers using the following methods.

i) Prime Factorization Method ii) Division Method

Prime Factorization Method

Previous Knowledge Check

Find the common factors of 45 and 60 by prime factorization method.

To find HCF by prime factorization method we follow these steps.

2	180
2	90
3	45
3	15
5	5
	1

2	222
3	111
37	37
	1

Note it down

If prime factors of two or more numbers are the same then their factors are called **common prime factors.**

Prime factorization of 180 = $2 \times 2 \times 3 \times 3 \times 5$

Prime factorization of 222 = $2 \times 3 \times 37$

Common prime factors = 2 and 3

Product of common prime factors = $2 \times 3 = 6$

So the greatest possible length of each piece of wire is 6 cm.

Example 1: Find the H.C.F of 340, 488 and 512 using prime factorization method.

Solution:

First, find the prime factors of all these numbers.

2	340
2	170
5	85
17	17
	1

2	488
2	244
2	122
61	61
	1

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of 340 $= ②\times②\times 5 \times 17 = 2^2 \times 5 \times 17$

Prime factors of 488 $= ②\times②\times 2 \times 61 = 2^3 \times 61$

Prime factors of 512 $= ②\times②\times 2 \times 2 \times 2\times 2 \times 2 \times 2 \times 2 = 2^9$

The common prime factors are 2, 2

The product of common prime factors $= 2^2 = 4$

So, HCF = 4

Quick Check

Find HCF of 215, 308 and 125 by prime factorization method.

Division Method

To find the HCF by division method, we follow these steps.

Step I: First, divide the greater number by the smaller number.

Step II: Then divide the smaller number with the remainder obtained in step I.

Step III: Divide the divisor of the second step with the remainder obtained in step II.

Step IV: Continue the above process till you get 0 as the remainder.

The last divisor will be the HCF of the given numbers.

TEACHING POINT: Write the terms 'prime factors', 'common factors' and 'HCF' on the board and ask the students to explain these. Encourage them for their responses. Place some number cards on the table and invite them randomly to come to the table and pick two cards and find the HCF.

Example 1:

Find the HCF of 278 and 132 by division method.

Solution:

smaller number ← 132) 278 (2 → greater number
−264
remainder ← 14) 132 (9
−126
6) 14 (2
−12
HCF ← 2) 6 (3
−6
0

Example 2:

Find the HCF of 122, 280 and 360 by division method.

Solution:

280) 360 (1
−280
80) 280 (3
−240
40) 80 (2
−80
0

So, the H.C.F of 280 and 360 is 40.

Now, find the H.C.F of 40 and 122.

40) 122 (3
−120
2) 40 (20
−4
0
−0
0

So, the H.C.F of 122, 280 and 360 is 2.

Quick Check

Find HCF of 728, 566 and 412 by division method.

TEACHING POINT: Put different 2-digit numbers flashcard on the table. Make groups of the students and ask each group to pick 2 or 3 numbers and find the HCF by division method. Then share their answer with the other groups to check if they have done it correctly.

Example 3:

Find HCF of 255, 345 and 495 by division method.

Solution:

```
         1
255 | 345
     -255       2
      ---------------
      90 | 255
          -180      1
          -----------
           75 | 90
               -75      5
               ----------
                15 | 75
                    -75
                    ----
                      0
```

Now we divide 495 by 15.

```
        33
15 | 495
    - 45
    ----
      45
    - 45
    ----
       0
```

So, 15 is the HCF of 255, 345 and 495.

1.3.2 Least Common Multiple

Asim goes to library after every 4 days while Amir goes to library after every 12 days. If both of them are in library today, find when they will next visit the library on the same day.

We can easily find it by finding the least common multiple of these numbers.

The **Least Common Multiple (LCM)** of two or more numbers is the smallest common multiple of given numbers.

To find the LCM of different numbers, use the following methods.

i) Prime Factorization Method

ii) Division method

Previous Knowledge Check

- How can we find the common multiples of two or more numbers?
- Find the common multiples of 36 and 48.

Note it down

A common multiple is a number that is a multiple of two or more numbers.

Prime Factorization

To find LCM by prime factorization we follow these steps:

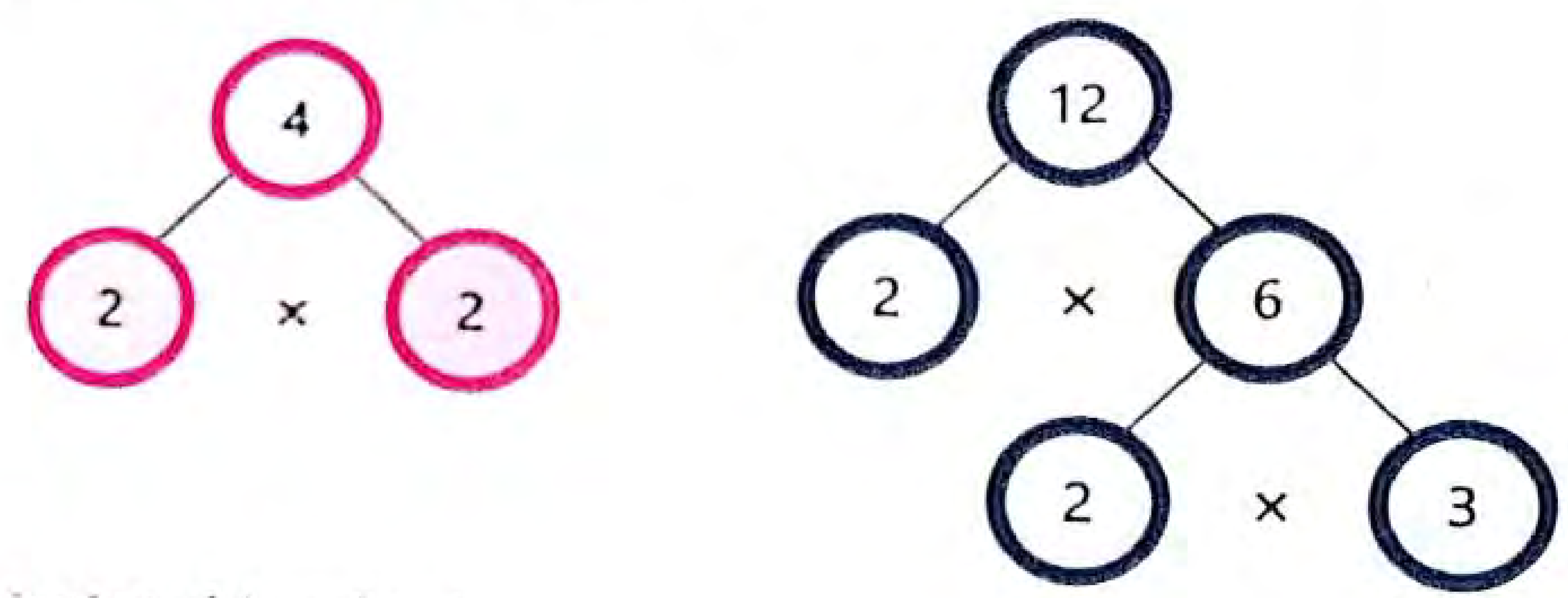
Step I: First find all prime factors of the given numbers.

Step II: find common prime factors of the given numbers.

Step III: Then find the non-common prime factors.

Step IV: Find the product of common and non-common prime factors.

Let us find the LCM of 4 and 12 by prime factorization method. For this here we will use factor trees to find the factors of 4 and 12.



Prime factorization of 4 $= 2 \times 2$

Prime factorization of 12 $= 2 \times 2 \times 3$

The common prime factors of 4 and 12 are 2 and 2.

The non-common prime factor of 4 and 12 is 3.

Now use the formula.

LCM = **Product of common prime factors of 2 or more numbers** × **Product of non-common prime factors**

LCM $= 2 \times 2 \times 3$

LCM $= 4 \times 3 = 12$.

So, the LCM of 4 and 12 is 12.

So, Asim and Amir will next visit the library together after 12 days.

Put different numbers on the table. Make groups of the students and ask each group to pick 2 or 3 numbers and find the LCM. Then share their answer with the other groups to check if they have done it correctly.

Example 1:
Find LCM of 128, 256 and 512 by prime factorization method.
Solution:
First find all the prime factors of 128, 256 and 512.

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factorization of 128 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
Prime factorization of 256 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
Prime factorization of 512 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
The common prime factors are of 128, 256 and 512 are 2, 2, 2, 2, 2, 2, 2.
The non-common prime factors of 128, 256 and 512 is 2, 2.

Quick Check
Find LCM of 114, 216, 425 and 322 by prime factorization method.

LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors

$LCM = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 512$
So, the LCM = 512

Put different 2-digit numbers in a basket. Make groups of the students and ask each group to pick 2 or 3 numbers and find the LCM. Then share their answer with the other groups to check if they have done it correctly.

Division Method

To find the LCM of two or more numbers by division method, we follow these steps:
Step I: Write the numbers in a horizontal line, separating them by commas.
Step II: Divide the numbers by a suitable prime number, which exactly divides at least two of the given numbers.
Step III: Continue the process until all quotients of all the numbers become '1'.
Step IV: Find the product of all the prime factors.

Example 1:
Find the LCM of 120, 140 and 130 by division method.
Solution:

2	120, 140, 130
2	60, 70, 65
3	30, 35, 65
5	10, 35, 65
2	2, 7, 13
7	1, 7, 13
13	1, 1, 13
	1, 1, 1

LCM = Product of prime factors
$LCM = 2 \times 2 \times 3 \times 5 \times 2 \times 7 \times 13$
LCM= 10920

Example 2:
Find the LCM of 160, 248, and 364 by division method.
Solution:

2	160 , 248 , 364
2	80 , 124 , 182
2	40 , 62 , 91
2	20 , 31 , 91
2	10 , 31 , 91
5	5 , 31 , 91
7	1 , 31 , 91
13	1 , 31 , 13
31	1 , 31 , 1
	1 , 1 , 1

Quick Check
Find LCM of 400, 456 and 356 by division method.

LCM = Product of prime factors
$LCM = 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 7 \times 13 \times 31$
LCM= 451360

Share the following online game with students to practice Highest Common Factor (HCF) and Least Common Multiple (LCM).
https://www.transum.org/software/SW/Starter_of_the_day/Students/HCF_LCM.asp
https://www.transum.org/software/SW/Starter_of_the_day/Students/HCF_LCM.asp?Level=2
https://www.tes.com/teaching-resource/lcm-and-hcf-treasure-hunt-11743277

1.3.3 Relationship between HCF and LCM

There is a very close relationship between HCF and LCM. Let two numbers 14 and 24 are given. The HCF of 14 and 24 is 2 and their LCM is 168.

If we find the product of the numbers:

$$14 \times 24 = 336$$

Now we find the product of the HCF and LCM:

$$HCF \times LCM = 2 \times 168 = 336$$

From this we find that the product of 14 and 24 is equal to the product of their LCM and HCF.

We can say that the product of two numbers is equal to the product of their HCF and LCM.

Product of the two numbers = Product of their HCF and LCM

In general if we represent the two numbers by "a" and "b".

Then we can represent formula as:

$$a \times b = HCF \times LCM$$

This formula is applicable if only two numbers are given.

Example 1:

Find the LCM of the number 256 and 128 if their HCF is 128.

Solution:

As we know the formula:

$$HCF \times LCM = a \times b$$

$$128 \times LCM = 256 \times 128$$

$$LCM = \frac{256 \times 128}{128}$$

$$LCM = 256$$

Quick Check

If LCM of two numbers is 24 and HCF is 12, find the product of the two numbers.

Example 2:

If HCF of the number 16 and LCM of the number is 60 then find the product of two numbers.

Solution:

As we know the formula:

$$HCF \times LCM = a \times b$$

$$16 \times 80 = a \times b$$

$$1280 = ab$$

TEACHING POINT: Guide the students about the relationship between the HCF and LCM. Make groups of the students and give flash cards of numbers to one group and HCF and LCM cards to the other group. Instruct the first group to show the two number cards and the other group show their LCM and HCF cards.

Exercise 1.3

1 Find the HCF of the following numbers using prime factorization method.

a) 224, 142 b) 520, 420, 124 c) 168, 400, 540
d) 366, 145, 780 e) 678, 424, 784 f) 905, 560
g) 248, 104, 565 h) 578, 456 i) 385, 855, 620

2 Find the HCF of the following numbers using division method.

a) 100, 350, 480 b) 678, 456 c) 400, 350, 250
d) 100, 245, 168 e) 332, 480, 290 f) 734, 202

3 Find the LCM of the given numbers using prime factorization method.

a) 120, 150,145 b) 108, 116, 128 c) 240, 412
d) 158, 250, 240 e) 296, 564, 444 f) 246, 378 and 984
g) 112, 142 h) 764, 986,130 i) 864, 789, 556

4 Find the LCM of the given numbers using division method.

a) 280, 250, 290 b) 456, 230, 900 c) 150, 560, 450
d) 789, 648, 832 e) 898, 986, 786 f) 446, 340, 850
g) 908, 682, 672 h) 408, 126, 522 i) 765, 635, 235

5 The LCM of two numbers is 456 and HCF is 6. If one number is 24 then find the other number.

6 The HCF of two numbers is 7 and LCM is 588. Find the product of two numbers.

7 The HCF of two numbers is 4 and the product of the two numbers is 512. Find the LCM of the numbers.

1.4 Real-Life Problems

Example 1:

What is the greatest length of meter tape that can exactly measure the three ropes of length 150 cm, 125 cm and 120 cm?

Solution:

To find the greatest length of meter tape, find the HCF.

2	150
5	75
5	15
3	3
	1

5	125
5	25
5	5
	1

2	120
2	60
2	30
3	15
5	5
	1

Prime factorization of $150 = 2 \times 3 \times 5 \times 5 = 2 \times 3 \times 5^2$

Prime factorization of $125 = 5 \times 5 \times 5 = 5^3$

Prime factorization of $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$

Common factors = 5

HCF = 5

So, the greatest length of the tape is 5 cm.

Example 2:

Find the minimum length of the pipe which can exactly be cut into pieces of length 140 cm, 180 cm and 60 cm.

Solution:

To find the minimum length of the pipe we have to find LCM.

2	140 , 180 , 60
2	70 , 90 , 30
3	35 , 45 , 15
3	35 , 15 , 5
5	35 , 5 , 5
7	7 , 1 , 1
	1 , 1 , 1

So, prime factorization is $= 2 \times 2 \times 3 \times 3 \times 5 \times 7$

LCM = 1260 cm

Quick Check

If the lengths of the three ribbons are 32 cm, 44 cm and 64 cm. Find the maximum length of the three ribbons.

TEACHING POINT: Tell the students clue word of LCM and HCF and then ask them by using these clue words make word problems of HCF and LCM. Share their word problems with their class- fellows..

(47)

Exercise 1.4

1. Find the minimum capacity of a water tank that can completely measure the amounts of 460 *l*, 245 *l* and 525 *l*.
2. Find the greatest number that can divide 460, 648 and 984.
3. Find the smallest number which exactly divides 406, 348 and 256.
4. The LCM of two numbers is 96 and HCF is 24. If one number is 32 then find the other number.
5. The HCF of two numbers is 18 and LCM is 75. Find the product of two numbers.
6. LCM of two numbers is 678 and the product of the two numbers is 4746. Find the HCF of the numbers.
7. Umar has two pieces of ropes. One is 18 metres long and the other is 112 metres long. He wants to cut it into pieces that are all of the same length, without any remainder. What is the greatest length that he can cut them into?
8. For an Iftar party, Aiza wants to serve chicken bread and nuggets to the guests invited. She wants to do it in such a way that each serving contains the same combination of chicken bread and nuggets. If there are 24 slices of chicken bread and 120 nuggets, find the maximum number of servings she can prepare.
9. Nida goes for a walk every 10 days and Sehrish goes for a walk every 25 days to the same park. If they both went for a walk today, then how many days from now will they next be in the park on the same day?
10. Find the smallest number which is exactly divisible by 40, 224 and 400 without leaving any remainder.

1.5 Squares and Perfect Squares

We know that we can find the area of a square by multiplying its length with its length. We also know about exponents where repeated multiplication of numbers is written in the form of power to make calculation simpler. Now we will learn about the square and perfect square of natural numbers.

1.5.1 Square of a Number

When we multiply a number by itself, we can write that number to the power of 2.
For example, when we multiply 2 by itself i.e. $2 \times 2 = 2^2$ we read it as 2 raised to the power of 2 or 2 to the power 2 or square of 2.
When a number is multiplied by itself, the value (result) we get is called the **square** of that number.

Previous Knowledge Check

- How can we find the multiplication of two numbers by repeated addition?

Example 1:
Observe the squares of the following numbers.

a) $4 \times 4 = 4^2 = 16$, $(-4) \times (-4) = (-4)^2 = 16$
b) $5 \times 5 = 5^2 = 25$, $(-5) \times (-5) = (-5)^2 = 25$
c) $(\frac{1}{7}) \times (\frac{1}{7}) = (\frac{1}{7})^2 = (\frac{1}{49})$, $(-\frac{1}{7}) \times (-\frac{1}{7}) = (\frac{-1}{7})^2 = (\frac{1}{49})$
d) $12 \times 12 = 12^2 = 144$, $(-12) \times (-12) = (-12)^2 = 144$

Note it down

The square of any two integers is always positive.

We can see that the square of every number (positive, negative, fractional, etc.) is always positive.

1.5.2 Perfect Squares

When a number is multiplied by itself, the product is called the **square** of that number. A number will be called a **perfect square** if it is a square of any number. A perfect square has a pair of the same factors.
In the above examples, 16, 25, 49, 81, etc., are perfect squares as they are squares of 4, 5, 7, 9, etc. respectively.

1^2	1×1	1
2^2	2×2	4
3^2	3×3	9
4^2	4×4	16
5^2	5×5	25
6^2	6×6	36
7^2	7×7	49
8^2	8×8	64
9^2	9×9	81
10^2	10×10	100
11^2	11×11	121
12^2	12×12	144

TEACHING POINT
Use the idea from the link to have the students practice square of numbers:
https://www.tes.com/teaching-resource/square-numbers-matchup-game-11059444

Exercise 1.5

1 **Find the square of the following numbers.**

a) 25	b) 77	c) 98	d) 11
e) 46	f) 69	g) 13	h) 37
i) 45	j) 79	k) 67	l) 50

2 **A glass is cut into square shape. If the length of the glass is 98 cm, what is the area of the surface of the glass? (Hint: Area of Square = Length × Length)**

Think Higher

- The length of a side of a square shaped park is 26 metres. What will be the cost of grassing the park at the rate of Rs 320 per metre square?
- Sehrish wants to distribute Rs 1000 among needy children on Eid. For this purpose, she needs change of Rs 1000 in smaller denominations. In how many ways can she get the change in smaller denominations?
- Hamza is thinking of a number between 1 to 100. Use the given clues to find the number.
  - The number is greater than 10.
  - The number is not a multiple of 4.
  - The number is not a multiple of 8 as well.
  - 6 is the factor of this number.
  - The number is a multiple of 9.
  - The number is even.
  - Its ones digit is greater than its tens digit.
  - Its tens digit ☐

Hint:
Use the hundreds chart to help you.

Summary

- The smallest factor of any number is 1 and the greatest factor of any number is the number itself.
- The numbers greater than 1 have at least two factors.

Vocabulary

- Factors
- Multiples
- Prime factorization
- HCF
- LCM

- 1 is a factor of every number and it is the smallest factor of the numbers.
- Every number is the factor of itself and it is the greatest factor.
- The multiple of a number is a number that is obtained by multiplying a number with any other number.
- The index of a number represents how many times a number is multiplied by itself. It is a small raised number written next to the number which is being multiplied by itself.
- The Highest Common Factor (HCF) of two or more numbers is the greatest number that divides the given numbers exactly without any remainder.
- The Least Common Multiple (LCM) of two or more numbers is the smallest number which is divisible by each of the given numbers.
- Product of the two numbers = Product of their HCF and LCM

Review Exercise

1 Circle the correct option.

a) The prime factorization of 24 is ______

i. 2 × 2 × 2 × 3   ii. 2 × 2 × 3 × 3   iii. 2 × 2 × 2 × 2   iv. 3 × 3 × 2 × 3

b) The HCF of two or more distinct prime numbers is equal to ______.

i. their sum   ii. their product   iii. their difference   iv. one

c) The LCM of two or more distinct prime numbers is equal to their ______

i. sum   ii. product   iii. difference   iv. quotient

d) The 2 × 2 × 2 × 2 × 3 × 3 × 5 in exponent notation is ______.

i. $2^5 \times 3^2 \times 5^2$ ii. $2^4 \times 3^2 \times 5$ iii. $2^5 \times 3 \times 5$ iv. $2^5 \times 3^2 \times 5$

e) The product of two numbers is equal to the product of their LCM and ______.

i. first number   ii. second number   iii. common multiple   iv. HCF

2 Define the following terms:

a) Factors   b) Multiples   c) Prime Factorization

d) Highest Common Factor   e) Least Common Multiple

3 Find the factors of the following numbers.

a) 78   b) 840   c) 144   d) 705

4 Find the multiples of the given number less than 250.

a) 18   b) 30   c) 55   d) 82

5 Find the prime factorization of the following numbers and write it in the form of index notation.

a) 566   b) 788   c) 1000   d) 3088

e) 8777   f) 4026   g) 6054   h) 5478

6 Write the following in expanded form.

a) 5^6 b) $2^3 \times 5^5$ c) $3^7 \times 7^2 \times 11^4$ d) $1^2 \times 5^3 \times 3^6$

7 Find the HCF of the following numbers using prime factorization method.

a) 455, 254   b) 569, 369, 545   c) 690, 468   d) 255, 751, 155

8 Find the HCF of the following numbers by division method.

a) 256, 164, 144   b) 178, 278, 240

c) 448, 385, 245   d) 212, 380, 180

9 Find the LCM of the following numbers using prime factorization method.

a) 600, 345, 476   b) 620, 434, 545

c) 156, 135, 215   d) 165, 425, 725

10 Find the LCM of the following numbers using division method.

a) 212, 404, 68   b) 306, 428, 845

c) 340, 146, 456   d) 560, 956, 890

11 Find square of the following numbers.

a) 16   b) 12   c) 44   d) 62

12 The length, breadth and height of a room is 680 cm, 510 cm and 340 cm respectively. Find the longest tape which can measure the dimensions of the room exactly.

13 A light flashes every 12 seconds, another every 18 seconds and a third every single minute. At 2:45 a.m, the three flashed simultaneously. What time the three lights will again flash simultaneously?

14 Find the greatest number which exactly divides 145, 360 and 578.

15 Find the smallest number which is exactly divisible by 200, 400 and 800.

16 Find the minimum length of ribbon that can be exactly cut into pieces of length 450 cm , 750 cm and 800 cm.

17 If LCM of two numbers is 678 and product of two numbers is 4986. Find HCF of the numbers.

18 If HCF of two numbers is 64 and LCM is 1024. Find the product of the two numbers.

19 The HCF of two numbers is 20 and LCM is 5604. If one number is 467 then find the other number.

Math Project

Material Required:

- Number cards (1-100)
- Paper
- Pencil

Procedure:

- Get into pairs.
- Put the number cards on the table.

- Member 1 picks up a card randomly.
- Member 2 has to find a factor or multiple of that number.

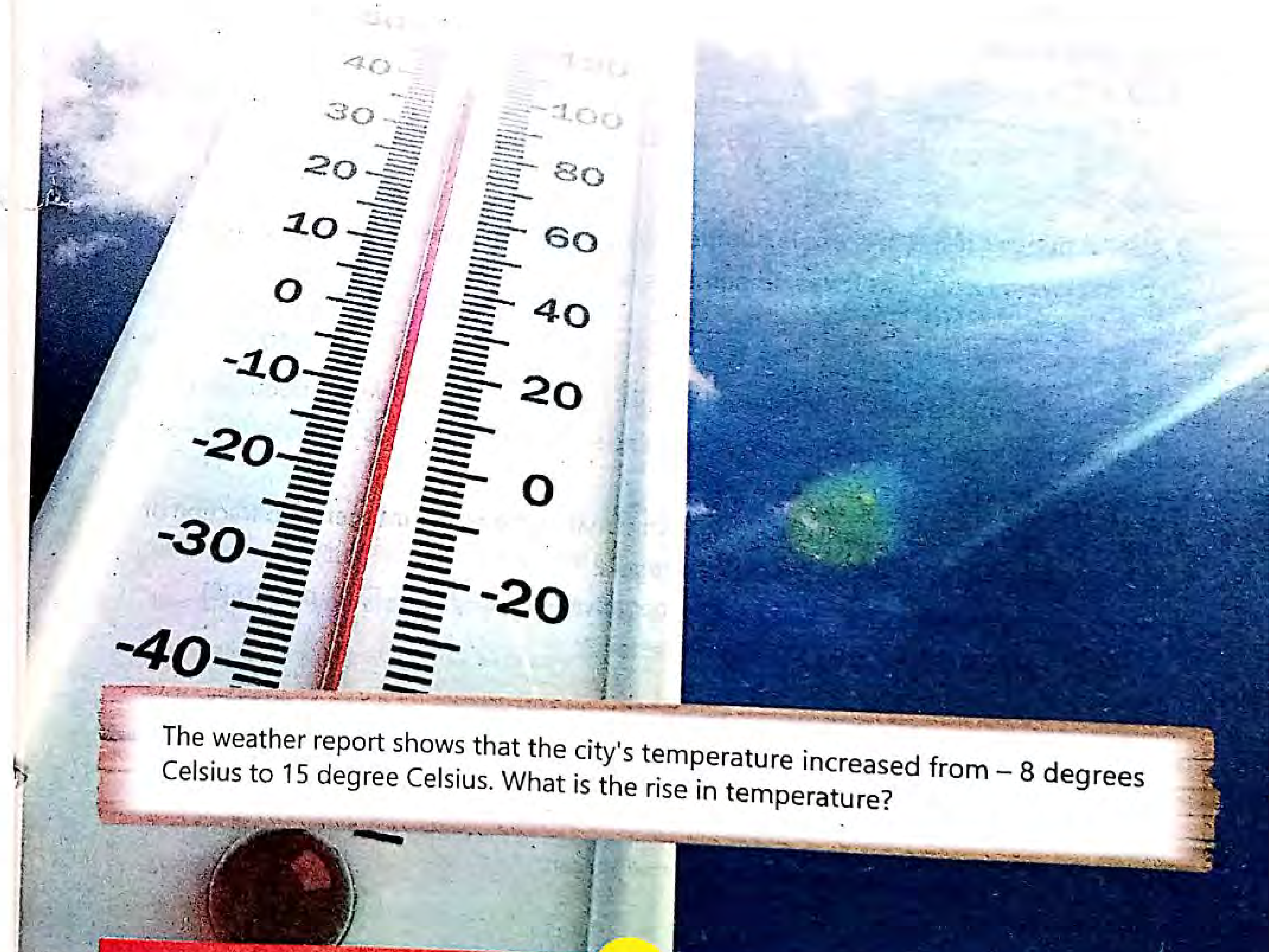
- The member who fails to find a factor or multiple loses a point.
- Repeat the activity several times with different numbers.

Unit 2 Integers

Student Learning Outcomes

After completing this unit, students will be able to:

- Recognise, identify, and represent integers (positive, negative, and neutral integers) and their absolute or numerical value.
- Arrange a given list of integers and their absolute value in ascending and descending order.



The weather report shows that the city's temperature increased from – 8 degrees Celsius to 15 degree Celsius. What is the rise in temperature?

Introduction

We have already learnt about the whole numbers and natural numbers in detail. In this unit we will learn about integers.

2.1 Integers

We know that the set of whole numbers consists of all **natural numbers** along with 0. **Whole numbers** are either positive numbers or zero. Also, when we add two whole numbers, the result is also a whole number according to closure law. But have you ever noticed why this law doesn't apply on subtraction of whole numbers? The reason is when we subtract two whole numbers, we do not always get a whole number as a result.

Quick Check

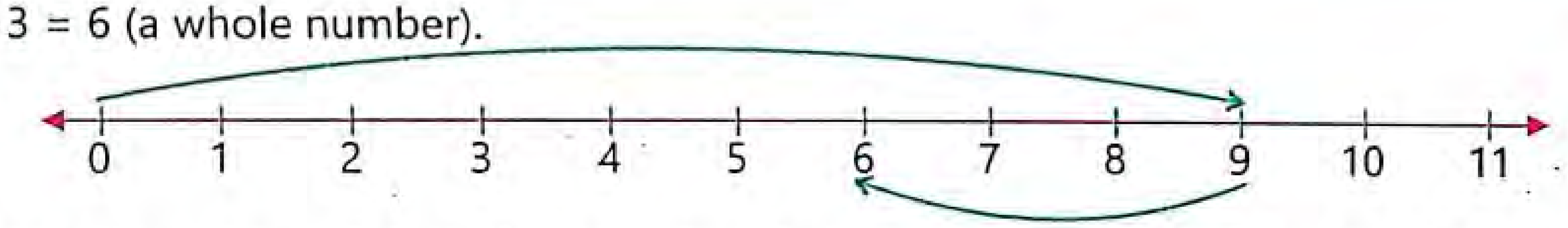
Natural numbers start at 1. Whole numbers start at 0. Can you guess what comes before 0?

Previous Knowledge Check

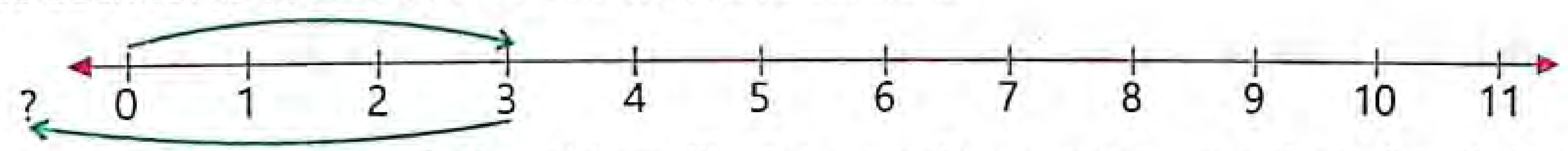
- Can you tell about set of whole numbers and natural numbers?

Let's take 9 and 3.

9 – 3 = 6 (a whole number).



But 3 – 9 doesn't result in a whole number. On the whole number line, we cannot go 9 steps backwards from 3 as it has no numbers before 0.



To address the mathematical problems like this, negative numbers were introduced. Every positive number has its opposite negative number. For example, –6 is opposite to 6, –88 is opposite to 88, etc.

All positive and negative numbers along with 0 makes the **set of integers**. No fraction or decimal number is included in the set of integers.

Integers can be positive {1, 2, 3, 4, 5, ...}, or negative {–1, –2,–3, –4, –5, ...}, or zero {0}.

A set of integers is denoted by the capital letter Z:

Z = {..., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}

or

Z = {0 , ± 1 , ± 2 , ± 3 , ± 4 , ...}

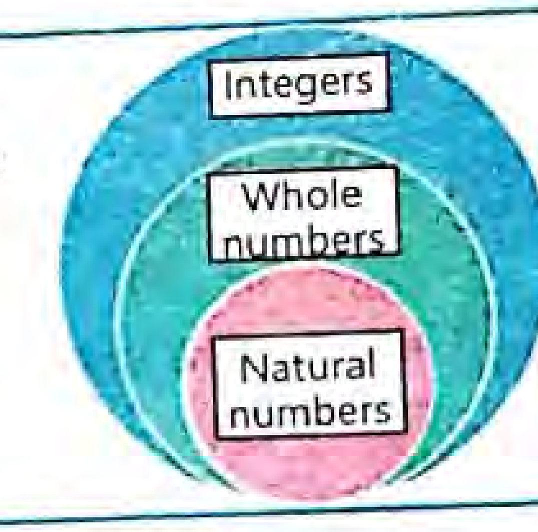
TEACHING POINT Using the number line explain the concept of integers to the students. Ask them to tell the difference between the positive and negative integers.

We usually don't put a plus sign with a positive integer. For example, 5 is same as +5. But for negative integers, it is essential to put the minus sign.

Note it down

The set of Natural numbers and Whole numbers are the subsets of the set of Integers.

$N \subseteq Z$ and $W \subseteq Z$



2.1.1 Zero

Zero is also an integer which is neither positive nor negative.

2.1.2 Positive Integers

Whole numbers that are greater than zero are called **positive integers**.

All natural numbers are positive integers. These numbers can be easily added, subtracted, multiplied and divided. For example, 3 + 4 = 7, 4 – 3 = 1 , 4 × 4 = 16, 15 ÷ 3 = 5.

A set of positive integers can be denoted by the capital letter Z^+.

Z^+ = {+1, +2, +3, +4, +5, ... }

Any positive integer may be written with or without the sign (+) before it.

So, Z^+ = {1, 2, 3, 4, 5, ... }

Note it down

The smallest positive integer is 1 but the greatest positive integers cannot be determined.

2.1.3 Negative Integers

Integers that are less than zero are called **negative integers**. A set of negative integers can be denoted by the capital letter Z^- . A minus sign is put before every negative integer e.g. –3, –5, etc.

So, Z^- = { –1, –2, –3, –4, –5 , ...}

Note it down

The greatest negative integer is –1 but the smallest negative integers cannot be determined.

- Suppose you are in a 5 storey building with 2 levels of basements. The ground floor acts as 0 of the number line. When you are going towards the first floor, you are moving towards +1. If you are at the 4th floor, you are at +4. If you are in the first level basement, you are at -1 and similarly for the 2nd level basement you are at –2.
- Huma needs to buy a chocolate but has no money to buy it. She borrows Rs 25 from her brother. This shows Huma is at –25.
- Ibrahim's grandfather gave him Rs 200 as pocket money. This means Ibrahim is at +200.
- A shopkeeper bought an item for Rs 500 and sold it for Rs 300 for some reason. So he is at –200.

Exercise 2.1

1 **Circle the positive integers from the following.**

+ 2, +10, –9, +11, –3, –5, +8, +9, –3, –23, 7, 10

2 **Write the opposite integers of each of the following.**

a) –5 b) +11 c) 0 d) –9
e) +14 f) –23 g) –19 h) –8

3 **Enlist all the integers which come between the following.**

a) –4 and +6 b) –7 and –1 c) –14 and +11 d) –8 and –18
e) –5 and +5 f) +6 and +8 g) –11 and +17 h) –19 and +20

4 **Write the six integers which are:**

a) < +5 b) > –6 c) < –12 d) > –16

5 **Write an integer for each situation.**

a) an earning of Rs 12000 b) a loan of Rs 700 c) 30° below 0°
d) a loss of Rs 720 e) a profit of Rs 100
f) 5695 metres below sea level g) two degrees above zero
h) fifteen degrees below zero i) 3 levels below ground level
j) 55 metres above sea level

6 **Akram bought an air conditioner for Rs 65,000 and sells it for Rs 62,000. Express his loss or profit as an integer.**

2.2 Comparing and Ordering Integers

Integers can be compared easily using a number line. After understanding the representation and ordering of integers on a number line, we can easily compare and order integers without the use of number line.

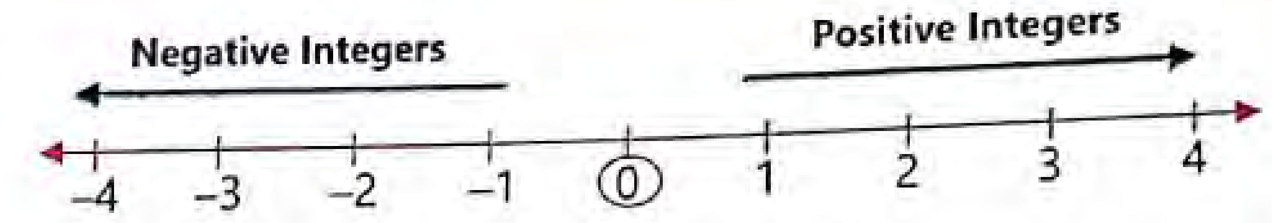
Previous Knowledge Check

Write any three whole numbers and then:
a) Compare and tell which one is the greatest and which one the smallest.
b) Arrange them in ascending and descending order.
c) Represent that numbers on number line.

2.2.1 Representation of Integers on a Number Line

A number line goes on forever in both directions and is indicated by arrows. Positive integers (greater than zero) are represented in the right direction after zero. Negative integers (less than zero) are represented in the left direction before zero.

Draw a line and mark a point 0 on it. Mark equal intervals (of about 1 cm) on both sides of 0. Label the integers to the right of 0 as 1, 2, 3, 4,... and the integers to the left of 0 as –1, –2, –3, –4,.... as shown in the figure.



The arrows at both sides indicate that the positive and negative integers keep continuing.

We can represent different groups of integers on the number line easily.

Let's consider these examples.

Note it down

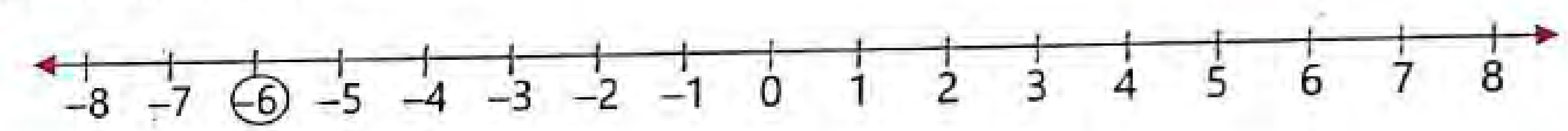
Positive and negative integers are represented on the number line in opposite direction.

Example 1:

Represent –6 on the number line.

Solution:

In order to represent –6 on the number line, count 6 points to the left of the zero.



Example 2:

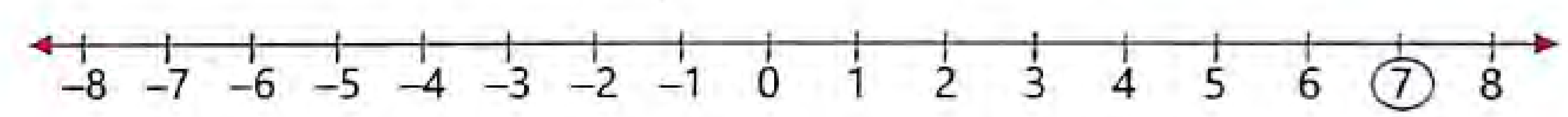
Represent 7 on the number line.

Solution:

In order to represent 7 on the number line, count 7 points to the right of the zero.

Quick Check

Write at least 3 numbers which are:
a) Less than 0
b) Greater than -9 and smaller than 0.
c) Between -3 and 1.

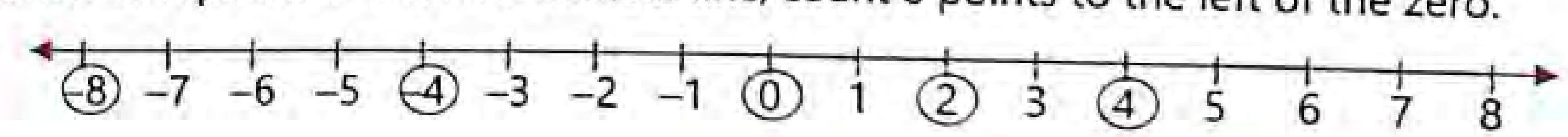


Example 3:

Represent –4, 2, 0, 4 and –8 on the number line.

Solution:

In order to represent –4 on the number line, count 4 points to the left of the zero.
In order to represent 2 on the number line, count 2 points to the right of the zero.
In order to represent 0 on the number line, mark exactly at point 0.
In order to represent 4 on the number line, count 4 points to the right of the zero.
In order to represent –8 on the number line, count 8 points to the left of the zero.



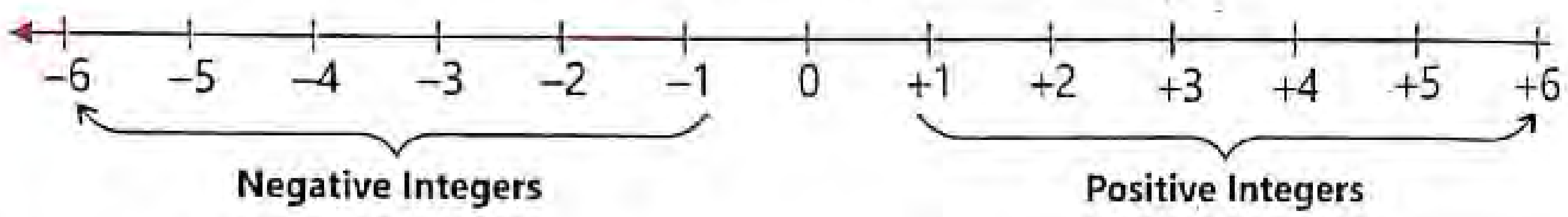
Example 4:

Represent integers from –6 to + 6 on a number line.

Solution:

Draw a number line and mark integers on it as learnt earlier.

Then mark –1, –2, –3, –4, –5, –6 on the left direction and +1, +2, +3, +4, +5, +6 on the right direction having equal distance between each integer as shown below.



Here are some facts about the integers on a number line.

- For any two integers on the number line, the integer lying to the right is always greater than the integer lying to the left. Also, for any integer on a number line, all the integers to its right are greater than it.

For example:

a) 9 > 8 b) –3 > –4 c) 0 > –1 d) –6 > –9 e) 0 >–10

- For any two integers on the number line, the integer lying to the left is always smaller than the integer lying to the right. Also, for any integer on a number line, all the integers to its left are smaller than it.

For example:

a) 4 < 5 b) –7 < –6 c) –1 < 0 d) –12< 5 e) – 3 < 3

- The values of integers increase when we move in the right direction on the number line and their values decrease when we move in the left direction on the number line.

So, +1 <+2, or +2 > +1,

Similarly, –1>–2 or –2<–1

- On a number line, zero is smaller than every integer to its right and greater than every integer to its left.

For example, 0 < 1, 0 < 5, 0 > – 1, 0 > – 6, etc.

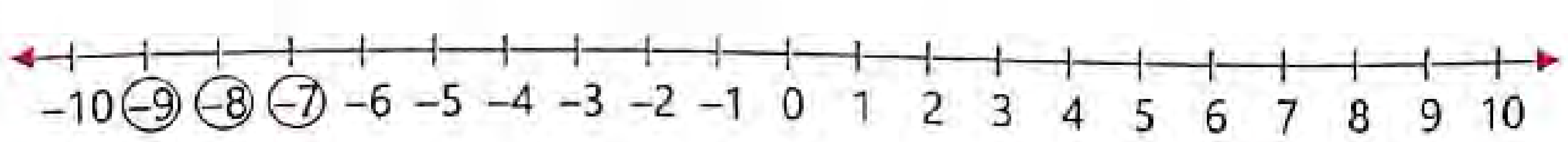
Quick Check

Represent the three positive integers on the number line whose sum is equal to their product.

Example 5:

What are the next three integers after –6 which are smaller than –6?

Solution:



From the number line we can see that –7, –8 and –9 are the next integers smaller than –6.

Example 6:

Which is greater; –10 or 3?

Solution:

On the number line, –10 lies to the left of 3. So, 3 is greater than –10.

Symbolically 3>–10.

Example 7:

Which is smaller; –9 or –8?

Solution:

On the number line, –9 lies to the left of –8. So, –8 is greater than –9.

Symbolically –8 > –9.

Example 8:

Which one is the smallest and which one is the greatest among 2, –5 and –1?

Solution:

On the number line, 2 lies to the right of both –1 and –5. So 2 is the greatest. Similarly, –1 lies to the right of –5. So –5 is smaller than –1. So, 2 is the greatest and –5 is the smallest among these integers.

Symbolically –5 < –1 < 2.

Let's learn a few rules for comparing integers without using the number line.

- Every positive integer is always greater than every negative integer.
- A greater integer with a positive sign is greater than every smaller integer with a positive sign.
- A greater integer with a negative sign is always smaller than every smaller integer with a negative sign.

Let's consider these examples.

Example 9:

Which is smaller, –3 or –6?

Solution:

As a greater integer with a negative sign is smaller than a smaller integer with a negative sign, so –6 is smaller than –3.

TEACHING POINT: Using number the line explain the concept of integers to the students. Help them to represent the positive integers and negative integers on the number line.

Example 10:

Which is greater, –4 or +4?

Solution:

As every positive integer is always greater than every negative integer, so +4 is greater than –4.

Example 11:

Arrange the following set of integers in ascending order.

–9, 0, +4, +6, –6, +5

Solution:

Compare the negative numbers separately.

–9 < –6 (A greater number with a negative sign is always smaller than every smaller number with a negative sign).

Similarly, compare the positive integers separately.

6 is the greatest and 4 is the smallest positive integer here.

So, 4 < 5 < 6.

0 is smaller than the positive integers and greater than the negative integers. So it will be between them.

Arranging the integers from the smallest to the greatest i.e. in ascending order, we get:

–9, –6, 0, 4, 5, 6.

Example 12:

Arrange the following set of integers in descending order.

10, –12, 155, –88, 52, –100, 12

Quick Check

Arrange the negative integers >-5 in ascending and descending order.

Solution:

Compare the negative integers separately.

–12 > –88 > –100 (as a greater integer with a negative sign is always smaller than every smaller integer with a negative sign).

Similarly, compare the positive integers separately.

155 is the greatest and 10 is the smallest positive integer here. Also 52 is greater than 12.

TEACHING POINT

Provide a handout with some blank spaces for 1-digit or 2-digit numbers with positive and negative signs and an empty box in between number blanks for comparison.

So, 155>52>12>10.

Arranging the integers from the greatest to the smallest i.e. in descending order, we get:

155, 52, 12, 10, –12, –88, –100.

Exercise 2.2

1 Represent the following sets of integers on a number line.

a) –4 to +4 b) –7 to + 9 c) –8 to 0 d) –5 to +13 e) –4 to +20

f) –2 to +7 g) –4 to + 5 h) –6 to 0 i) –3 to +20 j) –9 to +23

2 Which is greater? Give the reason.

a) –666 or –6666 b) –2340 or –234

c) –10000 or –1000 d) –55 or –555

3 Arrange the following integers in ascending order.

a) –5, +4, –13, –1, 0, +3, +7 b) –5, –3, +3, +2, +11, –12

c) –33, –12, 0, +11, +9, +5, –16 d) –12, 0, +17, +11, +6, –8, –5

e) –3, 0, +1, +8, +7 , –5, –4, +9 f) –15, –11, 0, +23, +24, +27, –14

4 Arrange the following integers in descending order.

a) –5, –1, 0, 7, 9, –3, –2 b) –11, –13, –8, +9, +12, +23

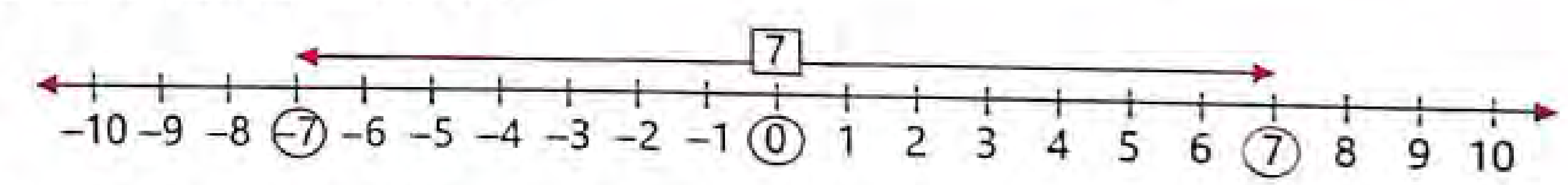
c) +24, +45, –17, 0, +10, –12, –20 d) –9, –7, 0, +16, +8, –8

e) –8, –1, +4, +1, +10 , –10, –9 f) –34, –78, 0, +56, +78, +90, –45, –11

2.3 Absolute or Numerical Value of an Integer

Let's observe how far –7 is from 0 and how far +7 is from 0 using the number line.

Look at the number line below.



Here, –7 is 7 steps away from 0 to its left. +7 is also 7 steps away from 0 but to its right. So the distance of –7 and +7 from zero is the same but in the opposite directions.

In Mathematics, this distance of –7 and +7 from 0 on the number line is known as its **absolute value**. Absolute value of an integer is always positive.

There are two features of integers.

a) Numeric value b) Signs (+ or –)

Numerical value is the value of integers which is measured from '0'. It is also called the **absolute value** of the integer.

To indicate the direction of integers we use the appropriate sign +or –, with 'numerical value' or 'absolute value'.

Note it down

The absolute or numerical value of any integer (positive or negative) is always positive.

So, – 7 indicates 7 steps towards the left of 0 while +7 indicates 7 steps towards the right of 0.

We represent the absolute value of any integer using the symbol "| |".

Thus, |+7| = |–7| = 7

Therefore, the absolute or numerical value of |+7| and |–7| is always 7.

Quick Check

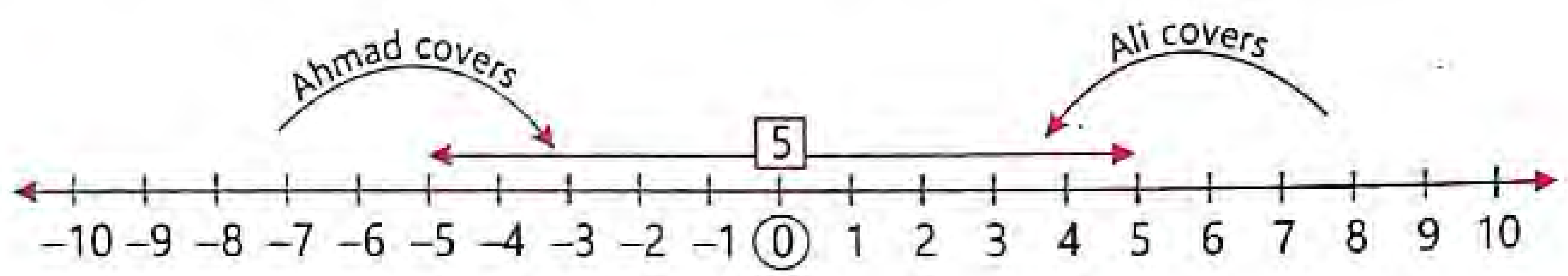
Write 8 negative and 8 positive numbers. Then find their absolute values.

Example 1:

Ali and Ahmad walked 5 kilometres away from the same place but in the opposite directions. Who covered more distance?

Solution:

Let's represent the distance on the number line.



We can observe that Ali and Ahmed are at the same distance from the starting point 0. So, we can say that the numerical or absolute value of 5 and –5 is the same because it gives us only distance but not direction.

So, |+5| = 5 and |–5| = 5

So, they covered the same distance.

Example 2:

What is the absolute value of –198?

Solution:

|–198| = 198

Quick Check

What is the absolute value of:
a) –23 b) –99 c) –100

Example 3:

What is the absolute value of +100?

Solution:

|+100| = 100

We can arrange the absolute or numerical value of integers in ascending and descending order. Let's consider the following examples.

Example 4:

Arrange the absolute or numerical values of the following integers in ascending order.

+3, –4, –6, +11, 0, –7, –5, +12

Solution:

Absolute or numerical values of the given integers in ascending order are;

0, 3, 4, 5, 6, 7, 11, 12

Quick Check

Compare the absolute value of -100 and +100. Which number has the greater absolute value? Give reason.

Example 5:

Arrange the absolute or numerical values of the following integers in descending order.

–2, –9, +6, –8, +1, –3, –12, +23

Solution:

Descending order of the absolute values of the given integers is: 23, 12, 9, 8, 6, 3, 2, 1.

Exercise 2.3

1 Write the absolute value of the following.

a) +5 b) +67 c) – 99 d) +90 e) – 80 f) – 40

TEACHING POINT

Use the video to explain the concept.
https://www.youtube.com/watch?v=zxaT8ArCKjQ
Have them play the online game for practicing.
https://www.math-play.com/Millionaire-Game-Absolute-Value/Millionaire-Game-Absolute-Value_html5.html

2 **Arrange the absolute or numerical values of the following integers in ascending order and descending order.**

a) +22, +33, 0, +50, +7, +23 b) +6, +11,– 3, +5, 0, +77

c) –14, 0, –11, –7, –8, +9 d) –11, –3, +5, +18, +13, –7

e) –16, +4, –7, –9, +2, +10 f) –6, +7, –8, –22, +25, –31, –56

g) +23, +12, +11, –14, –6,+8,– 9 h) +23, +13, –15, +6, –5, +7, +9, +3, –10, –8

3 **Aliya and Harris start at the same point. Aliya takes 11 steps upstairs and Harris comes 5 steps down the stairs. Write the integers for the two situations. Also write the absolute value of the integers.**

Think Higher

Which one of these doesn't belong to the rest?
Explain your reasoning.

- 3 feet above sea level
- A profit of Rs 3
- 3° below freezing point
- 3rd floor of a building

Summary

- Zero is also an integer which is neither positive nor negative.
- Whole numbers that are greater than zero are called positive integers.
- Integers that are less than zero are called negative integers.
- Positives or negative integers are represented on the number line in opposite direction.
- The absolute or numerical value of any integer (positive or negative) is always positive.

Vocabulary

- Integers
- Absolute value
- Positive integers
- Negative integers
- Comparing
- Ordering
- Ascending order
- Descending order

Review Exercise

1 **Choose the correct option**

a) The smallest positive integer is ______.

i. 0 ii. +1 iii.–1 iv. not determined

b) The greatest negative integer is ______.

i. 0 ii. +1 iii.–1 iv. not determined

c) The absolute value of –16 is ______.

i. –16 ii. +16 iii. –60 iv. +60

2 **Circle the negative integers from the following.**

+5, +15, – 78, +100, – 67, – 12, + 90

3 **Write the opposite integers for each of the following.**

a) – 7 b) + 101 c) –987 d) –50 e) +13 f) –56

4 **Represent the following sets of integers on a number line.**

a) – 6 to +6 b) – 5 to + 11 c) + 8 to +14 d) – 7 to +17 e) – 8 to –12

5 **Write the integers between the following.**

a) –5 and +5 b) –6 and +8 c) –11 and +17 d) –19 and +20

6 **Arrange the following integers in ascending and descending order.**

a) –6, –3, –1, 0, 5, 7 b) –12, –5, –3, +3, +2, +11

c) –23, –56, –13, 0, +14, +15, +23 d) –12, –16, –11, +10, +25, +34

7 **Represent the integers greater than –13 to 0 on a number line.**

8 **Write the absolute value of the following.**

a) –11 b) +45 c) – 99 d) +123 e) – 88 f) – 100

9 **Arrange the absolute or numerical values of the following integers in ascending order and descending order.**

a) +16, +8, 0, +2, +7, +19 b) +17, +11, – 3, +8, 0, +23

c) –17, 0, –16, –4,–1,+4 d) –18, –3,+4,+18, +19, – 9

e) –21, +5, +16, –5, –8, +5, +77

Math Project

Material Required:

- Chart paper
- Glue stick
- Paper slips
- Markers
- Scales

Procedure:

- Distribute paper slips among the groups.
- Each group will write at least 10 puzzles on their slips. (Like "I'm an integer greater than -7 but smaller than 1. My absolute value makes me the only even prime number. What integer am I?", or "I'm a set of 3 positive and 2 negative integers in ascending order" etc.
- Then they will exchange their puzzles with other groups.
- The groups will then paste each sum on their allocated chart paper and solve it on the chart paper using marker.
- The group with quick and accurate solutions wins.

Group A

I'm an integer greater than -7 but smaller than 1. My absolute value makes me the only even prime number. What integer am I?

Group B

I'm a set of 3 positive and 2 negative integers in ascending order.

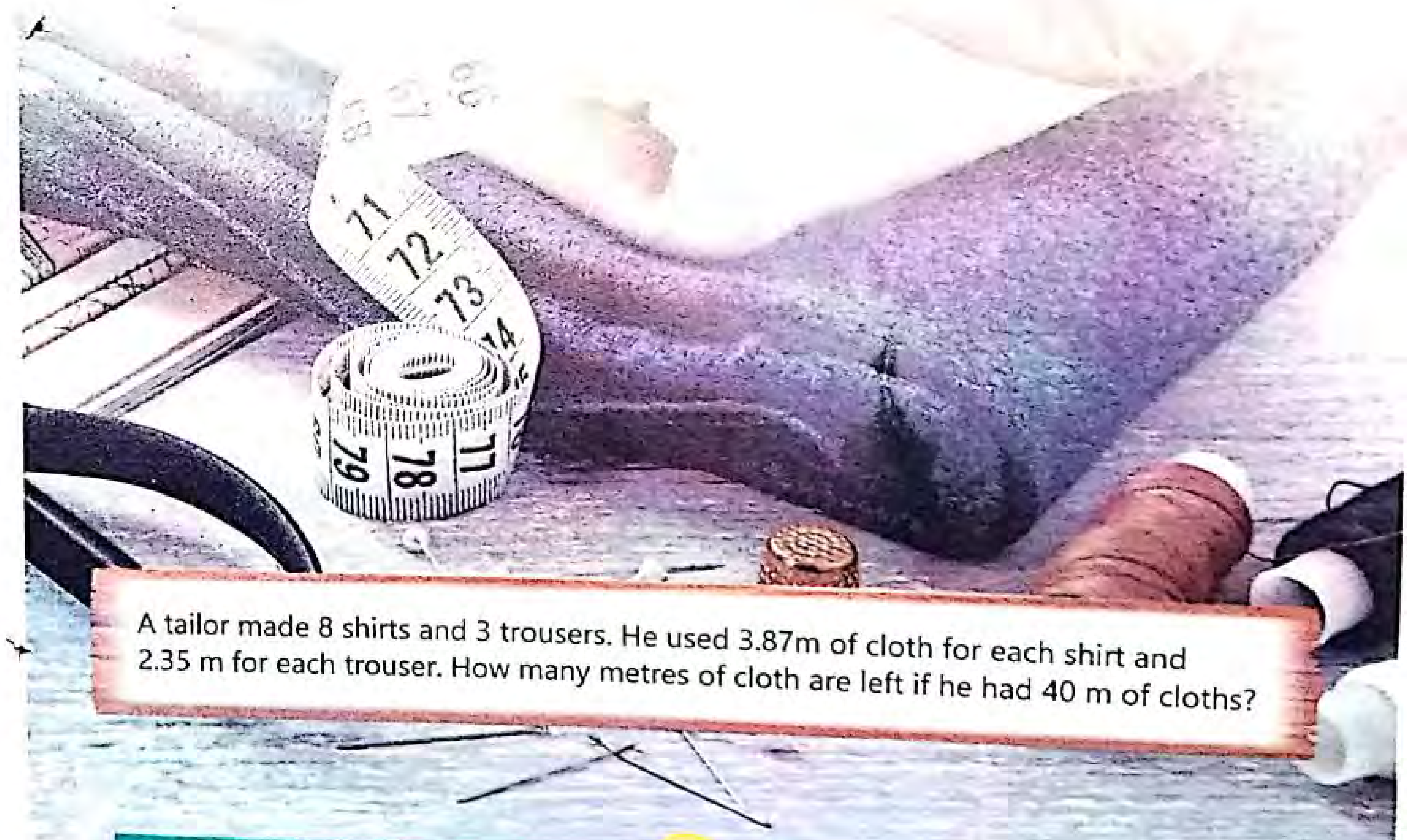
Unit 3

Laws of Integers and Order of Operations

Student Learning Outcomes

After completing this unit, students will be able to:

- Add and subtract upto-2-digit like and unlike integers and verify commutative and associative laws (where applicable).
- Multiply up to 2-digit like and unlike integers and verify commutative, associative, and distributive laws.
- Divide like and unlike integers.
- Recognise the order of operations and use it to solve mathematical expressions involving whole numbers, decimals, fractions, and integers.



A tailor made 8 shirts and 3 trousers. He used 3.87m of cloth for each shirt and 2.35 m for each trouser. How many metres of cloth are left if he had 40 m of cloths?

Introduction

The four basic operations of mathematics such as addition, subtraction, multiplication and division can also be performed on integers.

3.1 Addition of Integers

There are some rules for adding integers. Let's learn about these.

- When a positive integer is added to a positive integer, the result will be a positive integer.
- When a negative integer is added to a negative integer, the result will be a negative integer.
- When a positive integer is added to a negative integer, the result will have the sign of the greater integer.
- The sum of an integer and its opposite always results in 0.

Previous Knowledge Check

How can we add two whole numbers?
What are the steps of addition of two or more numbers?

Note it down

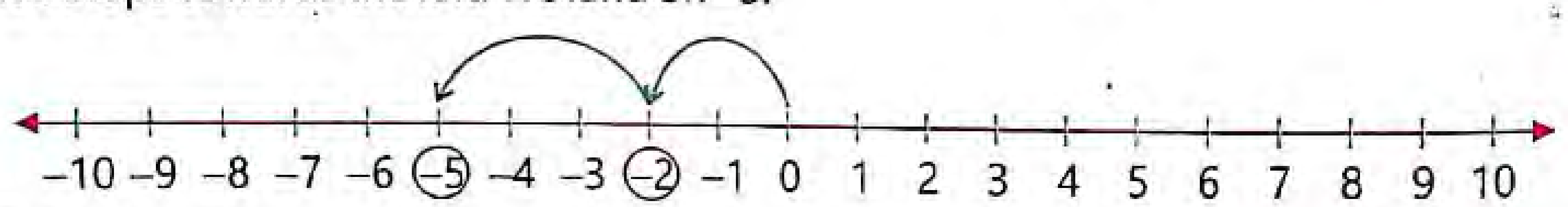
If we add two integers with positive signs, the sum will be positive i.e. (+) + (+) = +

Example 1:

Add the two negative integers –2 and –3 using the number line.

Solution:

Draw a number line. Start at 0 and reach –2. As the other integer is also negative i.e. –3, we will count 3 steps towards the left. We land on –5.



So, –2 + (–3) = –5.

Example 2:

Find the sum of –7, –5 and –1 on the number line.

Solution:

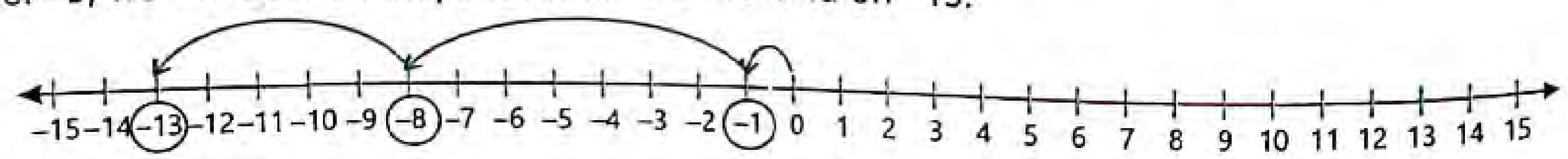
Draw a number line. First add any two of the given integers.

Let's take –1 and –7. Start at 0 and reach –1. As the other integer is also negative i.e. –7, we will count 7 steps towards the left. We land on –8.

Now add –8 to the remaining integer –5. Start at –8. As the other integer is also negative i.e. –5, we will count 5 steps towards left. We land on –13.

Quick Check

Find the sum of –5 and –6.



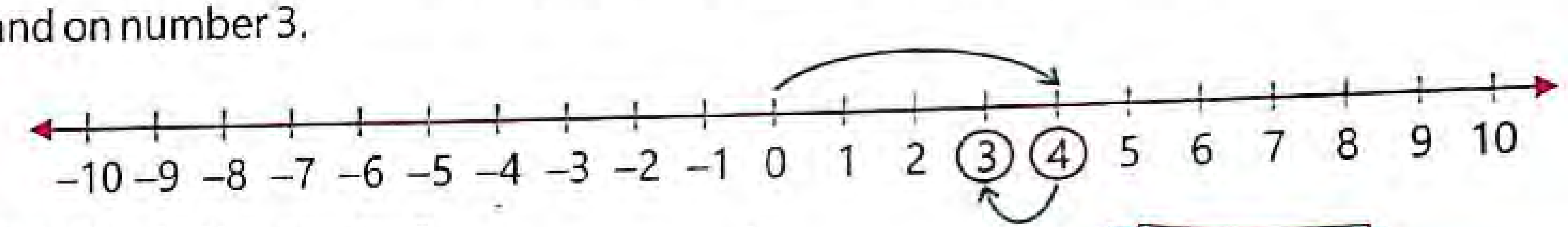
So, –1 + (–7) + (–5) = –13.

Example 3:

Find the difference of two positive integers +4 and +1 using the number line.

Solution:

Draw a number line. Start at 0 and reach 4. As both integers 4 and 1 are positive, we will find their difference as we do in whole number. Move 1 step backwards on the number line. We land on number 3.



So, (+4) – (+1) = 4 – 1 = 3.

Note it down

If we add two integers with negative signs, the sum will be negative i.e. (–) + (–) = –

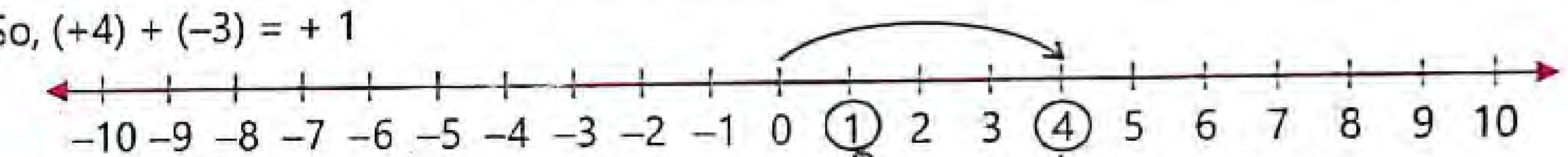
Example 4:

Find the sum of –3 and +4 using the number line.

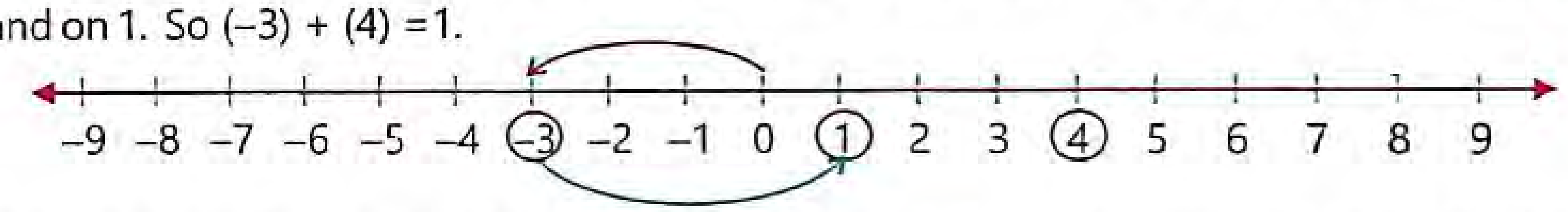
Solution:

Draw a number line. Start at 0 and reach 4. As the other integer is negative i.e. –3, we will count 3 steps towards the left. We land on 1.

So, (+4) + (–3) = + 1



We can also start at 0 and reach –3. 4 is positive so we will move 4 steps towards the right. We land on 1. So (–3) + (4) = 1.



We can also start at –3. 4 is positive so we will move 4 steps toward the right. We land on 1. So (–3) + (4) = 1.

3.1.1 Addition of Integers without using the Number Line

Addition of Integers having Same Signs

To find the sum of two integers with same signs, we have to follow three steps as below:

Step I: Take absolute values of given integers.

Step II: Add the absolute values.

Step III: Put the common sign of the two integers.

TEACHING POINT: Ask the students to use the number line explain the addition of integers. Help them to find the sum of integers using the rules.

Example 1:

The distance form Ali's Home to Park is 11 metres and from Park to masjid is 22 metres. What is the total distance from Ali's Homes to masjid?

Solution:

(+11) + (+22)

Take the absolute values.

|+11| = 11 and |+22| = 22

Add the absolute values: 11 + 22 = 33

Put the common sign of the two integers.

= + 33 (+ is the common sign).

Example 2:

Find the sum of (–23) + (–10).

Solution:

(–23) + (–10)

= |–23| +|–10|

= 23 +10

= – 33 (– is the common sign).

Note it down

Rules for addition:

a) (+) + (+) = +

b) (–) +(–) = –

Example 3:

Find the sum of (–7) + (–15).

Solution:

(–7) + (–15)

= |–7| + |–15|

= 7 + 15

= – 22 (– is the common sign)

Quick Check

Find the sum of:

a) –5 and –9 b) –4 and –7

Addition of Integers with Opposite Signs

To add two integers with opposite signs, we have to follow three steps as below.

Step I: Take absolute values of the given integers.

Step II: Subtract the smaller absolute value from the greater.

Step III: Put the sign of the integer with the greater absolute value.

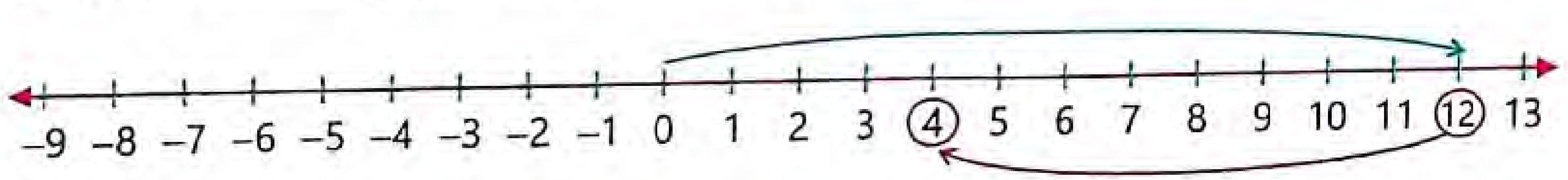
Example 1:

Ali had 12 cupcakes. He distributed 8 among the needy kids in his neighborhood. How many cupcakes does he have left?

Solution:

Ali had 12 cupcakes shows a quantity of +12. Distributed 8 means –8.

So we can find the amount of remaining cupcakes as: +12 + (–8)



Start at 0 and reach +12. Move 8 steps to the left. We land on +4. So, Ali had 4 cupcakes left.

Example 2:

Find the sum of (+8) and (–4).

Solution:

(+8) + (–4)

Take the absolute values

|+8| = 8 and |–4| = 4

Now subtract the smaller absolute value from the greater one.

8 – 4 = 4

Put the sign of greater absolute value. So, (+8) + (–4) = +4

Quick Check

Find the sum of +8 and –7.

Example 3:

Solve the following.

[(–2) + (+ 5)] + (–3)

Solution:

First find the absolute value of the given integers.

|–2| = 2, |+5| = 5, |–3| = 3

then solve brackets.

= [(–2) + (+5)] + (–3)

Now subtract the smaller absolute value from the greater one.

= [5 – 2] + (–3)

= (+3) + (–3)

= 0 (The sum of an integer and its opposite always results in 0)

Example 4:

Find the sum of (–7) and (+6).

Solution:

(+6) = 6, |–7| = 7

So 7 – 6 = 1

= (+6) + (–7) = –1

Exercise 3.1

1 Represent the sum of the integers on a number line.

a) (+1) + (+3) b) (–3) + (–5) c) (+4) – (+2)

d) (–3) + (–4) + (–2) e) (+7) + (–10) f) (–3) + (–7) + (–4)

g) (–1) + (–2) + (–3) h) (–3) + (–4) i) (+9) +(–10)

2 Solve the following.

a) (+9) – (+5) b) (–4) + (–7) c) (+4) – (+6)

d) (–9) + (–8) + (–2) e) (+10) – (–2) f) (–2) + (–5) + (+3)

g) (–5) + (–2) + (–7) h) (+3) – (+2) i) (+6) – (+4)

j) (–4) + (–6) + (–8) k) (+23) + (–2)+ (–34) l) (–7) + (+6)

TEACHING POINT: Ask the students to make a poster in their own creative way of Addition rules of integers with examples and present their working.

3 Write the missing integers in the given.

a) (+11) + (+3) = ____ b) (–4) + ____ = (–7) c) ____ – (+4) = (–7)

d) (–9) + ____ + (–2) = –16 e) ____ – (–3) = +5 f) (–1) + (–3) + (–7) = ____

g) (–5) + (–7) + (+2) = ____ h) (–5) + ____ + (–9) = –16

4 **The difference of two integers is +34. If minuend is –23. Find the subtrahend.**

5 **An elevator goes down 5 floors from the ground floor. Then it goes up 2 floors. What floor is the elevator on now?**

6 **The temperature in the night was –10°C. If the temperature rises 4°C in the morning, what is the new temperature?**

3.2 Subtraction of Integers

3.2.1 Relation of Addition and Subtraction

We have learned about addition and subtraction in previous grades. These two operations are inverse of each other. In Mathematics, an inverse operation means an operation that reverses the effect of earlier operation.

Previous Knowledge Check

How can we subtract two whole numbers?
What are the steps of subtraction of two or more whole numbers?

For example, when 2 is added to the number 5, we get 7. Now if 2 is subtracted from 7, we get the original number 5. So subtraction reversed the process of addition and the result is the original number 5.

3.2.2 Subtracting Integers

When we subtract two integers, we follow these steps.

Step I: First, change the operation from subtraction to addition and change the sign of the integer that is to be subtracted.

Step II: Then, add these according to rules of addition of integers.

Example 1:

A point A on the top of the mountain is 3500 m above the sea level and a point B is 2000 m below sea level. Find the vertical distance between points A and B.

Solution:

If we take sea level as 0, then:

Point A = +3500 m

Point B = –2000 m

Difference = 3500 – (–2000)

= 3500 + 2000

= +5500 m

So, the vertical distance between point A and B is +5500 m.

Example 2:

Solve (–5) – (+1).

Solution:

Keep the first number i.e. –5 unchanged. Next, change the operation from subtraction to addition and also change the sign of +1. It will become –1. Then add according to the rules of addition of integers.

(–5) – (+1) = (–5) + (–1)

Take the absolute values

|–5| = 5 and |–1| = 1

Add the absolute values

5+1 = 6

Put the common sign with the answer.

So, (–5) – (+1)= – 6.

Note it down

When we subtract two positive integers, if the 1st integer is greater then the answer will be positive.

Example 3:

Find (+9) – (–2).

Solution:

Keep the first number i.e. +9 unchanged. Next, change the operation from subtraction to addition and also change the sign of –2. It will become +2. Then add according to the rules of addition of integers.

(+9) – (–2) = (+9) + (+2)

Take the absolute values

|+9| = 9 and |+2| = 2

Add the absolute values

9 + 2 = 11

Put the common sign with the answer.

So, (+9) – (–2)= +11

Note it down

When we subtract two positive integers, if the 1st integer is smaller then the answer will be negative.

Example 4:

Find (+7) – (+3)

Keep the first number i.e. +7 unchanged. Next, change the operation from subtraction to addition and also change the sign of +3. It will become –3. Then add according to the rules of addition of integers.

TEACHING POINT: Ask the students to make a poster in their own creative way of Subtraction rules of integers with examples and present their working.

(+7) – (+3) = (+7) + (–3)

Take the absolute values

|+7| = 7 and |–3| = 3

Now subtract the smaller absolute value from the greater one.

7 – 3 = 4

Put the sign of the greater absolute value.

So, (+7) – (+3) = +4

Example 5:

Find (–4) – (–6)

Solution:

Keep the first number i.e. –4 unchanged. Next, change the operation from subtraction to addition and also change the sign of –6. It will become +6. Then add according to rules of addition of integers.

(–4) – (–6) = (–4) + (+6)

Take the absolute values

|–4| = 4 and |+6| = 6

Now subtract the smaller absolute value from the greater one.

6 – 4 = 2

Put the sign of the greater absolute value.

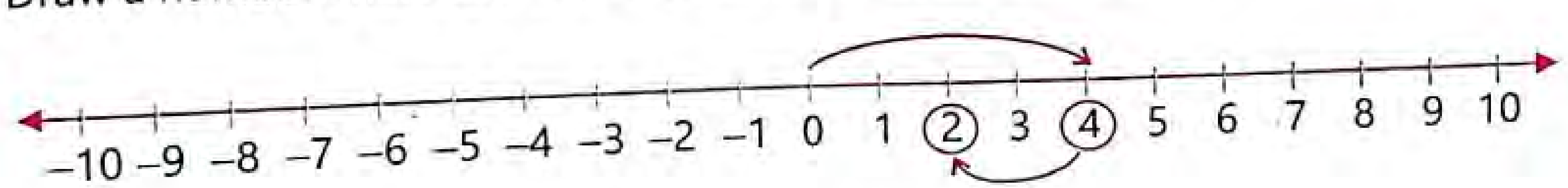
So, (–4) – (–6)= +2

Example 6:

Let's subtract +2 from +4 using the number line.

Solution:

Draw a number line. Start at 0 and reach +4 on the number line.



Now move 2 steps backwards to the left of 4.

So, (+ 4) – (+ 2) = + 4 – 2 = +2.

Use the following links to explain the concept of adding and subtracting integers.

https://www.youtube.com/watch?v=CfkaifC7tGY

https://www.youtube.com/watch?v=1DKWG5CBeek

Example 7:

Solve (–5) – (+1).

Solution:

Keep the first number i.e. –5 unchanged. Next, change the operation from subtraction to addition and also change the sign of +1. It will become –1. Then add according to the rules of addition of integers.

(–5) – (+1) = (–5) + (–1)

Take the absolute values

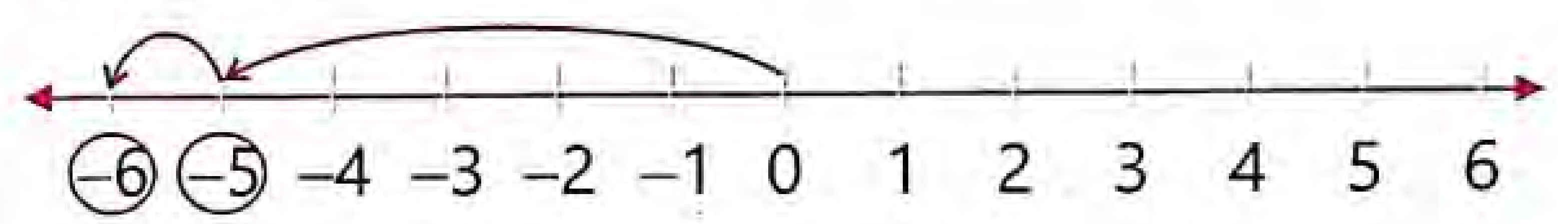
|–5| = 5 and |–1| = 1

Add the absolute values

5+1 = 6

Put the common sign with the answer.

So, (–5) – (+1)= – 6, We can also show the difference of the integers on the number line. Draw a number line and start at 0 and reach the point (–5) on the number line. Move 1 step to the left of –5, we get –6.



Example 8:

The sum of two integers is –45. If the smaller integer is –75, find the greater one.

Solution:

Sum of integers = –45

Smaller integer = –75

Greater integer =?

(–75) + ____ = –45

For finding the missing integer we subtract –75 from –45.

–45 –(–75) = –45 + 75 = 30

So, the greater integer is +30.

Quick Check

Find the difference between –75 and –34.

Exercise 3.2

1 **Represent the difference of the integers on the number line.**

a) (+3) – (+8) b) (–4) – (–6) c) (+2) – (+3)
d) (–9) – (+6) e) (–7) – (–5) f) (–1) – (–8)
g) (+7) – (–3) h) (–9) – (–3) i) (–3) – (–3)

2 **Solve the following without using a number line.**

a) (+4) – (+18) b) (–8) – (–3) c) (+5) – (+7) d) (–11) – (–5)
e) (–3) – (–2) f) (–5) – (–4) g) (–6) – (–4) h) (–7) – (–3)
i) (–4) – (+2) j) (+7) – (–8) k) (–9) – (+2) l) (+6) – (+8)

3 **Write the missing integers in the given.**

a) (+3) – (+8) = ____ b) (–3) – ____ = (–2) c) ____ – (+5) = (–9)
d) (–8) – ____ = (–3) e) ____ – (–8) = +7 f) (–9) – (–5) – (–2) = ____

4 **The sum of two integers is +54. One of them is –11. Find the other.**

5 **A place is 213 m above sea level and another is 114 m below sea level. What is the distance between the two places?**

6 **Two cars started from the same point. First car went towards the east and covered 44 km in an hour. The second car went towards the west and covered 62 km in 1 hour. Find the distance between the two cars after an hour.**

7 **Sifwa borrowed Rs 56 from her brother. She returned Rs 45. How much does she owe?**

3.3 Laws of Integers under Addition

There are the three laws of integers with respect to addition.

a) Closure Law b) Commutative Law c) Associative Law
d) Additive Identity e) Additive Inverse

Let's discuss these in detail.

TEACHING POINT: Write two or three numbers on the board and ask the students to verify the Commutative and Associate Law using these integers.

a) Closure Law

In addition, when we add two integers, the sum is always an integer. It is known as the **closure law of integers under addition**.

For example when we add -2 and +7, we get +5.

–2 + (+7) = –2 + 7 = +5

b) Commutative Law

In addition, when we change the order of two integers, the sum remains the same. It is known as the **commutative law of integers** under addition.

Let's suppose a and b are any two integers, then:

For example when we add -3 and +5, we get +2.

So, –3 + 5 = +2

Let's change the order of the addends, and add +5 and –3. We get +2

5 + (–3)= 2

So, –3 + 5 = +5 + (–3) = 2

It shows that changing the order of a whole numbers in addition does not change the sum.

Let's suppose a and b are any two integers, when we add we get: a + b = b + a

> **Note it down**
> Commutative and Associative Laws of integers with respect to subtraction are not valid, because the difference of two or more integers always change, when we change the order of integers.

This is the generalised form of Commutative Law of integers under addition.

c) Associative Law

In addition, when we add three integers, the order of grouping the numbers does not affect the sum. For example grouping the first two integers and adding their sum to the third integer is the same as grouping the last two numbers and adding their sum to the first integer.

Let's verify.

[(–2 + (+4)] + (–1)= (–2) + [(4)+ (–1)]

L.H.S =[(–2 + (+4)] + (–1)

=[(–2 + 4)] + (–1)

=+2 + (–1)

= 2 – 1

= +1

> **Note it down**
> When we add or subtract any integer from 0, we get the integer itself.

R.H.S = (–2) + [(4)+ (–1)]
= (–2) + [(4) + (–1)]
= (–2) + [4 – 1]
= (–2) + 3
= +1

So, L.H.S = R.H.S

[(–2 + (+4)] + (–1) = (–2) + [(4)+ (–1)]

1 = 1

Let's suppose a, b and c are any three integers, then: (a + b) + c = a + (b+ c)

This is the generalised form of the Associative Law of integers under addition.

d) Additive Identity

0 is the called the **additive identity** because when we add 0 to any integer the result is the integer itself.

Example 1:

If we add 0 and +7, we get +7.

0 + (+7) = 0 +7 = +7

Example 2:

If we add 0 and -5, we get -5.

0 + (-5) = 0 -5 = -5

e) Additive Inverse

If the sum of two integers is zero (the additive identity) , then both integers are called the **additive inverse** of each other.

Example 1:

If we add 3 and –3, we get 0.

3+ (–3) = 3 – 3 = 0

here, 3 and -3 are the additive inverse of each other.

Example 2:

If we add -4 and +4, we get 0.

–4+ (+4) = –4 +4 = 0

here, –4 and +4 are the additive inverse of each other.

TEACHING POINT

Following Online game links can be shared with students for practice of Commutative, Associative and Distributive property.
https://www.mathgames.com/skill/3.37-properties-of-addition
https://www.mathgames.com/skill/3.38-solve-using-properties-of-addition
https://www.mathgames.com/skill/7.96-properties-of-addition-and-multiplication

Exercise 3.3

1. Use laws of addition of integers and complete the following.

a) (–3) + 5 = ___ +(–3)
b) 7 + (–4) = –4 +___
c) (–38) + (–17) = ___ +(–38)
d) ___+ (–8) = –8 +(–16)
e) {(4 + (–25)} + 5 = 4 + (___+5)
f) (84 + 43) + (–9) = ___ + {(43 + (–9)}
g) 48 + (___ + 25) = (48 + 2) + 25
h) 38 + (48 + ___) = (38 + 48) + 12

2. Which of the following are true?

a) (–32) + 6 = 6 + (–32)
b) 18 + (–4) = –4 + (–18)
c) (–38) + 47 = 47 × (–38)
d) 86 + (–8) = (–8) + 86
e) (–7 + 4) + 6 = 7 + (4 + 6)
f) (44 + 93) + (–96) = 44 + {93 – (–96)}
g) 38 + [98 + (–86)] = (38 + 98) + (–86)
h) 79 + [(–63) + (–86)] = (79 – 63) – (–86)

3. Verify the Commutative Law for the following.

a) –6, +8 b) +9, +4 c) –7, –3 d) –4, +16 e) 3, –43 f) –8, +9

4. Verify the Associative Law for the following.

a) 8, –6, –9
b) 15, +5, –38
c) –34, 37, –48
d) –39, 18, 50
e) –16, –21, 98
f) –48, –96, –83

5. Verify the following.

a) (–2) + 5 = 5 + (–2)
b) 11 + (–3) = –3 + 11
c) (–8) + (–7) = (–7) + (–8)
d) 7 + (–8) = –8 + 7
e) {4 + (–7)} + 3 = 4 + (–7 + 3)
f) {(–14) + 9} + (–6) = –14 + {(9 + (–6)}
g) (–18) + {8 + (–4)} = {(–18) + 8} + (–4)
h) 50 + {(–13) + (–16)} = {50 + (–13)} + (–16)

3.4 Multiplication of Integers

When we multiply two integers, we follow some rules.

1. When we multiply two positive integers, the product will be a positive integer.
2. When we multiply two negative integers, the product will also be a positive integer.
3. When we multiply a positive and a negative integer, the product will be a negative integer.
4. The absolute value of the product of two or more integers is equal to the product of their absolute values.

Previous Knowledge Check

How we multiply two whole numbers?
Write any two whole numbers and multiply them step by step.

Multiplication of Integers with same signs

The product of two positive integers is always a positive integer.

Example 1:

Find the product of 2 and 4.

Solution:

As we know that multiplication is a process of repeated addition. The multiplicand is added the number of times as the multiplier.

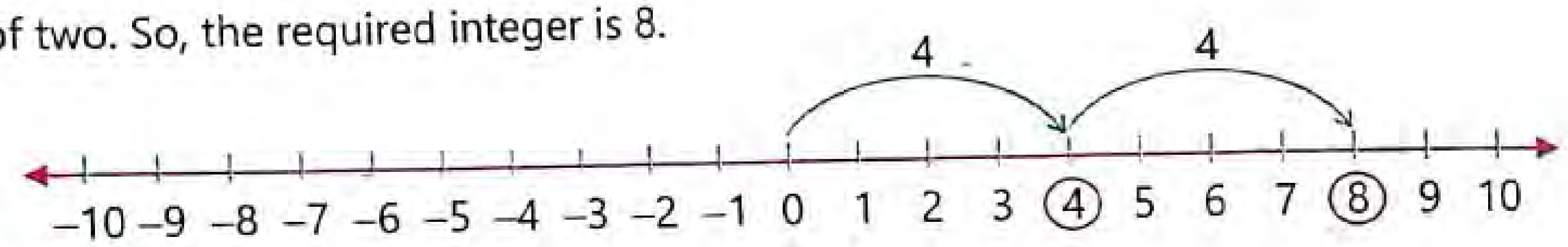
So, 2 × 4 means 2 times 4 is added.

i.e. 2 + 2 + 2 + 2 = 2 × 4 = 8

We can show 2 × 4 = 8 on the number line.

Mark a point 0 on the number line and going on to the right of the zero 4 times in steps of two. So, the required integer is 8.

> **Note it down**
>
> Positive × Positive = Positive
> (+) × (+) = +
> Negative × Negative = Positive
> (–) × (–) = +



Example 2:

Find the product of –3 and -2.

Solution:

Here, (–3) × (–2) means 2 times (–3) is added.

i.e. (–3) + (–3) = (–3) × (–2) = +6

It is difficult to show the product of two negative numbers on the number line.

Multiplication of Integers with opposite signs

The product of two integers with opposite signs is always negative.

Example 1:

Find the product of (-4) and 2.

Solution:

Here, (–4) × 2 means 2 times (–4) is added.

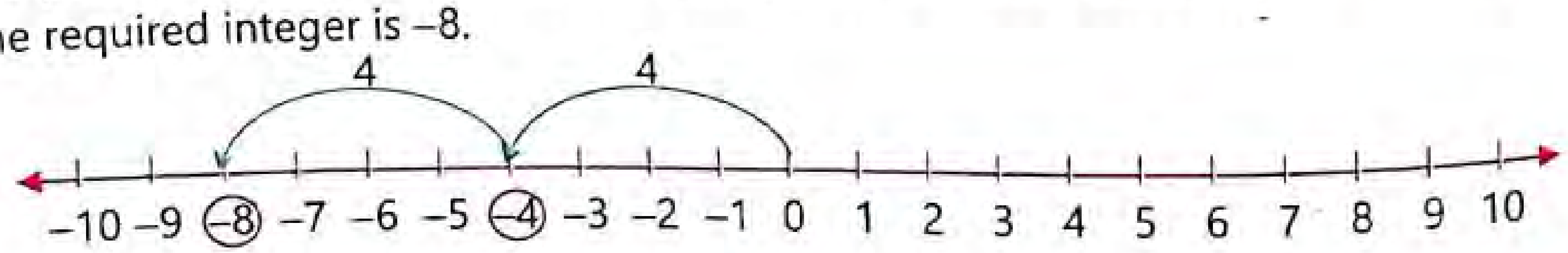
i.e. (–4) + (–4)) = (–4) × 2 = –8

We can show (–4) × 2 = –8 on the number line.

Mark a point 0 on the number line and going on to the left of the zero 2 times in steps of 4.

So, the required integer is –8.

> **Note it down**
>
> Negative × Positive = Negative
> (–) × (+) = –
> Positive × Negative = Negative
> (+) × (–) = –



Example 2:

Find the product of 3 and (–5).

Solution:

Here, 3 × (–5) means 3 times 5 is added.

i.e. (– 5) + (– 5) + (– 5) = 3 × (–5) = –15

We can show 3 × (–5) = –15 on the number line. Mark a point 0 on the number line and going on to the left of the zero 3 times in steps of 5. So, the required integer is –15.

Example 3:

| (+3) × (–5) | = |+3| × |–5|

Solution:

L.H.S = | (+3) × (–5) |
= | –15 |
= 15

R.H.S = |+3| × |–5|
= 3 × 5
= 15

So, L.H.S = R.H.S

Example 4:

| 4 × (–2) × (–3) | = |4| × |–2| × |–3|

L.H.S = | 4 × (–2) × (–3) |
= | 4 × (+6) |
= | +24 |
= 24

R.H.S = |4| × |-2| × |-3| ∴ |+4| = 4, |-2| = 2, |-3| = 3,
= 4 × 2 × 3
= 24

So, L.H.S = R.H.S

> **Quick Check**
>
> **Verify:**
> | (–4) × (–3) | = |–4| × |–3|

Exercise 3.4

1 **Find the product of the following without using a number line.**

a) +2, +5 b) –4, +7 c) +9, –2 d) +6, +5 e) +13, –10 f) +14, –20
g) –3, +22 h) –70, +21 i) –11, –22 j) +40, –29 k) –4, +5 l) –10, +10

2 **Find the product of the following using a number line.**

a) +5, +3 b) –3, –3 c) +2, –5 d) –4, +3 e) –1, –7 f) +3, –5
g) –4, –2 h) +1,–5 i) –2, +6 j) –3, +6 k) –2, +5 l) –7, +2

3 **Find the product of the greatest negative integer and the smallest positive integer.**

4 **Find the product of –100 and +100.**

3.5 Laws of Integers under Multiplication

There are three laws of integers with respect to multiplication.

a) Closure Law
b) Commutative Law
c) Associative Law
d) Multiplicative Identity
e) Multiplicative Inverse

f) Distributive Law of Integers over Addition

g) Distributive Law of Integers over Subtraction

Let's discuss these in detail.

a) Closure Law

In multiplication, when we multiply two integers, the product is always an integer. It is known as the Closure Law of integer under multiplication.

For example: When we multiply -4 and 6, we get -24. $-4 \times 6 = -24$

b) Commutative Law

In multiplication, when we change the order of two integers, the product remains the same. It is known as the Commutative Law of integers under multiplication.

For example: When we multiply -2 by 3, we get -6.

i.e. $-2 \times 3 = -6$

Or when we multiply 3 by (-2), we get -6.

i.e. $3 \times (-2) = -6$

So, $(-2) \times 3 = 3 \times (-2) = -6$

It shows that changing the order of integers in multiplication does not change the product.

Let's suppose a and b are any two integers, when we multiply we get:

$$a \times b = b \times a$$

This is the generalised form of the Commutative Law of integers under multiplication.

> **Note it down**
> Commutative and Associative Laws of integers with respect to division are not valid, because the answer of two or more numbers always changes, when we change the order of the integers.

c) Associative Law

In multiplication, when we multiply three integers, the order of grouping the integers does not affect the product. For example grouping the first two integers and multiplying their product to the third integer is the same as grouping the last two integers and multiplying their product to the first integer.

Let's verifty

$[(-3 \times 4)] \times (-5) = (-3) \times [(4) \times (-5)]$

L.H.S $= [(-3 \times 4)] \times (-5)$

$= -12 \times (-5)$

$= +60$

R.H.S $= (-3) \times [(4) \times (-5)]$

$= (-3) \times (-20)$

$= +60$

So, L.H.S = R.H.S

> **Quick Check**
> Verify the Associative Law under multiplication for -2, $+3$, -4.

TEACHING POINT: Using the number line explain to the students the concept of multiplication of integers. Help them to find the product of integers using rules.

$[(-3 \times 4)] \times (-5) = (-3) \times [(4) \times (-5)] = 60$

Let's suppose a, b and c are any three integers, then

$$(a \times b) \times c = a \times (b \times c)$$

This is the generalised form of the Associative Law of integers under multiplication.

d) Multiplicative Identity

When we multiply "1" to any integer the product will be the integer itself. So, "1" is known as the multiplicative identity of integers.

For example if we multiply 1 by -2, we get -2.

$(-2) \times 1 = 1 \times (-2) = -2$

e) Multiplicative Inverse

If the product of two integers is 1 (the multiplicative identity), then both integers are called the multiplicative inverse of each other.

> **Note it down**
> Multiplicative inverse of any integer is also known as the reciprocal of the integer.

Example 1:

If we multiply 4 and $\frac{1}{4}$, we get 1.

$4 \times \frac{1}{4} = 1$

Here, 4 and $\frac{1}{4}$ are the multiplicative inverse of each other.

> **Note it down**
> Except zero every integer has its multiplicative inverse.

Example 2:

If we multiply -5 and $\frac{1}{-5}$, we get 1.

$-5 \times \frac{1}{-5} = 1$

Here, -5 and $\frac{1}{-5}$ are the multiplicative inverse of each other.

f) Distributive Law of Integers over Addition

The Distributive Law says that multiplying an integer by a group of integers added together is the same as doing each multiplication separately and then adding their products.

$-5 \times [(-4) + (1)] = [(-5) \times (-4)] + [(-5) \times (1)]$

Here "-5" can be "distributed" across the "$(-4)+1$" as below.

$-5 \times [(-4) + (1)] = [(-5) \times (-4)] + [(-5) \times (1)]$

TEACHING POINT: Write two or three numbers on the board and ask the students to verify the Commutative and Associative Law under multiplication using these integers.

Look below:

Left Hand Side (L.H.S)

$-5 \times [(-4) + (1)]$

$= -5 \times (-3)$

$= 15$

Right Hand Side (R.H.S)

$[(-5) \times (-4)] + [(-5) \times (1)]$

$= 20 + (-5)$

$= 15$

We observe that L.H.S = R.H.S

Thus, $-5 \times [(-4) + (1)] = [(-5) \times (-4)] + [(-5) \times (1)]$

Let's suppose a, b and c are any three integers, then the Distributive Law of multiplication over addition is:

$$a \times (b + c) = (a \times b) + (a \times c)$$

This is the generalised form of the Distributive Law of integers of multiplication over addition.

g) Distributive Law of Integers over Subtraction

The Distributive Law says that multiplying an integer by a group of integers subtracted together is the same as doing each multiplication separately and then subtracting their products.

$[(-2) - 1)] \times (-3) = [(-3) \times (-2)] - [(1) \times (-3)]$

Here "-3" can be "distributed" across the "(–2) –1" as below.

$[(-2) - 1)] \times (-3) = [(-3) \times (-2)] - [(1) \times (-3)]$

Look below:

Left Hand Side (L.H.S)

$[(-2) - 1)] \times (-3)$

$= -3 \times (-3)$

$= +9$

Right Hand Side (R.H.S)

$[(-3) \times (-2)] - [(1) \times (-3)]$

$= +6 - (-3)$

$= +9$

We observe that L.H.S = R.H.S

Thus, $[(-2) - 1)] \times (-3) = [(-3) \times (-2)] - [(1) \times (-3)]$

Let's suppose a, b and c are any three integers, then the Distributive Law of multiplication over subtraction is:

$$a \times (b - c) = (a \times b) - (a \times c)$$

This is the generalised form of the Distributive Law of integers of multiplication over subtraction.

Exercise 3.5

1 **Verify the Commutative Law under multiplication for the following.**

a) –2, +6 b) 8, –4 c) –6, 3 d) –38, –94 e) –95, 34 f) 48, –82

2 **Verify the Associative Law under multiplication for the following.**

a) –4, +7, –8 b) –3, –2, 7 c) –13, 52, –68
d) –85, –44, 76 e) –63, –93, –81 f) 15, –38, –39

3 **Write the multiplicative inverse of each of the following.**

a) –3 b) –2 c) +7 d) +100 e) –77 f) –243
g) –48 h) –55 i) +60 j) $\frac{1}{-2}$ k) $\frac{3}{2}$ l) $\frac{-2}{7}$

4 **Verify the Distributive Law.**

a) $-4 \times [(1 + (-3)] = (-4 \times 1) + [(-4 \times (-3)]$

b $-4 \times [(-5) + 1] = [(-4) \times (-5)] + [(-4) \times 1]$

c) $[(-5) - 2)] \times (-2) = [(-5) \times (-2)] - [2 \times (-2)]$

d) $-8 \times [(1 + (-3)] = (-8 \times 1) + [(-8 \times (-3)]$

e) $[(-6) - 1)] \times (-4) = [(-6) \times (-4)] - [1 \times (-4)]$

f) $-2 \times [(1 + (-3)] = (-2 \times 1) + [(-2 \times (-3)]$

g) $[(-7) - 1)] \times (-3) = [(-7) \times (-3)] - [1 \times (-3)]$

h) $-4 \times [(7 + (-8)] = (-4 \times 7) + [(-4 \times (-8)]$

Previous Knowledge Check

- How we can relate the multiplication and division of whole numbers?
- How we can divide two whole numbers?
- Write any two numbers and divide them step by step.

3.6 Division of Integers

3.6.1 Relation of Multiplication and Division

We have learned how to multiply and divide numbers in previous grades. These two operations are inverse of each other. We know that an inverse operation in Mathematics means an operation that reverses the effect of an earlier operation.

For example, when 3 is multiplied to number 4, we get 12. Now if 12 is divided by 3, we get the original number 4. So the division reversed the process of multiplication and result is the original number 4.

3.6.2 Division of Integers

When we divide two integers, we follow these rules.

1. The quotient of two positive integers will be a positive integer.

2. The quotient of two negative integers will be a positive integer.
3. The quotient of a positive and a negative integer will be a negative integer.

Note it down

The absolute value of the quotient of two or more integers is equal to the quotient of their absolute values.

When dividing two positive integers, simply divide their absolute values and put the +sign with the answer.
When dividing two negative integers, simply divide their absolute values and put the +sign with the answer.
When dividing two integers with different signs, simply divide their absolute values and put the –sign with the answer.

Example 1:
Divide +6 by +2.
Solution:
Take absolute value and divide.
(+6) ÷ (+2) = |+6| ÷ |+2|
= 6 ÷ 2 = 3
As both the numbers are positive, so the answer will be positive.
So, (+6)÷(+2)= +3

Example 2:
Divide –10 by –5.
Solution:
Take absolute value and divide.
(–10) ÷ (–5) = |–10| ÷ |–5|
= 10 ÷ 5 = 2
As both the numbers are negative, so the answer will be positive.
So, (–10)÷(–5)= +2

Example 3:
Divide –16 by +4.
Solution:
Take absolute value and divide.
(–16) ÷ (+4) = |–16| ÷ |+4|
= 16÷ 4 = 4
As both the numbers have different signs, so the answer will be negative.
So, (–16) ÷ (+4) = –4

Example 4:
Divide +9 by –3.
Solution:
Take absolute value and divide.
(–19) ÷ (–3) = |+9| ÷ |–3|
= 9 ÷ 3 = 3
As both the numbers have different signs, so the answer will be negative.
So, (+9) ÷ (–3) = –3

Note it down

(+) ÷ (+) = +
(–) ÷ (–) = +
(–) ÷ (+) = –
(+) ÷ (–) = –

Note it down

• If 0 is divided by any integer the result is always 0.

Exercise 3.6

1 Solve the following.

a) (+45) ÷ (–5) b) (–63) ÷ (–9) c) (–80) ÷ (+4) d) (+44) ÷ (–11)
e) (–60) ÷ (+10) f) (–144) ÷ (+12) g) (+168) ÷ (– 8) h) (+390) ÷ (–39)
i) (–600) ÷ (–25) j) (–85) ÷ (+17)

2 Solve the following.

a) 0 ÷ 4 = ____ b) 0 ÷ 10 = ____ c) 0 ÷ (–90) = ____
d) 0 ÷ (–5) = ____ e) 0 ÷ (–130) = ____ f) 0 ÷ (+300) = ____

3.7 Simplification

In Mathematics we often need to show an expression in **"simplest terms"**.
An **expression** is a phrase that includes numbers and operators (+, –, ×, ÷) together to show the value of something in mathematical form.
For example Nida, bought 5 packets of cookies each having 10 cookies in it. Her mother gave her 3 more cookies.
The expression for this problem can be shown as "5 × 10 + 3" which can be further simplified to get the total number of cookies.

Previous Knowledge Check

What is the relationship of addition, subtraction, multiplication and division?

Note it down

An expression is a phrase that includes numbers, brackets and operators (+, –, ×, ÷) together to show the value of something in mathematical form.

To solve an expression which has more than one operation or brackets, it is very important to solve it in accurate order; otherwise we will get the wrong answer.
Let's observe the following.

4 + 6 × 2 (First add 4 and 6)
=10 × 2 (Next multiply 10 and 2)
= 20

4 + 6 × 2 (First multiply 6 and 2)
= 4 + 12 (Next add 4 and 12)
= 16

Teaching Point: Show the students with examples that BODMAS helps you remember the order in which to solve calculations. Also tell them that BODMAS helps us to easily solve more complicated expression.

We can see that same question has been solved in two ways and resulted in two different answers. Obviously both ways cannot be correct. For problems like this, there is a BODMAS rule to solve them. We will discuss it in detail. But before that let's know about different type of brackets.

There are four types of brackets.

—	Bar or Vinculum brackets
( )	Parentheses or round brackets or curved brackets
{ }	Braces or Curly brackets
[ ]	Box brackets or Square brackets

Note it down

Vinculum is represented by a bar on the top of the numbers.

3.7.1 BODMAS Rule

BODMAS stands for Bracket, Of, Division, Multiplication, Addition, and Subtraction. In this rule we learn about how to solve an operation in the correct order. If the expression contains Brackets, Multiplication, Addition, Division and Subtraction we follow the following rules to solve it.

1. Solve or simplify the operations within the brackets first.
2. Then solve division in order from left to right.
3. Now turn to multiplication in order from left to right.
4. Next carry out addition in order from left to right.
5. Finally subtract in order from left to right.

Always follow the BODMAS rules to simplify the sum. If we don't follow the BODMAS rules we will get the wrong answer.

If a mathematical expression has more than one bracket, follow the given order:

- First solve the operation(s) under the **vinculum**.
- Then solve the operation(s) within the **parentheses**.
- Next solve the operation(s) within the **curly brackets** (or braces).
- Finally solve the operation(s) within the **square brackets** (or box brackets).

The operations within a bracket should also be solved according to the above order.

Let's see the following examples to see how to solve the expressions containing more than one operation using BODMAS rule.

Example 1:

Simplify: $3 \times (12 + 23)$

Solution:

$= 3 \times (12 + 23)$

$= 3 \times 35 = 105$

Step 1: Solve the operation within the brackets.
Step 2: Multiply.

Example 2:

Simplify: $68 - [6 + 4 \times (35 - 2 \times 12)]$

Solution:

$= 68 - \{6 + 4 \times (35 - 2 \times 12)\}$

$= 68 - \{6 + 4 \times (35 - 24)\}$

$= 68 - \{6 + 4 \times 11\}$

$= 68 - \{6 + 44\}$

$= 68 - 50 = 18$

Step 1: First, multiply within parentheses.
Step 2: Next subtract within parentheses.
Step 3: Now multiply within curly brackets.
Step 4: Next add within curly brackets.
Step 5: Finally subtract.

Example 3:

Simplify: $1213 - [30 \times \{20 + 7 \times 21 \div 7 - (7 + \overline{5 - 3})\}]$

Solution:

$1213 - [30 \times \{20 + 7 \times 21 \div 7 - (7 + \overline{5 - 3})\}]$

$= 1213 - [30 \times \{20 + 7 \times 21 \div 7 - (7 + 2)\}]$

$= 1213 - [30 \times \{20 + 7 \times 21 \div 7 - 9\}]$

$= 1213 - [30 \times \{20 + 7 \times 3 - 9\}]$

$= 1213 - [30 \times \{20 + 21 - 9\}]$

$= 1213 - [30 \times \{41 - 9\}]$

$= 1213 - [30 \times 32]$

$= 1213 - 960 = 253$

Step 1: Subtract within vinculum.
Step 2: Add within parentheses brackets.
Step 3: Divide within curly brackets.
Step 4: Now multiply within curly brackets.
Step 5: Next add within curly brackets.
Step 6: Subtract within curly brackets.
Step 7: Now multiply square brackets.
Step 8: Finally subtract.

BODMAS rules are also applicable to fractions and decimals. We follow the same rules when solving fractions or decimal expressions which we use to solve simple expressions involving whole numbers.

Note it down

When dividing any number by a fraction, change the division sign to a multiplication sign and take the reciprocal of the fraction. Then simply multiply.

Example 4:

Simplify: $17\frac{1}{2} \div \{4\frac{1}{2} + (8 - \frac{3}{4})\}$

TEACHING POINT: Take a box in class containing mathematical expressions written on folded paper chits. Students will be asked to take out one paper chit and solve it.

Solution:

$= 17\frac{1}{2} \div \{4\frac{1}{2} + (8 - \frac{3}{4})\}$ Step 1: First subtract within parentheses.

$= 17\frac{1}{2} \div \{4\frac{1}{2} + (\frac{32-3}{4})\}$

$= 17\frac{1}{2} \div \{4\frac{1}{2} + \frac{29}{4}\}$ Step 2: Now add within curly brackets.

$= 17\frac{1}{2} \div \{\frac{9}{2} + \frac{29}{4}\}$

$= 17\frac{1}{2} \div \{\frac{18+29}{4}\}$

$= 17\frac{1}{2} \div \frac{47}{4}$ Step 3: Next divide.

$= \frac{35}{12} \times \frac{24}{47} = \frac{35 \times 2}{1 \times 47}$ Step 4: Division will become multiplication and $\frac{47}{4}$ will become $\frac{4}{47}$.

$= \frac{70}{47} = 1\frac{23}{47}$ Step 5: Finally change the fraction to a mixed number.

Example 5:

Simplify: 6.5 – {4 + 1.3 × (8.2 – 3.4 × 2.02)}

Hint
If there are no signs found between two numbers or brackets then multiply the numbers outside the brackets with the result of operations within the brackets.

Solution:

= 6.5 – {4 + 1.3 × (8.2 – 3.4 × 2.02)}

= 6.5 – {4 + 1.3 × (8.2 – 6.868)} Step 1: First multiply within parentheses.

= 6.5 – {4 + 1.3 × 1.332} Step 2: Now subtract within parentheses.

= 6.5 – {4 + 1.7316} Step 3: Multiply within curly brackets.

= 6.5 – 5.7316 Step 4: Next add within curly brackets.

= 0.7684 Step 5: Finally subtract.

Exercise 3.7

1. Simplify the following.

a) 1.6 + 3 × 7 – 5

b) 42 ÷ 2 + 45 – 22

c) $\frac{7}{9} \times 1\frac{1}{5} \div \frac{8}{15}$

d) $\frac{15}{21} \times \frac{45}{3} - \frac{1}{15} \div \frac{3}{7}$

e) $725 - [20 \times \{10 + 7 + (7 \times \overline{5-3})\}]$

f) $7 + \{\frac{1}{3} + \frac{2}{12}(\frac{7}{4} - \frac{5}{12})\}$

g) $\{(\overline{10\frac{1}{3} - \frac{2}{9}} \div \frac{5}{16}) \times \frac{5}{20} + \frac{1}{4}\}$

h) $\frac{1}{3} - \{\frac{17}{4} - (5\frac{1}{4} \times 2\frac{1}{2})\}$

i) $\frac{71}{3} - \frac{21}{4}(\frac{1}{5} \times \frac{3}{4})$

j) $(\frac{6}{17} \times \frac{34}{15}) - \frac{6}{35} \times \frac{2}{3}$

k) $\{(6\frac{1}{3} \div \frac{3}{5}) \times \frac{18}{9} - \frac{6}{3}\}$

l) 22.46 × 3 ÷ 2(7.82 – 2.42) + 9.5 × 3.4

m) 12.35 + [5.25 + 3(18.43 – 3.66 × 5.75)]

3.8 Real-life Problems involving Simplification

On every Sunday Ijaz goes to the Sunday Bazaar for groceries. Here is the grocery list.

Grocery list

Sugar	5.5 kg
Flour	8.3 kg
Milk	4 bottles
Meat	3.23 kg
Tomato	2.75 kg

Price list

Sugar	Rs 76 per kg
Flour	Rs 76 per kg
Milk	Rs 145.75 per bottle
Meat	Rs 567 per kg
Tomato	Rs 30 per kg

Example 1:

a) If Ijaz had a Rs 5000 currency note to pay the bill, how much amount will be left?

b) Find the total bill of the shopping.

c) Write in the single expression for Ijaz's shopping.

Solution:

Cost of 1 kg sugar = Rs 76
Cost of 1 kg flour = Rs 76
Cost of 1 bottle of milk = Rs 145.75
Cost of 1 kg meat = Rs 567
Cost of 1 kg tomato = Rs 30

Cost of 5.5 kg sugar = Rs 76 × 5.5 = Rs 418
Cost of 8.3 kg flour = Rs 76 × 8.3= Rs 630.8
Cost of 4 bottles of milk = Rs 145.75 × 4 = Rs 583
Cost of 3.23 kg meat = Rs 567 × 3.23 = Rs 1831.41
Cost of 2.75 kg tomato = Rs 30 × 2.75= Rs 82.5

a) The total bill = Rs (418 + 630.8 + 583 + 1831.41 + 82.5) = Rs 3545.71

b) Remaining = Rs 5000 – Rs 3545.71 = Rs 1454.29

So, the amount remaining is Rs 1454.29.

c) Expression for Ijaz's shopping

Teaching Point: Tell the students that simplification is a mathematical concept which is used very frequently, direct or indirect, in real- life. Give some example from real- life. We can also use simplification for the calculation of bills.

The total bill $= \text{Rs } 5000 - (76 \times 5.5) + (76 \times 8.3) + (4 \times 145.75) + (567 \times 3.23) + (30 \times 2.75)\}$

$= \text{Rs } 5000 - \{418 + 630.8 + 583 + 1831.41 + 82.5\}$

$= \text{Rs } 5000 - \{1048.8 + 2496.91\}$

$= \text{Rs } 5000 - \text{Rs } 3545.71 = \text{Rs } 1454.24$

Example 2:

Rabia bought $\frac{4}{6}$ kg chicken, $\frac{2}{3}$ kg mutton and $\frac{5}{12}$ beef from a butcher's shop. She also bought 2 packs of $\frac{4}{3}$ kg sugar and 3 packs of $\frac{3}{4}$ kg rice from a grocery store. Write the expression for her shopping and also find the total weight of the items she bought from both places.

Solution:

Weight of chicken $= \frac{4}{6}$ kg

Weight of mutton $= \frac{2}{3}$ kg

Weight of beef $= \frac{5}{12}$ kg

Weight of 2 packs of sugar $= 2 \times \frac{4}{3}$ kg

Weight of 3 packs of rice $= 3 \times \frac{3}{4}$ kg

Total weight she bought = ?

So the expression for the shopping is:

$$= \left(\frac{4}{6} + \frac{2}{3} + \frac{5}{12}\right) + \left(2 \times \frac{4}{3}\right) + \left(3 \times \frac{3}{4}\right)$$

$$= \frac{8+8+5}{12} + \frac{8}{3} + \frac{9}{4}$$

$$= \frac{21}{12} + \frac{32+27}{12} = \frac{80}{12} \text{ kg}$$

$$= \frac{20}{3} \text{ kg} = 6\frac{2}{3} \text{ kg}$$

So, the total weight of the items bought is $6\frac{2}{3}$ kg.

Exercise 3.8

1. **Afnan's monthly income is Rs 35,300. If he spent $\frac{2}{10}$ of his income on house rent, $\frac{3}{10}$ on food and $\frac{4}{10}$ on other expenditures, find the total amount that he saved.**

2. **The length of a piece of rope is 111.9 metres. If it is divided into 6 equal pieces and 10 metres of each rope is used, find the length of remaining rope.**

3. **Dania bought 6 boxes of cupcakes. Each box has 12 cupcakes in it. She distributed $\frac{2}{3}$ of the cupcakes among needy children and put the remaining in 8 jars equally. How many cupcakes did she put in each jar?**

4. **Marwa ordered 3 scarves for Rs 415 each and 2 abaya's for Rs 920 each online. There was a discount on these items and she paid half price for each item. She paid additional Rs 140 as courier charges. How much did she spend in total?**

5. **Ali has number blocks in three colours. He has 8 blue blocks. There are $\frac{3}{4}$ times as many green blocks as blue blocks, and there are 4 fewer red blocks than green blocks. How many blocks does Ali have?**

6. **Madiha needs 7.5 kg of apples and 5.75 kg of mangoes for an Iftar party. She already has 3.25 kg of mangoes and 2.25 kg apples at home. How many more apples and mangoes should she buy?**

Think Higher

Who is correct and why?
Who is wrong and why?
What could be a few other wrong ways to solve it?

$-9 - (-10) = -9 - 10$
$= -19$

$-9 - (-10) = -9 + 10$
$= -1$

Create and then solve a real life situation that involves the subtraction of integers with an answer of -12.

Summary

- In addition, when we change the order of two integers, the sum remains the same. It is known as the commutative law of integers under addition.
- In addition, when we add three integers, the order of grouping the numbers does not affect the sum.
- 0 is the called the additive identity because when we add 0 to any integer the result is the integer itself.
- In multiplication, when we multiply two integers, the product is always an integer. It is known as the Closure Law of integer under multiplication.
- In multiplication, when we change the order of two integers, the product remains the same. It is known as the Commutative Law of integers under multiplication.
- In multiplication, when we multiply three integers, the order of grouping the integers does not affect the product.
- If the product of two integers is 1 (the multiplicative identity), then both integers are called the multiplicative inverse of each other.
- The Distributive Law says that multiplying a integer by a group of integers added together is the same as doing each multiplication separately and then adding or subtracting their products.
- Zero can be divided by any integer or number but no number or integer cannot be divided by zero.
- BODMAS stand for Bracket, Of, Division, Multiplication, Addition, and Subtraction.

Vocabulary

- Closure law
- Commutative law
- Associative law
- Additive identity
- Additive inverse
- Multiplicative identity
- Multiplicative inverse
- Distributive law
- Simplification
- Expression
- BODMAS Rule
- Vinculum
- Brackets
- Braces
- Parentheses

Review Exercise

1 Encircle the correct option.

a) The smallest positive integer is ______.
i. 0 ii. +1 iii. –1 iv. not determined

b) The greatest negative integer is ______.
i. 0 ii. +1 iii. –1 iv. not determined

c) The greatest positive integer is ______.
i. +10 ii. +100 iii. +1 iv. not determined

d) The absolute value of –16 is ______.
i. –16 ii. 16 iii. –60 iv. 60

e) The product of two integers with opposite signs is ______.
i. always positive ii. always negative
iii. sometimes positive sometimes negative
iv. both positive and negative

f) What must be added to –135 to get –142?
i. –7 ii. 7 iii. 277 iv. –277

g) Additive inverse of 6 is ______.
i. –6 ii. 6 iii. $\frac{1}{6}$ iv. $-\frac{1}{6}$

h) The value of 4 × (–5) × (–2) is ______.
i. –16 ii) +20 iii. +40 iv. –40

i) Multiplicative inverse of –100 is ______.
i. –100 ii. +100 iii. $\frac{1}{100}$ iv. $-\frac{1}{100}$

j) ____________ is a phrase that includes numbers, brackets and operators together to show the value of something in mathematical form.
i. A set ii. An expression iii. An operation iv. A rule

k) We solve ________ bracket first in BODMAS rule
i. [ ] ii. { } iii. ( ) iv. Vinculum

l) In BODMAS rule we solve_____ operation first.
i. + ii. × iii. – iv. ÷

m) _____ is called Vinculum.
i. [ ] ii. { } iii. ( ) iv. _____

n) The correct order of operations is_____.
i. +, –, ÷, × ii. ÷, +, ×, – iii. ÷, ×, +, – iv. ÷, ×, –, +

2. **Solve the following**

a) 85 + (–96) = ____.
b) 80 + 57 = ____.
c) 86 + (–38) = ____.
d) 22 + (–41)+ (–8) = ____.
e) (–18) + (–45)+(-89) = _____
f) (–11) + (–5) + 6 = _____
g) (–4) – (6) = ____.
h) 80 – (–7) = ____.
i) 6 – (–8) = ____.
j) (–34) – (–40) = ____.
k) (–18) – (–45) – (56) = _____
l) (–101) – (–55) – 43 = _____
m) (+6) × (–9) = _____
n) (+8) × (–9) = _____
o) (–2) × (–34) = _____
p) (+134) × (–123) = _____
q) (+6) ÷ (–2) = _____
r) (+340) ÷ (–34) = _____
s) (–12) ÷ (–4) = _____
t) (+224) ÷ (–22) = _____

3. **The product of two integers is –75. If one of them is –15, then find the other one.**

4. **Find an integer which divides –90 to give –45.**

5. **Verify the following**

a) (–5) × 3 = 3 × (–5)
b) (–27) + (–97) = (–97) + (–27)
c) 17 + (–9) = –9 + 17
d) –101 + (–77) = –77 + (–101)
e) (–6) + [(–3) + (–4)] = [(–6) + (–3)] + (–4)
f) (–12) × [5 × (–4)] = [(–12) × 5] × (–4)
g) [(–6) – 2)] (–5) = [(–6) (–5)] – [(–6) × (–2)]
h) (–11) × [(5 +(–7)] = (–11 × 5) + [(–11 × (–7)]

6. **Simplify the following expressions.**

a) (2 × 7) + (12 ÷ 4)
b) 3 × (7 + 9) – 6
c) (8 × 5) + 5 – 3
d) 204 – [30 × {20 + (8 × 32 ÷ 4) – (9 ÷ 63)}]
e) $\{(9 \times \frac{1}{3} - \frac{3}{9}) \div \frac{5}{16}\} \times (\frac{5}{12} + \frac{3}{4})$

7. **While travelling Rohaan paid Rs 36.45 for a ricksaw, Rs 34.50 for a metro bus and Rs 56.56 for a metro train. If he had Rs 1300, find the total amount he was left with.**

8. **A shopkeeper has 42 $\frac{3}{4}$ m of cloths in his stock, if he sells 6 $\frac{2}{6}$ m of cloth, then how much cloth he has left?**

9. **Look at the following price list of the different fruits to answer the questions.**

Price list

 Rs 150 per kg

 Rs 200 per kg

 Rs 100 per dozen

 Rs 150 per dozen

a) If Rabia bought 3.45 kg of apples and 2.34 dozen of oranges. How much did she pay?
b) If the shopkeeper sold 34.45 kg of mangoes, 23.89 kg of apples, 7.67 dozen of oranges and 6.5 dozen banana, find his total sales.
c) Also write for it an expression.

10. **The pack of large erasers has 8 pieces; the pack of medium erasers has 15 pieces and the pack of small erasers has 20 erasers. Ibrahim has 3 small, 2 medium and 2 large packs of erasers. How many erasers does he have altogether?**

11. **Usman and his family use 13 $\frac{3}{9}$ litres of water for bathing, 10 $\frac{5}{15}$ litres for cleaning and 6 $\frac{2}{9}$ litres for cooking respectively. How much water do they use?**

12. **In a restaurant, a family ordered 3 sandwiches for Rs 250 each, 2 glasses of cold drinks for Rs 180 each and one cup of coffee for Rs 210. They paid additional tax of Rs 112. If they gave a Rs 5000 note to the cashier, how much was their total bill? How much amount did they get as change?**

13 **Ali and Fahad have Rs 564.32 and Rs 876.89 respectively. They want to buy a Tafseer book for Rs 2234.23. How many more rupees do they need to buy the Tafseer book?**

Math Project

Material Required:

- Integer cards
- Glue stick
- Chart papers
- 4 baskets
- Operations Symbol cards for addition, subtraction, multiplication, and division
- Markers

Procedure:

- Divide the students in groups of 4.
- Place 4 basket with cards of negative and positive integers in it.
- Label each basket with symbols of +, –, ÷ and ×.
- Students from each group will come forward and pick 2 integer cards from each of the 4 baskets.
- They will paste the relevant sum on their respective chart papers.
- Then they will solve the relevant sum of each operation on the chart paper.
- At the end each group will present their working and the other groups will check if they have solved it correctly or not.

Group A		
3	+	2

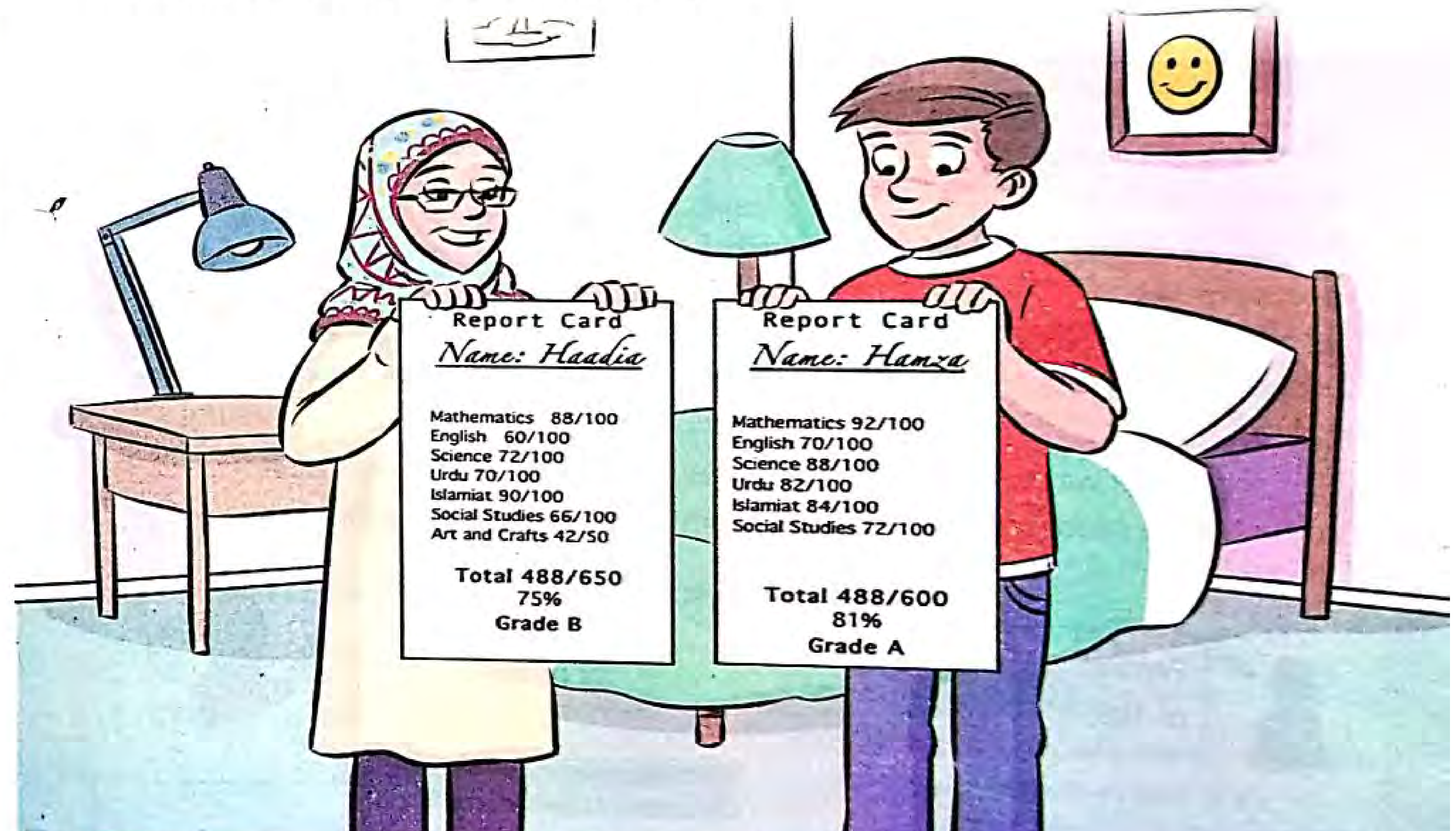
Group B		
10	÷	+5

Unit 4 Rate, Ratio and Percentage

Student Learning Outcomes

After completing this unit, students will be able to:

- Express one quantity as a percentage of another, compare two quantities by percentage and increase or decrease a quantity by a given percentage.
- Solve real-world word problems involving percentage.
- Explain rate as a comparison of two quantities where one quantity is 1.
- Calculate ratio of two numbers (up to 3-digit) and simplify ratios.
- Explain and calculate continued ratio.
- Solve real-world word problems involving ratio and rate.



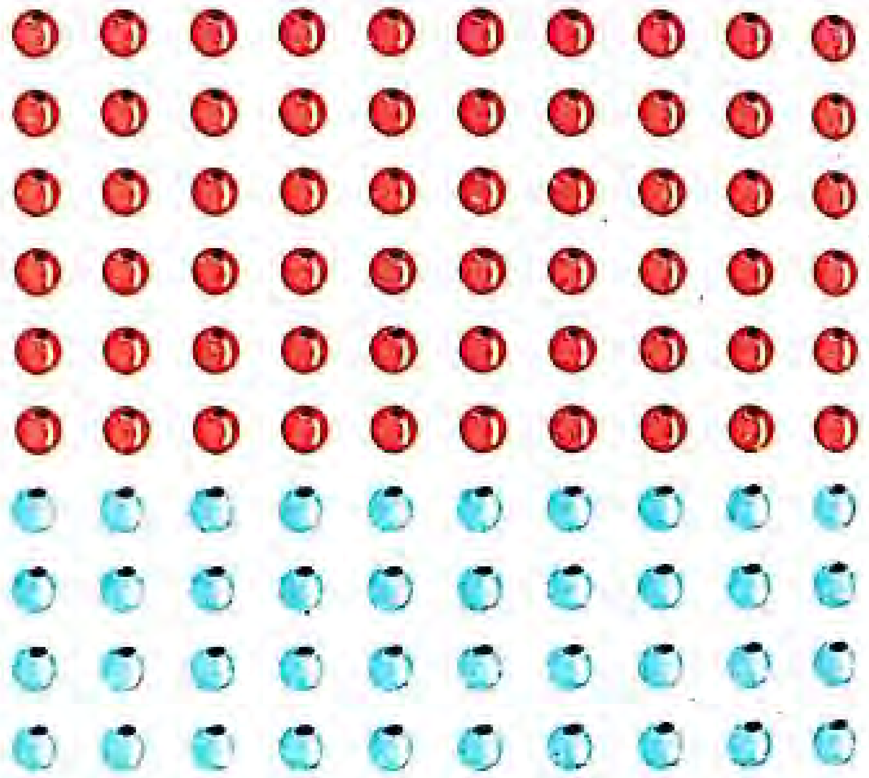
Hamza and Haadia got different grades although they got equal marks. Why?

Introduction

In the previous classes we have learnt about percentage, percentage as fraction and conversion of percentage to fraction and decimals and vice versa and their uses in our daily life. In this unit, we will learn about rate, ratio and a few more topics related to percentage which is used in daily life in many situations specifically when we compare quantities. For example, when we compare the ages of two children, length of two boxes, weight of fruits, expenditures and savings, rate of things, percentage marks of students, increase and decrease in price of objects in percentage.

4.1 Percent

Amna has 100 beads.
40 beads are in blue colour.



There are 100 beads.
40 beads are blue.
We can say that 40 out of 100 (40 hundredths) beads are blue.
As a fraction, we can write it as $\frac{40}{100}$. In decimal form, we can write it as 0.40.

There is another way to express it. Which is called '**percent**'. We can say that 40 percent of the beads are blue. In symbolic form, it can be written as 40%.

Percent (%) means out of 100.

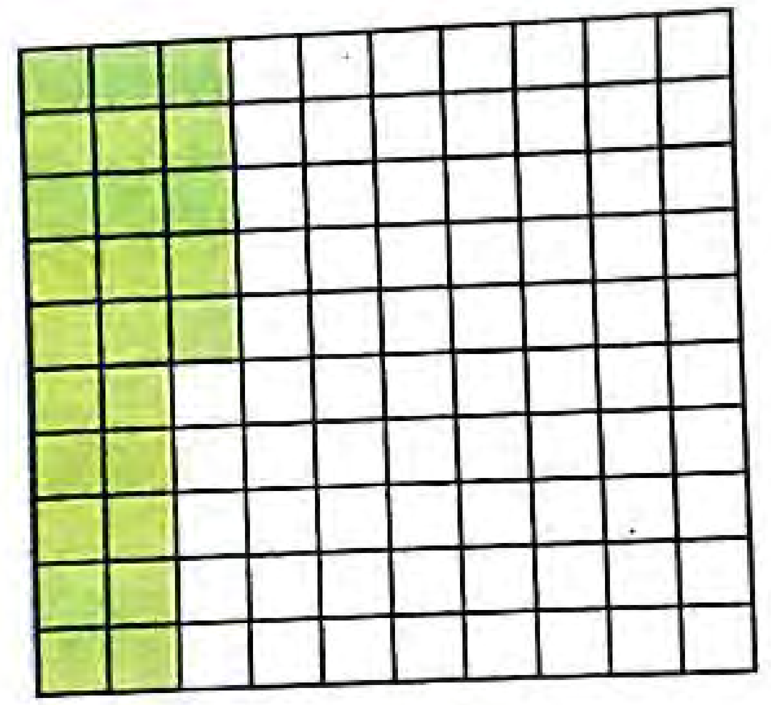
% is the symbol for percent.

So, 40% means 40 out of 100.

40 percent = 40% = $\frac{40}{100}$ = 0.40

Previous Knowledge Check

- What is meant by percentage?
- How can we convert fractions and decimal to percentage?
- Convert 0.56 into percentage.



Look at this square. It is divided into 100 equal parts.
25 out of 100 parts are coloured.
$\frac{25}{100}$ or 0.25 of the square is coloured.
So, 25% of the whole square is coloured.

Write: 25%

Read: Twenty-five percent

What **percentage** of the square is not coloured?

Now, have a look at this square. 8 out of 100 parts are pink.

So, 8% of the whole square is pink.

Write: 8%

Read: Eight percent

What percentage of the whole square is yellow?
What percentage of the whole square is blue?

4.1.1 Express one quantity as a percentage of another

Example 1: Saman used 8 eggs out of 40 eggs to make a few cupcakes. Express used eggs as a percentage and find what percentage of eggs, she used.

Solution:

First write it in the form of a fraction.

8 out of 40 = $\frac{8}{40}$

Now multiply the fraction by 100 and put the % symbol.

$\frac{8}{40} = \frac{8}{40} \times 100\%$

Simplify if possible.

$= \frac{1}{\cancel{5}} \times \overset{20}{\cancel{100}}\%$

= 20

So, she used 20% of the total eggs.

Note it down

When expressing one quantity as a percentage of another, both quantities must be in the same units.

TEACHING POINT: Explain to the students how to find quantity if percentage and total quantity is given. Write some questions of finding quantity if percentage and total quantity is given. Instruct them to solve them and share their findings with their class-fellows.

Example 2: 39 employees out of 65 use bus as conveyance. Express them as a percentage and find what percentage of total employees use bus.

Solution: First write it in the form of a fraction.

39 out of 65 $= \frac{39}{65}$

Now multiply the fraction by 100 and put the % symbol.

$\frac{39}{65} = \frac{39}{65} \times 100\%$

Simplify if possible.

$= \frac{3}{\cancel{15}} \times \overset{20}{\cancel{100}}\%$

$= 60\%$

So, 60% of total employees use bus.

Quick Check

- Express 5 out of 40 as a percentage.
- Express 7 out of 200 as a percentage.

Example 3:

Express 25 cm out of 285 cm as a percentage.

Solution:

First write it in the form of a fraction.

25 out of 285 $= \frac{25}{285}$

Now multiply the fraction by 100 and put the % symbol.

$\frac{25}{285} = \frac{25}{285} \times 100\%$

Simplify if possible.

$= \frac{5}{57} \times 100\% = \frac{500}{57}\%$

$= 8.77\%$

So, 25 cm out of 285 is 8.77%

Quick Check

- Find 30% of 50.
- Find 50% of 300.

We can also find a required percentage of any given quantity.

4.1.2 Percentage Part of a Whole

Example 1: Haadia baked 30 cupcakes. 30% of the cupcakes are chocolate flavoured. Find how many cupcakes are chocolate flavoured?

Solution: We can find the answer using two methods.

Method 1:

30% of 30 $= \frac{3\cancel{0}}{10\cancel{0}} \times 3\cancel{0}$

$= 3 \times 3$

$= 9$

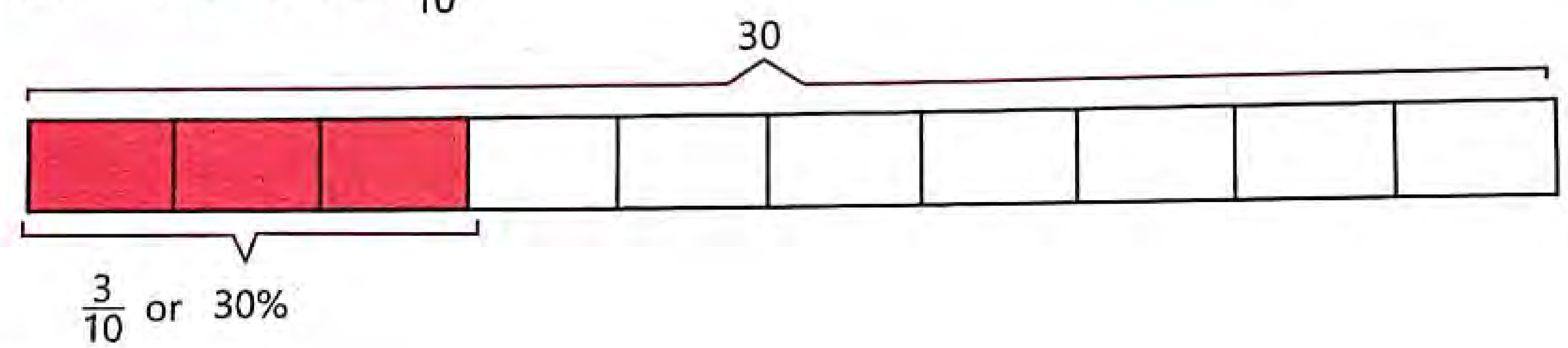
So, there are 9 chocolate flavoured cupcakes.

Method 2:

First, write 30% as a fraction in its simplest form.

$30\% = \frac{3\cancel{0}}{10\cancel{0}} = \frac{3}{10}$

Draw a model to show $\frac{3}{10}$.



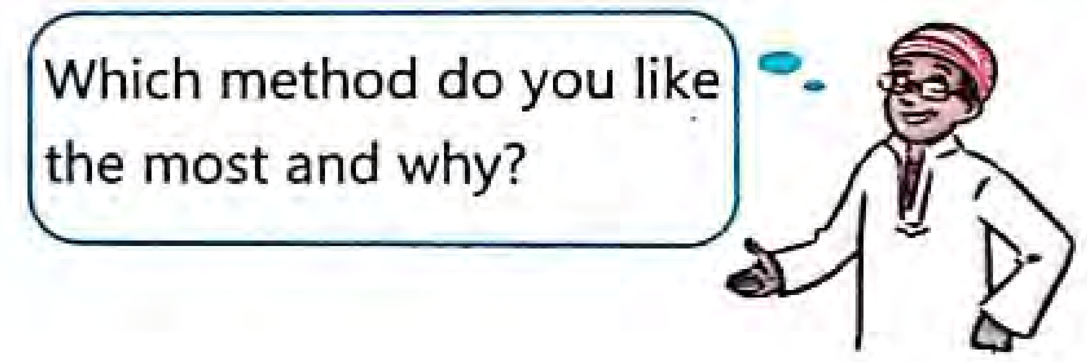
10 units ⟶ 30

1 unit ⟶ 30 ÷ 10 = 3

3 units ⟶ 3 × 3 = 9

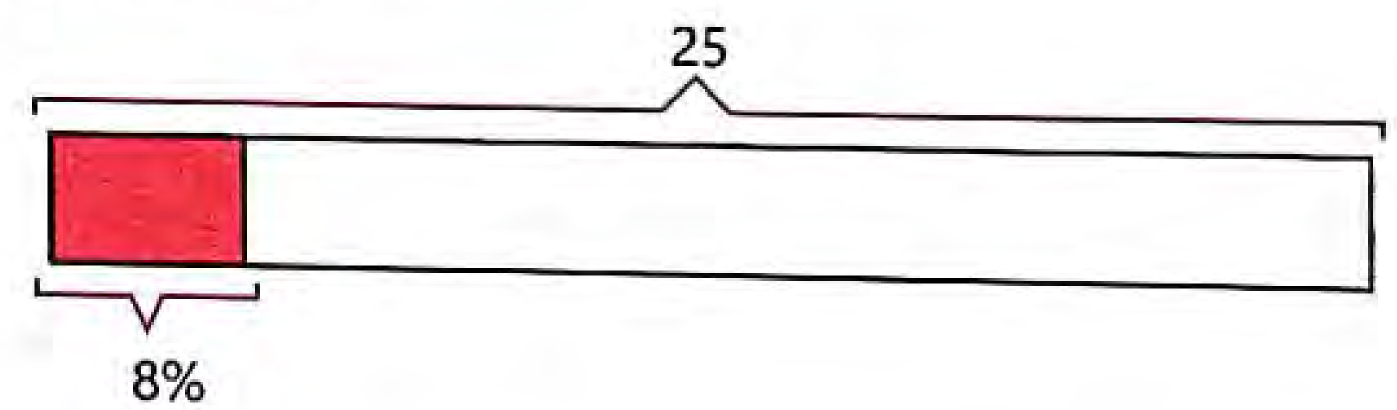
30% of 30 is 9.

So, there are 9 chocolate cupcakes.



Example 2: Find 8% of 25.

Solution:



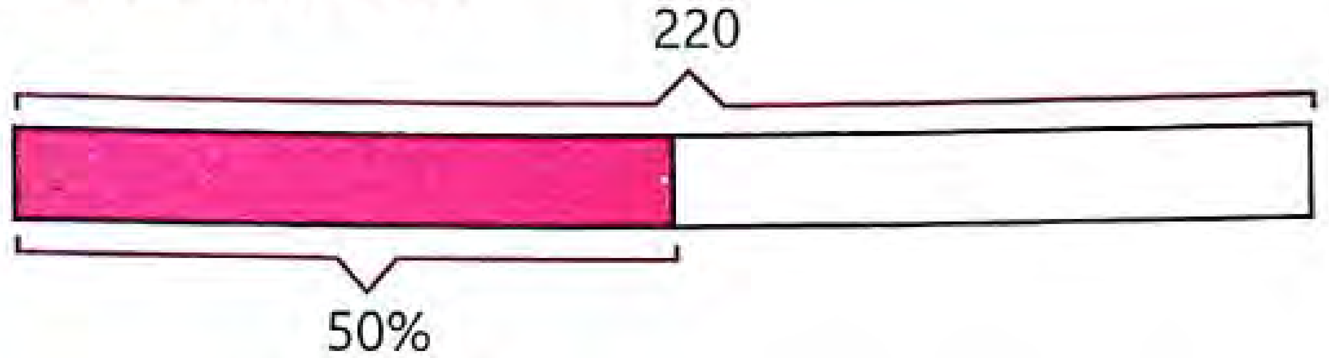
8% of 25 $= \frac{8}{\underset{4}{\cancel{100}}} \times \cancel{25}_1$

$= \frac{8}{4}$

$= 2$

So, 8% of 25 is 2.

Example 2: What is 50% of 220?



$$50\% \text{ of } 220 = \frac{5\cancel{0}}{1\cancel{00}} \times 22\cancel{0}$$

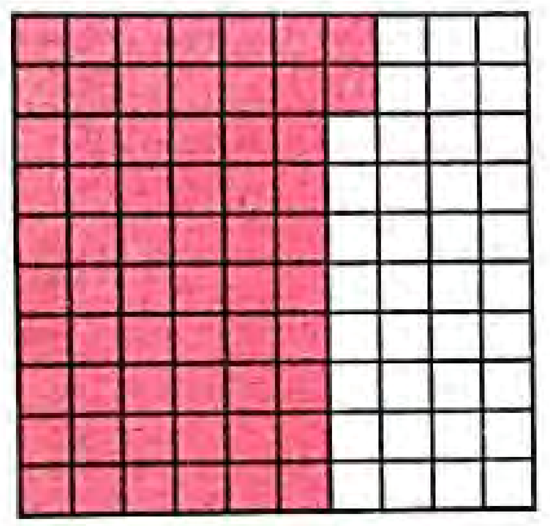
$$= 5 \times 22$$

$$= 110$$

So, 50% of 220 is 110.

Exercise 4.1

1 **The following squares are divided into 100 equal parts. Express the coloured parts as a percentage.**



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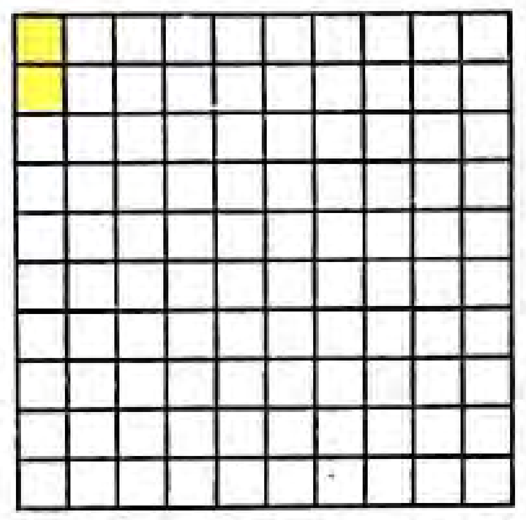
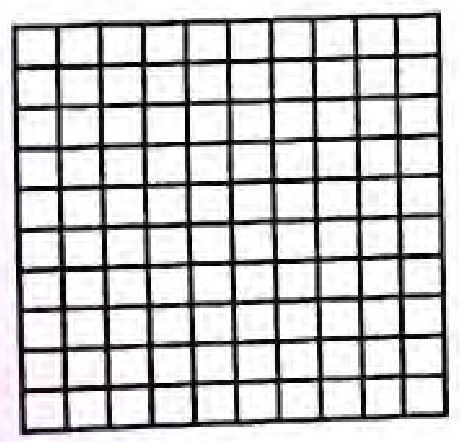
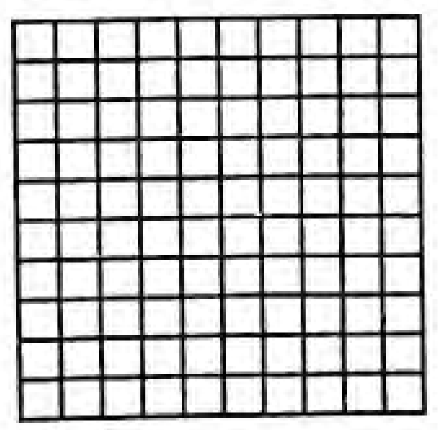


☐ %

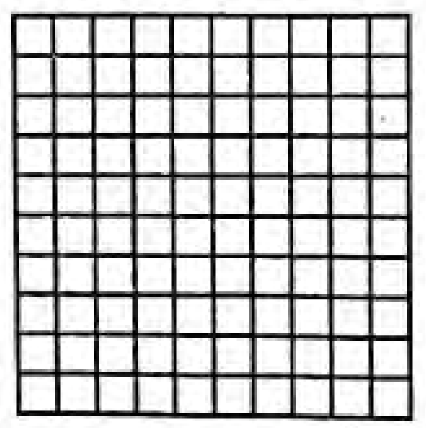
2 **Colour the squares to show the given percentage.**



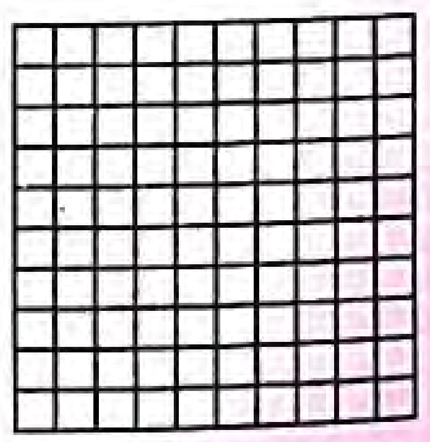
78%



13%



26%



50%

3 **Mahad got 92 marks out of 100 in a Mathematics test. Express the marks as a percentage.**

Write: ☐ **Read:** ☐

4 **Find the value of the following.**

a) 15 % of 35 b) 90% of 1000 c) 5.2% of 500 d) 25% of 175

e) $67\frac{1}{7}$% of 780 f) 20% of 1000 g) 87 % of 200 litres

h) 2.5% of 3200 grams i) 9.25% of Rs 300 j) 45% of 75 marks

k) 0.5 % of 50 mm l) 4.50 % of 100

5 **Find the percentage.**

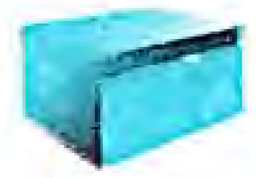
a) 50 out of 600 b) 175 out 900 c) 7.82 out of 40

d) 90 out of 100 e) 66 out of 190 f) 89 out of 200

g) 40 out of 75 marks h) 2 out of 10 children i) Rs 30 out of Rs 500

j) 78 m out of 560 m k) 3 months out of 5 years l) 1 day out of 1 week

6 **18 out of 90 boxes in a shop are blue. Express the number of blue boxes as percentage.**

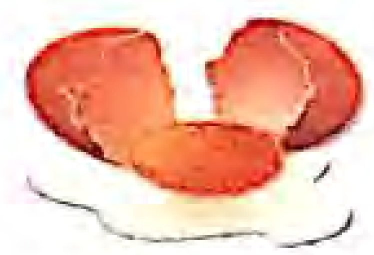


7 **Huma used 12 metres of cloth to stitch four shirts. If she had 30-meter cloth altogether, find the percentage of cloth she used.**

8 **11 students out of 25 students of a school participated in a marathon. Express this as percentage.**

9 **Fahad saved Rs 50 out of Rs 200 from his pocket money. What percentage of money did he save?**

10 **Out of 180 eggs, 25% are broken. How many eggs are broken?**



11 **In a match Imran scored 35% of the total 350 runs. How much runs he scored and how much runs the remaining team scored?**

12 **Sara donated 12% of her salary as a donation. If her salary is Rs 45000, find the amount she donated.**

4.2 Compare two quantities by percentage

Example 1:

Ali got 43 marks out of 50 and Madiha got 40 marks out of 50. Compare the marks obtained by using percentage.

Solution:

First find the percentage of marks Ali got.

Total marks = 50

Marks obtained by Ali = 43

Marks obtained by Ali = $\frac{43}{\cancel{50}_1} \times \cancel{100}^2\%$

= 86%

Now find the percentage of marks Madiha got.

Total marks = 50

Marks obtained by Ali = 40

Marks obtained by Ali = $\frac{40}{\cancel{50}_1} \times \cancel{100}^2\%$

= 80%

So, by looking at the percentage of their marks, we can see that Ali got 6% more marks than Madiha.

Previous Knowledge Check

- How can we compare two quantities?
- If Ali has Rs 459 and Ahmad has Rs 845. How can we compare and tell which one is greater?

Example 2:

Ahmad saved Rs 1600. He gave Rs 400 to some needy person. Saad has Rs 1200 and he gave Rs 624 to some needy person. Compare and tell who gave more percentage of his saving to the needy persons.

Solution:

By observing this we can see that we cannot compare these quantities simply as we compare the whole numbers because the total amounts are different. So, here we will use percentage to compare who donated more amount based on their individual savings.

Ahmad's savings = Rs 1600

He gave to needy persons = Rs 400

Percentage of money given to needy persons = $\frac{400}{1600} \times 100\%$

= 25%

Saad's savings = Rs 1200

He gave to needy persons = Rs 624

Percentage of money given to needy person = $\frac{\cancel{624}^{312}}{\cancel{1200}_2} \times \cancel{100}\%$

= 52%

So by comparing, we came to know that Saad gave more percentage of his individual savings to needy persons than Ahmad.

Example 3:

Compare both quantities as percentage:

40 out of 90 and 20 out of 50

Solution:

First we find the percentage of 40 out 90.

Percentage of 40 out of 90 = $\frac{40}{90} \times 100\%$

= 44.44%

Now we find the percentage of 20 out 50.

Percentage of 20 out of 50 = $\frac{20}{50} \times 100\%$

= 40%

As 44.44% is more than 40%, so 40 out 90 is more than 20 out of 50.

Quick Check

Compare the following as percentage.

a) 24 out of 70 and 13 out of 80

b) 112 out of 650 and 130 out of 700

Exercise 4.2

1 **Compare the following and tell which one is greater as percentage.**

a) 22 out of 60 and 54 out of 60.

b) 18 out of 20 and 16 out of 20.

c) 68 out of 120 and 66 out of 130

d) 440 out of 550 and 520 out of 710

e) 10 out 60 and 15 out of 70

f) 200 out of 400 and 150 out of 350.

2 **In town A, 200 out 500 houses are under construction and in town B 250 out of 600 houses are under construction. Compare and tell in which town more percentage of houses are under construction?**

3 In math test Anum got 40 marks out of 50. And in English test she got 35 marks out of 60. In which subject she got less percentage marks.

4 Mahad got 12 out of 15 marks, Hadia got 14 out of 15 marks and Samiha got 11 out of 15 marks in math weekly test. Compare their marks by finding percentage.

5 Omair's notebook has 82 empty pages out of 120 pages. Sara's notebook has 42 empty pages out of 54 pages. Compare by finding percentage to tell who has more empty pages in his/her notebook.

4.3 Increase or decrease a quantity by a given percentage

Example 1: Adnan bought a shirt in Rs 900. After some days the price of the shirt increased by 5%. How much amount increase? What is the price of the shirt now?

Solution:

Price of the shirt = Rs 900

Percentage increase = 5%

Increase in price = 5% of 900

$= \frac{5}{100} \times 900$

= Rs 45

So the price of the shirt increased by Rs 45.

Now we find the new price of the shirt.

Price of the shirt now = 900 + 45 = Rs 945

So, the price of the shirt is increased from Rs 900 to Rs 945.

> **Quick Check**
> Find 7% increase in price of rice that is Rs 146.

Example 2: The cost of the petrol per litre is Rs 160.

a) If the price of the petrol is increased by 5%, find the increased price of petrol.

b) If the price of petrol decreased by 3% of the new price, find the decreased price of the petrol.

Solution:

To find the increase in price of the petrol, first we find the percentage increase in petrol price.

Price of petrol = Rs 160

Percentage increase = 5%

Increase price of petrol = 5% of Rs 160

$= \frac{5}{100} \times 160$

= 8

New price of petrol = Rs 160 + Rs 8 = Rs 168

Now we find the decreased price of the petrol;

Price of petrol = Rs 168

Percentage decreased = 3%

Decreased price of petrol = 3% of Rs 168

$= \frac{3}{100} \times 168$

= 5.04

New price of petrol = Rs 168 – Rs 5.04 = Rs 162.96

Example 3:

625 decreased by 12 percent.

First we find the 12% of 625.

12% of 625 = $\frac{12}{100} \times 625$

So, 625 is decreased by 75.

625 – 75 = 550

> **Quick Check**
> Find 23% decrease of 820.

Exercise 4.3

1 **Find increase and decrease in quantity as given in percentage.**

a) 500 decreased by 17%　　b) 1400 increased by 9%

c) 18% increase in 1350　　d) 22% decrease in 760

e) 55% decrease in 2500　　f) 2% increase in 270

2 **Arhams' s salary is Rs 45900. If his salary increased by 12%. How much salary increased? Find his new salary?**

3 **On Monday 300 children visited the zoo. On Tuesday 15% less children visited the zoo. How many children visited the zoo on Tuesday?**

4 **In 2021, Ahads weight was 27 kg. His weight is decreased by 9% in 2022. Find:**

a) How much weight did he loose?　　b) What is his weight in 2022?

5 **Marwa's marks in monthly test in March were 56. In April, her marks increased by 24%. What are her marks in April?**

TEACHING POINT: Instruct the students to make their own word problem of percentage and share with their teacher and class-fellows.

4.4 Ratio

We frequently come across situations like: "Fahad's age is two times Mustafa's age" or "this pen is twice as costly as the other one". In such situations, we are actually comparing one thing, quantity or number with the other.

Math History

Abu Rayhan Al-Biruni (973-1048 AD), a Muslim Mathematician, made a significant contribution to the field of arithmetic, particularly irrational numbers, ratio and proportion.

There is another way to compare quantities which is called "**ratio**". We can say that the ratio of Fahad's age to Mustafa's age is 2 : 1. Here 2 and 1 are the terms of the ratio. We read this ratio as 2 is to 1.

A term that is used to compare two or more quantities of the same kind is called "Ratio". Symbol ":" is used to represent a ratio.

In general, If **a** and **b** are any two quantities of the same kind, then a : b is called the ratio between a and b. It is read as a is to b. The quantities **a** and **b** are called the terms of the ratio.

Previous Knowledge Check

- How can we find how much one quantity is more than the other quantity?
- How can we compare two or more quantities?

The first term of the ratio "**a**" is called the **antecedent** and the second term "**b**" is called the **consequent**. Look at these pencils and books.

There are 7 pencils and 4 books. The ratio of the number of pencils to the number of books can be expressed as 7 : 4 or 7 is to 4. Here, 7 is the antecedent and 4 is the consequent.

Number of pencils	to	Number of books
7	:	4

Also, the ratio of the number of books to the number of pencils can be expressed as 4 : 7 or 4 is to 7.

Number of books	to	Number of pencils
4	:	7

Here, 4 is the antecedent and 7 is the consequent. Hence, when we change the order of a ratio, the result will be changed. The following important points must be kept in mind while solving ratio related problems.

- The two quantities must be of the same kind.
- Ratio has no units.

TEACHING POINT: Explain the concept of ratios to the students by presenting different real-life examples and using manipulations.

Example 1:

The number of girls in a school is 340 and number of boys are 430. Find the ratio of boys to girls in school.

Solution:

Number of boys	to	Number of girls
430	:	340

The ratio of the number of boys to girls in school is 430 to 340 and it can be written in the form of a fraction as: $\frac{430}{340}$.

Note it down

To get the simplest form of the ratio we can divide both the terms of the ratio by their common factor until we get the simplified form of ratio.

The common factor of 430 and 340 is 10. Divide the antecedent (430) and consequent (340) by 10 so, we have

$$\frac{430 \div 10}{340 \div 10} = \frac{43}{34}$$

The ratio 43 : 34 has no common factor other than 1. So, it is the simplified form of 430 : 340.

To simplify a larger ratio is a very lengthy process. We find the common factor again and again and divide until we get the simplified form of the ratio.

Example 2:

Asim and Mahad have Rs 150 and Rs 190 respectively. Express the amount they have as ratio in the simplest form.

Solution:

Ratio of Asim's money to Mahad's money = 150 : 190

= 15 : 19 (divide antecedent and consequent by 10)

- While writing a ratio, the order of the terms matters. For example, 2 : 3 and 3 : 2 are not the same.
- A ratio can be written for more than two quantities, i.e. 1 : 2 : 3 or **a : b : c** etc.

Quick Check

What will happen when we change the order of the ratio 10 : 5?

4.4.1 Ratio and Fraction

We can also express ratios of two quantities as a **fraction**. Consider two ribbons of length, 14 cm and 16 cm. We can compare the length of these two ribbons with the help of a ratio.

Length of 1st ribbon	to	Length of 2nd ribbon
14	:	16

So, the ratio of the length of the 1st ribbon to the length of the 2nd ribbon can be expressed as 14 : 16 or 14 is to 16. This ratio can be expressed in the form of a fraction.

$$\frac{\text{Length of the 1st ribbon}}{\text{Length of the 2nd ribbon}} = \frac{14}{16} = \frac{7}{8}$$

Note it down

Every ratio can be expressed as a fraction.

Simplifying Ratios

A ratio can be reduced to its simplest form. To simplify a ratio a : b, we divide both antecedent and consequent by their common factor. It is called the **lowest** or simplest form of the ratio.

Example 1:

In a bus, there are 20 men and 14 women. Find the ratio of men to women in its simplest form.

Solution:

Number of men	to	Number of women
20	:	14

So, we can say that the number of men and women in the bus are in the ratio of 20 : 14.

We can also write it as fraction.

$$\frac{\text{Number of men}}{\text{Number of women}} = \frac{20}{14}$$

For finding the simplest form we divide the antecedent (20) and consequent (14) by their common factor that is 2, we get.

$$\frac{20}{14} = \frac{20 \div 2}{14 \div 7} = \frac{10}{7}$$

So, 20 : 14 = 10 : 7.

Quick Check

Can you tell the ratio of the number of women to the number of men?

TEACHING POINT: Explain the concept of simplification of ratios to the students by solving different examples.

Exercise 4.4

1 Find the ratio of:

a) 35 to 55 b) 15 min to 3 hours c) 8 km to 600 m
d) 500 gram to 4 kg e) Rs 240 to Rs 300 f)10 m to 100 cm

2 Write each of the following ratios in fraction form.

a) 4 : 5 b) 12 : 13 c) 19 : 25 d) 34 : 63
e) 55 : 78 f) 3 .1 : 6 . 2 g) 18 : 69 h) 72 : 81

3 Simplify the following ratios.

a) 4 : 8 b) 15 : 20 c) 30 : 45 d) 81 : 99
e) 16 : 39 f) 42 : 56 g) 35 : 100 h) 25 : 75
i) 8 : 12 j) 56 : 72 k) 18 : 36 l) 64 : 90

4 Write each of the following quantities into ratios and reduce into the simplest form (where possible).

a) Rs 500 and Rs 750 b) 45 m and 350 cm
c) 100 days and 35 weeks d) 4 kg and 12 grams
e) 8 weeks and 64 days f) 40 km and 90 km

5 There are 80 bulbs in a box. If 15 were found to be defective, find the ratio of defective to non-defective bulbs.

6 Ahmed and Saad bought a bat for Rs. 450. Ahmed paid Rs. 250 and Saad paid Rs. 200. Find the ratio of the amount Ahmed paid to the amount that Saad paid.

7 In a school there are 800 students and 80 teachers. Find the student to teacher ratio in the school.

8 The price of a pack of bread increased to Rs 50 from Rs 48. Find the ratio of the increase in price to the original price.

9 If Rs 340 is divided between Sara and Sonia according to the ratio 3 : 7, find the amount each gets.

4.5 Continued Ratio

We have learnt about ratios when comparing two quantities of the same kind. In some situations we need to compare more than two quantities. In such cases where two ratios x : y and y : z are expressed in the form of x : y : z, the ratio is called a continued ratio.

A **continued ratio** is a comparison of three or more quantities in a certain order.

In x : y and y : z we can see that y is common in both ratios. So we write it one time. Thus, the continued ratio will be x : y : z.

If a number is repeated two times in a ratio, then the number is called the common term of the given ratio.

We write the continued ratio as:

$$\begin{array}{ccccc} x & : & y & & \\ & & y & : & z \\ \hline x & : & y & : & z \end{array}$$

Example 1:

If a : b = 2 : 3 and b : c = 3 : 4, find the continued ratio between the quantities a, b and c.

Solution:

The given ratios are:

a : b = 2 : 3

b : c = 3 : 4

Here the common term is b, so write it in the middle of the continued ratio.

$$\begin{array}{ccccc} a & & b & & c \\ 2 & : & 3 & & \\ & & 3 & : & 4 \\ \hline 2 & : & 3 & : & 4 \end{array}$$

So, the required continued ratio a : b : c = 2 : 3 : 4

TEACHING POINT: Describe continued ratio with examples. Then ask the students to write some examples of continued ratio in their notebooks.

Example 2:

The ratio of Khadija's savings to Maryam's savings is 5 : 3 while the ratio of Maryam's savings to Ayesha's savings is 1 : 4. Find the continued ratio between their savings.

Solution:

Ratio of Khadija savings to Maryam's savings = 5 : 3

Ratio of Maryam's savings to Ayesha's savings = 1 : 4

To find the continued ratio, first we need to find the same value of the common term. For this we can use the following two methods.

Note it down

The ratio between the sides of the triangle are x : y = 3 : 4 and y : z = 6 : 7. Find the ratio x : y : z.

Method I:

In this method, multiply both ratios with a suitable number that can make the common term same.

$$\begin{array}{ccccc} \textbf{Khadija} & & \textbf{Maryam} & & \textbf{Ayesha} \\ 5\times 1 & : & 3\times 1 & & \\ & & 1\times 3 & : & 4\times 3 \\ \hline 5 & : & 3 & : & 12 \end{array}$$

Method II:

The product of the first terms of both ratios will become the first term of the continued ratio. The product of the last terms of two ratios will become the last term of the continued ratio. The product of the first term of the second ratio and the second term of the first ratio will become the middle term of the continued ratio.

$$\begin{array}{ccccc} \textbf{Khadija} & & \textbf{Maryam} & & \textbf{Ayesha} \\ 5 & : & 3 & & \\ & & 1 & : & 4 \\ \hline 5 & : & 3 & : & 12 \end{array}$$

So, 5 : 3 : 12 is the required continued ratio.

We can also divide a quantity in any given ratio a : b : c. Consider the following example.

Example 3:

Mr Adnan distributed Rs 87000 as charity among three orphanages A, B and C. He shared this amount in the following ratios.

A : B = 4 : 3

B : C = 2 : 5

Find the share of each orphanage.

Solution:

Total amount of charity = Rs 87000

Quick Check

Write the ratio for the following.

a) 50 days, 60 days and 30 days

b) 24 pencils, 15 erasers and 12 sharpeners.

Step I: First find the continued ratio.

A		B		C
4	:	3		
		2	:	5
8	:	6	:	15

Step II: Find the sum of the terms of the continued ratio.

Sum of the terms = 8 + 6 + 15 = 29

Step III: Divide each term of the ratio by the sum of the ratios and multiply it by the total amount to be shared.

Share of orphanage A = $\frac{8}{29} \times 87000$ = Rs 24000

Share of orphanage B = $\frac{6}{29} \times 87000$ = Rs 18000

Share of orphanage C = $\frac{15}{29} \times 87000$ = Rs 45000

So, the share of orphanage A, B and C is Rs 24000, Rs18000 and Rs 45000 respectively.

Exercise 4.5

1 Solve the following.

a) If a : b = 4 : 5 and b : c = 5 : 6, then find a : b : c.

b) If d : e = 2 : 6 and e : f = 7 : 8, then find d : e : f.

c) If g : h = 8 : 9 and h : i = 9 : 12, then find g : h : i.

d) If j : k = 6 : 7 and k : l = 10 : 1, then find j : k : l.

e) If m : n = 2 : 8 and n : o = 5 : 6, then find m : n : o.

2 The measurement of three angles of a triangle are in the ratio 2 : 2 : 5. Find the measurement of each angle.

3 The ratio of a to b is 8 : 9 and b to c is 4 : 6. Find the continued ratio a : b : c.

4 The two ratios of three quantities a, b and c are a : b = 2 : 3 and b : c = 4 : 5. Find their continued ratio.

5 The ratio of Ibrahim's marks to Ahad's marks is 5 : 3 and the ratio of Ibrahim's marks to Marwa's marks is 4 : 7. Find the continued ratio among their marks.

6 Divide Rs 7500 into the ratio 4 : 6 : 5.

7 Divide Rs 5376 between Ayesha, Sadia and Madiha in the ratio 8 : 7 : 6.

8 Divide 3416 among three persons a, b and c, such that the ratio between their shares is:

a : b = 4 : 5

b : c = 8 : 10

4.6 Rate

A **rate** is a ratio that compares two quantities with different units of measure.

Rates are used almost every day. For example, unit price of items, speed, service charges etc are often given as rates.

Previous Knowledge Check

- How can we find the price of 1 kg of apple if the price of 15 kg apple is given?

For example, the price of one kg of sugar is Rs 90. Here we compare the price in "Rs" with weight in "Kg" (two different units). In unit rate one of the two quantities is always one.

Example 1:

Rabia covered 6 kilometres in 3 hours. What is her per hour speed?

Solution:

Given is the distance covered in 3 hours.

$\frac{6 \text{ kilometres}}{3 \text{ hours}}$ = 6 kilometres : 3 Hours

Note it down

The units of the terms of the rate are different.

To find her per hour speed, we will simplify the ratio.

$6 : 3 = \frac{6 \div 3}{3 \div 3} = \frac{2}{1}$

So, rabia covered 3 kilometres in one hour or her speed is 2 km/hour.

Note it down

A rate that has 1 unit as its second term is called unit rate.

Teaching Point: Explain the concept of rates to students by giving different examples of rates from real life situation. For more practice visit the website: https://www.brainpop.com/games/unitrates/?topic_id

Example 2:

The price of 10 kg rice is Rs 2200. Find the unit rate (or per Kg rate) of rice.

Solution:

Here we are considering the mass of rice with its price.

We can write it as by removing units:

Rs 2200/10 Kg = 2200 : 10

To find per kg rate, we will simplify the ratio.

Note it down

Unit rate is the rate where one quantity is always 1.

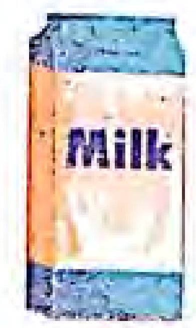
$$2200 : 10 = \frac{2200 \div 10}{10 \div 10} \text{ (reduce it in unit rate by dividing both terms by 10.)}$$

$$= \frac{220}{1}$$

So, the rate of per kilogram rice is Rs 220.

Example 3:

The price of 12 litre milk is Rs 2160. Find the unit rate (or per litre rate) of milk.



Solution:

Here we are considering the volume of milk with its price.

We can write it as by removing units:

Rs 2160/12 litre = 2160 : 12

To find per kg rate, we will simplify the ratio.

$$2160 : 12 = \frac{2160 \div 12}{12 \div 12} \text{ (reduce it in unit rate by dividing both terms by 12.)}$$

$$= \frac{180}{1}$$

So, the rate of per liter milk is Rs 180.

Exercise 4.6

1 Find the rate for each of the following.

a) 28 kilometers covered in 4 hours
b) 16 metre rope for Rs 1376.
c) 108 words typed in 6 minutes
d) 8 kg of mangoes in Rs 400
e) Rs 6000 for fencing 50 metre boundary

2 The entry ticket for 8 persons in the museum costs Rs 1760. What is the rate per ticket?

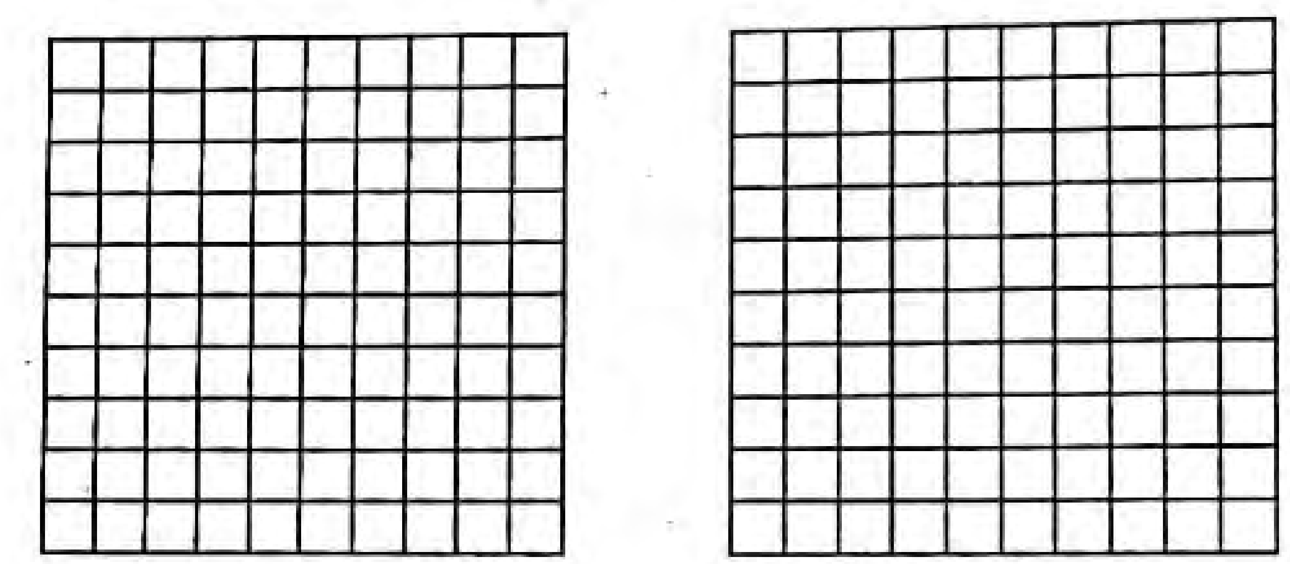
3 The cost of tiling a floor having an area of 234 meter square is Rs 205,920. What is the per square meter rate of tiling?

4 Dania bought 12 kg of apples in Rs 720. Find the rate of 1 kg of apples.

5 Aleem's income is Rs 3000 after working for 19 hours. What is his per hour pay?

Think Higher

Colour the following shapes and show that 0.5 is not equal to 5%.



0.5 5%

According to a criterion, for every 6 square metre of area, the garden should have 4 plants. A garden that has an area of 54 square metre has 32 plants. Is the number of trees is correct according to the set criterion? Justify your answer.

Summary

- Percent (%) means out of 100.
- % is the symbol for percent.
- A term used to compare two or more quantities of the same kind is called "Ratio".
- The first term of the ratio is called the antecedent and the second term is called the consequent.
- When two ratios x : y and y : z are expressed in the form of x : y : z, the ratio is called a continued ratio.
- A continued ratio is a comparison of three or more quantities in a certain order.
- A rate is a ratio that compares two quantities with different units of measure.

Vocabulary

- Percent
- Percentage
- Ratio
- Antecedent
- Consequent
- Continued ratio
- Rate

Review Exercise

1 Encircle the correct option.

a) When we compare two or more than two quantities of the same kind then that comparison is called ___________.

i) ratio ii) percentage iii) rate iv) competition

b) The value of one quantity in unit rate is always:

i) zero ii) two iii) three iv) one

c) There are 14 cars and 12 bikes on the road. The ratio of cars to bike is _____.

i) 12 : 7 ii) 6 : 7 iii) 7 : 6 iv) 7 : 12

d) A __________ is a comparison of three or more quantities in a certain order

i) ratio ii) continued ratio iii) rate iv) percentage

e) 50 out of 500 in percentage form is __________.

i) 1.2% ii) 10% iii) 15% iv) 5%

f) The ratio 20 : 50 in simplified form is _______.

i) 1 : 2 ii) 2 : 5 iii) 10 : 25 iv) 4 : 10

g) The rate of one week wages of Rs 4200. The rate of a one day wages:

i) 600 : 1 ii) 7 : 4200 iii) 1 : 4200 iv) 1 : 2800

h) If the cost of 12 notebooks is Rs 204, then the cost of 5 notebooks is

i) Rs 100 ii) Rs 102 iii) Rs 85 iv) Rs 120

i) The equivalent form of the ratio 3: 4 is ___________

i) 6 : 4 ii) 6 : 8 iii) 9 : 8 iv) 12 : 12

2 What is meant by ratio?

3 What is rate? Give examples.

4 Define percentage with examples.

5 Write the following in the form of ratio and simplify if possible.

a) 600 days, and 120 days b) 40 g and 120 g

c) 65 ml and 13 ml d) 14 men and 16 women

6 Aleena covers a distance of 40 m in 25 sec. How much time will she take to cover the distance of:

a. 50 m b. 120 m

7 There are 800 eggs. If 120 are rotten, find:

a) What percentage of eggs are fresh?

b) What is the ratio between the number of rotten and fresh eggs.

c) What is the ratio between the fresh and total number of eggs?

8 The ratio of Saad's salary to Sania's salary is 2 : 3 and the ratio of Sania's salary to Mauz's Salary is 4 : 3. Find the continued ratio among their salary.

9 An author charges Rs 82,500 for writing 55 pages. What is his per page rate for?

10 Umar got 15 marks out of 20 marks in Islamic study and 22 marks out of 25 marks in Urdu. Compare and tell in which subject he got more percentage marks?

11 Madeeha bought a table in Rs 1500. If its price decreased by 4% during sale, find the decreased price of table.

Math Project

Material Required:

- Dummy coins and notes
- food items

Procedure:

Instruct the students to play like customer and shopkeeper.

One student will open the tuck shop and play the character of shopkeeper. He will sell food items at decreased percentage prices from the original cost of the items.

Other students buy those items by finding the new prices of the items.

Students stand in queue and wait for their turn and one by one go to the tuck shop and buy the things with decreased percentage price.

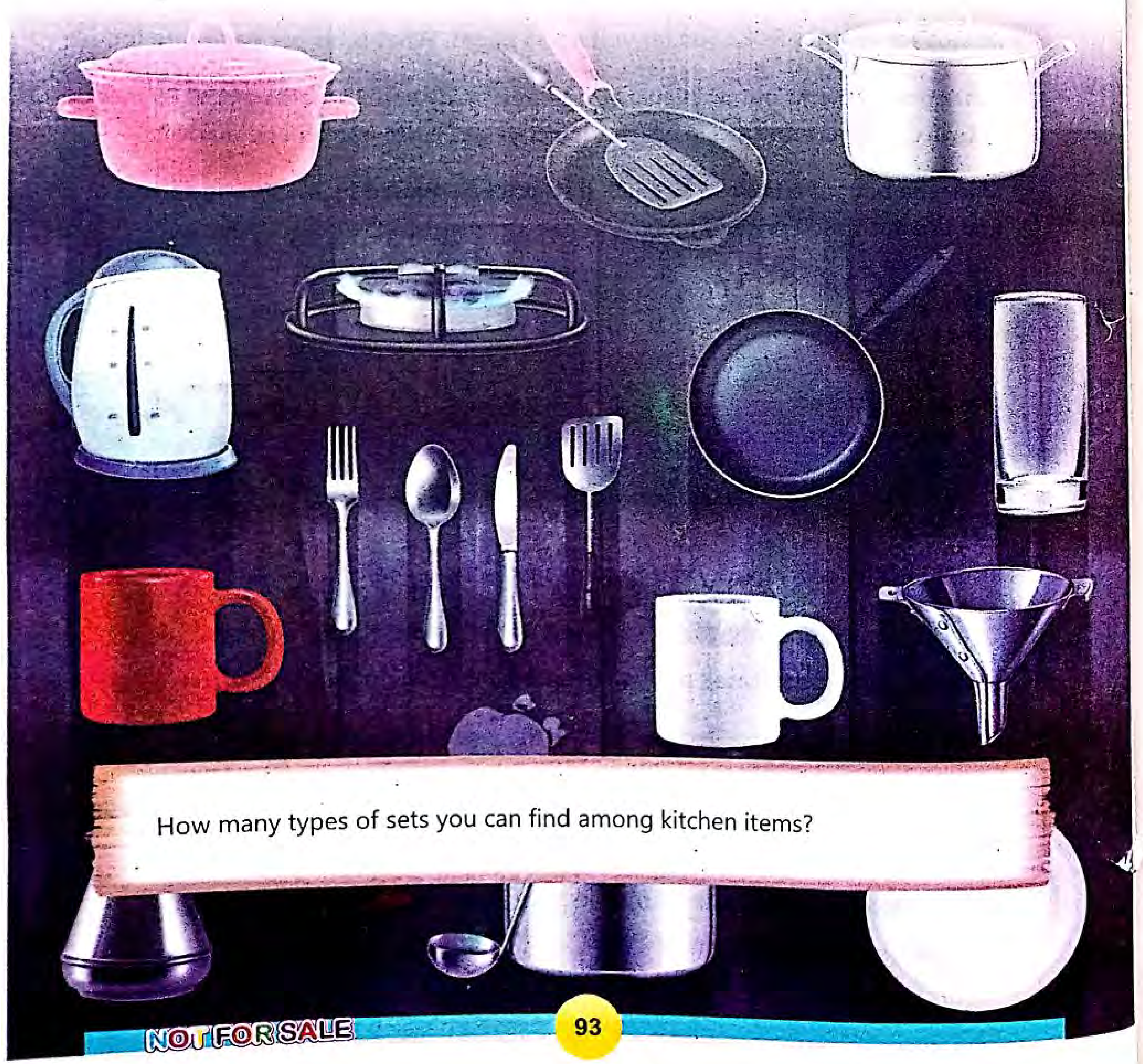
Instruct them to find the ratio of the old price and the new price of the food items they buy and also tell the rate of 1 item.

Unit 5 Sets

Student Learning Outcomes

After completing this unit, students will be able to:

- Use language, notation and Venn Diagrams to represent different types of sets and their elements.(finite, infinite, empty, singleton and universal set)



How many types of sets you can find among kitchen items?

Introduction

In the previous class we have learned about numbers, groups or collection of different objects or things. In this unit we will learn about how to represent this collection of objects, things or numbers in sets. Also we will learn about the uses of sets in our daily life.

5.1 Set and its Notation

In our daily life, we frequently use the term collection, group, set, list, etc. to mention different things of the same kind. For example:

a) A group of students of Grade 6.

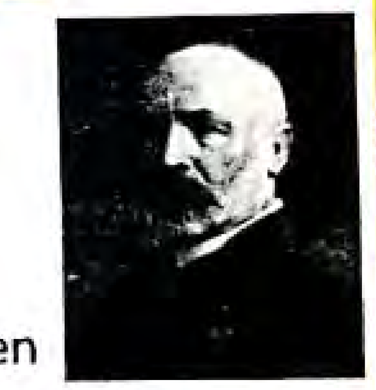
b) A list of cities of Pakistan that start with the letter 'G'.

c) A set of geometric instruments.

d) A bundle of books for Grade 6.

Math History

German mathematician Georg Cantor founded the set theory between the years 1874 and 1897.



Set of books for Grade 6

Set of geometric instruments

Previous Knowledge Check

- Write the days of week.
- Make a list of stationary items.
- Make groups of fruits and vegetables in such a way no fruit and vegetable

Similarly, to represent a collection of objects in Mathematics, we use the word "set". A well-defined collection of distinct objects is called a **set**.

Set is one of the basic concepts in Mathematics. The objects of the set are called "**members**" or "**elements**" of the set. Elements can be written in any order but they must not be repeated.

Sets can be unordered and we consider an element only once.

For example, {m, m, m, m}={m}, and {1,2, 3,4}={3,2,4,1}.

Note it down

"Well defined" is a characteristic of objects or elements which are clearly defined and there is no ambiguity about the objects of the set.

Note it down

"Distinct" means all the objects in a set must be unique and different and no object should be repeated.

Example 1:

1. **A set of months of the solar calendar.**

   It is a set because:
   - All of its elements are well-defined (there is no ambiguity about which months will be in this set).
   - All of its elements are distinct.

Quick Check

Is the set of English alphabets well- defined?

2. **The set of tasty food items.**

   It is not a set because:
   - Its elements are not well-defined. Tasty food means something different to everyone. If a person likes pasta, another person may not like it. So, the elements of this set are not clear.

3. **A set of names of your teachers of Grade 6.**

   It is a set because:
   - All of its elements are well-defined (there is no ambiguity about which names will be in this set).
   - All of its elements are distinct.

Note it down

Elements of a set may be limited or unlimited.

4. **The set of funny students in Grade 6.**

   It is not a set because:
   - Its elements are not well-defined. Funniness means something different to everyone.
   - A person may be funny to one and not funny to others. So, this is not a set.

A set is denoted by a capital letter such as A, B, J, K, M, N, etc.

Example 2:

1. **A set of 3 birds.**

Let's denote this set by the capital letter B.

So, B= {parrot, sparrow, crow}

2. **A set of the first 3 capital letters having only curved lines.**

Let's denote this set by the capital letter K.

So, K= {C, O, S}

3. **The set of odd numbers.**

Let's denote this set by the capital letter O.

O= {1, 3, 5, 7, 9, 11, 13, ...}

Here the three dots after 13 shows that the set keeps continuing in this pattern but the elements of this set are unlimited i:e. we cannot enlist all of its elements. These dots are known as ellipsis.

TEACHING POINT: List various collections of objects and ask the students to sort which of these are well-defined. Ask them to give the reason for their answer.

4. **A set of multiples of 2 greater than 10 but less than 100.**

Let's denote this set by the capital letter M.

So, M= {12, 14, 16, 18, ..., 98}

Here the ellipsis (the three dots) between 18 and 98 show that the set keeps continuing in this pattern and the elements of this set are limited (it ends at 98).

Elements of a Set

Every element of a set belongs to that set.

The symbol "$\in$" is used to show that an element "**belongs to**" the set. For example, for the set A= {10, 15, 20, 35}. we can say that 10 $\in$ A.

Note it down

Order of elements in a set doesn't matter.

We read it as 10 belongs to set A or 10 is an element/member of set A.

Similarly we can write that: 15 $\in$ A, 20 $\in$ A, 35 $\in$ A.

Also if an element doesn't belong to a set, we use the symbol "$\notin$" to denote it.

For example, in the above set A, 25 $\notin$ A.

We read it as: "25 does not belong to set A" or "25 is not a member/element of set A."

Consider the set E.

E = {set of stationery items} In this set, we can write that pencil $\in$ E, but apple $\notin$ E.

The elements of a set must be distinct. Distinct elements mean the elements which are unique in themselves when compared to the other objects of the set. In simple words, we can say that the objects are said to be distinct if they do not appear more than once in a set.

Example 3:

Let's consider a set of alphabets in the word "student" denoted by the letter R.

R = {s, t, u, d, e, n, t}

Here letter "t" appeared twice but we will write it one time as below:

R = {s, t, u, d, e, n}

If we write it twice then the elements of this set will not be distinct.

Cardinality of a Set

The cardinality of a set shows the number of the elements of the set.

For example, for the set B = {3, 6, 9, 12}, the **cardinality** of set B is 4 as it has 4 elements.

Symbolically we can write it as: n(B) = 4. We read it as the number of elements of set B is 4.

TEACHING POINT: Write 3 different sets on the board. Divide the board into 2 sections. Write' belongs to' at one side and 'does not belong to' at the other side. Now call the students one by one and ask them to point out if one number belongs to other sets or not using symbols.

Exercise 5.1

1 **Identify well-defined sets among these.**

a) A = the set of grades in your school
b) B = the set of interesting games
c) C = the set of the provincial capitals of Pakistan
d) D = the set of intelligent students in the class
e) E = the set of easy questions in the exams
f) H=the set of big animals
g) J=the set of the Prime Ministers of Pakistan
h) Set of small birds in the sky
i) Set of even numbers between 1 and 5
j) {+,-,+,-}
k) {1, 3, 5, 7}
l) Set of durable tables
m) Set of nice people in the neighbourhood
n) Set of beautiful people in a town

2 **List each element of these sets in symbolic form. (e.g. 1 ∈ a)**

a) A={1, 3, 5, 7, 9} b) B={a, b, c, d, e} c) C={s, t, a, r}
d) D={Monday, Wednesday, Friday} e) E = {chips, cookies, burgers, fries}

3 **Fill in the blanks using the symbols ∈ or ∉.**

a) 5 __ {1, 4, 5} b) 10_____ {set of even numbers} c) {u}_____ {a, i, o, e, u}
d) tiger _____ {set of birds} e) January _____ {set of months of a year}

4 **Write the following statements in symbolic form.**

a) 4 is an element of set W b) 4 is an element of set P
c) 0 is not the element of natural numbers

5 **If A= {1, 3, 5, 7, 11} and B= {2, 4, 6, 8, 10} then write true and false for the following statements.**

a) 3 ∈ A b) 5 ∉ B c) 11 ∈ A d) 0 ∉ B e) 0 ∉ A
f) 4 ∉ B g) 11 ∈ B h) 2 ∉ A i) 12 ∉ B j) {7} ∈ A

6 **Write the cardinality of the following sets.**

a) B = {1, 2, 3, 4, 5, 6, 7} b) Q = {a, e, i, o, u} c) N = { }
d) K = {maths, science, Urdu, English} e) R = {2,4,6} f) S={5, 10, 15, 20}
g) P = {k, l, m, n, o} h) E = {10, 20, 30, 40, 50}

5.2 Representation of a Set

We can represent a set in more than one form. Here we will discuss the following two forms: 1) Descriptive form 2) Tabular form

1) Descriptive form

In a descriptive form a set is represented in the form of statements using well-defined words.

Example 1:

a) A= set of prime numbers b) R= set of 4 provinces of Pakistan
c) P= set of the names of the solar months having 8 letters

In the above examples, we can see that all sets are written in the form of well defined statements without using brackets. These sets are in **descriptive form**.

2) Tabular form

In tabular form, elements of the set are placed within a curly bracket **{ }** separated by commas.

Example 2:

a) B= {Tuesday, Thursday}
b) W= {a, e, i, o, u}
c) F= {3, 9, 12, 15}

Quick Check

Is the following set in descriptive form or tabular form?
C= {10, 20, 30, 40}

Note it down

Tabular form is also called Roster form.

In the above example we see that all elements of the sets are separated by commas and written within the curly brackets. These sets are in **tabular form**. Set B is representing the names of the days of the week starting with the Letter T. Set W is representing the vowels of the English alphabet. Set F is representing the first 4 odd multiples of 3.

Exercise 5.2

1 **Write the following sets in descriptive form.**

a) A = {a, b, c, d, e, f} b) B = {1, 3, 5} c) D = {4, 8, 12, 16 ..., 32}
d) E = {10, 20, 30, ...} e) F = {2, 4, 6, 8, 10, 12} f) G = {Sunday, Saturday}

2 **Identify the well-defined sets and write them in tabular form.**

a) A set of months of the Islamic year. b) A set of all odd numbers.
c) A set of all provinces of Pakistan d) A set of famous Muslim countries.
e) A set of prime numbers less than 20 f) A set of the last seven English alphabets.
g) A set of even numbers between 55 and 77.
h) A set of composite numbers between 110 and 119.

TEACHING POINT: Write some examples of tabular and descriptive forms of sets on the board and ask the students to differentiate them as a tubular or descriptive set.

5.3 Types of Sets

We can classify sets in different types. Here we will discuss the following types of sets.

a) Finite set b) Infinite set c) Empty set
d) Singleton set e) Universal set

Hints

Following are some important sets which are often used in Mathematics.
E = {set of even numbers}
O = {set of odd numbers}
P = {set of prime numbers}
C = {set of composite numbers}

a) Finite set

A set that consists of a **limited** number of elements is called a **finite set**.

Example 1:

i) The set of natural numbers up to 50.

A= {1, 2, 3, 4, ..., 50}

In set A, the number of elements is limited and we can enlist all of them. So, it is a finite set.

ii) The set of English alphabets.

B= {a, b, c, d, e, ..., z}

Set B is also a finite set because its elements are limited and we can enlist them.

Quick Check

Is the set A= {1, 2, 3, 4, ...} finite or infinite?

Similarly, the following sets are finite sets as they have a limited number of elements and we can enlist their elements.

J = set of Islamic months
P = set of all prime numbers less than 220
K = set of names of people in a city
S = set of angles in a triangle

Note that the elements of K= the set of names of people in a city, are very difficult to count and enlist. However, we are eventually be able to enlist its elements. So the set R is a finite set.

b) Infinite sets

A set that consist of unlimited or an **uncountable number** of elements is called an infinite set.

Example 2:

i) N= Set of all natural numbers.

N= {1, 2, 3, 4, 5, ...}

Set N is an infinite set because its elements are unlimited and it is not possible to enlist or count them.

ii) M= Set of all multiples of 100.

M= {100, 200, 300, 400, 500, ...}

Set M is an infinite set because it has unlimited elements.

Similarly, the following sets are infinite sets as they have unlimited number of elements and we cannot enlist their elements.

J = set of all dust particles in the environment
O = set of all odd numbers
D = set of all drops of water in the ocean
L = the set of all prime numbers greater than 50

Quick Check

What is the cardinality of an empty set?

c) Empty set

When a set contains no elements, it is known as an **empty set**.

An empty set is denoted by curly brackets without any element. It can also be represented by using the symbol ø (read as phi). So, an empty set A can be denoted as: A = { } or A = ø

Note that A = { ø } is not an empty set because there is an element ø within the curly brackets. Also, A = {0} is not an empty set and has 0 as its single element.

Observe the following sets:

A = the set of odd multiples of 10.
B = the set of triangles having 4 sides.
C = the set of natural numbers which are even as well as odd.

What have you observed in these sets? Can we enlist them?

The answer is no. These sets have no elements in them because these sets describe the elements which do not exist at all.

- There is no multiple of 10 which is odd, as all multiples of 10 have 0 in their ones place.
- No triangle can have 4 sides. If a shape has 4 sides, it's a quadrilateral and is never a triangle.
- A natural number can either be even or odd. It can never be both.

d) Singleton set

When a set contains only **one element** (neither less nor more), it is known as a singleton set.

Examples:

i) F = Set of the names of the month having 3 letters

There is only one month which has 3 letters i.e. May. So, it is a singleton set.

ii) T= {9}

Set T contains only 1 element. So, it is a singleton set.

TEACHING POINT: Write some mixed examples of finite and infinite sets on the board. Ask the students to identify finite or infinite sets from the examples and then describe why it is a finite or infinite set.

iii) D= the set of numbers whose third multiple is 12.

Set D is also a singleton set as D = {4}.

iv) F = the set of LCM of 12, 18 and 30.

Set F is also a singleton set as LCM of 12, 18 and 30 is just one number i.e.130.

e) Universal Set

A universal set is the set of all elements of the sets which are under consideration in a particular context. The universal set is denoted by the capital letter "U". In other words, we can say that all the sets which are under consideration are the subsets of the universal set.

Let's consider the set of whole numbers, the set of composite numbers and the set of prime numbers. We can see that the set of whole numbers contains all the elements of the remaining two sets. So, in this case, the set of whole numbers is the universal set.

Note it down

Which set can be a universal set for the other two?

a) Set of even numbers

b) Set of odd numbers

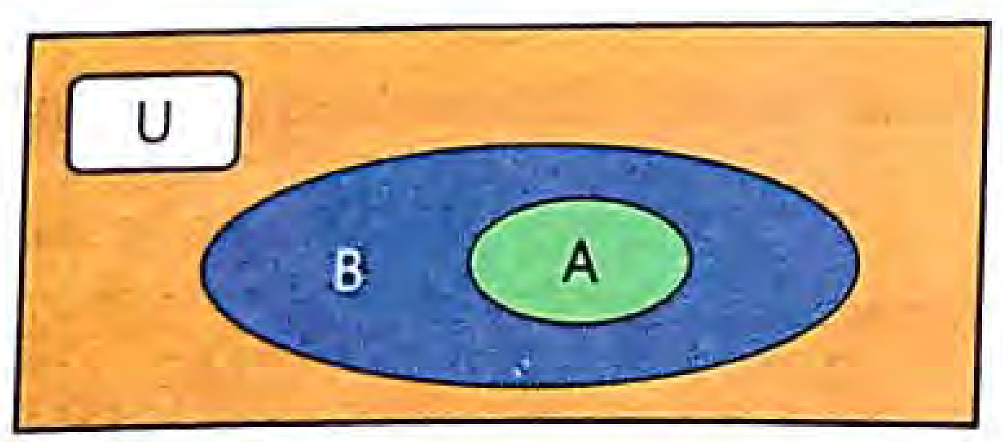
c) Set of natural numbers

5.3.1 Venn Diagram

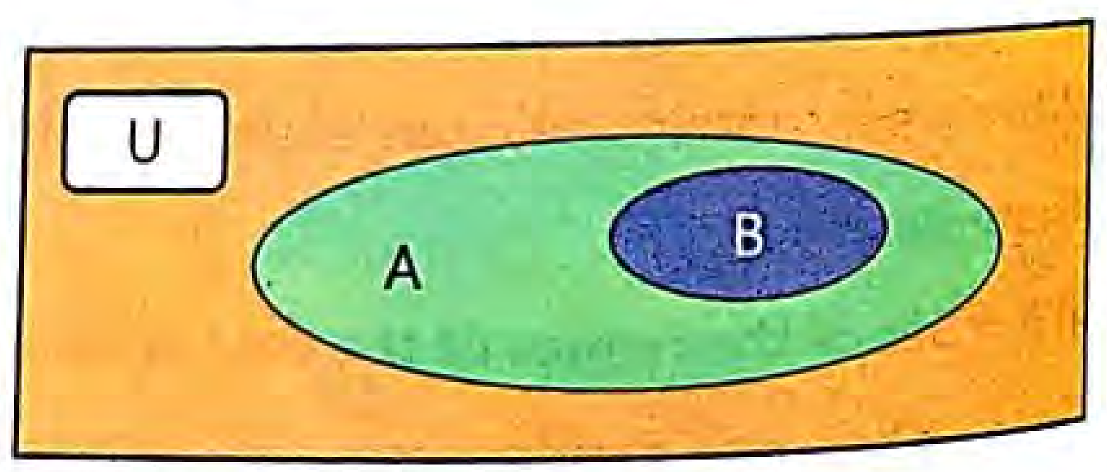
A diagram that represents mathematical sets and their relationships is called a **Venn diagram**. In a Venn diagram, the sets are shown using circles or closed curves bounded in a rectangle. The rectangle represents the universal set.

We can represent various set relationships with the help of Venn diagrams.

The following examples are given to understand the relation between sets using Venn diagrams.



A is a subset of B



B is a subset of A

TEACHING POINT: Write some mixed examples of different types of sets (null set, singleton set etc.) on the board. Ask the students to identify and describe them.

Exercise 5.3

1 **Separate the finite and infinite sets from the following.**

a) A = {10, 15, 20, 25, 30}.
b) B = set of numbers less than 20.
c) T = The set of all persons in Pakistan.
d) W = {0, 1, 2, 3, ...}.
e) The set of all animals in Lahore zoo.
f) Set of all positive integers which are multiple of 6.
g) N = {1, 2, 3, ...}

2 **Identify empty and non-empty sets. Also give the reason.**

a) The set of circles with 7 sides.
b) The set of numbers between 34 and 98.
c) The set of bicycles having 3 wheels.
d) The set of prime numbers.
e) The set of odd multiples of 4.
f) The set of numbers divisible by 2.
g) The set of prime numbers which have at least 5 factors.

3 **Identify the singleton sets.**

a) The set of composite numbers less than 5.
b) The set of prime numbers between 8 and 12.
c) The set of the provinces of Pakistan.
d) The set of days of the week starting with the letter F.
e) The set of common factors of 12, 40, 55.
f) The set of numbers which divide both 8 and 15.
g) The set of multiples of 9 between 10 and 20.

4 **Draw a Venn diagram which shows A is a subset of B.**

Think Higher

Think and create an infinite set and then write it in descriptive and tabular form. Then mention its element using correct symbols. Create a set and then mention at least two sets for which it can be a universal set

Summary

- A well defined collection of distinct objects is called a set.
- The objects of the set are called "members" or "elements" of the set.
- The symbol "∈" is used to show that an element "belongs to" the set.

Vocabulary

- Set
- Well-defind
- Element
- Distinct
- Cardinality
- Descriptive form
- Tabular form
- Finite set
- Infinite set
- Singleton set

- Distinct elements mean the elements which are unique in themselves when compared to the other objects of the set.
- The cardinality of a set shows the number of the elements of the set.
- In descriptive form a set is represented in the form of statements using well defined words.
- In tabular form, elements of the set are placed within a curly bracket { } separated by commas.
- A set that consists of a limited number of elements is called a finite set.
- A set which is not finite is called an infinite set.
- When a set contains no elements, it is known as an empty set.
- When a set contains only one element (neither less nor more), it is known as a singleton set.

Review Exercise

1 Choose the correction option.

a) A well-defined collection of distinct objects is called a ______.

i. set ii. subset iii. null set iv. infinite set

b) Empty set is denoted by______.

i. { ø } ii. { } or ø iii. { ø } or 0 iv. { 0 }

c) A set that consists of a limited number of elements is called a/an______ set.

i. equivalent ii. finite iii. empty iv. an infinite

d) The cardinality of set M = {5, 10, 15, 20, 25} is _____.

i. 2 ii. 3 iii. 4 iv. 5

e) Elements of a set are denoted by the symbol ____.

i. Ø ii. ∈ iii. ≠ iv. ⊃

2 Define the following:

a) Set
b) Element of a set
c) Tabular form of a set
d) Descriptive form of a set
e) Finite set
f) Infinite set
g) Empty set
h) Singleton set

3 Which of the following sets are well defined?

a) The set of intelligent boys.
b) The set of even numbers.
c) The set of cars in a showroom.
d) The set of prime numbers.
e) The set of grains in the sand.
f) The set of numbers which are divisible by 8.

4 List the elements of the following sets.

a) The set of 6 countries of world.
b) The set of vowels.
c) Set of the first six months of the year.
d) Set of colours in a rainbow.
e) Set of 5 pet animals.

5 Write the cardinality of the following sets.

a) the set of all factors of 45 which are even.
b) the set of prime numbers less than 30.
c) the set of odd numbers divisible by 4.

6 For A= {10, 20, 30, ...} and B= {5, 10, 15, 20, ...}, which of the following are correct?

a) 5 ∈ A b) 20 ∈ B c) 30 ∉ B d) 50 ∈ A e) 60 ∉ B f) 0 ∈ B

7 Identify the correct form of the following sets as tabular or descriptive.

a) A set of odd numbers between 5 and 15.
b) A set of prime numbers.
c) A={1,2, 3, 4, 6}.
d) B={2, 4, 6, 8, 10}.
e) A set of days of the week.

8 Write the following sets into tabular form.

a) The set of the first five natural numbers.
b) The set of all vowels of the English alphabet.
c) The set of all odd numbers less than 9.
d) The set of all numbers which divide 12.
e) The set of all letters in the word MATHEMATICS.
f) The set of the last four months of the year.

9 Write the following sets into descriptive form.

a) A= {7, 8, 9, 10}
b) B={6, 12, 18, 24, 30}
c) T={3, 5, 7,11}
d) D={r, e, s, t}
e) E={5, 10, 15, 20}

10 Classify the following sets as finite, infinite, empty or singleton set.

a) The set of stars in the sky.
b) The set of even prime numbers
c) The set of odd numbers.
d) The set of your friends.
e) The set of all women in the world.
f) The set of whole numbers between 88 and 89.

Math Project

Material Required:

- Basket
- Paper slips
- Board Marker

Procedure:

- Divide the students in groups.
- Place a basket with cards with variety of instructions regarding sets (for example:
  - Write a set which is infinite.
  - Write a set which is empty.
  - Write the statement" London is a member of set D." in symbolic form", etc.
- Students from each group will come forward and pick a card from the basket and follow the instruction written on it.
- The group with most accurate answers wins.

Write a set which is infinite.

Write a set which is empty.

Write the statement" Islamabad is a member of set P." in symbolic form".

Unit 6

Patterns and Algebraic Expressions

Student Learning Outcomes

After completing this unit, students will be able to:

- Recognise simple patterns from various number sequences.
- Continue a given number sequence and find:
  - term to term rule
  - position to term rule
- Solve real life problems involving number sequences and patterns.
- Explain the term algebra as an extension of arithmetic, where letters, numbers and symbols are used to construct algebraic expressions.
- Evaluate algebraic expressions by substitution of variables with numerical values.
- Manipulate simple algebraic expressions using addition and subtraction.
- Simplify algebraic expressions.

Factory workers are arranging packed boxes in stacks. They put 2 boxes in the first stack, 4 boxes on the second stack, 8 boxes on the third stack, 16 boxes on the fourth stack. If this pattern continues, how many boxes will the workers put in fifth and sixth stacks?

Introduction

In the previous classes we learn about patterns and number patterns. Algebraic expressions are commonly used in our daily life in many situations where we find some unknown quantity. We commonly use algebra where we find relationship between known and unknown quantity.

6.1 Number Patterns

Number patterns are sets of numbers that follow a pattern or a rule.
The rule can be add or subtract a number each time or to multiply or divide by a number each time. Each number in a sequence is called a **term**.

Previous Knowledge Check

What is meant by pattern of numbers, symbols, colours etc.?

Example 1: An author is writing a storybook. He wrote 2 pages on Monday, 5 pages on Tuesday, 8 pages on Wednesday and 11 pages on Thursday. If he keeps writing the pages in the same pattern, how many pages will he write on Friday, Saturday and Sunday?

Solution: Firstly, write the given pattern of pages written in a sequence:

2, 5, 8, 11, ____, ____, ____

Next, work out the difference in the terms. This pattern is going up by 3 each time, so we will add 3 to the last term to find the next term in the pattern.

2, 5, 8, 11, **14, 17, 20**

So, the author will write 14, 17 and 20 pages on Friday, Saturday and Sunday respectively.

Term to term rule

The term-to-term rule of a pattern describes how to get next term in that specific pattern.

Example 1:

Write down the **term-to-term rule** and then work out the next two terms in the following sequence.

Solution : 11, 17, 23, 29, ____, ____, ____

Firstly, work out the difference in the terms. This pattern is going up by 6 each time, so we will add 6 to the last term to find the next term in the pattern.

11, 17, 23, 29, **35, 41, 47**

To work out the term-to-term rule, give the starting number of the sequence and then describe the pattern of the numbers. Once the first term and term to term rule are known, all the terms in the pattern can be found.

Example 3:

Find the pattern if the first number is 5 and the term-to-term rule is 'add 10'.

Here we will start at 5 and keep adding 10 to each term to find the next term. So, the pattern will be:

5, 15, 25, 35, 45, ...

Example 4:

Write down the term-to-term rule and then work out the next two terms in the following sequence.

2, 4, 8, ____, ____, ____.

The first term is 2. The term-to-term rule is "multiply by 2". So, the pattern will continue like this:

2, 4, 8, 16, 32, 64, ...

6.1.1 Position to Term Rule

Look at the pattern.

4, 8, 12, 16, 20, ...

Here each term has a position. The first term 4 is at position 1, the second term 8 is at position 2, the third term 12 is at position 3 and so on. The **Position to terms rule** helps us to find out what number is in a pattern if its position in the pattern is given.

Example 1:

Find the position to term rule to find 18th term of the pattern: 6,7,8,9,10,11....

Solution:

First, write the terms and their positions of this pattern in a table.

Position	1	2	3	4	5	6
Term	6	7	8	9	10	11

Now find the rule for position to terms

Position	1	2	3	4	5	6
Term	6	7	8	9	10	11
Rule	1+5	2+5	3+5	4+5	5+5	6+5

We see that each term is obtained by adding 5 to the position of the term.
Now, to find the 18^{th} term of this pattern, we will add 5 to 18.
So, the 18^{th} Term is: 18 + 5 = 23.

Example 2:
Find the position to term rule to find 25^{th} term of the pattern:
7, 14, 21, 28, 35, 42, ...
Solution:

First, write the terms and their positions of this pattern in a table.

Position	1	2	3	4	5	6
Term	7	14	21	28	35	42

Now find the rule for position to terms

Position	1	2	3	4	5	6
Term	7	14	21	28	35	42
Rule	1×7	2×7	3×7	4×7	5×7	6×7

We see that each term is obtained by multiplying the position of the term by 7.
Now, to find the 25^{th} term of this pattern, we will multiply 25 by 7.
So, the 25^{th} term is: 25 × 7 = 175.

Exercise 6.1

1 **Identify the term-to-term rules for the following patterns and write the next three terms.**

a) 4, 9, 14 19, ___, ___, ___
The term-to-term rule of this pattern is: ___________
The next three terms are:___________________________

b)8, 16, 32, 64 ___, ___, ___, ___
The term-to-term rule of this pattern is: ___________
The next three terms are:___________________________

c)78125, 15625, 3125, ___, ___, ___
The term-to-term rule of this pattern is: ___________
The next three terms are:___________________________

d)122, 111, 100, 89, ___, ___, ___
The term-to-term rule of this pattern is: ___________
The next three terms are:___________________________

e)5, 10, 20, 40, ___, ___, ___
The term-to-term rule of this pattern is: ___________
The next three terms are:___________________________

f)26, 42, 58, 74, ___, ___, ___
The term-to-term rule of this pattern is: ___________
The next three terms are:___________________________

2 **Ahad daily read the storybook. On Monday, he read 3 pages. The next day, he read 6 pages. On Wednesday, he read 9 pages. If he keeps increasing the number of pages daily, how many pages will he read on Thursday and Friday?**

3 **Rida goes for a walk in the nearby park. On Monday, she completes 2 rounds of the walking track. The next day, she completes 4 rounds of the walking track. On Wednesday, she completes 6 rounds. If she keeps increasing the number of rounds daily, how many rounds does she complete over the next three days of the week?**

TEACHING POINT
Use the following online game link for practice of sequence.
https://mathsframe.co.uk/en/resources/resource/42/sequences
Paste a hundred square grid on board to ask students to color squares to explore different number patterns like odd numbers, even numbers, counting in multiples from times tables and ten more and ten less.

6.2 Algebra

Arithmetic and **Algebra** are two branches of Mathematics. Arithmetic is the basic branch of Mathematics which we are very familiar with. Algebra deals with the relation of known and unknown quantities along with numbers which help us to solve many Mathematical problems in an easy way. In algebra we represent a quantity by the symbol (Δ, □) or letter (a, b, c, x, y, z, etc.) without knowing its numerical value.

> **Maths History**
>
> The word "algebra" comes from the Arabic word "al-jabr" which means the reunion of broken parts. This word appeared in the book, "Hisab al-Jabr wal- Muqabala" written around the year 825 AD by the Muslim mathematician Muhammad ibn Musa al-Khwarizmi (780-850 AD). He is known as the "Father of Algebra".

As we know that numerals 1,2,3,... etc. and the basic operations (+, −, ×, ÷) are used in arithmetic, but in algebra, we use letters a, b, c,..., constants (numbers) along with the basic operations (+, −, ×, ÷).

Let's consider the following example.

The price of a bookmark is Rs 50. If Rohma wants to buy 12 bookmarks, how much will she pay?

To find the price of 12 bookmarks, we need to know the rule.

The table shows that:

The price of 1 bookmark is Rs 50 = 50 × 1

The price of 2 bookmarks is Rs 100 = 50 × 2

The price of 5 bookmarks is Rs 250 = 50 × 5

The price of 8 bookmarks is Rs 400 = 50 × 8

> **Previous Knowledge Check**
>
> - If one quantity is given how can we find the price of multiple quantities?
> - If price of multiple number of items are given how can we find the price of one item?

Hence, the rule of the pattern is multiply the number of bookmarks by 50.

So, the price of 12 bookmarks is = Rs 50 × 12 = Rs 600

In general, if we denote the number of bookmarks by x, the price of x bookmarks will be Rs 50 × x.

Here, the number of bookmarks is a changing quantity.

Yes. We use x to denote the changing quantity, so x is a variable here.

In the above examples, the number of bookmarks is changing quantities which cannot be given a fixed value. We have derived the general rules for finding these quantities using the letter x. The letters x is an example of variable.

The word **variable** means a quantity that can vary or change. The value of a variable is not fixed. It can take different values. A variable is a symbol that represents a number. Usually, we use letters x, y and z to represent variables.

Exercise 6.2

1 **Underline the variables in each of the following.**

a) 3 − y = 4 b) a + 3 = 30 c) 1 + b = 0 d) 10c − 4 = 15

e) 3 × 8 = z f) 4 + 5 = s g) 18 + x = 23 h) m × 6 = 18

2 **Use letters instead of symbols in each of these statements.**

a) 2 − □ = 3 b) Δ + 5 = 20 c) 4 + □ = 6

d) □ − 7 = 14 e) 3 × Δ = 6 f) 9 + □ = 1

6.3 Algebraic Expression

An algebraic expression is a combination of variables, constants, coefficients, exponents and symbols of operations i.e. + and −.

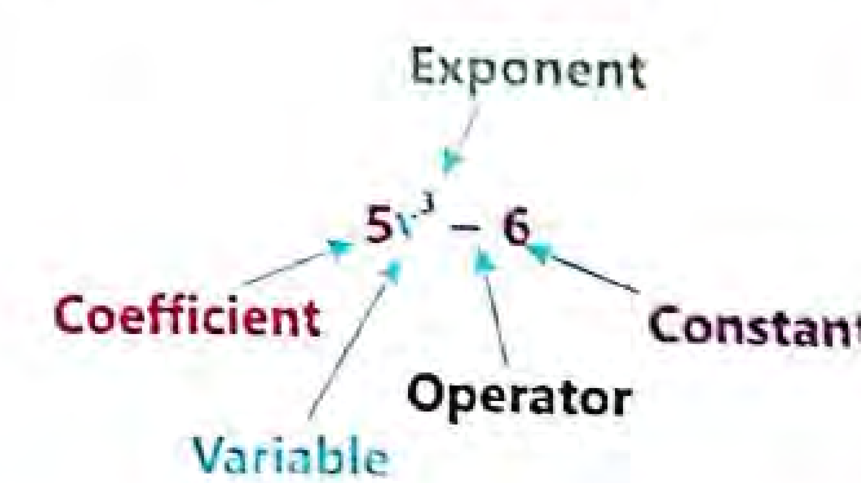
For example: a) $3x$ b) 9 c) $5x + 4y$ d) $2a - 3b + 4$, etc.

6.3.2 Coefficient

A number which is multiplied by a variable is called a **coefficient**.

For example: $4x$ means 4 times x. Here, x is a variable and 4 is the coefficient.

> **Note it down**
>
> The word variable means a quantity that can vary or change.



6.3.3 Exponent (power/index)

Exponent is the number which represents repeated multiplication of the same variable.

For example: In x^2, 2 is an **exponent or power** of variable x.

6.3.4 Terms of an expression

In an algebraic expression, the variables or numbers which are separated by operators (+ or –) are called the terms of the expression.

The division sign (÷) or multiplication sign (×) does not separate the terms of an algebraic expression.

An expression may have one or more terms separated by operators (+, –). The terms of the expression may be only variables, numbers or both.

In the expression **4a + 9b – c + 2**: 4a, 9b, –c and 2 are the four terms of the expression.

Note it down

In an algebraic expression a variable with no number has a coefficient of 1. Example: In b+3, the coefficient of b is 1.

Quick Check

Convert the following to algebraic expressions.

a) 4 is subtracted from y

b) 6 is multiplied by z and added to x

c) the product of 5, x, y

6.3.5 Like or Unlike Terms of an Algebraic Expression

In an algebraic expression, the terms whose variables as well as the exponents of those variables are the same are called **like terms**. The coefficient of the like terms may or may not be the same.

Like terms can be reduced to a single term through addition and subtraction.

On the other hand, **unlike terms** are those terms whose variables are different or they have the same variables but with different exponents.

Unlike terms cannot be added and subtracted together.

Example 1:

a) $3x - 4y + 5x$

Here $3x$ and $5x$ are like terms as they have the same variable.

b) $5a^2 - 2a + 3a^2$

Here $5a^2$ and $3a^2$ are like terms. In the term 2a, the variable is the same but its exponent is 1 which is different from the exponents of the other terms.

So, 2a and $5a^2$ as well as 2a and $3a^2$ are unlike terms.

Teaching Point

Share following online game links to practice algebraic expressions

https://www.mathgames.com/skill/4.94-write-variable-expressions

https://www.mathgames.com/skill/6.6-identify-terms-coefficients-and-monomials

Exercise 6.3

1 **Write the terms of the following algebraic expressions.**

a) 4a + 5b　b) $4y - 3x - 4$　c) $6x - y + 7$　d) $3x^2 + y^2$

e) abc – d　f) $11x^3 + xyz^2 + 4$　g) $4x^3 + 5y^3 + 4$　h) $a^2 - b^2 + 3$

2 **Identify variable, coefficient and constant in each algebraic expression.**

a) $x^2 + 3$　b) $7y^3 + z - 1$　c) $2a + 7b^2$　d) $\frac{1}{2}y^4 + 6$　e) $7a - 4b + c^2$

3 **Write the algebraic expressions whose terms are given below.**

a) 3a, –2b　b) 3y, – 4z, 1　c) $9x^2, -y^2, 5$

d) $x^2, -y^2$　e) 6abc, – bcd, 2de　f) $13x^4, xy^2, 7$.

4 **List out the like terms in each of the following sets.**

a) $7a - 5a + 8b - a + \frac{a}{3}$　b) $2p^3q^2 + 4p^2q^3 + 7q^2p^3 - 2p^3q^2$

c) $-xy + 3y + 5xy - x - xy - 11$　d) $2x^2y + 3x^3y + 2xy^2 + 4yx^2 - 2x^2y - 3yx^2$

e) $a^2b^3 - 5a^3b^2 + 7a^3b^2 + 11a^3b^3 - 3b^2a^3$

6.4 Evaluation of Algebraic Expressions

To evaluate an algebraic expression, we have to substitute a number for each variable and perform the arithmetic operations.

For example if a = 4 and b = 5, then a – b = 4 – 5 = –1.

Example 1:

Evaluate $xy - (x + yz)$, If $x = 1$, y = – 2 and z = – 3

Solution:

$xy - (x + yz)$

Replace x by 1 and y by –2 and z by – 3 in the given expression.

= 1(– 2) – {(1 + (– 2) (– 3)}

Then simplify.

= – 2 – {1 + 6}

= – 2 – {+ 7}

= – 2 – 7 = – 9

Example 2:

Evaluate $4(x^2 + 5x) - 2y$, if $x = 2$ and y = –3

Solution:

$4(x^2 + 5x) - 2y$

Replace x by 2 and y by –3 in the given expression.

$= 4[(2)^2 + 5(2)] - [2(-3)]$

Simplify using the order of operations.

= 4 [4 + 10] – (–6)

= 4 (14) – (–6)

= 56 + 6

= 62

Note it down

If a = –1, b = – 5 and c = 2, then solve:

a) $3a^2 - b^2 + 6c$

b) $-a^2 - b^2 + 7c^2$

6.3.3 Exponent (power/index)

Exponent is the number which represents repeated multiplication of the same variable.

For example: In x^2, 2 is an **exponent or power** of variable x.

6.3.4 Terms of an expression

In an algebraic expression, the variables or numbers which are separated by operators (+ or –) are called the terms of the expression.

The division sign (÷) or multiplication sign (×) does not separate the terms of an algebraic expression.

An expression may have one or more terms separated by operators (+, –). The terms of the expression may be only variables, numbers or both.

In the expression **4a+ 9b – c + 2**: 4a, 9b, –c and 2 are the four terms of the expression.

Note it down

In an algebraic expression a variable with no number has a coefficient of 1. Example: In b+3, the coefficient of b is 1.

Quick Check

Convert the following to algebraic expressions.

a) 4 is subtracted from y

b) 6 is multiplied by z and added to x

c) the product of 5, x, y

6.3.5 Like or Unlike Terms of an Algebraic Expression

In an algebraic expression, the terms whose variables as well as the exponents of those variables are the same are called **like terms**. The coefficient of the like terms may or may not be the same.

Like terms can be reduced to a single term through addition and subtraction.

On the other hand, **unlike terms** are those terms whose variables are different or they have the same variables but with different exponents.

Unlike terms cannot be added and subtracted together.

Example 1:

a) $3x - 4y + 5x$

Here $3x$ and $5x$ are like terms as they have the same variable.

b) $5a^2 - 2a + 3a^2$

Here $5a^2$ and $3a^2$ are like terms. In the term 2a, the variable is the same but its exponent is 1 which is different from the exponents of the other terms.

So, 2a and $5a^2$ as well as 2a and $3a^2$ are unlike terms.

Teaching Point

Share following online game links to practice algebraic expressions
https://www.mathgames.com/skill/4.94-write-variable-expressions
https://www.mathgames.com/skill/6.6-identify-terms-coefficients-and-monomials

Exercise 6.3

1 **Write the terms of the following algebraic expressions.**

a) 4a + 5b　b) $4y - 3x - 4$　c) $6x - y + 7$　d) $3x^2 + y^2$

e) abc – d　f) $11x^3 + xyz^2 + 4$　g) $4x^3 + 5y^3 + 4$　h) $a^2 - b^2 + 3$

2 **Identify variable, coefficient and constant in each algebraic experession.**

a) $x^2 + 3$　b) $7y^3 + z - 1$　c) $2a + 7b^2$　d) $\frac{1}{2}y^4 + 6$　e) $7a - 4b + c^2$

3 **Write the algebraic expressions whose terms are given below.**

a) 3a, –2b　b) 3y, – 4z, 1　c) $9x^2, -y^2, 5$

d) $x^2, -y^2$　e) 6abc, – bcd, 2de　f) $13x^4, xy^2, 7$.

4 **List out the like terms in each of the following sets.**

a) $7a - 5a + 8b - a + \frac{a}{3}$　b) $2p^3q^2 + 4p^2q^3 + 7q^2p^3 - 2p^3q^2$

c) $-xy + 3y + 5xy - x - xy - 11$　d) $2x^2y + 3x^3y + 2xy^2 + 4yx^2 - 2x^2y - 3yx^2$

e) $a^2b^3 - 5a^3b^2 + 7a^3b^2 + 11a^3b^3 - 3b^2a^3$

6.4 Evaluation of Algebraic Expressions

To evaluate an algebraic expression, we have to substitute a number for each variable and perform the arithmetic operations.

For example if a = 4 and b = 5, then a – b = 4 – 5 = –1.

Example 1:

Evaluate $xy - (x + yz)$, If $x = 1$, y = – 2 and z = – 3

Solution:

$xy - (x + yz)$

Replace x by 1 and y by –2 and z by – 3 in the given expression.

$= 1(-2) - \{(1 + (-2)(-3))\}$

Then simplify.

$= -2 - \{1 + 6\}$

$= -2 - \{+7\}$

$= -2 - 7 = -9$

Example 2:

Evaluate $4(x^2 + 5x) - 2y$, if $x = 2$ and y = –3

Solution:

$4(x^2 + 5x) - 2y$

Replace x by 2 and y by–3 in the given expression.

$= 4[(2)^2 + 5(2)] - [2(-3)]$

Simplify using the order of operations.

$= 4[4 + 10] - (-6)$

$= 4(14) - (-6)$

$= 56 + 6$

$= 62$

Note it down

If a = –1, b = – 5 and c = 2 , then solve:

a) $3a^2 - b^2 + 6c$

b) $-a^2 - b^2 + 7c^2$

Exercise 6.4

1 Substitute the given values for the variables to evaluate each expression.

a) $3y + 2y$ when $y = 5$
b) $2a - 2b + c$ when $a = 3$, $b = -2$ and $c = 4$
c) $3(x^2 + 4x) - 3y$ when $x = -5$ and $y = 3$
d) x^2y when $x = -3$ and $y = -4$
e) $(b - d)^2$ when $b = 9$ and $d = -3$
f) $m^4 + 2m^3 - m^2 + 8$ when $m = -2$
g) $\frac{3x}{2} + 1$ when $x = -5$
h) $5y^2 - 2(x^2 - 4x) - 5y$ when $x = 2$ and $y = -2$

2 Verify the following when; $p = 2$, $q = -3$, $s = 5$.

a) $p + (q + s) = (p + q) + s$
b) $p(q + s) = pq + ps$
c) $(p + s)q = pq - sq$
d) $q(p - s) = qp - qs$

3 If $x = 3$ and $y = 2$, then prove that $(x + y)^2 = x + 2xy + y^2$.

4 If $a = 1$, $b = 3$ and $c = 1$, then evaluate $4b^2 - 2ac$.

6.5 Addition and Subtraction of Algebraic Terms

In algebra, we can add or subtract like terms of two or more algebraic expressions. Unlike terms cannot be solved.

6.5.1 Addition

Ahmed had 2 cupcakes. He got 3 more cupcakes.

So, 2 cupcakes + 3 cupcakes = 5 cupcakes

If we denote cupcakes with the variable "a", then, by adding we can write that:

$2a + 3a = 5a$

Now Ahmed bought 6 nuggets.

So, 5 cupcakes + 6 nuggets = 5 cupcakes + 6 nuggets

We cannot add cupcakes to nuggets as they are not like items. If nuggets are denoted by "b", then, we can write that:

$5a + 6b = 5a + 6b$

Variables a and b are different, so terms 5a and 6b are unlike terms and hence they cannot be added together.

Horizontal Addition

We can add algebraic expressions horizontally by following these steps.

Step I: Write all the algebraic expressions in horizontal form.
Step II: Combine the like terms.
Step III: Add them.

Previous Knowledge Check

If Tahir has 9 balloons and his sister has 5 balloons. How many balloons did Tahir and his sister has. Who have more balloons and by how much?

Example 1:
Find the sum of 6ab, 3ab and 2ab.
Solution:
Here, 6ab, 3ab and 2ab are like terms.
The sum of the coefficients = 6 + 3 + 2 = 11
Thus, 6ab + 3ab + 2ab = 11ab

Note it down

When all the like terms are positive, add their coefficients. The variables and exponents remain the same. Also, keep the unlike terms unchanged.

Example 2:
Find the sum of x^2, $4x^2$ and $2y$.
Solution:
Here, x^2 and $4x^2$ are like terms and $2y$ is an unlike term.
The sum of the coefficients of like terms = 1 + 4 = 5 ($\because$ coefficient of $x^2 = 1$). So,

$x^2 + 4x^2 + 2y = (1 + 4)x^2 + 2y$
$= 5x^2 + 2y$

Unlike term 2y will remain unchanged.

Note it down

When all the like terms are negative, without considering their negative signs add their coefficients and then put the negative sign (–) to the sum.

Example 3:
Find the sum of $-4xy$, $-2xy$ and $-xy$.
Solution:
Without considering the negative signs, add the coefficients of the given terms i.e. 4 + 2 + 1 = 7.
Thus, the sum of $-4xy$, $-2xy$ and $-xy$
$= (-4xy) + (-2xy) + (-xy) = -7xy$

Note it down

When all the like terms are not with the same sign, then add them like integers with different signs.

Example 4:
Find the sum of $-4x - 3y$ and $-6y + x$.
Solution:
Here, $-4x$ and x are like terms and also $-3y$ and $-6y$ are also like terms.

$-4x - 3y + (-6y) + x = -4x + x + (-3y - 6y)$
$= -4x + x - 3y - 6y$
$= -3x - 9y$

Example 5:
Find the sum of $12x$ and $-4x$.
Solution:
$12x + (-4x)$
$= 12x - 4x$
$= 8x$

Example 6:
Add 9ab – 5cd and 7cd – ab
Solution:
$9ab - 5cd + 7cd - ab$
$= 9ab - ab + 7cd - 5cd$
$= 8ab + 2cd$

Vertical Addition
We can add algebraic expressions vertically by following these steps.
Step I: Write all the algebraic expressions in vertical form, such that the like terms are in the same column.
Step II: Add them.

Example 1:
Add the following
$4ab + b^2$, $ab + 2b^2 - 2$
Solution:

	$4ab + b^2$
+	$ab + 2b^2 - 2$
	$5ab + 3b^2 - 2$

Example 2:
Add the following
$2x + 7y - 3$, $-4x + 5y + 2$, $3x - y - 6$
Solution:

$2x + 7y - 3$
$-4x + 5y + 2$
$+3x - y - 6$
$x + 11y - 7$

Quick Check
Add the following.
a) $4a + b^2$, $a + 3b^2 - 2$
b) $5x^2 + 4y^2$, $7y^2 - x^2$

6.5.2 Subtraction
Ali bought 6 nuggets. He ate 2 nuggets.

So, 6 nuggets – 2 nuggets = 4 nuggets
If we denote nuggets with the variable "n", then by subtracting we can write that:
$6n - 2n = 4n$
Now can Ali eat 2 cupcakes from the remaining 4 nuggets? No, because nuggets and cupcakes are different items. Nuggets can be subtracted from nuggets not from cupcakes.
So, 4 nuggets – 2 cupcakes = 4 nuggets – 2 cupcakes
If cupcakes are denoted by "c", then we can write that:
$4n - 2c = 4n - 2c$
Variable n and c are different, so terms 4n and 2c are unlike terms, and hence they cannot be subtracted from each other.

TEACHING POINT: Explain the concept of addition of algebraic expressions to the students by telling the rules for addition of algebraic expressions.

An algebraic expression can be written for this situation as $6n - 2n - 2c$
Now we simplify it as:
$6n - 2n - 2c$ (6 nuggets, take away 2 is 4)
$= 4n - 2c$

Horizontal Subtraction
We can subtract expressions horizontally by following these steps.
Step I: Write all the algebraic expressions in horizontal form.
Step II: Combine the like terms.
Step III: Subtract them.

Example 1:
Subtract 5ab – ab.
Solution:
Here, 5ab, ab are like terms.
The difference of the coefficients = 5 – 1 = 4,
(∴ab means 1ab)
Therefore, 5ab – ab = 4ab

Example 2:
Subtract $2x^2$ from $-6x^2$.
Solution:
Here, $2x^2$ and $-6x^2$ are like terms.
$= -6x^2 - (+2x^2)$
$= -6x^2 - 2x^2$
$= -8x^2$

Example 3:
Subtract $-3x + 4x$ from $-7x$
Solution:
Here, $-3x + 4x$ and $-7x$ are like terms
$= -7x - (-3x + 4x)$
$= -7x + 3x - 4x$
$= -8x$

Note it down
If a bracket is preceded by a plus sign, remove it and write its terms without any change e.g.
$x + (y - z) = x + y - z$
If a bracket is preceded by a minus sign, remove it and change the signs of every term e.g.
$x - (y - z) = x - y + z$

Example 4:
Subtract $3x + 4y - z$ from $5x + 6y + z$.
Solution:
$(5x + 6y + z) - (3x + 4y - z)$
$= 5x + 6y + z - 3x - 4y + z$
Now arranging the like terms, we get
$= 5x - 3x + 6y - 4y + z + z$
$= 2x + 2y + 2z$

Quick Check
Subtract the following.
a) $-3a - b^2 + 5$ from $5b - 3b^2 - 2$
b) $-x^2 - y^2$ from $-7y^2 - 8x^2$

Example 5:
Subtract $a^3 - 4a^2 + 5a - 6$ from the sum of $3a^3 + a^2 + 1$ and $a^2 - 2$.
Solution:
$= [(3a^3 + a^2 + 1) + a^2 - 2)] - (a^3 - 4a^2 + 5a - 6)$
$= (3a^3 + a^2 + 1 + a^2 - 2) - (a^3 - 4a^2 + 5a - 6)$
$= (3a^3 + 2a^2 - 1) - (a^3 - 4a^2 + 5a - 6)$
$= 3a^3 + 2a^2 - 1 - a^3 + 4a^2 - 5a + 6$
$= 3a^3 - a^3 + 2a^2 + 4a^2 - 5a + 6 - 1$
$= 2a^3 + 6a^2 - 5a + 5$

Vertical Subtraction

We can subtract algebraic expressions vertically by following these steps.

Step I: Write all the algebraic expressions in vertical form. Such that the like terms are in the same column.

Step II: Change the sign of each term in the second expression from + to – and from – to + and solve, like integers.

Example 1:

Subtract $-3x^2 + 2xy^2 + 3y^2$ from $7x^2 - 2xy^2 + y^2$

Solution:

$$\begin{array}{rrr} 7x^2 & -2xy^2 & +y^2 \\ -3x^2 & +2xy^2 & +3y^2 \\ + & - & - \\ \hline 4x^2 & -4xy^2 & -2y^2 \end{array}$$

Example 2:

Subtract $4a + 5b - 3c$ from $6a - 3b + c$

Solution:

$$\begin{array}{rrr} 6a & -3b & +c \\ +4a & +5b & -3c \\ - & - & + \\ \hline 2a & -8b & +4c \end{array}$$

Exercise 6.5

1 **Add the following algebraic expressions horizontally.**

a) ab, ab b) $7x^3$, $9x^3$ c) $14x$, $3x$, $-7x$ d) 12y, 7y, y

e) 3yz, – 4yz, yz f) $3x + 2y$, $x + y$ g) $4x + 3y + 3$, $3x + 5y + 5$

h) $-9x^2 + 5x - 4$, $x^2 + 2x + 1$ i) $6x + 3y - 4z$, $6x + 8y + 9z$

j) $7x^2 + 8y$, $7 - 2x^2$, $6 + 4x^2$

2 **Add the following expressions vertically.**

a) $5x + y$, $x - 9y$ b) $5x^3 - 2y^3$, $7x^3 - 3y^3$ c) $x + 3y - 2z$, $x - y + z$

d) $2x + 9y - 7$, $x - y + 2$ e) $5x^2 + 4y^2$, $-7x^2 - 5$ f) $-3x^2 + 3$, $x^2 + 5$

g) $a^2 + b^2 + c^2 - 3abc$, $a^2 - b^2 + c^2 + abc$

h) $x^2 + 5x + 6$, $3x^2 + 4x$, $2x^2 - x - 3$

i) $(6a + 8b - 7c) + (2b + c - 4a) + (a - 3b - 2c)$

j) $8x^2 - 5xy + 3y^2$, $2xy - 6y^2 + 3x^2$, $y^2 + xy - 6x^2$

3 **Subtract the following algebraic terms horizontally.**

a) – ab from 3ab b) $4x^3$ from $-9x^3$ c) 8y from – 7y

d) –15y from y e) $3xz$ from $-4xz$ f) $7x + 4y$ from $-x - y$

g) $-6x + 2y^2 + 3$ from $3x - 5y^2 - 5$ h) $-10x^2 + 3x - 2$ from $x^2 - 5x + 9$

i) $5x - 2y - z$ from $6x + 8y - z$ j) $8x^2 - 8y$ from $7 - 2x^2$

4 **Subtract the following expressions vertically.**

a) $4a + 5b - 3c$ from $6a - 3b + c$ b) $3x^2 - 6x - 4$ from $5 + x - 2x^2$

c) $3x + y - 3z$ from $9x - 5y + z$ d) $2x + 9y - 7$ from $x - y + 2$

e) $-7x^2 - 5$ from $4x^2 - 5$ f) $-7x^2 + 5y - 4$ from $x^2 - 5y + 2$

g) $9x^2 - 5x + 6$ from $-3x^2 - 4x - 3$ h) $4a + 7b - 7c$ from $9b - c - 3a$

i) $10x^2 - xy + z^2$ from $z^2 + xy - 7x^2$

j) $7a^2 + 5b^2 - c^2 - 3abc$ from $4a^2 - b^2 + 6c^2 - 7abc$

5 **Subtract $x^2 + y^2 + 3xy$ from $4x^2 + 2xy - 3y^2$.**

6 **What should be subtracted from $a^3 - 4a^2 + 5a - 6$ to obtain $a^2 - 2a + 1$?**

7 **Subtract $5a^3 - 2a^2 + a - 6$ from the sum of $6a^3 + a^2 + 1$ and $a^2 - 4$.**

8 **If X = a – b + c, Y = a + b + c, Z = a – b – c then find;**

a) X + Y b) X – Z c) Y + Z d) Z – X

e) X + Y – Z f) X – Y – Z g) – X – Y + Z

6.6 Simplification of Algebraic Expressions

Sometimes, an expression contains more than one operation and brackets, etc. In these cases, it's important to remember the order of operations so that no miscalculations are made.

To simplify the algebraic expressions involving brackets, we follow the same order of preference as we do in whole numbers. When simplifying algebraic expressions the following points must be kept in mind.

- If there is no sign between the number and bracket, it means multiplication.

$x(5) = x \times 5 = 5x$

- If there is a number right before a bracket having more than 1 term, remove the bracket and multiply each term with the number or variable outside.

$x(y + z) = xy + xz$,

$-x(y + z) = -xy - xz$

Example 1:

$l - m - (l + m)$

Solution:

$l - m - (l + m)$ ← First solve the brackets

$= l - m - l - m$ ← Now subtract the like terms

$= -2m$

Note it down

When we solve expression involving brackets, addition and subtraction. First we solve the expression inside the brackets and then addition and at the end subtraction.

Example 2:

$3a - 2(a - c) + ab$

Solution:

$3a - 2(a - c) + ab$ ← First solve the brackets

$= 3a - 2a + 2c + ab$ ← Now add and subtract the like terms

$= a + 2c + ab$

Quick Check

Simplify the following.

a) $[5a + \{a - b + (a - 3b - a)\}]$

b) $[7x - \{4y - 2(5x - y)\}]$

Example 3:

$[12a - \{4b - 2(5a - b)\}]$

Solution:

$[12a - \{4b - 2(5a - b)\}]$ ← First solve the operation within parentheses

$= [12a - \{4b - 10a + 2b\}]$ ← Now solve the like terms in curly brackets

$= [12a - \{6b - 10a\}]$ ← Then open the curly bracket and change the sign

$= [12a - 6b + 10a]$ ← Next add the like terms

$= 22a - 6b$

Example 4:

$[4x + \{z - y + (x - \overline{3x + z})\}]$

Solution:

$[4x + \{z - y + (x - \overline{3x + z})\}]$ ← First solve the operation under the vinculum

$= [4x + \{z - y + (x - 3x - z)\}]$ ← Now solve the like terms in parentheses

$= [4x + \{z - y + (-2x - z)\}]$ ← Then remove the parentheses

$= [4x + \{z - y - 2x - z\}]$ ← Next solve the like terms in the curly brackets

$= [4x + \{-y - 2x\}]$ ← Remove the curly brackets

$= [4x - y - 2x]$ ← Next solve the like terms

$= 2x - y$

TEACHING POINT: Write different questions related to BODMAS on the board and ask the students to simplify the algebraic expressions according to the rules.

Exercise 6.6

1 **Simplify the following.**

a) $2(4x + 2)$
b) $2 + 2x\{2(3x + 2) + 2\}$
c) $[6x - \{3x + (2x - x)\}]$
d) $[7l - \{4m - (4m - 2m)\}]$
e) $3u + \{v - (2u - \overline{t + v})\}$
f) $2 + 3x\{4(x + 2) + 2\}$
g) $11a - \{5b - 3(2a + b)\}$
h) $8\{3(4a + 5b) - 2(6a - 5b)\}$
i) $[4y + 6z\{7(2y - \overline{y + 2}) + z\}]$
j) $8a + 4[(6a + \overline{2b + 3b}) - 2c]$

Think Higher

1. Complete the table.

Position	1			4		6
Term			11		13	
Position	1	2 + 8		4		6

2. Create a real life situation where you can use a pattern with term to term rule of subtracting 3.

3. Evaluate $2x + 10y$ and $2(x + 10y)$ if $x = 4$ and $y = 2$. Are both the values same? If yes, explain why? If no, explain why?

Summary

- Number patterns are sets of numbers that follow a pattern or a rule.
- The term-to-term rule of a pattern describes how to get next term in that specific pattern.
- The word "algebra" comes from the Arabic word "al-jabr" which means the reunion of broken parts.
- A sentence is a set of words that is complete in itself and conveying full meaning without any ambiguity.
- A statement is a sentence that is either true or false.
- The word variable means a quantity that can vary or change.
- A constant is a number without a variable.
- In an algebraic expression, the terms whose variables as well as the exponents of those variables are the same are called like terms.

Vocabulary

- Number pattern
- Term-to-term rule
- Algebra
- Arithmetic
- Variable
- Algebraic expression
- Expression
- Exponent
- Operator
- Constant
- Coefficient

Review Exercise

1 **Choose the correct option.**

a) In algebra, a fixed or unchanged value is called a ________.

i. constant ii. variable iii. term iv. coefficient

b) Unlike terms are those terms whose ________ are different with different exponents.

i. signs ii. variables iii. coefficients iv. constants

c) In $3x^2$, 2 is the ________.

i. variable ii. constant iii. coefficient iv. exponent

d) If a = 1, b = 2, the value of the expression $a^2 + 2a - b$ is ________.

i. 1 ii. 0 iii. –1 iv. 2

e) If $x = 3$, y = 2, then $3x^2 y$ = ________.

i. –54 ii. 54 iii. 36 iv. –36

2 **Identify the position-to-term rules for the pattern and find 20th term:**

11, 22, 33, 44, 55, ...

3 **Identify the position-to-term rules for the pattern and find 55th term:**

16, 17, 18, 19, 20, 21, ...

4 **Identify the term-to-term rules for the following patterns and write the next three terms.**

a) 15, 21, 27, 33, ___, ___, ___

The term-to-term rule of this pattern is: ________

The next three terms are: ________

b) 5, 10, 20, 40, 80, ___, ___, ___

The term-to-term rule of this pattern is: ________

The next three terms are: ________

5 **Subtract the following.**

a) $-3x - 5y - z$ from $3x - 6y - 6z$ b) $19p - q + r$ from $8p - 3q - 4r$

6 **Simplify.**

a) $[9x^2 - \{x^2 - 5y(5x - 2y)\}]$ b) $x - [2y - \{3x - (2y + 3z)\}]$

7 **If $x = -2$, y = 4 and z = 5, then find the value of the following.**

a) $9x - 5y + z$ b) $5x^2 + y - z$ c) $-2x^2 - x - 5$ d) $6xy - yz + z$

Math Project

Material Required:

- Chart papers
- Glue stick
- Paper slips
- Variable Value cards($x = -2$, $x = 3$, $x = -10$ etc.)
- Markers
- Scales

Procedure:

- Distribute paper slips among the groups.
- Each group will write at least 10 expressions on their slips. (like "Evaluate $x^4 + 2x^3 - x^2 + 8$", "Evaluate $4(x^2 + 5x) - 2x$" etc.)
- The teacher will then paste each sum on the chart paper.
- Teacher will call students from each group and ask him/her to choose a value card randomly.
- Then he/she will evaluate the expression according to the value of variable written on the value card she/he chooses.
- The group with quick and accurate solutions wins.

Group A

Evaluate $x^4 + 2x^3 - x^2 + 8$.

$4(x^2 + 5x) - 2x$

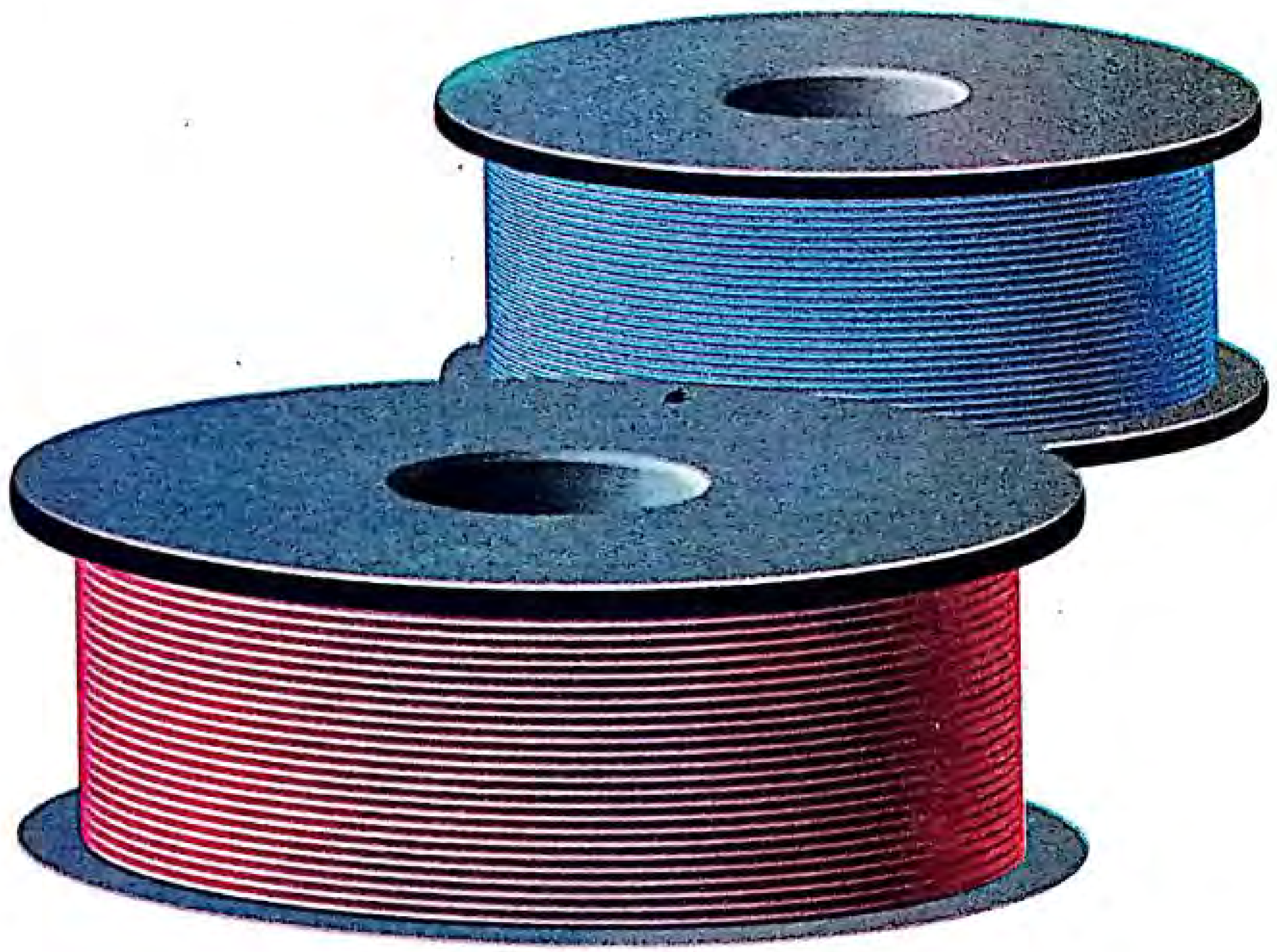
Unit 7

Linear Expressions and Equations

Student Learning Outcomes

After completing this unit, students will be able to:

- Recognise and construct linear equations in one variable.
- Solve linear equations involving integers, fractions and decimal coefficients.
- Solve real-world problems involving linear equations.



An electrician wants to cut a 92 metres long wire in two pieces such that the longer piece must be 3 times as long as the shorter piece. What should be the length of each piece?

Introduction

We come across various routine problems where directly or indirectly the use of linear equations is included. Any situation where there is an unknown quantity and some condition is given, we can easily solve it through linear equations.

7.1 Equations

Previous Knowledge Check

Zara has 68 beads. She wants to use equal beads on each shirt. How many beads will go on each shirt if she has 4 shirts?

A **mathematical sentence** which has equal sides separated by an equality sign "=" is called an **equation**.

a) $6 - 2 = 4$
b) $12 \div 4 = 3 \times 1$
c) $2 + 1 = 9 \div 3$
d) $35 = 40 - 5$

Note it down

In an equation both sides of the equation must be same, if one side is not equal to the other side then it is not an equation.

We see that all of the examples have two sides with an equality sign. All these are equations.

7.1.1 Algebraic Equations

Consider a weighing balance. We say that both sides of the balance are level if they have equal weight on both sides.

$x + 4 = 6$

An algebraic equation is an open mathematical statement that shows the equality of two or more expressions (with at least one variable). The equal sign between two expressions means that both sides of the equation are equal.

Use weighing balance in classroom to clarify the concept of equations. Put different number cards and blank card on the pans and ask the students to balance the pans using correct number for unknown card.

Algebraic equations can either be true or false based on the value allotted to the variable(s) involved.

Note it down

In an equation, changing the side of the expressions doesn't change the equation.
For example:
$2x + 3 = 43$ is the same as $43 = 2x + 3$.

Example 1:

a) $x + y = 12$
In this equation, $x + y$ and 12 are expressions.
b) $3x = 15 - y$
Here, $3x$ and $15 - y$ are expressions.
c) $a + 2b = c$
$a + 2b$ and c are expressions.
d) $P = 2L + 2B$
P and $2L + 2B$ are expressions.

Quick Check

Which of these are algebraic equations?
a) $2x + 5$ b) $5a + 3 = 21$
c) $2x - 7 = 12$ d) $8p - 3$
e) 71

In each of the above examples, we can see that there is a left-hand side expression, a right-hand side expression and an equals sign. So, these are representing algebraic equations. Let's observe the difference between algebraic expressions and algebraic equations.

Equations	Expressions
An equation has an equals sign.	An expression does not have an equals sign.
An equation has left and right sides.	An expression has only one side.
Equations are true for some	Expressions are true for all values of the variables.

Exercise 7.1

1 **Identify the following as algebraic equations or expressions.**

a) $2x + 3y = 13$ b) $5x = 3y$ c) $n = mc^2$ d) $6x - 3y$
e) $10b + 4 = 14$ f) $8x + 9y$ g) $3s + 4t$ h) $x^2 + 3x = 45$
i) $x^3 - 32x + 23 = 0$ j) $x^3 + y^3 - 3xy = 42$

7.2 Linear Equations

A **linear equation** is an algebraic equation where the highest power of the variables involved is 1. In other words, we can say that a linear equation contains the expressions having variables with highest power of 1.

Observe the following equations.

a) $z - 4 = 4$ b) $3y + 2 = 12$ c) $3s = 6$

We can see that all the variables involved in these equations have the power as 1 and hence all these are linear equations.

Quick Check

Which of these are linear equations?
a) $2x + 3 = 8$ b) $5x^2 + 3 = 60$
c) $3x^3 - 12 = 55$ d) $12x = 84$

Now look at the following equations:

a) $2x^2 + 3 = 4x$ b) $3x^3 + 1 = 9$

Here each equation has variables which have powers greater than 1. So these equations are not linear equations.

7.2.1 Linear Equation in One Variable

A linear equation that has only one variable is said to be a linear equation in one variable.

Observe these equations

a) $p - 7 = 9$ b) $2x = 8$ c) $5a + 2 = 32$

Each of these equations has only one variable; so these are linear equations in one variable.

Now look at the following equations:

a) $a + 2b + 3 = 4$ b) $m + 9n - 6p = 20$

Each of these equations has more than one variable. So, these linear equations are not in one variable.

Each of these equations has more than one variable. So, these linear equations are not in one variable.

Exercise 7.2

1 **Identify the linear equations from the following.**

a) $y = 2x + 1$ b) $5x^3 = 6 + 3y$ c) $\frac{y}{2} = 3 - x$ d) $y^3 - 2 = 0$
e) $\frac{x}{2} = 16$ f) $3 - y^3 = 6$ g) $y = 3 + z$ h) $2.2x + 5.7 = 22.5$

2 **Write at least 5 linear equations in one variable.**

7.3 Construction of a Linear Equation in One Variable

My age is one-fourth of my mother's age. How can I construct a linear equation from this sentence?

You can easily construct a linear equation for this if you know how to construct a linear expression.

Let's construct some expressions and equations.
Write the phrase "5 times a number" as an expression.

- First choose a variable to represent the unknown number; let's say x.
- 5 times a number means "**5 multiplied by a number**".
- So, the expression for this phrase is $5x$.

Previous Knowledge Check

- If we subtract 5 from 9 then what is the result?
- If Usman has 4 balls and Asif has 9 apples. How many apples and balls

We can choose any letter as the variable. Let's suppose we take x as the variable.
Now observe these examples:

Phrase	Chosen variable (unknown)	Expression
The sum of 6 and a number	x	$6 + x$
3 less than twice a number	a	$2a - 3$
A number decreased by 9	b	$b - 9$
12 increased by a number	z	$12 + z$
6 added to twice a number	y	$2y + 6$
5 added to half of a number	p	$\frac{1}{2}p + 5$

Now observe these examples:
In the same way, we can construct algebraic equations.

Let's write "Two less than a number is 5" as an equation.

- First choose a variable to represent the unknown number. Let's say the number is x.
- 2 less than a number means $x - 2$. This is the expression on one side of the equation.
- "is 5" means equal to 5. This shows the expression for the other side.

So, the equation for this is: $x - 2 = 5$.

Let's write "3 times a number is 20 more than half of the number" as an equation.

First choose a variable to represent the unknown number. Let's say the number is x.

- 3 times a number means $3x$.
- Half of a number means $\frac{x}{2}$.

Now $3x$ is 20 more than $\frac{x}{2}$. It means, to make them equal, we need to add 20 to the smaller expression or subtract 20 from the greater expression.

So, the equation for this is:

- $3x - 20 = \frac{x}{2}$ or $3x = \frac{x}{2} + 20$

Now observe these examples (using variable x):

Statement	Algebraic expression
3 times a number is 57	$3x = 57$
8 more than a number is 12	$x + 8 = 12$
30 divided by a number is 10	$\frac{30}{x} = 10$
The quotient of a number and 5 is 100	$\frac{x}{5} = 100$
6 is same as twice a number	$6 = 2x$
Sum of 4 and a number when divided by 10 equals 3	$\frac{(4 + x)}{10} = 3$
Sum of 7 and a number is 4 more than twice the number	$7 + x = 4 + 2x$
79 less than 6 times a number is equal to the number	$6x - 79 = x$
123 added to a number is 75 more than 4 times the number	$123 + x = 4x + 75$

Exercise 7.3

1 Write algebraic expressions for the following phrases.

a) 5 added to a number
b) A number subtracted from 8
c) 20 decreased by a number
d) Twice a number decreased by 2
e) Half of a number added to 7
f) 12 subtracted from the sum of 6 and a number

TEACHING POINT: Divide the students in two groups. Ask both groups to write 5 sentences on pieces of paper using a unique variable for each. Ask them to use different operations in each sentence. The group will then exchange their papers with each other and the other group will write the equation for those sentences. The group with correct equation in less time wins.

2 **Write algebraic equations for the following statements.**

a) 6 added to a number is 8.
b) 4 subtracted from a number is 13.
c) 6 times a number decreased by 3 is 5.
d) 3 times the sum of a number and 8 is 14.
e) A number decreased by 6 is 12.
f) Twice the number decreased by 33 is 66.
g) 2 times the number is 16 less than 10 times the number.
h) 6 subtracted from 5 times a number is 14 more than the number.
i) The product of a number and 2 is 26.

3 **Write the statements for the following algebraic equations.**

a) $8 - x = 1$ b) $9x + 4 = 22$ c) $3x - 7 = 23$ d) $x + 5 = 2x$
e) $(6 - x) + 2x = 8$ f) $48 - x = 7x$ g) $4x - 8 = 2x$ h) $\frac{36}{x} = 4x$

7.4 Solving Linear Equations

Hamza has two erasers. His sister has 3 erasers. There are 5 erasers altogether.

Consider the equation "2 + 3 = 5". Let's model this equation using the weighing scale.

We can see that the expressions on both sides of the equation are equal. Let's add the same number of erasers let's say 2, on both sides.

We can see that the equation is still "balanced." Both sides have equal erasers i.e. 7.

What will happen if we add 2 erasers to only one side of the scale? Will the equality still be maintained? The answer is no. We must add the same amount on both sides to maintain the balance or equality.

Same is the case for adding, subtracting, multiplying or dividing the number in equation. We must do the same operation on both sides of the equation to keep it equal.

7.4.1 Properties of Equality

Before solving equations, let's learn about some important properties of equality.

- If the same number is added to both sides of an equation, the equation still remains equal.
- If the same number is subtracted from both sides of an equation, the equation still remains equal.
- If both sides of an equation are multiplied by the same number, the equation still remains equal.
- If both sides of an equation are divided by the same non-zero number, the equation still remains equal.

To check if our solution is correct, put the value of the variable in the original equation. If the answer of the left hand side is equal to the right hand side, our solution is correct.

Example 1:

Khadija is thrice as old as her daughter. The sum of their ages is 48. Find her daughter's age.

Solution:

Suppose the present age of Khadija's daughter = x years

As she is thrice as old as her daughter then,

Khadija's present age = $3x$

According to the given condition:

$3x + x = 48$

$4x = 48$

$\frac{4x}{4} = \frac{48}{4}$ (dividing both sides by 4)

$x = 12$

So, her daughter's age is 12 years.

Verification: Let's put the value of x in the original equation.
$3 \times 12 + 12 = 48$
$36 + 12 = 48$
$48 = 48$
L.H.S = R.H.S
Hence, our solution is correct.

Example 2:

Solve $x - 5 = 4$

Solution:

$x - 5 = 4$

Add 5 to both sides.

$x - \cancel{5} + \cancel{5} = 4 + 5$

$x = 9$

Verification: Let's put the value of x in the original equation.
$x - 5 = 4$
$9 - 5 = 4$
$4 = 4$
L.H.S = R.H.S
So, our solution is correct.

TEACHING POINT: Guide the students to visit the given link for practicing the concept through online games:
https://www.education.com/game/treasure-diving-solving-one-step-addition-and-subtraction-equations/

Example 3:

Arham saved some amount out of his pocket money. He distributed $\frac{2}{7}$ of it among needy children. If he gave Rs 102 to the needy children, calculate the amount that Arham had initially.

Solution:

Let the amount Arham has = Rs x

According to the given condition

$\frac{2}{7}$ of $x = 102$

'Of' mean to multiply so,

$\frac{2x}{7} = 102$

$\frac{2x}{7} \times 7 = 102 \times 7$ (multiplying both sides by 7)

$2x = 714$

$\frac{2x}{2} = \frac{714}{2}$ (dividing both sides by 2)

$x = 357$

Therefore, Arham had Rs 357 initially.

Note it down

While verifying the equations, if both sides are equal than our solution is correct. Otherwise, it is incorrect.

Verification: Let's put the value of x in the original equation.

$\frac{2 \times 357}{7} = 102$

$\frac{714}{7} = 102$

$102 = 102$

L.H.S = R.H.S

Hence, our solution is correct.

a) $3x = 12$

Divide both sides by 3.

$3x \div 3 = 12 \div 3$

$\frac{3x}{3} = \frac{12}{3}$ $x = 4$

Verification:

$3x = 12$

$3 \times 4 = 12$

$12 = 12$

L.H.S = R.H.S

b) $\frac{x}{7} = 5$

Multiply both sides by 7.

$\frac{x}{7} \times 7 = 5 \times 7$

$x = 35$

Verification:

$\frac{x}{7} = 5$

$\frac{35}{7} = 5$

$5 = 5$

L.H.S = R.H.S

c) $5x + 0.5 = 6.0$

Subtract 0.5 from both sides.

$5x + 0.5 - 0.5 = 6.0 - 0.5$

$5x = 5.5$

Divide both sides by 5.

$\frac{5x}{5} = \frac{5.5}{5}$

$x = 1.1$

Verification:

$5x + 0.5 = 6.0$

$5(1.1) + 0.5 = 6.0$

$5.5 + 0.5 = 6.0$

$6 = 6$

L.H.S = R.H.S

Quick Check

Solve the following linear equations:

a) $\frac{1}{2}(y + 8) = 16$

b) $10(4x - 4) = 4(4x - 3)$

c) $3x - 4 = 4x + 12$

d) $\frac{4}{5}(x + 7) = 3(2x - 7)$

Multiply both sides by 5.

$5 \times \frac{4}{5}(x + 7) = 5 \times 3(2x - 7)$

$4(x + 7) = 15(2x - 7)$

$4x + 28 = 30x - 105$

Subtract 28 from both sides.

$4x + 28 - 28 = 30x - 105 - 28$

$4x = 30x - 133$

Separate the variables and numbers.

$4x - 30x = -133$

$-26x = -133$

Divide both sides by (–26) to get variable x.

$\frac{-26x}{-26} = \frac{-133}{-26}$

$x = \frac{133}{26}$

Verification:

$\frac{4}{5}(x + 7) = 3(2x - 7)$

$\frac{4}{5}(\frac{133}{26} + 7) = 3[2(\frac{133}{26}) - 7)]$

$\frac{4}{5}(\frac{133 + 182}{26}) = 3(\frac{133}{13} - 7)$

$\frac{4}{5}(\frac{315}{26}) = 3(\frac{133 - 91}{13})$

$\frac{252}{26} = \frac{3 \times 42}{13}$

$\frac{126}{13} = \frac{126}{13}$

LHS = RHS

Exercise 7.4

1 Solve the following linear equations. Also verify the solution.

a) $6x + 2 = -19$ b) $4x - \frac{1}{4} = 5$ c) $7x - 4 = 23$

d) $-6x = 21$ e) $18 - 7x = -3$ f) $5(x + 4) = 25$

g) $\frac{4x}{6} - \frac{3x}{5} = 12$ h) $3x - 5 = 5x - 4$ i) $4(x - 3) = 4(3x + 1)$

j) $\frac{3x}{12} - 10 = \frac{1}{2}$ k) $\frac{x}{2} - \frac{x}{3} = 8$ l) $6x - 9 - 2(1 - x) = x + 9$

m) $0.4x + 2.4 = 6.5$ n) $0.8x - 4 = 12$ o) $0.3x + 6.9 = 12$

p) $\frac{0.3x}{3.1} = 3$ q) $0.5x + 1.1 = 2.3$

r) $2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$

TEACHING POINT: Divide the students into groups and give them a word problem involving linear equations. Ask them to discuss the question, and come up with possible solutions to the problem. Ask them to share their answers and tell how they came to find that answer with the class.

2 Dania is 6 years older than her younger sister. After 10 years, the sum of their ages will be 48 years. Find their present ages.

3 The price of a shirt has decreased after a sale by Rs 215. Find the original price if the new price is Rs 345.

4 The total price of a paintbrush and a pen is Rs 156. Find the price of both the items if the price of the paintbrush is half the price of the pen.

5 Ali's weight is 13 kg less than Umar's weight. Find the weight of Umar if the sum of their weights is 59 kg.

6 The difference between 45 and 5 times a number is 15. Find the number.

7 Madiha thinks of a number. She multiplied it by 6.2 and added 8 to the product, she got the answer 39. Find the number.

8 Ayesha was asked to divide a number by 3 but by mistake she multiplied it by 3. If the correct number is 72 less than the wrong number, find the number.

Think Higher

Create and solve a real-world situation that can be represented by the given linear equation "$2x + 18 = 54$".

Summary

- A mathematical sentence which has equal sides separated by an equals sign is called an equation.
- An algebraic equation is an open mathematical statement that shows the equality of two or more expressions (with at least one variable).
- A linear equation is an algebraic equation where the highest power of the variables involved is 1.
- Solving linear equation is the process to find the value of the involved variable.
- Any situation where there is an unknown quantity and some condition is given, we can easily solve it through linear equations.

Vocabulary

- Expression
- Algebraic expression
- Equation
- Algebraic equation
- Linear equation
- Linear expression

Review Exercise

1 Encircle the correct option.

a) A mathematical sentence which has equal sides separated by an equal sign is called a/an ________.

i) equation ii) number iii) variable iv) power

b) An algebraic equation is an open mathematical statement that shows the equality of two or more ________.

i) numbers ii) expressions iii) variables iv) powers

c) The equals sign between two expressions means that both sides of the equation are ________.

i) greater ii) smaller iii) equal iv) nothing

d) A linear equation is an algebraic equation where the highest power of the variables involved is ________.

i) 0 ii) 1 iii) 3 iv) 4

e) The solution of $2x - 3 = 1$ is ________.

i) 5 ii) 4 iii) 3 iv) 2

f) If $\frac{2x}{5} = 3$, $x =$ ________.

i) $\frac{15}{2}$ ii) $\frac{11}{2}$ iii) $\frac{13}{5}$ iv) $\frac{16}{3}$

g) $2x + 0.06 = 0.4 + 1.22 =$ ____

i) 1 ii) 0.78 iii) 2.3 iv) 0

h) The solution of $5x - 20 = 3x + 2$ is ____

i) 15 ii) 3 iii) 10 iv) 11

i) "12 added to half of a number" means:

i) $\frac{x}{2} + 12$ ii) $12 + \frac{1}{2x}$ iii) $12x + \frac{1}{2}$ iv) $12x + \frac{1}{2x}$

j) "5 decreased by a number is 10" means:

i) $5 - 10 = x$ ii) $10x - 5 = x$ iii) $5 - x = 10$ iv) $x - 5 = 10$

2 **Answer the following questions.**

a) What is an algebraic equation?

b) What is the difference between an equation and an expression?

c) Define linear equation and linear equation in one variable.

d) What is meant by solving an equation?

3 **Identify the following as expressions or equations.**

a) $ab + 4$ b) $3x + 3y = 45$ c) $pq = 6$

d) $x - 5$ e) $3x = 9$ f) $x + 5$

4 **Solve the following linear equations. Also verify the solution.**

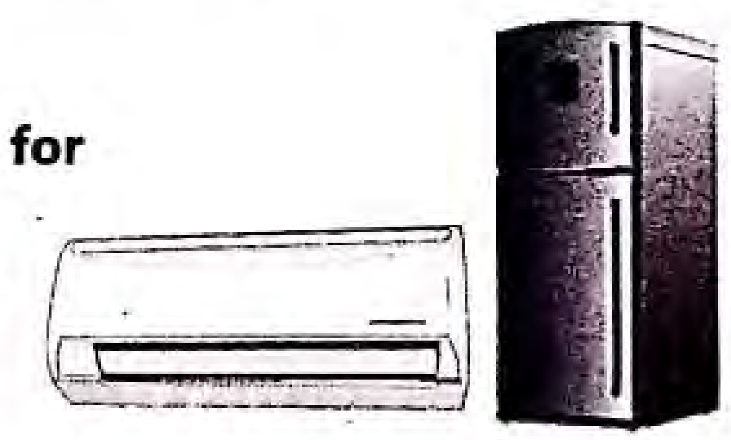
a) $\frac{(4x+3)}{(2x-3)} = -\frac{4}{2}$ b) $\frac{(x-9)}{3} = -\frac{(x-3)}{7}$ c) $2.5x + 3 = 7$

d) $4x + \frac{4}{8} = 3 - x$ e) $6x = \frac{3}{9}$ f) $5(a-6) - 3(4a+4) = 7(a+6)$

g) $3(y-2) + 4(y-3) = 5y + 14 - 4(y-8)$ i) $\frac{3(2x-4)}{15(x-2)} = 4$

j) $\frac{3}{5}(4a+5) = \frac{2}{7}(3x+5)$ k) $0.4x + 2.4 = 2.0x + 0.12$

5 **Mr Ali purchased a fridge and an air conditioner for Rs 95,230. If the cost of the air conditioner is Rs 23,000 more than the fridge, find the cost of both items.**



6 **After sixteen years, Asma will be 28 years old. Find her present age.**

7 **When five times a number is added to three times itself, the sum obtained is 48. Find the number.**

Math Project

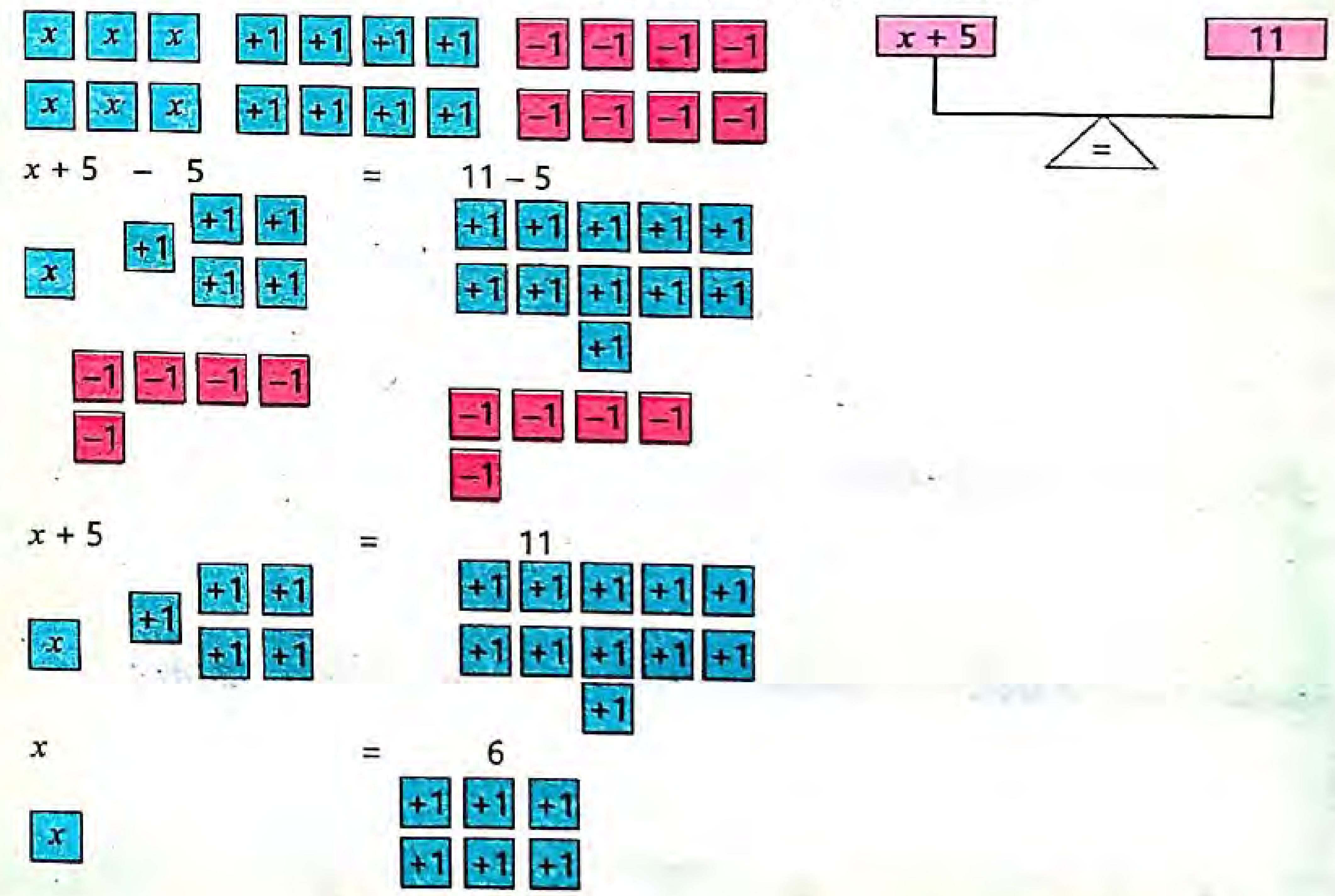
Material Required:

- Algebra tiles cut-outs (Made up of coloured paper)
- Cards with balance and equations written on it
- Scoring sheet

Note:
The teacher will create the equation cards in quantity sufficient for the groups (according to the class strength).

Procedure:

- Get in groups.
- Randomly choose at least 3 equation cards (like the ones given below; teacher can make more like this).
- Each group will solve the equation using the +1, -1 cards (as the one shown).
- Once done, each group will get the solution checked by the teacher or other groups.
- Record the result in scoring sheet.
- The group with most accurate results and quick response wins.

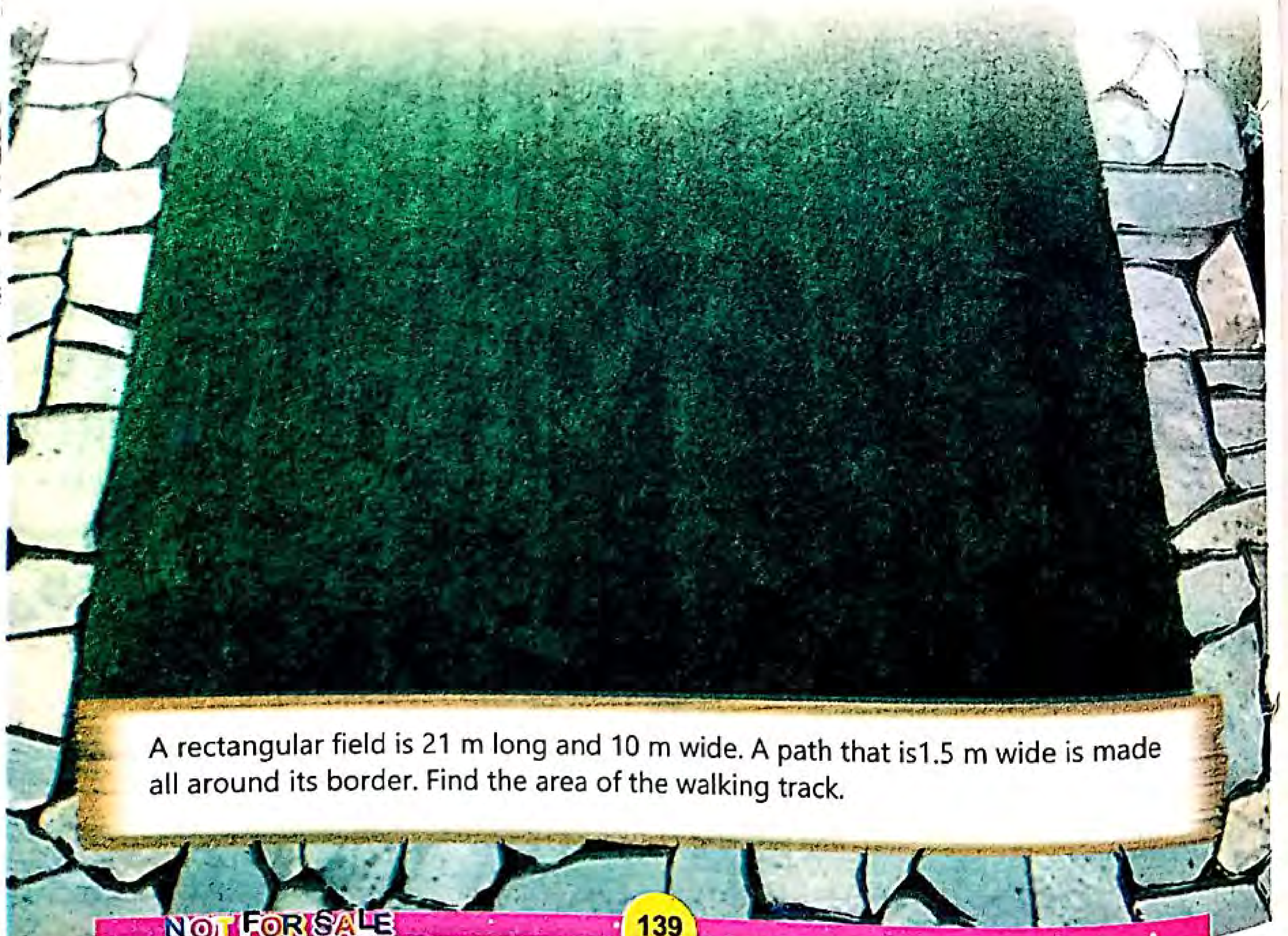


Unit 8 Surface Area and Volume

Student Learning Outcomes

After completing this unit, students will be able to:

- Calculate the area of; a path (inside or outside) a rectangle or square, parallelogram, triangle and trapezium.
- Solve real life word problems involving perimeter and area.
- Recognise and identify 3-D shapes (i.e., cube, cuboid, cone, cylinder, sphere, hemisphere and cone) with respect to their characteristics.
- Calculate the surface area and volume of cube and cuboids.
- Solve real life word problems involving the surface area and volume of cubes and cuboids.



A rectangular field is 21 m long and 10 m wide. A path that is1.5 m wide is made all around its border. Find the area of the walking track.

Introduction

Mensuration concepts are being used widely in different real life situations. For example, perimeter is used when:

- Measuring the length of wooden fence for fencing a plot, a garden or a playground
- Measuring the length of concrete border around a swimming pool or parking area
- Measuring the length of frames for different paintings or mirrors; etc.

Similarly, concept of area is used when:

- Measuring the space and cost for painting a wall
- Measuring the space and cost for ploughing a field
- Measuring the space and cost for growing grass in a field
- Measuring the space and cost for tiling a floor
- Measuring the space and cost for carpeting; etc.

In this unit we are going to learn more about area and perimeter of various shapes using formula.

8.1 Perimeter

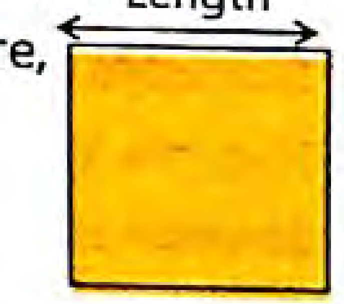
Previous Knowledge Check

- How can we find the space covered by a cupboard in a room?
- How can we find the length of the boundary wall around the playground?

We know that **perimeter** of a shape is measured by adding the length of all its sides. Its always measured in single units i.e. centimetres (cm), metres (m), etc.

8.1.1 Perimeter of square

A square is a four sided 2-D shape having all sides equal in length. Its perimeter can be measure by adding all 4 sides. If L is the length of a side of the square, we can write the perimeter as:

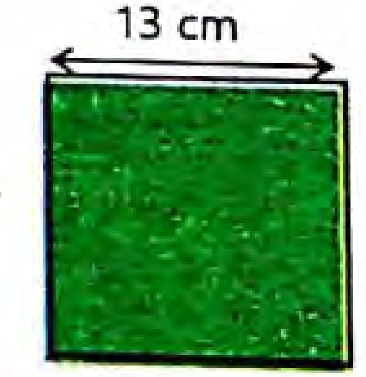


Perimeter of square = L + L + L + L

= 4 × L = 4 L

Example 1:

Find the perimeter of the square if the length of one side is 13 cm.



Solution:

Perimeter of a square = 4 × L

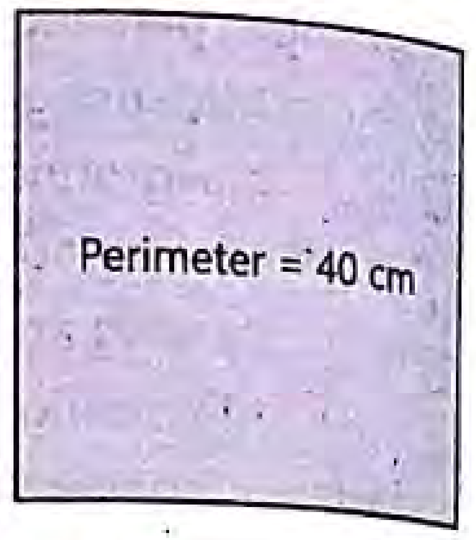
= 4 × 13 cm = 52 cm

The perimeter of the square is 52 cm.

TEACHING POINT: Explain to the students how to find the perimeter of a square. Draw a few squares on the board and ask them to find the perimeter.

Example 2:

Find the length of a side of the square, if the perimeter of the square is 40 cm.



Solution:

Perimeter of a square = 4 × L

As perimeter is given and we need to find the length, so we put the value of perimeter in this formula.

40 cm= 4 × L

Divide both sides by 4

$$\frac{\overset{10}{\cancel{40}}\text{ cm}}{\cancel{4}} = \frac{\cancel{4} \times L}{\cancel{4}} = 10 \text{ cm} = L$$

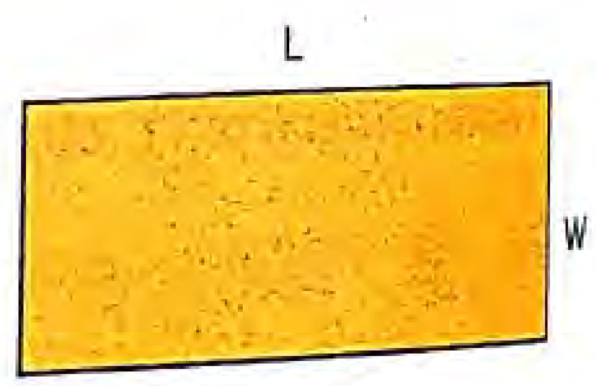
So, the length of the side of the square is 10 cm.

Quick Check

Find the perimeter of a square if the length of one side is 4.5 m.

8.1.2 Perimeter of Rectangle

We know that length of opposite sides of a rectangle are the same. So, its perimeter can be measured by adding all 4 sides. If L is the length of a side of the rectangle, and W is the width, we can write the perimeter as:



Perimeter of a rectangle = length + length + width + width

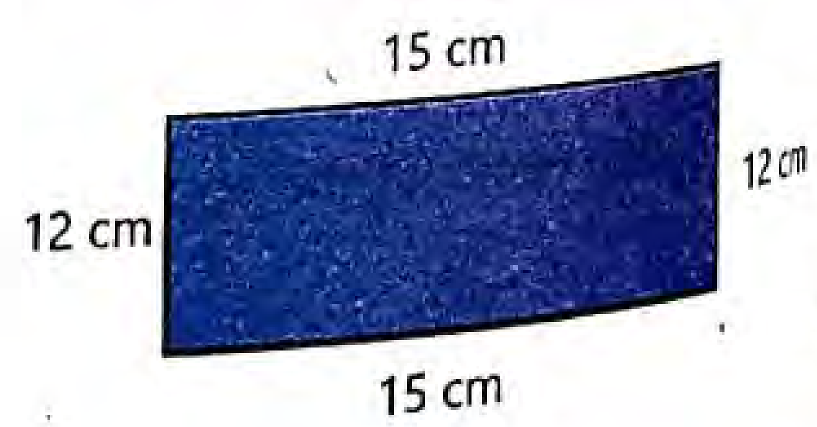
= 2 × length + 2 × width

= 2 (length + width)

= 2 (L + W)

Example 1:

Find the perimeter of the rectangle if its length is 15 cm and width is 12 cm.



Solution:

Perimeter of rectangle = 2 (length + width)

= 2 (15 cm + 12 cm)

= 2 × 27 cm

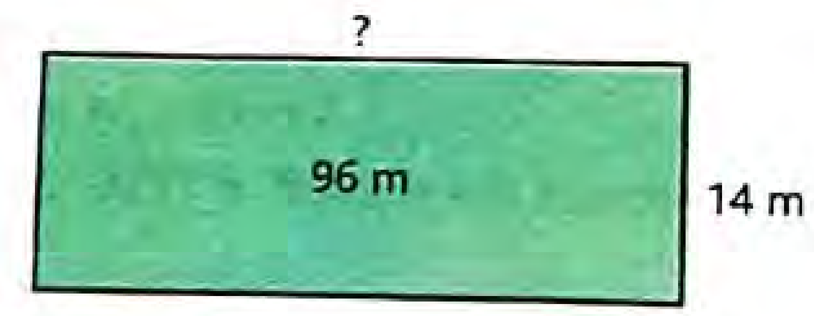
= 54 cm

So, the perimeter of the rectangle is 54 cm.

TEACHING POINT: Explain to the students how to find the perimeter of a rectangle. Draw a few rectangles on the board and ask them to find the perimeter.

Example 2:

Find the length of the rectangle if its perimeter is 96 m and its width is 14 m.



Solution:

As perimeter and width of the rectangle is given and we need to find the length, so we put the value of perimeter and width in the formula.

Perimeter of a rectangle = 2(L + W)

96 m = 2 (L + 14) m

96 m = 2 L+ 28 m

96 m – 28 m = 2 L

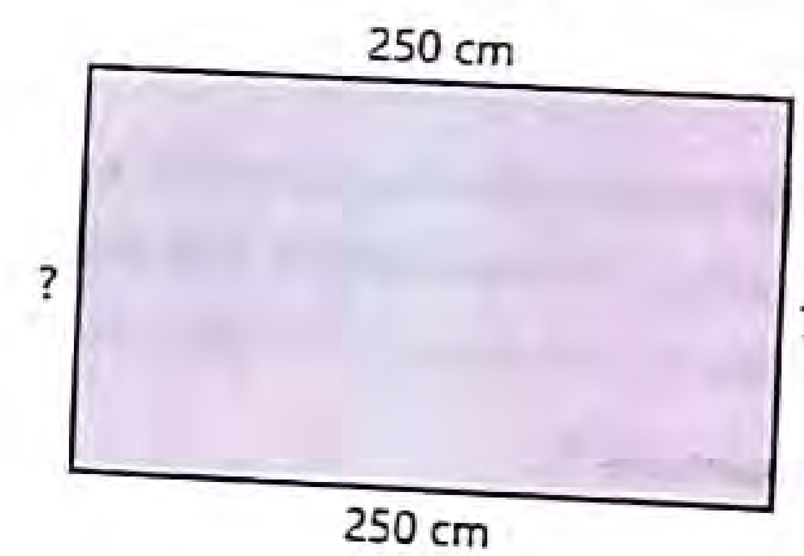
68 m = 2 L

34 m = L

So, the length of the rectangle is 34 m.

Example 3:

Find the width of a rectangular park if the length of the park is 250 m and perimeter is 900 m.



Solution:

Perimeter of rectangle = 2 (length + width)

= 2 (250m + width)

900 m = 500 m + 2 width

900 m – 500m = 2 width

400 m = 2 width

$\frac{400}{2}$ = width

200 m = width

The width of the rectangle is 200 m.

8.1.3 Area of the Square and rectangle

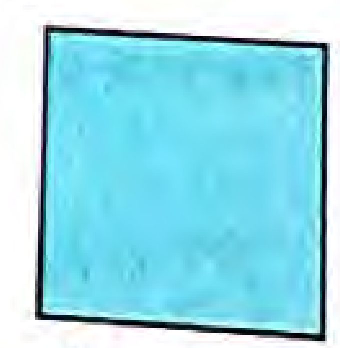
We have learnt that area of a figure is the surface covered by that figure. It is always measured in squared units' i.e. square centimetres cm^2, square meters m^2 etc.

Area of the Square

We can find the area of the square by multiplying its length and width.

Area of square = length × length

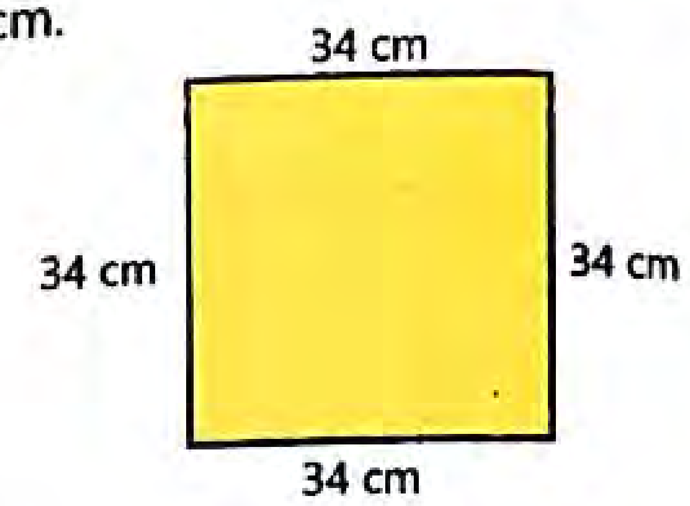
= L × L



Length = L

Example 1:

Find the area of the square whose length of one side is 34 cm.



Solution:

Area of a square = length × length

= 34 cm × 34 cm = 1156 cm^2

The area of the square is 1156 cm^2.

Example 2:

Find the length of a square, if the area of square is 144 m^2.

Solution:

Area of square = length length

As area of the square is given and we need to find the length, so we put the value of area in this formula.

144 m^2 = length × length

12m^2 = length

Length = 12m

Quick Check

Find area of a square if the length of one side is $\frac{5}{9}$ m.

Area of Rectangle

The total surface covered by a rectangle is called **its area**. The area of the rectangle is the product of their length and width.

Area of rectangle = length × width

Example 1:

If a length of a rectangular pond is 45 m and width is 2900 cm. Find the area of the rectangular pond in cm^2.

Solution:

Length = 45 m = 45 × 100 = 4500

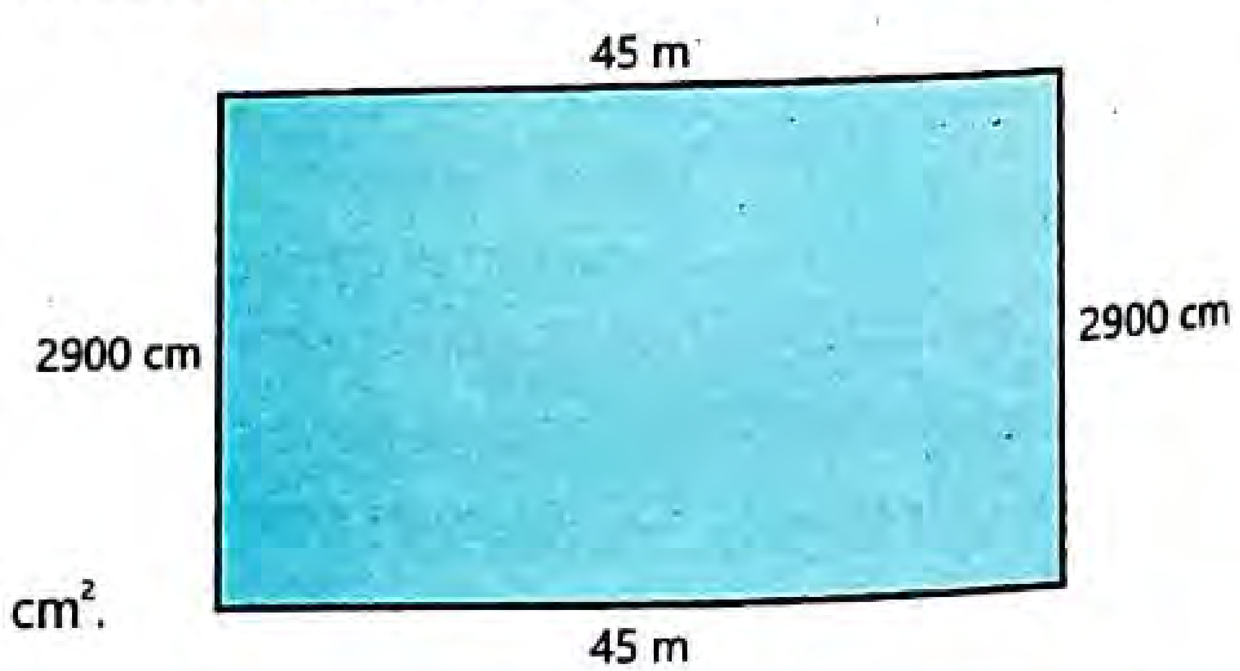
Width = 2900 cm

Area of rectangle= length × width

= 4500cm × 2900 cm

= 13050000 cm^2

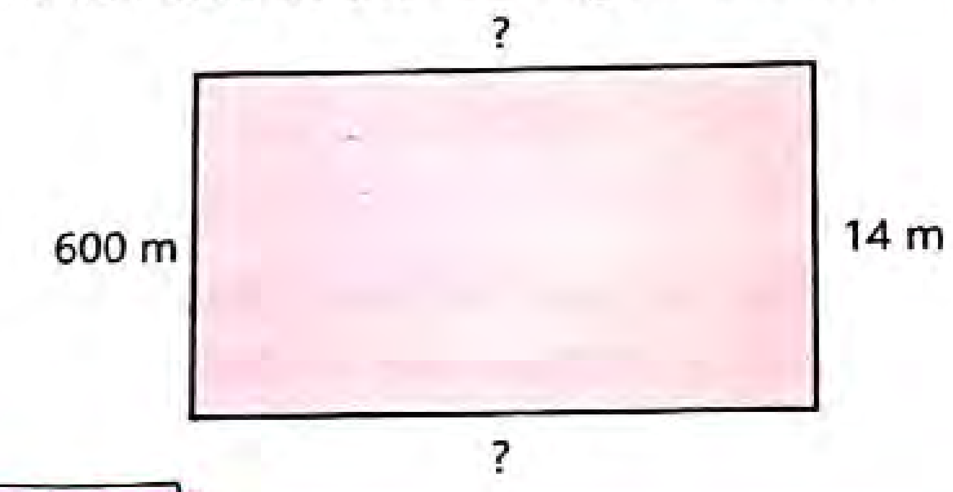
The area of a rectangle is 13050000 cm^2.



TEACHING POINT: Explain to the students how to find the area of square and rectangle. Draw a few squares and rectangles on the board and ask them to find their areas.

Example 2:

Find the length of the rectangular playground if the area of the rectangles 4800 m^2 and width is 600m.



Solution:

Area of a rectangle = 4800 m^2

Width of playground = 600 m

Area of rectangle= length × width

4800 m^2 = length 600 m

$\frac{4800 \text{ m}^2}{600 \text{ m}}$

800 m = length

The length of the playground is 800 m.

Quick Check

Find the area and perimeter of their lunch boxes.

Make the layout of their house on a grid sheet and find Area and perimeter of each room.

Count the tiles of the room and estimate the area and perimeter of the classroom.

Example 3:

A rectangular ground is 600 cm in length and 550 cm wide. The cost of fencing the ground Rs 75 per meter. Find the cost of fencing around the ground.

Solution:

Perimeter of ground is = 2 (length + width)

= 2 (600 cm + 550 cm)

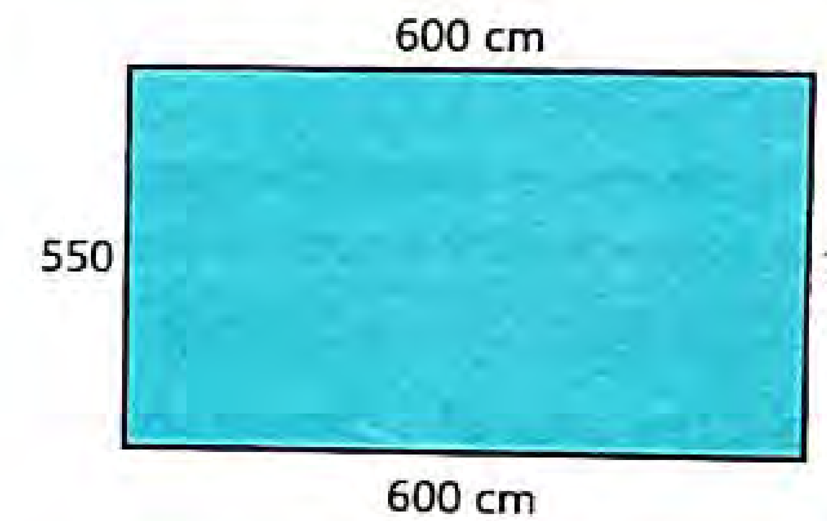
= 2 (1150 cm)

Perimeter of a ground = 2300 cm = 23 m

Cost of fencing per metre square = Rs 75 m

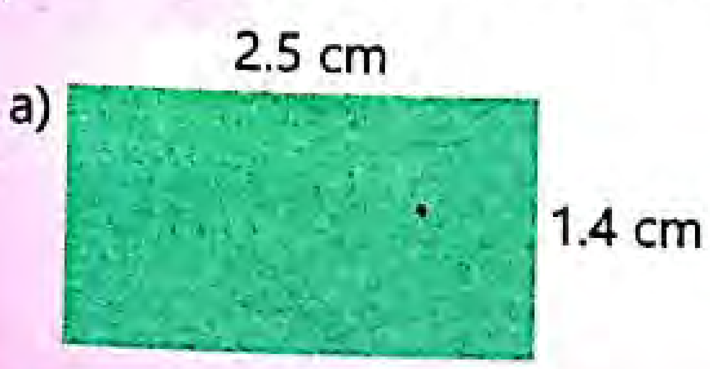
Cost of fencing 23 m = 23 × 75

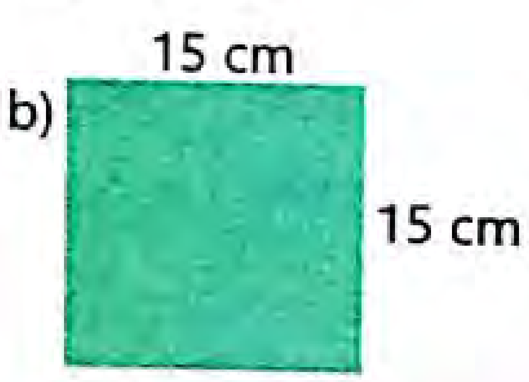
= Rs 1725 m



Exercise 8.1

1 **Find the perimeter of the following figures.**

a) 

b) 

c) 

2 Use the formula to find the area and perimeter of the squares whose lengths are given below:

a) 4 cm b) 7.2 cm c) 10.5 m d) 6.2 m

3 Use the formula to find the area and perimeter of the rectangles having the following lengths and widths.

a) Length = 4.7 cm and Width = 2 cm

b) Length = 16 m and Width = 15.5 m

c) Length = 23 cm and Width = 4 cm

d) Length = 20 m and Width = 12.2 m

4 Find the length of the rectangle if its area is 325 m^2 and width is 13 m.

5 Find the length of a square if its perimeter is 48 m.

6 A rectangular-shaped room is 24 metres long and 20 metres wide. Find the area of the room and the cost to cement the room at the rate of Rs 200 per metre square.

7 Find the area of a square shaped swimming pool if its length is 300 m.

8 The perimeter of a rectangular flowerbed is 244 cm. Find the length of the flowerbed if its width is 44 cm.

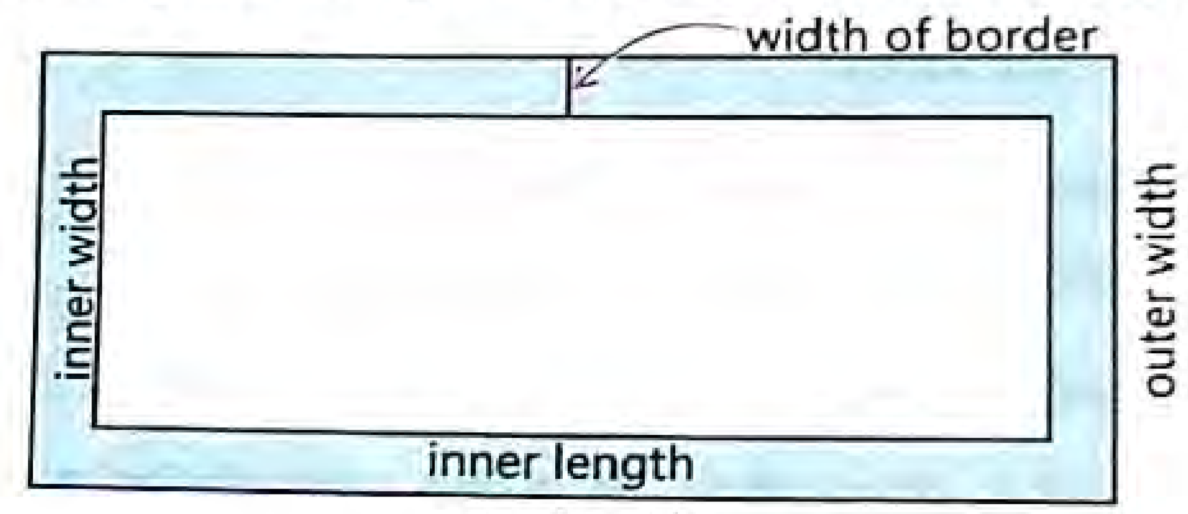
9 Find the length of a rectangular room if its area is 2048 cm^2 and width is 32 cm. Find the cost of flooring the room at the rate of Rs 250 per cm^2.

10 The cost of fencing a square garden at the rate of Rs. 25 per metre is Rs. 5625. Find the area of the garden.

8.2 Area of Paths around a Shape

Consider a rectangular shaped playground. It has a walking track. To find the area of the walking track around the playground, first we find the area of the playground, and then find the area of the whole space covered by the playground and the walking track. Then subtract the area of the playground from the whole area covered by the playground and walking track.

➤ **Area of border = Area of large rectangle – Area of small rectangle**



Area of the border = (outer length × outer width) – (inner length × inner width)

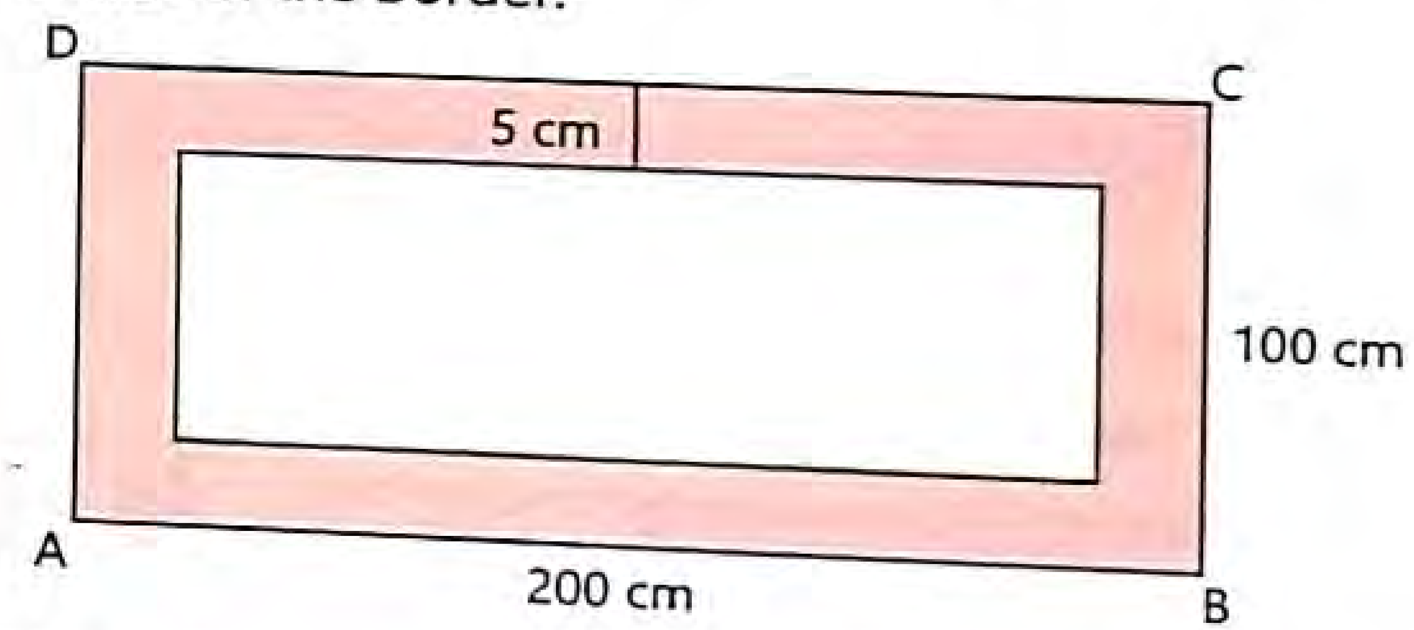
Length of large rectangle = Length of small rectangle + 2 × Width of the border

Width of large rectangle = Width of small rectangle + 2 × Width of the border

Example 1:

A rectangle ABCD is 200 cm long and 100 cm wide. There is a border towards the inner side of the rectangle that is 5 cm wide. Find the area of the border.

Solution:



Length of the rectangle = 200 cm

Width of the rectangle = 100 cm

Area of the rectangle = 200 × 100 = 20000 cm^2

Length of the rectangle without border = 200 – 5 – 5 = 200 – 10 = 190 cm

Width of the rectangle without border = 100 – 5 – 5 = 100 – 10 = 90 cm

Area of the rectangle without border = 190 cm × 90 cm = 17100 cm^2

Area of the border = 20000 cm^2 – 17100 cm^2 = 2900 cm^2

TEACHING POINT: Teachers can start the concept by asking students to make an anchor chart to mention the differences and similarities between area and perimeter measurements.

Exercise 8.2

1. A rectangle WXYZ is 50 m long and 40 m wide. A path is constructed towards the inner side of the rectangle that is 2 m wide. Find the area of the path.
2. A square is 1200 cm long. There is a 10 cm wide strip around the square. Find the area of the strip.
3. A 450 cm long and 150 cm wide rectangle is enclosed by a strip outside it that is 4 cm wide. Find the area of the strip.
4. A rectangle is 900 m long and 700 m wide. A path is constructed outside the rectangle that is 9 m wide. Find the area of the path.
5. A garden is 75 cm long and 82 cm wide. A 5 cm wide road is made all around the inside of the garden. Find the area of the road.
6. A playground is 60 m long and 50 m wide. A 1.5 m wide concrete path is made all around it towards the outer side of the playground. Find the area of the concrete path.

8.3 Area of Parallelogram, Trapezium and Triangle

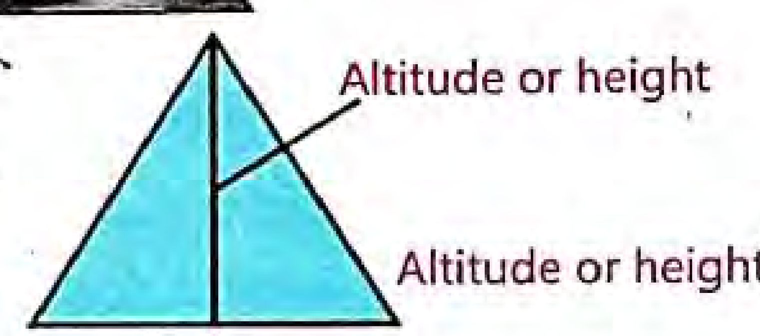
8.3.1 Altitude

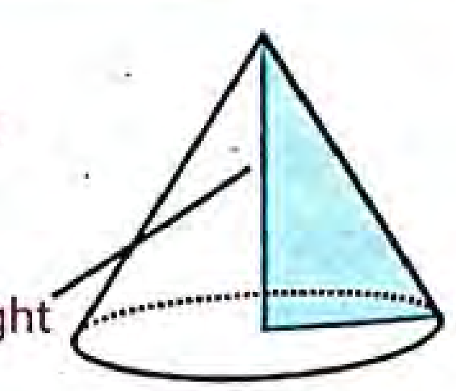
Fatima has a triangular mirror. We can find the area it occupies by finding its altitude and base.

Note it down

Altitude of a figure is also called the height of the geometric figure.

The altitude of any shape is the shortest distance between the top or vertex of the shape to the opposite side or base.

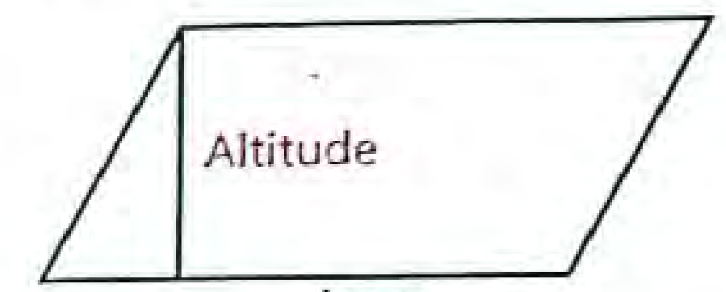




Teachers can share the following online game links to practice areas and perimeter.
https://www.splashlearn.com/s/math-games/find-the-perimeter-of-the-shapes-using-grids
https://www.splashlearn.com/s/math-games/find-the-perimeter-of-polygons
https://www.splashlearn.com/s/math-games/find-the-area-by-multiplying-the-side-lengths

8.3.2 Area of Parallelogram

A four-sided figure whose opposite sides are parallel is called a **parallelogram**.
If we draw a perpendicular line from any point of parallelogram to its opposite side. This perpendicular line is called height/altitude and opposite side is called base.

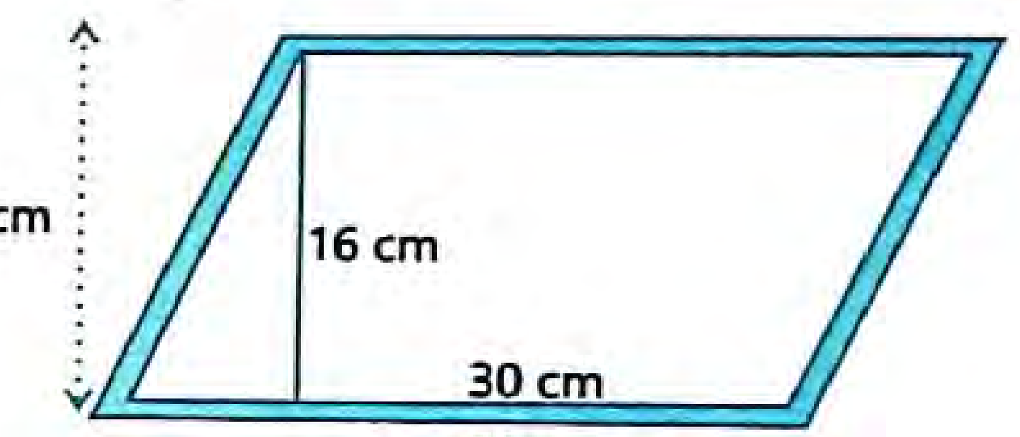


We can find the area of the parallelogram if we have its **base and altitude**.
The formula to find the area of the parallelogram is:

Area of Parallelogram = base × altitude

Example 1:

Find the area of the shaded path of parallelogram.



Solution:

Altitude = a_1 = 18 cm

Base = b_1 = 32 cm

Area of outer parallelogram = $b_1 \times a_1$

= 32 cm × 18 cm

= 576 cm^2

Altitude = a_2 = 16 cm

Base = b_2 = 30 cm

Area of inner parallelogram = $b_2 \times a_2$

= 30 cm × 16 cm

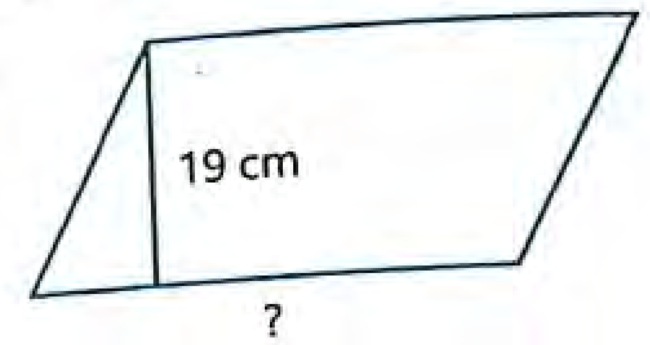
= 480 cm^2

Area of the shaded path of parallelogram = 576 cm^2 – 480 cm^2

= 96 cm^2

Example 2:

The altitude of the parallelogram is 19 cm and its area is 570 cm². Find the base of parallelogram.



Solution:

Altitude = 19 cm

Area = 570 cm²

Area of parallelogram = base × altitude

570 cm² = base × 19 cm

$\frac{570\ cm^2}{19\ cm}$ = base

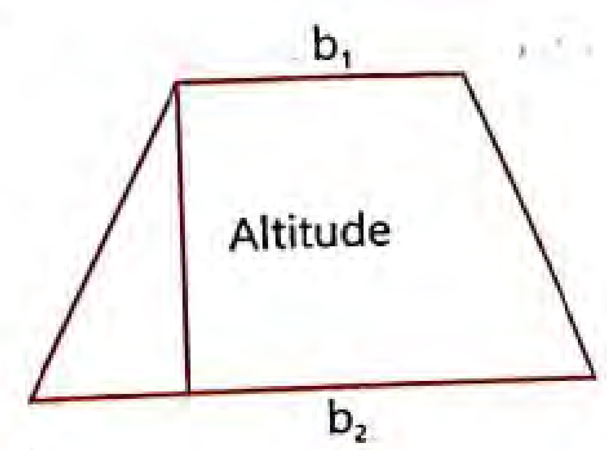
base = 30 cm

8.3.3 Area of trapezium

A trapezium is a 4-sided figure with one pair of parallel sides.

The two parallel sides of the trapezium are its two bases and altitude is the shortest distance between the two bases.

We can find the area of the trapezium if the length of its two bases and altitude is given:

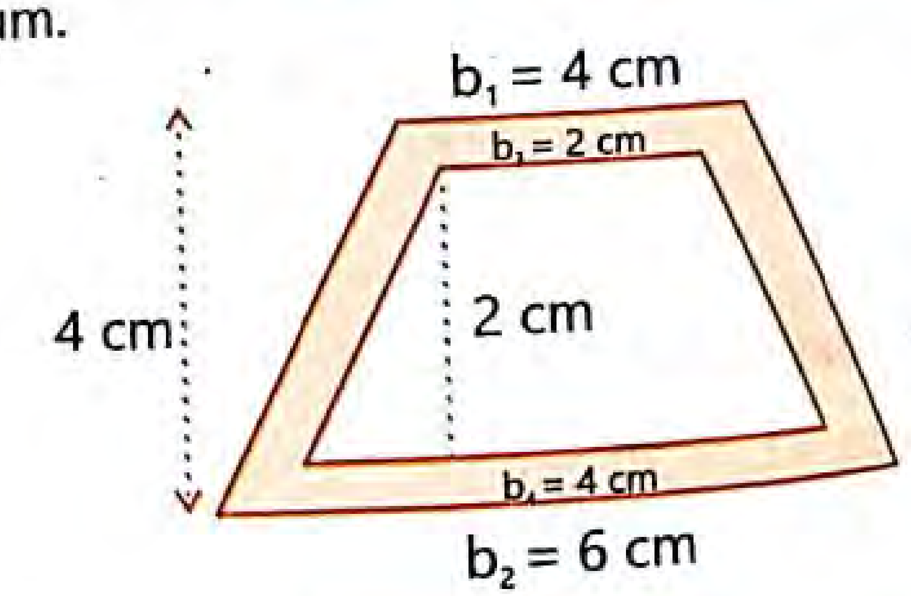


Area of trapezium = altitude × $(\frac{b_1 + b_2}{2})$

Note it down
A trapezium is a quadrilateral with one pair of parallel sides.

Example 1:

Find the area of the path around the given trapezium.



Solution:

Length of b_1 = 4 cm   Length of b_2 = 6 cm

Length of b_3 = 2 cm   Length of b_4 = 4 cm

Altitude = a_1 = 4 cm   Altitude = a_2 = 2 cm

Area of trapezium = $a_1 \times (\frac{b_1 + b_2}{2})$

Area of outer trapezium = 4 cm × $(\frac{4\ cm + 6\ cm}{2})$

= 4 cm × $(\frac{10\ cm}{2})$

= 20 cm²

Note it down
The space occupied by the triangle is called its area.

Area of inner trapezium = 2 cm × $(\frac{2\ cm + 4\ cm}{2})$

= 2 cm × $(\frac{6\ cm}{2})$

= 6 cm²

Area of the border = 20 cm² – 6 cm² = 14 cm²

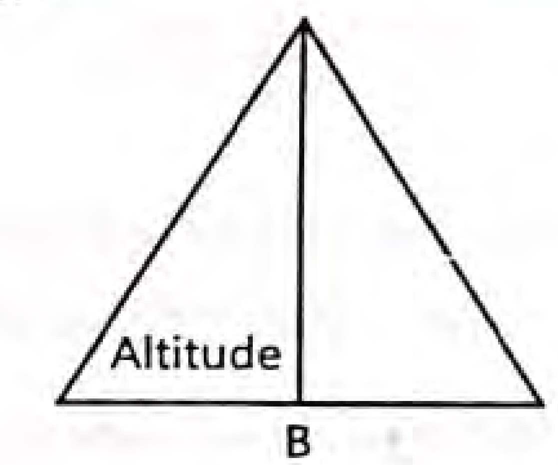
Note it down
In triangle if we choose one side as altitude the opposite side of the altitude is called base.

8.3.4 Area of triangle

A triangle is a 3-sided figure with one pair of parallel sides.

We can find the area of the triangle if the length of its base and altitude is given:

Area of triangle = $\frac{1}{2}$ (altitude × base)



Example 1:

Find the area of the path around the given triangle.

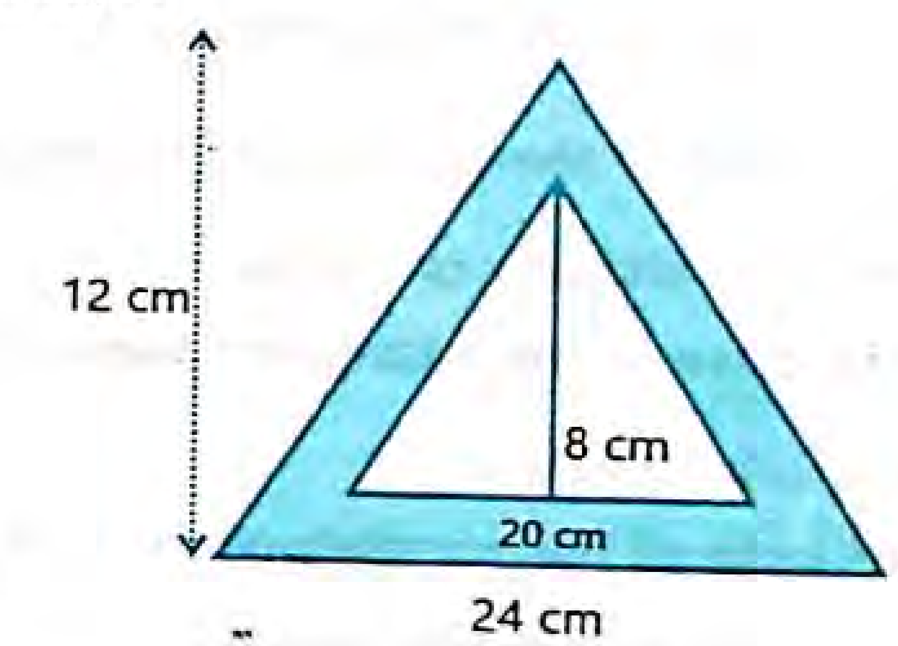
Solution:

Base of outer triangle = 24 cm

Base of inner triangle = 20 cm

Altitude of outer triangle = 12 cm

Altitude of inner triangle = 8 cm



Area of outer triangle = $\frac{1}{2}$ (altitude × base)

= $\frac{1}{2}$.(12 cm × 24 cm)

= 144 cm²

Area of inner triangle = $\frac{1}{2}$ (altitude × base)

= $\frac{1}{2}$ (8 cm × 20 cm)

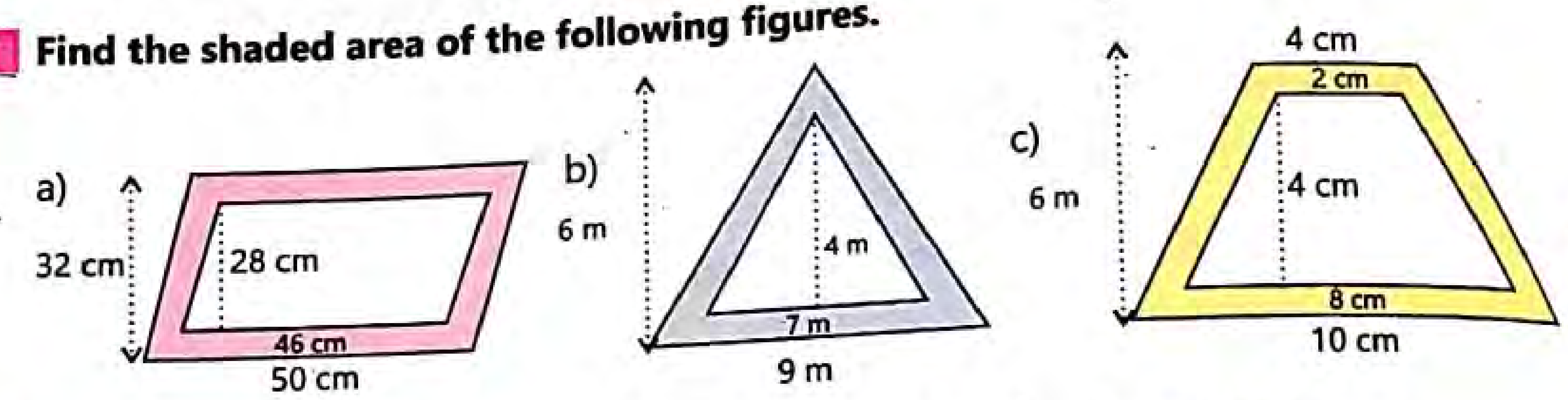
= 80 cm²

Area of the path around the triangle = 144 cm² – 80 cm² = 64 cm²

TEACHING POINT: Explain to the students about triangle by drawing some triangles on the board and instruct them to find its area.

Exercise 8.3

1 Find the shaded area of the following figures.



2 Find the altitude of a trapezium if the length of two bases are 5.6 cm and 2.9 cm respectively. The area of the trapezium is 42 m^2.

3 Find the area of a trapezium if the length of two bases are 8.8 m and 12.9 m respectively. The perpendicular distance between two bases are 12 m.

4 Find the altitude of the triangular field. If its area is 946cm^2 and base is 104cm.

5 Find the area of the triangle if the length of its base is 2.5 m and the length of its altitude is 1.2 m.

6 Find the base of the triangular piece of carpet. If its area is 1568 cm^2 and altitude is 9.04 m.

7 The altitude of the parallelogram is 56 m and its area is 6424 cm^2. Find the base of parallelogram.

8 The area of the parallelogram is 900 m^2 and base is 30 m. Find the altitude of parallelogram.

8.4 Three Dimensional Solids

In the previous class we have learnt about points, line segments, and 2-D shapes (flat shapes). Before learning about some other shapes, let's gain some knowledge of dimensions in geometry.

Previous Knowledge Check

- Define 2-D and 3-D shape? And give examples of some 2D and 3D shapes.
- Differentiate between 3-D and 2-D shapes?

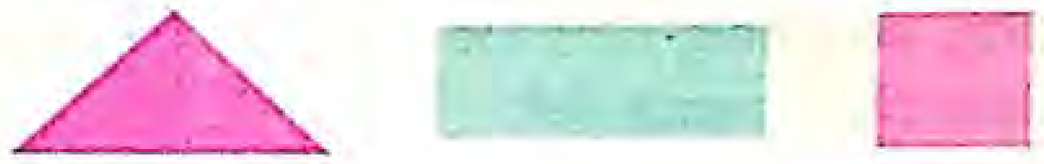
Dimension is the measurement of length in one direction.

A Point has no or zero dimensions. It has only its position. It has no length, width, or height. We can see a combination of points as lines, but on its own, a point has no dimension.

• P

A Line segment has one-dimension and that is its length. It is made up by a combination of points.

A flat shape or 2-D shape has two dimensions i.e. length and width. Squares, rectangles, triangles, etc. are examples of 2-D shapes.



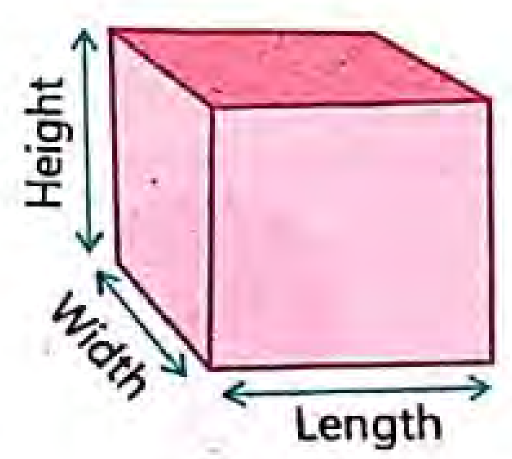
A solid shape or 3-D shape has three dimensions. The three dimensions of a solid are its length, width, and height. Let's learn about 3-D objects.

Consider these objects.

All these shapes are 3-D objects we use in our daily life. We call them 3-D shapes because they all have length, width and height.

Note it down

3-D figures are also called solids.



Note it down

- 3-D objects have faces, edges and vertices.
- 3-D shapes can slide, roll or both slide and roll.
- An edge of a 3-D shape is where two faces meet.
- A vertex of a 3-D shape is the corner where 3 edges (or more) meet.
- A face is space enclosed by the edges.

In this unit, we will learn about the following 3-D shapes:

a) Cube b) Cuboid c) Sphere d) Cylinder e) Cone

a) Cube

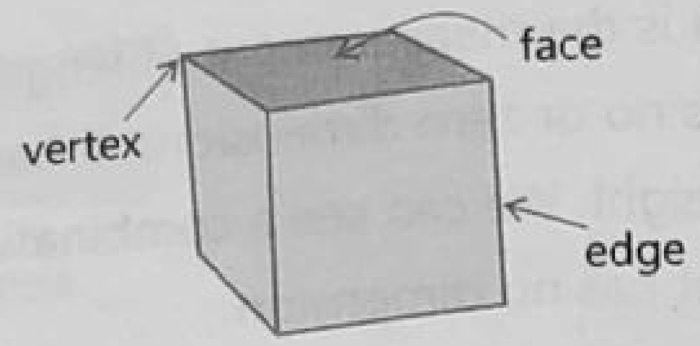
A 3-D shape which has six square shaped faces is called **a cube**. The length, width, and height of the cube is equal. It can slide but cannot roll.

A cube has:

i) 8 vertices (corners)

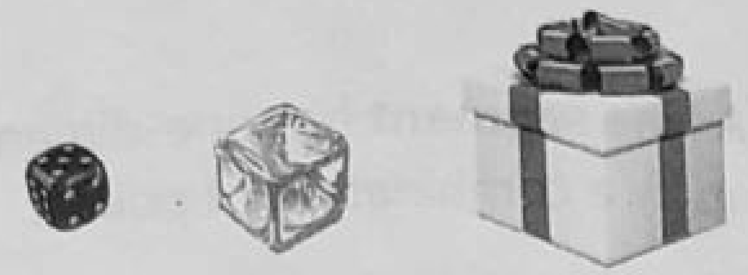
ii) 6 flat faces (all squares)

iii) 12 straight edges of equal length



Cubes Around Us:

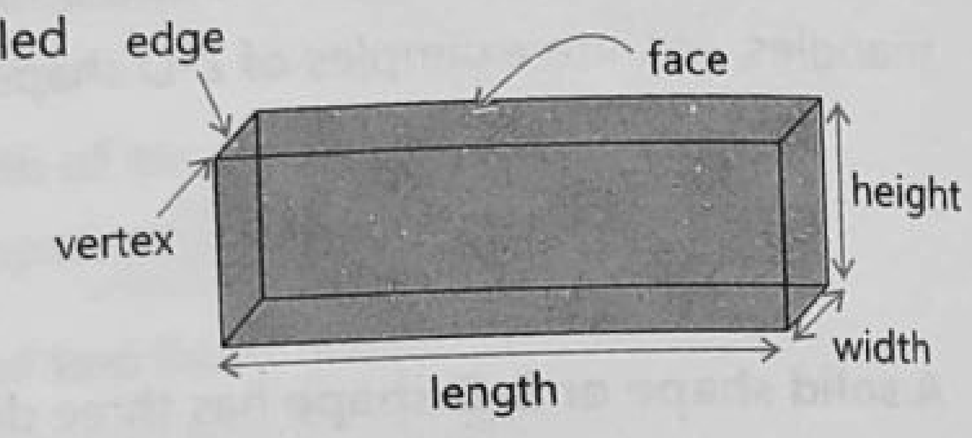
We can see different objects around us which have a cube shape, like dice, ice cubes, gift boxes, etc.



b) Cuboid

A 3-D shape which has six rectangular faces is called a **cuboid**.

We can see that length, width, and height of the cuboid is different. It can slide but cannot roll.

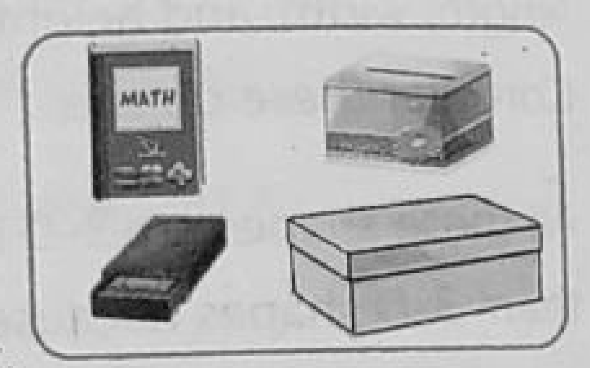


A cuboid has:

i) 8 vertices (corners)

ii) 6 flat faces (all rectangular)

iii) 12 straight edges



Cuboids Around Us:

We can see different objects around us which have a cuboid shape, like your Maths book, tissue box, matchbox, shoebox, etc.

c) Sphere

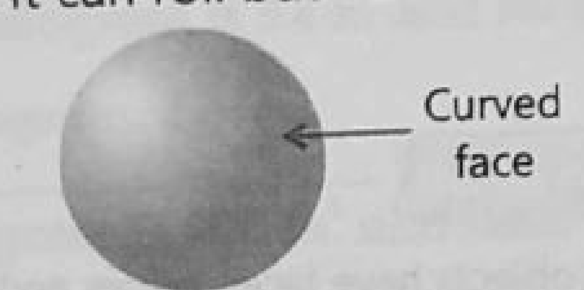
A sphere is a round 3-D shape. It can roll but cannot slide.

A sphere has:

i) 0 vertices (corners)

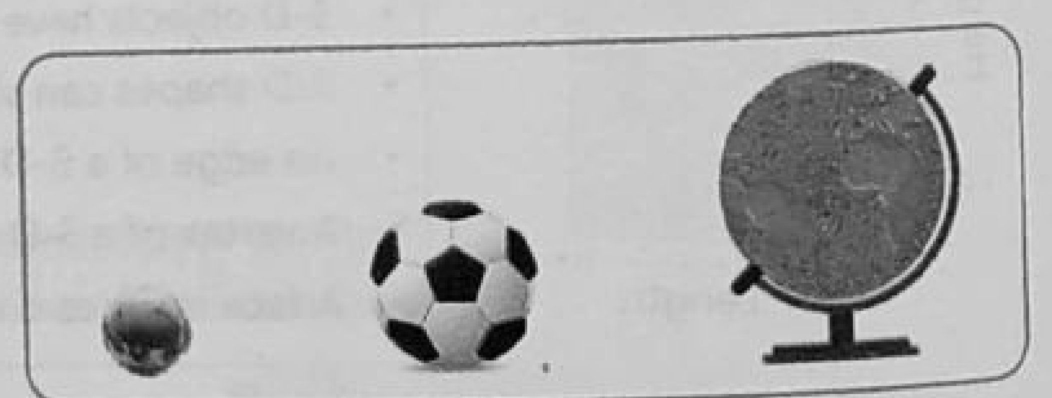
ii) 1 curved face

iii) 0 edges



Spheres Around Us:

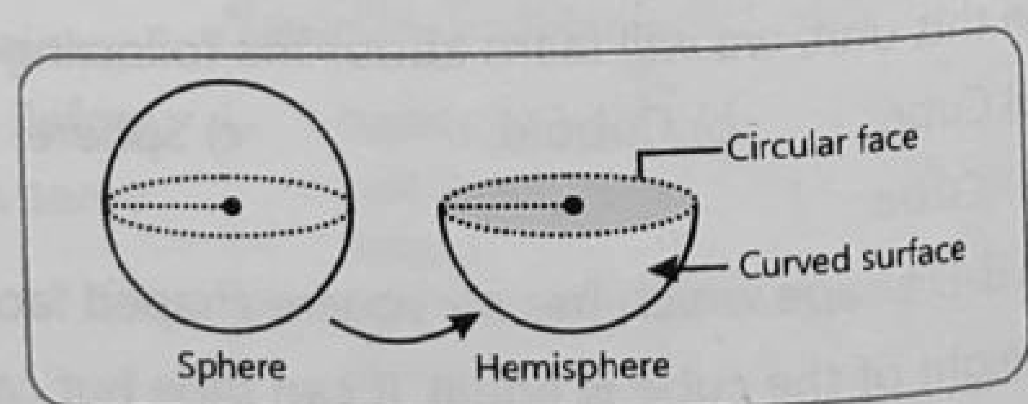
We can see various sphere shaped objects around us like marbles, balls, globes, etc.



d) Hemisphere

When a sphere is divided into two equal parts, each part is called a **hemisphere**.

It's a solid and can roll as well as slide.



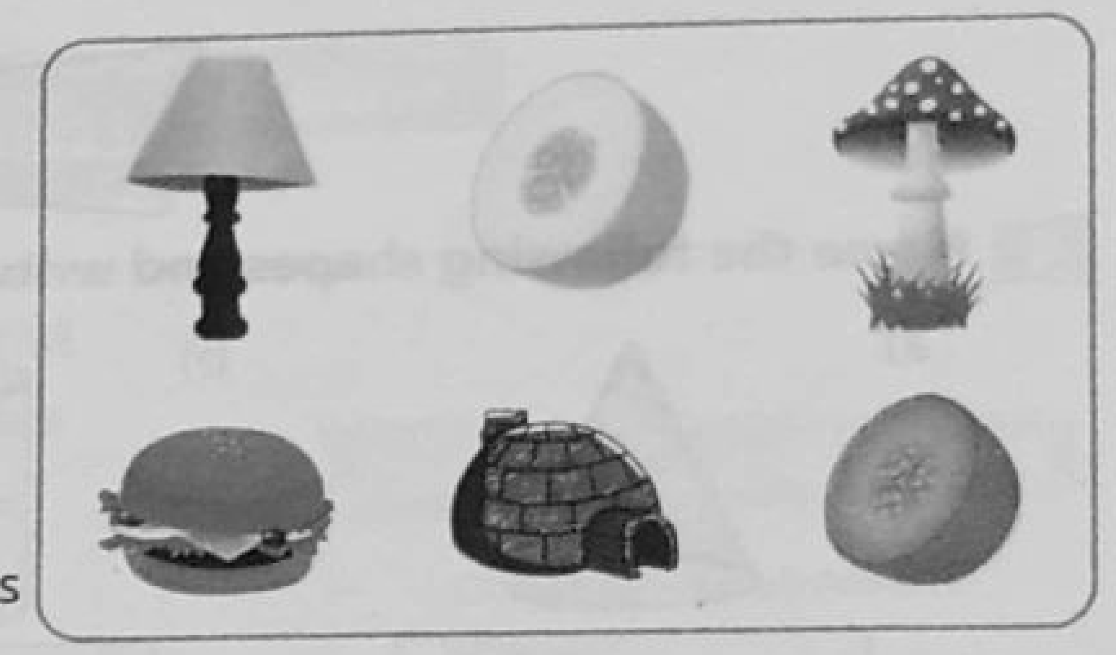
A hemisphere has:

i. 0 vertices (corners)

ii. 1 curved face

iii. 1 circular flat face (for non-hollow hemisphere)

iv. 1 curved edge



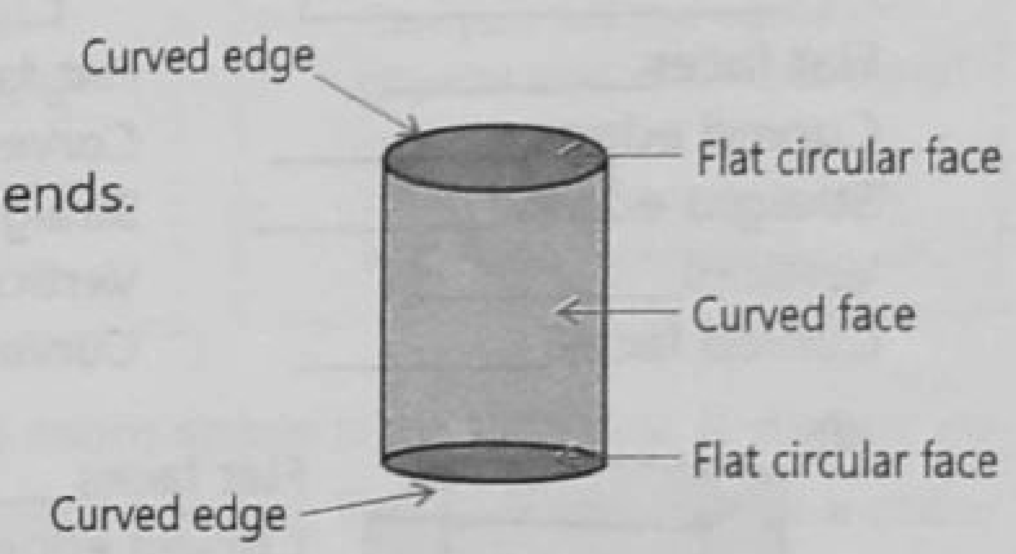
Hemisphere around Us:

We can see various hemisphere shaped objects around us e.g. bowls, half-watermelon, half-coconut, igloo etc.

e) Cylinder

Cylinder is a 3-D shape which has two circular flat ends.

It can roll as well as slide.



A cylinder has:

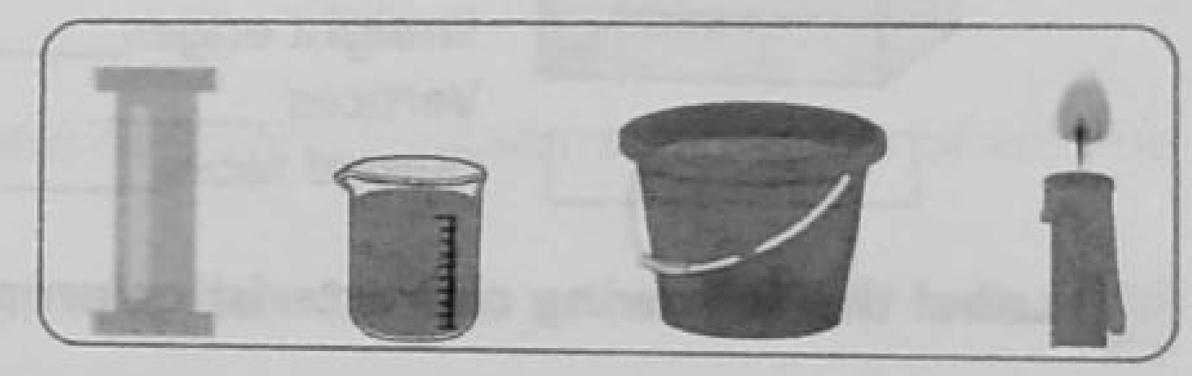
i) 0 vertices (corners)

ii) 3 faces (1 curved face and 2 flat faces)

iii) 2 curved edges

Cylinders Around Us:

We can see various examples of cylinders around us like, pipes, beakers, cans, buckets, candles, etc.



f) Cone

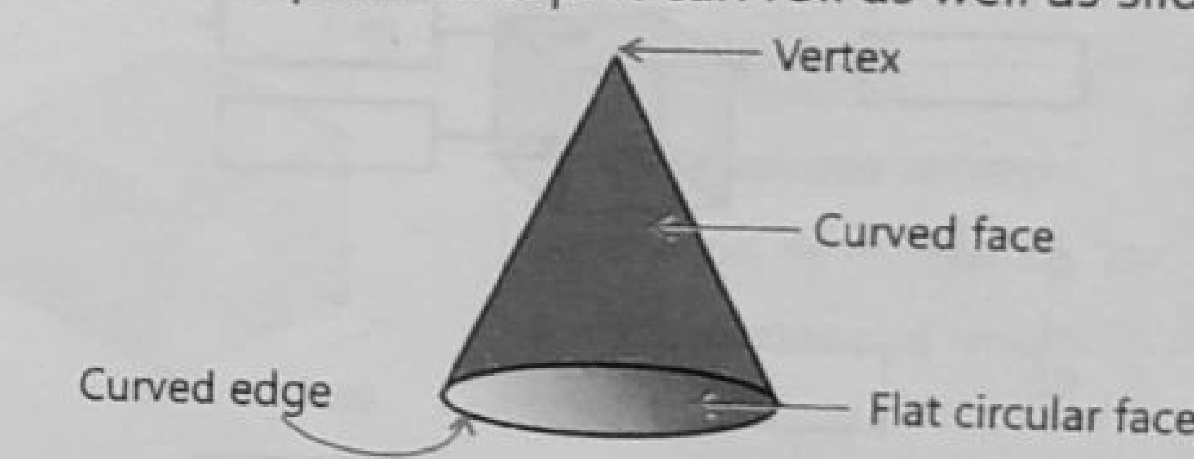
A cone is a 3-D shape that has a flat circular base and a pointed top. It can roll as well as slide.

A cone has:

i) 1 vertex (corner)

ii) 2 faces (1 curved face and 1 flat face)

iii) 1 curved edge



Cones Around Us:

We can see various examples of cones around us like, funnels, ice cream cones, barriers we see on roads, birthday hats, etc.

Quick Check

Tell the number of vertices, faces and edges of the following figures:

a) Cube b) Cuboid c) Sphere

d) Cylinder e) Cone

TEACHING POINT: Show the solid objects to the students and explain about their properties.

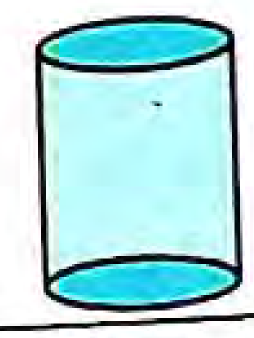
Exercise 8.4

1 **Name the following shapes and write their number of faces, edges and vertices.**

a)

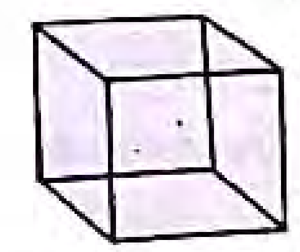
Flat faces________
Curved edges________
Straight edges________
Vertices________
Curved faces________

b)



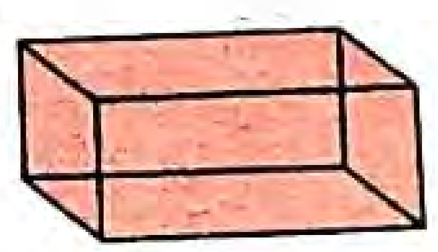
Flat faces________
Curved edges________
Straight edges________
Vertices________
Curved faces________

c)



Flat faces________
Curved edges________
Straight edges________
Vertices________
Curved faces________

d)



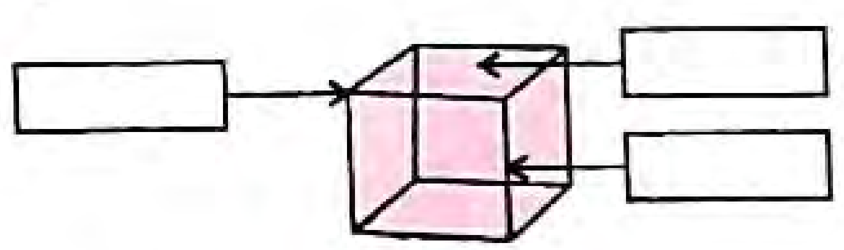
Flat faces______
Curved edges______
Straight edges______
Vertices______
Curved faces______

e)

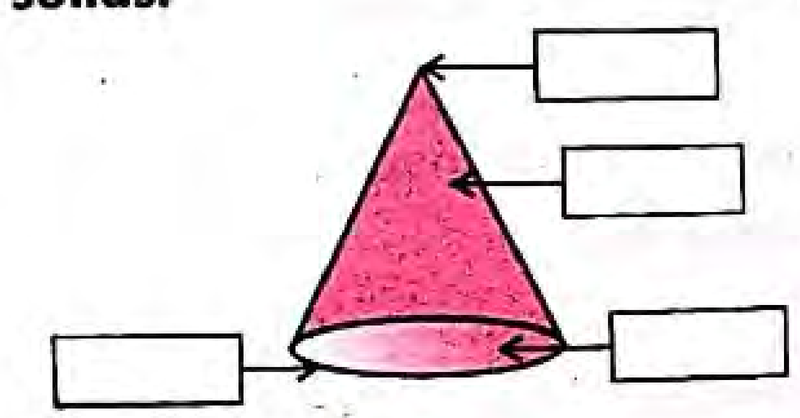
Flat faces______
Curved edges______
Straight edges______
Vertices______
Curved faces______

2 **Label the following characteristics/properties of solids.**

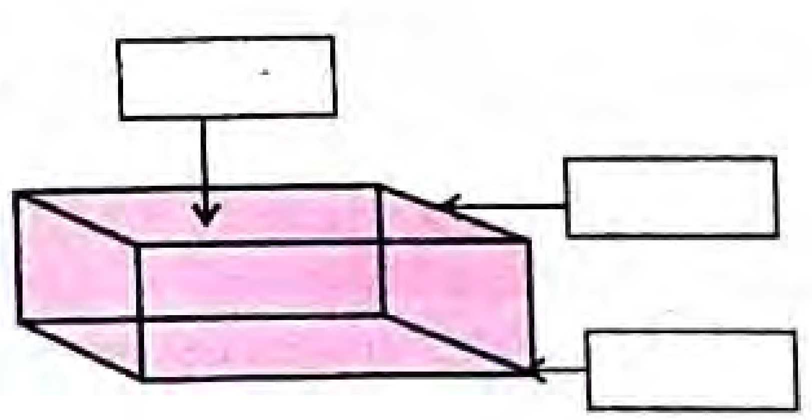
a)



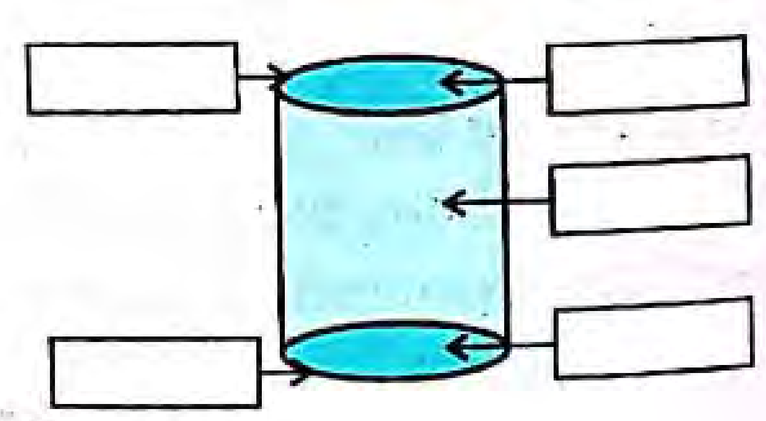
b)



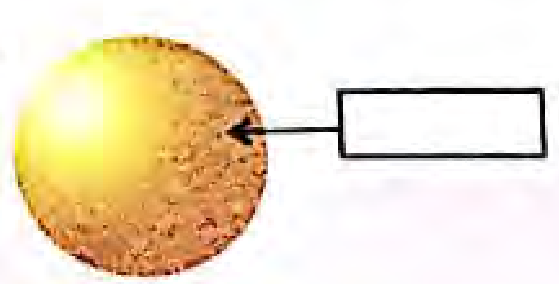
c)



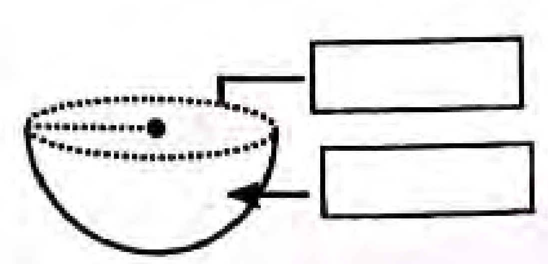
d)



e)



f)



8.5 Surface area and volume

8.5.1 Volume and Its Units

Look at the following 3-D shapes. We can see that these shapes occupy space. The amount of space which a 3-D shape occupies is called its **volume**.

Previous Knowledge Check

- Do you know about 3-D shapes?
- Can you tell the name of objects that are in cube and cuboid shape?
- What is the length, width and height of the cupboard?

Note it down

Volume of solids is always measured in cubic units.

Both shapes occupy space but the ice cube occupies more space than the dice. It means so dice has less volume than an ice cube. Now we will learn how we find the volume of a cube and cuboid.

Standard unit of volume is cubic metre or metre cube (m^3). Similarly, smaller unit of volume is cubic centimetre or centimetre cube (cm^3).

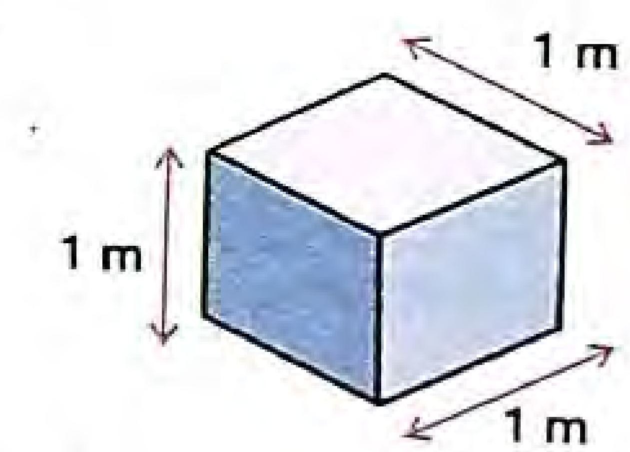
Look at the cube. Each edge of this cube is 1 m long. We can find its volume as:

Volume = length × width × height

$= 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m}$

$= 1 \text{ m}^3$

So, the volume of the cube is 1 cubic metre or 1 m^3.



Quick Check

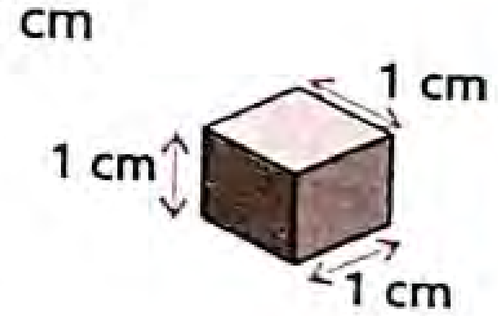
Find the volume of your notebooks by measuring its length, widths and height.

Similarly if each edge of this cube is 1 cm long, then its volume will be:

Volume = length × width × height

$= 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$

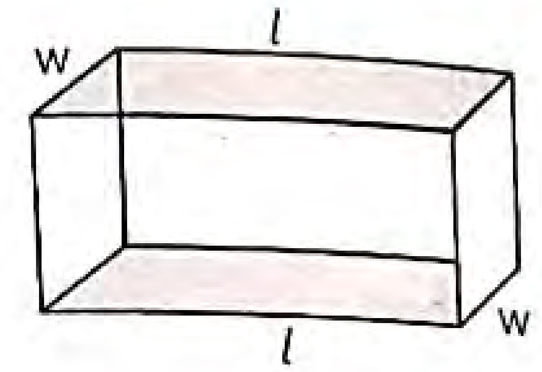
$= 1 \text{ cm}^3$



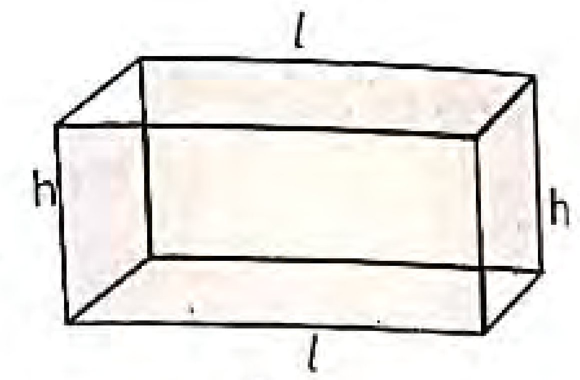
Note it down

To find the volume, length, width and height of the cube must be in same units.

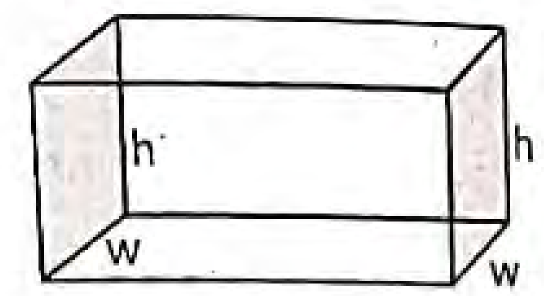
Total area of top and bottom faces = $2 \times (l \times w)$



Total area of front and back faces = $2 \times (l \times h)$



Total area of left and right faces = $2 \times (w \times h)$



If we add all the above areas, we get the total surface area of the cuboid.

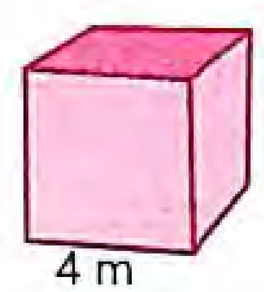
Total surface area of cuboid = $2 \times (l \times w) + 2 \times (l \times h) + 2 \times (w \times h)$

Take out 2 as common.

Total surface area of the cuboid = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

Example 1:

Find the surface area of the given cube.



Solution:

Length of the cube = 4 m

Surface area of the cube = $6\ (\text{length})^2$

$= 6 \times (4\ m)^2$

$= 6 \times (4\ m \times 4\ m)$

$= 6 \times 16\ m^2$

$= 96\ m^2$

Ask them to find out the surface area of their Maths book. Then call the students one by one and ask them how did they find the volume.
Ask students one day before to bring empty boxes of cube or cuboid shape in class then next day students will find the volume?surface area of those boxes.

Example 2:

Find the surface area of a cuboid if its length is 12.5 cm, width is 7 cm and height is 10 cm.

Solution:

Length of the cuboid = 12.5 cm

Height of the cuboid = 10 cm

Width of the cuboid = 7 cm

Total surface area of the cuboid = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

$= 2 \times [(12.5\ cm \times 7\ cm) + (12.5\ cm \times 10\ cm) + (7\ cm \times 10\ cm)]$

$= 2 \times [87.5\ cm^2 + 125\ cm^2 + 70\ cm^2]$

$= 2 \times 282.5\ cm^2$

$= 565\ cm^2$

Exercise 8.5

1 **Calculate the volume and surface area of the following cubes whose length of the edge are given below.**

a) 5 cm b) 8 cm c) 4.3 cm d) 6.4 m e) 2 m f) 3 m
g) 12 cm h) 90 cm i) 45.6 m j) 100 m k) 10 cm l) 55 m

2 **Calculate the volume and surface area of each of the following cuboids.**

a) l = 21 cm, w = 11 cm, h = 6 cm b) l = 43 cm, w = 31 cm, h = 9 cm
c) l = 34.5 m, w = 15.1 m, h = 800 m d) l = 19 cm, w = 20 cm, h = 21 cm

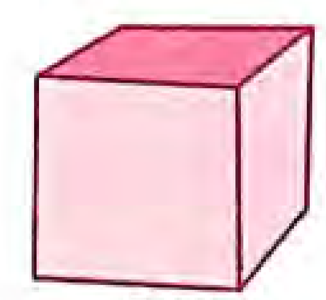
Ask the students to make a cube with card paper. Explain the surface area of the cube by cutting the edges of the cube.
Teachers can share the following online game links to practice volume and surface area.
https://www.splashlearn.com/s/math-games/find-volume-using-the-formula
https://www.splashlearn.com/s/math-games/solve-the-word-problems-related-to-volume

3 **Look at the following measurements of a cuboid and fill the missing values.**

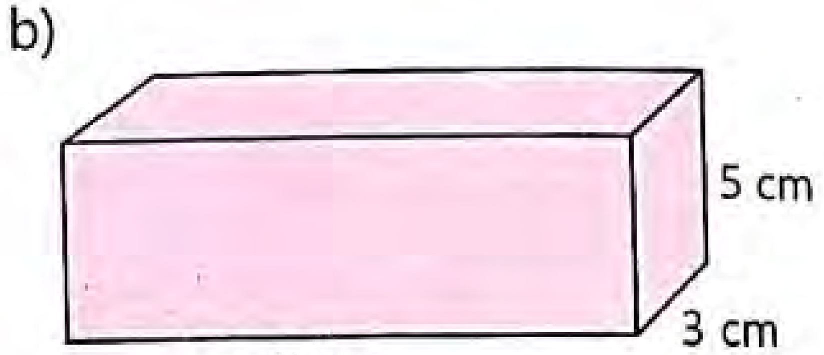
	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)	Surface area (cm^2)
a)	23		4		452
b)	21	14	7		
c)		5	9	321	
d)	45	3		453	

4 **Find the surface area and volume of the given cubes and cuboids.**

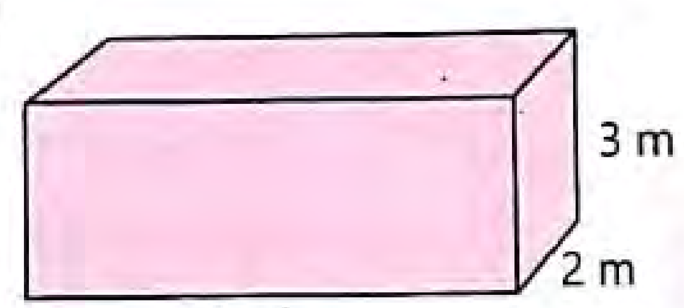
a)



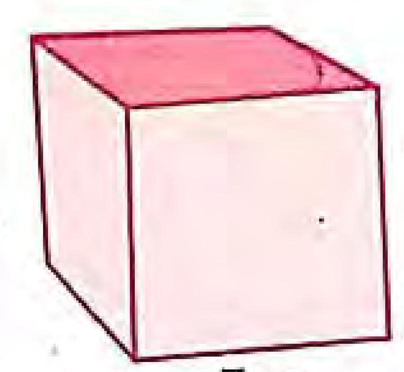
b)



c)



d)



8.6 Real-Life Problems

Example 1:

A swimming pool is 8 m long, 6 m wide and 1.5 m deep. Find the capacity of the swimming pool.

Solution:

Here the depth of the pool will be taken as height.

Length of pool = 8 m

Width of pool = 6 m

Height of pool = 1.5 m

Capacity of pool = length × width × height

$= 8 \times 6 \times 1.5 = 72\ m^3$

So, the capacity of the swimming pool is 72 cubic metres.

Example 2:

A classroom has dimensions 9 m × 5 m × 8 m. Find the total surface area of the classroom.

Solution:

Length of classroom = l = 9 m

Width of classroom = w = 5 m

Height of classroom = h = 8 m

Total surface area of classroom = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

$= 2 \times [(9 \times 5) + (9 \times 8) + (5 \times 8)]$

$= 2 \times [45 + 72 + 40]$

$= 2 \times 157\ m^2 = 314\ m^2$

So, the surface area of the classroom is 314 cubic metres.

Example 3:

A wooden cube shaped box has a length of 6.5 metres. Find the cost of painting the box at the rate of Rs 45 per m^2.

Solution:

To find the cost of painting the box, first we need to find the total surface area which is to be painted.

Surface area of cube $= 6 \times (\text{length})^2$

$= 6 \times (6.5)^2$

$= 6 \times 42.25\ m^2 = 253.5\ m^2$

Cost of painting 1 metre square area = Rs 45

Cost of painting 253.5 metre square area = Rs 45 × 253.5 = Rs 11407.5

So, the cost of painting the box is Rs 11407.5.

TEACHING POINT: Use variety of daily life examples to explain how to solve surface area and volume related questions. Then ask them to create some questions and solve them.

Example 4:

A chocolate box has a length of 0.2 m and height of 8 cm. Find the width of the box in cm if its surface area is 992 cm^2.

Solution:

First we need to make the units same.

Length = 0.2 m = 0.2 × 100 cm = 20 cm.

Height= 8 cm

Surface area = 992 cm^2

Width =?

As the surface area is given, we put the values of length, height and surface area in the formula.

Total surface area of the cuboid box $= 2 \times [(l \times w) + (l \times h) + (w \times h)]$

$992 = 2 \times [(20 \times w) + (20 \times 8) + (w \times 8)]$

$992 = 2 \times [20w + 160 + 8w]$

$992 = 40w + 320 + 16w$

$992 = 56w + 320$

$992 - 320 = 56w$

$672 = 56w$

$\frac{672}{56} = w$

$12 = w$

So, the width of the chocolate box is 12 cm.

Exercise 8.6

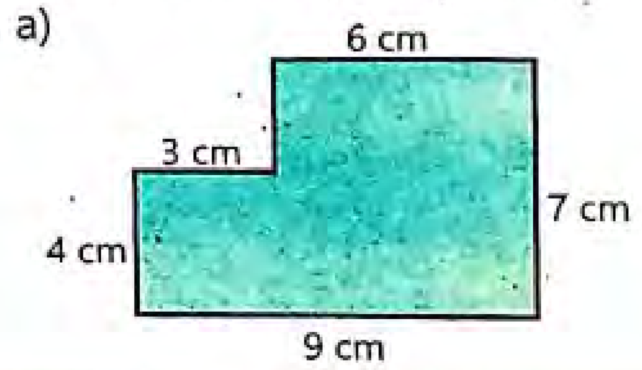
1. **A cuboid glass box has dimensions 3 m × 2 m × 1 m. Find the volume of water that it can hold.**
2. **A box of cereal is 22 cm high, 12 cm long and 6 cm wide. What is the capacity of the cereal box?**
3. **A cube shaped container has a length of 72 cm. If it is filled with blocks up to a height of 42 cm, find the remaining capacity of the container.**
4. **A Masjid is 22 m long, 12 m wide and 18 m high. Find the cost of painting its four walls at the rate of Rs 450 per square metre.**
5. **Find the cost of painting a cuboid container of length 4 m, width 3.5 m and height of 6 m at the rate of Rs 43.78 per square metre.**

6. **A cube shaped gift box has a length of 3 cm. Find whether the surface area of this box is greater or smaller than that of a cuboid box of dimension 5 cm × 6 cm × 3 cm.**
7. **The total surface area of a cuboid shoebox is 1296 cm^2. If its length and width is 18 cm and 9 cm respectively, find the height of the shoebox.**
8. **A wooden box has a length of 3 m, width of 2 m and height of 4 m. Find:**
   a) The volume of the box.
   b) The cost of polishing its surface if the rate of polishing is Rs 220 per square metre.
9. **A box is 0.56 m high and 21 cm wide. Its volume is 51744 cubic centimetres. Find:**
   a) The length of the box in cm. b) Total surface area of the box in cm^2.
   c) Cost of painting its complete outer surface at the rate of Rs 145 per square centimetres. (Hint: First make the units of its dimensions the same)

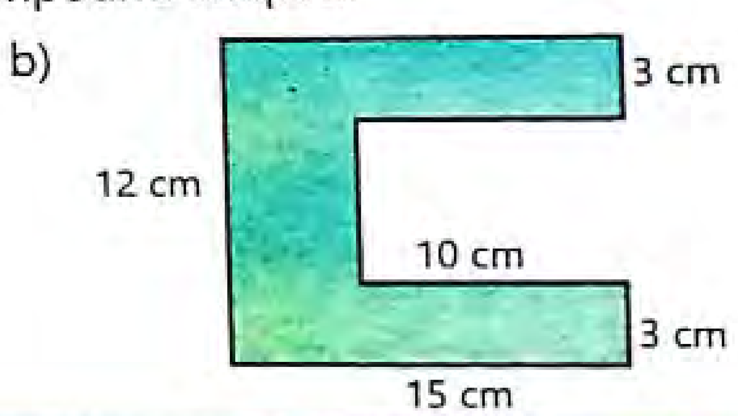
Think Higher

Find the area and perimeter of the following compound shapes.

a)



b)



Summary

- Perimeter of a shape is measured by adding the length of all its sides. Its always measured in single units i.e. centimetres (cm), metres (m), etc.
- Area of a figure is the surface covered by that figure. It is always measured in squared units i.e. square centimetres cm^2, square metres m^2, etc.
- Altitude of a figure is also called the height of the geometric figure.
- In a triangle if we choose one side as altitude the opposite side of the altitude is called the base.

Vocabulary

- Perimeter
- Area
- Surface area
- Volume
- Trapezium
- Parallelogram
- Triangle
- Altitude
- Base
- Cube
- Cuboid
- Dimensions
- Sphere
- Hemisphere
- Cylinder

- Diagonal is a line segment which is drawn between the opposite vertices (corners) of the rectangle.
- A solid shape or 3-D shape has three dimensions.
- The three dimensions of a solid are its length, width, and height.
- The amount of space which a 3-D shape occupies is called its volume.
- We can find its volume as: Volume = length × width × height
- The volume of the cube is 1 cubic metre or 1 m^3.
- Volume of cuboid is the product of its length, width, and height.

Review Exercise

1 Choose the correct option.

a) The ________of any shape is the space occupied by it.

i. perimeter ii. breadth iii. width iv. area

b) The sum of all the sides of a shape is called its______.

i. perimeter ii. space iii. width iv. area

c) The formula to find the area of a square is ________.

i. 4L ii. L + B iii. 2 (L + B) iv. L × L

d) _________ is the shortest distance between the top and bottom of a geometrical figure.

i. base ii. altitude iii. top iv. vertex

e) The formula to find the area of a trapezium is ____________.

i. $a + (\frac{b_1 + b_2}{2})$ ii. $a \times (\frac{b_1 - b_2}{2})$ iii. $a - (\frac{b_1 + b_2}{2})$ iv. $a \times (\frac{b_1 + b_2}{2})$

f) The formula for finding the area of a triangle is ______.

i. $\frac{1}{2}(b + h)$ ii. $\frac{1}{2} + (b \times h)$ iii. $\frac{1}{2}(b \times h)$ iv. $\frac{1}{2}(b - h)$

g) Surface area of cube =____.

i. 6 lenght^2 ii. lenght^3 iii. 4 lenght^5 iv. 8 lenght^3

h) Surface area of cuboid is _____.

i. 2[(l× w)×(l+ h)× (b+ h) ii. 4[(l× b)+(l× h)+ (b× h)

iii. 5[(l× b)+(l× h)+ (b× h) iv. 2[(l× W)+(l× h)+ (W× h)

i) Volume of a cuboid= _________

i. L+W+h ii. l×W×h iii. l−W−h iv. l÷W÷h

2 Define the following terms:

a) Perimeter of square and rectangle b) Area of square and perimeter d. Altitude

3 Find the perimeter and area of the squared shaped room whose length is 34 cm.

4 Find the perimeter and area of the rectangular pond whose length is 98 m and width is 88 m.

5 Find the length of the playground if its perimeter is 1250 cm and cost to construct a boundary around the play ground at the rate of Rs 90 per metre.

6 Find the length of squared shaped bed sheet if its perimeter is 14 cm.

7 Find the length of rectangular hall if its area is 980 m^2 if width is 20 m. Find the cost of cementing the room at the rate of Rs. 420 per cm^2.

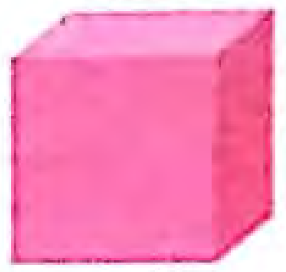
8 A pond is 98 cm long and 76 cm wide. A road 3.2cm wide is made all around it outside the pool. Find the area of the road.

9 A room is 7.8 m long and 6.5 m cm wide. Aliza wants to carpet 2 cm wide all round inside the room. Find the area of the carpeting part of the room.

10 Find the altitude of a trapezium if the length of two bases are 34.56 cm and 32.12 cm respectively. The area of the trapezium is 482cm^2.

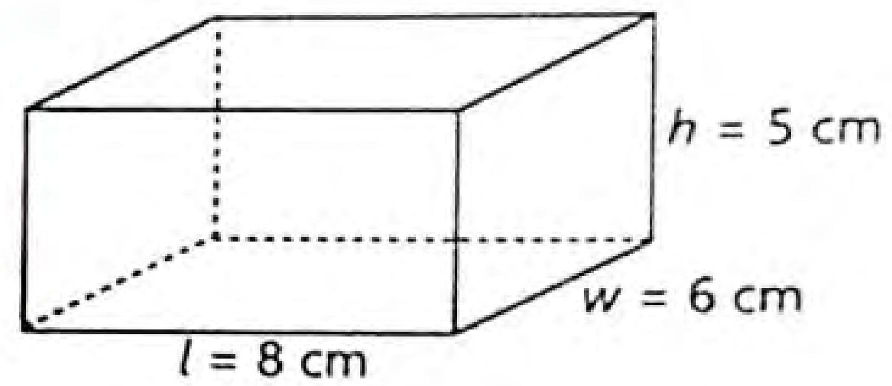
11 Find the area of a trapezium if the length of two bases are 44cm and 66cm respectively. The perpendicular distance between two bases are 52cm.

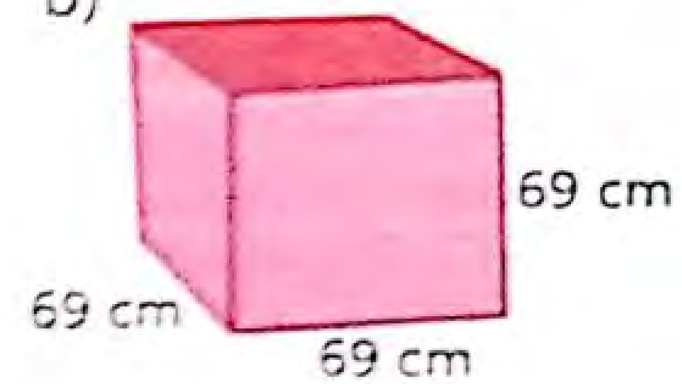
Write the names and write the properties of each of the given shape.

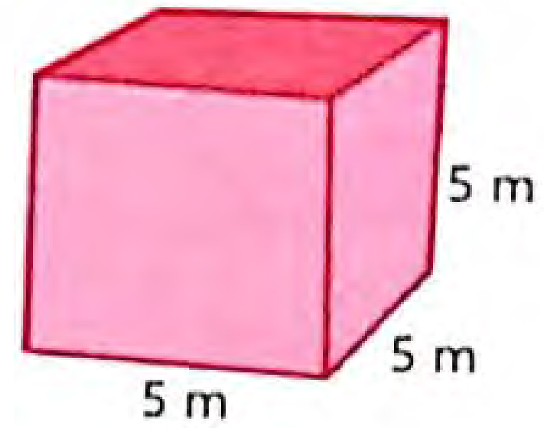
Shape	Circular edges	Straight edges	Vertices	Circular faces	Flat faces
Cone 					
Cuboid 					
Cube 					
Cylinder 					
Sphere 					
Semisphere 					

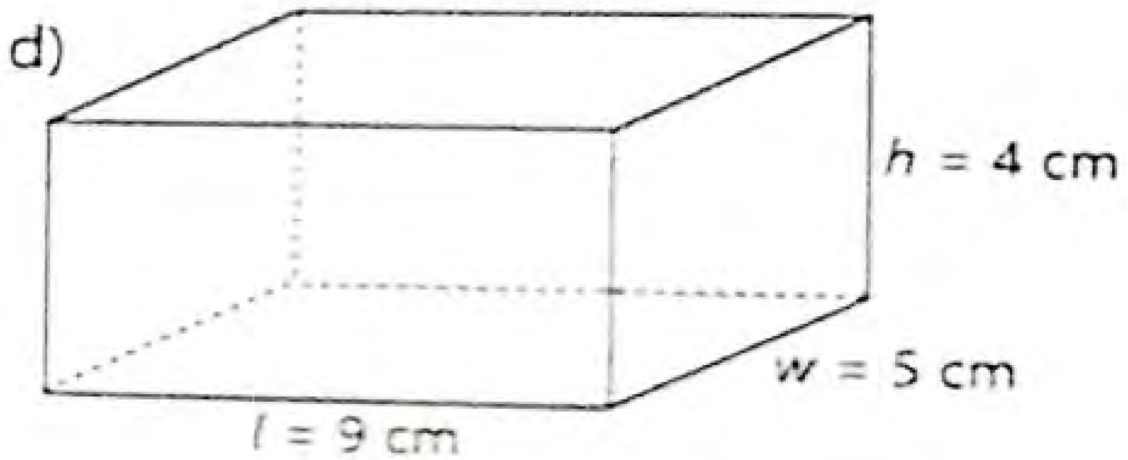
13 **Find the altitude of the triangular field. If its area is 99cm² and base is 9 cm.**

14 **Find the volume and surface area of the following cube.**

a) 

b) 

c) 

d) 

15 **Find the total surface area of the block whose length is 9cm, width 2.3cm and height is 3.3cm.**

16 **A classroom is 4m long, 5m wide and 3m high. Find the cost of whitewash the all four walls and ceiling the room at the rate of Rs 43 per m²**

17 **A water tank is cuboid in shape. The length, width and height of the tank are 3.5m, 4.2m and 5.3m respectively. If we completely fill it with water, find the volume of water in the tank in litre?**

18 **Find the capacity of a can whose length, width and height are 54cm, 35cm and 56cm respectively.**

19 **The altitude of the parallelogram is 2.4 cm and its area is 84.4 cm². Find the base of parallelogram.**

20 **Find the area of the triangle if the length of its base is 67.2 cm and the length of its altitude is 56.2 cm.**

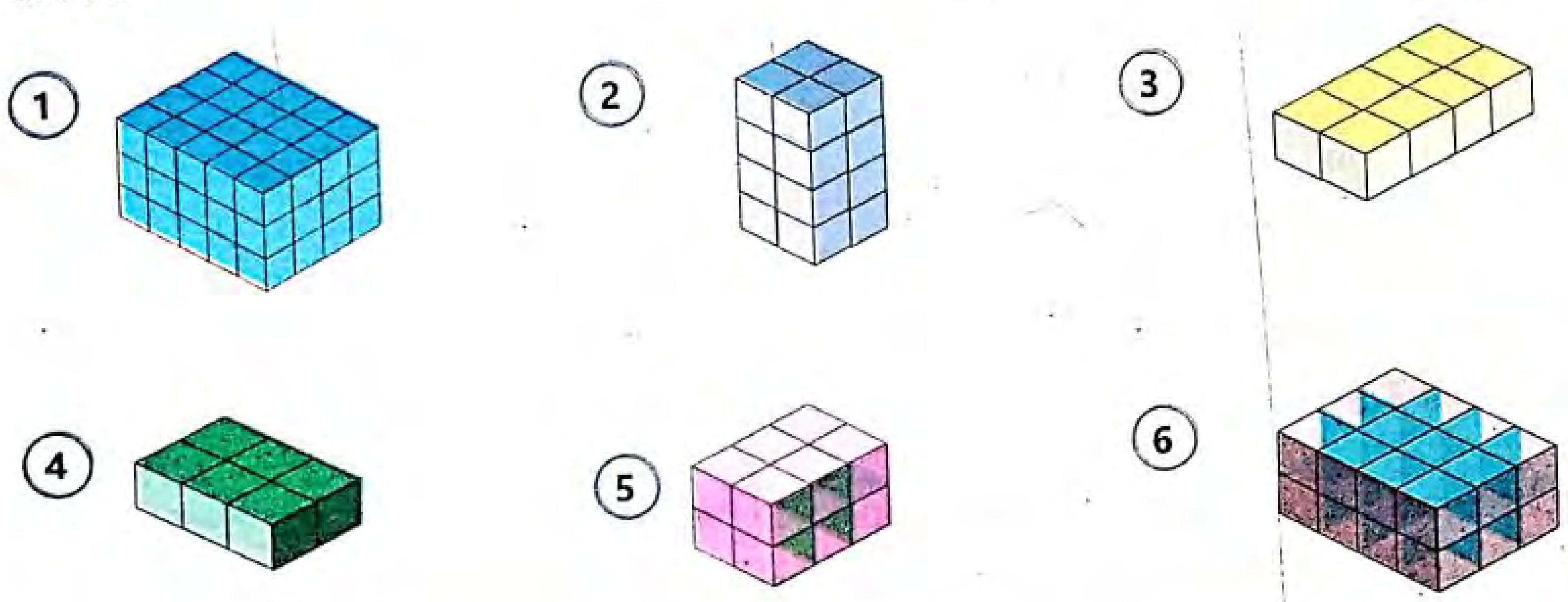
Math Project

Material Required:

- Unit cubes
- Recording Table

Procedure:

- Get in groups.
- Use the unit cubes and join them to build a cuboid.
- Count the number of squares on the outside of the cuboid to find the surface area.
- Record the lengths, widths heights to calculate the volume/surface area.
- Build two or three more cuboids using different number of unit cubes.
- Count the number of squares on the outside of the cuboid to find the surface area.
- Record the lengths, widths heights to calculate the volume/surface area in a table.
- Share your results by explaining which cuboid has the greatest and least surface areas.



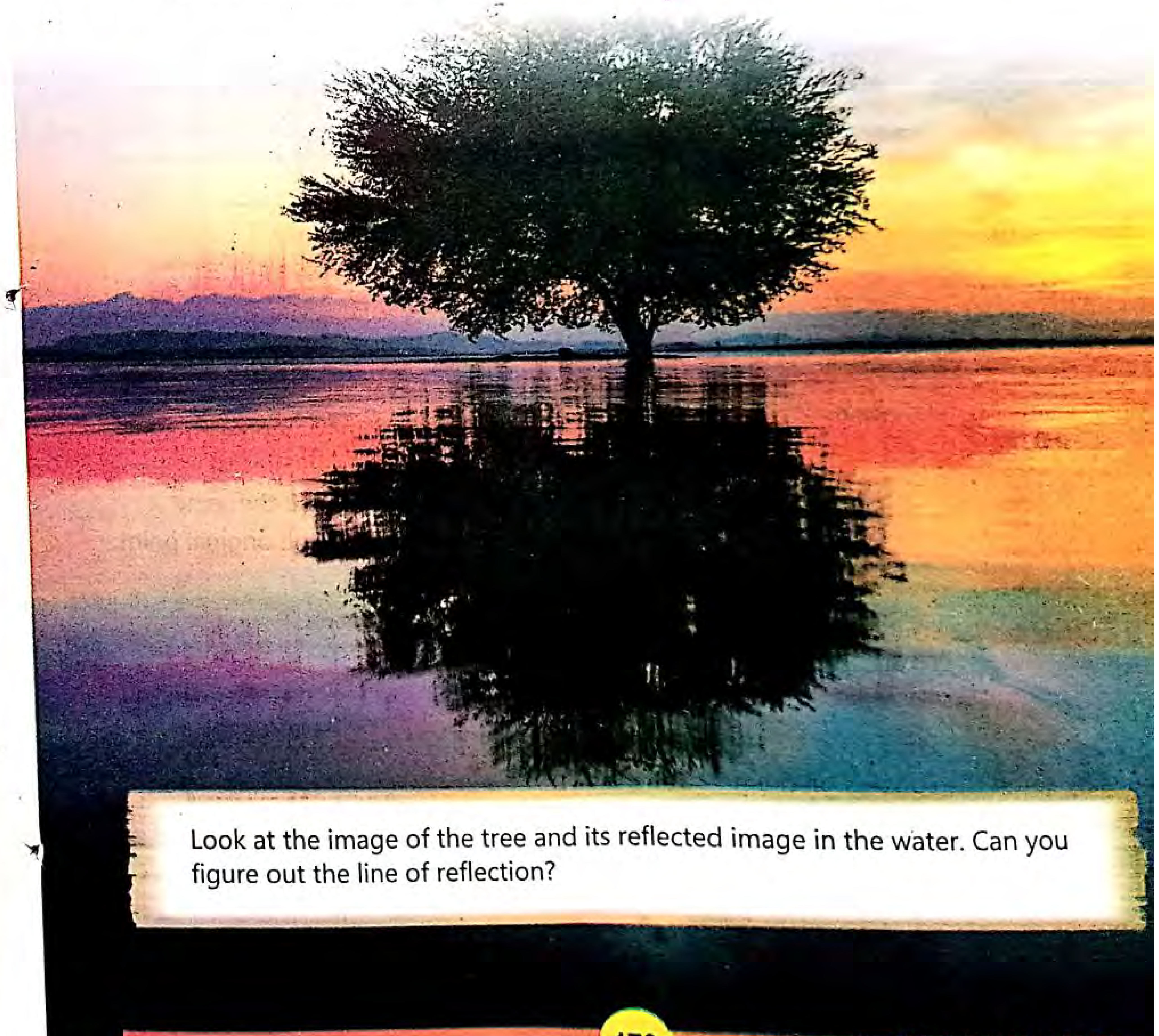
Cuboid Number	Length	Width	Height	Volume	Surface Area
1					
2					
3					
4					
5					
6					

Unit 9 Lines, Angles and Symmetry

Student Learning Outcomes

After completing this unit, students will be able to:

- Reflect an object using grid paper and compass and find the line of reflection by construction.
- Identify and differentiate between parallel lines, perpendicular lines and transversal.
- Identify adjacent angles and find unknown angles related to parallel lines and transversals.
- (corresponding, alternate and vertically opposite angles)
- Recognise rotational symmetry, find the point of rotation and order of rotational symmetry.



Look at the image of the tree and its reflected image in the water. Can you figure out the line of reflection?

Introduction

We have previously learnt about 3D shapes, rotational symmetry. In this unit we will recognise 3D shapes with respect to their characteristics, adjacent angles, reflect an object using grid paper and compass and find line of reflection by construction.

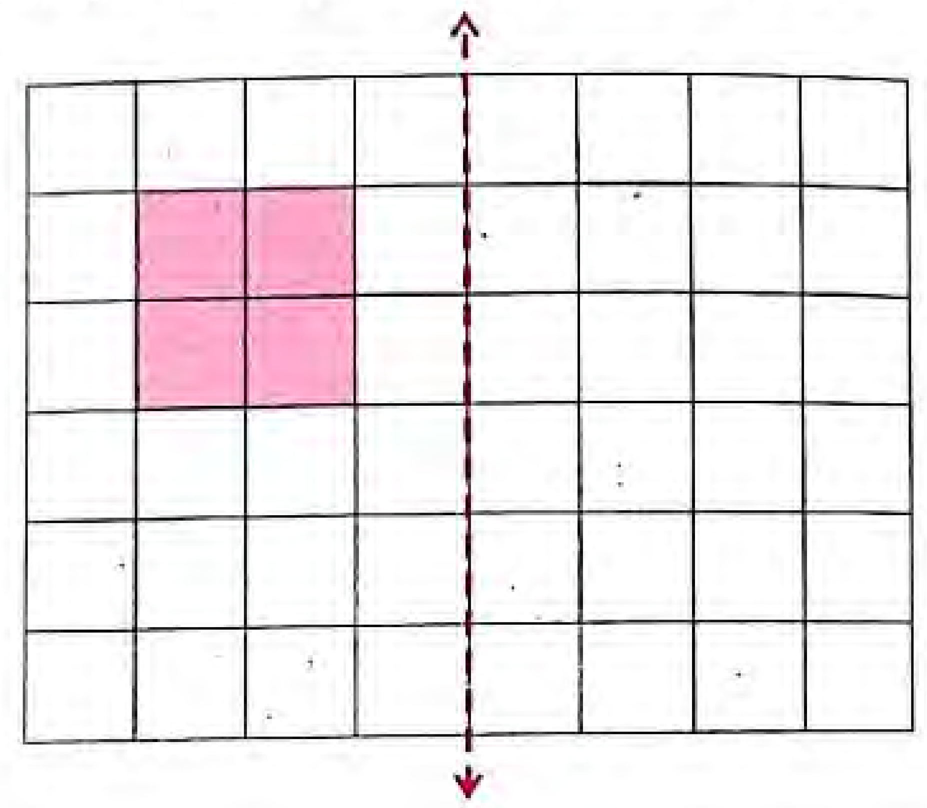
9.1 Reflection

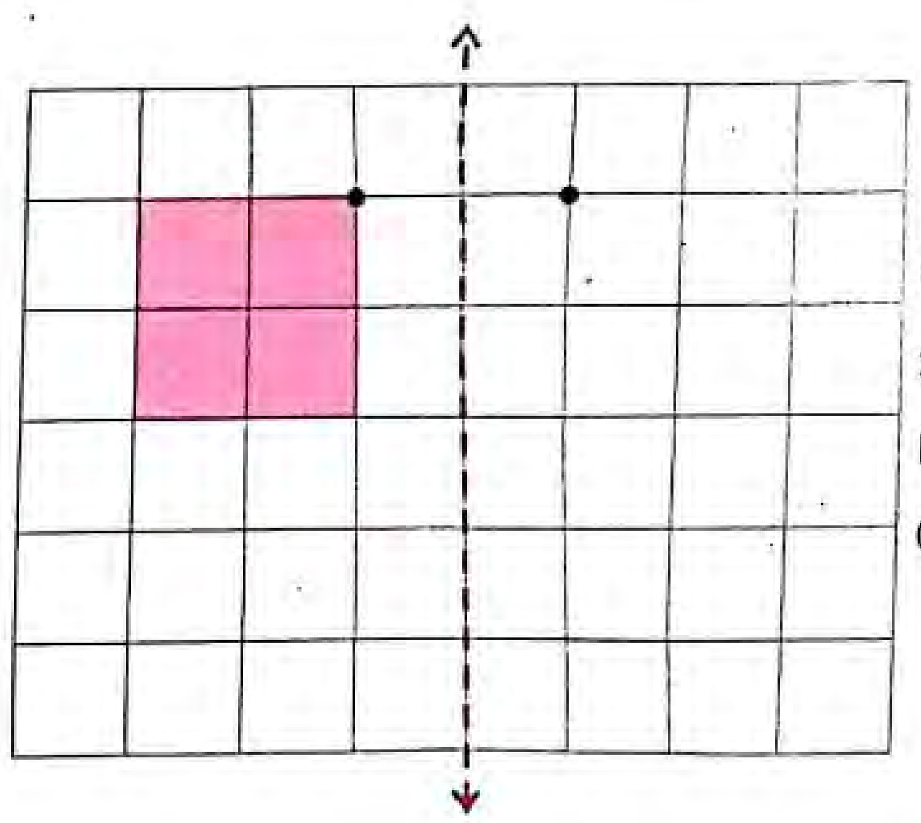
9.1.1 Reflecting an object on Grid Paper

A **reflection** is a transformation that uses a line like a mirror to reflect a figure. The line is called the **line of reflection**. When an object is reflected, it has the same shape and size. The difference is that the image formed is a flipped image appearing as **mirror image**.

We can easily reflect an object on a grid paper. Let's observe this example.

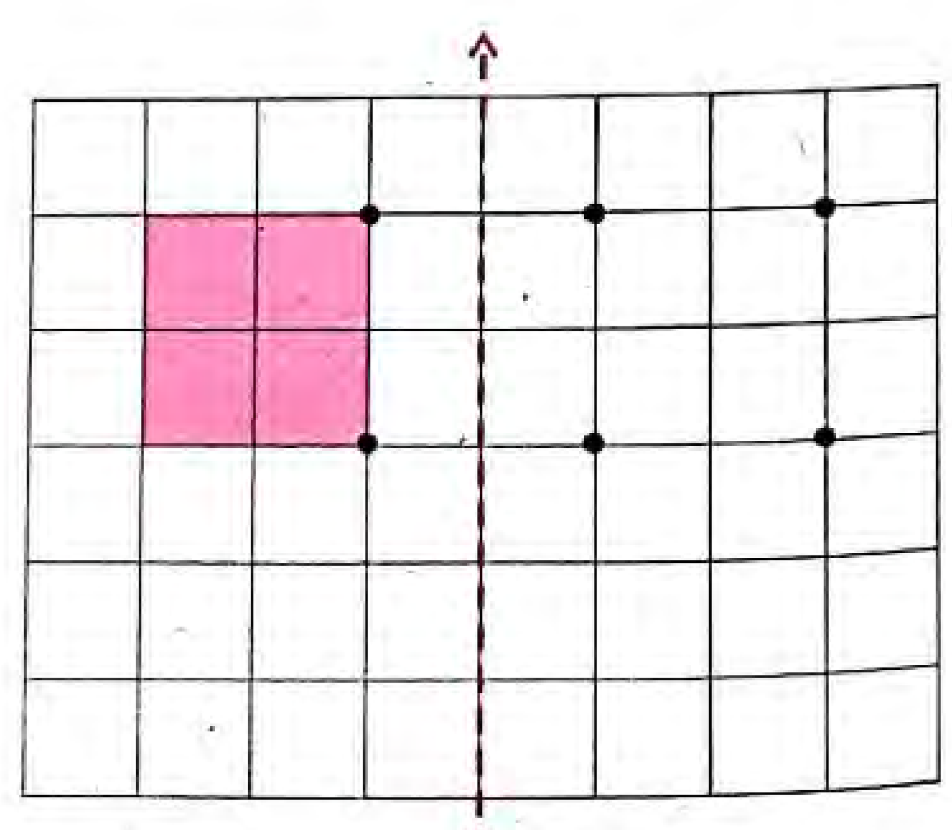
Example: Reflect the square on square grid.

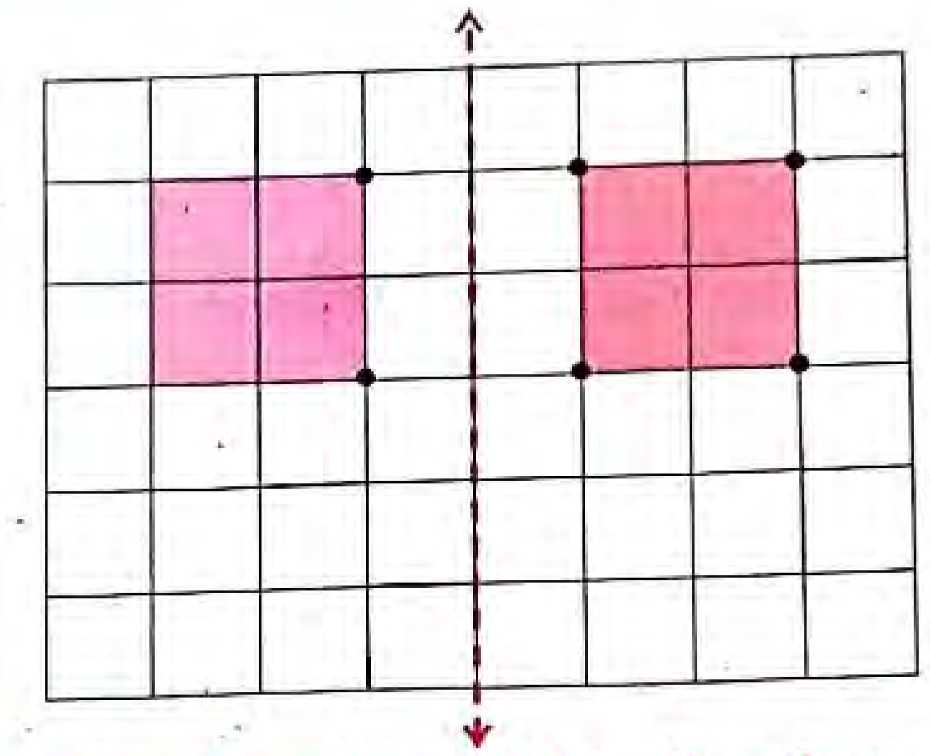




Choose the first point to reflect. It is easier to start with the closest point to the line of reflection. The new point must be the same distance away from the line as the original point.

Similarly count the squares on the grid and mark all the points.





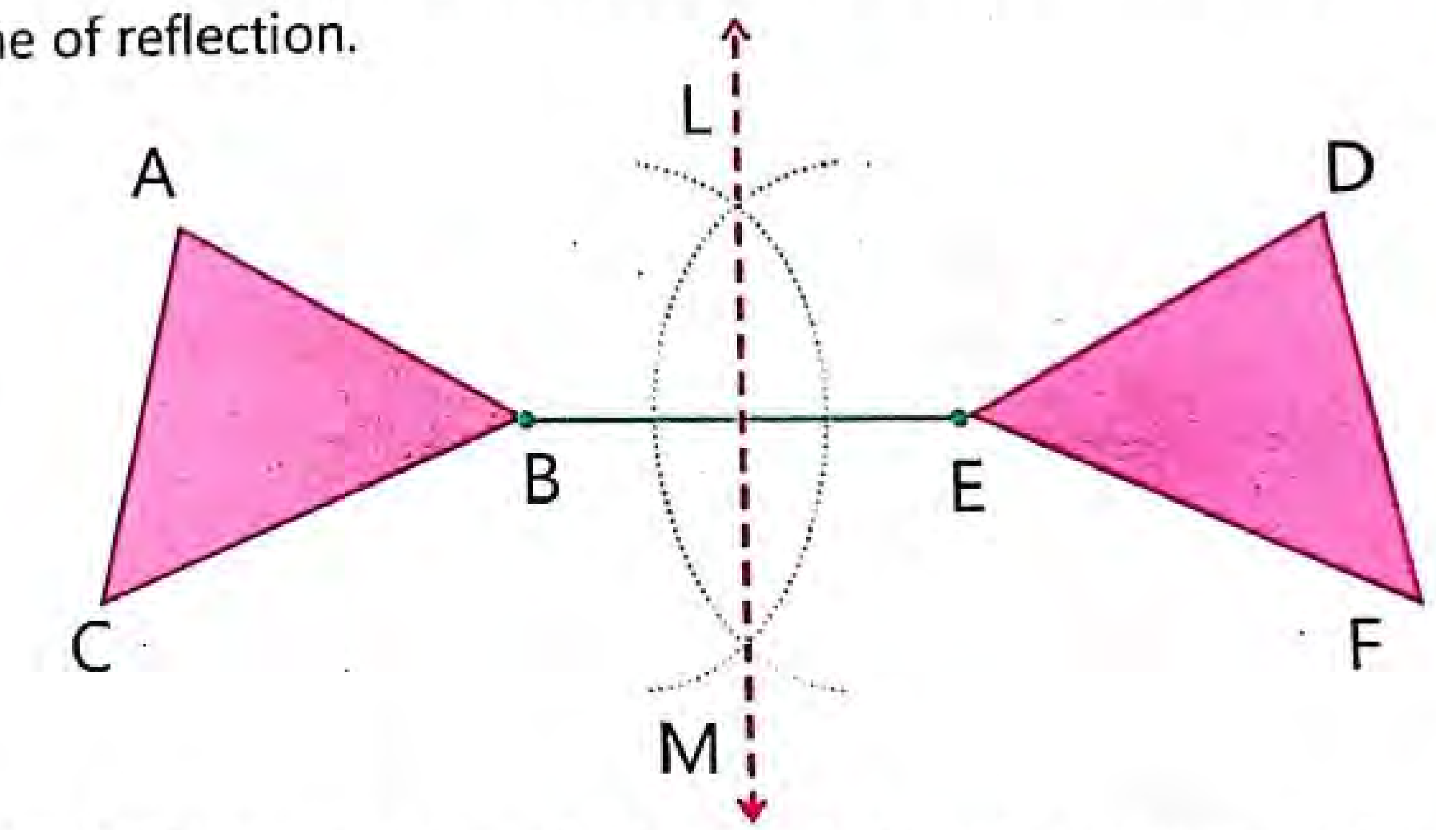
Join the marked points and the reflected object is obtained.

Quick Check

Draw a triangle on a square grid and reflect its image.

9.1.2 Finding Line of Reflection

When an **object** is reflected, the line of reflection is the perpendicular bisector of the corresponding line segments joining each point of the original object and **reflected image**. Let's observe the following object and its reflected image. Follow the steps to construct the line of reflection.



Previous Knowledge Check

What is symmetry? What is meant by line of symmetry? What is reflective symmetry?

Quick Check

Write your name in all capital letters and draw line of symmetry for each letter.

Step I: Join any vertex of triangle ABC to its corresponding vertex of the reflected image DEF. Let's join B to E through a straight segment.

Step II: Place the pointer of the compass at point B.

Step III: Open the compass with more than half of the measure of the line segment BE and draw an arc on the top and bottom of BE.

Step IV: Similarly, using the same opening of the compass, place the pointer of the compass at point E and draw two arc on the top and bottom of BE which intersects the previous arcs at point L and M respectively.

Step V: Join the point L and M by a line using a ruler.

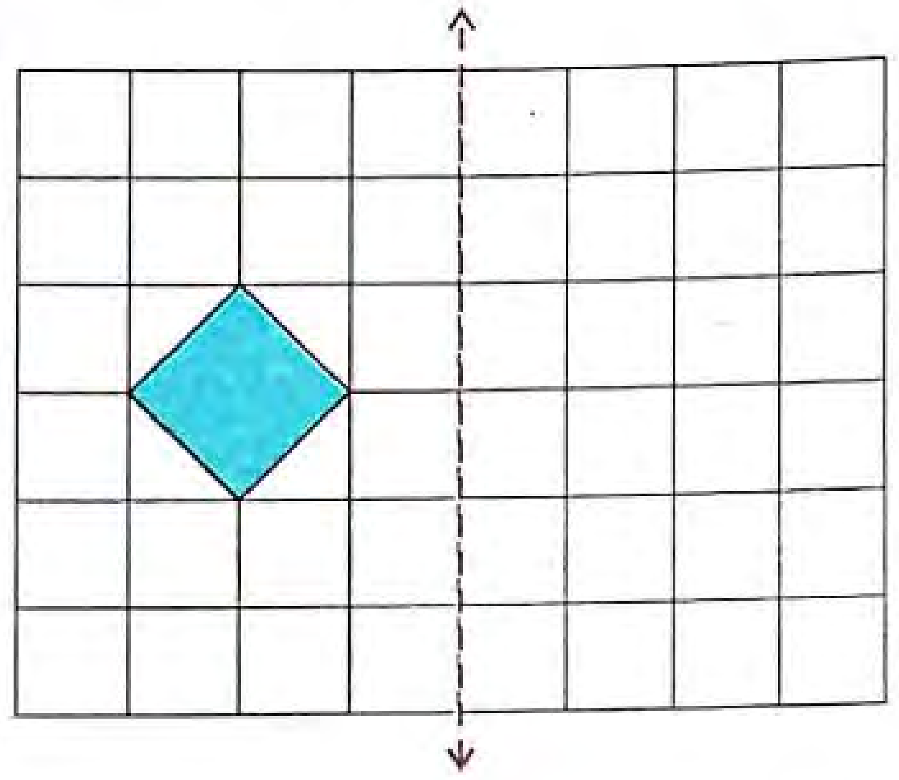
LM is the perpendicular bisector of BE.

The perpendicular bisector LM to the line BE is the required line of reflection.

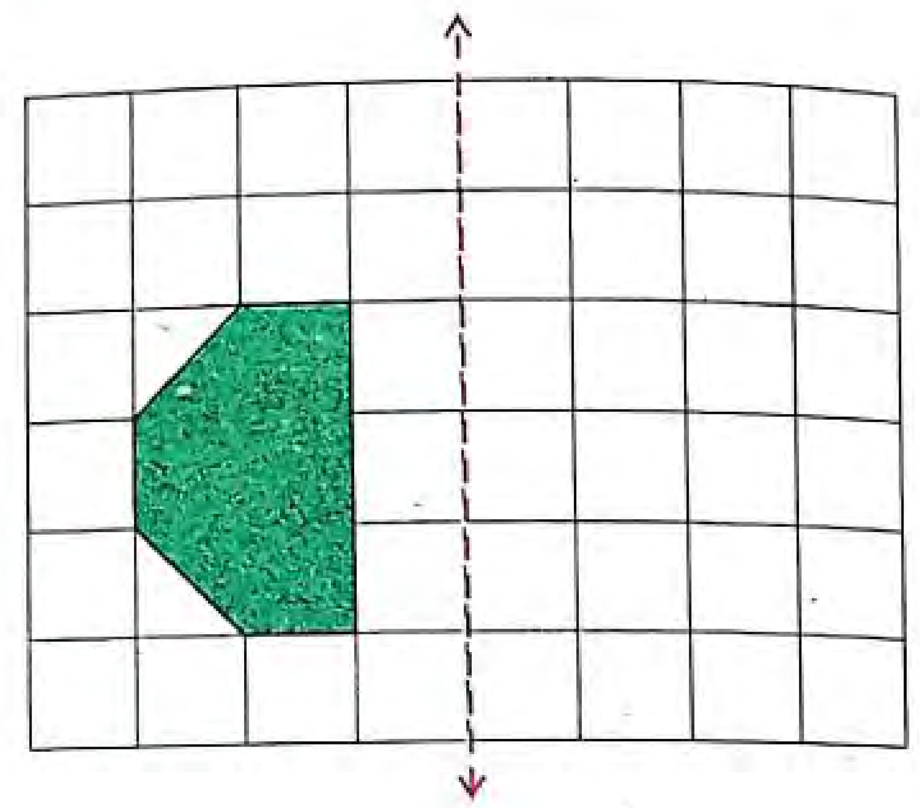
Exercise 9.1

1 Reflect the given images on square grid.

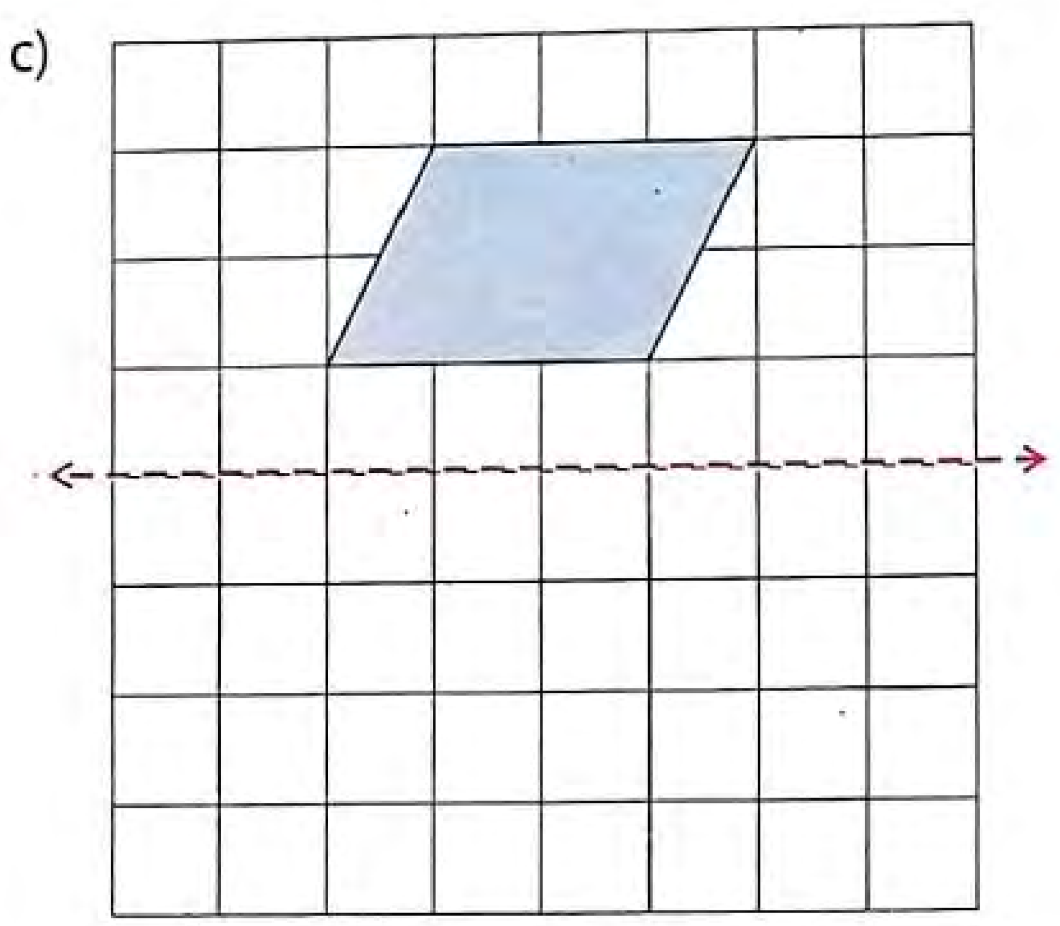
a)



b)

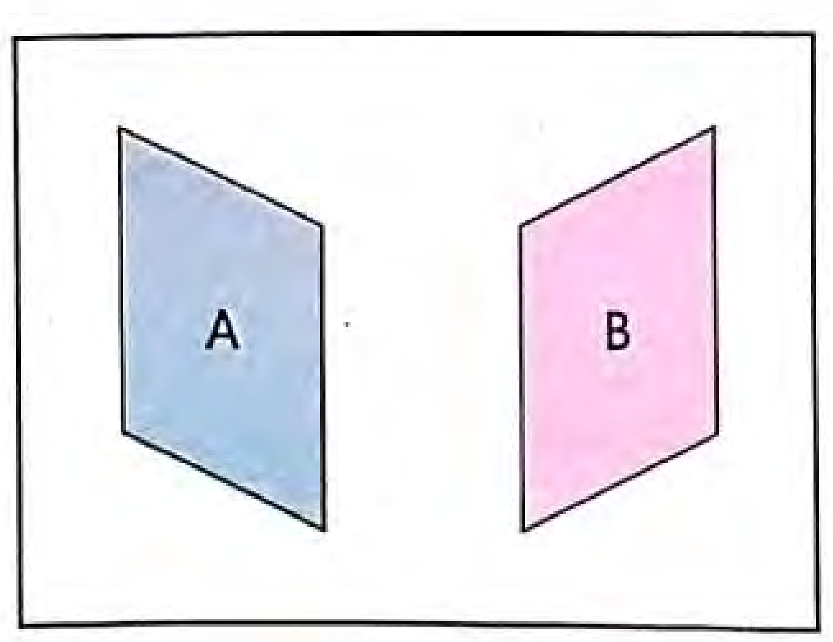


c)

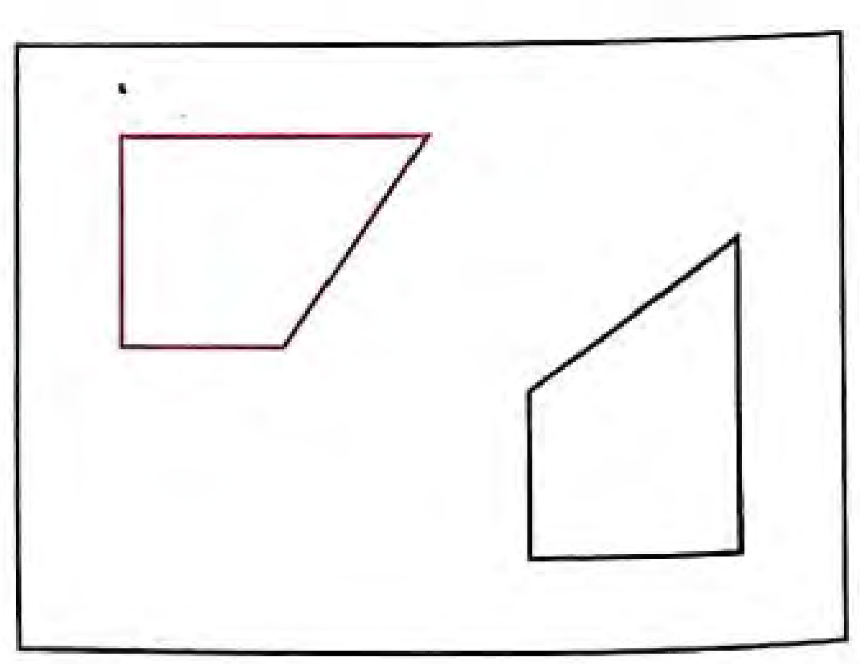


2 Find and draw the line of reflection for these images using compass.

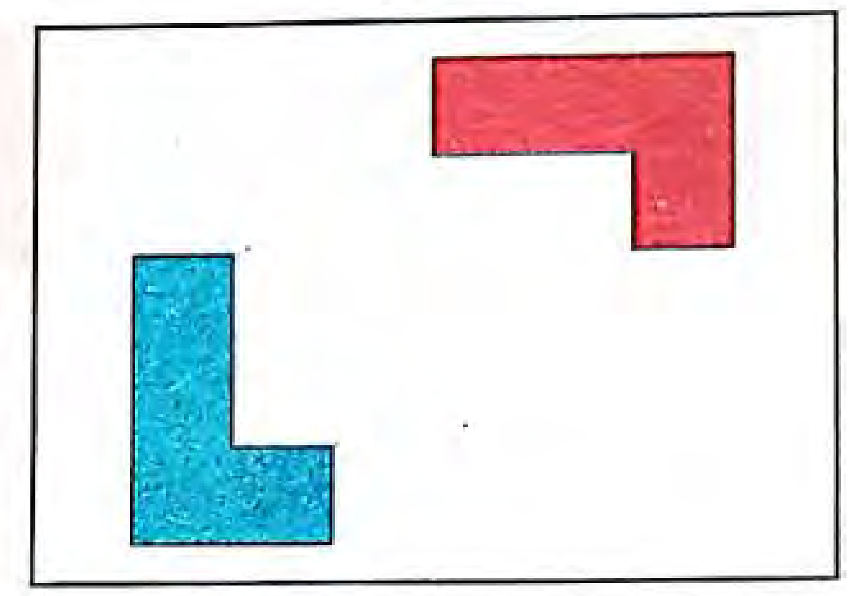
a)



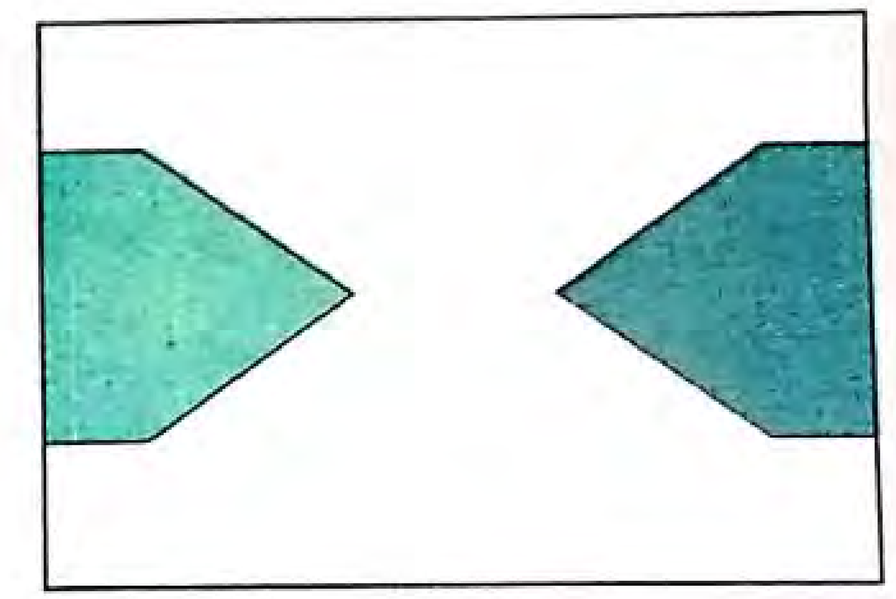
b)



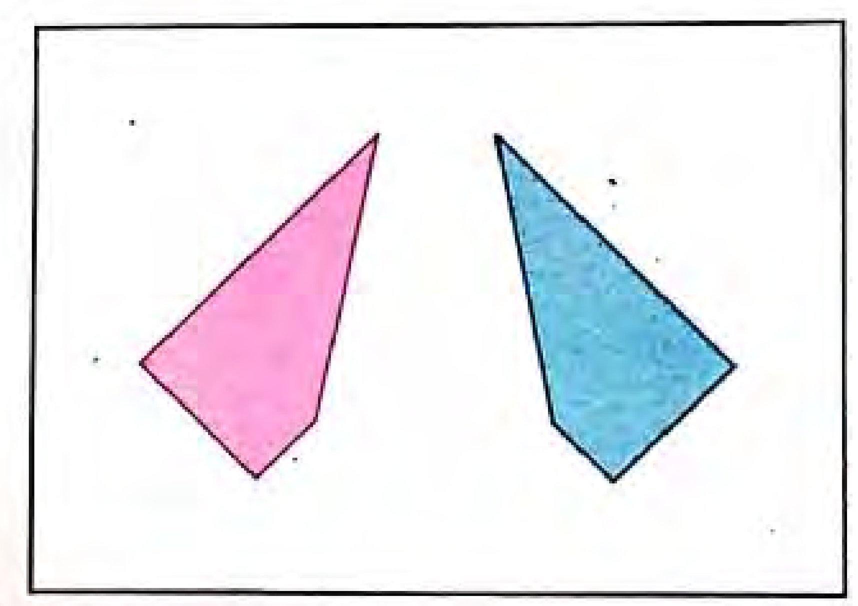
c)



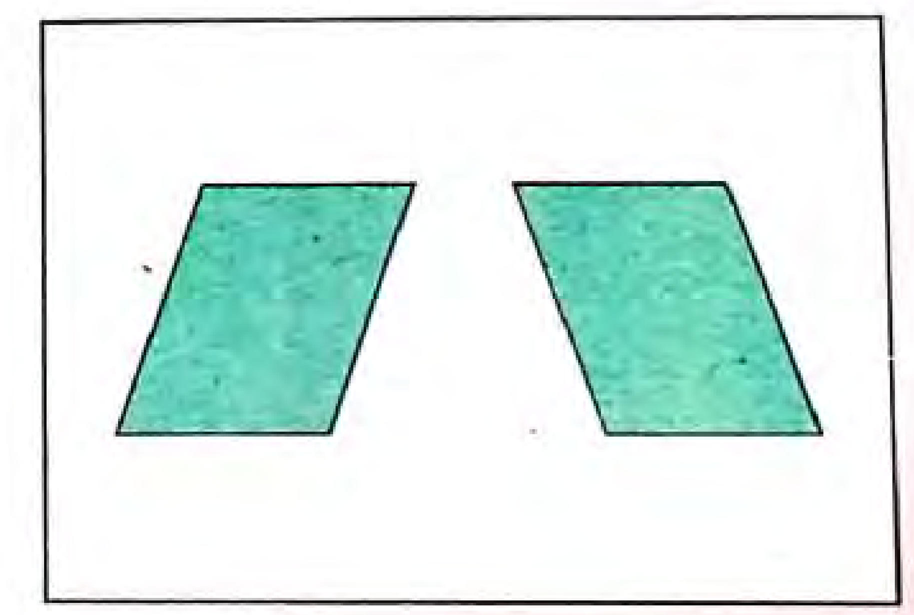
d)



e)

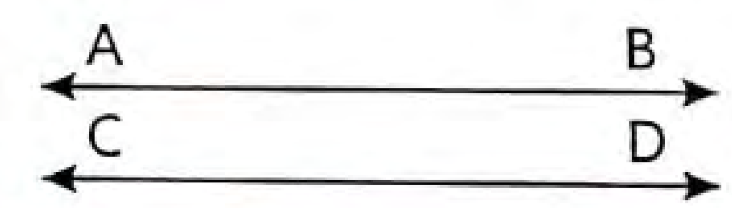


f)



9.2 Lines and Angles

9.2.1 Parallel Lines



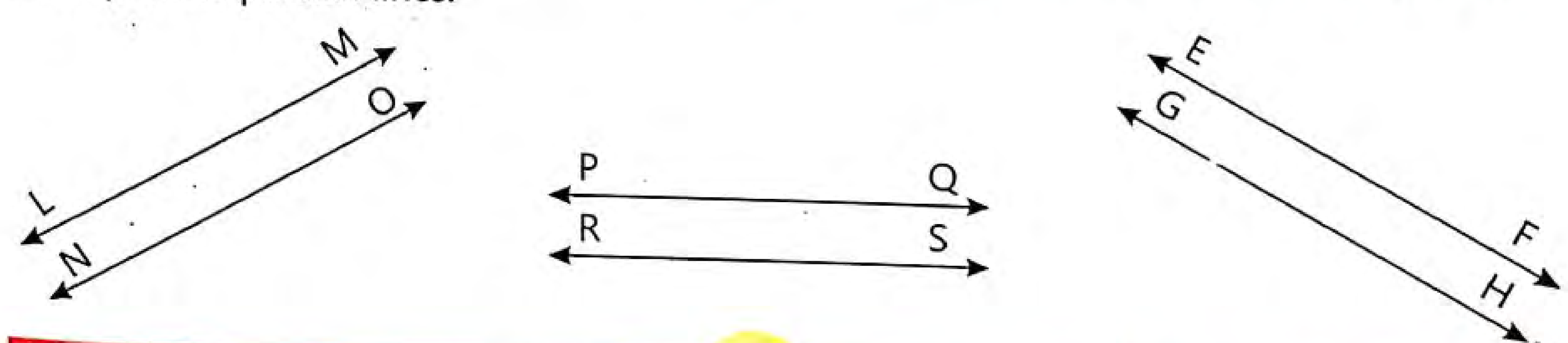
Observe the given pair of lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.
If we extend these lines in the same direction, we shall observe that these lines never intersect each other at any point.
The distance between them always remains the same.

Previous Knowledge Check

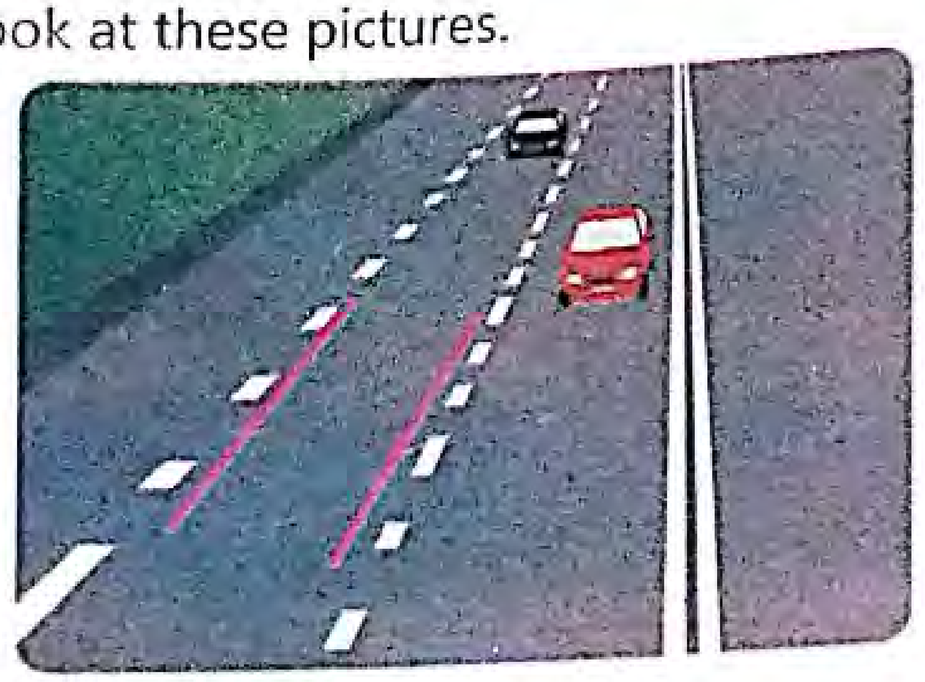
What are parallel and non-parallel lines?
What are perpendicular and vertical lines?

The lines which do not intersect each other at a point, when they are extended on either side in the same direction, are called **parallel lines**.

In real life, we see many parallel lines. e.g. lines in our note-books, railway lines, etc. See more pairs of parallel lines.

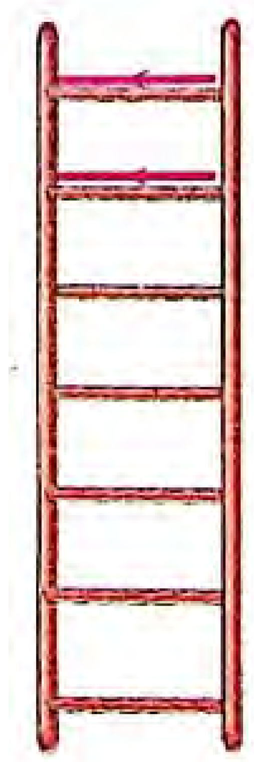


Look at these pictures.

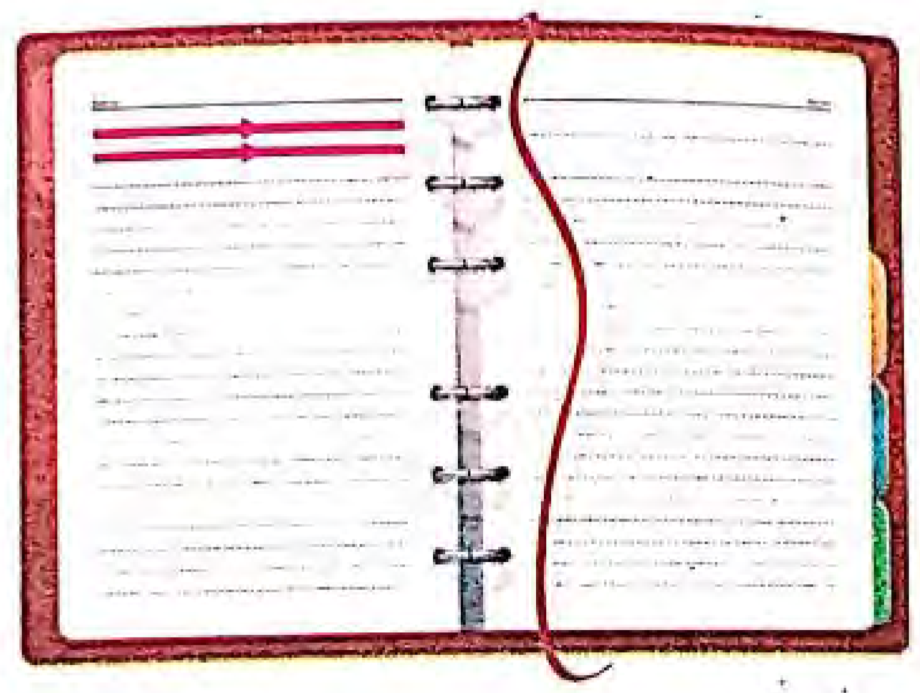


Two straight roads

Railway track



Steps on a ladder



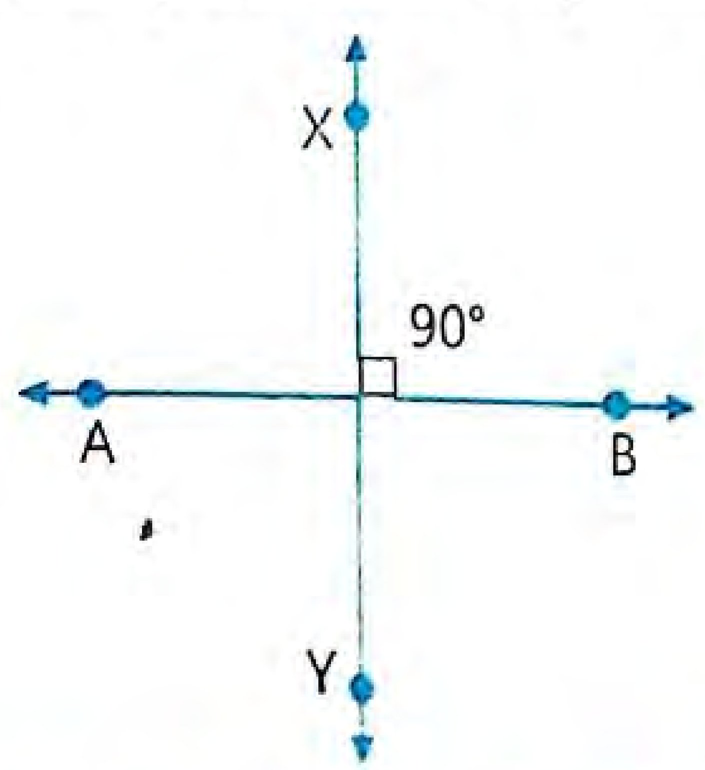
Lines on a notebook

9.2.2 Perpendicular Lines

The lines that intersect each other at 90° are called **perpendicular lines**.

Look at the hand of the clock, they are perpendicular to each other at 3 o'clock.

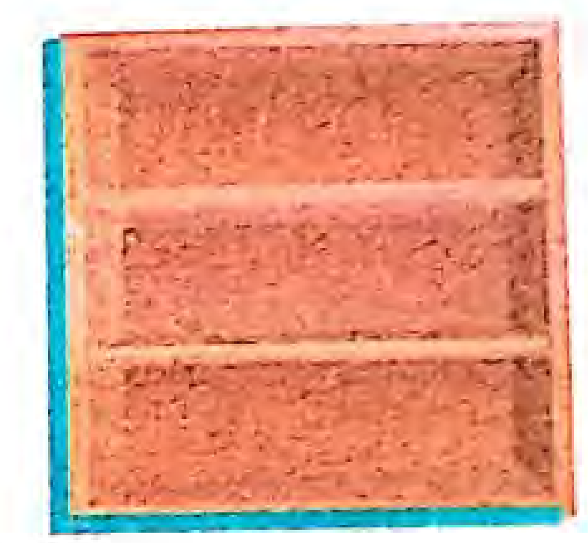
Similarly, the line XY and AB are perpendicular to each other as they are intersecting each other ar right angle.

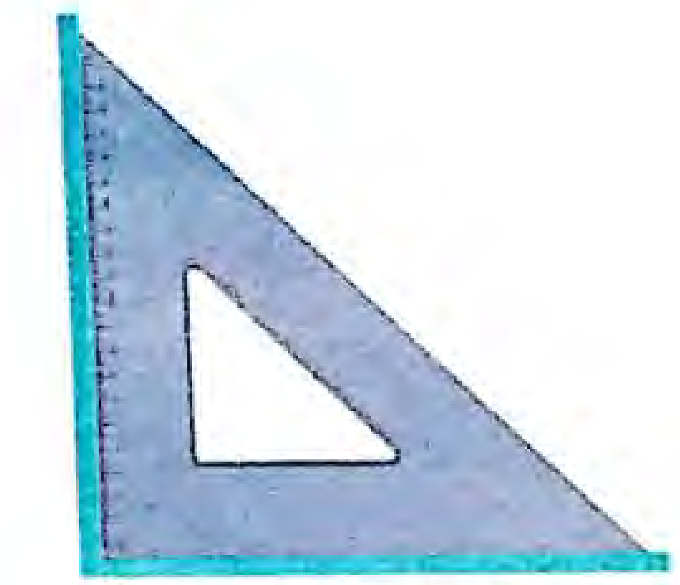


Note it down

When we extend the parallel lines on either side in the same direction, the distance between them remains the same.

We can find perpendicular lines in real-life objects.





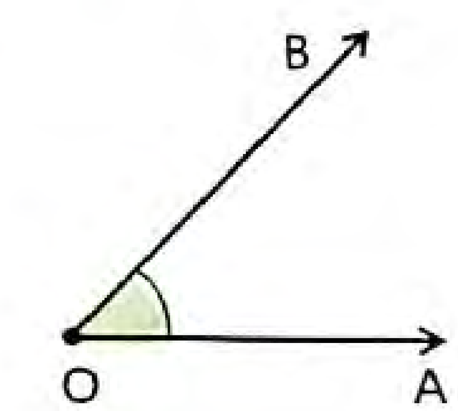
9.2.3 Angles

We have already learnt about different types of lines, line segments, angles and triangles.

Look at the figures. We can identify that these figures are made up of different types of lines and angles.

An angle is formed when two lines intersect each other.

Look at the figure $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are two rays that meet at a point to make an angle AOB or ∠O. Two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are called the arms of the angle AOB. Let's learn about some more concepts related to angles.



Adjacent Angles

Two angles are said to be **adjacent** to each other if they fulfil the following conditions:

i) They have a **common vertex**.

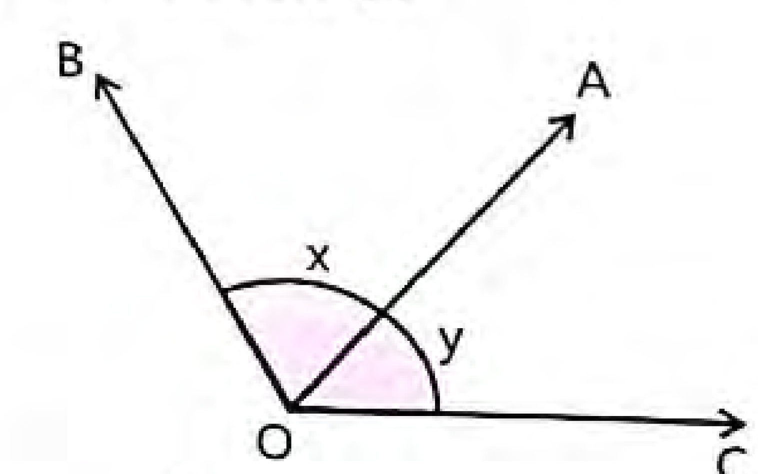
ii) They have a **common arm**.

iii) They lie on the opposite side of the common arm (i.e. they do not overlap)

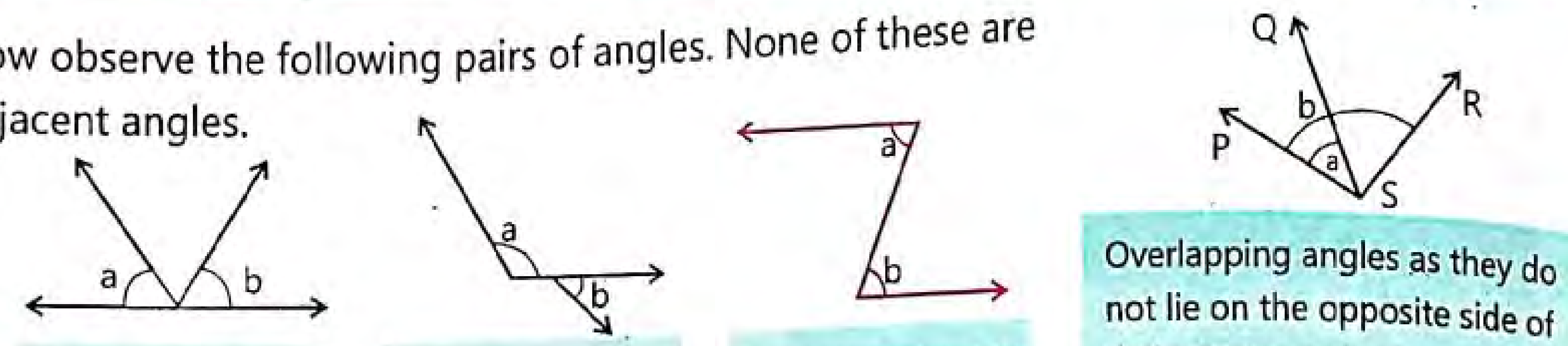
Look at the figure.

∠AOC and ∠AOB are adjacent angles as:

- They have a common vertex O.
- They have a common arm $\overrightarrow{OA}$.
- They lie on the opposite side of the common arm and are not overlapping.



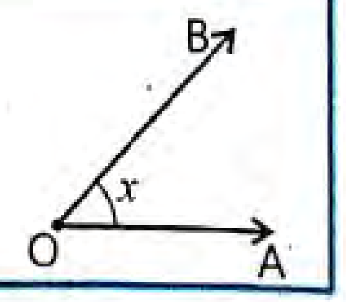
Now observe the following pairs of angles. None of these are adjacent angles.



No common arm | No common vertex | No common vertex | Overlapping angles as they do not lie on the opposite side of their common arm.

Note it down

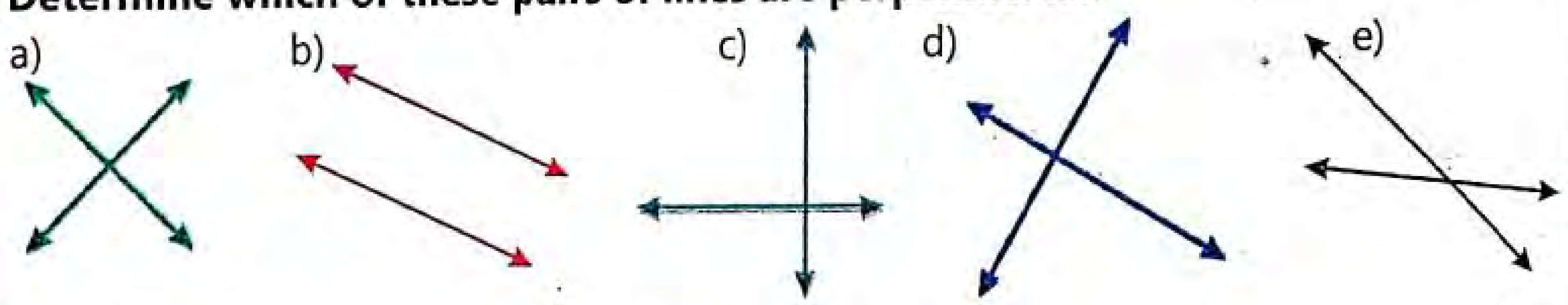
We can name an angle in three ways. For example in the figure, the angle can be named as: ∠AOB, ∠O or ∠x.



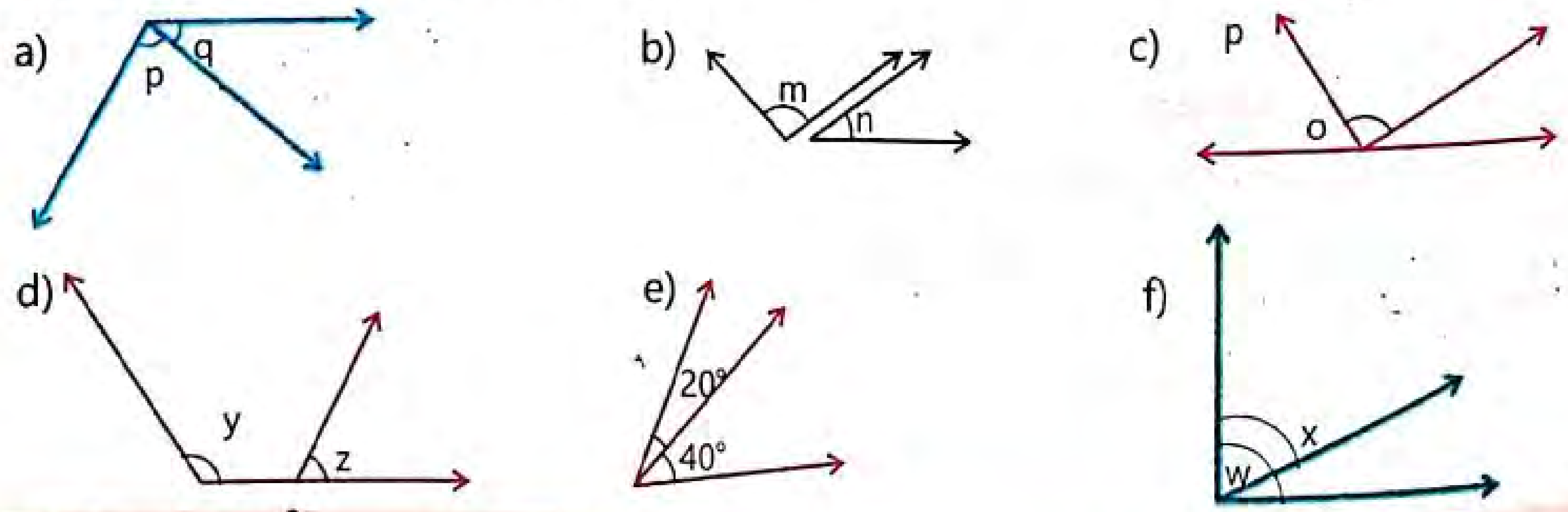
Exercise 9.2

1 **Identify and tick (✓) the parallel lines.**

2 **Determine which of these pairs of lines are perpendicular.**



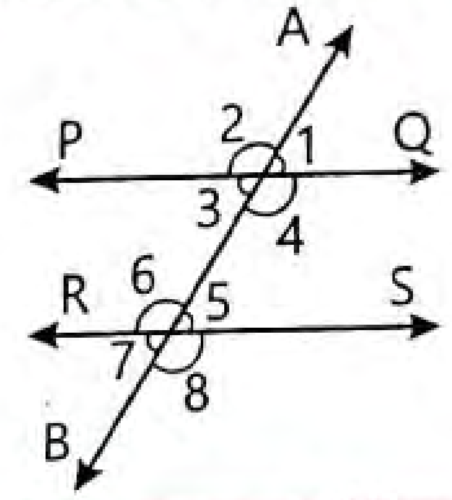
3 **Tick(✓) the pair of adjacent angles.**



Make two groups of the students and give each group adjacent and non-adjacent angle cards. Instruct them to separate out the adjacent and non-adjacent angle cards.

9.3 Transversals

Two Parallel lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ intersected by a line $\overleftrightarrow{AB}$ are shown in the figure. Here, the line AB is called transversal.



If a line passes through two or more given lines at different points, it is called a **transversal**.

a) In the given figure, the angles ∠2 and ∠6 are called the corresponding angles. Similarly, ∠3 and ∠7, ∠1 and ∠5, and ∠4 and ∠8 are also corresponding angles. Measure the pair of the corresponding angles. We shall observe that all the corresponding angles are equal.

b) Again observe figure 1, angles ∠3 and ∠5 are called alternate angles. Alternate angle are only between the parallel lines.

c) Alternate angles are also further described as, angles ∠3 and ∠5 and ∠4 and ∠6 are known as alternate interior angles.

d) While angles ∠1 and ∠7 and ∠2 and ∠8 are known as alternate exterior angles. Measure all the pair of alternate angles, we shall observe that all pair of alternate angles are equal. The angles ∠3, ∠4, ∠5 and ∠6 are called interior angles.

We observe that ∠5 and ∠6 are on the same side of $\overrightarrow{RS}$ and sum of their measurement is 180° because the angle of the straight line is equal to 180°. In the above figure ∠1 and ∠3, ∠2 and ∠4, ∠5 and ∠7, ∠6 and ∠8 are vertically apposite angles,

Form the above discussion, we conclude that when two parallel lines are intersected by a transversal, then:

i. each pair of corresponding angles are equal.

Note it down

The lines that never intersect each other at any point when extended in same direction are parallel lines while the lines that intersect each other at 90° are called perpendicular lines. On the other hand, the transversal is a line that passes through two or more given lines at different points.

Draw a table on the board. Write properties of parallel, perpendicular lines and transversal. Then ask students to write the differences among these.

ii. each pair of alternate interior (or exterior) angles are equal.

iii. each pair of interior angles on the same side of a transversal are supplementary.

We can solve different problems related to parallel lines using the above result.

Example 1:

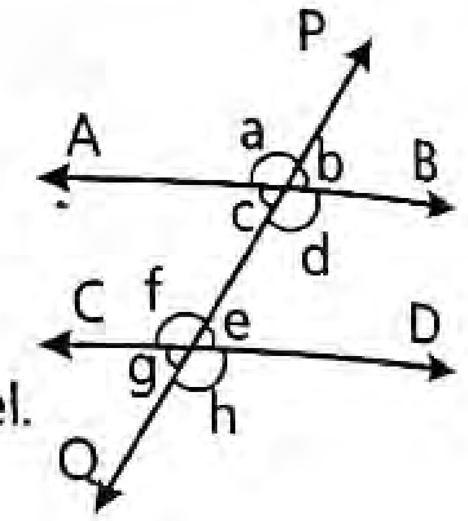
Observe figure and write answers of the following questions:

i. Write the names of parallel lines and a transversal.

ii. Write pairs of the corresponding angles.

iii. Write the name of alternate interior and exterior angles.

iv. Write the pairs of supplementary angles on same side of transversel.



Solution:

i. Parallel lines: AB and CD, Transversal: PQ

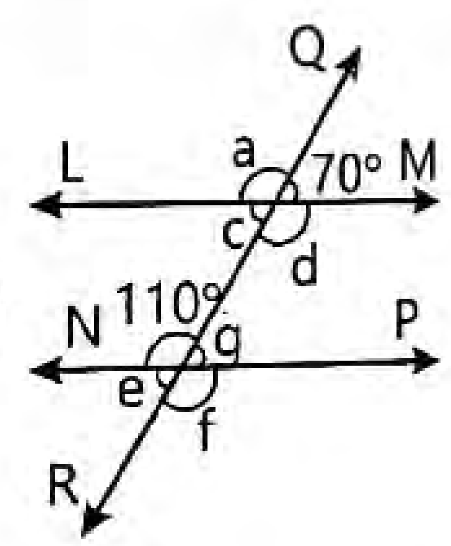
ii. Corresponding angles: $\angle a$ and $\angle f$; $\angle b$ and $\angle e$;

iii. Alternate exterior angles: $\angle a$ and $\angle h$; $\angle b$ and $\angle g$.

iv. Supplementary angles: $\angle d$ and $\angle e$; $\angle c$ and $\angle f$.

Example 2:

Two parallel lines are intersected by a transversal, as shown in the figure. Find the measurement of $\angle a$, $\angle b$, $\angle c$, $\angle d$, $\angle e$, and $\angle f$ using the properties of parallel lines.



Solution:

As we know that each pair of corresponding angles are equal.

So, $\angle a = 110°$ and $\angle d = 70°$

Similarly, we also know that each pair of alternate interior (or exterior) angles are equal.

So, $\angle b = \angle d = 70°$, $\angle d = 110°$

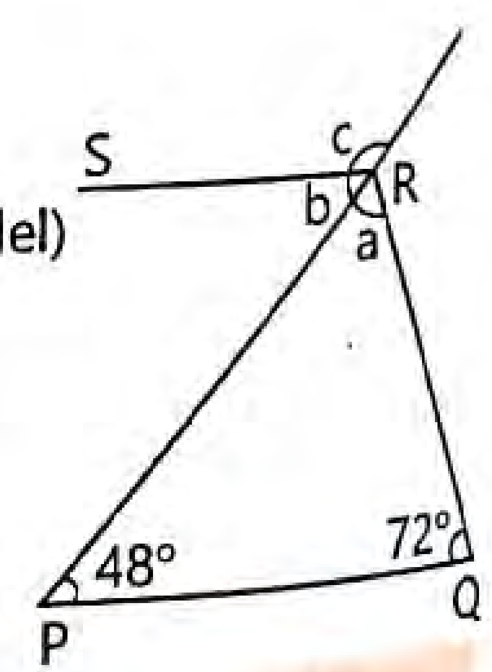
Similarly, $m\angle f = 110°$ (vertically opposite angle)

$\angle e = \angle d$ (vertically opposite angle)

$\angle e = 70°$ $(\because d = 70°)$

Example 3:

Find the measurement of angles a, b and c. (Here, $\overline{PQ}$ and $\overline{RS}$ are parallel)



Solution:

$48° + 72° + \angle a = 180°$

(Sum of angles of a triangle)

$\Rightarrow \quad \angle a = 180° - 48° - 72°$

$\angle a = 60°$

Share the following online game links to practice of angles formed by two parallel lines and a transversal.
https://www.mathgames.com/skill/8.46-transversal-of-parallel-lines
https://www.transum.org/software/SW/Starter_of_the_day/Students/AngleParallel.asp

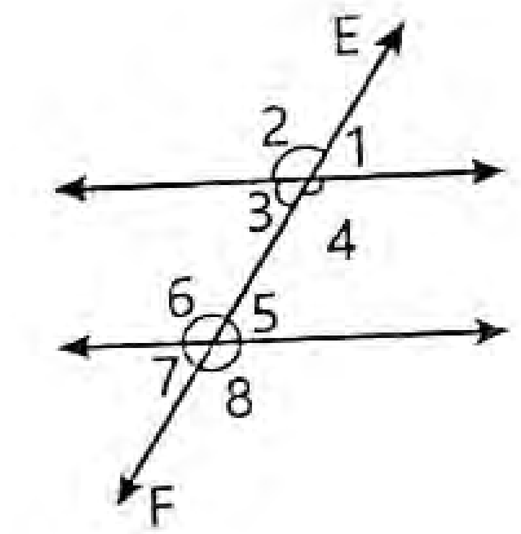
$\angle b = 48°$ (alternate angles)

$\angle b + \angle c = 180°$ (angles on a straight line)

$\Rightarrow \quad \angle c = 180° - 48° = 132°$

Example 4:

In the following figure, $m\angle l = 75°$. Find all the other angles.



Solution:

$m\angle l = m\angle 3$ (vertically opposite angles)

$\therefore \quad m\angle 3 = 75°$

$m\angle 3 = m\angle 5 = 75°$ (alternate interior angles)

So $\quad m\angle 5 = m\angle 7 = 75°$ (alternate opposite angles)

Also,

$m\angle 4 + m\angle 5 = 180°$ (supplementary angles)

$\Rightarrow \quad m\angle 4 + 75° = 180°$

$\Rightarrow \quad m\angle 4 = 180° - 75°$

$\Rightarrow \quad m\angle 4 = 105°$

and $\quad m\angle 2 = m\angle 4 = 105°$ (vertically opposite angles)

$m\angle 2 = m\angle 6 =$ (corresponding angles)

So $\quad m\angle 6 = 105°$

$\therefore \quad m\angle 6 = m\angle 8 = 105°$ (vertically opposite angles)

Exercise 9.3

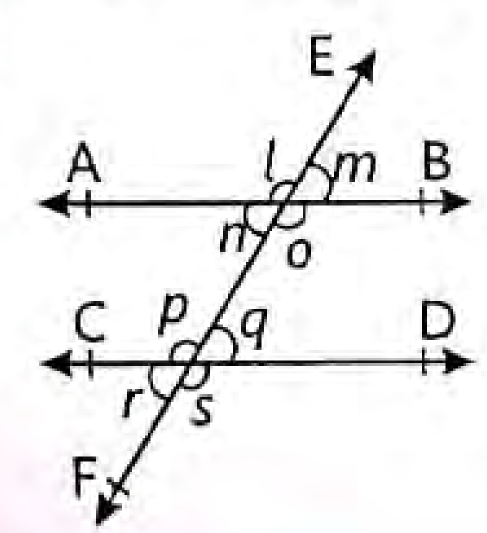
1 **Two parallel lines are intersected by a transversal, as shown in the given figures: Observe these figures and write answers of the following question:**

a) Write names of parallel lines and a transversal.

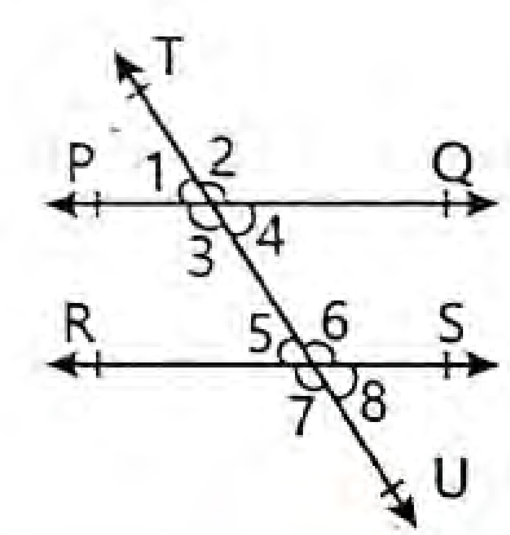
b) Write names of paris of :

- corresponding angles.
- alternate interior angles.
- alternate exterior angles.

i.

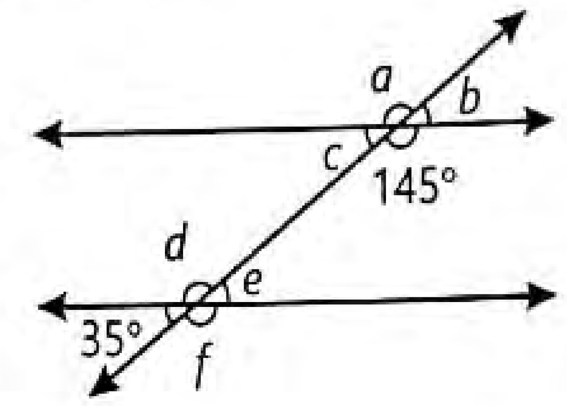


ii.

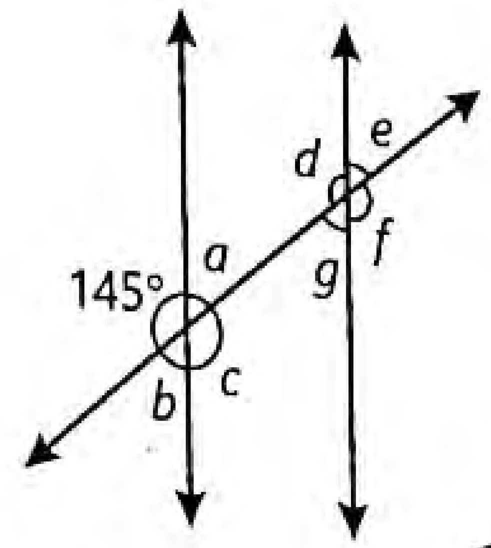


2 Using properties of parallel lines and a transversal, find unknown angles of the following figures:

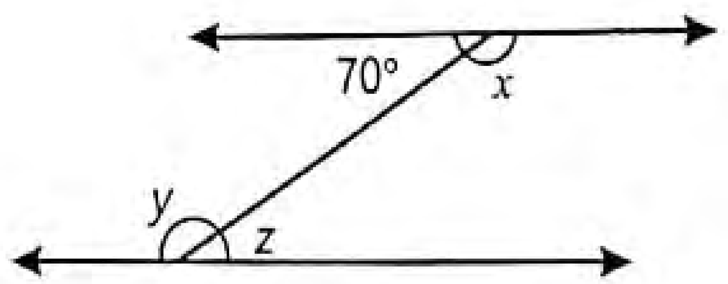
a)



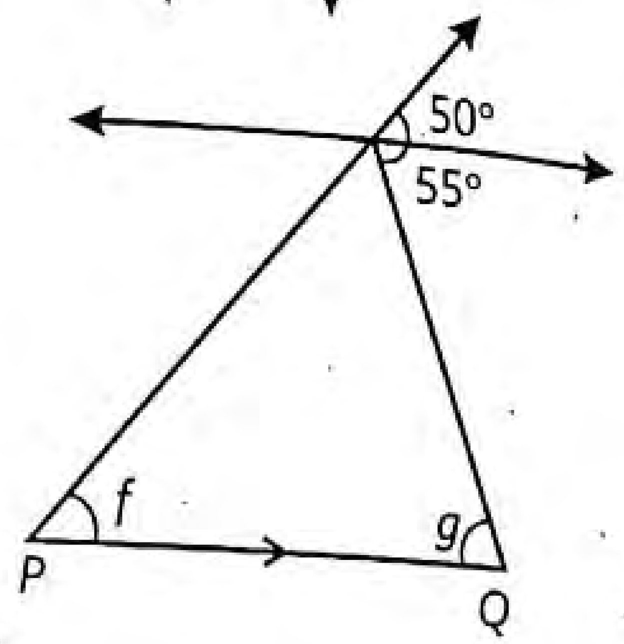
b)



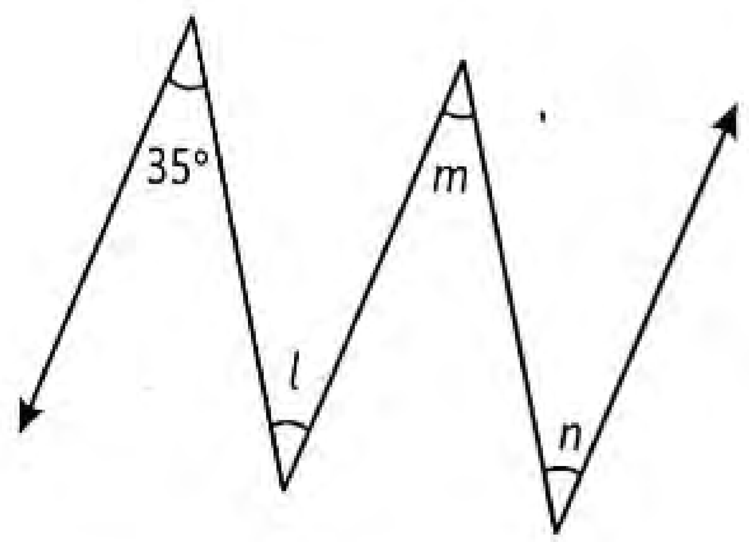
c)



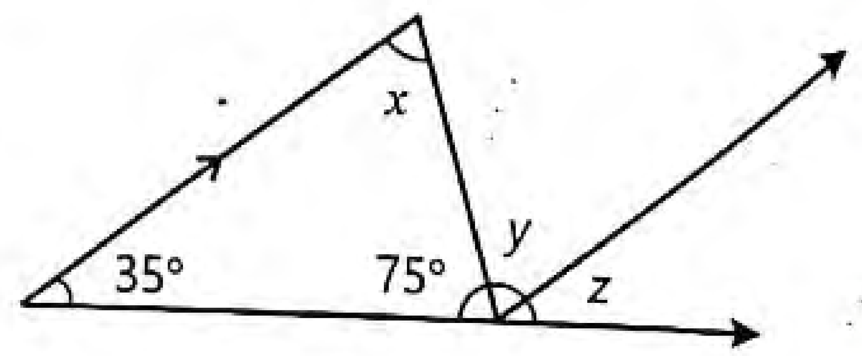
d)



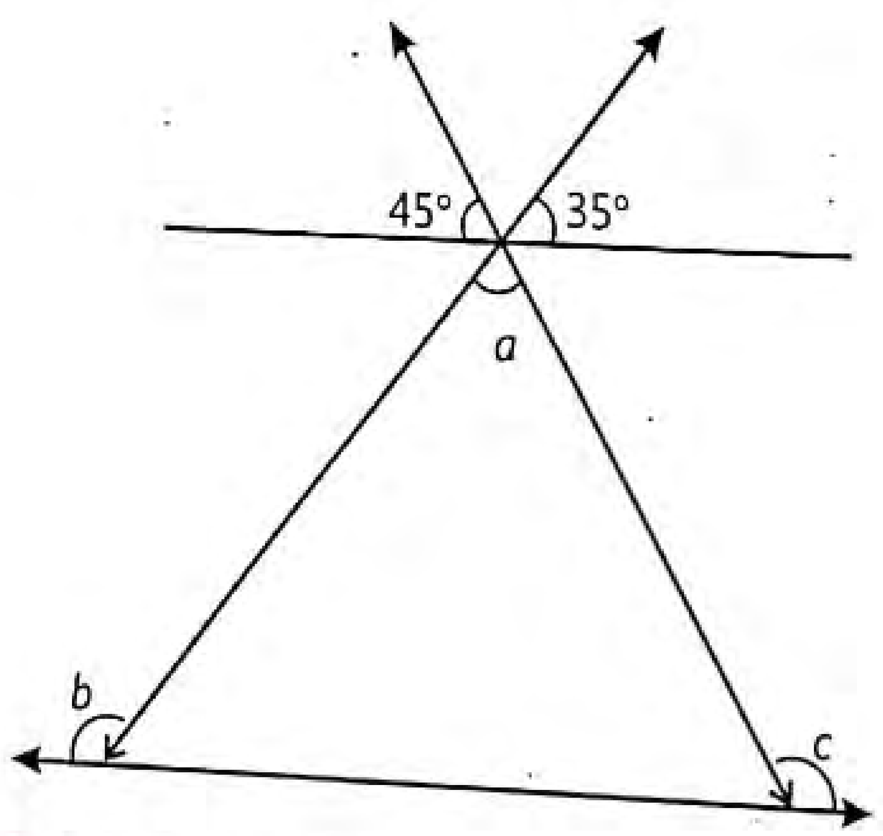
e)



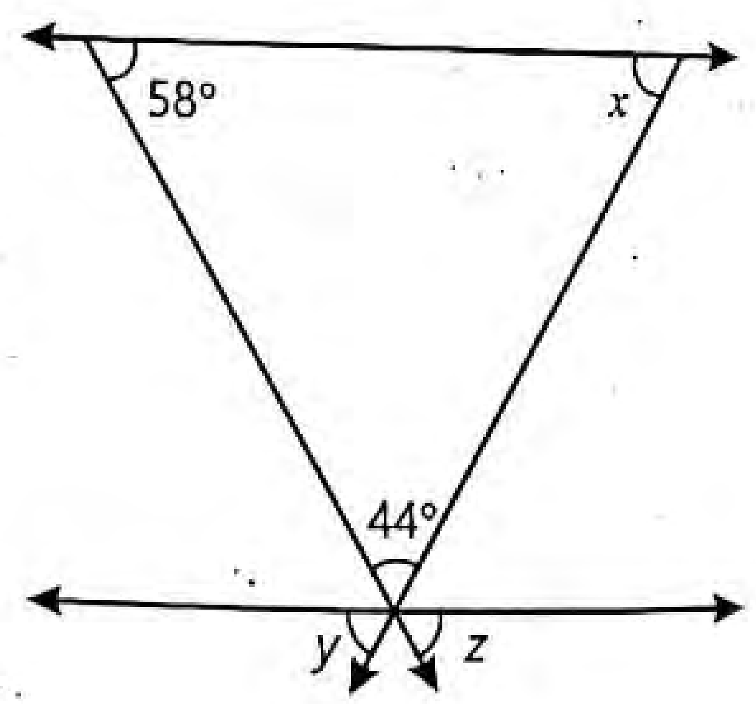
f)



g)

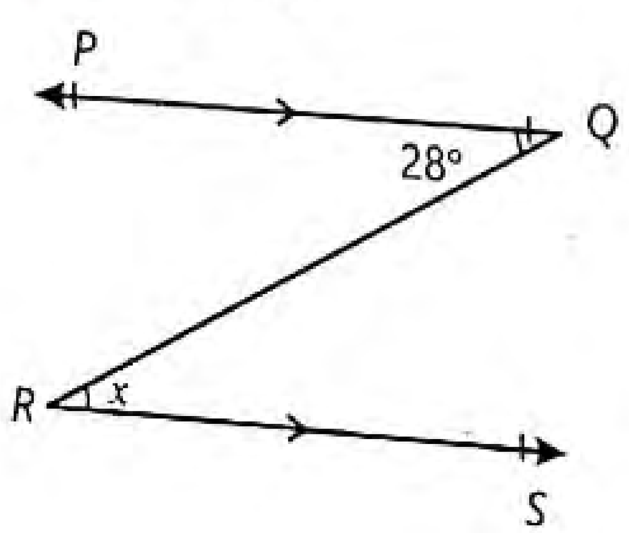


h)

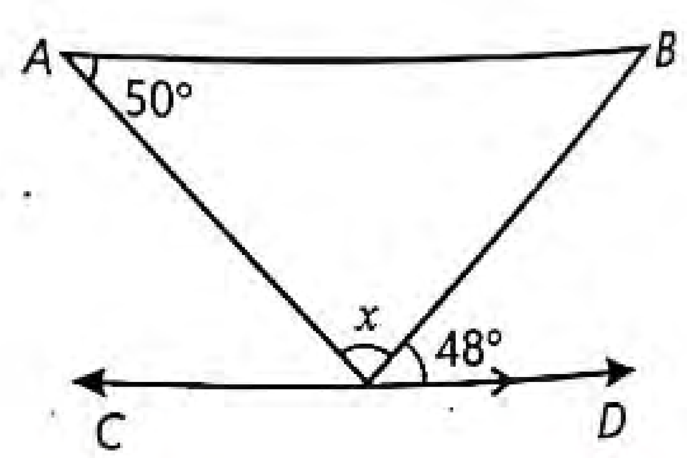


3 Find the value of x.

a)



b)



9.4 Rotational symmetry

If a shape or object is rotated around a central point and looks exactly the same at least 2 times in a full rotation, we say that it has **rotational symmetry**.

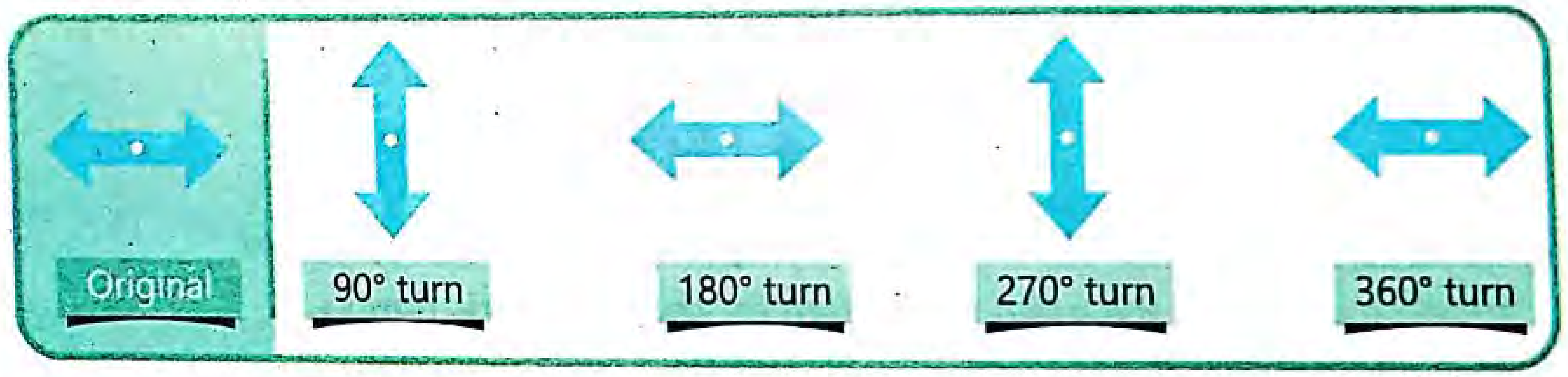
Previous Knowledge check

What is meant by symmetrical shapes?
Tell the names of some shapes of letters that are symmetric.

Note it down

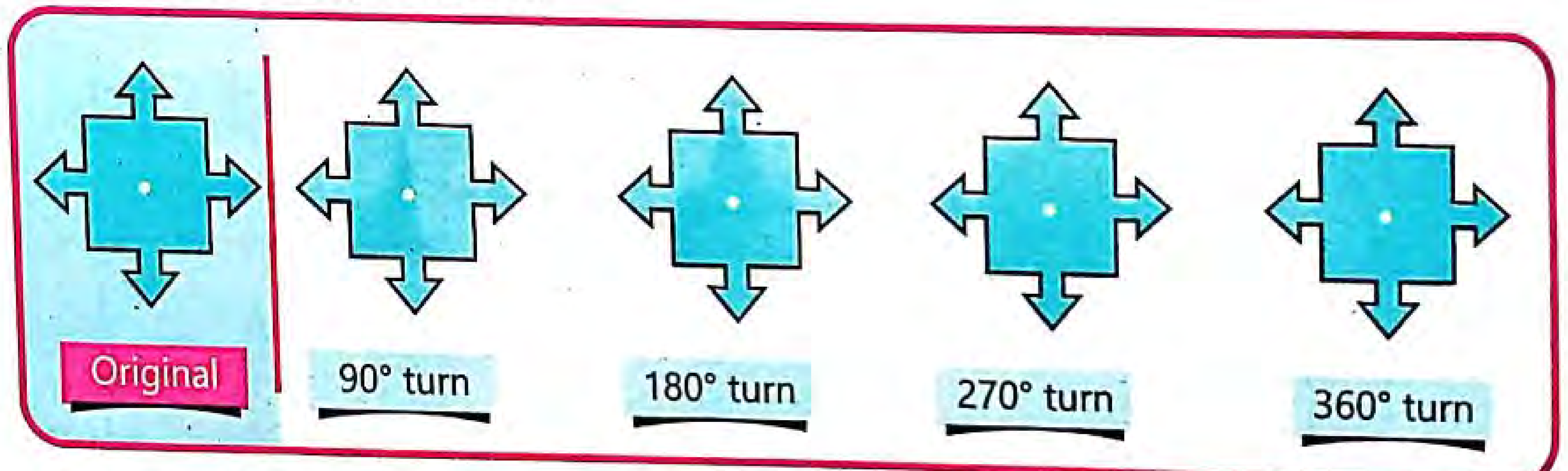
The number of times (minimum 2 times) a shape looks exactly the same in a full rotation is called its order of rotational symmetry. The center point is called the point of rotation.

Look at the figures below.



We can see that when the shape is rotated around its center, it looks exactly the same 2 times in a full rotation. It means it has a rotational symmetry and the order of its rotational symmetry is 2.

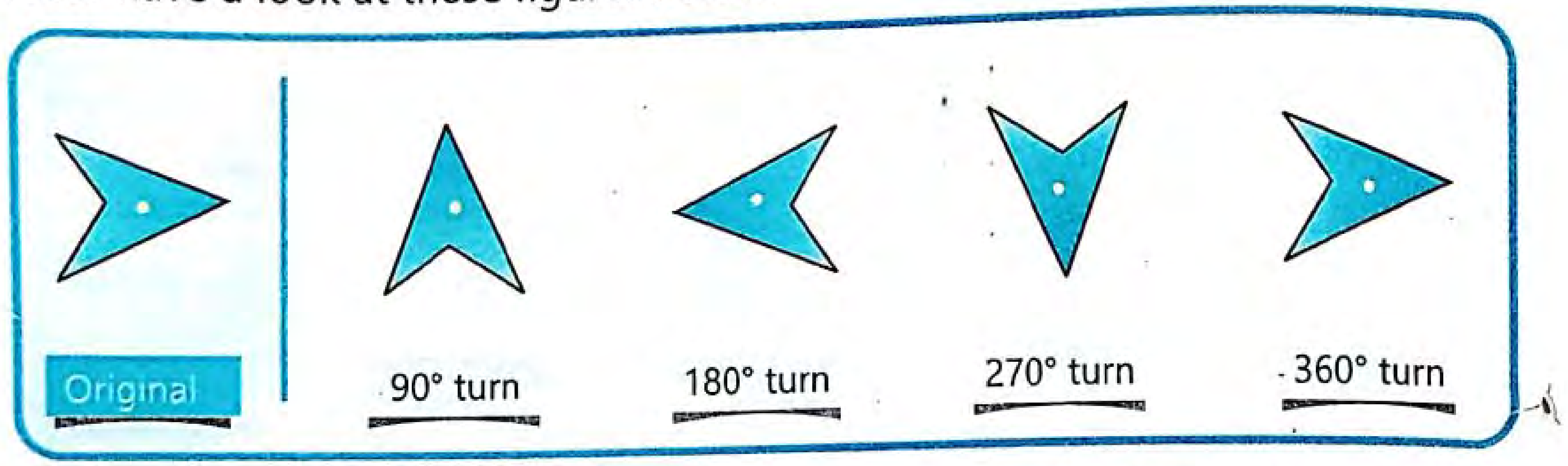
Now observe the figures below.



We can see that when the shape is rotated about its center point, it looks exactly the same 4 times during a full rotation. So, it has rotational symmetry of order 4.

Share the following online game links to practice symmetry.
https://www.topmarks.co.uk/symmetry/symmetry-atchinghttps://www.mathgames.com/skill/8.73-reflections-graph-the-image

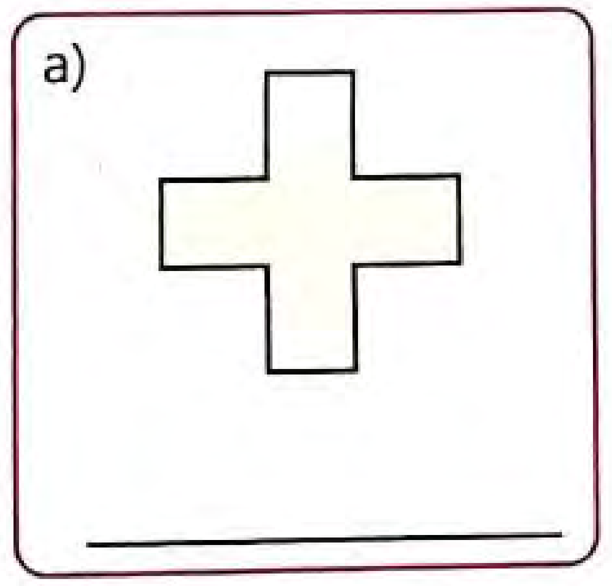
Now have a look at these figures below.

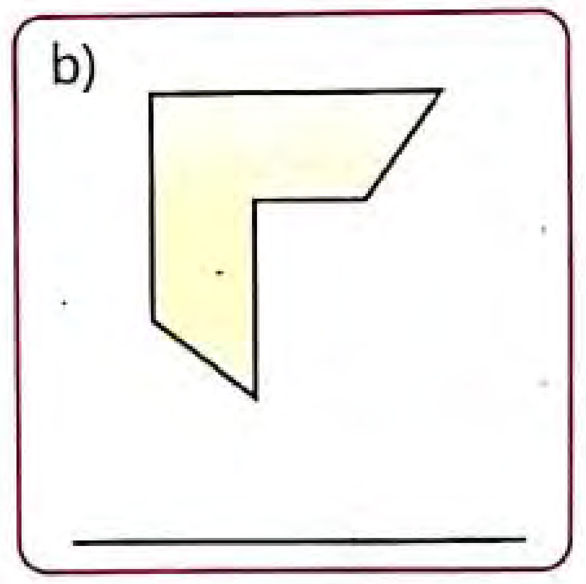


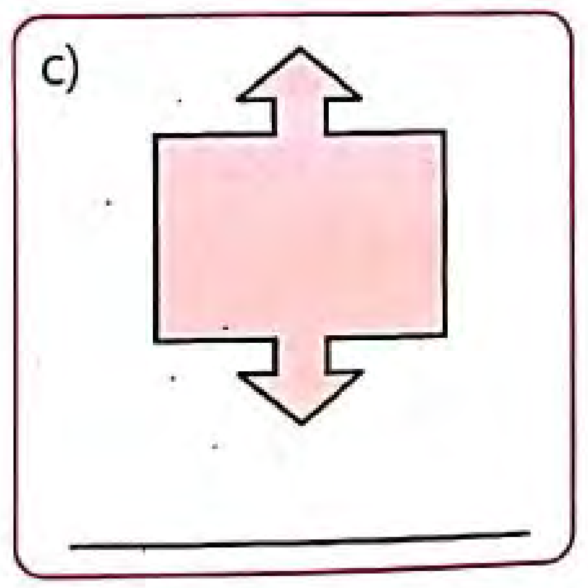
We can see that when the shape is rotated about its center point, it looks exactly the same only 1 time during a full rotation. So, it does not have rotational symmetry.

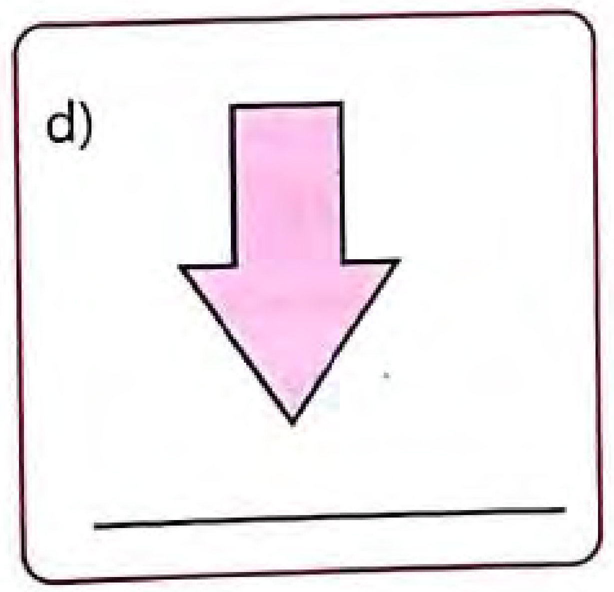
Exercise 9.4

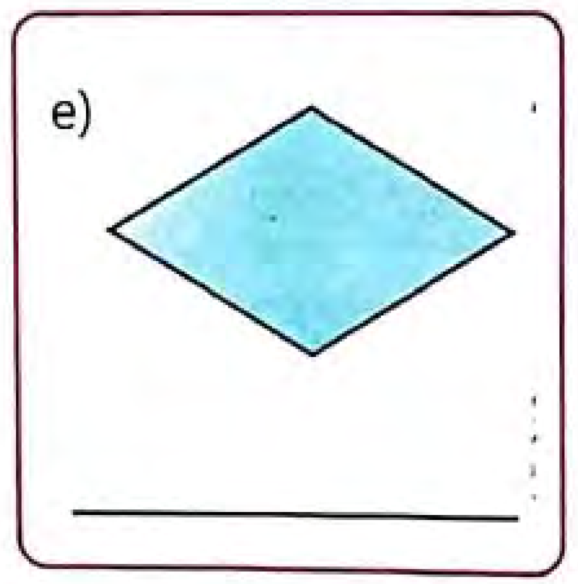
1 **Tick (✓) the figures that have rotational symmetry. Write the order of rotation and mark the point of rotation for the symmetric figures.**

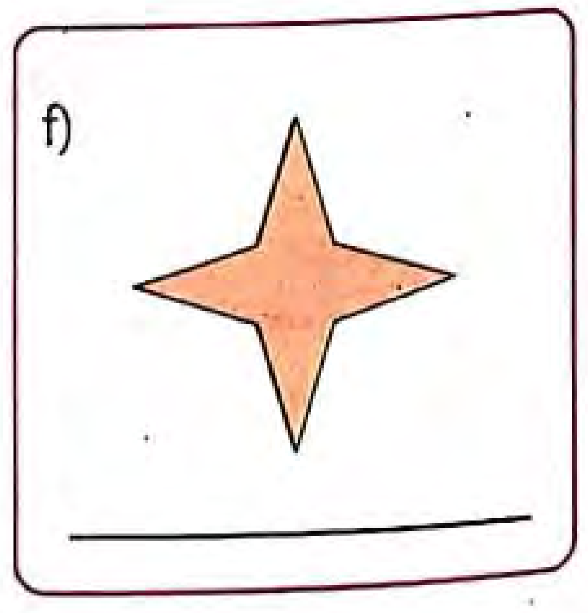


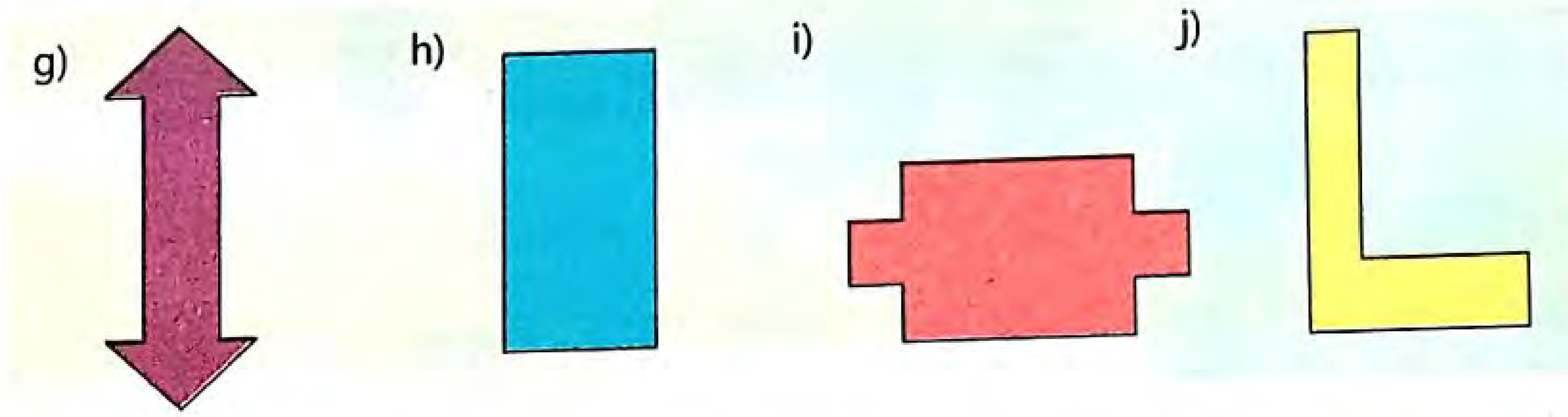






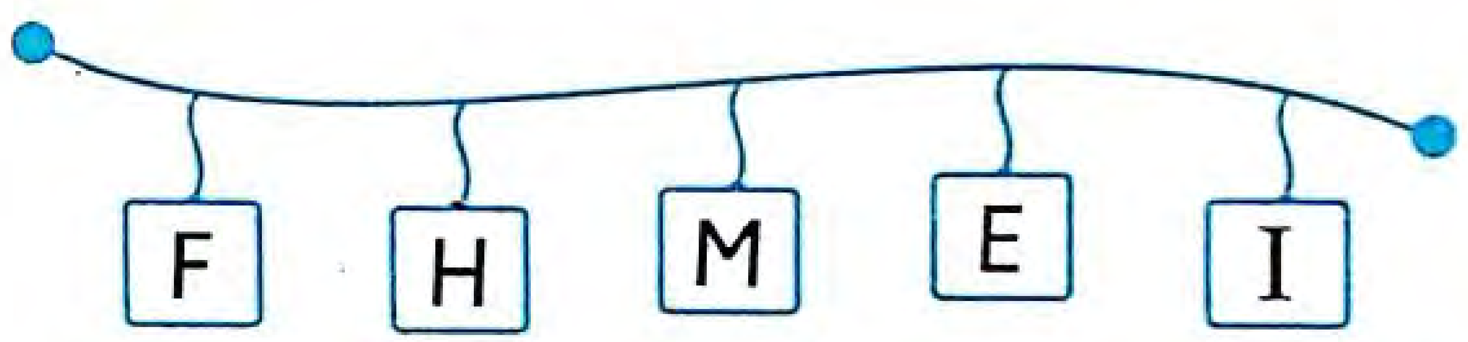






2 **Draw three shapes having rotational symmetry and mention the order of their rotational symmetry as well as point of rotation.**

Think Higher

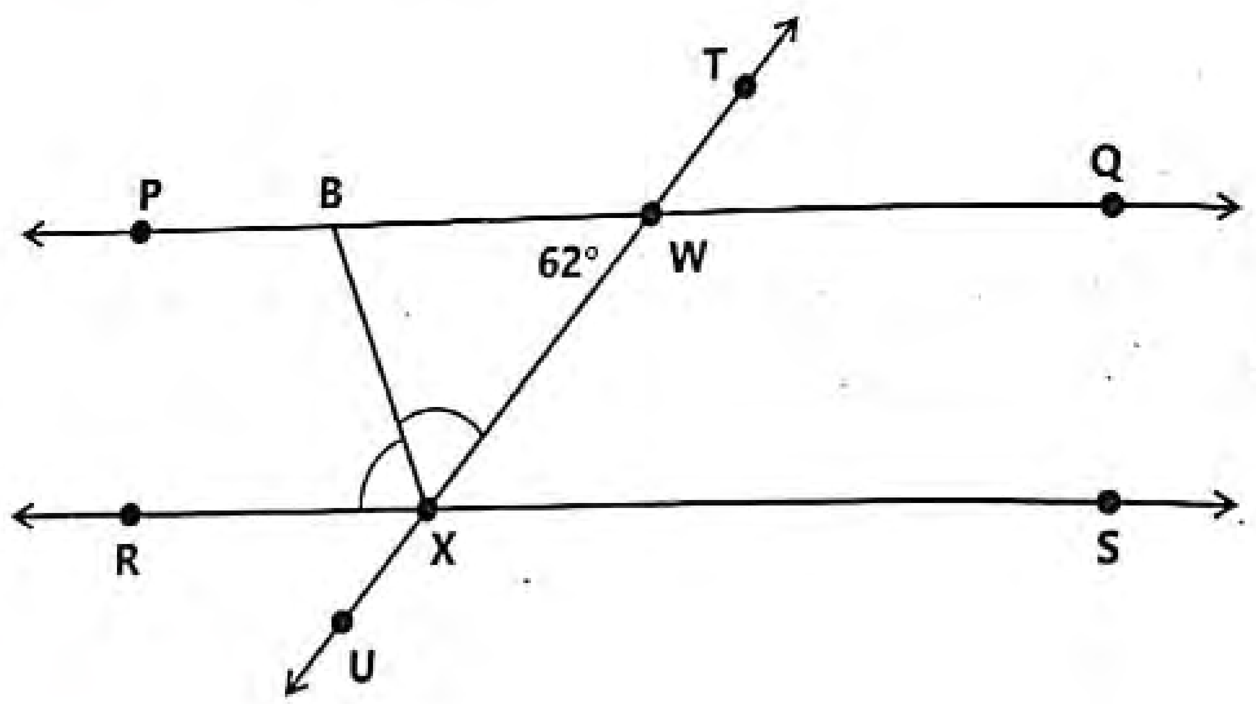


Which of these letters have:

a) One pair of parallel lines and two pairs of perpendicular lines.

b) Three pairs of perpendicular lines.

3 PQ is parallel to $\overleftrightarrow{RS}$. Transversal $\overleftrightarrow{TU}$ intersects lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ at points W and X respectively. $\overline{BX}$ is bisecting ∠RXW. If ∠BWX = 62°, find ∠BXR.



Draw a shape with exactly 2 lines of symmetry

Summary

- The lines which do not intersect each other at a point, when they are extended on either side in the same direction, are called parallel lines.
- The lines that intersect each other at 90° are called perpendicular lines.
- If a line passess through two or more given lines at different points, it is called transversal.
- A reflection is a transformation that uses a line like a mirror to reflect a figure.
- The number of times a shape looks exactly the same (at least two times) in a full rotation is called its order of rotational symmetry.

Vocabulary

- Reflection
- Line of reflection
- Reflected image
- Parallel lines
- Angle
- Adjacent angle
- Transversal
- Rotational symmetry

Review Exercise

1 Choose the correct option.

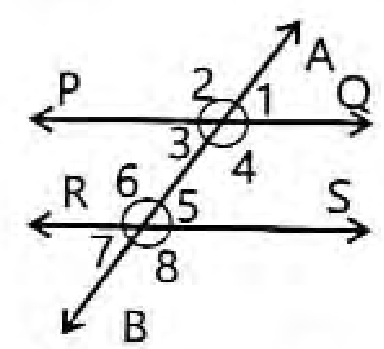
a) A point has ______ dimensions.
i. 1 ii. 2 iii. 3 iv. 0

b) A cuboid has ______ faces.
i) 8 ii. 6 iii. 5 iv. 4

c) The pair of lines that are not intersecting each other.
i. ii. iii. iv.

d) If a line passes through two or more given lines at different points, it is called:
i. Alternate ii. non-parallel
iii. Parallel iv. transversal

e) The interior angles are:



i. ∠1, ∠4, ∠5, ∠7 ii. ∠3, ∠4, ∠5, ∠6
iii. ∠1, ∠4, ∠2, ∠6 iv. ∠3, ∠4, ∠5, ∠8

f) The order of rotation of the given figure is:

i. 3 ii. 1 iii. 0 iv. 4

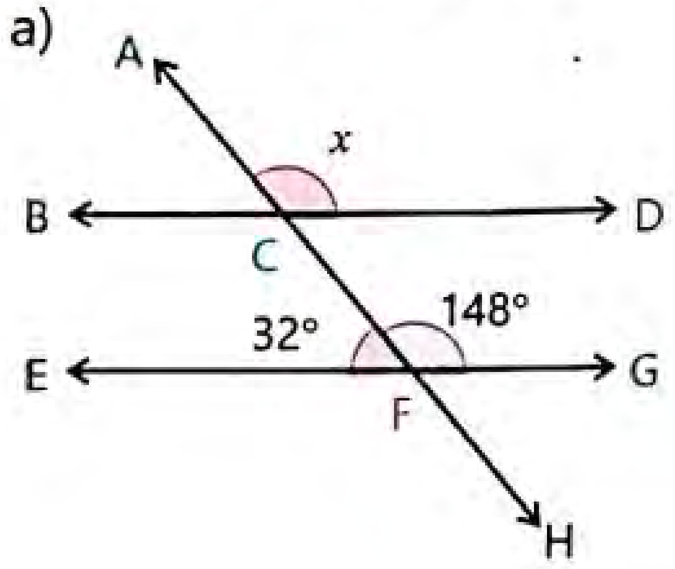
2 Define the following.

a) Transversal b) Corresponding angles c) Alternate angles
d) Parallel lines e) Perpendicular lines

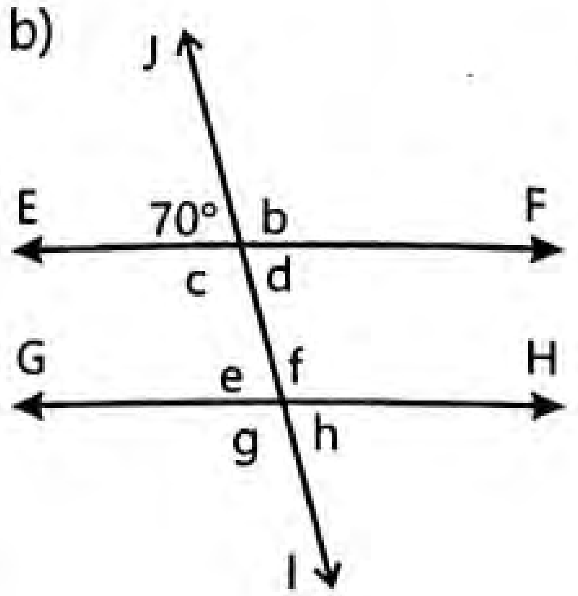
3 Differentiate between parallel lines, perpendicular lines and transversals.

4 Find unknown angles of the following figures.

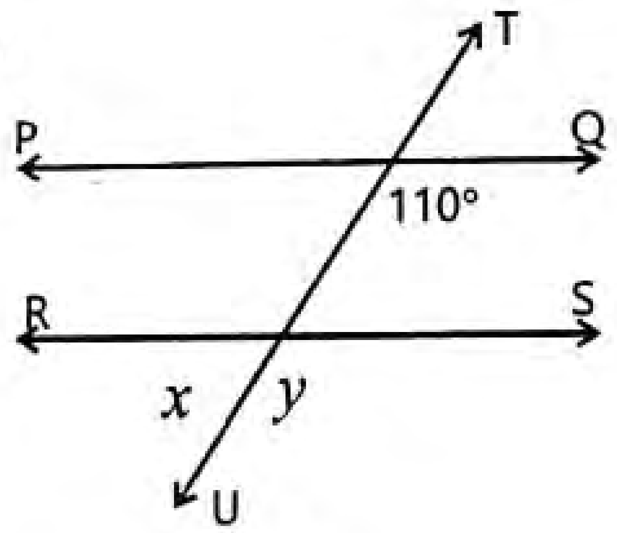
a)



b)

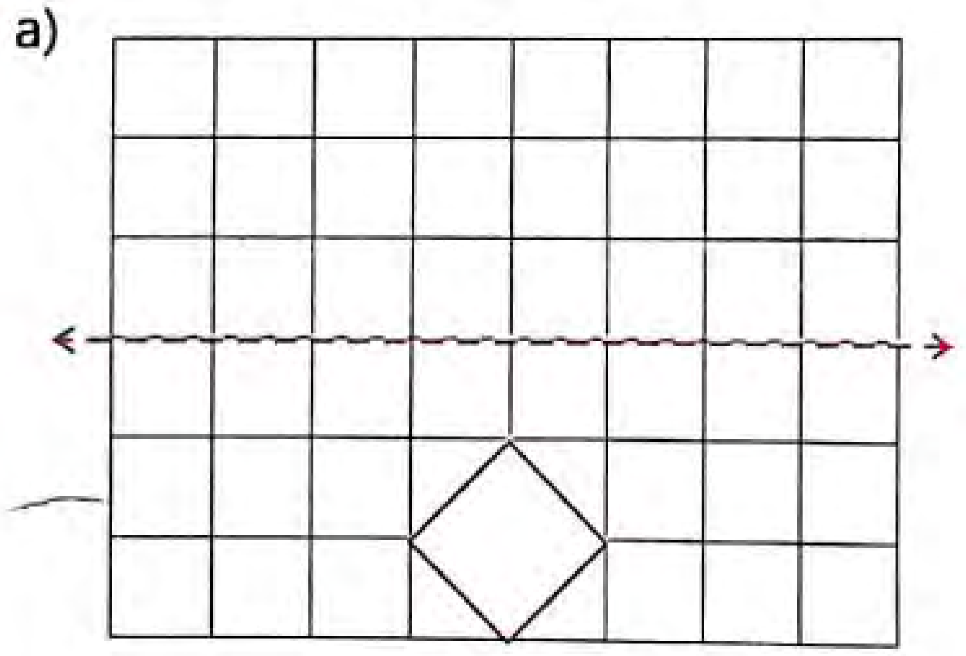


c)

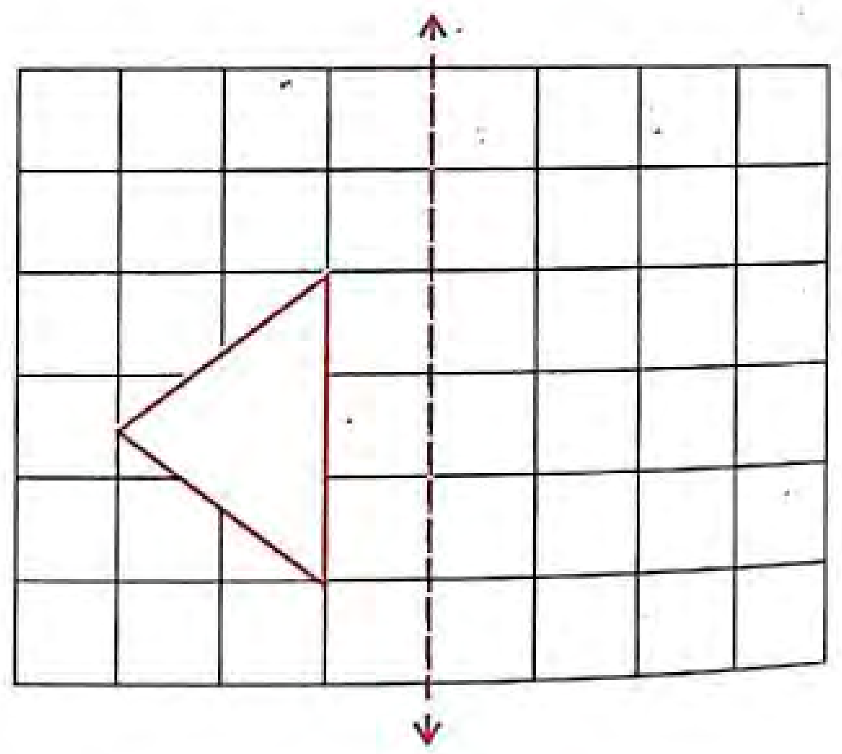


5 Reflect the images on grid paper.

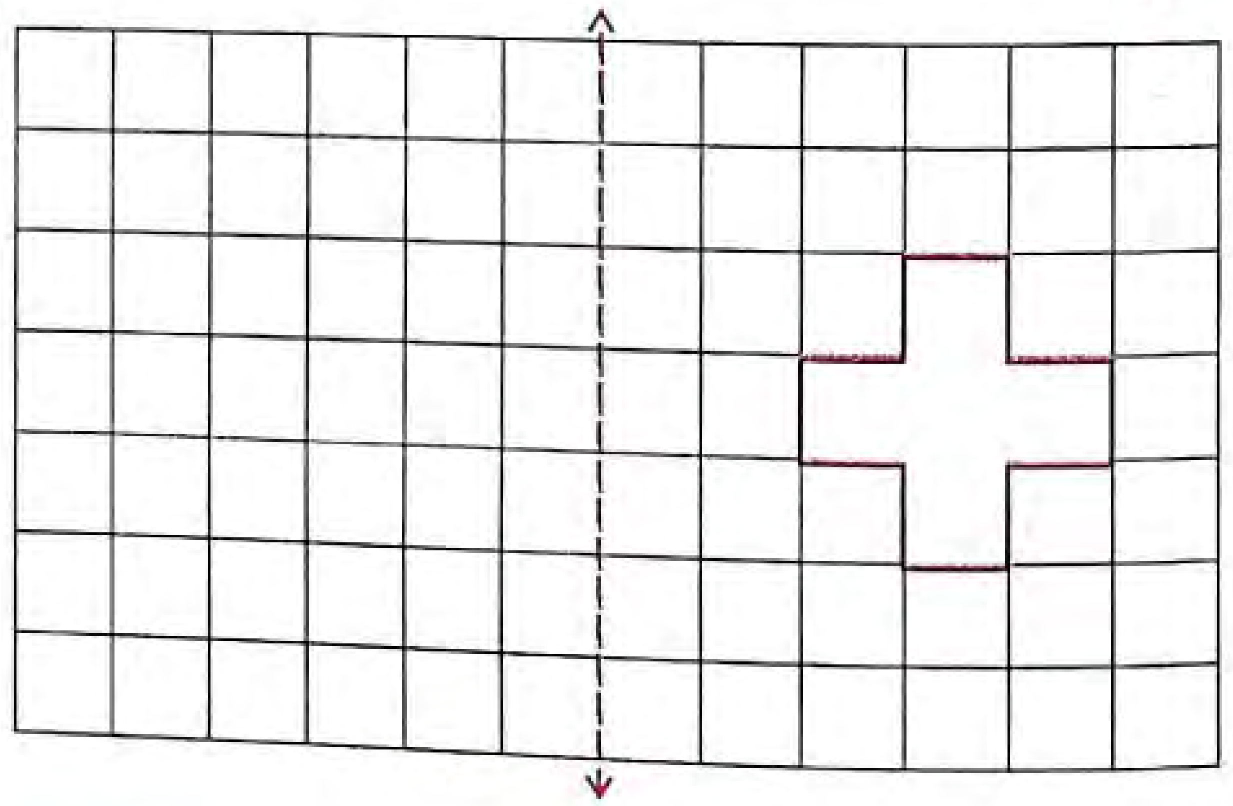
a)



b)

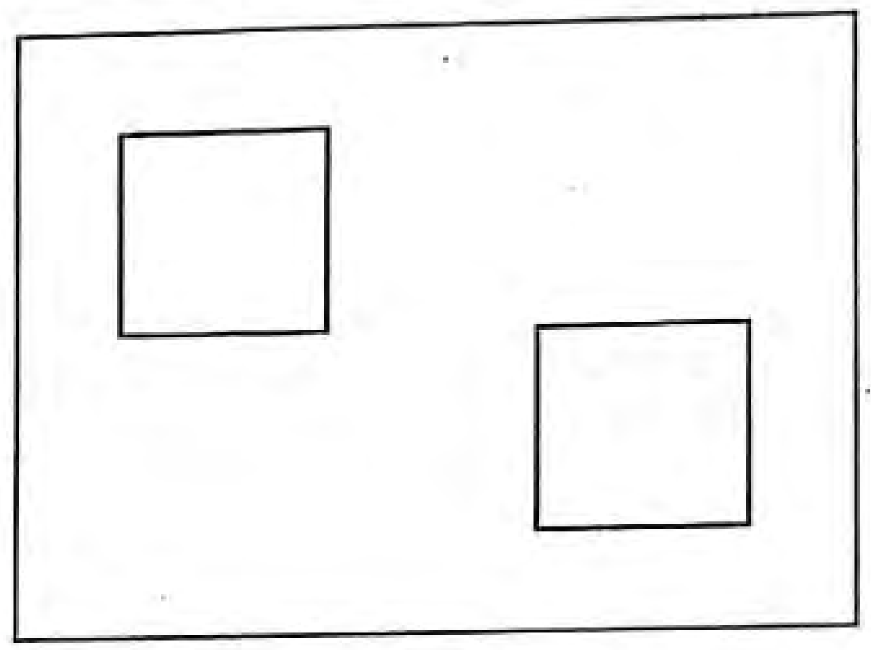


c)

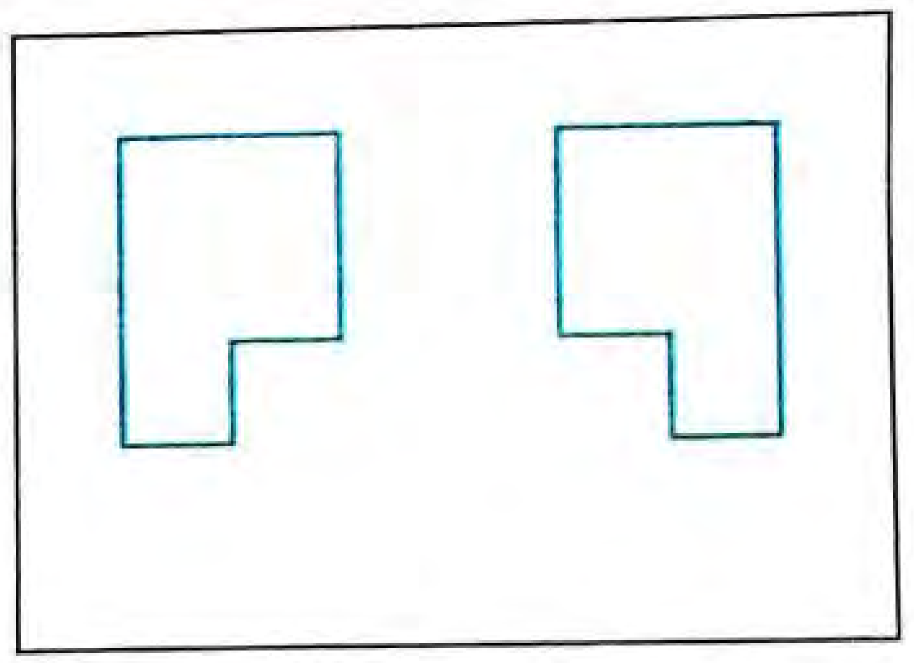


6 Find and draw the line of reflection for these images using compass.

a)

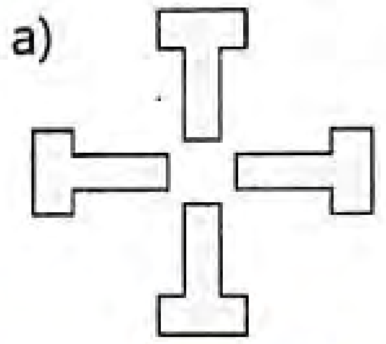


b)

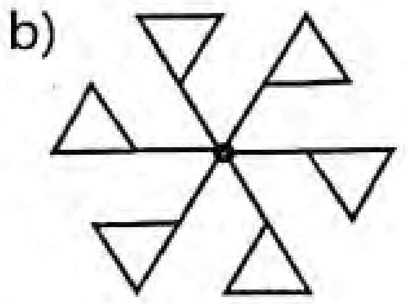


7 Write the order of rotation of each figure and mark the point of rotation for the symmetric figures.

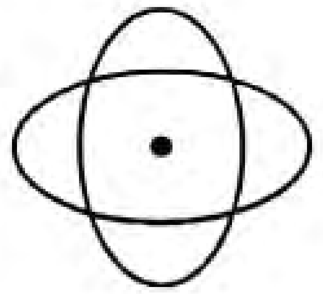
a)



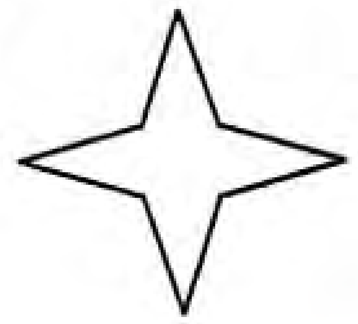
b)



c)



d)



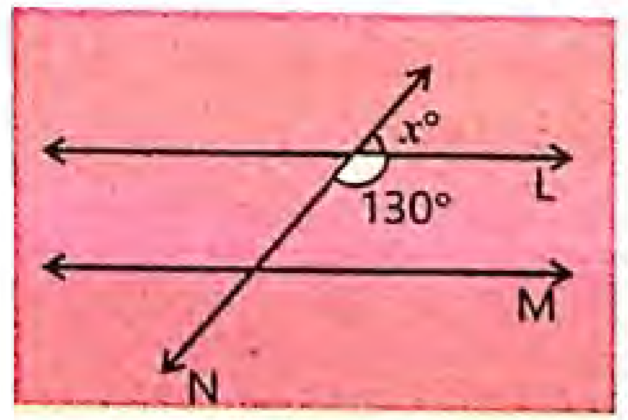
Math Project

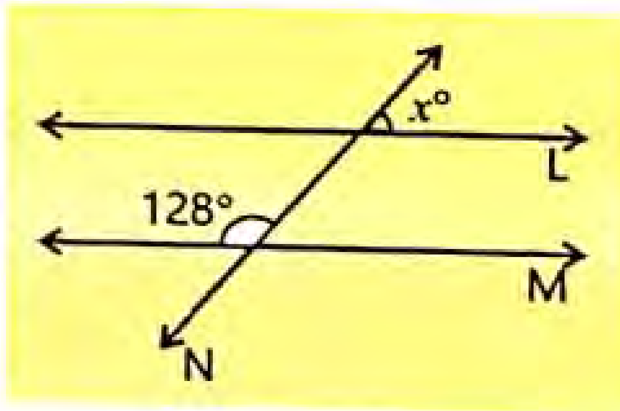
Material Required:

- Board marker/Chalk
- Scoring sheet

Procedure:

- Get into two groups.
- A member from group A will come to the whiteboard and draw parallel lines and transversal (not drawn to scale) and mark the angles and lines.
- He/she will leave at least one angle known.
- Then a member from group B will come and find the unknown angle(s).
- Repeat this and take turns.
- The group with accurate and quick answers wins.



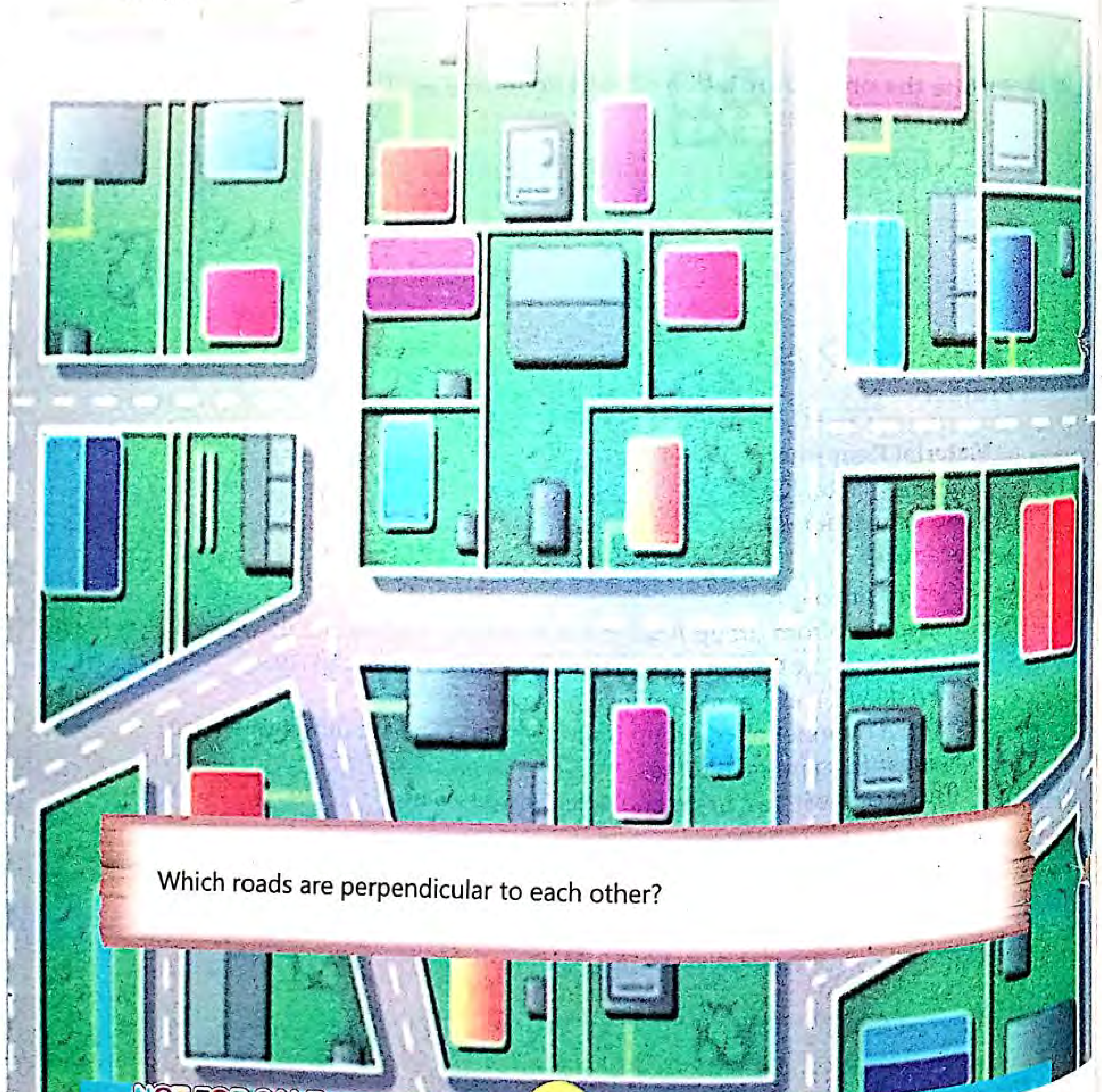


Unit 10 Geometrical Constructions

Student Learning Outcomes

After completing this unit, students will be able to:

- Construct a perpendicular (from a point on the line and outside the line) and a perpendicular bisector.
- Construct angles of specific measures (30, 45, 60, 75, 90, 105 and 120) and bisect angles using a compass.



Which roads are perpendicular to each other?

Introduction

In the previous classes we have learned about construction of angle using protractor. Now we will learn about construction of angle by compass and also learn about bisection of line segment and angle.

10.1 Line and Bisector

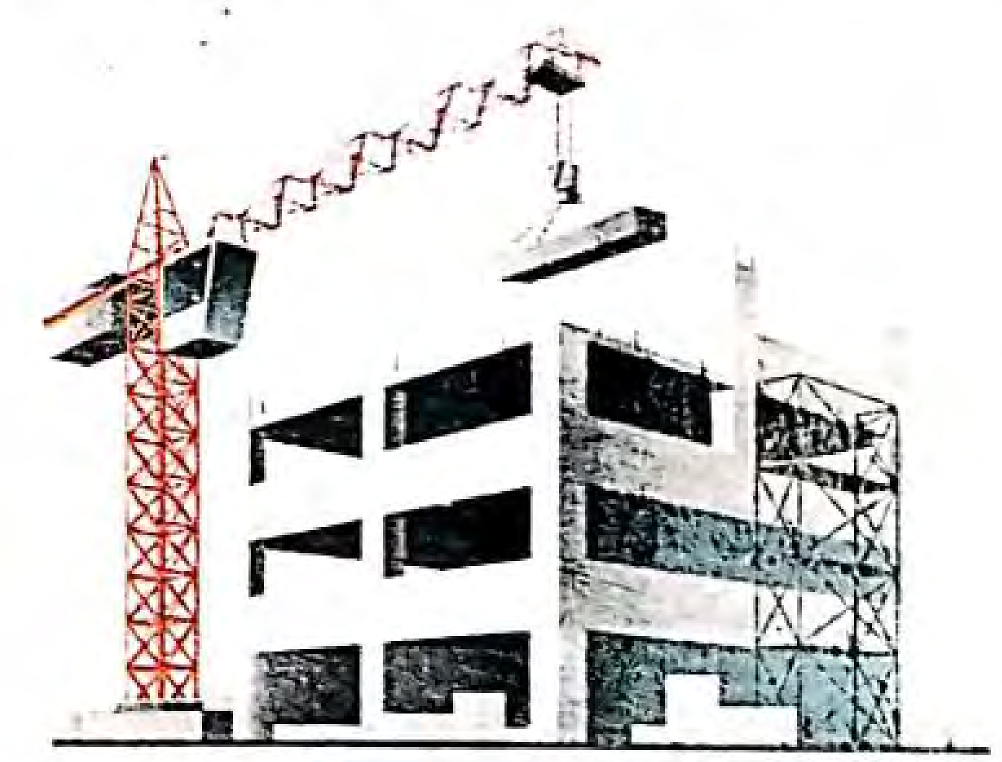
10.1.1 Line and Line Segment

A line is a **combination** of **points.** We cannot measure the length of a line. It continues in both directions and does not end. A line segment is part of a line. It has two end points. We read it as 'AB'.

Math History

Yaqub Ibn Ishaq Al-Kindi (801 AD- 873 AD), a Muslim mathematician, who worked on geometry, particularly on lines.

The concept of geometry is used in various daily life situations such as construction of buildings, bridges, stairs, etc. In geometry the concept of line is one of the basic concepts.

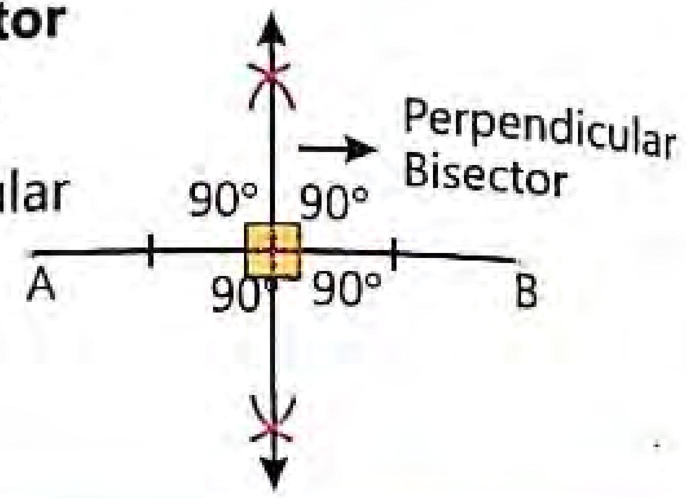


It helps to draw different angles and shapes. We have learnt to measure and draw line segments using a scale. Now, we shall learn how to draw a line segment using the compasses. We will also learn how to draw angles and triangles of different measures.

TEACHING POINT: Explain the concept of perpendicular bisectors by asking students to stand and make T using their arms. Teacher will explain how the body is acting as a perpendicular to their arms and the distance from their nose to tip of the finger on each side is equal.

10.1.2 Perpendicular Bisector

A bisector is a line that divides a given segment into two equal parts. It doesn't necessarily bisect at 90°. **Perpendicular bisector** is a line that divides a given line segment into two equal parts and make an angle of 90° at the point of bisection. Perpendicular bisector always passes through the midpoint of the line segment.



Note it down

- In Greek, "geo" means earth, and "metron" means measure. Egyptians were among the first people to use geometry to explore the land or earth.
- 'Bi' means two and bisection means to divide into two equal sections or parts.

Drawing of a right bisector of a given line segment

Draw a right bisector of a line segment $\overline{AB}$.

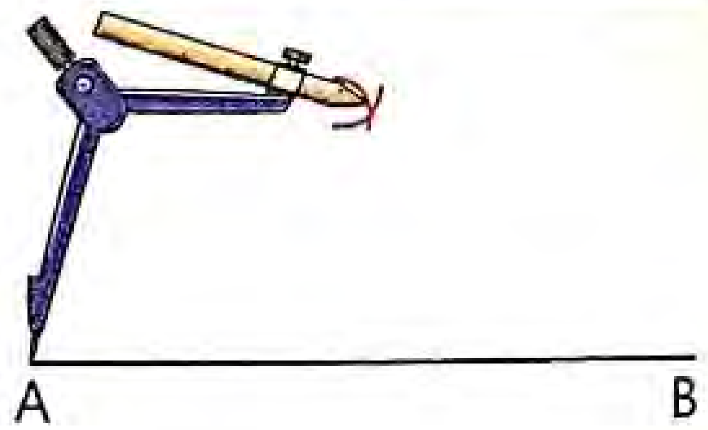
Steps of Construction:

A line segment $\overline{AB}$ is given.

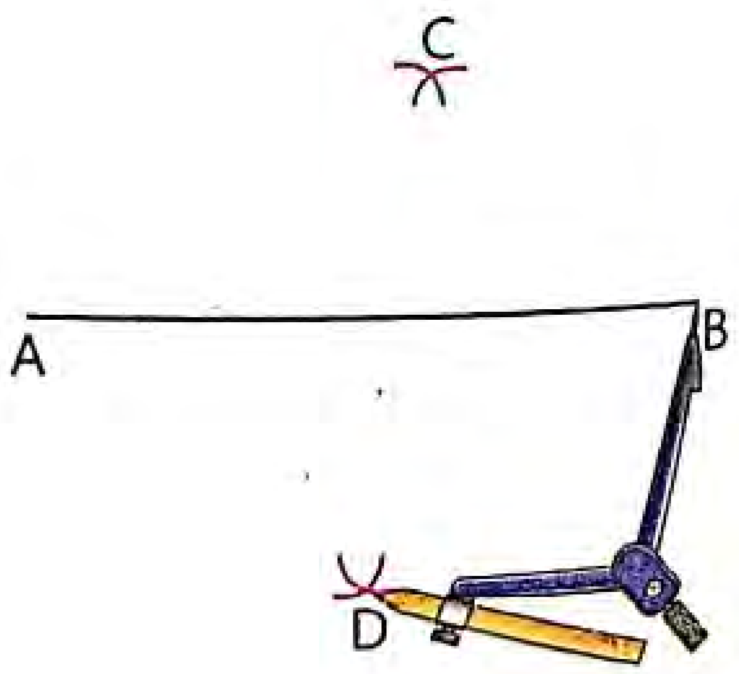
Previous Knowledge Check

What is point and how many dimensions a point has?
What is a perpendicular line?
What is right angle?

Step I: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment $\overline{AB}$ and draw an arc on the top and bottom of $\overline{AB}$.

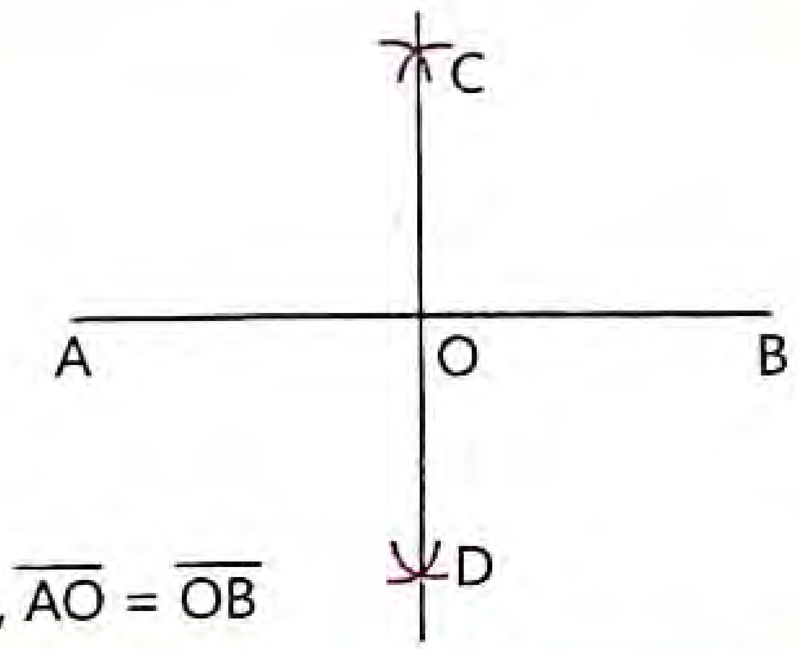


Step II: Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arc on the top and bottom of $\overline{AB}$ which intersects the previous arcs at point C and D respectively.



Teachers can use the following web link to show construction of angle and line bisector.
https://www.mathspad.co.uk/i2/construct.php

Step III: Join the point C and D by a line using a ruler that will cut the line segment $\overline{AB}$ at point O.



$\overline{CD}$ is the right bisector of $\overline{AB}$ and O is the mid point. So, $\overline{AO} = \overline{OB}$

Quick Check

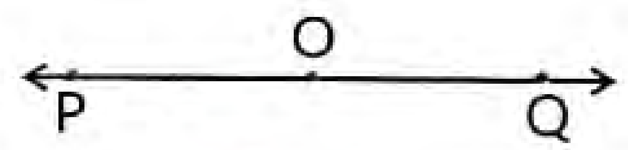
Draw a 7.2 cm line segment and draw a perpendicular bisector on it.

10.1.3 Drawing of a perpendicular to a given line on a point on it

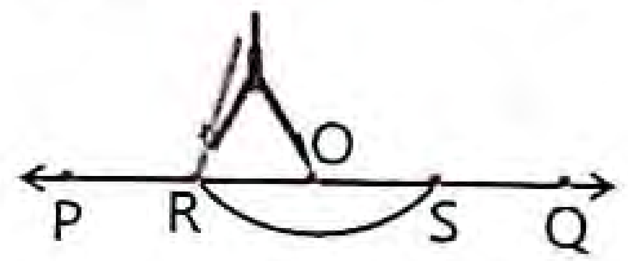
To draw a perpendicular on a given line from a point on it using the compass, we follow the steps given below.

Steps of Construction:

$\overleftrightarrow{PQ}$ is the given line and O is a point on it.

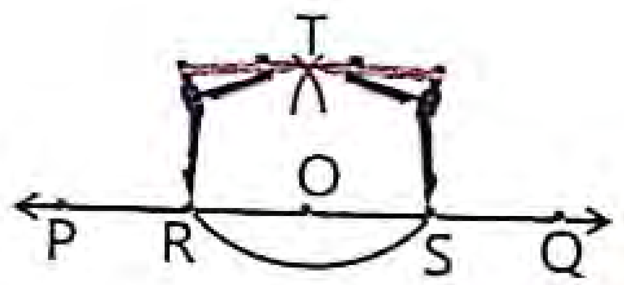


Step I: Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the line $\overleftrightarrow{PQ}$ at two points R and S respectively, such that $\overline{OR} = \overline{OS}$.



Step II: Place the pointer of the compass at point R and draw an arc of radius greater than $\overline{OR}$ as shown in the figure.

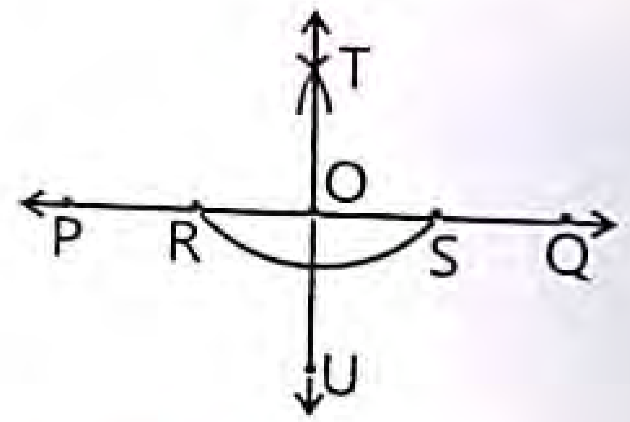
Step III: With the same opening of the compass, place the pointer of the compass at point S and draw an arc which cuts the previous arc at point T as shown in the figure.



Step IV: Join the point T to O and extend it to U.

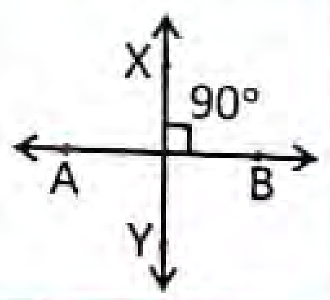
So, $\overleftrightarrow{TU}$ is the required perpendicular to the given line $\overleftrightarrow{PQ}$ at point O.

$\therefore \overleftrightarrow{TU} \perp \overleftrightarrow{PQ}$



Note it down

Two lines or rays are said to be perpendicular to each other if the angle formed between them is a right angle.



Quick Check

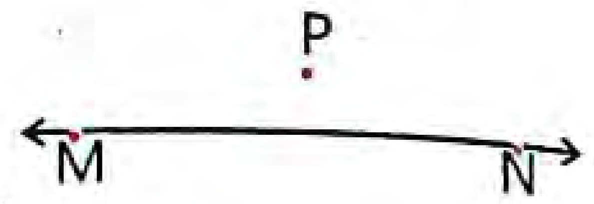
Draw perpendicular from the point P to the line segment $\overline{AB}$.

10.1.4 Drawing of a perpendicular to a given line from a point not on the line

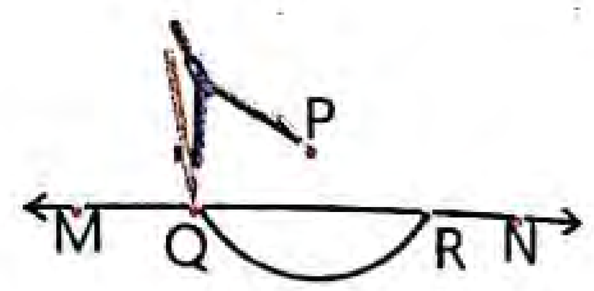
To draw a perpendicular on a given line and through a given point not on line using the compass, follow these steps.

Steps of Construction:

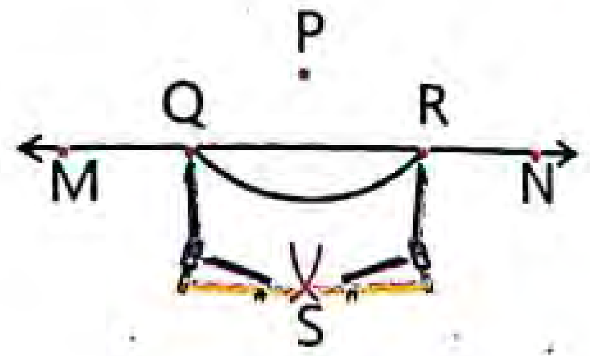
$\overleftrightarrow{MN}$ is the given line. A point P (not on it) is also given.



Step I: Place the pointer of the compass at point P and draw an arc that will cut the line $\overleftrightarrow{MN}$ at two points Q and R respectively.

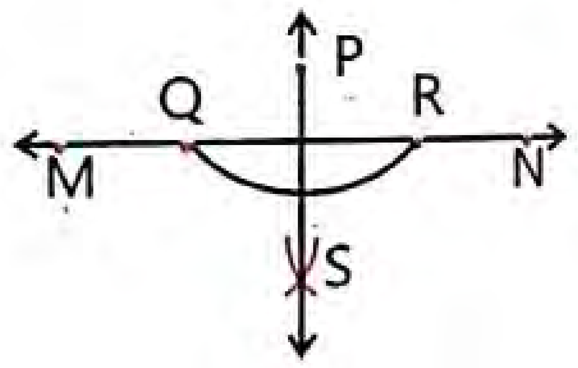


Step II: Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overleftrightarrow{MN}$ that cut each other at point S.



Step III: Join P to S. $\overleftrightarrow{PS}$ is the required perpendicular to the given line $\overleftrightarrow{MN}$.

$\therefore \overleftrightarrow{PS} \perp \overleftrightarrow{MN}$

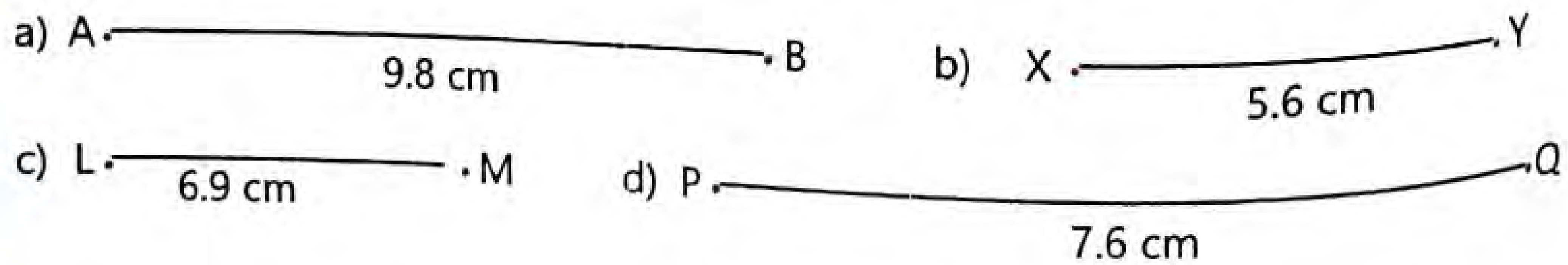


Quick Check

Draw perpendicular from the point Y to the line segment LM.

Exercise 10.1

1 Draw the right bisectors of the following line segments using a pair of compasses.

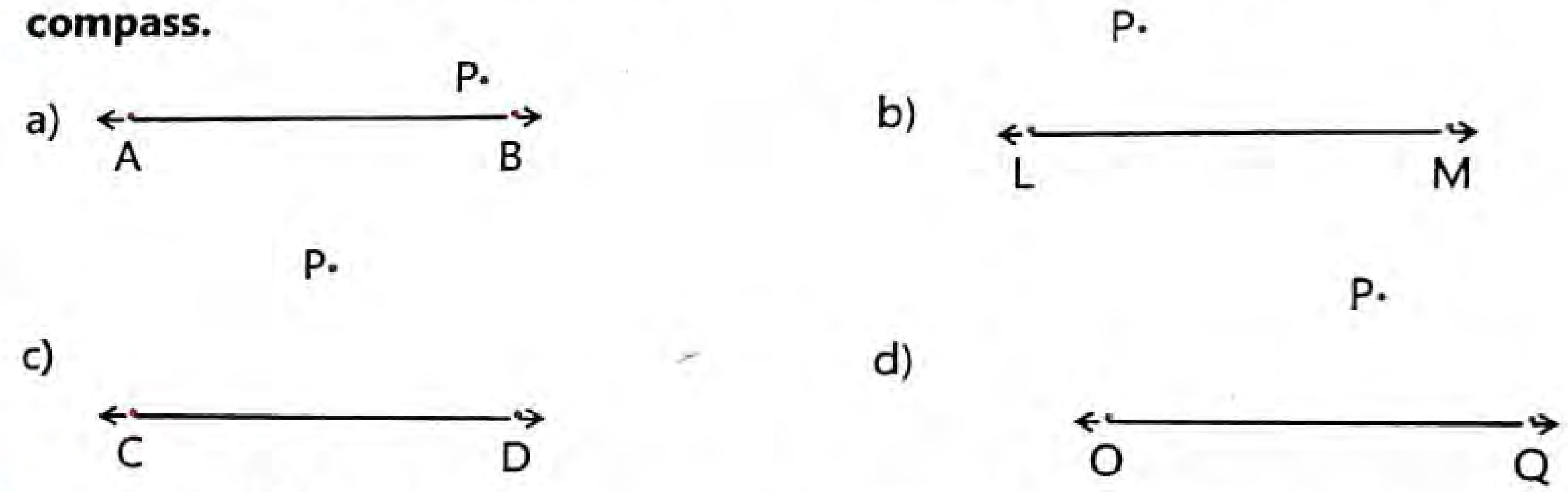


2 Draw the following line segments and bisect them.

a) 9 cm b) 7.9 cm c) 8.6 cm d) 9.5 cm

3 Draw perpendiculars to the lines from the given points on it using a compass.

4 Draw perpendiculars to the given lines from the point P not on it using a compass.



5 Draw a line $\overleftrightarrow{AB}$ and draw a perpendicular on it from a point C not on it.

6 Draw a line $\overleftrightarrow{LM}$ and draw a perpendicular on it from point F on it.

7 Draw a line segment m $\overline{PQ}$ = 7.5 cm. Draw a perpendicular from point S not on it.

8 Draw a line segment m $\overline{XY}$ = 8 cm. Construct a perpendicular at point Z on it such that $\overline{XZ}$ = 3 cm .

10.2 Angles and their construction

As we know that when two non-parallel lines meet at a common end point, they form **an angle**. We are familiar with the method of constructing an angle using a protractor. Now we will learn how to construct an angle using the compass.

> **Note it down**
> The line /ray dividing an angle in two equal angles is called bisector of that angle.

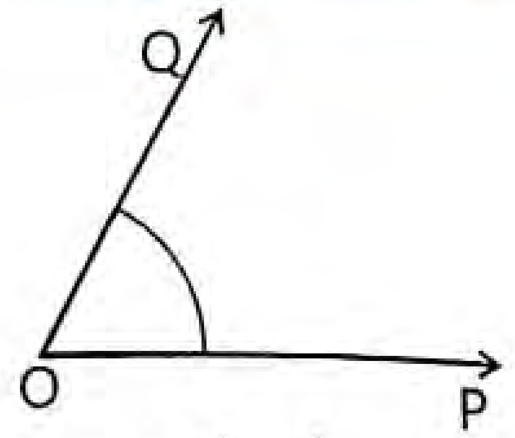
10.2.1 Bisection of an angle

When we divide an angle into two equal parts is called **bisection of the angle**.

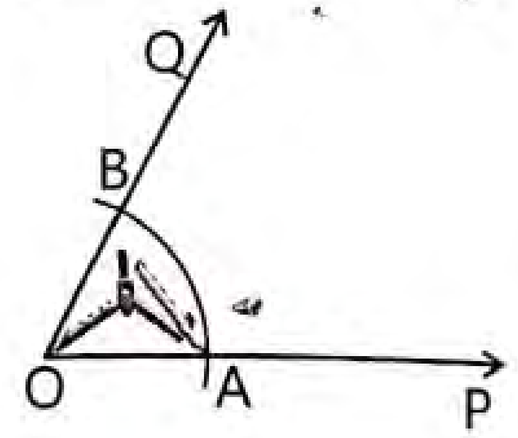
Example 1:

Bisect ∠POQ = 60° using a compass and ruler.

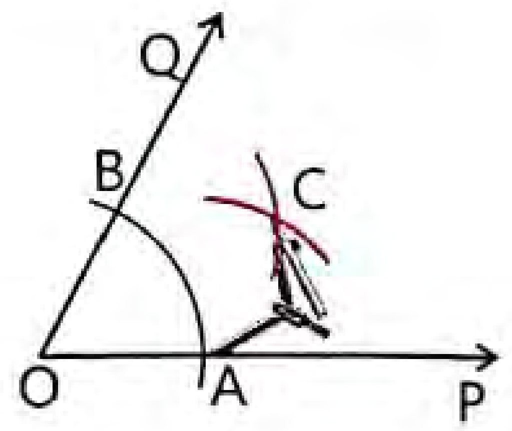
Solution:



Step I: Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OP}$ at any point A and cuts the ray $\overrightarrow{OQ}$ at any point B.

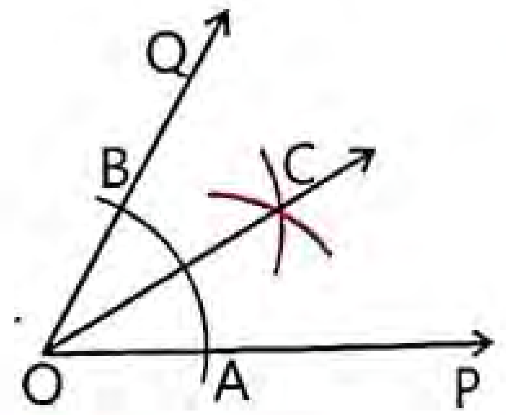


Step II: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



Step III: Using a ruler draw a ray $\overrightarrow{OC}$ passing through the point C this line is bisecting the angle ∠POQ into two equal parts.

So, m∠POC = m∠QOC.



Step IV: Using a ruler draw a ray OC passing through the point C and bisect the angle ∠POQ = 60° into two equal parts.

So, m∠POC = m∠QOC = $\frac{60^\circ}{2}$ = 30°

> **Quick Check**
>
> Construct a bisector of a 70° angle using a pair of compasses.

10.2.2 Construction of Angles using a compass

Example 1:

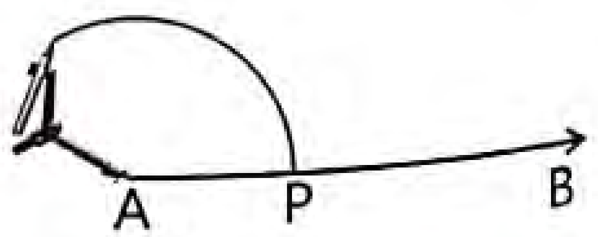
Draw ∠CAB = 60°.

Solution:

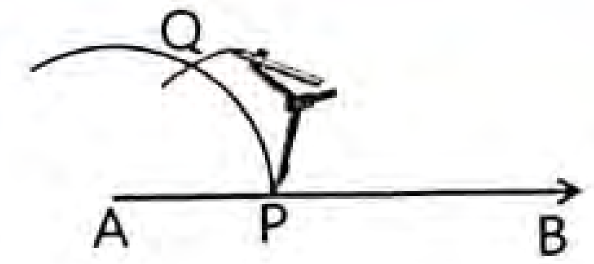
Steps of Construction:

Step I: Draw a ray $\overrightarrow{AB}$ using a ruler.

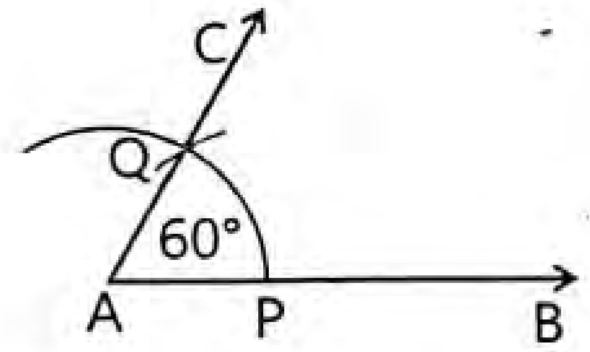
Step II: Place the pointer of the compass at point A and draw an arc of suitable radius that will cut the ray $\overrightarrow{AB}$ at a point P.



Step III: With the same opening of the compass, place the pointer of the compass at point P and draw an arc that will cut the previous arc at point Q.



Step IV: Using a ruler draw a ray $\overrightarrow{AC}$ passing through the point Q. ∠CAB = 60° is the required angle.



Example 2:

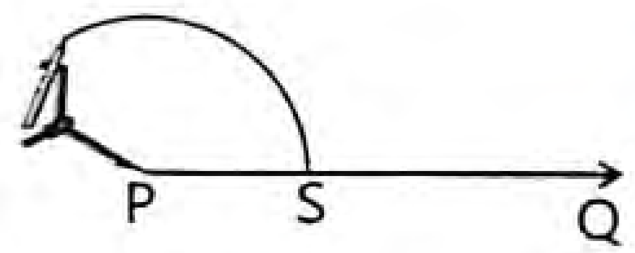
Draw ∠QPR = 30°.

Solution:

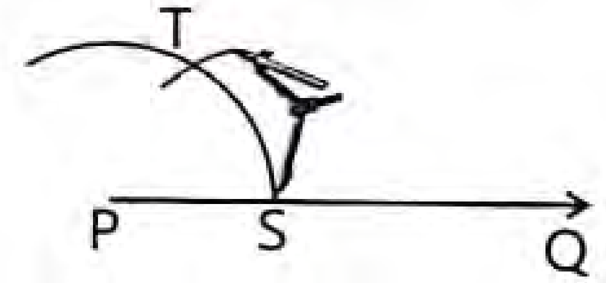
Steps of Construction:

Step I: Draw a ray $\overrightarrow{PQ}$ using a ruler.

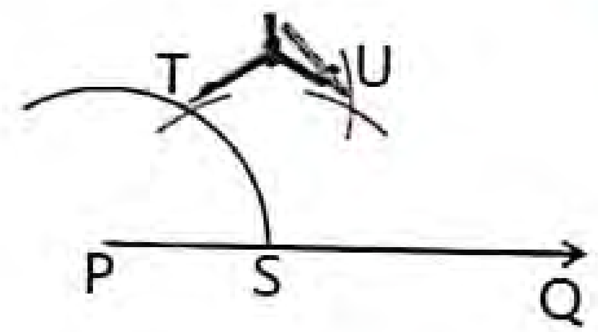
Step II: Place the pointer of the compass at point P and draw an arc of suitable radius that will cut the ray $\overrightarrow{PQ}$ at a point S.



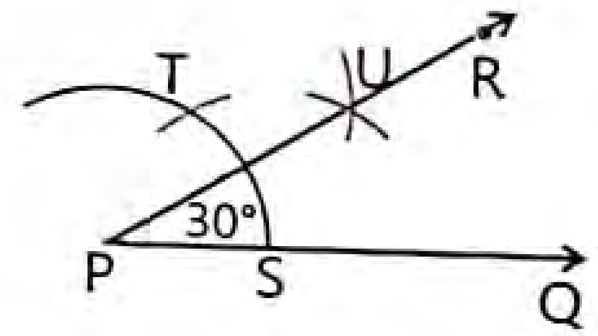
Step III: With the same opening of the compass, place the pointer of the compass at point S and draw an arc that will cut the previous arc at point T.



Step IV: Again with the same opening of the compass, place the pointer of the compass at point S and T and draw two arcs of same radius that will cut each other at point U.



Step V: Using a ruler draw a ray $\overrightarrow{PR}$ passing through the point U. ∠QPR = 30° is the required angle.



> **Note it down**
>
> If we bisect a 60° angle, we get two 30° angles.

TEACHING POINT: Share the following online quiz link of line and angle bisector.
https://www.mathgames.com/skill/8.111-measures-of-bisected-lines-and-angles

Example 3:

Draw ∠AOB = 90°.

Solution:

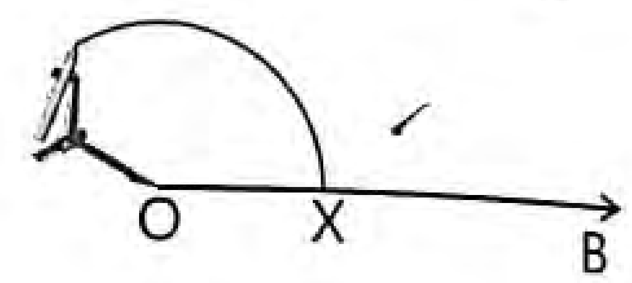
Steps of Construction:

Quick Check

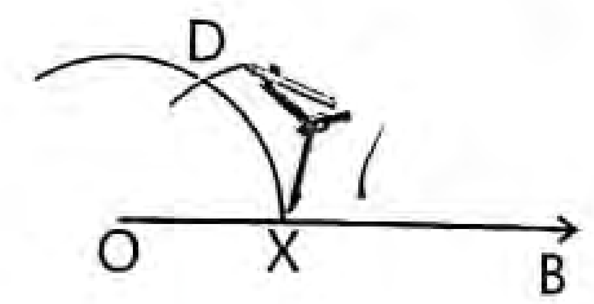
Construct 30° using a pair of compasses.

Step I: Draw a ray $\overrightarrow{OB}$ using a ruler.

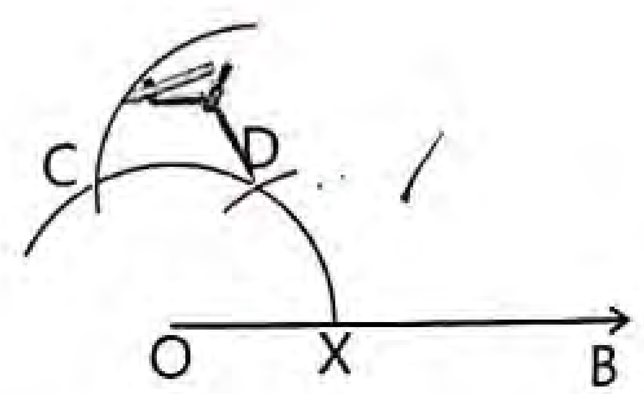
Step II: Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at a point X.



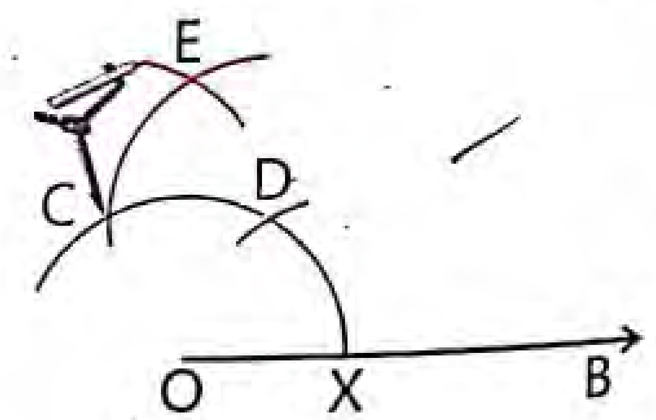
Step III: With the same opening of the compass, place the pointer of the compass at point X and draw an arc that will cut the previous arc at point D.



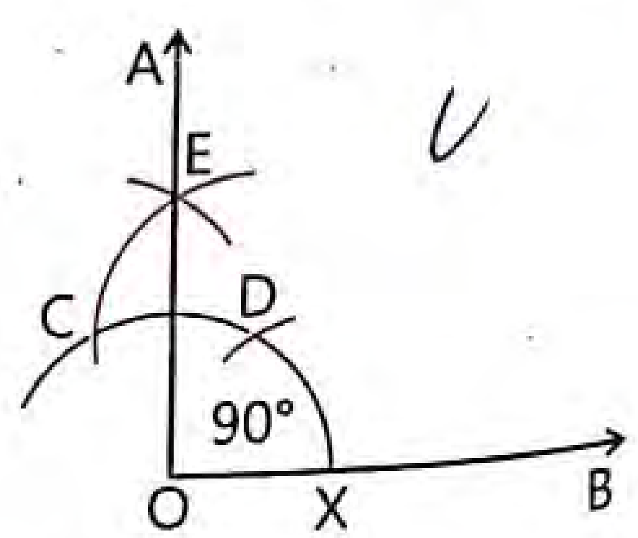
Step IV: Again using the same opening of the compass, place the pointer of the compass at point D and draw an arc that will cut the previous arc at point C.



Step V: Place the pointer of the compass at point D and C and draw two arcs of same radius that will cut each other at point E.



Step VI: Using a ruler draw a ray $\overrightarrow{OA}$ passing through the point E. ∠AOB = 90° is the required angle.



Note it down

If we bisect a 90° angle, we get two 45° angles.

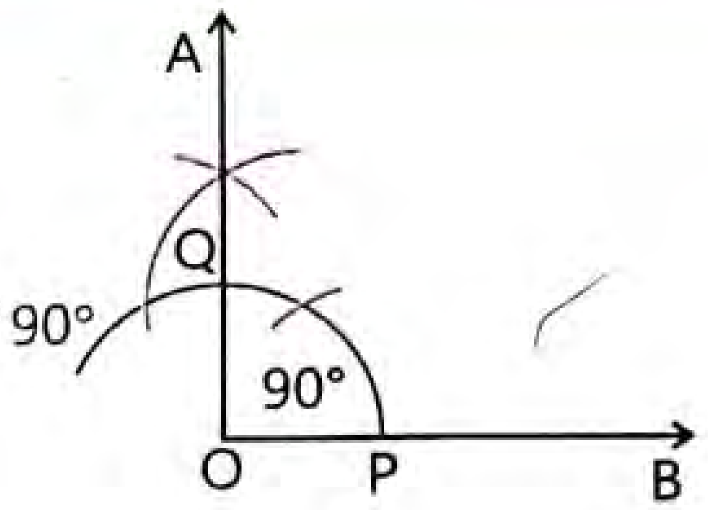
TEACHING POINT: Explain to the students by drawing bisectors we can draw angles which are half in measure of the angles.

Example 4:

Draw ∠BOE = 45°.

Solution:

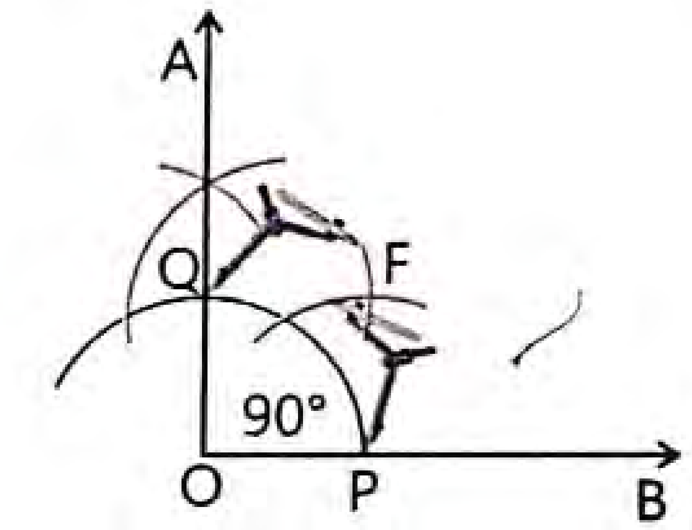
To construct the ∠BOE = 45°, first we construct an angle of 90°.



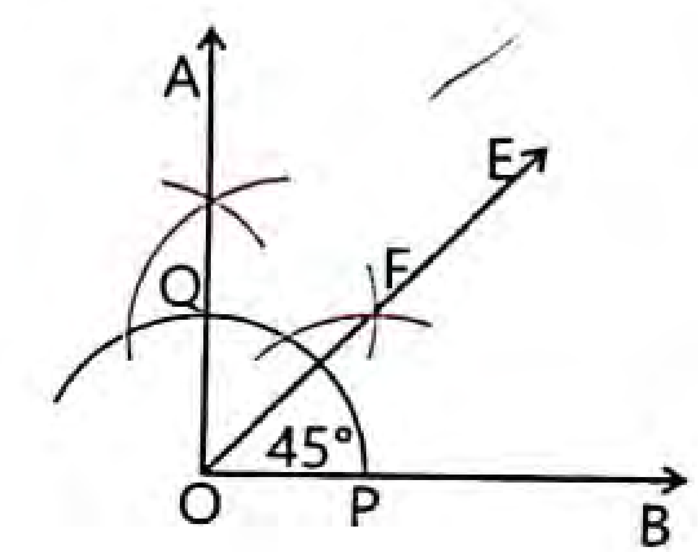
Steps of Construction:

Step I: Draw ∠AOB = 90°.

Step II: Place the pointer of the compass at point P and Q and draw two arcs of same radius that will intersect each other at point F.



Step III: Using a ruler draw a ray $\overrightarrow{OE}$ passing through the point F. ∠BOE = 45° is the required angle.



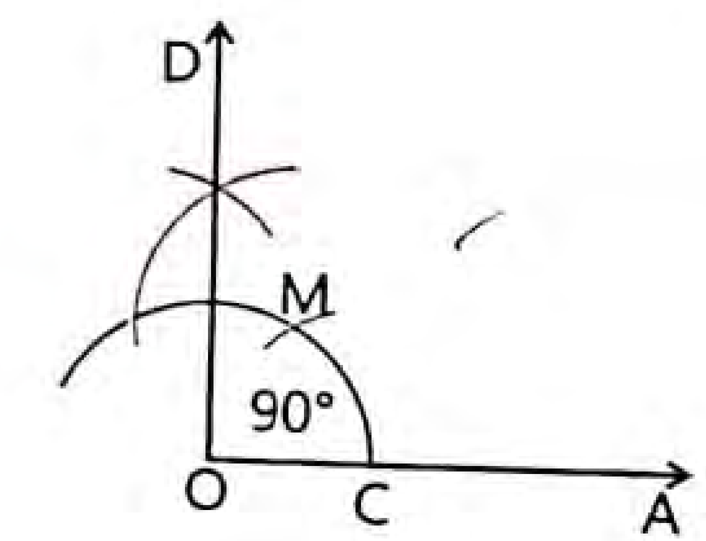
Example 5:

Draw ∠AOE = 75°.

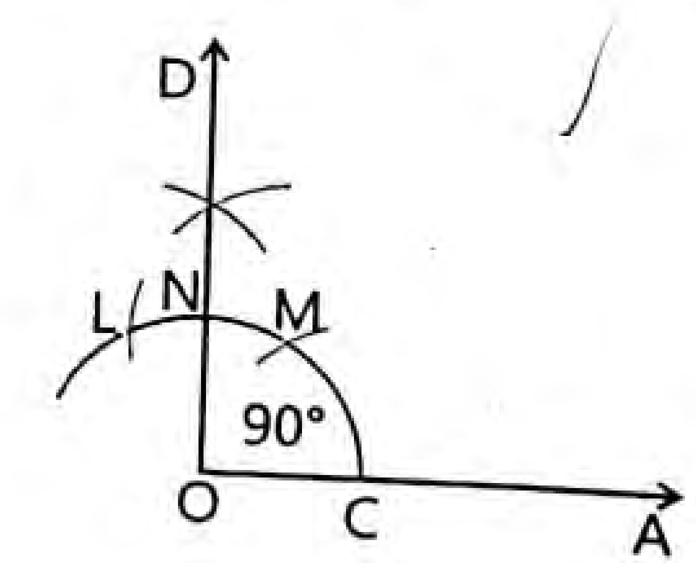
Solution:

Steps of Construction:

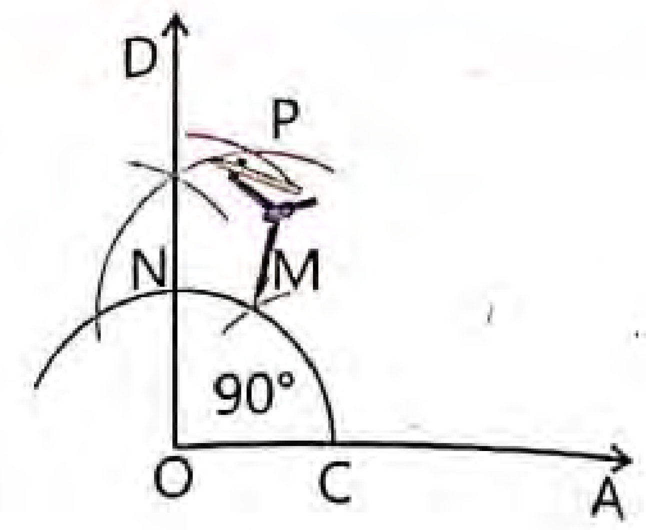
Step I: Draw an ∠AOD equal to 90°.



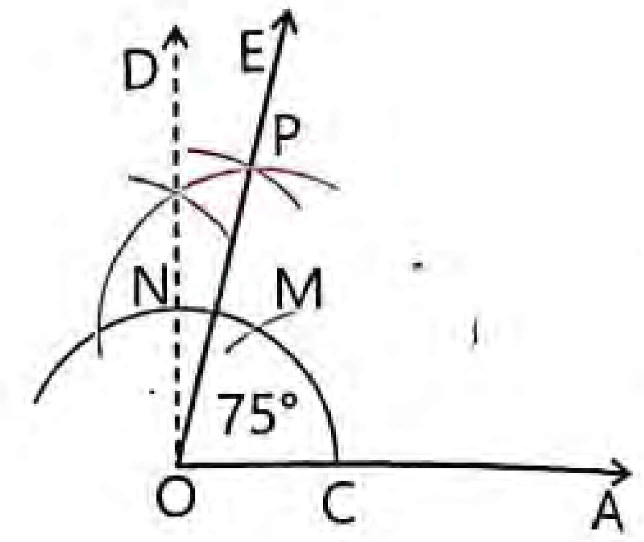
Step II: Mark a point N, where the arm $\overrightarrow{OD}$ of ∠AOD is cut by the arc $\overset{\frown}{CL}$.



Step III: Place the pointer of the compass at point M and N and draw two arcs of same radius that will cut each other at point P.



Step IV: Using a ruler draw a ray $\overrightarrow{OE}$ passing through the point P. ∠AOE = 75°.



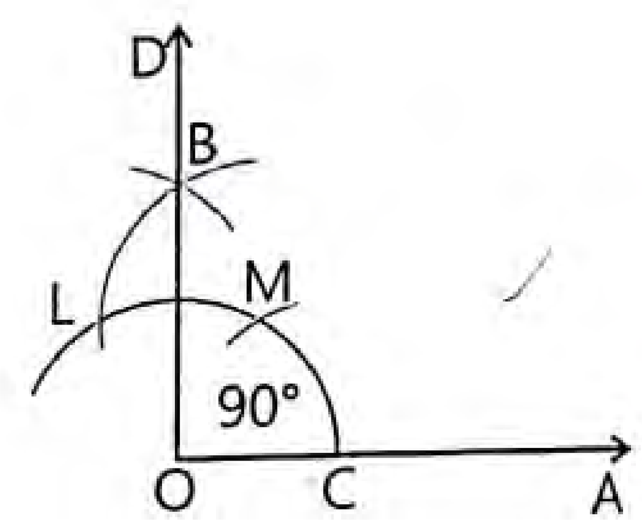
Example 6:

Draw ∠AOG = 105°.

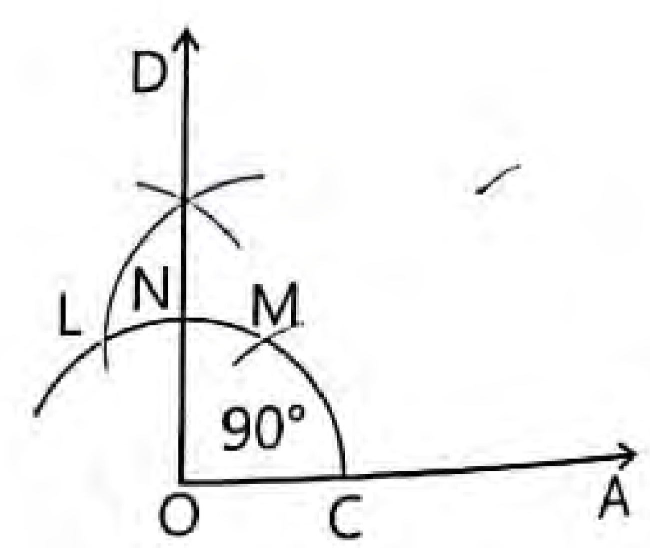
Solution:

Steps of Construction:

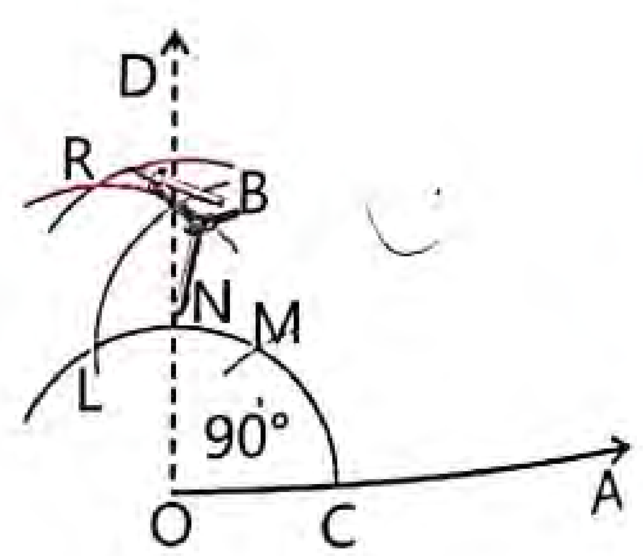
Step I: Draw an ∠AOD = 90°.



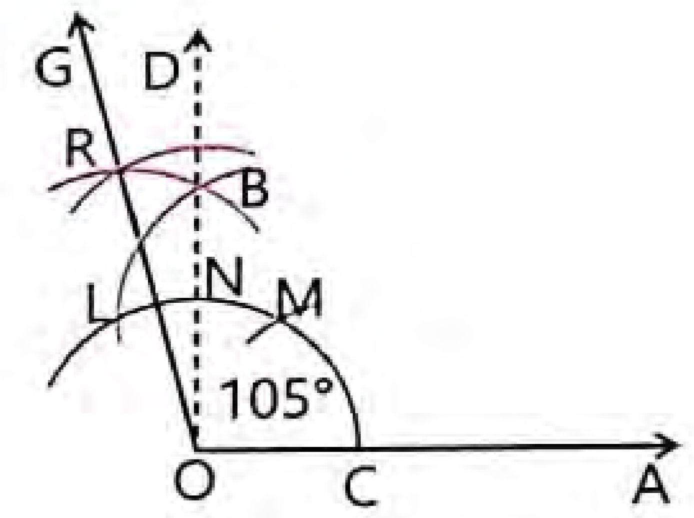
Step II: Mark a point N, where the arm $\overrightarrow{OD}$ of ∠AOD is cut by the arc $\overset{\frown}{CL}$.



Step III: Place the point of the compass at point N and L and draw two arcs of same radius that will cut each other at point R.



Step IV: Using a ruler draw a ray $\overrightarrow{OG}$ passing through the point R. So, ∠AOG= 105°



Example 7:

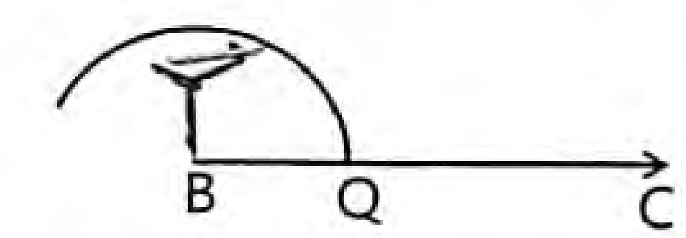
Let's draw ∠ABC = 120°.

Solution:

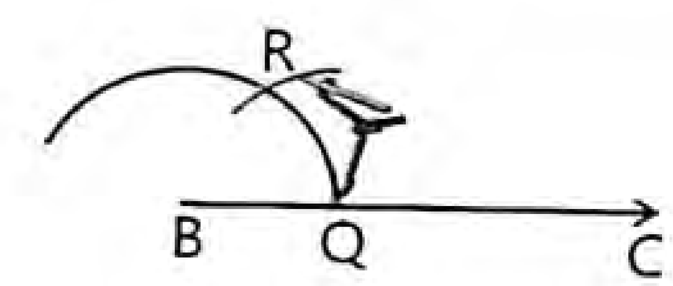
Steps of Construction:

Step I: Draw a ray $\overrightarrow{BC}$ using a ruler.

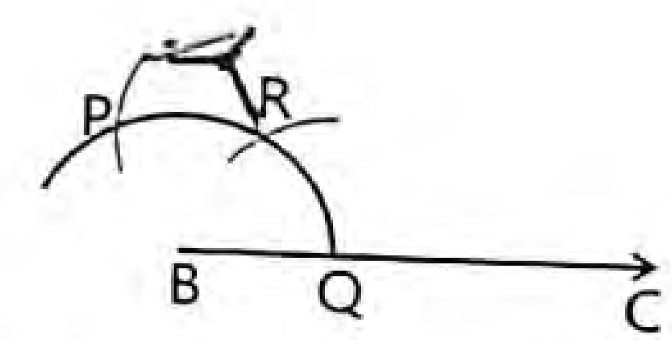
Step II: Place the pointer of the compass at point B and draw an arc of suitable radius that will cut the ray $\overrightarrow{BC}$ at any point Q.



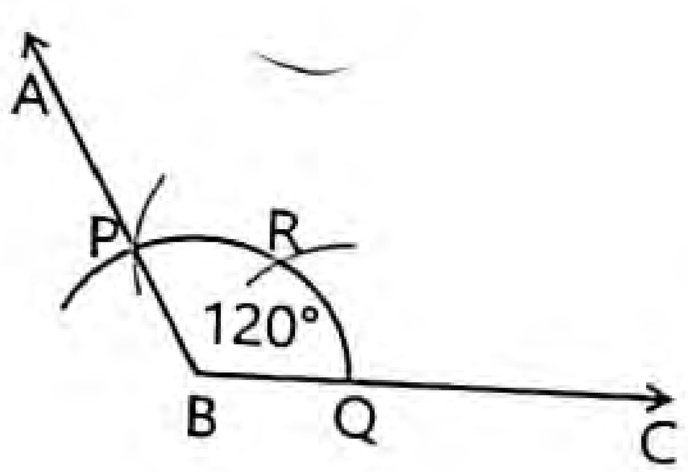
Step III: With the same opening of the compass, place the pointer of the compass at point Q and draw an arc that will cut the previous arc at point R.



Step IV: With the same opening of the compass, place the pointer of the compass at point R and draw an arc that will cut the initial arc at point P.



Step V: Using a ruler draw a ray $\overrightarrow{BA}$ passing through the point P. So, the ∠ABC = 120°.



Exercise 10.2

1 **Construct the following angles by using a protractor and bisect them by using a pair of compasses.**

a) 50° b) 80° c) 72° d) 110° e) 150°

2 **Construct the following angles by using a pair of compasses. Then bisect them.**

a) 30° b) 45° c) 60° d) 75°
e) 90° f) 105° g) 120°

Think Higher

Construct a 120° angle and write steps of construction. Then explains step by step how a 30° angle can be constructed from the 120° angle.

Summary

- A line segment is a part of a line which has two distinct end points and definite length.
- A perpendicular line segment which cuts the other line segment into two equal parts, is called a right bisector of the line segment.
- When two non-parallel lines meet at a common end point, they form an angle.
- When we divide an angle into two equal parts, It is called bisection of the angle.
- The line /ray dividing an angle in two equal angles is called the bisector of that angle.

Vocabulary

- Line
- Line segment
- Bisector
- Angle bisector
- Ruler
- Compass

Review Exercise

1 **Choose the correct option.**

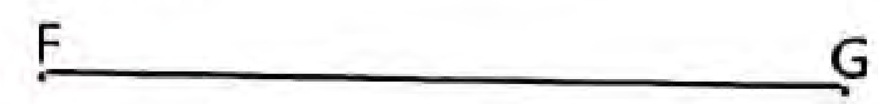
a) The word geometry comes from two __________ words.
i. Latin ii. Greek iii. German iv. Arabic

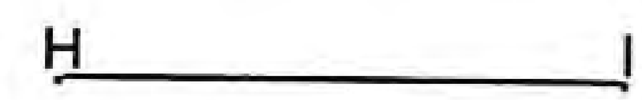
b) Bisection means to divide the line into __________ equal parts.
i. one ii. two iii. three iv. four

c) Two lines or rays are said to be perpendicular to each other if the angle formed between them is __________.
i. 60° ii. 45° iii. 30° iv. 90°

d) If we bisect a 90° angle, we get two __________ angles.
i. 30° ii. 15° iii. 40° iv. 45°

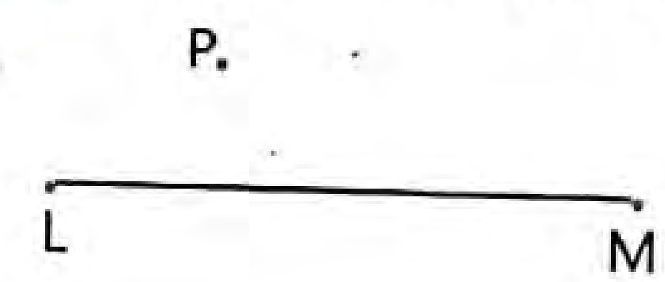
2 **Draw the right bisectors of the following line segments by using a pair of compasses.**

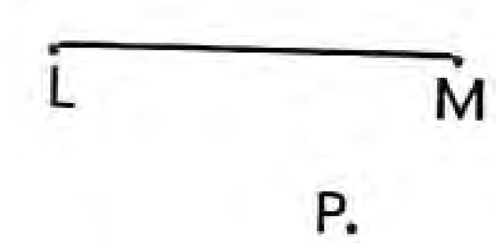
a) 

b) 

c) 

3 **Draw perpendiculars from the point P to the line segment LM.**

a) 

b) 

4 **Construct the following angles by using a protractor and bisect them by using a pair of compasses.**

a) 100° b) 85°

5 **Construct the following angles by using a pair of compasses. Then bisect them.**

a) 30° b) 45° c) 90° d) 120°

Math Project

Material Required:
- Basket
- Paper slips
- Board Marker
- Compass
- Ruler
- Pencil
- Protractor

Procedure:
- Divide the students in groups.
- Place a basket with cards with variety of instructions regarding angles, lines etc (for example:
  - "Draw a line segments 5.5 cm and bisect it.
  - Construct the angle 50° by using a protractor and bisect it by using a pair of compasses".
- Students from each group will come forward and pick a card from the basket and follow the instruction written on it.
- The group with most accurate answers wins.

Draw a line segments 5.5 cm and bisect it.

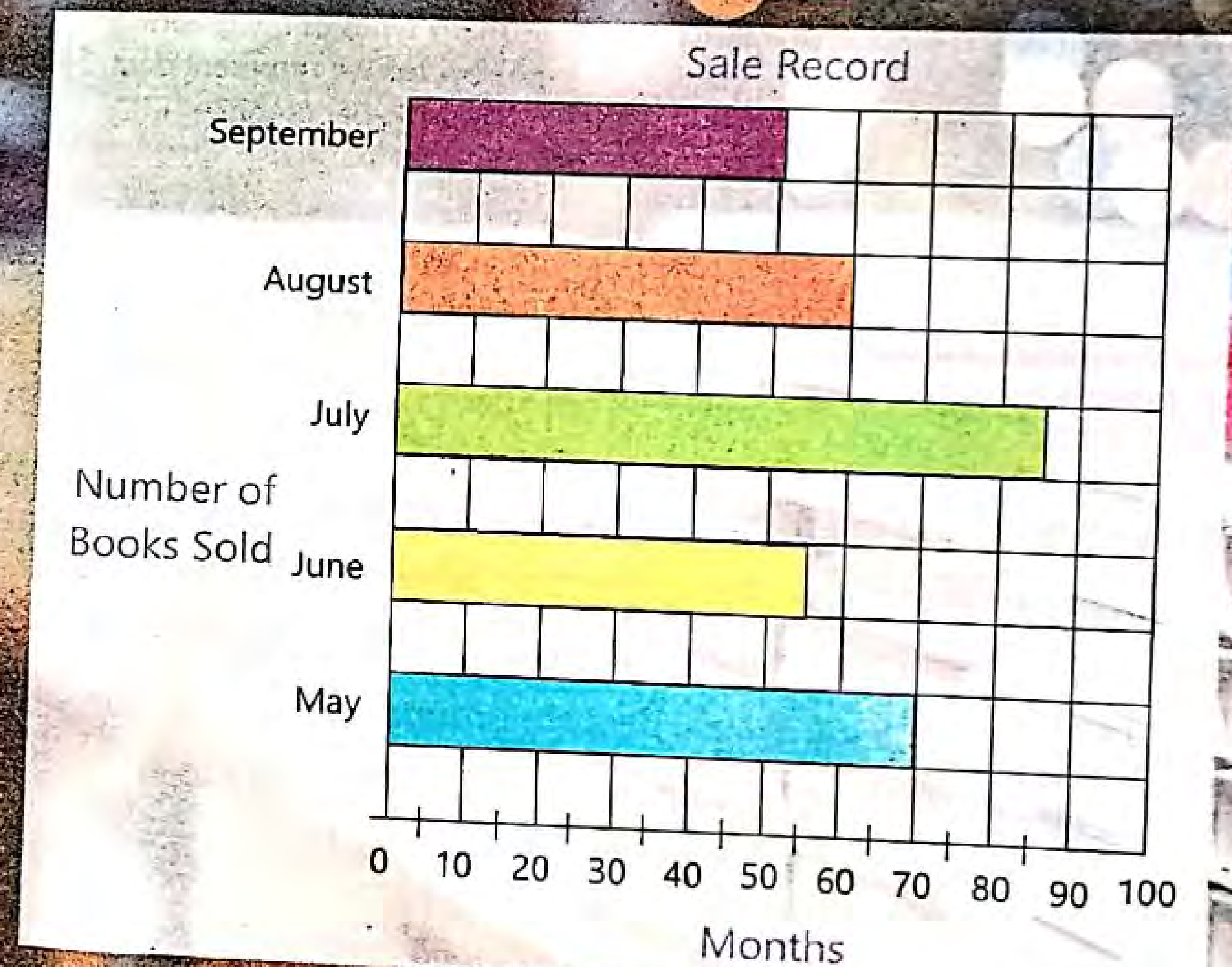
Draw perpendicular from the point P to the line segment LM.

Construct the angle 50° by using a protractor and bisect it by using a pair of compasses.

Unit 11 Data Management

Student Learning Outcomes

After completing this unit, students will be able to:
- Draw, read and interpret horizontal and vertical multiple bar graphs and pie charts. (including real-world problems)
- Identify and organize different types of data (i.e., discrete, continuous, grouped and ungrouped).
- Calculate the mean, median and mode for ungrouped data and solve related real-world problems.
- Explain experiments, outcomes, sample space, events, equally likely events and probability of a single event. Differentiate the outcomes that are equally likely and not equally likely to occur. (including real-world word problems).



Look at the graph showing the sale of books from May to September in a bookshop. Are these statements true? Justify your answer.
- The greatest number of books were sold in July.
- The difference between the number of books sold in June and September is 5.
- Altogether 130 books were sold in May and August.

Introduction

The information on the basis of which these graphs are drawn is called "data". We have already learnt to express the given information in pictographs, line graphs and simple bar graphs. Now we will learn more about graphs like drawing, reading and interpreting multiple or grouped bar graph, pie charts, organizing different types of data and measure of central tendency.

11.1 Graphs

Organising data in graphs gives us clear information and we can easily interpret and compare data. We know that a picture graph, bar graph, line graph etc. are used to organise collected data. Now we will learn about drawing bar graphs.

Previous Knowledge Check

- What is meant by data and organization of data?
- What is bar graph? How many types of bar graph?
- How can we represent data by using bar graph?

11.1.1 Multiple Bar Graph

To organize categories of data, we usually use a bar chart. A **bar graph** or **bar chart** is represented by the height or length of bars of equal width. One bar of the graph represents one observation or quantity.

Now we are going to learn how to draw and interpret a multiple bar graph. A multiple bar graph is an extension of simple bar graph. It shows the connection between different values of data. Each value is represented by a vertical or horizontal bar in the graph. Two or more than two categories of different kinds of data are drawn. Bars are grouped by position for values of data to be shown or compared. Different colours are assigned for clarity to distinguish categories of data.

Drawing a Multiple Bar Chart

The steps of drawing a vertical or **horizontal multiple bar chart** is similar to the one used in simple bar graph drawing. The difference is that different shades or colours are used to distinguish between different categories.

Let's learn how to draw a multiple bar chart by observing some examples.

Example 1:

The following data shows the number of girls and boys in 5 sections of grade 6 in a school. Let's draw a vertical multiple bar graph for this data.

Note it down

A graph is the pictorial representation of data that shows the relationship between two quantities.

Section	6A	6B	6C	6D	6E
No. of Boys	12	10	18	14	16
No. of Girls	16	20	8	15	12

Solution:

We use the following steps to draw the bar graph.

Step I: Draw an x-axis (horizontal line) and y-axis (vertical line) perpendicular to each other on graph paper.

Step II: Write the sections along the x-axis and the number of students along the y-axis.
Choose a colour for each category (girls and boys).

Note it down

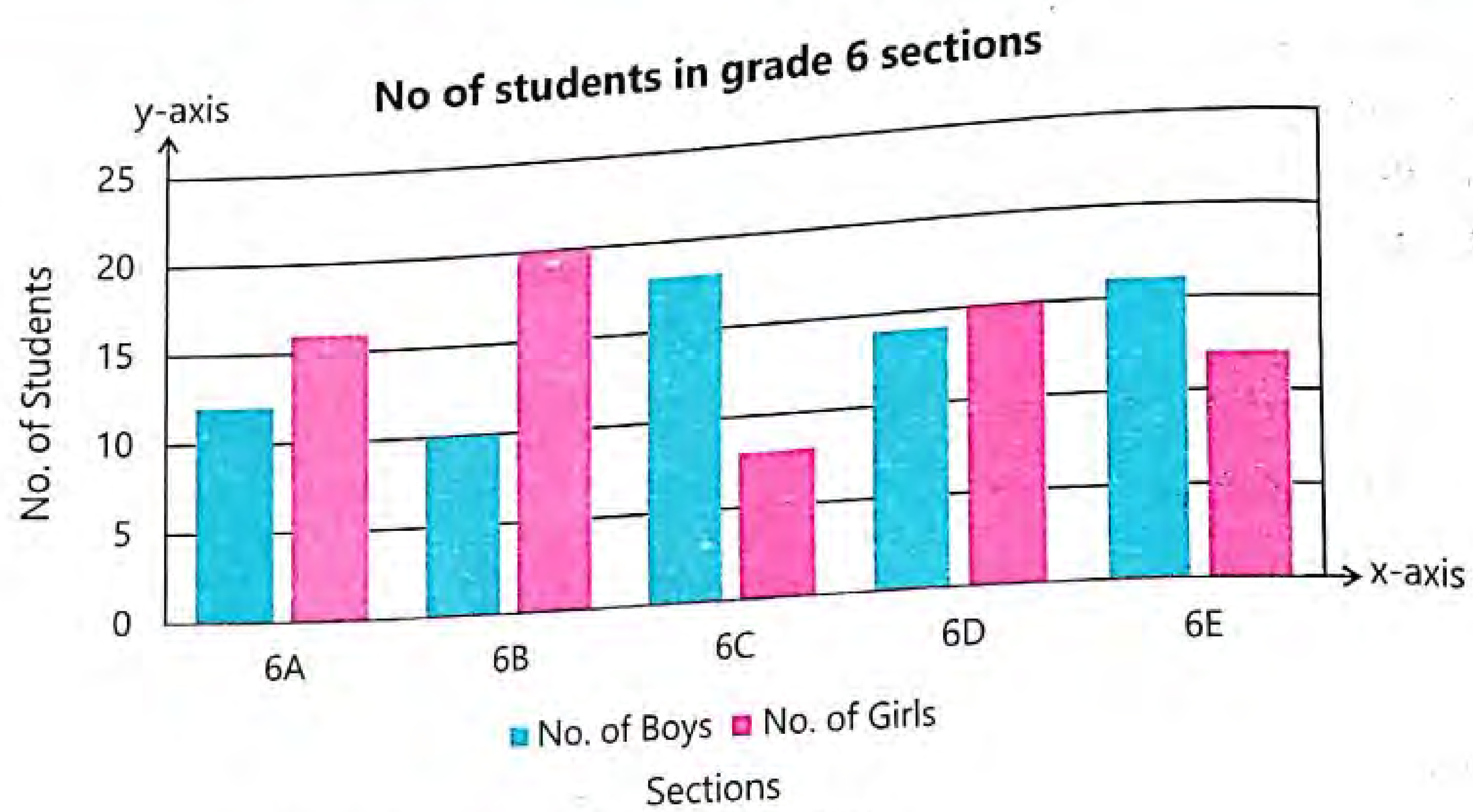
In a graph, the horizontal line represents the x-axis and the vertical line represents the y-axis.

Step III: Choose an appropriate scale. Here 1 mark represents 5 students along the y-axis.

Step IV: In section 6A, here are 12 boys and 16 girls according to the data given in the table. So we will colour 6 marks for boys(using the pre-assigned colour) and 8 marks for girls (using the pre- assigned colour).

Step V: In section 6B, here are 10 boys and 20 girls according to the data given in the table. So we will colour 5 marks for boys (using the pre-assigned colour) and 10 mark for girls (using the pre- assigned colour).

Similarly, keep following the steps and colour the correct numbers of marks for each section and draw the multiple-bars. The width of the bars must be the same throughout the multiple-bar graph.



This is the required multiple-bar graph drawn vertically.

Example 2:

The following data shows the sale of different numbers of cars (in different colours) sold during 5 months in a showroom. Let's draw a vertical multiple bar graph for this data.

Colours	May	June	July	August	September
Black	15	25	10	15	20
White	20	25	15	30	35
Silver	5	20	15	10	25

Solution:

We use the following steps to draw the bar graph.

Step I: Draw an X-axis (horizontal line) and Y-axis (vertical line) perpendicular to each other on graph paper.

Step II: Write the sections along the X-axis and the number of students along the Y-axis.

Choose a colour for each category (black, white, silver).

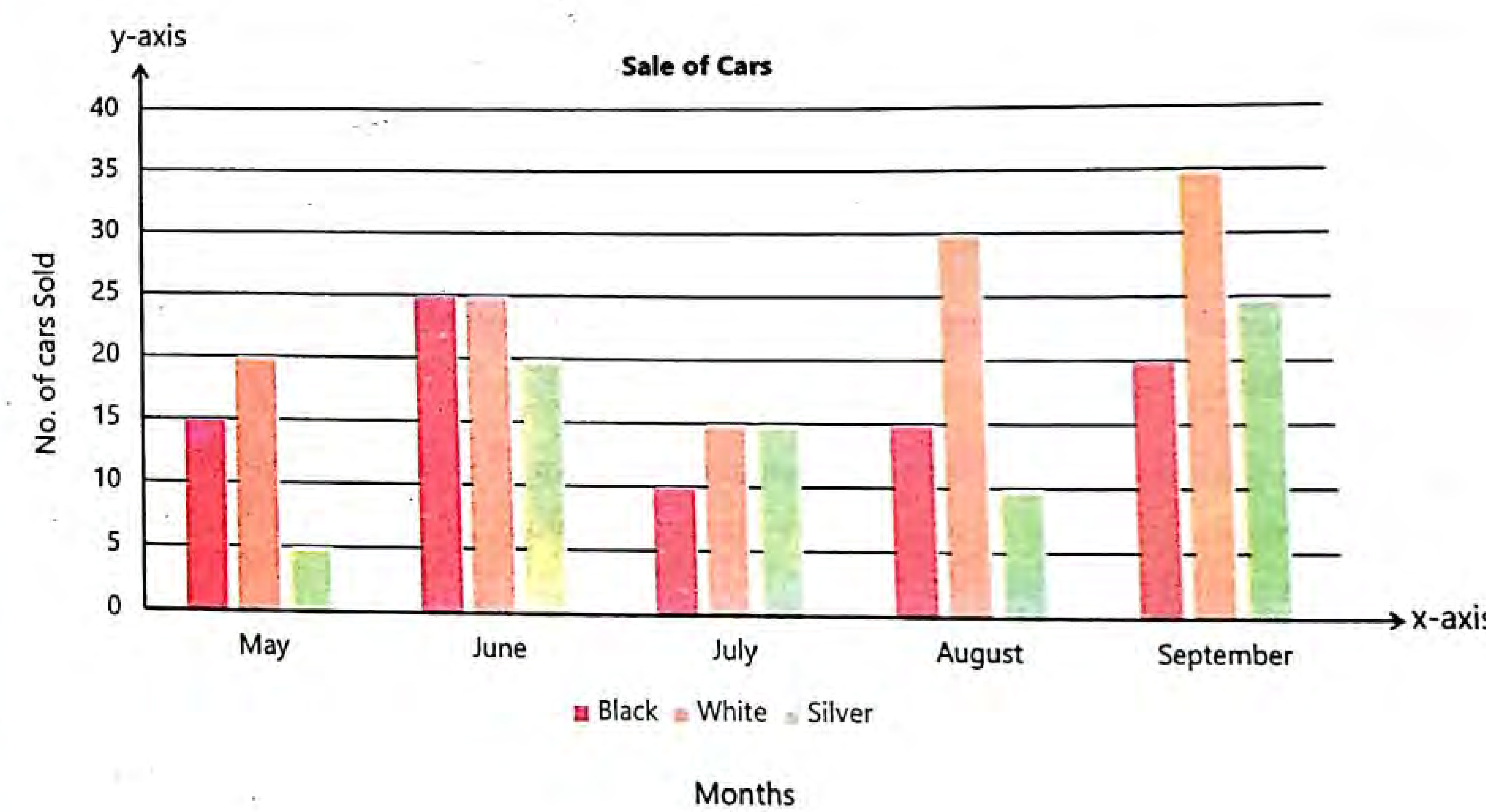
Step III: Choose an appropriate scale. Here one mark represents 5 cars along the Y-axis.

Step IV: In May, 15 black, 20 white and 5 silver cars were sold according to the data given in the table. So we will colour 3 marks for black cars (using the pre- assigned colour), 4 marks for white cars (using the pre- assigned colour) and 1 mark for silver cars (using the pre- assigned colour).

Step V: In June, 25 black, 25 white and 20 silver cars were sold according to the data given in the table. So we will colour 5 marks for both black and white cars(using the pre- assigned colour) and 4 marks for silver cars (using the pre- assigned colour).

Similarly, keep following the steps and colour the correct numbers of marks for each colour and draw the multiple-bars.

This is the required multiple-bar graph drawn vertically.



Divide the students in groups. Ask each group to create a table of information involving multiple bar graph based on some real-life situation. Then challenge the other groups to draw a multiple bar graph using the data.

Example 3:

The following data shows the number of pages of a storybook read by Arham and Nida during 5 days of a week. Let's draw a horizontal multiple bar graph for this data.

No. of pages	Saturday	Sunday	Monday	Tuesday	Wednesday
Arham	8	7.5	4	4	2
Nida	7.5	4	4	5	3

Solution:

We use the following steps to draw the bar graph.

Step I: Draw an X-axis (horizontal line) and Y-axis (vertical line) perpendicular to each other on graph paper.

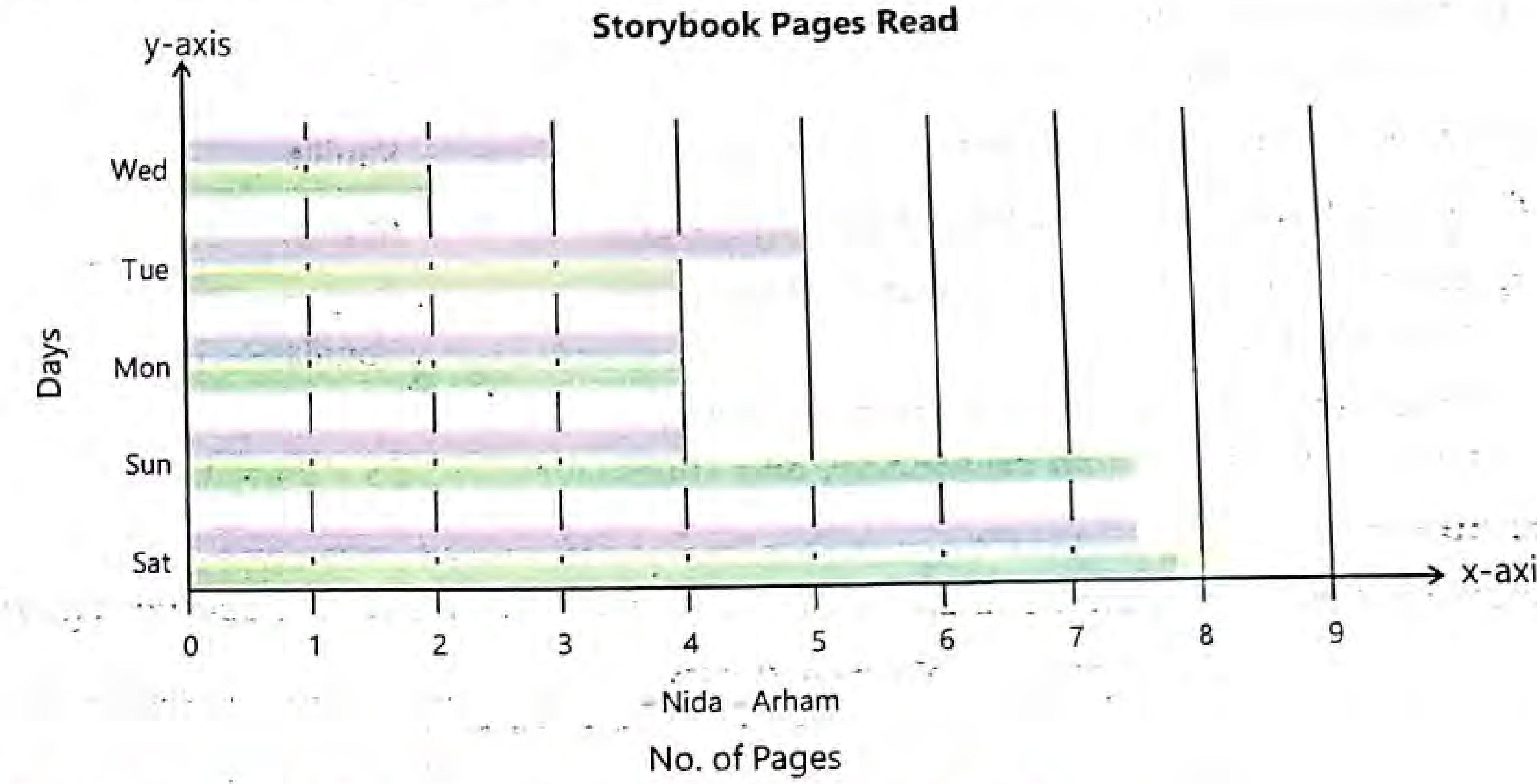
Step II: Write the number of pages along the X-axis and the days along the Y-axis. Choose a colour for each category (Nida and Arham).

Step III: Choose an appropriate scale. Here one mark represents 1 page along the X-axis.

Step IV: On Saturday, Arham read 8 pages and Nida read 7.5 pages. So we will colour 8 marks for Arham (using the pre- assigned colour) and 7.5 mark for Nida (using the pre- assigned colour).

Similarly, keep following the steps and colour the correct numbers of marks for each day and draw the multiple-bars. The width of the bars must be the same throughout the multiple-bar graph.

This is the required multiple-bar graph drawn horizontally.

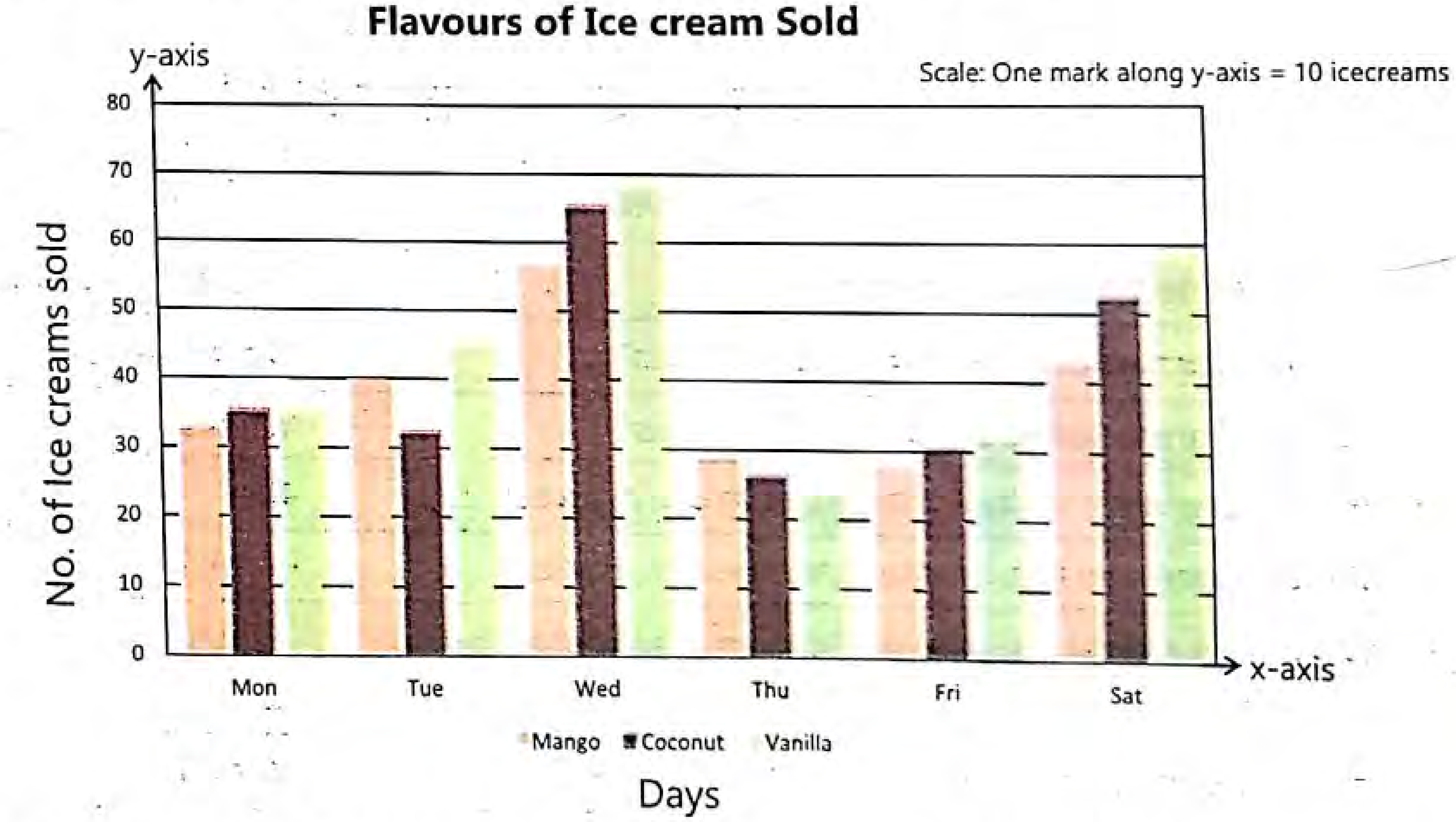


11.1.2 Reading and Interpreting Multiple bar graphs

We can easily read and interpret bar graphs if we know the scale of the graph. Let's observe the following multiple bar graphs, read and interpret them.

Example 1:

Observe the given multiple bar graph and answer the questions given.



a) What is the title of the graph? Flavours of Ice cream Sold

b) What information is shown on the X-axis? Days

c) What information is shown on the Y-axis? Number of ice-cream sold

d) How many coconut ice-creams were sold on Wednesday? 65

e) Which flavour was sold the most on Friday? Vanilla

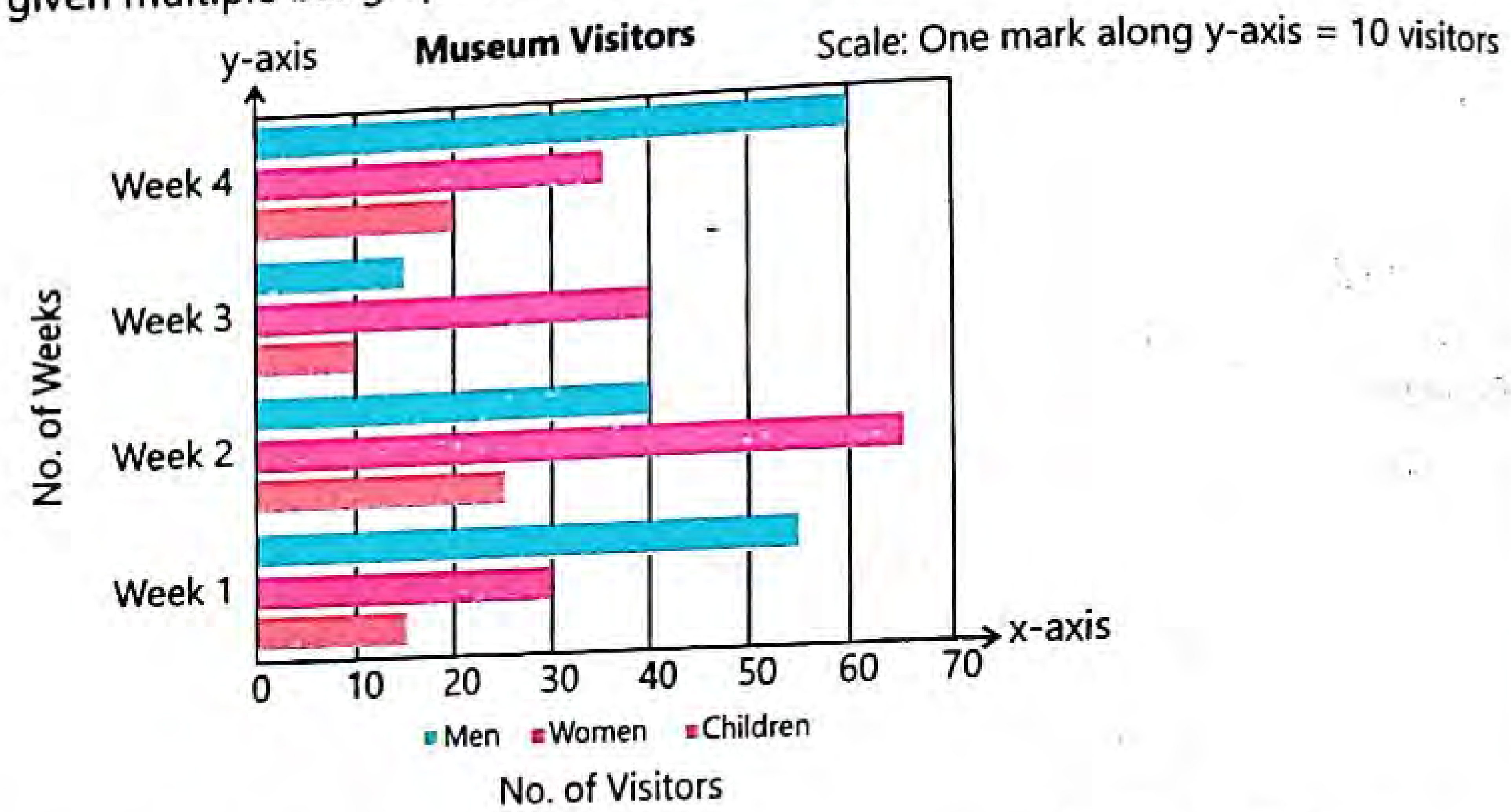
f) Which flavour was sold the least on Monday? Mango

g) Which flavour of ice-cream was sold the most from Mon to Sat? Vanilla

h) What is the difference between the number of Vanilla ice-creams sold on Monday and Thursday? 35 – 24 = 11

Example 2:

Observe the given multiple bar graph in horizontal form and answer the questions given.



a) What is the title of the graph? Museum Visitors

b) What information is shown on the X-axis? No. of visitors

c) What information is shown on the Y-axis? Weeks

d) How many men visited the museum during week 2?. 40

e) Which week has the greatest number of women? Week 2

f) What is the total number of children who visited the museum during all the four weeks altogether? 70

g) Which week has the least number of children? Week 3

h) How many visitors visited the museum during week 2 and 3 altogether? 195 visitors

i) What is the difference between the number of women during week 1 and week 3? 40 – 30 = 10

j) What is the sum of the number of men during week 2 and week 3? 40 + 15 = 55

Exercise 11.1

1 **The following data shows the number of new books added to a library during 5 months. Draw a vertical multiple bar graph for this data using appropriate scale.**

Language	July	August	September	October	November
English	20	60	80	30	20
Urdu	10	75	55	25	45

2 **The following data shows the number of girls and boys in different groups made for a mathematics competition. Draw a horizontal multiple bar graph for this data using appropriate scale.**

Students	Group A	Group B	Group C	Group D	Group E
Boys	20	60	80	30	20
Girls	10	75	55	25	45

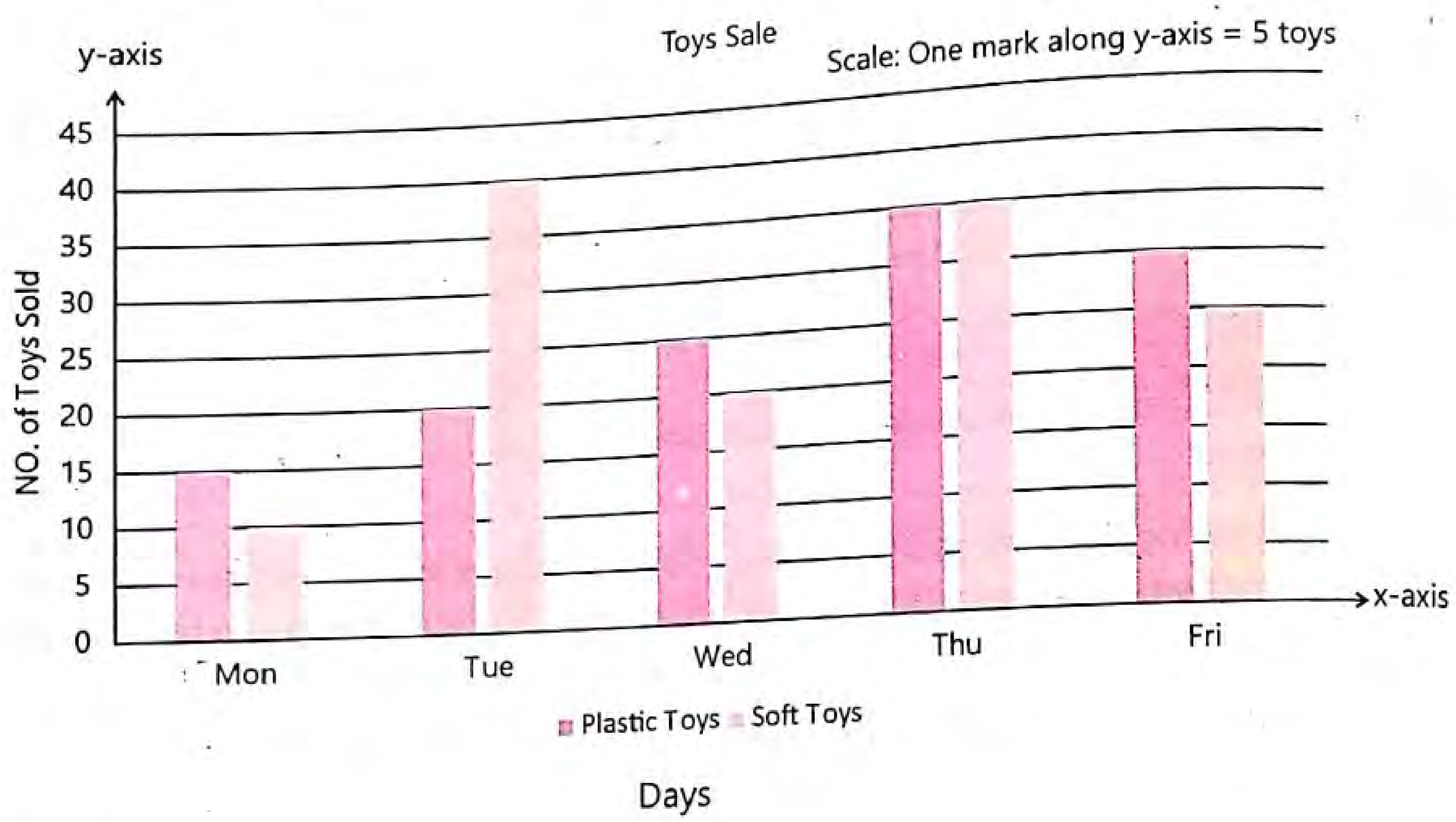
3 **The following data shows the average daily temperature of three cities. Draw a vertical multiple bar graph for this data using appropriate scale.**

Cities	Monday	Tuesday	Wednesday	Thursday	Friday
City A	12°	11°	14°	16°	13°
City B	7°	8°	10°	12°	10°
City C	22°	20°	22°	18°	24°

4 **The following data shows the amount collected by Marwa, Hadia, Mohid, Fatima and Hassan during three weeks to donate to the construction of a local Masjid. Draw a Horizontal multiple bar graph for this data using appropriate scale.**

Weeks	Marwa	Hadia	Mohid	Fatima	Hassan
Week 1	Rs 3200	Rs 1000	Rs 2000	Rs 1400	Rs 1000
Week 2	Rs 1200	Rs 1500	Rs 3000	Rs 2200	Rs 4000
Week 3	Rs 4000	Rs 1600	Rs 900	Rs 2000	Rs 1200

5 **Observe the given multiple bar graph in vertical form and answer the questions given.**



a) What is the title of the graph? ____________

b) What information is shown on the X-axis? ____________

c) What information is shown on the Y-axis? ____________

d) How many plastic toys were sold on Monday? ____________

e) How many soft toys were sold on Wednesday? ____________

f) On which day the least number of plastic toys were sold? ____________

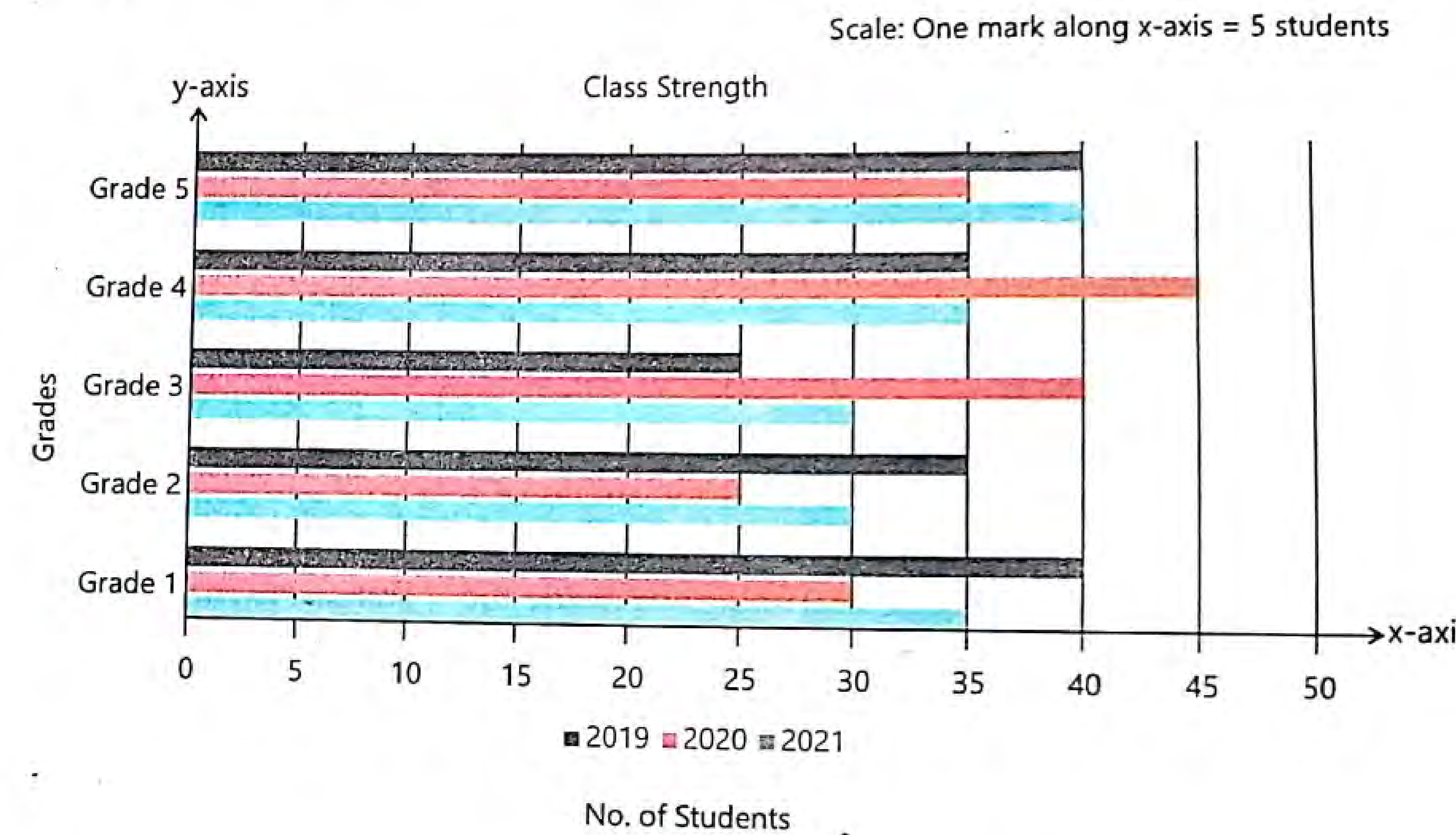
g) On which day the greatest number of soft toys were sold? ____________

h) What is the total number of soft toys sold on Tuesday and Wednesday altogether? ____________

i) On which day the greatest number of plastic toys were sold? ____________

j) What is the difference between the number of soft toys sold on Tuesday and Thursday? ____________

6 **Observe the given multiple bar graph in horizontal form and answer the questions given.**



a) What is the title of the graph? ____________

b) What information is shown on the X-axis? ____________

c) What information is shown on the Y-axis? ____________

d) What is the class strength of grade 5 students in 2021? ____________

e) What is the class strength of grade 3 students in 2019? ____________

f) What is the difference in class strength of grade 4 and Grade 5 in 2021? ____________

g) What is the difference in class strength of grade 1 and Grade 2 in 2020? ____________

h) Which year has the greatest number of grade 3 students? ____________

i) Which year has the least number of grade 2 students? ____________

j) In which year the greatest number of students were there in Grade 4? ____________

k) In which year the least number of students were there in Grade 5? ____________

11.2 Pie Graph

Previous Knowledge Check

- What is meant by data and organization of data?
- What is bar graph? How many types of bar graph are there?
- How can we represent data by using bar graph?

Pie graphs are a familiar way to display what information is composed of or divided into parts. In a Pie chart, different classes are shown in different sectors of a complete circle:

How to Construct a Pie Graph:

To construct a **pie graph**, we draw a circle with any radius. As, we know that the measurement of the total angles of a circle is 360°. In order to calculate the angle of different sectors or the measurement of central angle, we divide the numbers of observations of each class by the total number of observations in all classes and then multiply by 360°.

Maths History

William Playfair (1759-1823), a Scottish engineer, introduced pie graph in 1801.

i.e.

$$\text{Angle of the sector} = \frac{\text{number of observations of a class}}{\text{total number of observations in all classes}} \times 360^\circ$$

The procedure is illustrated by the example given below.

Example 1:

The following data gives the expenditure of a family on different items.

Items	Expenditures (Rs.)
House rent	24900
Food	30000
Education	32100
Clothing	21000

Note it down

The sum of central angles of a Pie graph must be equal to 360°.

Make a Pie graph for the above data.

Note it down

Pie graph is also known as Pie chart.

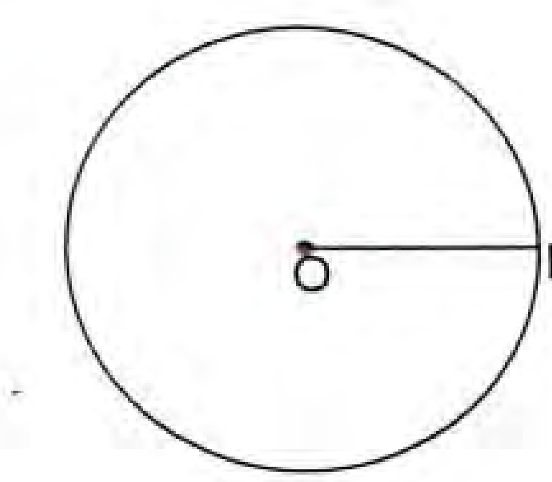
Solution:

First, we find the measurement of the central angles by using the following table:

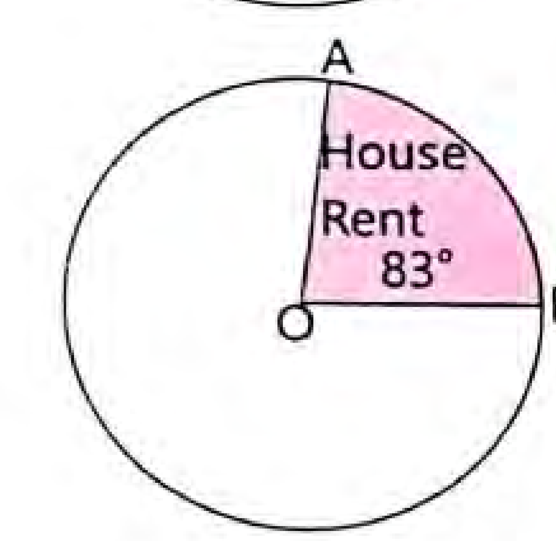
Items	Expenditure (Rs.)	Measurement of Central Angle
House rent	24900	$\frac{24900}{108000} \times 360^\circ = 83^\circ$
Food	30000	$\frac{30000}{108000} \times 360^\circ = 100^\circ$
Education	32100	$\frac{32100}{108000} \times 360^\circ = 107^\circ$
Clothing	21000	$\frac{21000}{108000} \times 360^\circ = 70^\circ$
Total	108000	Sum of central angles = 360°

Let us show information in a pie graph.

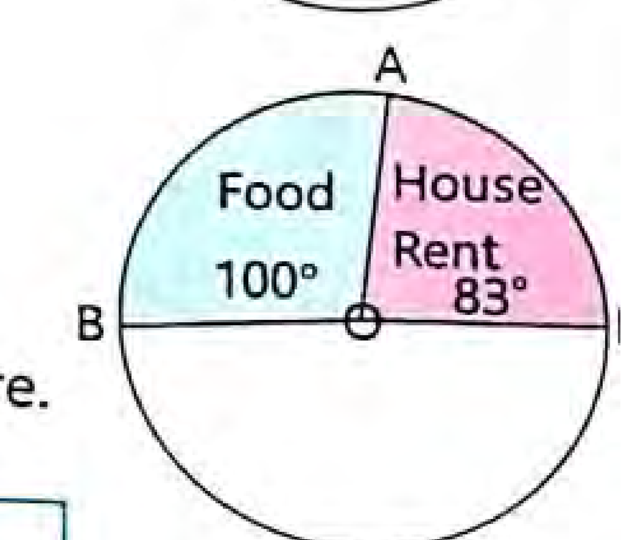
Step I: Draw a circle of any radius and draw radius $\overline{OP}$ of the circle at the right side of the circle.



Step II: Place the centre of a protractor at the centre of the circle and align it with radius $\overline{OP}$. Measure 83° angle and mark a point A on it. Join O to A. The sector of the circle formed by $\angle 83^\circ$ represents the house rent.



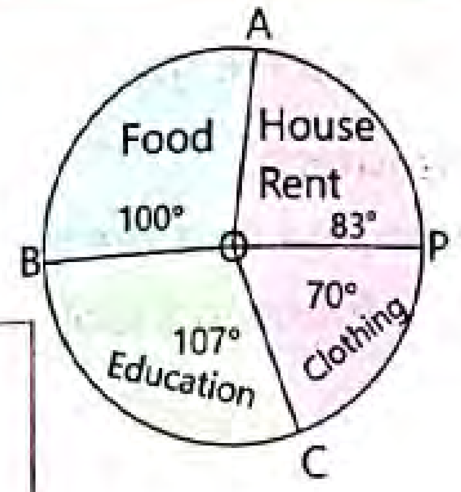
Step III: Place the centre of the protractor at the centre of circle and align it with radius $\overline{OA}$. Measure 100° angle and mark a point B on it. Join O to B. The sector AOB of the circle represent the food expenditure.



Note it down

The pie chart can be drawn anticlockwise by drawing angle using inner scale of protector.

Step IV: Continuing this procedure, we can make all angles of the pie graph representing the corresponding information.



Quick Check

Ali gives the following percentage of his time to different subjects. He gives 10% time to Islamiat, 20% to Science, 20% to English 20% to Mathematics and 30% to remaining subjects. Draw pie graph for the above information.

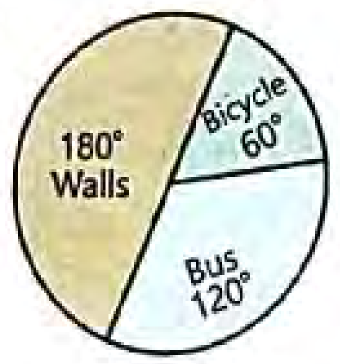
Example 2:

The pie graph shows the means of transportation to school. Find the answers of the following questions if total students of the school is 720.

i) How many students are going by bicycles?

ii) How many students are going by bus?

iii) How many students are going by foot?



Quick Check

Collect data of your own 12 hours activities and represent this data in figure.

Solution:

Total students = 720

i) Number of students going by cycle $= \frac{60°}{360°} \times 720 = 120$

ii) Number of students going by bus $= \frac{120°}{360°} \times 720 = 240$

iii) Number of students going by foot $= \frac{180°}{360°} \times 720 = 360$

Exercise 11.2

1 **Razia obtained the following marks in the different subjects:**

Subject	Marks
Maths	90
English	75
Social Studies	60
Urdu	70
Islamiat	65

TEACHING POINT: Share following online game link of interpreting and constructing pie chart https://www.transum.org/software/SW/Starter_of_the_day/Students/Pie_Charts.asp

2 **Draw pie charts for the following data:**

i) The population of 6 cities of Pakistan is given below.

City	Population (in Millions)
A	18
B	15
C	12
D	10
E	4
F	1

ii) The following table shows Amna's activities in a day:

Activities	Time (Hours)
Reciting Quran	2
School	6
Meal	1
Play	2
Study	4
Using Computer	1
Sleep	8

3 **The number of students in different sections of class VII are given below:**

Sections	Number of Students
A	73
B	65
C	50
D	42
E	30

Draw pie chart for the given data.

4 **The pie graph shows the sale of 180 litres of milk by a milkman in a day.**

Read this pie graph and answer the following questions:

i) How many litres does he supply to homes?

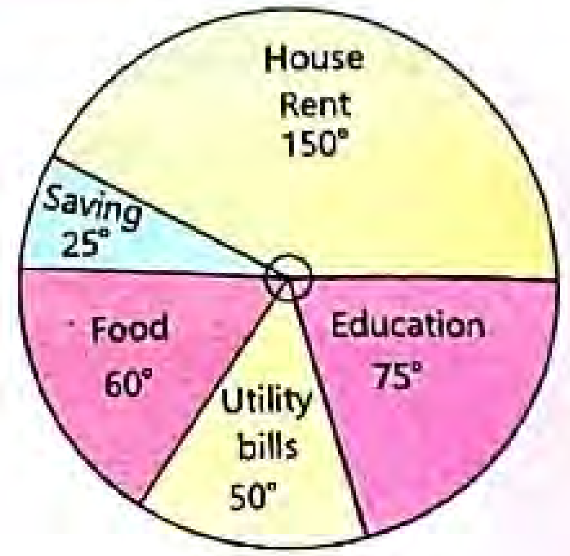
ii) How many litres does he sell to the customers?

iii) How many litres does he use for yogurt?

TEACHING POINT: Ask the students to work in groups and make a pie chart of the time they spend out of 24 hours in following activities: Studying, Playing, Eating, Sleeping, Watching TV, Spending time with family etc.

5 **The monthly expenditure of Rauf is shown by a pie graph. If the monthly income of Rauf is Rs. 7200, find the answers of the following questions:**

i) What is the largest expenditure of Rauf and how much?
ii) What amount does he spend on education and food?
iii) How much does he pay as utility bills?
iv) What amount does he save?



6 **Sana's activities in a day are shown by a pie graph. Read the graph and answer the following questions: (Hint: 1 day = 24 hours)**

i) How much time does she spend in school?
ii) How many hours does she sleep?
iii) How many hours does she study?
iv) In which activity does she spend the least time? and how much?
v) How much time does she spend in using computer and meal?
vi) How many hours does she spend on recitation of the Holy Quran and Namaz?

11.3 Data

The **information** collected in the form of numbers, words, figures, facts, or any other form is called **data**. For example, population of Peshawar, number of students who passed in exams, rainfall in a year, etc. All the information we show through graphs, tally charts, pie charts, etc. is based on data.

11.3.1 Data collection

The process of collecting information from any source related to problem is known as **data collection**. This information can be anything which we want to know about. For example:

- The favourite game of grade 6 students.
- The number of persons offering Isha prayer in a local Masjid.
- The number of different vehicles passing by a specific road/bridge roundabout in a day.
- The favourite breakfast food of a group of students.
- The daily temperature of a city in a week, etc.

Note it down
When we collect data ourself then the data is called primary data and when we get data from someone then that data is called secondary data.

Data collection must be done carefully as any mistaken data may lead to incorrect results.

Ways to collect data

There are many ways to collect data. For example:

- Observation
- Interview
- Questionnaire
- Newspapers/magazines
- Internet

11.3.1 Types of Data

There are four basic types of data.

a) Discrete data
b) Continuous data
c) Un-grouped data
d) Grouped data

a) Discrete data

The values or quantities that are in whole number form are fall under discrete data. The values in **discrete data are countable**. They cannot be subdivided into further parts. For example:

- The Number of students in Grade 6.
- The population of Islamabad.
- The number of players in a match.
- The number of animals in a forest.
- The number of fans in a school

Note it down
The term "discrete" means distinct or separate.

b) Continuous Data

The values or quantities that can be presented in fraction or decimal fall under **continuous data**. The values in continuous data are not countable. They can take any value within a range.

Note it down
Discrete data can be better organized using a bar chart or pie chart etc. While a continuous data can be better organized using line graphs or histograms.

- The weight of grade 6 students.
- The volume of liquid
- The height of a person
- The temperature of a city

Quick Check

Does any of these falls under continuous data?

- The speed of a car
- The number of cars in a showroom
- The mass of a bag of rice

c) Un-grouped data

When data is collected from any source it does not convey to us any meaningful information to understand. This data is called ungrouped data or raw data. Consider the following example.

Example 1:

The temperature in (°C) of a specific city in two weeks is shown below:

32°C	31°C	16°C	17°C	34°C	26°C	27°C
23°C	33°C	24°C	25°C	35°C	29°C	21°C

We can observe that we cannot come to a quick result after observing this data. This data is called **ungrouped data**. This type of data is difficult to analyse.

d) Grouped data

When collected data from any source is organised in groups/categories to show some meaning, this data is called grouped data.

We can group the above data of temperature (in °C) of a city in the following form.

Draw a table with three columns.

Divide the temperature of the two weeks into 4 groups.

Number of days when temperature lies between 15 - 20 = 2

Number of days when temperature lies between 21 - 25 = 4

Number of days when temperature lies between 26 - 30 = 3

Number of days when temperature lies between 31 - 35 = 5

TEACHING POINT: Explain to the students about types of data. Instruct them to write the ages of the students of their class and then organise the data in grouped form.

Groups	Temperature (°C)	Number of days	Tally Marks
15°C - 20°C	16°C, 17°C	2	//
21°C - 25°C	21°C, 23°C, 24°C, 25°C	4	////
26°C - 30°C	26°C, 27°C, 29°C	3	///
31°C - 35°C	31°C, 32°C, 33°C, 34°C, 35°C	5	~~////~~

We can see that the analysis of this type of data is much easier.

We can see that:

The temperature of most days lies between 31°C - 35°C.

The temperature of least days lies between 15°C - 20°C.

Example 2:

The marks obtained by students out of 75 in a Maths test are given below.

45	32	32	67	45	67	40	45	67	34	32	71	71
67	60	55	55	15	71	67	67	55	18	28	28	67
55	14	29	34	27	40	60	51	51	51	52	51	14

Show this data in grouped form.

Solution:

We can group the above data of marks obtained by students in the following form.

Draw a table with three columns.

Divide the marks obtained by students into 5 groups.

Note it down

We use a tally marks to show data in grouped form.

Now observe the values of the data. Look at the first value of the data that is 45, place a tally mark against this value in the group 31-45. Now read the second value and place a tally mark against this value in the third group 31-45. Continue this process until all the values of the data are placed in the form of tally marks in the table.

Quick Check

Find the data of heights of the students of your class and classify in a table.

Now count the number of marks in each group and write the number in the third column.

Number of marks obtained in Maths test lies between 1-15 = 3

Number of marks obtained in Maths test lies between 16-30 = 5

Number of marks obtained in Maths test lies between 31-45 = 10

Number of marks obtained in Maths test lies between 46-60 = 11

Number of marks obtained in Maths test lies between 61-75 = 10

Marks	Tally Marks	Number of students
1-15	///	3
16-30	~~////~~	5
31-45	~~////~~ ~~////~~	10
46-60	~~////~~ ~~////~~ /	11
61-75	~~////~~ ~~////~~	10

The maximum marks obtained by the students lies between the group of 46-60.

The minimum marks obtained by the students lies between the group of 1-15.

Exercise 11.3

1 **Azan makes a list of students about their favourite subjects. Show this in the form of grouped data and then answer the given questions.**

English	Science	Math	Math	Urdu	Urdu	English
Urdu	Math	Math	Science	English	English	Science
Science	Science	Science	Math	Math	English	Urdu

a) Which subject is the most favourite?

b) Which subject is the least favourite?

2 **The given data is the weight of class 6 students:**

20 kg	24 kg	21 kg	18 kg	20 kg	24 kg	21 kg	18 kg
19 kg	24 kg	19 kg	21 kg	24 kg	20 kg	18 kg	18 kg
20 kg	24 kg	18 kg	24 kg	26 kg	21 kg	19 kg	26 kg

Show it in the form of grouped data. Also:

a) Tell the number of students that have a weight less than 20 kg.

b) Tell the number of students that have a weight greater than 21 kg.

11.4 Measures of Central Tendency

When the data have been arranged into a **frequency distribution**, the information contained in the data is easily understood. We can also find a single value which will represent all the values of the distribution in some definite way.

A central value that represents all the values of a distribution is called an **average**.

Since the averages tend to lie in the centre of a distribution they are called measures of central tendency.

The most commonly used averages are;

i. Arithmetic Mean

ii. Median

iii. Mode

Quick Check

Arithmetic mean is the most commonly used average. In view of its common use, it is usually called average or simply mean.

11.4.1 Arithmetic Mean

The sum of values divided by the number of values is called **arithmetic** mean.

$$\text{i e, Arithmetic mean} = \frac{\text{sum of values}}{\text{number of values}}$$

If 'X' represent the values, Σ(sigma) represent the sum, 'n' represent the total number of values and $\bar{X}$ (read as X bar) represent the arithmetic mean, we can write.

$$\bar{X} = \frac{\Sigma X}{n}$$

If there are three observations X_1, X_2 and X_3 then,

$$\bar{X} = \frac{X_1 + X_2 + X_3}{n}$$

Previous Knowledge Check

How can we find the arithmetic mean of different quantities?

Example 1:

Find the mean of the values 2, 5, 6, 7, 8.

Solution:

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{2+5+6+7+8}{5}$$

$$\overline{X} = \frac{28}{5} \text{ or } \overline{X} = 5.6$$

Example 2:

The monthly income of five families is Rs. 5000, Rs. 8200, Rs. 9500, Rs. 9530 and Rs. 10000. Calculate their mean.

Solution:

$$\overline{X} = \frac{\sum X}{n}$$

$$\overline{X} = \frac{500+8200+9500+9530+10000}{5}$$

$$\overline{X} = \frac{42230}{5}$$

$$\overline{X} = \text{Rs. } 8446$$

Quick Check

Find the median of the first 20 even numbers.

11.4.2 Median

The **median** is the value which divides the values into two equal halves arranged either in ascending or descending order.

To find the median;

i. Arrange the values in ascending order.

ii. If you have odd numbers of values the median is the middle value.

iii. If you have even number of values, the median is the arithmetic mean of the two middle values i.e, the value of $\frac{(n+1)}{2}$ th item.

Let the solve an example:

Example 1:

Find the median for the following observations.

i. 5, 2, 7, 13, 8, 11, 9   ii. 4, 6, 8, 10, 12, 14

Solution:

i. First we arrange the observations.

2, 5, 7, 8, 9, 11, 13.

Here $n = 7$

then median $= (\frac{n+1}{2})$ th item.

$= (\frac{7+1}{2})$ th item

$= \frac{8}{2}$ th item

= 4th item in the given observations

Hence, Median = 8

ii. Here $n = 6$

So, Median $= \frac{(n+1)}{2}$ th item

$= \frac{6+1}{2} = \frac{7}{2}$

= (3.5) th item in the given observations.

3.5 th item is half way between the 3rd and 4th items.

3rd value = 8

4th value = 10

Hence, Median $= \frac{18+10}{2}$

$= \frac{18}{2}$

$\Rightarrow$ Median = 9

Quick Check

$\frac{(n+1)}{2}$ only tells us the position of the median.

11.4.3 Mode

The **frequently occurring value** (or values) in the data denoted by $\hat{X}$ is called **mode**.

To find the mode;

i. Calculate frequencies for all values in the data.

ii. The mode is the value with the greatest frequency.

If each value occurs the same number of times, then there is no mode. If two or more values occur the same number of times but more frequently than any of the other values, then there is more than one modes, which is the main difference between mode and mean and median because a data has only one mean and median.

Example 1:

Find the mode of the following set of observations.

i. 2, 3, 3, 3, 4, 4, 5, 5, 5, 5

ii. 8, 8, 9, 10, 11, 11, 13

Teaching Point

Share the following online quiz link of finding mean, median and mode.
https://www.mathgames.com/skill/7.90-interpret-charts-to-find-mean-median-mode-and-range

Solution:

i. Frequency of 5 is 4 which is the greatest frequency.

Hence,

Mode = $\hat{X}$ = 5

ii. Frequency of the values 8 and 11 is 2, hence there are two modes.

Mode = $\hat{X}$ = 8, 11

Quick Check

Find mode of the following;
2, 5, 6, 7, 8, 9.

Exercise 11.4

1 **Calculate the mean of the following data.**

i) 46, 50, 39, 50, 38   ii) 108, 115, 138, 190   iii) 210, 237, 340

2 **The monthly income of 10 employees of a factory is;**

Rs. 6000, Rs. 65230, Rs. 78600, Rs. 8000, Rs. 9350,
Rs. 9680, Rs. 1030, Rs. 1236, Rs. 14340, Rs. 15000.
Calculate the average of their monthly income.

3 **Find the median of the following sets of observations.**

i) 15, 12, 13, 10, 8   ii) 100, 106, 101, 108, 98, 93
iii) 84, 72, 95, 90, 65, 68, 69, 70

4 **Find out the mode for each of the following:**

i) 7, 8, 8, 13, 15, 15, 16   ii) 100, 120, 120, 120, 120, 150
iii) 357, 402, 427, 500

11.5 Probability

Many times in daily life we cannot be sure about the **outcome** (result) of an **event** (action or situation). We can only predict or guess about the result. In such situation we can talk about probability of certain outcomes (results). Let's discuss about some basic terms relevant to probability. It is the chance of an event taking place.

Previous Knowledge Check

- Can you tell about what is the weather forecast for next four days of a week?
- What are the chances that Pakistan will win the toss and bat first?

Note it down

Probability means how likely something is to happen. It is a measure of the likelihood or possibility of an event.

11.5.1 Experiment and Random Experiments

An **experiment** is a process which gives some results/outcomes. An experiment will be called a **random experiment** if it has two or more possible results.

An example of a random experiment is flipping a coin as we don't know the result (if it would be head or tail). Similarly rolling a dice is also a random experiment as again, as we don't know the result (would it be 1, 2, 3, 4, 5, or 6).
Similarly, when spinning a spinner, and picking an object randomly from a group of objects without looking, mean the result is not known.

Quick Check

When a random experiment is repeated several times, each one of them is called a **trial**.

11.5.2 Outcomes

The possible result of a random experiment is called an outcome.
For example, when rolling a dice, there are total six possible outcomes can be 1 or 2 or 3 or 4 or 5 or 6.

Similarly, when tossing a coin, there are total two possible outcomes; a head or a tail.

Note it down

An outcome is a result while chance that an outcome will occur is its probability.

11.5.3 Sample Space

The sample space is the set of all possible outcomes of an **experiment**.

- The sample space for rolling a die is:
  Sample space: S = {1, 2, 3, 4, 5, 6 }

Note it down

A sample space is like a **universal set** for possible outcomes.

TEACHING POINT: Use the following link to explain the concept of probability.
https://www.mathsisfun.com/data/probability.html

- The sample space for tossing a coin is:
  Sample space: S = {head, tail}

- The sample space for choosing a marble randomly is:
  Sample space: S = { Blue marble, red marble, green marble, yellow marble }

11.5.4 Event

An **event** is a subset of a sample space.
For example, tossing a coin and getting a head is an event, rolling a die and getting a 5 is an event.
There are different types of events in probability. Let's discuss a few of them.

11.5.5 Equally Likely and Not Equally Likely Outcomes

The outcomes in a sample space will be called **equally likely if** each outcome has the same chance of occurring. While the outcomes having different chances of occurring are not equally likely outcome.

- After tossing a coin, the chance of getting a head is same as the chance of getting a tail. So, these two outcomes are equally likely as the probability of each one is equal.

- In the spinner, the chance of stopping at 3 is same as the chance of stopping at 5. So the outcomes are equally likely.

After rolling a die:
- The chance of getting a 2 is same as the chance of getting a 4. So, these two outcomes are equally likely as the probability of each one is equal.
- The chance of getting an odd number is same as the chance of getting an odd number(as there are 3 odd and 3 even numbers on die) . So, the outcomes are equally likely.
- The chance of getting a number less than or equal to 3 is same as getting a number greater than or equal to 4. So, these two outcomes are also equally likely.

- In this spinner, the chance of stopping at 3 is not same as the chance of stopping at 5. So, the outcomes are not equally likely.

11.5.6 Probability of a Single Event

All possible outcomes of an experiment are represented by sample space. We can represent the possible outcome using a tree diagram. It can be used to represent all the possible groupings or outcomes of a sample space.
Let's show all the possible outcomes of tossing a coin using tree diagram .

Quick Check
We can also organize the outcomes in a list or chart. A

The **probability** of a single event is the ratio or comparison of favorable(desired) outcomes to the total number of possible outcomes. In this case, there are 2 possible outcomes, the head and the tail. If all the outcomes of an event are equally likely, the probability of the event occurring, denoted by P(E), will be is:

$$\text{Probability of an event} = P(E) = \frac{\textit{Number of favorable (desired) outcomes}}{\textit{Total number of possible outcomes}}$$

Example 1:

Umar and Nadia are tossing a coin to take the first turn in a game. Nadia chose tail. Find the probability of getting a tail (favorable outcome).

Solution:

Sample space or total number of possible outcomes= H, T
Number of possible outcomes = 2
Number of favorable (desired) outcomes = 1

TEACHING POINT: Use various suitable items from daily life and ask the students to identify the possible outcomes and sample space for them

$$\text{Probability of an event} = P(E) = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}}$$

$$\text{Probability of an event} = P(E) = \frac{1}{2}$$

$\frac{1}{2}$ shows that there are 1 out of 2 chances that the coin will show a tail.

Example 2:

Sadaf has a bag with eight marbles. Three are red, three are green, one is yellow, and one is blue. What is the probability that if she chooses one marble randomly without seeing, will be green?

Solution:

To find the probability of choosing a green marble will be:

Sample space or total number of possible outcomes= red, red, red, green, green, green, yellow, blue

Number of possible outcomes = 8

Number of favorable (desired) outcomes = 3 (green)

$$\text{Probability of an event} = P(E) = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{3}{8}$$

$$\text{Probability of an event} = P(E) = \frac{3}{8}$$

$\frac{3}{8}$ shows that there are 3 out of 8 chances that the chosen marble will be green.

Quick Check

What is the probability of choosing red marble?

Think Higher

Describe an event in which the probability is $\frac{1}{4}$.

A die is rolled.
Look at the results and tell:
Who is wrong and why?
Who is correct and why?

Favorable outcome: 4
Possible outcomes: 1, 2, 3, 4, 5, 6
$P(2) = \frac{1}{6}$

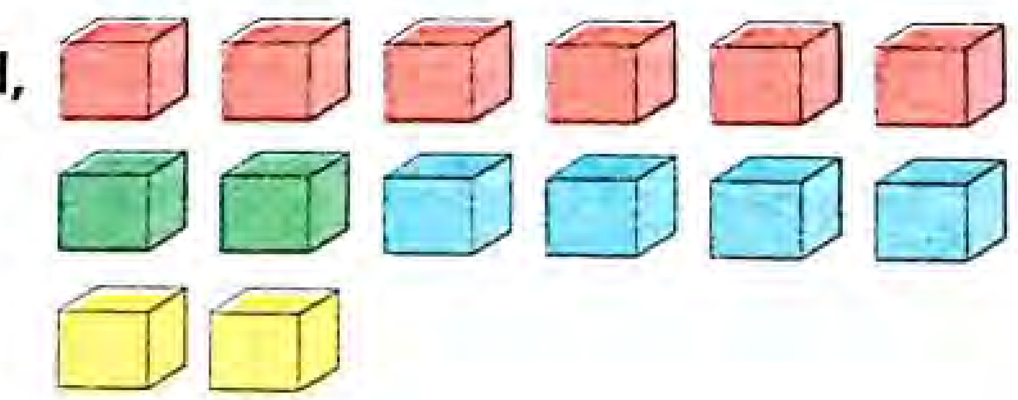
Favorable outcome: 4
Possible outcomes: 1, 2, 3, 4, 5, 6
$P(2) = \frac{1}{4}$

Exercise 11.5

1 **Describe the following with examples:**

a) Experiments
b) Random Experiments
c) Outcomes
d) sample space
e) Events
f) Equally Likely events

2 **Saad has a bag with 14 blocks. Six are red, two are green, four are blue and two are yellow. What is the probability that if he chooses one block, randomly without seeing, it will be red?**



3 **There are 14 candies in a jar. Hamza takes one candy randomly without looking at the candies.**

Find:

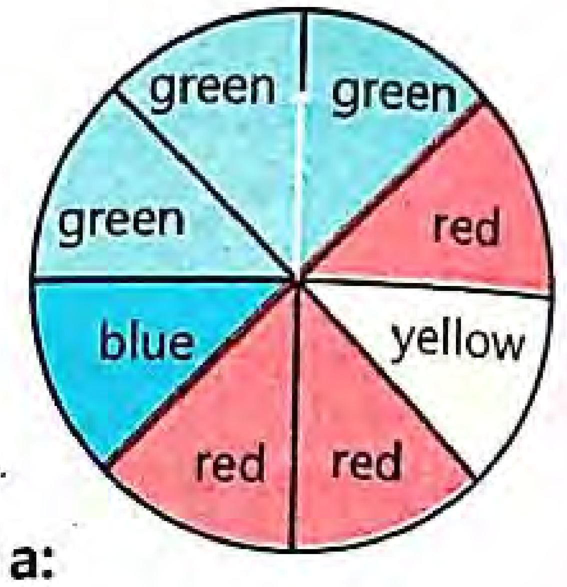
a) The sample space.
b) The probability of picking a chocolate candy.
c) The probability of picking a strawberry candy.
d) The probability of picking an orange candy.
e) What is more likely to be picked; an orange candy or a strawberry candy?

4 **Draw 2D shapes(as sample space) for each of these according to the below mentioned probabilities:**

a) P(squares)= $\frac{2}{4}$

b) P(circles)= $\frac{3}{8}$

c) P(triangles)= $\frac{1}{3}$



5 **Look at the spinner and find:**

a) The sample space

b) The probability:

i. P(red) ii. P(Green) iii. P(Blue) iv. P(yellow)

6 **The pond has 10 blue, 8 red, 7 green and 4 yellow fish.**
What is the probability that a randomly caught fish will be a:

a) Red fish b) Blue fish c) Green fish

7 **A box contains 8 buttons out of which 5 are transparent. What is the probability that a randomly selected button will not be a transparent one?**

8 **In a pack of randomly packed 14 pencils, 11 are blue ones.**
What is the probability that a randomly selected pencil will be blue?

9 **Ali picks a card randomly.**

4 7 7 0 3 8

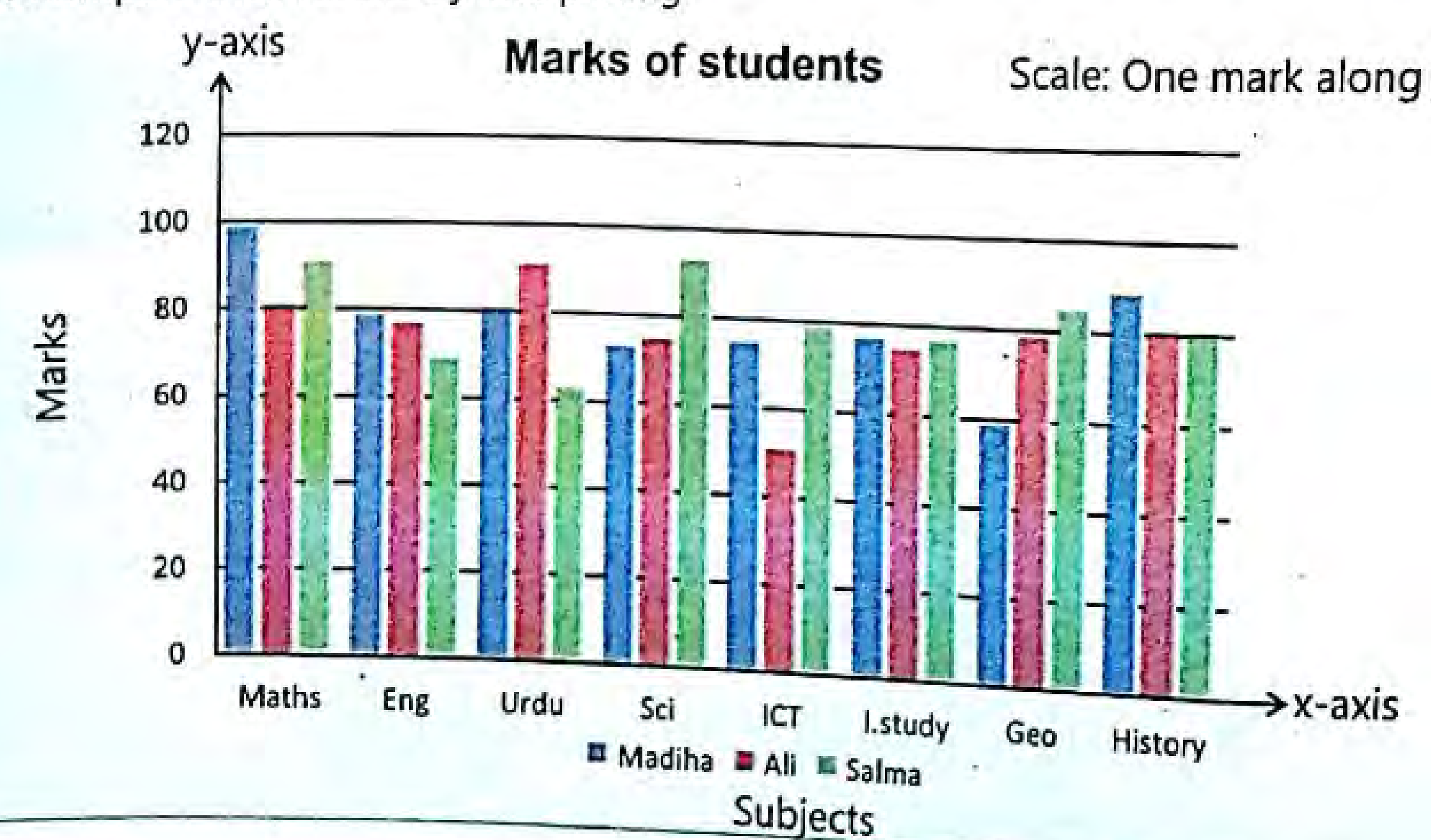
a) What are the possible outcomes?

b) What is the probability that a randomly selected card will have a 7?

c) What is the probability that a randomly selected card will have a 3?

Think Higher

Observe the bar graph. Interpret it and make a report about the marks of the students in different subjects. Compare their results by interpreting



Summary

- The information collected in the form of numbers, words, figures, facts, or any other form is called data.
- The sum of central angles of a Pie graph must be equal to 360.
- In a multiple bar graph, two or more related categories of data are represented.
- In a Pie chart, different classes are shown in different sectors of a complete circle.
- There are two basic classes of data.
  - Qualitative data
  - Quantitative data
- The values or quantities that are in integers or whole number form are fall under discrete data
- The values or quantities that can be presented in fraction or decimal fall under continuous data.
- When data is collected from any source it does not convey to us any meaningful information to understand. This data is called ungrouped data or raw data.
- When collected data from any source is organised in groups/categories to show some meaning, this data is called grouped data.
- The sum of values divided by the number of values is called arithmetic mean.
- The median is the value which divides the values into two equal halves, with half of values being lower than the median and half higher than it.
- The frequently occurring value (or values)in data is called mode.

Vocabulary

- Graph
- Multiple bar graphs
- Pie Graph
- Sector
- Data
- Data collection
- Observation
- Interview
- Qualitative data
- Quantitative data
- Discrete data
- Continuous data
- Grouped data
- Ungrouped
- Central tendency
- Average
- Arithmetic mean
- Median
- Mode
- Probability
- Sample Space
- Experiment
- Event

Review Exercise

1 Encircle the correct option.

a) The information on the basis of which these graphs are drawn is called:

i. Knowledge ii. Table iii. Data iv. Product

b) Two or more related categories of data are represented in a:

i. single bar graph ii. multiple bar graph

iii. pie bar graph iv. Line bar graph

c) The sum of central angles of a Pie graph must be equal to:

i. 160° ii. 380° iii. 360° iv. 180°

d) Which of these shows discrete data?

i. The mass of an animal ii. The number of rooms in a hotel

iii. The length of a chain iv. The capacity of a water container

e) The mode of the data 9, 10, 12, 13, 13, 13, 15, 15, 16, 16 is:

i. 9 ii. 15 iii. 16 iv. 13

f) The median of the data 23, 23, 23, 23, 45, 45, 56, 67, 73, 89 is:

i. 20 ii. 23 iii. 45 iv. 46

2 Define the following with examples:

a) Discrete data b) Continuous data c) Grouped fata

d) Ungrouped data e) Mean f) Median g) Mode

3 Fahad made a list of students about their favourite games. Show this in the form of grouped data:

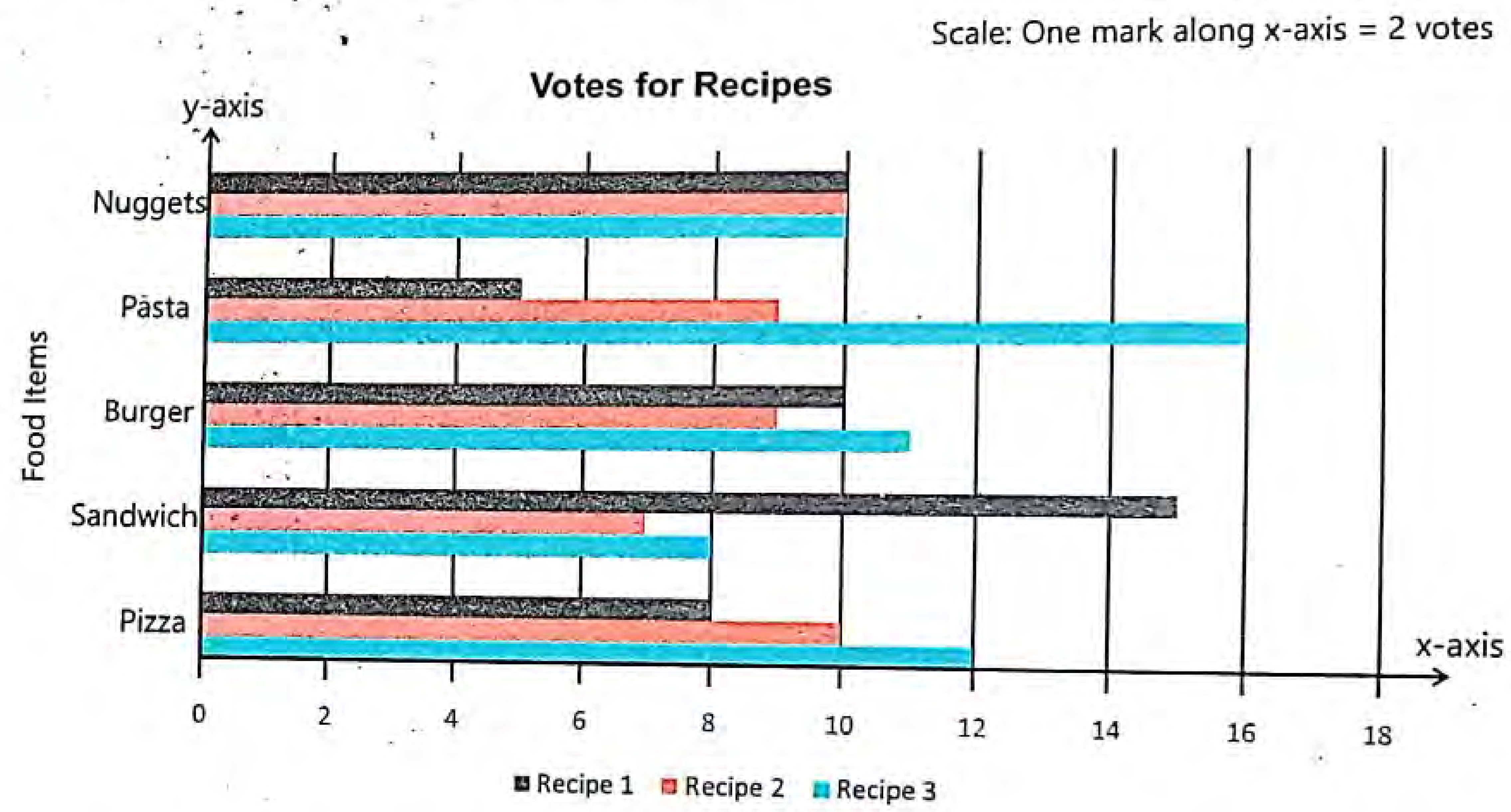
cricket	hockey	hockey	cricket	football	hockey	cricket	basketball
hockey	cricket	football	cricket	football	cricket	football	badminton
badminton	basketball	badminton	basketball	basketball			

a) Which game is the most favourite? b) Which game is the least favourite?

4 The following table shows the profit gained by 3 shopkeeper during five months. Draw a multiple bar graph for this data.

	April	May	June	July	August
Umar	Rs 20000	Rs 65000	Rs 55000	Rs 15000	Rs 35000
Ali	Rs 35000	Rs 45000	Rs 30000	Rs 20000	Rs 76000
Khalid	Rs 52000	Rs 19000	Rs 78000	Rs 25000	Rs 54000

5 Look at the bar graph and answer the question below.



a) Which items recipe got the greatest vote?

b) How many people vote for the recipe 3 of the pasta?

c) How many people vote for three recipes of sandwich?

d) How many people vote for the recipe 2 of the pizza?

e) How many people vote for recipe 1 of the burger?

6 The following table shows the money Ahmad spends on food in the first 5 months of the year.

Months	Januar	February	March	April	May
Spent on food	9000	12000	11780	1450	1340

Draw a pie chart for this data.

7 Find mean, median and mode for the following observations;

i. 52, 48, 50, 49, 47

ii. 207, 301, 205, 350, 322, 205

Math Project

Material Required:

- Sets of several number cards (may include repeated numbers)
- Paper
- Pencil

Procedure:

- Get into pairs.
- Each pair randomly chooses 5 cards from a set of cards and find the mean, median and mode of the chosen numbers.
- The pair with the smallest mean, median or mode gets a point.
- Repeat this at least 5 times.
- The pair with the greatest number of smallest average wins.

Unit 1: Multiples and factors

Exercise 1.1

1. a) 1, 2, 13, 26
   b) 1, 2, 4, 7, 8, 14, 16, 28, 56, 112
   c) 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150
   d) 1, 5, 7, 25, 35, 175
   e) 1, 2, 4, 17, 34, 68
   f) 1, 2, 7, 11, 14, 22, 77, 154
   g) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
   h) 1, 2, 4, 5, 10, 20, 25, 50, 100
   i) 1, 23
   j) 1, 47
   k) 1, 2, 29, 58
   l) 1, 2, 4, 8, 16, 32, 64
   m) 1, 2, 3, 6, 7, 14, 21, 42
   n) 1, 2, 3, 4, 6, 9, 12, 18, 36
   o) 1, 2, 5, 7, 10, 14, 35, 70
   p) 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
2. a) 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180, 192, 204, 216, 228, 240
   b) 25, 50, 75, 100, 125, 150, 175, 200, 225
   c) 37, 74, 111, 148, 185, 222
   d) 48, 96, 144, 192, 240
   e) 60, 120, 180, 240
   f) 75, 150, 225
   g) 45, 90, 135, 180, 225
   h) 57, 114, 171, 228
3. 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196, 210, 224, 238, 252, 266, 280, 294, 308, 322, 336, 350, 364, 378, 392, 406, 420, 434, 448, 462, 476, 490
4. 18 is not the factor of 256.

Exercise 1.2

1. a) $2\times2\times2\times2\times3\times3 = 2^{4}\times3^{2}$
   b) $2\times157 = 2^{1}\times157^{1}$
   c) $2\times2\times2\times19\times37 = 2^{3}\times19^{1}\times37^{1}$
   d) $2\times2\times3\times257 = 2^{2}\times3^{1}\times257^{1}$
   e) $2\times2\times2\times5\times197 = 2^{3}\times5^{1}\times197^{1}$
   f) $2\times2\times2\times3\times251 = 2^{3}\times3^{1}\times251^{1}$
   g) $2\times2\times3\times3\times5\times47 = 2^{2}\times3^{2}\times5^{1}\times47^{1}$
   h) $2\times1493 = 2^{1}\times1493^{1}$
2. a) $3^{1}\times5^{1}\times7^{1}\times23^{1}$
   b) $2^{4}\times13^{1}$
   c) $2^{5}\times7^{1}\times19^{1}$
   d) $2^{1}\times2417^{1}$
   e) $5^{1}\times19^{1}\times71^{1}$
   f) $2^{1}\times317^{1}$
   g) $2^{2}\times5^{1}\times229^{1}$
   h) $2^{1}\times4523^{1}$
3. a) 2x2x2x2x2x2
   b) 4x7x7x7x7x7
   c) 3x5x5x11x11x11x11
   d) 2x2x3x3x3x5x5x5x5x5x5
   e) 13x13x13x13x13x13x17 x17x17
   f) 4x7x7x7x7x7
   g) 3x3x5x5x5x5x5x5x17x17
   h) 7x7x11x11x11x29
4. a) 5^{3}
   b) $2^{2}\times5^{4}$
   c) $1\times2^{2}\times7^{3}$
   d) $11^{2}\times13^{3}\times19^{2}$
   e) $7^{3}\times11^{2}$

Exercise 1.3

1. a) 2
   b) 4
   c) 4
   d) 1
   e) 2
   f) 5
   g) 1
   h) 2
   i) 5

2. a) 10 b) 6 c) 50 d) 1 e) 2 f) 2 g) 341936 h) 24482380 i) 31585248

3. a) 17400 b) 100224 c) 173040 d) 474000 e) 41736 f) 61992 g) 52017504 h) 248472 i) 22562820

4. a) 203000 b) 786600 c) 781200 d) 17724096 e) 173986602 f) 379100

5. 114 6. 4116 7. 128

Exercise 1.4

1. 338100 *l* 2. 4 3. 155904 4. 72 5. 1350

6. 7 7. 9 pieces of 2 meters and 56 pieces of 2 meters

8. 24 servings with 1 chicken bread and 5 nuggets. 9. After 5 days 10. 5600

Exercise 1.5

1. a) 625 b) 5929 c) 9,604 d) 121 e) 2,116 f) 4,761 g) 169 h) 1,369 i) 2,025 j) 6,241 k) 4,489 l) 2,500

2. 9604cm^2

Review Exercise

1. a) i b) iv c) ii d) ii e) iv

3. a) 1, 2, 3, 6, 13, 26, 39, 78
   b) 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 20, 21, 24, 28, 30, 35, 40, 42, 56, 60, 70, 84, 105, 120, 140, 168, 210, 280, 420, 840
   c) 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
   d) 1, 3, 5, 15, 47, 141, 235, 705

4. a) 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198, 216, 234
   b) 30, 60, 90, 120, 150, 180, 210, 240
   c) 55, 110, 165, 220
   d) 82, 164, 246, 328

5. a) 2 x 283 b) 2^2 x 197 c) 2^3 x 5^3 d) 2^4 x 193 e) 1 x 8777 f) 2^1 x 3^1 x 11^1 x 61^1 g) 2^1 x 3^1 x 1009^1 h) 2^1 x 3^1 x 11^1 x 83^1

6. a) 5x5x5x5x5x5 b) 2x2x2x5x5x5x5x5 c) 3x3x3x3x3x3x3x7x7x11x11x11x11 d) 1x1x5x5x5x3x3x3x3x3x3

7. a) 1 b) 1 c) 6 d) 1

8. a) 4 b) 2 c) 7 d) 4

9. a) 1,642,200 b) 473,060 c) 301,860 d) 406,725

10. a) 364,004 b) 55,333,980 c) 2,829,480 d) 11,911,760

11. a) 256 b) 144 c) 1936 d) 3844

12. 170 cm 13. At 2:51 a.m. 14. 1 15. 800 16. 36,000

17. 7.35 18. 65536 19. 240

Unit 2: Integers

Exercise 2.1

2. a) +5 b) -11 c) 0 d) +9 e) -14 f) +23 g) +19 h) +8

3. a) -3, -2, -1, 0, +1, +2, +3, +4, +5
   b) -6, -5, -4, -3, -2
   c) -13, -12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +10
   d) -17, -16, -15, -14, -13, -12, -11, -10, -9, -8, -7
   e) -4, -3, -2, -1, 0, +1, +2, +3, +4
   f) +7
   g) -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +10, +11, +12, +13, +14, +15, +16
   h) -18, -17, -16, -15, -14, -13, -12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +10, +11, +12, +13, +14, +15, +16, +17, +18, +19

4. a) +4, +3, +2, +1, 0, -1 b) -5, -4, -3, -2, -1, 0 c) -13, -14, -15, -16, -17, -18 d) -15, -14, -13, -12, -11, -10

5. a) +12000 b) -700 c) -30° d) -720 e) +100 f) -5695 g) +2° h) -15° i) -3 j) +55

6. Loss of 3000, -3000

Exercise 2.2

2. a) -666 *(A greater integer with a negative sign is always smaller than every smaller integer with a negative sign)*
   b) *-234 (A greater integer with a negative sign is always smaller than every smaller integer with a negative sign)*
   c) -1000 *(A greater integer with a negative sign is always smaller than every smaller integer with a negative sign)*
   d) -55 (A greater integer with a negative sign is always smaller than every smaller integer with a negative sign)

3. a) -13, -5, -1, 0, +3, +4, +7 b) -12, -5, -3, +2, +3, +11 d) -12, -8, -5, 0, +6, +11, +17 e) -5, -4, -3, 0, +1, +7, +8, +9

c) -33, -16, -12, 0, +5, +9, +11

4. a) +9, +7, +0, -1, -2, -3, -5
b) +23, +12, 9, -8, -11, -13
c) +45, +24, +10, 0, -12, -17, -20

f) -15, -14, -11, 0, +23, +24, +27
d) +16, +8, 0, -7, -8, -9
e) +10, +4, +1, -1, -8, -9, -10
f) +90, +78, +56, 0, -11, 34, -45, -78

Exercise 2.3

1. a) 5 b) 67 c) 99 d) 90 e) 80 f) 40

2. a) A: 0, 7, 22, 23, 33, 50 D: 50, 33, 23, 22, 7, 0
b) A: 0, 3, 5, 6, 11, 77 D: 77, 11, 6, 5, 3, 0
c) A: 0, 7, 8, 9, 11, 14 D: 14, 11, 9, 8, 7, 0
d) A: 3, 5, 7, 11, 13, 18 D: 18, 13, 11, 7, 5, 3
e) A: 2, 4, 7, 9, 10, 16 A: 16, 10, 9, 7, 4, 2
f) A: 6, 7, 8, 22, 25, 31, 56 D: 56, 31, 25, 22, 8, 7, 6
g) A: 6, 8, 9, 11, 12, 14, 23 D: 23, 14, 12, 11, 9, 8, 6
h) A: 3, 5, 6, 7, 8, 9, 10, 13, 15, 23
D: 23, 15, 13, 10, 9, 8, 7, 6, 5, 3

3. +11, -5 absolute values are 11 and 5.

Review Exercise

1. a) ii b) iii c) ii

3. a) +7 b) -101 c) +987 d) +50 e) +13 f) +56

5. a) -4, -3,2, -1,0,+1,+2,+3,+4
b) -5, -4,3, -2, -1,0,+1,+2,+3,+4,+5,+6,+7
c) -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2,+3,+4,+5,+6,+7,+8,+9,10,+11,+12, +13, +14,+15,+16
d) -18, -17, -16, -15, -14, -13, -12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +10, +11, +12, +13, +14,+15,+16,+17,+18,+19

6. a) (-6, -3, -1, 0, +5, +7),
(+7, +5, 0, -1, -3, -6)
b) (-12, -5, -3, +2, +3, +11),
(+11, +3, +2, -3, -5, -12)
c) (-56, -23, -13, 0, +14, +15, +23),
(+23, +15, +14, 0, -13, -23, -56)
d) (-16, -12, -11, +10, +25, +34),
(+34, +25, +10, -11, -12, 16)

8. a) 11 b) 45 c) 99 d) 123 e) 88 f) 100

9. a) A: 0,2,7,8,16,19
D: 19,16,8,7,2,0
b) A: 0,3,8,11,17,23
D: 23,17,11,8,3,0
c) A: 0,1,4,16,17
D: 17,16,4,1,0
d) A: 3,4,9,18,19
D: 19,18,9,4,3
e) A: 5,8,15,21,77
D: 77,21,15,8,5

Unit 3: Laws of Integers and Order of Operations

Exercise 3.1

2. a) +4 b) -11 c) -2 d) -19 e) +12 f) -4 g) -14 h) +1 i) +2 j) -18 k) -13 l) -1

3. a) +14 b) -3 c) -3 d) -5 e) +2 f) -11 g) -10 h) -2

4. -57 5. 3 floors down from the ground floor 6. -6°C

Exercise 3.2

2. a) -14 b) -5 c) -2 d) -6 e) -1 f) -1 g) -2 h) -4 i) -6 j) +15 k) -11 l) -2

3. a) -5 b) -1 c) -4 d) -5 e) -1 f) -2

4. +65 5. 99m 6. 106km 7. Rs 11

Exercise 3.3

1. a) +5 b) +7 c) -17 d) -16 e) -25 f) +84 g) +2 h) +12

2. a) True b) False c) False d) True e) False f) False g) True h) False

Exercise 3.4

1. a) +10 b) -28 c) -18 d) +30 e) -130 f) -280 g) -66 h) -1470 i) +242 j) -1160 k) -20 l) -100

3. -1 4. -10,000

Exercise 3.5

3. a) $-\frac{1}{3}$ b) $-\frac{1}{2}$ c) $\frac{1}{7}$ d) $\frac{1}{100}$ e) $-\frac{1}{77}$ f) $-\frac{1}{243}$ g) $-\frac{1}{48}$ h) $-\frac{1}{55}$ i) $-\frac{1}{60}$ j) -2 k) $\frac{2}{3}$ l) $-\frac{7}{2}$

Exercise 3.6

1. a) -9 b) +7 c) -20 d) -4 e) -6 f) -12 g) -21 h) -10 i) +24 j) -5

2. a) 0 b) 0 c) 0 d) 0 e) 0 f) 0

Exercise 3.7

1. a) 22 d) $10\frac{176}{315}$ g) $16\frac{8}{45}$ j) $\frac{24}{35}$ m) 9.74

b) 44 e) 105 h) $9\frac{5}{24}$ k) $19\frac{1}{9}$

c) $1\frac{3}{4}$ f) $7\frac{5}{9}$ i) $22\frac{211}{240}$ l) 38.54

Exercise 3.8

1. Rs 3530 2. 51.9m 3. 3 4. Rs 1682.5 5. 24 blocks

6. 5.25 kg apples and 2.5 kg manoges

Review Exercise

1. a) ii b) iii c) iv d) ii e) ii f) i g) i h) iii i) iv j) ii k) iv

l) iv m) iv n) iii

3. 5 4. 2

6. a) 17 c) 42 d) $-2311\frac{5}{7}$ e) $9\frac{7}{10}$

b) 42

7. Rs. 1172.49 8. $36\frac{5}{12}$ 9. a) Rs 751.5 b) Rs 12215.5 10. 106 Erasers

11. $38\frac{3}{7}l$ 12. Rs 1432, Rs 3568 13. Rs.206.98

Unit 4: Rate, Ratio and Percentage

Exercise 4.1

1. 62% , 2% 3. 92%, 92 percent

4. a) 5.25 d) 43.75 g) 174 liters j) 33.75 marks

b) 900 e) 523.692 h) 80 grams k) 0.25 mm

c) 26 f) 200 i) Rs. 27.75 l) 4.5

5. a) 8.3% d) 90% g) 53.3% j) 13.9%

b) 19.4% e) 34.7% h) 20% k) 60%

c) 19.55% f) 44.5% i) 6% l) 14.3%

6. 20% 7. 40% 8. 44% 9. 25% 10. 45 eggs

11. 122.5 12. Rs. 5400

Exercise 4.2

1. a) 54 out of 60 c) 68 out of 120 e) 15 out of 17

b) 18 out of 20 d) 440 out of 550 f) 200 out of 400

2. In town B more percentage of houses are under construction.

3. In English she got less percentage marks.

4. Hadia got highest percentage and Samiha got lowest percentage marks.

5. Sara has more empty pages.

Exercise 4.3

1. a) 415 b) 1526 c) 1593 d) 592.8

e) 1125 f) 275.4

2. Rs. 5508, Rs. 51, 408 3. 255 4. 2.43 kg, 24.57kg 5. 69.44

Exercise 4.4

1. a) 7:11 c) 40:3 e) 4:5

b) 1:12 d) 1:8 f) 10:1

2. a) $\frac{4}{5}$ c) $\frac{19}{25}$ e) $\frac{55}{78}$ g) $\frac{6}{23}$

b) $\frac{12}{13}$ d) $\frac{34}{63}$ f) $\frac{1}{2}$ h) $\frac{8}{9}$

3. a) 1:2 d) 9:11 g) 7:20 j) 7:9

b) 3:40 e) 16:39 h) 1:3 k) 1:2

c) 2:3 f) 3:4 i) 2:3 l) 32:45

4. a) 2:3 c) 20:49 e) 7:8

b) 90:7 d) 1000:3 f) 4:9

5. 3:13 6. 5:4 7. 10:1 8. 25:24 9. 102:238

Exercise 4.5

1. a) 4:5:6 c) 8:9:12 e) 5:20:24

b) 7:21:24 d) 60:70:7

2. 40, 40, 100 3. 16:18:27 4. 8:12:15 5. 20:12:21

6. Rs 2000, Rs 3000, Rs 2500 7. Rs 2048, Rs 1792, Rs 1536 8. Rs 896, Rs 1120, Rs 1400

Exercise 4.6

1. a) 7 km/h c) 18w/min e) Rs.120/m

b) Rs.86/m d) Rs.50/kg

2. Rs. 220/ticket 3. Rs. 880/m 4. Rs. 60/kg 5. Rs. 157.89/h

Review Exercise

1. a) i b) iv c) iii d) ii e) ii f) ii g) i h) iii i) ii
5. a) 5:1 b) 1:3 c) 5:1 d) 7:8
6. a) $31\frac{1}{4}$ sec b) 75 sec
7. a) 85% b) 3:17
8. 8:12:9
9. 1500
10. In urdu he got more %.
11. Rs. 1440

Unit 5: Sets

Exercise 5.1

1. a, c, d, f, g, i, j, k
3. a) ∈ b) ∈ c) ∉ d) ∉ e) ∈
4. a) 4 ∈ W b) 4 ∈ P c) 0 ∉ natural numbers
5. a) T b) F c) T d) T e) T f) F g) F h) T i) T j) F
6. a) n(B) =7 b) n(Q)=5 c) n(N)=0 d) n(K)=4 e) n(R) =3 f) n(S)=4 g) n(P)=5 h) n(E) = 5

Exercise 5.2

1. a) A = set of first 6 small letters.
   b) B = set of first 3 odd numbers.
   c) D = set of first 8 multiples of 4.
   d) E = set of multiples of 10.
   e) F = set of first 6 multiples of 2.
   f) G = set of weekend days of a week.
2. a) {Muharram, Safar, Rabi-ul-Awwal, Rabi -us Sani, Jamadi-ul-Awwal, Jamadi-us-Sani, Rajab, Shaban, Ramadan, Shawal, Zil-Qadah, Zul-Hijah}
   b) {1, 3, 5, 7, 9, ...}
   c) {Punjab, Sindh, Balochistan, Khyber Pakhtunkhwa }
   d) {Pakistan, Saudi Arabia, Bangladesh, Turkey, Iran, Malaysia, Indonesia......}
   e) {2, 3, 5, 7, 11, 13, 17, 19}
   f) {t, u, v, w, x, y, z}
   g) {56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76}
   h) {111, 112, 114, 115, 116, 117, 118 }

Exercise 5.3

1. a, b, e, are finite.
   c, d, f, g are infinite.
2. a, b, c, d, f are non-empty sets.
   e is empty set because multiples of 4 are not odd.
   g is empty set because prime numbers have only two factors.
3. b, d, g are singleton sets.

Review Exercise

1. a) i b) ii c) ii d) iv e) ii
3. b, c, d, f
4. a) {Pakistan, Saudi Arabia, Malaysia, Turkey, Bangladesh, Iran}
   b) {a, e, i, o, u}
   c) {January, February, March, April, May, June}
   d) {Red, Orange, Yellow, Green, Blue, Violet, Indigo}
   e) {cat, dog, parrot, Horse, Hen}
5. a) { } b) {2, 3, 5, 7, 11, 13, 17, 19, 23} c) { }
6. b, & d
7. a) {7, 9, 11, 13}
   b) {2, 3, 5, 7, ...}
   c) Set of first 6 natural numbers
   d) Set of first 5 multiples of 2
   e) {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, and Sunday}
8. a) {1,2,3,4,5}
   b) {a,e,i,o,u}
   c) {1,3,5,7}
   d) {1,2,3,4,6,12}
   e) {M,A,T,H,E,I,C,S}
   f) {September, October, November, December}
9. a) Set of natural numbers between 6 and 11.
   b) Set of first 5 multiples of 6
   c) Set of odd nos between 1 and 13
   d) Set of letters to be used in word rest
   e) Set of first four multiples of 5
10. a) Infinite b) Singleton c) Infinite d) Finite e) Infinite f) Finite

Unit 6: Algebraic Expressions

Exercise 6.1

1. a) 24, 29,34, add 5 in previous term.
   b) 128, 256, 512, multiply previous term with2.
   c) 625, 125, 25, divide previous term with 5
   d) 78, 67, 56, subtract 11 from previous term
   e) 80, 160, 320, multiply previous term with 2
   f) 90, 106, 122, add 16 in previous term.
2. 12, 15
3. 8, 10, 12

Exercise 6.3

1. a) +4a, +5b
   b) +4y, $-3x$, -4
   c) $+6x$, -y, +7
   d) $+3x^2$, y^2
   e) +abc, -d
   f) $+11x^3$, $+xyz^2$, +4
   g) $+4x^3$, $+5y^3$, +4
   h) $+a^2$, $-b^2$, +3

2. a) x, 1, +3
   b) (y, z), (7, 1), -1
   c) (a, b), (2, 7), 0
   d) Y, ½ , +6
   e) (a, b, c), (7, -4, 1)

3. a) 3a – 2b
   b) 3y – 4z +1
   c) $9x^2 - y^2 + 5$
   d) $X^2 - y^2$
   e) 6abc -bcd+2de
   f) $13x^4 + xy^2 7$

4. a) $(7a, -5a, -a, +\frac{a}{3})$
   b) $(2p^3q^2, -2p^3q^2)$
   c) (-xy, +5xy,-xy)
   d) $(2x^2y, -2x^2y, +4yx^2, -3yx^2)$
   e) $(-5a^3b^2, 7a^3b^2, -3b^2a^3)$

Exercise 6.4

1. a) 25
   b) 14
   c) 6
   d) -36
   e) 144
   f) 4
   g) $-6\frac{1}{2}$
   h) 38

4. 34

Exercise 6.5

1. a) 2ab
   b) $16x^3$
   c) 10
   d) 20
   e) 0
   f) 4x+3y
   g) 7x+8y+8
   h) $-8x^2+7x-3$
   i) 12x+11y+5z
   j) $9x^2+8y+13$

2. a) 6x-8y
   b) $12x^3-5y^3$
   c) 2x+2y-z
   d) 3x+8y-5
   e) $-2x^2+4y-5$
   f) $-2x^2 + 8$
   g) $2a^2+2c^2-2abc$
   h) $5x^2+8x+3$
   i) 3a+7b-8c
   j) $5x^2-2xy-2y^2$

3. a) 4ab
   b) $-13x^3$
   c) -15y
   d) 16y
   e) -7xz
   f) -8x-5y
   g) $9x-7y^2-8$
   h) $11x^2-8x+11$
   i) x+10y
   j) $7-10x^2+8y$

4. a) 2a-8b+4c
   b) $9+7x-5x^2$
   c) 6x-6y+4z
   d) -x-10y+9
   e) $11x^2$
   f) $8x^2-10y+6$
   g) $-12x^2+x-9$
   h) 2b+6c-7a
   i) $2xy-17x^2$
   j) $-3a^2-6b^2+7c^2-4abc$

5. $3x^2-xy-4y^2$

6. a^3-5a^2+7a-7

7. a^3+4a^2-a+3

8. a) 2a+2c
   b) 2c
   c) 2a
   d) -2c
   e) a+b+3c
   f) -a-b+c
   g) -a-b-3c

Exercise 6.6

1. a) 8x+4
   b) $2+12x^2+12x$
   c) 2x
   d) 7l-2m
   e) u+2v+t
   f) $2+12x^2+30x$
   g) 17a-2b
   h) 200b
   i) $4y+42yz-84z+42z^2$
   j) 32a+20b-8c

Review Exercise

1. a) i b) ii c) iv d) i e) ii

2. 220 Q. 3. 70

4. a) 6 is added in previous term 39, 45, 51
   b) 2 is multiplied with previous term 160, 320, 640

5. a) 6x-y-5z
   b) -11p-2q-5r

6. a) $8x^2+25xy+10y^2$
   b) 4x-4y-3z

7. a) -33 b) 19 c) -11 d) -63

Unit 7: Linear expressions and Equations

Exercise 7.1

1. a, b, c, e, h, i, j are equations and d, f, g are expressions

Exercise 7.2

1. a, c, e, g, h are linear equations

Exercise 7.3

1. a) x+5
   b) 8-x
   c) X-20
   d) 2x-2
   e) $\frac{1}{2}x + 7$
   f) 6 + x - 2

2. a) x+6=8
   b) x-4=13
   c) 6x-3=5
   d) 3(x+8)=14
   e) x-6=12
   f) 2x-33=66
   g) 2x=10x-16
   h) 5x-6=14+x
   i) 2x=26

3. a) A number subtracted from 8 is one.
   b) 4 added to nine times the number is 22.
   c) 7 subtracted from three times the number is 23.
   d) 5 added to the number is twice the number.
   e) Sum of number subtracted from 6 and twice the number and is 8.
   f) Number subtracted from 48 is 7 times the number.
   g) 8 subtracted from 4 times the number is twice the number.
   h) Number divided by 36 is four times the number.

Exercise 7.4

1. a) $-3\frac{1}{2}$
 b) $1\frac{5}{16}$
 c) $3\frac{6}{7}$
 d) $-3\frac{1}{2}$
 e) 3
 f) 1
 g) 180
 h) $-\frac{1}{2}$
 i) -2
 j) 42
 k) 48
 l) $2\frac{6}{7}$
 m) $10\frac{1}{4}$
 n) 20
 o) 17
 p) 31
 q) $2\frac{2}{5}$
 r) $9\frac{4}{5}$

2. 11   3. Rs. 560   4. 104, 52   5. 36   6. 15, 12

7. 5   8. -27

Review Exercise

1. a) i
 b) ii
 c) iii
 d) ii
 e) iv
 f) i
 g) ii
 h) iv
 i) i
 j) iv

3. b,c,e are equations and a,d,f are expressions

4. a) $\frac{3}{8}$
 b) $7\frac{1}{5}$
 c) 1.6
 d) $\frac{1}{2}$
 e) 2
 f) -6
 g) $10\frac{2}{3}$
 h) 2
 i) $-1\frac{1}{54}$
 j) 1.425
 k) 6.5

5. Rs. 59115   6. 12 years   7. x=6

Unit 8: Surface area and Volume

Exercise 8.1

1. a) 7.8cm   b) 60cm   c) 168cm

2. a) $16cm^2$, 16cm
 b) $51.84cm^2$, 28.8cm
 c) $11.25cm^2$, 42cm
 d) $38.44m^2$, 24.8m

3. a) $9.4cm^2$, 13.4cm
 b) $248cm^2$, 63cm
 c) $92cm^2$, 54cm
 d) $244cm^2$, 64.4cm

4. l = 25m   5. l = 12m   6. $480m^2$, Rs.96000   7. $90000m^2$

8. 78cm   9. l = 64cm, Rs. 512000   10. $225m^2$

Exercise 8.2

1. $344m^2$ 3. $4864cm^2$ 5. $1470cm^2$

2. $1488400cm^2$ 4. $29124m^2$ 6. $339m^2$

Exercise 8.3

1. a) $312\ cm^2$ b) $13\ m^2$ c) $22\ cm^2$

2. 9.88cm   4. 18.19cm   6. 3.47cm   8. 30m

3. $130.2cm^2$ 5. $1.5m^2$ 7. 1.147cm

Exercise 8.4

1. a) 1, 1, 0, 1, 1
 b) 2, 2, 0, 0, 1
 c) 6, 0, 12, 8, 0
 d) 6, 0, 12, 0
 e) 0, 0, 0, 0, 1

Exercise 8.5

1. a) $150cm^2$, 125cm
 b) $384cm^2$, 512 cm
 c) $110.94cm^2$, 79.507 cm
 d) $245.76cm^2$, 262.144cm
 e) $24\ m^2$, 8m
 f) $54\ m^2$, 27m
 g) $864\ cm^2$, 1728cm
 h) $48600\ cm^2$, 72900cm
 i) $12476.16\ m^2$, 94818.816m
 j) $60000\ m^2$, 1000000m
 k) $600\ cm^2$, 1000cm
 l) $1815\ m^2$, 166375m

2. a) $846cm^2$, $1386cm^3$
 b) $3998cm^2$, $11997cm^3$
 c) $80401.9m^2$, $416760m^3$
 d) $2398cm^2$, $7980cm^3$

3. a) 4.96cm, $456.32cm^3$
 b) $2058cm^3$
 c) 7.13 cm, $289.64cm^2$
 d) 3.36cm, $592.56cm^2$

4. a) $150cm^2$, $125cm^3$
 b) $142cm^2$, $105cm^3$
 c) $52m^2$, $24m^3$
 d) $294m^2$, $343m^3$

Exercise 8.6

1. $6m^3$ 2. $1584cm^3$ 3. $299160cm^3$ 4. Rs.788400

5. Rs. 5166.04   6. Surface area of cube is smaller   7. 18cm

8. a) $24m^3$ b) Rs.11440

9. a) 44 cm   b) $9128cm^2$ c) Rs.1323560

Review Exercise

1. a) iv
 b) i
 c) iv
 d) ii
 e) iv
 f) iii
 g) i
 h) iv
 i) ii

3. $A=1156cm^2$, P=136cm   4. A=8624m, $P=372m^2$ 5. L = 400 m, Rs 8,100,000

6. 3.5 cm   7. 49 m, Rs 411,600   8. $1154.56\ cm^2$ 9. $73.2cm^2$ 10. 14.46 cm

11. $2860\ cm^2$ 13. 22 cm

14. a) $236\ cm^2$, $240 cm^3$
b) $28566\ cm^2$, $328509\ cm^3$
c) $150 cm^2$, $125\ cm^3$
d) $202\ cm^2$, $180\ cm^3$

15. $115.98\ cm^2$ 16. Rs.3182 17. $77.91\ m^3$ 18. $105840\ cm^3$ 19. 35.17 cm

20. $1888.32\ cm^2$

Unit 9: Lines, Angles and Symmetry

Exercise 9.2

1. a and d 2. a, c, d, e 3. a, e

Exercise 9.3

1. (i) a) $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EF}$ is transversal.
b) (l,p), (n,r), (m,q), (o,s) are corresponding angles.
(n,q), (o,p) are alternative interior angles.
(m,p), (l,s) are alternative exterior angles.

(ii) a) $\overleftrightarrow{PQ} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{TU}$ is transversal.
b) (1,5), (3,7), (2,6), (4,8) are corresponding angles.
(3,6), (4,5) are alternative interior angles.
(1,8), (2,7) are alternative interior angles.

2. a) b=c=e=35°, a=f=d=145°
b) c=d=f=145°, a=b=g=e=35°
c) z=70°, x=110°, y=110°
d) g=55°, f=50°
e) l=m=n=35°
f) x=70°, y=70°, z=35°
g) a=100°, b=145°, c=135°
h) x=78°, y=78°, z=58°

3. a) x=28° b) X=82°

Exercise 9.4

1. a) 4 b) 2 c) 2 d) 1 e) 2 f) 4 g) 2 h) 2 i) 2

Review Exercise

1. a) iv b) ii c) i d) iv e) ii f) iv

4. x=148°, h=d=e=70°, b=f=110°=g=c, x=70°, y=110°

7. Order of rotation of each figure is 4.

Unit 10: Geometrical Constructions

Review Exercise

1. a) ii b) ii c) iv d) iv

Unit 11: Data Management

Exercise 11.1

5. a) Multiple bar graph in vertical form
b) Days
c) No of toys sold
d) 15
e) 20
f) Monday
g) Tuesday
h) 40+20=60
i) Tuesday
j) 5

6. a) Multiple bar graph in horizontal form
b) No of students
c) Grades
d) 40
e) 25
f) 5
g) 5
h) 2020
i) 2020
j) 2020
k) 2020

Exercise 11.2

4. i) 90 ii) 60 iii) 30

5. i) 3000 ii) 1500, 1200 iii) 1000 iv) 500

6. i) 6h ii) 8h iii) 3h iv) Play time is 1h v) 2h vi) 4 h

Exercise 11.3

1. a) Math & Science b) Urdu 2. a) 8 b) 8

Exercise 11.4

1. i) 44.6 ii) 137.75 iii) 262.3 2. 20846.6

3. i) 12 ii) 100.5 iii) 71 4. i) 8,15 ii) 120 iii) no mode

Exercise 11.5

2. a) $\frac{3}{7}$ b) $\frac{1}{7}$ c) $\frac{1}{7}$

3. a) 14 candies b) $\frac{2}{7}$ c) $\frac{3}{14}$ d) $\frac{1}{2}$ e) Orange candy is more likely to be picked.

5. a) 5 b) i. $\frac{3}{8}$ ii. $\frac{3}{8}$ iii. $\frac{1}{8}$ iv. $\frac{1}{8}$

6. a) $\frac{8}{29}$ b) $\frac{10}{29}$ c) $\frac{25}{29}$ 7. $\frac{3}{8}$ Q. 8. $\frac{11}{14}$

9. a) 4, 7, 0, 3, 8,7 b) $\frac{1}{3}$ c) $\frac{1}{6}$

Review Exercise

1. a) iii b) ii c) iii d) ii e) iv f) iii

3. a) Cricket b) Badminton

5. a) Recipe 3 of pasta got the greatest vote b) 16 c) 30 d) 10 e) 10

7. a) Mean= 49.2, Median = 49, Mode= no mode b) Mean = 265, Median=254, Mode=205

Glossary

A	
Adjacent Angles	two angles with common vertex and common arm
Algebraic equation	An open mathematical statement that shows equality of ttwo or more expression
Altitude	Height of the geometric figure
Angle	Two non-parallel lines meet at a common end point
Antecedent	the first term of the ratio
Area	the surface covered by the shape
Arithmetic means	the sum of values divided by number of value
Average	The central value that represents all the values of a distribution
Bisector	A line that divides a given segment into two equal parts
C	
Consequent	The second term of the ratio
Continued Ratio	Comparison of three or more quantities in a certain order
Continuous Data	The value or quantities that are in form of fraction or decimal
Cube	A 3D shape with 6 square faces
Cuboid	A 3-D shape which has six rectangular faces
D	
Data collection	The process of collecting information
Diagonal	A line segment which is drawn between the opposite vertices of the rectangle
Dimension	the measurement of length in one direction
Discrete data	The value or quantities that are in form of integers or whole numbers
E	
Empty set	The set with zero element
Equation	A mathematical sentence which has equal sides separated by an equal sign.
Event	outcomes of collection of outcomes
F	
Factors	A number that divides the other number without leaving any leftover
Finite set	the elements of set limited
G	
Graph	The Pictorial representation of data that shows the relationship between two quantities
Grouped Data	the collection of data that is organised in group

H	
HCF	The greatest number that divides the two or more numbers exactly without any remainder.
Hemisphere	each part of the sphere when a sphere is divided into equal parts
I	
Index	A small number that show above the numbers that represents how many times
Infinite set	The elements of set are unlimited
Integers	Set of all positive and negative numbers with 0
L	
LCM	The smallest number that is divisible by each of the given number
Line	combination of points that extend in both directions
Linear equation	An algebraic equation where the highest power of the variable involved is 1.
M	
Median	the value which divides the value into two equal halves, with half of the values being lower than the value and half is higher than the value
Mode	the frequently occurring value in the data
Multiples	A number that is obtained by multiplying a number with any other number
N	
Number pattern	set of numbers that follow some pattern
O	
Outcomes	the possible result of random experiment
P	
Parallel lines	The lines which do not intersect each other at a point, when they are extended on either side in the same direction
Parallelogram	A four sided figure whose opposite sides are parallel and of equal lengths
percentage	out of 100
Perimeter	Sum of length of all sides of a shape
Perpendicular bisector	a line divides a given line segments into two equal parts and make an angle of 90o.
Perpendicular lines	The lines that intersect each other at 90o
Pie graph	the graph in form of circle
Point	zero dimensions
Prime factorization	a process in which a number can be written as a product of its prime factors

Probability	it is a measure of the likelihood or possibility of an even
R	
Rate	comparison of two quantities with different units
Ratio	The comparison of two quantities with same unit
Reflection	A transformation that uses a line like a mirror to reflect a figure.
Rotational symmetry	A shape or object is rotated around a central point and look exactly the same at least 2 times in a full rotation
S	
Sample space	the set of all possible outcomes of an experiment
Sentence	A set of word that is complete in itself and conveying full meaning without any ambiguity
set	A well defined collection of distinct object
Singleton set	The set with one element
Statement	A sentence that is either true or false
Surface	The outer most part of a shape
Surface area	The total area of all face of any solid shape
T	
Transversal	A line passes through two or more given lines at different points
Trapezium	A four-sided figures with one pair of parallel sides.
U	
Ungrouped data	raw data that does not convey meaning to us
Universal set	set of all elements which are under consideration in a particular context
V	
Variable	A quantity that varies or changes
Venn diagram	Representation of sets and their relationship in a diagram
Volume	The amount of space which a 3-D shape occupies

Important Formulas

LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors

$a \times b = HCF \times LCM$

$a + b = b + a$

$(a + b) + c = a + (b + c)$

$(a \times b) \times c = a \times (b \times c)$

$a \times (b + c) = (a \times b) + (a \times c)$

$a \times (b - c) = (a \times b) - (a \times c)$

$a \times (b - c) = (a \times b) - (a \times c)$

Perimeter of rectangle = 2(length + width)

Perimeter of a square = 4 × L

Area of a square = length × length

Area of a rectangle = length × width

Area of border = Area of large rectangle – Area of small rectangle

Area of parallelogram = base × altitude

Area of trapezium = altitude × $\left(\frac{b_1 + b_2}{2}\right)$

Area of triangle = $\frac{1}{2}$ (altitude × base)

Volume of cube = length × length × length

Volume of cuboid = $l \times w \times h$

Volume of cuboid = $l \times w \times h$

Area of cube = $6 \times l^2$

Total surface area of cuboid = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

Angle of the sector = $\frac{\text{number of observations of a class}}{\text{total number of observations in all classes}} \times 360°$

Arithmetic mean = $\frac{\text{sum of values}}{\text{number of values}}$

Probability of an event = P (E) = $\frac{\textit{Number of favorable (desired) outcomes}}{\textit{Total number of possible outcomes}}$

Web Links

https://www.mathplayground.com/factortrees.html
https://www.transum.org/software/SW/Starter_of_the_day/Students/HCF_LCM.asp?Level=2
https://www.tes.com/teaching-resource/lcm-and-hcf-treasure-hunt-11743277
https://www.tes.com/teaching-resource/square-numbers-matchup-game-11059444
https://www.mathplayground.com/mobile/numberballs_fullscreen.htm
https://www.teacherspayteachers.com/Product/Squared-and-Cubed-Numbers-Maze-4790446
https://www.youtube.com/watch?v=9w7gwFA1HNI
https://www.youtube.com/watch?v=1DKWG5CBeek
https://www.youtube.com/watch?v=CfkaifC7tGY
https://www.mathplayground.com/ASB_SpiderMatchIntegers.html
https://www.mathgames.com/skill/7.96-properties-of-addition-and-multiplication
https://www.mathgames.com/skill/3.38-solve-using-properties-of-addition
https://www.brainpop.com/games/unitrates/?topic_id
https://www.transum.org/software/SW/Starter_of_the_day/Students/Set_Notation/Default.asp
https://www.univie.ac.at/future.media/moe/tests/mengen/duv.html
https://mathsframe.co.uk/en/resources/resource/42/sequences
https://www.topmarks.co.uk/ordering-and-sequencing/chinese-dragon-ordering
https://www.mathgames.com/skill/6.6-identify-terms-coefficients-and-monomials
https://prodigy-website.cdn.prismic.io/prodigy-website/eb0ad3f4-7f29-4860-84dc-57932df3f9a6_hexagon.pdf
https://www.mathgames.com/skill/3.75-solve-for-the-variable-with-addition-and-subtraction
https://www.splashlearn.com/s/math-games/find-the-perimeter-of-the-shapes-using-grids
https://www.splashlearn.com/s/math-games/find-the-perimeter-of-polygons
https://www.splashlearn.com/s/math-games/find-the-area-by-multiplying-the-side-lengthshttps://www.splashlearn.com/s/math-games/find-volume-using-the-formula
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SYMBOLS

Symbol	Stands for	Symbol	Stands for
N	Set of natural numbers	$\sqrt{}$	square root
Z	Set of integers	Σ	summation
%	percent	$\in$	belongs to
φ	empty set	$\notin$	does not belong to
	since/as	$\therefore$	therefore/so
:	ratio	$\Rightarrow$	implies that
=	is equal to	$\cap$	intersection
$\neq$	is not equal to	$\cup$	union
$<$	is less than	ABC	arc ABC
$>$	is greater than	AB	ray AB
$\leq$	is less than or equal to	AB	line
$\geq$	is greater than or equal to	AB	line segment
$\approx$	is approximately equal to	$\angle$	angle
|	such that		and so on