

4. Show that in angular form, centripetal acceleration is:
 $\vec{a}_c = -\omega^2 \vec{r}$.

Answer:

As centripetal acceleration is given by,

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad \dots\dots\dots (1)$$

We can write linear velocity in form of angular velocity as,

$$v = r\omega$$

So putting this value in equation (1), we get

$$\begin{aligned} \vec{a}_c &= -\frac{(r\omega)^2}{r} \hat{r} = -\frac{r^2\omega^2}{r} \hat{r} \\ \Rightarrow \quad \vec{a}_c &= -\omega^2 r \hat{r} \end{aligned}$$

And as,

$$\vec{r} = r \hat{r}$$

So we get, centripetal acceleration as

$$\vec{a}_c = -\omega^2 \vec{r}$$

5. Show that centripetal force is also shown by

$$\vec{F}_c = \frac{-mv^2}{r} \hat{r} = -m\omega^2 \vec{r} = \frac{-mv^2}{r^2} \vec{r}$$

Answer:

According to Newton's first law of motion, we have

$$\vec{F} = m\vec{a}$$

But in form of centripetal acceleration and centripetal force, we can write above equation as

$$\vec{F}_c = m \left(-\frac{v^2}{r} \hat{r} \right) = -mv^2 \frac{\hat{r}}{r} \quad \dots\dots\dots (1)$$

And as according to the definition of vector we can write,

$$\vec{r} = r\hat{r} \quad \Rightarrow \quad \hat{r} = \frac{\vec{r}}{r}$$

So putting this in equation (1), we get

$$\vec{F}_c = -\frac{mv^2}{r} \vec{r} \quad \dots\dots\dots (2)$$

Also, we know the relation between linear and angular velocity given as,

$$v = r\omega$$

Putting this in equation (2), we get

$$\begin{aligned} \vec{F}_c &= -\frac{m(r\omega)^2}{r} \vec{r} = -\frac{mr^2\omega^2}{r} \vec{r} \\ \Rightarrow \quad \vec{F}_c &= -m\omega^2 \vec{r} \quad \dots\dots\dots (3) \end{aligned}$$

Comparing equations (1), (2) and (3), we get

$$\vec{F}_c = -\frac{mv^2}{r} \hat{r} = -m\omega^2 \vec{r} = \frac{-mv^2}{r^2} \vec{r}$$

6. Show that a satellite near the earth will have a greater velocity.

Answer:

Gravitational force 'F' on a satellite of mass 'm' moving in a radius of 'r' around the earth is given by,

$$F = \frac{GM_em}{r^2} \dots\dots\dots (1)$$

And as the satellite is moving around the earth in a circle and for motion in a circle we have centripetal force as,

$$\vec{F}_c = \frac{mv^2}{r} \dots\dots\dots (2)$$

Comparing equations (1) and (2), we get

$$\begin{aligned} \frac{mv^2}{r} &= \frac{GM_em}{r^2} \\ \Rightarrow v^2 &= \frac{GM_e}{r} \\ \Rightarrow v &= \sqrt{\frac{GM_e}{r}} \end{aligned}$$

And as 'GM_e' gives a constant value, so we can write

$$v = \sqrt{\frac{\text{Constant}}{r}}$$

This gives,

$$v \propto \frac{1}{\sqrt{r}} \dots\dots\dots (3)$$

Equation (3) tells us that the velocity and radius of satellite's orbit are inversely proportional to each other i.e. Greater the orbit of, lesser will be the speed of the satellite. Which mean, if the satellite's orbit is close to earth i.e. have a smaller value of 'r' then it will have a larger value of velocity. So, a satellite near the earth will have a greater velocity.

When we show up to the present moment with all of our senses, we invite the world to fill us with joy. The pains of the past are behind

us. The future has yet to unfold. But the now is full of beauty simply waiting for our attention.

7. What do you mean by weight of a body? Use examples to distinguish between real weight and the apparent weight of a body.

Answer:

Weight:

“The weight of an object is the gravitational pull of the earth on object”.

OR

“The gravitational force exerted on the body due to earth or other planet is known as the weight of that body”.

Real Weight:

The weight of an object due to the gravitational pull of the earth is known as real weight. The weight of a person, ball, stone etc. due to earth's gravity represents their real weight.

Apparent Weight:

Weight measured by a spring balance is called apparent weight. The weight of a person in a pound of water and in moving elevator represents their apparent weights.

Explanation:

Consider the apparent weight of the object of mass 'm' is suspended from a spring balance in an elevator. When the lift is at rest then according to Newton's second law of motion, the resultant force on the body will be zero.

If 'w' is the weight of the body and 'T' is the tension in spring, then according to Newton's law we have

$$\vec{F} = \vec{T} - \vec{w} = m\vec{a}$$

But as elevator is at rest so, the acceleration will be zero here. Thus we get,

$$\begin{aligned} \vec{T} - \vec{w} &= m(0) = 0 \\ \Rightarrow \vec{T} &= \vec{w} \end{aligned}$$

But tension will remain equal to weight of the body until and unless the lift is at rest. But when the lift is in motion and is moving upward, then

$$\begin{aligned} \vec{T} &= \vec{w} + m\vec{a} \\ \text{i.e. } \vec{T} &> \vec{w} \end{aligned}$$

In that case, apparent weight of the body will be greater than the real weight of the body.

But, when the lift is moving downward, then we have

$$\vec{T} - \vec{w} = -m\vec{a}$$

Negative sign is due to deceleration. SO we can write,

$$\vec{T} = \vec{w} - m\vec{a}$$

i.e.

$$\vec{T} < \vec{w}$$

So, in that case, the apparent weight of the body will be less than its real weight. But if the cable carrying the lift breaks, then the lift will start falling under the action of gravity. Then,

$$\vec{a} = \vec{g}$$

And as weight = mg, then,

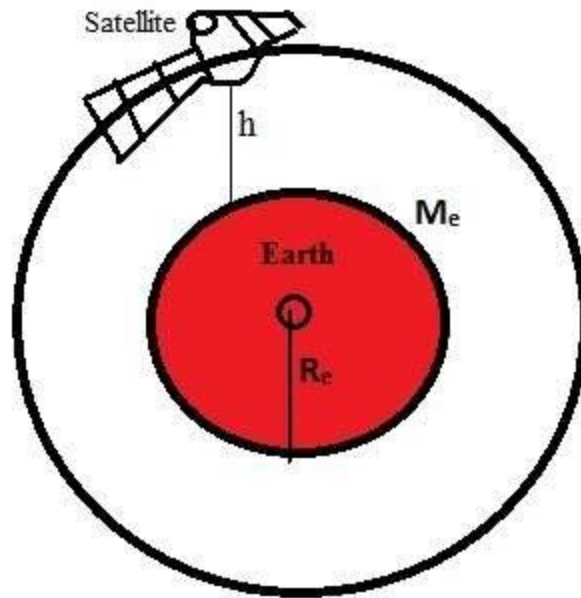
$$\vec{T} = m\vec{g} - m\vec{g} = 0$$

Which says that apparent weight shown by the scale will be zero there.

8. Explain, how gravity is provided to the occupants of the space ship.

Answer:

“In order to have a spaceship in space, we have to provide gravity to the occupants of the spaceship. Such provided gravity is known as artificial gravity because it does not exist naturally”.



When our earth spins around its own axis, it produces gravitational acceleration 'g' which is 9.8 ms^{-2} . This helps us to stay stable and walk on the surface of the earth. But in the case of satellite, there is no such spin so workers experience weightlessness in there which makes it difficult to work inside the satellite. In order to overcome this problem, artificial gravity is produced inside the satellite. For this purpose, some spin needs to be given to the satellite due to which the body inside the satellite will experience some centrifugal force which will push the body away from the axis of rotation and in turn the body will come into contact with the walls and ground of satellite.

To create this force, a centrifugal acceleration does also produce and is given by

$$\vec{a}_c = \frac{v^2}{r} \dots\dots\dots (1)$$

Where 'r' is the distance from the centre to the rim of a spaceship.

As, $v = r \omega$ and $a_c = g$

Then we can write equation (1) as,

$$g = \frac{(r\omega)^2}{r} = \frac{r^2 \omega^2}{r}$$

$$\Rightarrow g = r\omega^2 \dots\dots\dots (2)$$

As relation between frequency and angular velocity is given by,

$$f = \frac{\omega}{2\pi}$$

$$\Rightarrow \omega = 2\pi f$$

Putting this value of 'ω' in equation (2), we get

$$g = r (2\pi f)^2 = 4\pi^2 r f^2$$

Or we can write it as, $f^2 = g / 4\pi^2 r$

i.e.

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{r}}$$

This is the frequency with which we get gravity equal to the gravity of the earth.