

Biology Lesson 5 Problems

1. A string 1 m long is used to whirl a 100 g stone in a horizontal circle at a speed of 2 ms^{-1} . Find the tension in the string.

Answer:

Given Data:

Length of string = $l = 1\text{m}$

Mass of stone = $m = 100\text{g} = 0.1\text{kg}$

Speed of stone = $v = 2 \text{ ms}^{-1}$

To Find:

Tension in string = $T = ?$

Solution:

As the stone is whirling in a circle, so tension in the string will be equal to the centripetal force which is keeping stone in circular motion. So we can write,

$$T = F_c \dots\dots\dots$$

(1)

And as,

$$F_c = \frac{mv^2}{r}$$

Putting this value in equation (1), we get

$$T = \frac{mv^2}{r}$$

Putting values, we get

$$T = \frac{0.1kg \times (2ms^{-1})^2}{1m} = \frac{0.4kgm^2s^{-2}}{1m}$$

So tension in string is given as,

$T = 0.4 \text{ N}$

2. The moon revolves around the earth in almost a circle of radius 382400 km in 27.3 days. What is the centripetal acceleration?

Answer:

Given Data:

Radius of circle = $r = 382400 \text{ km} = 3.8 \times 10^7 \text{ m}$

Time = $T = 27.3 \text{ days} = 27.3 \times 24 \times 60 \times 60 \text{ s} = 2.4 \times 10^6 \text{ s}$

To Find:

Centripetal acceleration = $a_c = ?$

Solution:

As centripetal acceleration is given by,

$$a_c = \frac{v^2}{r} \dots \dots \dots (1)$$

As we know that, $v = r\omega$, so putting this value in equation (1) we get

$$a_c = \frac{(r\omega)^2}{r}$$

And as,

$$T = \frac{2\pi}{\omega} \quad \Rightarrow \quad \omega = \frac{2\pi}{T}$$

So putting this value, centripetal acceleration will become

$$a_c = \frac{r^2 (2\pi/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Putting values, we get

$$a_c = \frac{4 \times (3.14)^2 \times 3.8 \times 10^7 m}{(2.4 \times 10^6 s)^2} = \frac{1.5 \times 10^9 m}{5.76 \times 10^{12} s^2}$$

$$a_c = 0.27 \times 10^{-3} ms^{-2}$$

$a_c = 0.0027 ms^{-2}$

3. A ball of mass 'm' tied to one end of string and is whirled from the other end in a vertical circle of radius 'r' with constant speed. Find out the tension in the string, when the ball is at the highest point and the lowest point in its path of motion

a. (a) $T = m \left(\frac{v^2}{r} - g \right)$ (b) $T = m \left(\frac{v^2}{r} + g \right)$

Answer:

Given Data:

Mass of ball = m

Radius of circle = r

To Find:

(1) Tension in string when ball is at highest point = T_1

=?

(2) Tension in string when ball is at lowest point = T_2

=?

Solution:

(a) When the ball attain its maximum height then its weight will act in

the downward direction at that point. At that point, centripetal force will be

given by

$$F_c = T_1 + w$$

$\Rightarrow$

$$T_1 = F_c - w$$

Putting values of centripetal force and weight of a body under the action of

gravity, we get

$$T_1 = \frac{mv^2}{r} - mg$$

Taking 'm' common from left hand side, we get

$$T_1 = m\left(\frac{v^2}{r} - g\right)$$

(b) When the ball attains its maximum height then tension of string is acting upward and its weight will act in downward direction at that point. So taking direction of tension as reference, centripetal force will be given by

$$F_c = T_2 + (-w) = T_2 - w$$

$$\Rightarrow T_2 = F_c + w$$

Putting values of centripetal force and weight, we get

$$T_2 = \frac{mv^2}{r} + mg$$

Taking 'm' common from left hand side, we get

$$T_2 = m\left(\frac{v^2}{r} + g\right)$$

4. A motor car is traveling at a speed of 30 ms^{-1} . If its wheel has a diameter of 1.5m, find its angular speed in rads^{-1} and revs^{-1} .

Answer:

Given Data:

Speed of car = $v = 30 \text{ ms}^{-1}$

Diameter of wheel = $d = 2r = 1.5 \text{ m}$

$$\Rightarrow r = 0.75 \text{ m}$$

To Find:

(a) Angular speed in $\text{rad s}^{-1} = \omega = ?$

(b) Angular speed in $\text{rev s}^{-1} = \omega = ?$

Solution:

As the relation between angular velocity and linear velocity is given by,

$$v = r \omega$$

$$\Rightarrow \omega = \frac{v}{r}$$

(a):

Putting values, we get

$$\omega = \frac{30 \text{ ms}^{-1}}{0.75 \text{ m}} = 40 \text{ rad s}^{-1}$$

$\omega = 40 \text{ rad s}^{-1}$

(b):

As,

$$1 \text{ rev} = 2\pi \text{ rad}$$

$$\Rightarrow 1\text{rad} = \frac{1\text{rev}}{2\pi}$$

So angular velocity will become,

$$\omega = 40 \times \frac{1\text{rev}}{2\pi} \text{ s}^{-1} = \frac{40}{6.28} \text{ rev s}^{-1}$$

$\omega = 6.36 \text{ rev s}^{-1}$

5. An electric motor is running at $1800 \text{ rev min}^{-1}$. It comes to rest in 20 s. If the angular acceleration is uniform find the number of revolutions it made before stopping

Answer:

Given Data:

Initial speed of motor = $\omega_i = 1800 \text{ rev min}^{-1} = (1800 \times 2\pi/60) \text{ rad s}^{-1}$

$$\omega_i = 188.4 \text{ rad s}^{-1}$$

Final speed of motor = $\omega_f = 0 \text{ rad s}^{-1}$

Time = $t = 20\text{s}$

To Find:

Number of revolutions = $N = ?$

Solution:

We know that,

$$\text{No. of revolutions} = \frac{\text{Total angular displacement}}{2\pi}$$

i.e. $N = \frac{\theta}{2\pi}$ (1)

We have equation of angular motion as,

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$
 (2)

Also, $\omega_f = \omega_i + \alpha t$

$$\Rightarrow \alpha = \frac{\omega_f - \omega_i}{t}$$

Putting this value in equation (2), we get

$$\theta = \omega_i t + \frac{1}{2} \left(\frac{\omega_f - \omega_i}{t} \right) t^2 = \omega_i t + \frac{1}{2} (\omega_f - \omega_i) t$$

Putting values, we get

$$\theta = (188.4 \text{ rads}^{-1})(20s) + \frac{1}{2} (0 - 188.4 \text{ rads}^{-1})(20s) = 3768 \text{ rad} - 1884 \text{ rad}$$

$$\Rightarrow \theta = 1884 \text{ rad}$$

Putting this value in equation (1), we get

$$N = \frac{1884 \text{ rad}}{2 \times 3.14} = 300 \text{ rev}$$

$N = 300 \text{ rev}$

6. What is the moment of inertia of a 100 kg sphere whose radius is 50 cm?

Answer:

Given Data:

Mass of sphere = $m = 100 \text{ kg}$

Radius of sphere = $r = 50 \text{ cm} = 0.5 \text{ m}$

To Find:

Moment of inertia = $I = ?$

Solution:

As moment of inertia of a sphere is given by,

$$I = \frac{2}{5}mr^2$$

Putting values, we get

$$I = \frac{2}{5} \times 100\text{kg} \times (0.5\text{m})^2 = 0.4 \times 100\text{kg} \times 0.25\text{m}^2$$

So moment of inertia will become,

$I = 10\text{kgm}^2$

7. What is the kinetic energy of a 5.0 kg solid ball whose diameter is 15 m if it rolls across a level surface with a speed of 2 ms^{-1} ?

Answer:

Given Data:

Mass of ball = $m = 5.0 \text{ kg}$

Diameter of ball = $d = 2r = 15 \text{ m}$

$$r = 7.5 \text{ m}$$

Speed of ball = $v = 2 \text{ ms}^{-1}$

To Find:

Kinetic Energy of ball = K.E = ?

Solution:

As the ball is rolling then its K.E will be a sum of rotational kinetic energy and translational kinetic energy, given by

$$(K.E)_{\text{total}} = (K.E)_{\text{translational}} + (K.E)_{\text{rotational}}$$

Or,

$$(K.E)_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \dots\dots\dots (1)$$

Where ' ω ' is angular velocity and is given by, $\omega = v/r$

As the ball is a sphere, and inertia of a sphere is $I = \frac{2}{5}mr^2$. So, putting this, equation (1) will become

$$(K.E)_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

Putting values, we get

$$(K.E)_{\text{total}} = \frac{1}{2} \times 5.0\text{kg} \times (2\text{ms}^{-1})^2 + \frac{1}{5} \times 5.0\text{kg} \times (2\text{ms}^{-1})^2$$

$$\Rightarrow (K.E)_{\text{total}} = 10\text{kgms}^{-2} + 4\text{kgms}^{-2} = 14\text{joules}$$

$(K.E)_{\text{total}} = 14\text{joules}$
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8. A cylinder of 50 cm diameter at the top of an incline 29.4 cm high and 10 m long is released and rolls down the incline. Find its linear and angular speeds at the bottom. Neglect friction.

Answer:

Given Data:

Diameter of cylinder = $d = 50 \text{ cm} = 0.5 \text{ m}$

Radius of cylinder = $r = d/2 = 0.25 \text{ m}$

Height = $h = 29.4 \text{ cm} = 0.294 \text{ m}$

Length = $l = 10 \text{ m}$

To Find:

(a) Linear Speed = $v = ?$

(b) Angular Speed = $\omega = ?$

Solution:

According to the law of conservation of energy, we have

Loss in P.E = Gain in K.E

Again kinetic energy will be the sum of translational kinetic energy and rotational kinetic energy as the cylinder is rolling as well as moving forward. So the law of conservation of energy will give,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For a cylinder, inertia is $\frac{1}{2}mr^2$, so we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \left(\frac{v}{r}\right)^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

$$\Rightarrow gh = \left(\frac{1}{2} + \frac{1}{4}\right)v^2 = \frac{3}{4}v^2$$

$$\Rightarrow v = \sqrt{\frac{4}{3}gh}$$

(a)

Putting values, we get

$$v = \sqrt{\frac{4}{3} \times 9.8 \text{ms}^{-2} \times 0.294 \text{m}} = \sqrt{3.84 \text{m}^2 \text{s}^{-2}}$$

$$v = 1.96 \text{ms}^{-1}$$

(b)

$$\text{As, } v = r\omega \quad \Rightarrow \quad \omega = v/r$$

So we get angular velocity by putting values as,

$$\omega = \frac{1.96}{0.25} \text{rad s}^{-1}$$

$$\omega = 7.84 \text{ rad s}^{-1}$$

9. Show that when a hoop rolls down on an inclined plane, half of kinetic energy is rotational and the other half is translational.

Answer:

Consider a hoop of mass 'm' is rolling down an inclined plane with a linear velocity 'v' and angular velocity ' ω ', then according to law of conservation of energy we have,

$$\text{Loss in P.E} = \text{Gain in K.E}$$

i.e
$$\text{Loss in P.E} = (\text{K.E})_{\text{Translational}} + (\text{K.E})_{\text{Rotational}}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For hoop, inertia is, $I = mr^2$. So putting this value, we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\omega^2$$

And as, $v = r\omega$, so we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

In R.H.S of above equation, we can see that for rolling hoop, half of its kinetic energy is translational and other half is rotational

10. A disc without slipping rolls down a hill of vertical height 1000 cm. If the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom?

Answer:

Given Data:

$$\text{Height} = h = 1000 \text{ cm} = 10 \text{ m}$$

To Find:

Velocity at bottom= $v = ?$

Solution:

According to law of conservation of energy, we have

Loss in P.E = Gain in K.E

$$\Rightarrow \text{Loss in P.E} = (\text{K.E})_{\text{Translational}} + (\text{K.E})_{\text{Rotational}}$$

Or,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For disc, inertia is $(1/2)mr^2$ and also we know that

$$v = r\omega \quad \Rightarrow \quad \omega = v/r.$$

So putting these, we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}mv^2 + \frac{1}{4}mv^2$$

Cancelling 'm' from both sides, we get

$$gh = \left(\frac{1}{2} + \frac{1}{4}\right)v^2 = \frac{3}{4}v^2$$

So,

$$v = \sqrt{\frac{4}{3}gh}$$

Putting values, we get

$$v = \sqrt{\frac{4}{3} \times 9.8\text{ms}^{-2} \times 10\text{m}} = \sqrt{130.67\text{m}^2\text{s}^{-2}}$$

$v = 11.4\text{ms}^{-1}$
