

Required: Aerofoil lift $= F = ?$

Calculation: We know that,

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow P_1 - P_2 = \frac{F}{A} \Rightarrow F = A(P_1 - P_2) \quad (1)$$

From venturi meter, we have,

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) \quad (2)$$

Putting equation (2) in equation (1), we get,

$$F = A \times \frac{1}{2} \rho (v_2^2 - v_1^2) \quad (3)$$

Putting the values in equation (3), we get,

$$F = 0.01 \times \frac{1}{2} \times 1.28 (9^2 - 7^2) = 6.4 \times 10^{-3} (81 - 49)$$

$$\Rightarrow F = 6.4 \times 10^{-3} \times 32 \text{ N} = 204.8 \times 10^{-3}$$

$$\Rightarrow \boxed{F = 204.8 \times 10^{-3} \text{ N}} \text{ or } F = 0.2048 \text{ N} \text{ or } \boxed{F = 0.2 \text{ N}}$$

Q7: During a windstorm, a 25m/s wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is 1.28 kg/m³)

Solution:

Given Data: Speed of air $= v_2 = 25 \text{ m/sec}$

Density of air $= \rho = 1.28 \text{ kg/m}^3$

As the doors and windows of the house are closed, so we take the speed of inside air to be zero i.e. put $v_1 = 0$

Required: Pressure difference $= P_1 - P_2 = ?$

Calculation: We know that,

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) \quad (1)$$

Putting the value in equation (1), we get,

$$P_1 - P_2 = \left[\frac{1}{2} \times 1.28 (25^2 - 0^2) \right] \text{ N/m}^2$$

$$\Rightarrow P_1 - P_2 = 0.64 \times (625 - 0) \text{ N/m}^2$$

$$\Rightarrow \boxed{P_1 - P_2 = 400 \text{ N/m}^2} \text{ or } \boxed{P_1 - P_2 = 400 \text{ Pa}}$$

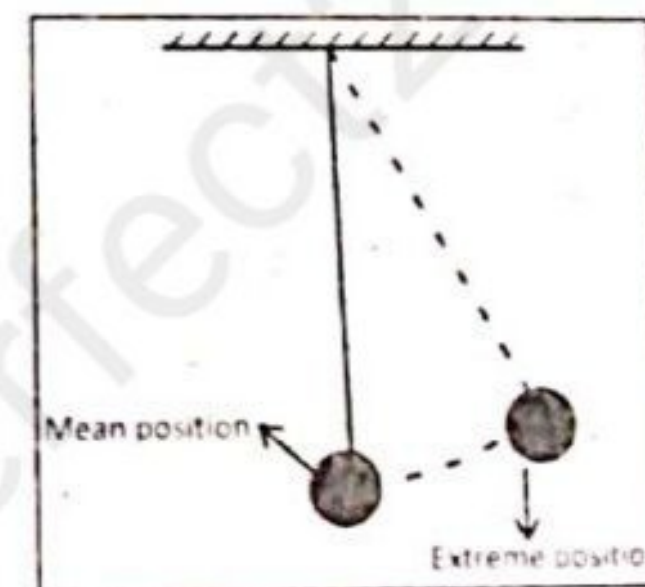
Q1: Define and explain oscillatory motion. Give examples of oscillatory motion.

Ans. Oscillatory Motion:

Definition: The back and forth motion of a body over the same path about its mean position is known as oscillatory motion.

Explanation:

The oscillatory [vibratory] motion can be initiated when a body is displaced from its mean position due to external applied force. When the external force is removed, the body moves back towards its mean position. On the way its velocity increases and becomes maximum at mean position. As a result the body does not stop at mean position and moves towards the other side of mean position. On the way its velocity decreases and finally comes to rest at extreme position. The body then moves back towards mean position due to restoring force. In this way, the body performs oscillatory motion about mean position.



Examples of Oscillatory Motion:

- Motion of a body attached to an elastic spring.
- Motion of wires of a sitar.
- Motion of simple pendulum.

Q2: Define the following terms: Periodic motion, vibratory motion, vibration, time period, frequency, displacement, amplitude and angular frequency

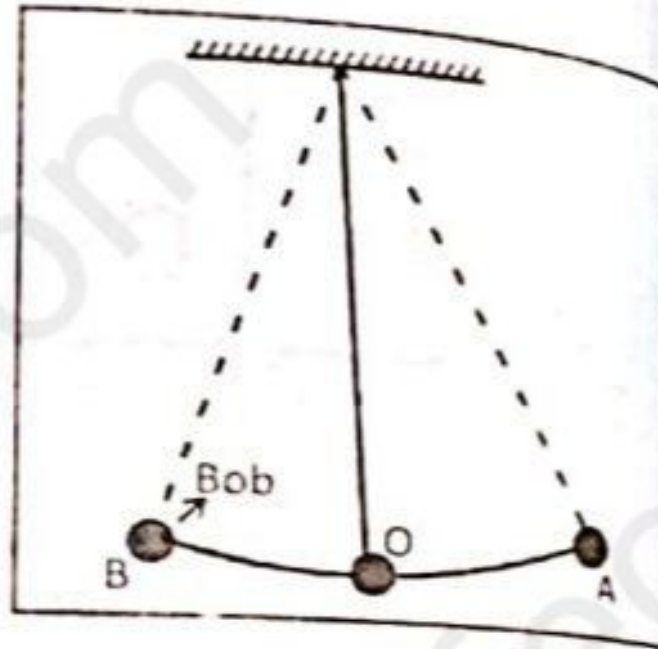
Ans. 1. Periodic Motion:

Motion, which is repeated in equal intervals of time is known as

periodic motion.

2. Vibratory Motion:

The to and fro [i.e. back and forth] motion of a body about the mean position is known as vibratory motion. In the given figure the motion of the bob from point "A" towards point "B" through mean position "O" and then from "B" towards "A" through "O" represents vibratory motion.



3. Vibration:

The complete round trip of a body about mean position is known as vibration. In the given figure, if the body moves from "A" towards "B" and then again reaches "A" from "B", then under such conditions, the body [i.e. bob] completes one vibration.

4. Time Period:

The time required by a body to complete one vibration is known as time period. It is represented by "T". It is usually measured in seconds.

5. Frequency:

The number of vibrations in one second is known as frequency. (OR)

The reciprocal of time period is known as frequency. i.e. $f = 1/T$

The S.I unit of frequency is Hertz or cycles per second.

$$1 \text{ Hertz} = 1 \text{ cycle/sec} \Rightarrow 1 \text{ Hz} = 1 \text{ c.p.s}$$

6. Angular Frequency:

The number of revolution per second of a body is known as angular frequency. If " ω " represents the angular velocity, then the angular frequency is given by,

$$f = \frac{\omega}{2\pi}$$

7. Displacement:

The distance of a vibrating body from its mean position at any instant is known as displacement. It is represented by "x".

8. Amplitude:

The maximum displacement of a vibrating body from mean position is known as amplitude. It is represented by " x_0 ".

Q3: (a) Define simple harmonic motion [S.H.M].

(b) Show that motion of a mass attached to an elastic spring executes S.H.M.

(OR) Show that mass-spring system execute S.H.M?

BISE Abbottabad 2019

Ans: (a) Simple Harmonic Motion [S.H.M]: ETEA 2010, 2013

Definition: The type of motion in which the acceleration of the body is always directly proportional to the displacement of the body from mean position and is always directed towards the mean position is known as simple harmonic motion [S.H.M]. i.e.

$$a \propto (-x)$$

The negative sign shows that the acceleration is always directed towards the mean position.

(b) Motion of a mass attached to an Elastic Spring:

Consider a body of mass "m" is attached to an elastic spring lying on a smooth frictionless surface as shown in the figure. Point "O" represents the mean position or equilibrium position of the mass.

Now if we displace the body towards right and bring it to point "A" and then released, the body moves back towards mean position

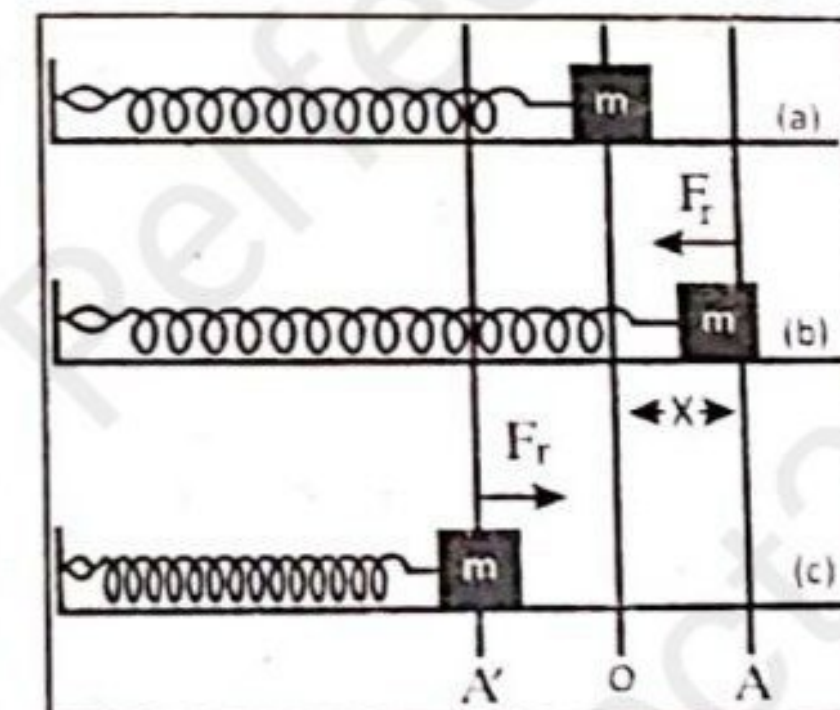
"O" due to restoring force " F_r ". In this way, the body begins to vibrate about its mean position "O".

Similarly, if we displace the body towards left and bring it to point "A'" and then released, the body moves back towards mean position

"O" due to restoring force " F_r ". In this way, the body begins to vibrate about its mean position "O". Now the restoring force which acts on the mass-spring system is given by,

$$F_{\text{restoring}} = kx \quad (1)$$

According to Newton's 2nd law, the applied force is given by,



"k" is called spring constant. Its value depends upon the physical shape and structure of the material.

$$F = ma \quad (2)$$

Here the applied force and restoring force are opposite in direction so we have,

$$F_{\text{applied}} = -F_{\text{restoring}} \quad (3)$$

Putting equation (1) and (2) in equation (3), we get,

$$ma = -kx$$

$$\Rightarrow a = -\left(\frac{k}{m}\right)x \quad (4)$$

As $\left(\frac{k}{m}\right) = \text{constant}$, so equation (4) becomes

$$a \propto (-x) \quad (5)$$

Relation (5) represents the mathematical form of S.H.M. Thus the motion of a body attached to an elastic spring executes simple harmonic motion [S.H.M].

Time Period of Mass-Spring System:

ETEA 2012

We know that,

$$S = vt \Rightarrow t = \frac{S}{v} \quad (6)$$

Here we put, $S = \theta = 2\pi$, $v = \omega$ & $t = T$

So equation (6) becomes,

$$T = \frac{2\pi}{\omega} \quad (7)$$

Put $\omega = \sqrt{\frac{k}{m}}$ in equation (7), we get,

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (8)$$

Equation (8) represents the time period of mass-spring system.

Conclusion about Time Period: Equation (8) shows that the time period depends upon the mass of the body attached to the spring. i.e. greater the mass, greater will be the time period and vice versa.

Frequency of Mass-Spring System:

We know that,

$$f = \frac{1}{T} \quad (9)$$

Putting equation (8) in equation (9), we get,

$$F = kx$$

ETEA 2013

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (10)$$

Equation (1) represents the frequency of mass-spring system.

Conclusion about Frequency: Equation (10) shows that frequency depends upon mass of the body inversely. i.e. greater then mass of the body, smaller will be the frequency and vice versa.

Q4: Discuss the relation between S.H.M and circular motion.

(OR) Show that if a particle perform circular motion then its projection executes S.H.M.

(OR) Show that motion of projection of a particle is simple harmonic motion.

ETEA 2009, BISE Peshawar 2017

Ans: Consider a body of mass "m" moving along a circular path of radius "r" as shown in the figure.

Now when the body is at point "A", its projection will be also at point "A". When the body moves from "A" towards "B", its projection moves from "A" to "O". Similarly, when the body moves from "B" towards "C" its projection moves from "O" towards "C".

Now when the body moves from "C" towards "A" through "D", its projection moves from "C" towards "A" through "O". In this way, the particle perform circular motion and its projection executes S.H.M along the diameter of the circle.

Now in $\triangle OPQ$ of figure (a), we have,

$$\cos \theta = \frac{x}{r}$$

$$\Rightarrow x = r \cos \theta \quad (1)$$

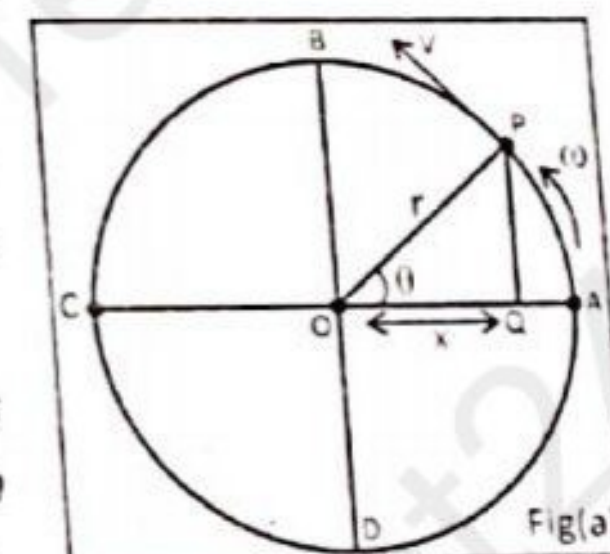
We know that, $\theta = \omega t$ (2)

Putting equation (2) in equation (1), we get,

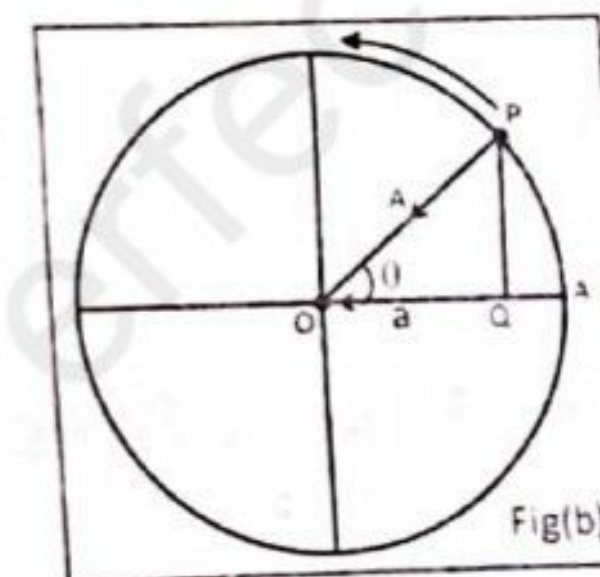
$$\Rightarrow x = r \cos(\omega t) \quad (3)$$

Equation (3) represents the displacement of projection from "O".

Now in $\triangle OPQ$ of figure (b), we have,



Fig(a)



Fig(b)

$$\cos \theta = \frac{a}{A} \Rightarrow a = A \cos \theta \quad (4)$$

a = acceleration
of projection
 A = centripetal
acceleration

Putting equation (2) in equation (4), we get,
 $a = A \cos(\omega t) \quad (5)$

In figure (b), "A" represents the centripetal acceleration, so we have,

$$A = \frac{v^2}{r} \quad (6)$$

centripetal acceleration
 $= a_c = \frac{v^2}{r}$

Putting equation (6) in equation (5), we get,

$$a = \frac{v^2}{r} \cos(\omega t) \quad (7)$$

We also know that, $v = r\omega \quad (8)$

Putting equation (8) in equation (7), we get,

$$a = \frac{r^2 \omega^2}{r} \cos(\omega t) \Rightarrow a = r\omega^2 \cos(\omega t)$$

$$\Rightarrow a = \omega^2 (r \cos \omega t) \quad (9)$$

Putting equation (3) in equation (9), we get,

$$a = \omega^2 x$$

(OR) $a = -\omega^2 x \quad (10)$

Equation (10) represents the acceleration of projection of the body at any instant. In equation (10), $\omega^2 = \text{constant}$

So equation (3) $\Rightarrow a = \text{constant} (-x)$

$$\Rightarrow a \propto (-x) \quad (11)$$

Relation (11) represents the mathematical form of S.H.M, so the motion of projection of a body is simple harmonic motion.

Time Period of Projection:

We know that, $S = vt$

$$\Rightarrow t = \frac{S}{v} \quad (12)$$

Here we put, $S = \theta = 2\pi \Rightarrow S = 2\pi \quad (13)$

Putting equation (13) in equation (12), we get,

$$t = \frac{2\pi}{v} \quad (14)$$

In case of circular motion
we put $S = \theta$
For one trip we have
 $\theta = 1 \text{ revolution} = 2\pi$
radian

Also we replace "v" by " ω " and "t" by "T", so equation (14) becomes,

$$T = \frac{2\pi}{\omega} \quad (15)$$

Equation (15) represents the time period of projection.

Conclusion about Time Period:

- Equation (15) shows that the time period depends upon the angular speed of the body. i.e. greater the angular speed, smaller will be the time period and vice versa.
- Equation (15) shows that the time period does not depend upon mass of the body.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

Also $V = r\omega$

$$V = r\omega$$

Put $r = x_0$
 $V = x_0 \omega$
ETEA 2009

Frequency of Projection: ETEA 2015

$$\text{As } f = \frac{1}{T} \quad (16)$$

Putting equation (15) in equation (16), we get,

$$f = \frac{\omega}{2\pi} \quad (17)$$

Equation (17) represents the frequency of projection in S.H.M.

Conclusion about Frequency: Equation (17) shows that:

- The frequency depends upon the angular velocity of the body i.e. greater the angular speed, greater will be the frequency and vice versa.
- The frequency does not depend upon the mass of the body.

Q5: Find an expression for the velocity of projection of a body in S.H.M. (OR) Show that, $v = \omega \sqrt{r^2 - x^2}$

Ans: If we resolve the velocity " V_0 " of the body into horizontal and vertical components, then we have,

$$V_{0x} = V_0 \cos \theta \quad (1)$$

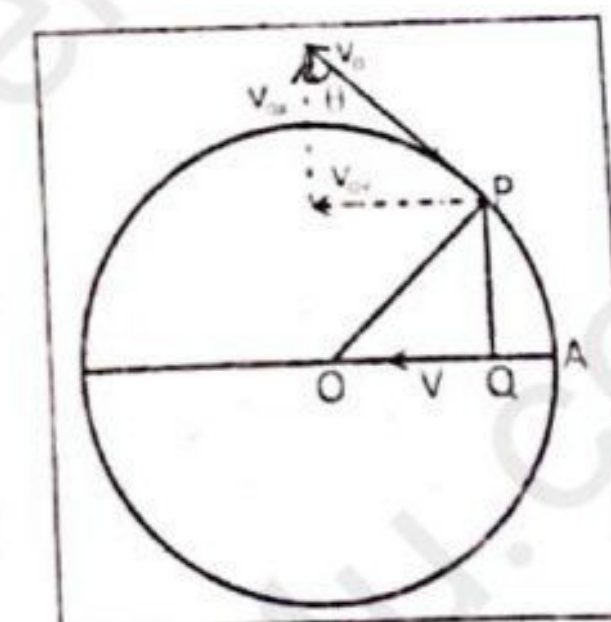
$$\& V_{0y} = V_0 \sin \theta \quad (2)$$

Now we see that the component " V_{0y} " is parallel to the velocity " V " of projection, so we have,

$$V = V_{0y} \quad (3)$$

Putting equation (2) in equation (3), we get,

$$V = V_0 \sin \theta \quad (4)$$



We know that,

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \quad (5)$$

$$\text{And } V_o = r\omega \quad (6)$$

Putting equation (5) and (6) in equation (4), we get,

$$V = r\omega \sqrt{1 - \cos^2 \theta} \quad (7)$$

As we know that the displacement of projection from mean position "O" at any instant is given by,

$$x = r \cos \theta \Rightarrow \cos \theta = \frac{x}{r} \quad (8)$$

Putting equation (8) in equation (7), we get,

$$V = r\omega \sqrt{1 - \frac{x^2}{r^2}}$$

$$\Rightarrow V = \frac{r\omega \sqrt{r^2 - x^2}}{r}$$

$$\Rightarrow V = \omega \sqrt{r^2 - x^2} \quad (9)$$

Equation (9) represents the velocity of projection of the body in S.H.M.

Q6: What is simple pendulum? Show that motion of simple pendulum is simple harmonic motion [S.H.M].

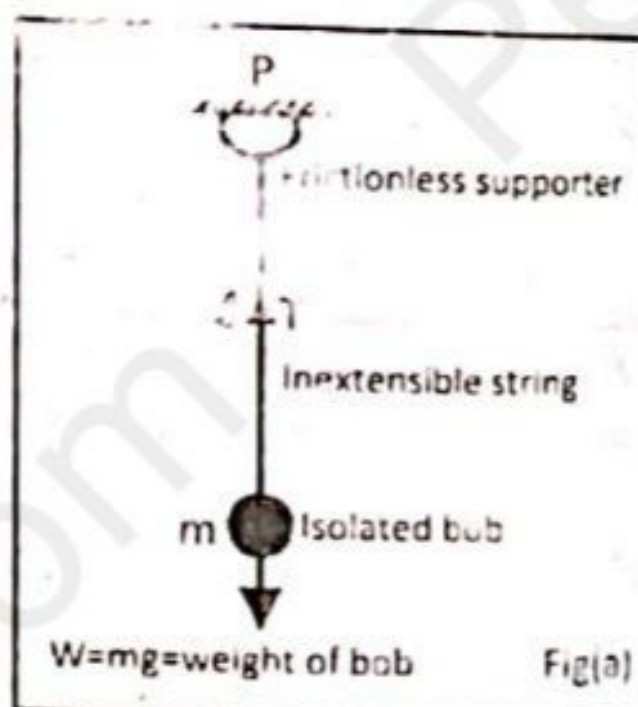
Ans. Simple Pendulum:

Definition: A simple pendulum consists of a small heavy mass "m" [called bob] suspended by a light string of length "l" fixed with its upper end, as shown in the figure (a).

Explanation:

Consider a simple pendulum as shown in figure (b). At point "A" the weight of the pendulum is acting downwards while the tension in the string is acting upwards. Both forces balance each other and thus the simple pendulum remains in state of equilibrium.

Now if we displace the bob from its mean position "A" towards point "B" through a displacement "x" due to external applied force



$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$\text{Also } V_o = r\omega$$

"F_{applied}" then at point "B" its weight "w" is resolved into two components i.e. "wcosθ" and "wsinθ".

The component "wcosθ" balance the tension "T" in the string, while the component "wsinθ" is directed towards the mean position and thus it acts as restoring force. So we have,

$$T = w \cos \theta \quad (1)$$

$$\& F_{\text{restoring}} = w \sin \theta \quad (2)$$

Now according to Newton's 2nd law, the applied force is given by,

$$F_{\text{applied}} = ma \quad (3)$$

Here "F_{restoring}" and "F_{applied}" are opposite in direction, so we have,

$$F_{\text{applied}} = -F_{\text{restoring}} \quad (4)$$

Putting equation (2) and equation (3) in equation (4), we get,

$$ma = -w \sin \theta \quad (5)$$

We know that, $w = mg$ (6)

Putting equation (6) in equation (5), we get,

$$ma = -mg \sin \theta \Rightarrow a = -g \sin \theta \quad (7)$$

Now for correct time period, we keep the amplitude to be very small. For smaller amplitude, we keep "θ" to be very small i.e. $\theta \ll 1$

Under such condition, we put $\sin \theta = \theta$, so equation (7) becomes,

$$a = -g \theta \quad (8)$$

Now according to the figure, the displacement "x" of the body is proportional to "θ", so we have,

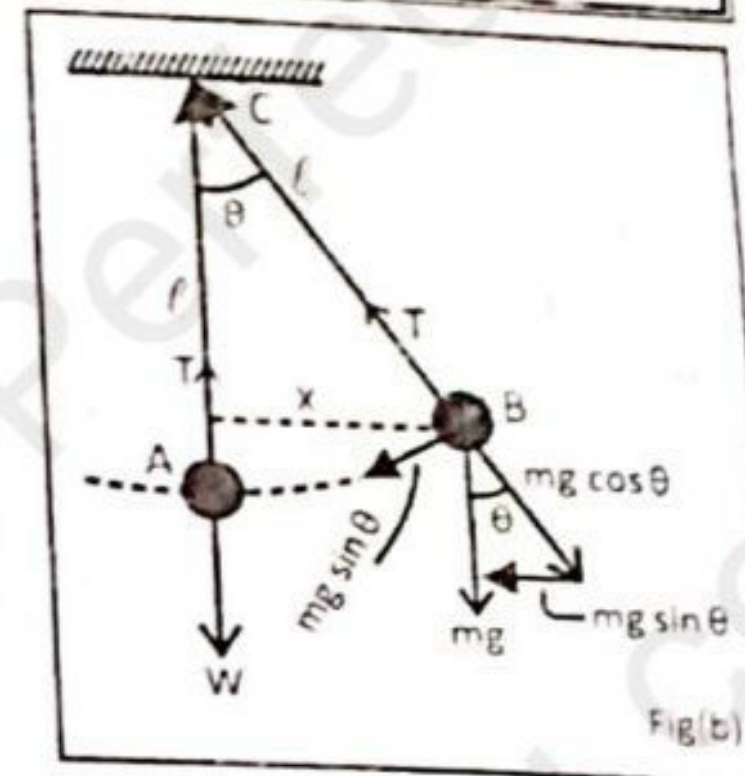
$$x \propto \theta \Rightarrow x = \text{constant } \theta \Rightarrow x = l \theta \quad (9)$$

In equation (9), "l" is constant which represents the length of the string. Now from equation (9), we have,

$$\theta = \frac{x}{l} \quad (10)$$

Putting equation (10) in equation (8), we get,

$$a = -g \cdot \frac{x}{l} \Rightarrow a = \frac{g}{l} (-x) \quad (11)$$



In equation (11), $\left(\frac{g}{l}\right) = \text{constant}$, so equation (11) becomes,

$$a = \text{constant} (-x) \Rightarrow a \propto (-x) \quad (12)$$

Equation (12) represents the mathematical form of simple harmonic motion, so the motion of simple pendulum is simple harmonic. The negative sign shows that the bob is always directed towards the mean position.

Time Period of Simple Pendulum: (ETE 2007, 9, 10, 11)

We know that,

$$T = \frac{2\pi}{\omega} \quad (a)$$

In case of circular motion, we have,

$$a = -\omega^2 x \quad (b)$$

In case of simple pendulum, we have,

$$a = -\left(\frac{g}{l}\right) x \quad (c)$$

Comparing equations (b), (c), we get,

$$\omega^2 = \frac{g}{l}$$

$$\Rightarrow \omega = \sqrt{\frac{g}{l}} \quad (d)$$

Putting equations (d) in equation (a), we get,

$$T = 2\pi \sqrt{\frac{l}{g}} \quad (e)$$

Equation (e) represents the time period of simple pendulum.

Conclusion about Time Period:

1. Equation (e) shows that the time period of simple pendulum depends upon the length of pendulum i.e. longer the pendulum, greater will be the time period and vice versa.
2. The time period of the pendulum does not depend on the mass of the bob.

Frequency of Simple Pendulum:

$$\text{As } f = \frac{1}{T} \quad (f)$$

Putting equation (e) in equation (f), we get,

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \quad (g)$$

Points to be Noted:

For a body executing S.H.M the following conditions must be satisfied:

1. The restoring force must be act on the system.
2. The system should have inertia.
3. S.H.M is performed along a straight line.
4. The system should be frictionless.
5. The Hook's law should be satisfied in case of mass-spring system.

Equation (g) represents the frequency of simple pendulum.

Conclusion about Frequency:

1. Equation (g) shows that longer the pendulum, smaller will be the frequency and vice versa.
2. The frequency of the pendulum does not depend on the mass of the bob.

Q7: (a) Derive equations for K.E and P.E of a body of mass "m" executing S.H.M. (b) Show that energy is conserved in case of S.H.M.

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Ans: (a) Kinetic Energy:

We know that,

$$K.E = \frac{1}{2} mv^2 \quad (1)$$

In case of circular motion, we know that,

$$v = \omega \sqrt{r^2 - x^2} \quad (2)$$

In case of mass-spring system, we put

$$r = x_0 \quad (3)$$

$$\text{And } \omega = \sqrt{\frac{k}{m}} \quad (4)$$

Putting equation (3) and equation (4) in equation (2), we get,

$$v = \sqrt{\frac{k}{m}} \sqrt{x_0^2 - x^2} \quad (5)$$

Putting equation (5) in equation (1), we get,

$$K.E = \frac{1}{2} m \left(\sqrt{\frac{k}{m}} \sqrt{x_0^2 - x^2} \right)^2$$

$$\Rightarrow K.E = \frac{1}{2} m \times \frac{k}{m} (x_0^2 - x^2) = \frac{1}{2} (kx_0^2 - kx^2)$$

$$\Rightarrow K.E = \frac{1}{2} k (x_0^2 - x^2) \quad (6)$$

Equation (6) represents the K.E of the body executing S.H.M.

Maximum Kinetic Energy:

The K.E will be maximum if we displace the body through maximum displacement " x_0 ". So under such conditions we consider " x " and ignore " x ". So we put $x = 0$ in equation (6), we get,

$$(K.E)_{\text{max}} = \frac{1}{2} k (x_0^2 - 0^2)$$

$$\Rightarrow (K.E)_{max} = \frac{1}{2} k x_0^2 \quad (7)$$

Equation (7) represents the maximum K.E of the body executing S.H.M.

(b) Potential Energy: (ETEA 2009)

Consider a body of mass "m" attached to an elastic spring, as shown in the figure. Initially when the body is at mean position "O", then the initial force acting on mass spring system will be zero i.e.

$$F_{initial} = 0 \quad (1)$$

Now if we increase the applied force and finally displace the body towards point "A" through a displacement "x", then the final force acting on mass-spring system is given by,

$$F_{final} = kx \quad (2)$$

Now the average force on mass-spring system is given by,

$$F_{ave} = \left(\frac{F_{final} + F_{initial}}{2} \right) \quad (3)$$

Putting equation (1) and equation (2) in equation (3), we get,

$$F_{ave} = \left(\frac{kx + 0}{2} \right) \Rightarrow F_{ave} = \frac{1}{2} kx \quad (4)$$

Now the workdone in compressing the spring through a displacement "x" is given by,

$$Work = F_{ave} \cdot x \quad (5)$$

Putting equation (4) in equation (5), we get,

$$\Rightarrow Work = \frac{1}{2} kx \cdot x \Rightarrow Work = \frac{1}{2} kx^2 \quad (6)$$

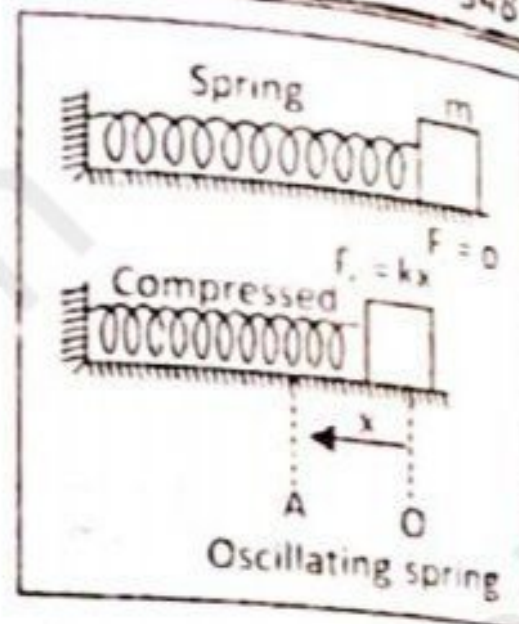
This work appears in the form of P.E in the spring, so equation (6) becomes,

$$P.E = \frac{1}{2} kx^2 \quad (7)$$

Equation (7) represents the P.E of the body attached to an elastic spring at any instant.

Maximum Potential Energy: (ETEA 2009)

The P.E will be maximum, if the spring is compressed through maximum displacement " x_0 ", under such condition, we replace "x" by " x_0 ", so equation (7) becomes,



$$(P.E)_{max} = \frac{1}{2} k x_0^2 \quad (8)$$

Equation (8) represents the maximum P.E at extreme position " x_0 ".

Conservation of Energy in S.H.M: (ETEA 2015, 2016)

The law of conservation of energy is satisfied in simple harmonic motion i.e. K.E may be converted into P.E and P.E may be converted into K.E but its total value remains the same.

Now the total energy in simple harmonic motion is given by,

$$E_t = K.E + P.E \quad (1)$$

We know that,

$$K.E = \frac{1}{2} k(x_0^2 - x^2) \quad (2)$$

$$\& P.E = \frac{1}{2} kx^2 \quad (3)$$

Putting equations (2), (3), in equation (1), we get,

$$E_t = \frac{1}{2} k(x_0^2 - x^2) + \frac{1}{2} kx^2$$

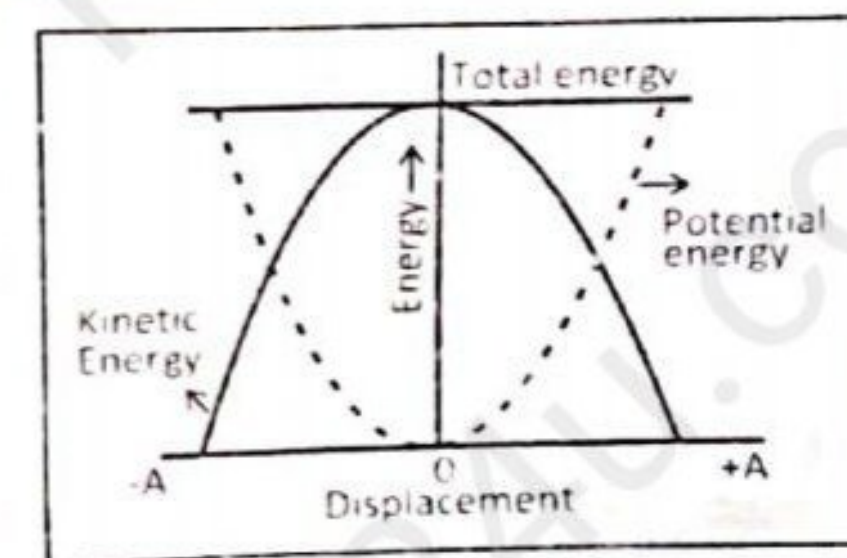
$$\Rightarrow E_t = \frac{1}{2} kx_0^2 - \frac{1}{2} kx^2 + \frac{1}{2} kx^2$$

$$\Rightarrow E_t = \frac{1}{2} kx_0^2 \quad (4)$$

Equation (4) represents the total energy of vibrating body in S.H.M. This energy is totally present in the form of K.E at mean position $\left[K.E_{max} = \frac{1}{2} kx_0^2 \right]$ and in the form of P.E at extreme position $\left[P.E_{max} = \frac{1}{2} kx_0^2 \right]$. Thus energy is conserved in simple harmonic motion.

Graphical Representation of Law of Conservation of Energy:

The graph between K.E and P.E is shown in the figure. The graph shows that when the body moves from mean position "O" toward extreme position "+A" or "-A", the P.E increases and K.E decreases. Similarly when the body moves



from extreme position "+A" or "-A" towards mean position "O", the P.E decreases and K.E increases.

Conclusion: The above discussion shows that during the motion of mass-spring system, the K.E is converted into P.E and P.E is converted into K.E, but the total energy remains constant. So the mass-spring system obeys the law of conservation of energy.

Q8: Differentiate between free and forced oscillations.

Ans: 1. Free Oscillation:

Definition: A body is said to be executing free oscillation if it oscillates with its natural frequency without the interference of an external force.

Explanation:

Free vibrations are oscillations where the total energy stays the same over time. This means that the amplitude of the vibration stays the same. For example, a simple pendulum vibrates freely with its natural frequency in the absence of external forces. This is a theoretical idea because in real systems the energy is dissipated to the surroundings over time and the amplitude decays away to zero. This dissipation of energy is called damping.

2. Forced Oscillation:

Definition: If a freely oscillating system is subjected to an external force, then forced oscillation will take place.

Explanation:

Forced oscillation takes place under the action of external applied force. For example, when the bob of vibrating pendulum is pushed repeatedly, then forced oscillations are produced. The vibrations of a factory floor due to running of heavy machinery, the loud music produced by sounding wooden boards of strings instruments are the examples of forced oscillations.

Q9: What is resonance? Give some applications of resonance in daily life.

Ans: Resonance:

Definition: The phenomenon in which large amplitude of vibration is

produced due to small periodic driving force is known as resonance.

Explanation:

It has been found that when the frequency of the external driving force becomes equal to the natural frequency of oscillating body, then the body oscillates with larger amplitude. Such phenomenon is known as resonance.

The resonance also occurs when the applied force has a frequency which are integral multiple of the natural frequency of the body i.e. $2f, 3f, 4f, 5f, \dots, nf$.

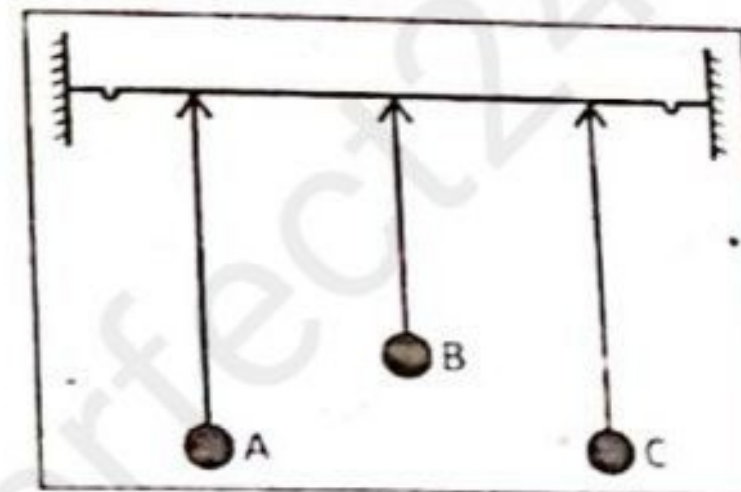
Experimental Demonstration of Resonance:

Consider three pendulums A, B, C which are suspended from a stretched rubber cord, as shown in the figure.

Now if the length of pendulum "A" and "B" are the same, then by disturbing pendulum "A" slightly, the pendulum

"B" also starts vibration due to the phenomenon of resonance.

Similarly, if the length of pendulum "A" and pendulum "C" are the same, then by disturbing "A" slightly, the pendulum "C" also begins to vibrate due to the phenomenon of resonance.



Applications of Resonance:

1. Radio and Resonance: Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio to tune a station, we are changing the natural frequency of the electric circuit of the receiver to make it equal to the transmission frequency of the radio station. When the two frequencies match each other, resonance will occur. Under such conditions, the absorption of energy becomes maximum and as a result we hear the program of concert station clearly.



2. Magnetic Resonance Image [M.R.I]: Magnetic resonance scan-

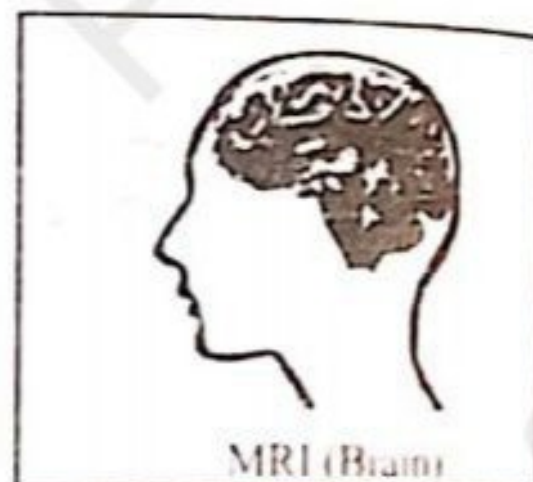
ning has greatly improved medical diagnoses. With the help of M.R.I machine, we can get photographic film of brain, backbone and other parts of the body. For this purpose, radio waves of high frequency are used to oscillate the nuclei of atoms. When resonance occurs, the molecules absorb the energy. The pattern of absorbed energy can be used to produce a computer enhanced photograph.

3. Cooking of Food: (BISE Mardan 2019)

The microwave oven is an important application of resonance. In a microwave oven, typical microwaves of wavelength 12cm and frequency of 2450 MHz is used. Such frequency of microwaves is similar to the natural frequency of vibration of water molecules and fat molecules in the food i.e.

Frequency of microwaves = Frequency of water molecules & fat molecules

Now when the food containing water is placed in the oven, the water molecules and fat molecules in the food resonates under the action of periodic force of microwaves. As a result, the molecules absorb energy from the microwaves due to resonance. The energy of microwaves heats up the food items. In this way, the food items are heated or cooked in the microwaves oven due to resonance.



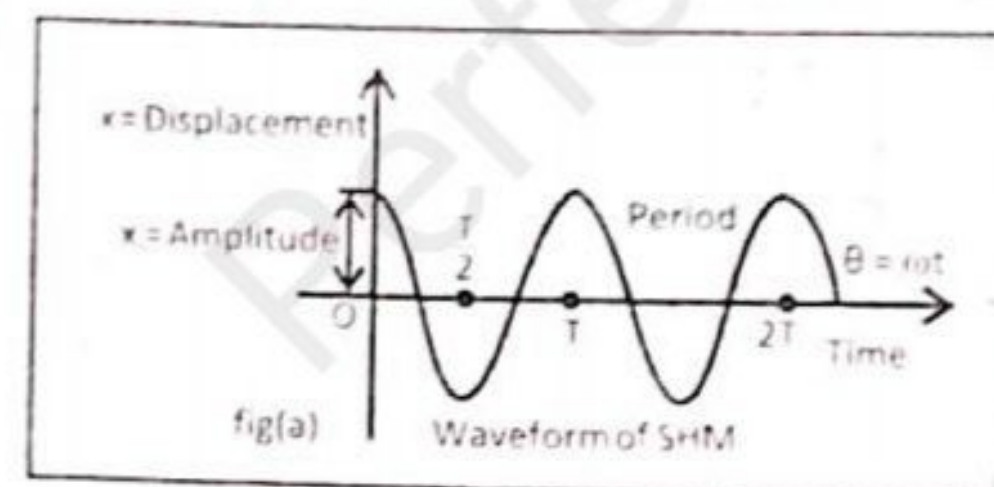
4. The amplitude of a swing can be increased due to the phenomenon of resonance by applying small periodic force.
5. We can use resonance to find natural frequency of different bodies.
6. The resonance phenomenon can be used in vehicles for comfortable journey.
7. With the help of resonance apparatus, we can find speed of sound.

Q10: Explain the waveform of S.H.M.

Ans: In case of circular motion, we know that the displacement of projection of a body from mean position at any instant is given by,

$$x = x_0 \cos(\omega t) \quad (1)$$

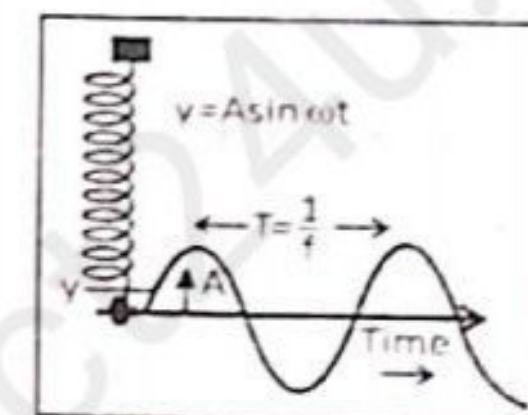
The waveform of equation (1) is shown in figure (a). If we determine the value of "x" at different instant and plot it against time, we get a curve as shown in figure (a).



This graph gives the displacement as a function of time and is known as waveform of S.H.M.

Experiment for getting the Wave form of S.H.M:

Consider a body of mass "m" is attached to an elastic spring as shown in the figure. A sheet of paper is placed behind the mass in such a way that it can move horizontally with constant speed. The mass is provided with a pen which can touch the paper slightly.



When the spring is compressed through a displacement "y" and then released, the mass execute S.H.M having amplitude "A" as shown in the figure. During oscillation, the pen would mark the position of the mass "m" at each instant on the paper. In this way, the pen will trace out the waveform of S.H.M on the paper, as shown in the figure.

Q11: What is phase? Explain.

ETEA 2011

Ans. Phase:

Phase is the quantity which shows the state of motion of an oscillator. (OR)

The angle $\theta = \omega t$ which satisfies the displacement "x" as well as the direction of motion of a point executing S.H.M is called phase.

Explanation:

We know that the equation for instantaneous displacement of an oscillating body executing S.H.M is given by,

$$x = x_0 \cos \theta \quad (1)$$

In equation (1), "x" represents the instantaneous displacement, "x₀" represents the amplitude, "θ" represents the phase angle. Generally, we put,

$$\theta = \omega t + \phi \quad (2)$$

In equation (2), " ω " represents the angular velocity, " ϕ " represents an angle which gives us information about the initial phase of an oscillator. The quantity " ϕ " represents the phase difference between the states of motion of two oscillators. Now putting equation (2) in equation (1), we get,

$$x = x_0 \cos(\omega t + \phi) \quad (3)$$

Case-1: If we put $\phi = 0$, then equation (3) becomes,

$$x = x_0 \cos \omega t \quad (4)$$

Now for different values of " t " we get a graph between " x " and " t " as shown in figure (a).

Case-2: If $\phi = 90^\circ$, then equation (3) can be written as,

$$x = x_0 \cos(\omega t + 90^\circ) \quad (5)$$

Now for different values of " t ", we get a graph between " x " and " t " as shown in figure (b).

Now comparing figure (a), (b), we see that the curve in figure (a) leads the curve in figure (b) by 90° [i.e. $\pi/2$].

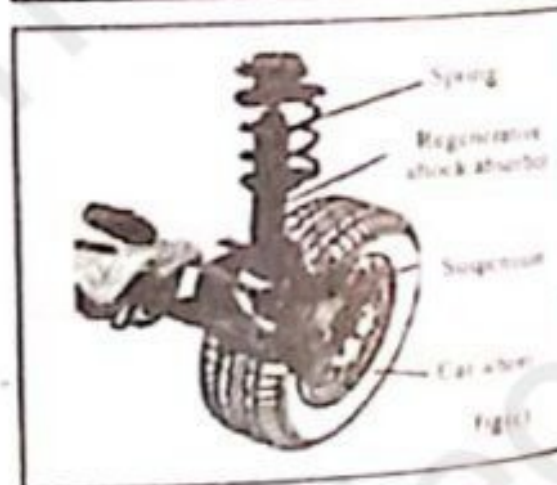
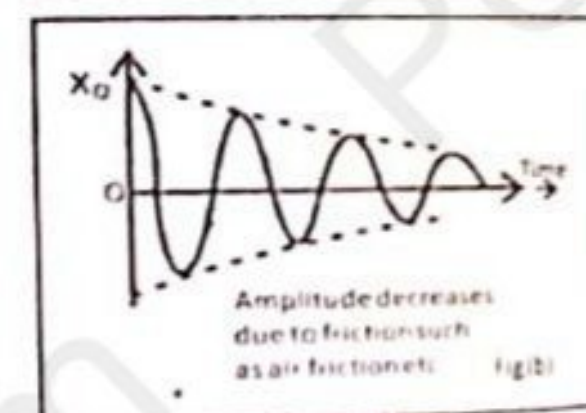
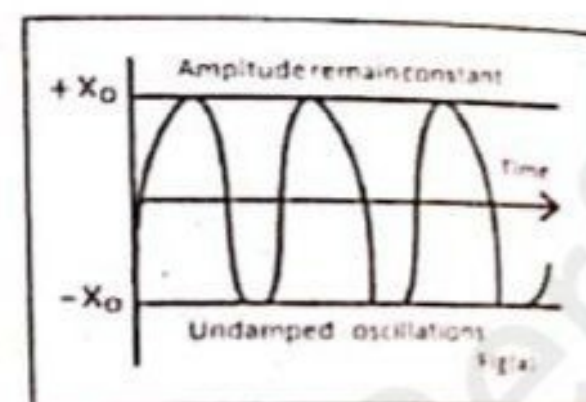
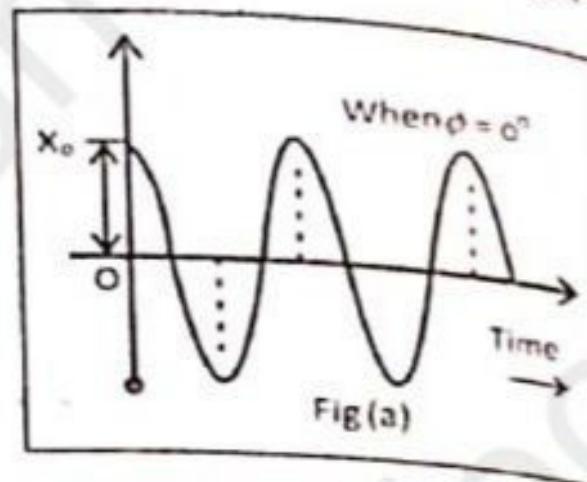
Case-3: If $\phi = 180^\circ$, then equation (3) becomes,

$$x = x_0 \cos(\omega t + 180^\circ) \quad (6)$$

Now for different values of " t ", we get a graph between " x " and " t " as shown in figure (c).

Now comparing the figures (a), (c), we see that the curve in figure (a) and curve in figure (c) are out of phase by 180° [i.e. π]. In other words, we can say that the phase difference between the two oscillations is 180° .

Thus the angle $\theta = \omega t$ which specifies the displacement " x " as well as the direction of motion of the point executing S.H.M is called phase.



Q12: Explain what is mean by damped oscillations?

BISE Abbottabad 2019

Ans: **Definition:** The oscillation in which the amplitude decreases steadily with time is known as damped oscillations. (OR)

A process in which energy is dissipated from the oscillating system is known as damped oscillation.

Explanation:

In case of mass-spring system, motion of a body along a circular path, motion of simple pendulum, we say that the total mechanical energy of the system remains constant. This is an ideal situation for which we ignore the frictional forces such as air friction.

In real situation, some part of mechanical energy of the oscillating system is used against the frictional forces. As a result, the amplitude of oscillation decreases steadily with time. Such oscillation is known as damped oscillations.

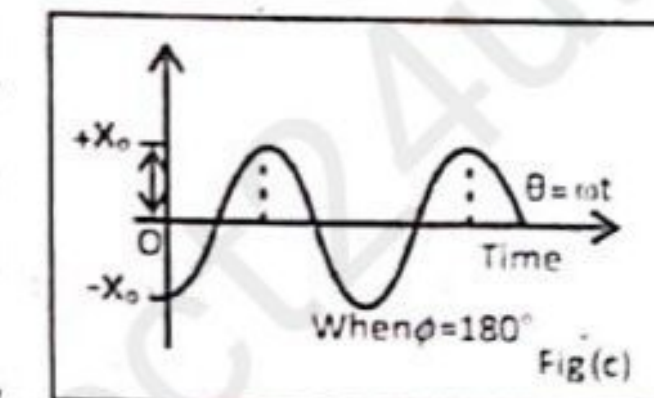
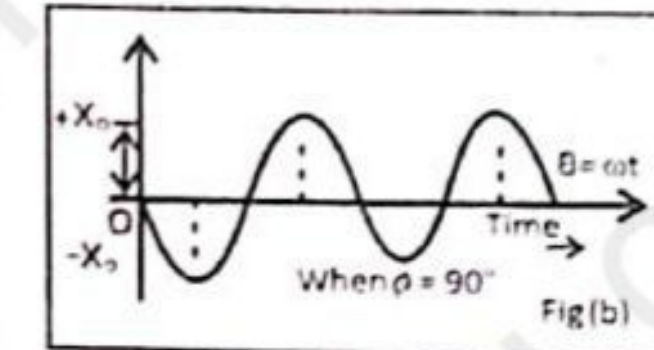
The undamped oscillation is shown in figure (a). The damped oscillation is shown in figure (b).

Applications of Damped Oscillation:

1. **Shock Absorber of Car:** The shock absorber of car is the application of damped oscillations which provides a damping force to prevent excessive oscillations as shown in figure (c). If the shock absorbers are worn badly, then the car becomes too bouncy and thus becomes uncomfortable.

2. **Shock Absorber System of Human Body:** Another application of damped oscillation is the shock absorber system of human body. Skier's body moves over the bumpy snow smoothly while his / her thighs and calves act like a damping spring. That is why skiing instructors always saying "bend zee knees".

3. The racing cars and hydrofoils can be fitted with active suspensions. These involve computer operated hydraulic systems in which bumps or waves are sensed and the suspension system is adjusted accordingly.



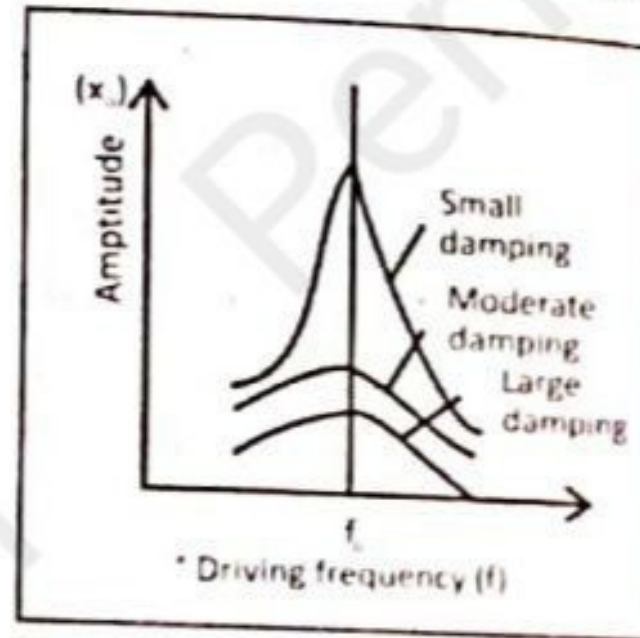
4. The computers are used to control the angle of the waves through the water to achieve remarkably stable movement of the hover craft.

Q13: Discuss the sharpness of resonance.

Ans: At resonance, the amplitude of vibrations becomes very large when damping is small. Thus the damping prevents the amplitude from becoming excessively large. The amplitude decreases rapidly at a frequency which is slightly different from the resonant frequency. The sharpness of the resonance curve of a resonating system depends on the frictional loss of energy. It can be explain with the help of following experiment:

Experiment:

Consider two bobs of simple pendulum that one is a pith - ball having lighter mass and the other is a lead bob having heavier mass. Let both are attached to a light, incompressible string to make two simple pendulums. These two pendulums are set into vibrations by a third pendulum of same length, attached to the same rod. It is observed that the amplitude of the lead bob is much greater than that of the pith - ball. It is because the damping effect for the lead bob due to air resistance is much smaller than that for the pith - ball. Thus, the sharpness of resonance curve of a resonating system depends on the frictional loss of energy.



④ زیادہ ہنسنے سے دل غمزدہ ہو جاتا ہے۔ (حضور ﷺ)

④ انسان کا سب سے بڑا کارنامہ یہ ہے کہ وہ اپنے دل اور زبان کو قابو میں رکھے۔ (حضرت امام غزالی)

④ جو شخص زمین کا سفر کرتا ہے اس کے پاؤں میں ابلے پڑتے ہیں۔ جو آسمان کا سفر کرتا ہے اس کے دل میں ابلے پڑتے ہیں۔ (حضرت ابوالحسن خرقانی)

④ غنی وہ ہے جس کا دل اللہ کے سوا ہر چیز سے خالی ہو۔ (حضرت داتا گنج بخش)

Solution of Examples & Assignments

EXAMPLE 7.1:

A mass of 0.5kg is suspended from a spring. The spring is stretched by 0.098m. Calculate the spring constant of the mass when it is given a small displacement.

Solution:

Given data: Mass of body = $m = 0.5\text{kg}$

Force acting on body = $F = w = mg = 0.5 \times 9.8 = 4.9\text{N}$

Extension in the spring = $x = 0.098\text{m}$

Required: Spring constant = $k = ?$

Calculation:

We know that, $F = kx \Rightarrow k = \frac{F}{x}$ (1)

Putting the values in equation (1), we get,

$$k = \frac{4.9}{0.098} = 50\text{N/m} \Rightarrow \text{Spring constant} = k = 50\text{N/m}$$

ASSIGNMENT 7.1:

A 2.0kg block hung from a vertical spring causes it to stretch by 20.0cm. If the 2.0kg block is replaced by a 0.50kg mass, and the spring is stretched and released, what are the frequency and period of the oscillations?

Solution:

Given data:

Mass of block = $m_1 = 2.0\text{kg}$

Displacement of block = $x = 20.0\text{cm} = 0.20\text{m}$

Spring constant = $k = \frac{F}{x} = \frac{w}{x} = \frac{m_1 g}{x}$

$$= \frac{2 \times 9.8}{0.20} \text{N/m} = 98\text{N/m}$$

$$F = w = mg$$

Mass of another block = $m_2 = 0.50\text{kg}$

Required:

1. Time period of oscillation = $T = ?$

2. Frequency of oscillation = $f = ?$

Calculation:

1. We know that,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_2}{k}} \Rightarrow T = 2\pi\sqrt{\frac{m_2}{k}} \quad (1)$$

Putting the values in equation (1), we get,

$$T = \left[2 \times 3.14 \times \sqrt{\frac{0.50}{98}} \right] \text{sec} = 0.0448 \text{sec} = 0.045 \text{sec}$$

$$\Rightarrow T = 0.045 \text{ sec}$$

2. We know that,

$$f = \frac{1}{T} \quad (2)$$

Putting the values of "T" in equation (2), we get,

$$f = \left(\frac{1}{0.45} \right) \text{Hz} = 2.2 \text{Hz} \Rightarrow f = 2.2 \text{Hz}$$

$$1. \text{ Time period of oscillation} = T = 0.45 \text{sec}$$

$$2. \text{ Frequency of oscillation} = f = 2.2 \text{Hz}$$

EXAMPLE 7-2:

A spider swings in the breeze from a silk thread with a period of 0.6s. How long is the spider's strand of silk?

Solution: Given data:

$$\text{Time period of spider} = T = 0.60 \text{sec}$$

$$\text{Acceleration due to gravity} = g = 9.8 \text{m/sec}^2$$

Required: Length of thread = ℓ = ?Calculation:

$$\text{We know that, } T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T^2 = 4\pi^2 \frac{\ell}{g} \Rightarrow \ell = \frac{gT^2}{4\pi^2} \quad (1)$$

Putting the values in equation (1), we get,

$$\ell = \left[\frac{9.8 \times (0.60)^2}{4 \times (3.14)^2} \right] \text{m} \Rightarrow \ell = 0.089293 \text{m}$$

$$\text{Length of thread} = \ell = 0.089293 \text{m}$$

ASSIGNMENT 7-2:

A pendulum extending from the ceiling almost touches the floor and that its period is 12s. How tall is the tower? ($\ell = 36 \text{m}$)

Solution:Given data:

$$\text{Time period of pendulum} = T = 12 \text{sec}$$

$$\text{Acceleration due to gravity} = g = 9.8 \text{m/sec}^2$$

Required:

$$\text{Length of tower} = \ell = ?$$

Calculation:

$$\text{We know that, } T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T^2 = 4\pi^2 \frac{\ell}{g} \Rightarrow \ell = \frac{gT^2}{4\pi^2} \quad (1)$$

Putting the values in equation (1), we get,

$$\ell = \left[\frac{9.8 \times (12)^2}{4 \times (3.14)^2} \right] \text{m} = 35.8 \text{m} = 36 \text{m} \Rightarrow \ell = 36 \text{m}$$

As the pendulum almost touches the floor, so the length of pendulum will give us the length of tower.

$$\text{Length of tower} = \ell = 36 \text{m}$$

EXAMPLE 7-3:

A mass at the end of a spring vibrates up and down with a frequency of 0.600Hz and an amplitude of 5cm. What is its displacement 2.56 sec after it reaches a maximum?

Solution:Given data:

$$\text{Frequency} = f = 0.600 \text{Hz},$$

$$\text{Amplitude} = x_0 = 5 \text{cm} = 0.05 \text{m}$$

$$\text{Time interval} = t = 2.56 \text{sec}$$

Required:

$$\text{Displacement of mass} = x = ?$$

Calculation:

$$\text{We know that, } x = x_0 \cos(2\pi ft) \quad (1)$$

Putting the values in equation (1), we get,

$$\begin{aligned} x &= x_0 \cos(\omega t) \\ x &= x_0 \cos(2\pi f t) \\ f &= \frac{\omega}{2\pi} \\ \Rightarrow \omega &= 2\pi f \end{aligned}$$

$$x = [0.05 \times \cos(2 \times 3.14 \times 0.600 \times 2.56)] \text{ m}$$

$$= 0.05 \times 0.9858615948 \text{ m}$$

$$\Rightarrow x = 0.04929307974 \text{ m} = 0.0493 \text{ m approximately}$$

$$\Rightarrow x = 0.0493 \text{ m}$$

$$\Rightarrow x = 4.93 \text{ cm}$$

$$\text{Displacement of mass} = x = 4.93 \text{ cm}$$

ASSIGNMENT 7.3:

Determine the period and frequency of a car whose mass is 1400 kg and whose shock absorbers have a spring constant of $6.5 \times 10^4 \text{ N/m}$ after hitting a bump. Assume the shock absorbers are poor, so the car really oscillates up and down.

Solution:Given data:

Mass of car = $m = 1400 \text{ kg}$, Spring constant = $6.5 \times 10^4 \text{ N/m}$

Required:

1. Time period = $T = ?$ 2. Frequency = $f = ?$

Calculation:

1. We know that,

$$T = 2\pi \sqrt{\frac{m}{k}} \quad (1)$$

Putting the values in equation (1), we get,

$$T = 2 \times 3.14 \times \sqrt{\frac{1400}{6.5 \times 10^4}} = \left[2 \times 3.14 \times \sqrt{\frac{1400}{65000}} \right] \text{ sec}$$

$$\Rightarrow T = (6.28 \times 0.147) \text{ sec} = 0.9216520284 \text{ sec}$$

$$\Rightarrow T = 0.921652 \text{ sec}$$

2. We know that,

$$f = \frac{1}{T} \quad (2)$$

Putting the values of "T" in equation (2), we get,

$$f = \left(\frac{1}{0.921652} \right) \text{ Hz} = 1.085 \text{ Hz} \Rightarrow f = 1.085 \text{ Hz}$$

اللہ تعالیٰ کا ذکر ہر قسم کے ذکر کو ختم کر دیتا ہے۔

ASSIGNMENT 7.4:

An object vibrates with an amplitude of 6 cm and a frequency of 0.490 Hz. Starting from maximum displacement in the positive direction, when will be the first time that its displacement is 2 cm?

Solution:Given data:

Amplitude = $x_0 = 6 \text{ cm} = 0.06 \text{ m}$,

Frequency = $f = 0.490 \text{ Hz}$

Displacement = $x = 2 \text{ cm} = 0.02 \text{ m}$

Required:Time interval = $t = ?$ Calculation:

We know that, $x = x_0 \cos(\omega t)$

$$\Rightarrow x = x_0 \cos(2\pi f t) \Rightarrow \cos(2\pi f t) = \frac{x}{x_0} \quad [\omega = 2\pi f]$$

$$\Rightarrow 2\pi f t = \cos^{-1}\left(\frac{x}{x_0}\right) \Rightarrow 2 \times 180 \times 0.490 \times t = \cos^{-1}\left(\frac{0.02}{0.06}\right)$$

$$\Rightarrow 176.4t = 70.5 \Rightarrow t = \left(\frac{70.5}{176.4}\right) \text{ sec}$$

$$\Rightarrow t = 0.399 \text{ sec} = 0.400 \text{ sec}$$

$$\Rightarrow t = 0.400 \text{ sec}$$

$$\pi \text{ radian} = 180^\circ$$

- ⊕ قلم سخی بھی باندی پہ ہوا اللہ کے پاس انصاف ضرور ہوتا ہے۔
- ⊕ بے شک زندگی تھکا دینے والی چیز ہے اور جب تم تھک جاؤ تو اپنے رب کا ذکر کیا کرو کیونکہ سکون تو صرف اللہ کے ذکر میں ہے۔
- ⊕ قابلیت اتنی بڑھاؤ کہ تمہیں ہر آنے کے لیے کوششیں نہیں سازشیں کرنی پڑیں۔
- ⊕ خوش رہنا چاہتے ہو تو دوسروں کو خوش کرو۔ (حضرت غوث اعظم)
- ⊕ بعض احمق اس غم میں گھٹتے ہیں کہ اللہ تعالیٰ نے فلاں کو دولت کیوں دی۔ (لام غزالی)
- ⊕ تم خوشی چاہتے ہو اور اسے نہیں پاتے یہ جنت میں ہوگی۔ (حضرت ابو تراب بخاری)

Solution of Quiz

QUIZ 7-1:

A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is measured (1) If the elevator moves with constant velocity, does the period (a) increase, (b) decrease, or (c) remain the same. (2) If the elevator accelerates upward, does the period (a) increase, (b) decrease, or (c) remain the same?

Ans: Part-1: We know that,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

When the elevator is moving with constant velocity, its acceleration "a" is zero. So there will be no change in value of "g". So according to equation (1), when "g" remain the same, time period will remain the same. Thus in part (1) the option (c) is correct i.e. "T" remain the same.

Part-2: When the elevator is moving upward with acceleration "a" then value of acceleration due to gravity becomes "g + a". So equation (1) can be written as,

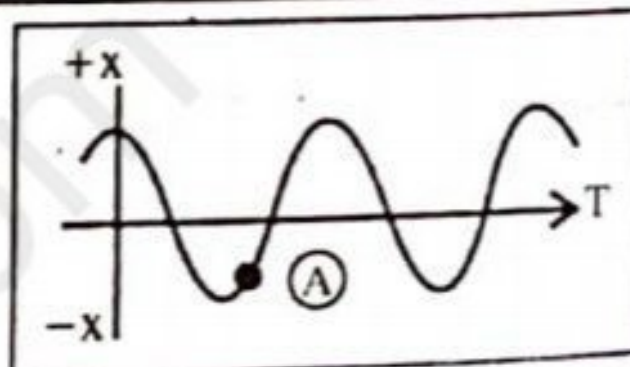
$$T = 2\pi\sqrt{\frac{l}{(g+a)}} \quad (2)$$

Equation (2) shows that "g + a" increases, so time period will decrease. Thus option (b) is correct i.e. "T" decreases.

QUIZ 7-2:

Consider a graphical representation of simple harmonic motion as described mathematically in equation 7.21 (ii).

When the particle is at point A on the graph, what can you say about its position and velocity? (a) The position and velocity are both positive. (b) The position and velocity are both negative. (c) The position is positive, and the velocity is zero. (d) The position is negative, and the velocity is zero.



ro. (e) The position is positive, and the velocity is negative. (f) The position is negative, and the velocity is positive.

Ans: 1. Conclusion about Position of Point "A":

It is clear from the figure that point "A" is in negative x-direction. Therefore, the position of point "A" is negative.

Conclusion about Velocity of Point "A":

Now the velocity of the particle is equal to the slope of position time graph at any point. So we have,

$$v = \frac{\Delta x}{\Delta t} \quad (1)$$

In equation (1), "Δx" represents the change in position and "Δt" represents the time interval. Now at point "A", the slope of the graph is positive. Therefore, the velocity at point "A" will be also positive. Thus option (f) is correct i.e. the position is negative and velocity is positive.

Exercise

Choose the best possible answer:

- Tuning of a radio set is an example of:
 - Mechanical resonance
 - Musical resonance
 - Electrical resonance
 - Free vibrations
- The heating and cooking of food evenly by microwave oven is an example of:
 - S.H.M.
 - Resonance
 - Damped oscillation
 - Free oscillation
- The time period of the same pendulum at Karachi and Murree are related as:
 - $T_k = T_m$
 - $T_k > T_m$
 - $T_k < T_m$
 - $2T_k = 3T_m$

Sol: The time period of a simple pendulum is given by,

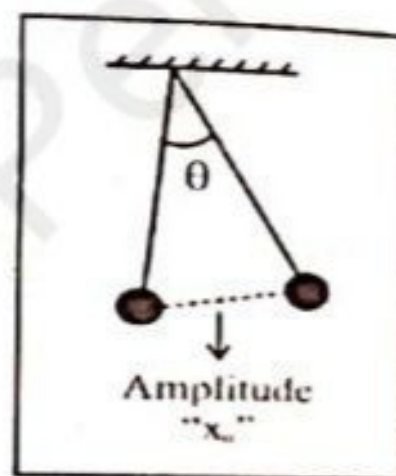
$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

Equation (1) shows that greater the value of "g", smaller will be the time period and vice versa. Now we know that value of "g" at

Karachi will be greater than the value of "g" at Murree. It is because the value of "g" decreases with altitude. Now according to equation (1), if "g" is greater than "T" will be smaller. Thus value of time period pendulum at Karachi will be smaller due to greater value of "g". While value of time period of pendulum at Murree will be greater due to smaller value of "g". Thus

$$T_k < T_m \text{ or } T_m > T_k$$

4. In an isolated system the total energy of vibrating mass and spring is:
 a. Variable b. Low c. High ✓ d. Constant
5. While deriving the equation of time period for simple pendulum which quantity should be kept small:
 a. Length of simple pendulum
 ✓ b. Amplitude
 c. Mass of simple pendulum
 d. Gravitational acceleration $\bar{g}$



Sol: When we are deriving equation for time period of simple pendulum, we put $\sin \theta \approx \theta$ i.e. we keep "θ" very very small. Now for keep "θ" to be small, we should keep the amplitude to be small as clear from the given figure.

6. If the period of oscillation of mass (M) suspended from a spring is 2s, then the period of mass 4M will be:
 a. 1s b. 2s c. 3s ✓ d. 4s

Sol: We know that

$$T = 2\pi\sqrt{\frac{M}{k}} \quad (1)$$

For mass "M" the time period is 2sec, so we have,

$$T = 2\text{sec}$$

Now for mass "4M", equation (1) can be written as,

$$T' = 2\pi\sqrt{\frac{4M}{k}} = 2\pi\sqrt{4}\sqrt{\frac{M}{k}} = \sqrt{4} \cdot 2\pi\sqrt{\frac{M}{k}}$$

$$\Rightarrow T' = 2 \left(2\pi\sqrt{\frac{M}{k}} \right) \quad (2)$$

Putting equation (1) in equation (2), we get,

$$T' = 2T \quad (3)$$

Putting $T = 2\text{sec}$ in equation (3), we get,

$$T' = 2 \times 2 \text{ sec} \Rightarrow T' = 4 \text{ sec}$$

7. The time period of a simple pendulum is 2 seconds. If its length is increased by 4 times, then its period becomes:
 a. 16s b. 12s c. 8s ✓ d. 4s

Sol: We know that,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

For length 'l', we have given that, $T = 2\text{sec}$
 Now length becomes four times, then we put $l' = 4l$, so equation (1) becomes

$$T' = 2\pi\sqrt{\frac{l'}{g}} \Rightarrow T' = 2\pi\sqrt{\frac{4l}{g}} = 2 \left(2\pi\sqrt{\frac{l}{g}} \right) \quad (2)$$

Putting equation (1) in equation (2), we get,

$$\Rightarrow T' = 2T \quad (3)$$

Putting $T = 2\text{sec}$ in equation (3), we get,

$$T' = 2 \times 2 \text{ sec} \Rightarrow T' = 4 \text{ sec}$$

8. To make the frequency double of a spring oscillation, we have to:
 ✓ a. Reduce the mass to one fourth
 b. Quadruple the mass
 c. Double the mass d. Half the mass

Sol: We know that, $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad (1)$

Now when frequency is doubled, then we have,

$$f' = 2f \quad (2)$$

For "f", we represent the mass by "m", so we get,

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m'}} \quad (3)$$

Putting equation (1) and equation (3) in equation (2), we get,

$$\frac{1}{2\pi}\sqrt{\frac{k}{m'}} = 2 \times \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{1}{m'}} = 2 \times \sqrt{\frac{1}{m}} \quad (4)$$

Squaring both sides, we get, $\frac{1}{m'} = \frac{4}{m} \Rightarrow m' = \frac{m}{4}$

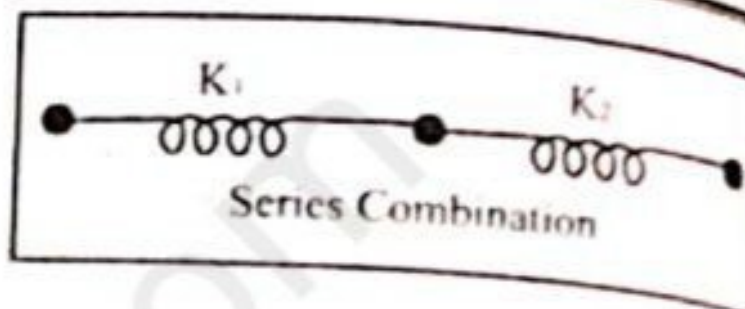
9. The restoring force of S.H.M is maximum when particle:
 ✓ a. Displacement is maximum b. Half way between them
 c. Crossing mean position d. At rest

10. Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by,

- a. $(k_1 + k_2) / 2$ b. $k_1 + k_2$
 ✓ c. $k_1 k_2 / (k_1 + k_2)$ d. $\sqrt{k_1 k_2}$

Sol: When two springs are connected in series then the effective spring constant 'K' is given by,

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} \Rightarrow \frac{1}{K} = \frac{K_2 + K_1}{K_1 K_2} \Rightarrow K = \frac{K_1 K_2}{K_1 + K_2}$$



Short Questions

Q1: Give two applications in which resonance plays an important role.

Ans. 1. **Microwave Oven:** BISE Mardan 2017

The microwave oven is an important application of resonance. In microwave oven, typical microwave with a wavelength 12cm frequency of 2450 MHz is produced. Such frequency of microwave is similar to the natural frequency of vibrations of water molecules and fat molecules in the food i.e.



Microwave oven

Frequency of microwaves = Frequency of water molecules & fat molecules

Now when food is placed in the oven, and the set is ON then the water molecules and fat molecules in the food resonates under the action of periodic force of microwaves. As a result, these molecules absorb energy of microwaves due to resonance. The energy of microwaves heats up the food item. In this way, the food items are heated or cooked in the microwave oven due to resonance.

2. **Radio Receiver:**

Tuning of radio is another important application of resonance. When we turn the knob of a radio to tune a station, we are changing the frequency of the electrical circuit of the receiver, to make it equal

to the transmission frequency of radio station. When the two frequencies match each other then resonance occurs. Under such conditions maximum absorption of energy takes place and thus we hear the concern station very clearly due to resonance.



Q2: What happens to the time period of a simple pendulum if its length is doubled?

Ans. The time period of a simple pendulum will increase by a factor $\sqrt{2}$ -times if its length is doubled.

Reason and Explanation:

We know that the time period of a simple pendulum is given by,

$$T = 2\pi \sqrt{\frac{l}{g}} \quad (1)$$

Now when the length is doubled, then we have,

$$l' = 2l \quad (2)$$

Now for " l' ", equation (1) can be written as,

$$T' = 2\pi \sqrt{\frac{l'}{g}} \quad (3)$$

Putting equation (2) in equation (3), we get,

$$T' = 2\pi \sqrt{\frac{2l}{g}} \Rightarrow T' = \sqrt{2} \left(2\pi \sqrt{\frac{l}{g}} \right) \quad (4)$$

Putting equation (1) in equation (4), we get,

$$T' = \sqrt{2} T \quad (5)$$

Equation (5) shows that the time of simple pendulum will increase by a factor $\sqrt{2}$ - times if its length is doubled.

Q3: What will be the frequency of a simple pendulum if its length is 1m?

Ans. The frequency of a simple pendulum will be 0.5 Hz if its length is 1m.

Reason and Explanation: We know that the frequency of simple pendulum is given by,

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \quad (1)$$

Now we put, $l = 1\text{m}$, $g = 9.8\text{m/sec}^2$ in equation (1), we get,

$$f = \frac{1}{2 \times 3.14} \sqrt{\frac{9.8}{1}} \Rightarrow f = \frac{3.13}{6.28} \Rightarrow f = 0.498\text{Hz}$$

$$\Rightarrow \boxed{f = 0.5\text{Hz}}$$

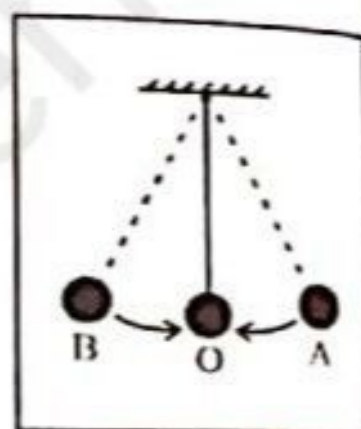
Conclusion: The frequency of a simple pendulum will be 0.5 Hz if its length is 1m.

Q4: Give one practical example each of free and force oscillation.

Ans. Free Oscillation:

When a system oscillates with its own natural frequency and no net force acts on it, then such oscillation of the system is known as free oscillation.

Example 1: When a simple pendulum is displaced from its mean position and then left free, it executes free oscillations, in the absence of external or resistive forces. For the oscillation, the time period of simple pendulum is given by,



$$\boxed{T_0 = 2\pi\sqrt{\frac{l}{g}}}$$

While the frequency of simple pendulum is given by,

$$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

Example 2: Oscillation of a swing in the absence of external force is another example of free oscillation.

Forced Oscillation:

When a system oscillates under the influence of a net external force, then such oscillation is known as forced oscillation.

Example 1: If the mass of a vibrating pendulum is struck again and again then such oscillations of the pendulum is called forced oscillations.

Example 2: The vibrations of a factory floor caused by the running of heavy machinery are another example of forced oscillations.

Example 3: Another example is a child's swing that is pushed on each downswing.

Q5: How can you compare the masses of two bodies by observing their frequencies of oscillation when supported by a spring?

Ans. Consider a mass-spring system as shown in figure (a). The frequency of mass-spring system is given by,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{--- (1)}$$

Now consider two bodies of mass " m_1 " and " m_2 " attached to an elastic spring as shown in figure (b) and figure (c).

Case-1: For " m_1 " equation (1) can be written as,

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}} \quad \text{--- (2)}$$

Case-2: For " m_2 " equation (1) can be written as,

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m_2}} \quad \text{--- (3)}$$

Now dividing equation (2) by equation (3), we get,

$$\frac{f_1}{f_2} = \frac{\frac{1}{2\pi}\sqrt{\frac{k}{m_1}}}{\frac{1}{2\pi}\sqrt{\frac{k}{m_2}}} = \frac{\sqrt{\frac{k}{m_1}}}{\sqrt{\frac{k}{m_2}}} = \sqrt{\frac{k}{m_1}} \times \sqrt{\frac{m_2}{k}} = \sqrt{\frac{m_2}{m_1}}$$

$$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}} \times \sqrt{\frac{m_2}{k}} = \sqrt{\frac{km_2}{m_1}} = \sqrt{\frac{m_2}{m_1}}$$

$$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}} \quad \text{--- (4)}$$

Squaring both sides of equation (4), we get,

$$\frac{f_1^2}{f_2^2} = \frac{m_2}{m_1} \Rightarrow \frac{m_2}{m_1} = \frac{f_1^2}{f_2^2} \Rightarrow \boxed{\frac{m_1}{m_2} = \frac{f_2^2}{f_1^2}} \quad \text{--- (5)}$$

Using equation (3), we can compare the masses two bodies by observing their frequencies of oscillation when supported by a spring.

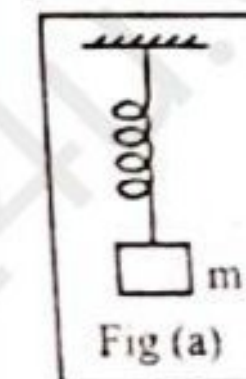


Fig (a)

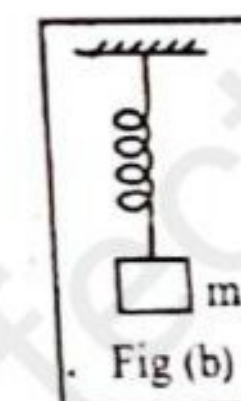


Fig (b)

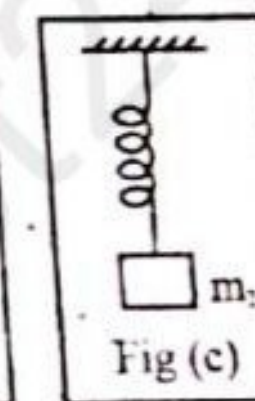


Fig (c)

Q6: A wire hangs from the top of a dark high tower, so that the top of the tower is not visible. How you would be able to determine the height of that tower?

Ans. We can determine the height of the tower by using the relation of simple pendulum. For this purpose we connect a bob to the lower end of the wire so that the arrangement becomes as a simple pendulum.

Reason and Explanation:

Consider a wire of length " l " is hanging from top of the tower as shown in the figure (a). Now for finding the height of the tower, we attach a bob to the lower end of the wire so that the arrangement behaves like a simple pendulum, as shown in figure (b).

The length of this pendulum will give us the length or height of the tower. Now the time period of the simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

Squaring both sides of equation (1), we get,

$$T^2 = 4\pi^2 \frac{l}{g} \Rightarrow l = \frac{gT^2}{4\pi^2} \quad (2)$$

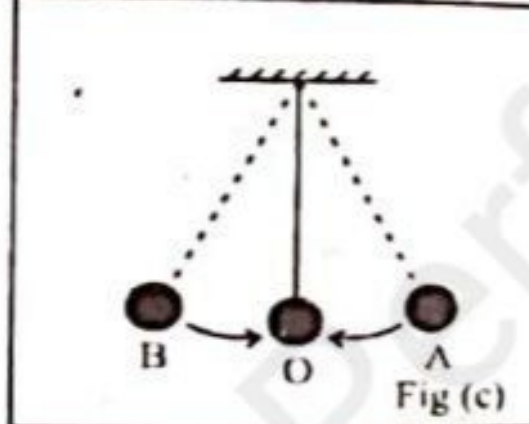
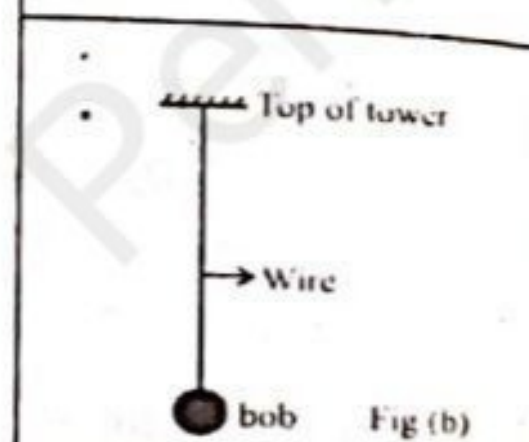
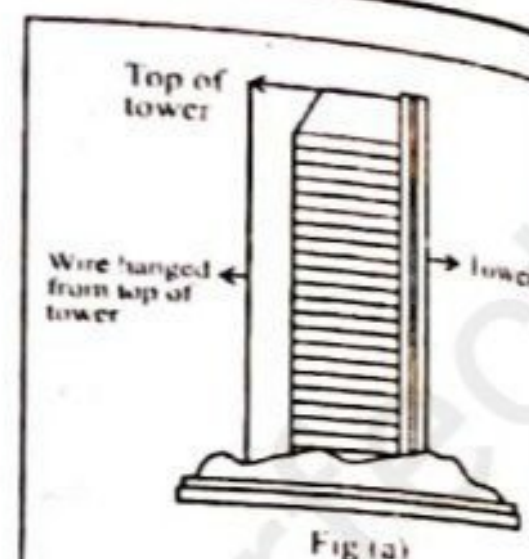
Now make the pendulum to vibrate about mean position " O " as shown in figure (c). Note the time for one vibration with the help of stopwatch. This will give us the time period " T " of the pendulum. Now by putting the values of " g ", " π " and " T " in equation (2), we can calculate the length of the pendulum. Such length of the pendulum will represent the height of the tower.

Q7: Why in S.H.M the acceleration is zero and the velocity is greatest?

Ans. In S.H.M when velocity is greatest, the acceleration is zero.

Reason and Explanation:

The velocity of a simple harmonic oscillator is given by,



$$V = \omega\sqrt{x_0^2 - x^2} \quad (1)$$

While the acceleration is given by,

$$a = -\omega^2 x \quad (2)$$

Now when the body is displaced to extreme position " x_0 " and then released, the body executes S.H.M and its velocity becomes maximum at its mean position. For maximum displacement, we consider " x_0 " and ignore " x " i.e. we put $x = 0$, so equation (1) becomes,

$$V_{\max} = \omega\sqrt{x_0^2 - 0^2} = \omega\sqrt{x_0^2}$$

$$\Rightarrow V_{\max} = \omega x_0 \quad (3)$$

Now put $x = 0$ in equation (2), we get,

$$a = -\omega^2 \times 0 \Rightarrow a = 0 \quad (4)$$

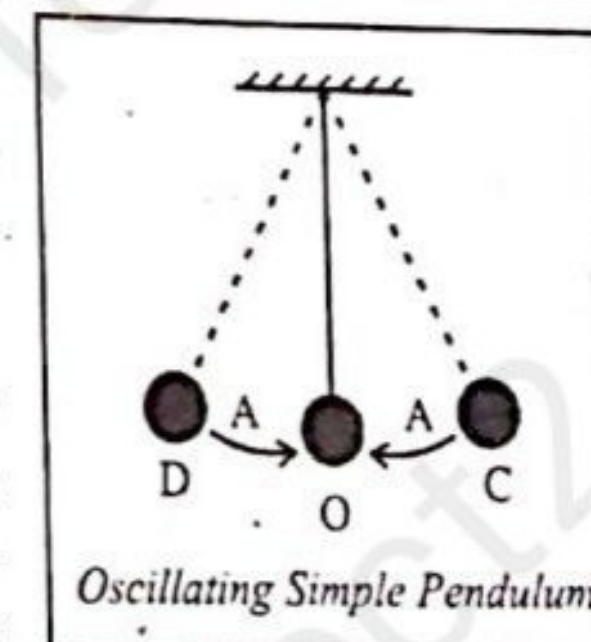
Equation (3) and equation (4) shows that in S.H.M, the acceleration is zero when the velocity is greatest.

Q8: What is the total distance covered by a simple harmonic oscillator in a time equal to its period? The amplitude of oscillation is A ?

Ans. The total distance travelled by a simple harmonic oscillator in a time equal to its time period is equal to " $4A$ ", where " A " is its amplitude.

Explanation:

In this question, the total distance in a time equal to period means the distance covered during one complete vibration. Consider the oscillation of simple pendulum as shown in the figure.



When the pendulum moves from mean position " O " towards extreme position " C " and then moves back from " C " towards " O ", it completes half vibration. During this half vibration, the pendulum covers a distance " $A + A = 2A$ ". Similarly, when the pendulum moves from " O " towards " D " and then from " D " towards " O ", it completes half vibration. During this half vibration, the pendulum covers a distance " $A + A = 2A$ ". So during one vibration, the time taken by the oscillator is equal to its time period and the total distance covered during one vibration is equal to " $2A + 2A = 4A$ ".

Q9: What happens to the frequency of a simple pendulum as its oscillations die down from large amplitude to small one?

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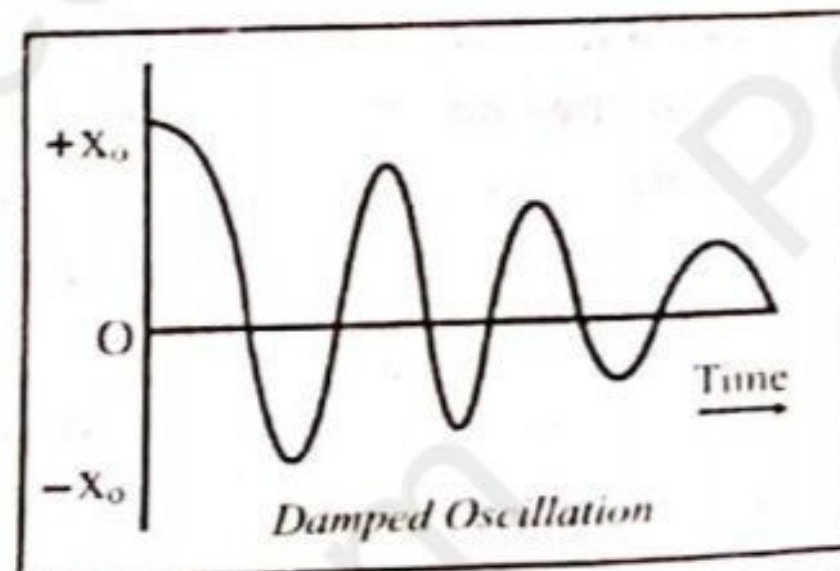
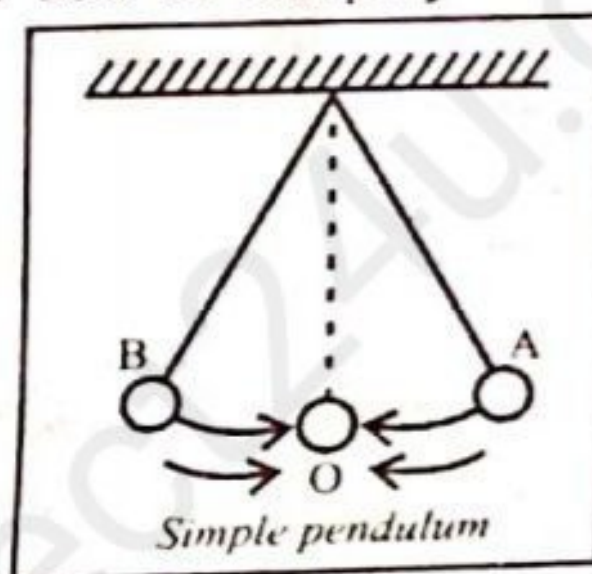
Ans. When the amplitude of a simple pendulum dies down from large amplitude to small, it behaves as a damped oscillator. Damping not only reduces the amplitude of the oscillator but also reduces the frequency of the oscillator while increasing the period of oscillation.

Explanation:

If we set a simple pendulum in a laboratory and put it into oscillations. Its amplitude decreases with time and finally comes to rest due to air friction. Since, the amplitude of the pendulum decreases as a result the energy of the oscillating system decreases due to work against the air resistive force. This energy is dissipated in the particles of the surrounding medium. Damping not only has effect of reducing the amplitude but also the effect of reducing the frequency or increasing the time period of oscillation. We know that,

$$f \propto \frac{1}{T} \quad (1)$$

Equation (1) shows that frequency decreases when time period increases due to damping.



Q10: A singer holding a note of right frequency, can shatter a glass? Explain.

Ans. A singer holding a note of right frequency can shatter a glass.

Reason and Explanation:

It is possible for a singer to shatter a glass, because every solid body has certain natural frequency of vibration depending upon its elasticity, mass, shape and size. A singer sings a note of particular

frequency ' f_s ' and when this frequency of the note becomes equal to the natural frequency ' f_0 ' of the air in glass i.e.

$$f_s = f_0 \quad (1)$$

So resonance takes place. Therefore, the air in glass will also begin to vibrate with large amplitude due to resonance. This large amplitude vibration may shatter the glass.

Comprehensive Questions

Q1: Show that motion of a mass attached with a spring executes S.H.M.

Ans. See answer of question no 3 in the text.

Q2: Prove that the projection of a body motion in a circle describes S.H.M.

Ans. See answer of question no 4 in the text.

Q3: Show that energy is conserved in case of S.H.M.

Ans. See answer of question no 7 in the text.

Q4: Differentiate free and forced oscillations.

Ans. See answer of question no 8 in the text.

Q5: What is resonance give three of its applications in our daily life?

Ans. See answer of question no 9 in the text.

Q6: Derive equations for kinetic and potential energy of a body of mass " m " executing S.H.M.

Ans. See answer of question no 7 in the text.

Q7: Explain what is mean by damped oscillations?

Ans. See answer of question no 12 in the text.

فرمانِ مصطفیٰ ﷺ: مسلمان کا صدقہ عمر میں زیادتی کا سبب ہے اور بڑی موت کو دفع کرتا ہے۔

کسی شخص کا آپ پر اعتماد کرنا آپ کے لیے باعثِ فخر ہے اور آپ پر جب کوئی اعتماد کرے تو کوشش کریں کہ اس کا نام رکھیں۔

Numerical Problems

Q1: A force of 0.4N is required to displace a body attached to a spring through 0.1m from its mean position. Calculate the spring constant of the spring.

Solution:

Given data:

Force = $F = 0.4\text{N}$, Displacement = $x = 0.1\text{m}$

Required:

Spring constant = $k = ?$

Calculation:

We know that,

$$F = kx \Rightarrow k = \frac{F}{x} \quad (1)$$

Putting the values in equation (1), we get,

$$k = \frac{0.4}{0.1} \text{N/m} = 4\text{N/m} \Rightarrow \boxed{\text{Spring constant} = 4\text{N/m}}$$

Q2: A pendulum clock keeps perfect time at a location where the acceleration due to gravity is exactly 9.8ms^{-2} . When the clock is moved to a higher altitude, it loses 80.0s per day. Find the value of 'g' at this new location. (9.78m/s^2)

Solution: Given data: Acceleration due to gravity $g = 9.8\text{m/sec}^2$

Loss in time per day = 80.0sec,

$$\text{Loss in time per second} = \frac{80}{86400}$$

$$\boxed{1\text{day} = 86400\text{sec}}$$

$$\text{Loss in time per second} = 0.000926\text{sec}$$

The loss in time shows that swinging of clock becomes slower. Thus if the time period "T" at the location where $g = 9.8\text{m/sec}^2$ is 1sec, then the time period "T'" at new altitude will be equal to $(1 + 0.000926)\text{sec}$ i.e. $T' = 1.000926\text{sec}$

Required:

Acceleration due to gravity at new altitude = $g' = ?$

Calculation:

We know that, the time period of pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

For new time period "T'", equation (1) can be written as,

$$T' = 2\pi\sqrt{\frac{l}{g'}} \quad (2)$$

Dividing equation (1) by equation (2), we get,

$$\frac{T}{T'} = \frac{2\pi\sqrt{\frac{l}{g}}}{2\pi\sqrt{\frac{l}{g'}}} = \frac{\sqrt{\frac{l}{g}}}{\sqrt{\frac{l}{g'}}} \Rightarrow \frac{T}{T'} = \sqrt{\frac{l}{g}} \div \sqrt{\frac{l}{g'}} = \sqrt{\frac{l}{g}} \times \sqrt{\frac{g'}{l}} = \sqrt{\frac{g'}{g}}$$

$$\Rightarrow \frac{T}{T'} = \sqrt{\frac{g'}{g}} \quad (3)$$

Squaring both sides of equation (3), we get,

$$\frac{T^2}{T'^2} = \frac{g'}{g} \Rightarrow g' = \frac{gT^2}{T'^2} \quad (4)$$

Putting the values of "g", "T" and "T'" in equation (4), we get,

$$g' = \frac{9.8 \times 1^2}{(1.000926)^2} \Rightarrow g = 9.78\text{m/sec}^2 \quad \boxed{g' = 9.78\text{m/sec}^2}$$

Q3: Calculate the length of a second pendulum having time period 2sec at a place where $g = 9.8\text{m/sec}^2$.

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Solution: Given data:

Time period = $T = 2\text{sec}$, Acceleration due to gravity = $g = 9.8\text{m/sec}^2$

Required:

Length of second pendulum = $l = ?$

Calculation:

$$\text{We know that, } T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring both sides, we get,

$$\Rightarrow T^2 = 4\pi^2 \left(\frac{l}{g} \right) \Rightarrow l = \frac{gT^2}{4\pi^2} \quad (1)$$

$$\Rightarrow l = \frac{9.8 \times (2)^2}{4 \times (3.14)^2} \Rightarrow l = 0.992\text{m}$$

$$\Rightarrow \boxed{l = 0.992\text{m}}$$

Q4: A body of mass "m" suspended from a spring with force constant "k" vibrates with " f_1 ". When its length is cut into half and the same body is suspended from one of the halves, the frequency is " f_2 ". Find out f_1/f_2 .

Solution:

Given data:

Mass attached = m, spring constant = k

Frequency of whole spring = f_1 , frequency of half spring = f_2

Displacement for whole spring = x_1

Displacement for half spring = $x_2 = \frac{x_1}{2}$

Required:

Relation of $\frac{f_1}{f_2} = ?$

Calculation:

For mass-spring system, we have,

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (1)$$

We know that, $F = kx$

$$\Rightarrow w = kx \Rightarrow mg = kx$$

$$\Rightarrow k = \frac{mg}{x} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$f = \frac{1}{2\pi} \sqrt{\frac{mg}{mx}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{x}} \quad (3)$$

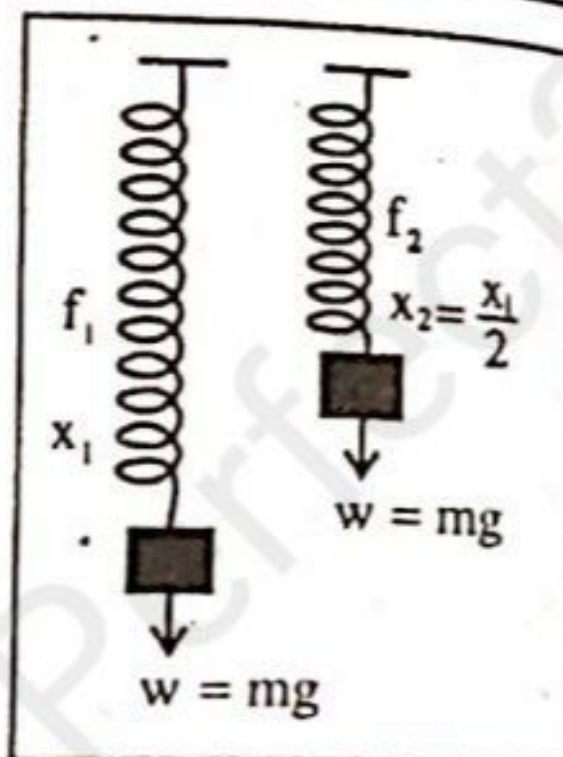
Now for 1st spring, equation (3) can be written as,

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{g}{x_1}} \quad (4)$$

While for 2nd spring, equation (3) can be written as,

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{g}{x_2}}$$

$$\Rightarrow f_2 = \frac{1}{2\pi} \sqrt{\frac{2g}{x_1}} \Rightarrow f_2 = \sqrt{2} \left(\frac{1}{2\pi} \sqrt{\frac{g}{x_1}} \right) \quad (5)$$



F = weight of suspended mass = $w = mg$

Putting equation (4) in equation (5), we get,

$$f_2 = \sqrt{2} f_1 \Rightarrow f_1 = \frac{1}{\sqrt{2}} f_2 \Rightarrow \frac{f_1}{f_2} = \frac{1}{\sqrt{2}} = 0.707$$

$$\Rightarrow \frac{f_1}{f_2} = 0.707$$

Q5: A mass at the end of spring describes S.H.M with $T = 0.40 \text{ sec}$. Find out acceleration when the displacement is 0.04 m .

Solution:

Given data:

Time period = $T = 0.40 \text{ sec}$, Displacement = $x = 0.04 \text{ m}$

Required:

Acceleration = $a = ?$

Calculation:

We know that, $a = -\omega^2 x$ (1)

$$\text{Also } T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$a = -\left(\frac{2\pi}{T}\right)^2 x \Rightarrow a = -\frac{4\pi^2 x}{T^2} \quad (3)$$

Putting the values in equation (3), we get,

$$a = \frac{-4 \times 3.14^2 \times 0.04}{(0.40)^2} = -9.86 \text{ m/sec}^2$$

$$\text{Acceleration} = a = -9.86 \text{ m/sec}^2$$

Q6: A block weighing 4 kg extends a spring by 0.16 m from its unstretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and released, what is its period of vibrations?

Solution:

Given data:

(i) Mass of block = $m_1 = 4 \text{ kg}$, Displacement = $x = 0.16 \text{ m}$

(ii) Mass of body = $m_2 = 0.50 \text{ kg}$

Required: Time period = $T = ?$

Calculation:

First we calculate the spring constant, we know that

$$F = kx \Rightarrow w = kx$$

$$F = w = mg$$

$$\Rightarrow mg = kx \Rightarrow k = \frac{mg}{x} \quad (1)$$

Putting the values in equation (1), we get,

$$k = \frac{4 \times 9.8}{0.16} \Rightarrow k = 245 \text{ N/m}$$

Now we know that,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_2}{k}} \Rightarrow T = 2\pi\sqrt{\frac{m_2}{k}} \quad (2)$$

Putting the values in equation (2), we get,

$$T = 2 \times 3.14 \sqrt{\frac{0.50}{245}} = 0.28 \text{ sec} \quad \boxed{T = 0.28 \text{ sec}}$$

Q7: What should be the length of simple pendulum whose time period is one second? What is its frequency?

Solution:

Given data:

Time period = $T = 1 \text{ sec}$

Required:

(a) Length of pendulum = $l = ?$

(b) Frequency of simple pendulum = $f = ?$

Calculation:

(a) We know that,

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2 \frac{l}{g} \Rightarrow l = \frac{gT^2}{4\pi^2} \quad (1)$$

Putting the values in equation (1), we get,

$$l = \frac{9.8 \times (1)^2}{4 \times (3.14)^2} = 0.25 \text{ m} \Rightarrow l = 0.25 \text{ m}$$

(b) We also know that,

$$f = \frac{1}{T} \quad (2)$$

Putting the values in equation (2), we get,

$$f = \frac{1}{1} \text{ cycles/sec} \Rightarrow f = 1 \text{ c.p.s}$$

$$l = 0.25 \text{ m} \quad \text{or} \quad f = 1 \text{ c.p.s}$$

Q8: A spring whose spring constant is 80 N/m vertically supports a mass of 1 kg is the rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with velocity of one meter per second?

Solution:

Given data: Spring constant = $k = 80 \text{ N/m}$

Mass = $m = 1 \text{ kg}$, Maximum velocity = $V_{\text{max}} = 1 \text{ m/sec}$

Required:

Amplitude = $x_0 = ?$

Calculation:

We know that, $V_{\text{max}} = \omega x_0$

$$\Rightarrow V_{\text{max}} = \sqrt{\frac{k}{m}} x_0 \quad (1)$$

Putting the values in equation (1), we get,

$$1 = \sqrt{\frac{80}{1}} x_0 \Rightarrow 1 = 8.944 x_0$$

$$\Rightarrow x_0 = \frac{1}{8.944} \Rightarrow \boxed{x_0 = 0.11 \text{ m}}$$

$$\Rightarrow \boxed{x_0 = 0.11 \text{ m}}$$

$$\begin{aligned} a &= \omega^2 x \\ a &= \frac{k}{m} x \\ \Rightarrow \omega^2 &= \frac{k}{m} \\ \omega &= \sqrt{\frac{k}{m}} \end{aligned}$$

Q9: A 8000 gm body vibrates S.H.M with amplitude 0.30 m . The restoring force is 60 N and the displacement is 0.30 m . Find out, (i) time period (ii) acceleration (iii) velocity (iv) K.E (v) P.E when the displacement is 12 cm .

Solution:

Given data:

Mass of body = $m = 8000 \text{ gm} = 8.0 \text{ kg}$

Amplitude = $x_0 = 0.3 \text{ m}$, Restoring force = $F = 60 \text{ N}$

Displacement = $x = 12 \text{ cm} = 0.12 \text{ m}$

Required:

- (i) Time period = $T = ?$
 (ii) Acceleration = $a = ?$
 (iii) Velocity = $v = ?$
 (iv) K.E = ? (v) P.E = ?

Calculation:

(i) As $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{mx_0}{F}} \quad (1)$$

Putting the values in equation (1), we get,

$$T = 2 \times 3.14 \sqrt{\frac{3.0 \times 0.3}{60}} \Rightarrow T = 1.3 \text{ sec}$$

(ii) As $a = \omega^2 x \Rightarrow a = \frac{k}{m} \cdot x \quad (2)$

Putting the values in equation (2), we get,

$$a = \frac{200}{2} \times 0.12 \Rightarrow a = 3 \text{ m/sec}^2$$

(iii) As $v = \omega\sqrt{x_0^2 - x^2} \Rightarrow v = \sqrt{\frac{k}{m}}\sqrt{x_0^2 - x^2} \quad (3)$

Putting the values in equation (3), we get,

$$v = \sqrt{\frac{200}{8}} \times \sqrt{0.3^2 - 0.12^2}$$

$$\Rightarrow v = (5 \times 0.274) \text{ m/sec} \Rightarrow v = 1.4 \text{ m/sec}$$

(iv) We know that,

$$K.E = \frac{1}{2}k(x_0^2 - x^2) \quad (4)$$

Putting the values in equation (4), we get,

$$K.E = \frac{1}{2} \times 200(0.3^2 - 0.12^2) = 100(0.0756) \text{ joules}$$

$$\Rightarrow K.E = 7.6 \text{ J}$$

(v) $P.E = \frac{1}{2}kx^2 \quad (5)$

Putting the values in equation (5), we get,

$$P.E = \frac{1}{2} \times 200 \times (0.12)^2 \Rightarrow P.E = 1.44 \text{ J}$$

$$F = kx_0$$

$$\Rightarrow k = \frac{F}{x_0}$$

$$\text{Also } \omega = \sqrt{\frac{k}{m}}$$

$$\omega^2 = \frac{k}{m}$$

$$F = 60 \text{ N}$$

$$x_0 = 0.3$$

$$K = \frac{F}{x_0}$$

$$K = \frac{60}{0.3} = 200 \frac{\text{N}}{\text{m}}$$

Q10: Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant "t" is given by, $x = 0.25 \cos(\frac{\pi}{8})t$. Find the displacement of the object after 2.0 seconds.

Solution:Given data:

The given equation is,

$$x = 0.25 \cos\left(\frac{\pi}{8}\right)t \quad (1)$$

Required:

- (i) Amplitude = $x_0 = ?$
 (ii) Frequency = $f = ?$
 (iii) Time period = $T = ?$
 (iv) Displacement after 2sec = ?

Calculation:

The instantaneous displacement in S.H.M is given by,

$$x = x_0 \cos \omega t \quad (2)$$

Comparing equation (1), (2), we get,

$$x_0 = 0.25 \text{ m}, \omega = \frac{\pi}{8}$$

(i) Amplitude = $x_0 = 0.25 \text{ m}$

(ii) We know that,

$$f = \frac{\omega}{2\pi} \quad (3)$$

Putting the values in equation (3), we get,

$$f = \frac{\pi}{8 \times 2\pi} = \left(\frac{1}{16}\right) \text{ Hz}$$

$$\Rightarrow f = \frac{1}{16} \text{ Hz}$$

(iii) We know that,

$$T = \frac{1}{f} \quad (4)$$

Putting the values in equation (4), we get,

$$T = \frac{1}{\left(\frac{1}{16}\right)} = 16 \text{ sec}$$

$$\Rightarrow T = 16 \text{ sec}$$

(iv) Put $t = 2 \text{ sec}$ in equation (1), we get,

$$x = 0.25 \times \cos\left(\frac{\pi \times 2}{8}\right) = 0.25 \times \cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow x = 0.25 \times \cos\left(\frac{180^\circ}{4}\right) = 0.25 \times \cos(45^\circ)$$

$$\pi \text{ rad} = 180^\circ$$

$$\Rightarrow x = 0.25 \times 0.707 \text{ m}$$

$$\Rightarrow x = 0.18 \text{ m}$$

+++

④ ہم کسی شخص کی طاقت سے زیادہ اس پر بوجھ نہیں ڈالتے، اگر تم اللہ پر ایمان رکھتے ہو تو شرط فرما تیر داری یہ ہے کہ اس پر بھروسہ رکھو۔ (فرمان خداوندی)

④ رسول اللہ ﷺ نے فرمایا: "دنیا سے بے رغبتی دل اور روح کو حقیقی راحت دیتی ہے اور دنیا کی رغبت و محبت زعمی میں رنج و الم کو بڑھاتی ہے۔"

④ رسول اللہ ﷺ نے فرمایا: "جب تم تین آدمی ساتھ ہو تو تم میں سے دو آدمی تیسرے کو اکیلا چھوڑ کر کان میں بات اور سرگوشی نہ کریں، کیونکہ یہ اسے رنج و غم میں مبتلا کر دے گا۔" (سنن ابوداؤد / سند صحیح)

④ رسول اللہ ﷺ نے فرمایا: "یاد رکھو! اللہ تعالیٰ کی مدد و نصرت مبر کے ساتھ ہی ہے اور ہر تکلیف و مصیبت کے بعد کشادگی ہے اور بلاشبہ ہر تنگی و مشکل کے بعد آسانی ہے۔" (مسند احمد: ۲۸۰۳، من حضرت ابن عباسؓ)

④ ایمان دو نصف ہیں، نصف مبر اور نصف شکر (ارشاد نبوی ﷺ)

④ ایمان دو نصف ہیں، جسے لوگ اپنے دل و جان کو محفوظ سمجھیں۔ (ارشاد نبوی ﷺ)

④ مبر ایمان سے ایسا ملا ہے جیسے سر جسم سے۔ (ارشاد نبوی ﷺ)

④ سچ بولنا ایمان کی پہلی علامت ہے۔ (حکیم بزرگمہر)

CHAPTER

08

WAVES

Q1: (a) What is a wave? (b) Define and explain wave motion.

Ans: (a) Wave:

Definition: Wave is a disturbance of some kinds that propagates through a medium and carries energy.

(b) Wave Motion:

Definition: The mechanism by which energy is transferred from one point to another point is known as wave motion.

Explanation:

When we throw a stone on the still surface of water in a pond, a disturbance is produced in water. In another words energy is transferred to the particles of water due to which they begins to oscillate about their mean position. The oscillating molecules of water transfer the energy to their neighboring molecules. This process continues and finally the energy is transferred to the edge of the pond. This motion of the disturbance is due to the transfer of energy from particle to particle is known as wave motion.

The waves transport energy from one place to another but the particles of the medium does not change their position permanently. They perform only vibratory motion about their mean position.

Q2: What is a periodic wave? Explain with examples.

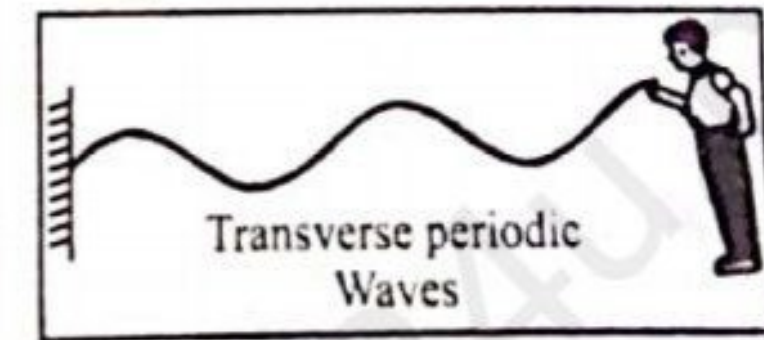
Ans: Periodic Waves:

Definition: If a medium is disturbed periodically [i.e. disturbed after equal intervals of time], then the waves with same frequency and amplitude are produced, which are known as periodic waves.

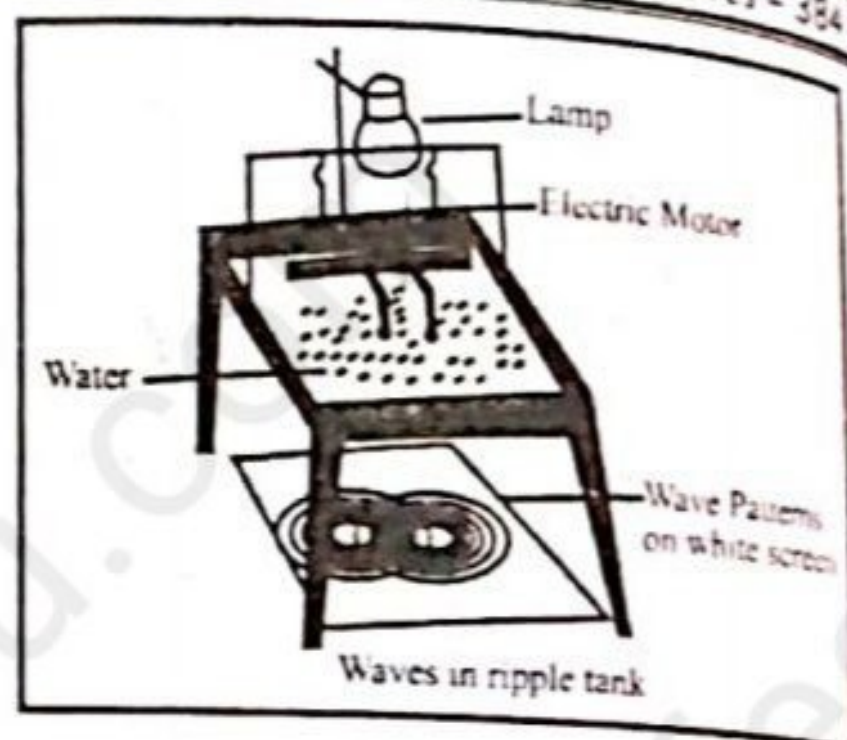
Examples of Periodic Waves:

1. Waves produced in a rope:

1. Consider an ordinary rope whose one end is fastened to a fixed



support, as shown in the figure. If we hold the other end of the rope in hand and give a sudden up and down motion periodically, then a train of transverse periodic waves are produced in the string, as shown in the figure. The waves travel towards the fixed end, while particles of the rope vibrate up and down about their mean position.



2. The circular periodic waves can be produced in a ripple tank when spherical dippers just touching the water surface. This is done by means of a mechanical arrangement driven by a small electric motor. The rate of dipping the rods is controlled by changing the speed of motor. The pattern of waves obtained at any instant of time is shown in the figure.

Q3: What are progressive waves? Explain.

Ans: Progressive Waves (or) Travelling Waves:

Definition: Those waves which transmit energy from one place to another place are known as progressive waves or travelling waves.

Explanation:

When we drop a stone on the still surface of water in a pond, a disturbance is produced in water. Due to this disturbance, the particles of water are set into motion. These particles transfer their energy to the neighboring particles. This process continues and finally, the energy reaches to the edge of the pond of water. Such waves which are produced in water and carry energy from one place to another are called transverse progressive waves.



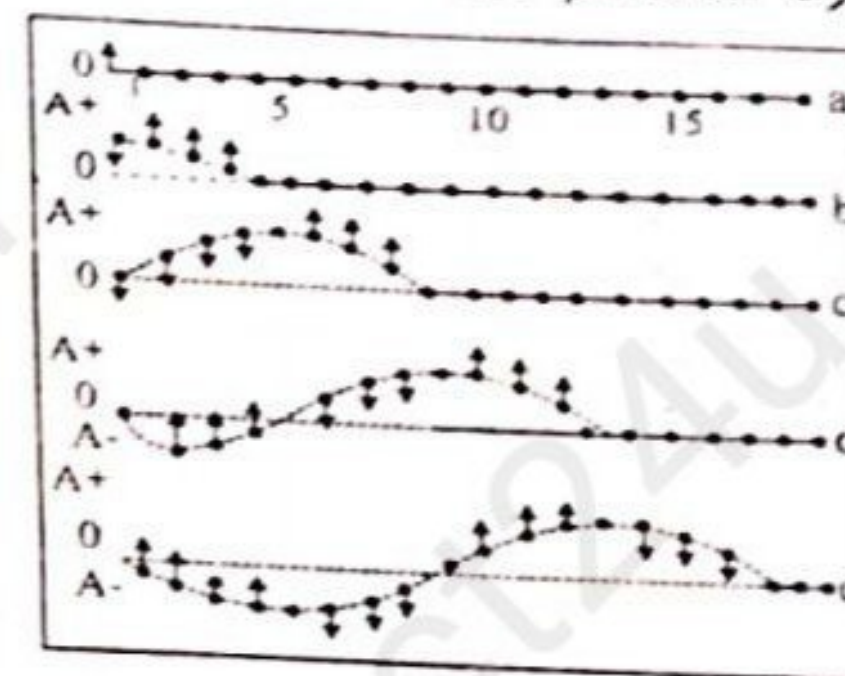
Photograph of shadows of circular waves on the water surface

Q4: Discuss the propagation of waves in detail.

Ans: Propagation of waves:

The propagation of mechanical waves occurs due to the interaction of the particles of the medium with each other. The particle by particle wave propagation can occur only in an elastic medium.

Now consider a mechanical mode of vibration of a stretched long elastic rod which consists of equally spaced particles such as 1, 2, 3, 4, 5, 6, along its length as shown in figure (a)



Now if the particle "1" is given an upward motion at right angle to the cord, the particle "2" will follow it due to elasticity of the cord. The particle "2" will lag behind the particle "1" due to its inertia.

Now when the particle "2" rises, the particle "3" will follow it but will lag behind the particle "2" due to its inertia. Similarly the particle "4" will follow particle "3" and so on.

When the particle "1" reaches to its highest position "+A", the particle "5" just begin to move up while at that instant particle "1" starts downwards motion. The particle "2" will continue its upward motion due to its inertia until it reaches to its highest position "+A".

When the particle "1" reach its mean position "O" it does not stop there and move downwards from mean position "O" as shown in figure (c).

When particle "1" reaches to position "-A", then it returns towards its mean position "O". When it reaches "O", one vibration is completed and as a result the other particles become disturbed as shown in figure (d). The particle "1" is now ready for second vibration. In this way, the waves propagate towards right and the particles of the medium [cord] oscillate only about their mean position.

Q5: Discuss the types of progressive mechanical waves. (OR)
Describe transverse and longitudinal waves with examples and clearly differentiate between them.

ETEA 2006, 2009, BISE Abbottabad 2019

Ans. 1. Transverse Waves:

Definition: Those waves in which the particle of the medium vibrates along a line perpendicular to the direction of propagation of the waves, are known as transverse waves.

Explanation:

Whenever a disturbance is propagating through a medium, then the particles of the medium are disturbed and start vibration. If the vibration of the particle is perpendicular to the direction of the propagation of the wave then it is called transverse wave pulse. The particle of medium does not change their position permanently and performs only vibratory motion about their mean position. In transverse waves, the particles vibrate with the period and frequency of the source.

The transverse waves consist of crests and troughs which are produced one after the other in a certain order. The distance between two consecutive crests or two consecutive troughs is known as wavelength. It is represented by " λ ", while the distance between a crest and nearest trough is equal to the half of the wavelength i.e. " $\lambda/2$ ".

The number of waves passing through a certain point is known as frequency of the waves. It is represented by " f ".

The velocity, frequency and wavelength of the waves are related to each other by the following equation i.e.

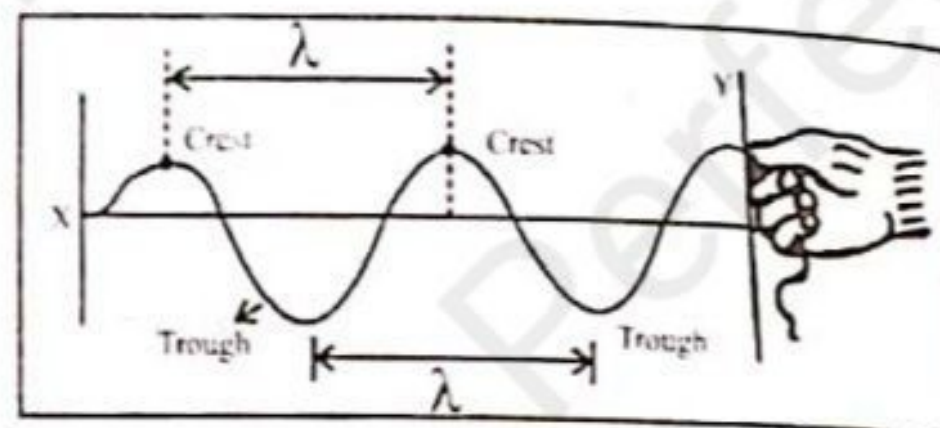
$$v = f\lambda$$

The transverse waves can propagate through a medium whose elasticity is larger. So these waves need a material medium for their propagation.

Examples of Transverse Waves:

The examples of transverse waves are given below:

- Water waves



- Light waves
- Radio waves
- Micro waves
- Waves produced in a stretched string

2. Longitudinal Waves [Compressional Waves]:

Those waves in which the particles of the medium vibrate about their mean position along the direction of propagation of the waves is known as longitudinal waves or compressional waves.

Explanation:

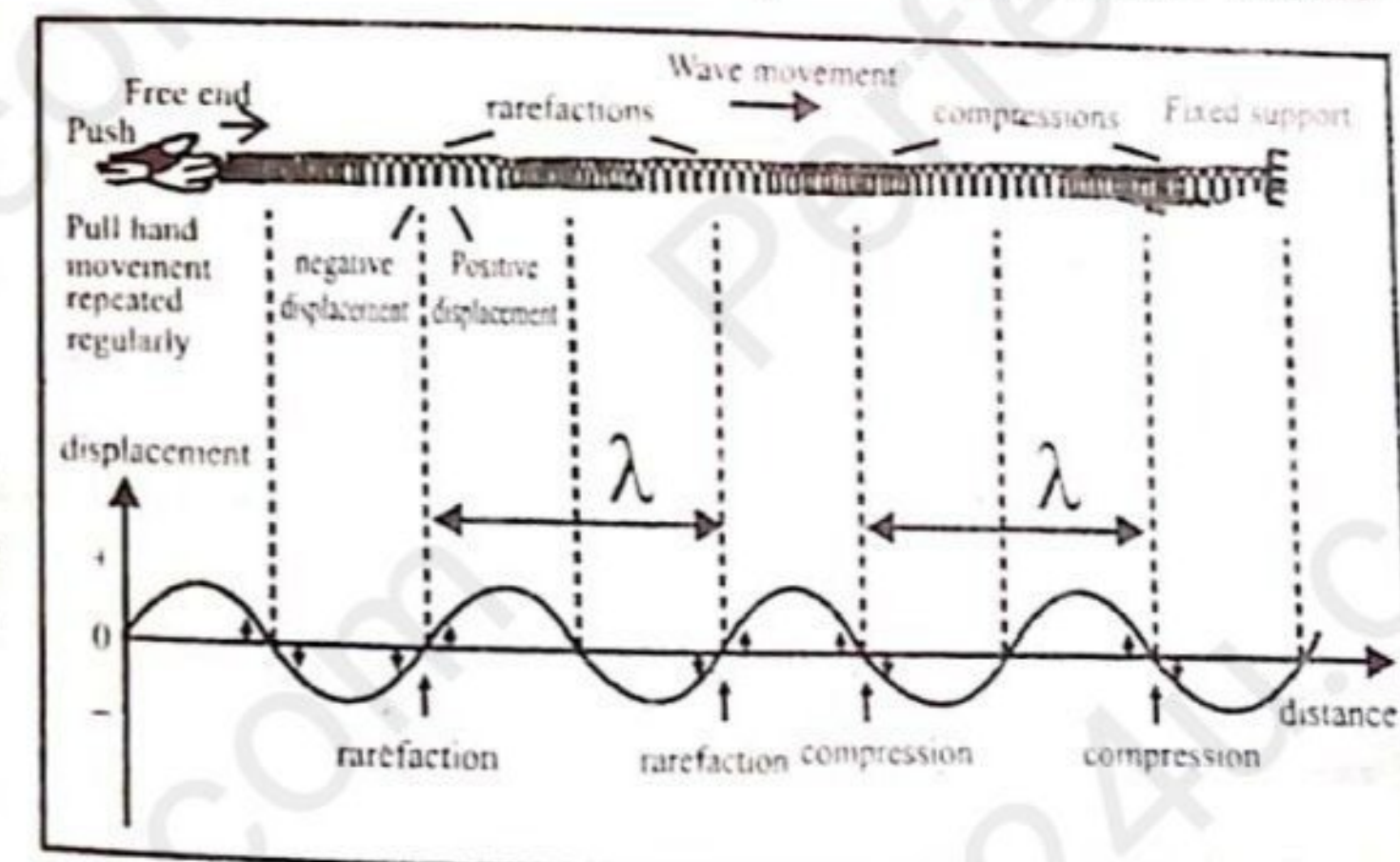
Consider a slinky spring placed on a smooth table top as shown in the figure. Let one end of the spring is free and the other end is connected to a fixed support, as shown in the figure. When we push the free end of the spring towards right, then the loops near the end are compressed towards right due to the applied force. The compressed loops then exert a force on the loops to the right side of them. In this way, a compressional pulse travels towards right along the spring.

Now when the spring is pulled suddenly in backward direction, [i.e. towards left], the loops near the end are rarefacted due to the applied force. The rarefacted loops then exert a force on the loops of the right of them. In this way, a rarefaction travels along the spring.

Now if the spring is pushed forward and backward at a constant rate, a series of longitudinal waves is setup. As the wave moves through the

slinky spring, compression and rarefaction can be seen along the spring.

The longitudinal waves consist of compression and rarefactions which are pro-



duced one after the other in a certain order. The distance between two consecutive compressions or two consecutive rarefactions is known as wavelength " λ ". The wavelength " λ ", frequency " f ", and velocity " v " of the waves are related to each other by the following equation i.e.

$$v = f \lambda$$

Examples of Longitudinal Waves:

- Sound waves
- Compressional waves in a spring
- Railway engine connected to boggies with buffer springs in between them.
- Shock waves [seismic waves] produced during an earthquake.

Q6: Define the following terms: Wave speed, frequency of waves, time period of waves, wavelength, amplitude of wave, intensity of wave, phase.

Ans: 1. Wave Speed: (ETEA 2006)

The distance travelled by a wave per unit time is known as speed of a wave. The speed of a wave depends upon the type of wave as well as the properties of the medium.

For example, the speed of transverse wave pulse in a stretched string depend upon the mass of string " M ", length of string " L " and tension in the string " T " and is given by,

$$v = \sqrt{\frac{TL}{M}} \quad (1)$$

If " m " represents mass per unit length, i.e.

$$m = \frac{M}{L} \Rightarrow M = mL \quad (2)$$

Putting equation (2) in equation (1), we get,

$$v = \sqrt{\frac{TL}{mL}} \Rightarrow v = \sqrt{\frac{T}{m}} \quad (3)$$

These equations show that the speed of transverse waves in well stretched and thin string is greater as compared to a loosely and thick one. While in case of longitudinal waves, the speed of wave depends upon the modulus of elasticity " E " and density " δ " of the medium, and is given by,

$$v = \sqrt{\frac{E}{\delta}} \quad (4)$$

As the gases are more compressible, so the elastic modulus " E " is smaller than solids. That is why, the longitudinal waves travel faster in solids as compared to gases. The unit of velocity is m/sec.

2. Frequency of Waves:

The number of waves passing through a certain point in unit time is known as frequency of the wave. When a wave progresses, each particle of the medium oscillates periodically with the time period and frequency of the source. Thus the frequency " f " of the wave is equal to the frequency of the source.

$$\text{Frequency} = \frac{\text{number of waves passing through a point}}{\text{time taken by waves}}$$

The unit of frequency is Hz or cycle per sec (c.p.s).

3. Time Period of Waves:

The time during which a wave passes through a certain point is called the time period of the wave. When a wave progresses, each particle of the medium oscillates periodically with the time period and frequency of the source. Thus the period " T " of the wave is equal to the time period of the source. The time period is equal to the reciprocal of frequency i.e. $T = \frac{1}{f}$

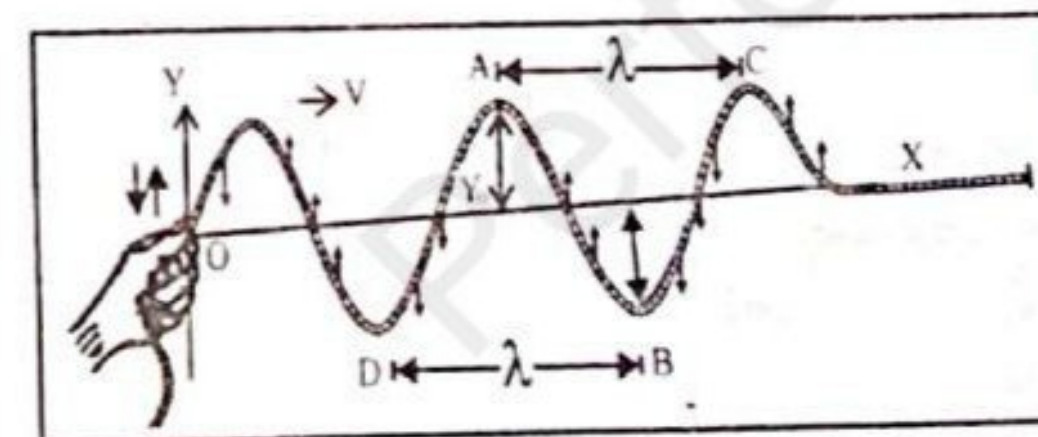
The unit of time period is sec.

4. Wavelength:

The distance between the two successive particles which are exactly in the same state of vibration is known as wavelength. It is represented by " λ ". Its

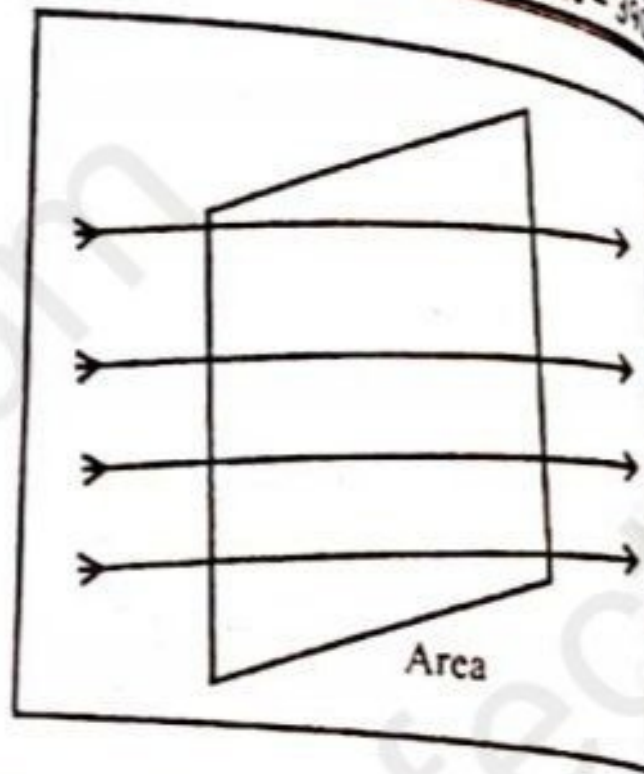
unit is meter. The relation between velocity, frequency and wavelength of a wave is given by,

$$v = f \lambda$$



5. Amplitude of Wave:

The maximum displacement covered by a vibrating particle from its mean position on either side is known as amplitude of the wave. In above figure, " y_0 " represents the amplitude.

**6. Intensity of Wave:**

The amount of energy transmitted per second per unit area placed perpendicular to the direction of propagation of waves is called the intensity of the waves. It is represented by " I ". Its unit is " $J/sec \cdot m^2$ ".

7. Phase:

The state of motion in a wave is known as phase.

Q7: Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace?

BISE Malakand 2019, BISE Mardan 2019, BISE Peshawar 2019

Ans. Speed of Sound:

The distance covered by sound wave per unit time is called speed of sound wave.

Newton's Formula for Speed of Sound:

The sound waves are longitudinal waves [mechanical waves] which need a material medium for their propagation. The speed of sound waves in a medium depends upon the density " δ " of the medium and elasticity " E " of the medium. i.e.

$$v = \sqrt{\frac{E}{\delta}} \quad (1)$$

According to Newton, the temperature of air does not change when sound waves pass through it. Newton told that heat which is produced in the compression region leaks to the surrounding, while the decrease in the temperature which occurs in the rarefaction region is compensated by the flow of heat from the surrounding.

In this way, the temperature of the medium [air] remains constant. So under such conditions, the elasticity " E " of air may be equal to the pressure of the air. i.e. $E = P$

Thus equation (1) becomes,

$$v = \sqrt{\frac{P}{\delta}} \quad (2)$$

To Show that, $E = P$:

Consider a layer of air whose volume is " V " and pressure is " P ". Let compression is produced in this layer due to which its volume decreases by " v " and pressure increases by " p ", so the final volume becomes " $V-v$ " and the final pressure becomes " $P+p$ ".

Now as the temperature was assumed to remain constant, so the process is isothermal for which we apply Boyle's law i.e., we have,

$$PV = (P+p)(V-v)$$

$$\Rightarrow PV = PV - Pv + pV - pv$$

$$\Rightarrow -Pv + pV - pv = 0 \Rightarrow -Pv + pV - 0 = 0$$

$$\Rightarrow -Pv = -pV$$

$$\Rightarrow P = \frac{pV}{v} = \frac{P}{(v/V)}$$

$$\Rightarrow P = \frac{P}{(v/V)} = \frac{\text{stress}}{\text{volumetric strain}} = \text{Elasticity} = E$$

$$\Rightarrow \boxed{P = E}$$

Which is the required proof.

Now to calculate the speed of sound in air, we put,

$$P = \delta'gh \quad (3)$$

In equation (3) " δ' " represents the density of standard fluid mercury and " h " represents height of mercury at atmospheric pressure.

Putting equation (3) in equation (2), we get,

$$v = \sqrt{\frac{\delta'gh}{\delta}} \quad (4)$$

Put $\delta' = 13.6 \text{ gm/cm}^3$, $g = 980 \text{ cm/sec}^2$, $h = 76 \text{ cm of Hg}$,

$\delta = \text{density of air} = 0.001293 \text{ gm/cm}^3$ in equation (4), we get,

$$v = \sqrt{\frac{13.6 \times 980 \times 76}{0.001293}} \Rightarrow v = 28000 \text{ cm/sec [approximately]}$$

$$\Rightarrow \boxed{v = 280 \text{ m/sec}}$$

But the above value is about 16% less than the experimental result which is about 332 m/sec. So we cannot ignore it as an experimental error. This error was then removed by Laplace and is known as Laplace correction.

Laplace Correction:

According to Laplace, Newton was wrong in assuming that the temperature of air does not change when sound waves pass through it. He told that sound waves consist of compression and rarefactions which are produced in air so rapidly, that the temperature of air does not remain constant i.e. the rise in temperature during compression cannot balance the fall in temperature during rarefaction. So Laplace assumes an adiabatic process instead of an isothermal process, for which he put, $E = \gamma P$.

Where " γ " is constant and its value depends upon the nature of the gas. For air, $\gamma = 1.42$.

To show that, $E = \gamma P$:

Consider a layer of air having pressure " P " and volume " V ". Let compression is produced in it, such that the pressure increases by a factor " p " and volume decreases by a factor " v ". Now we have,

$$PV' = (P + p)(V - v)$$

$$\Rightarrow PV' = (P + p)V' \left(1 - \frac{v}{V}\right)$$

$$\Rightarrow P = (P + p) \left[1 + \left(-\frac{v}{V}\right)\right] \quad \text{--- (5)}$$

Expanding the second term of R.H.S of equation (5), binomially, we get,

$$\left[1 + \left(-\frac{v}{V}\right)\right] = 1 + \gamma \left(-\frac{v}{V}\right) + \text{neglected terms}$$

$$\Rightarrow \left[1 + \left(-\frac{v}{V}\right)\right] = 1 - \frac{\gamma v}{V} \quad \text{--- (6)} \quad \left[\text{As } \frac{v}{V} \ll 1 \text{ so we neglect its high power terms} \right]$$

Putting equation (6) in equation (5), we get,

$$P = (P + p) \left(1 - \frac{\gamma v}{V}\right)$$

$$\Rightarrow P = P - \frac{\gamma P v}{V} + p - \frac{\gamma p v}{V}$$

$$\Rightarrow -\frac{\gamma P v}{V} + p - \frac{\gamma p v}{V} = 0$$

$$\Rightarrow -\frac{\gamma P v}{V} = -p$$

The term $\frac{\gamma p v}{V}$ is very very small, so we neglect it.

Binomial Formula:

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots$$

$$\Rightarrow \gamma P = \frac{P}{\left(\frac{v}{V}\right)} = \frac{\text{stress}}{\text{volumetric strain}} = \text{Elasticity} = E$$

$$\Rightarrow \gamma P = E \Rightarrow E = \gamma P \quad \text{--- (7)}$$

Putting equation (7) in equation (1), we get,

$$V = \sqrt{\frac{\gamma P}{\delta}}$$

$$P = \delta' gh$$

$$\Rightarrow V = \sqrt{\gamma \times \frac{\delta' gh}{\delta}} \Rightarrow V = \sqrt{\frac{1.42 \times 13.6 \times 980 \times 76}{0.001293}}$$

$$\Rightarrow V = 33353 \text{ cm/sec}$$

$$\Rightarrow V = 333.53 \text{ m/sec} \Rightarrow V = 333 \text{ m/sec}$$

This value agrees with experimental value which is 332 m/sec.

Q8: Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.

BISE Abbottabad 2019, BISE DI Khan 2019, BISE Mardan 2017

Ans: The speed of sound in a gas depends upon various factors which are given below:

1. Effect of Density:

We know that,

$$V = \sqrt{\frac{\gamma P}{\delta}} \quad \text{--- (1)}$$

Put $\gamma P = \text{constant}$ in equation (1), we get,

$$V \propto \sqrt{\frac{1}{\delta}} \quad \text{--- (2)}$$

Relation (2) shows that at constant temperature and pressure, the speed of sound in gases is inversely proportional to the square root of their density i.e. smaller the density of a gas, greater will be the speed of sound through it. As hydrogen is less denser than oxygen, so sound waves travel faster in hydrogen as compared to oxygen.

2. Effect of Moisture:

The speed of sound increases with the presence of moisture in the air. It is because the moisture reduces the resultant density of air. When the density decreases, the speed of sound will increase. Thus the velocity of sound in damp air is greater than its value in dry air.

3. Effect of Pressure: (ETEA 2007, 2011)

It has been found that the speed of air does not depend upon the

pressure of the air i.e. $V = \text{constant}$

Proof: We know that,

$$V = \sqrt{\frac{\gamma P}{\delta}} \quad (1)$$

We also know that,

$$\delta = \frac{m}{V} \quad (2)$$

Also $PV = nRT$ [Put $n = 1$]

$$\Rightarrow PV = RT \Rightarrow V = \frac{RT}{P} \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\delta = \frac{mP}{RT} \quad (4)$$

Putting equation (4) in equation (1), we get,

$$V = \sqrt{\gamma P \frac{RT}{mP}} = \sqrt{\frac{\gamma RT}{m}}$$

$$\Rightarrow V = \sqrt{\frac{\gamma RT}{m}} \quad (5)$$

Putting $\frac{\gamma PT}{m} = \text{constant}$, so equation (5) becomes,

$$V = \text{constant} \quad (6)$$

Equation (6) shows that the speed of sound is independent of the pressure of the air.

4. Effect of Temperature: (BISE Mardan 2019)

The speed of sound increases with increase in temperature. It has been found that the speed of sound in gas increases about 0.6m/sec for each 1°C rise in temperature.

We know that,

$$V = \sqrt{\frac{\gamma RT}{m}} \quad (1)$$

Put $\frac{\gamma R}{m} = \text{constant}$, so equation (1) becomes,

$$V = \text{constt.} \sqrt{T} \Rightarrow V \propto \sqrt{T} \quad (2)$$

Relation (2) shows that the speed of sound in a gas is directly proportional to the square root of the absolute temperature of the gas. i.e. when temperature of a gas increases, the speed of sound also increases and vice versa.

5. Effect of Wind:

If " V " is the velocity of sound and " V_w " is the velocity of wind, then if the wind is moving in the direction of sound waves, then the relative velocity becomes $(V + V_w)$. So relative speed of sound in such a case increases, and if the wind is moving in opposite direction of the sound waves, then the relative speed becomes $(V - V_w)$. So the relative speed of sound in such a case decreases.

Q9: How the speed of sound in the air varies with temperature and hence show that for each one degree centigrade rise in temperature the speed of sound increases by 0.61m/sec?

Ans: The speed of sound in air depends upon temperature of the air. If the temperature of air increases, then the velocity of speed of sound wave also increases. It has been found that for each one degree centigrade rise in temperature, the speed of sound increases by 0.61m/sec.

Proof:

Let " V_0 " represents the speed of sound at temperature " T_0 " and " V " represents the speed of sound at temperature " T ". Then we have,

$$\frac{V}{V_0} = \sqrt{\frac{T}{T_0}} \quad (1)$$

We have, $T_0 = (0^\circ\text{C} + 273) = 273\text{K}$ and $T = (t^\circ\text{C} + 273)\text{K}$

So equation (1) becomes,

$$\frac{V}{V_0} \sqrt{\frac{(t^\circ\text{C} + 273)\text{K}}{273\text{K}}} = \sqrt{\frac{t^\circ\text{C} + 273}{273}}$$

$$\Rightarrow \frac{V}{V_0} = \sqrt{\frac{t^\circ\text{C}}{273} + 1} = \left(1 + \frac{t^\circ\text{C}}{273}\right)^{1/2}$$

$$\Rightarrow \frac{V}{V_0} = \left(1 + \frac{t^\circ\text{C}}{273}\right)^{1/2} \quad (2)$$

[Binomial formula :

$$(1 + x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots]$$

Expanding the R.H.S of equation (2) binomially and neglecting the higher power terms, we get,

$$\left(1 + \frac{t^\circ\text{C}}{273}\right)^{1/2} = 1 + \frac{1}{2} \left(\frac{t^\circ\text{C}}{273}\right) + \dots$$

$$\Rightarrow \left(1 + \frac{t^{\circ}\text{C}}{273}\right)^{\frac{1}{2}} = 1 + \frac{t^{\circ}\text{C}}{546}$$

So equation (2) becomes,

$$\frac{V}{V_0} = \left(1 + \frac{t^{\circ}\text{C}}{546}\right) \Rightarrow V = V_0 + \frac{V_0 t^{\circ}\text{C}}{546} \quad (3)$$

At 0°C , we have, $V_0 = 332\text{m/sec}$

Put $V_0 = 332\text{m/sec}$ in 2nd term of R.H.S of equation (3), we get,

$$V = V_0 + \frac{332 t^{\circ}\text{C}}{546} \Rightarrow V = V_0 + 0.61 t^{\circ}\text{C} \quad (4)$$

Equation (4) shows that the speed of sound increases by 0.61m/sec for each one degree centigrade rise in temperature.

Q10: State and explain the principle of superposition of waves.

Ans. Statement:

It states that whenever two waves are travelling in the same region, the total displacement at any point is equal to the vector sum of their individual displacements at that point.

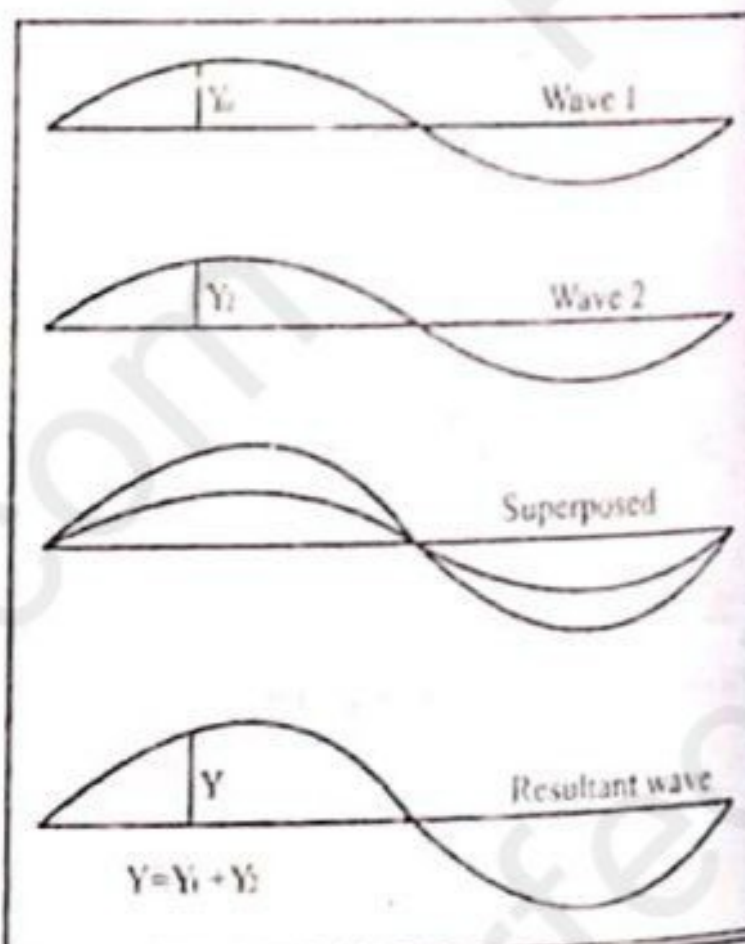
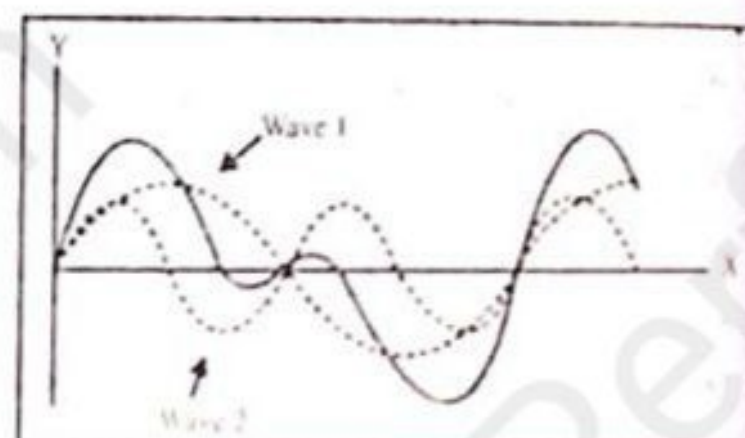
Explanation:

When two waves travelling in the same direction are superposed, the shape of the string at any instant can be determined by applying the principle of superposition.

In the given figure, the displacement " y_1 " arising from the wave "1" are given by the dotted curve. While the displacement " y_2 " arising from the wave "2" are given by the broken curve. The resultant displacements are given by the solid curve which is obtained by adding the displacement " y_1 " and " y_2 " for every point along the string i.e.

$$y = y_1 + y_2$$

The resultant wave will travel along the string with same speed as that



of component waves. The principle holds for mechanical wave and all electromagnetic waves.

The phenomena of interference, diffraction, beats and stationary waves are the consequences of superposition of waves.

1. If the two waves possess same frequency and same phase, then phenomenon of interference will take place.
2. If the two waves possess slightly different frequencies and are travelling in same direction, when phenomenon of beats will take place.
3. If the two waves of same frequency are travelling in opposite directions, then standing or stationary waves will be produced.

Q11: (a) What is meant by the interference of waves?

(b) Discuss the types of interference.

(c) Discuss the conditions for interference.

Ans. (a) Interference of Waves:

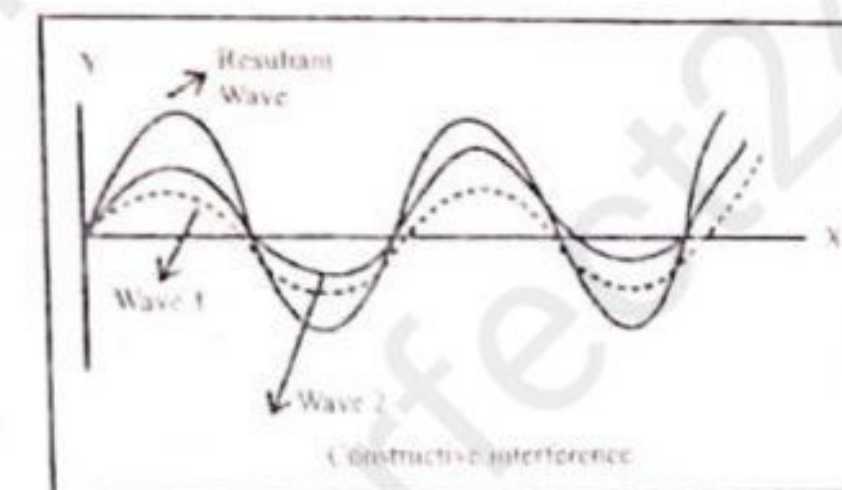
Definition: The superposition of two waves coming from two coherent sources at the same time through the same medium having the same frequency and amplitude is known as interference of waves.

(b) Types of Interference:

There are two types of interference, which are given below:

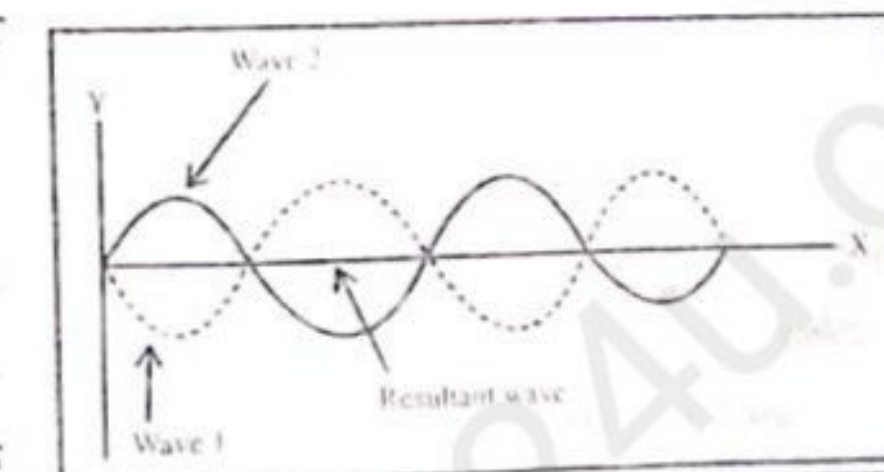
1. Constructive Interference:

When two waves arrive at a certain point in the same phase i.e. when the two crests and two troughs falls upon each other, they reinforce each other. As a result, the intensity of waves increases and a resulting wave with large amplitude is produced. In the given figure, wave 1 and wave 2 interfere each other constructively due to which a resultant wave of large amplitude is produced.



2. Destructive Interference:

When two waves arrive at a certain point in opposite phase i.e. when the crest of one wave falls



upon the trough of other wave, then they are said to be interfere destructively. In such a case, they cancel the effect of each other and the resultant intensity becomes minimum or zero. In given figure, wave 1 and wave 2 interfere destructively and as a result we get a resultant wave with zero amplitude.

(c) Conditions for Interference:

For interference, the following conditions must be satisfied.

1. The two waves must be phase coherent.
2. They must arrive at the same place at the same time.
3. The two waves must be travelling in the same direction.
4. The principle of linear superposition must be satisfied.
5. The constructive interference takes place only, if the path difference is zero i.e. an integral multiple of " λ ". i.e.
Path difference = $m\lambda$.

(OR) Path difference = $0\lambda, 2\lambda, 3\lambda, \dots, m\lambda$.

[Where $m = 0, 1, 2, 3, 4, \dots$]

6. The destructive interference takes place only, if the path difference between the two waves is an odd multiple of half of the wavelength i.e.

$$\text{Path difference} = m \frac{\lambda}{2}$$

(OR) $p.d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \dots, \left(m + \frac{1}{2}\right)\lambda$

[Where $m = 0, 1, 2, 3, 4, \dots$]

Q12: Explain the interference of sound waves. Also discuss its experimental verification.

Ans. The effect produced by the superposition of sound waves from two coherent sources, passing through the same region is known as interference of sound waves.

If the phase of the two waves is the same, i.e. when two compressions and two rarefactions falls upon each other, they reinforce each other and constructive interference will take place. The intensity and amplitude of the resultant wave increases and we hear a loud sound.

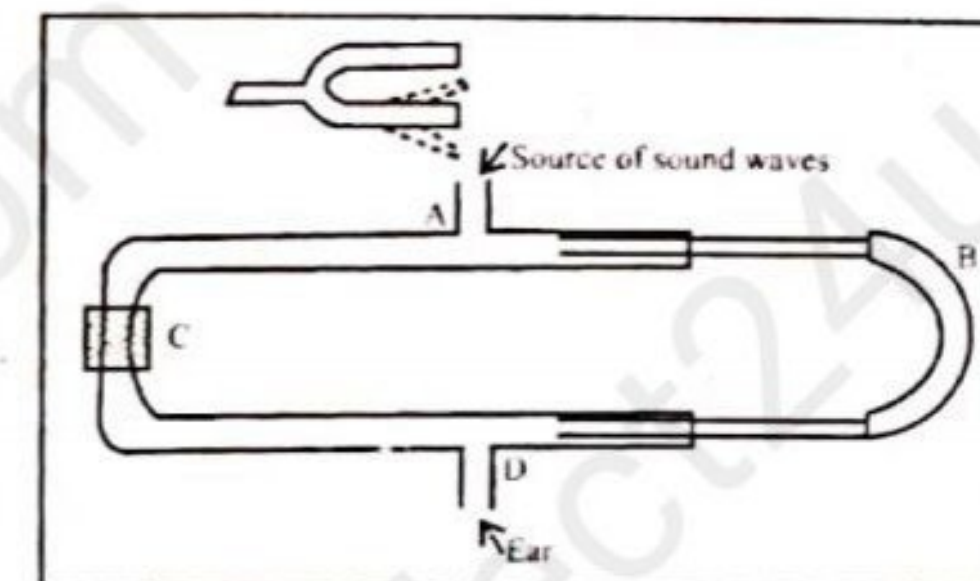
If the phase of the two sound waves is not the same, then the compression of one wave falls upon the rarefaction of the other

wave, then they cancel the effect of each other due to which the amplitude of the resultant wave becomes zero and thus we hear no sound. Such type of interference is known as destructive interference.

Experimental Verification of Interference of Sound Waves:

The interference of sound waves can be verified experimentally by using a U-shaped tube apparatus as shown in the figure.

The sound waves are produced with the help of tuning fork at "A". These waves reach the ear which is kept near at point "D" by travelling along the path "ACD" and "ABD", as shown in the figure.



Now if the lengths of the two paths are the same, then the waves reaching at "D" along the two paths will interfere constructively and a loud sound will be heard.

Now if the U-shaped tube is moved towards right such that the length of path "ABD" increases gradually, then a stage comes when the ear will hear no sound because at this stage destructive interference takes place at point "D". This happens when the length of the path "ABD" is greater than the length of the path "ACD" by " $\frac{\lambda}{2}$ ".

Now if the pinch cork at "C" is closed, then the ear will hear the sound which is due to the waves coming to the point "D" along the path "ABD" only.

Q13: What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources?

Ans. Beats: The periodic variation in the loudness of sound between maximum and minimum due to superposition of two waves of slightly different frequencies is known as beats. (OR)
The rise and fall in the intensity of sound waves per second is called beats.

Explanation:

One maximum and one minimum form one beat. The number of beats produced in a unit time is equal to the difference between the frequencies of the two sources.

For beats to be heard clearly, the difference between the frequencies should be small. The difference between beats and interference is that, in case of interference two tuning forks of equal frequencies are used, while in case of beats phenomenon, two tuning forks of slightly different frequencies are used.

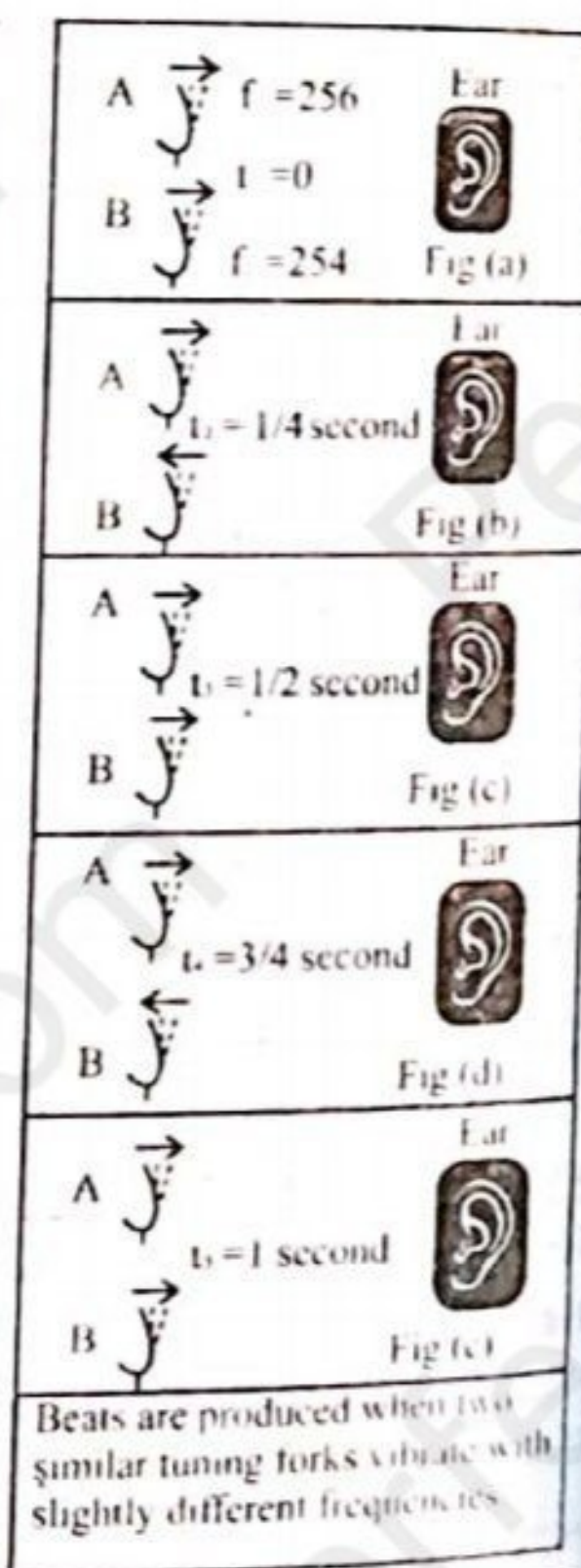
Experimental Demonstration of Beats:

Consider two tuning forks "A" and "B" having frequencies of 256 Hz and 254 Hz respectively. Both the tuning forks are placed at the same distance from ear, as shown in the figure. Let the two tuning forks are sounded together by striking them on a rubber pad.

Case-1: At $t = 0$, both the tuning forks send compression towards right as shown in figure (a). These compressions reinforce each other and as a result, a loud sound will be heard.

Case-2: At $t = \frac{1}{4}$ sec, the tuning fork "A" will send compression while the tuning fork "B" will send rarefaction, because at $t = \frac{1}{4}$ sec, tuning fork "A" will complete 64 vibrations while the tuning fork "B" complete 63.5 vibrations. The compression and rarefaction cancel the effect of each other and thus no sound is heard. Case number 2 is shown in figure (b).

Case-3: At $t = \frac{1}{2}$ sec, the tuning fork "A" complete 128 vibrations and thus it will send compression towards right. While the tuning fork "B" complete 127 vibrations. So it will also send compression towards



right. Both the compressions reinforce each other and as a result a loud sound is produced. Case number 3 is shown in figure (c).

Case-4: At $t = \frac{3}{4}$ sec, the fork "A" complete 192 vibrations and thus it will send compression towards right. While the tuning fork "B" complete 190.5 vibrations and thus it will send rarefaction towards right. The compression and rarefaction cancel the effect of each other and thus no sound will be heard. Case number 4 is shown in figure (d).

Case-5: At $t = 1$ sec, the tuning fork "A" will complete 256 vibrations and thus it will send compression towards right. While the tuning fork "B" will complete 254 vibrations and thus it will also send compression towards right. Both the compressions reinforced each other and as a result a loud sound is produced. Case number 5 is shown in figure (e).

Beats Frequency:

The difference between the frequencies of the two waves is called beat frequency. It is represented by "N" and is given by,

$$N = f_1 - f_2 \quad (1)$$

Now if "T" is the time interval between two successive loud sounds, then the number of oscillation of the first wave will be equal to " $f_1 T$ " and that of the second wave will be equal to " $f_2 T$ ". But the first wave should have made one oscillation more than the second one. So we have,

$$f_1 T - f_2 T = 1$$

$$\Rightarrow (f_1 - f_2) T = 1 \Rightarrow f_1 - f_2 = \frac{1}{T} \quad (2)$$

Comparing equations (1), (2), we get,

$$N = f_1 - f_2 = \frac{1}{T}$$

Uses of Beats:

1. We can use beats to tune a string instruments such as piano or violin by beating a note [sound] against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beat are heard.
2. Beats phenomenon is used for finding the unknown frequency.

3. Beats are used to produce variety in music.

Q14: Discuss the reflection of waves and phase changes that occurs during reflection.

Ans: Reflection of Waves:

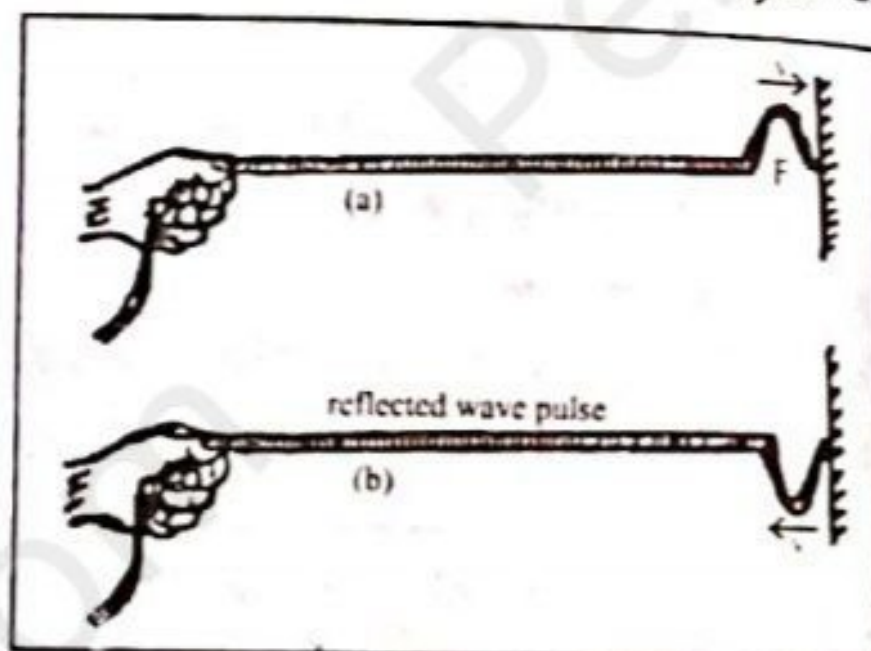
Definition: The bouncing back of waves from the boundary of a certain medium is known as reflection of waves.

(a) Reflection of Mechanical Waves:

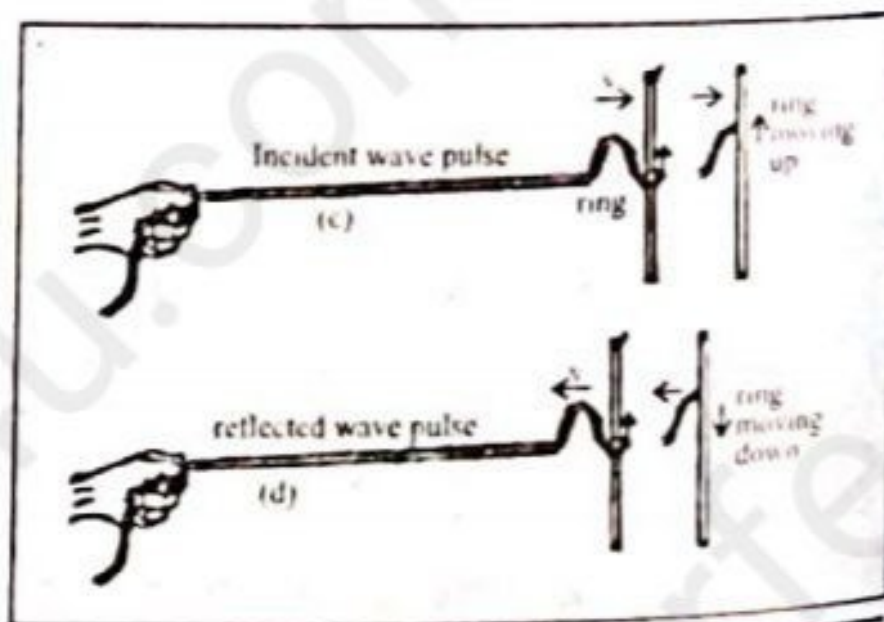
Consider a string whose one end is free and the other end is connected to a fixed support [wall] as shown in the figure.

If we hold the other end of the string in hand and move it up and down, then a crest [an upward pulse] is sent towards right as shown in the figure.

When the crest strike the fixed support [i.e. wall → denser medium], some part of its energy is absorbed and the rest is reflected. As the wall does not move up with the crest in same way as it pulls the string upwards, so the wall exert downward pull on the string. As a result the crest is reflected in the form of trough as shown in figure (b). In this way, a phase change of 180° or π radian occurs which is equal to half of the wavelength i.e. " $\frac{\lambda}{2}$ " between the incident crest and reflected trough.



On the other hand, if the end of the string is connected to a ring [rare medium], then when the crest strike the ring moves up and down and reflect the crest as crest, as shown in the figure. Thus there is no phase change in this case.



From the above discussion it is clear that when a transverse wave on a string is reflected from a denser medium, there is

a phase change of 180° or π radian. While when a transverse wave on a string is reflected from the boundary of a rare medium, it suffers no phase change. The same principle is applicable for longitudinal waves.

(b) Reflection of Electromagnetic Waves:

The phase change also occurs when electromagnetic waves such as light waves are reflected from the boundary of a denser medium. Various techniques have been developed for locating the position of objects by reflecting the waves of known speed from them as radar waves.

Q15: Discuss the reflection of sound waves.

Ans: Reflection of Sound Waves:

Definition: The bouncing back of sound waves from certain surface is known as reflection of sound waves.

Explanation:

The sound waves can be reflected from large flat and hard surfaces, such as wall, cliffs, clouds, ground etc. Like other waves, the sound waves also obey the laws of reflection.

Applications of Reflection of Sound Waves:

1. The reflection of sound waves can be used to determine the depth of the sea. For this purpose a sound pulse having frequency greater than 20KHz is sent out under water from a ship. When the sound waves reach the bottom of the sea, they reflect and moves back towards the receiver fitted on the ship. When the receiver detects the sound, the time interval " T " is noted. Now we take the half of " T " [i.e. $t = \frac{T}{2}$] which gives us the time taken by the sound waves in reaching the bottom of the sea. Now we use the following formula for determining the depth of the sea i.e.

$$S = vt$$

The velocity of sound in sea water is 1450m/sec.

2. The reflection of sound gives us information about an obstacle in the path of various insects and birds. When an insect travels in a dark night, it emits ultrasonic waves whose frequency is greater than 20KHz. These waves travel faster ahead the in-

sect and strike any obstacle in its way, they reflect and travels back along the same path. The insect detect the reflected sound and thus change their direction. In this way, the insect continue their journey in safe and sound conditions.

3. The reflection of sound is used in defence and detection departments for various purposes.
4. The reflection of sound waves gives us information about echo, reverberation etc.

Q16: Write a note on, (a) Echo, (b) Reverberation

Ans. (a) Echo:

The reflection of an original sound from a certain object is received at 0.1sec later than the direct sound is known as an echo.

Explanation:

Echo is a factor due to which we cannot hear the sound clearly. This phenomenon occurs in those halls, which having large size and smooth walls.

To avoid or minimize echo, the dimension and size of the hall should not be too large. Also the distance between source of sound and the wall of hall should be less than 17m. It is because, if the distance is greater than 17m, then the time intervals will be more than 0.1 seconds and under such conditions, our ear will hear two separate sounds. To avoid such two separate sounds, the time intervals should be less than 0.1 seconds.

(b) Reverberation:

It is that effect due to which we cannot distinguish between the original sound and reflecting sound and thus causes a general confusion of the sound impression on the air.

Q17: Write a detail note on stationary waves or standing waves.

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Ans. Standing Waves [Stationary Waves]:

When two waves of same frequency, period and amplitude are travelling along the same line in opposite direction and superimpose each other, then standing or stationary waves are produced.

Explanation:

Consider an elastic cord whose one end is fixed and the other is held

The speed of sound in air is mostly taken as 340m/s, so the effective distance for echo is $= \frac{340 \times 0.1}{2} = 17\text{m}$

in hand as shown in the figure.

Now if we wiggle the cord up and down then waves are set up in the cord. These waves travel towards the fixed end of the cord, suffers reflections and travels back towards the hand. The incoming waves and reflected waves overlap each other and thus form a system of standing waves.

There are certain points on stationary waves at which the amplitude of vibration is zero and tension is maximum. Such points are known as nodes. In given figure, N_1, N_2, N_3 etc represents the nodes. While the points where the amplitude of vibration is maximum and the tension is minimum, are known as antinodes. In the given figure, A_1, A_2 etc represents the antinodes.

The node and antinodes are produced in such a manner that there is one node between two antinodes and there is one antinode between two nodes.

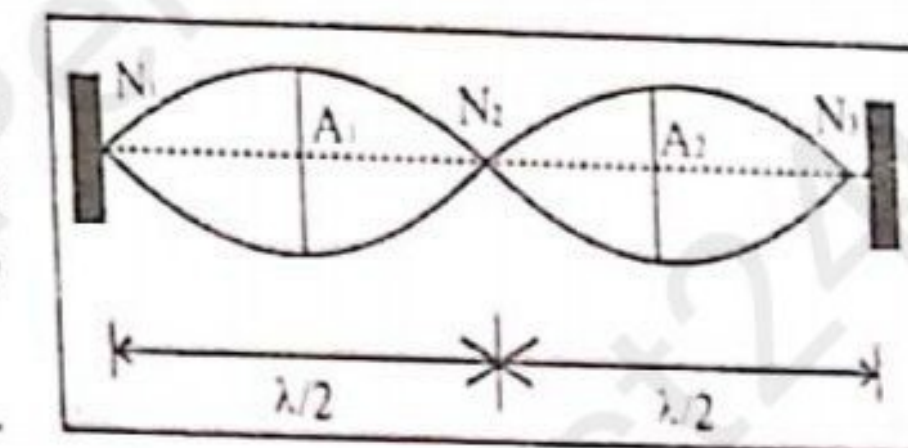
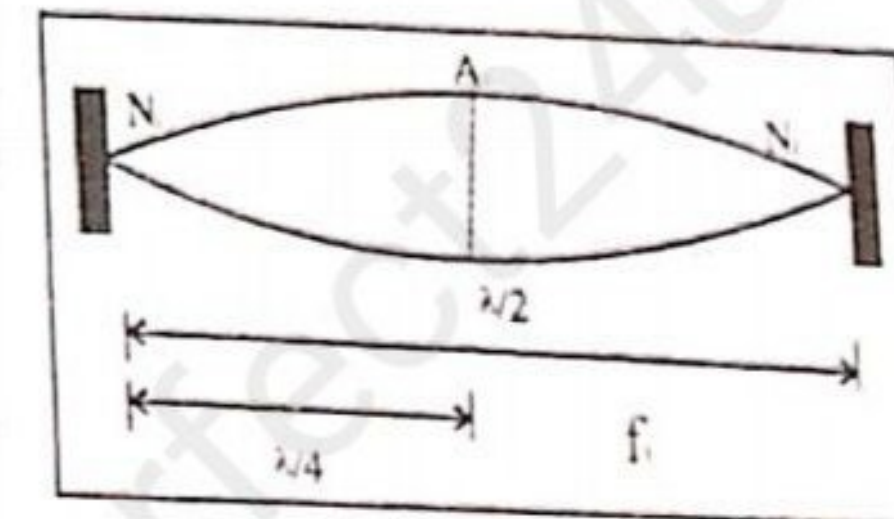
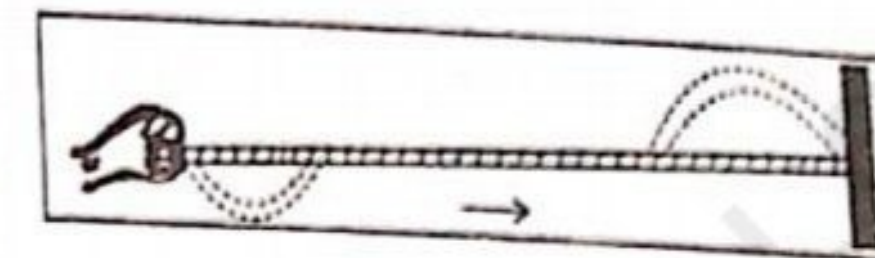
The distance between two consecutive nodes or two consecutive antinodes is equal to half of the wavelength i.e. $\frac{\lambda}{2}$. While the distance between a node and its neighboring antinode is equal to one fourth of the wavelength i.e. $\frac{\lambda}{4}$. In standing waves, the energy is not transferred from one particle to another.

When the cord vibrates in one loop, then the frequency is called 1st harmonic. It is represented by " f_1 ".

In case of two loops, the frequency is called second harmonic. It is represented by " f_2 " and is given by,

$$f_2 = 2f_1$$

Similarly, $f_3 = 3f_1, f_4 = 4f_1, f_5 = 5f_1, f_6 = 6f_1, \dots, f_n = nf_1$



Thus we can say that the frequencies with which the string can vibrate are quantized. That is the string can vibrate with discrete set of frequencies i.e. $f_1, 2f_1, 3f_1, 4f_1, \dots, nf_1$.

Q18: Discuss the formation of transverse stationary waves in a stretched string. (OR) What do you mean by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decrease.

Ans. Stationary Waves:

When two waves of same frequency, period, amplitude and velocity are travelling along the same line in opposite direction and overlap each other, then standing or stationary waves are produced.

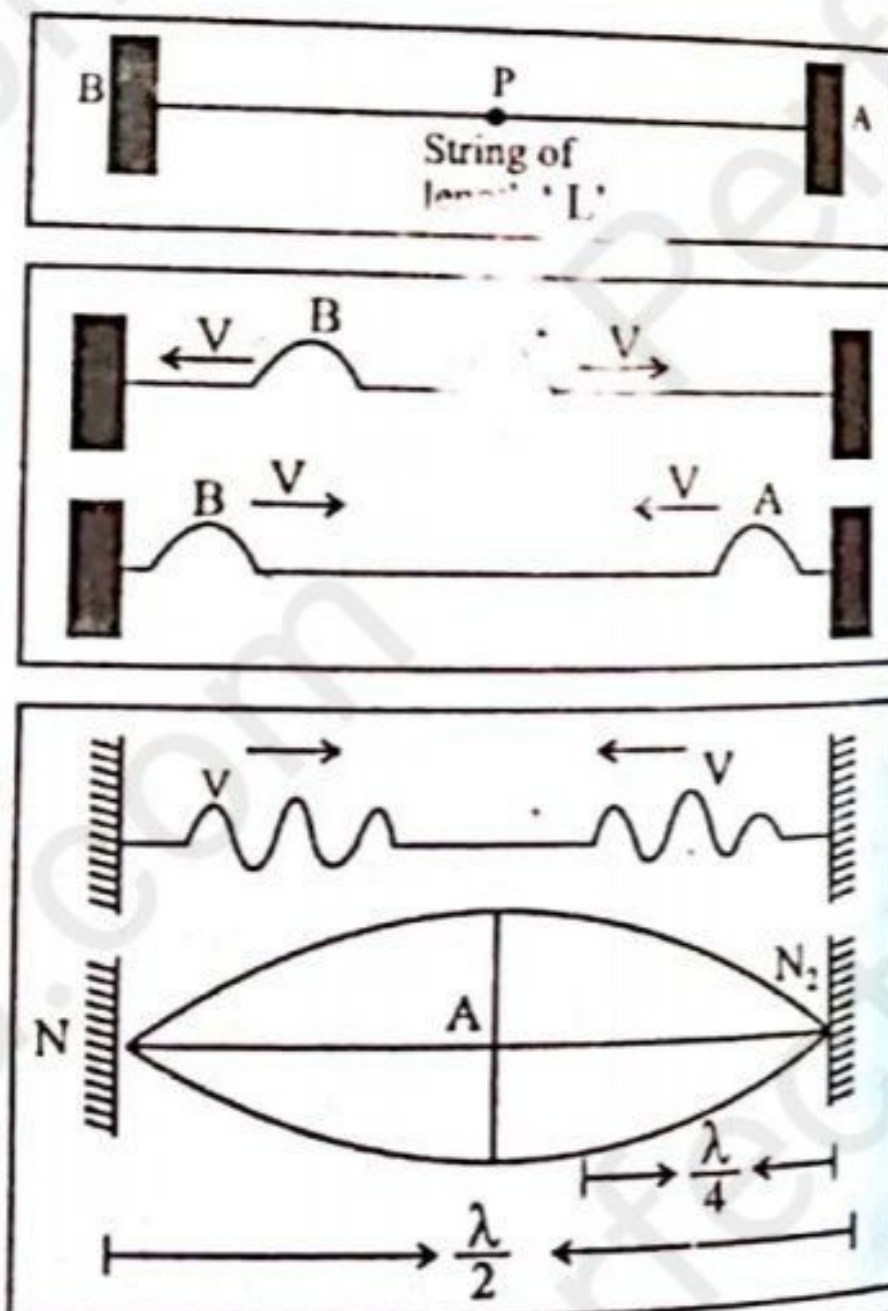
Transverse Stationary Waves in a Stretched String:

A standing wave obtained due to the superposition of transverse waves is known as transverse stationary wave.

Now consider a string of length " L " stretched between two supports " A " and " B " as shown in the figure. This string can be made to vibrate in any number of loops.

Case-1: Vibration of String in one Loop:

In this case, the string is made to vibrate in one loop. For this purpose, the string is plucked at its mid point " P " due to which two transverse waves originates from point " P ". One of these waves move towards the right end and the other move towards the left end. When these waves reach the fixed ends, they are reflected back. They meet at the middle where they superpose each other and as a result a stationary wave is setup. The whole string will vibrate in one loop, with two nodes and one antinode, as shown in the figure.



Now in case of one loop, the frequency is called fundamental frequency or first harmonic. It is represented by " f_1 " and is given by,

$$f_1 = \frac{V}{\lambda_1} \quad (1)$$

Now for one loop, we have,

$$L = \frac{\lambda_1}{2} = \frac{\lambda_1}{2}$$

$$\Rightarrow \lambda_1 = 2L \quad (2)$$

Putting equation (2) in equation (1), we get,

$$f_1 = \frac{V}{2L} \quad (3)$$

Equation (3) represents the lowest frequency which is called fundamental frequency or first harmonic.

Now we know that the velocity of wave in a stretched string in terms of tension " T " and mass per unit length " m " is given by,

$$V = \sqrt{\frac{T}{m}} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$f_1 = \frac{1}{2L} \sqrt{\frac{T}{m}} \quad (5)$$

Equation (5) represents the fundamental frequency or first harmonic with which the string vibrates in one loop.

Case-2: Vibration of String in two Loops:

In this case, the same string of length " L " is made to vibrate in two loops. For this purpose, the string is plucked from one quarter of its length and as a result the string vibrates with frequency " f_2 " in two loops as shown in the figure. Here we have,

$$f_2 = \frac{V}{\lambda_2} \quad (6)$$

In case of two loops, we see that,

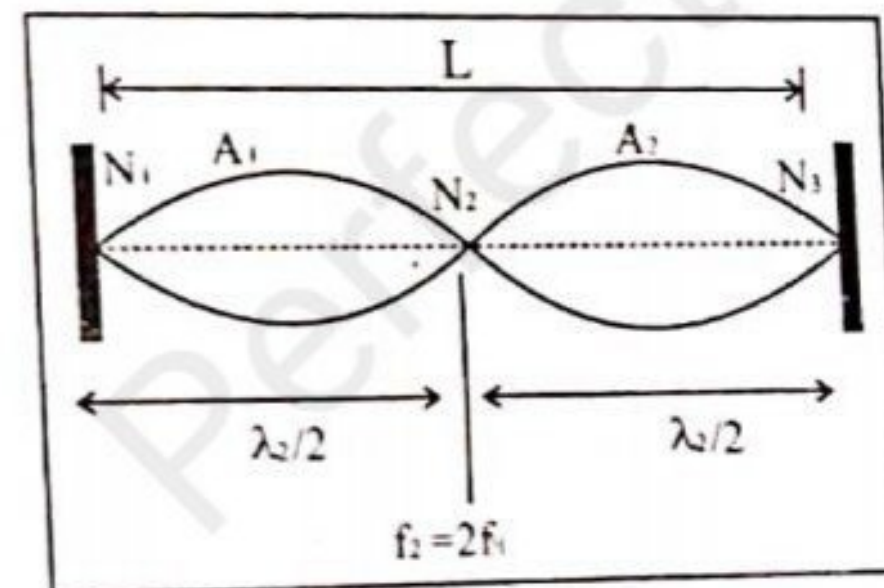
$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{2\lambda_2}{2}$$

$$\Rightarrow \lambda_2 = \frac{2L}{2} \quad (7)$$

$$\text{As } V = f \lambda$$

$$\Rightarrow f = \frac{V}{\lambda}$$

$$\Rightarrow f = \frac{V}{\lambda_1}$$



Putting equation (7) in equation (6), we get,

$$f_2 = \frac{v}{\left(\frac{2L}{2}\right)} = 2 \frac{v}{2L}$$

$$\Rightarrow f_2 = 2 \left(\frac{v}{2L}\right) \text{---(8)}$$

Putting equation (3) in equation (8), we get,

$$f_2 = 2f_1 \text{---(9)}$$

In equation (9) " f_2 " is called second harmonic.

Case-3: Vibration of String in three Loops:

In this case the same string is made to vibrate in three loops. For this purpose, the string is plucked at one sixth of its length and as a result the string vibrates with frequency " f_3 " in three loops, as shown in the figure.

Here we have,

$$f_3 = \frac{v}{\lambda_3} \text{---(10)}$$

In case of three loops, we have,

$$L = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} \Rightarrow L = \frac{3\lambda_3}{2}$$

$$\Rightarrow \lambda_3 = \frac{2L}{3} \text{---(11)}$$

Putting equation (11) in equation (10), we get,

$$f_3 = \frac{v}{\left(\frac{2L}{3}\right)} = 3 \frac{v}{2L} \Rightarrow f_3 = 3 \left(\frac{v}{2L}\right) \text{---(12)}$$

Putting equation (3) in equation (12), we get,

$$f_3 = 3f_1 \text{---(13)}$$

Similarly, in case of 4 loops, we have,

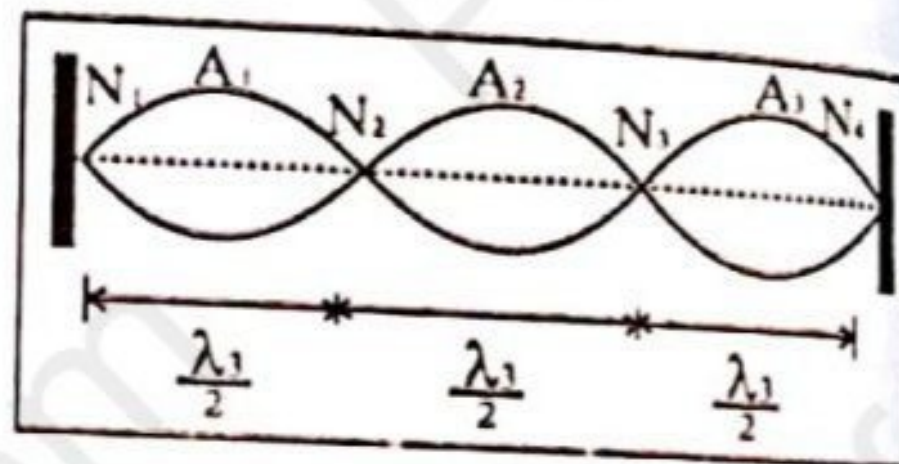
$$f_4 = 4f_1 \text{---(14)}$$

...

And in case of N-number of loops, we have,

$$f_n = nf_1$$

Conclusion: From above discussion it is clear that the frequencies with which the string can vibrate are quantized. That is the string



can vibrate with discrete set of frequencies i.e.
 $1f_1, 2f_1, 3f_1, 4f_1, 5f_1, 6f_1, \dots, nf_1$

Now comparing equations (2), (7), (11) etc, we see that $\lambda_3 < \lambda_2 < \lambda_1$

This shows that the wavelength decreases as the number of loops increases. Now comparing equations (9), (13), (14) etc we see that $f_4 > f_3 > f_2 > f_1$

This shows that, the frequency of vibrating string increases as the number of loop increases.

Q19: Explain the following: (a) Fundamental frequency, (b) Overtones, (c) Quantization of frequencies

Ans: (a) Fundamental Frequency or First Harmonic:

We know that a string fixed firmly at its two ends resonates to only certain very special frequencies i.e. $f_1, f_2, \dots, f_n$. The lowest characteristics frequency of vibration f_1 is called the fundamental frequency or first harmonic.

(b) Overtones or Harmonics:

The other possible modes of vibration whose frequencies are all integral multiple of a lowest frequency are called overtones or harmonics.

When a stretched string is excited by a small periodic force having a frequency equal to any of the quantized frequencies of the string, the phenomena of resonance will take place and stationary wave will be setup on the string.

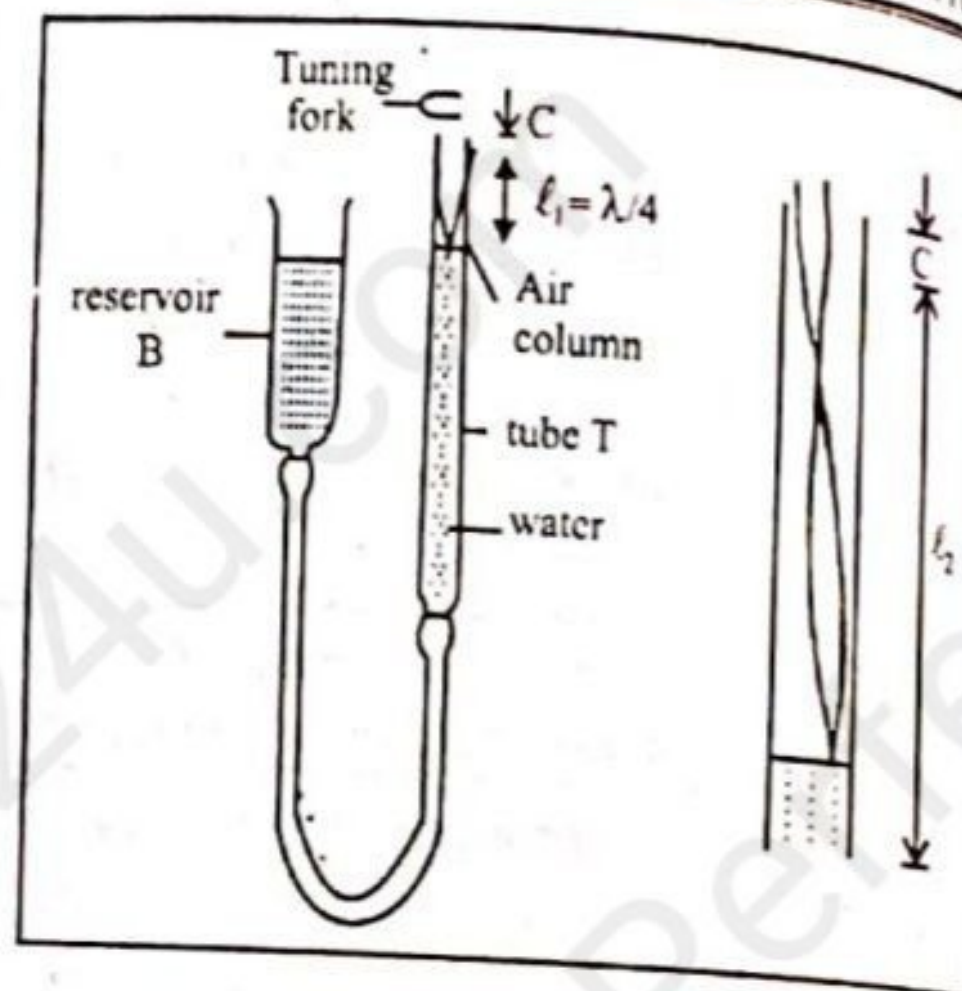
(c) Quantization of Frequencies:

It is observed that stationary waves on the string can be setup only with a discrete set of frequencies $f_1, f_2, f_3, \dots$. This shows that the resonant frequencies of the string are quantized, meaning that they are separated by frequency gaps. In other words quantum jumps in frequency exist between the resonance frequencies. This phenomenon is known as the quantization of frequency.

Q20: Discuss how stationary waves are produced in the air column?

Ans: The stationary wave can be also setup in the air column. In the given arrangement, a glass tube filled with water is connected to a reservoir.

Now if we bring a vibrating tuning fork over the open end of the glass tube and at the same time we slowly lower the water surface in the tube with the help of reservoir. During this process, a stage comes when at a certain height of water level, the air column in the tube will resonate loudly due to sound sent into it by the tuning fork.



The resonance occurs when the frequency " f " of the periodic force due to tuning fork becomes equal to the fundamental frequency " f_0 " of the air column. The resonance can occur several times by changing the height of water in the tube.

In above process, the waves produced in the air column are reflected from the surface of water. The reflected waves and incoming waves superpose each other and as a result standing or stationary waves are produced. The antinode is formed at open end of the tube while the node is formed at the other end.

Note: Using the resonance tube apparatus, we can also calculate the speed of sound through air column. For this purpose we consider the distance between node and antinode which is equal to " $\frac{\lambda}{4}$ ". This distance represents the length " l " of air column in the tube. So we have,

$$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l \quad (1)$$

We also know that, $v = f\lambda$ (2)

Putting equation (1) in equation (2), we get,

$$v = 4fl \quad (3)$$

In equation (3) f = frequency of air column = frequency of tuning fork and l = length of air column, which can be noted from graduated glass tube.

Putting the values of " f " and " l " in equation (3), we can calculate the speed of sound in air.

Q21: (i) What are organ pipes? Discuss the formation of stationary waves in an organ pipe. (ii) Also show that an open organ pipe is richer in harmonics than a closed organ pipe.

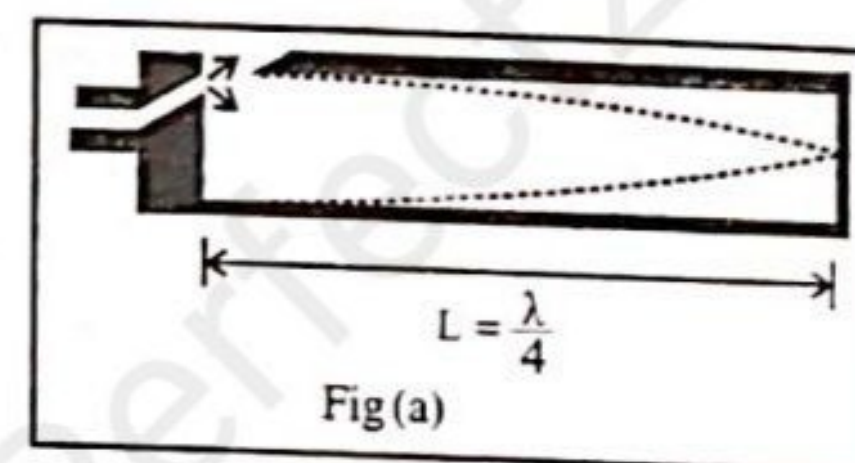
Ans: (i) Organ Pipes:

An organ pipe is the simplest example of an instrument which produces sound by means of vibrating air column. The organ pipes are of two types i.e.

- > Closed organ pipe
- > Open organ pipe

(a) Closed Organ Pipes:

A closed organ pipe is that which is closed at one end and open at the other end. When a jet of air is sent through the open end inside the pipe, then the columns of air are set into vibrations. These waves are reflected at the closed end and forms a system of standing waves. There is an antinode at open end and node at the closed end.



Case-1: In first case, a half loop is formed as shown in figure (a). We know that the distance between a node and its neighboring antinode is equal to " $\frac{\lambda}{4}$ " so we have,

$$L = \frac{\lambda}{4} \Rightarrow \lambda = 4L$$

$$\lambda_1 = 4L \quad (1)$$

Now the frequency in this case is given by,

$$f_1 = \frac{v}{\lambda_1} \quad (2)$$

Putting equation (1) in equation (2), we get,

$$f_1 = \frac{v}{4L} \quad (3)$$

Equation (3) represents the fundamental frequency or first harmonic.

Case-2: In second case one complete loop and one half loop is formed, as shown in figure (b). In this case, we have,

$$L = \frac{\lambda}{2} + \frac{\lambda}{4} \Rightarrow L = \frac{2\lambda + \lambda}{4}$$

$$\Rightarrow L = \frac{3\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{3} \Rightarrow \lambda_2 = \frac{4L}{3} \quad (4)$$

Now the frequency in this case is given by,

$$f_2 = \frac{V}{\lambda_2} \quad (5)$$

Putting equation (4) in equation (5), we get,

$$f_2 = \frac{V}{\left(\frac{4L}{3}\right)} = 3 \frac{V}{4L} \Rightarrow f_2 = 3 \left(\frac{V}{4L}\right) \quad (6)$$

Putting equation (3) in equation (3), we get,

$$f_2 = 3f_1 \quad (7)$$

Equation (7) represents the second harmonic or first overtone.

Case-3: In third case, two complete loops and one half loop is formed, as shown in figure (c). In this case, we have,

$$L = \frac{\lambda + 2\lambda + 2\lambda}{4}$$

$$\Rightarrow L = \frac{\lambda + 2\lambda + 2\lambda}{4} \Rightarrow L = \frac{5\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{5}$$

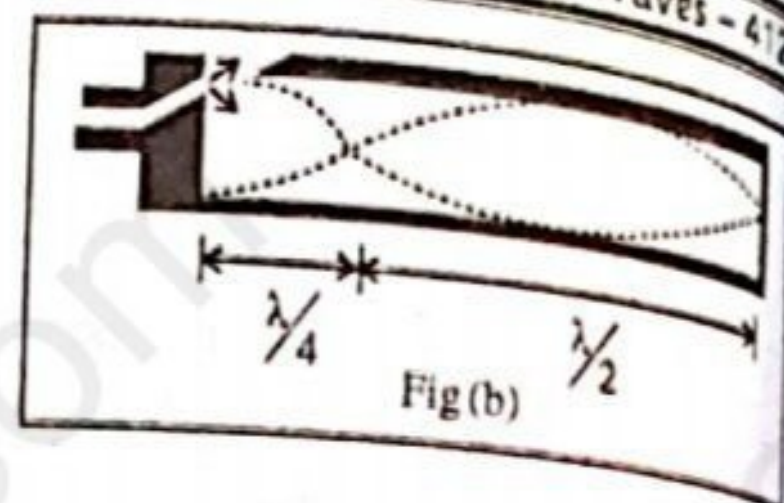
$$\Rightarrow \lambda_3 = \frac{4L}{5} \quad (8)$$

Now the frequency in this case is given by,

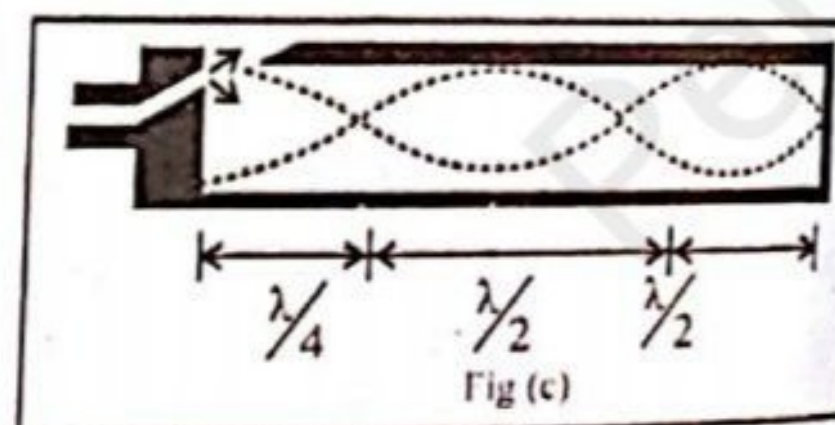
$$f_3 = \frac{V}{\lambda_3} \quad (9)$$

Putting equation (8) in equation (9), we get,

$$f_3 = \frac{V}{\left(\frac{4L}{5}\right)} = 5 \frac{V}{4L} \Rightarrow f_3 = 5 \left(\frac{V}{4L}\right) \quad (10)$$



For 2nd case, we put $\lambda = \lambda_2$



For 3rd case, we put $\lambda = \lambda_3$

Putting equation (3) in equation (10), we get,

$$f_3 = 5f_1 \quad (11)$$

$$\text{Similarly, } f_4 = 4f_1 \quad (12)$$

$$\text{And } f_n = (2n - 1)f_1 \quad (A)$$

As $f_1 = \frac{V}{4L}$ so equation (A) can be also written as,

$$f_n = (2n - 1) \frac{V}{4L} \quad (B)$$

Thus the possible frequencies are,

$$\frac{V}{4L}, 3 \frac{V}{4L}, 5 \frac{V}{4L}, \dots, (2n - 1) \frac{V}{4L}$$

(b) Open Organ Pipes:

(ETE 2009, 2013)

It is that pipe which is open at both ends. In such a case, antinodes will be formed at both ends.

Case-1: In this case, the air column

in the pipe oscillates for stationary wave and form certain loop with antinodes at both ends, as shown in figure (d). In this case, we have,

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = 2 \frac{\lambda}{4}$$

$$\Rightarrow L = \frac{\lambda}{2} \Rightarrow \lambda_1 = 2L \quad (1)$$

The frequency in this case is given by,

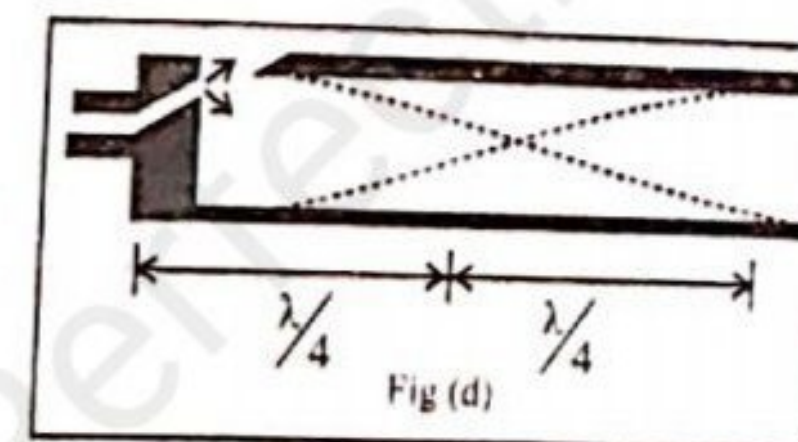
$$f_1 = \frac{V}{\lambda_1} \quad (2)$$

Putting equation (1) in equation (2), we get,

$$f_1 = \frac{V}{2L} \quad (3)$$

Equation (3) represents the first harmonic or fundamental frequency.

Case-2: In this case, the air column in the pipe oscillate for stationary waves and form certain loops, as shown in figure (e). In this case, we have,



$$L = \frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{4}$$

$$\Rightarrow L = \frac{(\lambda + 2\lambda + \lambda)}{4} = \frac{4\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{4}$$

$$\Rightarrow \lambda_2 = \frac{4L}{4} \quad (4)$$

Now the frequency in this case is given by,

$$f_2 = \frac{V}{\lambda_2} \quad (5)$$

Putting equation (4) in equation (5), we get,

$$f_2 = \frac{V}{\left(\frac{4L}{4}\right)} = 4 \left(\frac{V}{4L}\right) = 2 \left(\frac{V}{2L}\right)$$

$$\Rightarrow f_2 = 2 \left(\frac{V}{2L}\right) \quad (6)$$

Putting equation (3) in equation (6), we get,

$$f_2 = 2f_1 \quad (7)$$

Equation (7) represents the second harmonic or first overtone.

Case-3: In this case, the air column in the pipe oscillate for stationary waves and form certain loops as shown in figure (f). In this case, we have,

$$L = \frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4}$$

$$\Rightarrow L = \frac{(\lambda + 2\lambda + 2\lambda + \lambda)}{4} = \frac{6\lambda}{4} = \frac{3\lambda}{2} \Rightarrow L = \frac{3\lambda}{2}$$

$$\Rightarrow \lambda = \frac{2L}{3} \Rightarrow \lambda_3 = \frac{2L}{3} \quad (8)$$

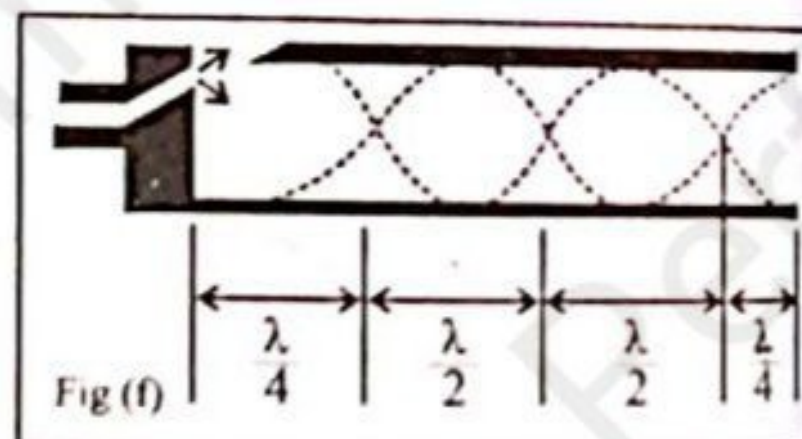
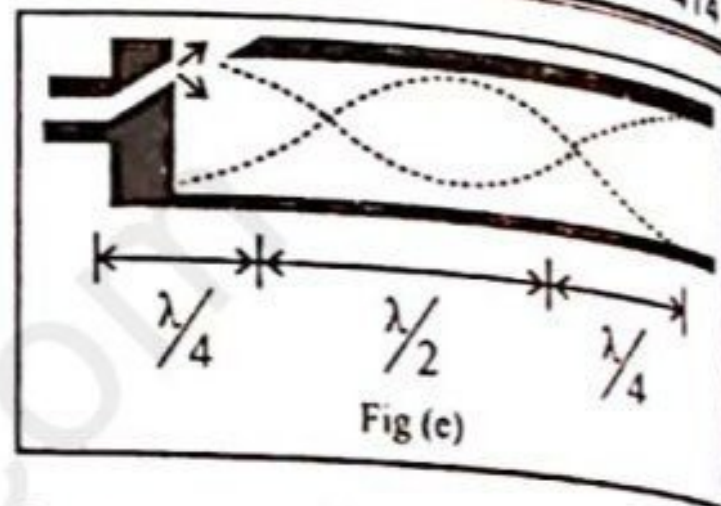
Now the frequency in this case is given by,

$$f_3 = \frac{V}{\lambda_3} \quad (9)$$

Putting equation (8) in equation (9), we get,

$$f_3 = \frac{V}{\left(\frac{2L}{3}\right)} = 3 \left(\frac{V}{2L}\right) \Rightarrow f_3 = 3 \left(\frac{V}{2L}\right) \quad (10)$$

Putting equation (3) in equation (10), we get,



$$f_3 = 3f_1 \quad (11)$$

$$\text{Similarly, } f_4 = 4f_1 \quad (12)$$

$$\text{And } f_n = nf_1 \quad (A)$$

Thus the possible frequencies are,

$$\frac{V}{2L}, 2 \frac{V}{2L}, 3 \frac{V}{2L}, 4 \frac{V}{2L}, \dots, n \left(\frac{V}{2L}\right)$$

NOTE:

The set of frequencies in case of closed organ pipe is given by,

$$f_2 = 3f_1, f_3 = 5f_1, f_4 = 7f_1, \dots, f_n = (2n-1)f_1$$

While the set of frequencies in case of open pipe is given by,

$$f_2 = 2f_1, f_3 = 3f_1, f_4 = 4f_1, f_5 = 5f_1, \dots, f_n = nf_1$$

Comparing the above sets of frequencies, we see that an open organ pipe is richer in harmonics than a closed organ pipe.

Q22: What is Doppler's effect? Derive expressions for the frequencies heard. (a) When the sounding source approaches a stationary listener? (b) When listener move towards a stationary sounding source? (c) When the source and listener are moving towards each other?

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Ans: (a) Doppler's Effect:

Definition: The apparent change in pitch of sound due to the relative motion between the source and listener [observer] is known as Doppler effect.

Explanation:

Consider a listener standing on a railway platform. An engine sounding whistle is approaching the listener. The listener will find that the pitch of the note of the whistle appears to increase as the engine approaches the listener. The pitch of sound will be maximum when the engine is just crossing the listener. This pitch will decrease as the engine moves away from the listener.

Let "f" be the actual frequency of the whistle and "f'" be the apparent frequency of the whistle appears to the listener. The value of

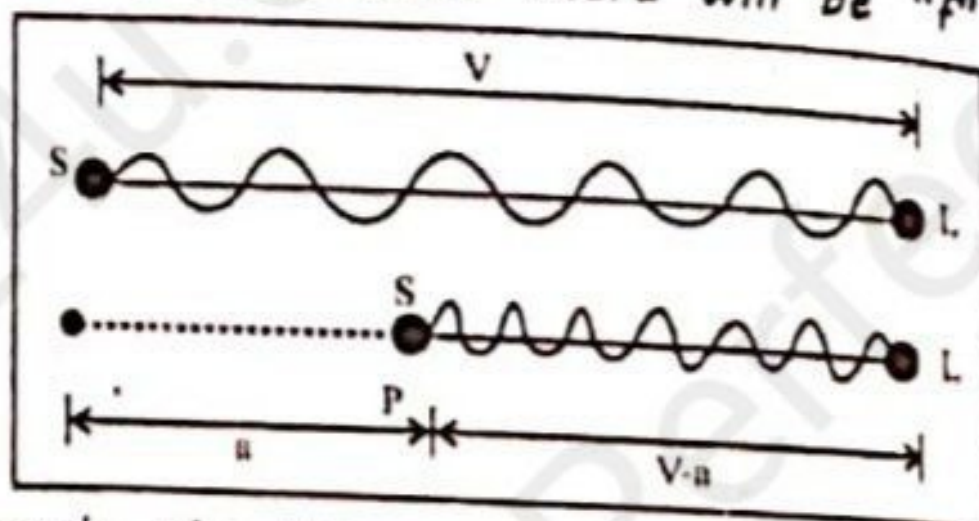
" f " will be different from " f " due to the relative motion between source and the listener. Let " v " be the velocity of sound in air.

Case 1: When the source is moving towards a stationary listener:
Consider a listener "L" situated at a distance " V " from the source "S" as shown in the figure.

Let the sound waves starting from "S" reach "L" in exactly one second. Now when the source is at rest, then there will be " f " waves in distance " V ". Now the wavelength of sound is given by,

$$\lambda = \frac{\text{total distance}}{\text{total no. of waves}}$$

$$\Rightarrow \lambda = \frac{V}{f} \quad (1)$$



Let the source is moving towards the listener with velocity " a " such that it reaches the point "P" in one second. Now the same number of waves " f " will be compressed in a distance " $V-a$ ". Now the wavelength " λ' " will be,

$$\lambda' = \frac{\text{distance}}{\text{no. of waves}} \Rightarrow \lambda' = \frac{(V-a)}{f} \quad (2)$$

Now the apparent frequency heard by the listener is given by,

$$f' = \frac{V}{\lambda'} \quad (3)$$

Putting equation (1) in equation (3), we get,

$$f' = \left(\frac{V}{V-a} \right) f \quad (4)$$

$$\text{Now as } (V-a) < V \Rightarrow \left(\frac{V}{V-a} \right) > 1$$

$$\text{So equation (4)} \Rightarrow f' > f$$

Thus the apparent frequency increases when the source of sound is moving towards the listener and as a result the pitch of sound increases. Similarly, when the source of sound is moving away from the listener, then equation (4) can be written as,

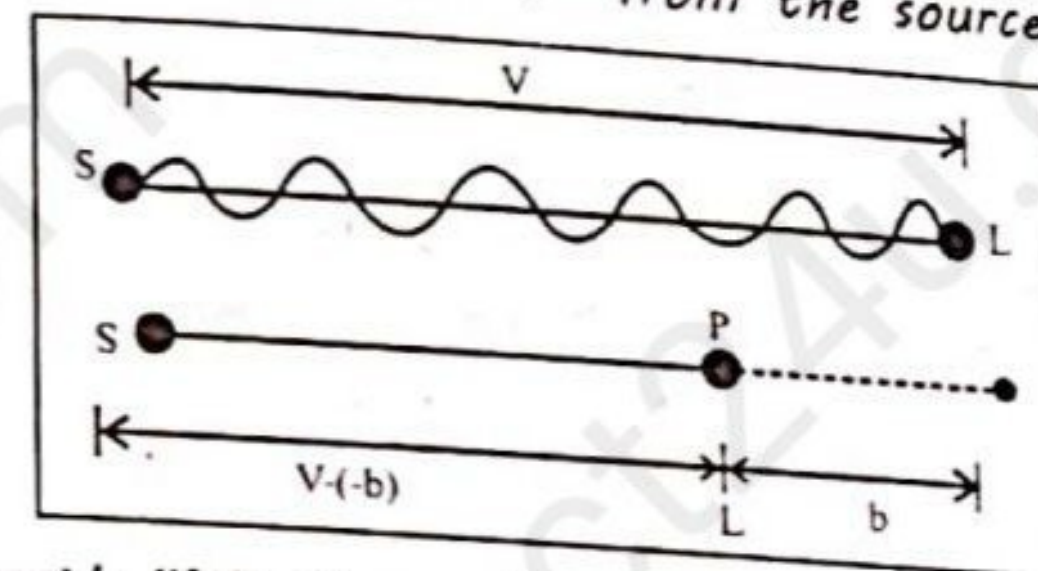
$$f' = \left(\frac{V}{V+a} \right) f \quad (5)$$

$$\text{As } (V+a) > V \Rightarrow \left(\frac{V}{V+a} \right) < 1, \text{ so equation (5)} \Rightarrow f' < f$$

Thus the apparent frequency decreases when the source of sound is moving away from the listener and as a result the pitch of sound decreases.

Case 2: When listener is moving towards stationary source:
Consider a listener "L" situated at a distance " V " from the source "S" as shown in the figure.

Let the sound waves starting from "S" and reaches "L" in exactly one second. When the listener is at rest, then there will be " f " waves in distance " V ". Now the wavelength " λ " of sound waves is given by,



$$\lambda = \frac{\text{total distance}}{\text{total no. of waves}}$$

$$\Rightarrow \lambda = \frac{V}{f} \quad (1)$$

Now let the listener is moving with velocity " b " towards the source and reaches the point "P", as shown in the figure. Now the relative velocity of sound waves with respect to the listener is given by,

$$V - (-b) = V + b$$

Now the apparent frequency in this case is given by,

$$f' = \frac{(V+b)}{\lambda} \quad (2)$$

Putting equation (1) in equation (2), we get,

$$f' = \left(\frac{V+b}{V} \right) f \quad (3)$$

$$\text{As } (V+b) > V \Rightarrow \left(\frac{V+b}{V} \right) > 1$$

$$\text{So equation (3)} \Rightarrow f' > f$$

Thus the apparent frequency increases when the listener is moving towards a stationary source of sound, and thus the pitch increases. Similarly, when the listener is moving away from the source, then equation (3) can be written as,

$$f' = \left(\frac{V - b}{V} \right) f \quad (4)$$

As $\left(\frac{V - b}{V} \right) < 1$, so equation (4) $\Rightarrow f' < f$

Thus the apparent frequency decreases when the listener is moving away from the stationary source and thus the pitch of sound decreases.

Case 3: When source and listener both are moving towards each other:

Let the source and listener both are moving towards each other with velocities 'a' and 'b' respectively. The apparent frequency in such a case is given by,

$$f' = \frac{v'}{\lambda'} \quad (1)$$

Here 'v' represents the speed of sound relative to the listener and is given by,

$$v' = v + b \quad (2)$$

While ' λ' ' represents the apparent wavelength and is given by,

$$\lambda' = \frac{v - a}{f} \quad (3)$$

Putting equation (2) and equation (3) in equation (1), we get,

$$f' = \frac{v + b}{\left(\frac{v - a}{f} \right)} = \left(\frac{v + b}{v - a} \right) f \Rightarrow f' = \left(\frac{V + b}{V - a} \right) f \quad (4)$$

In equation (4), $\left(\frac{V + b}{V - a} \right) > 1$, so equation (4) $\Rightarrow f' > f$

This shows that the apparent frequency increases when the source and listener both are moving towards each other.

Similarly, when the source and listener both are moving away from each other, then equation (4) can be written as,

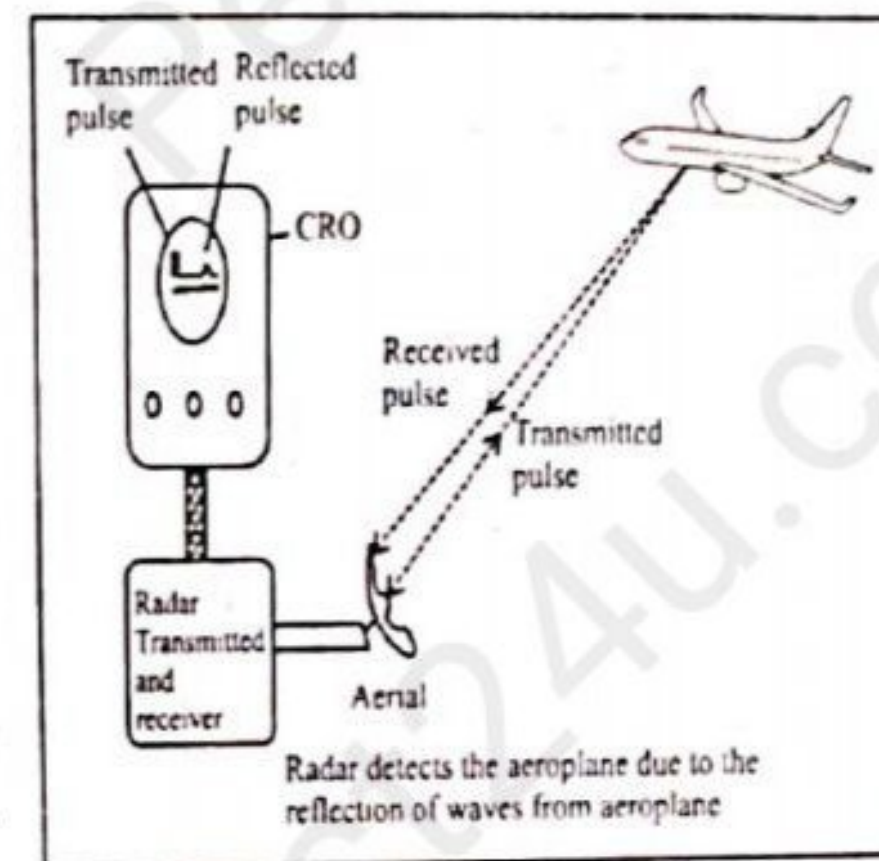
$$f' = \left(\frac{v - b}{v + a} \right) f \quad (5)$$

Here $\left(\frac{V - b}{V + a} \right) < 1$, so equation (5) $\Rightarrow f' < f$

This shows that the apparent frequency decreases when the source and listener both are moving away from each other.

Applications of Doppler Effect:

1. The Doppler Effect is not limited to only sound waves. It is also applicable to light waves. The frequency of light being received from certain stars which are moving towards earth or moving away from the earth is found to be slightly different than the frequency of the same light emitted by a source on the earth.



2. The reflection of radar waves from an aeroplane is another application of Doppler Effect.

The frequency of reflected waves is decreased, if the aeroplane is moving away from the source. The frequency of reflected waves is increased if the plane is moving towards the source. From this frequency shift ' Δf ', the speed and direction of the aeroplane can be determined.

3. When sound waves are reflected from a moving submarine, their frequency is changed. By this change in frequency, we can calculate the speed and direction of the submarine.

4. The Doppler shift in the frequency of radio waves [which they transmit] can be used to calculate velocities of earth satellites.

Q23: What are ultrasonic waves? Discuss the uses of ultrasonic waves.

Ans. Ultrasonic Waves:

Definition: Those sound whose frequencies are higher than 20 kHz are called ultrasonic waves.

Explanation:

A normal human ear can hear a sound whose frequency lies between 20 Hz and 20 kHz. If the frequency of sound is below 20 Hz or above 20 kHz, then human ear cannot hear it, because such sounds cannot vibrate the membrane of our ear.

The sounds whose frequencies below 20 Hz are known as infrasonic waves while the sound whose frequencies are above 20 kHz are

called ultrasonic waves.

The term ultrasonic means above or beyond sound. The ultrasonic waves can be produced by an object whose frequency of vibration is higher than 20 kHz. An ultrasonic wave is a pressure wave which has an extremely short wavelength because of its high frequency. In fact an ultrasonic wave of frequency 25 billion Hz has a wavelength of 10^{-8} m, which is smaller than the wavelength of visible light (10^{-6} m) and is comparable to the wavelength of x-rays (10^{-10} m).

Uses of Ultrasonic Waves:

As the ultrasonic waves possess shorter wavelength, higher frequency and energy, so these waves are widely used in various departments for various purposes. For example:

1. The ultrasonic waves have a great importance in bloodless surgery and diagnostic work in medicine. In diagnostic work an ultrasonic signal is transmitted through a patient. By the analysis of reflected or refracted signals, the cysts and tumor can be located in the body. It has also a great importance in surgical departments.
2. The ultrasonic waves are used in industry for the testing of materials. For this purpose a beam of ultrasonic waves of high frequency is allowed to penetrate through the material. The reflection of these waves from certain spots explores the physical defects in that material.
3. The ultrasonic waves are used for cleaning purposes. These waves can remove the traces of foreign matter sticking to the metal.
4. The ultrasonic waves of high intensity can be used to destroy bacteria and other microorganisms present in liquids and air.
5. The ultrasonic waves play an important role in neurosurgery. The ultrasonic wave of high intensity can be focused in the required region without disturbing the surrounding tissues. For this purpose, special type of ultrasonic equipments are used for the treatment of arthritis, muscular rheumatism and sciatic diseases.
6. The ultrasonic waves are used in the process of cavitations, which helps in degassing. When the ultrasonic waves are focused on a small space in liquids, very high intensity can be produced. The liquid is rapidly volatilized and a large number of bubbles are formed. This process is called cavitations. The collapse of these bubbles produces destructive forces near the surface in a liquid.

7. The ultrasonic waves can be used to determine the quantity of water in a container.
8. The ultrasonic waves can be used to determine the depth of the sea.
9. The ultrasonic waves are used to check an unborn baby in the womb [بطن] of mother. For this purpose, an ultrasonic transmitter is moved over the body of the mother. The ultrasonic waves which are reflected from different layers inside the body are picked up by a detector. The signals are then focused on the screen and thus the doctor can see its final image. From the final image, the doctor can get information about the nature, size, age, movement etc of the unborn baby in womb of the mother.

Q24: Discuss the following: (i) Generation of ultrasonic waves.
(ii) Detection of ultrasonic waves.

Ans: (a) Generation of Ultrasonic Waves:

Ultrasonic waves can be generated by any object which is capable of oscillation at a frequency higher than 20 kHz.

Method a:

In most of applications, ultrasonic waves are generated by applying an electric current (electric field) to a specialized kind of crystal known as piezoelectric crystal. This crystal converts electrical energy into mechanical energy, as a result the crystal is set to vibrate at high frequency, generating ultrasonic waves.

Method b:

In another technique a magnetic field is applied to a special crystal which causes it to oscillate at a frequency higher than 20 kHz, emitting ultrasonic waves.

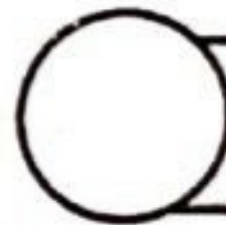
(b) Detection of Ultrasonic Waves:

There are so many methods by which ultrasonic waves can be detected. Some methods are given below:

(i) Piezoelectric Detection Method: As ultrasonic waves consisting of compressions and rarefaction so when they are allowed to fall on a quartz crystal, a certain potential difference is produced across the crystal faces. This difference is amplified by an amplifier and the ultrasonic waves are detected.

(ii) Kundt's Tube Method: Kundt's tube is a long glass tube supported horizontally with an air column in it. The lycopodium powder

is sprinkled in the tube. When ultrasonic waves are allowed to pass through this Kundt's tube, the lycomodium powder in the tube collects at the nodes and blown off at the antinodes. This method is used when the wavelength is not very short.



Solution of Examples & Assignments

EXAMPLE 8.1:

A wave generator produces 500 pulses in 10sec. Find the period and frequency of the pulses it produces.

Solution:

Given data:

Number of pulses = $n = 500$, Time taken = $t = 10\text{sec}$

Required: (i) Frequency of pulse = $f = ?$

(ii) Time period of pulse = $T = ?$

Calculation:

(i) We know that,

$$f = \frac{\text{No. of pulses}}{\text{time}} = \frac{500}{10} \Rightarrow \boxed{f = 50 \text{ pulse / sec}}$$

$$(ii) \text{ As } T = \frac{1}{f} = \frac{1}{50} \text{ sec} \Rightarrow \boxed{T = \frac{1}{50} \text{ sec}}$$

ASSIGNMENT 8.1:

Compute the speed of sound in sea water. if its density is 1025kg/m^3 and elastic modulus is $2.1 \times 10^9 \text{N/m}^2$

Solution:

Given data:

Density of sea water = $\delta = 1025 \text{kg / m}^3$

Elastic modulus = $E = 2.1 \times 10^9 \text{N/m}^2$

Required:

Speed of sound = $V = ?$

Calculation:

We know that,

$$V = \sqrt{\frac{E}{\delta}} \quad (1)$$

Putting the values in equation (1), we get,

$$V = \sqrt{\frac{2.1 \times 10^9}{1025}} = 0.1430 \times 10^4 \text{ m/sec}$$

$$\Rightarrow \boxed{V = 1430 \text{ m / sec}}$$

EXAMPLE 8.2:

In a ripple tank 40 waves passes through a certain point in one second. If the wavelength of the waves is 5cm, then find the speed of the waves?

Solution:

Given data:

Number of waves per second = $f = 40 \text{ wave/sec}$

Wavelength = $\lambda = 5 \text{cm} = 0.05 \text{m}$

Required: Speed of the mass = $V = ?$

Calculation:

We know that, $V = f \lambda$ _____ (1)

Putting the values in equation (1), we get,

$$V = (40 \times 0.05) \text{ m/sec} \Rightarrow \boxed{V = 2 \text{ m/sec}}$$

ASSIGNMENT 8.2:

How many beats per second are heard when two tuning forks of 256 Hz and 259 Hz are sounded together?

Solution: Given data:

Frequency of 1st tuning fork = $f_1 = 259 \text{Hz}$

Frequency of 2nd tuning fork = $f_2 = 256 \text{Hz}$

Required:

Beat per sec = $N = ?$

Calculation:

We know that, $N = f_1 - f_2$ _____ (1)

Putting the values in equation (1), we get,

$$N = 259 - 256 \Rightarrow \boxed{N = 3 \text{ beats per sec}}$$

EXAMPLE 8.3:

Find the speed of sound in a steel railway track, if the density of steel is 7800kg/m^3 and elastic modulus is $2 \times 10^{11} \text{N/m}^2$.

Solution: Given data:

Density of steel = $\delta = 7800 \text{kg / m}^3$,

$$\text{Elastic modulus} = E = 2 \times 10^{11} \text{ N/m}^2$$

Required:

$$\text{Speed of sound} = V = ?$$

Calculation:

$$\text{We know that, } V = \sqrt{\frac{E}{\delta}} \quad (1)$$

Putting the values in equation (1), we get,

$$V = \sqrt{\frac{2 \times 10^{11}}{7800}} = 0.0506 \times 10^5 \text{ m/sec} \Rightarrow V = 5.06 \times 10^3 \text{ m/sec}$$

ASSIGNMENT 8.3:

A 40g string 2m in length vibrates in three loops. The tension in the string is 270N. What is the wavelength and frequency? (1.33m, 87.1Hz)

Solution:

Given data:

$$\text{Length of string} = L = 2\text{m, Mass} = M = 40\text{gm} = 0.04\text{kg}$$

$$\text{Mass per unit length} = m = \frac{M}{L} = \frac{0.04}{2} = 0.02\text{kg/m}$$

$$\text{Number of loops} = n = 3, \text{ Tension} = T = 270\text{N}$$

Required:

$$(i) \text{ Wavelength} = \lambda_3 = ?$$

$$(ii) \text{ Frequency} = f_3 = ?$$

Calculation:

(i) We know that,

$$\lambda_n = \frac{2L}{n} \Rightarrow \lambda_3 = \frac{2 \times 2}{3} = \frac{4}{3} \text{ m} \Rightarrow \lambda_3 = 1.33\text{m}$$

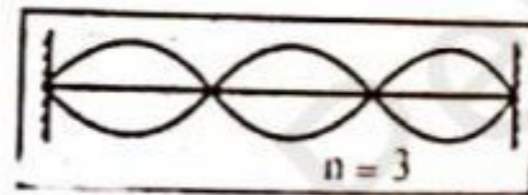
(ii) We know that, $v = f\lambda$

$$\Rightarrow v = f_3 \lambda_3 \Rightarrow f_3 = \frac{v}{\lambda_3} \quad (1)$$

$$\text{Also } v = \sqrt{\frac{T}{m}} = \sqrt{\frac{270}{0.02}} \text{ m/sec} = \sqrt{13500} \text{ m/sec} = 116.2 \text{ m/sec} = 116 \text{ m/sec}$$

Putting the values of "v" and " λ_3 " in equation (1), we get,

$$f_3 = \left(\frac{116}{1.33} \right) \text{ Hz} = 87.2 \text{ Hz} \Rightarrow f_3 = 87.2 \text{ Hz}$$



EXAMPLE 8.4:

Find the speed of sound in a neon gas at 0°C . γ for monatomic gas = 1.66. [Take $R = 8314 \text{ J/K}\cdot\text{mol}\cdot\text{K}$, molar mass of neon gas = $M = 20.18 \text{ kg/K}\cdot\text{mol}$]

Solution:

Given Data:

$$\gamma = 1.66, T = 0^\circ\text{C} = 0 + 273 = 273\text{K}$$

$$R = 8314 \text{ J/K}\cdot\text{mol}\cdot\text{K}, M = 20.18 \text{ kg/K}\cdot\text{mol}$$

Required:

$$\text{Speed of sound} = V = ?$$

Calculation:

$$\text{We know that, } V = \sqrt{\frac{\gamma RT}{M}} \Rightarrow V = \sqrt{\frac{1.66 \times 8314 \times 273}{20.18}}$$

$$\Rightarrow V = 432 \text{ m/sec}$$

ASSIGNMENT 8.4:

What length of closed pipe will produce a fundamental frequency of 256Hz at 20°C ?

Solution: Given data:

$$\text{Fundamental frequency} = f_1 = 256\text{Hz}$$

$$\text{Speed of sound at } 20^\circ\text{C} = 343 \text{ m/sec}$$

Required:

$$\text{Length of closed pipe} = L = ?$$

Calculation:

We know that for closed pipe,

$$f_1 = \frac{v}{4L} \Rightarrow L = \frac{v}{4f_1} \Rightarrow L = \frac{343}{4 \times 256} = 0.3349\text{m}$$

$$\Rightarrow L = 0.3349\text{m} = 0.3349 \times 100\text{cm} = 33.49\text{cm} \Rightarrow L = 33.5\text{cm}$$

EXAMPLE 8.5:

Two pianos sound the same note. If the vibration from one is 221.60Hz and that of the other is 221.40 Hz. What is the beat frequency between the two tones?

Solution: Given Data:

$$\text{Frequency of 1st piano} = f_1 = 221.60\text{Hz}$$

$$\text{Frequency of 2nd piano} = f_2 = 221.40\text{Hz}$$

Required: Beat frequency = $N = ?$

Calculation:

We know that, $N = f_1 - f_2$ (1)

Putting the values in equation (1), we get,

$$N = (221.60 - 221.40)\text{Hz} = 0.20\text{Hz}$$

$$\text{Beat frequency} = N = 0.20\text{Hz}$$

ASSIGNMENT 8-5:

(a) What frequency is received by a woman watching an oncoming ambulance moving at 110km/h and emitting a steady 800Hz sound from its siren? The speed of sound on this day is 345m/s. (b) What frequency does she receive after the ambulance has passed?

(a) 878Hz, (b) 735Hz

Solution: Given data:

Speed of sound = $v = 345\text{m/sec}$, Frequency of sound = $f = 800\text{Hz}$

Speed of source

$$= a = 110\text{km/hr} = \frac{110 \times 1000}{3600} \text{m/sec} = 30.5\text{m/sec}$$

Required:

(a) Apparent frequency when source is approaching = $f' = ?$

(b) Apparent frequency when source is receding = $f'' = ?$

Calculation:

(a) We know that, when source is approaching the listener, then

$$f' = \left(\frac{v}{v - a} \right) f = \left(\frac{345}{345 - 30.5} \right) 800 = 878\text{Hz} \Rightarrow \boxed{f' = 878\text{Hz}}$$

(b) When source is moving away, then we have,

$$f'' = \left(\frac{v}{v + a} \right) f = \left(\frac{345}{345 + 30.5} \right) 800 = 735\text{Hz} \Rightarrow \boxed{f'' = 735\text{Hz}}$$

EXAMPLE 8-6:

The speed of a wave on a particular string is 24m/sec. If the string is 6m long, to what driving frequency will it resonate?

Solution: Given Data: Length of string = $L = 6\text{m}$,

Speed of the wave = $V = 24\text{m/sec}$

Calculation:

$$\text{We know that, } \lambda_n = \frac{2L}{n} \quad (1)$$

Put $n = 1$, so equation (1) becomes,

$$\lambda_1 = \frac{2 \times 6}{1} = 12\text{m} \Rightarrow \lambda_1 = 12\text{m}$$

Put $n = 2$, in equation (1), we get,

$$\lambda_2 = \frac{2 \times 6}{2} \Rightarrow \lambda_2 = 6\text{m}$$

Put $n = 3$ in equation (1), we get,

$$\lambda_3 = \frac{2 \times 6}{3} \Rightarrow \lambda_3 = 4\text{m}$$

Now the possible frequencies are given below:

$$f_1 = \frac{V}{\lambda_1} = \frac{24}{12} \Rightarrow \boxed{f_1 = 2\text{Hz}}$$

$$f_2 = \frac{V}{\lambda_2} = \frac{24}{6} \Rightarrow \boxed{f_2 = 4\text{Hz}}$$

$$\& f_3 = \frac{V}{\lambda_3} = \frac{24}{4} \Rightarrow \boxed{f_3 = 6\text{Hz}}$$

EXAMPLE 8-7:

A string 4m long has a mass of 3gm. One end of the string is fasted to a stop and the other end hangs over a pulley with a 20kg mass attached. What is the speed of a transverse wave in this string?

Solution:

Given Data: Mass of pulley = $m = 20\text{kg}$,

Length of string = $L = 4\text{m}$

Mass of string = $M = 3\text{gm} = 0.003\text{kg}$

$$\text{Mass per unit length} = m' = \frac{M}{L} = \frac{0.003}{4} = 7.5 \times 10^{-4} \text{kg/m}$$

Tension in the string = $T = w = mg$

$$\Rightarrow T = 20 \times 9.8 = 19.6\text{N}$$

Required:

Speed of transverse waves = $V = ?$

Calculation:

$$\text{We know that, } V = \sqrt{\frac{T}{m'}} \Rightarrow V = \sqrt{\frac{19.6}{7.5 \times 10^{-4}}}$$

$$\Rightarrow V = 1.60 \times 10^2 \text{m/sec}$$

$$\Rightarrow \boxed{V = 160\text{m/sec}}$$

EXAMPLE 8-8:

The lowest resonance frequency for a guitar string of length 0.75m is 400Hz. Calculate the speed of a transverse wave on the string.

Solution: Given Data:

String's length = $L = 0.75\text{m}$, Lowest frequency = $f_1 = 400\text{Hz}$

Required: Speed of transverse waves = $V = ?$

Calculation: We know that, $\lambda_n = \frac{2L}{n} \Rightarrow \lambda_1 = \frac{2L}{1}$

$$\Rightarrow \lambda_1 = 2 \times 0.75 \Rightarrow \lambda_1 = 1.50\text{m}$$

$$\text{Now } V = f\lambda \Rightarrow V = f_1\lambda_1 = 400 \times 1.5 \Rightarrow \boxed{V = 600\text{m/sec}}$$

EXAMPLE 8-9:

A pipe open at one end and closed at the other end, is 82cm long. What are the three lowest frequencies to which it will resonate?

Take the speed of sound as 340m/sec.

Solution: Given Data:

Length of pipe = $L = 82\text{cm} = 0.82\text{m}$

Speed of sound = $V = 340\text{m/sec}$

Required:

Three lowest possible frequencies = ? i.e.

(i) $f_1 = ?$ (ii) $f_2 = ?$ (iii) $f_3 = ?$

Calculation:

(i) In case of closed pipe, we have,

$$f_1 = \frac{V}{4L} = \frac{340}{(4 \times 0.82)} \Rightarrow \boxed{f_1 = 104\text{Hz}}$$

$$\text{Now (ii) } f_2 = 3f_1 = 3 \times 104 \Rightarrow \boxed{f_2 = 312\text{Hz}}$$

$$\text{(iii) } f_3 = 5f_1 = 5 \times 104 \Rightarrow \boxed{f_3 = 520\text{Hz}}$$

EXAMPLE 8-10:

A car is moving at 20m/sec along a straight road with its frequency 500Hz horn sounding. You are standing at the road side. What receding from you at 20m/sec? Take the speed of sound as

340m/sec

Solution:

Given Data:

$$f = 500\text{Hz}, V = 340\text{m/sec}$$

Speed of sound source = $a = 20\text{m/sec}$

Required:

(i) Apparent frequency when car approaches = $f' = ?$

(ii) Apparent frequency when car receding = $f'' = ?$

Calculation:

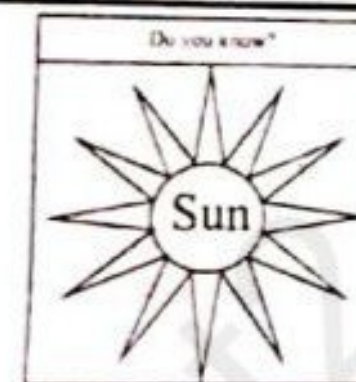
(i) We know that,

$$f' = \left(\frac{V}{V - a} \right) f = \left(\frac{340}{340 - 20} \right) \times 500 \Rightarrow \boxed{f' = 531\text{Hz}}$$

$$\text{(ii) And } f'' = \left(\frac{V}{V + a} \right) f = \left(\frac{340}{340 + 20} \right) \times 500$$

$$\Rightarrow \boxed{f'' = 472\text{Hz}}$$

Explosions, occurs on the surface of the sun due to fission and fusion reactions but we can't hear, why?



Ans: There is about 150 million kilometer distance between sun and earth. Sound waves are longitudinal waves which needs a material medium for their propagation. But the space between sun and the earth is mostly a vacuum through which sound waves cannot pass. There is not sufficient matter to transfer the sound of explosion on sun surface. Thus we cannot hear the sound of explosion occurs on sun's surface.

Quiz-1: Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

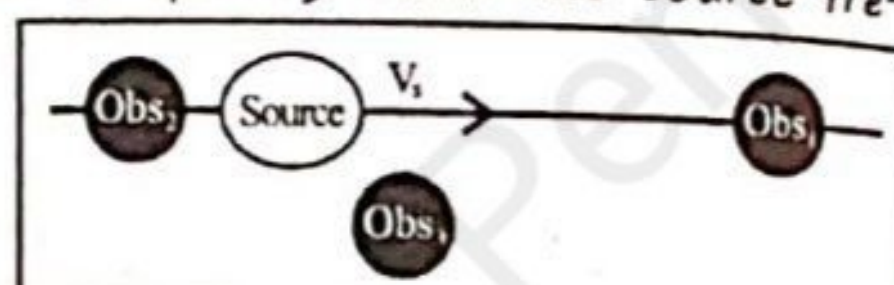
Ans: If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

Quiz-2: Three stationary observers observe the Doppler shift from a source moving at a constant velocity. The observers are stationed as shown below. Which observer will observe the highest frequency? Which observer will observe the lowest frequency? What can be said about the frequency observed by observer 3?

Ans. (1) Observer (1) will observe the highest frequency

(2) Observer (2) will observe the lowest frequency

(3) Observer (3) will hear a higher frequency than the source frequency, but lower than the frequency observed by observer (1), as the source approaches and a lower frequency than the source frequency, but higher than the frequency observed by observer (1), as the source moves away from observer (3).



Exercise

Choose the best possible answer:

1. When a wave goes from one medium to another medium, which one of the following characteristics of the wave remains constant?

- a. Velocity ✓ b. Frequency
c. Wavelength d. Phase

2. When a transverse wave is reflected from the boundary of a denser to a rarer medium, it undergoes a phase change of:

- a. 0 b. $\frac{\pi}{2}$ ✓ c. π d. 2π

3. If the tension in the string is doubled and its mass per unit length is reduced to half. Then the speed of transverse wave on it is:

- ✓ a. Doubled b. Halved
c. Constant d. One fourth

Sol: We know that,

$$v = \sqrt{\frac{T}{m}} \quad (1)$$

Now if tension becomes double and mass becomes half, then we have,

$$T' = 2T \text{ and } m' = \frac{m}{2}$$

For "T'" and "m'", equation (1) can be written as,

$$v' = \sqrt{\frac{T'}{m'}} \quad (2)$$

Putting the values of "T'" and "m'" in equation (2), we get,

$$v' = \sqrt{\frac{2T}{\frac{m}{2}}} = \sqrt{\frac{4T}{m}} = \sqrt{\frac{T}{m}} \Rightarrow v' = 2\sqrt{\frac{T}{m}} \quad (3)$$

Putting equation (1) in equation (3), we get,

$$v' = 2v$$

Thus the speed becomes double, so option (a) is correct.

4. Which one of the following properties is not exhibited by the longitudinal waves?

- a. Reflection b. Interference
c. Diffraction ✓ d. Polarization
5. A sounding source and a listener are both at rest relative to each other. If wind blows from the listener towards the source, then which one of the following of sound will change?
a. Frequency ✓ b. Speed
c. Phase d. Wavelength
6. Which one of the following factors has no effect on the speed of sound in a gas?
a. Humidity ✓ b. Pressure
c. Temperature d. Density
7. There is no net transfer of energy by particles of medium in:
a. Longitudinal wave b. Transverse wave
c. Progressive wave ✓ d. Stationary wave
8. Which one of the following could be the frequency of ultraviolet radiation?
a. $1.0 \times 10^6 \text{ Hz}$ b. $1.0 \times 10^9 \text{ Hz}$
c. $1.0 \times 10^{12} \text{ Hz}$ ✓ d. $1.0 \times 10^{15} \text{ Hz}$
9. When a stationary wave is formed then its frequency is:
✓ a. Same as that of the individual waves
b. Twice that of the individual waves
c. Half that of the individual waves
d. $\sqrt{2}$ that of the individual waves
10. The fundamental frequency of a closed organ pipe is f . If both the ends are opened then its fundamental frequency will be:
a. f b. $0.5f$ ✓ c. $2f$ d. $4f$

Sol: Case-1: When pipe is closed then

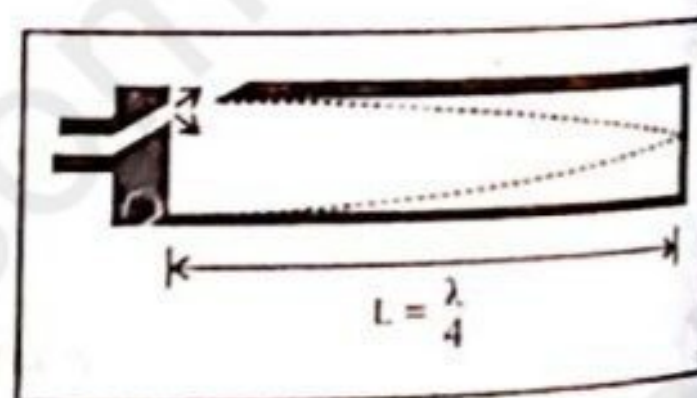
$$L = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4L$$

$$\text{Now } f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \Rightarrow f_1 = \frac{v}{4L} \quad (1)$$

Case-2: When pipe is open, then

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{2\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{2}$$



$$\text{Now } f_1 = \frac{v}{\lambda_1} = \frac{v}{(4L/2)}$$

$$\Rightarrow f_1 = 2 \left(\frac{v}{4L} \right) \quad (2)$$

Putting equation (1) in equation (2),

we get,

$$f_2 = 2f_1$$

11. If the amplitude of a wave is doubled, then its intensity is:

- a. Doubled b. Halved
✓ c. Quadrupled d. One fourth

Sol: Intensity is proportional to the square of amplitude. So when amplitude becomes double, then intensity will become four times i.e. quadrupled.

12. A sound source is moving towards stationary listener with $1/10^{\text{th}}$ of the speed of sound. The ratio of apparent to real frequency is:

- a. $\frac{11}{10}$ b. $\left[\frac{11}{10} \right]^2$ c. $\left[\frac{9}{10} \right]^2$ ✓ d. $\frac{10}{9}$

Sol: Speed of sound = v , speed of source = $a = \frac{v}{10}$

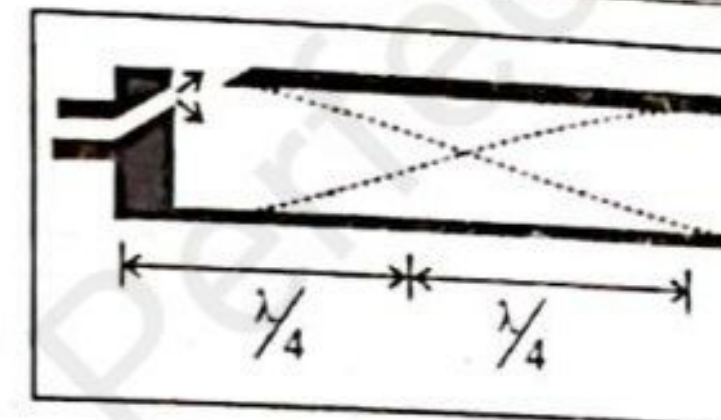
When source is moving towards listener, then we have,

$$f' = \frac{v}{(v - a)} f \quad (1)$$

Putting the values in equation (1), we get,

$$f' = \frac{v}{\left(v - \frac{v}{10} \right)} f \Rightarrow \frac{f'}{f} = \frac{v}{\left(\frac{10v - v}{10} \right)} = \frac{10v}{9v}$$

$$\Rightarrow \frac{f'}{f} = \frac{10v}{9v} \Rightarrow \boxed{\frac{f'}{f} = \frac{10}{9}} \text{ so option (d) is correct.}$$



Short Questions

Q1: What is the difference between progressive and stationary waves?

Ans: The difference between progressive and stationary waves are given below:

Progressive Waves	Stationary Waves
1. A disturbance of some kind produced due to a source as a result of which energy is transmitted from one place to another place is called progressive waves.	1. When two waves having the same amplitude and frequency with proper phase relationship travelling with the same speed in opposite direction through a medium are superposed and a wave obtained, is known as stationary or standing wave.
2. Progressive waves propagate in the medium and transmit energy from one point to another.	2. Stationary waves do not propagate, so they do not transmit energy from one place to another place. Stationary wave can occur only in a system which has boundaries such as string or spring stretched between two clamps.
3. Progressive waves are not confined in bounded system but travel unrestricted.	3. Stationary waves of discrete set of frequencies can be produced in any medium when its particles are disturbed.
4. In progressive waves we have no nodes and anti-nodes.	4. In stationary waves we have nodes and anti-nodes.
5. The frequency and wavelength of a progressive is equal to the frequency and wavelength of the source producing the wave.	5. The frequency and wavelength of a stationary wave are equal to that of either of the superposing wave.

Q2: Clearly explain the difference between longitudinal and transverse waves.

Ans: The difference between transverse and longitudinal waves is given below:

Transverse Waves	Longitudinal Waves
1. In transverse waves the particles of the medium execute SHM perpendicular to the direction of the propagation of the waves.	1. In longitudinal waves the particles of the medium execute SHM along the direction of the propagation of the waves.
2. Transverse waves consist of crests and troughs.	2. Longitudinal waves consist of compressions and rarefactions.
3. The distance between two consecutive crests or troughs is called wavelength ' λ '.	3. The distance between two consecutive compressions or rarefactions is called wavelength ' λ '.
4. Transverse waves may be mechanical or electromagnetic in nature.	4. Longitudinal waves are only mechanical waves.
5. Water waves, waves along a string, light waves, radio waves and T.V waves etc are the example of transverse waves.	5. Sound waves, waves along a spring or slinky etc are the examples of longitudinal waves.
6. Transverse waves can be polarized.	6. Longitudinal waves cannot be polarized.
7. Transverse wave equation is given by, $x = x_0 \sin \omega t$ _____ (1)	7. The longitudinal wave equation is given by, $x = x_0 \cos \omega t$ _____ (1)

Q3: How are beats useful in tuning a musical instrument?

Ans: Beats are useful in tuning a musical instrument.

Reason and Explanation:

Beats represents the periodic variation in the loudness of sound which can be heard when two notes of slightly different frequencies are played simultaneously. When two sound sources of slightly dif-

ferent frequencies are sounded at the same time, a single note is heard which rises and falls in intensity. As a result, a loud and fast sound is heard periodically. The phenomenon of beats plays an important role in tuning of musical instruments.

Tuning is the process of adjusting the pitch of one or more tones from musical instruments until they form a desired arrangement. Tuning may be carried out by sounding two instruments and adjusting one of them to match the frequency of the other. Thus when we are tuning, we are trying to match the frequency of one note to another.

Example: Consider a tuning fork and a guitar as shown in the figure. Let " f_A " represents the frequency of tuning fork and " f_B " represents the frequency of guitar. Let the two instruments are sounded simultaneously. In the beginning when " f_A " and " f_B " are slightly different, then beats occurs and we have,

$$N = f_A - f_B \quad (1)$$

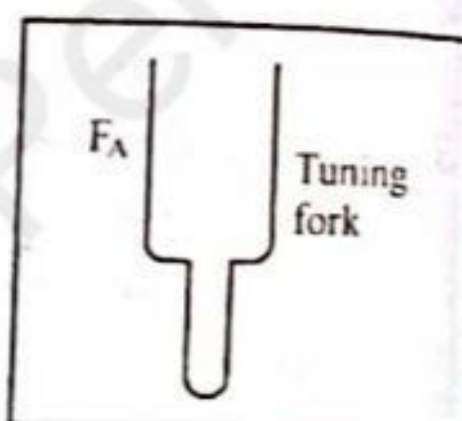
Now for tuning the guitar, we should match the frequency of guitar to the frequency of tuning fork. For this purpose, the tension in the string of guitar is adjusted until the beats disappear. Under such condition, we have,

$$f_A = f_B \text{ and } N = 0$$

Thus the frequencies of both instruments match each other and the guitar is tuned.

Q4: Two wave pulses travelling in opposite direction completely cancel each other as they pass. What happens to the energy possessed by the waves?

Ans: The energy remains conserved when two wave pulses travelling in opposite direction.



Reason and Explanation:

Consider two wave pulses "A" and "B" travelling in opposite direction, as shown in figure (1). Let the two pulses overlap each other as shown in figure (2). The two pulses cancel the effect of each other and the net displacement becomes zero at the point of superposition (overlapping). Thus the total energy becomes zero at the point of superposition. Now when the two pulses cross each other, they travel in their own direction with same shape and speed as shown in figure (3). Each pulse maintain its initial energy and thus the law of conservation of energy is satisfied in the above process.

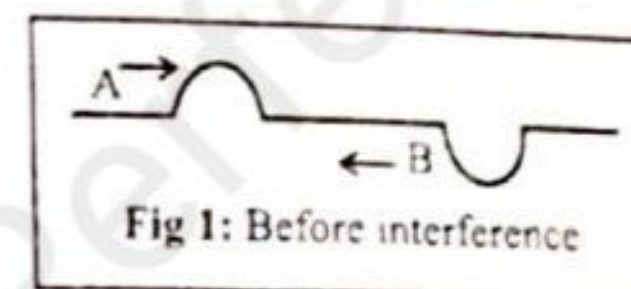


Fig 1: Before interference

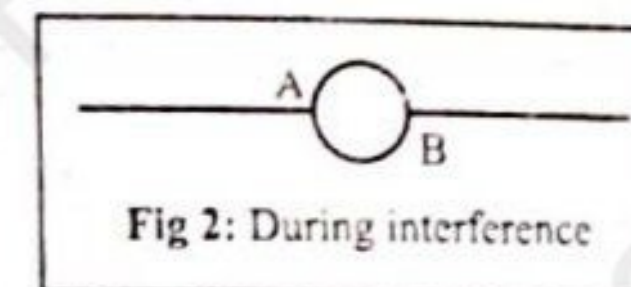


Fig 2: During interference

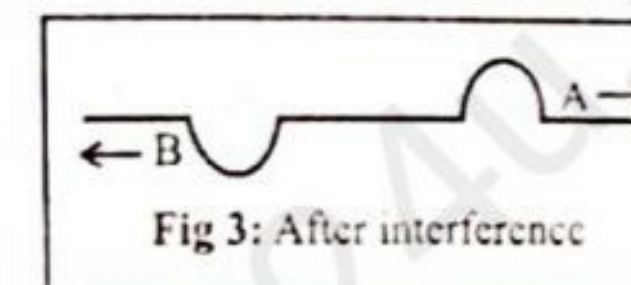


Fig 3: After interference

Q5: What are the conditions of constructive and destructive interference?

Ans: Conditions for Constructive and Destructive Interference:

For constructive and destructive interference, the following conditions must be satisfied.

7. The two waves must be phase coherent.
8. They must arrive at the same place at the same time.
9. The two waves must be travelling in the same direction.
10. The principle of linear super position must be satisfied.

Extra Special Conditions for Constructive Interference:

They constructive interference takes place only, if the path difference is zero i.e. an integral multiple of " λ ". i.e.

$$\text{Path difference} = m\lambda$$

$$(\text{OR}) \text{ Path difference} = 0, \lambda, 2\lambda, 3\lambda, \dots, m\lambda$$

[Where $m = 0, 1, 2, 3, 4, \dots$]

Extra Special Conditions for Destructive Interference:

The destructive interference takes place only, if the path difference between the two waves is an odd multiple of half of the wavelength i.e.

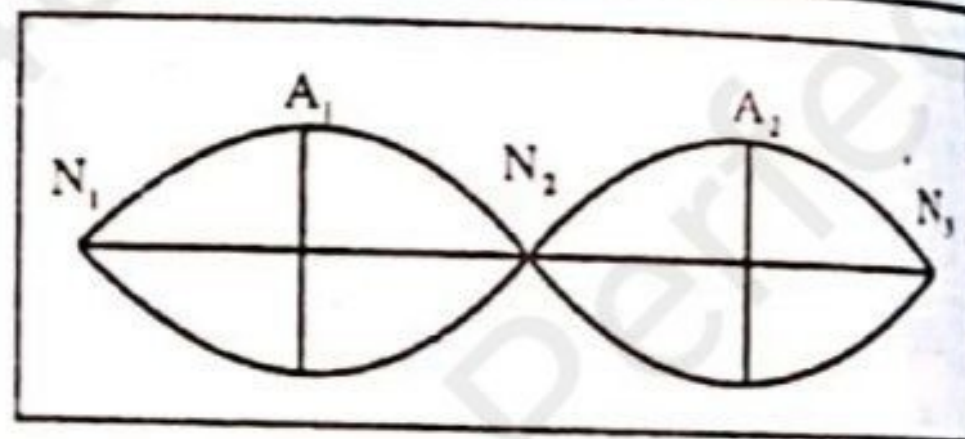
$$\text{Path difference} = m \frac{\lambda}{2}$$

$$(OR) \quad p.d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \dots, \left(m + \frac{1}{2}\right) \lambda$$

[Where $m = 0, 1, 2, 3, 4, \dots$]

Q6: How might one can locate the position of nodes and anti-nodes in a vibrating string?

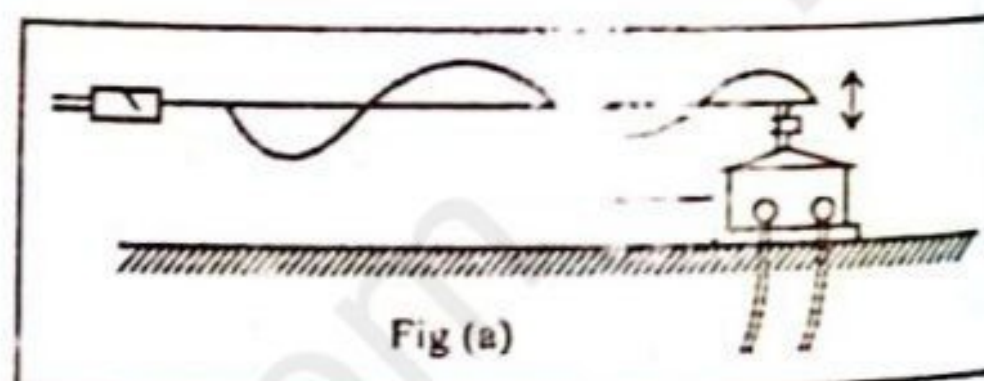
Ans No-1: We can locate the position of nodes in a vibrating string easily. It is because, at nodes point, the tension is maximum and the amplitude of vibration is zero.



The nodes are at rest and the string appears to us in the form of loops with nodes which are located at the ends of each loop as shown in the figure.

Similarly, we can locate the position of antinodes in a vibrating string easily. It is because, at antinodes point, the tension is minimum and amplitude of vibration is maximum. The string appears to us in the form of loops. At midpoint of each loop, the amplitude is maximum and thus the antinodes are located at the midpoint of each loop above or below the mean position, as shown by "A₁" and "A₂" in the given figure.

Ans No-2: We can locate the position of nodes and antinodes in a vibrating string with the help of Melde's experiment. In this experiment,



stationary waves are produced in the string by means of electric vibrator as shown in figure (a).

Location of Nodes: The points of the string which do not vibrate represent the nodes. The displacement of the particles is zero at the nodes and can be located easily. The nodes are denoted by "N".

Location of Antinodes: The points of the string at which the amplitude of vibration is maximum represents the antinodes. The dis-

placement of the particles is maximum at antinodes and can be located easily. The antinodes are denoted by "A".

Q7: Is it possible for an object which is vibrating transversely to produce sound wave?

Ans: Yes, it is possible for an object which is vibrating transversely to produce sound wave.

Reason and Explanation:

All the stringed and percussion instruments, when played, produce transverse waves. And the energy from these vibrating instruments travel through air in the form of sound waves. The human ear can detect sound whose frequency range is from 20Hz to 20kHz. So if the frequency range of the transverse waves of the musical instrument is in between 20Hz and 20kHz, we can hear the sound of that instrument which were vibrated transversely.

Q8: Why does a sound wave travel faster in solid than in gases?

Ans: Sound waves travel faster in solids than in gases, because for solids the ratio of elastic modulus to density is greater than for gases.

Reason and Explanation:

We know that the speed of sound waves in a medium of elasticity "E" and density "δ" is given by,

$$v = \sqrt{\frac{E}{\delta}} \quad (1)$$

In case of solids, "E" is taken as its Young's modulus while in case of gases, "E" is taken as its bulk modulus.

It is true that density "δ" of solids is greater than that of a gas, but at the same time, the Young modulus of a solid or elasticity of solids is far greater than the bulk modulus of a gas or elasticity of gases. So the term "E/δ" becomes greater for solids than in gases i.e. sound will travel faster in solids than gases.

Q9: Why does the speed of a sound wave in a gas changes with temperature?

Ans: The speed of sound in a gas changes with temperature.

Reason and Explanation:

We know that the speed of sound in a gas is given by,

$$v = \sqrt{\frac{E}{\delta}} \quad (1)$$

Equation (1) shows that the speed of sound is inversely proportional to the square root of the density i.e.

$$v \propto \sqrt{\frac{1}{\delta}} \quad (2)$$

Now we know that when temperature increases, the volume of the gas increases and its density decreases. So according to relation (2), the speed of sound will increase. And when the temperature of gas decreases, its volume decreases and density increases. So according to relation (2), the speed of sound decreases, that is why the speed of a sound wave in a gas change with temperature.

Ans-2: We know that the speed of sound in the gas at temperature "T" is given by,

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (3)$$

In equation (1) $\frac{\gamma R}{M} = \text{constant}$, so equation (1) becomes,

$$v = \sqrt{\text{constant} \times T}$$

$$\Rightarrow v = \text{constant} \times \sqrt{T} \Rightarrow v \propto \sqrt{T} \quad (2)$$

Relation (2) shows that the speed of sound is directly proportional to the square root of absolute temperature of the gas. i.e. when temperature increases, speed of sound also increases. It is because at higher temperature, the molecules of the gas have greater mobility to transport energy from particle to particle.

Relation (2) shows that the speed of sound increases by a factor 0.6m/sec for each one degree rise in temperature. Thus the speed of sound wave in a gas changes with temperature.

Q10: Is it possible for two astronauts to talk directly to one another even if they remove their helmets?

Ans: It is not possible for two astronauts to talk directly to one another even if they remove their helmets.

Reason and Explanation:

It is not possible for two astronauts to talk directly to one another when they remove their helmets in space, because ordinary sound waves are longitudinal mechanical waves. They always require medium for their propagation. Since in space there is no atmospheric gas, so it is not possible for astronaut to talk directly to one another, even if they could remove their helmets in the space. They can use electronic communication system for contact with each other.



Q11: Estimate the frequencies at which a test tube 15cm long resonates when you blow across its lips.

Ans: Length of tube = $L = 15\text{cm} = 0.15\text{m}$

Speed of sound in air = $v = 332\text{m/sec}$

Here the test tube behave like a closed pipe for which the fundamental frequency is given by,

$$f_1 = \frac{v}{4L} \quad (1)$$

Putting the values in equation (1), we get,

$$f_1 = \left(\frac{332}{4 \times 0.15} \right) \text{Hz} \Rightarrow f_1 = 553\text{Hz}$$

The second harmonic is given by,

$$f_2 = 3f_1 = 3 \times 553\text{Hz} \Rightarrow f_2 = 1659\text{Hz}$$

$$\text{Similarly, } f_3 = 5f_1 = 5 \times 553\text{Hz} \Rightarrow f_3 = 2765\text{Hz}$$

$$\& f_n = (2n - 1)f_1 = (2n - 1)553\text{Hz}$$

رسول اللہ ﷺ نے ارشاد فرمایا: جس کی زندگی کا آخری لمحہ لا الہ الا اللہ ہو گا وہ جنت میں ضرور جائے گا۔

حضرت علی کا فرمان ہے: تمہارا ایک رب ہے پھر بھی تم اسے یاد نہیں کرتے لیکن اس کے کتنے بندے ہیں پھر بھی تم کو نہیں بھولتا۔

oscillation of the particles is maximum is known as anti-node. It is represented by $A_1, A_2, A_3, \dots$, as shown in the figure.

Q4: What do you mean by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decreases.

Ans. See answer of question no 18 in the text.

Q5: Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace.

Ans. See answer of question no 7 in the text.

Q6: Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.

Ans. See answer of question no 8 in the text.

Q7: How the speeds of sound in the air varies with temperature and hence show that for each one degree centigrade rise in temperature the speed of sound increases by 0.61 ms^{-1} ?

Ans. See answer of question no 9 in the text.

Q8: What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources.

Ans. See answer of question no 13 in the text.

Q9: What is Doppler's Effect? Derive expression for the frequencies heard. (a) When the sounding source approaches a stationary listener. (b) When listener move towards a stationary sounding source.

Ans. See answer of question no 21 in the text.

Q10: What are organ pipes? Show that an open organ pipe is richer in harmonics than a closed organ pipe.

Ans. See answer of question no 20 in the text.

Q11: Explain the vibrations in a closed organ pipe and show that the frequency of third harmonic is $5v/4L$.

Ans. See answer of question no 20 part (a) in the text.

Numerical Problems

Q1: What are the wavelengths of a television station which transmits vision on 500 MHz and sound on 505 MHz respectively? Take speed of electromagnetic wave as $3 \times 10^8 \text{ ms}^{-1}$. (60.0 cm, 59.4 cm)

Solution:

Given data:

$$\text{Frequency} = f_1 = 500 \text{ MHz} = 500 \times 10^6 \text{ Hz}$$

$$\text{Mega} = 10^6$$

$$\text{Frequency} = f_2 = 505 \text{ MHz} = 505 \times 10^6 \text{ Hz}$$

$$\text{Speed of electromagnetic waves} = c = 3 \times 10^8 \text{ m/sec}$$

Required:

$$(a) \text{ Wavelength} = \lambda_1 = ? \quad (b) \text{ Wavelength} = \lambda_2 = ?$$

Calculation:

(a) We know that,

$$v = f\lambda \Rightarrow c = f\lambda \Rightarrow \lambda = \frac{c}{f} \Rightarrow \lambda_1 = \frac{c}{f_1} \quad (1)$$

$$1 \text{ m} = 100 \text{ cm}$$

Putting the values in equation (1), we get,

$$\lambda_1 = \frac{3 \times 10^8}{500 \times 10^6} \Rightarrow \lambda_1 = 0.006 \times 10^2 \text{ m} = 0.6 \text{ m} = 60 \text{ cm}$$

$$\Rightarrow \lambda_1 = 60 \text{ cm}$$

$$(b) \text{ Now } \lambda_2 = \frac{c}{f_2} \quad (2)$$

Putting the values in equation (2), we get,

$$\lambda_2 = \frac{3 \times 10^8}{505 \times 10^6} \Rightarrow \lambda_2 = 0.00594 \times 10^2 \text{ m} = 0.594 \text{ m} = 59.4 \text{ cm}$$

$$\Rightarrow \lambda_2 = 59.4 \text{ cm}$$

$$\lambda_1 = 60 \text{ cm} \text{ and } \lambda_2 = 59.4 \text{ cm}$$

Q2: A person on the sea shore observes that 48 waves reach the shore in one minute. If the wavelength of the waves is 10 meter, then find the velocity of the waves. (8 ms^{-1})

Solution: Given data:

$$\text{Number of waves} = 48, \text{ Time} = t = 1 \text{ min} = 60 \text{ sec}$$

$$\text{Frequency of waves} = \frac{\text{no. of waves}}{\text{time}}$$

$$\Rightarrow f = \frac{48}{60} = 0.8 \text{ Hz} \Rightarrow f = 0.8 \text{ Hz}$$

Wavelength of the waves = $\lambda = 10 \text{ m}$

Required:

Velocity of waves = $v = ?$

Calculation:

We know that, $v = f\lambda$ _____ (1)

Putting the values in equation (1), we get,

$$v = 0.8 \times 10 \text{ m/sec} = 8 \text{ m/sec} \Rightarrow \boxed{v = 8 \text{ m/sec}}$$

Q3: In a ripple tank 500 waves pass through a certain point in 10 seconds, if the speed of the wave is 3.5 ms^{-1} , then find the wavelength of the waves. (7cm)

Solution:

Given data:

Number of waves = 500, Time = $t = 10 \text{ sec}$

Frequency of waves = $f = \frac{\text{no. of waves}}{\text{time}} \Rightarrow f = \frac{500}{10} = 50 \text{ Hz}$

Speed of waves = $v = 3.5 \text{ m/sec}$

Required: Wavelength of the waves = $\lambda = ?$

Calculation:

We know that, $v = f\lambda \Rightarrow \lambda = \frac{v}{f}$ _____ (1)

$$\boxed{1 \text{ m} = 100 \text{ cm}}$$

Putting the values in equation (1), we get,

$$\lambda = \frac{3.5}{50} \Rightarrow \lambda = 0.07 \text{ m} \Rightarrow \boxed{\lambda = 7 \text{ cm}}$$

$$\boxed{\text{Wavelength} = \lambda = 7 \text{ cm}}$$

Q4: A string of a guitar is 1.3m long vibrates with 4 nodes, 2 of them at the two ends. Find the wavelength and speed of the wave in the string if it vibrates at 500Hz.

Solution:

Given data:

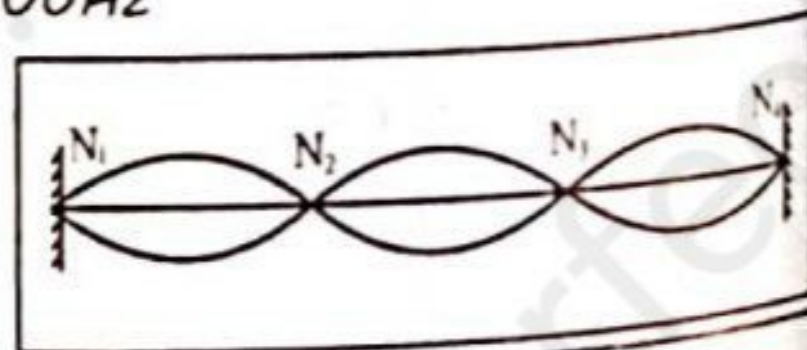
In case of 4 nodes, the string will vibrate in three loops, i.e. $n = 3$

Length of string = $L = 1.3 \text{ m}$, $f_3 = 500 \text{ Hz}$

Required:

(i) Wavelength = $\lambda_3 = ?$

(ii) Speed of the wave = $v = ?$



Calculation:

(i) We know that,

$$\lambda_n = \frac{2L}{n} \Rightarrow \lambda_3 = \frac{2L}{3} \text{ _____ (1)}$$

Put $n = 3$, for 3 loops

Putting the values in equation (1), we get,

$$\lambda_3 = \frac{2 \times 1.3}{3} = 0.866 \text{ m} \Rightarrow \lambda_3 = 0.866 \text{ m}$$

(ii) As $v = f\lambda \Rightarrow v = f_3 \lambda_3$ _____ (2)

Putting the values in equation (2), we get,

$$v = (500 \times 0.866) \text{ m/sec} \Rightarrow v = 433 \text{ m/sec}$$

$$(i) \boxed{\lambda_3 = 0.866 \text{ m}} \quad (ii) \boxed{v = 433 \text{ m/sec}}$$

Q5: A tension of 400N causes a 300g wire of length 1.6m to vibrate with a frequency of 40Hz. What is the wavelength of the transverse waves? (1.15m)

Solution:

Given data:

Tension in wire = $T = 400 \text{ N}$,

Mass of wire = $m = 300 \text{ gm} = 0.3 \text{ kg}$

Length of wire = 1.6 m , Frequency of vibration = $f = 40 \text{ Hz}$

Required:

Wavelength of the waves = $\lambda = ?$

Calculation:

We know that, $v = f\lambda \Rightarrow \lambda = \frac{v}{f}$ _____ (1)

We also know that speed of transverse waves in a string is given by,

$$v = \sqrt{\frac{T}{m}} \text{ _____ (2)}$$

Putting the values in equation (2), we get,

$$v = \sqrt{\frac{400 \times 1.6}{0.3}} = \sqrt{\frac{640}{0.3}} = 46 \text{ m/sec} \Rightarrow v = 46 \text{ m/sec}$$

Now putting the values of " f " and " v " in equation (1), we get,

$$\lambda = \left(\frac{46}{40}\right) \text{ m} = 1.15 \text{ m} \Rightarrow \lambda = 1.15 \text{ m}$$

$$\Rightarrow \boxed{\text{Wavelength} = \lambda = 1.15 \text{ m}}$$

Q6: Compare the theoretical speeds of sound in hydrogen ($M_H = 2.0\text{g/mol}$, $\gamma_H = 1.4$) with helium ($M_{He} = 4.0\text{g/mol}$, $\gamma_{He} = 1.66$ and $R = 8334\text{J/kmol}$) at 0°C . ($v_{He} = 0.77v_H$)

Solution:

Given data:

Mass of hydrogen $M_H = 2.0\text{g/mol}$

Value of " γ " for hydrogen $= \gamma_H = 1.4$

Mass of helium $= M_{He} = 4.0\text{g/mol}$

Value of " γ " for helium $= \gamma_{He} = 1.66$

Gas constant $= R = 8334\text{J/kmol}$

Temperature $= 0^\circ\text{C} = 0 + 273 = 273\text{K}$

Required: Comparison of speed of "H" with "He" = ?

Calculation:

We know that the speed of sound is given by,

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (1)$$

For hydrogen, equation (1) becomes,

$$v_H = \sqrt{\frac{\gamma_H RT}{M_H}} \quad (2)$$

For helium, equation (1) becomes,

$$v_{He} = \sqrt{\frac{\gamma_{He} RT}{M_{He}}} \quad (3)$$

Dividing equation (3) by equation (2), we get,

$$\frac{v_{He}}{v_H} = \sqrt{\frac{\gamma_{He} RT}{M_{He}}} \times \sqrt{\frac{M_H}{\gamma_H RT}} \Rightarrow \frac{v_{He}}{v_H} = \sqrt{\frac{M_H \gamma_{He}}{M_{He} \gamma_H}} \quad (4)$$

Putting the values in equation (4), we get,

$$\frac{v_{He}}{v_H} = \sqrt{\frac{2 \times 1.66}{4 \times 1.4}} = \sqrt{\frac{3.32}{5.6}} = 0.77$$

$$\Rightarrow \frac{v_{He}}{v_H} = 0.77 \Rightarrow v_{He} = 0.77v_H \Rightarrow \boxed{v_{He} = 0.77v_H}$$

Q7: The speed of sound in air at 0°C is 332ms^{-1} . What will be the speed of sound at 22°C ? (345.2ms^{-1})

Solution:

Given data:

Initial temperature $= T_0 = 0^\circ\text{C} = 273\text{K}$

Speed of sound at $0^\circ\text{C} = v_0 = 332\text{m/sec}$

Final temperature $= T = 22^\circ\text{C} = 22 + 273 = 295\text{K}$

Required: Speed of sound at $22^\circ\text{C} = v = ?$

Calculation: We know that,

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}} \Rightarrow v = \sqrt{\frac{T}{T_0}} \times v_0 \quad (1)$$

Putting the values in equation (1), we get,

$$v = \sqrt{\frac{295}{273}} \times 332 \Rightarrow \boxed{v = 345.2\text{m/sec}}$$

$$\text{Velocity} = v = 345.2\text{m/sec}$$

Q8: Two tuning forks "P" and "Q" give 4 beats per second. On loading "Q" lightly with wax, we get 3 beats per second. What is the frequency of "Q" before and after loading if the frequency of "P" is 512Hz . (516Hz , 515Hz)

Solution:

Given data: Number of beats before loading $= n = 4$

Number of beats after loading $= n' = 3$

Frequency of tuning fork "P" $= f_p = 512\text{Hz}$

Required:

(i) Frequency of tuning fork "Q" before loading $= f_q = ?$

(ii) Frequency of tuning fork "Q" after loading $= f'_q = ?$

Calculation:

(i) We know that, $n = f_2 - f_1$

$$\Rightarrow n = f_q - f_p$$

$$\text{Put } n = 4, f_p = 512\text{Hz}$$

$$\Rightarrow 4 = f_q - 512 \Rightarrow f_q = 4 + 512$$

$$\Rightarrow \boxed{f_q = 516\text{Hz}}$$

(ii) After loading, we put $n' = 3, f_p = 512\text{Hz}$

So we get, $n' = f'_q - f_p$

$$\Rightarrow 3 = f'_q - 512 \Rightarrow f'_q = 3 + 512 \Rightarrow \boxed{f'_q = 515\text{Hz}}$$

Q9: On a sunny day, the speed of sound in the air is 340ms^{-1} , tuning forks A and B are sounded simultaneously. The wavelength of the sounds emitted are 1.5m and 1.68m respectively. How many beats will produce per second? (24 beats approx)

Solution: Given data:

Speed of sound $= v = 340\text{m/sec}$

Wavelength of tuning fork "A" = $\lambda_A = 1.5\text{m}$

Wavelength of tuning fork "B" = $\lambda_B = 1.68\text{m}$

Required: Number of beats = $n = ?$

Calculation: We know that, $n = f_A - f_B$

$$\Rightarrow n = \frac{v}{\lambda_A} - \frac{v}{\lambda_B} = \left[\frac{1}{\lambda_A} - \frac{1}{\lambda_B} \right] \Rightarrow n = 340 \left[\frac{1}{1.5} - \frac{1}{1.68} \right]$$

$$\Rightarrow n = 340 [0.666 - 0.595] = 340(0.071)$$

Ans. $n = 24$ beat approximately

$$v = f\lambda, f = \frac{v}{\lambda}$$

Q10: A sound source vibrates at 200Hz and is receding from a stationary observer at 18ms^{-1} . If the speed of sound is 331ms^{-1} then what frequency does the observer hear? (189.68Hz)

Solution:

Given data: Frequency of sound source = $f = 200\text{Hz}$

Velocity of sound = $v = 331\text{m/sec}$,

Velocity of source = $a = 18\text{m/sec}$

Required: Apparent frequency = $f' = ?$

Calculation: When sound source is moving away from the stationary observer [listener], then we have,

$$f' = \left(\frac{v}{v + a} \right) f \quad (1)$$

Putting the values in equation (1), we get,

$$f' = \frac{331 \times 200}{(331 + 18)} \Rightarrow f' = 189.68\text{Hz}$$

Ans. $f' = 189.68\text{Hz}$

Q11: Suppose a train that has a 150Hz horn is moving at 35.0m/s in still air on a day when the speed of sound is 340m/s . (a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes? (b) What frequency is observed by the train's engineer traveling on the train? (167Hz, 136Hz)

Solution:

Given data: Frequency of horn = $f = 150\text{Hz}$,

Velocity of train = $a = 35\text{m/sec}$

Velocity of sound = $v = 340\text{m/sec}$

Required:

(a) Apparent frequency of horn, when

(i) Train approaches the person = $f' = ?$

(ii) Train moving away from person = $f'' = ?$

(b) Frequency observed by engineer traveling on the train = ?

Calculation:

(a) (i) When train approaches the person, then the apparent frequency is given by,

$$f' = \left(\frac{v}{v - a} \right) f \quad (1)$$

Putting the values in equation (1), we get,

$$f' = \left(\frac{340}{340 - 35} \right) \times 150\text{Hz} = \left(\frac{340 \times 150}{305} \right) \text{Hz} \Rightarrow f' = 167\text{Hz}$$

(ii) When train is moving away from person, then equation (1) can be written as,

$$f'' = \left(\frac{v}{v + a} \right) f \quad (1)$$

Putting the values in equation (2), we get,

$$f'' = \left(\frac{340}{340 + 35} \right) 150\text{Hz} = \left(\frac{340 \times 150}{375} \right) \Rightarrow f'' = 136\text{Hz}$$

(b) As engineer is present on the train, so both are the part of the same system. So there will be no apparent change for engineer in frequency of horn.

Q12: The first overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe 3.6m in length. What is the length of the open organ pipe? (7.2m)

Solution:

Let the frequency of 1st overtone of an open organ pipe = f_1 and let the frequency of 1st overtone of the closed pipe = f'_1

Length of open organ pipe = $L = ?$

Length of closed organ pipe = $L' = 3.6\text{m}$

By given condition,

$$f_1 = f'_1 \Rightarrow \frac{v}{\lambda_1} = \frac{v}{\lambda'_1} \Rightarrow \frac{v}{2L} = \frac{v}{4L'} \Rightarrow \frac{1}{2L} = \frac{1}{4L'} \Rightarrow 2L = 4L' \Rightarrow L = 2L' = 2 \times 3.6 \Rightarrow L = 7.2\text{m}$$

For open pipe,
 $\lambda_1 = 2L$
For closed pipe,
 $\lambda'_1 = 4L'$

Q13: What length of open pipe will produce a frequency of 1200Hz as its first overtone on a day when the speed of sound is 340ms⁻¹? (28.3cm)

Solution: Given data: Speed of sound = $v = 340\text{m/sec}$

First overtone = $f_2 = 1200\text{Hz}$

Required:

Length of open pipe = L ?

Calculation:

In case of 1st overtone, we have,

$$f_2 = 2 \left(\frac{v}{2L} \right) \Rightarrow f_2 = \frac{v}{L} \Rightarrow L = \frac{v}{f_2} \quad (1)$$

Putting the values in equation (1), we get,

$$L = \left(\frac{340}{1200} \right) \text{m} = 0.283\text{m}$$

$$\Rightarrow L = (0.283 \times 100)\text{cm}$$

$$100\text{cm} = 1\text{m}$$

$$\Rightarrow L = 28.3\text{cm}$$

Ans: Length of open pipe = $L = 28.3\text{cm}$

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- ① رسول اللہ ﷺ نے ارشاد فرمایا: اللہ تعالیٰ نے تمہاری شفاعت میں چیزیں نہیں رکھی۔
- ② رسول اللہ ﷺ نے ارشاد فرمایا: جس شخص کے جسم سے روح اس حال میں نکلی کہ وہ تین چیزوں سے بڑی تھا تو وہ جنت میں داخل ہوگا: تکبر، قرض اور غیبت۔
- ③ ہم تکلیف کے جس مقام پر بھی ہوں، ہم سے آگے ضرور کچھ لوگ ہوتے ہیں۔ انہیں دیکھ کر ہمیں شکر کی لاطی تمام لینی چاہیئے تاکہ راستہ سہولت سے کٹ سکے۔
- ④ اگر تم اس وقت مسکرا سکتے ہو جب تم پوری طرح ٹوٹ چکے ہو تو یقین جانو کہ دنیا میں تمہیں کبھی کوئی تڑپ نہیں سکتی۔
- ⑤ خوبصورتی تو قسم اور تبدیل ہو جاتی ہے لیکن سیرت نہ تو قسم ہوتی ہے نہ تبدیل۔
- ⑥ مشکوں سے لڑنا نہیں بلکہ سنبھل کر گزرنا سیکھو ورنہ آپ کی شخصیت تباہ ہو جائے گی۔
- ⑦ حضرت عثمان غنی کا قول ہے "محبوبین کی کمالیں بھی دل کی طرح نرم ہو جاتی ہیں۔ ان کے روگئے کھڑے ہو جاتے ہیں۔ ان کے دل اور جلد نرم ہو جاتے ہیں اور یاد خدا سے ان کو راحت ہوتی ہے۔"

Q1: What is meant by the dual nature of light? Discuss the history about the nature of light in detail.

Ans: Dual Nature of Light:

Light is a form of energy with the help of which we can see objects in our surrounding. The light shows dual nature i.e. in some cases, it behaves like a wave and in some cases it behaves like a particle. In case of Compton effect, photoelectric and pair production phenomenon's, the light behaves like a particle. While in case of interference, polarization, reflection, diffraction, the light behaves like a wave. Thus the light shows dual nature.

History about the Nature of Light:

As the light shows dual nature, so in the beginning it was a great problem for the scientists to explain its real nature. The scientists gave their ideas about light in the form of following theories:

1. Newton's Corpuscular Theory of Light:

The corpuscular theory of light was put forward by Newton. It is also known as particle theory of light. According to Newton, light travels from one place to another in the form of extremely small particles known as corpuscles. On the basis of this theory, Newton was able to explain the phenomenon of reflection and refraction of light.

2. Wave Theory of Light:

This theory was put forward by Huygen. According to this theory, light travels from one place to another in the form of waves. This theory was able to explain reflection and refraction. However, this theory was failed to explain the phenomenon of polarization, diffraction and interference of light.

3. Maxwell's Theory of Light:

This theory was put forward by Maxwell in 1873. According to