$(K.E)_{linear} + (K.E)_{rot}$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad \text{\_\_\_\_\_\_\_\_(1)}$$

For disc, we have, $I = \frac{1}{2}mr^2$ ________(2)

Also, $v = r\omega \Rightarrow \omega = \frac{v}{r}$ ________(3)

Putting equations (2), (3) in equation (1), we get,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \frac{v^2}{r^2}$$

$$\Rightarrow gh = \frac{v^2}{2} + \frac{v^2}{4} = \frac{3v^2}{4} \Rightarrow gh = \frac{3v^2}{4} \Rightarrow v^2 = \frac{4gh}{3}$$

$$\Rightarrow v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4 \times 9.8 \times 10}{3}}$$

$$\Rightarrow \boxed{v = 11.4m / sec}$$

Q12: A motor car is travelling at a speed of 30m/sec. If its wheel has a diameter of 1.5m, find its angular speed in rad/sec and in rev/sec.

Solution:

Speed of the car = v = 30m/sec, diameter of wheel = d = 1.5m

Radius of wheel = $r = \frac{d}{2} = \frac{1.5}{2} = 0.75m$

Required:

(a) Angular speed in rad/sec = ω = ?

(b) Angular speed in rev/sec = ω = ?

Calculation:

(a) We know that, $v = r\omega$

$$\Rightarrow \omega = \frac{v}{r} = \frac{30}{0.75} \Rightarrow \boxed{\omega = 40rad / sec}$$

(b) $\omega = 40rad / sec$

$$\boxed{1rev = 2\pi rad \Rightarrow 1rad = \frac{1}{2\pi}rev}$$

$$\Rightarrow \omega = 40 \times \frac{1}{2\pi} rev / sec$$

$$\omega = 6.369 rev/sec \Rightarrow \boxed{\omega = 6.37rev / sec}$$

+++

- اچھی صورت کے مقابلے میں اچھی سیرت کا رتبہ بلند ہوتا ہے۔
- جو کسی کا برا نہیں چاہتے، ان کے ساتھ کوئی برا نہیں کر سکتا، یہ میرے رب کا وعدہ ہے۔

CHAPTER 06 FLUID DYNAMICS

Q1: Define the following terms: (a) Fluid dynamics, hydrostatics, (c) Fluid friction, (d) Ideal fluid

Ans: **(a) Fluid:**

A substance which can flow is known as fluid. For example, oil, milk, water, gas etc are fluids.

(b) Hydrostatics and Fluid Dynamics:

The study of fluid at rest is called hydrostatic. While the study of properties and effect of moving fluids is known as fluid dynamics. In fluid dynamics, 'hydrodynamics' deals with the study of liquids in motion, while 'aerodynamics' deals with the study of gases in motion.

(c) Fluid Friction:

When a body moves through a fluid, then an opposing force acts on the body. This opposing force is known as fluid friction or dragging force. This force depends upon the:

- Size of the body
- Geometric shape of the body
- Direction of velocity of the body and
- Viscosity of the fluid

(d) Ideal Fluid: (ETEA 2016)

An incompressible, non-viscous fluid having constant density at any pressure, is called an ideal fluid.

Q2: What is a fluid? Discuss briefly, the viscosity of a fluid and explain how the flow of a fluid is characterized by viscosity.

Ans. **Fluid:**

A substance which flows is known as fluid. For example, oil, water, gas etc.

Viscosity of a Fluid:

The resistance offered by different layers of a fluid to its flow is known as viscosity. When fluids are in motion, the viscosity is a key characteristic of fluids, which determines the rate of flow of a fluid.

Coefficient of Viscosity:

The numerical value of resistance to the flow of fluids between its different layers is called coefficient of viscosity. Greater the coefficient of viscosity of a fluid, the fluid is said to be more viscous and will offer greater dragging force. The coefficient of viscosity is represented by the symbol "η". Its numerical values for different fluids are given in the table given below:

Fluid Name	Viscosity (Pa·S)	Fluid Name	Viscosity (Pa·S)
Air	1.8×10^{-5}	Acetone	2.9×10^{-4}
Methanol	5.1×10^{-4}	Benzene	1.00×10^{-3}
Water	8.91×10^{-4}	Ethanol	1.00×10^{-3}
Blood	1.6×10^{-3}	Honey	1.42

Effect of Temperature on Viscosity:

The viscosity of a fluid depends upon the temperature of fluid. In case of liquids, the viscosity is produced due to intermolecular forces of attraction. If we increase the temperature, the intermolecular forces decrease and hence viscosity decreases.

In case of gases, the viscosity depends upon the velocity of molecules. Greater the velocity of molecules of gases, smaller will be the viscosity and vice versa. If we increase the temperature of the gas, the velocity of molecules of the gas increases and hence viscosity decreases.

Units of Viscosity:

The SI unit of viscosity is Pascal second (Pa·S), Dyne/cm². Poise is another unit of viscosity.

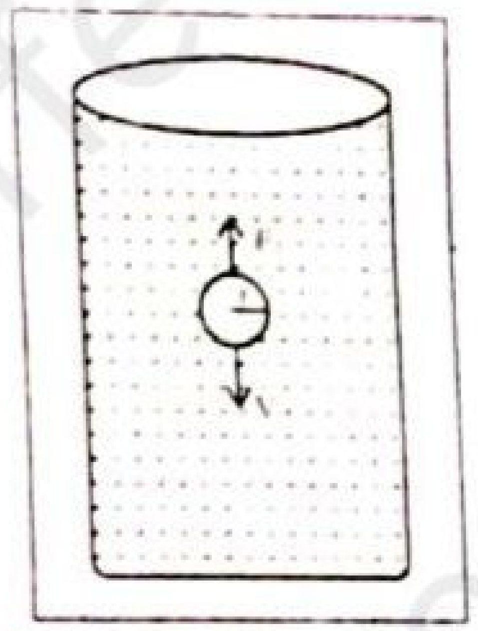
$\boxed{1Pa\cdot S = 1\text{ poise},\ 1\text{centipoise} = 10^{-3}Pa\cdot S}$

Q3: (a) State and explain stokes law. (b) Derive a mathematical equation for acceleration and terminal velocity by using stokes law.

Ans. (a) Stokes Law:

Statement:

This law states that the dragging force "F_D" acting on a spherical body inside a fluid is directly proportional to the radius of spherical body, velocity of spherical body and viscosity of the fluid.



Explanation:

Stoke did his research and study on spherical shaped objects of uniform density. Let a spherical body of radius "r" is allowed to move slowly with velocity "v" inside a fluid of viscosity "η" as shown in the figure.

Now according to stokes law, we have,

$F_D \propto r$ ________(1)

$F_D \propto v$ ________(2)

And $F_D \propto \eta$ ________(3)

Combining relations (1), (2), (3), we get,

$F_D \propto \eta r v \Rightarrow F_D = \text{constant}\cdot\eta r v$

$\Rightarrow \boxed{F_D = 6\pi\eta r v}$ ________(4) $[6\pi = \text{constant}]$

Dimension of viscosity:

$F_D = 6\pi\eta r v$ (ETEA 2011)

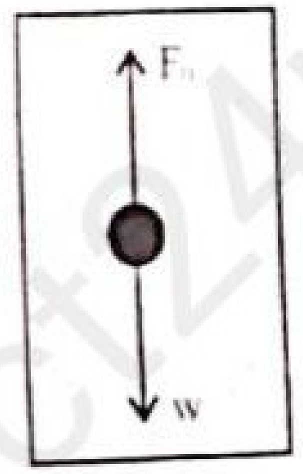
$$\Rightarrow \eta = \frac{F_D}{6\pi r v} = \frac{kg\cdot\frac{m}{sec^2}}{m\,\frac{m}{sec}}$$

$$\Rightarrow \eta = kg\cdot\frac{m}{sec^2}\cdot\frac{sec}{m^2}$$

$$\Rightarrow \eta = kg/sec\cdot m$$

$$\Rightarrow \eta = [MT^{-1}L^{-1}]$$

Equation (4) represents the dragging force acting on spherical body inside a fluid.



(b) Calculation of Acceleration of Spherical Droplet:

(ETEA 2009)

When a spherical body [droplet] is moving through a fluid, then its weight "w" is acting downwards while the dragging "F_D" is acting upwards. So the net force on the body is given by,

$F_{net} = w - F_D \Rightarrow ma = mg - 6\pi\eta r v$

$$\Rightarrow a = \frac{mg - 6\pi\eta r v}{m}$$

$$\Rightarrow \boxed{a = g - \frac{6\pi\eta r v}{m}} \quad \text{________(5)}$$

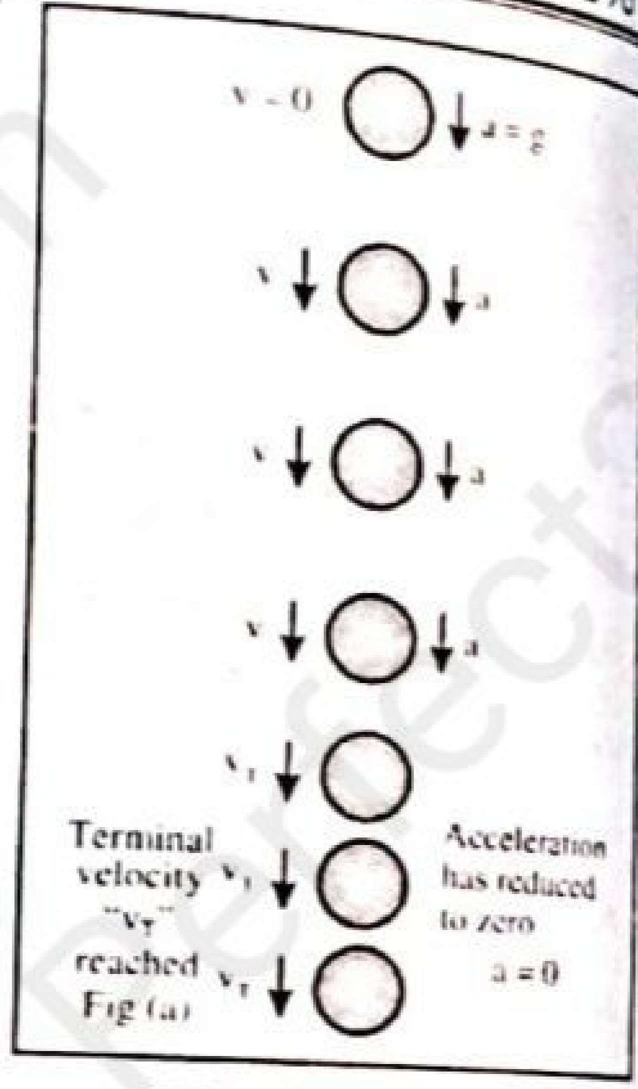
$\left[\begin{array}{l} F_{net} = ma \\ w = mg \\ F_D = 6\pi\eta r v \end{array}\right]$

Equation (5) represents the acceleration of the droplet moving through the fluid.

Calculation of Terminal Velocity:

The maximum constant velocity of a body, when forward directed forces are balanced by backward directed forces is known as terminal velocity of that body.

When a droplet is moving downward its velocity increases until a stage comes, when the weight of the droplet and dragging force balance each other. At this stage, there is no further increase in velocity and the body moves down with constant velocity. Such velocity is known as terminal velocity. At this stage, we put $a = 0$ and $v = v_T$ so equation (5) becomes,



$$0 = g - \frac{6\pi\eta r v_T}{m}$$

$$\Rightarrow \frac{-6\pi\eta r v_T}{m} = -g \Rightarrow 6\pi\eta r v_T = mg$$

$$\Rightarrow \boxed{v_T = \frac{mg}{6\pi\eta r}} \text{———(6)}$$

For spherical body, volume $= \frac{4}{3}\pi r^3$

Equation (6) represents the terminal velocity in terms of mass of the droplet.

Rain drops falling from 5kg reach the ground with terminal velocity. ETEA 2010

Terminal Velocity in terms of Density:

We know that,

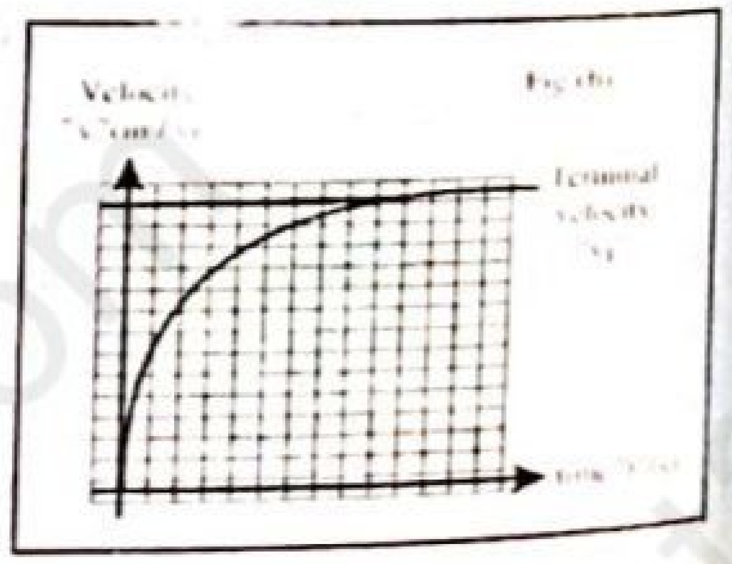
Mass = density x volume

$$\Rightarrow m = \delta \times \frac{4}{3}\pi r^3 \text{———(7)}$$

Putting equation (7) in equation (6), we get,

$$v_T = \frac{4\pi\delta r^3 g}{18\pi\eta r}$$

$$\Rightarrow \boxed{v_T = \frac{2\delta g r^2}{9\eta}} \text{———(8)}$$



Equation (8) represents the terminal velocity of the droplet in terms of density.

Graphical Representation of Terminal Velocity:

The graphical representation of terminal velocity is shown in the figure (b). The graph shows that the velocity increases with the passage of time and finally becomes constant.

Object	Mass (kg)	Cross-sectional Area (m^2)	Terminal speed (m/s)
Sky diver	70	0.70	54
Base ball (radius 3.7cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1cm)	0.046	1.4×10^{-3}	44
Hail stone (radius 0.5cm)	4.8×10^{-4}	7.9×10^{-5}	14
Rain drop (radius 0.2cm)	3.4×10^{-5}	1.3×10^{-5}	09

Q4: Discuss the terminal velocity of a paratroopers?

ETEA 2009

Ans. Terminal Velocity of a Paratrooper:

The constant velocity gained by a paratrooper falling under the action of gravity at the stage when drag force on his body become equal to his weight is known as terminal velocity of the paratrooper. It is denoted by v_T. Paratrooper gains terminal velocity twice.

Explanation:

At $t = 0$ when a paratrooper jumps from an aeroplane, at the beginning the air drag force acting on his body is less than his weight and the net force is given by,

$$F_n = w - F_d \text{———(1)}$$

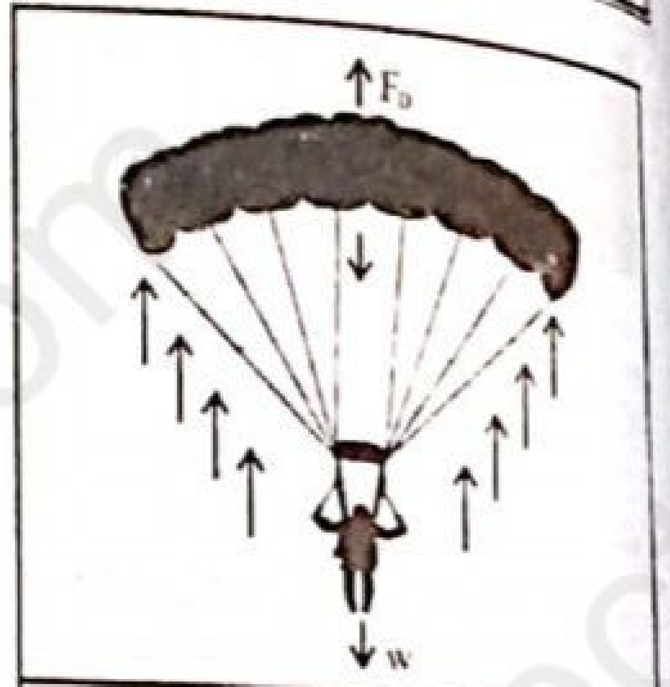
$$\Rightarrow ma = w - F_d$$

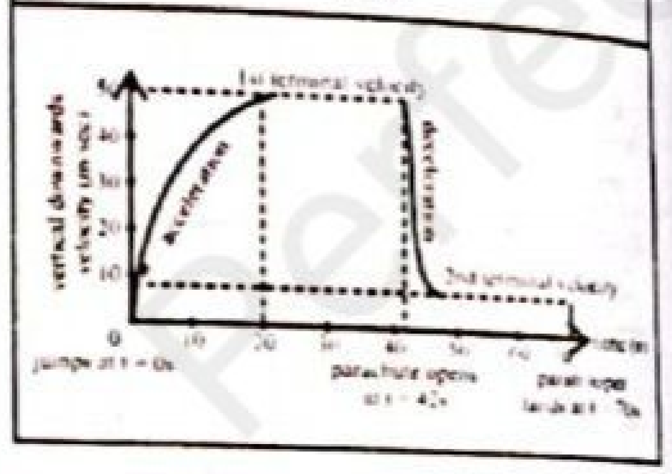
$$a = \frac{w - F_d}{m} \text{———(2)}$$

With this acceleration the paratrooper moves down which is shown in the v-t graph.

1st Terminal Velocity:

But as the paratrooper moves down, his velocity increases due to the above acceleration. Since drag force is directly proportional to velocity, so drag force also increases. A stage is reached when drag force becomes equal to the weight of paratrooper. At this stage the paratrooper gains maximum constant velocity which is called 1st terminal velocity. In the graph the paratrooper gains first terminal velocity of 50m/s after 20 seconds.





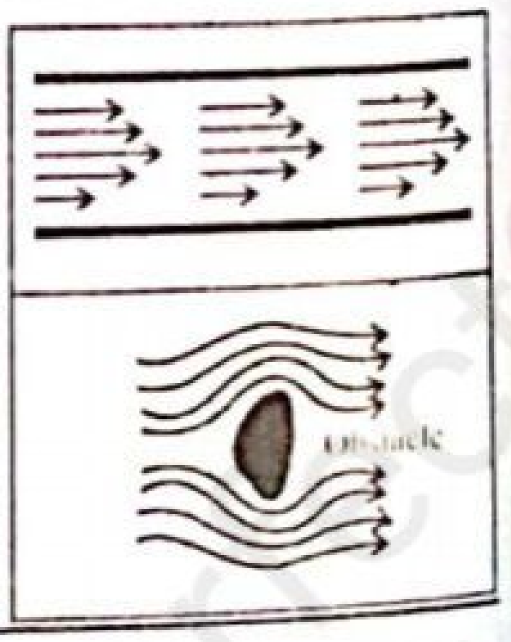
2nd Terminal Velocity:

When a paratrooper opens his parachute, it dramatically increases the area of intercept and the air resistance force becomes much greater than the weight of paratrooper. This causes a rapid deceleration in the 1st terminal velocity of the paratrooper as shown in the graph. But as the speed of the paratrooper decreases, the air drag force also decreases and once again a stage is reached when drag force becomes equal to the weight of paratrooper. This time the paratrooper gains 2nd terminal velocity which is slow enough and he lands safely on the ground. In graph the parachute is opened after 40sec and large drag force decelerates his motion. Within 50sec or so the paratrooper gains 2nd terminal velocity which is quite low and nearly 8m/sec.

Q5: Explain the difference between stream line flow and turbulent flow of fluids.

Ans. 1. **Stream Line Flow (or) Laminar Flow:**

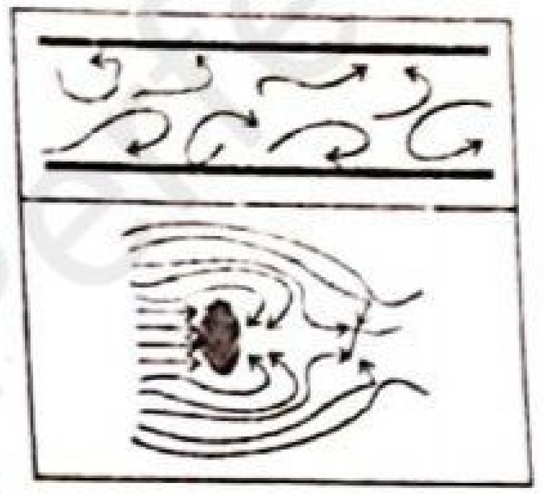
The flow of a fluid in which layers of fluid slide smoothly past each other is known as stream line flow or laminar flow.



The pattern of laminar flow is shown in the figure. Such flow occurs in steady state. If an obstacle is placed in the path of the fluid, then the fluid does not cross the obstacle and its adjacent layer slide smoothly past each other.

2. **Turbulent Flow:**

The flow of a fluid in which there is a greater disorder and the layer of fluid does not slide smoothly past each other is known as turbulent flow.



The pattern of turbulent flow is shown in the figure. If an obstacle is placed in front of the fluid, then the adjacent layers of the fluid do not slide smoothly past each other. The turbulent flow does not occur in steady state and continuous changes occur in the pattern of the flow.

Q6: State the equation of continuity. Show that how it is based on law of conservation of mass? Give two examples from daily life which are related with the principle of equation of continuity.

ETEA 2015,2016, BISE Mardan 2019, BISE Bannu 2017

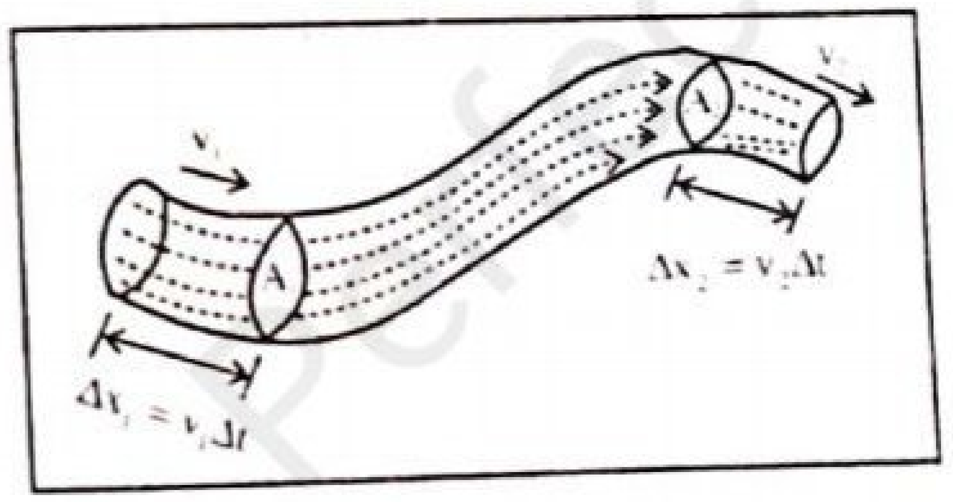
Ans. **Statement:** It states that for an incompressible fluid in steady flow, the net rate of flow of mass inward across any closed surface is equal to the net rate of flow of mass outward.

Explanation:

Consider a pipe having non-uniform area of cross-section "A_1" and "A_2", as shown in the figure.

Let "V_1" and "V_2" be the speed of a steady fluid at "A_1" and "A_2" respectively. Now the distance covered by the fluid across "A_1" during time interval "Δt" is given by,

$$\Delta x_1 = V_1 \Delta t \quad \text{------} (1)$$



Now the volume of the fluid through the cross-sectional area "A_1" is given by,

$$\Delta V_1 = A_1 \cdot \Delta x_1 \quad \text{------} (2)$$

Putting equation (1) in equation (2), we get,

$$\Delta V_1 = A_1 \cdot V_1 \Delta t \quad \text{------} (3)$$

Similarly, the volume of the fluid across "A_2" during time interval "Δt" is given by,

$$\Delta V_2 = A_2 \cdot V_2 \Delta t \quad \text{------} (4)$$

Now the mass of the fluid across "A_1" is given by,

$\Delta m_1 = \delta \Delta V_1$ ______________(5)

Putting equation (3) in equation (5), we get,

$\Delta m_1 = \delta A_1 V_1 \Delta t$ ____________(6)

Similarly, the mass of the fluid across "A_2" is given by,

$\Delta m_2 = \delta A_2 V_2 \Delta t$ ____________(7)

Now in case of steady flow, the total mass of the fluid in the pipe remains constant. So we have,

$\Delta m_1 = \Delta m_2 \Rightarrow \delta A_1 v_1 \Delta t = \delta A_2 v_2 \Delta t$

$\Rightarrow \boxed{A_1 v_1 = A_2 v_2}$ ____________(8)

Equation (8) represents the equation of continuity. Equation (8) shows that the volume flow rate across any cross-section of flow tube remains constant. The equation of continuity is the mathematical expression of the principle of conservation of mass. This principle states that for steady state flow, the mass flow into the volume must be equal to the mass flow rate out.

Examples related to equation of continuity:

- Flowing of fluid in pipes
- Flowing of gases in pipes and tubes
- Flowing of blood in aorta and capillaries

Q7: Show that the speed of fluid through any pipe is inversely proportional to the cross-sectional area of the pipe.

BISE Malakand 2019, BISE Peshawar 2019

Ans. From equation of continuity, we have,

$A_1 v_1 = A_2 v_2 \Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1}$ ____________(1)

In general form, equation (1) can be written as,

$\boxed{v \propto \frac{1}{A}}$ ____________(2)

Relation (2) shows that the speed of fluid through any pipe is inversely proportional to the cross-sectional area of the pipe.

The deeper part of a river has larger cross-sectional area as compare to shallow part of the river. So according to relation (2), the deep water flows slowly in comparison to shallow water. However, the volume flow rates in both cases are the same.

Q8: Show that the volume flow rate (or) time rate of flow of volume is constant.

ETEA 2015

Ans. According to the equation of continuity, we have,

$A_1 V_1 = A_2 V_2 =$ constant ____________(1)

Generally equation (1) can be written as,

$\Rightarrow AV =$ constant ____________(2)

In equation (2), we have,

$V =$ velocity = rate of change of displacement

$\Rightarrow V = \frac{\Delta x}{\Delta t}$ ____________(3)

Putting equation (3) in equation (2), we get,

$A \cdot \frac{\Delta}{\Delta t} x =$ constant $\rightarrow \frac{\Delta}{\Delta t}(A \cdot x) =$ constant ____________(4)

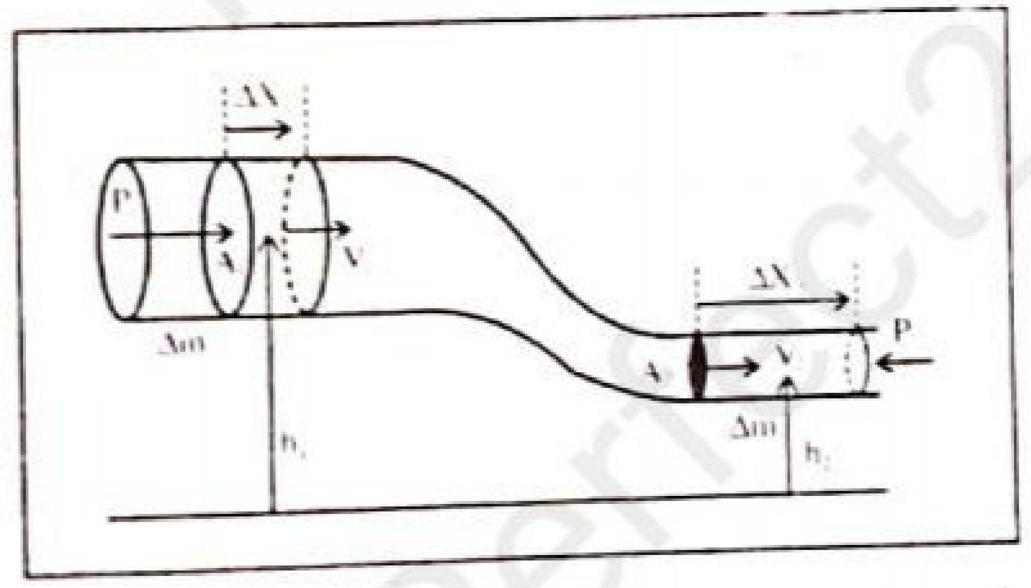
In equation (4), $A \cdot x =$ volume $= V$, so equation (4) becomes,

$\boxed{\frac{\Delta}{\Delta t} V = \text{constant}}$ ____________(5)

Equation (5) shows that the volume flow rate [time rate of flow of volume] is constant. In other words we can say that, the volume of an incompressible fluid passing through any point in unit time through a pipe of non-uniform cross-section is constant in the steady flow.

Q9: What is Bernoulli's equation? Show that how it is based on the law of conservation of energy?

ETEA 2010, BISE Mardan 2019, BIE DI Khan 2019



Ans. The Bernoulli's equation was derived by a Swiss physicist Daniel Bernoulli in 1738. This equation represents the relationship between pressure, speed and elevation of an ideal fluid.

Explanation:

Consider an incompressible, non-viscous fluid flowing in a pipe in steady state manner. Let "v_1" and "v_2" be the speed of fluid at cross-sectional area "A_1" and "A_2" respectively. The workdone on the fluid at "A_1" and "A_2" are respectively given as,

$\Delta w_1 = F_1 \cdot \Delta x_1$ ____________(1)

& $\Delta w_2 = F_2 \cdot \Delta x_2$ ____________(2)

Now the net workdone on the fluid is given by,

$\Delta w = \Delta w_1 - \Delta w_2$ ____________(3)

Putting equations (1), (2) in equation (3), we get,

$$\Delta w = F_1\Delta x_1 - F_2\Delta x_2 \text{\_\_\_\_\_\_\_\_\_\_} (4)$$

Now we know that,

$$P = \frac{F}{A} \Rightarrow P_1 = \frac{F_1}{A_1} \Rightarrow F_1 = P_1A_1 \text{\_\_\_\_\_} (5)$$

Similarly, $F_2 = P_2A_2$ __________ (6)

Putting equations (5), (6) in equation (4), we get,

$$\Delta w = P_1A_1\Delta x_1 - P_2A_2\Delta x_2 \text{\_\_\_\_\_\_\_\_\_\_} (7)$$

Here, $A_1\Delta x_1$ = volume = v_1

Similarly, $A_2\Delta x_2 = v_2$

So equation (7) becomes,

$$\Delta w = P_1v_1 - P_2v_2 \text{\_\_\_\_\_\_\_\_\_\_} (8)$$

For incompressible fluid, we have,

$$v_1 = v_2 = v$$

So equation (8) becomes,

$$\Delta w = P_1v - P_2v$$

$$\Rightarrow \Delta w = (P_1 - P_2)v \text{\_\_\_\_\_\_\_\_\_\_} (9)$$

We know that, mass = density × volume

$$\Rightarrow m = \delta \times v \Rightarrow v = \frac{m}{\delta}$$

So equation (9) becomes,

$$\Delta w = (P_1 - P_2)\frac{m}{\delta} \text{\_\_\_\_\_\_\_\_\_\_} (10)$$

Now according to work-energy theorem, we have,

$$\Delta w = \Delta K.E + \Delta P.E \text{\_\_\_\_\_\_\_\_\_\_} (11)$$

Here $\Delta K.E = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ __________ (12)

& $\Delta P.E = mgh_2 - mgh_1$ __________ (13)

Putting equations (12), (13) in equation (11), we get,

$$\Delta w = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1 \text{\_\_\_\_\_\_\_\_\_\_} (14)$$

Comparing equations (10), (14), we get,

$$(P_1 - P_2)\frac{m}{\delta} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

$$\Rightarrow P_1 - P_2 = \delta\left[\frac{v_2^2}{2} - \frac{v_1^2}{2} + gh_2 - gh_1\right]$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\delta v_2^2 - \frac{1}{2}\delta v_1^2 + \delta gh_2 - \delta gh_1$$

$$\Rightarrow \boxed{P_1 + \frac{1}{2}\delta v_1^2 + \delta gh_1 = P_2 + \frac{1}{2}\delta v_2^2 + \delta gh_2} \text{\_\_\_\_\_\_\_\_\_\_} (15)$$

Unit of pressure:

Hint: $P = \frac{F}{A} = \frac{ma}{A}$

$$\Rightarrow P = \frac{kg \cdot m}{m^2 \cdot sec^2}$$

$$\Rightarrow P = kg \cdot m^{-1} \cdot sec^{-2}$$

ETEA 2013

In general form, equation (15) can be written as,

$$\boxed{P + \frac{1}{2}\delta v^2 + \delta gh = constant} \text{\_\_\_\_\_\_\_\_\_\_} (16)$$

Equations (15), (16) represent the Bernoulli's equation. As the Bernoulli's equation is derived by using work-energy theorem, so it is not a free standing law of physics. It is a consequence of the law of conservation of energy, applied to an ideal fluid.

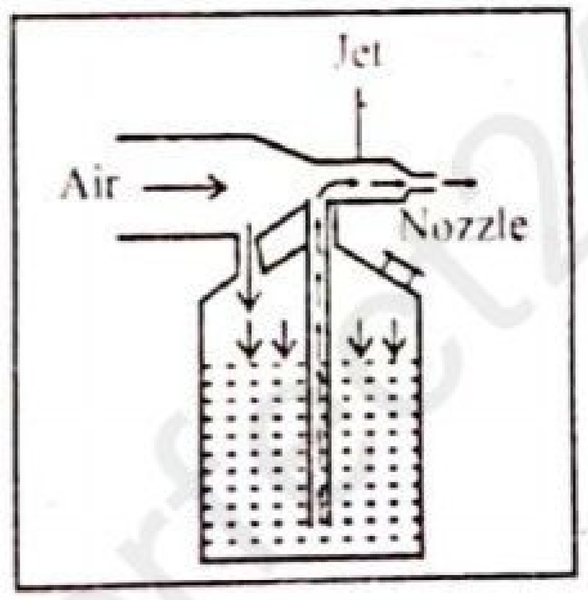
In other words, we can say that the Bernoulli's equation is based on the law of conservation of energy.

Importance of Bernoulli's Equation:

The Bernoulli's equation plays an important role in analyzing plumbing system, hydroelectric generations and flight of aeroplanes. The "jet and nozzle" works on the principle of Bernoulli's equation. Using Bernoulli's equation, we can determine the speed of flow of a fluid through a pipe. The device which is used for the measurement of blood pressure is based on Bernoulli's equation.

Q10: Discuss the applications of Bernoulli's equation.

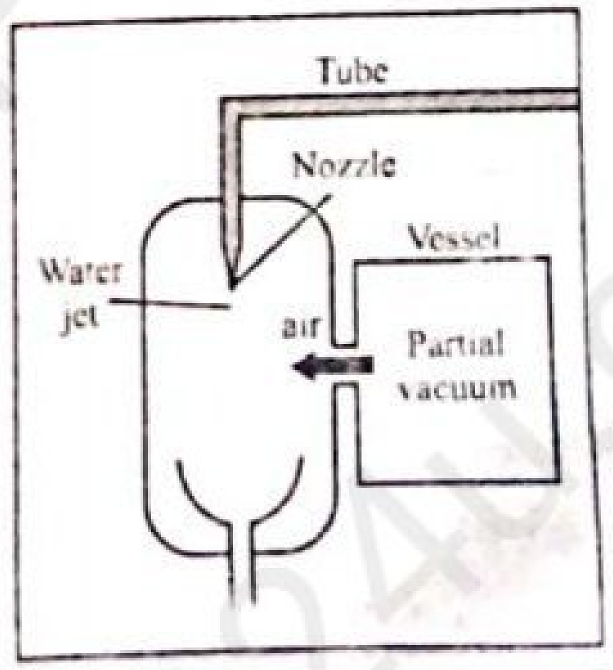
Ans: **1. Jets and Nozzles (or) Atomizers:**



The Bernoulli's equation plays an important role in our daily life. We can apply its principle to many devices like jets and nozzles. For this purpose, the air from the pipe is evacuated due to which the pressure in the pipe decreases. As a result, the velocity of the fluid increases.

The velocity of the fluid can be further increased by minimizing the area of cross-section of the exit opening of the device. Using the above phenomenon, we can draw water, oil etc from a container.

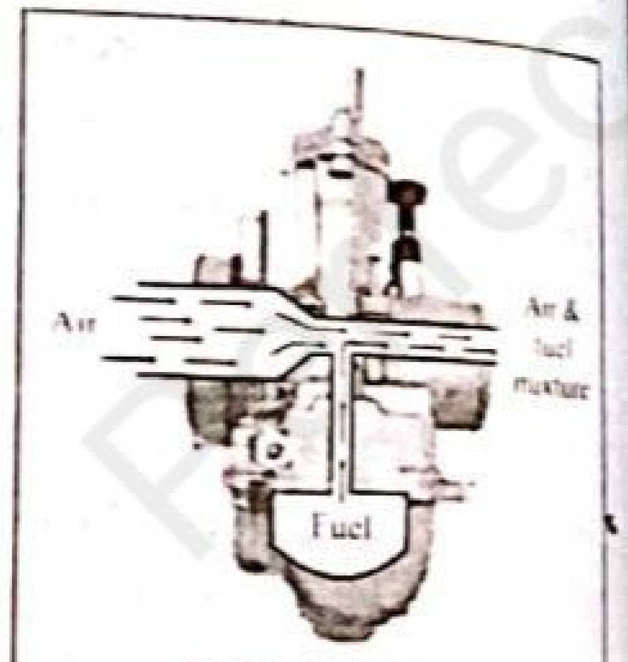
2. Filter Pumps:



Pumps are used to transfer liquids from low-pressure zones to high pressure zones. "A filter pump is a device used to produce partial vacuum in vessel attached to it".

A filter pump consists of a tube with jet attached to it, in which water flows from the tube toward the jet as shown in figure. When water reaches the jet section its speed increases, as a result the pressure drops near it. This drop in pressure allows air to flow in from the side tube to which the vessel is connected, thus air and water are forced together at the bottom of the filter pump. In this way a partial vacuum is created in the vessel attached to it.

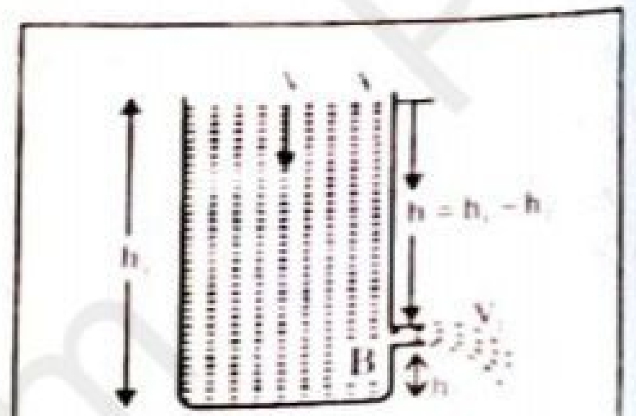
3. Engine Carburetor:



Engine Carburetor

The principle of engine carburetor is the same as that of filter pump. The carburetor of an engine uses the Bernoulli's theorem to feed a suitable amount of mixture of air and fuel to the cylinder. The engine draws air through a horizontal pipe to the cylinder. This pipe is connected to the fuel tank by a side tube. When the air through the pipe is flowing with high speed, the pressure of air inside the pipe decreases. As a result, the fuel moves to the pipe and a proper mixture of the fuel vapours and air is formed which is then sprayed to the engine.

4. Torricelli's Theorem [Speed of Efflux]: ETEA 2011, 2015, BISE Malakand 2019, BISE Peshawar 2019



Statement: This theorem states that the velocity of efflux from an opening at a depth "h" below the top surface of the fluid is equal to the vertical velocity of a freely falling body from height "h".

Explanation:

Torricelli's theorem is a special case of Bernoulli's equation with the help of which we can determine the velocity of a fluid coming out through narrow opening of a container.

Consider a fluid inside a container as shown in the figure. Let there is a narrow opening at point "B" through which the fluid flows out of the container and let "h" be the height of the fluid above point "B". Now we apply Bernoulli's equation to point "A" and "B" of the given figure.

At Point A: We have, P_1 = atmospheric pressure = P

Height $= h_1, v_1 = 0$

At Point B: We have,

$P_2 = P$, height $= h_2$

v_2 = velocity of efflux of the fluid = v

> We put $v_1 = 0$, because the upper surface of fluid move downwards very very slowly.

Now the Bernoulli's equation is given by,

$$P_1 + \frac{1}{2}\delta v_1^2 + \delta g h_1 = P_2 + \frac{1}{2}\delta v_2^2 + \delta g h_2$$

$$\Rightarrow P = 0 + \delta g h_1 = P + \frac{1}{2}\delta v^2 + \delta g h_2$$

$$\Rightarrow \frac{1}{2}\delta v^2 = \delta g h_1 - \delta g h_2 \Rightarrow \frac{1}{2}\delta v^2 = \delta g(h_1 - h_2)$$

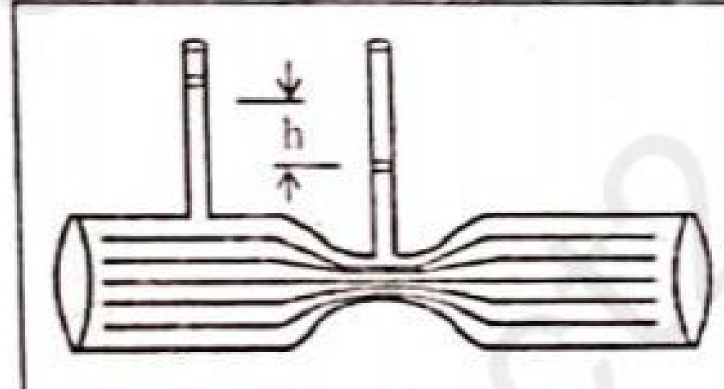
$$\Rightarrow \frac{v^2}{2} = g(h_1 - h_2) \qquad \boxed{h_1 - h_2 = h}$$

$$\Rightarrow v^2 = 2gh \Rightarrow \boxed{v = \sqrt{2gh}} \quad \text{———— (1)}$$

Equation (1) represents the speed of efflux of a fluid. This expression was first obtained by Torricelli and is known as Torricelli's theorem.

The speed of efflux is the same as the speed of a ball that falls freely through a height "h".

5. Flow Meter (or) Venturi Meter:



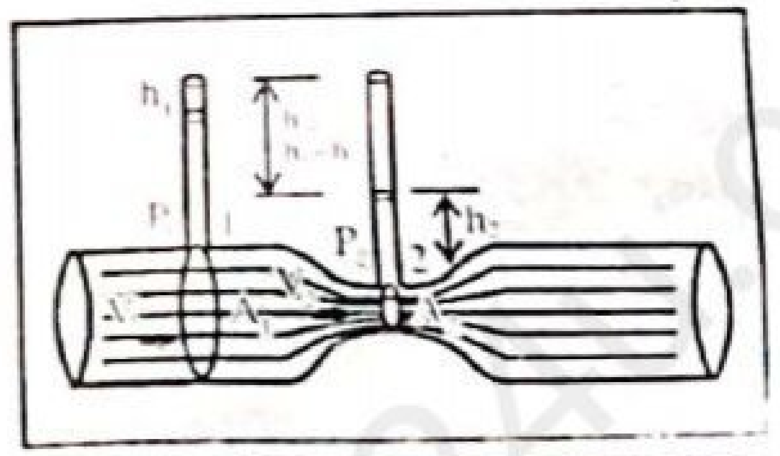
Venturi meter

The venturi meter is a device which works on the principle of Bernoulli's equation and with the help of which we can determine the speed of flow of a fluid through a pipe.

Explanation:

The venturi meter consist of a horizontal tube having a narrow part in the middle, which is known as throat. Now from Bernoulli's equation, we have,

$$P_1 + \frac{1}{2}\delta v_1^2 + \delta g h_1 = P_2 + \frac{1}{2}\delta v_2^2 + \delta g h_2 \quad \text{———— (1)}$$



Here both ends are the same height from ground level, so we put $h_1 = h_2 = h$, so equation (1) becomes

$$P_1 + \frac{1}{2} + \cancel{\delta g h} = P_2 + \frac{1}{2}\delta v_2^2 + \cancel{\delta g h}$$

$$\Rightarrow \quad P_1 + \frac{1}{2}\delta v_1^2 = P_2 + \frac{1}{2}\delta v_2^2$$

$$\Rightarrow \quad P_1 - P_2 = \frac{1}{2}\delta(v_2^2 - v_1^2)$$

$$\Rightarrow \quad v_2^2 - v_1^2 = \frac{2}{\delta}(P_1 - P_2) \text{———} (2)$$

From equation of continuity, we have,

$$A_1v_1 = A_2v_2 \quad \Rightarrow v_2 = \frac{A_1v_1}{A_2} \text{———} (3)$$

Putting equation (3) in equation (2), we get,

$$\frac{A_1^2v_1^2}{A_2^2} - v_1^2 = \frac{2}{\delta}(P_1 - P_2) \Rightarrow v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) = \frac{2}{\delta}(P_1 - P_2)$$

$$\Rightarrow \quad \frac{v_1^2(A_1^2 - A_2^2)}{A_2^2} = \frac{2}{\delta}(P_1 - P_2) \Rightarrow v_1^2 = \frac{2A_2^2}{\delta}\frac{(P_1 - P_2)}{(A_1^2 - A_2^2)}$$

$$\Rightarrow \quad \boxed{v_1 = A_2\sqrt{\frac{2}{\delta}\frac{(P_1 - P_2)}{(A_1^2 - A_2^2)}}} \text{———} (4)$$

Equation (4) represents the speed of fluid in terms of pressure difference.

We also know that,

$$P_1 - P_2 = \delta gh \text{———} (5)$$

Putting equation (5) in equation (4), we get,

"h" represents the difference of liquid level in the two tubes at point "1" and point "2" of the given figure.

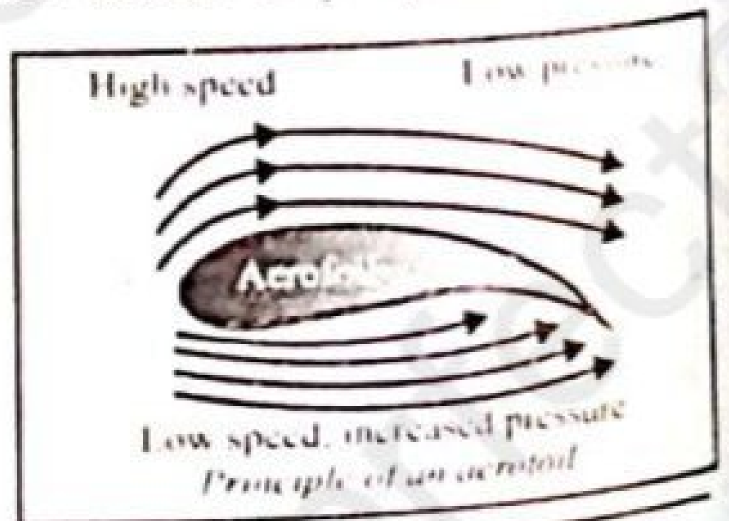
$$v_1 = A_2\sqrt{\frac{2}{\delta}\frac{\delta gh}{(A_1^2 - A_2)}} = A_2\sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}$$

$$\Rightarrow \quad \boxed{v_1 = A_2\sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}} \text{———} (6)$$

Equation (6) represents the speed of fluid in terms of difference of liquid level in the two tubes.

6. Aerofoil [Lift on an Aeroplane Wing]:

It is a device which is designed in such a way that the relative motion between it and a fluid produce a force which is perpendicular to the flow. One of the examples of aerofoil is the aeroplane wing which is designed in such a way that fluid above it flows faster than the fluid flowing below it. As a result, the pressure above the wing becomes minimum and below the wing becomes maximum.

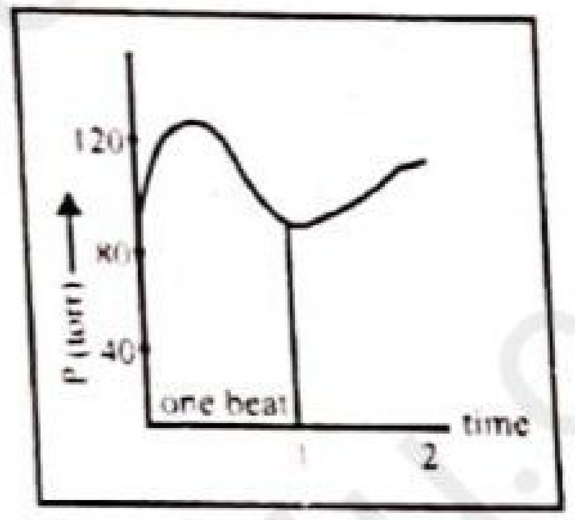


Principle of an aerofoil

In this way, an upwards force exists which is perpendicular to the flow of the fluid. This provides "lift" for the aeroplane.

7. Blood Flow:

Blood is an incompressible fluid having a density nearly equal to that of water. The red blood cells increase the viscosity of blood from 3 to 5 times than that of water. Blood vessels are not rigid but they stretch like the rubber hose. Under normal circumstances the volume of blood is sufficient to keep the vessels in flattened all the times. This means that there is tension in the walls of the blood vessels. As a result the pressure of blood inside the vessel is greater than the external atmospheric pressure.



Variation of Blood Pressure with Heart Beats:

The variation in blood pressure as the heart beats is shown in the figure. The pressure varies from high [systolic pressure] of 120 torr to a low pressure [diastolic pressure] of about 75 – 80 torr between beats in a normal healthy person. The number tends to increase with age due to the decrease in flexibility of vessel walls.

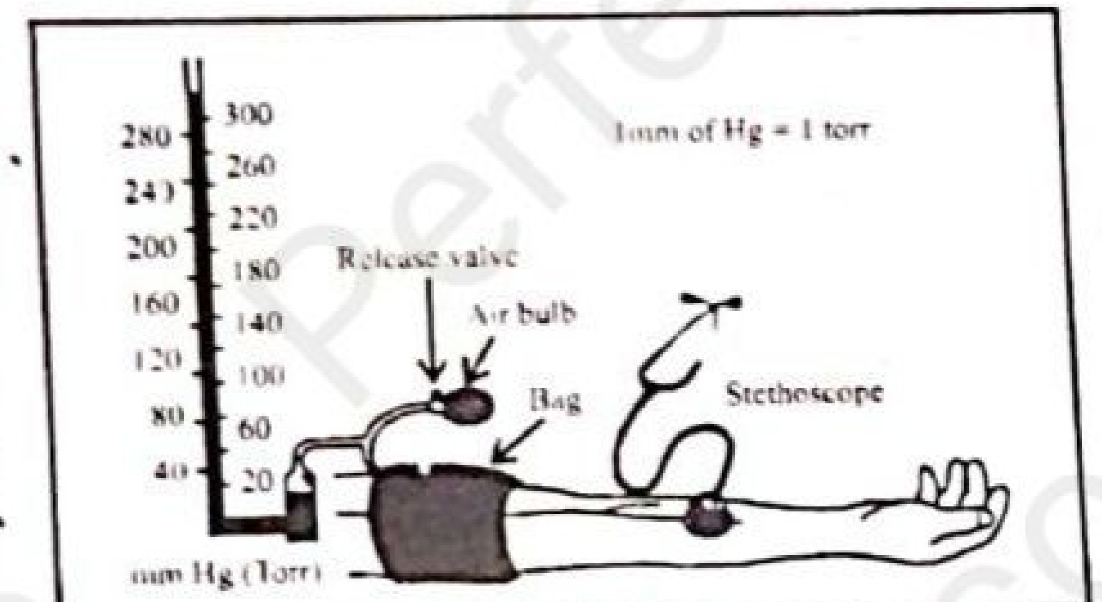
Unit of Blood Pressure:

Torr or millimeter [mm] of Hg [mercury] are the units of blood pressure due to their extensive use in medical equipments. Although the unit of pressure in SI system is Pascal or N/m^2.

$$1\,\text{torr} = 133.3\,N/m^2$$

Blood Flow Meter:

The human blood flows in flexible blood vessels. The blood posses certain pressure due to heart beats. The range of this pressure lies between 75 mm of Hg to 120 mm of Hg. Its high value is known as systolic pressure and its low value is known as diastolic pressure.



For measuring the blood pressure, an inflatable bag is wound around the arm of a person. The external pressure is increased by inflating the bag, due to which the arm squeezes and the blood is allowed to compress inside. When the external pressure becomes larger than systolic pressure, then the vessels collapse and the flow of blood cuts off at this stage.

Now the external pressure is allowed to decrease gradually by opening the release valve. When the external pressure and systolic pressure becomes equal, then at this point the first surges of the blood flow through the narrow stricture with very high speed. This flow occurs in turbulent manner. In this way, the systolic pressure of the person is recorded.

Now when the external pressure decreases further, then a stage comes when the external pressure becomes equal to the diastolic pressure. At this stage, the vessels become fully relaxed and the flow of blood occurs in steady state. i.e. in laminar form. The gurgle in the stethoscope disappears. In this way, the diastolic pressure is recorded.

Solution of Examples & Assignments

EXAMPLE 6.1:

The radius of small fog droplet in air is found to be 5.1×10^{-6} m. The coefficient of viscosity of air is 1.9×10^{-5} kg/m.sec. Find out the settling speed of the droplet in air.

Solution:

Given data: Radius of droplet $= r = 5.1 \times 10^{-6}\, m$

Coefficient of viscosity $= \eta = 1.9 \times 10^{-5}\, kg\, m^{-1} s^{-1}$

Density of water $= \delta = 10^3\, kg/m^3$

Required:

Settling speed of the droplet $= v_T = ?$

Calculation:

We know that, $v_T = \dfrac{2\delta g r^2}{9\eta}$ ——— (1)

Putting the values in equation (1), we get,

$\Rightarrow v_T = \dfrac{[2 \times 10^3 \times 9.8 \times (5.1 \times 10^{-6})^2]}{9 \times 1.9 \times 10^{-5}}\, m/sec$

$\Rightarrow v_T = 29.8 \times 10^{-4}\, m/sec \Rightarrow \boxed{v_T = 2.98 \times 10^{-3}\, m/sec}$

ASSIGNMENT 6.1:

A certain globular protein particle has a density of $1246\, kgm^{-3}$. It falls through water (having coefficient of viscosity 8.91×10^{-4} Pa.s) with a terminal speed of $8.33 \times 10^{-6}\, ms^{-1}$. Find the radius of the particle.

Solution:

Given data: Density of protein particle $= \delta = 1246\, kg.m^{-3}$

Coefficient of viscosity of water $= \eta = 8.91 \times 10^{-4}\, Pa.s$

Terminal speed of protein particle $= v_T = 8.33 \times 10^{-6}\, m/sec$

Required:

Radius of the particle $= r = ?$

Calculation:

We know that, $v_T = \dfrac{2\delta g r^2}{9\eta} \Rightarrow r^2 = \dfrac{9\eta v_T}{2\delta g}$ ——— (1)

Putting the values in equation (1), we get,

$r^2 = \dfrac{9 \times 8.91 \times 10^{-4} \times 8.33 \times 10^{-6}}{2 \times 1246 \times 9.8} = \dfrac{667.9 \times 10^{-10}}{24421.6}\, m^2$

$\Rightarrow r^2 = 0.0273 \times 10^{-10}\, m^2 \Rightarrow r = \sqrt{0.0273 \times 10^{-10}\, m^2}$

$\Rightarrow r = 0.165 \times 10^{-5}\, m \Rightarrow \boxed{r = 1.65 \times 10^{-6}\, m}$

EXAMPLE 6.2:

A garden hose of inner radius 1.25cm carries water at 2.60m/s. The nozzle at the end has radius 0.30cm. How fast does the water emerge out through the nozzle?

Solution:

Given data:

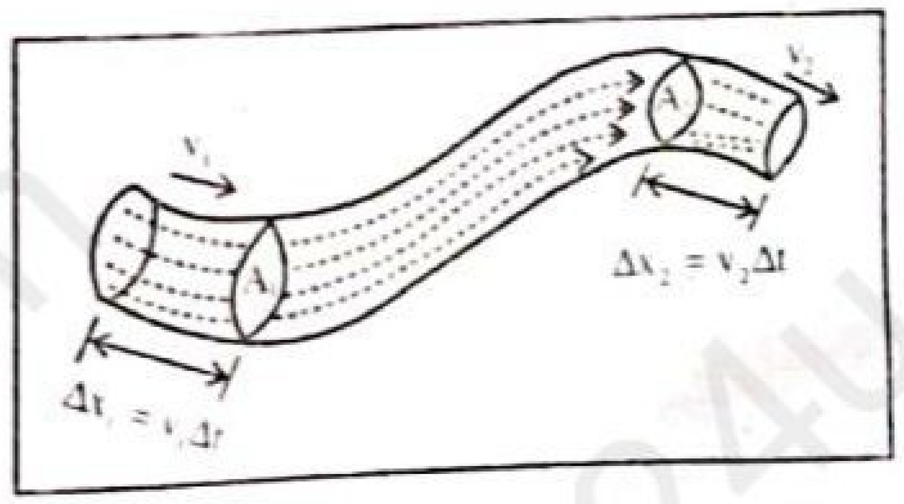
Radius of garden hose = "r_1" = 1.25cm = 0.0125m

Radius of the nozzle = "r_2" = 0.30cm = 0.0030m

Speed through garden hose

Required:

Speed out of nozzle = "v_2" = ?



Calculation:

The equation of continuity is,

$A_1 v_1 = A_2 v_2$ ——— (1)

We know that area $= \pi r^2$

$\Rightarrow A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

So equation (1) becomes,

$$\pi r_1^2 v_1 = \pi r_2^2 v_2 \Rightarrow r_1^2 v_1 = r_2^2 v_2$$

$$\Rightarrow v_2 = \frac{r_1^2 v_1}{r_2^2} \quad \text{———} (2)$$

Putting values in equation (2), we get,

$$v_2 = \frac{(0.0125m)^2 \times 2.60ms^{-1}}{(0.0030m)^2}$$

Hence $\boxed{v_2 = 45.14ms^{-1}}$

The speed of the water from the nozzle is 45.14m/s.

ASSIGNMENT 6.2:

The heart pumps blood into the aorta, which has an inner radius of 1.0cm. The aorta feeds 32 major arteries (each have an inner radius of 0.21cm). If blood in the aorta travels at a speed of 25cm/s, at approximately what average speed does it travel in the arteries? Assume that blood can be treated as an ideal fluid?

Solution:

Given data:

Radius of aorta = $r_1 = 1cm = 0.01m$

Speed of blood in aorta = $v_1 = 25cm/sec = 0.25m/sec$

Radius of artery = $r_2 = 0.21cm = 0.0021m$

Area of cross-section of aorta = $A_1 = \pi r_1^2$

Area of cross-section of 32 arteries = $A_2 = 32\pi r_2^2$

Required:

Speed of blood in artery = $v_2 = ?$

Calculation:

From equation of continuity, we have,

$$A_1 v_1 = A_2 v_2 \Rightarrow \pi r_1^2 v_1 = 32\pi r_2^2 v_2$$

$$\Rightarrow r_1^2 v_1 = 32 r_2^2 v_2$$

$$\Rightarrow v_2 = \frac{r_1^2 v_1}{32 \times r_2^2} \quad \text{———} (1) \qquad \boxed{Area = \pi r^2}$$

Putting the values in equation (1), we get,

$$v_2 = \left[\frac{(0.01)^2 \times 0.25}{32(0.0021)^2}\right] m/sec = 0.18 m/sec$$

$$\Rightarrow \boxed{v_2 = 0.18 m/sec}$$

EXAMPLE 6.3:

Water is flowing smoothly through a pipe. At one point the pressure is 33.2 kPa and the speed of water is 2m/s. While at another point 2.3m higher the pressure is 3.7 kPa, at what speed is the water flowing through this point?

Solution:

Given data:

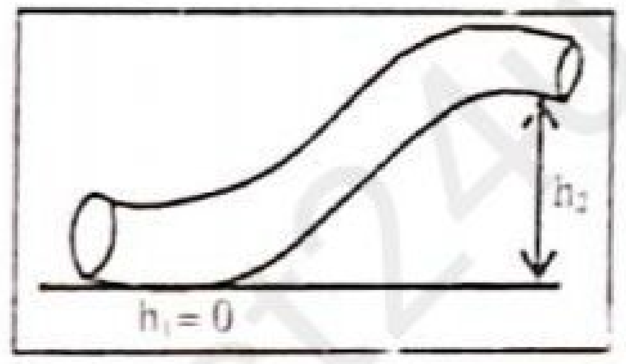
Pressure = "P_1" = $33.2kPa = 33.2 \times 10^3 Nm^{-2}$

Pressure = "P_2" = $3.7kPa = 3.7 \times 10^3 Nm^2$

Speed of water = "v_1" = $2ms^{-1}$,

Height = "h_1" = $0m$, Height = "h_2" = $2.3m$

Density of water = "δ" = $1000 kgm^{-3}$



Required:

Speed of water = "v_2" = ?

Calculation:

We know that the Bernoulli's equation is given by,

$$P_1 + \frac{1}{2}\delta v_1^2 + \delta g h_1 = P_2 + \frac{1}{2}\delta v_2^2 + \delta g h_2$$

$$\Rightarrow \frac{1}{2}\delta v_2^2 = P_1 + \frac{1}{2}\delta v_1^2 + \delta g h_1 - P_2 - \delta g h_2$$

$$\Rightarrow v_2^2 = \frac{2}{\delta}\left[P_1 + \frac{1}{2}\delta v_1^2 + \delta g h_1 - P_2 - \delta g h_2\right] \quad \text{———} (1)$$

Putting the values in equation (1), we get.

$$v_2^2 = \frac{2}{1000}\left[(33.2\times10^3) + (\frac{1}{2}\times1000\times2^2) + (1000\times9.8\times0) - (3.7\times10^3) - (1000\times9.8\times2.3)\right]$$

$$\Rightarrow v_2^2 = 2\times10^{-3}\left[(33.2\times10^3) + (2\times10^3) + 0 - (3.3\times10^3) - (22.5\times10^3)\right]$$

$$\Rightarrow v_2^2 = 2\times10^{-3}\times10^3\left[33.2 + 2 - 3.3 - 22.5\right] = 2[9.4]$$

$$= 18.8m^2/sec^2 \qquad \left[10^3 = kilo, \quad \frac{N}{m^2} = Pascal\right]$$

$$\Rightarrow v_2^2 = 18.8m^2/sec^2 \Rightarrow \sqrt{v_2^2} = \sqrt{18.8m^2/sec^2}$$

$$\Rightarrow v_2 = 4.33m/sec \Rightarrow \boxed{v_2 = 4m/sec} \text{ Approximately}$$

ASSIGNMENT 6.3:

Water is flowing smoothly through a closed pipe system. At one point the speed of water is $3ms^{-1}$, while at another point 3m higher, the speed is $4ms^{-1}$. At lower point the pressure is 80 kPa. Find the pressure at the upper point.

Solution:

Given data:

Speed of water at one point of pipe $= v_1 = 3m/sec$

Speed of water at other point of pipe $= v_2 = 4m/sec$

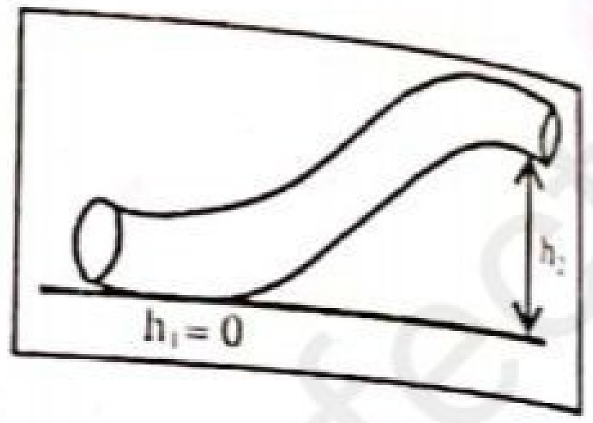
Initial height $= h_1 = 0$,

Height at other end $= h_2 = 3m$

Pressure at lower point

$= P_1 = 80kPa = 80\times10^3 Pa = 80\times10^3 N/m^2$

Density of water $= \delta = 1000kg/m^3$



Required:

Pressure at upper point $= P_2 = ?$

Calculation:

We know that Bernoulli's equation is given by,

$$P_1 + \frac{1}{2}\delta v_1^2 + \delta g h_1 = P_2 + \frac{1}{2}\delta v_2^2 + \delta g h_2$$

$$\Rightarrow P_2 = P_1 + \frac{1}{2}\delta v_1^2 + \delta g h_1 - \frac{1}{2}\delta v_2^2 - \delta g h_2 \text{ ——— (1)}$$

Putting the values in equation (1), we get,

$$P_2 = (80\times10^3) + \left(\frac{1}{2}\times1000\times3^2\right) + (1000\times9.8\times0) - \left(\frac{1}{2}\times1000\times4^2\right) - (1000\times9.8\times3)$$

$$\Rightarrow P_2 = \left[(80\times10^3) + \left(\frac{9}{2}\times10^3\right) + 0 - (8\times10^3) - (29.4\times10^3)\right] N/m^2$$

$$\Rightarrow P_2 = 10^3[80 + 4.5 - 8 - 29.4] N/m^2 = 10^3[47.1] N/m^2 \quad \boxed{10^3 = kilo}$$

$$\Rightarrow \boxed{P_2 = 47.1\times10^3 N/m^2} \text{ or } \boxed{P_2 = 47.1 KPa}$$

EXAMPLE 6.4:

A cylindrical water storage tank has a horizontal spigot near the bottom, at a depth of 1.2m beneath the water surface. (a) When the spigot opened, how fast does the water come out? (b) If the radius of spigot is $6.0 \times 10^{-3}m$, what will be the volume flow rate?

Solution:

Given data:

Height of water in tank $= h = 1.2m$

Radius of spigot $= r = 6.0\times10^{-3} m$

Acceleration due to gravity $= g = 9.8 ms^{-1}$

Required: (i) Speed of water $= v = ?$

(ii) Volume flow rate $= \frac{\Delta V}{\Delta t} = ?$

Calculation:

(i) According to Torricelli's theorem, we have,

$$V = \sqrt{2gh} \text{ ——— (1)}$$

Putting the values in equation (1), we get,

$$V = \sqrt{2\times9.8\times1.2} = \sqrt{23.52} = 4.85m/s$$

$$\Rightarrow \boxed{V = 4.85m/sec}$$

(ii) We know that,

Volume flow rate $= \frac{\Delta V}{\Delta t} = Av$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \pi r^2 v \text{ ——— (2)} \quad \boxed{A = \pi r^2}$$

Putting the values in equation (2), we get,

$$\frac{\Delta V}{\Delta t} = \left[3.14\times(6\times10^{-3})^2\times4.85\right] m^3/sec = (3.14\times36\times10^{-6}\times4.85) m^3/sec$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = 548.2\times10^{-6} m^3/sec = 5.48\times10^{-4} m^3/sec$$

$$\Rightarrow \boxed{\frac{\Delta V}{\Delta t} = 5.48\times10^{-4} m^3/sec}$$

EXTENSION EXERCISE:

If the opening from spigot points upward, how high does the resulting fountain go?

Sol: Speed of efflux $= V = 4.85m/sec$

Height of resulting fountain $= h = ?$

We know that,

$$V = \sqrt{2gh} \Rightarrow V^2 = 2gh$$

$$\Rightarrow h = \frac{V^2}{2g} \Rightarrow h = \frac{(4.85)^2}{2\times9.8} = \frac{23.5}{19.6} \Rightarrow \boxed{h = 1.2m}$$

ASSIGNMENT 6.4:

A tank full of water has a (small) hole near its bottom at a depth of 2.0m from the top surface, which is open to air. What is the speed of the stream of water emerging from the hole?

Solution:

Given data:

Depth of hole $= h = 2m$

Required:

Speed of efflux of water $= v = ?$

Calculation: According to Torricelli's theorem, we have,

$$V = \sqrt{2gh} \quad \text{———} (1)$$

Putting the values in equation (1), we get,

$$V = \left[\sqrt{2 \times 9.8 \times 2}\right] m/sec = \left(\sqrt{39.2}\right) m/sec$$

$$\Rightarrow \boxed{V = 6.3\, m/sec}$$

EXAMPLE 6.5:

A venturi meter measures the flow speed in the pipe. Suppose water flows through the pipe with cross-sectional area of $0.001257 m^2$, if the throat has an area of $0.000314 m^2$, and the fluid heights in the vertical tubes are $h_1 = 1.20 m$ and $h_2 = 0.80 m$. (a) Find the gauge pressure P_1 and P_2. (b) Find the flow speed in the pipe.

Solution:

Given data:

Fluid height $= h_1 = 1.2 m$, Fluid height $= h_2 = 0.80 m$

Cross-sectional area $= A_1 = 0.001257 m^2$,

Cross-sectional area $= A_2 = 0.000314 m^2$,

Density $= \delta = 1000\, kg m^{-3}$

Required:

(a) Gauge pressures $= P_1$ and $P_2 = ?$

(b) Flow speed $= v_1 = ?$

Calculation: (a) We know that, $P = \delta g h$

$$\Rightarrow P_1 = \delta g h_1 \quad \text{———} (1)$$

$\boxed{10^3 = kilo}$

Putting the values in equation (1), we get,

$$P_1 = (1000 \times 9.8 \times 1.2) Pa = 11760 Pa$$

$$\Rightarrow P_1 = 11.76 \times 10^3 Pa = 11.76 kPa \Rightarrow \boxed{P = 11.76 kPa}$$

Similarly [illegible] ——— (2)

Putting the [illegible] in equation (2) we get,

$$P_2 = [1000 \times 9.8 \times 0.80] Pa = 7\text{[illegible]} \times 10^3 Pa$$

$$\Rightarrow \boxed{P_2 = 7.84 \text{[illegible]}Pa}$$

(b) For venturi meter relation, [illegible]

$$V_1 = A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}} \quad \text{———} (3)$$

Putting the values in equation (3), we get,

$$V_1 = 0.314 \times 10^{-3} \sqrt{\frac{2 \times 9.8 \times 0.4}{(1.257 \times 10^{-3})^2 - (0.314 \times 10^{-3})^2}}$$

$$\Rightarrow V_1 = \left[0.314 \times 10^{-3} \sqrt{\frac{7.84}{(1.6 \times 10^{-6} - 0.1 \times 10^{-6})}}\right] m/sec$$

$h = h_1 - h_2$
$h = 1.2m - 0.8m$
$\Rightarrow h = 0.4m$

$$\Rightarrow V_1 = \left[0.314 \times 10^{-3} \sqrt{\frac{7.84}{(1.6 - 0.1) \times 10^{-6}}}\right] m/sec$$

$$\Rightarrow V_1 = \left[\frac{0.314 \times \cancel{10^{-3}}}{\cancel{10^{-3}}} \sqrt{\frac{7.84}{1.5}}\right] m/sec$$

$$\Rightarrow V_1 = \left[0.314 \sqrt{5.23}\right] m/sec = (0.314 \times 2.3) m/sec$$

$$\Rightarrow \boxed{V_1 = 0.722\, m/sec}$$

ASSIGNMENT 6.5:

A venturi meter is measuring the flow of water in a pipe having cross-sectional area of $0.0038 m^2$, a throat will cross-sectional area of $0.00031 m^2$ is connected to it. If the pressure difference is measured to be 2.4 kPa, what is the speed of the water in the pipe?

Solution:

Given data:

Cross-sectional area of pipe $= A_1 = 0.0038\, m^2$

Cross-sectional area of throat $= A_2 = 0.00031\, m^2$

Pressure difference $= P_1 - P_2 = 2.4 kPa = 2.4 \times 10^3 Pa$

Density of water $= \delta = 1000 kg/m^3 = 10^3 kg/m^3$

Required:

Speed of water in the pipe $= v = ?$

Calculation:

We know that speed of water in a pipe in terms of pressure difference is given by,

$$V_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\delta(A_1^2 - A_2^2)}}$$

$$\Rightarrow V_1 = 0.00031 \sqrt{\frac{2 \times 2.4 \times 10^3}{10^3\left[(0.0038)^2 - (0.00031)^2\right]}}$$

$$\Rightarrow V_1 = 0.00031\sqrt{\frac{4.8}{(1.4\times10^{-5} - 9.61\times10^{-8})}}$$

$$= 0.00031\sqrt{\frac{4.8}{(1.4\times10^{-5} - 0.00961\times10^{-5})}}$$

$$\Rightarrow V_1 = 0.00031\sqrt{\frac{4.8}{10^{-5}(1.4 - 0.00961)}} = 0.00031\sqrt{\frac{4.8\times10^{5}}{1.39}}$$

$$\Rightarrow V_1 = 0.00031\sqrt{3.45\times10^{5}} = 0.00031\sqrt{34.5\times10^{4}}$$

$$\Rightarrow V_1 = 0.00031\times10^{2}\sqrt{34.5} = 0.031\times5.9\,m/sec$$

$$\Rightarrow \boxed{V_1 = 0.18\,m/sec}$$

EXAMPLE 6.6:

What is the aerofoil's lift (in Newtons) on a wing of area $88m^2$ if the air passes at speed over its top surface at 280m/s and bottom surface at 150m/s?

Solution:

Given data: Surface area $= A = 88m^2$,

Speed at top of wing $= v_2 = 280m/s$

Speed at bottom of wing $= v_1 = 150m/s$

Density of the air $= \delta = 1.28kg/m^3$

Required: Force $= F = ?$

Calculation: We know that, Pressure $= \frac{Force}{Area} \Rightarrow P_1 - P_2 = \frac{F}{A}$

$$\Rightarrow F = A(P_1 - P_2) \text{ ———— (1)}$$

From venturimeter, we have,

$$P_1 - P_2 = \frac{1}{2}\delta(v_2^2 - v_1^2) \text{ ———— (2)}$$

Putting equation (2) in equation (1), we get,

$$F = A\times\frac{1}{2}\delta(v_2^2 - v_1^2) \text{ ———— (3)}$$

Putting the values in equation (3), we get,

$$F = 88\times\frac{1}{2}\times1.28\left[(280)^2 - (150)^2\right]N$$

$$\Rightarrow F = 56.32[78400 - 22500]N = 56.32(55900)N$$

$$\Rightarrow F = 56.32\times55900N = 3148288N$$

$$\Rightarrow F = 3.148288\times10^6N = 3.2\times10^6N \Rightarrow \boxed{F = 3.2\times10^6N}$$

ASSIGNMENT 6.6:

An airplane wing is designed so that the speed of the air across the top of the wing is 240m/s when the speed of the air below the wing is 215m/s. The density of the air is $1.28kg/m^3$. What is the lifting force on a wing of area $24.0m^2$?

Solution: **Given data:** Speed below the wing $= v_1 = 215m/sec$

Speed of air above the wing $= v_2 = 240m/sec$

Density of air $= \delta = 1.28kg/m^3$, Area of wing $= 24m^2$

Required: Lift force on wing $= F = ?$

Calculation:

We know that,

Pressure $= \frac{Force}{Area} \Rightarrow P_1 - P_2 = \frac{F}{A}$

$$\Rightarrow F = A(P_1 - P_2) \text{ ———— (1)}$$

From venturimeter, we have,

$$P_1 - P_2 = \frac{1}{2}\delta(v_2^2 - v_1^2) \text{ ———— (2)}$$

Putting equation (2) in equation (1), we get,

$$F = A\times\frac{\delta}{2}(v_2^2 - v_1^2) \text{ ———— (3)}$$

Putting the values in equation (3), we get,

$$F = 24\times\frac{1.28}{2}\left[(240)^2 - (215)^2\right]N$$

$$\Rightarrow F = 15.36[57600 - 46225] = 15.36\times11375N$$

$$\Rightarrow F = 174720.0N = 1.75\times10^5N \Rightarrow \boxed{F = 1.75\times10^5N}$$

Exercise

Choose the best possible answer:

1. For substances that do not flow easily [like honey] have the............... value for the coefficient of viscosity.

   a. Low ✓b. High c. Zero d. Negative

2. A unit for viscosity, the centipoises, is equal to which of the following?

   ✓a. $10^{-3} Ns/m^2$ b. $10^{-2} Ns/m^2$
   c. $10^{2} Ns/m^2$ d. $10^{3} Ns/m^2$

Sol: We know that,

1 centipoises $= 10^{-3}$ Pascal·sec $\boxed{1\,Pascal = 1N/m^2}$

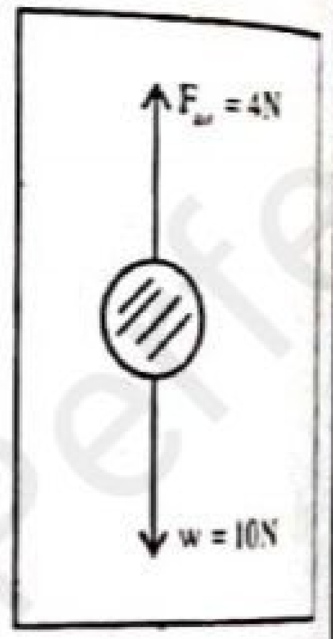
$\Rightarrow$ 1 centipoises $= 10^{-3}(N/m^2)\cdot sec$

$\Rightarrow$ $\boxed{1\,centipoise = 10^{-3} N\cdot sec/m^2}$

So option (a) is correct.

3. Stokes law is applicable only if a body is moving through a liquid with slow speed and has...shape.

   a. A cubical ✓b. A spherical
   c. A rough d. Any



Sol: According to Stokes law,

$F_{dragging} = 6\pi\eta r v$ ________ (1)

In equation (1), "r" represents the radius of spherical body, so option (b) is correct.

4. The net force that acts on a 10-N falling object, when it encounters 4N of air resistance is:..

   a. 0N b. 4N ✓c. 6N d. 10N

Sol: In given figure, the weight "w" of the object is acting downward while force of friction "F_{air}" is acting upwards, so net force is given by,

$F_{net} = w - F_{air} = 10N - 4N = 6N$

So equation (c) is correct.

5. A skydiver jumps from a high-flying helicopter. Before reaching terminal velocity, her acceleration:

   a. Increase ✓b. Decrease
   c. Remain the same d. Is zero

Sol: We know that terminal velocity of an object is given by,

$v_t = \dfrac{mg}{6\pi\eta r}$ ________ (1)

Equation (1) shows that for terminal velocity, we take value of "g" equal $+9.8 m/sec^2$.

6. At terminal velocity the acceleration of a falling object is:

   ✓a. $0\ ms^{-2}$ b. $1\ ms^{-2}$
   c. $-9.8\ ms^{-2}$ d. $+9.8\ ms^{-2}$

7. According to equation of continuity Av = constant. This constant is equal to:

   a. Volume of fluid b. Mass of fluid
   c. Density of fluid ✓d. Volume flow rate

Sol: According to equation of continuity, we have,

$\dfrac{\Delta V}{\Delta t} = AV = constant$ ________ (1)

In equation (1) $\dfrac{\Delta V}{\Delta t}$ represents the volume flow rate of a fluid.

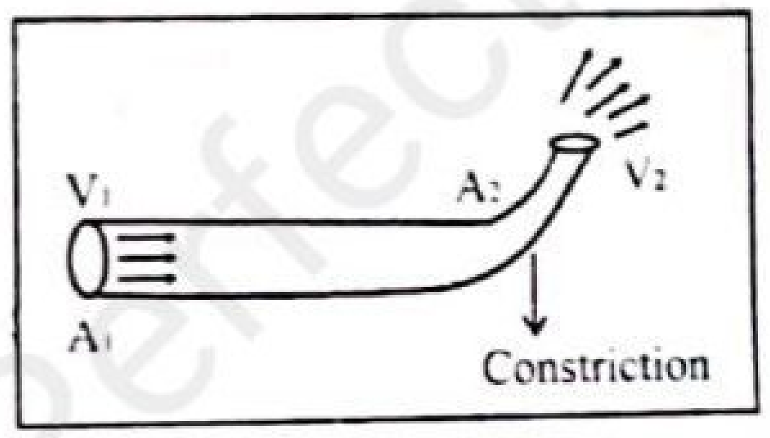
So option (d) is correct.

8. At the constriction in the cross-section for ideal flow, from equation of continuity it follows that, the speed of fluid is:

   ✓a. Greater b. Less c. Same d. Zero

Sol: According to equation of continuity, we have,

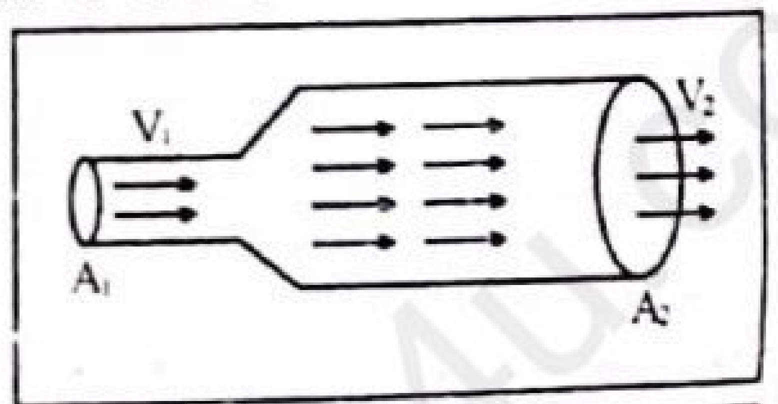
$A_1v_1 = A_2v_2 \Rightarrow v_2 = \dfrac{A_1v_1}{A_2}$ ________ (1)



Due to constriction, the area of cross-section "A_2" of the pipe decreases. Now according to equation (1) when "A_2" decreases, the speed "v_2" at that end increases. So option (a) is correct.

9. As water in a level pipe passes from a narrow cross-section of pipe to a wider cross-section, the pressure against the wall:

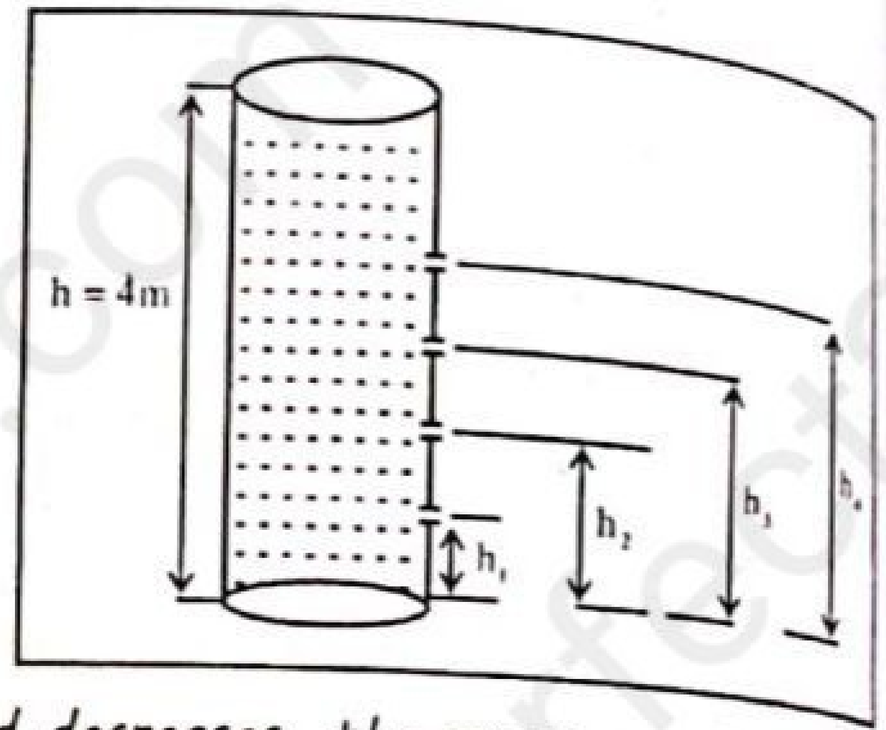
   ✓a. Increases b. Decrease
   c. Remains the same d. Is zero



Sol: We know that, $A_1v_1 = A_2v_2$

$\Rightarrow v_2 = \dfrac{A_1 v_1}{A_2}$

Equation (1) shows that greater the area of cross-section, smaller will be speed of the fluid and vice versa. As in the given figure, "A_2" > "A_1", so speed of fluid in the wider region will reduce. Now according to Bernoulli's principle, when speed of fluid decreases, the pressure of fluid in that region increases. So option (a) is correct.



10. A 4m high tank filled with water is drilled with four identical small holes at 1m, 1.5m, 2m and 2.5m from the bottom of tank, the speed of efflux will be greatest from the hole at:

✓a. 1m b. 1.5m c. 2m d. 2.5m

Sol: Height of tank = $h = 4m$

Let $h_1 = 1m$ from bottom, $h_2 = 1.5m$ from bottom, $h_3 = 2m$ from bottom $h_4 = 2.5m$ from bottom

Now speed at 1st hole is given by,

$v = \sqrt{2gh_{net}}$ ______(1)

$\Rightarrow v_1 = \sqrt{2g(h-h_1)} = \sqrt{2\times 9.8\times(4-1)}\ m/sec$

$\Rightarrow \boxed{v_1 = 7.7m/sec}$

Similarly, for 2nd hole, $v_2 = \sqrt{2g(h-h_2)}$

$\Rightarrow v_2 = \sqrt{2\times 9.8(4-1.5)}m/sec \Rightarrow \boxed{v_2 = 7m/sec}$

Similarly, for 3rd hole, $v_3 = \sqrt{2g(h-h_3)}$

$= \sqrt{2\times 9.8(4-2)}m/sec$

$\Rightarrow \boxed{v_3 = 6.3m/sec}$

Now for 4th hole, $v_4 = \sqrt{2g(h-h_4)} = \sqrt{2\times 9.8(4-2.5)}$

$\Rightarrow \boxed{v_4 = 5.4m/sec}$

As $v_1 > v_2 > v_3 > v_4$

So the speed at height of 1m from bottom of the tank will be greater. So option (a) is correct.

11. Venturimeter is a device used to measure the:

a. Mass of fluid b. Viscosity of fluid

✓c. Speed of fluid d. Density of fluid

12. A certain pipe has a cross-sectional area of $0.0001m^2$ in which water is flowing at 10m/s. The volume flow rate is:

a. $0.00001\ m^3/s$ ✓b. $0.001\ m^3/s$

c. $1\ m^3/s$ d. $10.0001\ m^3/s$

Sol: Area $= A = 0.0001m^2$, speed $= v = 10m/sec$

Volume flow rate is given by,

$\dfrac{\Delta V}{\Delta t} = AV \Rightarrow \dfrac{\Delta V}{\Delta t} = 0.0001\times 10$

$\Rightarrow \dfrac{\Delta V}{\Delta t} = 0.001m^3/sec$, so option (b) is correct.

13. At sufficiently high speeds the flow of viscous fluid becomes:

a. Unexpected b. Stream line

c. Non-viscous ✓d. Turbulent

14. The water in the tank is 10m above the leak point. The speed with which the water emerges from the leak is:

a. 10m/s ✓b. 14m/s

c. 194m/s d. 0.1m/s

Sol: $h = 10m$, speed $= v = ?$

As $v = \sqrt{2gh} = \sqrt{2\times 9.8\times 10} = 14m/sec \Rightarrow v = 14m/sec$

So option (b) is correct

15. When the radius of the artery is reduced, the blood pressure:

a. Increased ✓b. Decreased

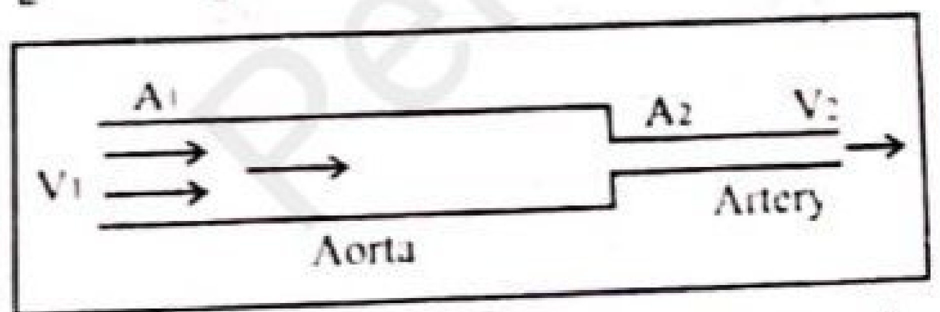
c. Remain the same d. Is zero

Sol: From equation of continuity, we have,

$A_1 v_1 = A_2 v_2$

$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 \Rightarrow r_1^2 v_1 = r_2^2 v_2$ $\quad [A = \pi r^2]$

$\Rightarrow v_2 = \dfrac{r_1^2 v_1}{r_2^2}$ ______(1)



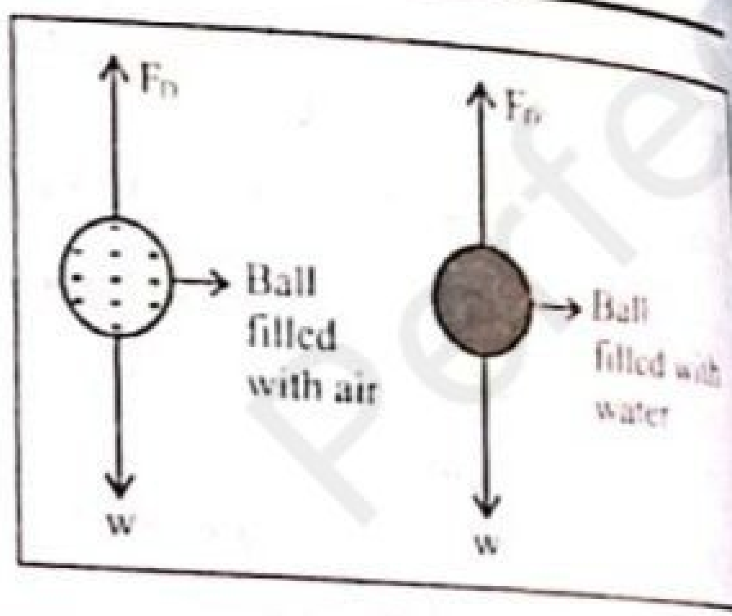
Equation (1) shows that when radius "r_2" of artery is reduced, velocity of blood in artery will increase. Now according to the Bernoulli's principle, when velocity of fluid increases, the pressure of fluid decreases. Thus the pressure of blood in artery will decrease if its radius is reduced, so option (b) is correct.

Conceptual Questions

Give short answers to the following questions:

Q1: From the top of a tall building, you drop two table-tennis balls, one filled with air and the other with water. Which ball reaches terminal velocity first and why?

Ans. When we drop two table-tennis balls from the top of a tall building such that one is filled with air and the other with water, then the ball filled with air will reaches to its terminal velocity first.



Reasons and Explanation:

Consider two ball such that one is filled with air and other is filled with water as shown in the figure.
Now we know that the dragging force is given by,

$$F_D = 6\pi\eta r v \text{\_\_\_\_\_\_\_\_} (1)$$

Now when velocity becomes terminal velocity, then we put $v = v_T$ and $F_D = w$

So equation (1) becomes,

$w = 6\pi\eta r v_T \Rightarrow 6\pi\eta r v_T = w$

$$\Rightarrow \boxed{v_T = \frac{w}{6\pi\eta r}} \text{\_\_\_\_\_\_\_\_} (2)$$

Equation (2) shows that the terminal velocity of the body is directly proportional to the weight of the object. i.e. greater the weight of the body, greater will be its velocity for attaining terminal velocity and vice versa. As the ball filled with water is more heavier than ball filled with air. So water filled ball will take more time for attaining its terminal velocity while the air filled ball will take less time for attaining its terminal velocity. Thus the ball filled with air will reaches to its terminal velocity first.

Q2: Why can squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Ans. A squirrel jumps from a tree branch to the ground and run

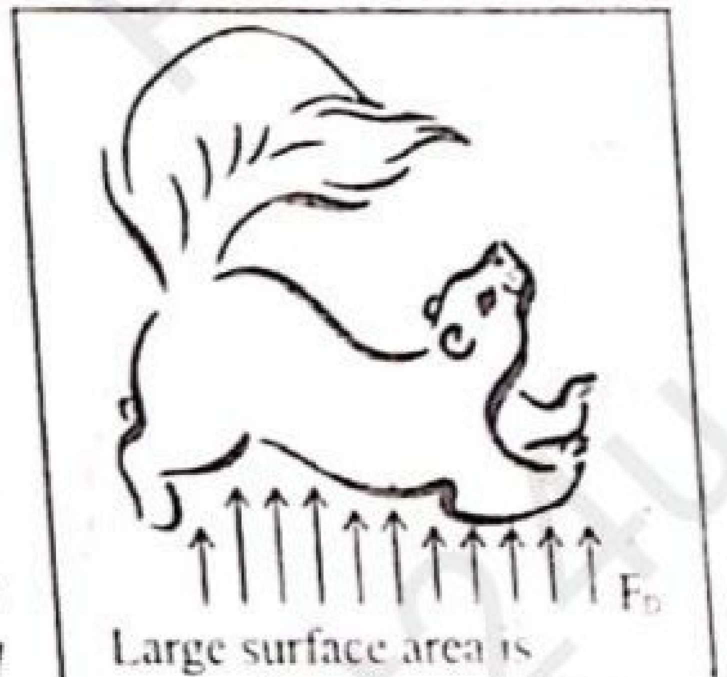
away undamaged while a human could break a bone in such a fall.

Reasons and Explanation:

We know that the terminal velocity is given by,

$$v_T = \frac{mg}{6\pi\eta r} \text{\_\_\_\_\_\_\_\_} (1)$$

Equation (1) shows that the terminal velocity of a falling body depends upon its mass. i.e. greater the mass, high will be its terminal velocity and vice versa.



Large surface area is exposed to dragging force

Now the mass of squirrel is less than the mass of man. Also when it jumps from the tree, it falls down with extended legs and tail. Due to less mass and extended legs its terminal velocity remains low. As a result, the squirrel gains its terminal velocity before reaching the ground. Thus it lands on ground in safe and sound conditions.

Less surface area is exposed to dragging force

On the other hand, the mass of the man is greater than the mass of squirrel. So its terminal velocity remains high due to its heavier mass. So when the man jumps from the tree he cannot gain its terminal velocity before reaching the ground. As a result, he feels great pain while landing on ground due to accelerated motion.

Q3: How does the terminal speed of a parachutist before opening a parachute compare to the terminal speed afterward? Why is there a difference?

Ans. The terminal velocity of a parachutist before opening the parachute is different from its terminal velocity after opening the parachute.

Reasons and Explanation:

The dragging force depends upon the expose surface area of the falling object. i.e. greater the exposed surface area of the body, greater will be the dragging force and low will be its terminal velocity and vice versa.

Case-1: Now when the parachutist is falling down without opening parachute, the exposed surface of parachutist to air resistance is minimum. So the parachutist experience less dragging force than its weight. Thus it posses high terminal velocity as shown in figure (a).

Fig a: Parachutist is falling without opening parachute

Case-2: When the parachutist is falling down with open parachute, large surface area is exposed to air resistance. As a result the paratrooper experience large dragging force and hence posses low terminal velocity as shown in figure (b). Thus there is a difference in terminal velocities of paratrooper before opening the parachute and after opening the parachute due to different values of dragging force.

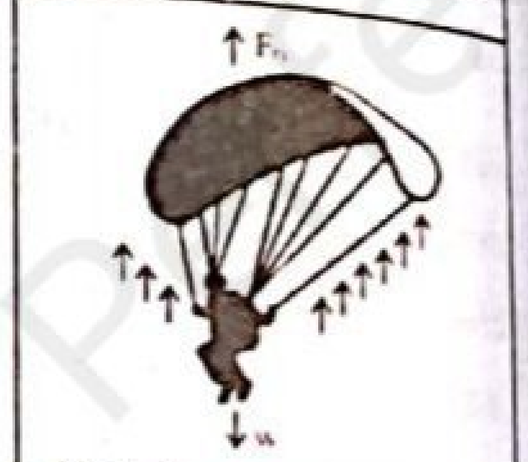


Fig b: Parachutist is falling with open parachute

Q4: You can squirt water over a greater distance by placing your thumb over the end of a garden hose, than by leaving it completely uncovered. Explain how this works?

Ans. Water squirt with greater distance by placing a thumb on the end of garden hose than by leaving it completely uncovered.

Reasons and Explanation:

According to the equation of continuity, we have,

$$A_1v_1 = A_2v_2 \Rightarrow v_2 = \frac{A_1v_1}{A_2} \quad \text{———} (1)$$

Equation (1) shows that the speed of fluid emerging out from an opening is inversely proportional to the area of cross-section of that opening. i.e. smaller the area of cross-section of the opening, greater will be the speed of outgoing fluid and vice versa.

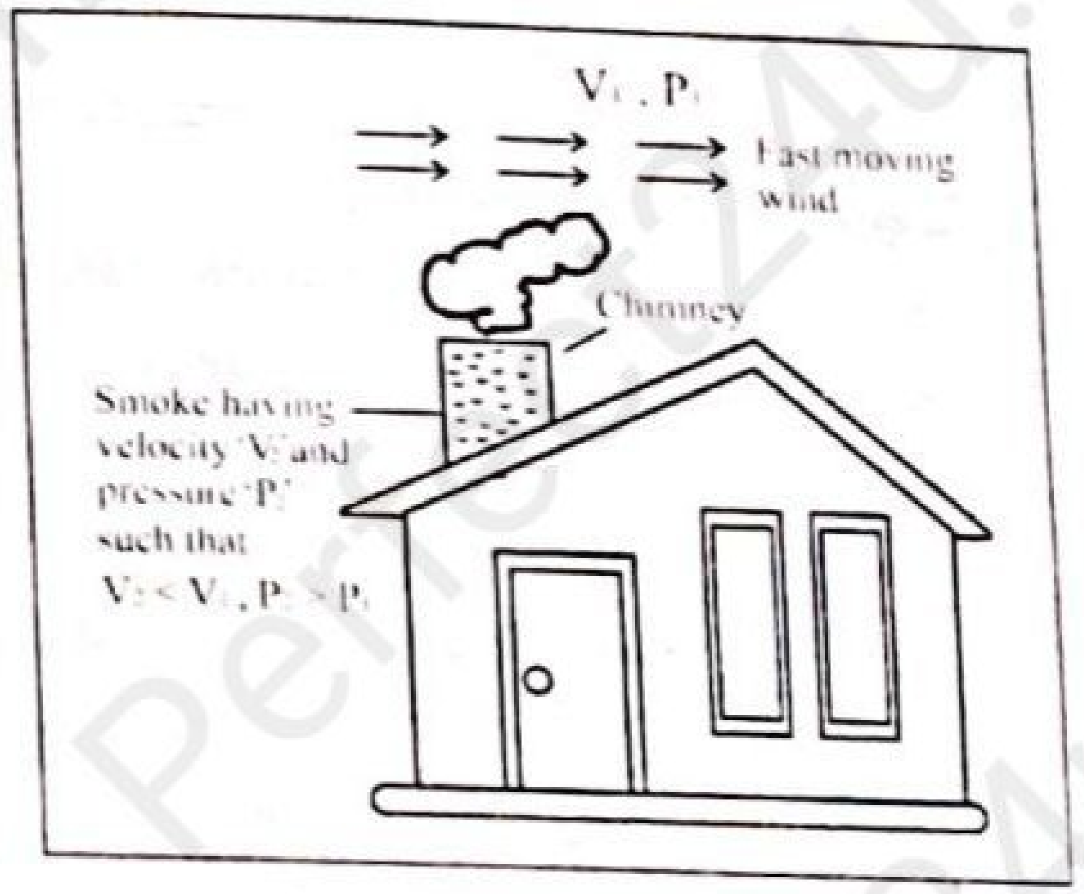
Now when we places our thumb over the end of a garden hose, its cross-sectional area decreases. As a result the speed of outgoing water increases. In this way, the water can be squirt over a greater distance.

Q5: Why does smoke rise faster in a chimney on a windy day?

Ans. The smoke rises faster in a chimney on a windy day due to difference in pressure inside the chimney and outside the chimney.

Reasons and Explanation:

Let "v_1" represents the velocity of wind outside the chimney and "v_2" represents the velocity of smoke inside the chimney. Let "P_1" represents the pressure of fast moving wind and "P_2" represents the pressure of smoke inside the chimney.



Now when wind passes over the chimney with high speed "v_1", then according to Bernoulli's theorem the pressure "P_1" in that region remains minimum. While the speed of "V_2" of the smoke inside the chimney is low, so the pressure "P_2" inside is high as compare to the pressure outside the chimney. i.e.

When '$v_1 > v_2$' and '$P_2 > P_1$'

Due to the pressure difference, the smoke rises in chimney faster on windy day, i.e. when wind are moving fast over the chimney.

Q6: Two boats moving in parallel paths closed to one another risk colliding. Why?

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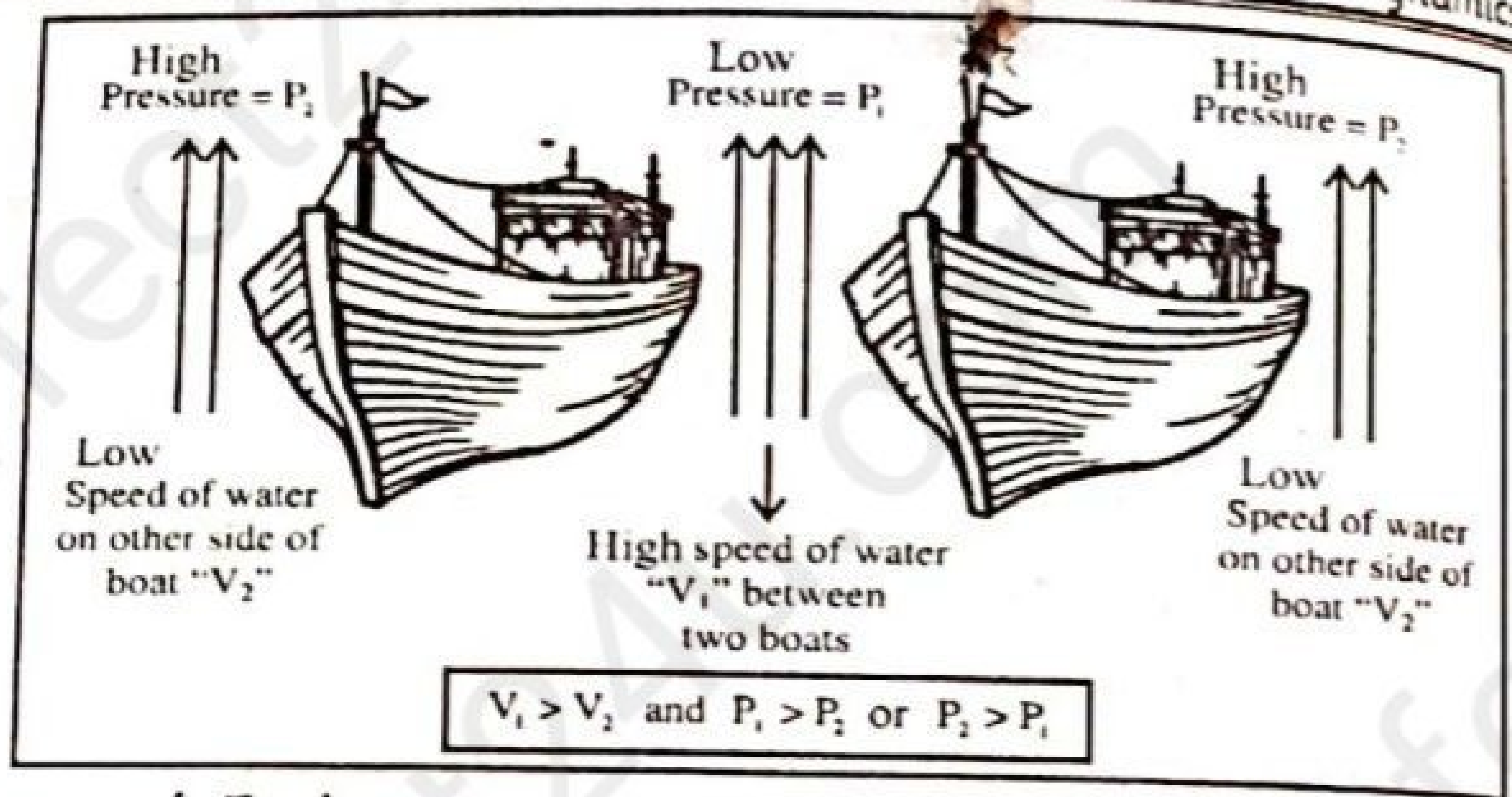
Ans. Two boats moving in parallel path closed to one another risk colliding. It is due to pressure difference between the two boats and outside the two boats.



Reasons and Explanation:

When two boats are moving parallel in water, the water between the two boats is moving with high speed "v_1". While the water present on the other side of both boats are moving with low speed "v_2". Due to difference in velocities, a pressure difference exists between the boats and on other sides of the boats. The pressure "P_1" between boats is minimum and pressure "P_2" on the outer sides of the boats is maximum. So both the boats are push towards low pressure region i.e. towards each other. In this way risk of collision between the two boats increases due to pressure difference. According to Bernoulli's principle, this difference in pressure is given by,

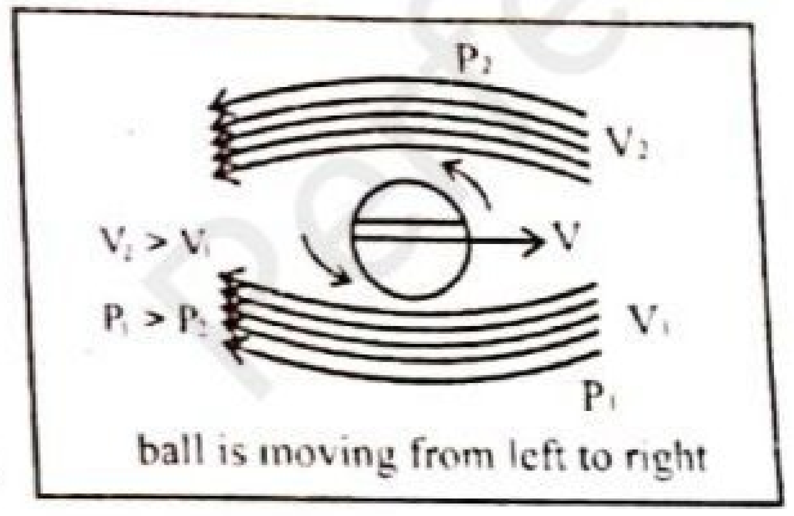
$$\boxed{P_1 - P_2 = \frac{1}{2}\delta(v_2^2 - v_1^2)}$$

Q7: A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?

Ans. When a cricket ball moves past an observer from left to right, spinning counter clockwise, then the ball will move away from the observer, due to pressure difference above and below the ball.

Reasons and Explanation:

Let a cricket ball is moving with velocity "v" from left to right and spinning counter clockwise as shown in the figure.



ball is moving from left to right

Now it is clear from the figure, that the air above the ball will passes with greater speed "v_2". It is because the spin motion of the ball and motion of air are parallel. While the air below the ball will passes with low speed "v_1". It is because the spin motion of the ball and motion of air are opposite in direction. In this way, pressure difference exist on the top of ball and below the ball such that $v_2 > v_1$ and $P_2 < P_1$ or $P_1 > P_2$.

Now according to Bernoulli's theorem, the difference in pressure is given by,

$$P_1 - P_2 = \frac{1}{2}\delta\left(v_2^2 - v_1^2\right)$$

$$\Rightarrow \Delta P = \frac{1}{2}\delta\left(v_2^2 - v_1^2\right) \Rightarrow \frac{F}{A} = \frac{1}{2}\delta\left(v_2^2 - v_1^2\right) \qquad \left[\Delta P = \frac{F}{A}\right]$$

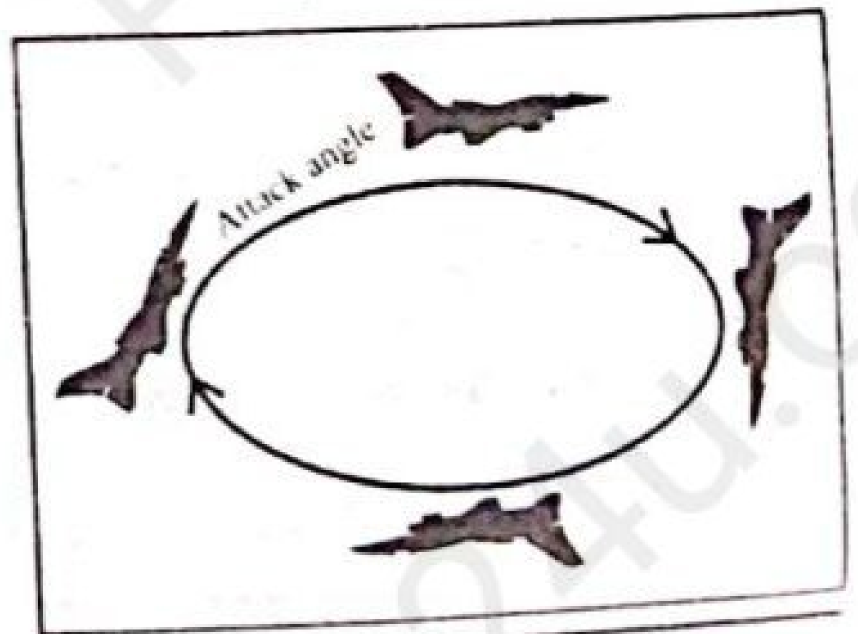
$$\Rightarrow \boxed{F = \frac{1}{2}\delta A\left(v_2^2 - v_1^2\right)} \text{ ——— (1)}$$

Equation (1) represents the force which gives an extra curvature to the cricket ball and as a result the ball will be deflected from high pressure side towards low pressure side. Thus the ball will move away from the observer.

Q8: If aerofoil lift the aeroplane in upright position, how do the pilots make the aeroplane's fly upside down?

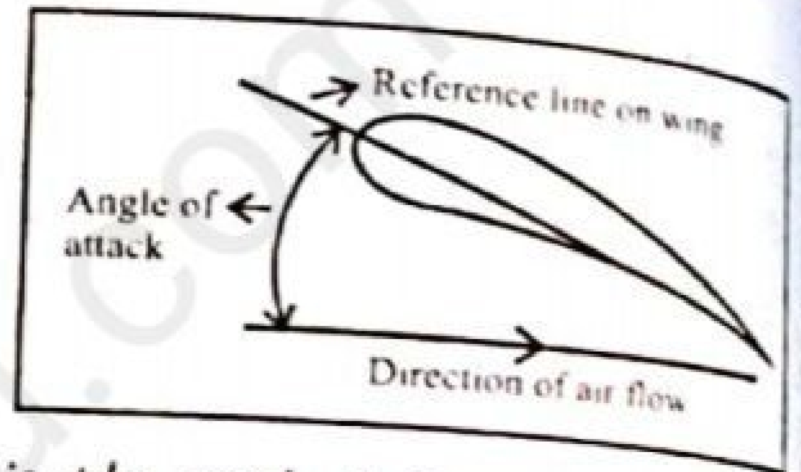
Ans. The pilots make the aeroplane fly upside down by increasing the angle of attack by pointing their nose up or down.

Reason and Explanation:



To fly upside down, we need a wing which is designed in such a way that it can still provide lift even when inverted. On a conventional aircraft, the aerofoil is curved on the upper side and flat on the underside. In such a case, the air over the top flows more quickly as compare to the air below the wing. The difference in pressure gives the wing lift upwards.

The wings of aerobatic planes are curved on both upper and lower sides. With this symmetric design, the plane can fly either normally or inverted. The pilot can flip from one to the other by altering the angle of attack. The angle of attack is the angle between the flowing air and a reference line on the wing. The angle of attack can increase the lift as well as the drag and thus provide more thrust with the aircraft engine and thus keep the aircraft in safe condition.



Q9: Why do the golf balls have dimples?

Ans. Golf balls have dimples, so that can move up in the air very high and can travel larger distance.

Reasons and Explanation:

Consider two balls as shown in the figure. Figure (a) represents a ball without dimples while figure (b) represents a ball with dimples. In case of ball without dimples, the air passes smoothly with greater speed. While in case of ball with dimples, the air bends more around the backside of ball turbulently due to dimples. In this way, a thin layer of air clings to the surface of the ball, which reduces the drag force. Due to reduction of dragging force, the golf ball cannot be slowed down easily. As a result, the golf ball moves up in the air easily and travel larger distance as compare to a ball without dimples.

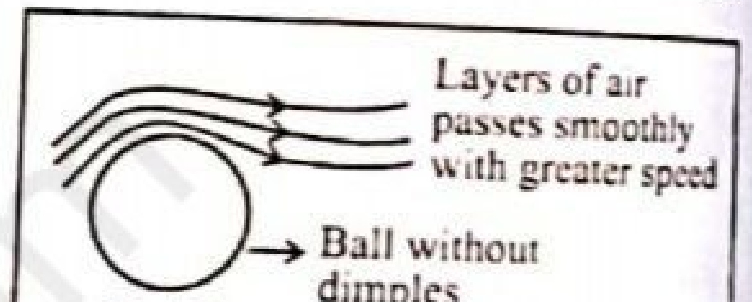


Fig (a)

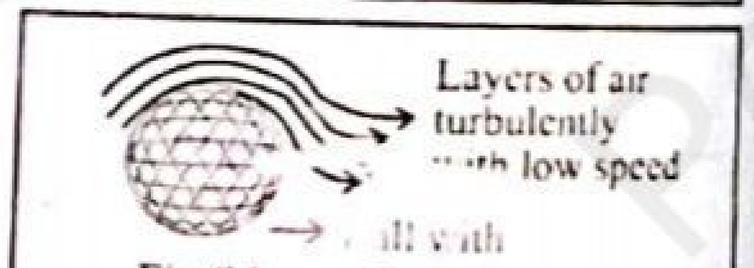


Fig (b)

Q10: How by using wind deflectors on the top truck cabs reduce fuel consumption?

Ans. By using wind deflectors on the top of truck cabs reduce dragging force and the consumption of fuel.

Reasons and Explanation:

In figure (a) a truck is moving on highway without having wind deflectors. The air cannot passes smoothly over the truck and as result the truck faces larger air resistance. Thus the engine of the truck needs to do more work against air resistance and hence consume more fuel.

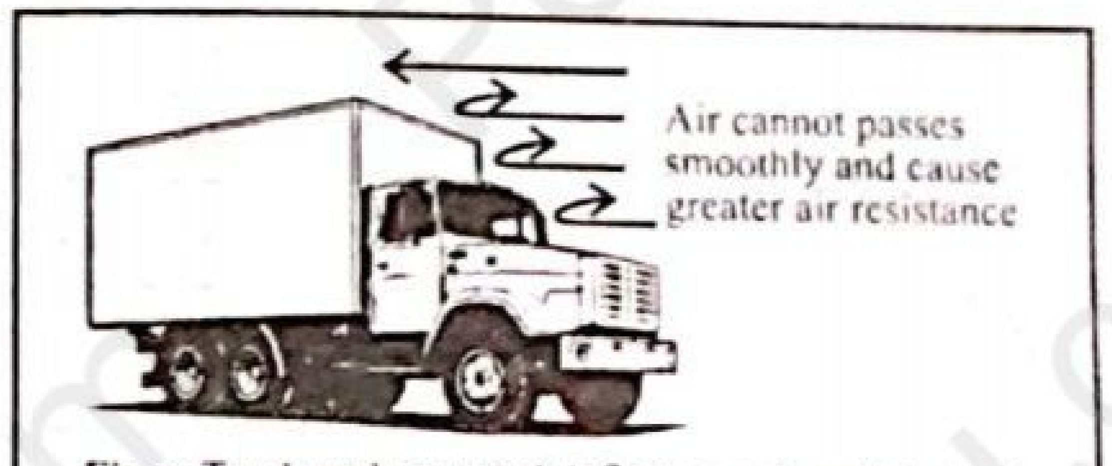


Fig a: Truck without wind deflector on top of their cab

Fig b: Truck with wind deflector on top of their cab

In figure (b) a truck is moving on highway having wind deflectors on the top of their cabs. The deflectors enable the air to pass over the truck smoothly. As a result, the truck faces less forces of friction of air. So the engine do less work against the low air resistance and thus consume less amount of fuel. Thus by using wind deflectors on the top of truck cabs reduce the consumption of fuel.

- حضرت علیؑ کا قول ہے "احسان کی خوبی اس کے نہ جتلانے پر منحصر ہے۔"
- حضرت علیؑ کا قول ہے "جو شخص کسی کے احسان کا شکر گزار نہیں ہے، وہ آئندہ ضرور اس سے محروم ہو جاتا ہے۔"
- حضرت علیؑ کا قول ہے "جب کسی احسان کا بدلہ ادا کرنے سے تیرے ہاتھ قاصر ہوں تو زبان سے اس کا شکریہ ضرور ادا کر۔"
- حضرت علیؑ کا قول ہے "معافی نہایت اچھا احسان ہے اور احسان انسان کو غلام بنا لیتا ہے۔"

Comprehensive Questions

Give extended response to the following questions:

Q1: What is viscous drag? State and explain Stokes law.

Ans. See answer of question no 3 in the text.

Q2: What is terminal velocity? Derive mathematical relation for terminal velocity by using Stokes law.

Ans. See answer of question no 3 in the text.

Q3: Derive mathematically the equation of continuity, and relate it to the time rate of volume flow. How equation of continuity is based on conservation of mass?

Ans. See answer of question no 6 in the text.

Q4: Derive mathematical expression for the Bernoulli's equation. How Bernoulli's equation is based on conservation of energy?

Ans. See answer of question no 9 in the text.

Q5: Using Bernoulli's equation, what is the speed of efflux from a leak at the bottom of large storage tank?

Ans. See Torricelli's theorem in the text.

Q6: By Bernoulli's equation, how we can determine the speed of the fluid in a pipe?

Ans. See Venturi meter in the text.

Q7: What is aero-foil? Explain aero-foil lift on the wing of an aeroplane.

Ans. See article aerofoil in the text.

Q8: Use Bernoulli's equation to explain the working of engine carburetor and perfume bottle spray.

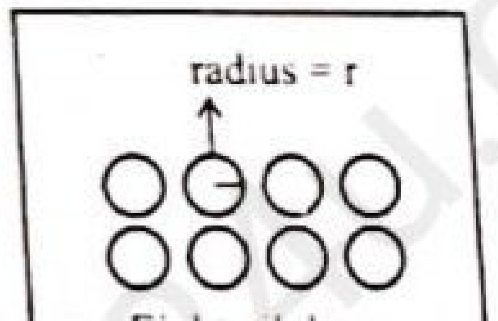
Ans. See article engine carburetor and jet and nozzle in the text.

- حضرت علی رضی اللہ عنہ کا فرمان ہے کہ صبر ایک ایسی سواری ہے جو اپنے سوار کو گرنے نہیں دیتی نا کسی کے قدموں میں نہ کسی کی نظروں سے . . .

Numerical Problems

Q1: Eight equal drops of oil are falling through air with steady velocity of $0.1 ms^{-1}$. The drops recombine to form a single drop, what should be the new terminal velocity?

Solution:

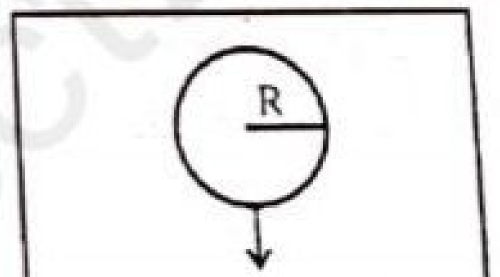


Eight oil drops

Terminal velocity of single oil drop $= v_T = 0.1 m/sec$

Let "m" represents the mass of each individual oil drop and "M" represents the mass of single drop after the combination of eight drops. So we have,

$$M = 8m \quad \text{———} (1)$$



Single drop of radius "R" after the combination of eight drops

Similarly, if "V" represents the volume of each individual drop, then the volume of the single drop after the combination of eight drops is given by,

$$V' = 8V$$

$$\Rightarrow \frac{4}{3}\pi R^3 = 8\left[\frac{4}{3}\pi r^3\right]$$

$$\Rightarrow R^3 = 8r^3 \Rightarrow R^3 = (2r)^3$$

$$\Rightarrow R = 2r \quad \text{———} (2)$$

Now the terminal velocity of big drop of radius "R" is given by,

$$V_T' = \frac{Mg}{6\pi\eta R} \quad \text{———} (3)$$

Putting equation (1) and equation (2) in equation (3), we get,

$$V_T' = \frac{8m \times g}{6\pi\eta \times 2r} = \frac{8}{2}\left[\frac{mg}{6\pi\eta r}\right] \quad \text{———} (4)$$

In equation (4), $\frac{mg}{6\pi\eta r} =$ terminal velocity of individual drop $= v_T$

So equation (4) becomes,

$$V_T' = \frac{8}{2}(V_T) \Rightarrow V_T' = 4v_T \quad \text{———} (5)$$

Putting the value of "v_T" in equation (5), we get,

$$V_T' = (4 \times 0.1) m/sec \Rightarrow \boxed{V_T' = 0.4 m/sec}$$

Q2: Water travels through a 9·6 cm diameter fire hose with a speed of 1·3 m/s· At the end of the hose, the water flows out through a nozzle whose diameter is 2·5 cm· (a) What is the speed of the water coming out of the nozzle? (b) What diameter nozzle is required to give water speed of 21 m/s?

Solution:

Given Data:

Diameter of fire hose $= d_1 = 9.6cm = 0.096m$

Speed of water at one end $= v_1 = 1.3m/sec$

Diameter of fire hose at other end $= d_2 = 2.5cm = 0.025m$

Required:

(a) Speed of water at other end $= v_2 = ?$

(b) Diameter of nozzle [other end] for speed of 21m/sec $= d_2 = ?$

Calculation:

(a) From equation of continuity, we have,

$$A_1v_1 = A_2v_2 \Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 \qquad \left[A = \pi r^2\right]$$

$$\Rightarrow r_1^2 v_1 = r_2^2 v_2 \Rightarrow v_2 = \frac{r_1^2 v_1}{r_2^2} \text{ ——— (1)}$$

Here $r_1 = \frac{d_1}{2}$ and $r_2 = \frac{d_2}{2}$ so equation (1) becomes,

$$v_2 = \frac{\frac{d_1^2}{4} \times v_1}{\frac{d_2^2}{4}} = \frac{d_1^2 v_1}{d_2^2} \Rightarrow v_2 = \frac{d_1^2 v_1}{d_2^2} \text{ ——— (2)}$$

Putting the values in equation (2), we get,

$$v_2 = \left[\frac{(0.096)^2 \times 1.3}{(0.025)^2}\right] m/sec = 19.2m/sec$$

$$\Rightarrow \boxed{v_2 = 19.2m/sec}$$

(b) For speed $v_2 = 21m/sec$, the diameter of nozzle "d_2" is given by,

$$d_2^2 = \frac{d_1^2 v_1}{v_2} \text{ ——— (3)}$$

Putting the values in equation (3), we get,

$$d_2^2 = \frac{(0.096)^2 \times 1.3}{21} \Rightarrow d_2^2 = 5.7 \times 10^{-4}$$

$$\Rightarrow \sqrt{d_2^2} = \sqrt{5.7 \times 10^{-4}} \qquad \boxed{100cm = 1m}$$

$$\Rightarrow \boxed{d_2 = 2.4 \times 10^{-2} m} \text{ or } \boxed{d_2 = 0.024m} \text{ or } \boxed{d_2 = 2.4cm}$$

Q3: A fish tank has dimensions 0·30m wide by 1·0m long by 0·60m high· If the filter should process all the water in the tank once every 3 hour, what should the flow speed be in the 3·0cm diameter input tube for the filter·

Solution: Given Data:

Width of tank $= w = 0.30m$, Length of tank $= L = 1m$

Height of tank $= h = 0.6m$, Volume of tank

$= \Delta V = L \times w \times h = (1m \times 0.30m \times 0.6m)$

$\Rightarrow \Delta V = 0.18m^3$

Time $= 3hr = 3 \times 3600 = 10800sec$

$\Rightarrow \Delta t = 10800\,sec$

Diameter $= d = 3.0cm = 0.03m$

Required:

Flow speed $= v = ?$

Calculation:

We know that volume flow rate is given by,

$$\frac{\Delta V}{\Delta t} = Av \qquad \boxed{A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}}$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \frac{\pi d^2}{4} \times v \Rightarrow v = \frac{4}{\pi d^2}\left(\frac{\Delta v}{\Delta t}\right) \text{ ——— (1)}$$

Putting the values in equation (1), we get, $\boxed{100cm = 1m}$

$$\Rightarrow v = \frac{4}{3.14 \times (0.03)^2}\left[\frac{0.18}{10800}\right] m/sec = 0.02359m/sec$$

$$\Rightarrow \boxed{V = 0.024m/sec} \text{ or } \boxed{V = 2.4\ cm/sec}$$

Q4: A venturi meter is measuring the flow of water; it has a main diameter of 3·5cm tapering down to a throat diameter of 1·0cm· If the pressure difference is measured to be 18mm Hg, what is the speed of the water entering the venturi throat?

Solution: Given Data:

Diameter of one end $= d_1 = 3.5cm = 0.035m$

Diameter of other end $= d_2 = 1.0cm = 0.01m$

Pressure difference $= P_1 - P_2 = 18mm\,Hg$

$\Rightarrow P_1 - P_2 = (133 \times 18)Pascal = 2394Pascal \qquad \boxed{1mm - Hg = 133\,Pascal}$

Area of one end $= A_1 = \pi r_1^2 = \pi \frac{d_1^2}{4}$

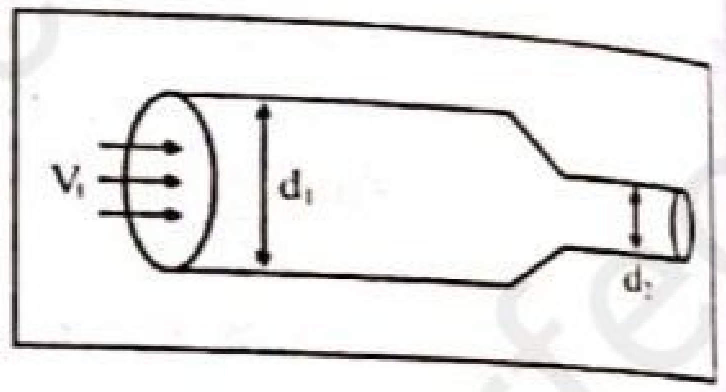
$$\Rightarrow A_1 = \left(\frac{3.14\times(0.035)^2}{4}\right)m^2 = 9.62\times10^{-4}m^2$$

$\boxed{A=\pi r^2, r=\frac{d}{2}}$

Area of other side $= A_2 = \frac{\pi d_2^2}{4} = \frac{3.14\times(0.01)^2}{4}$

$\Rightarrow A_2 = 7.85\times10^{-5}m^2$

Density of water $= \delta = 1000 kg/m^3$



Required:

Velocity of water $= v_1 = ?$

Calculation:

We know that the speed of water by venturi meter in terms of pressure is given by,

$$v_1 = A_2\sqrt{\frac{2(P_1-P_2)}{\delta(A_1^2-A_2^2)}} \text{______}(1)$$

Putting the values in equation (1), we get,

$$v_1 = 7.85\times10^{-5}\sqrt{\frac{2(2394)}{1000\left[(9.62\times10^{-4})^2-(7.85\times10^{-5})^2\right]}}$$

$$\Rightarrow v_1 = 7.85\times10^{-5}\sqrt{\frac{4788}{1000\left[92.6\times10^{-8}-61.6\times10^{-10}\right]}}$$

$$\Rightarrow v_1 = 7.85\times10^{-5}\sqrt{\frac{4.8}{10^{-8}(92.6-61.6\times10^{-2})}}$$

$$\Rightarrow v_1 = 7.85\times10^{-5}\sqrt{\frac{4.8\times10^{8}}{(92.6-0.616)}}$$

$$\Rightarrow v_1 = 7.85\times10^{-5}\sqrt{\frac{4.8\times10^{8}}{91.98}} \Rightarrow v_1 = 7.85\times10^{-5}\times10^{4}\sqrt{\frac{4.8}{92}}$$

$$\Rightarrow v_1 = 7.85\times10^{-1}\sqrt{0.05} \Rightarrow v_1 = 7.85\times10^{-1}\times0.23 = \frac{7.85\times0.23}{10}$$

$$\Rightarrow \boxed{v_1 = 0.18 m/sec}$$

Q5: A small circular hole 6.00mm in diameter is cut in the side of a large water tank, 14.0m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water and (b) the volume discharged per second.

Solution:

Given Data:

Diameter of hole $= d = 6mm = 6\times10^{-3}m$

Height of tank $= h = 14m$

Required:

(a) Speed of efflux of water $= v = ?$

(b) Volume discharge per second $= \frac{\Delta V}{\Delta t} = ?$

$\boxed{10^{-3} = milli}$

Calculation:

(a) We know that,

$V = \sqrt{2gh}$ ______(1) [Torricelli's theorem]

Putting the values in equation (1), we get,

$$V = \left(\sqrt{2\times9.8\times14}\right)m/sec \Rightarrow \boxed{V = 16.6m/sec}$$

(b) We know that

Volume flow rate $= \frac{\Delta V}{\Delta t} = Av$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \pi r^2 v \Rightarrow \frac{\Delta V}{\Delta t} = \pi\left(\frac{d}{2}\right)^2 v$$

$\boxed{A=\pi r^2, r=\frac{d}{2}}$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \frac{\pi d^2 v}{4} \text{______}(1)$$

Putting the values in equation (1), we get,

$$\frac{\Delta V}{\Delta t} = \frac{3.14\times(6\times10^{-3})^2\times16.6}{4}$$

$$\Rightarrow \boxed{\frac{\Delta V}{\Delta t} = 469.12\times10^{-6}m^3/sec}$$

or $\frac{\Delta V}{\Delta t} = 469.12\times10^{-6}\times10^{6}cm^3/sec$

or $\boxed{\frac{\Delta V}{\Delta t} = 0.469cm^3/sec}$

$1m = 100cm$
$1m^3 = (100cm)^3$
$\Rightarrow 1m^3 = 10^6 cm^3$

Q6: What is the aerofoil lift (in newtons) due to Bernoulli's principle on a paper plane of wing area $0.01m^2$ if the air passes over the top and bottom surfaces at speeds of 9m/s and 7m/s respectively? (Take the density of air as $1.28kg/m^3$)

Solution:

Given Data: Area of wing $= 0.01m^2 \Rightarrow A = 0.01m^2$

Speed at top of wing $= v_2 = 9m/sec$

Speed below the wing $= v_1 = 7m/sec$

Density of air $= \delta = 1.28kg/m^3$

Required: Aerofoil lift $= F = ?$

Calculation: We know that,

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow P_1 - P_2 = \frac{F}{A} \Rightarrow F = A(P_1 - P_2) \_\_\_\_\_\_ (1)$$

From venturi meter, we have,

$$P_1 - P_2 = \frac{1}{2}\delta\left(v_2^2 - v_1^2\right) \_\_\_\_\_\_ (2)$$

Putting equation (2) in equation (1), we get,

$$F = A \times \frac{1}{2}\delta\left(v_2^2 - v_1^2\right) \_\_\_\_\_\_ (3)$$

Putting the values in equation (3), we get,

$$F = 0.01 \times \frac{1}{2} \times 1.28\left(9^2 - 7^2\right) = 6.4 \times 10^{-3}(81-49)$$

$$\Rightarrow F = 6.4 \times 10^{-3} \times 32N = 204.8 \times 10^{-3}$$

$$\Rightarrow \boxed{F = 204.8 \times 10^{-3} N} \text{ or } F = 0.2048N \text{ or } \boxed{F = 0.2N}$$

Q7: During a windstorm, a 25m/s wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is 1.28 kg/m³)

Solution:

Given Data: Speed of air $= v_2 = 25m/sec$

Density of air $= \delta = 1.28kg/m^3$

As the doors and windows of the house are closed, so we take the speed of inside air to be zero i.e. put $v_1 = 0$

Required: Pressure difference $= P_1 - P_2 = ?$

Calculation: We know that,

$$P_1 - P_2 = \frac{1}{2}\delta\left(v_2^2 - v_1^2\right) \_\_\_\_\_\_ (1)$$

Putting the value sin equation (1), we get,

$$P_1 - P_2 = \left[\frac{1}{2} \times 1.28\left(25^2 - 0^2\right)\right] N/m^2$$

$$\Rightarrow P_1 - P_2 = 0.64 \times (625 - 0) N/m^2 \qquad \left[\frac{N}{m^2} = Pascal\right]$$

$$\Rightarrow \boxed{P_1 - P_2 = 400N/m^2} \text{ or } \boxed{P_1 - P_2 = 400Pa}$$

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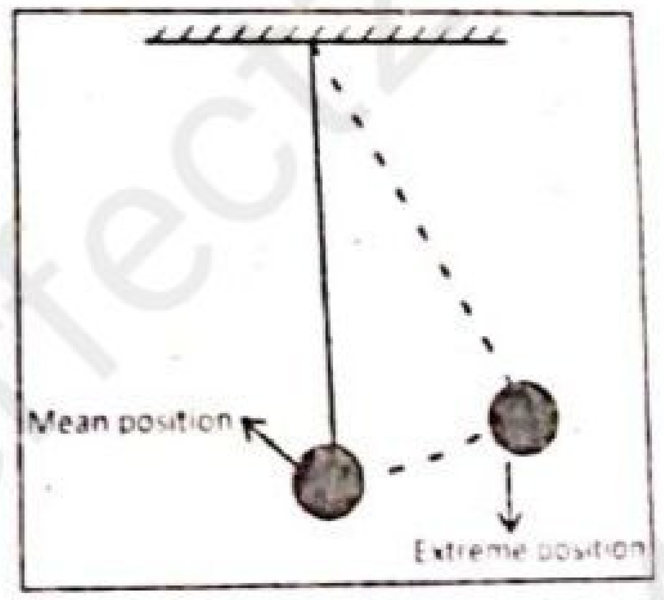
CHAPTER 07 OSCILLATION

Q1: Define and explain oscillatory motion. Give examples of oscillatory motion.

Ans. Oscillatory Motion:

Definition: The back and forth motion of a body over the same path about its mean position is known as oscillatory motion.

Explanation:



The oscillatory [vibratory] motion can be initiated when a body is displaced from its mean position due to external applied force. When the external force is removed, the body moves back towards its mean position. On the way its velocity increases and becomes maximum at mean position. As a result the body does not stop at mean position and moves towards the other side of mean position. On the way its velocity decreases and finally comes to rest at extreme position. The body then moves back towards mean position due to restoring force. In this way, the body performs oscillatory motion about mean position.

Examples of Oscillatory Motion:

- Motion of a body attached to an elastic spring.
- Motion of wires of a sitar.
- Motion of simple pendulum.

Q2: Define the following terms: Periodic motion, vibratory motion, vibration, time period, frequency, displacement, amplitude and angular frequency

Ans. 1. Periodic Motion:

Motion, which is repeated in equal intervals of time is known as