

$$v_{as} = \left[\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{10 \times 6.4 \times 10^6} \right] \text{m/sec}$$

$$\Rightarrow v_{as} = \left[\sqrt{0.63 \times 10^{-11+24-6}} \right] \text{m/sec} = \sqrt{0.63 \times 10^7}$$

$$\Rightarrow v_{as} = \left[\sqrt{6.3 \times 10^6} \right] \text{m/sec} = 2.5 \times 10^3 \text{m/sec}$$

$$\Rightarrow v_{as} = 2.5 \times 10^3 \text{m/sec}$$

$$(OR) v_{as} = 2.5 \text{km/sec}$$

$$10^3 = \text{kilo}$$

+++

- جس نے طلب علم میں وفات پائی وہ شہید مراد (رسول اللہ ﷺ)
- خاموشی عالم کے لیے زیور اور جاہل کے لیے پردہ ہے۔ (حضرت عین)
- تم بڑوں کی عزت کرو، چھوٹے تمہاری عزت کریں گے۔ (حضرت عین)
- جس نے انسان کا حق نہیں پہچانا اس نے اللہ تعالیٰ کا حق نہیں پہچانا۔ (حضرت عین)
- جاہلوں کی بات پر تحمل کرنا عقل کو زکوۃ دیتا ہے۔ (حضرت علی)
- دنیا کی سب سے بڑی خیانت قوم کے ساتھ غداری ہے۔ (حضرت علی)
- پرہیز گاری سے بڑھ کر کوئی عزت نہیں۔ (حضرت علی)

CHAPTER

05

ROTATIONAL & CIRCULAR MOTION

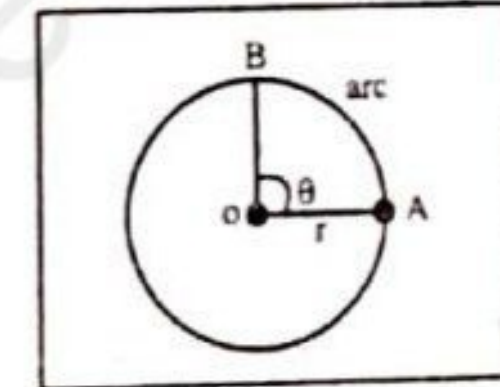
Q1: Define rotational motion, angular displacement, angular velocity and angular acceleration?

Ans: (i) Rotational Motion:

The motion of a body about its axis of rotation is known as rotational motion or angular motion.

(ii) Angular Displacement:

The angle subtended by an arc at the centre during angular motion is known as angular displacement. In given figure, " θ " represents the angular displacement, such that, $\theta = 1$ radius = 1 rad. The units of angular displacement are degree, revolution and radian [rad].



$$1 \text{ revolution} = 360^\circ$$

$$1 \text{ revolution} = 2\pi \text{ radian}$$

$$360^\circ = 2\pi \text{ radian} \quad [1 \text{ rad} = 57.3^\circ]$$

NOTE: If the length of an arc is equal to the radius of the circle, then the angle subtended at the centre of the circle is equal to 1 radian.

(iii) Angular Velocity:

The rate of change of angular displacement is known as angular velocity. It is represented by " ω " and is given by,

$$\omega = \frac{\Delta\theta}{\Delta t}$$

The units of angular velocity are rad/sec, rev/sec and degree/sec.

Average Angular Velocity:

The average angular velocity can be obtained by dividing total angular displacement by total time taken. i.e.

$$\langle \omega \rangle = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Velocity:

The angular velocity at any particular instant of time is known as instantaneous angular velocity. (OR)

The angular velocity for which time intervals approaches to zero is known as instantaneous angular velocity. i.e.

$$\bar{\omega}_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$$

Dimension of angular velocity:

$$\text{As } \omega = \frac{\Delta \theta}{\Delta t} = \frac{\text{rad}}{\text{sec}} = \frac{\text{rad}}{\text{time}} = [T^{-1}]$$

(iv) Angular Acceleration:

The rate of change of angular velocity is known as angular acceleration. It is represented by " α " and is given by,

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

The units of angular acceleration are, rad/sec^2 , rev/sec^2 , degree/sec^2 .

Average Angular Acceleration:

The average angular acceleration can be obtained by dividing total change in angular velocity by total time taken. i.e.

$$\langle \alpha \rangle = \frac{\Delta \omega}{\Delta t}$$

Instantaneous Angular Acceleration:

The angular acceleration of a body at any particular instant of time is known as instantaneous angular acceleration. i.e.

$$\alpha_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}$$

Dimension of angular acceleration: (ETEA 2007)

$$\text{As } \alpha = \frac{\Delta \omega}{\Delta t} = \frac{\text{rad}}{\text{sec}^2} = [T^{-2}]$$

Q2: Discuss the relation between linear displacement and angular displacement. (OR) Show that, $S = r\theta$

■ ISE Malakand 2019

Ans. Consider a body of mass "m" is moving along a circular path of radius "r" as shown in the figure.

Let "AB" and "AC" represents the two arc making an angle $\angle AOB$ and $\angle AOC$ with the centre respectively. Now we have,

$$\angle AOB \propto \text{Arc AB} \Rightarrow \angle AOB = \text{constant arc AB} \quad (1)$$

$$\text{Similarly, } \angle AOC = \text{constant arc AC} \quad (2)$$

Dividing equation (1) by equation (2), we get

$$\frac{\angle AOB}{\angle AOC} = \frac{\text{Arc AB}}{\text{Arc AC}} \quad (3)$$

$$\text{Here } \angle AOB = \theta$$

$$\angle AOC = 1 \text{ radian}$$

$$\text{Arc AB} = S$$

$$\text{Arc AC} = r$$

So equation (3) becomes,

$$\frac{\theta}{1} = \frac{S}{r} \Rightarrow S = r\theta \quad (4)$$

Equation (4) represents the relation between linear displacement and angular displacement.

Q3: Discuss the relation between linear velocity and angular velocity. (OR) Show that $\vec{v} = \vec{\omega} \times \vec{r}$

Ans. We know that,

$$S = r\theta \quad (1)$$

We also know that,

$$S = vt \quad (2) \text{ Comparing equations (1), (2), we get,}$$

$$vt = r\theta$$

$$\Rightarrow v = r \frac{\theta}{t} \quad (3)$$

$$\text{As } \frac{\theta}{t} = \text{angular velocity} = \omega$$

So equation (3) becomes,

$$v = r\omega \quad (4)$$

In vector form equation (4) can be written as,

$$\vec{v} = \vec{\omega} \times \vec{r} \quad (5)$$

Equations (4), (5) represent the relation between linear velocity and angular velocity.

Q4: Discuss the relation between linear acceleration and angular acceleration. (OR) Show that, $\vec{a} = \vec{\alpha} \times \vec{r}$

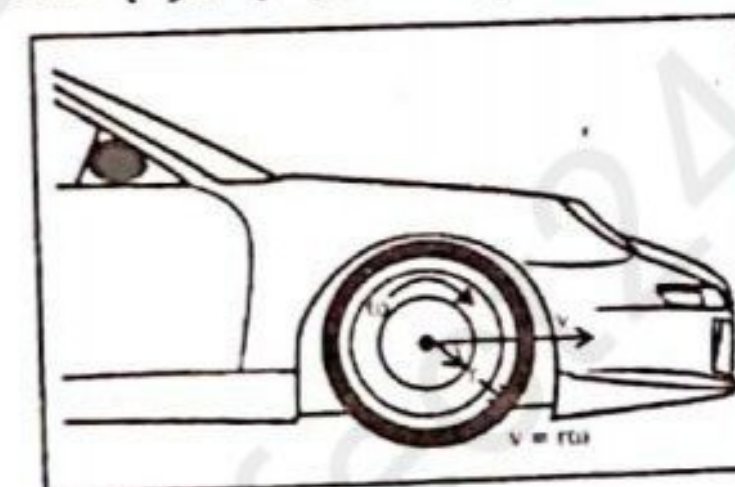
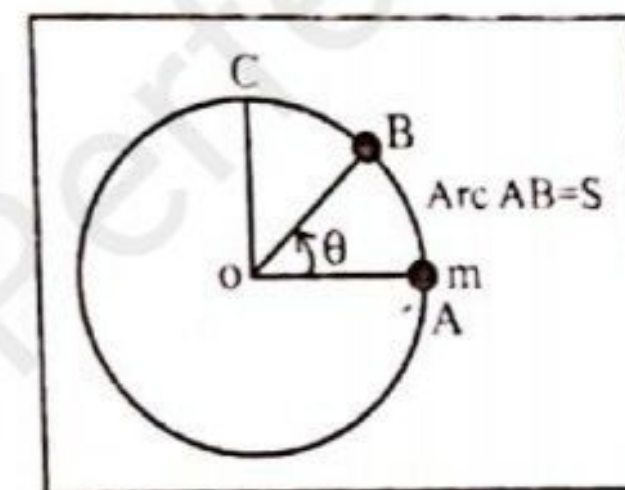
Ans. By definition, we have,

$$\text{Linear acceleration} = \frac{(\vec{v}_f - \vec{v}_i)}{t}$$

$$\Rightarrow \vec{a} = \frac{(\vec{v}_f - \vec{v}_i)}{t} \quad (1)$$

Here we have,

$$[\vec{v} = \vec{\omega} \times \vec{r}]$$



$$\vec{v}_f = \vec{\omega}_f \times \vec{r} \text{ and } \vec{v}_i = \vec{\omega}_i \times \vec{r}$$

So equation (1) becomes,

$$\vec{a} = \frac{(\vec{\omega}_f - \vec{\omega}_i)}{t} \times \vec{r} \quad (2)$$

We know that,

$$\frac{(\vec{\omega}_f - \vec{\omega}_i)}{t} = \text{rate of change of angular acceleration}$$

$$\Rightarrow \frac{(\vec{\omega}_f - \vec{\omega}_i)}{t} = \vec{\alpha} \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\vec{a} = \vec{\alpha} \times \vec{r} \quad (4)$$

Equation (4) represents the relation between linear acceleration and angular acceleration.

Q5: Discuss the linear kinematic equation in rotational equation form.

LINEAR MOTION	ROTATORY MOTION
1. $S = vt$ $\longrightarrow$	$\theta = \omega t$
2. $v_f = v_i + at$ $\longrightarrow$	$\omega_f = \omega_i + \alpha t$
3. $\langle v \rangle = \frac{(v_f + v_i)}{2}$ $\longrightarrow$	$\langle \omega \rangle = \frac{(\omega_f + \omega_i)}{2}$
4. $2aS = v_f^2 - v_i^2$ $\longrightarrow$	$2\alpha\theta = \omega_f^2 - \omega_i^2$
5. $S = v_i t + \frac{1}{2}at^2$ $\longrightarrow$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

Q6: (a) Define circular motion. (b) Define centripetal force and centripetal acceleration. Derive mathematical relations for centripetal force and centripetal acceleration.

BISE Peshawar 2017

Ans. (a) Circular Motion: The motion of a body along a circular path is known as circular motion.

(b) Centripetal Force and Centripetal Acceleration: (ETEA 2006-8)

Centripetal Force: The force which compels the body to move in circle is known as centripetal force. It is also known as centre seeking force.

Centripetal Acceleration: The acceleration of the body during circular motion is known as centripetal acceleration. It is always directed towards the centre of the circle.

Mathematical Form:

Consider a body of mass "m" moving along a circular path of radius "r" as shown in the figure.

Let " $\vec{v}_1$ " and " $\vec{v}_2$ " represents the velocity of the body at points "A" and "B" respectively. Here changes occur in velocity due to change in its direction, which is given by,

$$\Delta \vec{v} = (\vec{v}_2 - \vec{v}_1) \quad (1)$$

We also know that,

$$S = r\theta \quad (2)$$

Here we put,

$$S = \Delta v, r = v, \theta = \Delta\theta$$

So equation (2) becomes,

$$\Delta v = v\Delta\theta \quad (3)$$

Dividing both sides of equation (3) by " Δt " and taking the limit i.e. $\lim_{\Delta t \rightarrow 0}$ we get,

$$\lim_{\Delta t \rightarrow 0} \left(\frac{\Delta v}{\Delta t} \right) = (v) \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta\theta}{\Delta t} \right)$$

$$\Rightarrow \boxed{a = v\omega} \quad (4)$$

We also know that, $v = r\omega$ so equation (4)

$$a = r\omega \cdot \omega \Rightarrow \boxed{a = r\omega^2} \quad (5)$$

As $v = r\omega \Rightarrow \omega = \frac{v}{r}$, so equation (4) becomes,

$$a = v \cdot \frac{v}{r} \Rightarrow \boxed{a = \frac{v^2}{r}} \quad (6)$$

Equations (4), (5), (6) represent the magnitude of centripetal acceleration. In vector form, equations (4), (5), (6) can be written as,

$$\boxed{\vec{a} = -v\omega \hat{r}} \quad (7)$$

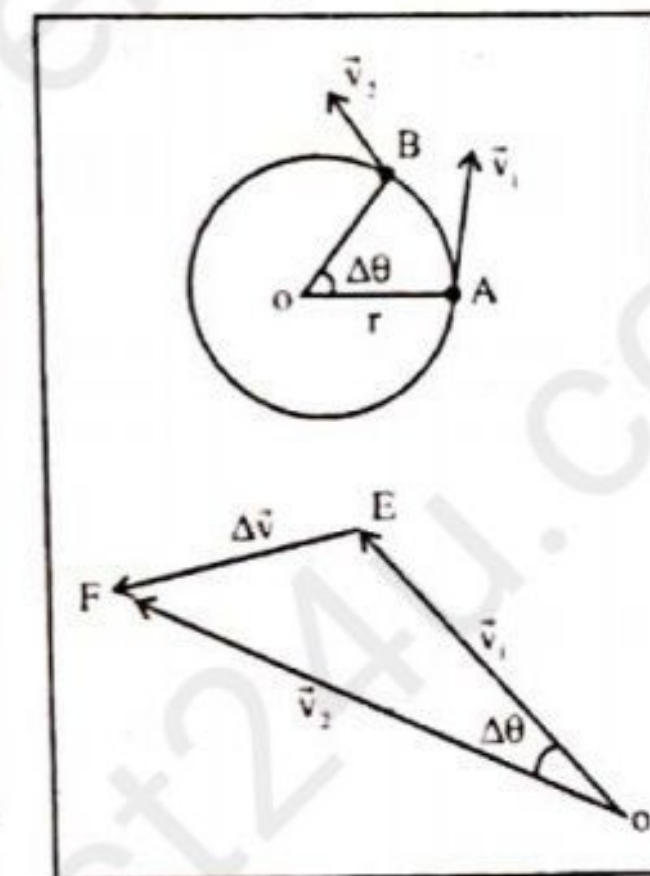
$$\boxed{\vec{a} = -r\omega^2 \hat{r}} \quad (8)$$

$$\& \boxed{\vec{a} = -\frac{v^2}{r} \hat{r}} \quad (9)$$

The negative sign shows that the acceleration is directed towards the centre of the circle.

Expressions for Centripetal Force: (ETEA 2007)

We know that, $F = ma$ $\quad (10)$



$$\left[\begin{aligned} \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta v}{\Delta t} \right) &= a \\ \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta\theta}{\Delta t} \right) &= \omega \end{aligned} \right]$$

Putting equations (4), (5), (6) in equation (10) one by one, we get,

$$F = mv\omega \quad (11)$$

$$F = mr\omega^2 \quad (12)$$

$$\& \quad F = \frac{mv^2}{r} \quad (13)$$

Equations (11), (12), (13) represent the magnitude of centripetal force. In vector form, equations (11), (12), (13) can be written as,

$$\vec{F} = -v\omega \hat{r} \quad (14)$$

$$\vec{F} = -mr\omega^2 \hat{r} \quad (15)$$

$$\& \quad \vec{F} = -\frac{mv^2}{r} \hat{r} \quad (16)$$

Q7: Define and explain centrifugal force or reaction force.

Ans: Centrifugal Force (or) Reaction Force:

Definition: The force which is equal in magnitude but opposite in direction to the centripetal force is known as centrifugal force.

Explanation:

Consider a piece of stone is tied to a string as shown in the figure.

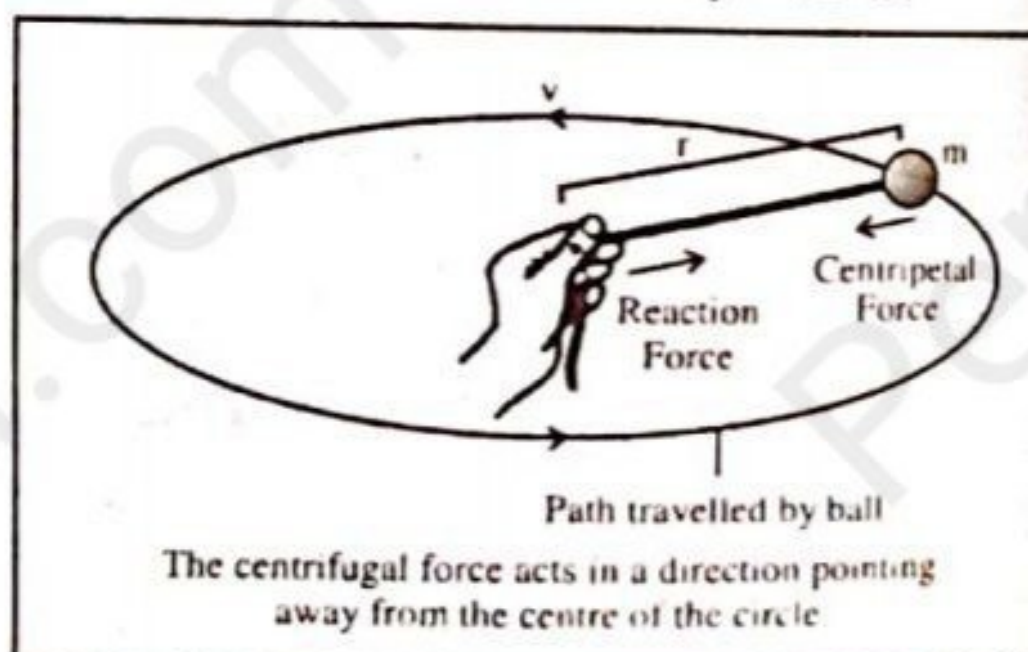
Now if we whirl the stone, then we provide centripetal force to the stone. During this process, our hands experience a push in the outwards direction. Such outwards push or force is known as centrifugal force. Its magnitude is equal to that of centripetal force but opposite in direction. Mathematically, we have,

$$F_{\text{centrifugal}} = \frac{mv^2}{r}$$

Q8: Explain the "bending of road" or curved road?

Ans: Consider a car is moving along a circular path [banked curve] of radius "r" as shown in the figure.

During the turn, the following forces are acting on the car.



1. The weight of the car "w" is acting downwards.

2. The normal force " F_N " is acting at angle " θ " with normal in upwards direction.

3. The centripetal force " F_c " is acting towards the centre.

Now if we resolve " F_N " into its components i.e. " $F_N \cos \theta$ " and " $F_N \sin \theta$ ", then we see that " $F_N \cos \theta$ "

balance the weight "w" of the car. While the component " $F_N \sin \theta$ " is acting towards the centre. So " $F_N \sin \theta$ " acts as centripetal force. So we have,

$$F_c = F_N \sin \theta$$

$$\Rightarrow \frac{mv^2}{r} = F_N \sin \theta \quad (1)$$

$$\& \quad w = F_N \cos \theta$$

$$\Rightarrow mg = F_N \cos \theta \quad (2)$$

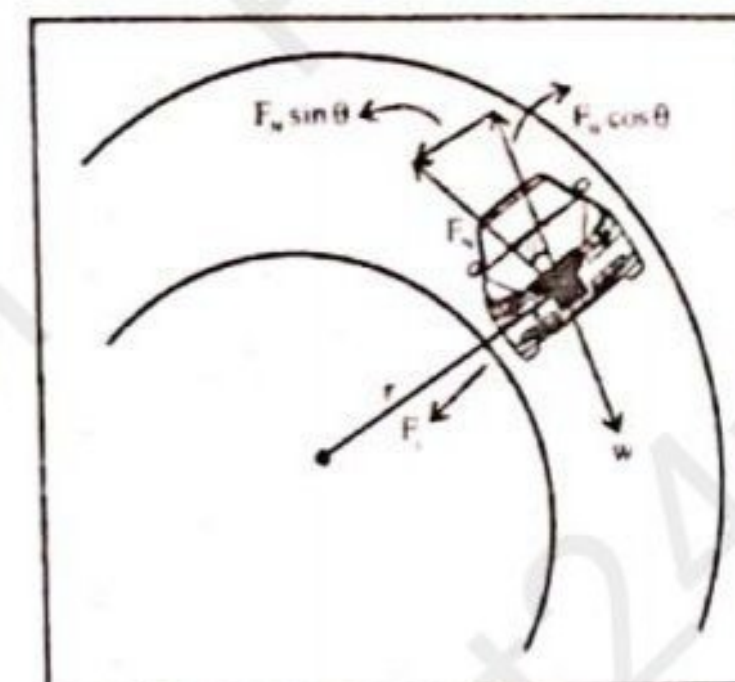
Now dividing equation (1) by equation (2), we get,

$$\frac{mv^2}{rmg} = \frac{F_N \sin \theta}{F_N \cos \theta} = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \frac{v^2}{rg} = \tan \theta \Rightarrow v^2 = rg \tan \theta$$

$$\Rightarrow v = \sqrt{rg \tan \theta} \quad (3)$$

Equation (3) shows that for given values of r, g and θ we can determine the speed of the car during the turn.



$$F_c = mv^2 / r \quad w = mg$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

Q9: What is moment of inertia? Discuss the importance of moment of inertia.

ETEA 2015

Ans: Moment of Inertia:

The reluctance of a rotating body to change its angular velocity is known as moment of inertia. (OR)

The property of a rotating body which maintains its state of rest or uniform rotational motion is called moment of inertia.

Significance of Moment of Inertia:

We know that the mass of a body plays an important role in linear motion. In the same way, the moment of inertia plays an important role in rotatory motion. The moment of inertia is the reluctance

tance of a rotating body to change its angular velocity. It opposes the angular acceleration of a rotating body. It depends upon the distribution of mass from the axis of rotation. The moment of inertia is represented by "I" and is given by,

$$I = mr^2 \quad (1)$$

In equation (1), "m" represents the mass of the rotating body and "r" represents the distance of the body from the axis of rotation.

Q10: Discuss the relation between torque and moment of inertia of a rotating body. (OR) Show that $\tau = I\alpha$

Ans. Consider a body of mass "m" attached to the end of a light rod, as shown in the figure.

Now if some force is applied on the body, then the body begins to rotate about the origin "O". In this way, torque is produced which is given by,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\Rightarrow \tau = rF \sin \theta \Rightarrow \tau = rF \sin \theta \quad (1)$$

Now if the force "F" is perpendicular to "r", then we put $\theta = 90^\circ$, so equation (1)

$$\Rightarrow \tau = rF \sin 90^\circ \quad [\sin 90^\circ = 1]$$

$$\Rightarrow \tau = rF \quad (2)$$

We also know that,

$$F = ma \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\tau = rma \quad (4)$$

We also know that,

$$a = r\alpha \quad (5)$$

Putting equation (5) in equation (4), we get,

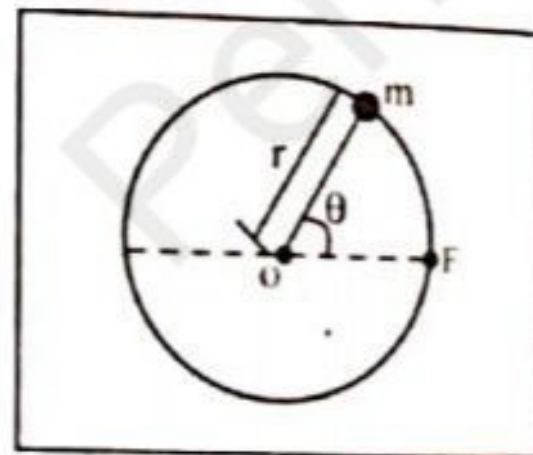
$$\tau = rm \times r\alpha$$

$$\Rightarrow \tau = mr^2\alpha$$

$$[mr^2 = \text{moment of inertia} = I]$$

$$\Rightarrow \tau = I\alpha \quad (6)$$

Equation (6) represents the relation between torque and moment of inertia during rotatory motion. Equation (6) shows that the product of moment of inertia and angular acceleration is equal to torque.



Torque produces angular acceleration in a body. (ETEA 2010-11)

Torque and Moment of Inertia of a Non-Uniform Rigid Body:

In case of a rigid body, we suppose the whole body to be consists of large number of small parts having masses $m_1, m_2, m_3, m_4, \dots$, at distance $r_1, r_2, r_3, \dots, r_n$ from the axis of rotation respectively. Now the torque acting on " m_1 " is given by,

$$\tau_1 = m_1 r_1^2 \alpha$$

Similarly,

$$\tau_2 = m_2 r_2^2 \alpha$$

$$\tau_3 = m_3 r_3^2 \alpha$$

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Now the total torque is given by,

$$\tau_t = \tau_1 + \tau_2 + \tau_3 + \dots + \tau_n$$

$$\Rightarrow \tau_t = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \dots + m_n r_n^2 \alpha$$

$$\Rightarrow \tau_t = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2) \alpha$$

$$\Rightarrow \tau_t = \left(\sum_{i=1}^n m_i r_i^2 \right) \alpha \quad (A)$$

In equation (A), we have,

$$\sum_{i=1}^n m_i r_i^2 = \text{total inertia} = I$$

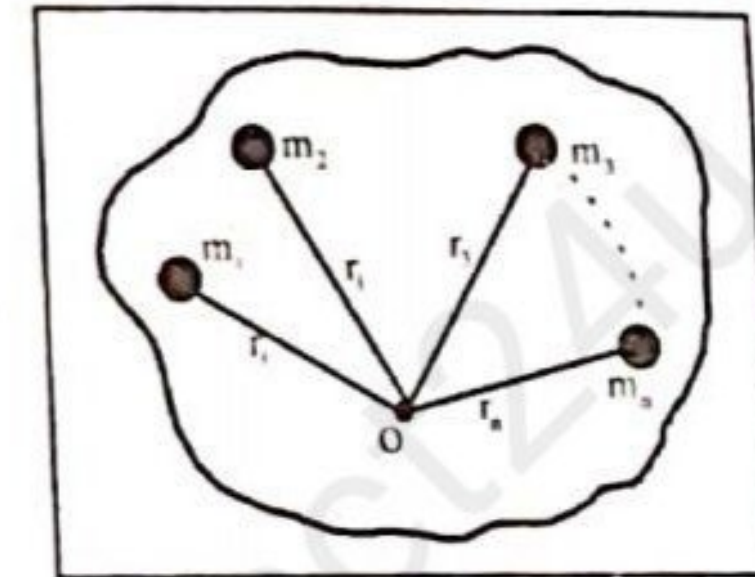
So equation (A) becomes,

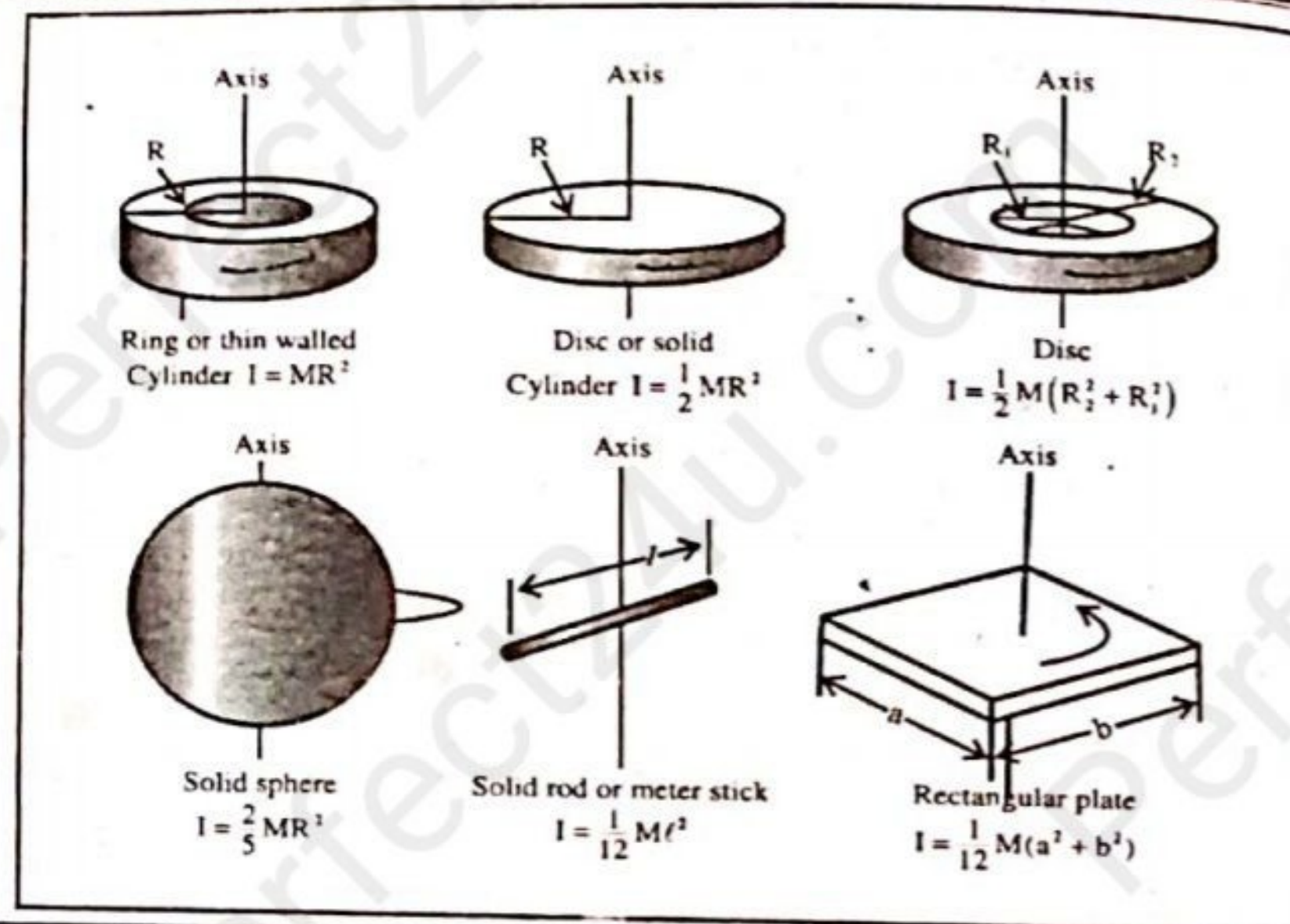
$$\tau_t = I\alpha \quad (B)$$

Equation (B) represents the torque and moment of inertia of a rigid body.

NOTE:

The mathematical expression of moment of inertia for bodies of various geometrical shapes are given below:





ACTIVITY:

Find the torque acting on bike wheel of the given figure.

Ans: Let "F" represents the applied force on the wheel which rotates with angular acceleration " α " as shown in the figure. Then the torque produced is given by,

$$\tau = rF \quad (1)$$

$$\text{Also } F = ma \quad (2)$$

Putting equation (2) in equation (1), we get,

$$\tau = rma \quad (3)$$

$$\text{Also } a = r\alpha \quad (4)$$

Putting equation (4) in equation (3), we get

$$\tau = rm \cdot r\alpha = r^2 m\alpha \quad (5)$$

In equation (5), $mr^2 = \text{moment of inertia} = I$

So equation (5) becomes,

$$\tau = I\alpha \quad (6)$$

Equation (6) represents the magnitude of torque acting on the bike wheel of radius "r" and having angular acceleration " α ".



Q11: Define and explain angular momentum.

BISE DI Khan 2019

Ans: Definition:

The momentum of a body due to rotatory motion is known as angular momentum. (OR)

The cross product of moment arm and linear momentum is known as angular momentum.

Explanation:

The angular momentum is represented by " $\vec{L}$ " and is given by,

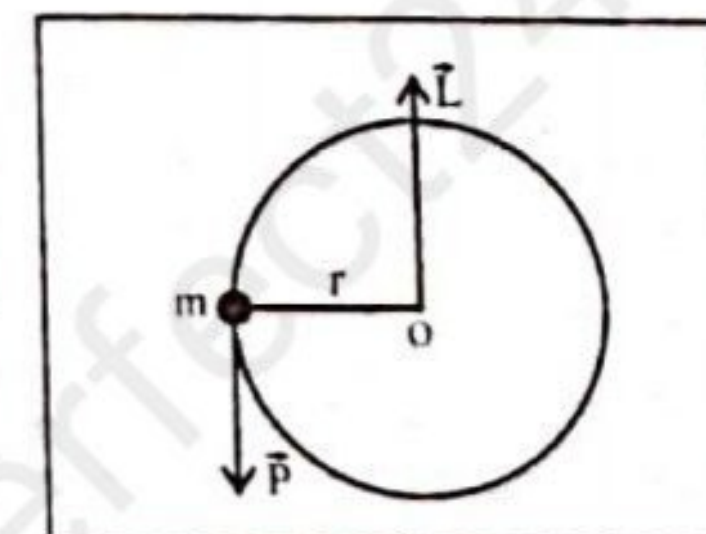
$$\vec{L} = \vec{r} \times \vec{p} \quad (1)$$

$$\Rightarrow \vec{L} = r p \sin \theta \hat{n}$$

$$\Rightarrow L = r p \sin \theta \quad (2)$$

Equation (2) represents the magnitude of angular momentum. Equation (2) shows that for given values of r, p, the angular momentum depends upon " θ ".

The angular momentum is a vector quantity. Its unit is $\text{kg} \cdot \text{m}^2 / \text{sec}$. It is always perpendicular to the plane determined by " $\vec{r}$ " and " $\vec{p}$ ".



Dimension of Angular Momentum:

$$\text{As } L = \text{kg} \cdot \text{m}^2 / \text{sec}$$

$$\Rightarrow L = \frac{\text{mass} \cdot \text{length}^2}{\text{time}}$$

$$\Rightarrow L = [ML^2T^{-1}]$$

Dimension of angular momentum and Planck's constant are same. (ETEA 2009, 10)

$$\text{Hints: } L = \frac{nh}{2\pi} \Rightarrow h = \frac{2\pi L}{n} = \frac{2\pi}{n} \cdot \frac{\text{kgm}^2}{\text{sec}} \Rightarrow h = [ML^2T^{-1}]$$

Q12: Discuss the relation between angular momentum and moment of inertia. (OR) Show that, $L = I\omega$

Ans: Consider a particle of mass "m" having linear momentum " $\vec{p}$ " moving along a circular path of radius "r" as shown in the figure. Now the angular momentum " $\vec{L}$ " of the particle is given by,

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Rightarrow \vec{L} = r p \sin \theta \hat{n} \Rightarrow L = r p \sin \theta \quad (1)$$

For maximum angular momentum, we put $\theta = 90^\circ$ so equation (1) becomes,

$$L = r p \sin 90^\circ$$

$$[\sin 90^\circ = 1]$$

$$\Rightarrow L = rp \quad (2)$$

We know that,

$$p = mv \quad (3)$$

Putting equation (3) in equation (2), we get,

$$L = rmv \quad (4)$$

We also know that,

$$v = r\omega \quad (5)$$

Putting equation (5) in equation (4), we get,

$$L = rm \times r\omega$$

$$\Rightarrow L = mr^2\omega \quad [mr^2 = \text{moment of inertia} = I]$$

$$\Rightarrow \boxed{L = I\omega} \quad (6)$$

Equation (6) represents the relation between angular momentum and moment of inertia.

Angular Momentum of a Rotating Rigid Body:

In case of rigid body, we suppose the body to be consist of large number of parts having masses $m_1, m_2, m_3, m_4, \dots, m_n$ at distance $r_1, r_2, r_3, \dots, r_n$ from axis of rotation "O" respectively. Now the angular momentum of " m_1 " is given by,

$$L_1 = m_1 r_1^2 \omega$$

Similarly,

$$L_2 = m_2 r_2^2 \omega$$

$$L_3 = m_3 r_3^2 \omega$$

...

...

...

$$L_n = m_n r_n^2 \omega$$

Now the total angular momentum of the rigid body is given by,

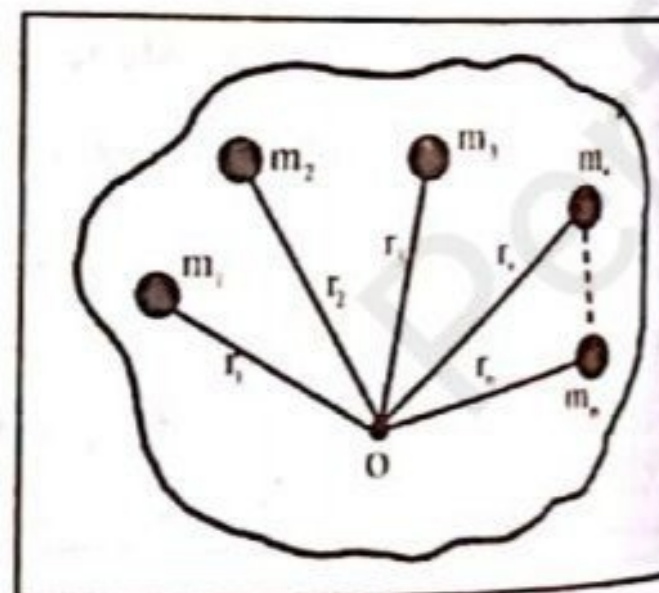
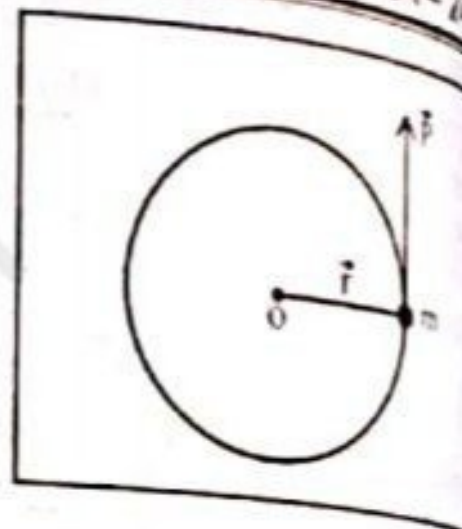
$$L_t = L_1 + L_2 + L_3 + \dots + L_n$$

$$\Rightarrow L_t = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + \dots + m_n r_n^2 \omega$$

$$\Rightarrow L_t = (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2) \omega$$

$$\Rightarrow L_t = \left[\sum_{i=1}^n m_i r_i^2 \right] \omega$$

$$\Rightarrow \boxed{L_t = I\omega} \quad (A) \quad \left[\sum_{i=1}^n m_i r_i^2 = \text{total moment of inertia} = I \right]$$



Equation (A) represents the angular momentum of a rotating rigid body.

Q13: Discuss the relation between torque and angular momentum.

(OR) Show that, $\tau = \frac{\Delta L}{\Delta t}$

BISE Malakand 2019, DI Khan 2019, BISE Peshawar 2019

Ans. We know that the angular momentum is given by,

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \vec{L} = rp \sin \theta \hat{n} \Rightarrow L = rp \sin \theta$$

$$\Rightarrow L = rp \sin 90^\circ = rp \times 1 \quad [\text{put } \theta = 90^\circ]$$

$$\Rightarrow L = rp \Rightarrow \Delta L = r \Delta p \quad (1)$$

Dividing both sides of equation (1) by " Δt ", we get,

$$\frac{\Delta L}{\Delta t} = r \left(\frac{\Delta p}{\Delta t} \right) \quad (2)$$

According to Newton's 2nd law, we get,

$$\frac{\Delta p}{\Delta t} = F \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\frac{\Delta L}{\Delta t} = rF \quad (4)$$

As $rF = \text{torque}$, so equation (4) becomes

$$\frac{\Delta L}{\Delta t} = \tau \Rightarrow \tau = \frac{\Delta L}{\Delta t} \Rightarrow \boxed{\tau = \frac{\Delta L}{\Delta t}} \quad (5)$$

Equation (5) represents the relation between torque and angular momentum. Equation (5) shows that the torque is equal to the rate of change of angular momentum.

Q14: What do you mean by conservation of a physical quantity?

Ans. The conservation of a physical quantity means that a physical quantity remains conserved during any change of an isolated system. For example, mass, charge, energy, momentum etc are the physical quantities which remain constant or conserved during any change taking place in an isolated system.

Q15: State and explain the law of conservation of angular momentum with examples.

ETEA 2005

Ans. **Statement:**

This law states that, the angular momentum of a system remains constant in the absence of external torque.

Explanation: We know that,

$$\dot{L} = \frac{\Delta L}{\Delta t} \quad (1)$$

Now in the absence of external torque, we have,

$$\dot{L} = 0 \quad (2)$$

Putting equation (2) in equation (1), we get,

$$0 = \frac{\Delta L}{\Delta t} \Rightarrow \frac{\Delta L}{\Delta t} = 0 \Rightarrow \Delta L = 0 \quad (3)$$

Equation (3) shows that change in angular momentum is equal to zero. In other words, we can say that, there is no change in angular momentum i.e. the angular momentum remains constant. So equation (2) can be written as,

$$L = \text{constant} \quad (4)$$

Equation (4) represents the mathematical form of law of conservation of angular momentum.

Examples: The examples of law of conservation of angular momentum are:

1. Motion of planets around the sun.
2. Motion of moon and artificial satellites around the earth.
3. Orbital motion of electrons around the nucleus.
4. Motion of divers, ice skaters, dancers etc.

Q16: Discuss the K.E of a rotating rigid body.

BISE Bannu 2017

Ans. Consider a rigid body which consists of large number of parts having masses $m_1, m_2, m_3, \dots, m_n$ at distance $r_1, r_2, r_3, \dots, r_n$ from fixed origin "O" as shown in the figure. Now the K.E of " m_1 " is given by,

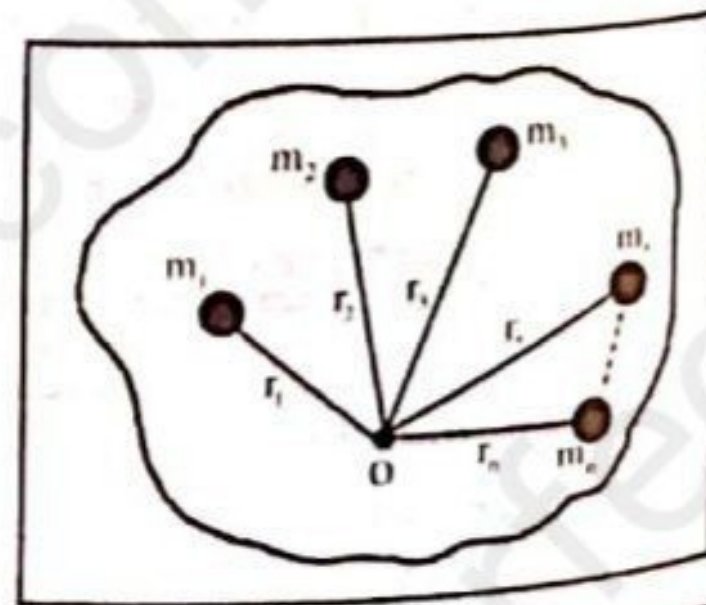
$$(K.E)_1 = \frac{1}{2} m_1 v_1^2 \quad [v = r \omega]$$

$$\Rightarrow (K.E)_1 = \frac{1}{2} m_1 r_1^2 \omega^2$$

$$\text{Similarly, } (K.E)_2 = \frac{1}{2} m_2 r_2^2 \omega^2$$

$$(K.E)_3 = \frac{1}{2} m_3 r_3^2 \omega^2$$

$\vdots$



$$(K.E)_n = \frac{1}{2} m_n r_n^2 \omega^2$$

$$(K.E)_n = \frac{1}{2} m_n r_n^2 \omega^2$$

Now the total K.E of the rotating rigid body is given by,

$$(K.E)_t = (K.E)_1 + (K.E)_2 + \dots + (K.E)_n$$

$$\Rightarrow (K.E)_t = (K.E)_1 + (K.E)_2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$\Rightarrow (K.E)_t = \frac{1}{2} \omega^2 [m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2]$$

$$\Rightarrow (K.E)_t = \frac{1}{2} \omega^2 \left[\sum_{i=1}^n m_i r_i^2 \right] \quad \left[\sum_{i=1}^n m_i r_i^2 = \text{total moment of inertia} = I \right]$$

$$\Rightarrow (K.E)_t = \frac{1}{2} \omega^2 I$$

$$\Rightarrow (K.E)_t = \frac{1}{2} I \omega^2 \quad (1)$$

Equation (1) represents the K.E of a rotating rigid body.

Q17: Discuss the K.E of a rolling body.

Ans. When an object rolls down on an inclined plane, it possesses linear motion as well as rotational motion. Thus the body possesses transla-

tional K.E due to linear velocity " $\vec{v}$ " and rotational K.E due to rotational velocity " $\vec{\omega}$ ".

Thus the total K.E of the rolling body is given by,

$$(K.E)_{\text{total}} = (K.E)_{\text{linear}} + (K.E)_{\text{rotational}}$$

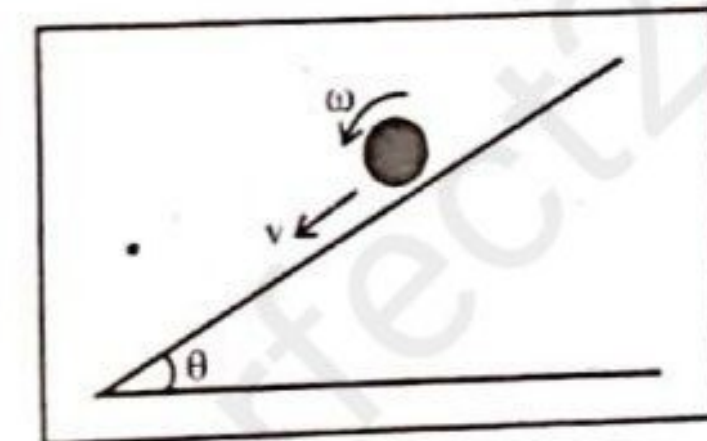
$$\Rightarrow (K.E)_t = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \quad (1)$$

Equation (1) represents the K.E of a rolling body.

Q18: A disc and a hoop of the same radius starts moving down from the top of an inclined plane at the same time. Which will be moving faster on reaching the bottom of the inclined?

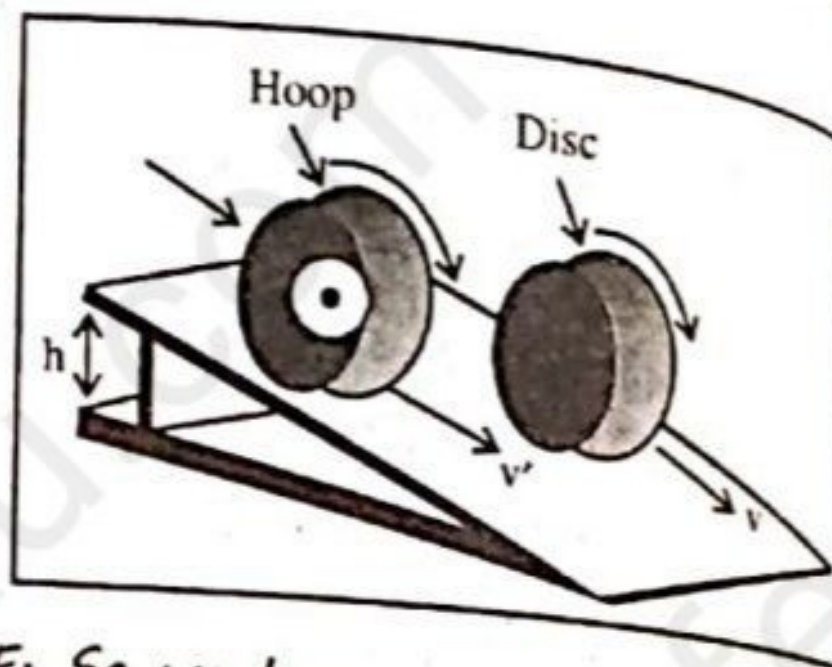
BISE Mardan 2019

Ans. A piece of hollow sphere is known as hoop or thin ring. While a



piece of solid cylinder is known as disc.

Consider a hoop and disc of the same radius start moving down from the top of an inclined plane at the same time as shown in the figure. At the top, the hoop and disc possess P.E. At the bottom of the inclined plane, their P.E. is totally converted into translational K.E. and rotational K.E. So we have,



$P.E = (K.E)_{trans} + (K.E)_{rot}$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{v^2}{2r^2}I \quad (1) \quad \left[v = r\omega, \omega = \frac{v}{r} \right]$$

For Disc: For disc, the moment of inertia is given by,

$$I = \frac{1}{2}mr^2 \quad (2)$$

Putting equation (2) in equation (1), we get,

$$mgh = \frac{1}{2}mv^2 + \frac{v^2}{2r^2} \times \frac{1}{2}mr^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{2mv^2 + mv^2}{4}$$

$$\Rightarrow mgh = \frac{3mv^2}{4} \Rightarrow gh = \frac{3v^2}{4}$$

$$\Rightarrow v^2 = \frac{4gh}{3} \Rightarrow v = \sqrt{\frac{4}{3}gh}$$

$$\Rightarrow v = \sqrt{1.33gh} \quad (3)$$

For Hoop:

For hoop, the moment of inertia is given by,

$$I = mr^2 \quad (4)$$

Putting equation (4) in equation (1), we get,

$$mgh = \frac{1}{2}mv^2 + \frac{v^2}{2r^2} \times mr^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{2mv^2}{2} = mv^2$$

$$\Rightarrow mgh = mv^2 \Rightarrow v^2 = gh$$

$$\Rightarrow v = \sqrt{gh} \quad (5)$$

Comparing equation (3), (5), we get,
 $v > v'$ i.e. $(v)_{disc} > (v)_{hoop}$

Thus the disc will reach the bottom of inclined plane faster than hoop.

Q19: (a) What is an artificial satellite? (b) Discuss the critical velocity of satellite. (c) Discuss the orbital velocity of satellite.

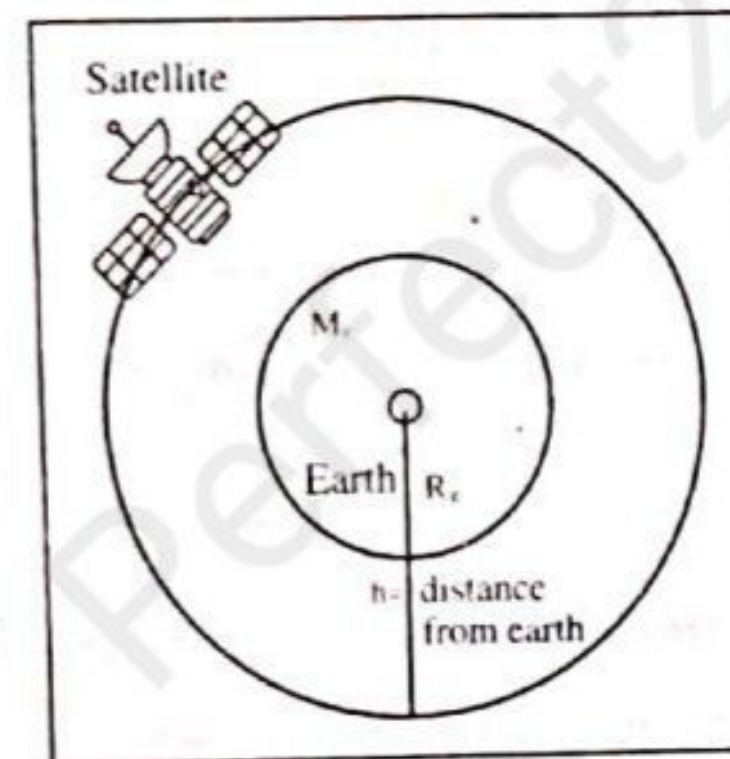
ETEA 2006, 2009, 2010, 2013, 2015, BISE Peshawar 2015, BISE Mardan 2017

Ans. (a) Artificial Satellite:

Artificial satellites are spacecraft that stay in orbit around the earth.

Applications of Artificial Satellites:

1. With the help of artificial satellites, we can take the pictures of the earth.
2. Artificial satellites relay communications around the earth.
3. Some satellites are used for navigation. The signals they send out are used by the crews of ships and aircraft to keep on course as they travel around the earth.
4. The global positioning system [GPS] is a system of 24 satellites which can tell a person his or her position on the earth very accurately.
5. Some satellites are used to investigate space.



(b) Critical Velocity of Satellite:

The minimum velocity required to put a satellite into the orbit is known as critical velocity of satellites.

When a satellite is moving in a circle of radius "R" from centre of earth of mass "M_e", it has centripetal acceleration, which is given by,

$$a_c = \frac{v^2}{R}$$

$$[a_c = g]$$

$$\Rightarrow g = \frac{v^2}{R}$$

$$\Rightarrow v^2 = gR \Rightarrow v = \sqrt{gR} \quad (1)$$

Put $g = 9.8 \text{ m/sec}^2$, $R = 6.4 \times 10^6 \text{ m}$ [radius of earth] in equation (1), we get,

$$v = \sqrt{9.8 \times 6.4 \times 10^6} = \sqrt{62.72 \times 10^3} \text{ m/sec}$$

$$\Rightarrow v = 7.9 \times 10^3 \text{ m/sec}$$

$$\Rightarrow v = 7.9 \text{ km/sec} \quad [10^3 = \text{kilo}]$$

This is the minimum velocity required to put a satellite into the orbit and is known as critical velocity.

(c) Orbital Velocity of Satellite:

A satellite of mass "m" moving around the earth in a circular orbit due to gravitational force between earth and satellite. So this force acts as centripetal force. So we have,

$$F_c = F_g$$

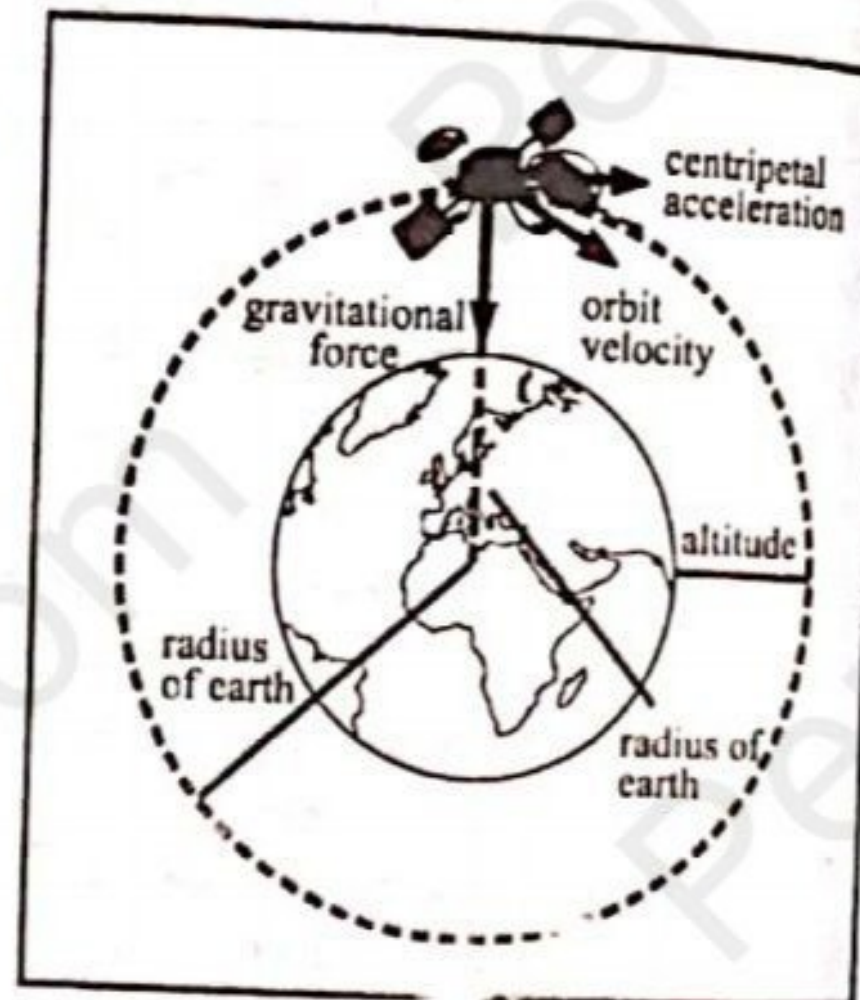
$$\Rightarrow \frac{mv^2}{r} = \frac{GM_e m}{r^2} \Rightarrow v^2 = \frac{GM_e}{r}$$

$$\Rightarrow v = \sqrt{\frac{GM_e}{r}} \quad (2)$$

In equation (2) $\sqrt{GM_e} = \text{constant}$, so equation (2) becomes,

$$\Rightarrow v \propto \frac{1}{\sqrt{r}} \quad (3)$$

Equation (2) shows that the velocity of satellite is inversely proportional to the distance of satellite from the planet. i.e. a satellite in orbit will move faster when it is closer to the planet and slower when it is farther away. When a satellite falls from high altitude to lower altitude, it gains speed, and when it rises from low altitude to higher altitude, it loses speed.



Q20: Write a note on geostationary satellite.

Ans: It is a satellite which moves around the earth about its own axis. Its time period is 24 hours and appears stationary as seen from the earth. These satellites are very useful for world wide communication, weather observations and military purposes.

Now we know that the orbital speed of satellite is given by,

$$v = \sqrt{\frac{GM_e}{r}} \quad (1)$$

We also know that,

$$S = vt \Rightarrow v = \frac{S}{t}$$

$$\Rightarrow v = \frac{2\pi r}{T} \quad (2)$$

Comparing equations (1), (2), we get,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM_e}{r}}$$

Squaring both sides, we get,

$$\Rightarrow \left(\frac{2\pi r}{T}\right)^2 = \left(\sqrt{\frac{GM_e}{r}}\right)^2$$

$$\Rightarrow \frac{4\pi^2 r^2}{T^2} = \frac{GM_e}{r} \Rightarrow r^3 = \frac{GM_e T^2}{4\pi^2} \quad (3)$$

Time Period of Satellite:

From equation (3), we have,

$$T^2 = \frac{4\pi^2 r^3}{GM_e} = \frac{4\pi^2 r}{\left(\frac{GM_e}{r^2}\right)} = \frac{4\pi^2 r}{g} \quad \left[\frac{GM_e}{r^2} = g\right]$$

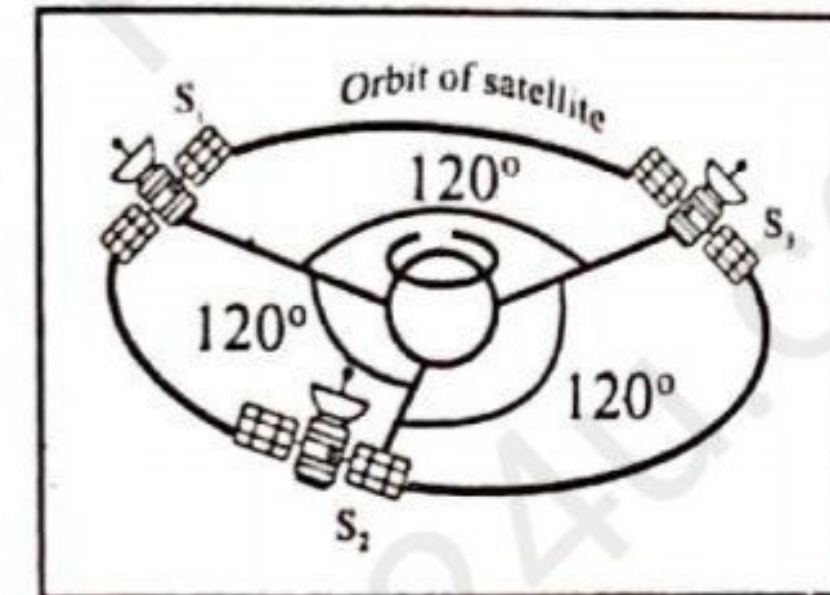
$$\Rightarrow T^2 = 4\pi^2 \frac{r}{g} \Rightarrow T = 2\pi \sqrt{\frac{r}{g}} \quad (4)$$

Equation (4) represents the time period of satellite.

Frequency of Satellite:

$$\text{As } f = \frac{1}{T} \Rightarrow f = \frac{1}{2\pi \sqrt{\frac{r}{g}}} \quad (5)$$

Equation (5) represents the frequency of satellite.



$$\left[\begin{array}{l} S = \text{circumference} = 2\pi r \\ t = \text{time period} = T \end{array} \right]$$

Radius of Satellite:

From equation (3), we have,

$$r^3 = \frac{GM_e T^2}{4\pi^2} \Rightarrow r = \left[\frac{GM_e T^2}{4\pi^2} \right]^{1/3} \quad (3)$$

Put $G = 6.673 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$, $M_e = 6 \times 10^{24} \text{ kg}$

$T = 365 \text{ days} = 365 \times 24 \text{ hr} = 365 \times 24 \times 3600 \text{ sec}$

$\pi = 3.14$ in equation (3), we get,

$$r = \left[\frac{6.673 \times 10^{-11} \times 6 \times 24 \times (365 \times 24 \times 3600)^2}{4 \times (3.14)^2} \right]^{1/3}$$

$$\Rightarrow r = 4.23 \times 10^7 \text{ m} \quad [10^3 = \text{kilo}]$$

$$\Rightarrow r = 4.23 \times 10^4 \times 10^3 \text{ m} = 4.23 \times 10^4 \text{ km}$$

$$\Rightarrow r = 4.23 \times 10^4 \text{ km} \quad (4)$$

Equation (4) represents the orbital radius for geostationary satellite which is measured from the centre of the earth. At this height, a satellite will always stay over a particular point on the surface of earth. This height above the equator comes to be 36000 km. The whole surface of the earth can be covered by using three geostationary satellites as shown in the figure. Each satellite covers a longitude of 120° .

Q21: (a) What do you mean by real weight and apparent weight?

(b) Discuss the weight of a body in a lift or elevator?

Ans: (a) Real and Apparent Weight:

The earth's gravitational force on the body represents a real weight. While the force needed to support the body against gravity, i.e. to prevent the body from falling represent its apparent weight.

(b) Weight of a Body in a Lift or Elevator:

Let Anmol weight a fish with a spring scale attached to the ceiling of lift as shown in the figure.

Case-1: When the lift is at rest: When the lift is at rest, then the weight of fish is acting in downwards direction while the tension in the string is acting in upward direction. Both the forces balance each other and the net force on the fish is zero. So we have,

$$T = w \Rightarrow T = mg \quad (1)$$

Case-2: When the lift is moving upward: When the lift is moving upward, then the net force on the fish is given by,

$$F_{\text{net}} = T - w$$

$$\Rightarrow ma = T - mg \Rightarrow T = ma + mg \quad (2)$$

Equation (2) shows that when the lift is moving upward, then the spring balance measures the weight which is greater than actual weight "W". This increase in weight of the fish is called apparent weight of the fish.

Case-3: When the lift is moving downwards: When the lift is moving downwards then the net force on the fish is given by,

$$F_{\text{net}} = w - T$$

$$\Rightarrow ma = mg - T \Rightarrow -T = ma - mg$$

$$\Rightarrow T = mg - ma \quad (3)$$

Equation (3) shows that when the lift is moving downwards, the spring balance measures the weight which is less than actual weight "w" of the fish. This decrease in weight of the fish represents the apparent weight of the fish.

Case-4: When the lift is falling freely: When the lift is falling freely, then we put $a = g$, so equation (3) becomes,

$$T = mg - mg \Rightarrow T = 0 \quad (4)$$

Equation (4) shows that the spring balance shows zero reading of weight when the lift falls freely. Thus the apparent weight of the fish becomes zero when the lift is falling freely under gravity.



Fig. a: When the elevator is at rest, the spring scale reads the true weight of the fish.



Fig. b: When the elevator accelerates upward, the spring scale reads a value greater than the weight of the fish.



Fig. c: When the elevator accelerates downward, the spring scale reads a value less than the weight of the fish.

Q22: Discuss the weightlessness in a satellite.

Ans. Consider a satellite of mass "m" moving around the earth in a circular orbit of radius "r" as shown in the figure.

Here the weight of satellite is responsible for providing centripetal force. So we have,

$$w = F_c \Rightarrow mg = \frac{mv^2}{r}$$

$$\Rightarrow g = \frac{v^2}{r} \quad (1)$$

Now consider a body of mass "m" suspended by a spring from the ceiling of satellite. Here two forces are acting on the body. One is the force of gravity and the other is the normal reaction or tension in the spring, which acts in upwards direction. Thus the net force on the body is given by,

$$F_{net} = w - T$$

$$\Rightarrow F_{net} = mg - T \quad (2)$$

This force provides the necessary centripetal force to keep the body in circular orbit. So equation (2) becomes,

$$\frac{mv^2}{r} = mg - T \quad (3)$$

Putting equation (1) in equation (3), we get,

$$\frac{mv^2}{r} = \frac{mv^2}{r} - T \Rightarrow -T = 0$$

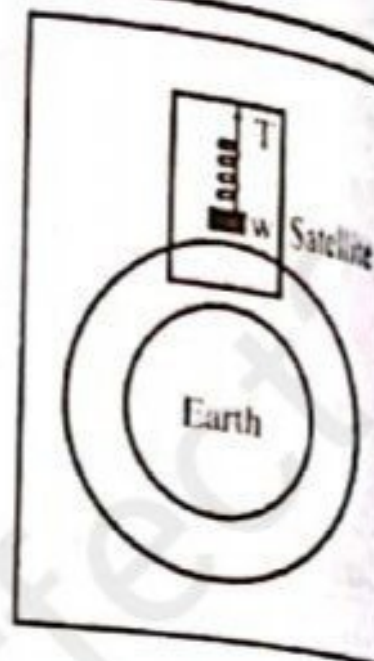
$$\Rightarrow T = 0 \quad (4)$$

Equation (4) shows that the tension or normal reaction on the body is zero, which shows that the body inside the satellite is weightless. Thus the body inside the satellite will appear in state of weightlessness.

Q23: Write a note on artificial gravity.

BISE Kohat 2017, 2019

Ans. As we know that the body inside the satellite is weightless, so the body faces certain problems while working inside the satellite. To overcome these problems, an artificial gravity is produced inside the satellite. For this purpose, some spin motion is given to the



satellite due to which the body inside the satellite experiences some centrifugal force. Due to centrifugal force, the body experiences a push away from the axis of rotation and thus finally comes in contact with the walls of satellite. In this way, an artificial acceleration is produced which is equal to the centrifugal acceleration and is given by,

$$a = \frac{v^2}{r}$$

$$\Rightarrow g = \frac{(r\omega^2)}{r}$$

$$\Rightarrow g = r\omega^2 \quad (1)$$

We know that,

$$f = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f \quad (2)$$

Putting equation (2) in equation (1), we get,

$$g = r \times (2\pi f)^2$$

$$\Rightarrow g = 4\pi^2 r f^2 \Rightarrow f^2 = \frac{1}{4\pi^2} \frac{g}{r}$$

$$\Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{r}} \quad (3)$$

Equation (3) represents the frequency of satellite with which we get gravity equal to earth gravity and as a result the astronaut will feel no difficulty in performing their duties.

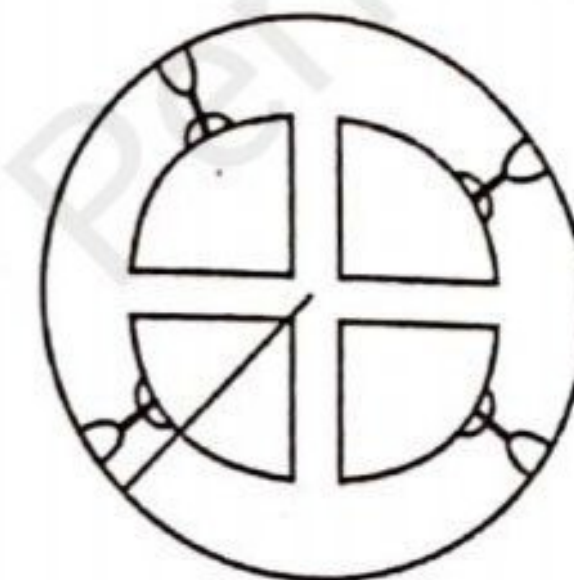
Q24: Discuss the various types of artificial satellite.

Ans. Types of Satellites:

There are several types of satellite which are given below:

1. Navigation Satellites:

The GPS (global positioning system) is made up of 24 satellites that orbit at an altitude of 20,000km above the surface of the earth. The difference in time for signals received from four satellites is used to calculate the exact location of a GPS receiver on earth.



The spaceship rotates with angular velocity " ω " and frequency $f = \frac{1}{2\pi} \sqrt{\frac{g}{r}}$ giving us the artificial gravity like earth.

For producing gravity equal to earth gravity, we put $a = g$

$$\text{As } \omega = \frac{\theta}{t}$$

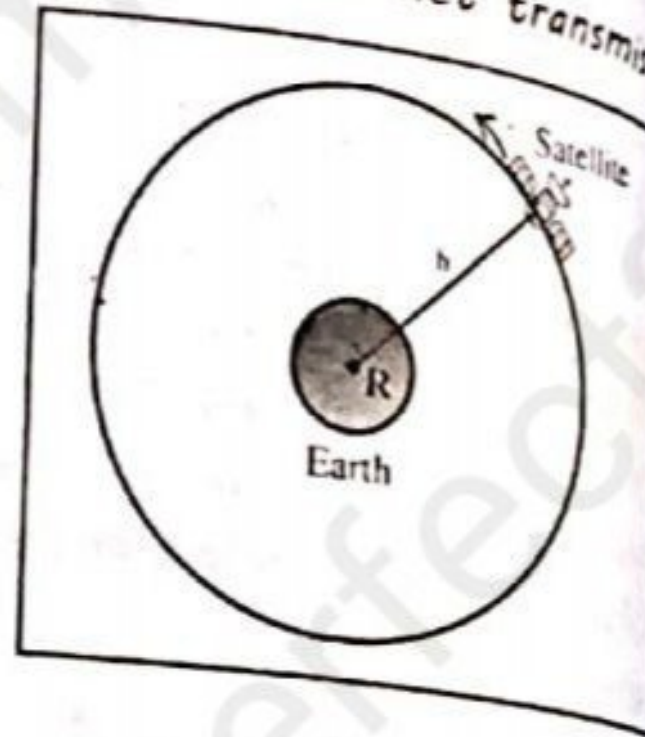
$$\Rightarrow \omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

$$\Rightarrow f = \frac{1}{T} = \frac{\omega}{2\pi}$$

2. Communication Satellites:

These satellites are used for television, phone or internet transmissions, for example, the Optus D1 satellite is in a geostationary orbit above the equator and has a coverage footprint to provide signals to all of Australia and New Zealand.



Communications satellites allow radio, television and telephone transmissions to be sent live anywhere in the world. Before satellites, transmissions were

difficult or impossible at long distances. The signals, which travel in straight lines, could not bend around the round earth to reach a destination far away. Because satellites are in orbit, the signals can be sent instantaneously into space and then redirected to another satellite or directly to their destination.

Communications satellites are often in geostationary orbit. At the high orbital altitude of 35,800 kilometers, a geostationary satellite orbits the earth in the same amount of time it takes the earth to revolve once. From earth, therefore, the satellite appears to be stationary, always above the same area of the earth. The area to which it can transmit is called a satellite's footprint.

3. Weather Satellites:

These satellites are used to image clouds and measure temperature and rainfall. Both geostationary and low earth orbits are used depending on the type of weather satellite. Weather satellites are used to help with more accurate weather forecasting.

مشکلات میں گھبراہٹ نہ کیجئے کیونکہ ستارے ہمیشہ اندھیرے میں ہی چمکتے ہیں۔

اصل طاقت برداشت کرنے میں ہے انتقام لینے میں نہیں۔

Solution of Examples & Assignments

EXAMPLE 5.1:

An electric motor turns at 400rpm. What is the angular velocity in rad/sec? What is the angular displacement after 4s?

Solution:

Given data:

Angular velocity = $\omega = 400 \text{ rev/min}$, time = $t = 4 \text{ sec}$

Required:

(a) Angular velocity in rad/sec = $\omega = ?$

(b) Angular displacement = $\theta = ?$

Calculation:

(a) Angular velocity = $\omega = 400 \text{ rev/min}$

$$\Rightarrow \omega = 400 \times \frac{2\pi}{60} \text{ rad/sec}$$

$$\Rightarrow \omega = 41.9 \text{ rad/sec}$$

$$1 \text{ revolution} = 2\pi \text{ radian}$$

(b) We know that, $\theta = \omega t$ (2)

Putting the values in equation (2), we get,

$$\theta = (41.9 \times 4) \text{ rad} \Rightarrow \theta = 167.6 \text{ rad}$$

ASSIGNMENT 5.1:

A rotating pulley completes 12rev in 4s. Determine the average angular velocity in rev/s, rpm, and in rad/s? (3.00 rev/s, 28.6 rpm, and 18.8 rad/s)

Solution:

Given data:

Total angular displacement = $\theta = 12 \text{ rev}$

Total time = $t = 4 \text{ sec}$

Required:

(a) Average angular velocity in rev/sec = ?

(b) Average angular velocity in rev/min = ?

(c) Average angular velocity in rad/sec = ?

Calculation:

(a) We know that average angular velocity is given by,

$$60 \text{ sec} = 1 \text{ min}$$

$$1 \text{ sec} = \frac{1}{60} \text{ min}$$

$$1 \text{ rev} = 2\pi \text{ rad}$$

$$\omega_{avg} = \frac{\text{total angular displacement}}{\text{total time taken}}$$

$$\Rightarrow \omega_{avg} = \frac{12 \text{ rev}}{4 \text{ sec}} \Rightarrow \boxed{\omega_{avg} = 3 \text{ rev / sec}}$$

$$(b) \omega_{avg} = 3 \text{ rev / sec} = \frac{3 \text{ rev}}{\left(\frac{1}{60}\right) \text{ min}} = 3 \times 60 \text{ rev / min}$$

$$\Rightarrow \boxed{\omega_{avg} = 180 \text{ rev / min}}$$

$$(c) \omega_{avg} = 180 \text{ rev / min} = 180 \times \frac{2\pi}{60} \text{ rad / sec}$$

$$\Rightarrow \boxed{\omega_{avg} = 18.8 \text{ rad / sec}}$$

EXAMPLE 5.2:

In a carnival ride, the passenger travel in a circle of radius 5.0m, making one complete cycle in 4.0s. What is its acceleration?

Solution:Given data:

Radius of circle = $r = 5.0\text{m}$, time = $t = 4.0\text{s}$

Required:

Acceleration = $a = ?$

Calculation:

We know that,

$$a_c = \frac{v^2}{r} \quad (1)$$

We also know that,

$$v = \frac{S}{t} \quad (2)$$

Putting equation (2) in equation (1), we get

$$a_c = \frac{S^2}{t^2 \cdot r}$$

$$\Rightarrow a_c = \frac{(2\pi r)^2}{t^2 \cdot r} = \frac{4\pi^2 r^2}{t^2 \cdot r}$$

$$\Rightarrow a_c = \frac{4\pi^2 r}{t^2} \quad (3)$$

Putting the values in equation (3), we get,

$$a_c = \frac{4 \times (3.14)^2 \times 5}{4^2} = 12.3 \text{ rad/sec}^2 \Rightarrow \boxed{a_c = 12.3 \text{ rad / sec}^2}$$

For complex cycle,
 $S = 2\pi r$

ASSIGNMENT 5.2:

An airplane dives along a curved path of radius "R" and velocity "v". The centripetal acceleration is 10ms^{-2} . If both the velocity and the radius are doubled, what will be the new acceleration? (20ms^{-2})

Solution:Given data:

Centripetal acceleration = $a_c = 10\text{m / sec}^2$

Radius of curved path = R, Velocity of airplane = v

If velocity and radius are doubled, then, $R' = 2R$ and $v' = 2v$

Required:

New centripetal acceleration = $a'_c = ?$

Calculation:

We know that,

$$a_c = \frac{v^2}{R} \quad (1)$$

For new acceleration equation (1) becomes,

$$a'_c = \frac{v'^2}{R'} \quad (2)$$

Putting the values of "v" and "R" in equation (1), we get,

$$a_c = \frac{(2v)^2}{2R} = \frac{4v^2}{2R} = 2 \left(\frac{v^2}{R} \right)$$

$$\Rightarrow a'_c = 2 \left(\frac{v^2}{R} \right) \quad (3)$$

From equation (1),

$$\frac{v^2}{R} = a_c = 10 \text{ m/sec}^2 \Rightarrow \frac{v^2}{R} = 10 \text{ m/sec}^2 \quad (4)$$

Putting equation (4) in equation (3), we get,

$$a'_c = 2 \times 10 \text{ m/sec}^2 \Rightarrow \boxed{a'_c = 20 \text{ m / sec}^2}$$

$$\Rightarrow \boxed{a'_c = 20 \text{ m / sec}^2}$$

EXAMPLE 5.3:

The curved roadway is designed in such a way that a car will not have to rely on friction to round the curve even when the road is covered with ice. Suppose the designated speed for the road is to be 12m/s and the radius of the curve is 36.0m . At what angle

should the curve be banked?

Solution:

Given data:

Radius of circle = $r = 36.0\text{m}$, speed = $v = 12\text{m/s}$

Required: Angle of bank = $\theta = ?$

Calculation: We know that, $v = \sqrt{rg \tan \theta}$

$$\Rightarrow v^2 = rg \tan \theta \Rightarrow \tan \theta = \frac{v^2}{rg}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{v^2}{rg} \right) = \tan^{-1} \left[\frac{12^2}{36 \times 9.8} \right] = \tan^{-1} \left[\frac{144}{352.8} \right]$$

$$\Rightarrow \theta = \tan^{-1}(0.408) = 22.1^\circ = 22^\circ \Rightarrow \boxed{\theta = 22^\circ}$$

ASSIGNMENT 5.3:

At what speed (in km/h) is a bank angle of 45° required for an aeroplane to turn on a radius of 60m? (87.34km/h)

Solution:

Given data:

Bank angle = $\theta = 45^\circ$, radius of turn = $r = 60\text{m}$

Required:

Speed of aeroplane = $v = ?$

Calculation:

We know that,

$$v = \sqrt{rg \tan \theta} \quad (1)$$

Putting the values in equation (1), we get,

$$v = \sqrt{60 \times 9.8 \times \tan(45^\circ)} = \sqrt{588 \times 1}$$

$$= 24.3 \text{ m/sec}$$

$$\Rightarrow \boxed{v = 24.3 \text{ m/sec}}$$

$$\text{OR } v = 24.3 \times \frac{3600}{1000} \text{ km/hr} = 87.3 \text{ km/hr}$$

$$\Rightarrow \boxed{\text{Speed} = v = 87.3 \text{ km/hr}}$$

EXAMPLE 5.4:

A 2kg mass swings in a circle of radius 50cm at the end of a light rod. What resultant torque is required to give an angular acceleration of 2.5 rad/s^2 .

Solution:

Given data: Radius = $r = 50\text{cm} \Rightarrow r = 0.5\text{m}$,
Mass = $m = 2\text{kg}$

Angular acceleration = $\alpha = 2.5 \text{ rad/sec}^2$

Required: Torque = $\tau = ?$

Calculation:

We know that, $\tau = I\alpha$ _____ (1)

Also we know that, moment of inertia of a circular hoop is given by,

$$I = mr^2 \quad (2)$$

Putting equation (2) in equation (1), we get,

$$\tau = mr^2 \alpha \quad (3)$$

Putting the values in equation (3), we get,

$$\tau = [2 \times (0.5)^2 \times 2.5] \text{ N}\cdot\text{m} = 1.25 \text{ N}\cdot\text{m}$$

$$\Rightarrow \boxed{\tau = 1.25 \text{ N}\cdot\text{m}}$$

ASSIGNMENT 5.5:

A cord is wrapped around the rim of a cylinder that has a mass of 10kg and a radius of 30cm. If the rope is pulled with a force of 60N, what is the angular acceleration of the cylinder? (40 rad/s^2)

Solution:

Given data: Mass = $m = 10\text{kg}$, Radius = $30\text{cm} = 0.3\text{m}$

Force = $F = 60\text{N}$

Required:

Angular acceleration = $\alpha = ?$

Calculation:

For cylinder, we have,

$$I = \frac{1}{2} mr^2 \quad (1)$$

We know that, $\tau = I\alpha$ _____ (2)

Also $\tau = rF$ _____ (3)

Comparing equation (1), (2), (3), we get,

$$I \propto rF \Rightarrow \alpha = \frac{rF}{I} \quad (4)$$

Putting equation (1) in equation (4), we get,

$$\alpha = \frac{rF}{\frac{1}{2} mr^2} = \frac{2rF}{mr^2} = \frac{2F}{mr}$$

$$\Rightarrow \alpha = \frac{2F}{mr} \quad (5)$$

Putting the values in equation (5), we get,

$$\alpha = \left[\frac{2 \times 60}{10 \times 0.3} \right] \text{rad/sec}^2 = 40 \text{rad/sec}^2 \Rightarrow \alpha = 40 \text{rad/sec}^2$$

ASSIGNMENT 5.6:

A belt is wrapped around the edge of a pulley that is 40cm in diameter. The pulley rotates with a constant angular acceleration of 3.50 rad/s^2 . At $t = 0$, the rotational speed is 2 rad/s . What is the angular displacement and angular velocity of the pulley 2s later? ($\theta = 238 \text{ rad}$, 9.00 rad/s)

Solution: Given data:

Diameter of pulley = $d = 40 \text{ cm} = 0.4 \text{ m}$

Angular acceleration = $\alpha = 3.5 \text{ rad/sec}^2$

Initial speed = $\omega_i = 2 \text{ rad/sec}$, time = $t = 2 \text{ sec}$

Required:

(a) Final angular velocity = $\omega_f = ?$

(b) Angular displacement = $\theta = ?$

Calculation:

(a) We know that, $\omega_f = \omega_i + \alpha t$ (1)

Putting the values in equation (1), we get,

$$\omega_f = (2 + 3.5 \times 2) \text{ rad/sec} = 9 \text{ rad/sec}$$

$$\Rightarrow \omega_f = 9 \text{ rad/sec}$$

(b) We also know that,

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2 \quad (2)$$

Putting the values in equation (2), we get,

$$\theta = \left[2 \times 2 + \frac{1}{2} \times 3.5 \times 2^2 \right] \text{ rad} = (4 + 7) \text{ rad}$$

$$\Rightarrow \theta = 11 \text{ rad}$$

EXAMPLE 5.5:

Find earth angular momentum using earth-sun distance and mass of earth. Earth-sun distance $149.6 \times 10^9 \text{ m}$, mass of the earth $5.9742 \times 10^{24} \text{ kg}$

Solution:

Given data: Mean distance from earth to sun = $r = 149.6 \times 10^9 \text{ m}$

Mass of the earth
= $M_e = 5.9742 \times 10^{24} \text{ kg}$

Required:

Angular momentum = $L = ?$

Calculation:

We know that,

$$L = mvr \quad (1)$$

We also know that, $S = vt \Rightarrow v = \frac{S}{t}$

$$\Rightarrow v = \frac{2\pi r}{T} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$L = m \times \frac{2\pi r}{T} \times r$$

$$\Rightarrow L = \frac{2\pi mr^2}{T} \quad (3)$$

Putting the values in equation (3), we get,

$$L = \left[\frac{2 \times 3.14 \times 5.9742 \times 10^{24} \times (149.6 \times 10^9)^2}{31536000} \right] \text{ kgm}^2 / \text{sec}$$

$$\Rightarrow L = \left[\frac{839658.3058 \times 10^{24} \times 10^{18}}{31536000} \right] = 0.02663 \times 10^{42} \text{ kgm}^2 / \text{sec}$$

$$\Rightarrow L = 2.663 \times 10^{40} \text{ kgm}^2 / \text{sec}$$

ASSIGNMENT 5.7:

A DVD disc has a radius of 0.0600 m , and a mass of 0.0200 kg . The moment of inertia of a solid disc is $I = (1/2) MR^2$, where M is the mass of the disc, and R is the radius. When a DVD in a certain machine starts playing, it has an angular velocity of 160.0 rads^{-1} . What is the angular momentum of this disc? ($0.00576 \text{ kgm}^2/\text{s}^{-1}$)

Solution:

Given data:

Radius = $r = 0.0600$, Mass = $m = 0.0200 \text{ kg}$

Angular velocity = $\omega = 160 \text{ rad/sec}$

Required:

Angular momentum of disc = $L = ?$

Earth complete one rotation around the sun in 365 day, so
Time period = $T = 365 \text{ days}$
 $\Rightarrow T = 365 \times 24 \times 3600 \text{ sec}$
 $\Rightarrow T = 31536000 \text{ sec}$
And distance covered in one rotation
= $S = \text{circumference}$
 $\Rightarrow S = 2\pi r$

Calculation:

We know that, $L = I\omega$ _____ (1)

For disc, $I = \frac{1}{2}mr^2$ _____ (2)

Putting equation (2) in equation (1), we get,

$$L = \frac{1}{2}mr^2 \times \omega$$
 _____ (3)

Putting the values in equation (3), we get,

$$L = \left[\frac{1}{2} \times 0.0200 \times (0.0600)^2 \times 160 \right] \text{ kg} \cdot \text{m}^2 / \text{sec}$$

$$\Rightarrow L = 5.76 \times 10^{-3} \text{ kg} \cdot \text{m}^2 / \text{sec}^2$$

$$\Rightarrow L = 0.005763 \text{ kg} \cdot \text{m}^2 / \text{sec}$$

EXAMPLE 5.6:

A body of moment of inertia $0.80 \text{ kg} \cdot \text{m}^2$ about a fixed axis, rotates with constant angular velocity of 100 rad/s^{-1} . Calculate (i) its angular momentum = $\bar{L}$ (ii) Torque to sustain this position = $\bar{\tau}$

Solution:Given data:

Moment of inertia = $I = 0.80 \text{ kg} \cdot \text{m}^2$

Angular velocity = $\omega = 100 \text{ rad/s}^{-1}$

Required:

(i) Angular momentum = $L = ?$, (ii) Torque = $\tau = ?$

Calculation:

We know that, $L = I\omega$ _____ (1)

Putting the values in equation (1), we get,

$$L = [0.80 \times 100] \text{ kg} \cdot \text{m}^2 / \text{sec}^2 \Rightarrow L = 80 \text{ kg} \cdot \text{m}^2 / \text{sec}$$

(ii) Also $\tau = I\alpha$ _____ (2)

As the body is rotating with constant angular velocity so angular acceleration will be zero. So put $\alpha = 0$ in equation (2), we get,

$$\tau = [0.80 \times 0] \text{ N} \cdot \text{m} \Rightarrow \tau = 0$$

EXAMPLE 5.7:

A child of mass 25 kg stands at the edge of a rotating platform of mass 150 kg and radius 4.0 m . The platform with the child on it rotates with an angular speed of 6.2 rad/s . The child jumps off in a radial direction. What happens to the angular speed of the plat-

form? Treat the platform as a uniform disk.

Solution:Given data:

Mass of child = $m_1 = 25 \text{ kg}$, mass of platform = $m_2 = 150 \text{ kg}$,
Radius = $r = 4.0 \text{ m}$, initial angular speed = $\omega_i = 6.2 \text{ rad/s}^{-1}$

Required:

Final angular speed = $\omega_f = ?$

Calculation:

By law of conservation of angular momentum, we have,
Total initial angular momentum = Total final angular momentum

$$\Rightarrow L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f \Rightarrow \omega_f = \frac{I_i \omega_i}{I_f}$$
 _____ (1)

$$L = I\omega$$

Here $I_i = (I_{\text{platform}} + I_{\text{child}})$

$$\Rightarrow I_i = \frac{1}{2}m_2 r^2 + m_1 r^2$$

$$\Rightarrow I_i = \left(\frac{1}{2} \times 150 \times 4^2 \right) + (25 \times 4^2)$$

$$\Rightarrow I_i = (1200 + 400) \text{ kg} \cdot \text{m}^2$$

$$\Rightarrow I_i = 1600 \text{ kg} \cdot \text{m}^2$$

$$\text{While } I_f = \frac{1}{2}m_2 r^2 = \frac{1}{2} \times 150 \times 4^2 = 1200 \text{ kg} \cdot \text{m}^2$$

Putting the values of I_f , I_i and ω_i in equation (1), we get,

$$\omega_f = \left(\frac{1600 \times 6.2}{1200} \right) \text{ rad/sec} = 8.27 \text{ rad/sec}$$

$$\Rightarrow \omega_f = 8.27 \text{ rad/sec}$$

EXAMPLE 5.8:

A 70 kg man is standing on a scale in an elevator which is accelerating, as it heads for the top floor of a building at 4 m/s^2 . What apparent weight will show on the scale?

Solution:Given data:

Mass of man = $m = 70 \text{ kg}$, Acceleration = $a = 4 \text{ m/s}^2$

Required: Apparent weight = $T = ?$

Calculation:

For platform

$$I_{\text{platform}} = \frac{1}{2}m r^2$$

For child

$$I_{\text{child}} = m r^2$$

For upward motion, we have,

$$T = mg + ma \quad (1)$$

Putting the values in equation (1), we get,

$$T = (90 \times 9.8 + 70 \times 4) \text{ N} = 686 \text{ N} + 280 \text{ N}$$

$$\Rightarrow T = 966 \text{ N}$$

$$\Rightarrow \text{Apparent weight on the scale} = 966 \text{ N}$$

ASSIGNMENT 5.8:

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2 , what will be the reading of the spring balance? (24N) [Take $g = 10 \text{ m/sec}^2$]

Solution:

Given data: Weight of bag = $w = 49 \text{ N}$,

Acceleration = $a = 5 \text{ m/sec}^2$

Required:

Apparent weight of bag = $T = ?$

Calculation:

First we calculate the mass of the bag, at stationary condition, we have,

$$w = 49 \text{ N} \Rightarrow mg = 49 \text{ N}$$

$$\Rightarrow m = \frac{49}{10} \text{ kg} \Rightarrow m = 4.9 \text{ kg}$$

Now when the lift is moving downwards, then the apparent weight is given by,

$$T = mg - ma = (4.9 \times 10 - 4.9 \times 5) \text{ N}$$

$$\Rightarrow T = 49 - 24.5 \Rightarrow T = 24 \text{ N}$$

ASSIGNMENT 5.9:

In an elevator that has a constant acceleration upward, Abdur Rahman is standing on a scale. The scale reads 960N. When Abdur Rahman picks up a 20kg box, the scale reads 1200N. The acceleration of the elevator stays the same. (a) Find the acceleration of the elevator (b) Find Abdur-Rahman's weight. (784N, 2.2 m/s^2)

Solution:

Given data:

Let mass of Abdur-Rahman = $m \text{ kg}$

Normal force before picking box = $F_N = 960 \text{ N}$

Normal force after picking box = $F'_N = 1200 \text{ N}$

Required:

(a) Mass of Abdur-Rahman = $m = ?$

(b) Weight of Abdur-Rahman = $w = ?$

(c) Acceleration = $a = ?$

Calculation:

(a) For upward motion, we have,

$$F_N = mg + ma$$

$$\Rightarrow 960 = mg + ma \quad (1)$$

When Abdur-Rahman picks up 20 kg box, then we have,

$$F'_N = (m + 20)g + (m + 20)a \quad (2)$$

From equation (1), we have,

$$960 = mg + ma \Rightarrow ma = 960 - mg$$

$$\Rightarrow a = \frac{960}{m} - g \quad (3)$$

Putting equation (3) in equation (2), we get,

$$F'_N = (m + 20)g + (m + 20)\left(\frac{960}{m} - g\right)$$

$$\Rightarrow 1200 = \cancel{mg} + 20g + m \times \frac{960}{m} - \cancel{mg} + \frac{20 \times 960}{m} - 20g$$

$$\Rightarrow 1200 = 960 + \frac{19200}{m} \Rightarrow \frac{19200}{m} = 1200 - 960 = 240$$

$$\Rightarrow \frac{19200}{m} = 240 \Rightarrow m = \frac{19200}{240} \text{ kg} \Rightarrow m = 80 \text{ kg}$$

$$\Rightarrow \text{Mass of Abdur - Rahman} = m = 80 \text{ kg}$$

$$(b) \text{ Weight of Abdur - Rahman} = w = mg = 80 \times 9.8 \text{ N} = 784 \text{ N}$$

(c) For upward motion, we have,

$$F_N = mg + ma \Rightarrow a = \frac{F_N - mg}{m}$$

$$= \left(\frac{960 - 80 \times 9.8}{80} \right) \text{ m/sec}^2$$

$$\Rightarrow a = \left(\frac{960 - 784}{80} \right) \text{m/sec}^2 \Rightarrow a = 2.2 \text{m/sec}^2$$

EXAMPLE 5.9:

Determine the orbital speed for the international space station (ISS). If its orbit $4.0 \times 10^2 \text{km}$ above the earth surface.

Solution:Given data:

Height of space station from earth surface

$$= h = 4 \times 10^2 \text{km} = 4 \times 10^2 \times 10^3 \text{m} = 4 \times 10^5 \text{m}$$

$$\text{Radius of earth} = R = 6.36 \times 10^6 \text{m}$$

Total distance of space station from centre of earth is given by,

$$R = R_e + h = (6.36 \times 10^6 + 0.4 \times 10^6) \text{m}$$

$$\Rightarrow R = 10^6 (6.36 + 0.4) \text{m} = 6.76 \times 10^6 \text{m}$$

Value of gravitational constant

$$= G = 6.673 \times 10^{-11} \text{Nm}^2 / \text{kg}^2$$

$$\text{Mass of earth} = M_e = 6 \times 10^{24} \text{kg}$$

Required:

$$\text{Orbital speed} = v = ?$$

Calculation:

The orbital speed is given by,

$$v = \sqrt{\frac{GM_e}{R}} \quad (1)$$

Putting the values in equation (1), we get,

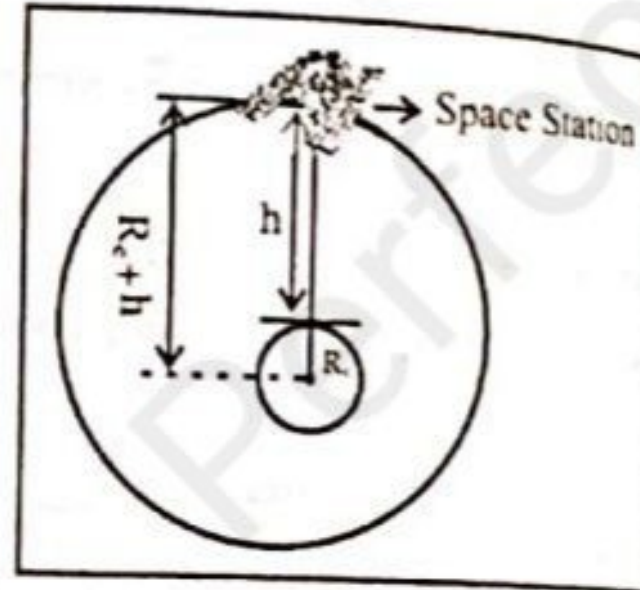
$$v = \sqrt{\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{6.76 \times 10^6}} \text{m/sec}$$

$$\Rightarrow v = \sqrt{5.9 \times 10^{-11+24-6}} \text{m/sec} = \sqrt{5.9 \times 10^7} \text{m/sec}$$

$$\Rightarrow v = \sqrt{59.0 \times 10^6} \text{m/sec} = 7.67 \times 10^3 \text{m/sec}$$

$$\Rightarrow v = 7.67 \times 10^3 \text{m/sec}$$

$$\Rightarrow \text{Orbital speed} = v = 7.67 \times 10^3 \text{m/sec}$$

**EXAMPLE 5.10:**

What should be the orbital speed to launch a satellite in a circular orbit 900km above the surface of the earth?

Solution:Given data:

$$\text{Height above the surface of earth is } = h = 900 \text{km} \\ = 900 \times 10^3 \text{m} = 9 \times 10^2 \times 10^3 \text{m} = 9 \times 10^5 \text{m}$$

$$\text{Radius of earth} = R_e = 64 \times 10^5 \text{m}$$

$$\text{So radius of orbit will be } r_o = R_e + h$$

$$= (64 \times 10^5 + 9 \times 10^5) \text{m}$$

$$= (64 + 9) \times 10^5 \text{m} = 73 \times 10^5 \text{m}$$

$$\text{Here } G = 6.673 \times 10^{-11} \text{Nm}^2 \text{kg}^{-2}$$

$$\text{And mass of earth} = M_e = 6 \times 10^{24} \text{kg}$$

$$\text{Required: Orbital speed} = v_o = ?$$

Calculation:

We know that,

$$v_o = \sqrt{\frac{GM_e}{r_o}} \quad (1)$$

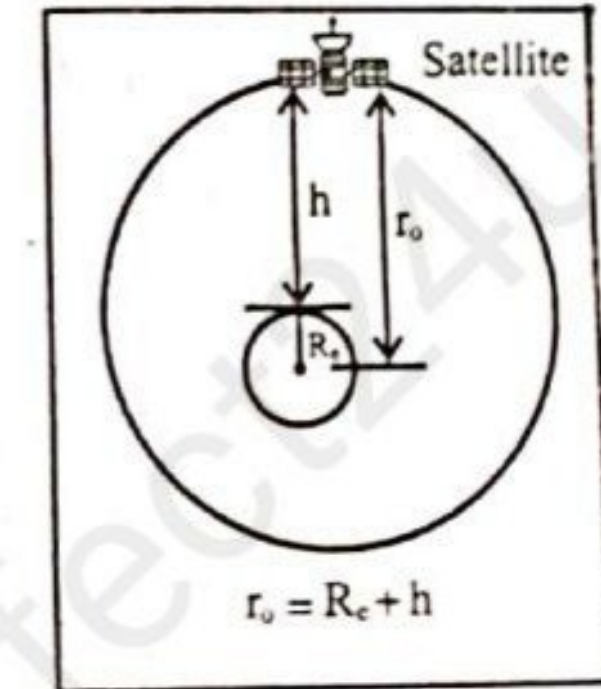
Putting the values in equation (1), we get,

$$v_o = \sqrt{\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{73 \times 10^5}}$$

$$\Rightarrow v_o = \sqrt{0.548 \times 10^8} \text{m/sec}$$

$$\Rightarrow v_o = 0.740 \times 10^4 \text{m/sec}$$

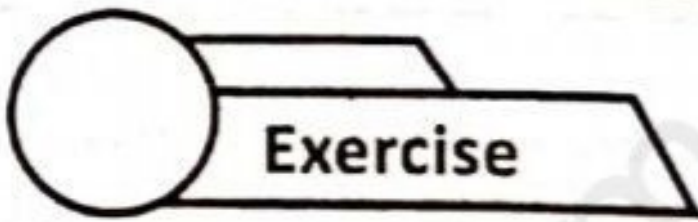
$$\Rightarrow v_o = 7.40 \times 10^3 \text{m/sec}$$



⊕ حضرت علی کا قول ہے: "خدا روئی سے پیش آنا سب سے پہلی نیکی ہے۔"

⊕ حضرت علی کا قول ہے: "کارخانہ قدرت میں فکر کرنا بھی عبادت ہے۔"

⊕ حضرت علی کا قول ہے: "بے موقع حیا بھی باعثِ محرومی ہے، قابلِ محبت بہت کم لوگ ہیں۔"



Exercise

Choose the best possible answer:

1. The angular speed in radians / hours for daily rotation of our earth is:

a. 2π b. 4π c. $\frac{\pi}{6}$ ☒ d. $\frac{\pi}{12}$

Sol: As $T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$ _____ (1)

Put $T = 24$ hour in equation (1), we get,

$$\omega = \frac{2\pi}{24} \text{ rad/hour} \Rightarrow \omega = \frac{\pi}{12} \text{ rad/hr}$$

2. Linear acceleration $a = r\alpha$ when " θ " is:

a. 0° b. 180° c. 360° ☒ d. 90°

3. What is moment of inertia of a sphere?

a. MR^2 b. $\frac{1}{2}MR^2$ ☒ c. $\frac{2}{5}MR^2$ d. $\frac{1}{2}M^2R$

4. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of particle. The motion of the particle takes place in a horizontal plane. It follows:

a. Linear momentum is constant b. Velocity is constant

☒ c. It moves in a circular path d. None of these

5. A body moving in a circular path with constant speed has:

a. Constant acceleration

b. Constant retardation

☒ c. Variable acceleration

d. Variable speed and constant velocity

- Sol: The body is moving with constant speed but its direction is changes continuously due to which the body posses variable acceleration.

6. Astronauts appear weightless in space because:

a. There is no gravity in space

b. There is no floor pushing upwards

☒ c. Satellite is freely falling

d. There is no air in space

7. Which one is constant for a satellite in orbit?

a. Velocity

☒ c. Angular momentum

b. $K \cdot E$

d. Potential energy

8. If the earth suddenly stops rotating the value of "g" at equator would:

a. Decrease

b. Remain unchanged

☒ c. Increase

d. Become zero

- Sol: If the earth stops rotating then the value of "g" at equator will increase because when the earth rotates the gravity at every point remains uniform. And when it stops rotating, the gravity increases because the pressure is exerted on only one point at the equator.

9. If solid sphere and solid cylinder of same mass and density rotate about their own axis, the moment of inertia will be greater for:

a. Solid sphere

☒ b. Solid cylinder

c. The one that has the largest mass arrives first

d. The one that has the largest radius arrives first

Sol: For sphere, $I_{\text{sphere}} = \frac{2}{5}MR^2$ _____ (1)

For cylinder, $I_{\text{cylinder}} = \frac{1}{2}MR^2$ _____ (2)

Dividing equation (2) by equation (1)

$$\frac{I_{\text{cylinder}}}{I_{\text{sphere}}} = \frac{\frac{1}{2}MR^2}{\frac{2}{5}MR^2} = \frac{1}{2} \cdot \frac{5}{2} = \frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$$

$$\Rightarrow \frac{I_{\text{cylinder}}}{I_{\text{sphere}}} = \frac{5}{4} \Rightarrow I_{\text{cylinder}} = \left(\frac{5}{4}\right) I_{\text{sphere}} \text{ _____ (3)}$$

As $\frac{5}{4} > 1$, so equation (3) $\Rightarrow I_{\text{cylinder}} > I_{\text{sphere}}$

So option (b) is correct.

10. The gravitational force exerted on an astronaut on earth's surface is 650N down. When she is in the international space station, is the gravitational force on her:

a. Larger

b. Exactly the same

☒ c. Smaller

d. Nearly but not exactly zero

e. Exactly zero

Sol: Gravitational force is inversely proportion to the square of the distance between two bodies. The distance of internal space station is situated very far from earth, so distance is greater and force will be smaller.

11. A solid cylinder of mass "M" and radius "R" rolls down an incline without slipping. Its moment of inertia about an axis through its center of mass is $MR^2/2$. At any instant while in motion, its rotational kinetic energy about its center of mass is what fraction of its total kinetic energy?

(a) $\frac{1}{2}$ b. $\frac{1}{4}$ ✓ c. $\frac{1}{3}$ d. $\frac{2}{5}$ e. None of these

Sol: According to the question, the fraction of rotational K.E and total K.E is given by,

$$\frac{(K.E)_r}{(K.E)_{total}} = \frac{K.E_r}{K.E_r + K.E_{trans}}$$

$$K.E_{total} = K.E_{rot} + K.E_{trans}$$

$$\Rightarrow \frac{(K.E)_r}{(K.E)_{total}} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} m v^2} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} M R^2 \omega^2} \quad [v = R\omega]$$

$$\Rightarrow \frac{(K.E)_r}{(K.E)_{total}} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + I \omega^2} \quad \text{For cylinder}$$

$$I = \frac{1}{2} M R^2$$

$$\Rightarrow \frac{(K.E)_r}{(K.E)_{total}} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 [1 + 2]} = \frac{1}{1 + 2} = \frac{1}{3}$$

So option (c) is correct.

④ حضرت علی کا قول ہے: "شکر نعمت حصول کا باعث ہے اور ناشکری حصول زحمت کا باعث ہے۔"

④ حضرت علی کا قول ہے: "ادب بہترین کمالات اور خیرات افضل ترین عبادات سے ہے۔"

④ حضرت علی کا قول ہے: "موت ایک بے خبر ساتھی ہے۔"

Conceptual Questions

Give short answers to the following questions:

Q1: Why is the fly wheel of an engine made heavy in the rim?

Ans: The fly wheel of an engine is made heavy in the rim for increasing the moment of inertia of fly wheel.

Reasons and Explanation:

When a fly wheel is put into motion, torque is acting on it which is given by,

$$\tau = I \alpha \Rightarrow \alpha = \frac{\tau}{I} \quad (1)$$

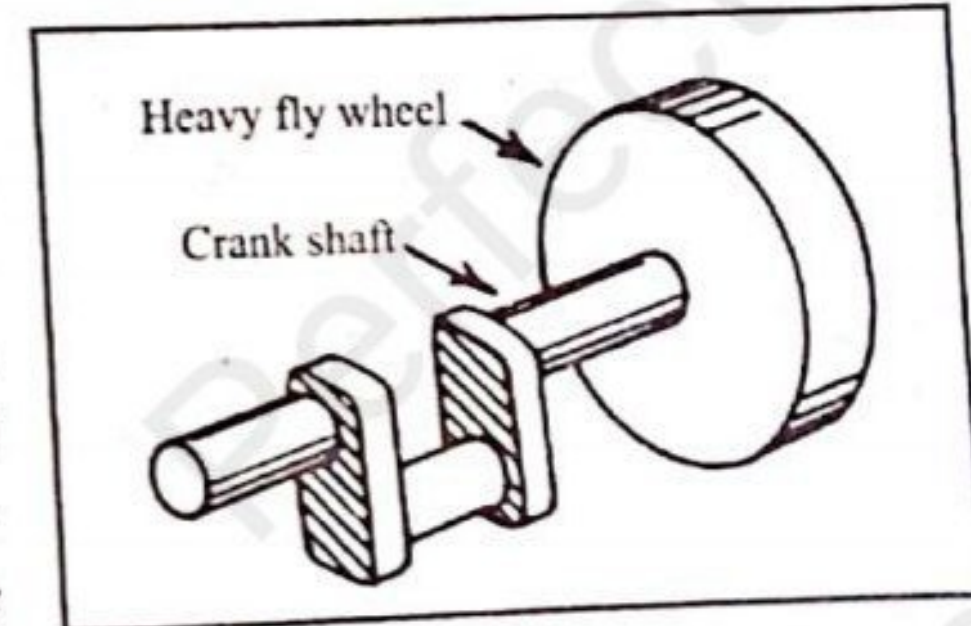
We also know that the moment of inertia is given by,

$$I = m r^2 \quad (2)$$

When the engine is started, the crank gets energy from the engine in the form of jerks. The crank is connected to the heavy flywheel. Now to control the jerky motion, the flywheel is needed to rotate with steady angular acceleration " α ".

Now according to equation (1), for getting smaller value of angular acceleration, we need greater moment of inertia. Equation (2) shows that keeping radius "r" constant, we need greater mass for getting more moment of inertia.

Thus for getting more moment of inertia the flywheel is made heavy or massive in the rim. Such heavy flywheel posses large moment of inertia which avoid the jerky motion of the crank. In this way the engine remains in working conditions and remains safe from serious damages.



Q2: Why is a rifle barrel "rifled"?

Ans: The barrel of a rifle is rifled to keep the bullet along the re-

quired direction and to hit the target accurately.

Reasons and Explanation:

Rifling is the process in which helical grooves are made inside the barrel of a rifle. When the rifle is fired, the helical grooves provide spin to the bullet around its long axis. The spin motion improves the projectile motion of the bullet. It also improves the aerodynamic stability and accuracy of the bullet.

The bullet fired from rifled rifle possesses rotational K.E as well as translation K.E. Thus the total energy of the bullet is given by,

$$(K.E)_{\text{total}} = (K.E)_{\text{trans}} + (K.E)_{\text{rot}}$$

$$\Rightarrow (K.E)_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Now due to rotational and translational K.E, the bullet possesses both spin angular momentum as well as linear momentum. Both momentums are parallel to each other and as a result the bullet gains large momentum. As a result the bullet travels fast and can hit the target with accuracy. Such a bullet imparts greater momentum to the target.

Q3: Is it possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on a table? Explain.

Ans. Yes, it is possible for a person to distinguish between a raw egg [uncooked] and a hard boiled one by spinning each on a table.

Reasons and Explanation:

We know that the angular momentum of a spinning body is given by,

$$L = I\omega \Rightarrow \omega = \frac{L}{I} \quad (1)$$

Equation (1) shows the angular speed of a spinning object is inversely proportional to the moment of inertia of the spinning object i.e. greater the moment of inertia of the object, smaller will be its angular velocity and vice versa.

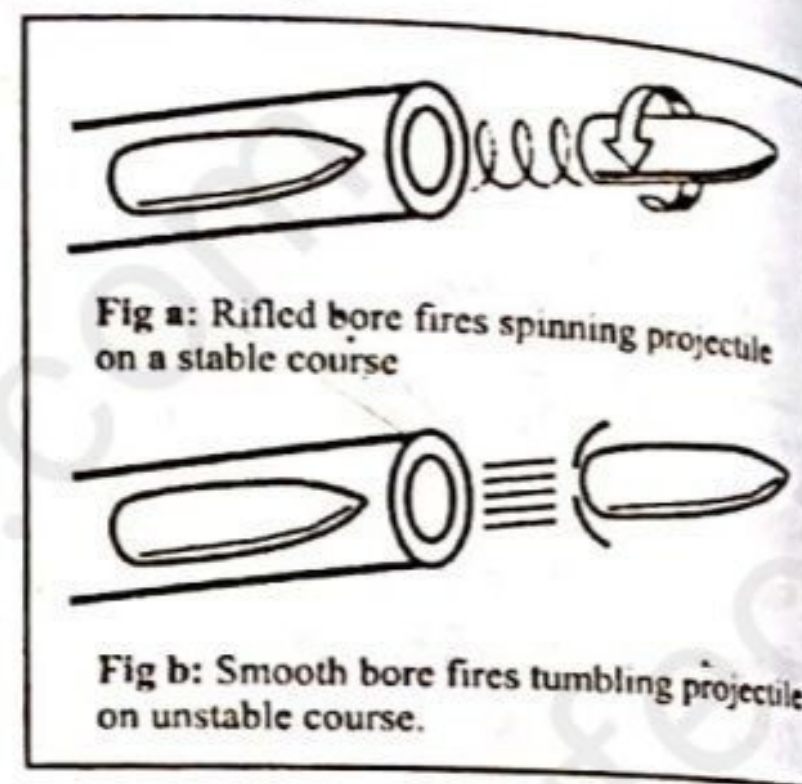


Fig a: Rifled bore fires spinning projectile on a stable course

Fig b: Smooth bore fires tumbling projectile on unstable course.

Now we can distinguish between a raw egg and a boiled hard egg on the basis of their moment of inertia and angular velocities.

Case1: Spinning of raw egg [uncooked egg]:

When a raw egg is allowed to spin, the liquid molecules inside the shell move away from the axis of rotation towards the inner surface of the shell as shown in figure (a). As a result the mass distribution is disturbed and the egg becomes more massive near the shell. We also know that the moment of inertia increases with increase in mass. So the moment of inertia of a raw egg increases due to non-uniform distribution of the mass of liquid inside the shell.

Now according to equation (1), the angular velocity of the raw egg will decrease due to greater moment of inertia. Thus the raw egg will rotate slowly and will come to rest sooner.

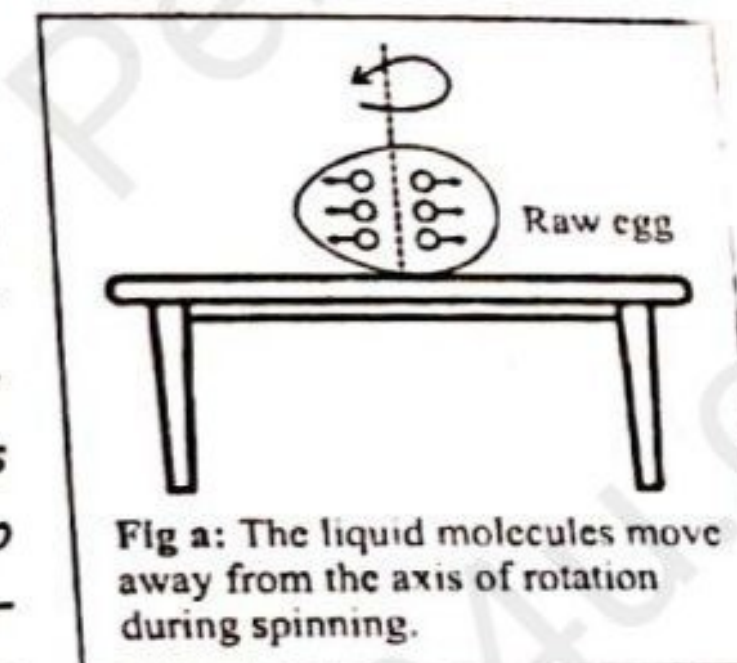


Fig a: The liquid molecules move away from the axis of rotation during spinning.

Case2: Spinning of hard boiled egg:

The liquid inside the shell appears in solid state after boiling the egg. Now when the egg is allowed to spin, the solid molecule does not move towards the shell. As a result mass distribution remains the same. Due to same mass distribution, the moment of inertia of the boiled egg remains low as compared to a raw egg.

Now according to equation (1), the angular speed of the boiled egg will be greater due to its low moment of inertia i.e. the boiled egg will rotate with greater angular speed and will come to rest later as compared to a raw egg.

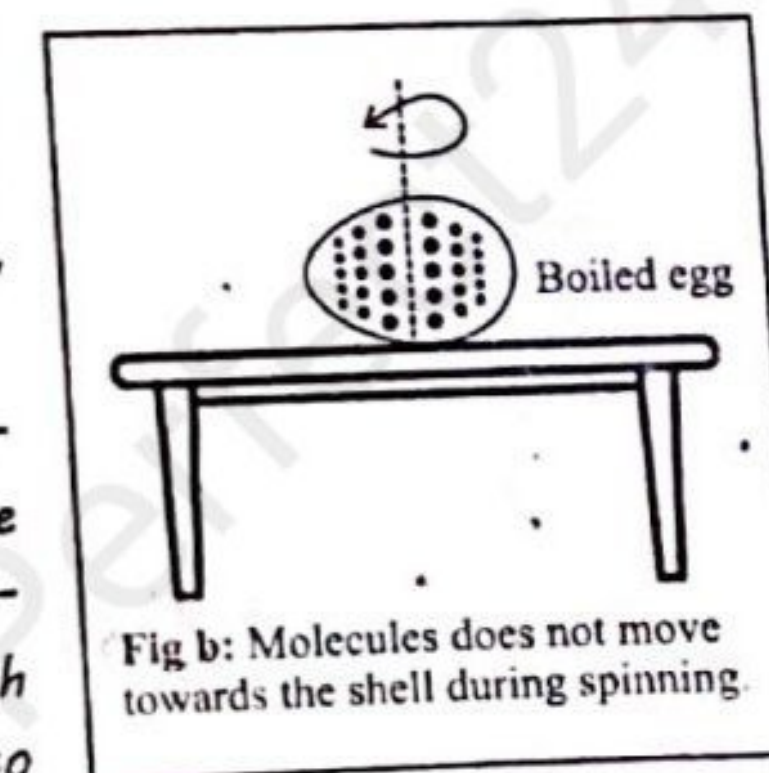


Fig b: Molecules do not move towards the shell during spinning.

Conclusion:

From the above discussion, it is clear that a boiled egg will possess more angular velocity and will come to rest later as compared to a raw egg.

raw egg. Thus we can distinguish between a raw egg and hard boil egg due to difference in moment of inertia and angular speed of the two eggs.

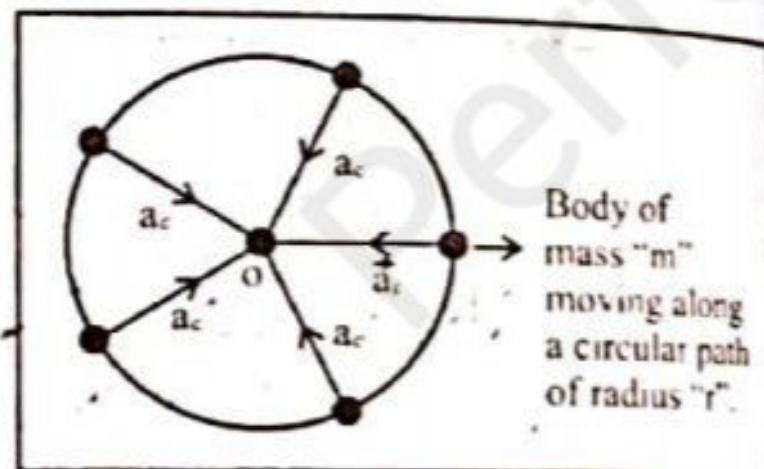
Q4: Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?

Ans: The acceleration of a body moving uniformly in a circle is always directed towards the centre of the circle.

Reasons and Explanation:

When a body of mass "m" is moving along a circle uniformly, then the magnitude of the velocity of the body (i.e. speed) remains constant while the direction of its velocity changes at each point.

Now when the magnitude of velocity remains constant, then there will be no tangential acceleration in the motion of the body. Only radial acceleration is produced due to change in the direction of velocity.



The change in the direction of velocity is due to centripetal force. So the radial acceleration is produced due to action of centripetal force. Thus the radial acceleration is also known as centripetal acceleration. As the centripetal force is directed towards the centre of the circular path, so the centripetal acceleration is also directed towards the centre of the circle.

Mathematical Form:

We know that,

$$F_{\text{centripetal}} = -m \frac{v^2}{r} \quad (1)$$

We also know that,

$$F_{\text{centripetal}} = ma_c \quad (2)$$

Comparing equation (1) and equation (2), we get,

$$ma_c = -m \frac{v^2}{r} \Rightarrow a_c = -\frac{v^2}{r} \quad (3)$$

The negative sign in equation (3) shows that the centripetal acceleration "a_c" is directed towards the centre of the circular path.

Q5: A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?

Ans: A swinging ball can break the string due to the influence of centrifugal force.

Reasons and Explanation:

When a ball is just supported by a string without breaking, then under such condition, the weight "w" of the ball is acting in downward direction while the tension in the string "T" is acting in upward direction. So we have,

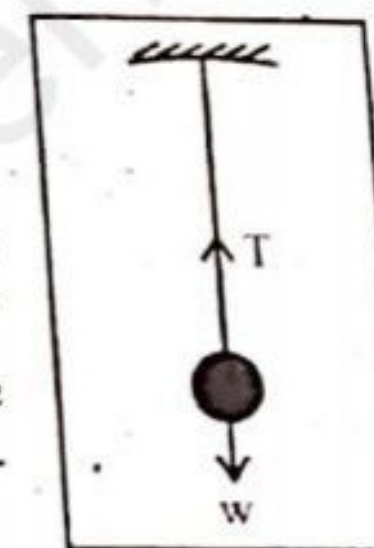
$$T = w \quad (1)$$

Equation (1) shows that the tension in the string remains within its elastic limit and hence remains un-breaking.

Now when the string is set swinging, then centripetal and centrifugal forces exist. The tension in the string acts as centripetal force. The centrifugal force is acting in opposite direction of centripetal force. So centrifugal force is added to the weight. So equation (1) becomes,

$$T = w + \text{centripetal force} \quad (2)$$

Equation (2) shows that the tension in the swinging string increases. Such increases in tension cross the elastic limit of the string which causes the string to break.

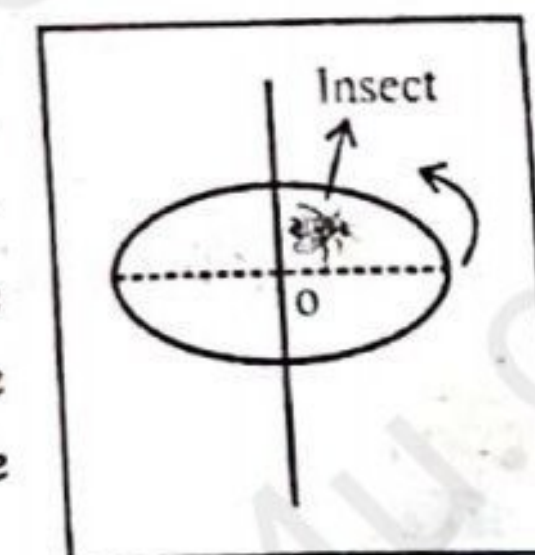


Q6: An insect is sitting close to the axis of a wheel. If the friction between the insect and the wheel is very small, describe the motion of the insect when the wheel starts rotating.

Ans: When the wheel starts rotation, the insect will fly away from the axis of rotation due to lack of necessary centripetal force.

Reasons and Explanation:

Let the insect is sitting on the wheel near the axis of rotation. Now if the wheel starts rotation, and there is sufficient force of friction between surface of wheel and insect, then the force of friction in this case will act as centripetal force and as a result the insect will remain stick to the surface of the wheel during rotation.



But here we are given that the force of friction between the insect and wheel is very very small. So when the wheel begins to rotate, the smaller force of friction will be unable to provide necessary centripetal force to the insect. As a result the insect will not remain stick to the surface of the wheel and will move away from the axis of rotation of the wheel due to centrifugal force. The insect will follow a spiral motion and will fall off the rotating wheel tangentially after a short period of time.

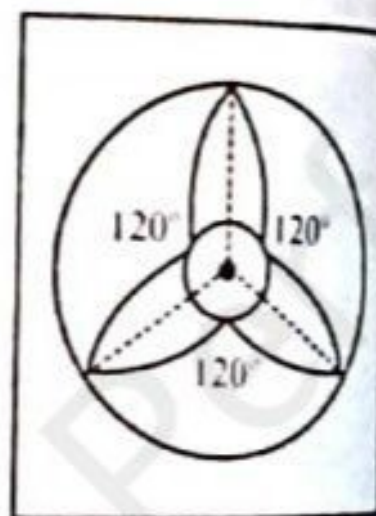


Q7: Explain how many minimum number of geo-stationary satellites are required for global coverage of T-V transmission?

Ans: At least three geo-stationary satellites are required for the global coverage of T-V transmission.

Reasons and Explanation:

We know that the geo-stationary satellites move around the earth from west to east in the equatorial plane at the same rate as that of the rotation of the earth about its own axis. A single geo-stationary satellite covers a longitude of 120° [$360^\circ/3=120^\circ$] so at least three geo-stationary satellites will be required to the whole earth longitude. These satellites are placed at three proper position in such a way that they can transmit T-V signals from any part of the world. These satellite have made the whole world as global village.



Q8: Explain the significance of moment of inertia in rotatory motion?

Ans: The moment of inertia play the same role in rotary motion as the mass perform in the translatory motion.

Reasons and Explanation:

The Newton, 2nd Law for translatory motion is given by,

$$F = ma \quad (1)$$

The Newton's 2nd law for rotatory motion is given by,

$$\tau = I\alpha \quad (2)$$

Using equation (1), we can explain various phenomenon related to translatory motion. In equation (1), 'm' represents the mass which is also known as "measure of inertia". Equation (1) shows that massive objects possess more reluctance and greater force will be required to change the state of rest or motion of a that object. Equation (1) also shows that acceleration produced in a body is inversely proportional to its mass i.e. heavier body can be accelerated by applying larger force and lighter body can be accelerated by applying smaller force. Equation (1) can be used to explain various phenomenon like momentum, energy, weight, and many other interaction b/w various objects.

In the same way, in equation (2) 'I' represents the moment of inertia of rotating objects. The moment of inertia of various rotating objects depends upon their mass, geometrical shape and distance from the axis of rotation. Heavier objects possess greater moment of inertia and vice versa.

The moment of inertia plays an important role in daily life. Their applications are use in vehicles for comfortable journey. We can express the torque, angular velocity, angular acceleration, angular momentum, rotational K.E of various objects in terms of moment of inertia. These parameters are used to explain various phenomenon in case of satellite and planetary motion.

Q9: Why does the coasting rotating system slow down as water drops into the beaker?

Ans: The coasting rotating system becomes slowed down when water is added to the beaker. It is because the moment of inertia of the rotating system increases which opposes the rotation of system.

Reasons and Explanation:

We know that the angular momentum of a system having moment of inertia 'I' rotating with certain angular velocity ' ω ' is given by,

$$L = I\omega \Rightarrow \omega = \frac{L}{I} \quad (1)$$

We also know that the moment of inertia of a rotating system depends upon its mass. i.e. greater the mass, greater will be the its

moment of inertia and vice versa.

Now when water is added into the beaker, the mass of rotating system increases. Due to increase in mass of the rotating system, its moment of inertia " I " increases.

Now according to equation (1), when moment of inertia increases, then the angular velocity " ω " of the system decreases. Thus by adding water into the beaker, the angular velocity of coasting rotating system decreases and becomes slowed down.

Q10: A body will be weightless when the elevator falls down just like a free falling body. Explain.

Ans: A body will be weightless when the elevator falls down just like a free falling body.

Reasons and Explanation:

Let a fish is hanged from the ceiling of a lift with the help of spring balance. Let a man is standing on the floor of lift and observe the weight of the fish with the help of the spring balances.

We know that when the lift is moving downwards with certain acceleration " a ", then the tension in spring balance is given by,

$$T = mg - ma \quad (1)$$

In equation (1), the tension in the spring represents the apparent weight of the fish. Equation (1) shows that the apparent weight of the fish decreases when the lift is falling down with certain acceleration " a ".

Now when the elevator is falling freely, then we put $a = g$ so equation (1) becomes,

$$T = mg - mg \Rightarrow T = 0 \quad (2)$$

Equation (2) shows that the spring balance will read zero tension if the lift is falling freely. In other words we can say that the fish becomes weightless under the influence of free fall motion. Thus the body will be weightless when the elevator (lift) falls down just like

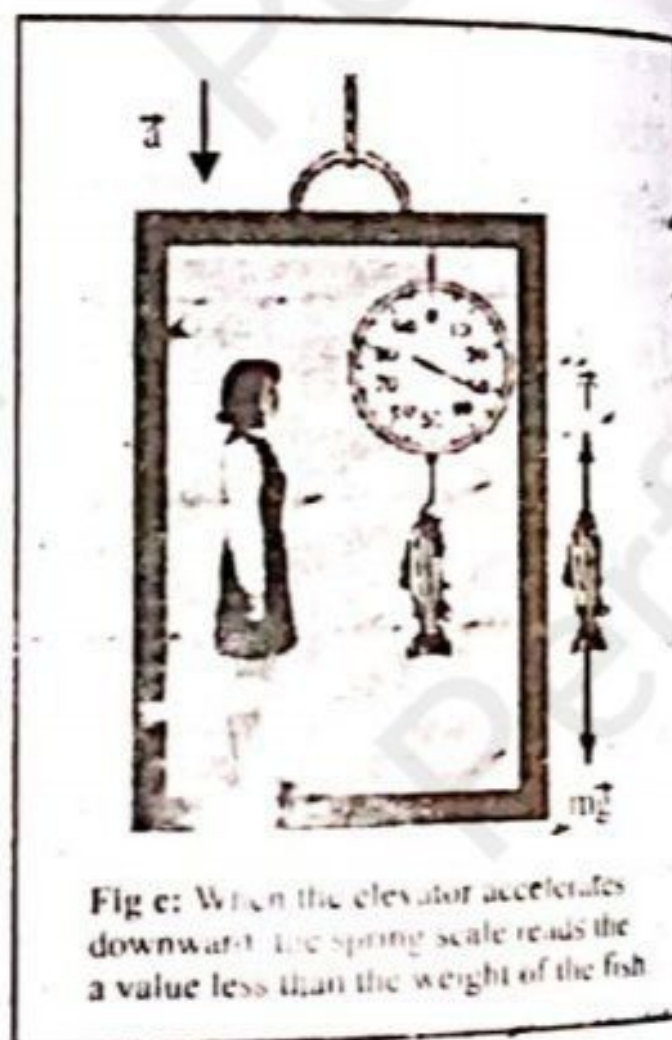


Fig c: When the elevator accelerates downward, the spring scale reads the a value less than the weight of the fish

a free falling body.

Q11: When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel, why? Explain.

Ans: When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel.

Reason and Explanation:

First Answer: Let " I_1 ", " ω_1 " represents the moment of inertia and angular velocity of lighter wheel and " I_2 ", " ω_2 " represents the moment of inertia and angular velocity of heavier wheel respectively.

Now the angular momentum of lighter wheel is given by,

$$L_1 = I_1 \omega_1 \quad (1)$$

Similarly, the angular momentum of heavier wheel is given by,

$$L_2 = I_2 \omega_2 \quad (2)$$

Now by law of conservation of momentum we have,

$$L_1 = L_2 \Rightarrow I_1 \omega_1 = I_2 \omega_2$$

$$\Rightarrow \omega_1 = \left(\frac{I_2}{I_1} \right) \omega_2 \quad (3)$$

As the heavier wheel has greater mass than lighter wheel, so its moment of inertia " I_2 " will be also greater than the moment of inertia " I_1 " of the lighter wheel. So we

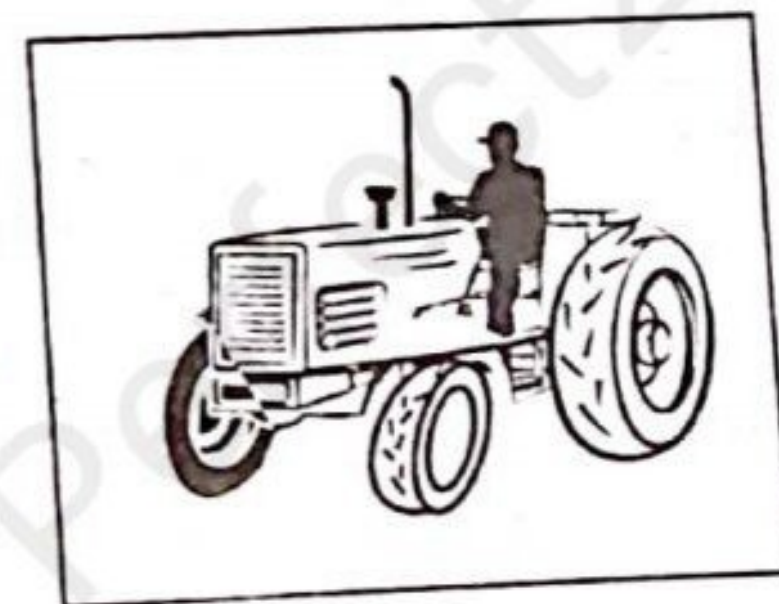
$$\text{have, } \frac{I_2}{I_1} > 1$$

So equation (3) becomes, $\omega_1 > \omega_2$ or $\omega_2 < \omega_1$

This shows that due to greater mass and greater moment of inertia, the heavier wheel will rotate slowly as compare to the lighter wheel.

Second Answer: Let " r " represents the radius of the lighter wheel and " R " represents the radius of heavier wheel. Now the distance covered by the lighter wheel during one revolution is equal to the circumference, so we have,

$$S_1 = 2\pi r \quad (1)$$



Similarly, distance covered by the heavier wheel during one rotation is given by,

$$S_2 = 2\pi R \quad (2)$$

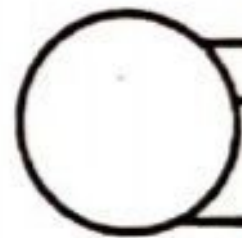
Dividing equation (1) by equation (2), we get,

$$\frac{S_1}{S_2} = \frac{2\pi r}{2\pi R} \Rightarrow \frac{S_1}{S_2} = \frac{r}{R}$$

$$\Rightarrow S_1 = \left(\frac{r}{R}\right) S_2 \quad (3)$$

As $R > r$, so $\frac{r}{R} < 1$, so equation (3) shows that $S_1 < S_2$

This shows that the distance covered by lighter wheel during one rotation is less than the distance covered by heavier wheel during one rotation. Now for covering the same distance in same time the lighter wheel needs to make more rotation than the heavier wheel. i.e. the lighter wheel will rotate more quickly as compare to heavier wheel. In other words we can say that, when a tractor moves with a uniform velocity, its heavier wheel rotates slowly than its lighter wheel.



Comprehensive Questions

Give extended response to the following questions:

Q1: What are centripetal acceleration and centripetal force? Derive their equations.

Ans: See answer of question no 6 part-b in text.

Q2: Show that angular momentum in magnitude is given by,

$$|\vec{L}| = |\vec{r} \times \vec{p}| = mvr = mr^2\omega$$

Ans: The cross product of moment arm " $\vec{r}$ " and linear momentum " $\vec{p}$ " is known as angular momentum. i.e.

$$\vec{L} = \vec{r} \times \vec{p} \quad (1)$$

In magnitude form, equation (1) can be written as,

$$|\vec{L}| = |\vec{r} \times \vec{p}| \quad (2)$$

As $\vec{p} = m\vec{v}$, so equation (2) becomes,

$$|\vec{L}| = |\vec{r} \times m\vec{v}| \Rightarrow |\vec{L}| = mvr \quad (3)$$

As $v = r\omega$, so equation (3) becomes,

$$\Rightarrow |\vec{L}| = m(r\omega)r = mr^2\omega \quad (4)$$

Equations (2), (3), (4) represent the magnitude form of angular momentum.

Q3: Show that role playing by mass in linear motion is playing by moment of inertia in rotatory motion.

Ans: See answer of short question no. 8.

Q4: What do you mean by "INTELSAT". At what frequencies it operates? For how many T.V stations this system is used?

Ans: "INTELSAT" means international telecommunication satellite organization. It is the largest satellite system which is managed by 126 countries. It operates at microwaves of frequencies 4, 6, 11 and 14 GHz and has a capacity of 30000 two way telephone circuits plus three T.V channels.

There are 200 earth stations which transmit the signals to satellites and receive signals via satellites from other countries. We can pick up the signals from the satellite using a dish antenna on our houses.

Q6: Show that in angular form, centripetal acceleration is $\vec{a}_c = -\omega^2 \vec{r}$

Proof: We know that,

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad (1)$$

We also know that, $v = r\omega$ (2)

Putting equation (2) in equation (1), we get,

$$\vec{a}_c = -\frac{r^2\omega^2}{r} \hat{r} = -r\omega^2 \hat{r}$$

$$\Rightarrow \vec{a}_c = -\omega^2(r\hat{r}) \quad (3)$$

The unit vector " $\hat{r}$ " can be expressed as,

$$\hat{r} = \frac{\vec{r}}{r} \Rightarrow \vec{r} = r\hat{r} \Rightarrow r\hat{r} = \vec{r} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$\vec{a}_c = -\omega^2 \vec{r}$$

Which is the required proof.

Q7: Show that the centripetal force is also shown by,

$$\vec{F}_c = -\frac{mv^2}{r} \hat{r} = -\frac{mv^2}{r^2} \vec{r} = -m\omega^2 \vec{r}$$

Ans. We know that,

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad (1)$$

We also know that,

$$\vec{F}_c = m\vec{a}_c \quad (2)$$

Putting equation (1) in equation (2), we get,

$$\vec{F}_c = -m \frac{v^2}{r} \hat{r} \quad (3)$$

We know that,

$$\hat{r} = \frac{\vec{r}}{r} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$\vec{F}_c = -m \frac{v^2}{r^2} \vec{r} \quad (5)$$

We also know that, $\omega = \frac{v}{r}$ (6)

Putting equation (6) in equation (5), we get,

$$\vec{F}_c = -\frac{mr^2\omega^2}{r^2} \vec{r} \Rightarrow \vec{F}_c = -m\omega^2 \vec{r} \quad (7)$$

Comparing equations (3), (5), (7), we get,

$$\vec{F}_c = -\frac{mv^2}{r} \hat{r} = -\frac{mv^2}{r^2} \vec{r} = -m\omega^2 \vec{r}$$

Which is the required proof.

Q8: Show that a satellite near the earth will have greater velocity.

Ans. Consider a satellite of mass "m" is moving around the earth in a circular orbit of radius "r" as shown in the figure. The gravitational force between earth and satellite is given by,

$$F = \frac{GM_e m}{r^2} \quad (1)$$

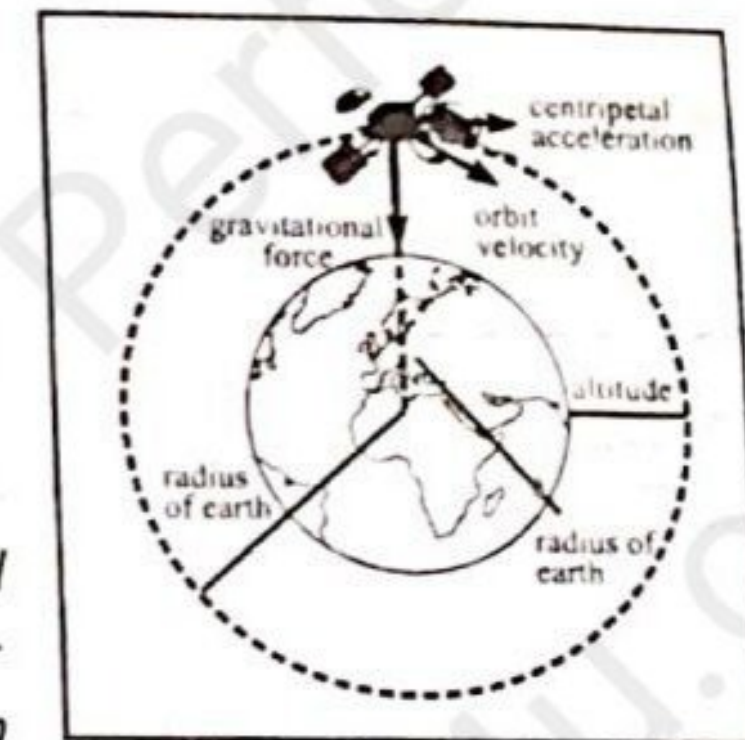
Due to this gravitational force, the satellite performs circular motion around the earth, so this force acts as centripetal force. So we have,

$$F = \frac{mv^2}{r} \quad (2)$$

Comparing equations (1), (2), we get,

$$\begin{aligned} \frac{mv^2}{r} &= \frac{GM_e m}{r^2} \\ \Rightarrow v^2 &= \frac{GM_e}{r} \Rightarrow v = \sqrt{\frac{GM_e}{r}} \\ \Rightarrow v &= \sqrt{\frac{\text{constant}}{r}} \quad [GM_e = \text{constt}] \\ \Rightarrow v &\propto \frac{1}{\sqrt{r}} \quad (3) \end{aligned}$$

Relation (3) shows that the orbital speed of satellite is inversely proportional to the distance "r" of satellite from the centre of earth. That is, the higher the satellite, the slower will be its orbital velocity and will take longer time to complete one revolution around the earth. In other words, we can say that a satellite near the earth will have greater velocity.



Q9: What do you mean by weight of a body? Using examples to distinguish between real weight and the apparent weight of a body.

Ans. **Weight:** The gravitational force exerted on the body due to earth or other planet is known as weight of that body. Weight is a vector quantity. Its SI unit is Newton.

Dimensions of Weight: As weight = Newton = kg·m/sec²

$$\Rightarrow \text{Weight} = \frac{\text{mass} \times \text{length}}{\text{time}^2} \Rightarrow \text{Weight} = [MLT^{-2}]$$

Real Weight and Apparent Weight:

The weight of an object due to the gravitational pull of the earth is called real weight. While the weight measured by a spring balance is called apparent weight. The weight of a person, ball, stone etc due to earth's gravity represents their real weight. While the weight of a person in a pond of water, and in moving elevator represents their apparent weights.

Q10: Explain how gravity is provided to the occupants of the space ship?

Ans. See answer of question 23 in the text.

Q11: Give three examples to illustrate the phenomenon of conservation of angular momentum.

Ans. See answer of question 15 in the text.

Q12: Explain why mud guards are used on the wheels of cycles, motor cars and other driving vehicles.

Ans: The mud guards are used on the wheels of driving vehicles to prevent the falling of mud on the surrounding parts of vehicles near the wheels.

Explanation:

When the tyre rotates, each point on the tyre, including mud clinging to the tyre has a tangential velocity " v " as well as angular velocity " ω ". There is adhesive force [چپکے والی قوت] between the tyre and mud. When the tyre rotates, a centripetal force acts on the mud which is equal to the adhesive force between tyre and mud.

When the angular speed of the tyre increases, the centripetal force on the mud also increases. When the centripetal force becomes greater than adhesive force, the mud leaves the tyre and fly off tangentially along a straight line due to centrifugal force which is simply the reaction of the centripetal force. The mud strikes the mud guard and thus the other parts of the vehicle is prevented from falling of mud on them.



Numerical Problems

Q1: If the plate microwave oven has a radius of 0.15m and rotates at 6.0 rev/min, calculate the total distance travelled by the fly during a 2.0m cooking period. (11m)

Solution:

Radius = $r = 0.15\text{m}$

Angular velocity = $\omega = 6\text{ rev/min} = 6 \times \frac{2\pi}{60} \text{ rad/sec} = 0.628\text{ rad/sec}$

Time period = $T = 2\text{ min} = 2 \times 60\text{ sec} = 120\text{ sec}$

Required:

Distance travelled = $S = ?$

Calculation:

We know that, $S = r\theta$ (1)

We also know that, $\theta = \omega T$ (2)

Putting equation (2) in equation (1), we get,
 $S = r\omega T$ (3)

Putting the values in equation (3), we get,
 $S = [0.15 \times 0.628 \times 120]\text{m} \Rightarrow S = 11.3\text{m}$

OR $S = 11\text{m}$ approximately

Distance travelled by fly = $S = 11\text{m}$

Q2: A circular drum of radius 40cm is initially rotating at 400 revolution/min. It is brought to stop after making 50 revolutions. What is the angular acceleration and the stopping time? (2.79 rad/s^2 , 15.0s)

Solution: Radius of drum = $r = 40\text{cm} = 0.4\text{m}$

Initial angular velocity

$$= \omega_i = 400\text{ rev/min} = \frac{400 \times 2\pi}{60} \text{ rad/sec}$$

$$\Rightarrow \omega_i = 41.866 \text{ rad/sec}$$

Angular displacement = $\theta = 50\text{ revolution}$
 $= 50 \times 2\pi \text{ rad}$

$$\Rightarrow \theta = 314\text{ rad, final angular velocity} = \omega_f = 0$$

Required:

(a) Angular acceleration = $\alpha = ?$

(b) Stopping time = $t = ?$

Calculation:

(a) We know that,

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \quad (1)$$

Putting the values in equation (1), we get,

$$2 \times \alpha \times 314 = 0^2 - (41.866)^2 \Rightarrow 628\alpha = -1752.76$$

$$\Rightarrow \alpha = \left[\frac{-1752.76}{628} \right] \text{ rad/sec}^2 = -2.79 \text{ rad/sec}^2 \quad \alpha = -2.79 \text{ rad/sec}^2$$

(b) We know that,

$$\omega_f = \omega_i + \alpha t \quad (2)$$

Putting the values in equation (2), we get,

$$0 = 41.866 - 2.79t$$

$$\Rightarrow -2.79t = -41.866$$

$$\Rightarrow t = \left(\frac{41.866}{2.79} \right) \text{ sec} \Rightarrow t = 15.0 \text{ sec}$$

Drum comes to rest,
so put $\omega_f = 0$

$$2as = v_f^2 - v_i^2$$

$$2\alpha\theta = \omega_f^2 - \omega_i^2$$

The negative sign shows
that the drum becomes
slowed down.

Stopping time = 15.0 sec

Q3: A string 1m long is used to whirl a 100g stone in a horizontal circle at a speed of 2ms^{-1} . Find the tension in the string. ($T=0.4\text{N}$)

Solution:Length of string = $l = r = 1\text{m}$ Mass of stone = $m = 100\text{g} = 0.1\text{kg}$ Speed of stone = $v = 2\text{m/sec}$ Required:Tension in the string = $T = ?$ Calculation:

Here the tension in the string must be equal to the centripetal force i.e.

$$T = F_c \Rightarrow T = \frac{mv^2}{r} \quad (1)$$

Putting the values in equation (1), we get,

$$T = \left(\frac{0.1 \times 2^2}{1} \right) \text{N} = 0.4\text{N} \Rightarrow T = 0.4\text{N}$$

Tension in the string = $T = 0.4\text{N}$

Q4: The moon revolves around the earth in almost a circle of radius 382400 km in 27.3 days. What is the centripetal acceleration?

Solution:Given Data:Radius of circle = $r = 382400\text{km} \Rightarrow r = 382400 \times 10^3\text{m}$ Time period = $T = 27.3 \text{ day} = 27.3 \times 24 \times 3600 = 2358720\text{sec}$ Required: Centripetal acceleration $a_c = ?$ Calculation: We know that, $a_c = \frac{v^2}{r} = \frac{r^2 \omega^2}{r}$

$$\Rightarrow a_c = r \omega^2 \Rightarrow a_c = r \left(\frac{2\pi}{T} \right)^2 = \frac{4\pi^2 r}{T^2}$$

$$\Rightarrow a_c = \frac{4\pi^2 r}{T^2} \quad (1)$$

Putting the values in equation (1), we get,

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ \Rightarrow \omega &= \frac{2\pi}{T} \\ v &= r\omega \end{aligned}$$

$$\begin{aligned} a_c &= \left[\frac{4 \times 3.14^2 \times 382400 \times 10^3}{(2358720)^2} \right] \text{m/sec}^2 \\ &= 0.00271 \text{m/sec}^2 \\ \Rightarrow a_c &= 0.00271 \text{m/sec}^2 \end{aligned}$$

Q5: A modern FI car can accelerate from 0 to 62 mile/h (100km/h) in 2.50s. What is the angular acceleration of its 170mm radius wheels? (65.17 rad/s^2)

Solution:Given Data:Initial velocity = $v_i = 0\text{m/sec}$ Final velocity = $v_f = 100\text{km/h} = 100 \times \frac{10^3\text{m}}{3600\text{sec}}$ $\Rightarrow v_f = 27.77\text{m/sec} = 27.8\text{m/sec}$ Time taken = $t = 2.5\text{sec}$ Radius of wheel = $r = 170\text{mm} = 170 \times 10^{-3}\text{m} = 0.170\text{m}$

$$\begin{aligned} 1\text{km} &= 10^3\text{m} \\ 1\text{hr} &= 3600\text{sec} \end{aligned}$$

Required:Angular acceleration = $\alpha = ?$ Calculation:

We know that,

$$\omega_f = \omega_i + \alpha t \Rightarrow \alpha t = \omega_f - \omega_i \quad (1)$$

$$\text{Also } v = r\omega \Rightarrow v_i = r\omega_i \Rightarrow \omega_i = \frac{v_i}{r} \quad (2)$$

$$\text{Similarly, } \omega_f = \frac{v_f}{r} \quad (3)$$

Putting equation (2) and equation (3) in equation (1), we get,

$$\alpha t = \frac{v_f}{r} - \frac{v_i}{r} = \frac{1}{r}(v_f - v_i)$$

$$\Rightarrow \alpha = \frac{1}{rt}(v_f - v_i) \quad (4)$$

Putting the values in equation (4), we get,

$$\alpha = \frac{1}{(0.170 \times 2.5)} [27.8 - 0] = \left(\frac{27.8}{0.425} \right) \text{rad/sec}^2 \Rightarrow \alpha = 65.4 \text{rad/sec}^2$$

Q6: An electric motor is running at 1800rev/m. It comes to rest in 20sec. If the angular acceleration is uniform, find the number of revolutions it made before stopping?

Solution: Initial angular speed = $\omega_i = 1800\text{rev/min}$

$$\Rightarrow \omega_i = \frac{1800}{60} \text{ rev/sec} = 30 \text{ rev/sec}$$

Final angular speed = $\omega_f = 0$, time taken = $t = 20 \text{ sec}$

Required:

Number of revolutions = $\theta = ?$

Calculation:

We know that,

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \theta = \omega_i t + \frac{1}{2} (\omega_f - \omega_i) t$$

$$\Rightarrow \theta = \omega_i t + \frac{1}{2} (\omega_f - \omega_i) t \quad (1)$$

Putting the values in equation (1), we get,

$$\theta = 30 \times 20 + \frac{1}{2} (0 - 30) 20$$

$$\Rightarrow \theta = 600 - \frac{600}{2} = 600 - 300 = 300 \text{ rev}$$

$$\boxed{\text{Number of revolution} = \theta = 300 \text{ rev}}$$

Q7: What is the moment of inertia of a 100kg sphere whose radius is 50cm?

Solution:

Mass of sphere = $M = 100 \text{ kg}$

Radius of sphere = $R = 50 \text{ cm} = 0.5 \text{ m}$

Required:

Moment of inertia = $I = ?$

Calculation:

We know that the moment of inertia of a solid sphere is given by,

$$I = \frac{2}{5} MR^2 \quad (1)$$

Putting the values in equation (1), we get,

$$\Rightarrow I = \left[\frac{2}{5} \times 100 \times (0.5)^2 \right] \text{ kg} \cdot \text{m}^2 \Rightarrow \boxed{I = 10 \text{ kg} \cdot \text{m}^2}$$

$$\Rightarrow \boxed{I = 10 \text{ kg} \cdot \text{m}^2}$$

$$S = v_i t + \frac{1}{2} a t^2$$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

Q8: A rope is wrapped several times around a cylinder of radius 0.2m and mass 30kg. What is the angular acceleration of the cylinder if the tension in the rope is 40N and it turns without friction? (13.3 rad/s^2).

Solution: Radius of cylinder = $r = 0.2 \text{ m}$, mass of cylinder = $m = 30 \text{ kg}$
Tension in the rope = $T = F = 40 \text{ N}$

Required:

Angular acceleration = $\alpha = ?$

Calculation:

We know that,

$$\tau = I \alpha \quad (1)$$

We also know that,

$$\tau = r F \quad (2)$$

Comparing equation (1) and equation (2), we get,

$$I \alpha = r F \Rightarrow \alpha = \frac{r F}{I} \quad (3)$$

The moment of inertia of cylinder is given by,

$$I = \frac{1}{2} m r^2 \quad (4)$$

Putting equation (4) in equation (3), we get,

$$\alpha = \frac{r F}{\left(\frac{1}{2} m r^2 \right)} = \frac{2 r F}{m r^2} = \frac{2 F}{m r} \Rightarrow \alpha = \frac{2 F}{m r} \quad (5)$$

Putting the values in equation (5), we get,

$$\alpha = \left(\frac{2 \times 40}{30 \times 0.2} \right) \text{ rad/sec}^2 \Rightarrow \boxed{\alpha = 13.3 \text{ rad/sec}^2}$$

Q9: What is the kinetic energy of a 5.0kg solid ball whose diameter is 15cm if it rolls across a level surface with a speed of 2 ms^{-1} ? (14J)

Solution: Given Data:

Mass of solid ball = $M = 5 \text{ kg}$, diameter of ball

$$= D = 15 \text{ cm} = 0.15 \text{ m}$$

Radius of solid ball

$$= R = \frac{D}{2} = \frac{0.15}{2} = 0.075 \text{ m}$$

Speed of ball = $v = 2 \text{ m/sec}$

Required:

$$K.E \text{ of rolling ball} = K.E = ?$$

Calculation:

We know that rolling body possesses both linear K.E and rotational K.E, so its total K.E is given by,

$$(K.E)_{\text{total}} = (K.E)_{\text{linear}} + (K.E)_{\text{rot}}$$

$$\Rightarrow (K.E)_{\text{total}} = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 \quad (1)$$

For solid sphere or ball, we have,

$$I = \frac{2}{5} MR^2 \quad (2)$$

$$\text{Also } v = R\omega \Rightarrow \omega = \frac{v}{R} \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$K.E = \frac{1}{2} Mv^2 + \frac{1}{2} \times \left(\frac{2}{5} MR^2 \right) \times \frac{v^2}{R^2}$$

$$\Rightarrow K.E = \frac{1}{2} Mv^2 + \frac{1}{5} Mv^2 \quad (4)$$

Putting the values in equation (4), we get,

$$\Rightarrow K.E = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{5} \times 5 \times 2^2$$

$$\Rightarrow K.E = (10 + 4)J \Rightarrow \boxed{K.E = 14 \text{ joules}}$$

Q10: A cylinder of 50cm diameter at the top of an incline 29.4cm high and 10m long is released and rolls down the incline. Find its linear and angular speed at the bottom. Neglect friction.

Solution: Given Data:

$$\text{Diameter of cylinder} \\ = D = 50\text{cm} = 0.5\text{m}$$

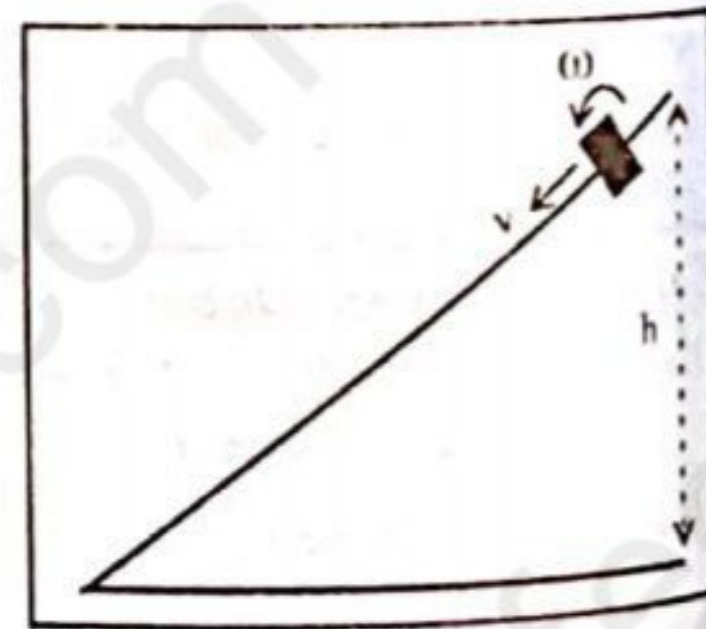
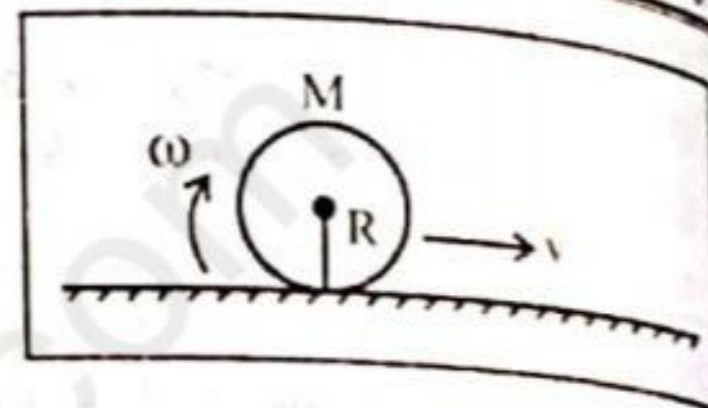
$$\text{Radius of solid cylinder} \\ = R = \frac{D}{2} = \frac{0.5}{2} = 0.25\text{m}$$

$$\text{Height} = h = 29.4\text{cm} = 0.294\text{m}$$

Required:

$$(a) \text{ Linear speed} = v = ?$$

$$(b) \text{ Angular speed} = \omega = ?$$



Calculation:

(a) As the cylinder is rolling down from top to bottom, so according to the law of conservation of energy, we have,

$$\text{Loss in P.E} = \text{Gain in K.E}$$

$$\Rightarrow mgh = (K.E)_{\text{linear}} + (K.E)_{\text{rot}}$$

$$\Rightarrow mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \quad (1)$$

For solid cylinder, we have,

$$I = \frac{1}{2} mR^2 \quad (2)$$

$$\text{Also we know that, } v = R\omega \Rightarrow \omega = \frac{v}{R} \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{1}{2} mR^2 \times \frac{v^2}{R^2}$$

$$\Rightarrow gh = \frac{v^2}{2} + \frac{v^2}{4} \Rightarrow gh = \frac{3v^2}{4} \Rightarrow v^2 = \frac{4}{3} gh$$

$$\Rightarrow v = \sqrt{\frac{4}{3} gh} \quad (4)$$

Putting the values in equation (4), we get,

$$v = \sqrt{\frac{4 \times 9.8 \times 0.294}{3}} \text{ m/sec} = 1.96 \text{ m/sec}$$

$$\Rightarrow \boxed{v = 1.96 \text{ m/sec}}$$

$$(b) \text{ As } v = R\omega \Rightarrow \omega = \frac{v}{R} = \frac{1.96}{0.25} \Rightarrow \boxed{\omega = 7.84 \text{ rad/sec}}$$

Q11: A disc without slipping rolls down a hill of vertical height of 1000cm. If the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom?

Solution:

Given Data:

$$\text{Height of hill} = h = 1000\text{cm}$$

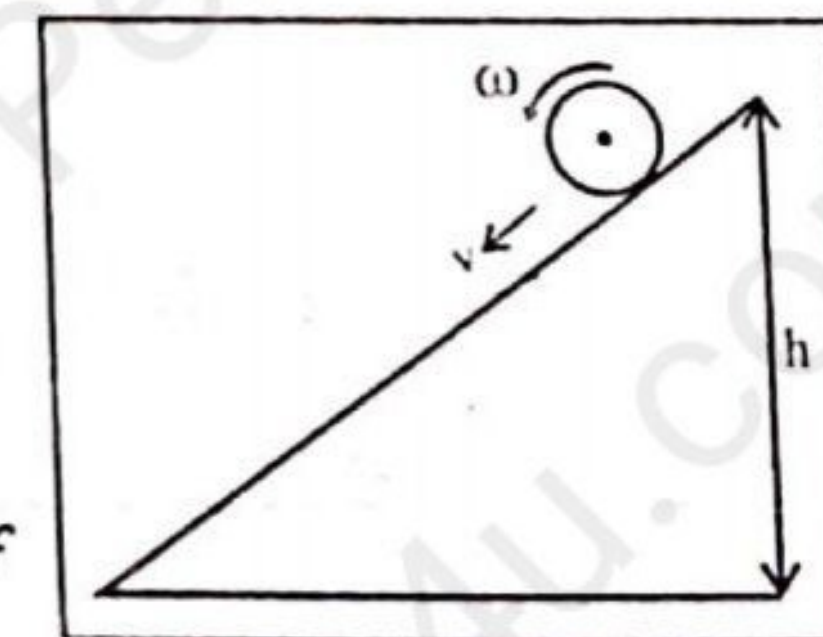
$$\Rightarrow h = 10\text{m}$$

Required:

$$\text{Speed at the bottom} = v = ?$$

According to the law of conservation of energy, we have,

$$\text{Loss in P.E} = \text{Gain in}$$



$$(K.E)_{linear} + (K.E)_{rot} \\ \Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad (1)$$

For disc, we have, $I = \frac{1}{2}mr^2 \quad (2)$

Also, $v = r\omega \Rightarrow \omega = \frac{v}{r} \quad (3)$

Putting equations (2), (3) in equation (1), we get,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \frac{v^2}{r^2} \\ \Rightarrow gh = \frac{v^2}{2} + \frac{v^2}{4} = \frac{3v^2}{4} \Rightarrow gh = \frac{3v^2}{4} \Rightarrow v^2 = \frac{4gh}{3} \\ \Rightarrow v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4 \times 9.8 \times 10}{3}} \\ \Rightarrow v = 11.4 \text{ m/sec}$$

Q12: A motor car is travelling at a speed of 30m/sec. If its wheel has a diameter of 1.5m, find its angular speed in rad/sec and in rev/sec.

Solution:

Speed of the car = $v = 30 \text{ m/sec}$, diameter of wheel = $d = 1.5 \text{ m}$

Radius of wheel = $r = \frac{d}{2} = \frac{1.5}{2} = 0.75 \text{ m}$

Required:

(a) Angular speed in rad/sec = $\omega = ?$

(b) Angular speed in rev/sec = $\omega = ?$

Calculation:

(a) We know that, $v = r\omega$

$$\Rightarrow \omega = \frac{v}{r} = \frac{30}{0.75} \Rightarrow \omega = 40 \text{ rad/sec}$$

(b) $\omega = 40 \text{ rad/sec}$

$$1 \text{ rev} = 2\pi \text{ rad} \Rightarrow 1 \text{ rad} = \frac{1}{2\pi} \text{ rev}$$

$$\Rightarrow \omega = 40 \times \frac{1}{2\pi} \text{ rev/sec}$$

$$\Rightarrow \omega = 6.369 \text{ rev/sec} \Rightarrow \omega = 6.37 \text{ rev/sec}$$

• اچھی صورت کے مقابلے میں اچھی سیرت کا رتبہ بلند ہوتا ہے۔
• جو کسی کا رتبہ نہیں چاہتے، ان کے ساتھ کوئی بُرا نہیں کر سکتا، یہ میرے رتب کا وعدہ ہے۔

Q1: Define the following terms: (a) Fluid dynamics, hydrostatics, (c) Fluid friction, (d) Ideal fluid

Ans: (a) Fluid:

A substance which can flow is known as fluid. For example, oil, milk, water, gas etc are fluids.

(b) Hydrostatics and Fluid Dynamics:

The study of fluid at rest is called hydrostatic. While the study of properties and effect of moving fluids is known as fluid dynamics. In fluid dynamics, 'hydrodynamics' deals with the study of liquids in motion, while 'aerodynamics' deals with the study of gases in motion.

(c) Fluid Friction:

When a body moves through a fluid, then an opposing force acts on the body. This opposing force is known as fluid friction or dragging force. This force depends upon the:

- Size of the body
- Geometric shape of the body
- Direction of velocity of the body and
- Viscosity of the fluid

(d) Ideal Fluid: (ETEA 2016)

An incompressible, non-viscous fluid having constant density at any pressure, is called an ideal fluid.

Q2: What is a fluid? Discuss briefly, the viscosity of a fluid and explain how the flow of a fluid is characterized by viscosity.

Ans: Fluid:

A substance which flows is known as fluid. For example, oil, water, gas etc.