Physics Chapter 7 Oscillation Numericals Problems pdf

Q.1 A force of 0.4 N is required to displace a body attached to spring through 0.1 m from its mean position. Calculate the spring constant of spring.

Answer:

Given Data:

Force = F = 0.4 N

Displacement = x = 0.1 m

To Find:

Spring Constant = k = ?

Solution:

According to Hook's law, we have

F = kx

$\Rightarrow \quad k = \frac{F}{x}$

Putting values, we get

$$k = \frac{0.4N}{0.1m} = 4Nm^{-1}$$

So spring constant will be,

$k = 4\ Nm^{-1}$

Answer

Q.2 A body of mass 0.025 kg attached to a spring is displaced through 0.10 m to the right of mean position. If the spring constant of spring is 0.4 Nm^{-1} and its velocity at the end of this displacement be 0.4 ms^{-1}. Calculate

(i) Time period 'T'

(ii) Frequency 'f'

(iii) Angular speed 'ω'

(iv) The total energy

(v) the amplitude of its motion

(vi) the maximum velocity

(vii) the maximum accelerations

Answer:

Given Data:

Mass of the body = m = 0.025 kg

Displacement = x = 0.10 m

Spring Constant = k = 0.4 Nm^{-1}

Velocity = v = 0.4 ms^{-1}

To Find:

(i) Time Period = T = ?

(ii) Frequency = f = ?

(iii) Angular Speed = ω = ?

(iv) Total Energy = E_{total} = ?

(v) Amplitude = x_0 = ?

(vi) Maximum Velocity = v_{max} = ?

(vii) Maximum Acceleration = a_{max} = ?

Solution:

(i) As time period of a mass-spring system is given by,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Putting values, we get

$$T = 2 \times 3.14\sqrt{\frac{0.025kg}{0.4Nm^{-1}}} = 6.28 \times 0.25s$$

So time period will become,

$$\boxed{T = 1.57s}$$

Answer

(ii) We know that frequency is given as,

$$f = \frac{1}{T}$$

Putting values, we get

$$f = \frac{1}{1.57s} = 0.638s^{-1}$$

$$\boxed{f = 0.638 \text{ cps}}$$

Answer

(iii) Relation between time period and angular frequency is given by,

$$T = \frac{2\pi}{\omega} \qquad \Longrightarrow \qquad \omega = \frac{2\pi}{T}$$

Putting values, we get

$$\omega = \frac{2 \times 3.14}{1.57s} = \frac{6.28}{1.57} s^{-1}$$

$$\boxed{\omega = 4 \text{ rad s}^{-1}}$$

Answer

(iv) We know that total energy of a system is the sum of kinetic energy and potential energy. So we can write it as,

$$E_{total} = K.E + P.E$$

$$i.e. \qquad E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

Putting values, we get

$$E_{total} = \left[\frac{1}{2} \times 0.025kg \times (0.4ms^{-1})^2\right] + \left[\frac{1}{2} \times 0.4Nm^{-1} \times (0.10m)^2\right]$$

$$\Longrightarrow \qquad E_{total} = 0.002kgm^2s^{-2} + 0.002kgm^2s^{-2} = 0.004\, joules$$

Or,

$$\boxed{E_{total} = 4 \times 10^{-3} \text{ J}}$$

Answer

(v) We can also write total energy as,

$$E_{total} = \frac{1}{2}kx_0{}^2 \qquad \Longrightarrow \qquad x_0{}^2 = \frac{2E_{total}}{k}$$

Putting values, we get

$$x_0{}^2 = \frac{2 \times 4 \times 10^{-3} j}{0.4Nm^{-1}} = 2 \times 10^{-2} m^2$$

Or,

$\boxed{X_0 = 0.1414\ m}$

Answer

(vi) As maximum velocity is given by, $v_{max} = \omega x_o$, so putting values

we get

$V_{max} = 4\ rad\ s^{-1} \times 0.1414\ m$

$\boxed{V_{max} = 0.57ms^{-1}}$

(vii) We know that maximum acceleration for a mass-spring

system is given by,

$$a_{max} = -\omega^2 x_o$$

Putting values, we get

$$a_{max} = -(4\ rad\ s^{-1})^2\ (0.1414m)$$

So,

$\boxed{a_{max} = -2.26\ ms^{-2}}$

Answer

Q.3 A simple pendulum completes one vibration in one second calculate its length when g = 9.81 ms^{-2}

Answer:

Given Data:

Time Period = T = 1 s

Gravitational Acceleration = g = 9.8 ms^{-2}

To Find:

Length of pendulum = l = ?

Solution:

As time period of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow \quad T^2 = 4\pi^2\frac{l}{g}$$

So we get,

$$l = \frac{T^2 g}{4\pi^2}$$

Putting values, we get

$$l = \frac{(1s)^2 \times 9.8ms^{-2}}{4(3.14)^2} = \frac{9.8m}{39.43}$$

$\Rightarrow$ l = 0.248 m = 24.8×10^{-2} m

$\boxed{l = 24.8\ cm}$

Answer

Q.4 Calculate the length of a second pendulum having time period 2 s at a plate where g = 9.8 ms^{-2}.

Answer :

Answer:

Given Data:

Time Period = T = 2 s

Gravitational Acceleration = g = 9.8 ms^{-2}

To Find:

Length of second pendulum = l = ?

Solution:

As time period of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \Longrightarrow \quad T^2 = 4\pi^2\frac{l}{g}$$

So we get,

$$l = \frac{T^2 g}{4\pi^2}$$

Putting values, we get

$$l = \frac{(2s)^2 \times 9.8ms^{-2}}{4(3.14)^2} = \frac{39.2m}{39.43}$$

$$\boxed{l = 0.994 \text{ m}}$$

Answer

Q.5 A body of mass 'm', suspended from a spring with a force constant k, vibrates with 'f_1' When its length is cut into half and the same body is suspended from one of the halves, the frequency is 'f_2' Find out $f_1f_2^{-1}$.

Answer:

Given Data:

Mass of the body = m

Spring Constant = k

Displacement from whole spring = x_1

Frequency of whole spring = f_1

Displacement from half of spring = $x_2 = x_1/2$

Frequency of half spring = f_2

To Find:

Value of $f_1 f_2^{-1}$ = ?

Solution:

We know that time period for a mass-spring system is given by,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

And as frequency is given by, f = 1/T, so putting value of 'T' we get

$$f = {}^{1}\Big/_{2\pi\sqrt{\frac{m}{k}}} \qquad \Rightarrow \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \ldots\ldots\ldots\ldots (1)$$

According to Hook's law, we have

$$F = kx \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Here force acting is the weight of the body, so

$$F = w = mg \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Comparing equation (1) and (2), we get

$$ma = kx \qquad \Rightarrow \qquad k = ma/x$$

Putting this value of 'k' in equation (1), we get

$$f = \frac{1}{2\pi}\sqrt{\frac{mg/x}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{x}} \qquad \text{.....................................} (4)$$

For full whole spring, equation (4) gives

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{g}{x_1}}$$

Similarly, for half spring, equation (4) gives

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{g}{x_2}}$$

We have to find the value of $f_1 f_2^{-1}$, so putting values of f_1 and f_2, we get

$$f_1 f_2^{-1} = \left(\frac{1}{2\pi}\sqrt{\frac{g}{x_1}}\right) \times \left(\frac{1}{2\pi}\sqrt{\frac{g}{x_2}}\right)^{-1}$$

$$\Rightarrow \qquad f_1 f_2^{-1} = \left(\frac{1}{2\pi}\sqrt{\frac{g}{x_1}}\right) \times \left(2\pi\sqrt{\frac{x_2}{g}}\right) = \sqrt{\frac{x_2}{x_1}} \qquad \text{............................} (5)$$

And as,

$$x_2 = x_1/2 \qquad \Rightarrow \qquad x_1 = 2\,x_2$$

Putting this value in equation (5), we get

$$f_1 f_2^{-1} = \sqrt{\frac{x_2}{2x_2}} = \frac{1}{\sqrt{2}}$$

$$\boxed{f_1 f_2^{-1} = 0.707}$$

Answer

Q.6 A mass at the end of spring describes S.H.M with T = 0.40 s. Find out 'a' when the displacement is 0.04m.

Answer:

Given Data:

Time Period = T = 0.40 s

Displacement = x = 0.04m

To Find:

Acceleration = a = ?

Solution:

Acceleration for a S.H.M is given by,

$a = -\omega^2 x$ .... (1)

Where the relation between time period and angular frequency is,

$$T = \frac{2\pi}{\omega} \quad \Longrightarrow \quad \omega = \frac{2\pi}{T}$$

Putting values, we get

$$\omega = \frac{2 \times 3.14}{0.40s} = \frac{6.28}{0.40} s^{-1}$$

$\Rightarrow$ $\omega = 15.7\ s^{-1}$

Putting this value in equation (1), we get

$a = -(15.7\ s^{-1})^2\ (0.04m) = -\ 9.86\ ms^{-2}$

$\boxed{a = -\ 9.8\ ms^{-2}}$

Answer

Q.7 A block weighing 4.0 kg extends a spring by 0.16 m from its unstretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and the released, What is its period of vibration?

Answer:

Given Data:

Mass of block = m_1 = 4.0 kg

Displacement = x = 0.16 m

Mass of second body = m_2 = 0.50 kg

To Find:

Time Period = T = ?

Solution:

Time period of a mass-spring system is given by,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_2}{k}} \quad \text{.................................. (1)}$$

Where 'm' is the mass of the body and 'k' is spring constant.

According to Hooks law, we have

$$F = kx = w$$

$$\Rightarrow \quad m_1 g = kx$$

$$\Rightarrow \quad k = m_1 g/x$$

Putting this value in equation (1), we get

$$T = 2\pi\sqrt{\frac{m_2}{m_1 g/x}} = 2\pi\sqrt{\frac{m_2 x}{m_1 g}}$$

Putting values, we get

$$T = 2 \times 3.14\sqrt{\frac{0.50kg \times 0.16m}{4.0kg \times 9.8ms^{-2}}} = 6.28 \times \sqrt{\frac{0.08kgm}{39.2kgms^{-2}}}$$

$$T = 6.28 \times 0.0452s$$

$$\boxed{T = 0.28\ s}$$

Answer

Q.8 What should be the length of simple pendulum whose time period is one second? What is its frequency?

Answer:

Given Data:

Time Period = T = 1 s

To Find:

(a) Length of simple pendulum = l = ?

(b) Frequency = f = ?

Solution:

(a) As time period of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \Rightarrow \quad \sqrt{\frac{l}{g}} = \frac{T}{2\pi}$$

Or,

$$l = \frac{T^2 g}{4\pi^2}$$

Putting values of time period and gravitational acceleration, we get

$$l = \frac{(1s)^2(9.8ms^{-2})}{4(3.14)^2} = \frac{9.8m}{39.4}$$

$$\boxed{l = 0.25\ m}$$

Answer

(b) As relation between frequency and time period is given as,

$$f = \frac{1}{T}$$

Putting value of time period, we get

$$f = \frac{1}{(1s)^2} = 1s^{-2}$$

$$\boxed{f = 1\ cpr}$$

Answer

Q.9 A spring, whose spring constant is 80.0 Nm^{-1} vertically supports a mass of 1.0 kg is at rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with a velocity of one meter per second.

Answer:

Given Data:

Spring Constant = k = 80.0 Nm^{-1}

Mass of the body = m = 1.0 kg

Maximum velocity of motion = v_{max} = 1 ms^{-1}

To Find:

Amplitude = x_o = ?

Solution:

As maximum velocity for a mass-spring system is given by,

$$v_{max} = \omega x_o$$

.... (1)

Where 'ω' is given as,

$$\omega = \sqrt{\frac{k}{m}}$$

Putting this value in equation (1), we get

$$v_{max} = \sqrt{\frac{k}{m}} x_0 \quad \Longrightarrow \quad x_0 = v_{max}\sqrt{\frac{m}{k}}$$

Putting values, we get

$$x_0 = 1ms^{-1} \times \sqrt{\frac{1.0kg}{80.0Nm^{-1}}} = 1ms^{-1} \times 0.1118s$$

$$\boxed{X_0 = 0.11 \text{ m}}$$

Answer

Q.10 A 8000 g body vibrates S.H.M with amplitude 0.30 m. The restoring force is 60 N and the displacement is 0.30 m. Find out
(i) Time period

(ii) Acceleration
(iii) Velocity
(iv) Kinetic Energy
(v) Potential Energy when the displacement is 12cm.

Answer:

Given Data:

Mass of body = m = 8000 g = 8.0 kg

Amplitude = x_0 = 0.30 m

Restoring Force = F = 60 N

Displacement = x = 0.30 m

To Find:

(i) Time period = T = ?

(ii) Acceleration = a = ?

(iii) Velocity = v = ?

(iv) Kinetic Energy = K.E = ?

(v) Potential energy = P.E = ? for x = 12 cm = 0.12m

Solution:

(i) As time period of a mass-spring system is given by,

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{.................................} (1)$$

According to Hook's law we have,

$$F = kx_0 \quad \Rightarrow \quad k = F/x_0$$

Putting this value in equation (1), we get

$$T = 2\pi\sqrt{\frac{m}{F/x_0}} = 2\pi\sqrt{\frac{mx_0}{F}}$$

Putting values, we get

$$T = 2 \times 3.14\sqrt{\frac{8kg \times 0.30m}{60N}} = 6.28 \times 0.2s$$

Or,

$$\boxed{T = 1.3\ s}$$

Answer

(ii) Acceleration for a mass-spring system is given as,

$$\mathbf{a} = -\omega^2 x$$

Where '$\omega^2 = k/m$' and '$k = \mathbf{F}/x = 200\ Nm^{-1}$', so putting this we get

$$a = -\left(\frac{k}{m}\right)x = -\frac{kx}{m}$$

Putting values, we get

$$a = -\frac{200Nm^{-1} \times 0.12m}{8kg} = -\frac{24}{8}ms^{-2}$$

Or,

$$\boxed{a = 3\ ms^{-2}}$$

Answer

(iii) Velocity of a mass-spring system is,

$$v = \omega\sqrt{{x_0}^2 - x^2} = \sqrt{\frac{k}{m}({x_0}^2 - x^2)}$$

Putting values, we get

$$v = \sqrt{\frac{200Nm^{-1}[(0.3m)^2 - (0.12m)^2]}{8kg}} = \sqrt{1.89m^2}$$

So, velocity will be

$\boxed{v = 1.4\ ms^{-1}}$

Answer

(iv) As kinetic energy for a mass-spring system is given by,

$$K.E = \frac{1}{2}k({x_0}^2 - x^2)^2$$

Putting values, we get

$$K.E = \frac{1}{2} \times 200Nm^{-1} \times [(0.3m)^2 - (0.12m)^2]^2 = 7.56Nm$$

Or,

$\boxed{K.E = 7.56\ J}$

Answer

(v) Potential energy of a mass-spring system is,

$$P.E = \frac{1}{2}kx^2$$

Putting values, we get

$$P.E = \frac{1}{2} \times 200Nm^{-1} \times (0.12m)^2 = 1.44Nm$$

Or,

$\boxed{P.E = 1.44\ J}$

Answer

Q.11 Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant t is given by $x = 0.25 \cos(\pi/8)\, t$. Find the displacement of the object after 2.0s

Answer:

Given Data:

Equation for position:

$$x = 0.25 \cos\left(\frac{\pi}{8}\right) t \qquad \text{.........................} (1)$$

To Find:

(a) Amplitude = x_0 = ?

(b) Frequency = f = ?

(c) Time Period = T = ?

(d) Displacement = x = ? at t = 2.0 s

Solution:

(a) We know that equation of displacement for a S.H.M is given as,

$$x = x_0 \cos \omega t \qquad \text{.................................}(2)$$

Comparing equation (1) and (2), we get

$$\boxed{X_0 = 0.25 \text{ m}}$$

Answer

(b) Relation between linear frequency and angular frequency is given by,

$$\omega = 2\pi f$$

Putting the value of “ ω ” from above we get,

$$f = \frac{\omega}{2\pi} = \frac{\pi}{2\pi 8} = \frac{1}{16}$$

$$\boxed{f = 0.06Hz}$$

Answer

(c) Relation between time period and frequency is given as,

$$T = \frac{1}{f} = \frac{1}{0.06}$$

$$\boxed{T = 16 \text{ s}}$$

Answer

(d) Displacement after 2 sec:

As equation of position as a function of time is given here,

$$x = 0.25\cos\left(\frac{\pi}{8}\right) t$$

So, putting t = 2 s in it we get,

$$x = 0.25cos\left(\frac{\pi}{8}\,2\right) = 0.25\cos\left(\frac{\pi}{4}\right)$$

$$x = 0.25cos45 = 0.25 \times 0.707$$

$$\boxed{X = 0.177 \text{ m}}$$

Answer

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