Here $m = [M^1]$, $a = \frac{L}{T^2} = [L^1T^{-2}]$ and $d = [L^1]$

So equation (2) becomes

$$K\cdot E = [M^1][L^1T^{-2}][L^1] = [M^1L^{1+1}T^{-2}]$$

$$\Rightarrow K\cdot E = [M^1L^2T^{-2}] \_\_\_\_\_\_ (3)$$

Now taking R·H·S, we have,

$$\frac{1}{2}mv^2 = \frac{1}{2}\left[M^1\cdot\left(\frac{L}{T}\right)^2\right] = \frac{1}{2}\left[M^1\,\frac{L^2}{T^2}\right]$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}[M^1L^2T^{-2}] \_\_\_\_\_\_ (4)$$

Comparing equation (3) and equation (4), we get,

$$\boxed{\text{Dimension of } K\cdot E = \text{Dimension of } \frac{1}{2}mv^2}$$

Thus the expression $\left(K\cdot E = \frac{1}{2}mv^2\right)$ is dimensionally correct·

Part (b): $P\cdot E = mgh \_\_\_\_\_\_ (1)$

Taking L·H·S, we have,

$$P\cdot E = \text{workdone} = Fd = mad$$

$$\Rightarrow P\cdot E = mad$$

$$\Rightarrow P\cdot E = \left[M^1\cdot\frac{L^1}{T^2}\cdot L^1\right] = [M^1L^2T^{-2}]$$

$$\Rightarrow P\cdot E = [M^1L^2T^{-2}] \_\_\_\_\_\_ (2)$$

Now taking R·H·S, we have,

$$mgh = \left[M^1\cdot\frac{L^1}{T^2}\cdot L^1\right] \Rightarrow mgh = [M^1L^{1+1}\cdot T^{-2}]$$

$$\Rightarrow mgh = [M^1L^2T^{-2}] \_\_\_\_\_\_ (3)$$

Comparing equation (2) and equation (3), we get,

$$\boxed{\text{Dimension of } P\cdot E = \text{Dimension of } mgh}$$

Thus the expression $P\cdot E = mgh$ is dimensionally correct·

✦✦✦

CHAPTER 02 VECTORS AND EQUILIBRIUM

Q1: Define (a) physical quantities, (b) magnitude?

Ans· (a) ***Physical Quantities:***

Those quantities which can be measured are known as physical quantities· For example, length, mass, temperature, speed, velocity, acceleration, energy, torque etc·

(b) ***Magnitude:***

A number with proper unit is known as magnitude· For example 100kg, 27°C, 50N, 1000m etc· Here 100, 27, 50, 1000 represent number which kg, °C, N, m represent the unit·

Q2: Define and explain scalar and vector quantities· Give examples in each case?

Ans· (a) ***Scalar Quantities:***

Definition: Those physical quantities which are completely specified by their magnitude only are known as scalar quantities·

Explanation:

For the complete explanation of scalar quantities, the unit and number both are necessary· If we ignore any one of them, then our explanation will be incomplete· The scalars can be added, subtracted, multiplied and divided by ordinary mathematical method·

Examples: Speed, mass, energy, work, power, distance, temperature, electric flux etc are the examples of scalar quantities·

(b) ***Vector Quantities:***

Definition: Those physical quantities which are completely specified by their magnitude and proper direction are known as vector quantities·

Explanation:

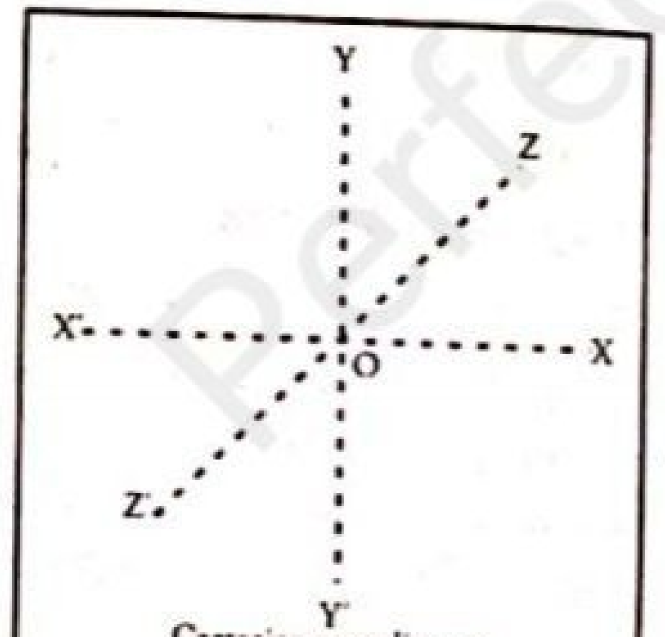
For the complete explanation of vector quantities, the number, unit and proper direction all are necessary· If we ignore any one of them, then our explanation will be incomplete. The vectors can be added, subtracted, divided and multiplied by graphical method or geometrical method.

Examples: Momentum, acceleration, torque, displacement, velocity, weight, force etc are the examples of vector quantities.

Q3: (a) What is Cartesian coordinate system? (b) Write the steps to represent a vector in Cartesian coordinate system.

Ans. (a) Cartesian Coordinate System:

The Cartesian coordinate system is also called number plane. It consists of lines xox', yoy', zoz' which are mutually perpendicular to each other. The xox' [x-axis] and yoy' [y-axis] lines are usually drawn in the plane of the page. While the line zoz' [z-axis] comes directly out of the page at the origin "o" as shown in the figure. All the three axis [i.e. x-axis, y-axis, z-axis] are making an angle of 90° with each other.



Cartesian coordinates

(b) Steps to represent a Vector in Cartesian Coordinate System:

The following steps are taken for representing a vector in Cartesian coordinate system:

Step-1: Draw a Cartesian coordinate system.

Step-2: Select a suitable scale in case of large values.

Step-3: Draw a line according to the scale selected.

Step-4: Put an arrow at the end of the line, which represent the required vector.

Q4: Discuss the representation of a vector in Cartesian coordinate system?

Ans. There are two methods for representing a vector in Cartesian coordinate system. These methods are given below:

- Representing a vector in plane
- Representing a vector in space

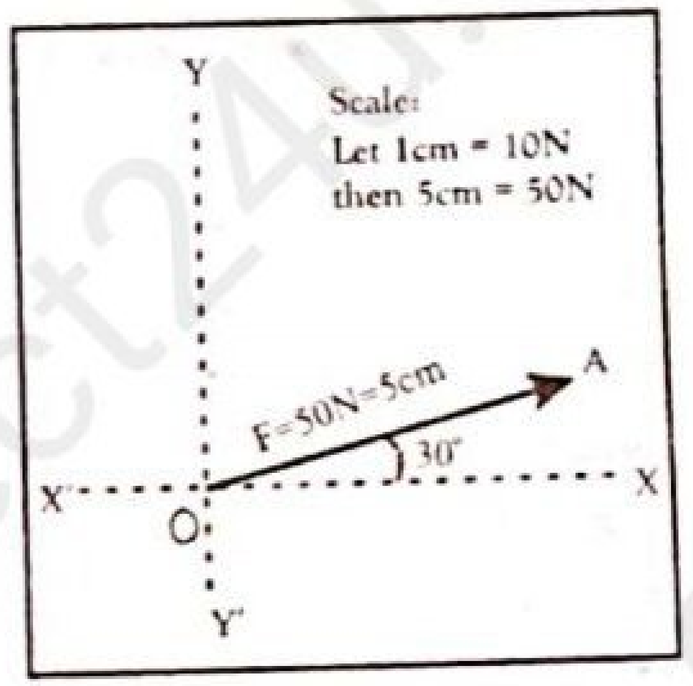
1. Representing a Vector in Plane:

To represent a vector in plane, we draw horizontal and vertical lines [i.e. xox' and yoy'] perpendicular to each other. Then we select a suitable scale. We draw the representative line according to the given scale. We put arrow at the end of the line which represent the required vector.

Example: A force of 50N is acting on xy-plane by making an angle of 30° with x-axis. Draw the representative vector of the given force.

Solution:

Step-1: First we draw the Cartesian coordinate system. i.e. we draw x-axis and y-axis perpendicular to each other as shown in the figure.



Step-2: Now we select a suitable scale. Let 10N = 1cm then 50N = 5cm

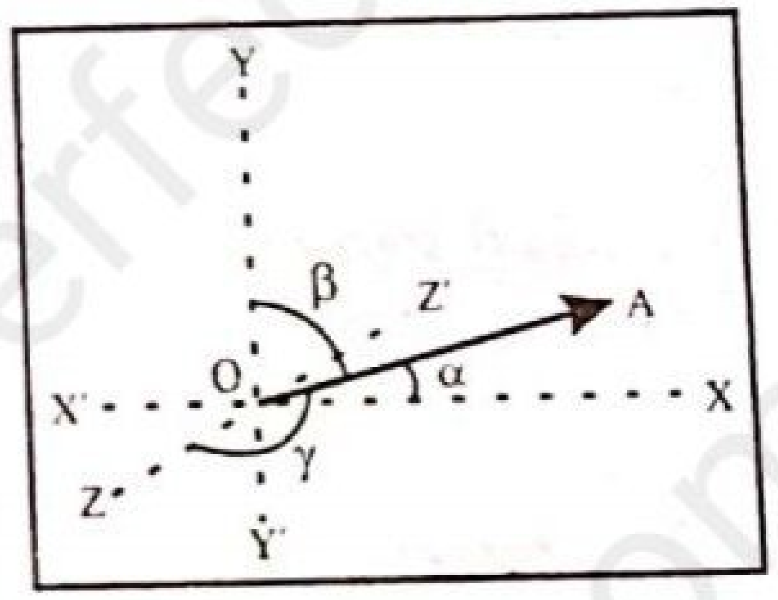
Step-3: Now we draw a line OA = 5cm at an angle of 30° with x-axis.

Step-4: Now we put an arrow at the end of the line OA. Thus $\vec{OA}$ = 5cm [at 30° with x-axis] represents the force of 50N acting in xy-plane.

2. Representing of a Vector in Space:

To represent a vector in space, we draw x-axis, y-axis and z-axis [i.e. xox', yoy' and zoz'] which are mutually perpendicular to each other.

Now we draw a vector $\vec{OA}$ in space making an angle "α" with x-axis, "β" with y-axis and "γ" with z-axis respectively as shown in the figure.



Note: The line zoz' cannot be draw actually on the paper. It is directed out of the paper at the origin "o". All these axis i.e. x-axis, y-axis and z-axis are mutually perpendicular to each other.

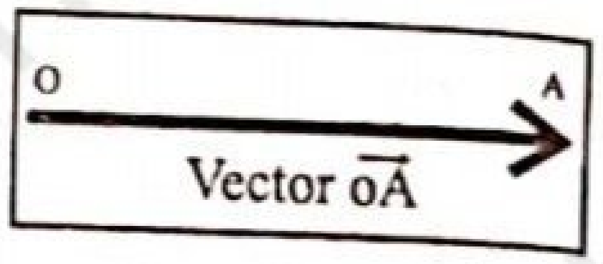
﴿اہل ایمان کے دل اللہ کی یاد سے سکون پاتے ہیں۔ خبردار رہو کہ دلوں کا چین اللہ کے ذکر ہی میں ہے۔﴾

Q5: Discuss how a vector is represented?

Ans. A vector is usually represented by the following two methods:

1. Graphical Representation:

Graphically, a vector is represented by a bold straight line having an arrow head at its one end. The arrow indicates the direction of the given vector. For example, in the given figure, the line "$\vec{OA}$" represents a vector.



2. Symbolic Representation:

Symbolically, a vector is represented by any English alphabet having an arrow head upon it. For example, $\vec{A}, \vec{a}, \vec{B}, \vec{b}, \vec{S}, \vec{T}$ etc represents the vector symbolically.

Q6: Define the following: (i) Parallel vectors, (ii) Anti-parallel vectors, (iii) Equal vectors, (iv) Negative of a vector, (v) Null vector (or) Zero vector?

Ans. (i) **Parallel Vectors:**

Vectors are said to be parallel to each other if they are acting in the same direction, as shown in the figure (a). The angle between parallel vectors is 0°.

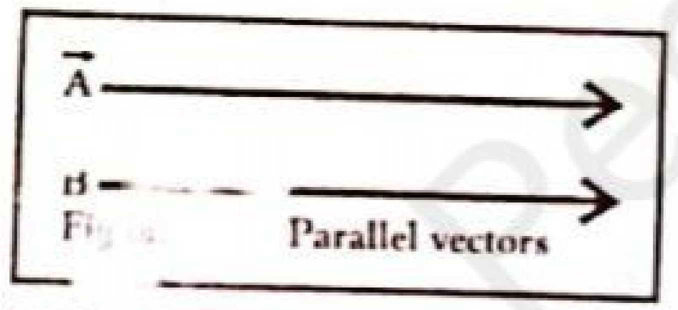


Fig (a) Parallel vectors

(ii) **Anti-Parallel Vectors:**

Vectors are said to be anti-parallel to each other if they are acting in opposite direction, as shown in the figure (b). The angle between anti-parallel vectors is 180°.

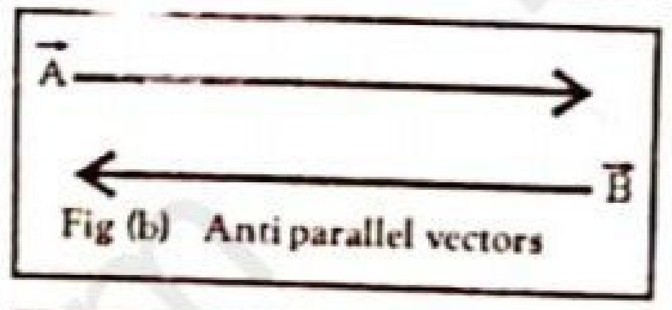


Fig (b) Anti parallel vectors

(iii) **Equal Vectors:**

Two vectors are said to be equal, if they have the same magnitude and same direction, as shown in the figure (c).

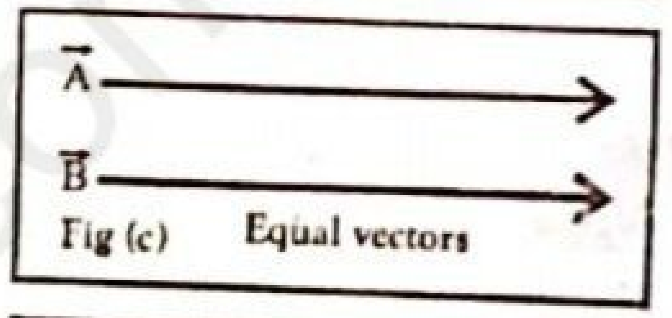


Fig (c) Equal vectors

(iv) **Negative of a Vectors:**

Negative of a vector is that vector whose magnitude is the same to that of the given vector but opposite in direction, as shown in the figure (d).

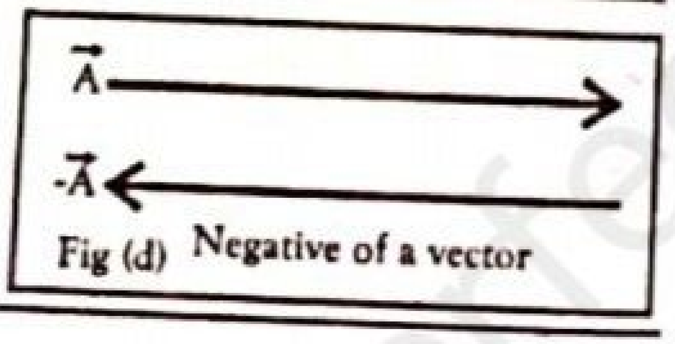


Fig (d) Negative of a vector

(v) **Null Vector or Zero Vectors:**

A vector whose magnitude is zero is known as null vector or zero vector. It is arbitrary in direction.

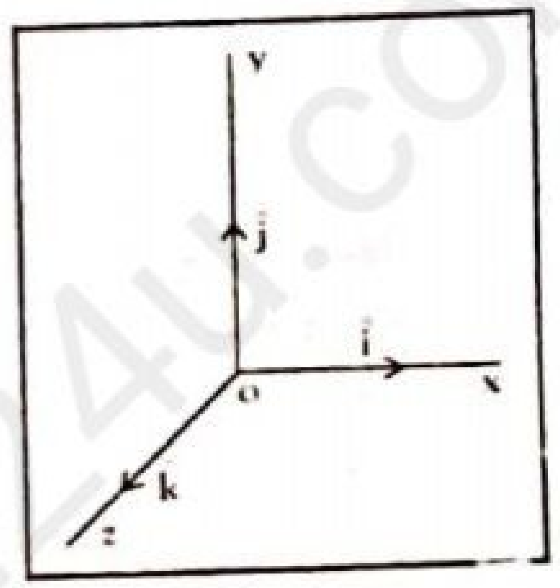
Q7: Define and explain unit vector?

ETEA 2009-2015

Ans. **Definition:** A vector whose magnitude is one is known as unit vector.

Explanation:

A unit vector is represented by an English alphabet having a cap upon it. For example, $\hat{a}, \hat{b}, \hat{c}$ etc represents unit vectors. A unit vector can be obtained by dividing a vector with its magnitude i.e.

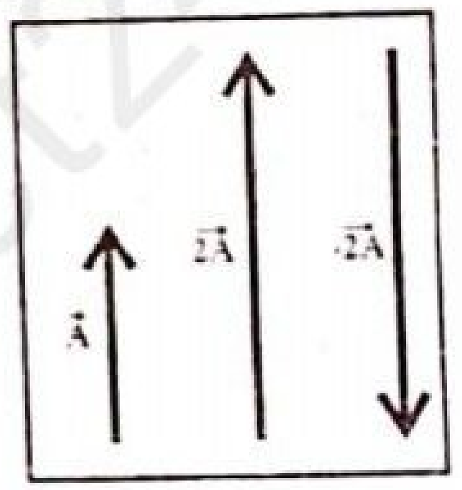


$$\boxed{\hat{a} = \frac{\vec{a}}{a}}$$

The unit vector is used for the specification of directions. In Cartesian coordinates, we take unit vector "$\hat{i}$" along x-axis, "$\hat{j}$" along y-axis and "$\hat{k}$" along z-axis.

Q8: Discuss the multiplication of a vector by a number?

Ans. We can multiply a vector by a number. If the number is positive, then the direction of given vector and product vector will remain the same. If the number is negative, then the given vector and product vector will be acting in opposite directions to each other, as shown in the figure.



Generally, if a vector "$\vec{A}$" is multiplied by a number "n", then its magnitude will be equal to "nA".

Q9: Discuss the addition of vectors by graphical method (or) geometrical method?

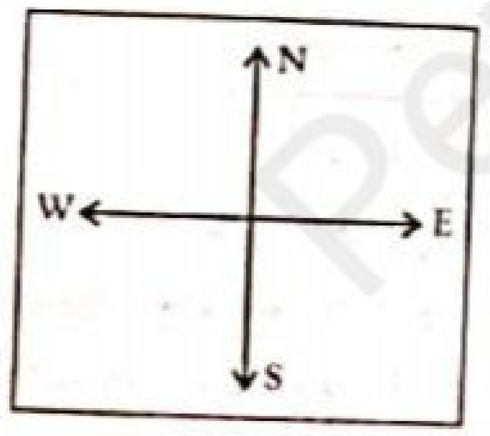
Ans. We can add two or more than two vectors by geometrical method or graphical method to get their resultant vector. We use "head-to-tail rule" for addition of vectors by graphical method or geometrical method.

Steps to add two Vectors by Head-to-Tail Rule:

The following steps must be followed to add vectors by head-to-tail rule:

1. Select a suitable scale for the representation of given vectors.
2. Draw the 1st vector according to the selected scale in the given direction.
3. Draw the 2nd vector according to the selected scale in the specified direction such that its tail touches the head of the first vector.
4. Now draw the 3rd vector according to the selected scale in the specified direction such that its tail touches the head of the second vector.
5. Continue the above process for given number of vectors.
6. For getting the resultant vector, combine the tail of the first vector to the head of the last vector. Represent the resultant vector by the symbol "$\vec{R}$".
7. Measure the length of "$\vec{R}$" which will given its magnitude.
8. To determine the direction of "$\vec{R}$", measure the angle of "$\vec{R}$" with respect to positive x-direction in anticlockwise sense.

Example: Consider two vectors $\vec{A}$ and $\vec{B}$ such that $\vec{A} = 25km$ and directed towards East. $\vec{B} = 20km$ and directed towards North. Draw their resultant vector and find its magnitude and direction?



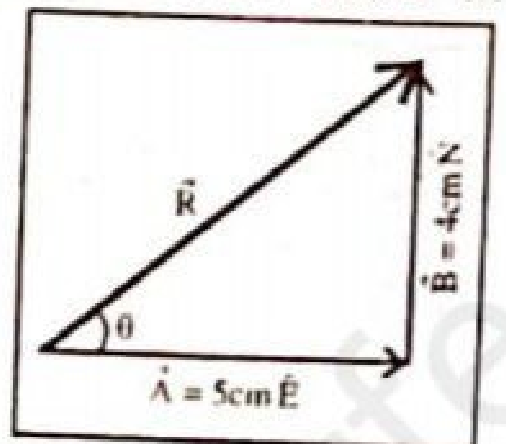
Solution: (i) First we select a suitable scale

Let $5km = 1cm$ then $25km = 5cm$ and $20km = 4cm$

(ii) Now we specify the directions as shown in the figure.

(iii) Now we draw vector "$\vec{A}$" according to the selected scale in the specified direction.

(iv) Now we draw vector "$\vec{B}$" according to the selected scale in the specified direction such that its tail touches the head of $\vec{A}$



(v) Now we combine the tail of $\vec{A}$ with the head of $\vec{B}$ which gives us the resultant vector "$\vec{R}$".

(vi) Now measure the length of "$\vec{R}$" with the help of meter rod which is about 6.4cm. This represents the magnitude of vector "$\vec{R}$", which is infact 32km according to the selected scale.

(vii) Now measure the angle made by vector "$\vec{R}$" with x-axis or East direction with the help of protractor which is about 38.6°. This gives the direction of resultant vector $\vec{R}$.

Q10: Discuss the addition of more than two vectors.

Ans. We can add any number of vectors by "head-to-tail rule" method.

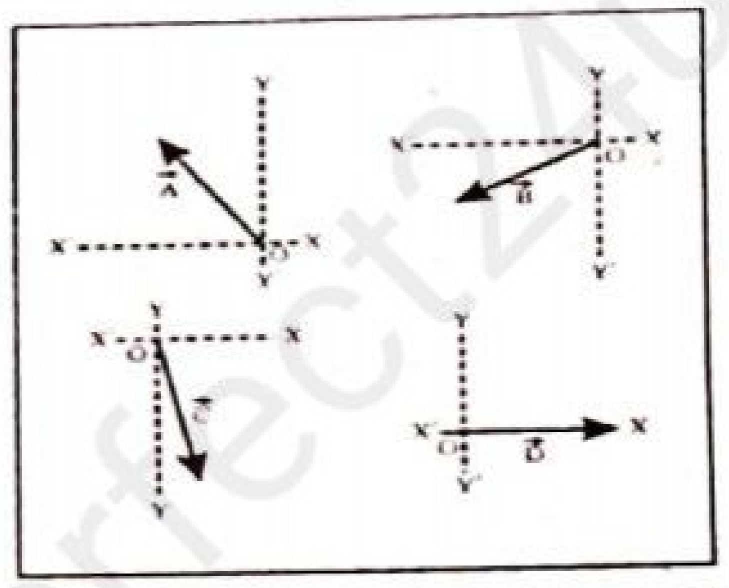
Example: Consider four vectors $\vec{A}, \vec{B}, \vec{C}$ and $\vec{D}$ in xy-plane as shown in the figure:

To add these vectors, we take the following steps:

Step-1: We draw x-axis and y-axis perpendicular to each other.

Step-2: We select a suitable scale.

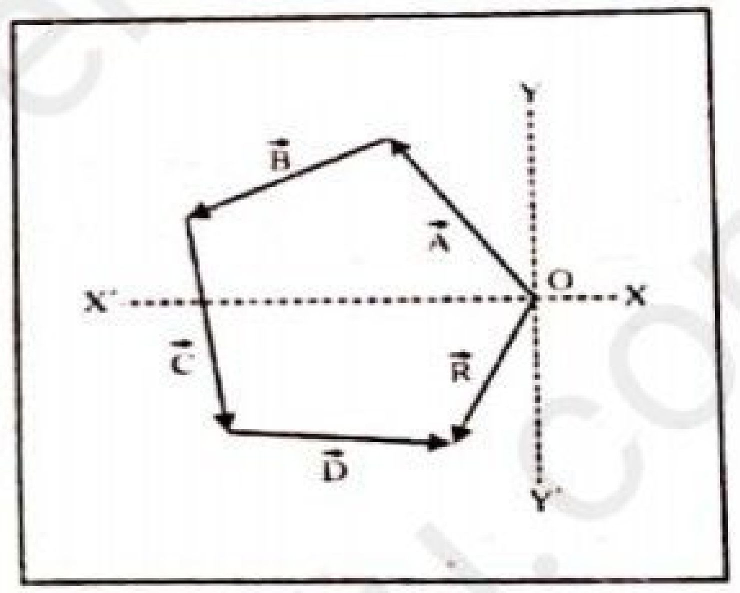
Step-3: We combine the tail of "$\vec{B}$" with the head of "$\vec{A}$" according to the specified direction.



Step-4: We combine the tail of "$\vec{C}$" with the head of "$\vec{B}$" according to the specified direction.

Step-5: We combine the tail of "$\vec{D}$" with the head of "$\vec{C}$" according to the specified direction.

Step-6: Now for finding the resultant, we combine the tail of 1st vector "$\vec{A}$" with the head of last vector "$\vec{D}$" as shown in the figure. Thus vector "$\vec{R}$" represents the resultant vector due to combination of four vectors.

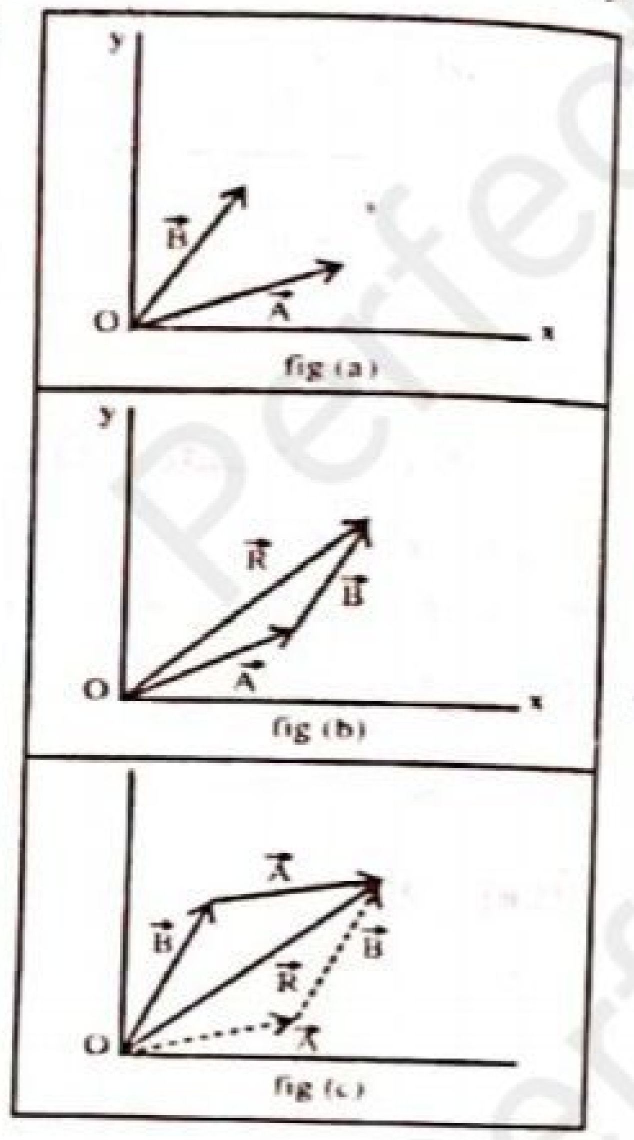


$$\boxed{\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}}$$

Using this method we can add any number of vectors.

Q11: Discuss the commutative property of addition of vectors. (OR) Show that, $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Ans. Consider two vectors $\vec{A}$ and $\vec{B}$ lying in xy-plane in such a way that both represents the adjacent sides of a parallelogram as shown in the figure (a).



Let we add vector $\vec{B}$ to vector $\vec{A}$ by head-to-tail rule and get their resultant vector $\vec{R}$ as shown in figure (b).

So we have,

$\vec{R} = \vec{A} + \vec{B}$ ______________ (1)

Now if we add vector $\vec{A}$ to vector $\vec{B}$ we get the same resultant vector $\vec{R}$ as shown in figure (c). So we have,

$\vec{R} = \vec{B} + \vec{A}$ ______________ (2)

$(\vec{A} + \vec{B})$ and $(\vec{B} + \vec{A})$ construct a parallelogram as shown in figure (c).

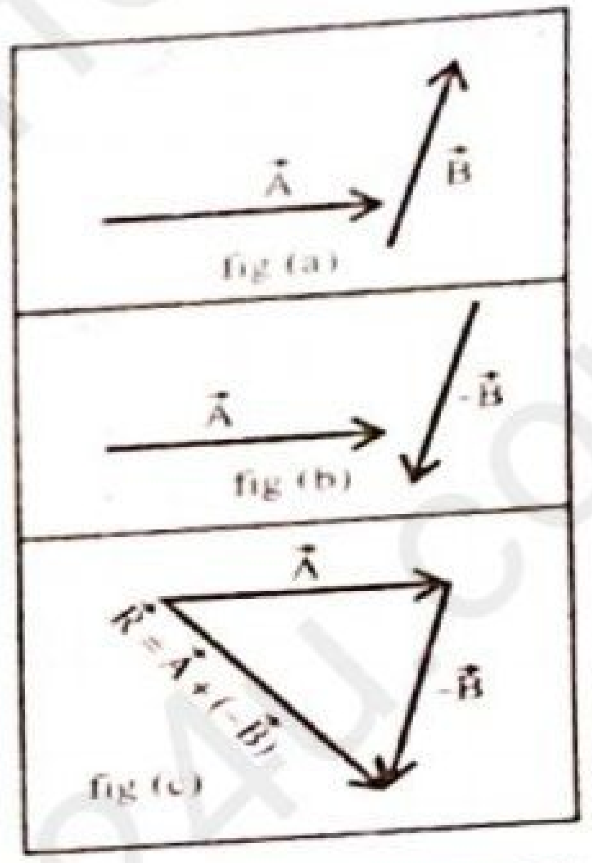
Now comparing equation (1) and equation (2), we get,

$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A}}$ ______________ (3)

Equation (3) shows that vectors obey the commutative property of addition.

Q12: Discuss the subtraction of vectors?

Ans. There is no direct method for determining the result of subtraction of two vectors. Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in figure (a).



Now if we want to subtract vector $\vec{B}$ from vector $\vec{A}$ then first we find the negative of vector $\vec{B}$ as shown in figure (b).

Now to subtract $\vec{B}$ from $\vec{A}$ we add the negative of $\vec{B}$ to $\vec{A}$ by head-to-tail rule as shown in figure (c). For getting their resultant, we combine the tail of $\vec{A}$ to the head of $(-\vec{B})$.

Thus $\vec{R} = \vec{A} + (-\vec{B})$ represents the resultant due to subtraction of two vectors.

Q13: Define and explain position vector?

ETEA 2007

Ans. **Position Vector:**

Definition:

The vector with the help of which we can locate the position of a point with respect to some fixed paint is known as position vector.

Explanation:

In the given figure, $\vec{OP} = \vec{r}$ represents the position vector. In two dimensions, the position vector can be expressed as,

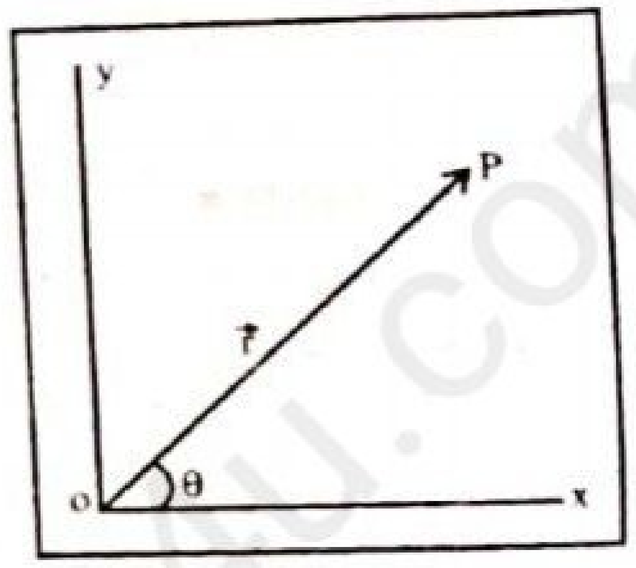
$$\boxed{\vec{r} = x\hat{i} + y\hat{j}}$$

In three dimensions, it can be expressed as,

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

Q14: Define and explain the resolution of a vector? (ETEA 2005,2006)

Ans. **Definition:** The splitting of a vector into its components is known as resolution of a vector.

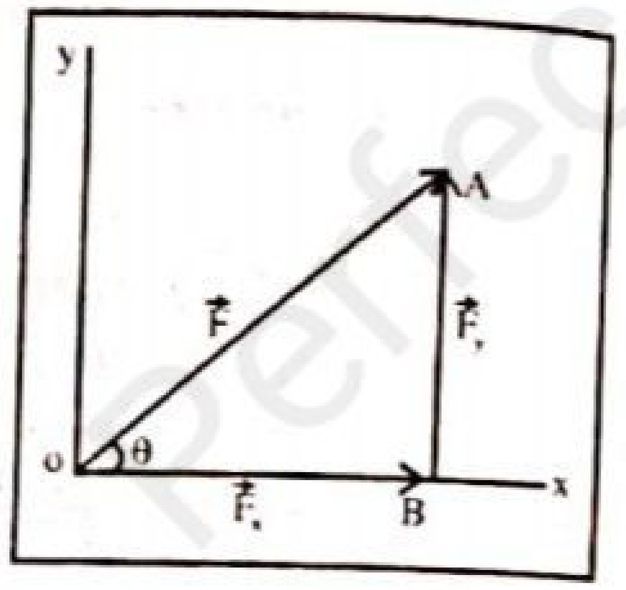


Explanation:

Consider a vector $\vec{F}$ lying in xy-plane by making an angle "θ" with x-axis, as shown in the figure.

In order to resolve this vector into its components, we draw a perpendicular "AB" from the head of vector $\vec{F}$ on x-axis.

Thus $\vec{OB}=\vec{F}_x$ represents the x-component or horizontal component of vector $\vec{F}$ while $\vec{BA}=\vec{F}_y$ represents the y-component or vertical component of vector $\vec{F}$. Now in ΔAOB, we have,



$$Cos\theta = \frac{OB}{OA} \Rightarrow Cos\theta = \frac{F_x}{F}$$

$$\Rightarrow \boxed{F_x = FCos\theta} \text{ ——— (1)}$$

Similarly, in ΔAOB, we have,

$$Sin\theta = \frac{AB}{OA} \Rightarrow Sin\theta = \frac{F_y}{F}$$

$$\Rightarrow \boxed{F_y = F\sin\theta} \text{ ——— (2)}$$

Equation (1) represents the magnitude of horizontal component of vector $\vec{F}$. While equation (2) represents the magnitude of vertical component of vector $\vec{F}$.

Magnitude of Vector $\vec{F}$:

Applying pathagorem theorem to ΔAOB, we get,

$$(Hypotenuse)^2=(Base)^2+(Perpendicular)^2$$

$$\Rightarrow (OA)^2 = (OB)^2 + (BA)^2$$

$$\Rightarrow F^2 = F_x^2 + F_y^2$$

$$\Rightarrow \boxed{F = \sqrt{F_x^2 + F_y^2}} \text{ ——— (3)}$$

Equation (3) represents the magnitude of vector $\vec{F}$

Direction of Vector $\vec{F}$: (ETEA 2005)

We know that,

$$\tan\theta = \frac{perpendicular}{base}$$

$$\Rightarrow \tan\theta = \frac{AB}{OB}$$

$$\Rightarrow \tan\theta = \frac{F_y}{F_x} \Rightarrow \boxed{\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)} \text{ ——— (4)}$$

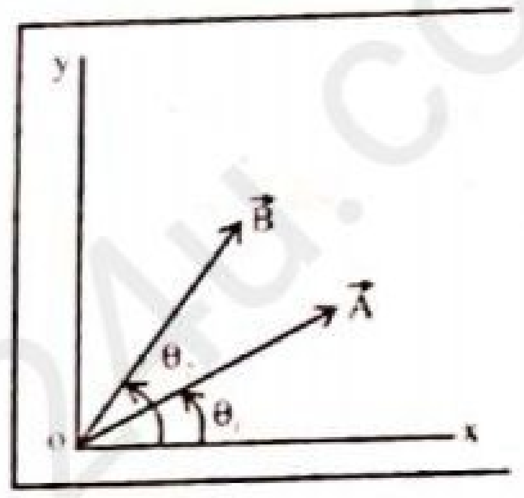
> In three dimension the magnitude of $\vec{F}$ is $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ (ETEA 2010)

Equation (4) represents the direction of vector $\vec{F}$.

Q15: Discuss the addition of vectors by rectangular components method. (OR) Discuss the addition of vectors by analytical method? (BISE Mardan 2019, BISE Kohat 2019, BISE DI Khan 2019)

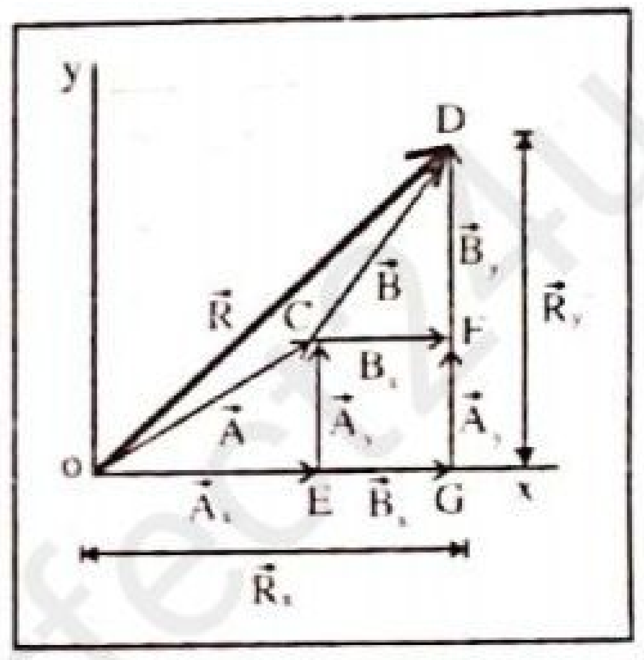
Ans. **Definition:** The rectangular components of a vector are those components which are making an angle of 90° with each other.

Explanation:



The analytical method gives us more accurate result as compare to geometrical method. It is because, the analytical method is more mathematical method than geometrical method. The result obtained by geometrical method [head-to-tail rule method] is always approximately correct, because drawing sketches can never be perfect.

Here we will consider two vectors and will find an expression for their addition. Then we will generalize the expression for addition of "n" number of vectors.



Now let us consider two vectors $\vec{A}$ and $\vec{B}$ making an angle 'θ_1' and 'θ_2' with x-axis respectively, as shown in the figure.

Now we add $\vec{B}$ to $\vec{A}$ by geometrical method. For this purpose, we combine the tail of $\vec{B}$ with the head of $\vec{A}$ as shown in the figure. Now to get their resultant, we combine the tail of $\vec{A}$ with the head of $\vec{B}$ as shown in the figure. Thus $\vec{OD}=\vec{R}$ represents the resultant of $\vec{A}$ and $\vec{B}$ so,

$$\boxed{\vec{R} = \vec{A}+\vec{B}} \text{ ——— (1)}$$

Now for finding the rectangular components, we resolve $\vec{A}$ and $\vec{B}$ into their horizontal and vertical components. For this purpose we draw perpendiculars from "C" and "D" on x-axis as shown in the figure. We also draw a perpendicular from "C" on "DG" which cuts "DG" at point "F" as shown in the figure.

Now we see that, $\vec{OE}=\vec{A}_x$ and $\vec{CE}=\vec{A}_y$ represent the horizontal and vertical components of $\vec{A}$ respectively. While $\vec{CE} = \vec{B}_x$ and $\vec{FD} = \vec{B}_y$ represents the horizontal and vertical components of $\vec{B}$ respectively.

Now from the figure, we have, $\left[\vec{CF} = \vec{EG} = \vec{B}_x \text{ and } \vec{CE} = \vec{FG} = \vec{A}_y\right]$

$$\vec{R}_x = \vec{A}_x + \vec{B}_x$$

$$\Rightarrow R_x\hat{i} = A_x\hat{i} + B_x\hat{i} \Rightarrow \boxed{R_x = A_x + B_x} \text{ ——— } (2)$$

Equation (2) represents the magnitude of x-component of $\vec{R}$.

Similarly, $\vec{R}_y = \vec{A}_y + \vec{B}_y$

$$\Rightarrow R_y\hat{j} = A_y\hat{j} + B_y\hat{j} \Rightarrow \boxed{R_y = A_y + B_y} \text{ ——— } (3)$$

Equation (3) represents the magnitude of y-component of $\vec{R}$.

Now for any number of co-planar vectors $(\vec{A}, \vec{B}, \vec{C}, \vec{D} \cdots)$ equation (2) and equation (3) can be written as,

$$\boxed{R_y = A_y + B_y + C_y + D_y + \cdots} \text{ ——— } (4)$$

Similarly,

$$\boxed{R_y = A_y + B_y + C_y + D_y + \cdots} \text{ ——— } (5)$$

Magnitude of $\vec{R}$:

The magnitude of $\vec{R}$ is given by,

$$R = \sqrt{(R_x^2 + R_y^2)} \text{ ——— } (6)$$

Putting equation (4) and equation (5) in equation (6), we get,

$$\boxed{R = \sqrt{(A_x + B_x + C_x + D_x + \cdots)^2 + (A_y + B_y + C_y + D_y + \cdots)^2}} \text{ ——— } (7)$$

Equation (7) represents the magnitude of $\vec{R}$

Direction of $\vec{R}$:

We know that,

$$\tan\theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \text{ ——— } (8)$$

Putting equations (4), (5) in equation (8), we get,

$$\boxed{\theta = \tan^{-1}\left[\frac{A_y + B_y + C_y + D_y + \cdots}{A_x + B_x + C_x + D_x + \cdots}\right]} \text{ ——— } (9)$$

Equation (9) represents the direction of vector $\vec{R}$

Steps for Vector Addition by Rectangular Components Method:

1. Draw the individual vectors being summed and the axis being used.
2. Find the x-components [horizontal component] and y-components [vertical component] of all the given vectors.
3. Add the x-components of all the given vectors to get the x-component "R_x" of the resultant vector $\vec{R}$.
4. Add the y-components of all the given vectors to get the y-component "R_y" of the resultant vector $\vec{R}$.
5. Find the magnitude of the resultant vector $\vec{R}$ i.e.
   $R = \sqrt{R_x^2 + R_y^2}$
6. Find the direction of the resultant vector $\vec{R}$ i.e.
   $\theta = \tan^{-1}\left[\frac{R_y}{R_x}\right]$

Where θ represents the angle which is made by the resultant vector with positive x-axis in anti-clockwise sense.

Procedure for determining "θ":

Let "ϕ" be the angle made by the vector $\vec{R}$ in any quadrant and θ be the angle made by the vector $\vec{R}$ with positive x-axis in anti-clockwise sense. We can determine the value of θ by using the following procedure:

1. Find the value of $\tan^{-1}\left(\frac{R_y}{R_x}\right)$ by using calculator and call this angle as "ϕ".
2. If "R_x" and "R_y" both are positive, then the resultant "$\vec{R}$" will lies in 1st quadrant as shown in the figure (a). In such a case, we have
   $\boxed{\theta = \phi}$

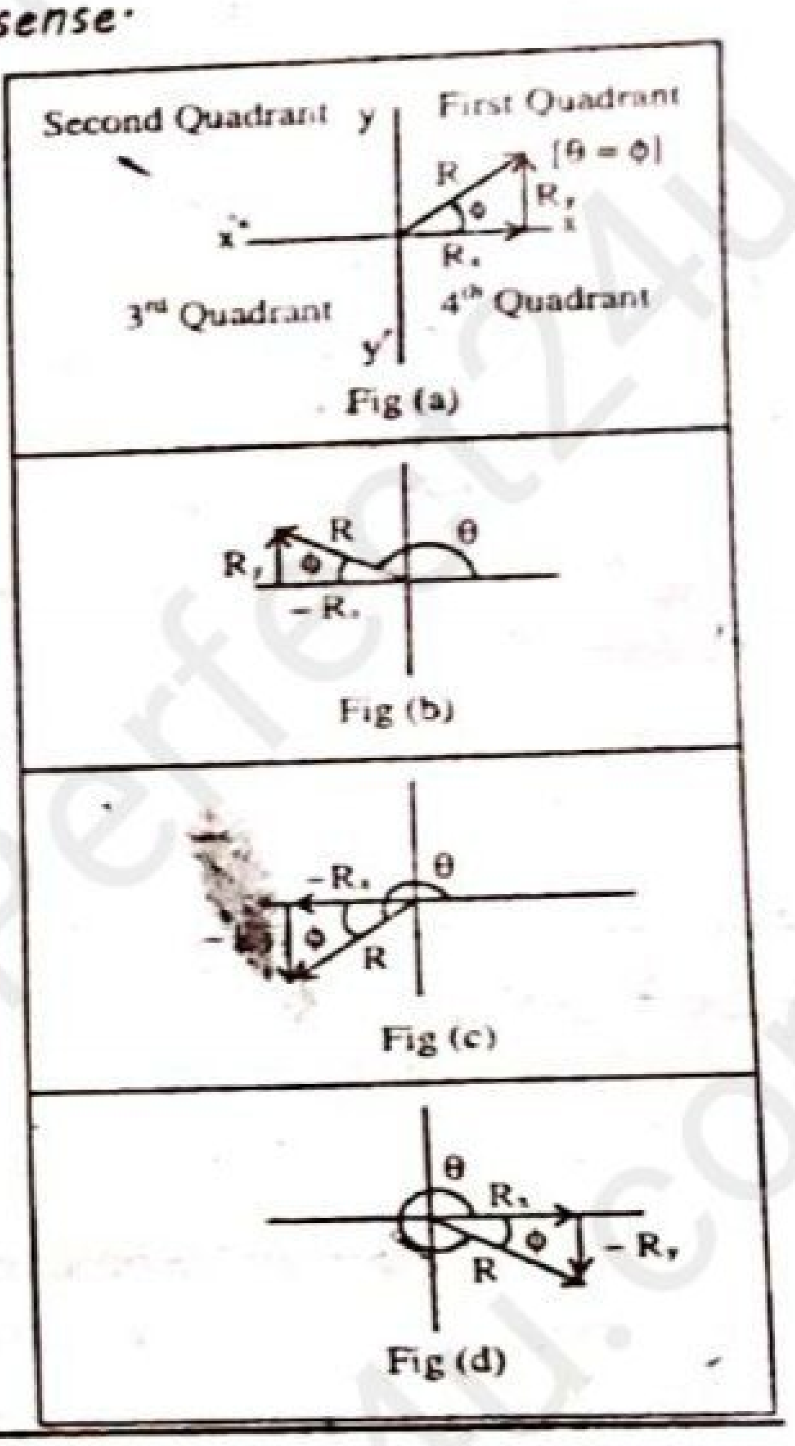


Fig (a)
Fig (b)
Fig (c)
Fig (d)

3. If "R_x" is negative and "R_y" is positive, then "$\vec{R}$" will lies in 2^{nd} quadrant, as shown in figure (b). In such a case, θ can be calculated as,

$\boxed{\theta = 180^\circ - \phi}$

4. If "R_x" and "R_y" both are negative, then "$\vec{R}$" will lie in 3^{rd} quadrant, as shown in figure (c). In such a case, θ can be calculated as,

$\boxed{\theta = 180^\circ + \phi}$

5. If "R_x" is positive and "R_y" is negative, then "$\vec{R}$" will lie in the 4^{th} quadrant, as shown in figure (d). In such a case, θ can be calculated as,

$\boxed{\theta = 360^\circ - \phi}$

Q16: Define and explain scalar product (or) dot product?

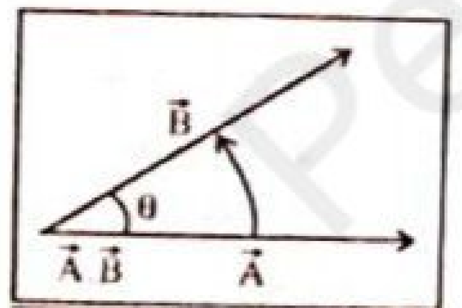
(ETEA 2007-2017, BISE DI Khan 2019)

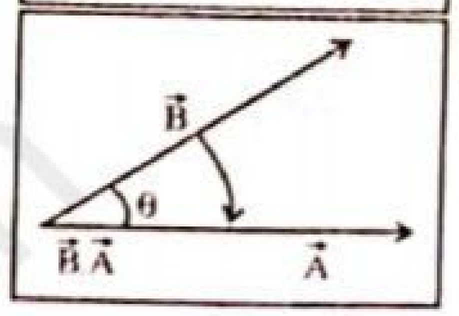
Ans. **Scalar Product:**

Definition: If we multiply two vectors and we get a scalar quantity, then such product of two vectors is known as scalar product or dot product.

Explanation:

The scalar product of two vectors is equal to the product of the magnitude of the two vectors and the cosine of the angle between them.





Let $\vec{A}$ and $\vec{B}$ represents the two vectors, then we have,

$\vec{A}.\vec{B} = AB\cos\theta$ ________ (1)

Similarly,

$\vec{B}.\vec{A} = BA\cos\theta$ $\quad [AB = BA]$

$\Rightarrow \vec{B}.\vec{A} = AB\cos\theta$ ________ (2)

Comparing equation (1) and equation (2), we get,

$\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$ ________ (3)

Equation (3) shows that the scalar product (or) dot product obeys commutative law. Now in case of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ we have,

$\hat{i}\cdot\hat{i} = ii\cos\theta$ $\quad$ [Magnitude of unit vector = i = 1]

$\Rightarrow \hat{i}\cdot\hat{i} = 1\times 1\times\cos 0^\circ$

$\Rightarrow \hat{i}\cdot\hat{i} = 1\times 1\times 1$ $\quad [\cos 0^\circ = 1]$

$\Rightarrow \boxed{\hat{i}\cdot\hat{i} = 1}$

Similarly,

$\boxed{\hat{j}\cdot\hat{j} = 1}$ and $\boxed{\hat{k}\cdot\hat{k} = 1}$

Now $\hat{i}\cdot\hat{j} = ij\cos\theta$

$\Rightarrow \hat{i}\cdot\hat{j} = 1\times 1\times\cos 90^\circ$ $\quad [\cos 90^\circ = 0]$

$\Rightarrow \hat{i}\cdot\hat{j} = 1\times 1\times 0 = 0$

$\Rightarrow \boxed{\hat{i}\cdot\hat{j} = 0}$

Similarly,

$\boxed{\hat{j}\cdot\hat{i} = 0,\ \hat{j}\cdot\hat{k} = 0,\ \hat{k}\cdot\hat{j} = 0,\ \hat{k}\cdot\hat{i} = 0,\ \hat{i}\cdot\hat{k} = 0}$

Example: If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then find $\vec{A}\cdot\vec{B} = ?$

Solution:

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}, \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Now $\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$$\Rightarrow \vec{A}\cdot\vec{B} = \begin{bmatrix} A_xB_x(\hat{i}\cdot\hat{i}) + A_xB_y(\hat{i}\cdot\hat{j}) + A_xB_z(\hat{i}\cdot\hat{k}) + \\ A_yB_x(\hat{j}\cdot\hat{i}) + A_yB_y(\hat{j}\cdot\hat{j}) + A_yB_z(\hat{j}\cdot\hat{k}) + \\ A_zB_x(\hat{k}\cdot\hat{i}) + A_zB_y(\hat{k}\cdot\hat{j}) + A_zB_z(\hat{k}\cdot\hat{k}) \end{bmatrix}$$

$$\Rightarrow \vec{A}\cdot\vec{B} = \begin{bmatrix} A_xB_x\times 1 + A_xB_y\times 0 + A_xB_z\times 0 + \\ A_yB_x\times 0 + A_yB_y\times 1 + A_yB_z\times 0 + \\ A_zB_x\times 0 + A_zB_y\times 0 + A_zB_z\times 1 \end{bmatrix}$$

$\Rightarrow \vec{A}\cdot\vec{B} = A_xB_x = 0 + 0 + 0 + A_yB_y + 0 + 0 + 0 + A_zB_z$

$\Rightarrow \boxed{\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = A_zB_z}$ ________ (4)

The R.H.S of equation (4) represents a scalar quantity, so such product of two vector $\vec{A}$ and $\vec{B}$ is known as scalar product (or) dot product.

Characteristics of Scalar Product:

1. The scalar product obeys the commutative law i.e.

$\boxed{\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}}$

2. The scalar product obeys the condition of perpendicularity i.e.

$\vec{A}.\vec{B} = AB\cos\theta = AB\cos 90^\circ = AB \times 0 = 0 \Rightarrow \boxed{\vec{A}\cdot\vec{B} = 0}$

3. The scalar product obeys the condition of parallelism i.e.

$\vec{A}\,\vec{B} = AB\cos 0^\circ AB \times 1 \Rightarrow \boxed{\vec{A}\cdot\vec{B} = AB}$

4. The scalar product obeys the condition of anti parallelism i.e.

$\vec{A}.\vec{B} = AB\cos\theta = AB\cos 180^\circ = AB \times (-1) \Rightarrow \boxed{\vec{A}\cdot\vec{B} = -AB}$

5. If $\vec{A}.\vec{B} = 0$ and $\theta \neq 90^\circ$, then either $A = 0$ or $B = 0$.

6. If $\vec{A}.\vec{B} = 0$ and $\theta \neq 90^\circ$, also $A \neq 0$ then $\vec{B}$ must be a null vector i.e. $B = 0$.

7. The scalar product obeys distributive law i.e.

$\boxed{\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B}+\vec{A}\cdot\vec{C}}$

Examples of Scalar Product:

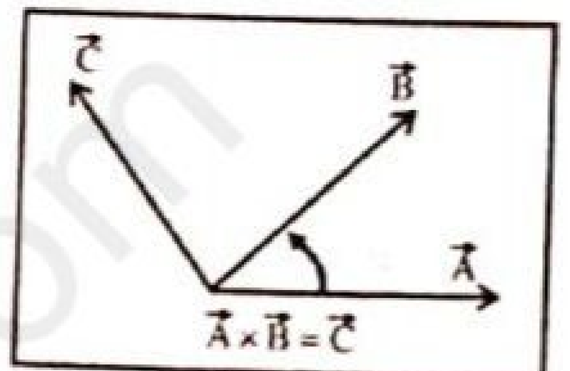
1. Workdone $= w = \vec{F}\cdot\vec{d} \Rightarrow \boxed{w = \vec{F}\cdot\vec{d}}$
2. Electric flux $= \phi_e = \vec{E}\cdot\vec{A} \Rightarrow \boxed{\phi_e = \vec{E}\cdot\vec{A}}$
3. Power $= P = \vec{F}\cdot\vec{v} \Rightarrow \boxed{P = \vec{F}\cdot\vec{v}}$
4. Magnetic flux $= \phi_m = \vec{B}\cdot\vec{A} \Rightarrow \boxed{\phi_m = \vec{B}\cdot\vec{A}}$

Q17: Define and explain vector product (or) cross product?

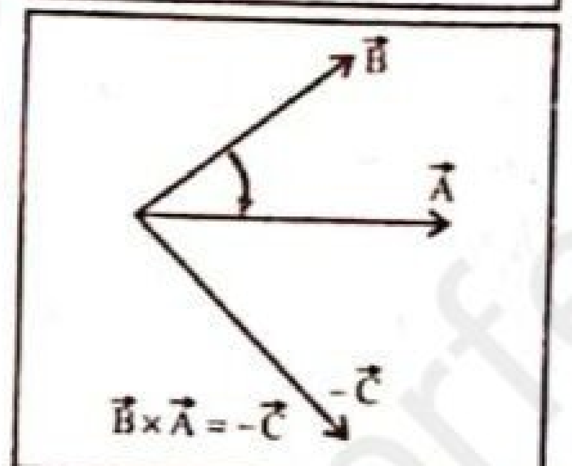
ETEA 2006-2008, BISE Malakand 2019, BISE Peshawar 2019

Ans. **Vector Product:**

Definition: If we multiply two vectors and we get a vector quantity, then such product of two vectors is known as vector product (or) cross product.



Explanation: The vector product of two vectors is equal to the product of the magnitude of the two vectors and the sine of the angle between them. Let $\vec{A}$ and $\vec{B}$ represents the two vectors, then $(\vec{A}\times\vec{B})$ gives us another vector $\vec{C}$ which is perpendicular to both $\vec{A}$ and $\vec{B}$ i.e.



$\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$ _____ (1)

(OR) $\vec{A}\times\vec{B} = \vec{C}$ _____ (2)

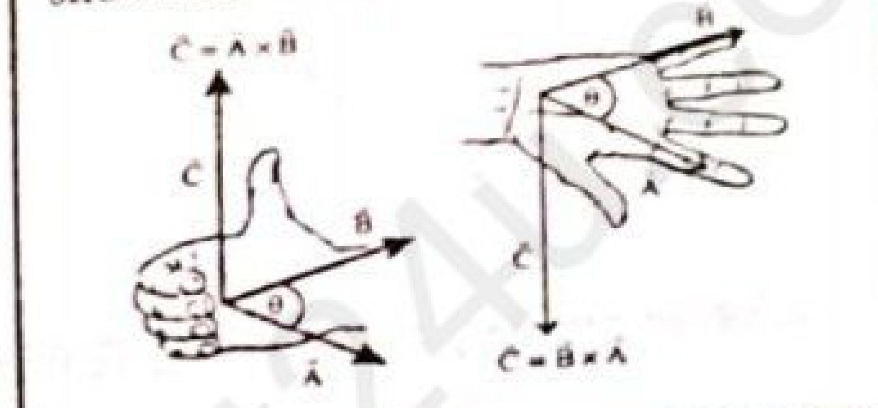
But $\vec{B}\times\vec{A} = -\vec{C}$ _____ (3)

Comparing equation (2) and equation (3), we get,

$\boxed{\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}}$ _____ (4)

For your information:

You can determine the direction of vector "$\vec{C}$" by Right Hand Rule as shown in the figure.



Equation (4) shows that the vector product do not obeys the commutative law. Now in case of unit vectors $\hat{i}, \hat{j}, \hat{k}$ we have,

$\hat{i}\times\hat{i} = ii\sin\theta\hat{n}$

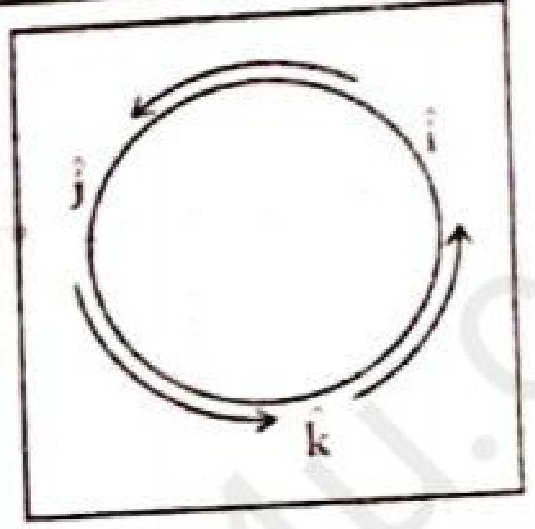
$\Rightarrow \hat{i}\times\hat{i} = ii\sin\theta\hat{n}$

$= 1\times 1\times 0\times\hat{n}$

$\Rightarrow \boxed{\hat{i}\times\hat{i} = 0}$

Similarly,

$\boxed{\hat{j}\times\hat{j} = 0,\ \hat{k}\times\hat{k} = 0}$



Now $\hat{i}\times\hat{j} = ij\sin\theta\hat{k}$

$= ij\sin 90^\circ(\hat{k}) = 1\times1\times1(\hat{k})$

$\Rightarrow \boxed{\hat{i}\times\hat{j} = \hat{k}}$

Similarly,

$\boxed{\hat{j}\times\hat{i} = -\hat{k},\ \hat{j}\times\hat{k} = \hat{i},\ \hat{k}\times\hat{j} = -\hat{i},\ \hat{k}\times\hat{i} = \hat{j},\ \hat{i}\times\hat{k} = -\hat{j}}$

Example: If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then find $(\vec{A}\times\vec{B})$?

Solution: $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k},\ \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

Now $\vec{A}\times\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\times\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right)$

$$\Rightarrow \vec{A}\times\vec{B} = \begin{bmatrix} A_xB_x(\hat{i}\times\hat{i}) + A_xB_y(\hat{i}\times\hat{j}) + A_xB_z(\hat{i}\times\hat{k}) + \\ A_yB_x(\hat{j}\times\hat{i}) + A_yB_z(\hat{j}\times\hat{k}) + A_yB_z(\hat{j}\times\hat{k}) + \\ A_zB_x(\hat{k}\times\hat{i}) + A_zB_y(\hat{k}\times\hat{j}) + A_zB_z(\hat{k}\times\hat{k}) \end{bmatrix}$$

$$\Rightarrow \vec{A}\times\vec{B} = \begin{bmatrix} A_xB_x\times 0 + A_xB_y\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{j} + A_yB_y\times 0 \\ A_yB_z\hat{i} + A_zB_x\hat{j} - A_zB_y\hat{i} + A_zB_z\times 0 \end{bmatrix}$$

$$\Rightarrow \vec{A}\times\vec{B} = \begin{bmatrix} 0 + A_xB_y\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{k} + 0 + A_yB_z\hat{i} + \\ A_zB_x\hat{j} - A_zB_y\hat{i} + 0 \end{bmatrix}$$

$$\Rightarrow \vec{A}\times\vec{B} = A_xB_y\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{k} + A_yB_z\hat{i} + A_zB_x\hat{j} - A_zB_y\hat{i}$$

$$\Rightarrow \vec{A}\times\vec{B} = A_yB_z\hat{i} - A_zB_y\hat{i} + A_zB_x\hat{j} - A_xB_z\hat{j} + A_xB_y\hat{k} - A_yB_x\hat{k}$$

$$\Rightarrow \boxed{\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}} \quad \text{\_\_\_\_(5)}$$

The R·H·S of equation (5) represents a vector quantity. So such product of the two vectors is known as vector product (or) cross product.

Characteristics of Vector Product:

1. The vector product do not obeys the commutative law i·e·
   $\vec{A}\times B \neq \vec{B}\times\vec{A}$
2. The vector product obeys the condition of parallelism i·e·
   $\vec{A}\times B = AB\sin\theta\hat{n} = AB\sin 0^\circ\hat{n}$ $\quad [\sin 0^\circ = 0]$
   $\Rightarrow \vec{A}\times\vec{B} = 0$
3. If $\vec{A}$ and $\vec{B}$ are anti-parallel, then we put $\theta = 180^\circ$, so we get,
   $\vec{A}\times\vec{B} = AB\sin\theta\hat{n} = AB\sin\theta\hat{n} = AB\sin 180^\circ\hat{n}$ $\quad [\sin 180^\circ = 0]$
   $\Rightarrow \vec{A}\times\vec{B} = 0$
4. If $\vec{A}\times\vec{B} = 0$ and $A \neq 0, \theta \neq 0^\circ$ or $\theta \neq 180^\circ$, then $\vec{B}$ must be a null vector.
5. The vector product obeys the distributive law i·e·
   $\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B}) + (\vec{A}\times\vec{C})$

Examples of Vector Product:

1. Angular momentum $= \vec{L} = \vec{r}\times\vec{p} \Rightarrow \vec{L} = \vec{r}\times\vec{p}$
2. Linear velocity in terms of angular velocity i·e· $\vec{v} = \vec{\omega}\times\vec{r}$
3. Torque $= \vec{r}\times\vec{F} \Rightarrow \vec{\tau} = \vec{r}\times\vec{F}$
4. Magnetic force $= q(\vec{v}\times\vec{B}) \Rightarrow \vec{F}_m = q(\vec{v}\times\vec{B})$

Physical Significance of Vector Product:

1. Using vector product, we can determine the area of a parallelogram.

2. Using vector product, we can determine the torque or moment of force i·e· $\vec{\tau} = \vec{r}\times\vec{F}$.
3. Using vector product, we can determine the angular momentum of a body during angular motion i·e· $\vec{L} = \vec{r}\times\vec{p}$.

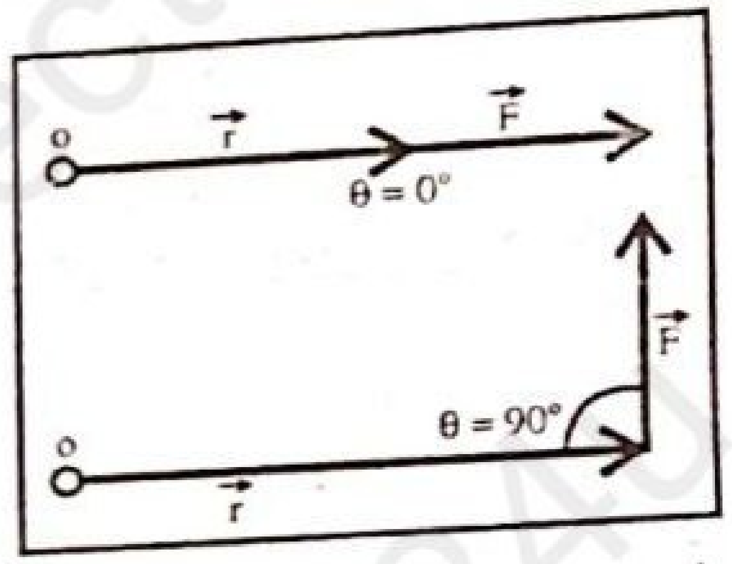
Q18: Define and explain torque (or) moment of force.

Ans· Torque (or) Moment of Force:

Definition: The turning effect produced in a body about a fixed point due to an applied force is known as torque (or) moment of force.

Explanation:

The torque is an agent which produces the angular acceleration in a body. It depends upon the moment arm $\vec{r}$ and applied force $\vec{F}$. The moment arm represents the perpendicular distance between the line of action of force and the point of rotation. Mathematically, we have,



Torque = Moment arm x effective force

$\Rightarrow \vec{\tau} = \vec{r}\times\vec{F}$

$\Rightarrow \boxed{\vec{\tau} = rF\sin\theta\hat{n}}$ ____(1)

Equation (1) represents the vector form of torque. The magnitude of torque is given by,

$\boxed{\tau = rF\sin\theta}$ ____(2)

Equation (2) shows that for given values of "r" and "F", the torque also depends upon "θ".

(i) If $\theta = 0^\circ$ i·e· $\vec{r}$ and $\vec{F}$ are parallel to each other, then,

$\tau = rF\sin 0^\circ$

$\Rightarrow \tau = rF\times 0$ $\quad [\sin 0^\circ = 0]$

$\Rightarrow \boxed{\tau = 0}$ ____(3)

Equation (3) shows that, there will be no torque, if $\vec{r}$ and $\vec{F}$ are parallel to each other.

(ii) If $\theta = 90^\circ$ i·e· $\vec{r}$ and $\vec{F}$ are perpendicular to each other, then equation (2)

$\Rightarrow \tau = rF\sin 90^\circ$ $\quad [\sin 90^\circ = 1]$

$\Rightarrow \tau = rF \times 1$

$\Rightarrow \boxed{\tau_{max} = rF}$ ________ (4)

Equation (4) shows that torque will be maximum if $\vec{r}$ and $\vec{F}$ are perpendicular to each other. The torque is a vector quantity. Its unit is N·m. Clockwise torque is taken as negative while anti-clockwise torque is taken as positive.

Dimension of Torque: (ETEA 2006-2008)

As torque $= \tau = rF$

$\Rightarrow \tau = rma$ $\quad [F = ma]$

$\Rightarrow \tau = m \times kg = m / sec^2$

$\Rightarrow \tau =$ length x mass x length/(time)^2

$\Rightarrow \boxed{\tau = [L^2MT^{-2}]}$

> Torque is perpendicular to the plane of $\vec{r}$ and $\vec{F}$. (ETEA 2009)

Q19: Discuss the application of torque?

Ans.

1. When we apply equal and opposite forces on the steering wheel of a vehicle with both hands, torque is produced and steering wheel can be rotated easily.
2. When we apply forces on the paddles of a cycle by means of our feet, then these forces acts as couple forces. As a result the wheels of cycle begins to rotate and cycle moves in the forward direction.
3. Similarly in an electrical instrument such as galvanometer, ammeter and voltmeter torque is exerted on the rectangular coil by means of electric current flowing through it. Under the action of this torque the coil rotates and comes to rest where the torque is balanced by opposing torque. In this way needle move on the calibrated scale.
4. Applying torque we can open a tightly closed bottle having threaded closures on the neck of the bottle.
5. A cross bar wrench is used to change a tyre of a car. A certain force is being exerted on each handle of the wrench in opposite direction. These forces constitute a couple due to which the

net torque is equal to the product of any one force and the couple arm.

$\tau = FL$ ________ (1)

Where "L" is called a couple arm.

6. Similarly applying torque on a pulley by means of a rope we can take water from a well in daily life.

Q20: Define equilibrium. Explain with examples?

ETEA 2012

Ans. **Equilibrium:**

Definition: The state of a body, under the action of several forces acting together and there is no change in the translational motion as well as its rotational motion is known as equilibrium. (OR)

A body is said to be in state of equilibrium, if it is at rest or moving with uniform velocity.

Explanation:

There are two types of equilibrium which are given below:

1. Static Equilibrium:

When a body is at rest then its acceleration will be zero and it will be in state of static equilibrium.

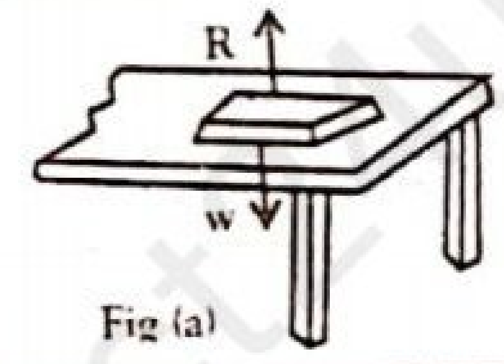


Fig (a)

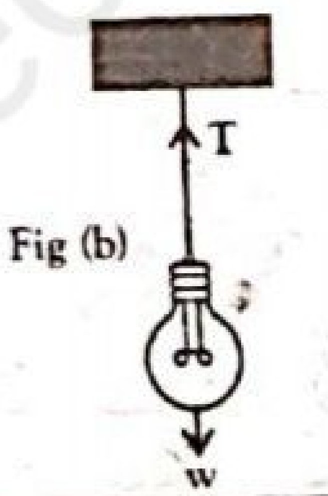


Fig (b)

Examples of Static Equilibrium:

A book lying on the table is the example of static equilibrium. In such a case, the weight of the book "w" is acting in downwards direction while the normal reaction "R" of the table on the book is acting in upwards direction, as shown in figure (a). Both forces balance each other and as a result, the book remains in state of static equilibrium.

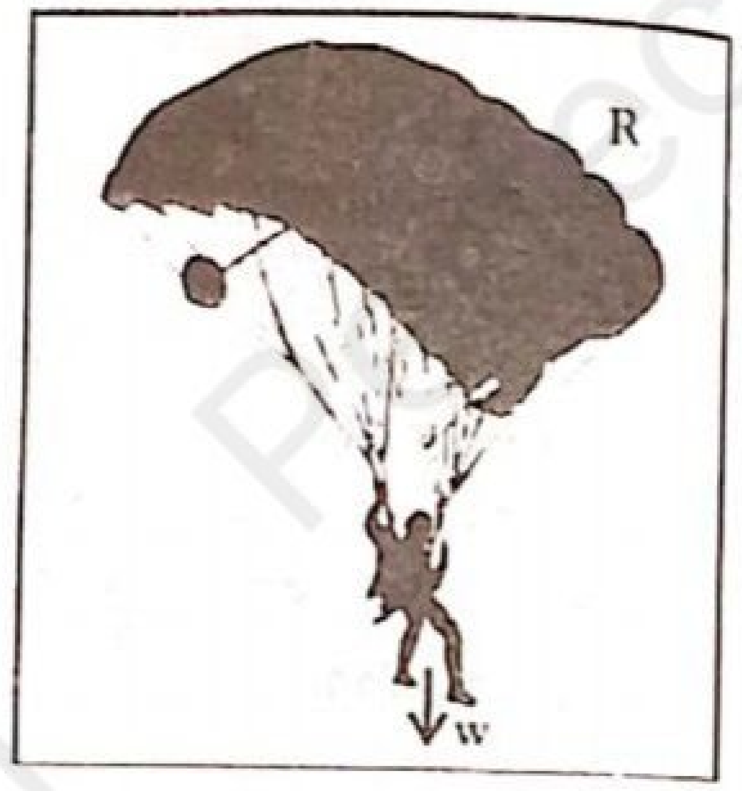
A bulb hanging from a rigid support by means of an inextensible thread is another example of static equilibrium. In this case, the weight of the bulb "w" is acting in downwards direction while the tension [reaction] "T" in the string [thread] is acting in upwards direction. Both forces balance each other and as a result the bulb remains in state of static equilibrium.

2. Dynamic Equilibrium:

When a body is moving with uniform velocity, then the body will have no acceleration and the body will be in state of dynamic equilibrium.

Types of Dynamic Equilibrium:

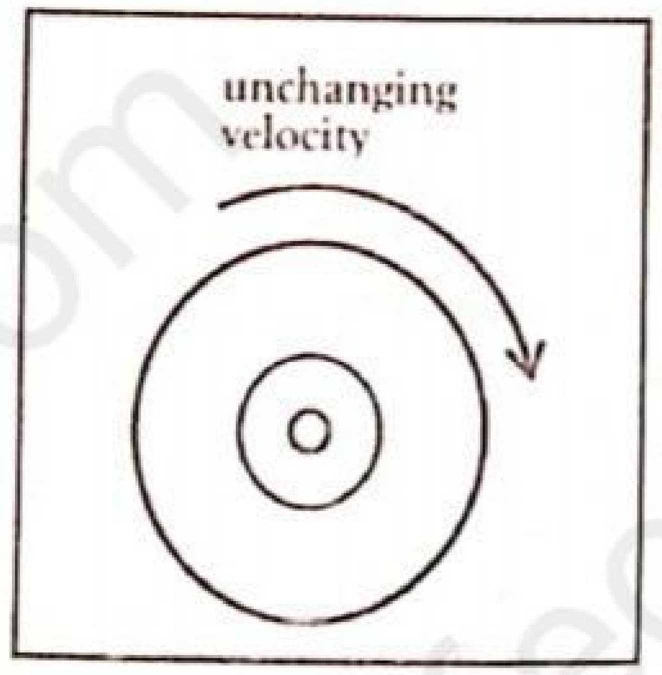
1. Dynamic Translational Equilibrium: When a body is moving uniform linear velocity then the body is said to be in state of dynamic translational equilibrium.



Example: A paratrooper falling down with constant velocity is the example of translational dynamic translational equilibrium.

When a paratrooper jumps from an aeroplane in air, its weight "w" is acting in downwards direction while the reaction "R" of air is acting in upwards direction. Both forces balance each other and as a result the paratrooper moves downwards with uniform velocity. Such velocity is known as terminal velocity of paratrooper. In this way, the paratrooper remains in state of translational dynamic equilibrium.

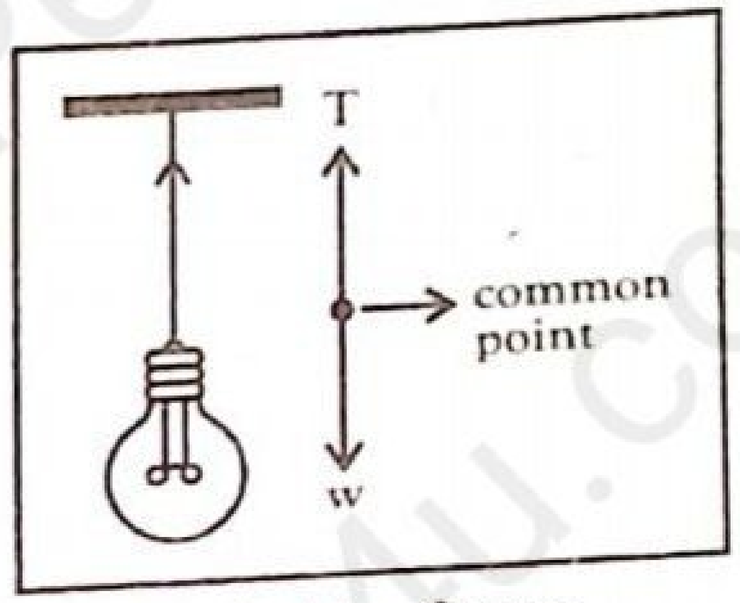
2. Dynamic Rational Equilibrium: When a body is moving with uniform angular velocity, then the body is said to be in state of dynamic rotational equilibrium.



Examples: A compact disk [CD] rotating in CD-player with constant angular velocity is the example of dynamic rotational equilibrium. A rotating wheel and grinding rotating about its axis with constant angular speed are also the examples of dynamic rotational equilibrium.

Q21: What do you mean by concurrent forces? Explain with examples.

Ans. Concurrent Forces:



Definition: Two or more than two forces are said to be concurrent forces, if they are acting upon a body and their lines of action passes through a common point.

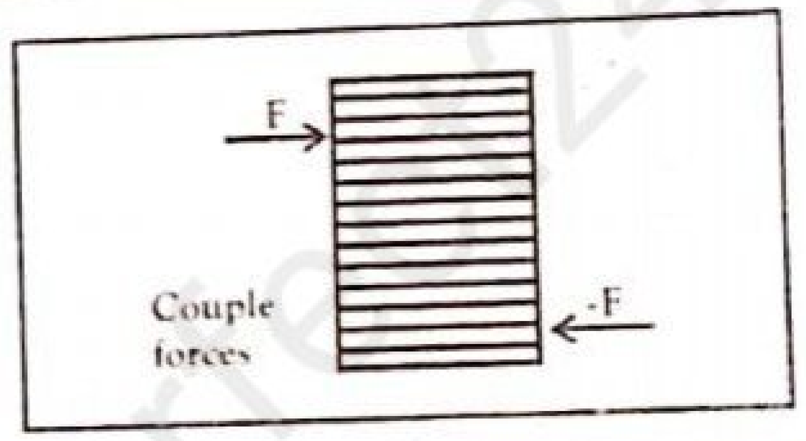
Explanation:

Consider a bulb which is suspended by means of a thread from a rigid support as shown in the figure. In this case, the weight "w" of the bulb is acting in downwards direction while the tension [reaction] "T" in the thread is acting in upwards direction. The line of action of both forces passes through the same point or common point as shown in the figure. Both forces balance each other and as a result, the bulb remains in state of equilibrium. Such forces are known as concurrent forces.

Q22: Explain couple forces?

ETEA 2006-2008, 2012

Ans. Couple Forces:

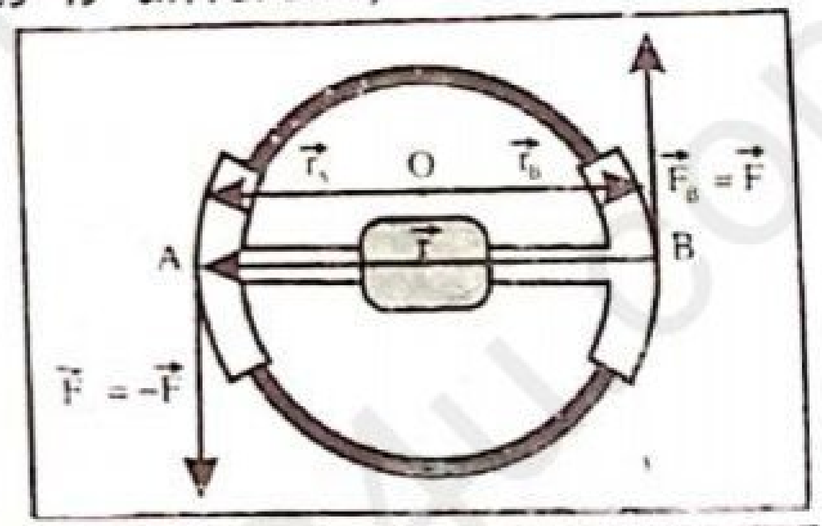


Definition: Two forces having same magnitude, acting in opposite direction having different line of action are called couple force.

Explanation:

Consider two forces "F" and "-F" of equal magnitude acting on a body in opposite direction as shown in the figure.

The line of action of both these forces is different, due to which the body begins to rotate and thus produce torque. Such forces are known as couple forces.



Similarly, when we are driving a car, with both hands on the steering wheel, we push one side of the wheel upward with one hand and push the other side downward with the other hand, as shown in the figure. Both forces are equal in magnitude but opposite in direction. So as a result, the steering wheel begins to rotate due to couple forces. The couple forces produce angular motion in the body and results in torque. Such forces are always non-collinear and non-concurrent.

Q23: Discuss the conditions of equilibrium?

(BISE Peshawar 2017)

Ans. There are two conditions of equilibrium which are given below:

1. First Condition of Equilibrium:

According to the first condition of equilibrium, a body is said to be in equilibrium, if the vector sum of all external forces acting on the body is zero i.e. they cancel the effect of each other i.e.

$$\boxed{\sum \vec{F} = \vec{0}}$$

In components form, we have,

$$\vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x} + \cdots\cdots + \vec{F}_{nx} = \vec{0}$$

$$\Rightarrow \boxed{\sum \vec{F}_x = \vec{0}}$$

Similarly,

$$\vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y} + \cdots\cdots + \vec{F}_{ny} = \vec{0}$$

$$\Rightarrow \boxed{\sum \vec{F}_y = \vec{0}}$$

A book lying on the table and a paratrooper moving downwards with uniform velocity satisfies the first condition of equilibrium.

2. Second Condition of Equilibrium:

According to the second condition of equilibrium, a body is said to be in equilibrium if the vector sum of all the torques acting on a body is equal to zero. i.e. $\boxed{\sum \vec{\tau} = \vec{0}}$

Explanation:

There are certain situations, in which an extended body will not be in equilibrium even when the first condition of equilibrium is satisfied.

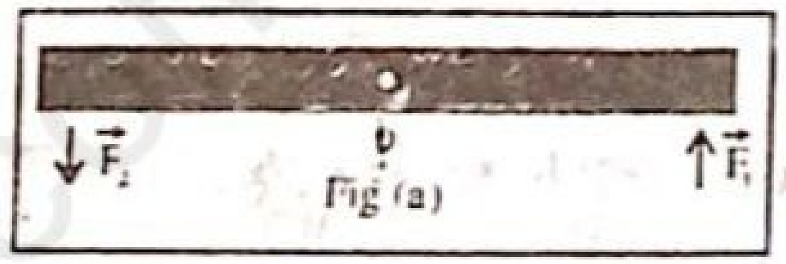
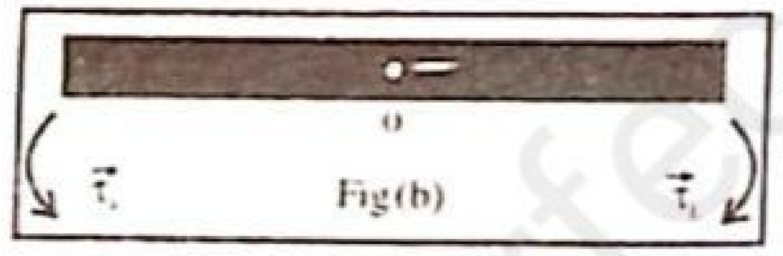


Fig (a)

Example: In the figure (a) two equal forces are acting on a meter rod in opposite direction. As the



Fig(b)

lines of action of these forces are not the same, so the meter rod do not remain in state of equilibrium and begins to rotate in anti-clockwise direction. So the first condition of equilibrium is not sufficient in such cases. Under such situations we should arrange the forces on the body in such a way, that they causes clockwise torque and anti-clockwise torque acting on the body simultaneously and cancel the effect of each other, as shown in figure (b). Thus we have,

Sum of clockwise torque = Sum of anti-clockwise torque

$$\Rightarrow \boxed{\sum \vec{\tau}_{left} = \sum \vec{\tau}_{right}} \quad \text{\_\_\_\_\_\_(1)}$$

In general form, equation (1) can be written as,

$$\boxed{\sum \vec{\tau} = \vec{0}} \quad \text{\_\_\_\_\_\_(2)}$$

NOTE: When the first condition is satisfied, there will be no linear acceleration and the body will be in state of translational equilibrium.
When the second condition of equilibrium is satisfied, there will be no rotational acceleration and the body will be in state of rotational equilibrium.
Thus we can say that, a body will be in complete equilibrium, if both conditions are satisfied. Thus in general, a body is said to be in state of equilibrium if it has no acceleration.

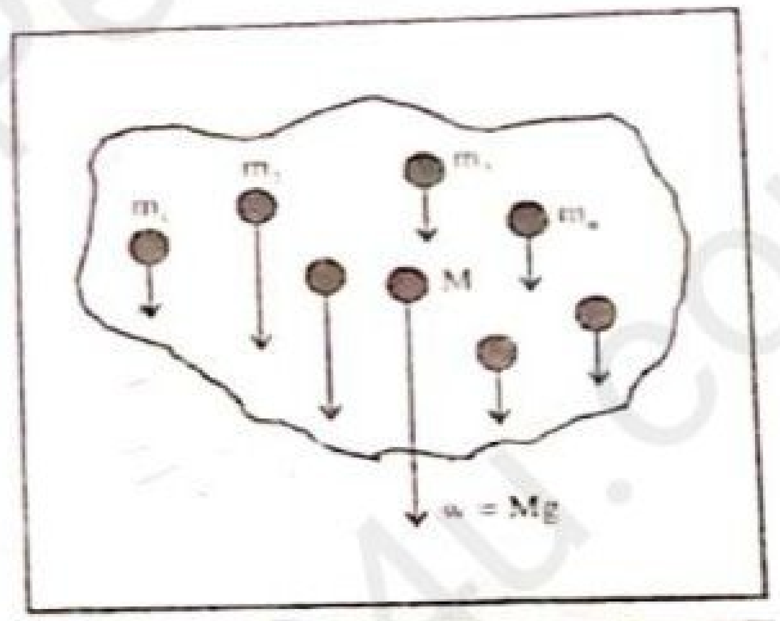
Q24: Explain centre of gravity and centre of mass.

ETEA 2011

Ans. Definition: The centre of gravity is that point in a rigid body, where total weight of the body act, while the point at which the external force can produce translatory motion in the body is known as centre of mass. Both these points exists in the same region of the rigid body.

Explanation:

Consider a rigid body having total weight "w" and total mass "M" as shown in the figure.



Let the body is divided into large number of small elements each having mass m_1, m_2, m_3, ..., m_n respectively.

Now due to the force of gravity, each element will experience gravitational force, i·e· $w_1, w_2, w_3, \ldots\ldots, w_n$ respectively·

Now the total resultant force experienced by this body will be equal to the sum of forces experienced by the individual elements i·e·

Total weight $= w = w_1 + w_2 + w_3 + \cdots + w_n$

$\Rightarrow w = m_1g + m_2g + m_3g + \cdots + m_ng$

$\Rightarrow w = [m_1 + m_2 + m_3 + \cdots + m_n]g$

$\Rightarrow w = [\text{total mass}]\ g$ [total mass = M]

$\Rightarrow \boxed{w = Mg}$ ______(1)

In equation (1), "w" represents the resultant weight which acts at a single point· Such point is known as centre of gravity·

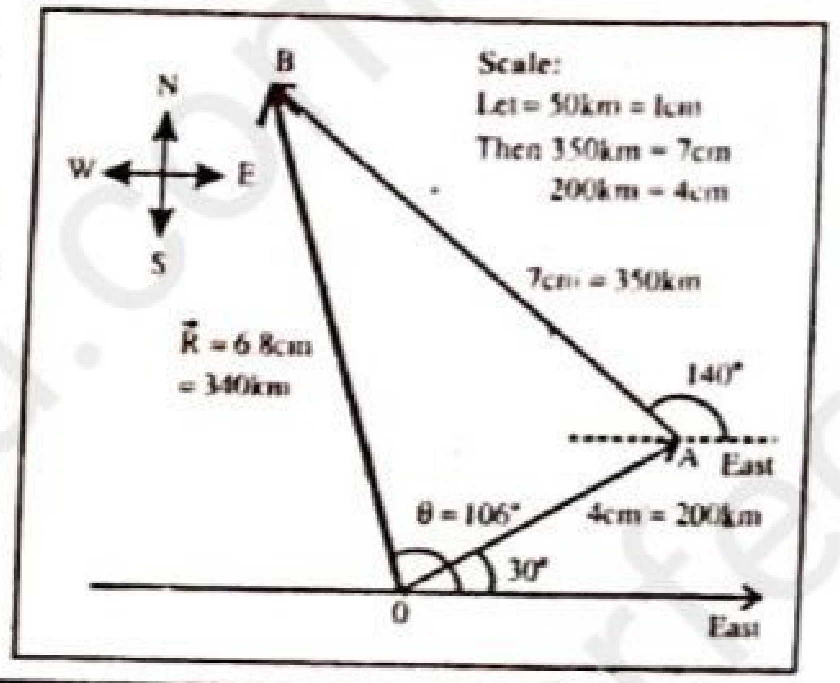
Solution of Examples & Assignments

EXAMPLE 2·1:

A ship leaves part and travels 200km at 30° North of East· Then it changes its direction and travels 350km in a direction 140° North of East to reach destination· Calculate straight line distance covered by ship·

Solution:

1· First we specify the direction·
2· Now we select a suitable scale· Let 50km = 1cm then 200km = 4cm, 350km = 7cm
3· Now we draw a vector which is 4cm long and making an angle of 30° with east· Now we draw another vector which is 7cm long and making an angle of 140° with east as shown in the figure·



4· Now for finding the resultant vector, we combine the tail of first vector $\vec{OA}$ with the head of last vector $\vec{AB}$· Thus $\vec{OB} = \vec{R}$ represent the straight line distance covered by the ship·

5· We measure the length of $\vec{OB}$ with the help of meter rod which is about 6·8cm [340km]·
6· We measure the angle made by vector $\vec{OB}$ with east with the help of protector which is 106° with east·

Ans· (a) $\boxed{\text{Straight line distance} = 6{\cdot}8\text{cm} = 340\text{km}}$

(b) $\boxed{\text{Angle of straight line with east} = 106^\circ}$

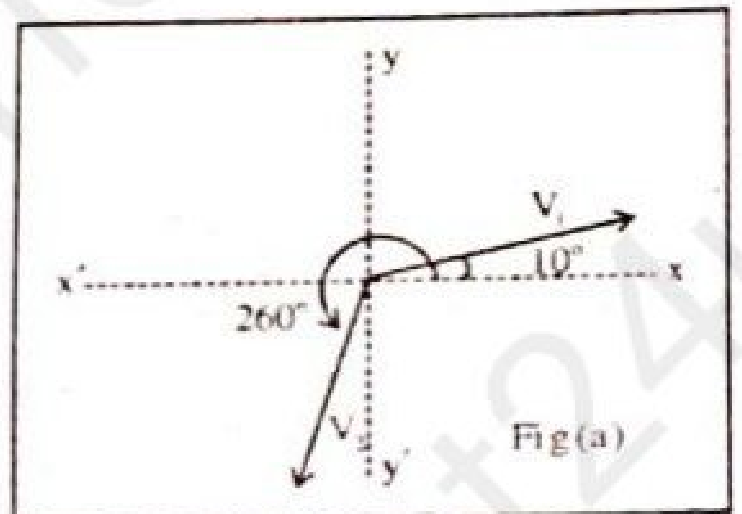
ASSIGNMENT 2·1:

An airplane is moving at 120m/s at an angle of 10° with x-axis, through a 30m/s cross wind, blowing at angle of 260° with x-axis· Determine the resultant velocity of the airplane·

Solution:

Given Data:

Let $\vec{v_1}$ = 120m/sec making an angle of 10° with x-axis and $\vec{v_2}$ = 30m/sec making an angle of 260° with x-axis·

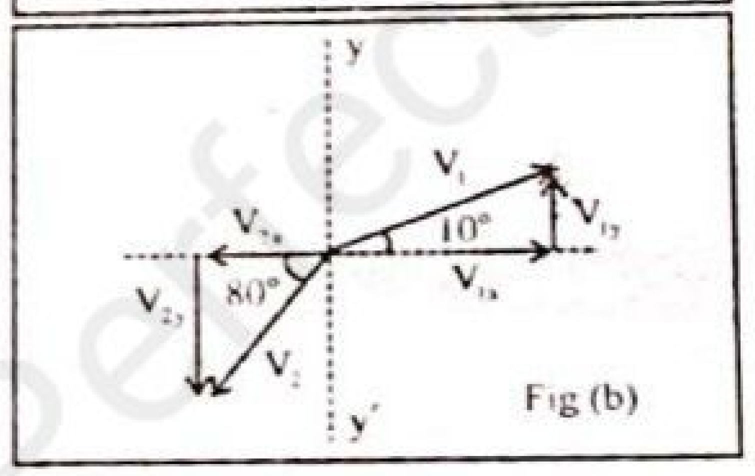


Required: Resultant velocity of airplane = $\vec{v}$ = ?

Calculation: The resultant velocity is given by,

$v = \sqrt{v_x^2 + v_y^2}$ ______(1)

First we resolve $\vec{v_1}$ and $\vec{v_2}$ into its horizontal and vertical components as shown in figure (b)· Now we have,



$v_{1x} = v_1 \cos\theta_1 = 120 \times \cos 10^\circ$ m/sec

$\Rightarrow v_{1x} = 120 \times 0{\cdot}985$ m/sec

$\Rightarrow \boxed{v_{1x} = 118{\cdot}2\ m/sec}$

And $v_{1y} = v_1 \sin\theta_1 = 120 \times \sin 10^\circ = 120 \times 0{\cdot}174$ m/sec

$\Rightarrow \boxed{v_{1y} = 20{\cdot}88\ m/sec}$

Similarly,

$v_{2x} = v_2\cos\theta_2 = 30\times\cos 80^\circ$

$= 30\times 0.174$ m/sec

$\Rightarrow \boxed{v_{2x} = 5.22\ m/sec}$

And $v_{2y} = v_2\sin\theta_2 = 30\times\sin 80^\circ$

$= 30\times 0.985$ m/sec

$\Rightarrow \boxed{v_{2y} = 29.55\ m/sec}$

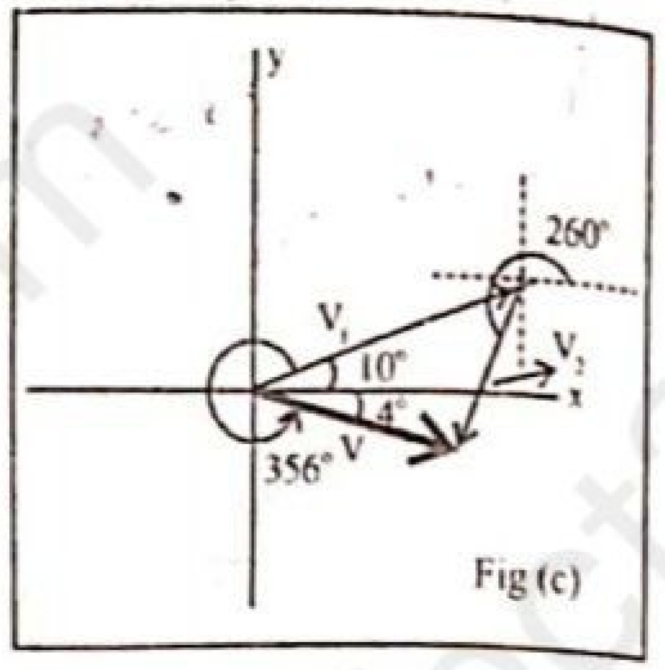


Fig (c)

Now the x-component of resultant velocity is given by,

$v_x = v_{1x} + (-v_{2x}) = v_{1x} - v_{2x} = (118.2 - 5.22)\ m/sec$

$\Rightarrow v_x = 112.98\ m/sec$

Now the y-component of resultant velocity is given by,

$v_y = v_{1y} + (-v_{2y}) = v_{1y} - v_{2y} = (20.88 - 29.55)\ m/sec$

$\Rightarrow v_y = -8.67\ m/sec$

Putting the values of "v_x" and "v_y" in equation (1)

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(112.98)^2 + (-8.67)^2}$

$\Rightarrow v = \sqrt{12764.5 + 75.2} = \sqrt{12839.7}$

$\Rightarrow \boxed{v = 113.3\ m/sec}$

The direction of "v" is given by,

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{8.67}{112.98}\right)$

$\Rightarrow \theta = \tan^{-1}(0.077) \Rightarrow \theta = 4^\circ$

Now the actual angle of "v" with positive x-axis is given by,

$\theta' = 360^\circ - 4^\circ \Rightarrow \boxed{\theta' = 356^\circ}$

EXAMPLE 2.2:

Two forces $\vec{F}_1 = 15N$ making an angle $\theta_1 = 70^\circ$ with positive x-axis and force $\vec{F}_2 = 25N$ making an angle $\theta_2 = 220^\circ$ with positive x-axis, act at a point, calculate the resultant force $\vec{F}_R$.

Solution:

Given Data:

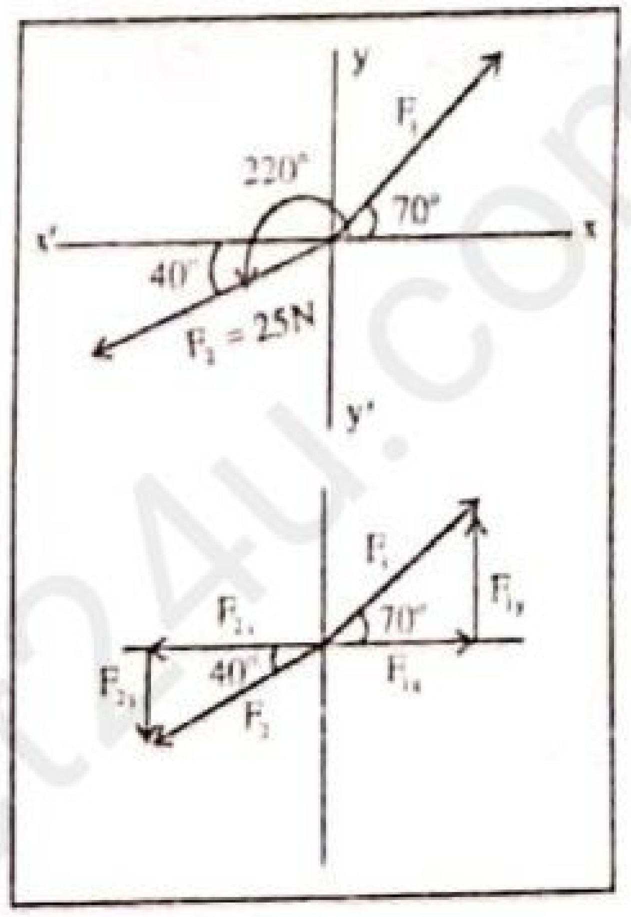
Force $F_1 = 15N$, angle $\theta_1 = 70^\circ$

Force $F_2 = 25N$, angle $\theta_2 = 220^\circ$

6 **Required:** Resultant force $\vec{F}_R = ?$

Calculation: The magnitude of the resultant force is given by,

$F = \sqrt{F_x^2 + F_y^2}$ ______(1)



First we resolve "$\vec{F}_1$" and "$\vec{F}_2$" into its components. For "F_1", we have,

$F_{1x} = F_1\cos\theta_1 = 15\times\cos 70^\circ = 15\times 0.342N$

$\Rightarrow F_{1x} = 5.13N$

And $F_{1y} = F_1\sin\theta_1 = 15\times\sin 70^\circ = 15\times 0.939N$

$\Rightarrow F_{1y} = 14.09N$

For F_2 we have,

$F_{2x} = -F_2\cos\theta_2 = -25\times\cos 40^\circ = -25\times 0.766N$

$\Rightarrow F_{2x} = -19.15$

$\Rightarrow F_{2x} = -19.15N$ [negative sign shows that "F_{2x}" is acting along x'-axis]

And $F_{2y} = -F_2\sin\theta_2 = -25\times\sin 40^\circ$

$= -25\times 0.643N$

$\Rightarrow F_{2y} = 16.07N$

Now the x-component of the resultant is given by,

$F_x = F_{1x} + F_{2x} = 5.13N + (-19.15)N = 5.13N - 19.15N$

$\Rightarrow F_x = -14.02N$

And the y-component of the resultant is given by,

$F_y = F_{1y} + F_{2y} = 14.09N + (-16.07N) = 14.09N - 16.07N$

$\Rightarrow F_y = -1.98N$

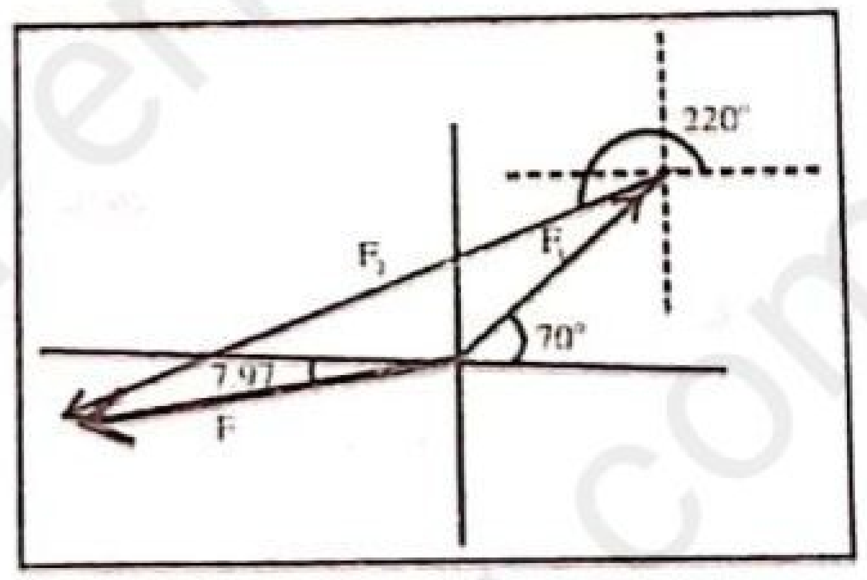
Putting the values of "F_x" and "F_y" in equation (1) we get,

$F_R = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(-14.02N)^2 + (-1.98N)^2}$

$\Rightarrow F_R = \left[\sqrt{196.6 + 3.9}\right]N = \sqrt{200.5N}$

$\Rightarrow \boxed{F_R = 14.16N}$



The direction of "F_R" is given by,

$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{1.98}{14.02}\right) = \tan^{-1}(0.14)$

$\Rightarrow \boxed{\theta = 7.97^\circ}$

As "F_x" and "F_y" both are negative so the resultant force lies in third quadrant. So the actual angle of "F" with positive x-axis is given by,

$\theta' = 180^\circ + \theta = 180^\circ + 7.97^\circ \Rightarrow \boxed{\theta = 187.97^\circ}$

ASSIGNMENT 2.2:

A force $F_1 = 20N$ making an angle $\theta_1 = 30^\circ$ with positive x-axis and force $F_2 = 30N$ making an angle $\theta_2 = 150^\circ$ with positive x-axis, acts at a point, calculate the resultant force?

Solution:

Given Data:

$F_1 = 20N$ at angle of $\theta_1 = 30^\circ$ with positive x-axis

$F_2 = 30N$ at angle of $\theta_2 = 150^\circ$ with positive x-axis

Required: Resultant force = F = ?

Calculation: The magnitude of resultant force is given by,

$F = \sqrt{F_x^2 + F_y^2}$ ____________(1)

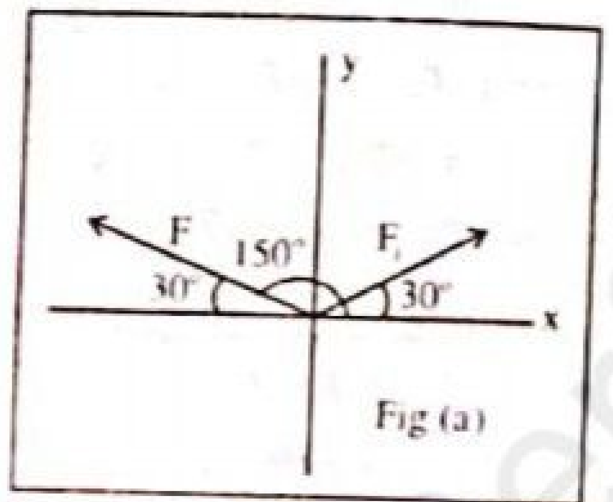


Fig (a)

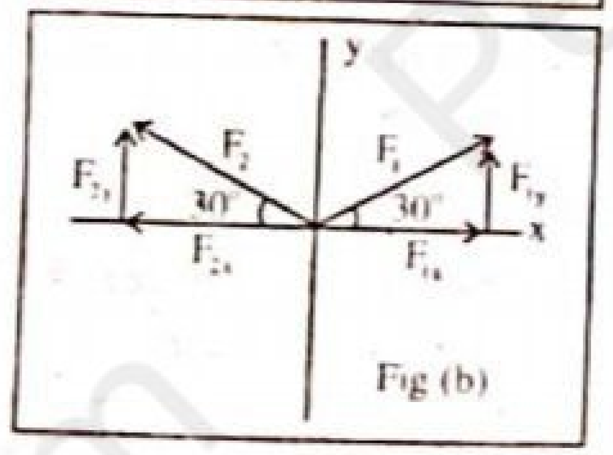


Fig (b)

First we resolve $\vec{F_1}$ and $\vec{F_2}$ into its components, as shown in figure (b). For "F_1", we have,

$F_{1x} = F_1\cos\theta_1 = 20\times\cos30^\circ = 20\times0.866N$

$\Rightarrow \boxed{F_{1x} = 17.32N}$

And $F_{1y} = F_1\sin\theta_1 = 20\times\sin30^\circ = 20\times0.5N$

$\Rightarrow \boxed{F_{1y} = 10N}$

For F_2, we have,

$F_{2x} = -F_2\cos\theta_2 = -30\times\cos30^\circ = -30\times0.866N$

$\Rightarrow \boxed{F_{2x} = -25.98N}$

And $F_{2y} = F_2\sin\theta_2 = 30\times\sin30^\circ = 30\times0.5N$

$\Rightarrow \boxed{F_{2y} = 15N}$

Now the horizontal component [x-component] of the resultant "F" is given by,

$F_x = F_{1x} + F_{2x} = [17.32 + (-25.98)]N = (17.32 - 25.98)N$

$\Rightarrow \boxed{F_x = -8.66N}$

Now the y-component [vertical component] of the resultant force "F" is given by,

$F_y = F_{1y} + F_{2y} = 10N + 15N \Rightarrow \boxed{F_y = 25N}$

Putting the values of "F_x" and "F_y" in equation (1), we get,

$F = \sqrt{(-8.66N)^2 + (25)^2} = \left[\sqrt{74.9956 + 625}\right]N$

$\Rightarrow F = \left[\sqrt{699.9956}\right]N \Rightarrow \boxed{F = 26.46N}$ or $\boxed{F = 26.5N}$

Direction of $\vec{F}$:

We have, $\theta = \tan^{-1}\left(\dfrac{F_y}{F_x}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{25}{8.66}\right) = \tan^{-1}(2.886)$

$\Rightarrow \boxed{\theta = 70.8^\circ}$

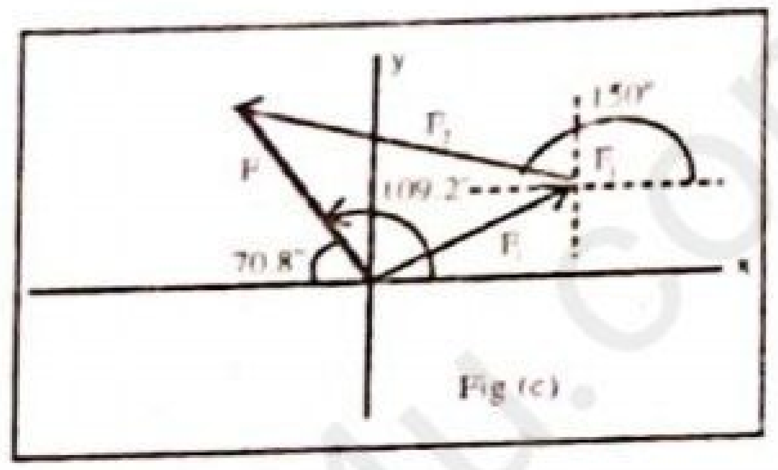


Fig (c)

As x-component of $\vec{F}$ is negative, y-component is positive, so resultant force $\vec{F}$ will be acting in second quadrant by making an angle of 70.8° with negative x-axis, as shown in the figure (c). So the actual angle made by $\vec{F}$ with positive x-axis is given by,

$\theta' = 180^\circ - 70.8^\circ = 109.2^\circ \Rightarrow \boxed{\theta' = 109.2^\circ}$

EXAMPLE 2.3:

Find the angle between two forces of equal magnitude such that the magnitude of their resultant is also equal to either of them.

Solution:

Let $\vec{F_1}$ and $\vec{F_2}$ represents the two vectors such that $\vec{F_1}$ is acting along x-axis while $\vec{F_2}$ is making an angle θ with positive x-axis, as shown in the figure. [θ also represents the angle between the two vectors]. First we resolve $\vec{F_1}$ and $\vec{F_2}$ into their horizontal and vertical components.

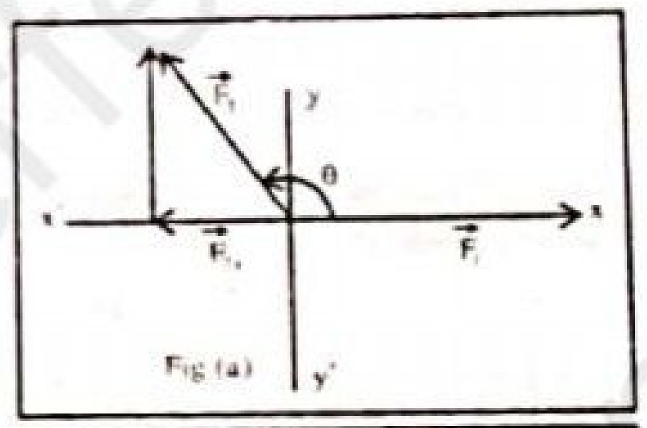


Fig (a)

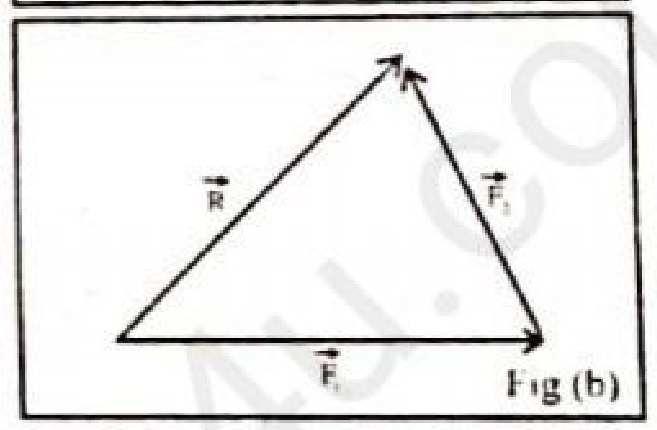


Fig (b)

As $\vec{F_1}$ is acting along x-axis, so it posses horizontal component and no vertical component. So we have,

$F_{1x} = F_1\cos\theta = F_1\cos0^\circ = F_1 \times 1 = F_1$

$\Rightarrow F_{1x} = F_1$

While $F_{1y} = 0N$

Now, $F_{2x} = F_2\cos\theta, F_{2y} = F_2\sin\theta$

Now the horizontal component of resultant is given by,

$R_x = F_{1x} + F_{2x}$

$\Rightarrow R_x = (F_1 + F_2 \cos\theta)$ ______ (1)

While the vertical component of the resultant is given by,

$R_y = F_{1y} + F_{2y} = 0 + F_2 \sin\theta$

$\Rightarrow R_y = F_2 \sin\theta$ ______________ (2)

Now the magnitude of $\vec{R}$ is given by,

$R = \sqrt{R_x^2 + R_y^2}$ _______ (3)

Putting equations (1), (2) in equation (3), we get,

$R = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2 \sin\theta)^2}$

$\Rightarrow R = \sqrt{F_1^2 + F_2^2\cos^2\theta + 2F_1F_2\cos\theta + F_2^2\sin^2\theta}$ $[\cos^2\theta + \sin^2\theta = 1]$

$\Rightarrow R = \sqrt{F_1^2 + F_2^2(\cos^2\theta + \sin^2\theta) + 2F_1F_2\cos\theta}$

$\Rightarrow R = \sqrt{(F_1^2 + F_2^2 + 2F_1F_2\cos\theta)}$ _________ (4)

As in equation, we have given that, $R = F_1 = F_2$

So we generally put $R = F_1 = F_2 = F$

So equation (4) becomes,

$\Rightarrow F = \sqrt{F^2 + F^2 + 2F^2\cos\theta}$

$\Rightarrow F = F\sqrt{1 + 1 + 2\cos\theta} = F\sqrt{2 + 2\cos\theta}$

$\Rightarrow F = F\sqrt{2 + 2\cos\theta}$

$\Rightarrow 1 = \sqrt{(2 + 2\cos\theta)}$ ____ (5)

Squaring both sides of equation (5), we get,

$1 = 2 + 2\cos\theta$

$\Rightarrow 2\cos\theta = -1 \Rightarrow \cos\theta = \frac{-1}{2} = -0.5$ $[\cos(-x) = \cos x]$

$\Rightarrow \cos\theta = -0.5 \Rightarrow \theta = \cos^{-1}(0.5) \Rightarrow \boxed{\theta = 120^\circ}$

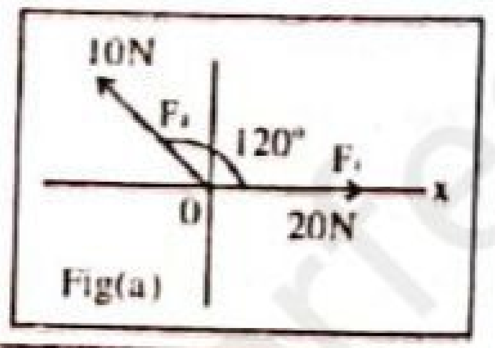
Thus the angle between the two forces is equal to 120°.

ASSIGNMENT 2.3:

Two forces of 20N and 10N are making an angle of 120° with each other. Find a single pull that would (a) replace the given forces system (b) balance the given forces system.

Solution:

Let $\vec{F}_1 = 20N$ acting along x-axis, $\vec{F}_2 = 10N$ making an angle of 120° with positive x-axis



Fig(a)

(a) (i) Magnitude of resultant of $\vec{F}_1$ and

$\vec{F}_2 = R = ?$

(ii) Direction of $\vec{R} = \theta = ?$

(b) Balance force of $\vec{R} = \vec{F} = ?$

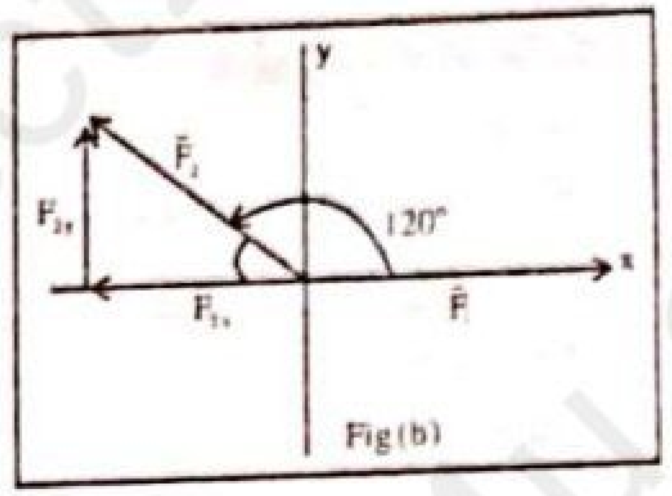
(a) First we resolve $\vec{F}_1$ and $\vec{F}_2$ into its horizontal and vertical components. As $\vec{F}_1$ is lying only along x-axis, so it has only horizontal component and its vertical component will be equal to zero. So we have,

$F_{1x} = F_1 \cos\theta = 20 \times \cos 0^\circ$

$\Rightarrow F_{1x} = 20 \times 1N$

$\Rightarrow F_{1x} = 20N$ and $F_{1y} = 0N$

Now we resolve $\vec{F}_2$ into its horizontal and vertical components, as shown in figure (b). So we have,



Fig(b)

$F_{2x} = -F_2 \cos\theta = -10 \times \cos 60^\circ$

$\Rightarrow F_{2x} = -10 \times 0.5 = -5N$

$\Rightarrow F_{2x} = -5N$

While, $F_{2y} = F_2 \sin\theta = 10 \times \sin 60^\circ$

$\Rightarrow F_{2y} = 10 \times 0.866N$

$\Rightarrow F_{2y} = 8.66N$

Now the x-component of the resultant is given by,

$R_x = F_{1x} + F_{2x}$

$\Rightarrow R_x = 20 + (-5) = 20 - 5 = 15N$

$\Rightarrow R_x = 15N$

F_{2x} is in second quadrant so it is taken as negative.

Now y-component of $\vec{R}$ [resultant] is given by,

$R_y = F_{1y} + F_{2y}$

$\Rightarrow R_y = 0N + 8.66N$

$\Rightarrow R_y = 8.66N$

(i) Now the magnitude of the resultant is given by,

$R = \sqrt{R_x^2 + R_y^2}$

$\Rightarrow R = \sqrt{15^2 + (8.66)^2} = \sqrt{225 + 74.9956}$

$\Rightarrow R = \sqrt{299.9956}$

$\Rightarrow \boxed{R = 17.32N}$

(ii) The direction of $\vec{R}$ is given by,

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{8\cdot66}{15}\right) = \tan^{-1}(0\cdot577)$

$\Rightarrow = 29\cdot98^\circ = 30^\circ$

$\Rightarrow \boxed{\theta = 30^\circ}$

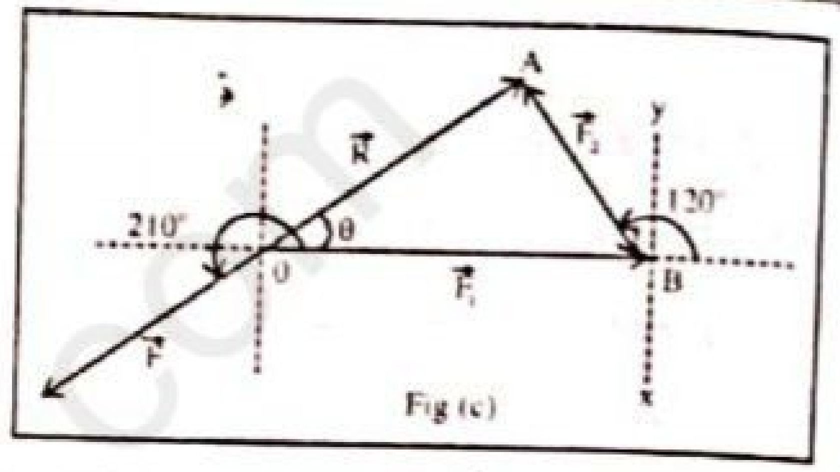


Fig (c)

(b) Let $\vec{F}$ be the force which balance the resultant $\vec{R}$ [i·e· resultant of given forces] then $\vec{F}$ must have the same magnitude as $\vec{R}$ but opposite to $\vec{R}$ as shown in the figure· So $F = 17\cdot32N$ and making an angle of 210° with positive x-axis, as shown in the figure (c)· $\vec{R}$ and $\vec{F}$ are opposite, so the angle between them is 180°· As $\vec{R}$ make 30° with x-axis, so the angle made by $\vec{F}$ with x-axis = 180° + 30° = 210°·

EXAMPLE 2·4:

Vector 'A' having magnitude 3·2 makes 50° with x-axis and vector 'B' with magnitude 5·1 makes 110° with x-axis· What is the magnitude of their dot and cross products?

Solution:

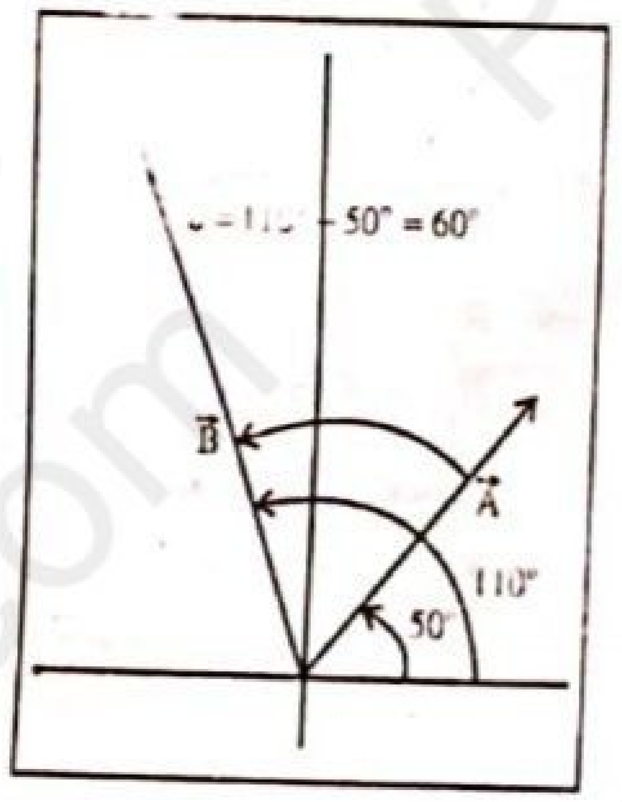
Given Data:

Vector $|\vec{A}| = 3\cdot2$, angle $\theta_1 = 50^\circ$ with x-axis

Vector $|\vec{B}| = 5\cdot1$, angle $\theta_2 = 110^\circ$ with x-axis

Required: (a) $|\vec{A} \bullet \vec{B}| = ?$

(b) $|\vec{A} \times \vec{B}| = ?$



Calculation: (a) Let 'θ' be angle between two vectors, then we have,

$\theta = \theta_2 - \theta_1 = 110^\circ - 50^\circ = 60^\circ$

We know that the magnitude of dot product is given by,

$|\vec{A} \bullet \vec{B}| = AB\cos\theta$ ________(1)

Putting the values in equation (1), we get,

$|\vec{A} \bullet \vec{B}| = (3\cdot2)(5\cdot1)(\cos 60^\circ)$

$\Rightarrow |\vec{A} \bullet \vec{B}| = 3\cdot2 \times 1\cdot5 \times 0\cdot5 = 8\cdot16 = 8\cdot2$

$\Rightarrow \boxed{|\vec{A} \bullet \vec{B}| = 8\cdot2}$

(b) The magnitude of cross-produce of two vectors is given by,

$|\vec{A} \times \vec{B}| = AB\sin\theta$ ________(2)

Putting the values in equation (2), we get,

$|\vec{A} \times \vec{B}| = 3\cdot2 \times 5\cdot1 \times \sin 60^\circ = 3\cdot2 \times 5\cdot1 \times 0\cdot866 \Rightarrow \boxed{|\vec{A} \times \vec{B}| = 14\cdot1}$

ASSIGNMENT 2·4:

Show that $|\vec{A}.\vec{B}|^2 + |\vec{A}\times\vec{B}|^2 = A^2B^2$

Proof:

The magnitude of dot product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$|\vec{A} \times \vec{B}| = AB\cos\theta$ ________(1)

Squaring both sides of equation (1), we get,

$|\vec{A} \cdot \vec{B}|^2 = A^2B^2\cos^2\theta$ ________(2)

The magnitude of cross-produce of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$|\vec{A} \times \vec{B}| = AB\sin\theta$ ________(3)

Squaring both sides of equation (3), we get,

$|\vec{A} \times \vec{B}|^2 = A^2B^2\sin^2\theta$ ________(4)

Adding equation (2) and equation (4), we get,

$|\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2B^2\cos^2\theta + A^2B^2\sin^2\theta$

$\Rightarrow |\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2B^2[\cos^2\theta + \sin^2\theta]$ $\boxed{\cos^2\theta + \sin^2\theta = 1}$

$\Rightarrow |\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2B^2 \times 1$

$\Rightarrow |\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2B^2 \times 1$

$\Rightarrow \boxed{|\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2B^2}$

EXAMPLE 2·5:

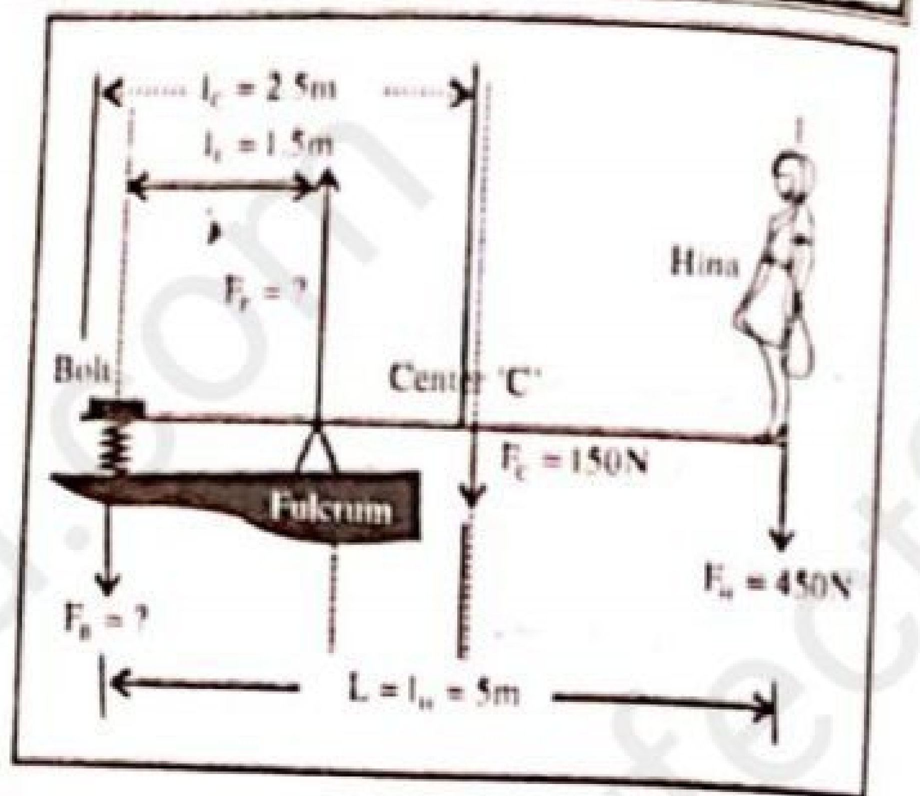
Hina weighing 450N is standing at the edge of the uniform diving board 5m in length· Weight of the board is 150N, and is bolted down at the left end, while being supported 1·50m away by a fulcrum, as in figure 2·35· Find the forces and that the bolt and the fulcrum, exert on the board·

Solution:

Given Data:

Weight of Hina = F_H = 450N

Weight of board = F_C = 150N



Center of gravity of board
$= l_c = 25m$

Distance of fulcrum $= l_F = 1.5m$

Length of board
$L = l_H = 5m$

Required: (a)
Force exerted by fulcrum $= F_F = ?$
(b) Force exerted by bolt $= F_B = ?$

Calculation: For the axis of rotation at point A, let the torque produced by support fulcrum is "τ_F" the torque produced by weight of board is "τ_c" and the torque produced by weight of girl Hina is "τ_H".

By second condition of equilibrium,

$$\Sigma \tau = 0$$

$$\Rightarrow \tau_F - \tau_c - \tau_H = 0$$

$$\Rightarrow (F_F)(l_F) - (F_c)(l_c) - (F_H)(l_H) = 0$$

$$\Rightarrow (F_F)(l_F) = (F_c)(l_c) + (F_H)(l_H) \Rightarrow F_F = \frac{(F_c)(l_c) + (F_H)(l_H)}{(l_F)} \quad \text{\_\_\_\_(1)}$$

Putting the values in equation (1), we get,

$$F_F = \frac{(150N)(2.5m) + (450N)(5m)}{1.5m}$$

$$\Rightarrow F_F = \frac{375Nm + 2250Nm}{1.5m} \Rightarrow F_F = \frac{2625Nm}{1.5m} \quad \boxed{F_F = 1750N}$$

(b) Now the force due to bolt "F_B" can be easily found out by solving first condition of equilibrium along y-axis. Such that

$$\Sigma F_y = 0 \Rightarrow -F_B + F_F + (-F_c) + (-F_H) = 0$$

$$\Rightarrow -F_B + F_F - F_c - F_H = 0$$

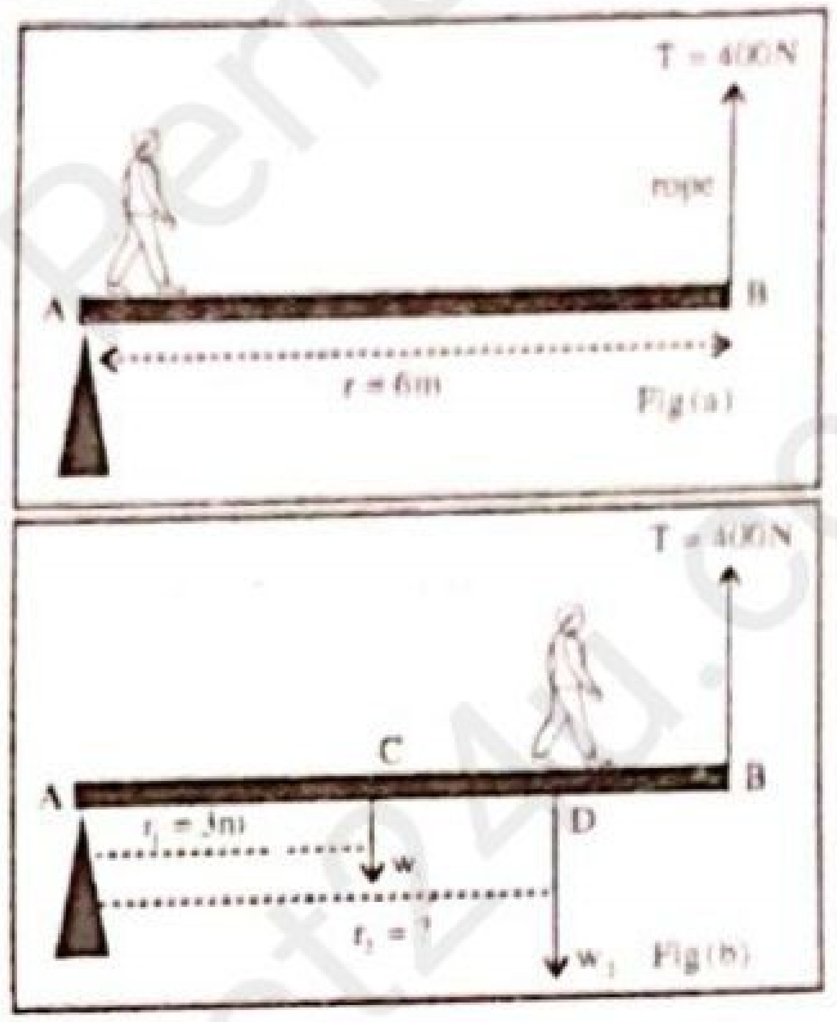
$$\Rightarrow F_B = F_F - F_c - F_H \quad \text{\_\_\_\_\_\_\_\_(2)}$$

Putting the values in equation (3), we get,

$$F_B = 1750N - 150N - 450N \Rightarrow \boxed{F_B = 1150N}$$

ASSIGNMENT 2.6:

A uniform plank of 200N and length 6m is supported horizontally by a rope as shown in figure (a). If the breaking tension of the rope is 400N, how far can a boy of weight 400N walk from "A" to "B" i.e. towards the support?



Solution: Given Data:

Weight of beam $= w_1 = 200N$
Weight of man $= w_2 = 400N$
Tension of the rope $= T = 400N$
Length of beam $= r = 6m$

Required: Distance of boy from "A" $= r_2 = ?$

Calculation: Now for balance condition, let the weight of the beam is acting at point "C" and weight of the man is acting at point "D" as shown in figure (b). The torque due to "w_1" is given by,

$$\tau_1 = r_1 \times w_1 = 3 \times 200 \Rightarrow \tau_1 = 600N{\cdot}m$$

The torque due to "w_2" is given by,

$$\tau_2 = r_2 \times w_2 = r_2 \times 400 \Rightarrow \tau_2 = 400r_2N$$

The torque due to tension "T" is given by,

$$\tau_3 = r \times T = 6 \times 400 \Rightarrow \tau_3 = 2400N{\cdot}m$$

Here we see that "w_1" and "w_2" produces torque towards left, so total torque towards left is given by,

$$\tau_L = \tau_1 + \tau_2 = 600N{\cdot}m + 400 \times r_2N$$

While "τ_3" represents the torque towards right, so

$$\tau_R = \tau_3 = 2400N{\cdot}m$$

Now for balance conditions, we have, $\tau_L = \tau_R$

$$\Rightarrow 600N{\cdot}m + 400 \times r_2N = 2400N{\cdot}m \Rightarrow 400 \times r_2N = 2400N{\cdot}m - 600N{\cdot}m$$

$$\Rightarrow 400 \times r_2N = 1800N{\cdot}m \Rightarrow r_2 = \left(\frac{1800}{400}\right)m = 4.5m \Rightarrow \boxed{r_2 = 4.5m}$$

- قرآن مجید میں ارشاد ہے "والدین، رشتہ داروں، مسکینوں اور اور مسافروں سے نیک سلوک کرو"۔
- قرآن میں ارشاد ہے "بے شرمی کی باتیں کھلی ہوں یا چھپی ان کے پاس تک نہ بھٹکو"۔
- قرآن میں ارشاد ہے "اللہ ہی تمہارا مالک ہے اور وہ سب سے اچھا مددگار ہے"۔

Exercise

Choose the best possible answer:

1. Two vectors lie with their tails at the same point. When the angle between them is increased by 20° their scalar product has the same magnitude but changes from positive to negative. The original angle between them was:

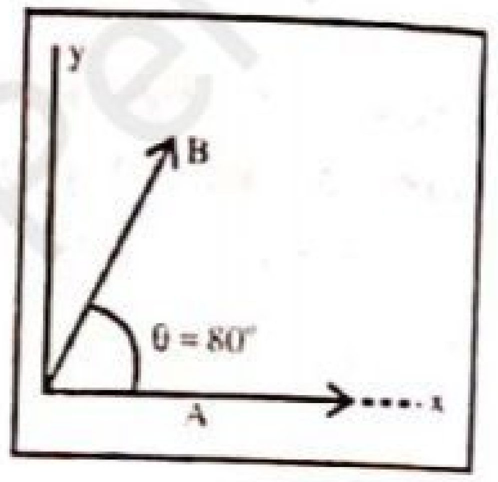
a. 0 b. 60° c. 70° ✓d. 80°

Sol: We know that, $\vec{A} \cdot \vec{B} = AB\cos\theta$ ______ (1)

Putting $\theta = 80°$ we get,

$\vec{A} \cdot \vec{B} = AB\cos 80° = 0.174AB$

$\Rightarrow \vec{A} \cdot \vec{B} = 0.174AB$ ______ (2)



Now put $\theta = 80° + 20° = 100°$ in equation (1),

$\vec{A} \cdot \vec{B} = AB\cos 100° AB(-0.174)$

$\Rightarrow \vec{A} \cdot \vec{B} = -0.174AB$ ______ (3)

Equation (2) shows that the dot product for 80° and 100° has the same magnitude but changes from positive to negative, so the original angle is 80°.

2. The minimum number of vectors of unequal magnitude required to produce a zero resultant is:

a. 2 ✓b. 3 c. 4 d. 5

3. If the resultant of two vectors, each of magnitude "A" is also a magnitude of "A". The angle between the two vectors will be:

a. 30° b. 45° c. 60° ✓d. 120°

Sol: The resultant of two force "F_1" and "F_2" is given by,

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$ ______ (1)

According to given condition, we have,

$R = F_1 + F_2 = F$, so equation (1) becomes

$F = \sqrt{F^2 + F^2 + 2F^2\cos\theta} = F\sqrt{(1 + 1 + 2\cos\theta)}$

$\Rightarrow 1 = \sqrt{2 + 2\cos\theta} \Rightarrow 1^2 = (\sqrt{2 + 2\cos\theta})^2$

$\Rightarrow 1 = 2 + 2\cos\theta \Rightarrow 2\cos\theta = -1 \Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{-1}{2}\right) \Rightarrow \boxed{\theta = 120°}$

4. The magnitude of vector $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k}$ is:

a. 9 b. 5 ✓c. 3 d. 1

Sol: $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k} \Rightarrow |\vec{A}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$

5. When $F_x = 3N$ and $F = 5N$ then F_y =

a. 6N ✓b. 4N c. 2N d. 0N

Sol: $F_x = 3N, F = 5N$, we know that

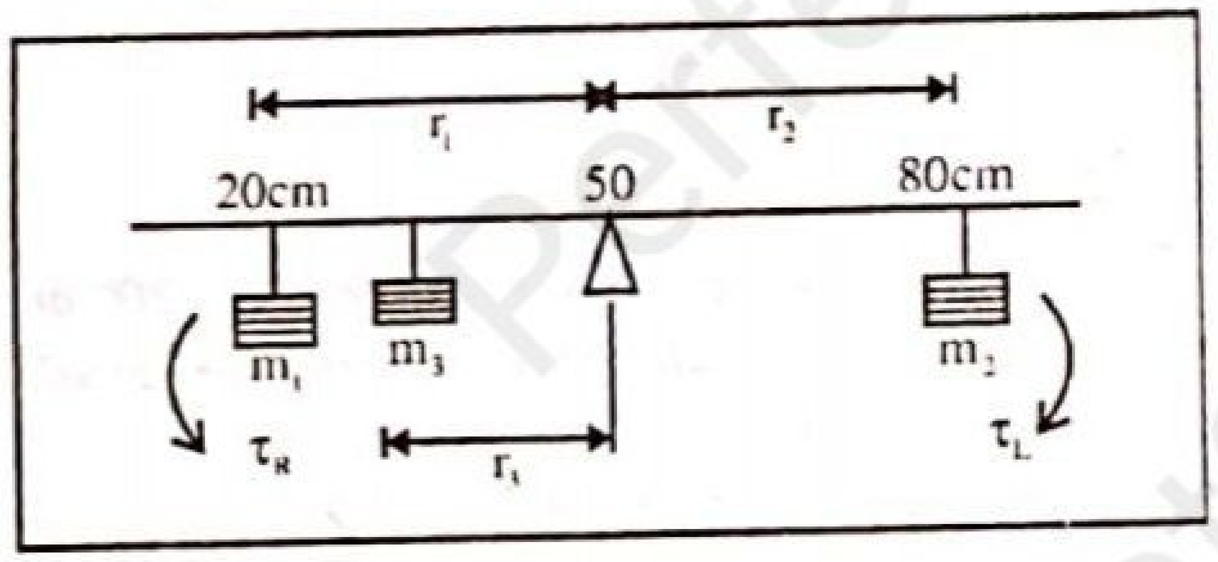
$F^2 = F_x^2 + F_y^2 \Rightarrow F_y^2 = F^2 - F_x^2 = (5N)^2 - (3N)^2 = 25N^2 - 9N^2$

$\Rightarrow F_y^2 = 16N^2 \Rightarrow \boxed{F_y = 4N}$

6. A meter stick is supported by a knife-edge at the 50cm mark. Arif hangs masses of 0.40 and 0.60kg from the 20cm and 80cm marks, respectively. Where should Arif hang a third mass of 0.30kg to keep the stick balanced?

✓a. 20cm b. 70cm c. 30cm d. 25cm

Sol: $m_1 = 0.40kg, m_2 = 0.60kg, m_3 = 0.3kg$



$r_1 = 30cm, r_2 = 30cm, r_3 = ?$

For balance conditions, we have, $\tau_R = \tau_L$

$\Rightarrow (w_1 \times r_1) + (w_3r_3) = w_2r_2$

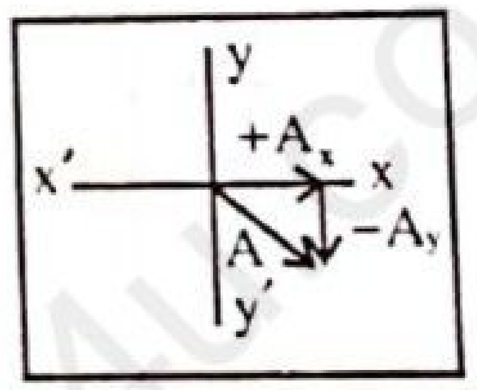
$\Rightarrow (m_1gr_1) + (m_3gr_3) = m_2gr_2$

$\Rightarrow m_1r_1 + m_3r_3 = m_2r_2$

$\Rightarrow r_3 = \frac{m_2r_2 - m_1r_1}{m_3} = \left(\frac{0.6 \times 30 - 0.4 \times 30}{0.3}\right)cm = \left(\frac{18 - 12}{0.3}cm\right)$

$= \left(\frac{6}{0.3}\right)cm \Rightarrow \boxed{r_3 = 20cm}$

7. If $A_x = 1.5cm, A_y = -1.0cm$, into which quadrant do the vector "A" point?

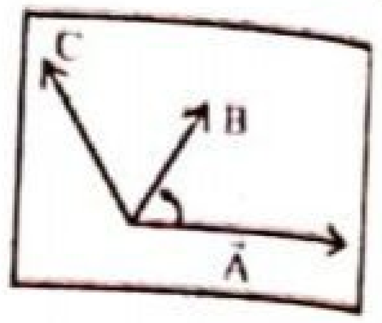


a. I b. II c. III ✓d. IV

Sol: $A_x = 1.5cm, A_y = -1.0cm$

8. $\vec{A}\cdot(\vec{A}\times\vec{B}) = ?$

✓a. 0 b. 1 c. AB d. A^2B



Sol: We know that $(\vec{A}\times\vec{B}) = \vec{C}$ ____(1)

Now $\vec{A}\cdot(\vec{A}\times\vec{B}) = \vec{A}\cdot\vec{C}$

$\Rightarrow$ $\vec{A}\cdot(\vec{A}\times\vec{B}) = AC\cos\theta$ ____(2)

As vector $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$ so put $\theta = 90°$ in equation (1), we get,

$\vec{A}\cdot(\vec{A}\times\vec{B}) = AC\times 0 = 0$

$\Rightarrow$ $\boxed{\vec{A}\cdot(\vec{A}\times\vec{B}) = 0}$

9. Two forces of magnitude 20N and 50N act simultaneously on a body. Which one of the following forces cannot be a resultant of the two forces?

✓a. 20N b. 30N c. 40N d. 70N

Sol: $F_1 = 20N, F_2 = 50N$

We have, $R_{min} \le R \le R_{max}$

$\Rightarrow$ $(F_2 - F_1) \le R \le (F_2 + F_1)$

$\Rightarrow$ $(50 - 20) \le R \le 50 + 20$

$\Rightarrow$ $\boxed{30 \le R \le 70}$

This shows that the range of resultant will be 30 or 70 or between them. So 20N does not comes among the possible values of resultant.

10. If the dot product of two non-zero vectors "A" and "B" is zero then the magnitude of their cross product is......

a. 0 b. 1 ✓c. AB d. –AB

Sol: $\vec{A}\cdot\vec{B} = AB\cos\theta$ ____(1)

For $\vec{A}\cdot\vec{B} = 0$, put $\theta = 90°$, so equation (1) becomes

$\vec{A}\cdot\vec{B} = AB\cos 90° AB\times 0 \Rightarrow \vec{A}\cdot\vec{B} = 0$ ____(1)

Now $\vec{A}\times\vec{B} = AB\sin\theta = AB\sin 90° = AB\times 1 = AB$

11. The sum of magnitude of two forces is 16N. If the resultant force is 8N and its direction is perpendicular to minimum force then the force are:

✓a. 6N and 10N b. 8N and 8N

c. 4N and 12N d. 2N and 14N

Sol: We have given that, resultant = R= 8N and

$F_1 + F_2 = 16 \Rightarrow F_2 = (16 - F_1)$ ____(1)

According to equation, we have,

$R^2 = F_2^2 - F_1^2 \Rightarrow F_2 = \sqrt{R^2 + F_1^2}$ ____(2)

Comparing equation (1), equation (2), we get,

$(16 - F_1) = \sqrt{R^2 + F_1^2} \Rightarrow (16 - F_1)^2 = R^2 + F_1^2 = 8^2 + F_1^2$

$\Rightarrow$ $256 + F_1^2 - 32F_1 = 64 + F_1^2 \Rightarrow 256 - 64 - 32F_1 = 0$

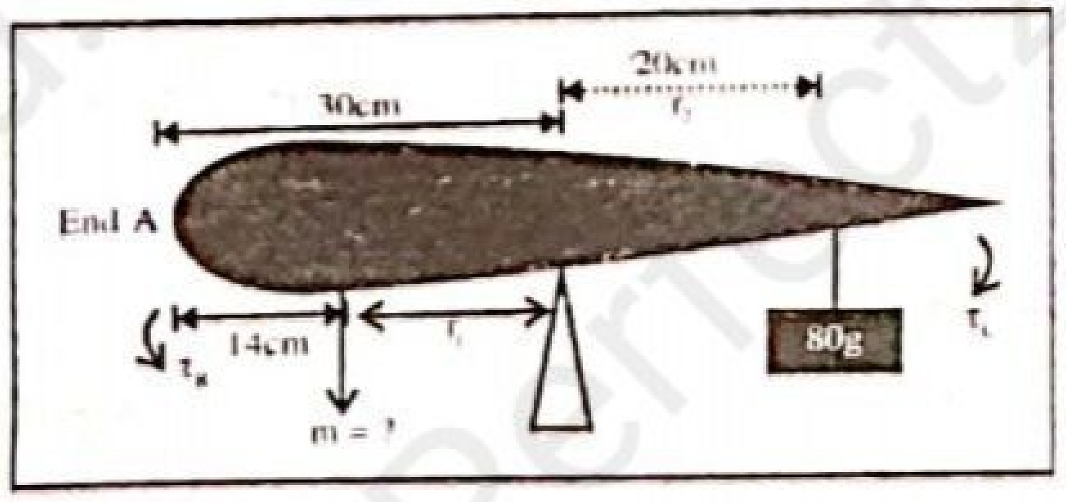
$\Rightarrow$ $192 - 32F_1 = 0 \Rightarrow -32F_1 = -192 \Rightarrow \boxed{F_1 = 6N}$

Putting $F_1 = 6N$ in equation (1), we get,

$F_2 = (16 - 6)N \Rightarrow \boxed{F_2 = 10N}$

Thus the two forces are (6N, 10N).

12. Find the mass of the uneven rod shown in the figure. If its center of gravity is 14cm from end A is.........



✓a. 100g b. 150g c. 80g d. 5g

Sol: Mass of rod= $m_1 = ?$, $r_1 = 30cm - 14cm = 16cm$

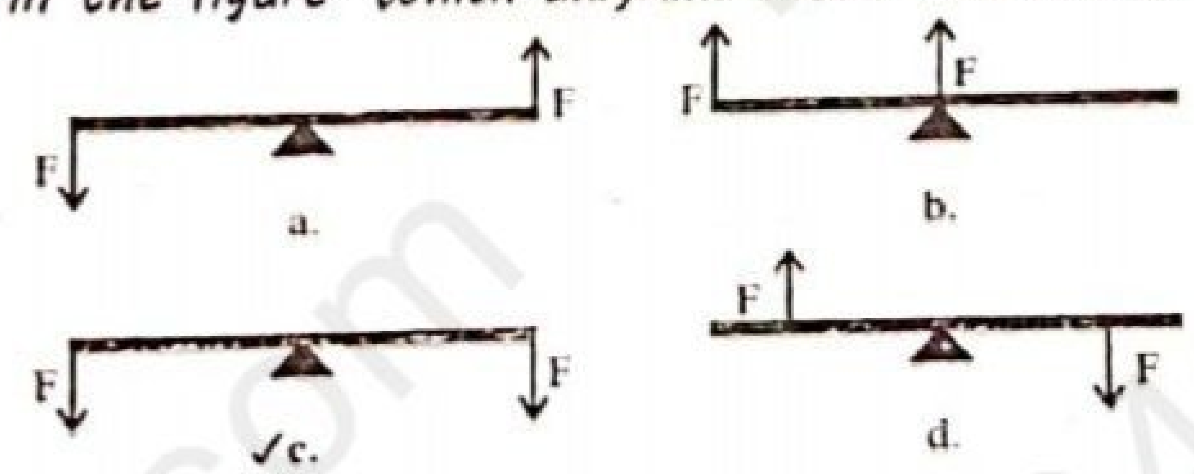
Mass of object $= m_2 = 80gm, r_2 = 20cm$

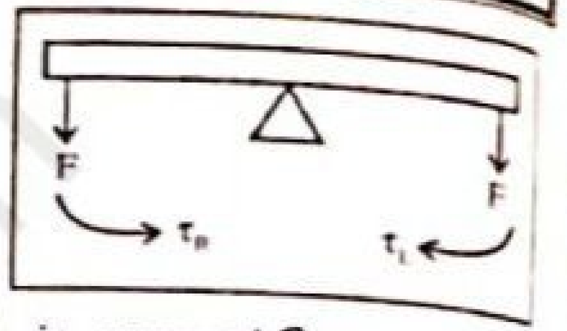
For balance condition, we have, $\tau_{right} = \tau_{left}$

$\Rightarrow$ $w_1\times r_1 = w_2\times r_2 \Rightarrow m_1gr_1 = m_2gr_2 \Rightarrow m_1r_1 = m_2r_2$

$\Rightarrow$ $m_1 = \dfrac{m_2r_2}{r_1} = \left(\dfrac{80\times20}{16}\right)gm = 100gm \Rightarrow \boxed{m_1 = 100gm}$

13. The following diagrams show a uniform rod with its midpoint on the pivot. Two equal forces "F" are applied on the rod, as shown in the figure. Which diagram shows the rod in equilibrium?





Sol: The torque produce towards left balance the torque produce towards right i.e. $\tau_R = \tau_L$ or $\sum \tau = 0$

14. For which angle the equation $|\vec{A} \bullet \vec{B}| = |\vec{A} \times \vec{B}|$ is correct?

a. 30° ✓ b. 45° c. 60° d. 90°

Sol: $|\vec{A} \bullet \vec{B}| = AB\cos\theta = AB\cos 45° = AB(0.707) = 0.707AB$

$\Rightarrow$ $|\vec{A} \bullet \vec{B}| = 0.707AB$ ________(1)

Now $|\vec{A} \times \vec{B}| = AB\sin\theta AB\sin 45° = AB(0.707) = 0.707AB$

$\Rightarrow$ $|\vec{A} \times \vec{B}| = 0.707AB$ ________(2)

Comparing equation (1), equation (2), we get,

$|\vec{A} \bullet \vec{B}| = |\vec{A} \times \vec{B}|$ for $\theta = 45°$

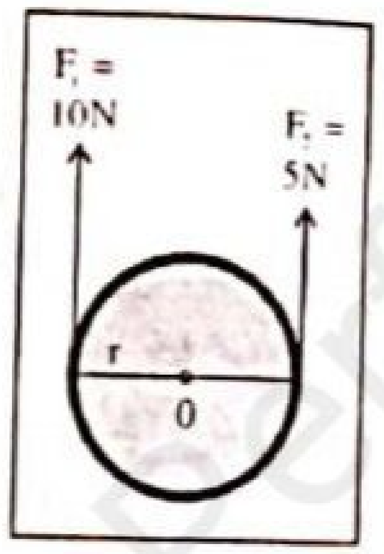
15. What is the net torque on wheel radius 2m as shown:

a. 10N anti-clockwise

b. 10Nm anti-clockwise

✓ c. 10Nm clockwise

d. 5Nm clockwise



Sol: Clockwise torque

$= \tau_c = r \times F_1 = 2 \times 10 = 20N{\cdot}m$

Anti-clockwise torque

$= \tau_{anti} = r \times F_2 = 2 \times 5 = 10N{\cdot}m$

Net torque

$= \tau_c - \tau_{anti} = 20N{\cdot}m - 10N{\cdot}m = 10N{\cdot}m$ clockwise

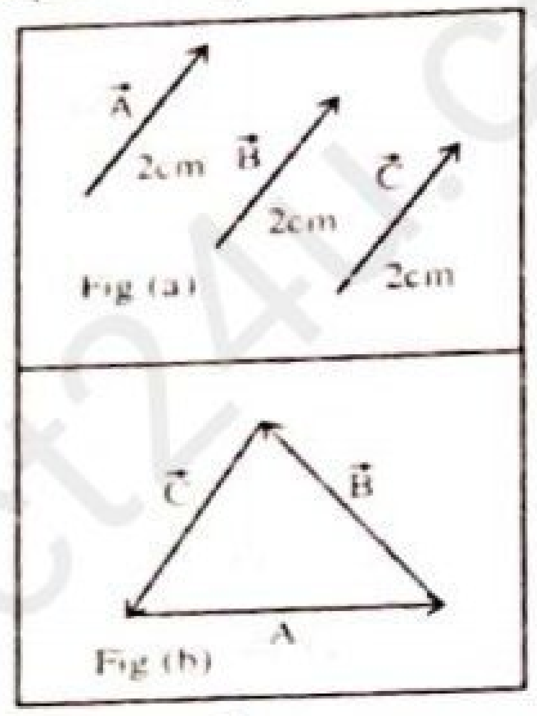
$\Rightarrow$ $\boxed{\tau_{net} = 10N{\cdot}m \text{ clockwise}}$

- قرآن مجید میں ارشاد ہے "اچھی بات کہنا اور درگزر کرنا ایسی خیرات سے بہتر ہے جس کے بعد آزار پہنچایا جائے۔"
- نبی کریم ﷺ کا ارشاد ہے "بُری مجلس سے احتراز کرو۔"
- نبی کریم ﷺ کا ارشاد ہے "انسان کی سمجھداری یہ ہے کہ وہ کفایت شعار ہو۔"
- نبی کریم ﷺ کا ارشاد ہے "مکر، دھوکہ اور خیانت کرنے والا دوزخ میں جائے گا۔"

Conceptual Questions

Q1: Is it possible to add three vectors of equal magnitude but in different directions to get a null vector? Illustrate with a diagram.

Ans. Yes, it is possible to add three vectors of equal magnitude but different in directions to get a null vector or zero vector.



Example and Explanation:

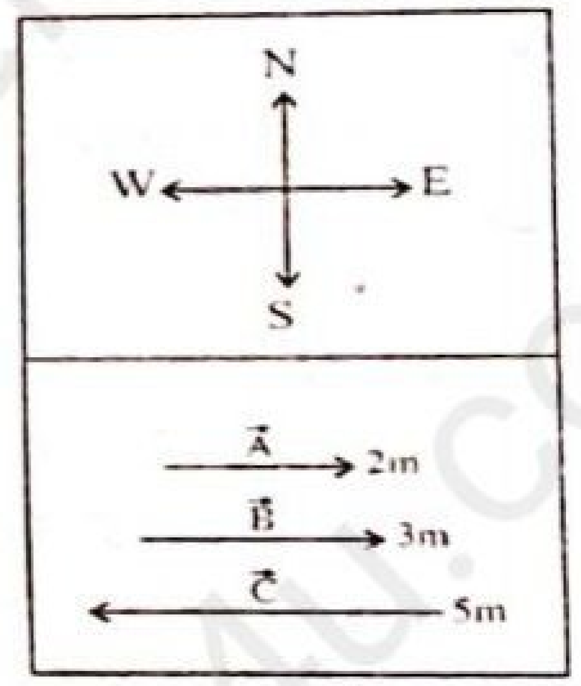
Consider three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ having same magnitude as shown in the figure (a). Let we add these vectors by head to tail rule. For this purpose we combine the head of $\vec{A}$ with the tail of $\vec{B}$. Then we combine the head of $\vec{B}$ with the tail of $\vec{C}$ as shown in the figure (b). Now for resultant, we will combine the tail of first vector [i.e. $\vec{A}$] to the head of last vector [i.e. $\vec{C}$]. But we see that the tail of $\vec{A}$ already touches the head of $\vec{C}$. We can put only a dot at their point of contact. So the resultant vector in this case will be a null vector or zero vector. Thus we have

$\vec{R} = \vec{A} + \vec{B} + \vec{C} = 0$

Conclusion: Thus it is possible to add three vectors of equal magnitude but different in directions to get a null vector.

Q2: The magnitudes of three vectors are 2m, 3m and 5m respectively. The directions are at your disposal. Can these vectors be added to yield zero? Illustrate with a diagram.

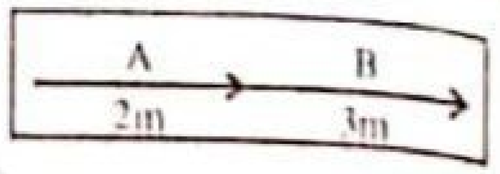
Ans. Yes, it is possible to add these vectors to yield zero or null vector.



Reason and Explanation:

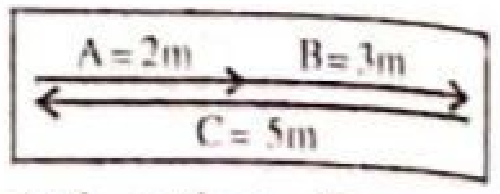
Consider three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ having magnitude 2m, 3m and 5m respectively. Let $\vec{A}$ and $\vec{B}$ are directed towards East and $\vec{C}$ is directed towards West as shown in the figure. Now by adding $\vec{A}$ and $\vec{B}$ according to head to tail rules, we get,

$\vec{A} + \vec{B} = 5m$ (East)



Now adding vector $\vec{C}$ with $\vec{A} + \vec{B}$ in opposite direction, i·e· towards West, we get

$\vec{A} + \vec{B} + (-\vec{C}) = 0$



Thus we get null vector or zero vector because the tail of $(\vec{A} + \vec{B})$ and head of $\vec{C}$ just touches each other· For resultant we can put only a dot which will give zero magnitude·

Mathematically we have,

$\vec{R} = \vec{A} + \vec{B} + (-\vec{C})$

$\Rightarrow \vec{R} = 2\hat{i}m + 3\hat{i}m + (-5\hat{i}m)$

$\Rightarrow \vec{R} = 5\hat{i}m - 5\hat{i}m$

$\Rightarrow \boxed{\vec{R} = 0}$

Which is thus proved·

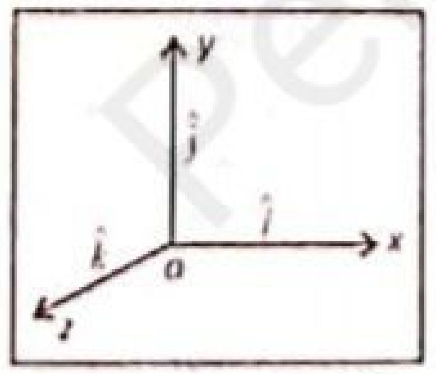
Q3· What units are associated with the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$?

Ans· The unit vectors $\hat{i}, \hat{j}, \hat{k}$ are unitless because they are used for the specification of directions only·

Reason and Explanation:

A unit vector in that vector whose magnitude is "1"· The unit vectors along x, y and z-axis are represented by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively·

The magnitude of $\hat{i}, \hat{j}$ and $\hat{k}$ is given by,



$|\hat{i}| = 1$ _____ (1)

$|\hat{j}| = 1$ _____ (2)

$|\hat{k}| = 1$ _____ (3)

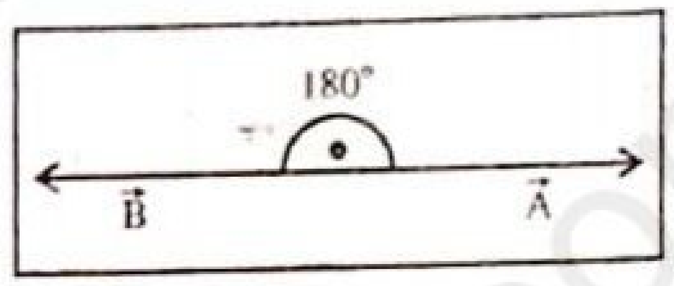
Thus no unit is associated with $\hat{i}, \hat{j}$ and $\hat{k}$· The unit vectors specifies the direction of the given vector only· It do not change the magnitude or dimensions of the vectors·

Example: Let there is a force of $20N\hat{i}$ i·e· $F = 20\hat{i}N$· Here the unit "N" is not the unit of "$\hat{i}$"· The unit "N" goes with magnitude and not with direction· Here "$\hat{i}$" tells us that the vector, $F = 20N$ is acting along x-axis· Similarly, $F = 30N\hat{j}$ tells us that force of 30N is acting along y-axis· Similarly, $F = 30N\hat{k}$ tells us that force of 30N is acting along z-axis· Thus the unit vectors $\hat{i}, \hat{j}, \hat{k}$ are used for the specification of direction only·

Q4: Can a scalar product of two vectors be negative? Provide a proof and give an example·

Ans· Yes a scalar product of two vectors may be negative if they are antiparallel·

Proof and Explanation:



Consider two vectors $\vec{A}$ and $\vec{B}$ making an angle of 180° with each other, as shown in the figure·

The scalar product of these two vectors is given by,

$\vec{A} \cdot \vec{B} = AB\cos\theta$ _____ (1)

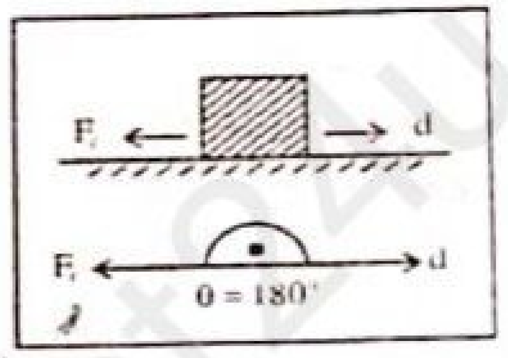
Putting $\theta = 180°$ in equation (1), we get,

$\vec{A} \cdot \vec{B} = AB\cos 180°\ AB(-1) = -AB$

$\Rightarrow \boxed{\vec{A} \cdot \vec{B} = -AB}$ _____ (2)

Which is the required proof· Equation (2) shows that the scalar product of two vectors will be negative if they are antiparallel i·e· the angle between them is 180°·

Example: Workdone against force of friction is given by,



$w = \vec{F}_f \cdot \vec{d}$

$\Rightarrow w = F_f d\cos\theta$

$\Rightarrow w = F_f d\cos 180° = F_f d(-1)$

$\Rightarrow \boxed{w = -F_f d}$ _____ (3)

Equation (3) shows that the scalar product of two vectors i·e· force of friction and displacement is negative· Thus the workdone against the force of friction is the example of negative scalar product·

Q5: $\vec{A}$ and $\vec{B}$ are two non-zero vectors· (a) How can their scalar product be zero? And (b) How can their vector product be zero?

Ans· The scalar product of two vectors $\vec{A}$ and $\vec{B}$ will be equal to zero if they are perpendicular to each other· While their vector product will be equal to zero if they are parallel or anti-parallel to each other·

Reason and Explanation:

(a) The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$|\vec{A}\cdot\vec{B}| = AB\cos\theta$ ______ (1)

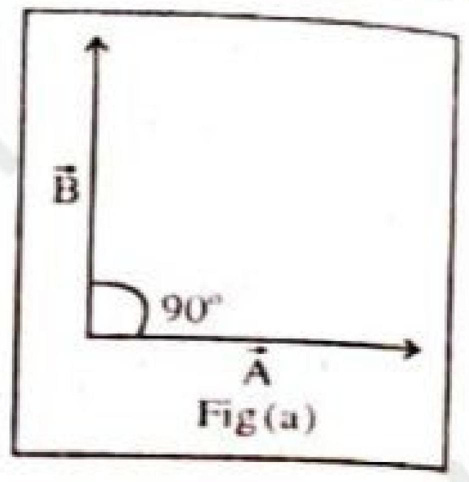
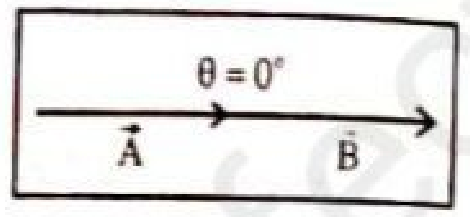


Fig (a)

As we have given that $\vec{A}$ and $\vec{B}$ are non-zero vectors, so for $|\vec{A}\cdot\vec{B}| = 0$, the other possibility is that we must put $\theta = 90°$. i·e· when the two vectors are perpendicular to each other· So equation (1) becomes,

$|\vec{A}\cdot\vec{B}| = AB\cos 90°\ AB \times 0 \Rightarrow \boxed{|\vec{A}\cdot\vec{B}| = 0}$



(b) The vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$|\vec{A}\times\vec{B}| = AB\sin\theta$ ________ (2)

As we have given that $\vec{A}$ and $\vec{B}$ are non-zero vectors, so for getting $|\vec{A}\times\vec{B}| = 0$ the value of θ must be "0" i·e· the two vectors must be parallel to each other· Put $\theta = 0°$ in equation (2), we get,

$|\vec{A}\times\vec{B}| = AB\sin 0°\ AB \times 0 \Rightarrow \boxed{|\vec{A}\times\vec{B}| = 0}$

Also if the two vectors are anti-parallel to each other, then we put $\theta = 180°$, so equation (2) becomes

$|\vec{A}\times\vec{B}| = AB\sin 180° = AB \times 0 = 0$

Q6: Suppose your are given a known non-zero vector A· The scalar product of A with an unknown vector B is zero· Likewise, the vector product of A with B is zero· What can you conclude about B?

Ans· In such a case vector $\vec{B}$ must be a null vector· i·e· $|\vec{B}| = 0$

Reason and Explanation:

Case-1: We know that the scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$|\vec{A}\cdot\vec{B}| = AB\cos\theta$ ______________ (1)

Equation (1) shows that the scalar product will be equal to zero, if $\vec{A}$ or $\vec{B}$ is a null vector or if the two vectors are perpendicular to each other· As we have given that $\vec{A}$ is non-zero vector, so if $|\vec{A}\cdot\vec{B}| = 0$, then $\vec{B}$ will be a null vector or both vectors will be perpendicular to each other $[\cos 90° = 0]$·

Case-2: We know that the vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$|\vec{A}\times\vec{B}| = AB\sin\theta$ ______________ (2)

According to equation (2), if $|\vec{A}\times\vec{B}| = 0$, then $\vec{A}$ will be a null vector, or $\vec{B}$ will be a null vector or the two vectors will be parallel to each other· But we have given that $\vec{A}$ is non-zero vector· So if $|\vec{A}\times\vec{B}| = 0$, then $\vec{B}$ will be a null vector or both vectors will be parallel to each other·

Conclusion about Case-1 and Case-2:

In case (1) there is condition of perpendicularly and case (2) there is condition of parallelism· These two conditions are contradictory to each other· That is we cannot imagine the two vectors to be perpendicular and parallel at the same time· So we ruled out the condition of perpendicularity and parallelism· So under such conditions, there is only one possibility that is vector $\vec{B}$ must be a null vector or zero vector·

Q7: Why a particle experiencing only one force cannot be in equilibrium?

Ans· A particle experiencing only one force cannot be in state of equilibrium, because it does not satisfied the condition of equilibrium·

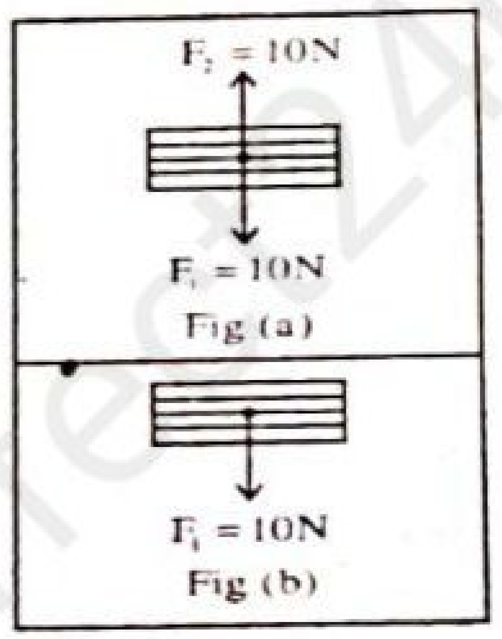


Fig (a)
Fig (b)

Reason and Explanation:

According to Newton's second law, we have,

$F = ma$ ______________ (1)

According to first condition of equilibrium, a body will be in state of equilibrium if the algebraic sum of forces acting on the body is equal to zero i·e·

$\Sigma F = 0$ __________ (2)

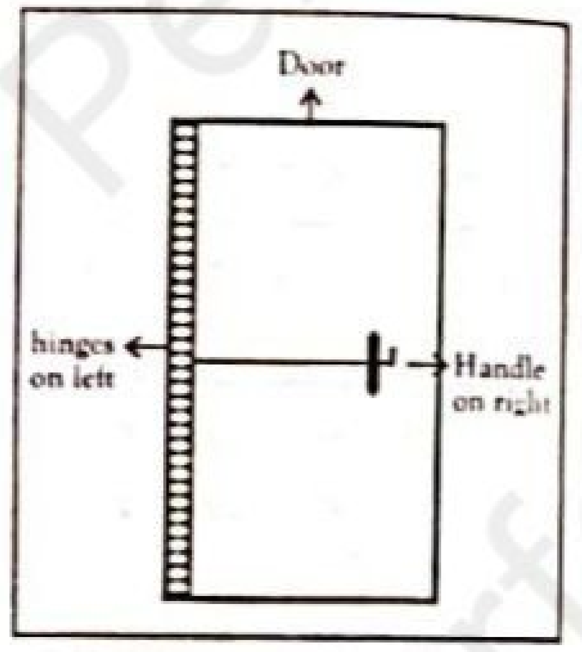
Equation (2) will be satisfied, if forces of equal magnitude are acting at the same line in opposite direction· Now in case of single force, equation (2) will not be satisfied· According to equation (1), such single force will produce certain acceleration in the body and as a result the body will not remain in state of equilibrium·

Example: In case figure (a) two equal and opposite forces are acting along the same line on a uniform body, so the body remain in state of equilibrium.

In case of figure (b), only a single force is acting on body. There is no equal and opposite force which balance the effect of "F_1". So the body will move with certain acceleration and thus will not remain in state of equilibrium.

Q8: To open a door that has the handle on the right and the hinges on the left a torque must be applied. (a) Is the torque clockwise or counterclockwise when viewed from above? (b) Does your answer depend on whether the door opens toward or away from you?

Ans. (a) The direction of the torque depends on whether the door opens outside or inside, when viewed from above. For example, if the door opens from inside of the room then anticlockwise torque will be produced.



(b) Yes the direction of torque depends on whether the door open towards or away from us.

Reason and Explanation:

Consider a door having handle on the right and hinges on the left as shown in the figure.

Case-1: When we pull the door towards us:

If we apply the force $\vec{F}$ on the door towards ourselves for opening, then according to figure, clockwise torque will be produced.

Case-2: When we push the door away from us:

If we apply the force $\vec{F}$ on the door away from ourselves for opening, then according to figure, counterclockwise torque will be produced.

Q9: Explain the warning, never use a large wrench to tighten a small bolt?

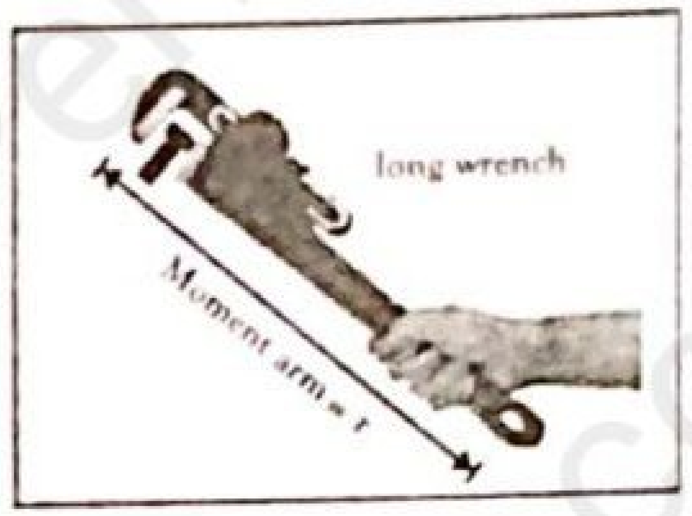
Ans. We should never use a large wrench to tighten a small bolt. It is because the bolt will be damaged due to large torque.

Reasons and Explanation:

We know that the magnitude of torque depends upon the magnitude of applied force, magnitude of moment arm and value of θ

between them i.e.

$\tau = rF\sin\theta$ ________ (1)



Equation (1) shows that, for given values of "F" and θ the torque depends upon moment arm 'r' i.e. greater the moment arm, greater will be the torque and vice versa.

Now if we tight a small bolt with large wrench, then greater torque will act on the bolt due large moment arm. The bolt may be damaged due to excessive torque. So we should avoid the use of large wrench to tighten a small bolt. To keep the bolt in safe condition, we should use a wrench of proper size.

Q10: A central force is one that is always directed towards the same point. Can a central force give rise to a torque about that point?

Ans. A central force cannot give rise to a torque because it is anti-parallel or parallel to the moment arm at that point.

Reasons and Explanation:

Consider a central force "$F_{central}$" is acting on a body at point "P" at a distance "r" from the origin "o" as shown in the figure.

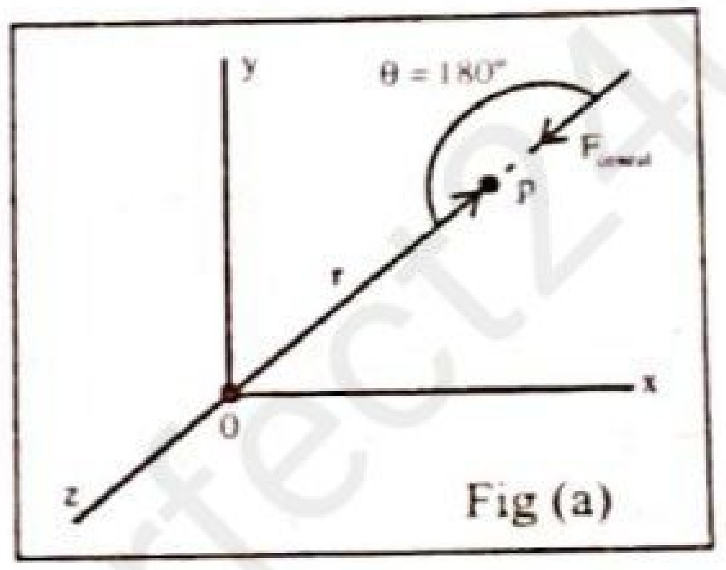


Fig (a)

Case-1:

We know that the torque acting on a body is given by,

$\tau = rF\sin\theta$

$\Rightarrow \tau = rF_{central}\sin\theta$ ________ (1)

Now in the given figure, we see that the angle between "$F_{central}$" and moment arm "r" is 180°, so equation (1) becomes

$\tau = rF_{central} \times \sin 180°$

$\Rightarrow \tau = rF_{central} \times 0 = 0$ $\quad [\sin 180° = 0]$

$\Rightarrow \boxed{\tau = 0}$ ________ (2)

Equation (2) shows that the central force produce no torque.

Case-2:

When "$F_{central}$" and "r" are parallel, then we put θ = 0° so, equation

(1) becomes,

$$\tau = rF\sin 0^\circ = rF \times 0 = 0$$

$$\Rightarrow \boxed{\tau = 0}$$

Answers 2: The central force produce no torque because the line of action of force passes through the pivot point.

Comprehensive Questions

Give extended response to the following questions:

Q1: How are vectors added and subtracted geometrically?

Ans. See answer of questions no 7 and question no 9 in the text.

Q2: If a vector is multiplied by a positive scalar, how is the result related to the original vector? What if the scalar is zero? Negative?

Ans. We can multiply a vector by any number.

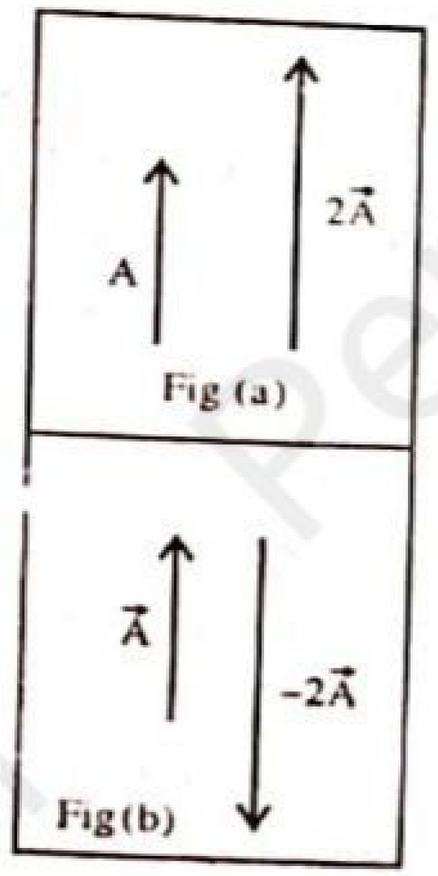


Fig (a)
Fig (b)

Case-1: If the number is positive, then the product vector and original vector will be acting along the same direction. For example in figure (a), the original vector $\vec{A}$ is multiplied with positive number "2". As a number we get a product vector which is twice than original vector and both are acting along the same direction.

Case-2: If we multiply a given vector with zero, then we will get a null vector or zero vector.

Case-3: If we multiply a given vector with negative number, then the original vector and product vector will be acting in opposite direction. In figure (b) an original vector $\vec{A}$ is multiplied with "-2". As a result we get a product vector which is twice than original vector but opposite in direction.

Q3: What are rectangular components of a vector? How rectangular components are used to represent a vector?

Ans. See answer of question no 11 in the text.

Q4: Explain addition of vectors by rectangular components.

Ans. See answer of questions no 12 in the text.

Q5: Define the dot product [scalar product] of two vectors. What geometric interpretation does the dot product have? Give examples.

Ans. See answer of question no 13 in the text.

Q6: Define the cross product [vector product] of two vectors. What geometric interpretation does the cross product have? Give examples.

Ans. See answer of question no 14 in the text.

Q7: Define torque. Show that torque is the vector product of force and position vector.

Ans. See answer of question no 15 in the text.

Q8: What is mechanical equilibrium? Explain different types of equilibrium.

Ans. See answer of question no 16 in the text.

Q9: What type of equilibrium is guaranteed by each condition of equilibrium?

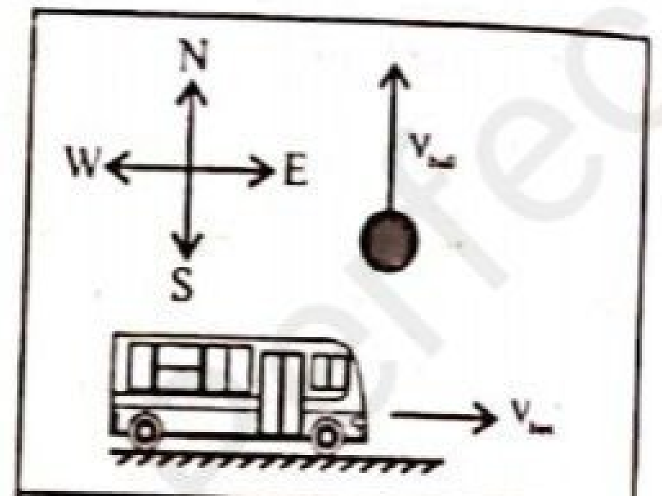
Ans. Static and dynamic equilibrium is guaranteed by each condition of equilibrium. For detail see "types of equilibrium" in equations no 16 in the text.

- ارشادِ نبوی ﷺ ہے "عورت کی عزت شریف الطبع ہی کرتے ہیں اور اس کی اہانت کمینے لوگوں کے سوا کوئی نہیں کرتا۔"
- ارشادِ نبوی ﷺ ہے "عورتوں میں سب سے اچھی عورت وہ ہے جسے اس کا شوہر دیکھے تو خوش ہو جائے۔"
- ارشادِ نبوی ﷺ ہے "اللہ کو ماننے کے بعد بہترین دانائی انسانوں سے محبت کرنا ہے۔"
- ارشادِ نبوی ﷺ ہے "دولت مند پر حسد نہ کرو دولت کی لذت فانی و عارضی ہیں۔"
- ارشادِ نبوی ﷺ ہے "اللہ کے نزدیک بہترین دوست وہ ہے جو اپنے دوست کا خیر خواہ ہو۔"

Numerical Problems

Q1: A person throws a ball straight up with a speed of 12m/s. If the bus is moving at 25m/s, what is the velocity of the ball to an observer on ground?

Solution:



Given Data:

Velocity of ball

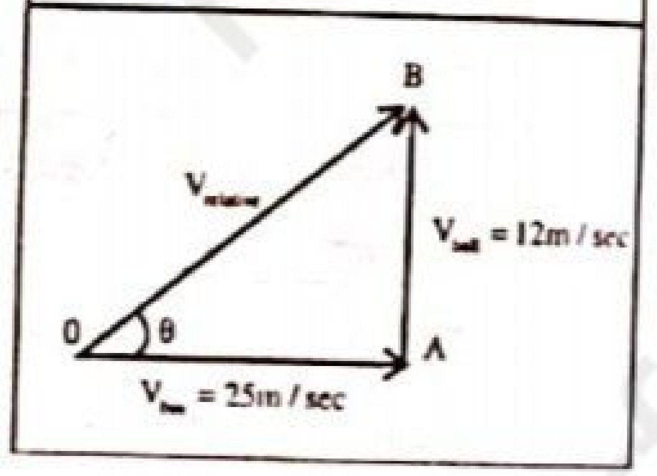
$= V_{ball} = 12m / sec$

Velocity of bus

$= V_{bus} = 25m / sec$

Required: Velocity of ball to an observer on ground

$= V_{relative} = ?$



Calculation: Let the ball is thrown straight North and the bus is moving East. Here we add the velocities of ball and bus by head to tail rule as shown in the figure. Now for finding the resultant velocity, we combine the tail of first vector $\vec{OA}$ with the head of last vector $\vec{AB}$. Thus $\vec{OB}$ gives us the resultant velocity or relative vector of the ball to an observer on ground. Now

$(OB)^2 = (OA)^2 + (AB)^2$

$\Rightarrow v_{rel}^2 = v_{bus}^2 + v_{ball}^2 = (25m / sec)^2 + (12m / sec)^2$

$\Rightarrow v_{rel}^2 = (625m^2 / sec^2) + (144m^2 / sec)^2$

$\Rightarrow v_{rel}^2 = 769m^2 / sec^2 \Rightarrow v_{rel} = \sqrt{769m^2 / sec^2}$

$\Rightarrow \boxed{v_{rel} = 27.7m / sec}$

OR $\boxed{v_{rel} = 28m / sec}$

Perpendicular = V_{ball}
Base = V_{bus}

Direction of V_{rel}:

As $\theta = \tan^{-1}\left(\frac{V_{ball}}{V_{bus}}\right)$ $\theta = \tan^{-1}\left(\frac{12}{25}\right) = 25.6°$

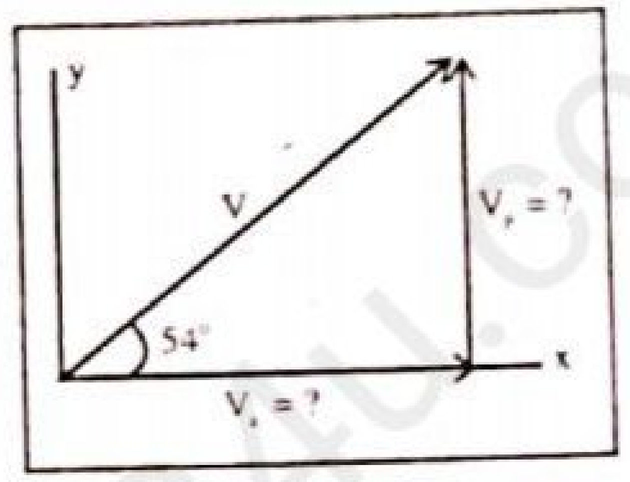
$\Rightarrow \boxed{\theta = 25.6°}$ or $\boxed{\theta = 26°}$

Thus the relative velocity of ball will make an angle of 25.6° with

horizontal [i.e. with direction of bus].

Q2: A football leaves the foot of a punter at an angle of 54° positive x-direction at a speed of 21m/s. Determine the horizontal and vertical components of the velocity.

Solution:



Given Data: Velocity of football

$= V = 21m / sec$

Angle of football with x-axis $= \theta = 54°$

Required:

(a) Horizontal component of velocity $= V_x = ?$

(b) Vertical component of velocity $= V_y = ?$

Calculation: (a) We know that,

$V_x = V\cos\theta$ ______ (1)

Putting the values in equation (1), we get,

$V_x = [21 \times \cos 54°]\, m / sec = [21 \times 0.588]\, m / sec$

$\Rightarrow \boxed{V_x = 12.4m / sec}$ or $\boxed{V_x = 12m / sec}$

(b) We also know that,

$V_y = V\sin\theta$ ______ (2)

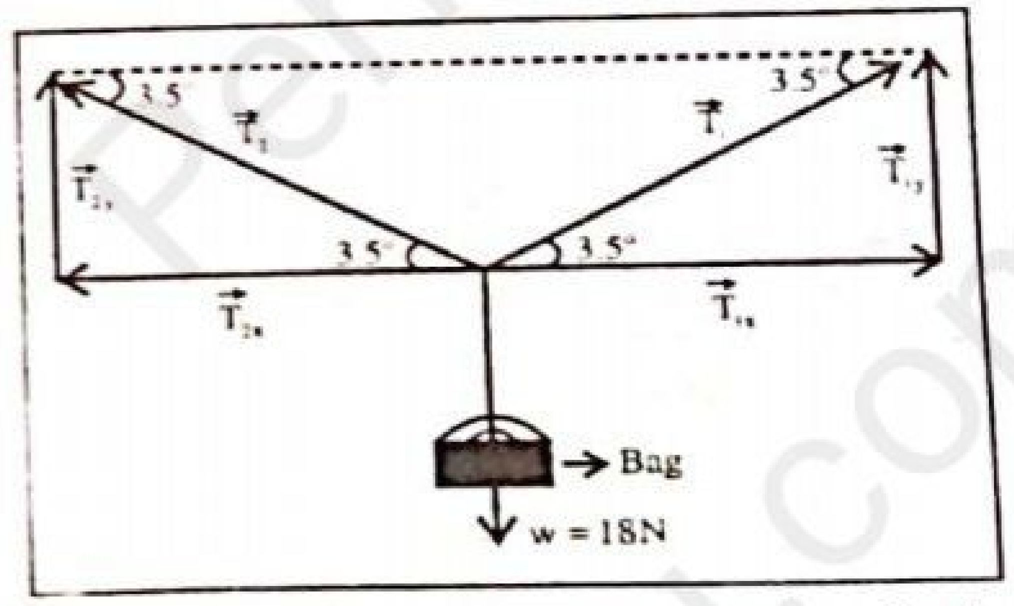
Putting the values in equation (2), we get,

$V_y = (21 \times \sin 54°)m / sec = (21 \times 0.809)m / sec$

$\boxed{V_y = 16.99m / sec}$ or $\boxed{V_y = 17m / sec}$

Q3: A 1.84kg school bag hangs in the middle of a clothesline, causing it to sag by an angle θ=3.50°. Find the tension, T in the clothesline.

Solution:



Given Data:

Mass of bag $= m = 1.84kg$

Weight of bag

$= w = mg = 1.84 \times 9.8N = 18N$

Angle $= \theta = 3.50°$

Required: Tension in the clothesline

Calculation: Let $T_1 = T_2 = T$ be the tension in each part of the cloth line. Here we resolve the tension in each part into their horizontal and vertical components, as shown in the figure. As the system is in equilibrium state, so we apply the first condition of equilibrium i.e.

$$\Sigma F_x = 0 \Rightarrow T_{1x} + (-T_{2x}) = 0 \Rightarrow T_{1x} = T_{2x} \text{ ____ } (1)$$

And $\Sigma F_y = 0 \Rightarrow \Sigma T_y = 0 \Rightarrow T_{1y} + T_{2y} + (-w) = 0$

$$\Rightarrow T_{1y} + T_{2y} - w = 0 \Rightarrow T_1 \sin\theta_1 + T_2 \sin\theta_2 = w$$

$$\Rightarrow T\sin\theta + T\sin\theta = w \Rightarrow 2T\sin\theta = w \qquad \begin{bmatrix} \theta_1 = \theta_2 = \theta \\ T_1 = T_2 = T \end{bmatrix}$$

$$\Rightarrow T = \frac{w}{2\sin\theta} = \left(\frac{18}{2 \times \sin 3.5^\circ}\right)N$$

$$\Rightarrow T = \left(\frac{18}{2 \times 0.061}\right)N = \left(\frac{18}{0.122}\right)N = 147.5N$$

$$\Rightarrow \boxed{T = 147.5N} \text{ or } \boxed{T = 148N}$$

Q4: Find the magnitude and direction of vector represented by the following pair of components. (a) $A_x = -2.3cm$, $A_y = 4.1cm$ (b) $A_x = 3.9m$, $A_y = -1.8m$.

Solution:

Given Data:

(a) $A_x = -2.3cm, A_y = +4.1cm$

(b) $A_x = +3.9m, A_y = -1.8m$

Required: Magnitude of vector in each case $= A = ?$

Direction of $\vec{A}$ in each case $= \theta' = ?$

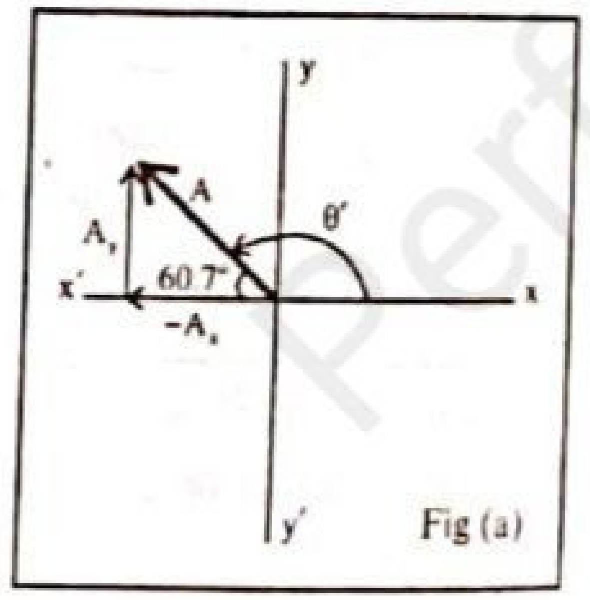


Fig (a)

Calculation: (a) We know that the magnitude of a vector is given by,

$$A = \sqrt{A_x^2 + A_y^2} \text{ ____ } (1)$$

Putting $A_x = -2.3cm$ and $A_y = +4.1cm$ in equation (1), we get,

$$A = \sqrt{(-2.3cm)^2 + (4.1cm)^2} = \sqrt{5.29cm^2 + 16.81cm^2}$$

$$\Rightarrow A = \sqrt{(5.29 + 16.81)cm^2} = \sqrt{22.1cm^2} = 4.7cm$$

$$\Rightarrow \boxed{A = 4.7cm}$$

We also know that,

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{4.1}{2.3}\right) = \tan^{-1}(1.78) \Rightarrow \boxed{\theta = 60.7^\circ}$$

As the x-component of $\vec{A}$ is negative and y-component is positive, so vector $\vec{A}$ lies in second quadrant as shown in figure (a). Thus the actual angle made by the vector $\vec{A}$ with positive x-axis is given by,

$$\theta' = 180^\circ - \theta \Rightarrow \theta' = 180^\circ - 60.7^\circ \Rightarrow \boxed{\theta' = 119.3^\circ}$$

Part (b): Now putting $A_x = +3.9m$ and $A_y = -1.8m$ in equation (1), we get,

$$A = \sqrt{(3.9m)^2 + (-1.8m)^2} = \sqrt{15.21m^2 + 3.24m^2}$$

$$\Rightarrow A = \sqrt{18.45m^2} \Rightarrow \boxed{A = 4.3m}$$

Now the direction of vector $\vec{A}$ is given by,

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1.8}{3.9}\right) \Rightarrow \boxed{\theta = 24.8}$$

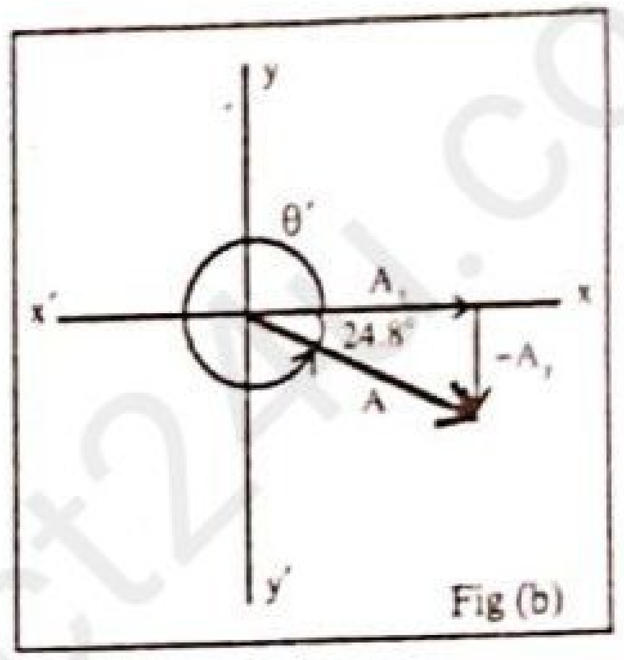


Fig (b)

As the x-component of $\vec{A}$ is positive and y-component is negative, so the vector $\vec{A}$ lies in fourth quadrant as shown in figure (b). Thus the actual angle made by the vector $\vec{A}$ with positive x-axis is given by,

$$\theta' = 360^\circ - \theta \Rightarrow \theta' = 360^\circ - 24.8^\circ$$

$$\Rightarrow \boxed{\theta' = 335.2^\circ}$$

Q5: Vector "F" having magnitude 5.5N makes 10° with x-axis and vector "r" with magnitude 4.3m makes 80° with x-axis. What is the magnitude of their dot and cross products?

Solution:

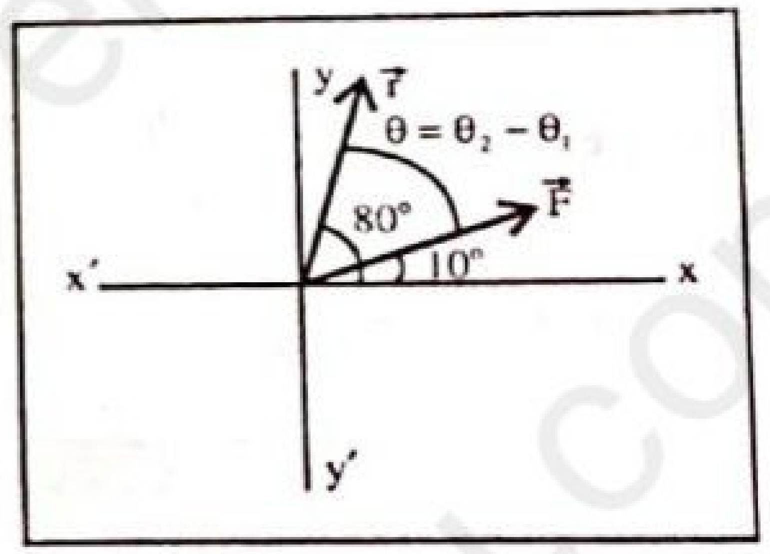
Given Data: Magnitude of vector $\vec{F} = F = 5.5N, \theta_1 = 10^\circ$

Magnitude of vector $\vec{r} = r = 4.3m, \theta_2 = 80^\circ$

Required:

(a) $|\vec{F} \cdot \vec{r}| = ?$

(b) $|\vec{F} \times \vec{r}| = ?$



Calculation: The angle between $\vec{F}$ and $\vec{r}$ is given by,

$$\theta = \theta_2 - \theta_1 = 80^\circ - 10^\circ \Rightarrow \theta = 70^\circ$$

(a) The magnitude of dot product of $\vec{F}$ and $\vec{r}$ is given by,

$$|\vec{F} \cdot \vec{r}| = Fr\cos\theta \text{ ____ } (1)$$

Putting the values in equation (1), we get,

$|\vec{F}\cdot\vec{r}| = 5.5N \times 4.3m \times \cos 70^\circ$

$= 5.5N \times 4.3m \times 0.342$

$\Rightarrow \boxed{|\vec{F}\cdot\vec{r}| = 8.09N\cdot m}$ or $\boxed{|\vec{F}\cdot\vec{r}| = 8.1N\cdot m}$

(b) The magnitude of cross product of $\vec{F}$ and $\vec{r}$ is given by,

$|\vec{F}\times\vec{r}| = Fr\sin\theta$ ______________(2)

Putting the values in equation (2), we get,

$|\vec{F}\times\vec{r}| = 5.5N \times 4.3m \times \sin 70^\circ = 5.5N \times 4.3m \times 0.939$

$\Rightarrow \boxed{|\vec{F}\times\vec{r}| = 22.2N\cdot m}$

Q6: The magnitude of dot and cross product of two vectors is $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.

Solution:

Given Data:

$|\vec{A}\cdot\vec{B}| = 6\sqrt{3},\ |\vec{A}\times\vec{B}| = 6$

Required:

Angle between two vectors $= \theta = ?$

Calculation: We know that

$|\vec{A}\cdot\vec{B}| = AB\cos\theta$ ______________(1)

And $|\vec{A}\times\vec{B}| = AB\sin\theta$ ______________(2)

Dividing equation (2) by equation (1), we get,

$$\frac{|\vec{A}\times\vec{B}|}{|\vec{A}\cdot\vec{B}|} = \frac{AB\sin\theta}{AB\cos\theta}$$

$$\Rightarrow \frac{|\vec{A}\times\vec{B}|}{|\vec{A}\cdot\vec{B}|} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\Rightarrow \tan\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}\cdot\vec{B}|} = \frac{6}{6\sqrt{3}}$$

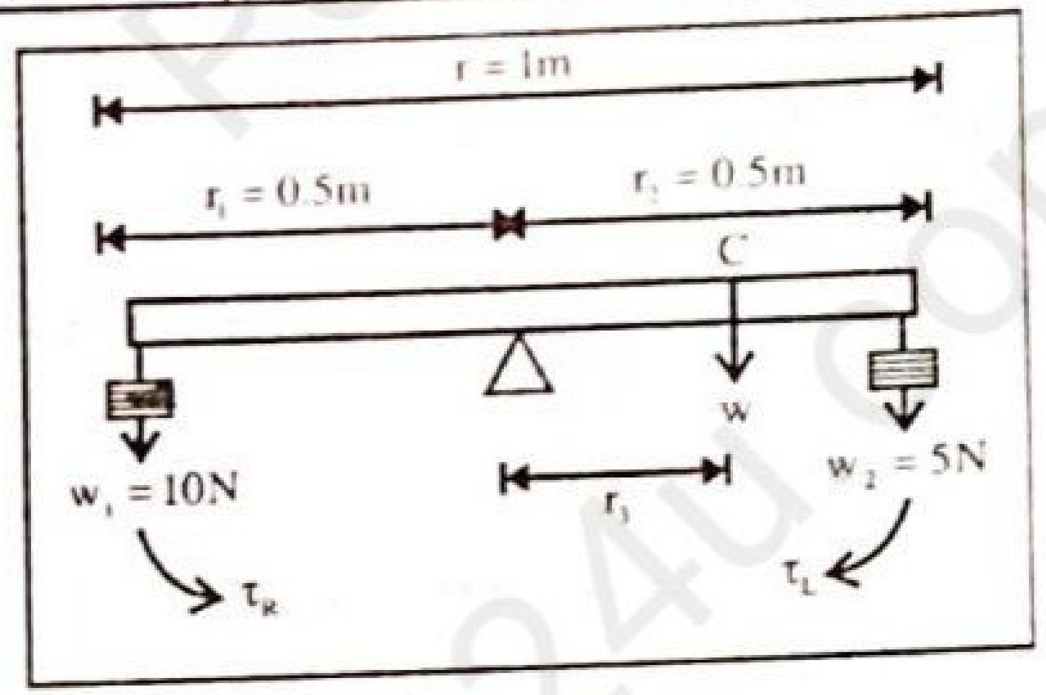
$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \boxed{\theta = 30^\circ}$$

Q7: A uniform rod 1m long with weight 6N can be supported in a horizontal position on a sharp edge with weights of 10N and 5N suspended from its ends. What is the position of point of balance?

Solution:



Given Data:

Length of rod $= r = 1m$

Weight of rod $= w = 6N$

Weight suspended from one end of rod $= w_1 = 10N$

Weight suspended from other end of rod $= w_2 = 5N$

Let the midpoint of the rod represents the pivot point, then

$r_1 = \left(\frac{1}{2}\right)m \Rightarrow r_1 = 0.5m$ and $r_2 = \left(\frac{1}{2}\right)m \Rightarrow r_2 = 0.5m$

Let the center of gravity is situated at a distance "r_3" from pivot point.

Required: Position of point of balance $= r_3 = ?$

Calculation: For balance condition, we have, $\tau_{right} = \tau_{left}$

$\Rightarrow \tau_R = \tau_L \Rightarrow w_1r_1 = wr_3 + w_2r_2$

$\Rightarrow wr_3 = w_1r_1 - w_2r_2 \Rightarrow r_3 = \dfrac{w_1r_1 - w_2r_2}{w}$ ______(1)

Putting the values in equation (1), we get,

$$r_3 = \left(\frac{10\times0.5 - 5\times0.5}{6}\right)m \Rightarrow r_3 = \left(\frac{5-2.5}{6}\right)m$$

$$\Rightarrow r_3 = \left(\frac{2.5}{6}\right)m = 0.416m$$

$\Rightarrow \boxed{\text{Position of point of balance} = r_3 = 0.42m}$

Q8: A 4.0m long uniform ladder with weight of 120N leans against a wall making 70° above a cement floor as shown in figure. Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

Solution:

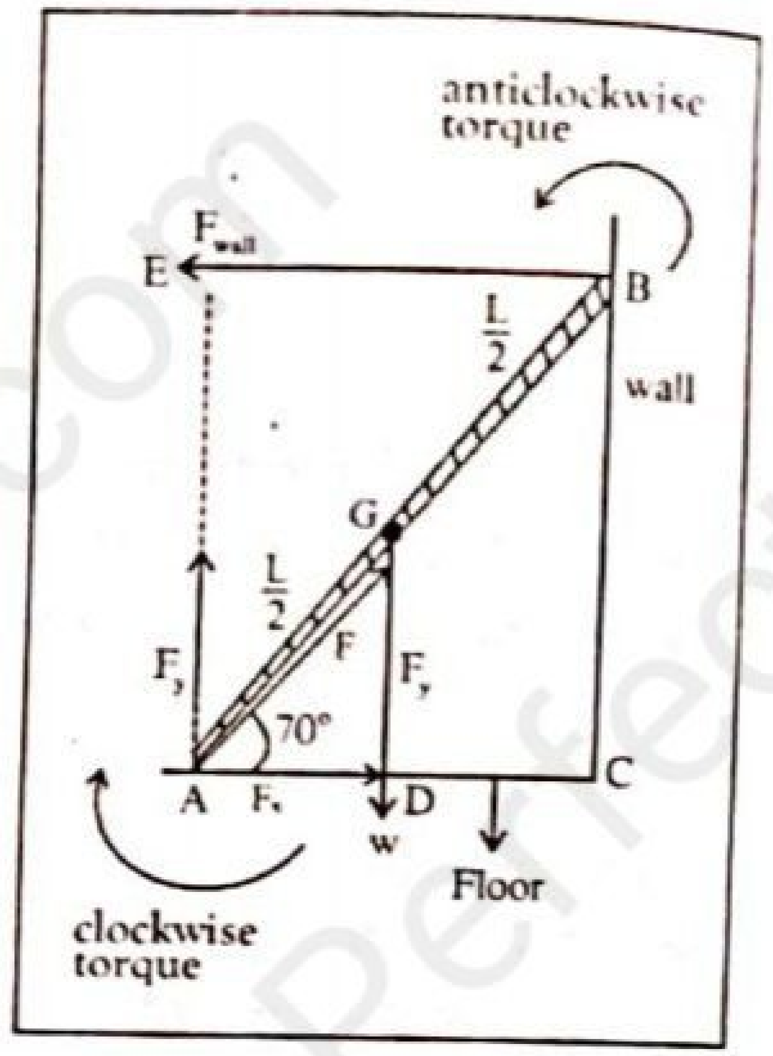
Given Data:

Length of ladder $= AB = L = 4m$

Weight of ladder $= w = 120N$

Centre of gravity from ground

$= AG = \frac{L}{2}$



Required: (a) Force of reaction on the ladder due to wall $= F_{wall} = ?$

(b) Force of reaction on the ladder due to ground $= F = ?$

Calculation: (a) According to first condition of equilibrium, we have,

$\Sigma F_x = 0 \Rightarrow F_x + (-F_{wall}) = 0$

$\Rightarrow F_x - F_{wall} = 0 \Rightarrow F_x = F_{wall}$ ______ (1)

& $\Sigma F_y = 0 \Rightarrow F_y + (-w) = 0 \Rightarrow Fy - w = 0$

$\Rightarrow F_y - 120N = 0 \Rightarrow F_y = 120N$ ______ (2)

Now applying second condition of equilibrium, we get,

Sum of anticlockwise moments = Sum of clockwise moments

$\Rightarrow$ ~~AE~~ $\times F_{wall} = AD \times w$

$\Rightarrow F_{wall} = \frac{AD \times w}{AE}$ ______________ (3)

In ΔAGD,
$\cos\theta = \frac{AD}{AG}$
$\Rightarrow AD = AG\cos\theta$

Now from the figure, we have, $AD = AG\cos\theta$

$\Rightarrow AD = \frac{L}{2}\cos 70° = \frac{4}{2}\cos 70° = 2\cos 70° = 2 \times 0.342 = 0.684$

$\Rightarrow AD = 0.684m$

$AG = \frac{1}{2}$
$L = 4m$

And $AE = CB = AB\sin\theta = L\sin 70° = 4 \times 0.939m$

$\Rightarrow AE = 3.756m$

Putting the values of "AD", "AE" and "w" in equation (3), we get,

$F_{wall} = \left(\frac{0.684 \times 120}{3.756}\right) N \Rightarrow \boxed{F_{wall} = 21.8N}$

In ΔABC,
$\sin\theta = \frac{BC}{AB}$
$\Rightarrow BC = AB\sin\theta$
$\Rightarrow BC = AE = AB\sin\theta$

Putting the value of "F_{wall}" in equation (1), we get,

$F_x = 21.8N$ ________ (4)

(b) Now $F = \sqrt{F_x^2 + F_y^2}$ ______ (5)

AB = length of Ladder = L

Putting equation (2) and equation (4) in equation (5), we get,

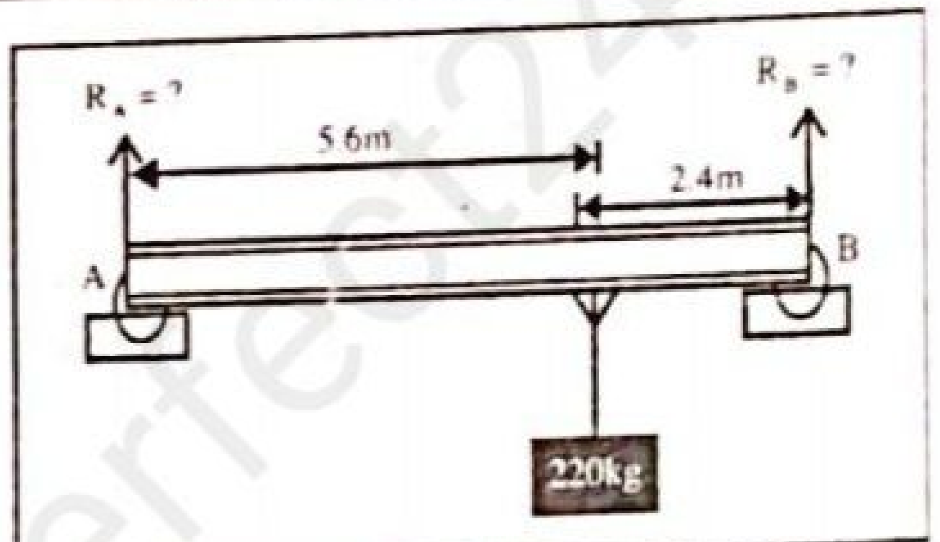
$F = \left[\sqrt{(21.8)^2 + (120)^2}\right] N$

$\Rightarrow F = \left[\sqrt{475.24 + 14400}\right] N = \left[\sqrt{14875.24}\right] N$

$\Rightarrow \boxed{F = 121.96N}$ or $\boxed{F = 122N}$

Q9: The 450kg uniform I-beam supports the load of 220kg as shown. Determine the reactions at the supports.

Solution:



Given Data:

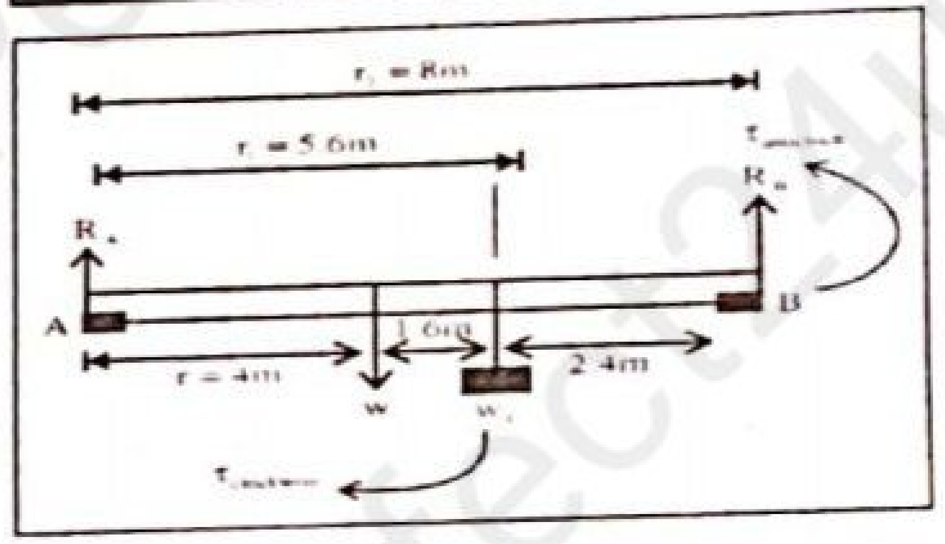
Mass of beam $= M = 450kg$

Weight of beam

$= w = mg = 450 \times 9.8N$

$\Rightarrow w = 4410N$

Mass of load $= m = 220kg$



Weight of load

$= w_1 = mg = 220 \times 9.8N$

$\Rightarrow w_1 = 2156N$

Moment arm of "w" $= r = 4m$

Moment arm of "w_1" $= r_1 = 5.6m$

Moment arm of "R_B" $= r_2 = 8m$

Required:

(a) Reaction at "B" $= R_B = ?$

(b) Reaction of "A" $= R_A = ?$

Calculation:

(a) Applying second condition of equilibrium, we get,

$\tau_{anticlock} = \tau_{clockwise}$

$\Rightarrow R_B \times r_2 = (w \times r) + (w_1 \times r_1)$

$$\Rightarrow R_B = \frac{(wr) + (w_1 r_1)}{r_2} \text{ ——— } (1)$$

Putting the values in equation (1), we get,

$$R_B = \left(\frac{(4410 \times 4) + (2156 \times 5{\cdot}6)}{8}\right)N = \left(\frac{17640 + 12073{\cdot}6}{8}\right)N$$

$$\Rightarrow R_B = \left(\frac{29713{\cdot}6}{8}\right)N \Rightarrow \boxed{R_B = 3714{\cdot}2N} \text{ or } \boxed{R_B = 3714N}$$

Now applying first condition of equilibrium, we get,

$$\Sigma F = 0 \Rightarrow R_A + R_B + (-w) + (-w_1) = 0$$

$$\Rightarrow R_1 + R_B - w - w_1 = 0 \Rightarrow R_1 = w + w_1 - R_B$$

$$\Rightarrow R_A = (4410 + 2156 - 3714)N = (6566 - 3714)N$$

$$\Rightarrow \boxed{R_A = 2852N}$$

+++

- ارشاد نبوی ﷺ ہے "جو شخص عیب جوئی کرتا ہے اور لوگوں پر آوازیں کستا ہے اس کیلئے بڑی تباہی ہے۔"
- ارشاد نبوی ﷺ ہے "وہ شخص کامل مومن نہیں ہو سکتا جو خود تو سیر ہو کر کھائے لیکن اس کا ہمسایہ بھوکا رہے۔"
- ارشاد نبوی ﷺ ہے "جو شخص جھوٹی قسم کھائے اپنا ٹھکانہ جہنم میں بنائے۔"
- ارشاد نبوی ﷺ ہے "تم میں سے اچھے لوگ وہی ہیں جو اپنی بیویوں کے ساتھ اچھا سلوک کرتے ہیں۔"
- ارشاد نبوی ﷺ ہے "دولت خرچ کرتے رہو روکو مت، ورنہ ہو سکتا ہے کہ اللہ بھی دولت کے دروازے تم پر بند کر دے۔"
- ارشاد نبوی ﷺ ہے "دو شخصوں پر رشک کرنا چاہیے، ایک وہ دولت مند جو اللہ کے راستے میں مال خرچ کرتا ہے، دوسرے وہ عالم جو علم کے ذریعے فیصلے کرتا ہے۔"
- ارشاد نبوی ﷺ ہے "اپنے گھر کی دیوار اتنی بلند نہ کرو کہ پڑوسی کی ہوا رک جائے۔"

CHAPTER 03 MOTION & FORCE

Q1: Define rest and motion?

Ans· (i) **Rest:**

A body is said to be at rest if it do not change its position with respect to the surrounding·

(ii) **Motion:**

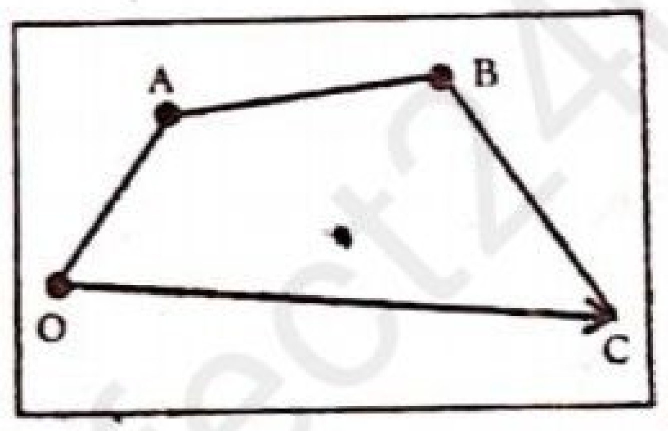
A body is said to be in motion if it changes its position with respect to the surrounding·

> **Point to Ponder:**
> When sitting on a chair, your speed is zero relative to earth. But you are moving at 30km/sec relative to the sun, because the earth is revolving around sun continuously.

Q2: Define distance and displacement?

Ans· (i) **Distance:**

The length of actual path traversed by a body in motion is known as distance· The actual path may be straight or curved· The distance is a scalar quantity· Its SI unit is meter·



(ii) **Displacement:**

The shortest distance between two points is known as displacement·

Explanation:

In given figure, a body reaches from point "O" to "C" through "A" and "B"· Thus the path "OABC" represents the actual path traversed by the body during its motion· So it is called distance· While $\vec{OC}$ represents the shortest distance between the initial point "O" and final point "C"· Thus $\vec{OC}$ is known as displacement· It is a vector quantity· Its SI unit is meter·

Q3: Define the following terms: position, speed, uniform speed, variable speed, average speed, instantaneous speed·

Ans· 1· **Position:**

The location of an object relative to some reference point [origin] is