

MARKING SCHEME MATHEMATICS MODEL PAER CLASS 9

SCORING KEYS SECTION: A (MCQs)

Marks: 15

MCQs No.	Correct Option
1	B
2	B
3	A
4	D
5	C
6	A
7	A
8	D
9	B
10	B
11	A
12	D
13	C
14	B
15	A

RUBRICS

SECTION- B	
Question No.2- Attempt any Eight of the following.	
i. If $A = \begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 7 \\ 9 & 2 \end{bmatrix}$ are matrices show that $A+B=B+A$.	
Checking Hints:	
Total Marks:4=1+1+1+1	
Solution:	
$A+B = \begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} -5 & 7 \\ 9 & 2 \end{bmatrix}$	
$A+B = \begin{bmatrix} 2-5 & 1+7 \\ 0+9 & 7+2 \end{bmatrix} \longrightarrow \textcircled{1}$	
$A+B = \begin{bmatrix} -3 & 8 \\ 9 & 9 \end{bmatrix} \longrightarrow \textcircled{1}$	
Now,	
$B+A = \begin{bmatrix} -5 & 7 \\ 9 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix}$	
$B+A = \begin{bmatrix} -5+2 & 7+1 \\ 9+0 & 2+7 \end{bmatrix} \longrightarrow \textcircled{1}$	
$B+A = \begin{bmatrix} -3 & 8 \\ 9 & 9 \end{bmatrix} \longrightarrow \textcircled{2}$	
From eq $\textcircled{1}$ and eq $\textcircled{2}$, it is proved: $A+B=B+A$. $\longrightarrow \textcircled{1}$	
ii. Find the product $(a-1)(a^2+a+1)$	
Checking Hints:	
Total Marks:4=1+1+1+1	
Solution:	
$(a-1)(a^2+a+1)$	
By Using Formula: $[(a-b)(a^2+ab+b^2)=a^3-b^3]$ $\longrightarrow \textcircled{1}$	
$(a-1)(a^2+(a)(1)+1^2)$ $\longrightarrow \textcircled{1}$	
$(a)^3-(1)^3 \longrightarrow \textcircled{1}$	
$a^3-1 \longrightarrow \textcircled{1}$	
iii. Factorize $4x^4+81$	
Checking Hints:	
Total Marks:4=1+1+1+1	
Solution:	
$(2x^2)^2 + (9)^2 + 2(2x^2)(9) - 2(2x^2)(9) \longrightarrow \textcircled{1}$	
$(2x^2+9)^2 - 36x^2 \longrightarrow \textcircled{1}$	
$(2x^2+9)^2 - (6x)^2 \longrightarrow \textcircled{1}$	
$(2x^2+9+6x)(2x^2+9-6x)$	
$(2x^2+6x+9)(2x^2-6x+9) \xrightarrow{\diamond (a+b)(a-b)=a^2-b^2} \textcircled{1}$	
iv. Divide $Z_1=2+3i$, by $Z_2=5-i$	
Checking Hints:	
Total Marks:4=1+1+1+1	
Solution:	
$\frac{Z_1}{Z_2} = \frac{2+3i}{5-i}$	
$\frac{Z_1}{Z_2} = \frac{2+3i}{5-i} \times \frac{5+i}{5+i} \longrightarrow \textcircled{1}$	
Multiplying and Dividing by(5+i)	

$$\begin{aligned} \frac{Z_1}{Z_2} &= \frac{10+2i+15i+3i^2}{5^2-i^2} \longrightarrow (1) \\ \frac{Z_1}{Z_2} &= \frac{10+17i+3(-1)}{25-(-1)} \longrightarrow (1) \quad \diamondsuit (i^2 = -1) \\ \frac{Z_1}{Z_2} &= \frac{10+17i-3}{25+1} \\ \frac{Z_1}{Z_2} &= \frac{7+17i}{26} \\ \frac{Z_1}{Z_2} &= \frac{7}{26} + \frac{17}{26}i \longrightarrow (1) \end{aligned}$$

v. If $x = \sqrt{3} - \sqrt{2}$, find the value of $x - \frac{1}{x}$.

Checking Hints:

Total Marks: 4=1+1+1+1

Solution:

$$\begin{aligned} x &= \sqrt{3} - \sqrt{2} \\ \frac{1}{x} &= \frac{1}{\sqrt{3}-\sqrt{2}} \\ \frac{1}{x} &= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} \longrightarrow (1) \\ \frac{1}{x} &= \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} \\ \frac{1}{x} &= \frac{\sqrt{3}+\sqrt{2}}{3-2} \longrightarrow (1) \\ \frac{1}{x} &= \frac{\sqrt{3}+\sqrt{2}}{1} \\ \frac{1}{x} &= \sqrt{3} + \sqrt{2} \longrightarrow (1) \end{aligned}$$

Now,

$$\begin{aligned} x - \frac{1}{x} &= \sqrt{3} - \sqrt{2} - (\sqrt{3} + \sqrt{2}) \\ x - \frac{1}{x} &= \sqrt{3} - \sqrt{2} - \sqrt{3} - \sqrt{2} \\ x - \frac{1}{x} &= -2\sqrt{2} \longrightarrow (1) \end{aligned}$$

vi. Find L.C.M by factorization of $x+y$, x^2-y^2

Checking Hints:

Total Marks: 4=1+1+1+1

Solution:

$$\begin{aligned} x + y &= x + y \longrightarrow (1) \\ x^2 - y^2 &= (x + y)(x - y) \longrightarrow (1) \\ \text{L.C.M} &= \text{Common Factors} \times \text{Non-Common Factors} \longrightarrow (1) \\ &= (x + y)x(x - y) \\ &= (x + y)(x - y) \\ \text{L.C.M} &= x^2 - y^2 \longrightarrow (1) \end{aligned}$$

vii. Sum of three consecutive integers is 39, find the integers

Checking Hints:

Total Marks:4=1+1+1+1

Solution:

$$1^{\text{st}} \text{ Positive Integer} = x$$

$$2^{\text{nd}} \text{ Positive Integer} = x+1$$

$$3^{\text{rd}} \text{ Positive Integer} = x+2$$

According to given Condition:

$$x + (x + 1) + (x + 2) = 39$$

$$x + x + x + 1 + 2 = 39$$

$$3x + 3 = 39$$

$$3x = 39 - 3$$

$$3x = 36$$

$$x = \frac{36}{3}$$

$$x = 12$$

$$1^{\text{st}} \text{ Positive Integer} = x=12$$

$$2^{\text{nd}} \text{ Positive Integer} = x+1=12+1=13$$

$$3^{\text{rd}} \text{ Positive Integer} = x+2=12+2=14$$

1

1

1

1

viii. Find the solution set of the equation $6x-5=2x+9$

Checking Hints:

Total Marks:4=1+1+1+1

Solution:

$$6x - 5 = 2x + 9$$

$$6x - 2x = 9 + 5$$

$$4x = 14$$

$$\frac{4x}{4} = \frac{14}{4}$$

$$x = \frac{7}{2}$$

$$\text{Solution Set} = \left\{ \frac{7}{2} \right\}$$

1

1

1

1

ix. Show that A(-1, 2), B(7, 5) and C(2,6) are the vertices of scalene triangle.

Checking Hints:

Total Marks:4=1+1+1+1

Solution:

$$|\overline{AB}| = \sqrt{[7 - (-1)]^2 + (5 - 2)^2}$$

$$|\overline{AB}| = \sqrt{[7 + 1]^2 + (3)^2}$$

$$|\overline{AB}| = \sqrt{8^2 + 9}$$

$$|\overline{AB}| = \sqrt{64 + 9}$$

$$|\overline{AB}| = \sqrt{73} \rightarrow \text{①}$$

Now,

1

$$|\overline{BC}| = \sqrt{(2-7)^2 + (6-5)^2}$$

$$|\overline{BC}| = \sqrt{(-5)^2 + (1)^2}$$

$$|\overline{BC}| = \sqrt{25+1}$$

$$|\overline{BC}| = \sqrt{26} \rightarrow \textcircled{2}$$

And,

$$|\overline{CA}| = \sqrt{(-1-2)^2 + (2-6)^2}$$

$$|\overline{CA}| = \sqrt{(-3)^2 + (-4)^2}$$

$$|\overline{CA}| = \sqrt{9+16}$$

$$|\overline{CA}| = \sqrt{25}$$

$$|\overline{CA}| = 5 \rightarrow \textcircled{3}$$

From eq $\textcircled{1}$, $\textcircled{2}$ and $\textcircled{3}$.

$$|\overline{AB}| \neq |\overline{BC}| \neq |\overline{CA}| \rightarrow \textcircled{1}$$

Hence points A, B, and C are the vertices of scalene triangle.

x. Prove that $\log_b pq = \log_b p + \log_b q$.

Checking Hints:

Total Marks: 4=1+1+1+1

Solution:

Let,

$$\log_b p = x \rightarrow \textcircled{1} \text{ and } \log_b q = y \rightarrow \textcircled{2} \rightarrow \textcircled{1}$$

In Exponential Form:

$$b^x = p \text{ and } b^y = q \rightarrow \textcircled{1}$$

Now,

$$pq = b^x \times b^y$$

$$pq = b^{x+y} \text{ or}$$

$$b^{x+y} = pq \rightarrow \textcircled{1}$$

Now in logarithmic form:

$$\log_b pq = x + y$$

$$\log_b pq = \log_b p + \log_b q \rightarrow \textcircled{1} \quad \spadesuit (\text{by eq } \textcircled{1} \text{ and } \textcircled{2})$$

xi. If two angles of a triangle are congruent then the sides opposite to them are also congruent.

Checking Hints:

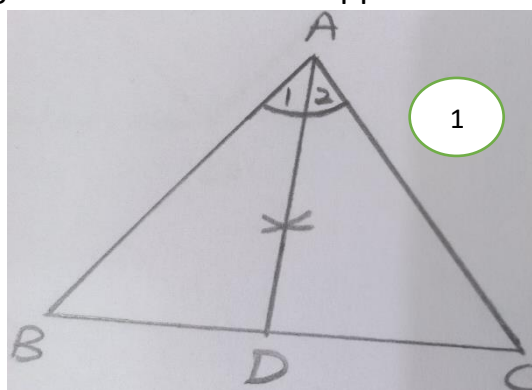
Total Marks: 4=1+1+1+1

Solution:

In $\triangle ABC$, $\angle B \cong \angle C$

To prove:

$$\overline{AB} \cong \overline{AC} \rightarrow \textcircled{1}$$



Construction:

Draw bisector of $\angle A$ cutting $\overline{BC}$ at point D.

Proof:

Statements	Reasons
In $\triangle ABD \leftrightarrow \triangle ACD$ $\overline{AD} \cong \overline{AD}$ $\angle 1 \cong \angle 2$ $\angle B \cong \angle C$ $\triangle ABC \cong \triangle ACD$	Common Side Construction Given A.A.S $\cong$ A.A.S
Hence $\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

xii. Prove that each diagonal of a parallelogram divides it into two congruent triangles.

Checking Hints:

Total Marks:4=1+1+1+1

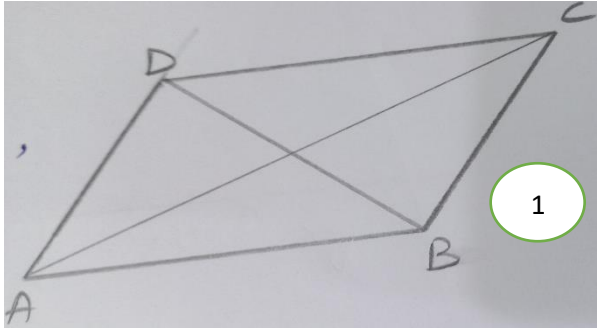
Solution:

In parallelogram ABCD
 $\overline{AC}$ and $\overline{BD}$ are two diagonals.

To Prove:

$\triangle ABC \cong \triangle ACD$
& $\triangle ABD \cong \triangle BCD$

Proof:



Statements	Reasons
In $\triangle ABC \leftrightarrow \triangle ADC$ $\overline{AC} \cong \overline{AC}$ $\overline{AB} \cong \overline{CD}$ $\overline{AD} \cong \overline{BC}$ $\triangle ABC \leftrightarrow \triangle ADC$	Common Side Opposite sides of the parallelogram Opposite sides of the parallelogram S.S.S $\cong$ S.S.S
Hence Diagonal $\overline{AC}$ divides parallelogram ABCD into two congruent triangles $\triangle ABC$ and $\triangle ADC$. Similarly diagonal $\overline{BD}$ divides the parallelogram ABCD into two congruent triangles $\triangle ABD$ and $\triangle BCD$.	

SECTION-C

Attempt any 4 of the following.

Q2. The bisectors of angles of triangle are concurrent.

Checking Hints:

Total Marks:6=1+1+1+1+1+1

Solution:

Given:

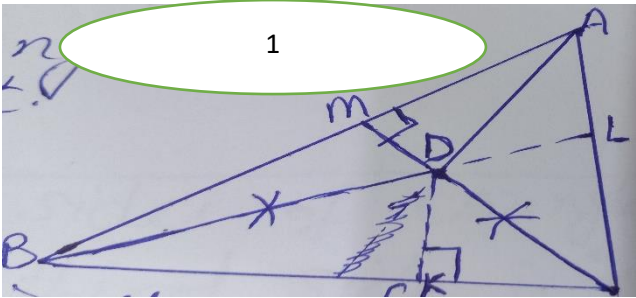
$\overline{BD}$ and $\overline{CD}$ are the bisectors of $\angle B$ and $\angle C$ of $\triangle ABC$, which intersect each other at D. D is joined to A.

To Prove:

$\overline{DA}$ is the bisector of $\angle A$.

Construction:

From D, draw $\overline{DK} \perp \overline{BC}$ and $\overline{DL} \perp \overline{CA}$ and $\overline{DM} \perp \overline{AB}$.



Proof:

Statements	Reasons
Since D lies on the bisector of $\angle B$.	Given
$\overline{DK} \cong \overline{DM} \rightarrow \textcircled{1}$	Distance of D from the arms of $\angle B$.
Similarly, D lies on the bisector of $\angle C$.	
$\overline{DK} \cong \overline{DL} \rightarrow \textcircled{2}$	Distance of D from the arms of $\angle C$.
Hence	
$\overline{DL} \cong \overline{DM}$	From $\textcircled{1}$ and $\textcircled{2}$.
i.e. D lies on the bisector of $\angle A$. OR AD is the bisector of $\angle A$. OR the bisectors of the angles of the angles of the Triangle ABC are concurrent.	D is equidistant from L and M.

Q3. The lengths of two sides of triangle are 11 and 23 and the length of third side is X. Find the range of possible values of X.

Checking Hints:

Total Marks: 6=1+1+1+1+1+1

Solution:

We use triangle inequality theorem to write and solve inequalities.

Small Values	Large Values
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$$\begin{aligned} x+11 &> 23 \\ x &> 23-11 \\ x &> 12 \end{aligned}$$

$$\begin{aligned} 11+23 &> x \\ 34 &> x \\ x &< 34 \end{aligned}$$

The length of the third side must be greater than 12 and less than 34.

Q4. If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to third side.

Checking Hints:

Total Marks: 6=1+1+1+1+1+1

Solution:

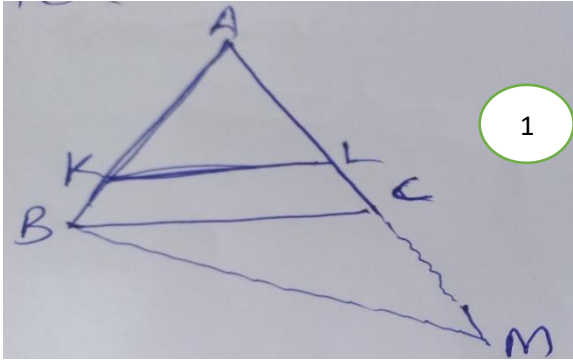
Given:

$\overleftrightarrow{KL}$ intersects two sides $\overline{AB}$ and $\overline{AC}$ of $\triangle ABC$ at K and L such that:

$$\frac{m\overline{AK}}{m\overline{KB}} = \frac{m\overline{AL}}{m\overline{LC}}$$

To Prove:

$$\overline{KL} \parallel \overline{BC}$$



Proof:

Statements	Reasons
Suppose $\overleftrightarrow{KL}$ is not parallel to $\overline{BC}$, then let $\overline{BM}$ be drawn through B, parallel to $\overline{KL}$, meeting $\overline{AC}$ at M.	
$\frac{m\overline{AK}}{m\overline{KB}} = \frac{m\overline{AL}}{m\overline{LM}} \longrightarrow \textcircled{1}$	Proportional segments are cut by line parallel to one side of the triangle.
But	
$\frac{m\overline{AK}}{m\overline{KB}} = \frac{m\overline{AL}}{m\overline{LC}} \longrightarrow \textcircled{2}$	Given
$\frac{m\overline{AL}}{m\overline{LM}} = \frac{m\overline{AL}}{m\overline{LC}}$	From $\textcircled{1}$ and $\textcircled{2}$.

$m\overline{LM} = m\overline{LC}$ This is possible only when C and M coincide, hence. Hence our supposition is wrong and $\overline{KL} \parallel \overline{BC}$.	Denominators of equal fractions.
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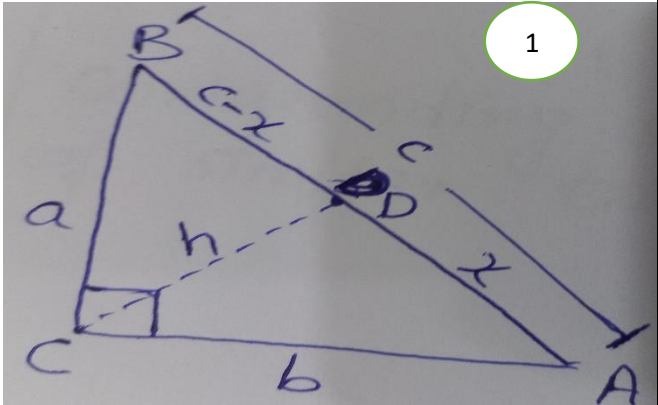
Q5. In a right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Checking Hints:

Total Marks:6= 1+1+ 1
+1+1+1

Solution:

Given
ABC is a right-angled triangle,
having right angle at ‘C’,
where ‘a’, ‘b’ and ‘c’ are the
measures of sides $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$ respectively.



To Prove:

$$c^2 = a^2 + b^2$$

Construction:

Draw $\overline{CD} \perp \overline{AB}$,let $m\overline{CD}=h$ and $\overline{AD}=x$, therefore $\overline{BD}=c-x$

Proof:

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta CDA$ $\angle BCA \cong \angle CDA$ $\angle A \cong \angle A$	Right Angles Self-Congruent
And $\angle ABC \cong \angle ACD$ $\Delta BCA \cong \Delta CDA$	Complement of $\angle A$ By Definition
Hence $\frac{c}{b} = \frac{b}{x}$	Corresponding Sides of similar triangles.
i.e $cx= b^2 \longrightarrow \textcircled{1}$	
Again in $\Delta ABC \leftrightarrow \Delta BDC$ $\angle BCA \cong \angle BDC$ $\angle B \cong \angle B$	Right Angles Self-Congruent
And $\angle CAB \cong \angle DCB$ $\Delta BCA \cong \Delta BDC$	Complement of $\angle B$ by definition.
Hence	

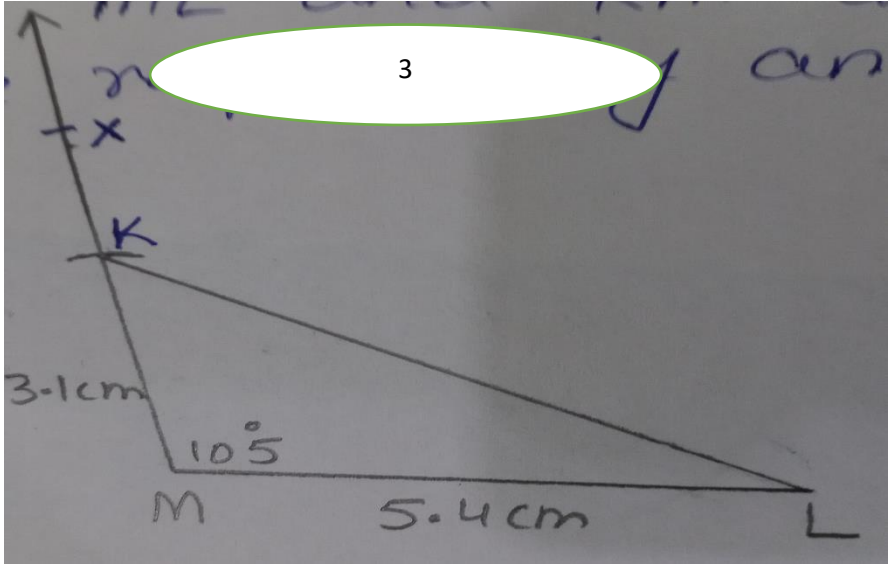
$\frac{c}{a} = \frac{a}{c-x}$ $c(c-x) = a^2$ $c^2 - cx = a^2 \rightarrow \textcircled{2}$ Adding $\textcircled{1}$ and $\textcircled{2}$ we get, $cx + c^2 - cx = b^2 + a^2$ $c^2 = a^2 + b^2$	Corresponding of similar angles.
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Q8: Construct triangle **KML** when length of its two sides **ML** and **KM** are 5.4 cm and 3.1 cm respectively and **m < M = 105°**

Checking Hints:

Total Marks: 6=1+1+1+1+1+1

Solution:



Steps of Construction:

1. Draw a line segment m $\overline{ML}=5.4\text{cm}$.
2. Draw an angle m $\angle M=105^\circ$
3. Taking M as center, draw an arc of 3.1cm which cuts $\overrightarrow{MX}$ at point K.
4. Join point K and L.

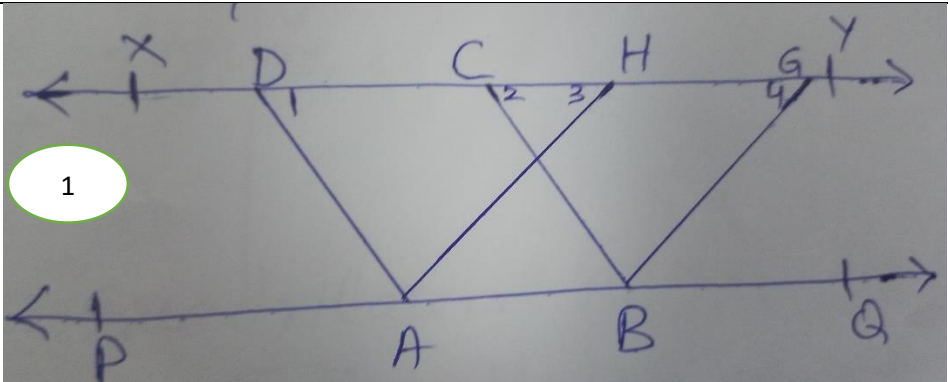
ΔKLM is formed which the required triangle.

Q9: Parallelogram on the same base and lying between the same parallel lines (or of the same altitude) are equal in area.

Checking Hints:

Total Marks: 6=1+1+1+1+1+1

Solution:



Given:
ABCD and ABGH are two parallelograms having the same base $\overline{AB}$ and lying between two parallel lines $\overleftrightarrow{XY}$ and $\overleftrightarrow{PQ}$ (i.e., both have same altitude).

To Prove:
Parallelogram ABCD and ABGH are equal in area.

Statements	Reasons
In $\triangle ADH \leftrightarrow \triangle BCG$ $\angle AH \cong \angle BG$ $\angle 1 \cong \angle 2$ $\angle 3 \cong \angle 4$ $\therefore \triangle ADH \cong \triangle BCG$ Or $\triangle ADH$ and $\triangle BCG$ are equal in area. Subtracting each triangle from the whole figure ABDG, we get $\text{Quadrilateral ABDG} - \triangle ADH \cong \text{Quadrilateral ABDG} - \triangle BCG$. Parallelogram ABCD $\cong$ Parallelogram ABGH Or Both the Parallelograms are equal in area.	Opposite sides of Parallelogram Corresponding Angles Corresponding Angles A.A.S $\cong$ A.A.S