Exercise 8.1

1- *If* $A = \{1,4,7,8\}, B = \{4,6,8,9\}$ *and* $C = \{3,4,5,7\}$

find:

(i) $A \cup B$ (ii) $B \cup C$

(iii) $A \cap C$ (iv) $A \cap (B \cap C)$

(v) $(A \cup B) \cup C$ (vi) $(A \cap B) \cap C$

Solution:1(i)

$$A = \{1, 4, 7, 8\}$$
$$B = \{4, 6, 8, 9\}$$
$$A \cup B = ?$$
$$A \cup B = \{1, 4, 7, 8\} \cup \{4, 6, 8, 9\}$$
$$= \{1, 4, 6, 7, 8, 9\}$$

Solution:1(ii)

Taking common elements once a time

$$B = \{4, 6, 8, 9\}$$
$$C = \{3, 4, 5, 7\}$$
$$B \cup C = ?$$
$$B \cup C = \{4, 6, 8, 9\} \cup \{3, 4, 5, 7\}$$
$$B \cup C = \{3, 4, 5, 6, 7, 8, 9\}$$

Solution:1(iii)

$$A = \{1, 4, 7, 8\}$$
$$C = \{3, 4, 5, 7\}$$
$$A \cap C = ?$$
$$A \cap C = \{4, 7\}$$

Solution:1(iv)

$$A = \{1, 4, 7, 8\}$$
$$B = \{4, 6, 8, 9,\}$$
$$C = \{3, 4, 5, 7\}$$
$$A \cap (B \cap C) = ?$$
$$B = \{4, 6, 8, 9\}$$
$$C = \{3, 4, 5, 7\}$$

$B \cap C = \{4, 6, 8, 9\} \cap \{3, 4, 5, 7\}$
$= \{4\}$
$A \cap (B \cap C) = \{1, 4, 7, 8\} \cap \{4\}$
$= \{4\}$

Solution:1(v)

$(A \cup B) \cup C = ?$

$A = \{1, 4, 7, 8\}$
$B = \{4, 6, 8, 9\}$
$C = \{3, 4, 5, 7\}$
$A = \{1, 4, 7, 8\}$
$B = \{4, 6, 8, 9\}$
$A \cup B = \{1, 4, 7, 8\} \cup \{4, 6, 8, 9\}$
$A \cup B = \{1, 4, 7, 8, 9\}$
$(A \cup B) \cup C = \{1, 4, 6, 7, 8, 9\} \cup \{3, 4, 5, 7\}$
$(A \cup B) \cup C = \{1, 3, 4, 5, 6, 7, 8, 9\}$

Solution:1(vi)

$(A \cap B) \cap C = ?$

$A = \{1, 4, 7, 8\}$
$B = \{4, 6, 8, 9\}$
$C = \{3, 4, 5, 7\}$
$A = \{1, 4, 7, 8\}$
$B = \{4, 6, 8, 9\}$
$A \cap B = \{1, 4, 7, 8\} \cap \{4, 6, 8, 9\}$
$= \{4\}$
$(A \cap B) \cap C = \{4\} \cap \{3, 4, 5, 7\}$
$= \{4\}$

2- *If* $A = \{1, 7, 11, 15, 17, 21\}, B = \{11, 17, 19, 23\}$ ***and*** $C = \{2, 3, 5\}$ ***verify that:***

$(A \cap B) \cap C = A \cap (B \cap C)$

Solution: L.H.S

$(A \cap B) \cap C = ?$

$A \cap B = \{1,7,11,15,17,21\} \cap \{11,17,19,23\}$

$= \{11,17\}$ (iv)

$(A \cap B) \cap C = \{11,17\} \cap \{2,3,5\}$ (iv),(iii)

$= \{\ \}$

$= \phi$ empty set (No element is common)

R.H.S

$= A \cap (B \cap C) = ?$

$B \cap C = \{11,17,19,23\} \cap \{2,3,5\}$

$= \{\ \}$ (v) *No element is common*

$A \cap (B \cap C) = \{1,7,11,15,17,21\}\ \{\cap\}$ (i)(v)

$= \{\ \}$

$= \phi$

L.H.S = R.H.S

$(A \cap B) \cap C = A \cap (B \cap C)$

3- *If A = {2, 4, 6}, B = {3, 6, 9, 12} and C = {4, 6, 8, 10} verify that:*

$$A \cup (B \cup C) = (A \cup B) \cup C$$

Solution:

$(A \cup B) \cup C = ?$

$A \cup B = \{2, 4, 6\} \cup \{3, 6, 9, 12\}$

$= \{2, 3, 4, 6, 9, 12\}$

Taking common elements once a time

$(A \cup B) \cup C = \{2, 3, 4, 6, 9, 12\} \cup \{4, 6, 8, 10\}$

$= \{2, 3, 4, 6, 8, 9, 10, 12\}$

$A \cup (B \cup C) = ?$

$(B \cup C) = \{2,4,6\} \cup \{3,4,6,8,9,10,12\}$

$= \{3,4,6,8,9,10,12\}$

$A\cup(B\cup C) = \{2,4,6\}\cup\{3,4,6,8,9,10,12\}$

$= \{2,3,4,6,8,9,10,12\}$

L.H.S =R.H.S

4- *If* $A=\{2,3,5,7,9\}$, $B=\{1,3,5,7\}$ *and* $C=\{2,3,4,5,6\}$ *verify that:*

$(A\cap B)\cap C = A\cap(B\cap C)$

$(A\cap B)\cap C = ?$

$(A\cap B) = \{2, 3, 5, 7, 9\}\cap\{1, 3, 5, 7\}$

(Only common elements) $= \{3, 5, 7\}$

$(A\cap B)\cap C = \{3, 5, 7\}\cap\{2, 3, 4, 5, 6\}$

(Only common elements)

$= \{3, 5\}$

$A\cap(B\cap C) = ?$

$A\cap B = \{1, 3, 5, 7\}\cap\{2, 3, 4, 5, 6\}$

(Only common elements)

$= \{3, 5\}$

$A\cap(B\cap C) = \{2,3,5,7,9\}\cap\{3, 5\}$

$A\cap(B\cap C) = \{3, 5\}$

$A\cap(B\cap C) = (A\cap B)\cap C$

5- *If* $U=\{7,8,9,10,11,12,13,14\}$

$A=\{7,10,13,14\}$ *and* $B=\{7,8,11,12\}$ *verify*

$(A\cap B)' = A'\cup B'$

Solution:-

$U = \{7, 8, 9, 10, 11, 12, 13\}$

$A = \{7,10,13,14\}$

$B = \{7,8,11,12\}$

(Only common elements)

$A \cap B = \{7, 10, 13, 14\} \cap \{7, 8, 11, 12\}$

$A \cap B \quad \{7\}$

$(A \cap B)' \equiv \cup - (A \cap B)$

$= \{7,8,9,10,11,12,13,14\} - \{7\}$

$= \{8, 9, 10, 11, 12, 13, 14\}$

$A' \cup B'$

$A = \{7,10,13,14\}$

First we find $A' = u - B$

$= \{7,8,9,10,11,12,13,14\} - \{7,10,13,14\}$

elements of A are left

$= \{8,9,11,12\}$

Now $B = \{7,8,11,12\}$

$B' = u - A$

$= \{7,8,9,10,11,12,13,14\} - \{7,8,11,12\}$

elements of B are left

$B' = \{9,10,13,14\}$

$A' \cup B'$

$= \{8,9,11,12\} \cup \{9,10,13,14\}$

$= \{8,9,10,11,12,13,14\}$ (M)

$(A \cap B)' = A' \cup B'$

6- $U = \{4,6,8,9,10\}$, $A = \{4,6\}$ and $B = \{6,8,9\}$ **then verify De Morgan's Law**

Solution:-

First we write De Morgan's Laws

(i) $(A \cup B)' = A^c \cap B^c$

(ii) $(A \cap B)' = A' \cup B'$

(i) $u = \{4,6,8,9,10\}$

(ii) $A=\{4,6\}$
(iii) $B=\{6,8,9\}$

1st Law

$$(A\cup B)^c = A^c \cup B^c$$

L.H.S $A\cup B = \{4,6\}\cup\{6,8,9\}$
$= \{4,6,8,9\}$ (iv)

Now $(A\cup B)^c = u-(A\cup B)$
$= \{4,6,8,9,10\}-\{4,6,8,9\}$
$= \{10\}$

$A^c = u-A$
From (i) and (ii) $= \{4,6,8,9,10\}-\{4,6\}$
$= \{8,9,10\}$

$B^c = u-B$
$= \{4,6,8,9,10\}-\{6,8,9\}$
$B^c = \{4,10\}$

$A^c \cap B^c = \{8,9,10\}\cap\{4,10\}$
$= \{10\}$

Result (v) and (viii)

$$(A\cup B)^c = A^c \cap B^c$$

$$(A\cap B)^c = A^c \cup B^c$$

2nd Law

L.H.S $A = \{4,6\}$
$B=\{6,8,9\}$

$A\cap B = \{4,6\}\cap\{6,8,9\}$
$= \{6\}$

Now $(A\cap B)^c = u-(A\cap B)$
$= \{4,6,8,9,10\}-\{6\}$

$=\{4,8,9,10\}$

$A^c = u - A$

R.H.S $=\{4,6,8,9,10\}-\{4,6\}$

$=\{8,9,10\}$

and $B^c = u - B$

$=\{4,6,8,9,10\}-\{6,8,9\}$

$=\{4,10\}$

$A^c \cup B^c = \{8,9,10\} \cup \{4,10\}$

$P = \{4,8,9,10\}$

Proved

$$(A \cap B)^c = A^c \cup B^c$$

7- **If** $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,3,6,9\}$ **and** $B = \{1,3,6,7,8\}$ **then verify** $(A \cup B)' = A' \cap B'$

L.H.S

$A \cup B = \{2,3,6,9\} \cup \{1,3,6,7,8\}$

$=\{1,2,3,6,7,8,9\}$

$(A \cup B)' = u - (A \cup B)$

$=\{1,2,3,4,5,6,7,8,9\}-\{1,2,3,6,7,8,9\}$iv,i

$=\{4,5\}$

R. H. S $A' = u - A$

$=\{1,2,3,4,5,6,7,8,9\}-\{2,3,6,9\}$

$=\{1,4,5,7,8\}$

$B' = u - B$

$=\{1,2,3,4,5,6,7,8,9\}-\{1,3,6,7,8\}$

$B' = \{2,4,5,9\}$

Now $A' \cap B' = \{1,4,5,7,8\} \cap \{2,4,5,9\}$

$=\{4,5\}$....

$(A \cup B)' = A' \cap B'$

L. H. S = R. H. S

8- *Fill in the blanks:*

(i) $A \cup A =$ ________ (ii) $A \cap A =$ ________

(iii) $A \cup \Phi =$ ________ (iv) $A \cap \Phi =$ ________

(v) $\Phi \cup \Phi =$ ________ (vi) $(A \cap B)' =$ ________

(vii) $(A \cup B)' =$ ________ (viii) $(A')' =$ ________

(ix) $\phi \cap \phi' =$ ________ (x) $A \cap A' =$ ________

Answers:

(i) A *(ii)* A *(iii)* A *(iv)* ϕ *(v)* ϕ

(vi) $A' \cup B'$ *(vii)* $A' \cap B'$ *(viii)* A *(ix)* ϕ *(x)* ϕ

Exercise 8.2

1- *If* $A=\{3,5,6\},\ B=\{1,3\}$, ***Find*** $A\times B$ ***and*** $B\times A$ ***and also the domains and range of the two binary relations established at our own for each case.***

Solution:-

$$A=\{3,5,6\}$$
$$B=\{1,3\}$$
$$A\times B=\{3,5,6\}\times\{1,3\}$$
$$=\{(3,1),(3,3),(5,1),(5,3),(6,1),(6,3)\}$$
$$R_1=\{(3,1),(5,3)\}$$
$$Dom\ R_1=\{3,5\}$$
$$Range\ R_1=\{1,3\}$$

Let $R_2=\{(3,1),(3,3),(5,3)\}$

$$Dom\ R_2=\{3,5\}$$
$$Range\ R_2=\{1,3\}$$

Now $A=\{3,5,6\}$

$$B=\{1,3\}$$

Then $B\times A=\{1,3\}\times\{3,5,6\}$

$$=\{(1,3),(1,5),(1,6),(3,3),(3,5),(3,6)\}$$

Let $R_3=\{(1,3),(3,3)\}$

$$Dom\ R_3=\{1,3\}$$
$$Range\ R_3=\{3\}$$

Let $R_4=\{(1,5),(1,6),(3,5),(3,6)\}$

$$Dom\ R_4=\{1,3\}$$
$$Range\ R_4=\{5,6\}$$

2- *If* $A=\{-2,1,4\}$, ***then write two binary relations A also write their domains and range.***

Solution:-

$$A=\{-2,1,4\}$$
$$A\times A=\{-2,1,4\}\times\{-2,1,4\}$$

$= \{(-2,-2),(-2,1),(-2,4),(1,-2),(1,1),(1,4),$
$(4,-2),(4,1),(4,4)\}$

$R_1 = \{(-2,-2),(1,-2),(4,4)\}$

Now $Dom\ R_1 = \{-2,1,4\}$

$Range\ R_1 = \{-2,4\}$

Let $R_2 = \{(-2,1),(-2,4),(1,1),(4,1)\}$

Now $Dom\ R_2 = \{-2,1,4\}$

$Range\ R_2 = \{1,4\}$

3- *Write the number of binary relations possible in each of following cases.*

(i) In $C \times C$ when the number of elements in C is 3.

(ii) In $A \times B$ if the number of elements in set A is 3 and in set B is 4.

Solution:3(ii)

Number of elements C $= 3$

Number of Binary relation $C \times C = ?$

Number of elements $C \times C = 3 \times 3 = 9$

Formula

If number of elements in characteristic in cartesian product is 'n' then number of binary relation will be 2^n

Number of Binary relation $C \times C = 2^9$

Number of elements of $A = 3$

Number of elements of $B = 4$

Number of elements $A \times B = 3 \times 4 = 12$

Number of Binary relation $A \times B = 2^{12}$

4- *If* $L = \{1,2,3\}$ ***and*** $M = \{2,3,4\}$ ***then, write a binary relation R such that*** $R = \{(x,y) \setminus x \in L, y \in M \wedge y \leq x\}$. ***Also write Dom (R) and Range (R).***

Solution:-

$L = \{1,2,3\}$

$M = \{2,3,4\}$

$R = \{(x,y) / x \in L, y \in M \wedge y \leq x\}$

Now $R = \{(x,y) / x \in L, y \in M \wedge y \leq x\}$

The explanation of this condition is that we have to write such kind of binary relation, whose first elements are the number of L. And in the 2nd condition is that, second elements should be member of M. According to the condition 1st elements are smaller or equal.

Now $L \times M = \{1,2,3\} \times \{2,3,4\}$

$$= \begin{Bmatrix} (1,2),(1,3),(1,4),(2,2),(2,3), \\ (2,4),(3,2),(3,3),(3,4) \end{Bmatrix}$$

$R = \{(2,2),(3,2),(3,3)\}$

Dom $R = \{2,3\}$

Range $R = \{2,3\}$

5- If $X = \{0,3,5\}$ and $Y = \{2,4,8\}$, then establish any four binary relations in $X \times Y$.

Solution:-

$X \times Y = \{0,3,5\} \times \{2,4,8\}$

$= \{(0,2),(0,4),(0,8),(3,2),(3,4),$
$(3,8),(5,2),(5,4),(5,8)\}$

Now $R_1 = \{(0,2)\}$

$R_2 = \{(0,8),(3,4)\}$

$R_3 = \{(3,8)\}$

$R_4 = \{(0,4),(3,2),(3,8),(5,2),(5,8)\}$

6- ***If*** $A = \{a,b,c\}$ ***and*** $B = \{2,4,6\}$ ***and*** $f = \{(a,4),(b,4),(c,4)\}$ ***is a binary relation***

from $A \times B$, then show that 'f' is A into B function.

Solution:- Proof

$A = \{a, b, c\}$
$B = \{2, 4, 6\}$
$f = \{(a, 4), (b, 4), (c, 4)\}$
$Dom\ f = \{a, b, c\} = A$

i) *The first condition of function is proved.*

ii) *There is no repetition in the first element of ordered pairs contained in "f". i.e. a, b, c is not repeated.*
The second condition of function is proved.
The "f" is called a function from A to B and expressed as $f : A \rightarrow B$.

$Range\ f = \{4\}$

$Range\ f = \{4\} \neq B$

$Range\ f = \{4\} \subset C$ *Hence 'f' A into B is function*

7- ***If $A = \{l, m, n\}$ and $B = \{1, 2, 3\}$ and $g = \{(l, 3), (m, 1), (n, 1)\}$ is a binary relation from $A \times B$, then show that 'g' is A into B function.***

Solution:-

$A = \{l, m, n\}$
$B = \{1, 2, 3\}$
$g = \{(l, 3), (m, 1), (n, 1)\}$
$Dom\ g = \{l, m, n\} = A$

i) *The first condition of function is proved.*

ii) *There is no repetition in the first element of ordered pair contained in "g". i.e. l, m, n is not repeated.*

iii) *The second condition of "f" is proved.*
Then "g" is called function from A to B and expressed as $f: A \rightarrow B$.

Range $g = \{3, 1\}$

Range $g = \{3,1\} \neq B$

Range $g = \{3,1\} \subset B$

But Hence 'g' is A into B function.

8- ***If*** $A = \{1,3,5\}$ ***and*** $B = \{x, y, z\}$ ***and*** $g = \{(1,x),(3,y),(5,z)\}$ ***is a binary relation from*** $A \times B$***, then show that 'g' is A onto B function.***

Solution:-

$A = \{1,3,5\}$

$B = \{x,y,z\}$

$g = \{(1, x),(3, y),(5, z)\}$

Dom $g = \{1,3,5\} = A$

i) *The first condition of function is proved.*
There is no repetition in the first element of ordered pair contained in 'g' i.e. 1, 2, 3 is not repeated.
Thus second condition of function is proved.

Range $g = \{x,y,z\}$

Range $g = \{x,y,z\} = B$

Thus "g" is A on to B function.
Hence "g" is called function from A to B and expressed as $f: A \rightarrow B$.