## Exercise 7.1

**1- Write the first three terms of the following:**

*(i)* $a_n = n + 3$ *(ii)* $a_n = (-1)^n n^3$ *(iii)* $a_n = 3n + 5$

*(iv)* $a_n = \frac{n+1}{2n+5}$ *(v)* $a_n = \frac{1}{(2n-1)^2}$ *(vi)* $a_n = n + 3 = 2$

*(vii)* $a_n = \frac{1}{3^n}$ *(viii)* $a_n = 3n - 5$ *(ix)* $a_n = (n+1)a_{n-1}, a_1 = 1$

***Solution:(i)***

$$a_n = n + 3$$

*Putting values* $n = 1, 2, 3$

$$a_1 = 1 + 3 = 4$$
$$a_2 = 2 + 3 = 5$$
$$a_3 = 3 + 3 = 6$$

First three terms 4,5,6

***Solution:(ii)***

$$a_n = (-1)^n n^3$$

*Putting values* $n = 1, 2, 3$

$$a_1 = (-1)^1(1)^3$$
$$= (-1)(1) = -1$$
$$a_2 = (-1)^2(2)^3$$
$$= (+1)(8) = 8$$
$$= (+1)(8) = 8$$
$$a_3 = (-1)^3(3)^3$$
$$= (-1)(27) = -27$$

First three terms $-1, 8 -27$

***Solution:(iii)***

$$a_n = 3n + 5$$

Putting $n = 1, 2, 3$

$$a_1 = 3(1) + 5$$
$$= 3 + 5 = 8$$
$$a_2 = 3(2) + 5$$
$$= 6 + 5 = 11$$
$$a_3 = 3(3) + 5$$
$$= 9 + 5 = 14$$

First three terms are 8, 11, 14.

**Solution:(iv)**

$$a_n = \frac{n+1}{2n+5}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1+1}{2(1)+5} = \frac{2}{2+5} = \frac{2}{7}$$

$$a_2 = \frac{2+1}{2(2)+5} = \frac{3}{4+5} = \frac{3}{9}$$

$$a_3 = \frac{3+1}{2(3)+5} = \frac{4}{6+5} = \frac{4}{11}$$

First three terms are $\frac{2}{7}, \frac{3}{9}, \frac{4}{11}$

**Solution:(v)**

$$a_n = \frac{1}{(2n-1)^2}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1}{\{2(1)-1\}^2} = \frac{1}{(2-1)^2} = \frac{1}{(1)^2} = 1 \quad (i)$$

$$a_2 = \frac{1}{\{2(2)-1\}^2} = \frac{1}{(4-1)^2} = \frac{1}{(3)^2} = \frac{1}{9} \qquad (ii)$$

$$a_3 = \frac{1}{\{2(3)-1\}^2} = \frac{1}{(6-1)^2} = \frac{1}{(5)^2} = \frac{1}{25} \qquad (iii)$$

First three terms are $1, \frac{1}{9}, \frac{1}{25}$

***Solution:(vi)***

$$a_n = n+3$$

*Putting* $n = 1, 2, 3$

$$a_1 = 1 + 3 = 4$$
$$a_2 = 2 + 3 = 5$$
$$a_3 = 3 + 3 = 6$$

First three terms are $4, 5, 6$

***Solution:(vii)***

$$a_n = \frac{1}{3^n}$$

*Putting* $n = 1, 2, 3$

$$a_1 = \frac{1}{3^1} = \frac{1}{3}$$

$$a_2 = \frac{1}{3^2} = \frac{1}{9}$$

$$a_3 = \frac{1}{3^3} = \frac{1}{27}$$

First three terms are $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

***Solution:(viii)***

$$a_n = 3n - 5$$

*Putting* $n = 1, 2, 3$

$$a_1 = 3(1) - 5 = 3 - 5 = -2$$

$a_2 = 3(2) - 5 = 6 - 5 = 1$

$a_3 = 3(3) - 5 = 9 - 5 = 4$

First three terms are $-2, 1, 4$

**Solution:(ix)**

$a_n = (n+1)$

*Putting* $n = 2, 3$

$a_1 = 1$

$a_2 = 2 + 1 = 3$

$a_3 = 3 + 1 = 4$

First three terms are $1, 3, 4$

2- ***Find the terms indicated in the following sequences.***

(i) $2, 6, 11, 17, \ldots a_8$ (ii) $1, 3, 12, 60, \ldots a_7$ (iii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots a_6$

(iv) $-1, 1, 3, 5, \ldots, a_9$ (v) $\frac{1}{3}, \frac{2}{5}, \ldots, a_5$ (vi) $1, -3, 5, -7, \ldots, a_9$

**2(i) 2, 6, 11, 17, ... $a_8$**

**Solution:**

$a_1 = 2$

$a_2 = 6$

$a_3 = 11$

$a_4 = 17$

$a_2 - a_1 = 6 - 2 = 4$

$a_3 - a_2 = 11 - 6 = 5$

$a_4 - a_3 = 17 - 11 = 6$

$a_5 - a_4 = 24 - 17 = 7$

$a_6 - a_5 = 32 - 24 = 8$

$a_7 - a_6 = 41 - 32 = 9$

$a_8 - a_7 = 51 - 41 = 10$

$$a_8 = 51$$

**2(ii)** $1, 3, 12, 60, \ldots a_7$

**Solution:**

$$\begin{aligned}
a_1 &= 1 \\
a_2 &= 3 = 1\times 3 \\
a_3 &= 12 = 3\times 4 \\
a_4 &= 60 = 12\times 5 \\
a_5 &= 360 = 60\times 6 \\
a_6 &= 2520 = 360\times 7 \\
a_7 &= \quad = 2520\times 8 = 20160
\end{aligned}$$

**2(iii)** $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots a_6$

**Solution:-**

$$\begin{aligned}
a_1 &= 1 \\
a_2 &= \frac{1}{3} = 1\times\frac{1}{3} \\
a_3 &= \frac{1}{9} = \frac{1}{3}\times\frac{1}{3} \\
a_4 &= \frac{1}{27} = \frac{1}{9}\times\frac{1}{3} \\
a_5 &= \frac{1}{81} = \frac{1}{27}\times\frac{1}{3} \\
a_6 &= \frac{1}{243} = \frac{1}{81}\times\frac{1}{3} = \frac{1}{243}
\end{aligned}$$

**2(iv)** $-1, 1, 3, 5, \ldots a_9$

*Solution:*

$a_1 = -1$
$a_2 = 1$
$a_3 = 3$
$a_4 = 5$

$a_2 - a_1 = 1 - (-1)$
$= 1 + 1 = 2$
$a_3 - a_2 = 3 - 1 = 2$
$a_4 - a_3 = 5 - 3 = 2$
$a_5 = 5 + 2 = 7$
$a_6 = 7 + 2 = 9$
$a_7 = 9 + 2 = 11$
$a_8 = 11 + 2 = 13$
$a_9 = 13 + 2 = 15$

2(v) $\frac{1}{3}, \frac{2}{5}, \ldots, a_5$

*Solution:*

$\frac{1}{3}, \frac{2}{5}, \ldots, a_5$

$a_1 = \frac{1}{3}$

$a_2 = \frac{2}{5}$

$a_3 = \frac{3}{7}$

$a_4 = \frac{4}{9}$

$a_5 = \frac{5}{11}$

2(vi) $1, -3, 5, -7, \ldots, a_9$

*Solution:*

$a_1 = 1$

$a_2 = -3$

$a_3 = 5$

$a_4 = -7$

$a_5 = 9$

$a_6 = -11$

$a_7 = 13$

$a_8 = -15$

$a_9 = 17$

**3- Find the next four terms of the following sequences.**

(i) 12,16,21,27,... (ii) 1,3,7,15,31,... (iii) -1,2,12,40,...
(iv) 9,11,14,17,19,22,... (v) 4,8,12,16,... (vi) -2,0,2,4,6,8,10,...

**3(i) 12, 16, 21, 27,...**

**Solution:**

$16 - 12 = 4$
$21 - 16 = 5$
$27 - 21 = 6$
$34 - 27 = 7$
$42 - 34 = 8$
$51 - 42 = 9$
$61 - 51 = 10$

$27 + 7 = 34$
$34 + 8 = 42$
$42 + 9 = 51$
$51 + 10 = 61$
34,42,51,61

**3(ii) 1,3,7,15,31,...**

**Solution:**

$3 - 1 = 2$
$7 - 3 = 4$
$15 - 7 = 8$
$31 - 15 = 16$
$63 - 31 = 32$
$127 - 63 = 64$

$31 + 32 = 63$
$63 + 64 = 127$
$127 + 128 = 255$

$225 - 127 = 128$

$511 - 255 = 256$ $\qquad$ $255 + 256 = 511$

$1, (1+2), (3+2^2)+(7+2^3)+(15+2^4)$
$+ (31+2^5)+(63+2^6)+(127+2^7)....$

**3(iii) -2,2,12,40,......**

**Solution:**

$2-(-2)=2+2=4=3^1+1$
$12-2=10=3^2+1$
$40-12=28=3^3+1$

$112-40=82=3^4+1$
$356-112=244=3^5+1$
$1086-356=730=3^6+1$
$3274-1086=2188=3^7+1$

$112, 356, 1086, 3274$

**3(iv) 9,**

3 3 3 3 3 3
9 11 14 17 19 22 25 27 30 33
2 2

**3(v) 4, 8, 12, 16,.........**

**Solution:**

$8 - 4 = 4$
$12 - 8 = 4$
$16 - 12 = 4$
$20 - 16 = 4$
$24 - 20 = 4$
$28 - 24 = 4$
$32 - 28 = 4$

**3(vi) -2, 0, 2, 4, 6, 8, 10**

**Solution:**

$0-(-2) = 0 + 2 = 2$
$2-0 = 2$
$4-2 = 2$
$6-4 = 2$
$8-6 = 2$
$10-8 = 2$
$12-10 = 2$
$14-12 = 2$
$16-14 = 2$
$18-16 = 2$

-2, 0, 2, 4, 6, 8, 10, ......

## Exercise 7.2

**1- *Find the specified term of the following A.P.***

(i) $3, 7, 11, \ldots 61st$ term. (ii) $-4, -7, -10, \ldots a_{19}$

(iii) $6, 4, 2, \ldots 45th$ term. (iv) $9, 14, 19, \ldots a_{14}$

(v) $11, 6, 1, \ldots a_{18}$

***Solution:-***

$61^{st}$ $3, 7, 11, \ldots\ldots$

Here $a_1 = 3$

$d = 7 - 3 = 4$

$a_{61} = ?$

$n = 61$

We know that $a_n = a + (n-1)d$

Putting values $a$, $d$ and $n$ in the formula

$$a_{61} = 3 + (61 - 1)(4)$$
$$= 3 + 60 \times 4$$
$$= 3 + 240$$
$$= 243$$

***1(ii) -4, -7, -10, ..... $a_{19}$***

***Solution:***

*Here* $a = a_1 = -4$

$d = (-7) - (-4)$

$= -7 + 4 = -3$

$a_{19} = ?$

$n = 19$

*We know that* $a_n = a + (n-1)d$

Putting values of $a$, $d$ and $n$ in the formula

$$a_{19} = -4 + (19 - 1)(-3)$$
$$= -4 + 18(-3)$$
$$= -4 - 54$$
$$= -58$$
$$a_{19} = -58$$

***Solution:***

*Here* $a = a_1 = 4$

$d = 4 - 6 = -2$

$a_{45} = ?$

$n = 45$

We know that $a_n = a+(n-1)d$

Putting values of $a$, $d$ and $n$ in the formula

$$\begin{aligned} a_{45} &= 6+(45-1)(-2) \\ &= 6+(44)(-2) \\ &= 6-88 \\ &= -82 \end{aligned}$$

Hence 45th term is -82

***1(iv)*** $9, 14, 19, \ldots a_{14}$

***Solution:***

*Here* $a = a_1 = 9$

$d = 14 - 9 = 5$

$a_{14} = ?$

$n = 14$

We know that $a_n = a+(n-1)d$

Putting values of $a$, $d$ and $n$ in the formula

$$\begin{aligned} a_{14} &= 9+(14-1)(5) \\ &= 9+(13)(5) \\ &= 9+65 \\ &= 74 \end{aligned}$$

***1(v)*** $11, 6, 1, \ldots a_{18}$

***Solution:***

$a = a_1 = 11$

$d = 6 - 11 = -5$

$a_{18} = ?$

$n = 18$

We know that $a_n = a+(n-1)d$

Putting values in $a$, $d$ and $n$ in the formula

$$\begin{aligned} a_{18} &= 11+(18-1)(-5) \\ &= 11+(17)(-5) \end{aligned}$$

$= 11 - 85$

$= -74$

**2- Find the missing element using the formula of A.P**

$a_n = a + (n - 1)\,d$

(i) $a = 2,\ a_n = 402,\ n = 26$

(ii) $a_n = 81,\ d = -3,\ n = 18$

(iii) $a = 5,\ a_n = 61,\ n = 15$

(iv) $a = 16,\ a_n = 0,\ d = -\dfrac{1}{4}$

(v) $a = 10,\ a_n = 400,\ d = 5$

(vi) $a_n = 261,\ d = 4,\ n = 18$

**Solution: (i)** $a_n = a + (n - 1)\,d$

$a = 2$

$a_n = 402$

$n = 26$

$d = ?$

Here we find $d$?

$a_n = a + (n - 1)\,d$

Putting values of $a$, $a_n$, $n$

$402 = 2 + (26 - 1)\,d$

$402 = 2 + 25\,d$

$2 + 25\,d = 402$ OR

$25\,d = 402 - 2$

$25\,d = 400$

*Hence* $d = \dfrac{400}{25} = 16$

**Solution: (ii)** $a_n = a + (n - 1)\,d$

$a_n = 81$

$d = -3$

$n = 18$

Here $a$ is unknown

$a_n = a + (n - 1)\,d$

*Putting values of* $a$, $a_n$, $n$

$$81 = a + (18 - 1)(-3)$$
$$81 = a + (17)(-3)$$
$$81 = a - 51$$
$$a - 51 = 81 \quad OR$$
*Therefore* $\quad a = 81 + 51$
$$= 132$$

**Solution: (iii)** $a_n = a + (n - 1)\,d$ *formula*

$a_n = 5$ *and*

$a_n = 61$

$n = 15$

Here $d$ is unknown

$$a_n = a + (n - 1)\,d$$

Putting values of $a$, $a_n$, and $n$.

$$61 = 5 + (15 - 1)\,d$$
$$61 = 5 + 14\,d$$
$$5 + 14\,d = 61 \quad OR$$
$$14\,d = 61 - 5$$
$$14\,d = 56 \quad \textit{Therefore}$$
$$d = \frac{56}{14}$$
$$d = 4$$

**Solution: (iv)** $a_n = a + (n - 1)\,d$ *formula*

$a = 16$ *and*

$a_n = 0$

$$d = -\frac{1}{4}$$

Here $n$ is unknown.

$$a_n = a + (n - 1)\,d$$

Putting values of $a$, $a_n$ and $d$.

$$0 = 16 + (n - 1)\left(-\frac{1}{4}\right)$$

Multiplying by 4.

$$0 = 64 + (n - 1)(-1)$$

$0 = 64 - n + 1$

$n = 64 + 1$ OR

$n = 65$ OR

***Solution:* (v)** $a_n = a + (n - 1)\,d$ formula

$a = 10$ *and*

$a_n = 400$

$d = 5$

Here $n$ is unknown

$a_n = a + (n - 1)\,d$

Putting values of $a$, $an$ and $d$.

$400 = 10 + (n - 1)(5)$

$400 - 10 = 5n - 5$

$5n - 5 = 390$ OR

$5_n = 390 + 5$

$5_n = 395$

$n = \dfrac{395}{5}$

$= 79$

***Solution:* (vi)** $a_n = a + (n - 1)\,d$ fromula

$a_n = 261$ *and*

$d = 4$

$n = 18$

Here a is unknown

$a_n = a + (n - 1)\,d$

Putting values of $a_n$, $d$ and $n$.

$261 = a + (18 - 1)(4)$

$= a + 17 \times 4$

$261 = a + 68$

$261 - 68 = a$

$193 = a$ OR

$a = 193$

3- ***Find the $15^{th}$ terms of an A.P where the $3^{rd}$ term is 8 and the common difference is $\frac{1}{3}$.***

***Solution:***

$$a_3 = 8$$
$$d = \frac{1}{3}$$
$$a_{15} = ?$$

First we find a

$$a_n = a + (n - 1)\, d$$
$$a_3 = a + (3 - 1)\, d$$
$$8 = a + (2)\left(\frac{1}{3}\right)$$
$$8 = a + \left(\frac{2}{3}\right)$$
$$a = 8 - \frac{2}{3} \quad OR$$
$$a = 8 - \frac{2}{3}$$
$$a = \frac{24-2}{3} = \frac{22}{3} \quad (A)$$
$$a_n = a + (n - 1)\, d \quad \text{formula}$$
$$a_{15} = \frac{22}{3} + (15 - 1)(\frac{1}{3}) \quad \text{from A}$$
$$= \frac{22}{3} + (14)\frac{1}{3}$$

$$= \frac{22}{3} + \frac{14}{3}$$

$$= \frac{22+14}{3}$$

$$= \frac{36}{3} = 12$$

**4- *Which term of an A.P 6, 2, −2...... is −146?***

***Solution:***

***Here term ...... 6, 2, -2, ......***

$a = 6$

$d = 2 - 6 = -4$

$an = -146$

$n = ?$

$a_n = a + (n - 1)\,d$ *formula*

Putting values $a$, $a_n$ and $d$ in formula.

$-146 = 6 + (n-1)(-4)$ *formula*

$-146 = 6 - 4n + 4$

$4n = 6 + 4 + 146$ *OR*

$4n = 156$

$$n = \frac{156}{4}$$

$n = 39$

**5- *Which term of an A.P 5,2,−1 .... is −134?***

***Solution:***

$a = 5$

$d = 2 - 5 = -3$

$n = -118$

$n = ?$

$a_n = a + (n-1)\,d$ formula

Putting values of $a$, $d$ and $n$.

$-118 = 5 + (n-1)(-3)$

$-118 = 5 + 3n + 3$

$3n = 5 + 3 + 118$ Or

$3n = 126$

$n = \frac{126}{3}$

$n = 42$

6- **How many terms are there in an A.P, in which $a_1 = a = 11$, $a_n = 68$, $d = 3$.**

**Solution:-** Here term.......

$d = 3$

$a_n = 68$

$a = 11$

$n = ?$

$a_n = a + (n-1)d$ *formula*

Putting values of $a$, $a_n$ and $d$ in the formula.

$68 = 11 + (n-1)(3)$

$68 = 11 + 3n - 3$

$68 = 8 + 3n$

$8 + 3n = 68$

$3n = 68 - 8$ Or

$3n = 60$

$n = \frac{60}{3}$

$n = 20$

7- **Find the $11^{th}$ term of $a_n$ A.P $2 - x, 3 - 2x, 4 - 3x$........**

**Solution:-** *In term of $a_n$ A.P $2 - x, 3 - 2x, 4 - 3x$......*

$a = 2 - x$

$d = (3 - 2x) - (2 - x)$

$d = 3 - 2x - 2 + x = 1 - x$

$a_{11} = ?$

$\therefore n = 11$

$a_n = a + (n-1)d$ *formula*

$a_{11} = (2 - x) + (11 - 1)(1 - x)$

$= 2 - x + 10(1 - x)$

$= 2 - x + 10 - 10x$

*Hence* $a_{11} = 12 - 11x$

8- *Find the $n^{th}$ term of an A.P, where $a_{n-5} = 3n + 9$*

$a_n = ?$

**Solution:-**

$a_{n-5} = 3n + 9$ We put $n + 5$ in place of $n$.

$a_{n+5-5} = 3(n + 5) + 9$

$a_n = 3n + 15 + 9$

$a_n = 3n + 24$

9- *Find the $n^{th}$ term of an A.P:* $\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2 \ldots\ldots$

**Solution:**

$a = 4$

$d = 7 - 4 = 3$

$a_n = ?$

$a_n = a + (n - 1)\, d$ *formula*

$= 4 + (n - 1)(3)$

$= a + 3n - 3$

$= 4 + 3n - 3$

$= 3n + 1$

*The nth term of the sequence* $\left(\frac{3}{3n+1}\right)^2$

10- *If the $n^{th}$ term of an A.P is $3n - 5$. Find the A.P*

**Solution:-**

$a_n = 3n - 5$

*Putting* $n = 1, 2, 3, 4, \ldots$

$a_1 = 3(1) - 5 = 3 - 5 = -2$

$a_2 = 3(2) - 5 = 6 - 5 = 1$

$a_3 = 3(3) - 5 = 9 - 5 = 4$

$a_4 = 3(4) - 5 = 12 - 5 = 7$

$-2, 1, 4, 7 \ldots\ldots\ldots$ *is required*

## Exercise 7.3

**1- Find A.M between:**

(i) $-3, 7$ (ii) $x-1, x+7$

(iii) $\sqrt{7}, 3\sqrt{7}$ (iv) $x^2+x+1, x^2-x+1$

**Solution:-(i)** $-3, 7$

*Here* $a=-3$

$b=7$

*(A. M)* $A=\dfrac{-3+7}{2}$

*Here* $A=\dfrac{4}{2}=2$

**Solution:-(ii)** $x-1, x+7$

*Here* $a=x-1$

$b=x+7$

*(A. M)* $A=\dfrac{a+b}{2}$

$=\dfrac{(x-1)+(x+7)}{2}$

$=\dfrac{x-1+x+7}{2}$

$=\dfrac{2x+6}{2}$

$=\dfrac{2(x+3)}{2}$

$A=x+3$

***Solution:-(iii)*** $\sqrt{7}, 3\sqrt{7}$

*Here* $a = \sqrt{7}$

$b = 3\sqrt{7}$

*(A.M)* $A = \dfrac{a+b}{2}$

$= \dfrac{\sqrt{7} + 3\sqrt{7}}{2}$

$= \dfrac{\sqrt{7}(1+3)}{2}$

$= \dfrac{\sqrt{7}\cdot 4}{2}$

*Hence* $A = 2\sqrt{7}$

***Solution:-(iv)*** $x^2 + x + 1, x^2 - x + 1$

*Here* $a = x^2 + x + 1$

$b = x^2 - x + 1$

*(A.M)* $A = \dfrac{a+b}{2}$

$= \dfrac{(x^2 + x + 1) + (x^2 - x + 1)}{2}$

$= \dfrac{x^2 + x + 1 + x^2 - x + 1}{2}$

$= \dfrac{2x^2 + 2}{2}$

$= \dfrac{2(x^2 + 1)}{2}$

***Hence*** $A = x^2 + 1$

2- ***If 3 and 6 are two A.Ms between a and b, find a and b.***

***Solution:-***

Here A. Ms between $a, 3, 6, b$

$d = 6 - 3 = 3$

$b = a_4 = a_3 + d$

$= 6 + 3$

$b = 9$

*and* $a = a_2 - d$

$= 3 - 3$

$a = 0$

$a = 0, b = 9$

We can also find $a$ and $b$ using this method.

$a + d = 3$

$a + 3 = 3$

$a = 3 - 3$

$a = 0$

$6 + d = b$

*and* $6 + 3 = b$

$9 = b$

$b = 9$

3- ***Find three A.Ms between 11 and 19.***

***Solution:-***

*Here* $a_1 = 11$

$a_5 = 19$

$n = 5$

Now $a_n = a + (n - 1)d$

$a_5 = a + (5 - 1)d$

Now $a_n = 11 + 4d$

$19 = 11 + 4d$

$19 - 11 = 4d$

$8 = 4d$

$d = 2$

$A_1 = a + d$

$= 11 + 2 = 13$

$A_2 = A_1 + d$

$= 13 + 2 = 15$

$A_3 = A_2 + d$

$= 15 + 2 = 17$

*required three A. Ms are 13, 15, 17.*

4- ***Find three A.Ms between $2\sqrt{3}$ and $6\sqrt{3}$***

***Solution:-***

**Let** $A_1, A_2, A_3$ be three A.Ms between $2\sqrt{3}$ and $6\sqrt{3}$. Such that $2\sqrt{3}, A_1, A_2, A_3, 6\sqrt{3}$ is an A.P.

$a_1 = 2\sqrt{3}$

$a_5 = 6\sqrt{3}$

$n = 5$

*Now* $a_n = a + (n - 1)d$

$6\sqrt{3} - 2\sqrt{3} = 4d$

Or $4d = 6\sqrt{3} - 2\sqrt{3}$

$= \sqrt{3}(6 - 2)$

$4d = \sqrt{3}(4)$

$d = \sqrt{3}$

$A_1 = a + d$

Now $= 2\sqrt{3} + \sqrt{3}$

$\sqrt{3} + (2 + 1)$

$A_1 = 3\sqrt{3}$

and $A_2 = a + 2d$

$= 2\sqrt{3} + 2\sqrt{3}$

$= \sqrt{3}(2 + 2) = 4\sqrt{3}\ A_2$

$A_3 = a + 3d$

$$= 2\sqrt{3} + 3\sqrt{3}$$

$$= \sqrt{3}(2+3)$$

$$A_3 = 5\sqrt{3}$$

$$3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$$

Thus $3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$ are the required three A.Ms between $6\sqrt{3}$ and $2\sqrt{3}$.

5- ***Find six A.Ms between 5 and 8.***

***Solution:-***

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six A.Ms between 5 and 8. Such that $5, A_1, A_2, A_3, A_4, A_5, A_6, 8$ *is an A.P.*

***Here*** $a_1 = 5$

$a_8 = 8$

$n = 8$

$$a_n = a + (n-1)d$$

$$8 = 5 + (8-1)d$$

$$8 - 5 = 7d$$

$$3 = 7d$$

$7d = 3$ Or

$d = \dfrac{3}{7}$ Or

***Now*** $A_1 = a + d$

$$= 5 + \frac{3}{7} = \frac{38}{7}$$

$$A_2 = a + 2d$$

$$= 5 + 2 \times \frac{3}{7}$$

$$= 5 + \frac{6}{7} = \frac{41}{7}$$

$$A_3 = a + 3d$$

$$= 5 + 3 \times \frac{3}{7}$$

$$= 5 + \frac{9}{7} = \frac{44}{7}$$

$$A_4 = a + 4d$$

$$= 5 + 4 \times \frac{3}{7}$$

$$= 5 + \frac{12}{7} = \frac{47}{7}$$

$$A_5 = a + 5d$$

$$= 5 + 5\left(\frac{3}{7}\right)$$

$$= 5 + \frac{15}{7} = \frac{50}{7}$$

$$A_6 = a + 6d$$

$$= 5 + 6\left(\frac{3}{7}\right)$$

$$= 5 + \frac{18}{7}$$

$$= \frac{35 + 18}{7} = \frac{53}{7}$$

Thus $\frac{38}{7}, \frac{41}{7}, \frac{44}{7}, \frac{47}{7}, \frac{50}{7}, \frac{53}{7}$ are six required A.Ms, between 5 and 8.

**6- *Find seven A.Ms between 8 and 12.***

***Solution:-***

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ be seven required A.Ms, between 8 and 12.

Such that $8\ A_1, A_2, A_3, A_4, A_5, A_6, A_7, 12$ is an A. P.

$$a_1 = 8$$
$$a_9 = 12$$
$$n = 9$$
$$a_n = a + (n-1)d$$
$$12 = 8 + (a-1)d$$
$$12 - 8 = 8d$$
$$4 = 8d$$
$$8d = 4 \qquad \text{Or}$$
$$d = \frac{4}{8}$$
$$d = \frac{1}{2} \qquad \text{Or}$$
$$A_1 = a + d$$
$$= 8 + \frac{1}{2}$$
$$= \frac{16+1}{2} = \frac{17}{2}$$
$$A_2 = 8 + 2d \qquad \text{Now}$$
$$= 8 + 2(\frac{1}{2})$$
$$= 8 + \frac{2}{2}$$
$$= \frac{16+2}{2} = \frac{18}{2}$$
$$A_3 = a + 3d$$
$$= 8 + 3(\frac{1}{2})$$
$$= 8 + \frac{3}{2}$$

$$=\frac{16+3}{2}=\frac{19}{2}$$

$A_4 = a + 4d$

$$=8+4(\frac{1}{2}) \quad =\frac{8}{1}+\frac{4}{2}$$

$$=\frac{16+4}{2}=\frac{20}{2}$$

$A_5 = a + 5d$

$$=8+5(\frac{1}{2})$$

$$=\frac{16+5}{2}=\frac{21}{2}$$

$A_6 = 8 + 6d$

$$=8+6(\frac{1}{2})$$

$$=\frac{16+6}{2}=\frac{22}{2}$$

$A_7 = a + 7d$

$$=8+7(\frac{1}{2})$$

$$=\frac{16+7}{2}=\frac{23}{2}$$

Thus $\frac{17}{2},\frac{18}{2},\frac{19}{2},\frac{20}{2},\frac{21}{2},\frac{22}{2},\frac{23}{2}$, are the required

seven A.Ms between 8 and 12.

7- ***If the A.M between 5 and b is 10, then find the value of b.***

***Solution:-***

$a, A, b$ is an A. P.

$$A = \frac{a+b}{2}$$

*Here* $a = 5$

$b = b$

$A = 10$

$$A = \frac{a+b}{2}$$

$$10 = \frac{5+b}{2}$$

Or $20 = 5 + b$

Or $5 + b = 20$

$b = 20 - 5$

$b = 15$

8- ***If the A.M between a and 10 is 40, then find the value of "a".***

***Solution:-***

$a, A, b$ is an A. P.

$$A = \frac{a+b}{2}$$

*Here* $a = a$

$b = 10$

$A = 40$

$$A = \frac{a+b}{2} \quad \text{and}$$

$$40 = \frac{a+10}{2}$$

$$40 \times 2 = a + 10$$

$$80 = a + 10$$

$$a + 10 = 80$$

$$a = 80 - 10 \qquad \text{Or}$$

$$a = 70$$

9. ***If the three A.Ms between a and b are 5,9 and 13, find a and b.***

***Solution:-***

5, 9 and 13 is an A.P between $a$ and $b$.

$a, 5, 9, 13, b$ is A. Ms

$$d = 9 - 5 = 4$$

$$= 13 - 9 = 4$$

$$a + d = 5$$

$$a + 4 = 5$$

$$a = 5 - 4$$

$$a = 1 \qquad (i)$$

$$13 + d = b \qquad \text{and}$$

$$13 + 4 = b$$

$$17 = b$$

$$b = 17 \qquad (ii)$$

$$a = 1, b = 17$$

## Exerise 7.4

**1- *Find the 7th term of a G.P 2,8,32.....***

***Solution:*** *G. P sequence is 2, 8, 32, .......*

Given $a = 2$

$$r = \frac{8}{2} = 4$$

$$n = 7$$

$$a_n = ar^{n-1} \quad \textit{formula}$$

$$a_7 = ?$$

$$a_7 = 2(4)^{7-1}$$

$$= 2(4)^6$$

$$= 2 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$$

$$= 2 \times 4096$$

$$a_7 = 8192$$

**2- *Find the 11th term of a G.P 2,6,18 ....***

***Solution:-*** *G. P sequence is 2, 6, 18,......*

*Given* $a = 2$

$$r = \frac{6}{2} = 3$$

$$n = 11$$

$$a_{11} = ?$$

$$a_n = ar^{n-1} \quad \textit{formula}$$

$$a_{11} = 2 \times 3^{11-1}$$

$$= 2 \times 3^{10}$$

$$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= 2 \times 9 \times 9 \times 9 \times 9 \times 9$$

$$= 2 \times 81 \times 81 \times 9$$

$$a_{11} = 118098$$

**3- Find the $6^{th}$ term of a G.P $-\frac{3}{2}, 3, -6.....$**

***Solution:-***

In term of a G. P $-\frac{3}{2}, 3, -6.....$

$a = -\frac{3}{2}$

$r = -\frac{6}{3} = -2$

$n = 6$

$a_6 = ?$

$a_n = ar^{n-1}$ ***formula***

$a_6 = \left(-\frac{3}{2}\right)(-2)^{6-1}$ Now

$= \left(-\frac{3}{2}\right)(-2)(-2)(-2)(-2)(-2)$

$= \left[\left(-\frac{3}{2}\right)(-2)\right]\left[(-2)(-2)\right]\left[(-2)(-2)\right]$

$= 3 \times 4 \times 4$

$= 48$

**4- Find the $5^{th}$ term of a G.P 4, –12, 36....**

***Solution:-***

*In term of a G. P 4, -12, 36,....*

$a = 4$

$d = \frac{-12}{4} = -3$

$n = 5$

$a_5 = ?$

$a_n = ar^{n-1}$ *formula*

$a_5 = 4(-3)^{5-1}$

$= 4(-3)^4$

$= 4(-3)(-3)(-3)(-3)$

$= 4 \times 9 \times 9$

$a_5 = 324$

**5- *Find the missing elements of the G.P***

(i) $r = 10, a_n = 100, a = 1$

(ii) $a_n = 400, r = 2, a = 25$

(iii) $a = 128, r = \frac{1}{2}, a_n = \frac{1}{4}$

***Solution:5(i)***

$r = 10$

$a_n = 100$

$a = 1$

$n = ?$

$a_n = ar^{n-1}$ *formula*

*Putting values of* $a$ ' $a_n$ ' $r$

$100 = (1)(10)^{n-1}$

$(10)^2 = (10)^{n-1}$

$(10)^2 = (10)^{n-1}$

***Therefore*** $n-1 = 2$

$n = 2 + 1 = 3$

***Solution:5(ii)***

$a_n = 400$

$r = 2$

$a = 25$

$n = ?$

$a^n = ar^{n-1}$ *formula*

*Putting values* $a$ ' $r$ ' $a_n$

$400 = 25(2)^{n-1}$

$$\frac{400}{25} = 2^{n-1}$$

$$16 = 2^{n-1}$$
$$2^4 = 2^{n-1}$$

*Therefore* $n - 1 = 4$

$$n = 4 + 1 = 5$$

***Solution:5(iii)***

$$a = 128$$
$$r = \frac{1}{2}$$
$$a_n = \frac{1}{4}$$
$$n = ?$$
$$a^n = ar^{n-1} \quad \textit{Formula}$$

*Putting values a, r,* $a_n$

$$\frac{1}{4} = 128\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{4 \times 128} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{32 \times 4 \times 4} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^2 \times 2^2 \times 2^2 \times 2^3} = \left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{2^9} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$$

$$n-1=9$$
$$n=9+1$$
Hence $n=10$

6- ***Find the $11^{th}$ term of a G.P whose $5^{th}$ term is 9 and common ratio is 2.***

**Solution:**

$$r=2$$
$$a_5=9$$
$$a_{11}=?$$
$$a_n=ar^{n-1} \quad \text{formula}$$
$$a_5=ar^{5-1}=ar^4 \quad \text{Putting values of } a_5 \text{ and } r.$$
$$a_5 r$$
$$9=a(2)^4$$
$$9=a\times 16$$
$$\frac{9}{16}=a$$
$$a=\frac{9}{16}$$

Now we want to find $a_{11}=ar^{11-1}$

putting values of $a$ and $r$.

$$a_{11}=\frac{9}{16}\times 2^{10}$$
$$a_{11}=\frac{9}{16}\times 1024$$
$$a_{11}=9\times 64$$
$$=576$$

7- ***Find the $13^{th}$ term of a G.P whose $7^{th}$ term is 25 and common ratio is 3.***

**Solution:**

$$a_7=25$$
$$r=3$$

$a_{13} = ?$

$a_n = ar^{n-1}$ *formula*

$a_7 = ar^{7-1} = ar^6$ putting values of $a_7$ and $r$.

$25 = a(3)^6$

$25 = a \times 729$

$$\frac{25}{729} = a$$

$$a = \frac{25}{729}$$

$a_{13} = ar^{13-1}$ *Now*

*Putting values of a,* $a_{13}$

$$= \frac{25}{729}(3)^{13-1}$$

$$= \frac{25}{729} \times 3^{12}$$

$$= \frac{25}{3^6} \times 3^6 \times 3^6$$

$$= 25 \times 3^6$$

$$= 25 \times 729 = 18225$$

**8- If a, b, c, d are in G.P, show that, $a-b, b-c, c-d$, are in G.P**

**Solution:-**

*a, b, c, d is G.P* *therefore*

$$(A)\quad \frac{b}{a}=\frac{c}{b}=\frac{d}{c}$$

$$\frac{c}{b}=\frac{d}{c} \quad \textit{and}$$

$$\frac{b}{a}=\frac{d}{c}$$

If $a-b, b-c, c-d$ is G.P.

Then $$\frac{b-c}{a-b}=\frac{c-d}{b-c}$$

$(b-c)(b-c)=(a-b)(c-d)$

$$b^2-bc-bc+c^2=ac-ad-bc+bd$$

(i) $b^2=ca \quad \left(\frac{b}{a}=\frac{c}{b}\right)$

$\left(\frac{c}{b}=\frac{d}{c}\right) \quad c^2=bd \quad$ (ii)

$\left(\frac{b}{a}=\frac{d}{c}\right) \quad bc=ad$ (iii)

$$b^2-bc-bc+c^2$$

$$=ca-bc-ad+bd \quad \text{From (i),(ii),(iii)}$$

$a-b, b-c, c-d$ is G.P

***Second Law:***

$a, b, c, d$ is G.P

$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}$ Therefore

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d} \quad (A) \quad \text{Or}$$

Now if $a-b, b-c, c-d$ is G.P

Then $\frac{b-c}{a-b}=\frac{c-d}{b-c}$ prove

Now $\frac{a}{b}=\frac{b}{c}$ from A

$$\frac{a}{b}-1=\frac{b}{c}-1$$

$$\frac{a-b}{b}=\frac{b-c}{c}$$

$$\frac{a-b}{b-c}=\frac{b}{c}$$

$$\frac{b-c}{a-b}=\frac{c}{b} \quad (i)$$

$$\frac{b}{c}=\frac{c}{d}$$

$$\frac{b}{c}-1=\frac{c}{d}-1$$

$$\frac{b-c}{c}=\frac{c-d}{d}$$

$$\frac{b-c}{c-d}=\frac{c}{d}$$

$$\frac{c-d}{b-c}=\frac{d}{c}$$

$$\frac{c-d}{b-c}=\frac{c}{b} \, (ii) \left(\frac{c}{b}=\frac{d}{c}\right) \quad (A)$$

$$\frac{b-c}{a-b}=\frac{c-d}{b-c} \qquad (i),(ii)$$

9- ***Find the n$^{th}$ term of a G.P, if $\frac{a_5}{a_3}=\frac{4}{9}$ and $a_{\,}$***

***Solution:***

$$\frac{a_5}{a_3}=\frac{4}{9}$$

$$\frac{ar^{5-1}}{ar^{3-1}}=\frac{4}{9}$$

$$\frac{r^4}{r^2}=\frac{4}{9}$$

$$r^2=\left(\frac{2}{3}\right)^2$$

$$r=\frac{2}{3} \qquad (A)$$

$$a_2=ar^{2-1} \qquad (i)$$

$$a_2=\frac{4}{9} \qquad (ii)$$

$$ar=\frac{4}{9} \qquad \text{from } (i) \text{ and } (ii)$$

$$a\times\frac{2}{3}=\frac{4}{9} \qquad \text{from } (A)$$

$$a=\frac{4}{9}\times\frac{3}{2}$$

$$a=\frac{2}{3}$$

Now $a_n = ar^{n-1}$

$$= \frac{2}{3}\left(\frac{2}{3}\right)^{n-1}$$

$$a_n = \left(\frac{2}{3}\right)^{n-1+1} = \left(\frac{2}{3}\right)^n$$

**10- *Find three consecutive numbers in G.P, whose sum is 26 and their product is 216.***

***Solution:-***

Three consecutive numbers according to the given condition $a, ar, ar^2$.

$$a + ar + ar^2 = 26$$

$$(a)(ar)(ar^2) = 216 \quad \text{and}$$

$$a^3r^3 = 216$$

$$(ar)^3 = (6)^3$$

$$ar = 6 \quad (A)$$

$$a = \frac{6}{r} \quad \text{Putting } (i)$$

$$a + ar + ar^2 = 26$$

$$a(1 + r + r^2) = 26$$

$$a = \frac{6}{r} \quad (i)$$

$$(1 + r + r^2) = 26\frac{6}{r}$$

$$6(1 + r + r^2) = 26r$$

$$6 + 6r + 6r^2 - 26r = 0$$

$$6r^2 - 20r + 6 = 0 \quad \text{Dividing by 2}$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r - 1) - 3(3r - 1) = 0$$

$$(3r - 1)(r - 1) = 0$$

$$3r-1=0$$
$$3r=1$$
$$r=\frac{1}{3} \quad (ii)$$

and $r-3=0$

$$r=3 \quad \text{then } (B)$$
$$ar=6 \quad \text{Now } (A)$$
$$a(3)=6 \quad \text{Now } (B)$$
$$a=\frac{6}{3}$$
$$a=2 \quad (C)$$

Take 3 or $a=2, r=\frac{1}{3}$

$$a=2$$
$$ar=2\times\frac{1}{3}=\frac{2}{3}$$
$$ar^2=2\times\left(\frac{1}{3}\right)^2=2\times\frac{1}{9}$$
$$=\frac{2}{9}$$

Three consecutive numbers are $2, \frac{2}{3}, \frac{2}{9}$

If $r=3$

Then $a=2$

$$ar=2\times 3=6$$
$$ar^2=2\times 3^2=2\times 9=18$$

Consecutive number $\boxed{2, 6, 18}$

**11-** ***Find the $30^{th}$ term of a G.P*** $x, 1, \frac{1}{x}....$

*Solution:*

**G. P is** $x, 1, \frac{1}{x} ....$

$a = x \qquad (i)$

$r = \frac{1}{x} \qquad (ii)$

$a_n = ar^{n-1}$

$a_{30} = ar^{30-1}$

from (i) and (ii)

$= x \times \left(\frac{1}{x}\right)^{29}$

$= x \times x^{-29}$

$= x^{1-29}$

$= x^{-28}$

$= x^{\frac{1}{28}}$

12- ***Find the $p^{th}$ term of a G.P $x, x^3, x^5,$.......***

**Solution:-**

**G.P is** $x, x^3, x^5,$.......

*Therefore* $a = x$

$r = \frac{x^3}{x}$

$x^2 =$

$a_n = ar^{n-1}$ formula

$a_p = (x)(x^2)^{p-1}$

$= (x)x^{2(p-1)}$

$= x\, x^{2p-2}$

$= x^{2p-2+1}$

$a_p = x^{2p-1}$

## Exercise 7.5

G.M between $a$, $b$ is **G**. $G = \pm\sqrt{ab}$

**1-** ***Find G.M between: (i) 9 and 5 (ii) 4 and 9 (iii) −2 and −8.***

(i) 9 and 5 (ii) 4 and 9

(iii) -2 and -8

***Solution:(i)***

**Given** 9 and 5

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{9\times 5} \\ &= \pm 3\sqrt{5} \end{aligned}$$

***Solution:- (ii)***

4 and 9

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{(4)(9)} \\ &= \pm\sqrt{36} \\ &= \pm 6 \end{aligned}$$

***Solution:- (iii)***

-2 and -8

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{(-2)(-8)} \\ &= \pm\sqrt{16} \\ &= \pm 4 \end{aligned}$$

**2-** ***Insert two G.Ms between:(i) 1 and 8 (ii) 3 and 81***

**(i) 1 and 8 (ii) 3 and 81**

*Solution:-*

Let $G_1$ and $G_2$ be the two G.Ms between $1$ and $8$ such that $1, G_1, G_2\ 8$ is a $G.P.$

*Here* $a = 1$

$n = 4$

$a_4 = 8$

*Since* $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$8 = ar^3$

$8 = 1 \times r^3$

$2^3 = r^3$

$r = 2$

*Thus* $G_1 = ar$

$= 1 \times 2 = 2$

$G_2 = ar^2$

$= 1 \times 2^2$

$= 1 \times 4 = 4$

**Solution:- (ii)** ***3 and 81***

**Let $G_1$ and $G_2$ be the two GMs between 3 and 81, such that 3, $G_1$, $G_2$, 81 is a G.P.**

$n = 4$

*Here* $a = 3$

$n = 4$

$a_4 = 81$

*Since* $a_n = ar^{n-1}$

$a_4 = ar^{4-1}$

$81 = 3r^3$

$r^3 = \frac{81}{3} = 27$

$r^3 = 3^3$

$r = 3$

*Thus* $G_1 = ar$

$= 3 \times 3$

$= 9$

$G_2 = ar^2$
$= 3(3)^2$
$= 3 \times 9 = 27$

3- *Insert three G.Ms between:(i) 1 and 16 (ii) 2 and 32*
(i) **1 and 16**
(ii) **2 and 32**

***Solution:-***

Let $G_1$, $G_2$ and $G_3$ be the three G.Ms between 1 and 16, that are required. G.Ms 1, $G_1$, $G_2$, $G_3$ 16 is a G. P.

*Here* $a = 1$
$n = 5$
$a_5 = 16$
*Since* $a_n = ar^{n-1}$
$a_5 = (1)r^{5-1}$
$16 = r^4$
$r^3 = r^4$
*Thus* $2^4 = r^4$
$r = 2$
$G_1 = ar$
$= 1 \times 2 = 2$
$G_2 = ar^2 = 1 \times 2^2 = 4$
$G_3 = ar^3 = 1 \times 2^3 = 8$

*2, 4, 8 are required three G. Ms*

***Solution:- 2 and 32***

Let $G_1$, $G_2$, $G_3$ be the three G.Ms between 2 and 32 that are required. G.Ms 2, $G_1$, $G_2$ and $G_3$ 32 is a G. P.

*Here* $a = 2$
$n = 5$
$a_5 = 32$
*Since* $a_n = ar^{n-1}$
$a_5 = ar^{5-1}$
$32 = 2r^4$
$r^4 = 16$
*Thus* $r^4 = 2^4$

$r = 2$

*Now* $G_1 = ar$

$= 2 \times 2 = 4$

$G_2 = ar^2$

$= 2 \times 2^2 = 8$

$G_3 = ar^3$

$= 2 \times 2^3 = 16$

4, 8 and 16 are required three G.Ms.

4- ***Insert four real geometric means between 3 and 96.***

***Solution:-*** ***3 and 96***

Let $G_1$, $G_2$, $G_3$ and $G_4$ be the four G.Ms between 3 and 96 that are required. G. Ms 3, $G_1$, $G_2$, $G_3$ and $G_4$, 96 is a G. P.

*Here* $a = 3$

$n = 6$

$a_6 = 96$

*Since* $a_n = ar^{n-1}$

$a_6 = ar^{6-1}$

$96 = 3r^5$

$\frac{96}{3} = r^5$

$r^5 = \frac{96}{3} = 32$

$\therefore r^5 = 2^5$

$r = 2$

$G_1 = ar$

$= 3 \times 2 = 6$

$G_2 = 3r^2$

$= 3(2)^2 = 12$

$G_3 = 3r^3$

$= 3(2)^3 = 24$

$G_4 = 3(r)^4$

$= 3(2)^4 = 48$

6,12,24,48 are four required G.Ms.

5- ***The A.M between two numbers is 5 and their positive G.M is 4. Find the numbers.***

***Solution:-***

Let numbers are $a$ and $b$.

(i) A is A.M $A = \frac{a+b}{2}$

(ii) G is G.M $G = \sqrt{ab}$

According to condition A = 5, G = 4

$5 = \frac{a+b}{2}$ from (i) and (ii)

$4 = \sqrt{ab}$

$16 = ab$ *from (iii)*

$10 = a + b$ *from (iv)*

$10 - b = a$ Or

$a = 10 - b$ Or

Putting values of $a$ in *(iii)*

$16 = (10 - b)(b)$

$16 = 10b - b^2$

$b^2 - 10b + 16 = 0$ Or

$b^2 - 2b - 8b + 16 = 0$

$b(b-2) - 8(b-2) = 0$

$(b-2)(b-8) = 0$

$b - 2 = 0$

$b = 2$

$b - 8 = 0$

$b = 8$

$a + b = 10$ Now from *(iv)*

$a + 2 = 10$

$a = 10 - 2 = 8$

$a + 8 = 10$ b = 8

$a = 10 - 8 = 2$

*Hence Numbers are* 2,8 or 8,2

6- ***The positive G.M between two numbers is 6 and the A.M between them is 10. Find the numbers.***

***Solution:-***

$$\text{A is A.M} = \frac{a+b}{2} \qquad \text{(i)}$$

$$\text{G is G.M} = \sqrt{ab} \qquad \text{(ii)}$$

*From (i)* and *(ii)*

$$10 = \frac{a+b}{2} \quad \text{Or}$$

$$20 = a + b \qquad \text{(iii)}$$

$$6 = \sqrt{ab} \quad \text{Or}$$

$$36 = ab \qquad \text{(iv)}$$

$$a = 20 - b \qquad \text{from (iii)}$$

*Putting values of a in (iv)*

$$36 = (20 - b)(b)$$

$$36 = 20b - b^2$$

$$b^2 - 20b + 36 = 0 \quad \text{Or}$$

$$b^2 - 2b\, b - 18b + 36 = 0$$

$$b(b-2) - 18(b-2) = 0$$

$$(b-2)(b-18) = 0$$

$$\text{If} \quad b-2 = 0$$

$$b-18 = 0$$

$$b-2 = 0$$

$$b = 2$$

*Putting b = 2 in (iv)*

[illegible] $= 2a$

$$a = 18$$

$$b-18 = 0 \quad \text{and}$$

$$b = 18$$

*Putting b = 18 in (iv)*

$$36 = 18a$$

$a = 2$

*Thus numbers are* 2, 18 or 18, 2

7- ***Show that the A.M between the two numbers 4 and 8 is greater than their geometric mean.***

**Solution:-**

***Here*** $a = 4$

$b = 8$

$$(A.M) = \frac{4+8}{2} = 6$$

$$(G.M) = \sqrt{ab}$$

$$G = \sqrt{4 \times 8}$$

$$= \sqrt{32}$$

*Now* $\sqrt{32}$ *is less than 6.*

*Proved* $6 > \sqrt{32}$

*Therefore* $A > G$

8- ***Insert four geometric means between 160 and 5.***

**Solution:-**

numbers are 5 and 160.

Let $G_1$, $G_2$, $G_3$ and $G_4$ the four G.Ms between 160 and 5.

*Here* $n = 6$

$a = 5$

$a_6 = 160$

*Since* $a_n = ar^{n-1}$

$a_6 = 5r^{6-1}$

$160 = 5r^5$

$$r^5 = \frac{160}{5}$$

$r^5 = 32 = 2^5$

$r = 2$

$G_1 = ar$

$$= 5 \times 2 = 10$$
$$G_2 = ar^2$$
$$= 5 \times 2^2 = 20$$
$$G_3 = ar^3$$
$$= 5 \times 2^3 = 40$$
$$G_4 = 5 \times 2^4 = 80$$

Hence required G.Ms are 10, 20, 40, 80.

9- ***Insert three geometric means between 486 and 6.***

***Solution:-*** *Let $G_1$, $G_2$ and $G_3$ the three G. Ms between 186 and 6.*

*Here* $n = 5$
$$a = 486$$
$$a_5 = 6$$
*Since* $a_n = ar^{n-1}$
$$a_5 = ar^{5-1}$$
$$6 = 486r^4$$
$$\frac{6}{486} = r^4$$
$$\frac{1}{81} = r^4$$
$$r^4 = \frac{1}{81} = \left(\frac{1}{3}\right)^4$$
$$r = \frac{1}{3}$$
$$G_1 = ar$$
$$= 486 \times \frac{1}{3} = 162$$
$$G_2 = 486 \times \left(\frac{1}{3}\right)^2$$

$$= 486 \times \frac{1}{9} = 54$$

$$G_3 = 486 \times \left(\frac{1}{3}\right)^3$$

$$= 486 \times \frac{1}{27} = 18$$

Hence required G.Ms are 162, 54, 18.

**10- *Insert four geometric means between $\frac{1}{8}$ and 128.***

***Solution:-*** $\frac{1}{8}$, 128

Let $G_1, G_2, G_3, G_4$ be the four G.Ms between $\frac{1}{8}$ and 128.

*Here* $n = 6$

$$a = \frac{1}{8}$$

$$a_6 = 128$$

$$a_n = ar^{n-1}$$

$$a_6 = ar^{6-1}$$

$$128 = \frac{1}{8} r^5$$

$$128 \times 8 = r^5$$

$$1024 = r^5$$

$$4^5 = r^5$$

$$r = 4$$

$$G_1 = ar$$

$$= \frac{1}{8} \times 4 = \frac{1}{2}$$

$$G_2 = ar^2$$

$$= \frac{1}{8} \times 4^2$$

$$= \frac{1}{8} \times 16 = 2$$

$$G_3 = ar^3$$

$$= \frac{1}{8} \times 4^3$$

$$= \frac{1}{8} \times 64 = 8$$

$$G_4 = \frac{1}{8} r^4$$

$$= \frac{1}{8} \times 4^4 = 32$$

$\frac{1}{2}$, 2, 8, 32 are required G.Ms.

**11- *Insert six geometric means between 56and $-\frac{7}{16}$***

***Solution:-*** 56, $-\frac{7}{16}$

*Let $G_1, G_2, G_3, G_4, G_5, G_6$ be the six G.Ms between 56 and $-\frac{7}{16}$*

*Here* $n = 8$

$a = 56$

$a_6 = -\frac{7}{16}$

*Since* $a_n = ar^{n-1}$

$a_6 = ar^{a-1}$

$$-\frac{7}{16} = 56r^7$$

$$r^7 = -\frac{7}{16 \times 56}$$

$$= -\frac{1}{16 \times 8}$$

$$= -\frac{1}{128}$$

$$r^7 = \left(-\frac{1}{2}\right)^7$$

$$r = -\frac{1}{2}$$

$$G_1 = ar$$

$$= 56\left(-\frac{1}{2}\right) = -28$$

$$G_2 = ar^2$$

$$= 56\left(-\frac{1}{2}\right)^2$$

$$= 56 \times \frac{1}{4} = 14$$

$$G_3 = ar^3$$

$$= 56 \times \left(-\frac{1}{2}\right)^3$$

$$= 56 \times \left(-\frac{1}{8}\right) = -7$$

$$G_4 = ar^4$$

$$= 56\left(-\frac{1}{2}\right)^4$$

$$= 56 \times \frac{1}{16} = \frac{7}{2}$$

$$G_5 = ar^5$$

$$= a\left(-\frac{1}{2}\right)^5$$

$$= 56 \times \left(-\frac{1}{32}\right) = -\frac{7}{4}$$

$$G_6 = ar^6$$

$$= 56 \times \left(-\frac{1}{2}\right)^6$$

$$= 56 \times \frac{1}{64} = \frac{7}{8}$$

Six required G. Ms are $-28, 14, -7, \frac{7}{2}, -\frac{7}{4}, \frac{7}{8}$.

**12-** ***Insert five geometric means between $\frac{32}{81}$ and $\frac{9}{2}$***

***Solution:-***

Numbers are $\frac{32}{81}, \frac{9}{2}$

Let $G_1 G_2 G_3 G_4 G_5$ *be the five G. Ms between* $\frac{32}{81}$ and $\frac{9}{2}$.

*Here* $n = 7$

$$a = \frac{32}{81}$$

$$a_6 = +\frac{9}{2}$$

*Since* $a_n = ar^{n-1}$

$$a_n = ar^{6-1}$$

$$\frac{9}{2} = \frac{32}{81}r^5$$

$$r^5 = \frac{9}{2} \times \frac{32}{81}$$

$$r^5 = \frac{729}{64} = \left(\frac{3}{2}\right)^5$$

*Therefore* $r = \frac{3}{2}$

$$G_1 = ar$$

$$= \frac{32}{81} \times \frac{3}{2} = \frac{16}{27}$$

$$G_2 = ar^2$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^2$$

$$= \frac{32}{81} \times \frac{9}{4} = \frac{8}{9}$$

$$G_3 = ar^3$$

$$=\frac{32}{81}\times\left(\frac{3}{2}\right)^3$$

$$=\frac{32}{81}\times\frac{27}{8}=\frac{4}{3}$$

$$G_4 = ar^4$$

$$=\frac{32}{81}\times\left(\frac{3}{2}\right)^4$$

$$=\frac{32}{81}\times\frac{81}{16}=2$$

$$G_5 = ar^5$$

$$=\frac{32}{81}\times\left(\frac{3}{2}\right)^5$$

$$=\frac{32}{81}\times\frac{243}{32}=3$$

Five required G.Ms are $\frac{16}{27},\frac{8}{9},\frac{4}{3},2,3$