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EX 6.5

① i) Foci $(\pm 3, 0)$

Length of minor Axis = 10

Here $c = 3$ and $2b = 10 \Rightarrow b = 5$

Now $c^2 = a^2 - b^2 \Rightarrow 9 = a^2 - 25 \Rightarrow a^2 = 34$

Thus Required Equation of the

Ellipse is $\frac{x^2}{34} + \frac{y^2}{25} = 1$ — ①

From ①

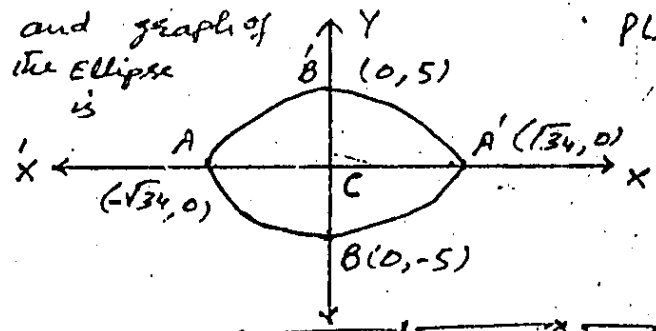
$$a^2 = 34 \Rightarrow a = \pm\sqrt{34}$$

$$b^2 = 25 \Rightarrow b = \pm 5$$

$\therefore$ vertices of the ellipse on the x-axis are $(\pm\sqrt{34}, 0)$ and

co-vertices are $(0, \pm 5)$

and graph of the ellipse is



ii) Foci $(0, -1)$ & $(0, -5)$

Length of major Axis = 6

Here Centre of the ellipse

$$= \left(\frac{0+0}{2}, \frac{-1-5}{2} \right) = (0, -3)$$

and c is the distance from the centre to each focus. So

$$c = \sqrt{(0-0)^2 + (-3+1)^2} = \sqrt{0+4} = 2$$

Also given that $2a = 6 \Rightarrow \boxed{a=3}$

$$\Rightarrow \boxed{a=3}$$

Now using $c^2 = a^2 - b^2$

$$\Rightarrow 4 = 9 - b^2 \Rightarrow b^2 = 9 - 4 \Rightarrow b^2 = 5$$

From the foci we see that Major Axis is along the y-axis

Thus Required equation of the ellipse is

$$\frac{(y+3)^2}{9} + \frac{x^2}{5} = 1 \quad \text{--- ①}$$

From ① $a^2 = 9 \Rightarrow a = \pm 3$

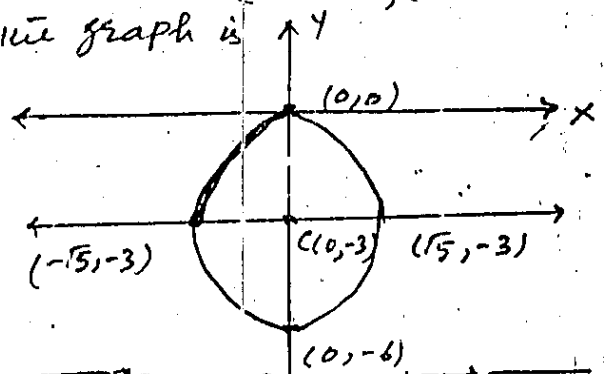
$$b^2 = 5 \Rightarrow b = \pm\sqrt{5}$$

$\therefore$ vertices are $(0, -3 \pm 3)$

i.e. $(0, 0)$, $(0, -6)$ and

co-vertices are $(\sqrt{5}, -3)$, $(-\sqrt{5}, -3)$

and the graph is



iii) Foci $(\pm 3\sqrt{3}, 0)$ vertices $(\pm 6, 0)$

Here $c = 3\sqrt{3}$ and $\boxed{a=6}$

Now using $c^2 = a^2 - b^2$

$$\Rightarrow 27 = 36 - b^2 \Rightarrow \boxed{b^2 = 9}$$

Thus Required Equation of the Ellipse

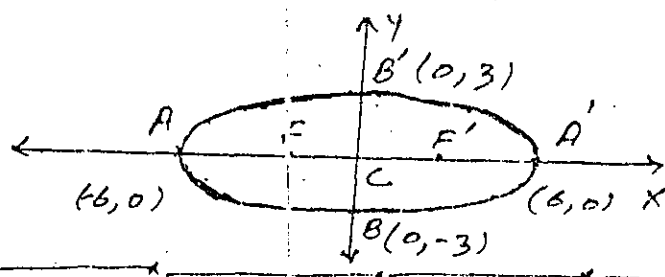
$$\frac{x^2}{36} + \frac{y^2}{9} = 1 \quad \text{--- (1)}$$

Here $a^2 = 36 \Rightarrow a = \pm 6$

$\therefore$ vertices $(6, 0), (-6, 0)$

and $b^2 = 9 \Rightarrow b = \pm 3$

so co-vertices are $(0, 3), (0, -3)$
and the graph of (1) is



(iv) vertices $(-1, 1), (5, 1)$

Foci $(4, 1), (0, 1)$

Here mid point of the Foci
= Centre of the ellipse

$$= \left(\frac{4+0}{2}, \frac{1+1}{2} \right) = (2, 1)$$

Distance between the vertices

$$= 2a = \sqrt{(5+1)^2 + (1-1)^2} = \sqrt{36} = 6$$

$$\Rightarrow 2a = 6 \Rightarrow \boxed{a = 3}$$

Distance between the foci $= 2c$

$$= \sqrt{(4-0)^2 + (1-1)^2} = \sqrt{16} = 4$$

$$\Rightarrow 2c = 4 \Rightarrow \boxed{c = 2}$$

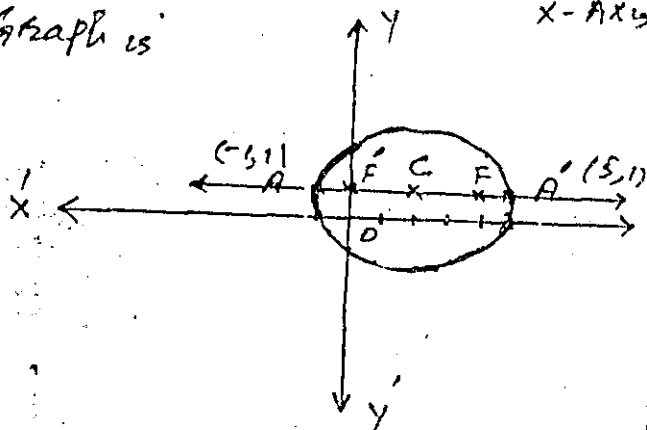
Now using $c^2 = a^2 - b^2 \Rightarrow b^2 = a^2 - c^2$

$$\Rightarrow b^2 = 9 - 4 = 5 \Rightarrow \boxed{b^2 = 5}$$

Thus Required equation of the Ellipse

$$\frac{(x-2)^2}{9} + \frac{(y-1)^2}{5} = 1 \quad \because \text{Axis is } \parallel \text{ to the } x\text{-axis}$$

Graph is



(v) Foci $(\pm\sqrt{5}, 0)$ and through

$$\left(\frac{3}{2}, \sqrt{3} \right)$$

Here

Foci are $(\sqrt{5}, 0)$ & $(-\sqrt{5}, 0)$

Centre is $= \left(\frac{\sqrt{5}-\sqrt{5}}{2}, \frac{0+0}{2} \right) = (0, 0)$

and $c = \sqrt{5}$.

Now using $c^2 = a^2 - b^2 \Rightarrow 5 = a^2 - b^2$

$$\Rightarrow \boxed{b^2 = a^2 - 5} \quad \text{--- (i)}$$

From the foci we see that Major Axis of the Ellipse is along the x-axis

So the Required equation is of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - 5} = 1 \quad \because b^2 = a^2 - 5$$

As this Ellipse passes through $\left(\frac{3}{2}, \sqrt{3} \right)$

$$\text{Then } \frac{9}{4a^2} + \frac{3}{a^2 - 5} = 1$$

$$\Rightarrow 9(a^2 - 5) + 12a^2 = 4a^2(a^2 - 5)$$

$$\Rightarrow 9a^2 - 45 + 12a^2 = 4a^4 - 20a^2$$

$$21a^2 - 45 - 4a^4 + 20a^2 = 0$$

$$-4a^4 + 41a^2 - 45 = 0$$

$$\Rightarrow 4a^4 - 41a^2 + 45 = 0$$

which is quadratic in a^2

$$\therefore a^2 = \frac{41 \pm \sqrt{1681 - 720}}{8} = \frac{41 \pm 31}{8}$$

$$a^2 = \frac{41+31}{8}, \quad \frac{41-31}{8} = \frac{72}{8}, \quad \frac{10}{8}$$

$$a^2 = 9, \quad \frac{5}{4}$$

For $a^2 = 9$

From (i) $b^2 = 9 - 5 \Rightarrow \boxed{b^2 = 4}$

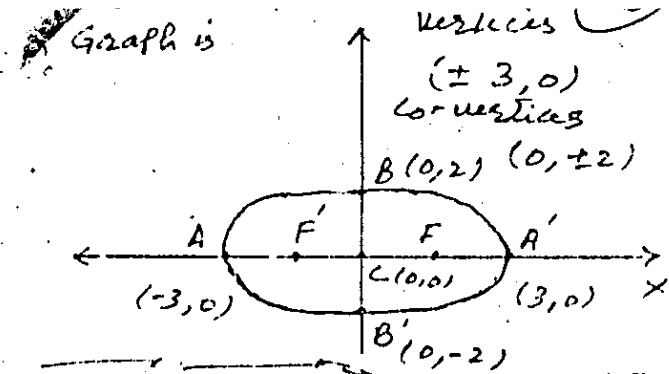
For $a^2 = \frac{5}{4}$

From (i) $b^2 = \frac{5}{4} - 5 = \frac{5-20}{4} = -\frac{15}{4} < 0$

i.e. $b^2 = -\frac{15}{4} < 0$ Neglecting (which is not possible)

$\therefore$ Required equation is $a^2 = 9 \Rightarrow a = \pm 3$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \quad \left| \quad b^2 = 4 \Rightarrow b = \pm 2 \right.$$



vi) vertices $(0, \pm 5)$, $e = \frac{3}{5}$
Here $a = 5$, $e = \frac{3}{5}$, Centre $(0, 0)$

Now $\therefore c = ae \Rightarrow c = 5\left(\frac{3}{5}\right) = 3$
 $c = 3$

Again using $c^2 = a^2 - b^2$

$\Rightarrow 9 = 25 - b^2 \Rightarrow b^2 = 25 - 9$

$\Rightarrow b^2 = 16$

From the vertices $(0, 5), (0, -5)$
we see that Axis of the Ellipse
is along y-Axis

$\therefore$ Required Equation of the Ellipse

is $\frac{x^2}{16} + \frac{y^2}{25} = 1$

i.e. $\frac{y^2}{25} + \frac{x^2}{16} = 1$ — (1)

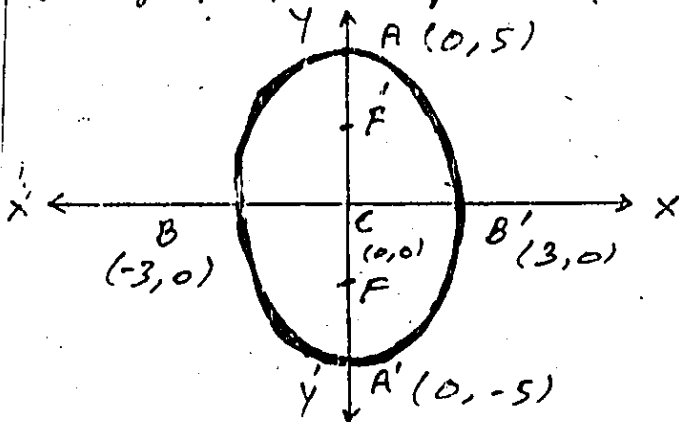
From (1) $a^2 = 25 \Rightarrow a = \pm 5$

$\therefore$ vertices are $(0, 5), (0, -5)$

and $b^2 = 16 \Rightarrow b = \pm 4$

$\therefore$ Co-vertices $(4, 0), (-4, 0)$

Thus graph of the Ellipse (1) is



vii) Centre $(0, 0)$ Focus $(0, -3)$

Vertex $(0, 4)$

Here $c = 3, a = 4$

Now using $c^2 = a^2 - b^2$

$\Rightarrow b^2 = a^2 - c^2 \Rightarrow b^2 = 16 - 9 = 7$

$b^2 = 7$

Thus required eq. of the Ellipse

$\frac{y^2}{16} + \frac{x^2}{7} = 1$ — (1)

$\therefore a^2 = 16 \Rightarrow a = \pm 4$

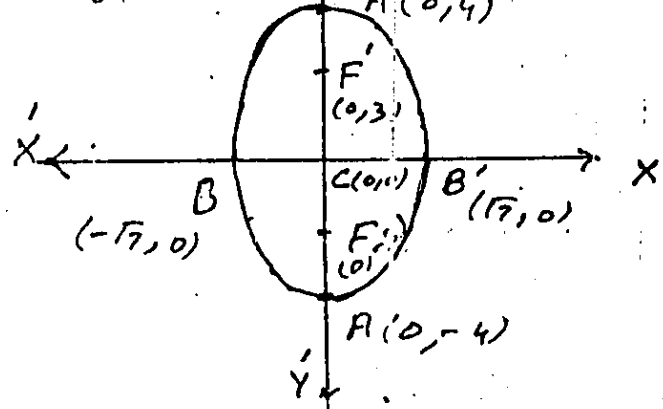
$\therefore$ vertices $(0, 4), (0, -4)$

and $b^2 = 7 \Rightarrow b = \pm \sqrt{7}$

$\therefore$ Co-vertices $(\sqrt{7}, 0), (-\sqrt{7}, 0)$

Centre $(0, 0)$, Foci $(0, \pm 3)$

So graph is



viii) Centre $(2, 2)$

$2a = 8 \Rightarrow a = 4$ || to y-Axis

and $2b = 6 \Rightarrow b = 3$ || to x-Axis

Required equation is

$\frac{(y-2)^2}{16} + \frac{(x-2)^2}{9} = 1$ — (1)

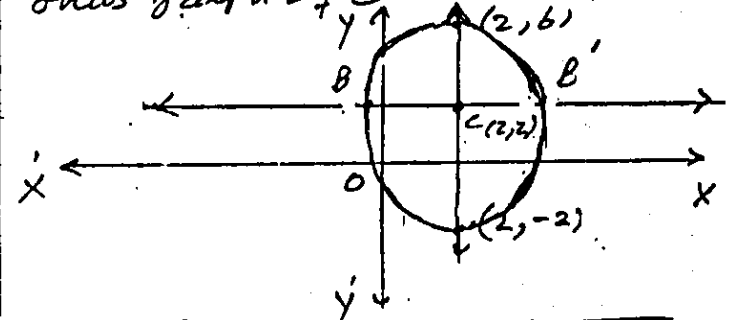
vertices are $(2, 2 \pm 4)$

i.e. $(2, 6), (2, -2)$

Co-vertices are $(2 \pm 3, 2)$

i.e. $(5, 2), (-1, 2)$, Centre $(2, 2)$

Thus graph of (1) is



(ix) $C(0,0)$, $(2,3)$, $(6,1)$

Let Equation of the Ellipse is

$$\frac{x^2}{d_1^2} + \frac{y^2}{d_2^2} = 1 \quad \text{--- (1)}$$

$\therefore$ The Ellipse passes through $(2,3)$ and $(6,1)$

$$\therefore \frac{4}{d_1^2} + \frac{9}{d_2^2} = 1 \quad \text{--- (2)}$$

$$\text{and } \frac{36}{d_1^2} + \frac{1}{d_2^2} = 1 \quad \text{--- (3)}$$

By multiplying Eq. (2) by 9 and then subtracting (3) from it.

$$\frac{36}{d_1^2} + \frac{81}{d_2^2} = 9$$

$$-\frac{36}{d_1^2} + \frac{1}{d_2^2} = 1$$

$$\frac{80}{d_2^2} = 8 \Rightarrow 8d_2^2 = 80$$

$$\Rightarrow d_2^2 = 10$$

putting $d_2^2 = 10$ in (2) we get

$$\frac{4}{d_1^2} + \frac{9}{10} = 1 \Rightarrow \frac{4}{d_1^2} = 1 - \frac{9}{10}$$

$$\Rightarrow \frac{4}{d_1^2} = \frac{1}{10} \Rightarrow d_1^2 = 40$$

Thus Required equation of the Ellipse is

$$\frac{x^2}{40} + \frac{y^2}{10} = 1$$

Here $C(0,0)$

$$a^2 = 40 \Rightarrow a = 2\sqrt{10}$$

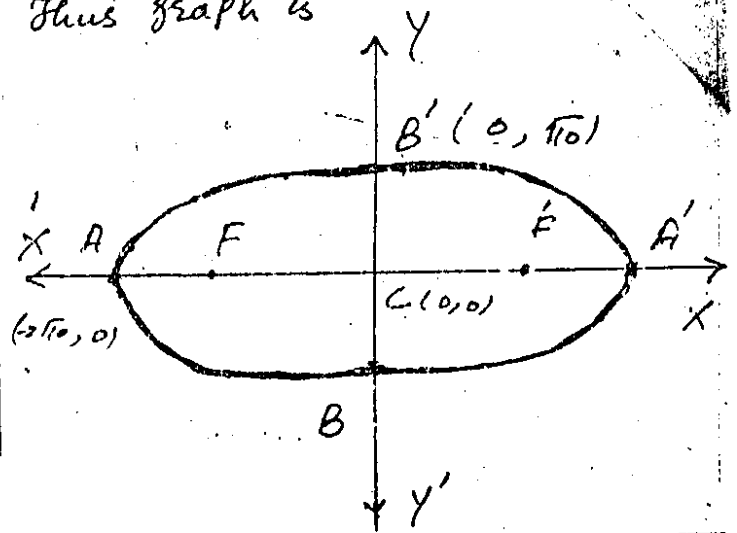
$\therefore$ vertices $(\pm 2\sqrt{10}, 0)$

and $b^2 = 10 \Rightarrow b = \pm \sqrt{10}$

$\therefore$ co-vertices $(0, \pm \sqrt{10})$

points on the Ellipse are $(2,3)$ & $(6,1)$

Thus graph is



(x) Centre $(0,0)$, $(3,1)$, $(4,0)$

Let the required equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{--- (1)}$$

As the points $(3,1)$ and $(4,0)$ lie on (1)

$$\therefore \frac{9}{a^2} + \frac{1}{b^2} = 1 \quad \text{--- (2)}$$

$$\text{and } \frac{16}{a^2} + 0 = 1 \Rightarrow \frac{16}{a^2} = 1$$

$$\Rightarrow a^2 = 16$$

putting $a^2 = 16$ in (2) we get

$$\frac{9}{16} + \frac{1}{b^2} = 1 \Rightarrow \frac{1}{b^2} = 1 - \frac{9}{16} = \frac{7}{16}$$

$$7b^2 = 16 \Rightarrow b^2 = \frac{16}{7}$$

$\therefore$ Required equation of the Ellipse is

$$\frac{x^2}{16} + \frac{y^2}{\frac{16}{7}} = 1$$

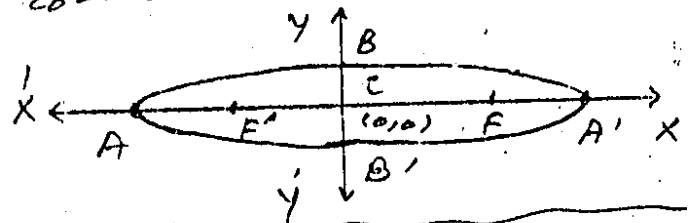
$$\Rightarrow \frac{x^2}{16} + \frac{7y^2}{16} = 1 \quad \text{--- (3)}$$

$$\therefore a^2 = 16 \Rightarrow a = \pm 4$$

$\therefore$ vertices $(\pm 4, 0)$

$$b^2 = \frac{16}{7} \Rightarrow b = \pm \frac{4}{\sqrt{7}}$$

co-vertices $(0, \pm \frac{4}{\sqrt{7}})$ graph is



$$\begin{aligned} \text{Q6) } x^2 + 4y^2 &= 16 \quad (36) \quad \text{Case} \\ \Rightarrow \frac{x^2}{16} + \frac{4y^2}{16} &= \frac{16}{16} \quad e = \frac{c}{a} \\ \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} &= 1 \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \text{Here } a^2 &= 16 \Rightarrow a = 4 \\ b^2 &= 4 \Rightarrow b = 2 \\ \text{and } c^2 &= a^2 - b^2 = 16 - 4 \Rightarrow c^2 = 12 \\ &\Rightarrow c = 2\sqrt{3} \\ \text{Now Centre is } (0, 0) \end{aligned}$$

$$\text{Foci are } (\pm 2\sqrt{3}, 0)$$

$$\text{Eccentricity: } e = \frac{c}{a} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\text{Vertices: } (\pm 4, 0)$$

$$\text{Directrices: } x = \pm \frac{c}{e^2} = \pm \frac{ae}{e^2}$$

$$\Rightarrow x = \pm \frac{a}{e} = \pm \frac{4}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow x = \pm \frac{8}{\sqrt{3}}$$

$$\begin{aligned} \text{ii) } 9x^2 + y^2 &= 18 \\ \Rightarrow \frac{9x^2}{18} + \frac{y^2}{18} &= \frac{18}{18} \\ \frac{x^2}{2} + \frac{y^2}{18} &= 1 \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \text{Here } a^2 &= 18 \Rightarrow a = 3\sqrt{2} \\ b^2 &= 2 \Rightarrow b = \sqrt{2} \\ \text{and } c^2 &= a^2 - b^2 = 18 - 2 = 16 \\ c &= 4, \quad e = \frac{c}{a} = \frac{4}{3\sqrt{2}} \\ \text{Now Centre: } (0, 0) \end{aligned}$$

$$\text{Foci: } (0, \pm c) = (0, \pm 4)$$

$$\text{Eccentricity: } e = \frac{c}{a} = \frac{4}{3\sqrt{2}}$$

$$\Rightarrow e = \frac{4}{3\sqrt{2}}$$

$$\text{Vertices: } (0, \pm a) = (0, \pm 3\sqrt{2})$$

$$\text{Directrices } y = \pm \frac{c}{e^2} = \pm \frac{ae}{e^2}$$

$$\Rightarrow y = \pm \frac{a}{e} \Rightarrow y = \pm \frac{3\sqrt{2}}{\frac{4}{3\sqrt{2}}} = \pm \frac{18}{4}$$

$$\Rightarrow y = \pm \frac{9}{2}$$

$$\begin{aligned} \text{iii) } 25x^2 + 9y^2 &= 225 \quad P(36) \\ \Rightarrow \frac{25x^2}{225} + \frac{9y^2}{225} &= \frac{225}{225} \\ \frac{x^2}{9} + \frac{y^2}{25} &= 1 \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \text{Here } a^2 &= 25 \Rightarrow a = 5 \quad \text{Case} \\ b^2 &= 9 \Rightarrow b = 3 \\ \text{Using } c^2 &= a^2 - b^2 \\ \Rightarrow c^2 &= 25 - 9 = 16 \Rightarrow c = 4 \\ \text{Now Centre of the Ellipse (1) is } (0, 0) \end{aligned}$$

$$\text{Foci: } (0, \pm 4)$$

$$\text{Eccentricity: } e = \frac{c}{a} = \frac{4}{5}$$

$$\text{Vertices: } (0, \pm a) = (0, \pm 5)$$

$$\text{Directrices: } y = \pm \frac{c}{e^2} = \pm \frac{ae}{e^2}$$

$$\Rightarrow y = \pm \frac{a}{e} = \pm \frac{5}{\frac{4}{5}} = \pm \frac{25}{4}$$

$$\Rightarrow y = \pm \frac{25}{4}$$

$$\text{iv) } \frac{(2x-1)^2}{16} + \frac{(y+2)^2}{16} = 1 \quad \text{--- (1)}$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{16} = 1 \quad \text{--- (1)} \quad \begin{matrix} X = 2x-1 \\ Y = y+2 \end{matrix}$$

$$a^2 = 16 \Rightarrow a = 4, \quad b^2 = 4 \Rightarrow b = 2$$

$$\text{and } c^2 = a^2 - b^2 = 16 - 4 = 12 \Rightarrow c = 2\sqrt{3}$$

$$\text{For Centre } X = 0, \quad Y = 0$$

$$\Rightarrow 2x-1 = 0$$

$$x = \frac{1}{2}$$

$$\Rightarrow y+2 = 0$$

$$y = -2$$

$$\therefore \text{Centre is } \left(\frac{1}{2}, -2\right)$$

$$\text{For Foci: } X = 0, \quad Y = \pm c$$

$$\Rightarrow 2x-1 = 0$$

$$x = \frac{1}{2}$$

$$y+2 = \pm 2\sqrt{3}$$

$$y = -2 \pm 2\sqrt{3}$$

$$\therefore \text{Foci: } \left(\frac{1}{2}, -2 \pm 2\sqrt{3}\right)$$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{2\sqrt{3}}{4}$$

$$\Rightarrow e = \frac{\sqrt{3}}{2}$$

$$\text{Directrices } Y = \pm \frac{c}{e^2} = \pm \frac{ae}{e^2} = \pm \frac{a}{e}$$

$$\Rightarrow y+2 = \pm \frac{4}{\frac{\sqrt{3}}{2}}$$

$$\Rightarrow y = -2 \pm \frac{8}{\sqrt{3}}$$

$$\begin{aligned}
 \text{(V)} \quad & x^2 + 16x + 4y^2 - 16y + 76 = 0 \quad \text{--- (1)} \\
 \Rightarrow & x^2 + 16x + 64 + 4(y^2 - 4y) + 76 = 64 \\
 \Rightarrow & (x+8)^2 + 4(y^2 - 4y + 4 - 4) = 64 - 76 \\
 & (x+8)^2 + 4(y-2)^2 - 16 = -12 \\
 & (x+8)^2 + 4(y-2)^2 = -12 + 16 \\
 & (x+8)^2 + 4(y-2)^2 = 4 \\
 \Rightarrow & \frac{(x+8)^2}{4} + \frac{4(y-2)^2}{4} = \frac{4}{4}
 \end{aligned}$$

$$\frac{(x+8)^2}{4} + \frac{(y-2)^2}{1} = 1 \quad \text{--- (1)}$$

which is of the form

$$\frac{X^2}{4} + \frac{Y^2}{1} = 1 \quad \left. \begin{array}{l} \text{Major Axis} \\ \text{(11 to X-Axis)} \end{array} \right\}$$

where $X = x+8$, $Y = y-2$

$$a^2 = 4 \Rightarrow a = 2$$

$$b^2 = 1 \Rightarrow b = 1$$

$$\text{Now using } c^2 = a^2 - b^2 = 4 - 1 = 3$$

$$c = \sqrt{3}$$

For Centre $X = 0$, $Y = 0$

$$\Rightarrow x+8=0, \quad y-2=0$$

$$x = -8, \quad y = 2$$

$\therefore$ Required Centre of the Ellipse is $(-8, 2)$

For Foci

$$X = \pm c, \quad Y = 0$$

$$\Rightarrow x+8 = \pm \sqrt{3}, \quad y-2=0$$

$$x = -8 \pm \sqrt{3}, \quad y = 2$$

$\therefore$ Foci $(-8 \pm \sqrt{3}, 2)$

$$\text{For Eccentricity } e = \frac{c}{a} = \frac{\sqrt{3}}{2}$$

For Vertices

$$X = \pm a, \quad Y = 0$$

$$\Rightarrow x+8 = \pm 2, \quad y-2=0$$

$$x = -8 \pm 2 = -6, -10 \quad \boxed{y=2}$$

$\therefore$ vertices are $(-6, 2), (-10, 2)$

$$\text{Directrices } X = \pm \frac{c}{e^2} = \pm \frac{\sqrt{3}}{\frac{3}{4}}$$

$$\Rightarrow x+8 = \pm \frac{4\sqrt{3}}{3} = \pm \frac{4}{\sqrt{3}}$$

$$\Rightarrow x = -8 \pm \frac{4}{\sqrt{3}}$$

$$\text{vi) } 25x^2 + 4y^2 - 250x - 16y + 541 =$$

$$\Rightarrow 25x^2 - 250x + 4y^2 - 16y = -541$$

$$25(x^2 - 10x) + 4(y^2 - 4y) = -541$$

$$25(x^2 - 10x + 25 - 25) + 4(y^2 - 4y + 4 - 4) = -541$$

$$25[(x-5)^2 - 25] + 4[(y-2)^2 - 4] = -541$$

$$25(x-5)^2 - 625 + 4(y-2)^2 - 16 = -541$$

$$25(x-5)^2 + 4(y-2)^2 = -541 + 625 + 16$$

$$25(x-5)^2 + 4(y-2)^2 = 100$$

$$\Rightarrow \frac{25(x-5)^2}{100} + \frac{4(y-2)^2}{100} = \frac{100}{100}$$

$$\frac{(x-5)^2}{4} + \frac{(y-2)^2}{25} = 1 \quad \text{--- (1)}$$

which is of the form

$$\frac{X^2}{4} + \frac{Y^2}{25} = 1 \quad \text{--- (1)}$$

where $X = x-5$, $Y = y-2$

$$a^2 = 25 \Rightarrow a = 5$$

$$b^2 = 4 \Rightarrow b = 2$$

Now using $c^2 = a^2 - b^2$

$$\Rightarrow c^2 = 25 - 4 = 21 \Rightarrow c = \sqrt{21}$$

Now for Centre

$$X = 0 \quad \text{and} \quad Y = 0$$

$$\Rightarrow x-5=0, \quad y-2=0$$

$$x = 5, \quad y = 2$$

$\therefore$ Centre of the Ellipse $(5, 2)$

For Foci

$$X = 0, \quad Y = \pm c$$

$$\Rightarrow x-5=0, \quad y-2 = \pm \sqrt{21}$$

$$x = 5, \quad y = 2 \pm \sqrt{21}$$

$$\therefore \text{foci} : (5, 2 \pm \sqrt{21})$$

$$\text{Eccentricity } e = \frac{c}{a} = \frac{\sqrt{21}}{5}$$

For vertices

$$\begin{aligned} X=0, & \quad Y = \pm a \\ \Rightarrow x-5=0 & \quad y-2 = \pm 5 \\ x=5 & \quad y = 2 \pm 5 = 7, -3 \end{aligned}$$

$\therefore$ vertices are

$$(5, -3), (5, 7).$$

$$\text{Directrices } Y = \pm \frac{c}{e^2}$$

$$\Rightarrow y-2 = \pm \frac{\sqrt{21}}{\frac{21}{25}} = \pm \frac{25\sqrt{21}}{21}$$

$$y = 2 \pm \frac{25\sqrt{21}}{21}$$

$$\textcircled{3} \quad 0 < c < a, \quad F(-c, 0), F(c, 0)$$

$$P(x, y)$$

$$\text{Given That } |PF| + |PF'| = 2a$$

$$\Rightarrow \sqrt{(x+c)^2 + (y-0)^2} + \sqrt{(x-c)^2 + (y-0)^2} = 2a$$

$$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2} \quad \textcircled{1}$$

Squaring both sides of ①

$$(x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

$$x^2 + 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + x^2 - 2cx + c^2 + y^2$$

$$\Rightarrow 2cx + 2cx - 4a^2 = -4a\sqrt{(x-c)^2 + y^2}$$

$$4cx - 4a^2 = -4a\sqrt{x^2 - 2cx + c^2 + y^2}$$

$$4(cx - a^2) = -4a\sqrt{x^2 + y^2 + c^2 - 2cx}$$

$$cx - a^2 = -a\sqrt{x^2 + y^2 + c^2 - 2cx}$$

$$\Rightarrow a^2 - cx = a\sqrt{x^2 + y^2 + c^2 - 2cx} \quad \textcircled{2}$$

Again Squaring both sides of ②

we get

$$a^4 + c^2x^2 - 2a^2cx = a^2x^2 + a^2y^2 + a^2c^2 - 2a^2cx$$

$$\Rightarrow c^2x^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$$

$$\Rightarrow a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$$

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2) \quad P(38)$$

Dividing both sides by $a^2(a^2 - c^2)$

$$\frac{(a^2 - c^2)x^2}{a^2(a^2 - c^2)} + \frac{a^2y^2}{a^2(a^2 - c^2)} = \frac{a^2(a^2 - c^2)}{a^2(a^2 - c^2)}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

$$\text{i.e. } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{where } a^2 - c^2 = b^2$$

which is an Ellipse

$$\textcircled{4} \quad P(x, y) \quad (0, 0), (1, 1), 2$$

we Name the given points

$$O(0, 0) \text{ \& } A(1, 1)$$

Now Given That

$$|OP| + |AP| = 2$$

$$\text{i.e. } \sqrt{(x-0)^2 + (y-0)^2} + \sqrt{(x-1)^2 + (y-1)^2} = 2$$

$$\Rightarrow \sqrt{(x-1)^2 + (y-1)^2} = 2 - \sqrt{x^2 + y^2} \quad \textcircled{1}$$

Squaring both sides of ① we have

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 4 + x^2 + y^2 - 4\sqrt{x^2 + y^2}$$

$$-2x - 2y + 2 - 4 = -4\sqrt{x^2 + y^2}$$

$$-2x - 2y - 2 = -4\sqrt{x^2 + y^2}$$

$$\Rightarrow x + y + 1 = 2\sqrt{x^2 + y^2}$$

$$\text{or } 2\sqrt{x^2 + y^2} = x + y + 1 \quad \textcircled{2}$$

Again Squaring both sides of ②

$$4(x^2 + y^2) = x^2 + y^2 + 1 + 2xy + 2x + 2y$$

$$4x^2 + 4y^2 - x^2 - y^2 - 2xy - 2x - 2y - 1 = 0$$

$$3x^2 + 3y^2 - 2xy - 2x - 2y - 1 = 0$$

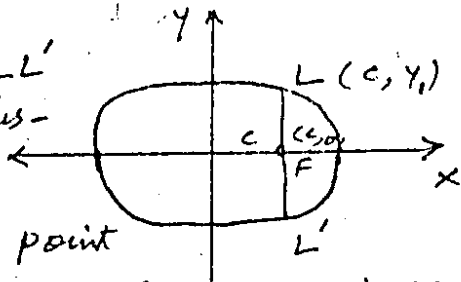
$$3x^2 - 2xy + 3y^2 - 2x - 2y - 1 = 0 \quad \textcircled{3}$$

(5) Latusrectum: The focal Chord perpendicular to the Major Axis is called Latusrectum of the Ellipse.

Let us consider the Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \textcircled{1} \quad a > b$$

Suppose L, L' be the Latus-rectum



Then the point

L is $L(c, y_1)$ when focus is $F(c, 0)$

$\therefore L(c, y_1)$ lies on ①

$$\therefore \frac{c^2}{a^2} + \frac{y_1^2}{b^2} = 1 \Rightarrow y_1^2 = b^2 \left(1 - \frac{c^2}{a^2}\right)$$

$$\Rightarrow y_1^2 = b^2 \left(\frac{a^2 - c^2}{a^2}\right)$$

$$y_1^2 = b^2 \left(\frac{b^2}{a^2}\right) \quad \because a^2 - c^2 = b^2$$

$$\Rightarrow y_1^2 = \frac{b^4}{a^2} \Rightarrow y_1 = \pm \frac{b^2}{a}$$

$\therefore$ The points L and L' are $L(c, \frac{b^2}{a})$ and $L'(c, -\frac{b^2}{a})$

$$\text{Now } |LL'| = \sqrt{(c-c)^2 + \left(\frac{b^2}{a} + \frac{b^2}{a}\right)^2}$$

$$= \sqrt{\left(\frac{2b^2}{a}\right)^2} = \frac{2b^2}{a}$$

Thus $|LL'| = \frac{2b^2}{a}$ Hence Proved.

(b) Given that $2a = 4\sqrt{2}$

$$\Rightarrow a = 2\sqrt{2} \Rightarrow a^2 = 8$$

Also given that $2c = 2b$

$$\Rightarrow c = b$$

Now using $c^2 = a^2 - b^2$

$$\Rightarrow b^2 = a^2 - b^2 \Rightarrow 2b^2 = a^2$$

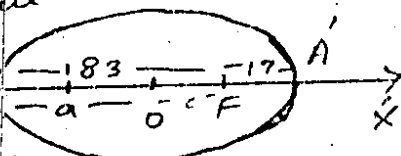
$$\Rightarrow 2b^2 = 8 \Rightarrow b^2 = 4$$

Thus required equation of the Ellipse is $\frac{x^2}{8} + \frac{y^2}{4} = 1$ Ans.

⑦ Let the Sun be at F .
Then we have

$$a - c = 17 \text{ --- ①}$$

$$a + c = 183 \text{ --- ②}$$



Adding ① and ②

$$2a = 200 \Rightarrow a = 100$$

Putting $a = 100$ in ② we get

$$100 + c = 183 \Rightarrow c = 83$$

Now using $c^2 = a^2 - b^2$

$$\Rightarrow b^2 = a^2 - c^2 = (100)^2 - (83)^2$$

$$= 10000 - 6889 = 3111$$

$$\boxed{b^2 = 3111}$$

$\therefore$ The equation is

$$\frac{x^2}{100^2} + \frac{y^2}{3111} = 1 \quad \text{Ans.}$$

⑧ Here

$$2a = 90 \Rightarrow a = 45$$

and

$$b = 30$$

$\therefore$ Equation of the ellipse is

$$\frac{x^2}{(45)^2} + \frac{y^2}{(30)^2} = 1 \quad \text{--- ①}$$

At the height $20\sqrt{2}$ m let x_1 be the distance from the centre. Then the point $(x_1, 20\sqrt{2})$ lies on the Ellipse ①

$$\therefore \frac{x_1^2}{(45)^2} + \frac{(20\sqrt{2})^2}{(30)^2} = 1$$

$$\frac{x_1^2}{2025} + \frac{800}{900} = 1 \Rightarrow \frac{x_1^2}{2025} = 1 - \frac{8}{9}$$

$$\Rightarrow \frac{x_1^2}{2025} = \frac{1}{9} \Rightarrow 9x_1^2 = 2025$$

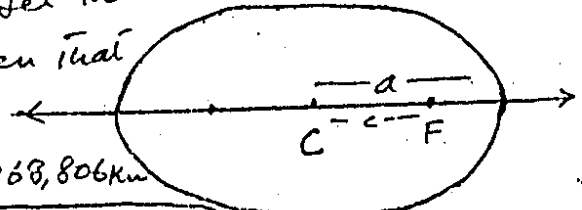
$$\Rightarrow x_1^2 = 225 \Rightarrow x_1 = \pm 15$$

$$\Rightarrow x_1 = 15 \text{ m} \quad (\text{Neglecting Negative value of } x_1)$$

$\therefore$ Required distance from the Centre = 15 m.

⑨ Let the earth be at F .

Given that



$$2a = 768,806 \text{ km}$$

$$\Rightarrow a = 384,403 \text{ km}$$

$$2b = 767,746 \text{ km}$$

$$\begin{aligned}
 & b = 383873 \text{ km} \\
 & \text{Using } c^2 = a^2 - b^2 \\
 & \Rightarrow c^2 = (a-b)(a+b) \\
 & \Rightarrow c^2 = 530(768276) \\
 & c^2 = 407186280 \\
 & c = 20178.86 \\
 & \text{Now Required greatest distance} \\
 & = a + c = 404582 \text{ km (Approx.)} \\
 & \text{and} \\
 & \text{Least distance} = a - c \\
 & = 364224 \text{ km (Approx.)}
 \end{aligned}$$

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If you have any suggestion question or query,

Exercise 6.5

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