

5

PROBABILITY

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5.1 INTRODUCTION

In this chapter, we introduce the concept of chance or probability. The subject of probability is of great importance these days and it is being applied in almost all branches of science and technology. The idea of probability was first applied in gambling games in France. In 17th century, the theory of probability was developed by mathematicians like **Pascal, Format Bernoulli**. The subject was further developed and put on a sound footing by **Laplace, Kolmogoroff, Markoff, Lagrange, Bayes, Gauss Morgan and Fisher** among others. To day the application of the theory of probability is the most important in making decisions in business, insurance, management, sociology, psychology, physics, etc.

Before defining probability, we explain some algebraic concepts or mathematical preliminaries or technologies of counting.

5.2 FACTORIAL NOTATION

n -factorial ($n!$) denotes the products of first n positive integers, i.e.,

$$n! = n(n-1)(n-2)(n-3) \dots 3 \cdot 2 \cdot 1$$

For example $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ is the product of first seven positive integers. Thus this product is denoted by 7 factorial, that is

$$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7!$$

$n!$ may also be written as

$$\begin{aligned} n! &= n(n-1)(n-2)(n-3)! \\ &= n(n-1)(n-2)! \\ &= n(n-1)! \end{aligned}$$

If $n = 0$, then we define $0! = 1$

EXAMPLE 5.1

Solve the following:

$$(i) 7! \quad (ii) \frac{10!}{8!} \quad (iii) \frac{12!}{8!0!} \quad (iv) \frac{n!}{(n-2)!} \quad (v) \frac{(n+1)!}{(n-1)!}$$

SOLUTION

$$(i) 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040 \quad (ii) \frac{10!}{8!} = \frac{10 \times 9 \times 8!}{8!} = 10 \times 9 = 90$$

$$(iii) \frac{12!}{8!0!} = \frac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 1} = 12 \times 11 \times 10 \times 9 = 11880$$

$$(iv) \frac{n!}{(n-2)!} = \frac{n(n-1)(n-2)!}{(n-2)!} = n(n-1) = n^2 - n$$

$$(v) \frac{(n+1)!}{(n-1)!} = \frac{(n+1)n(n-1)!}{(n-1)!} = (n+1)n = n^2 + n$$

5.3 PERMUTATIONS

Permutation is defined as the number of arrangements which can be made by taking some or all of the number of objects. Thus the number of

arrangements which can be made by taking three objects a, b and c in a row are given below:

abc, acb, bca, bac, cab, cba

Thus we see that any three objects can be arranged in six ($6=3!$) different ways.

5.3.1 Permutations of Different Objects

1. The number of permutations of n different objects, taken all at a time denoted by ${}^n P_n$, is given by

$${}^n P_n = n!$$

2. The number of permutations of n different objects, taken r at a time, denoted by ${}^n P_r$, is given by

$${}^n P_r = \frac{n!}{(n-r)!}$$

3. The number of permutations of n objects of which n_1 are of one type, n_2 are of second kind, n_3 are third type and n_s of the s th type, denoted by P , is given by

$$P = \frac{n!}{(n_1! n_2! n_3! \dots n_s!)}$$

EXAMPLE 5.2

Find the total number of permutations of the set of four letters taken

- (a) All at a time, (b) Three at a time, (c) Two at a time.

SOLUTION

- (a) Here $n = 4$, therefore,

$${}^n P_n = {}^4 P_4 = 4! = 4 \times 3 \times 2 \times 1 = 24$$

- (b) Here $n = 4$, $r = 3$, therefore,

$${}^n P_r = \frac{n!}{(n-r)!} \text{ gives, } {}^4 P_3 = \frac{4!}{(4-3)!} = 4! = 24$$

- (c) Here $n = 4$, $r = 2$, therefore,

$${}^n P_r = \frac{n!}{(n-r)!} \text{ gives, } {}^4 P_2 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$$

EXAMPLE 5.3

In how many ways can 7 men form a line?

SOLUTION

Here $n = 7$, therefore,

$${}^7 P_7 = 7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

EXAMPLES 5.4

How many telephone numbers of four different digits each can be made from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

SOLUTION

There are ten digits to choose from and four different digits are to be chosen at a time. Here

$n = 10$, $r = 4$, and the desired number is

$${}^{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \times 9 \times 8 \times 7 = 5040$$

EXAMPLE 5.5

Five tires on a car (one as spare) are to be rotated so that all will be used equally. How many different ways are there of putting the five tires on the car without removing the tires from the wheels?

SOLUTION

We have a permutation of five tires taken all at a time, then the desired number is

$$\begin{aligned} {}^nP_n &= {}^5P_5 = 5! \\ &= 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways} \end{aligned}$$

EXAMPLE 5.6

A company's labour union wants to elect 4 office bearers viz., President, Secretary, Joint secretary and a Treasurer out of eight people. In how many different ways these posts can be filled up?

SOLUTION

Here $n = 8$, $r = 4$

$$\begin{aligned} {}^nP_4 &= {}^8P_4 = \frac{8!}{(8-4)!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} \\ &= 8 \times 7 \times 6 \times 5 = 1680 \text{ ways} \end{aligned}$$

EXAMPLE 5.7

In how many ways 8 chairs can be arranged in a row if there are 4 ordinary chairs, 3 arm chairs and one easy chair?

SOLUTION

Required number is given by

$$P = \frac{8!}{4!3!1!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} = 8 \times 7 \times 5 = 280 \text{ ways}$$

EXAMPLE 5.8

In how many ways the letters in the word TOMORROW can be arranged?

SOLUTION

The word Tomorrow contains 8 letters in all, of these the 3 O's are alike of one kind, the 2 R's are alike of second kind. There is one T, one M and one W. Thus, we have

Total No. n	= 8	No. of T's	= 1
No of O's	= 3	No. of M's	= 1

No of R's = 2 No. of W's = 1
Hence required number of arrangements are given by

$$P = \frac{8!}{3! 2! 1! 1! 1!} = 3360$$

EXAMPLE 5.9

How many different arrangements are there of the letters in the word **MISSISSIPPI**, taken all together?

SOLUTION

These arrangements are permutations of 11 letters, with some alike. There are 4S, 4I's and 2 P's. Hence

$$P = \frac{11!}{4! 4! 2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1 \times 2 \times 1} = 34650 \text{ arrangements}$$

5.4 COMBINATION

A combination is a subset or part of the objects of a single set without regarding the order of the object, the total number of possible combinations of a set of objects, taken all at a time, is clearly one. The total number of possible combinations of a set of a different object, taken r at a time, denoted by nC_r , is given by

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

EXAMPLE 5.10

Find ${}^{12}C_8$

SOLUTION

$$\begin{aligned} \text{We know that } {}^nC_r &= \frac{n!}{r!(n-r)!} = {}^{12}C_8 = \frac{12!}{8!(12-8)!} = \frac{12!}{8!4!} \\ &= \frac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 4 \times 3 \times 2 \times 1} = 11 \times 5 \times 9 = 495 \end{aligned}$$

EXAMPLE 5.11

Three from 20 numbers are to be selected to form a committee. In how many ways can the committee be formed?

SOLUTION

The committee formed by members, say A, B and C, is the same as the committee formed by B, C, A or any other such arrangement problem, since we disregard the order of arrangement.

Here $n = 20$, $r = 3$, therefore,

$${}^nC_r = \frac{n!}{r!(n-r)!} \text{ gives}$$

$${}^{20}C_3 = \frac{20!}{2!(20-3)!} = \frac{20!}{3! 17!} = \frac{20 \times 19 \times 18 \times 17!}{17! \times 3 \times 2 \times 1} = 20 \times 19 \times 3 = 1140.$$

EXAMPLE 5.12

A bag contains 10 balls of which 4 are white and 6 are black. How many selections consisting of 5 balls can be made if two are white and 3 are black.

SOLUTION

Two white balls out of 4 white balls can be drawn n_1 ways as:

$$n_1 = {}^4C_2 = \frac{4!}{2! 2!} = 6$$

Three black balls out of six black balls can be drawn in n_2 ways as:

$$n_2 = {}^6C_3 = \frac{6!}{3! 3!} = 20$$

Five balls of which 2 white and 3 black balls out of 10 balls of which 4 are white and 6 black balls can be drawn n ways as:

$$n = n_1 \times n_2 = 6 \times 20 = 120$$

The desired number can be obtained as:

$$n = {}^4C_2 \times {}^6C_3 = 6 \times 20 = 120$$

EXAMPLE 5.13

A bag contains 8 white and 8 black balls. 4 balls are drawn. Find the number of possibilities is which

- (i) All are white.
- (ii) All are black.
- (iii) 2 white and 2 black.

SOLUTION

(i) Let n_1 be the number of possibilities of drawing four white balls and zero black balls out of 8 white and 8 black balls, then

$$n_1 = {}^8C_4 \times {}^8C_0 = \frac{8!}{4! 4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4 \times 3 \times 2 \times 1 \times 4!} = 70$$

(ii) Let n_2 be the number of possibilities to draw four black balls out of 8 white and 8 black balls. Then

$$n_2 = {}^8C_0 \times {}^8C_4 = 70$$

(iii) Let n_3 be the number of possibilities to draw two white and two black balls out of 8 white and 8 black balls. Then

$$n_3 = {}^8C_2 \times {}^8C_2 = \left(\frac{8!}{2! 6!} \right) \left(\frac{8!}{2! 6!} \right) = 28 \times 28 = 784$$

5.5 BINOMIAL EXPANSION

By actual multiplication the product $(a + b)(a + b)$ may be written as

$$\begin{array}{c} a + b \\ a + b \end{array}$$

$$\begin{array}{r} ab + b^2 \\ a^2 + ab \\ \hline a^2 + 2ab + b^2 \end{array}$$

Binomial expansion is an algebraic expression containing two terms which are connected by positive or negative sign. If the binomial $(a+b)$ where a and b are real numbers is raised to successive integral powers, the expanded expressions are as follows:

$$(a+b)^0 = 1$$

$$(a+b)^1 = a + b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

.....
.....

In general, when the binomial is raised to n th power, the expression is known as the binomial expansion given by

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_{n-1} ab^{n-1} + {}^nC_n b^n$$

From the above expansion, when n represents the power of $(a+b)$, we observe the following properties:

- 1) The number of terms in the expansion is $(n+1)$
- 2) The letter a in the first term of the expansion has the power n , in the second term has $(n-1)$, in the third term has $(n-2)$ and so on.
- 3) The letter b in the second term of the expansion has power one, in the third term has power two, in the fourth term has power three and so on.
- 4) The sum of powers of a and b in any term is n .
- 5) The coefficients of the terms equally placed from the ends of the expansion are the same. For example, in the expansion of $(a+b)^4$, the co-efficient of the first and the last term is one, the co-efficient of second term and the last second term is 4.

Substitution the values of nC_0 , nC_1 , nC_2 , etc. we have

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2} a^{n-2}b^2 + \dots + b^n$$

5.5.1 Pascal Triangle

If the coefficients in the expansion of $(a+b)^n$ for $n = 0, 1, 2, 3, 4, \dots$ are arranged triangular form, the result is known as the Pascal triangle.

n=0										1									
n=1										1		1							
n=2										1		2		1					
n=3										1		3		3		1			
n=4										1		4		6		4		1	
n=5										1		5		10		10		5	
n=6										1		6		15		20		15	
n=7										1		7		21		35		35	
n=8										1		8		28		56		70	
n=9										1		9		36		84		126	

5.5.2 General Term

From the binomial expansion $(r + 1)$ th term or general term is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

Putting $r = 0, 1, 2$, etc. We can get the first, second, third, etc. terms.

EXAMPLE 5.14

Expand $(a + x)^6$ by using binomial expansion.

SOLUTION

$$(a + x)^6 = a^6 + 6a^5x + 15a^4x^2 + 20a^3x^3 + 15a^2x^4 + 6ax^5 + x^6$$

EXAMPLE 5.15

Expand $(2x + 3y)^4$

SOLUTION

$$\begin{aligned} (2x + 3y)^4 &= (2x)^4 + 4(2x)^3(3y) + \frac{4 \times 3}{1 \times 2}(2x)^2(3y)^2 + \frac{4 \times 3 \times 2}{1 \times 2 \times 3}(2x)(3y)^3 + (3y)^4 \\ &= 16x^4 + 96x^3y + 216x^2y^2 + 216xy^3 + 81y^4 \end{aligned}$$

EXAMPLE 5.16

Expand $(x - 2y)^5$

SOLUTION

$$\begin{aligned} (x - 2y)^5 &= x^5 + {}^5C_1x^4(-2y) + {}^5C_2x^3(-2y)^2 + {}^5C_3x^2(2y)^3 + {}^5C_4x(-2y)^4 + (-2y)^5 \\ &= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5 \end{aligned}$$

EXAMPLE 5.17

Find the 6th term of $(x + 2y)^{10}$

SOLUTION

Here $r = 5$, $n = 10$, $a = x$, $b = 2y$

$$\begin{aligned} \text{Now } T_{r+1} &= {}^nC_r a^{n-r} b^r \text{ gives} \\ T_6 &= T_{5+1} = {}^{10}C_5 x^{10-5} (2y)^5 = 8064 x^5 y^5 \end{aligned}$$

EXAMPLE 5.18

Find the middle term of $(x - 3y)^6$

SOLUTION

Here $n = 6$, hence total number of terms will be 7 and the middle term, is 4th. Therefore, $T_4 = T_{3+1} = {}^6C_3 x^{6-3} (-3y)^3 = -540 x^3 y^3$.

5.6 SETS

The concept of a set is essential for getting a clear concept of several branches of mathematics. A set is a well-defined collection of objects, called elements or members. We may talk about a set of books, a set of dishes, or a set of students. We generally use capital letters for sets and small letters or numbers for elements of sets.

The symbol

$$a \in x$$

means "the object a is member of the set x " or "the object a belong to the set x ", or simply " a is in x ". We shall also read this as " a is an element of the set x ". the symbol

$$a \notin x$$

represent the negation " a is not an element of x ". The following are the methods which are frequently used of describing a set.

(1) Tabular Method

In this method elements of a set are written within brackets and the elements are separated by commas, i.e. $A = \{1, 2, 3, 4, 5, 6, 7\}$

(2) Set Builder Form

In this form a sentence describing the elements of a set is written within brackets. For example:

$$A = \{x / x \text{ is an odd natural number}\}$$

5.6.1 Types of Sets

(i) Universal Set

A set consisting of all the elements under consideration and is denoted by ' U '

(ii) Nul Set

A set having no element is called nul.set. It is denoted by $\emptyset$.

(iii) Equal Sets

Two sets having same and equal number of elements are called equal sets.

(iv) Disjoint Sets

Two sets having no elements in common are called disjoint set.

(v) Over Lapping Set

Sets having elements in common are called overlapping sets.

(vi) Finite Set

Set having finite number of elements.

(vii) Infinite Set

Set having infinite number of elements.

(viii) Difference of Sets

The difference of two sets $A - B$ is a set consisting of all the elements of A other than B .

(ix) Compliment of a Set:-

If U is a universal set, then compliment of set A is a set consisting of all elements of U other than A and is denoted by A' .

(x) Sub Sets:-

The set A is a subset of the set B if every elements of A is an element of B . We use the symbol $A \subseteq B$ to denote this.

Let $A = \{a, b, c\}$, then

$\{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \phi$

are subsets of A .

The subset $\{a, b, c\}$ is an improper subset whereas the remaining subsets are proper subsets.

(xi) Union of Sets:-

The union of two sets, A and B , is the set of all elements that are in A , or in B or in both, we denote this $A \cup B$. For example, if

$A = \{a, b, c, f\}$, and $B = \{a, e, f\}$ then $A \cup B = \{a, b, c, e, f\}$

(xii) Intersection of Sets:-

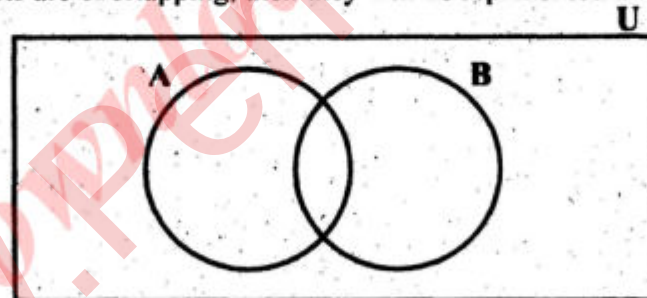
The intersection of two sets A and B , is the set of all elements that are in both A and B . We denote this $A \cap B$. For example, if

$A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7\}$, then $A \cap B = \{4, 5\}$

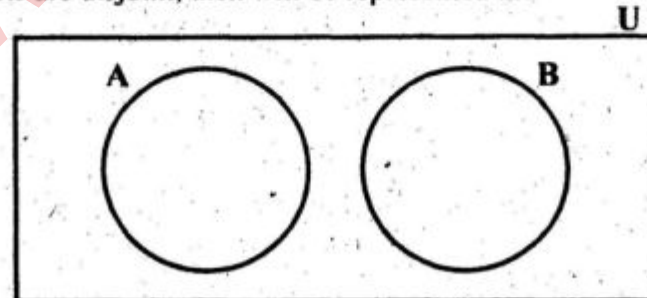
5.6.2 Venn - Diagram

A useful way of picturing the operations union, intersection, and compliment on sets is by depicting the sets as circles contained in a box. This type of diagram is called a Venn - diagram.

If sets are overlapping, then they will be represented as:

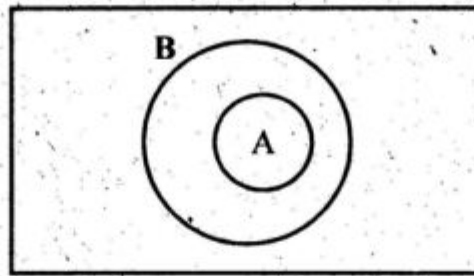


If sets are disjoint, then will be represented as:

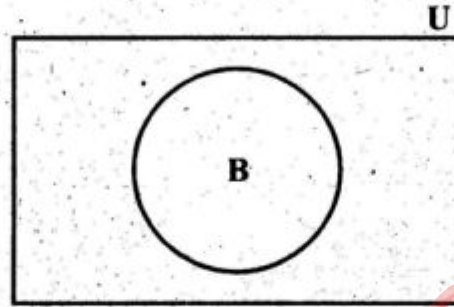


If set A is the subset of B , then it will be represented as:

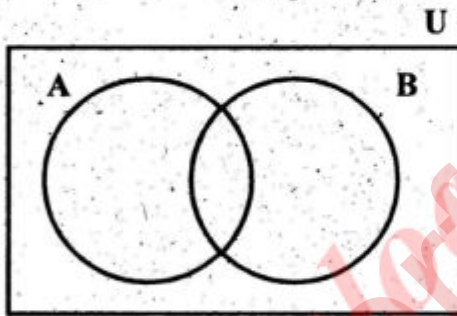
$$A \subseteq B$$



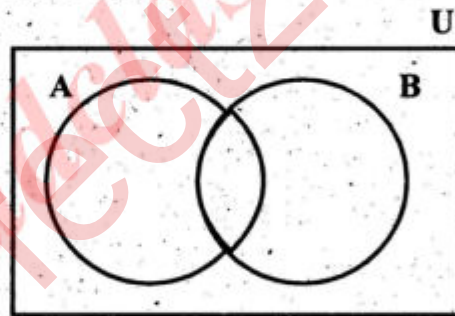
compliment of a set B is represented as:



Compliment of B = Shaded Area = B'



$A \cup B$ = Shaded Area



$A \cap B$ = Shaded Area

The following properties of the operations on sets may be noted.

- (i) $A \cup B = B \cup A$
- (ii) $A \cap B = B \cap A$
- (iii) $A \cup (B \cap C) = (A \cup B) \cap C$
- (iv) $A \cap (B \cup C) = (A \cap B) \cup C$
- (v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- (vi) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- (vii) (a) $(A \cup B)' = A' \cap B'$
 (b) $(A \cap B)' = A' \cup B'$

5.7. DEFINITIONS AND NOTATIONS OF TERMS

In the study of probability, we are concerned with experiments and their outcomes. The terms to be used in the theory of probability are defined below:

5.7.1 Experiment

An act or the process of obtaining an observation is called experiment. Performing of an experiment is called a trail. For example, a single performance of an experiment is called a trail: If we toss a coin or throw a die we are performing trails. The result obtained from an experiment is called an outcome.

5.7.1.1 Random Experiment.

An experiment is said to be random, if it provides different results, if it is repeated a large number of times under essentially identical conditions. The process of performing the trails is frequently referred to as a random, stochastic or chance process. Examples of a random experiment are, tossing of coin, rolling of a die, drawing a card from a deck of playing cards, drawing a ball from a bag containing balls of different colours, selecting a bolt from a lot and observing whether it is defective or not, etc. A random experiment has three properties (i) The experiment can be repeated (ii) The experiment always has two or more possible outcomes (iii) The outcomes of each trail is unpredictable.

5.7.2 Sample Space

A sample space is formed by a set of sample points which represent all possible outcomes of the random experiment. An outcome of a trail is called a sample point. Sample space is denoted by S . In tossing a coin, for example, the possible outcomes are H (a head) and T (a tail). These are two sample points. The two sample points form a sample space.

$$S = \{H, T\}$$

If two coins are tossed together, the sample space consists of four sample points.

$$S = \{HH, HT, TH, TT\}$$

The sample space in rolling a die is

$$S = \{1, 2, 3, 4, 5, 6\}$$

The sample space for the random experiment of rolling two six-sided dice consists 36 sample points.

$$S = \{(1, 1), (1, 2), (1, 3), \dots, (6, 6)\}$$

The sample space S in drawing a card from a well shuffled pack of 52 playing card is $S = \{1, 2, \dots, 52\}$

5.7.3 Event

Any subset of the sample space is called an event. It is denoted by A , B , C , D , etc.

(I) Simple Event

A simple event is one which contains only one sample point of the sample space: The occurrence of 4 when a die is thrown, is a simple event.

(II) Compound Event

It is an event that contains two or more sample points of the sample space. Compound event can be further decomposed into simple events. the occurrence of 9 with a pair of dice, is a compound event, as it can be decomposed into four simple events. (3, 6), (4, 5), (5, 6) and (6, 3)

(III) Impossible Event

The empty set ϕ is also an event as it is a subset of sample space. An event is called impossible event, if it contains no sample point of the sample space. Hence ϕ is impossible event.

(IV) Sure Event

It is an event that contains all the sample points of the sample space.

(V) Complementary Event

If A is an event of the sample space S, then the complementary event A^c is an event that contains all those sample points of S which are not in A.

(VI) Mutually Exclusive Events

Two or more events are said to be mutually exclusive, if they cannot occur together. Thus the occurrence of head or tail in a single throw of a coin are mutually exclusive events. When a die is rolled, the outcomes are mutually exclusive.

(VII) Not Mutually Exclusive Events

If two or more event can occur together, they are not mutually exclusive. If we draw a card from an ordinary deck of 52 cards, it can be both a queen and card of spade, Therefore, queens and cards of spade are not mutually exclusive.

(VIII) Equally Likely Events

Two or more events are said to be equally likely events if they have same chance to occur. For example, if we toss a coin, the head and tail have the same chance to be occur. Similarly, the six events when we throw a die are equally likely.

(IX) Independent Events

Two or more events are considered to be independent, if the events in no way affect each other. For example, when we toss two unbiased coins we have independent events because the probability of occurrence of head or tail on the second coin does not depend on the occurrence of head or tail on the first coin. Suppose event A is the drawing of king from a pack of 52 cards and event B is head on tossing a fair coin. The events are independent because the drawing of a card does not affect the head when tossing a coin.

(X) Dependent Events

Two or more events are said to be dependent when the probability of happening of any one event is affected by the happening or non happening of the others. For example, the results of two drawings of a ball from a bag are dependent, if the ball is not returned to the bag after the first draw.

(XI) Favourable Cases

The cases which entail the occurrence of an event are said to be favourable to the event.

(XII) Exhaustive Cases

The set of all possible outcomes in a trial constitutes the set of exhaustive cases. For example, in the case of tossing a coin there are two exhaustive cases head or tail.

5.8 DEFINITIONS OF PROBABILITY

(I) Mathematical or a Priori or Classical Definition.

In some cases, we can count exactly the different ways in which a given event may or may not happen and can assume that all the possible ways will occur on an equally likely basis. The probability under such cases and assumption is called classical probability or priori probability. For equally likely, exhaustive and mutually exclusive cases, the probability of the happening of an event A is defined as the ratio of the favourable cases for the event to the total number of cases. Thus, if the number of cases favourable to event a is m out of n equally likely, mutually exclusive and exhaustive cases, then.

$$P(A) = \frac{\text{Favourable cases for A}}{\text{Total number of cases}} = \frac{m}{n}$$

The probability of the non occurrence of the event A (failure) is

$$P(\text{not } A) = P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$$P(\bar{A}) = 1 - P(A)$$

The probability of success and the probability of failure is always equal to one i.e.

$$P(A) + P(\bar{A}) = 1$$

The classical definition of probability fails when the events are not equally likely, mutually exclusive and exhaustive. For example, if we are interested in knowing the probability that a ceiling fan will fall, the above definition breaks down because the two events that the fan stays in the ceiling or fall down are not equally likely.

EXAMPLE 5.19

A fair coin is tossed. What is the probability of getting (i) head (ii) tail.

SOLUTION

These are two possible outcomes head and tail. The number of equally, likely, mutually exclusive outcomes are 2 i.e; n = 2

(i) Let the event head = A

There is only one head i.e. only one outcome favourable to event A. Thus the probability of getting a head is.

$$P(\text{Head}) = P(A) = \frac{1}{2}$$

(ii) Let the event tail = B

There is only one tail i.e. only one outcome favourable to event B.
Thus the probability of getting a tail is

$$P(\text{Tail}) = P(B) = \frac{1}{2}$$

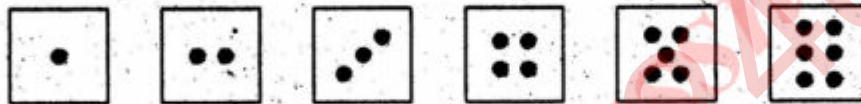
EXAMPLE 5.20

Find the probability that one throw of a die will

- (a) Have "3" on top (b) Have "5" on top
(c) Greater than 4 (d) An odd number

SOLUTION

A die is a cube-shaped gambling instrument with dots on each of the six sides



Assume that it is a well-balanced die; that is, each of the six sides will have an equal chance to turn on the top when the die stops in a throw. Then the sample points are the six dots and the sample space is

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$(a) \quad P(\text{Having a "3"}) = \frac{\text{Number of Favourable cases}}{\text{Number of Possible cases}} = \frac{m}{n} = \frac{1}{6}$$

$$(b) \quad P(\text{Having a "5"}) = \frac{\text{Number of Favourable cases}}{\text{Number of Possible cases}} = \frac{m}{n} = \frac{1}{6}$$

(c) Of these 6 outcomes 2 outcomes result the event more than 4 i.e. the face 5 or 6

$$P(\text{More than 4}) = \frac{\text{The Favourable cases}}{\text{Number of Possible cases}} = \frac{m}{n} = \frac{2}{6} = \frac{1}{3}$$

(d) Of these 6 outcomes 3 outcomes are in the favour of event an odd number i.e. 1 or 3 or 5.

$$P(\text{An odd}) = \frac{\text{Number of Favourable cases}}{\text{Number of Possible cases}} = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$$

EXAMPLE 5.21

An urn contains 8 balls of which 5 are white and 3 are black. A ball is drawn at random. What is the probability that the ball drawn is white.

SOLUTION

Let the balls are numbered as 1 to 8 and the selection of a ball at random can produce 8 different outcomes as any of the balls may turn up.

Let an event A be defined as the ball is white of these 8 different mutually exclusive and equally likely outcomes. 5 outcomes result in the favour of event A.

Here $n = 8$, $m = 5$

$$P(A) = \frac{\text{Favourable to event A}}{\text{Number of Possible cases}} = \frac{m}{n} = \frac{5}{8}$$

(II) Statistical or Empirical or posterior definition

The definition of a priori or mathematical probability given in article (I) can be used only if we have a complete knowledge of all possibilities and their equal likelihood. For example, with that definition we cannot determine the probability that Mr. Aslam, aged 40, will die within one year. In this situation we do not know all the possible contingencies that will affect Mr. Aslam during the year.

Mortality records are kept for the population and these records show that on the average 5 out of every 1000 alive at age 40 die from some cause during the year. Thus we would say that the probability that Aslam will die during the year is $5/1000$. A probability obtained in this way is called statistical or a posteriori probability. Consequently, we have the following definition of statistical probability.

The limiting value of the ratio of favourable cases for an event to the total number of cases when the total number of cases is very large is defined as the probability of happening of the event. It is assumed that this limiting value is unique and finite. Symbolically,

$$P(A) = \lim_{n \rightarrow \infty} \frac{m}{n}$$

(III) Axiomatic Definition

If A is an event that belongs to sample space S of a given random experiment then the probability of A must satisfies the following basic axioms

(a) $P(A) \geq 0$

(b) $P(S) = 1$

(c) if A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

EXAMPLE 5.22

A student solved 135 questions from 333 questions of a book. What probability should be associated that he will solve the next question.

SOLUTION

Here $n = 333$, $m = 135$

$$P(A) = \frac{m}{n} = \frac{135}{333} = 0.40541$$

5.9 PROBABILITY OF AN EVENT

The probability of an event A which belongs to a sample space S is defined as

$$P(A) = \frac{\text{Number of sample point in A}}{\text{Number of sample points in S}} = \frac{n(A)}{n(S)}$$

EXAMPLE 5.23

A card is drawn from an ordinary deck of 52 playing cards, find the probability that (i) The card is a black card. (ii) The card is a diamond (iii) The card is a 9 (iv) The card is a king (v) Pictured card (vi) Ace card.

(Lahore Board, 2008)

SOLUTION

The total number of possible outcomes is 52, and we assume that all possible outcomes are equally likely.

- (i) Let A represent the event that the card drawn is a black card. The number of black cards is 26. The number of outcomes favourable to the event A are 26.

$$\text{Here } n(S) = 52, \quad n(A) = 26$$

$$\text{Hence } P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

- (ii) Let B be the event that the card drawn is diamond. The number of diamond cards are 13. The number of outcomes favourable to the event B are 13.

$$\text{Here } n(S) = 52, \quad n(B) = 13$$

$$\text{Hence } P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

- (iii) Let C denote the event that the card drawn is a 9. There are 4 cards of 9. The number of outcomes favourable to event C are 4.

$$\text{Here } n(S) = 52, \quad n(C) = 4$$

$$\text{Hence } P(C) = \frac{n(C)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

- (iv) Let D represent the event that the card drawn is a King. There are 4 cards of King. The number of outcomes favourable to event D are 4.

$$\text{Here } n(S) = 52, \quad n(D) = 4$$

$$\text{Hence } P(D) = \frac{n(D)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

- (v) Let E represent the event that the card drawn is a pictured card. There are 12 pictured card. The number of outcomes favourable to event E are 12.

$$\text{Here } n(S) = 52, \quad n(E) = 12$$

$$\text{Hence } P(E) = \frac{n(E)}{n(S)} = \frac{12}{52}$$

- (vi) Let F represent the event that the card drawn is a ace. There are 4 cards of ace. The number of outcomes favourable to event F are 4.

$$\text{Here } n(S) = 52, \quad n(F) = 4$$

$$\text{Hence } P(F) = \frac{n(F)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

EXAMPLE 5.24

A coin is tossed twice or two coins are tossed once. Find the probability that

- (i) Exactly one head appears.
(ii) At least one head appears. (Faisalabad Board, 2008 & 2010)

SOLUTION

The sample space in tossing a coin twice is

$$S = \{HH, HT, TH, TT\} \quad \text{Thus } n(S) = 4$$

- (i) Let A represents the event that "Exactly one head occur":

$$A = \{HT, TH\}, \quad n(A) = 2 \quad n(S) = 4$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

- (ii) Let B denotes the event that "At least on head" occur.

$$B = \{HH, HT, TH\} \quad n(B) = 3 \quad n(S) = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{4}$$

EXAMPLE 5.25

Three coins are tossed. What is the probability of getting (i) Exactly 2 heads (ii) At most 2 heads (iii) At least 2 heads (iv) At least one head (v) At most one head (vi) No head.

(Faisalabad Board, 2009)

SOLUTION

The sample space in tossing of three coins is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}, \quad n(s) = 8$$

- (i) Let A represents the event that exactly 2 heads

$$A = \{HHT, HTH, THH\}, \quad n(A) = 3, \quad n(S) = 8$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

- (ii) Let B denotes the event that at most 2 heads

$$B = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}, \quad n(B) = 7, \quad n(S) = 8$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

- (iii) Let C represent the event of getting at least 2 heads

$$C = \{HHT, HTH, THH, HHH\}, \quad n(C) = 4, \quad n(S) = 8$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

- (iv) Let D be the event of getting at least one head

$D = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$, $n(D) = 7$, $n(S) = 8$

$$P(D) = \frac{n(D)}{n(S)} = \frac{7}{8}$$

(v) Let E be the event of getting at most one head

$E = \{HTT, THT, TTH, TTT\}$, $n(E) = 4$, $n(S) = 8$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$$

(vi) Let F denote the event of getting no head

$F = \{TTT\}$, $n(F) = 1$, $n(S) = 8$

$$P(F) = \frac{n(F)}{n(S)} = \frac{1}{8}$$

EXAMPLE 5.26

Make the sample space of the random experiment "Toss a coin twice" and given the probability of obtaining (i) two tails (ii) No tail

(Sargodha Board, 2010)

SOLUTION

The sample space in tossing of a coins twice is

$S = \{HH, TH, HT, TT\}$, $n(s) = 4$

(i) Let A represents the event of two tails

$A = \{TT\}$, $n(A) = 1$, $n(S) = 4$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{4}$$

(ii) Let B represent the event of getting no tail

$B = \{HH\}$, $n(B) = 1$, $n(S) = 4$

$$P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$$

(iii) Let C represent the event of getting at least one tail

$C = \{HT, TH, TT\}$, $n(C) = 3$, $n(S) = 4$

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{4}$$

EXAMPLE 5.27

A fair die is rolled. What is the probability of getting (i) An even number (ii) An odd number (iii) at least one tail (iv) A prime number (v) A number divisible by 3

(Gujranwala Board, 2010)

(Rawalpindi Board, 2010)

SOLUTION

When we throw a six sided die, the sample space is

$S = \{1, 2, 3, 4, 5, 6\}$ $n(S) = 6$

(i) Let A represent the event of getting an even number

$$A = \{2, 4, 6\} \quad n(A) = 3, \quad n(S) = 6$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

- (ii) Let B represent the event of getting an odd number

$$A = \{1, 3, 5\} \quad n(B) = 3 \quad n(S) = 6$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

- (iv) Let D represent the event of getting a prime number

$$D = \{2, 3, 5\}, \quad n(D) = 3, \quad n(S) = 6$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

- (v) Let E represent the event of getting a number divisible by 3

$$E = \{3, 6\}, \quad n(E) = 2, \quad n(S) = 6$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

EXAMPLE 5.28

Two fair dice are thrown. What is the probability of getting.

- The sum of the faces is 5.
- The sum of the faces is 9.
- The sum of faces is 10 or more.
- The sum of faces is 8 or more.
- A product of the numbers on two dice is 6.
- A product of the numbers on two dice is 12.

SOLUTION

The dice can be thrown in $6 \times 6 = 36$ ways. Sample space S consists the following 36 outcomes.

$$S = \begin{aligned} &(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ &(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ &(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ &(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ &(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ &(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{aligned}$$

- (i) Let A represent event that the sum of the faces is 5.

$$A = \{(1, 4), (2, 3), (3, 2), (4, 1)\}, \quad n(A) = 4, \quad n(S) = 36$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

- (ii) Let B denote the event that sum is 9.

$$B = \{(3, 6), (4, 5), (5, 4), (6, 3)\}, \quad n(B) = 4, \quad n(S) = 36$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

- (iii) Let C represent the event of getting a sum of the number on both the sides as 10 or more.

$$C = \{(4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)\} \quad n(C) = 6 \quad n(S) = 36$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(iv) Let D represent the event of getting a sum of 8 or more.

$$D = \{(6, 2), (5, 3), (4, 4), (3, 5), (2, 6), (6, 3), (5, 4), (4, 5), (3, 6), (6, 4), (5, 5), (4, 6), (5, 6), (6, 5), (6, 6)\}, \quad n(D) = 15, \quad n(S) = 36$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

(v) Let E be the event of getting a product of the numbers on the both faces as 6.

$$E = \{(1, 6), (2, 3), (3, 2), (6, 1)\}, \quad n(E) = 4, \quad n(S) = 36$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(iv) Let F represent the event of getting a product of the numbers on the both faces as 12.

$$F = \{(2, 6), (3, 4), (4, 3), (6, 2)\}, \quad n(F) = 4, \quad n(S) = 36$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

EXAMPLE 5.29

Find the probability of throwing at least 8 in a single throw of 2 dice.

SOLUTION

Two dice can be thrown in $6 \times 6 = 36$ ways. Sample space consists 36 outcomes, $n(S) = 36$

Throwing at least eight means, throwing 8, 9, 10, 11, 12 with 2 dice.

Let A represents the event of getting at least 8.

Possible outcomes of getting 8 = (6, 2), (5, 3), (4, 4), (3, 5), (2, 6) = 5 ways

Possible outcomes of getting 9 = (6, 3), (5, 4), (4, 5), (3, 6) = 4 ways

Possible outcomes of getting 10 = (6, 4), (5, 5), (4, 6) = 3 ways

Possible outcomes of getting 11 = (6, 5), (5, 6) = 2 ways

Possible outcomes of getting 12 = (6, 6) = 1 ways

Possible outcomes of getting at least 8 = 15 ways
= $n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

EXAMPLE 5.30

Find the probability of getting 8 with 3 dice.

SOLUTION

Three dice can be thrown in $6 \times 6 \times 6 = 216$ ways.

Possible outcomes to get 8 by 3 dice are

Outcome	No. of outcomes
(1, 1, 6)	3
(1, 2, 5)	6
(1, 3, 4)	6
(2, 2, 4)	3
(2, 3, 3)	3
Total:	21

Let A represent the event of getting 8 with 3 dice.

$$n(A) = 21 \quad n(S) = 216$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{21}{216} = \frac{7}{72}$$

EXAMPLE 5.31

Find the probability of getting 11 with 3 dice.

SOLUTION

Three dice can be thrown in $6 \times 6 \times 6 = 6^3 = 216$ ways

Sample space contains 216 sample points. $n(S) = 216$

Possible outcomes of getting 11 by 3 dice are.

Outcome	No. of Outcomes
(1, 4, 6)	6
(1, 5, 5)	3
(2, 3, 6)	6
(2, 4, 5)	6
(3, 3, 5)	3
(3, 4, 4)	3
Total:	27

Let A represent the event of getting 11 with 3 dice.

$$n(A) = 27$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{27}{216} = \frac{9}{72}$$

EXAMPLE 5.32

A bag contains 4 red and 6 green balls, 3 balls are drawn. Find the probability of drawing.

- (i) 2 red and one green ball.
- (ii) All red balls.
- (iii) No. red ball.

SOLUTION

There are 10 balls of which 4 are red and 6 are green. Three balls are drawn at random. The number of sample points in the sample space are.

$$n(S) = {}^{10}C_3 = 120$$

- (i) Let A represent the event of drawing 2 red and one green ball

$$n(A) = {}^4C_2 \times {}^6C_1 = 6 \times 6 = 36$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{36}{120} = \frac{3}{10}$$

- (ii) Let B represent the event of drawing all red balls

$$n(B) = {}^4C_3 \times {}^6C_0 = 4 \times 1 = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{120} = \frac{1}{30}$$

- (iii) Let C represent the event of drawing no red balls.

$$n(C) = {}^4C_0 \times {}^6C_3 = 1 \times 20 = 20$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{20}{120} = \frac{1}{6}$$

EXAMPLE 5.33

A bag contains 3 white, 4 black and 5 blue balls. 4 balls are drawn from the bag. Find the probability that they are (i) One white, Two black and one blue. (ii) Two white (iii) No black (iv) All are black.

SOLUTION

Four balls are drawn from 12 balls of which 3 white, 4 black 5 blue balls, the number of possible sample points are

$$n(S) = {}^{13}C_4 = 495$$

- (i) Let A represent the event of drawing one white, two black and one blue balls.

$$n(A) = {}^3C_1 \times {}^4C_2 \times {}^5C_1 = 3 \times 6 \times 5 = 90$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{90}{495} = \frac{2}{11}$$

- (ii) Let B represent the event of drawing two white balls

$$n(B) = {}^3C_2 \times {}^9C_2 = 3 \times 36 = 108$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{108}{495} = \frac{12}{55}$$

- (iii) Let C represent the event of drawing no white balls

$$n(C) = {}^4C_0 \times {}^8C_4 = 1 \times 70 = 70$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{70}{495} = \frac{14}{99}$$

- (iv) Let D represent the event of drawing all black balls

$$n(D) = {}^4C_4 \times {}^8C_0 = 1 \times 1 = 1$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{1}{495}$$

EXAMPLE 5.34

52 well-shuffled playing cards are distributed at random among 4 players. Find the probability that a hand contains (i) 3 aces (ii) 2 kings and one queen (iii) No card of diamond (iv) 5 pictured cards.

SOLUTION

The number of sample points in the sample space are

$$n(S) = {}^{52}C_{13} = \frac{52!}{13! 39!}$$

- (i) Let A represent the event of having 3 aces.

$$n(A) = {}^4C_3 \times {}^{48}C_{10}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^4C_3 \times {}^{48}C_{10}}{{}^{52}C_{13}} = \frac{858}{20825} = 0.0412$$

- (ii) Let B represent the event of having 2 kings and one queen.

$$n(B) = {}^4C_2 \times {}^4C_1 \times {}^{44}C_{10}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{{}^4C_2 \times {}^4C_1 \times {}^{44}C_{10}}{{}^{52}C_{13}} = 0.0938$$

- (iii) Let C represent the event of having no card of diamond.

$$n(C) = {}^{13}C_0 \times {}^{39}C_{13}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{{}^{13}C_0 \times {}^{39}C_{13}}{{}^{52}C_{13}} = 0.0128$$

- (iv) Let D represent the event of having 5 pictured card.

$$n(D) = {}^{12}C_5 \times {}^{40}C_8$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{{}^{12}C_5 \times {}^{40}C_8}}{{}^{52}C_{13}} = 0.0959$$

EXAMPLE 5.35

6 cards are drawn at random from a deck of 52 cards. What is the probability: That 3 will be red and 3 black.

SOLUTION

The number of sample points of drawing 6 cards out of 52 cards are

$$n(S) = {}^{52}C_6$$

Let A represent the event of drawing 3 red and 3 black cards.

$$n(A) = {}^{26}C_3 \times {}^{26}C_3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^{26}C_3 \times {}^{26}C_3}{{}^{52}C_6} = \frac{6760000}{20358520} = 0.332$$

EXAMPLE 5.36

4 cards are drawn at random from a deck of 52 cards. What is the probability that there will be one card of each suit.

SOLUTION

The number of sample points of drawing 4 cards out of 52 cards are

$$n(S) = {}^{52}C_4$$

Let A represent the event of drawing one card from each suit.

$$n(A) = {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4} = \frac{2197}{208225} = 0.1055$$

EXAMPLE 5.37

An urn contains 12 balls of which 3 are marked. If 5 balls are drawn out together. What is the probability that all 3 of the marked balls are among these 5.

SOLUTION

The number of sample points of sample space are

$$n(S) = {}^{12}C_5 = 792$$

Let A represent the event of drawing 5 balls of which 3 are marked balls.

$$n(A) = {}^3C_3 \times {}^9C_2 = 1 \times 36 = 36$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{{}^3C_3 \times {}^9C_2}{{}^{12}C_5} = \frac{36}{792} = \frac{1}{22}$$

5.10 ADDITION LAWS OF PROBABILITY

(I) Addition Law for Mutually Exclusive Events.

If A and B are two mutually exclusive events; then the probability that event A or B occur is equal to the probability that A occurs plus the probability B occurs. that in,

$$\begin{aligned} P(A \text{ or } B) &= P(A \cup B) \\ &= P(A) + P(B) \end{aligned}$$

EXAMPLE 5.38

A pair of fair dice is rolled. Find the probability that the sum of the two upper most dots is either 6 or 9

SOLUTION

Two dice can be thrown in $6 \times 6 = 36$ ways.

$$n(S) = 36$$

Let A represent the event that the sum of the upper dots is 6, then

$$A = (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)$$

$$n(A) = 5, \quad P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

Let B be the event that the sum of upper dots is 9

$$B = (3, 6), (4, 5), (5, 4), (6, 3)$$

$$n(B) = 4, \quad P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

A and B are mutually exclusive, therefore, the probability that sum of upper dots is either 6 or 9

$$P(A \cup B) = P(A) + P(B) = \frac{5}{36} + \frac{4}{36} = \frac{9}{36} = \frac{1}{4}$$

EXAMPLE 5.39

A bag contains 3 white 5 black and 7 red balls. One ball is drawn at random, what is the probability that it is either white or black.

(Lahore Board, 2009)

SOLUTION

The number of sample points in sample space are $n(S) = {}^{15}C_1 = 15$

Let A represent the event of drawing a white ball

$$n(A) = {}^3C_1 = 3, \quad P(A) = \frac{n(A)}{n(S)} = \frac{3}{15}$$

Let B represent the event of drawing a black ball.

$$n(B) = {}^5C_1 = 5, \quad P(B) = \frac{n(B)}{n(S)} = \frac{5}{15}$$

The Probability that it is either white or black ball

$$P(A \cup B) = P(A) + P(B) = \frac{3}{15} + \frac{5}{15} = \frac{8}{15}$$

EXAMPLE 5.40

100 students appeared in I. Com examination from a certain college. 65 percent failed the exam, 5% passed in the third division; 20% passed in 2nd division and 10% passed in first division. Find the probability that a student selected at random from the list of the students who appeared in the examination either failed or got first division.

SOLUTION

There are mutually exclusive events.

Let A represent a student failing the exam.

$$P(A) = \frac{65}{100}$$

and B represent a student getting first division

$$P(B) = \frac{10}{100}$$

$$P(A \text{ or } B) = P(A) + P(B) = \frac{65}{100} + \frac{10}{100} = 0.65 + 0.10 = \frac{75}{100} = 0.75 = 75\%$$

EXAMPLE 5.41

A person approaching a traffic light find the light is either red, yellow, or green. Construct a probability space corresponding to this situation if the light follows the cycle, red for 35 seconds, green for 45 seconds, yellow for 4 seconds. What is the probability that the light is either green or yellow?

SOLUTION

The number of sample points in seconds of sample space,

$$n(S) = 35 + 45 + 4 = 84 \text{ second}$$

Let R represent the red light, G represent the green light and Y represent the yellow light.

Here

$$n(R) = 35, \quad P(R) = \frac{35}{84}, \quad n(G) = 45, \quad P(G) = \frac{45}{84}$$

$$n(Y) = 4, P(Y) = \frac{4}{48}$$

Probability that light is either green or yellow

$$P(G \text{ or } Y) = P(G \cup Y) = P(G) + P(Y) = \frac{45}{84} + \frac{4}{84} = \frac{49}{84} = \frac{7}{12}$$

EXAMPLE 5.42

Three horses A, B and C are in a race. A is twice as likely to win as B and B is twice as likely to win as C.

- (i) What are their respective chances of winning.
- (ii) What is the probability that A or B win A or C win.

SOLUTION

- (i) Let $P(C) = P$.

Since B is twice as likely to win as C then $P(B) = 2P$ and as A is twice as likely to win as B then

$$P(A) = 2P, P(B) = 4P$$

Since A, B, and C are mutually exclusive events, Therefore, the sum of the probabilities of their winning must be equal to 1. hence

$$P(A) + P(B) + P(C) = 1$$

$$4P + 2P + P = 1$$

$$7P = 1$$

$$P = \frac{1}{7}$$

$$\text{Therefore } P(C) = \frac{1}{7}, P(B) = \frac{2}{7}, P(A) = \frac{4}{7}$$

- (ii) The probability of winning A or B is

$$P(A \cup B) = P(A) + P(B) = \frac{4}{7} + \frac{2}{7} = \frac{6}{7}$$

The probability of winning A or C

$$P(A \cup C) = P(A) + P(C) = \frac{4}{7} + \frac{1}{7} = \frac{5}{7}$$

(II) Addition Law For Not Mutually Exclusive Events

If A and B are two not mutually exclusive events belonging to the same sample space, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

EXAMPLE 5.43

In drawing one card from a deck of 52 cards, find the probability that a single draw will be either a pictured card or a heart card.

SOLUTION

There are 13 cards in each of the suits: heart, spade, diamond and club. The number of sample points in the sample space are $n(S) = {}^{52}C_1 = 52$

Let A represent the event of drawing a pictured card. There are three pictured cards (jack, queen, king) in each of the four suits.

thus $A = \{J_h, Q_h, K_h, J_s, Q_s, K_s, J_d, Q_d, K_d, J_c, Q_c, K_c\}$

$$n(A) = 12$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{12}{52}$$

Let B represent the event of drawing a heart card. There are 13 cards with hearts.

$B = \{J_h, Q_h, K_h, A_h, 2_h, 3_h, 4_h, 5_h, 6_h, 7_h, 8_h, 9_h, 10_h\}$

$$n(B) = 13$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52}$$

Thus A and B are not mutually exclusive events.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$$

EXAMPLE 5.44

A class contains 14 boys and 28 girls of which half of the boys and half of the girls have brown eyes. find the probability that a person chosen at random is (i) a boy or has brown eyes. (ii) a girls or has brown eyes.

SOLUTION

The number of sample points in the sample space when a person is chosen from 42 are

$$n(S) = {}^{42}C_1 = 42$$

(i) Let A represent the event that person chosen is a boy.

$$n(A) = {}^{14}C_1 = 14$$

$$P(A) = \frac{14}{42}$$

Let B represent the event that the person chosen has brown eyes.

$$n(B) = {}^{21}C_1 = 21$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{21}{42}$$

$A \cap B$ represent the event that the person chosen is a boy having brown eyes.

$$n(A \cap B) = 7$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{7}{42}$$

As A and B are not mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{14}{42} + \frac{21}{42} - \frac{7}{42} = \frac{28}{42} = \frac{2}{3}$$

(ii) Let A represent the event that person chosen is a girl.

$$n(A) = {}^{28}C_1 = 28$$

$$P(A) = \frac{28}{42}$$

Let B represent the event that the person chosen has brown eyes.

$$n(B) = {}^{21}C_1 = 21$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{21}{42}$$

$A \cap B$ represent the event that the person chosen is a girl having brown eyes.

$$n(A \cap B) = 14$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{14}{42}$$

As A and B are not mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{28}{42} + \frac{21}{42} - \frac{14}{42} = \frac{35}{42} = \frac{5}{6}$$

EXAMPLE 5.45

A fair die is tossed. Find the probability that the outcome is either an odd number or a multiple of 3.

SOLUTION

The number of sample points in sample space in tossing a fair die are

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

Let A represent the event that the outcome is an odd number.

$$A = \{1, 3, 5\}, n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6}$$

Let B represent the event that outcome is multiple of 3.

$$B = \{3, 6\}, n(B) = 2$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{6}$$

The outcome which is an odd number and also multiple of 3 is

$$A \cap B = \{3\}, n(A \cap B) = 1$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$$

As A and B are not mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

EXAMPLE 5.46

An integer is chosen at random from the first 80 positive integers. Find the probability that the chosen integer is divisible either by 8 or by 10.

SOLUTION

An integer is chosen at random from the first 80 positive integers as

$$n(S) = 80$$

Let A represent the event that the integer chosen is divisible by 8.

$$A = \{8, 16, 24, 32, \dots, 80\}, \quad n(A) = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{80}$$

Let B represent the event that the integer chosen is divisible by 10.

$$B = \{10, 20, 30, \dots, 80\}, \quad n(B) = 8$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{80}$$

$$A \cap B = \{40, 80\}, \quad n(A \cap B) = 2$$

$$P(A \cap B) = \frac{2}{80}$$

Therefore,

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{10}{80} + \frac{8}{80} - \frac{2}{80} = \frac{16}{80} = \frac{1}{5} \end{aligned}$$

5.11 CONDITIONAL PROBABILITY

Probabilities can be used to indicate whether two events are related. for example, the incidence of lung cancer is much higher among smokers than in the general population. This difference is used as evidence of connection between smoking and lung cancer. A probability computed using a subset of the sample space is called a conditional probability. the probability that one contracts lung cancer computed using only the set of smokers instead of the general population is an example of a conditional probability. if L represents contracts lung cancer and S represents smoker, then the conditional probability that a smoker gets lung cancer is written $P(L/S)$ and is read: the probability of L given S. from above discussion we define the conditional probability as:

If A and B are two events with probabilities $P(A)$ and $P(B)$ respectively, then the conditional probability of A given B, that is, the probability of occurs of A on the condition that B has already occurred denoted by $P(A/B)$ and is defined as:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

The $P(B/A)$, stands for the conditional probability of occurring B on condition that A has already occurred.

$$P(B / A) = \frac{P(A \cap B)}{P(A)}$$

When the events A and B are independent then obviously we have

$$P(A/B) = P(A)$$

$$P(B/A) = P(B)$$

EXAMPLE 5.47

A die is thrown. What is the probability of getting an even number that the outcome is greater than 2.

SOLUTION

The sample space is $S = \{1, 2, 3, 4, 5, 6\}$

Let A represent the event of getting an even number.

$$A = \{2, 4, 6\}, n(A) = 3$$

Let B represent the event of getting a number greater than 2,

$$B = \{3, 4, 5, 6\}, n(B) = 4, P(B) = 4/6$$

$$\text{Then } A \cap B = \{4, 6\}, n(A \cap B) = 2, P(A \cap B) = 2/6$$

$$P(A / B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{2/6}{4/6} = \frac{2}{4} = \frac{1}{2}$$

EXAMPLE 5.48

Three coins are tossed. What is the conditional probability that two heads result, given that there is at least one head.

SOLUTION

As three coins are tossed so the sample space is

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}, n(S) = 8$$

Let A represent the event that two heads appear

$$A = \{HHT, HTH, THH\}, n(A) = 3$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$$

Let B represent the event that there is at least one head appears.

$$B = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}, n(B) = 7$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{7}{8}$$

$$(A \cap B) = \{HHT, HTH, THH\}, n(A \cap B) = 3$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{8}$$

$$\text{Hence } P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{3/8}{7/8} = \frac{3}{7}$$

EXAMPLE 5.49

A bond issue for the construction of a new public library is before the voters. A pool showed that 85% of those with a college education favoured the construction of a new library, but only 20% of those not having a college education did so. Suppose that 90% of the voting population does not have a college education what is the probability that a voter selected at random who favours the bond issue will be one with a college education.

SOLUTION

Sample space is the set of all voters.

Let A denote the event that voter has a college education and B denote the event that voter favours the library construction.

For a voter selected at random, then

$$P(A) = 0.1 \text{ and } P(\bar{A}) = 0.9$$

For event B , we are given that

$$P(B/A) = 0.85 \text{ and } P(B/\bar{A}) = 0.2$$

We have

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\text{Also } P(B) = P(B/A)P(A) + P(B/\bar{A})P(\bar{A})$$

$$P(B) = (0.85)(0.1) + (0.2)(0.9) = 0.265$$

Since

$$\begin{aligned} P(A \cap B) &= P(B/A) \cdot P(A) \\ &= (0.85)(0.1) = 0.085 \end{aligned}$$

$$\text{Thus } P(A/B) = \frac{0.085}{0.265} = 0.32$$

5.12 MULTIPLICATION LAWS OF PROBABILITY**(I) Multiplication Law for Independent Events**

If A and B are two independent events belonging to the same sample space S , then the probability that both happen is equal to the product of their respective probabilities; that is,

$$P(A \cap B) = P(A) \cdot P(B)$$

EXAMPLE 5.50

A bag contains five white balls and three red balls. One ball is drawn from the bag and then is replaced. Another ball is drawn after the replacement. Find the probability that both drawings are white balls.

SOLUTION

The happening on the first draw certainly has nothing to do with the second draw, since the second ball is drawn after the first ball is replaced. Thus the two drawings are independent events.

$$N(S) = 8$$

Let A represent the event of the first draw that will have a white ball

$$n(A) = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{8}$$

Let B represent the event of the second draw that will have a white ball also $n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{8}$$

$$P(A \cap B) = P(A) \cdot P(B) = \frac{5}{8} \cdot \frac{5}{8} = \frac{25}{64}$$

EXAMPLE 5.51

A card is drawn from a well shuffled pack of 52 cards and it is replaced. Now a second card is drawn. What is the probability that both cards are kings.

SOLUTION

The number of sample points in sample space when we draw one card out of 52 cards are

$$n(S) = {}^{52}C_1 = 52$$

The two drawings are independent

Let A represent the event first card is a king.

$$n(A) = 4, P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$$

The first card is replaced before drawing the second card.

Let B be the event that second card is king.

$$n(B) = 4, P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$$

$$P(A \cap B) = P(A) \cdot P(B) = \frac{4}{52} \cdot \frac{4}{52} = \frac{16}{2704} = \frac{1}{169}$$

EXAMPLE 5.52

A can solve 60% of the problems in a book and B can solve 80% of the problems. What is the probability that A and B can solve a problem chosen at random.

SOLUTION

Since A can solve the 60% of the problems = $P(A) = 60\% = 0.60$ and

B can solve the 80% problems = $P(B) = 80\% = 0.80$

As the events A and B are independent, therefore

$$P(A \cap B) = P(A) \cdot P(B) = 0.60 \times 0.80 = 0.48$$

EXAMPLE 5.53

A fair coin is tossed twice. Find the probability of getting 4 or 6 on the first throw and an odd number on the second throw.

SOLUTION

The number of sample points in sample space are $n(S) = 6$

Let A represent the event of getting 4 or 6 on the first throw, then

$$n(A) = 2, P(A) = \frac{n(A)}{n(S)} = \frac{2}{6}$$

Let B represent the event of getting an odd number on the second throw, then

$$n(B) = 3, P(B) = \frac{n(B)}{n(S)} = \frac{3}{6}$$

A and B are independent, so

$$P(A \cap B) = P(A) \cdot P(B) = \frac{2}{6} \cdot \frac{3}{6} = \frac{6}{36} = \frac{1}{6}$$

EXAMPLE 5.54

Three groups of children contain respectively, 2 girls and 3 boys, 1 girl and 4 boys, 3 girls and 2 boys. One child is selected at random from each group. Find the probability that all are boys.

SOLUTION

Let A represent the event of selecting a boy from first group containing 2 girls and 3 boys.

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{5}$$

Let B represent the event of selecting a boy from second group containing 1 girl and 4 boys.

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{5}$$

Let C represent the event of selecting a boy from third group containing 3 girls and 2 boys.

$$P(C) = \frac{n(C)}{n(S)} = \frac{2}{5}$$

A, B and C are independent, therefore,

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = \frac{3}{5} \cdot \frac{4}{5} \cdot \frac{2}{5} = \frac{24}{125}$$

(II) Multiplication Law for Not Independent Events

If A and B are two events with probabilities $P(A)$ and $P(B)$ respectively, then the $P(A \cap B)$ is equal to the probability of A multiplied by the conditional probability of B or the probability of A multiplied by the conditional probability of A i.e.

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B/A) \\ \text{or} \quad &= P(B) \cdot P(A/B) \end{aligned}$$

EXAMPLE 5.55

Refer to example 5.50. assume that the first ball is not returned to the bag when the second ball is drawn. Find the probability that both balls of the two drawings are white.

SOLUTION

Let A represent the event of the first draw that will have a white ball

$$n(A) = 5, \quad P(A) = \frac{n(A)}{n(S)} = \frac{5}{8}$$

Let B represent the event of the second draw that will have a white ball also. Event B depends on the occurrence of event A. If the first ball is white, the probability of the second draw that will have a white ball is

$$P(A / B) = \frac{4}{7}$$

(Since there are only four white balls left in the bag of seven balls after the white ball is drawn).

The probability that both balls of the two drawings are white is

$$P(A \cap B) = P(A) \cdot P(B) \cdot P(B / A) = \frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$$

EXAMPLE 5.56

A card is drawn from a well shuffled pack of 52 cards and it is not replaced. Now, a second card is drawn. What is the probability that both cards are aces.

SOLUTION

Let A represent the event that first draw is an ace.

$$P(A) = \frac{4}{52}$$

Let B represent the event of the second draw that will have an ace also. Event B depends on the occurrence of event A, so

$$P(B / A) = \frac{3}{51}$$

Now

$$P(A \cap B) = P(A) \cdot P(B) \cdot P(B / A) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$$

5.13 DISTRIBUTION OF A DECK OF PLAYING CARDS

An ordinary deck of playing cards contains 52 cards arranged in 4 suits of 13 cards in each. The four suits are called diamonds, hearts, clubs and spades. The diamonds and spades are black card. The face values of the 13 cards in each suit are ace, 2, 3, 4,, 10, jack, queen and king. There are 12 pictured cards and 16 face cards. Face cards are comprising of 12 pictured cards and 4 aces. The tabular form of distribution of 52 cards is given below:

Deck of Playing Cards (52 cards)

Suit - I Diamond	Suit - II Heart	Suite - III Club	Suit - IV Spade
Λ_d	Λ_h	Λ_c	Λ_s
2_d	2_h	2_c	2_s
3_d	3_h	3_c	3_s
-	-	-	-
-	-	-	-
-	-	-	-
10_d	10_h	10_c	10_s
J_d	J_h	J_c	J_s
Q_d	Q_h	Q_c	Q_s
K_d	K_h	K_c	K_s
13	13	13	13
Red	Red	Black	Black

EXERCISE NO. 5

5.1:- Solve the following:

- | | | |
|---------------------------|----------------------------|----------------------------|
| (i) $11!$ | (ii) $10!$ | (iii) $\frac{12!}{8!0!}$ |
| (iv) $\frac{12!8!}{9!6!}$ | (v) $\frac{10!8!}{9!5!}$ | (vi) $\frac{13!4!}{15!2!}$ |
| (vii) $\frac{10!3!}{13!}$ | (viii) $\frac{13!}{10!3!}$ | |

5.2:- Evaluate the following:

- (i) $^{10}P_3$ (ii) 9P_4 (iii) $^{10}P_6$ (iv) 8P_7 (v) $^{25}P_5$

5.3:- How many numbers of 3 digits can be formed with the digits of the set (1, 2, 3, 4, 6).

5.4:- Three travelers arrive at Faisalabad city where there are four hotels. How many ways can they take up their quarters each at a different hotel?

5.5:- Four students enter the laboratory in which there are six doors. In how many ways can they enter from different doors?

5.6:- In how many ways can 8 seats be occupied by 4 students?

5.7:- In how many ways can the letters of the word STATISTICS be arranged?

5.8:- In how many ways can the letters of the word FAISALABAD be arranged?

5.9:- find out the number of ways in which 6 flags can be arranged in a row if 3 are red, 2 green and 3 yellow?

5.10:- Solve the following:

- (i) 8C_5 (ii) 7C_3 (iii) $^{24}C_1$ (iv) $^{19}C_4$ (v) $^{52}C_{32}$ (vi) $^{20}C_{r-1}$

5.11:- In how many ways 6 persons can be chosen out of 8?

5.12:- One of 12 books in how many ways a selection of 5 be made when one specified book is always included?

5.13:- In how many ways can two balls be drawn from a bag containing 4 white and 6 black balls?

5.14:- In how many ways can a committee consisting of 3 men and 2 women be formed out of 6 men and 4 women.

5.15:- A bag contains 9 balls, 5 of which are yellow and 4 are black. In how many ways 3 yellow balls and 2 black balls can be drawn?

5.16:- Expand $(x+y)^7$

5.17:- Expand $(x-2y)^4$

5.18:- Expand $(x-3y)^5$

5.19:- Expand $(2x-y)^5$

5.20:- Expand $(1-3a^2)^6$

5.21:- Expand $(a-1/a)^6$

5.22:- Find the 4th term of $(x-5)^{13}$

5.23:- Find the 5th term of $(a+3x^3)^{17}$

5.24:- Find the middle term of $\left(\frac{a}{x} + \frac{x}{a}\right)^{10}$

- 5.25:-** Find the 7th term in the expansion of $(x-2)^{15}$.
- 5.26:-** Find an approximate value of $\sqrt{1.07}$ by using the first four terms in the expansion of $(1+0.07)^{1/2}$.
- 5.27:-** Find the value of $\sqrt{23}$ correct to 3 places of decimals using binomial expansion.
- 5.28:-** Evaluate $\sqrt{0.97}$ to 6 places of decimals.
- 5.29:-** By the binomial formula, find the value of $(1.2)^7$ correct to four decimal places, taking $(1.2)^7 = (1+0.2)^7$.
- 5.30:-** Use $\in$ or $\notin$ to indicate whether the given object is an element of the given set in the following problems.
- | | | |
|-----|----|--|
| (1) | x | {x, y, z, a} |
| (2) | 3 | {1, 2, 4, 5, 6} |
| (3) | 12 | {1, 2, 3, 4, ...} |
| (4) | 5 | {x/x is a natural number greater than 5} |
| (5) | 6 | {x/x is a natural number less than 6} |
| (6) | 3 | ϕ |
- 5.31:-** If A is the set of all counting numbers less than or equal to 16, indicate which of the following are true and which are false.
- | | | |
|---------------|----------------|----------------|
| a) $11 \in A$ | b) $16 \in A$ | c) $100 \in A$ |
| d) $5 \in A$ | e) $1/3 \in A$ | f) $81 \in A$ |
- 5.32:-** Which of the following sets is finite and infinite:
- The set of existing colleges in Punjab.
 - The set of natural numbers between 100 and 210.
 - The set of sand particles in the desert.
 - The set of students in a class.
 - The set of stars in the sky.
 - The set of natural numbers.
- 5.33:-** Write the following sets in tabular form.
- $A = \{x/x \text{ is a natural number less than } 8\}$
 - $B = \{x/x \text{ is a natural number greater than } 6 \text{ and less than } 12\}$
 - $C = \{x/x \text{ is an even number}\}$
- 5.34:-** If $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$ is A subset of B.
- 5.35:-** Is $A \subseteq B$, if $A = \{a, b, c, d\}$ and $B = \{a, b, d\}$
- 5.36:-** Is $A \subseteq B$, if $A = \{6, 8, 10, 12\}$ and $B = \{6, 8, 10, 14, 18\}$
- 5.37:-** Use notation to indicate which set is a subset of the other in the following problems.
- | | | |
|----|---------------------------|----------------------|
| a) | $C = \{a, b, 1, 2, 3\}$, | $D = \{a, b, 1\}$ |
| b) | $A = \{1, \pi, e, 3\}$ | $D = \{1, \pi, 3\}$ |
| c) | $A = \{6, 8, 7, 4\}$ | $B = \{8, 7, 6, 4\}$ |
- 5.38:-** Indicate whether the following pairs of sets are equal or not.
- $A = \{a, b, \pi, \sqrt{3}\}$ $B = \{a, \pi, \sqrt{3}, b\}$

- ii) $A = \{x, g, a, b\}$ $B = \{x, a, b, y\}$
 iii) $D = \{x/x \text{ is a natural number less than } 4\}$, $E = \{1, 2, 3, 4\}$
 iv) $F = \{x/x \text{ is a natural number greater than } 6\}$
 $G = \{7, 8, 9, \dots\}$

5.39:- From the following list of sets, indicate which pairs of sets are disjoint.

- $A = \{1, 2, 3, 4\}$
 $B = \{x/x \text{ is a natural number greater than } 4\}$
 $C = \{4, 5, 6, \dots\}$, $D = \{1, 2, 3\}$

5.40:- Find the union of sets A and B.

- i) $A = \{x/x \text{ is a natural number greater than } 6\}$ and
 $B = \{x/x \text{ is a natural number less than } 7\}$
 ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c, d\}$

5.41:- Find the intersection of set A and B.

- i) $A = \{2, 3, 4, 5, 6\}$ $B = \{4, 6, 8, 10, 12, 14\}$
 ii) $A = \{a, b, c, d, e\}$ $B = \{a, d, e, f, g, h\}$

5.42:- If U is the universal set as

- $U = \{1, 2, 3, \dots, 25\}$
 $A = \{2, 4, 6, \dots, 18\}$, $B = \{1, 3, 5, \dots, 21\}$ then find
 a) A' b) B' c) $(A \cup B)'$ d) $A' \cap B'$

5.43:- What is meant by random experiment. Also define sample space and event.

5.44:- Explain the following terms.

- (a) Mutually exclusive events.
 (b) Equally likely events.
 (c) Independent and dependent events.

5.45:- Find the probability of each of the following.

- i) An odd number appears in a single throw of a fair die.
 ii) Ace appear in a single throw of the fair die.
 iii) A five appears in a throw of a fair die
 iv) The sum 8 appears in a toss of a pair of dice
 v) A head appears in tossing a fair coin
 vi) At least one head appears in three tosses of a fair coin
 vii) A king, ace, jack of clubs or queen of diamonds appears in drawing a single card from a well-shuffled ordinary deck of 52 cards.
 viii) An even number appears in a throw of a fair die.

5.46:- A card is drawn from a well-shuffled pack of 52 playing cards. What is the probability that it is

- | | |
|---------------------|----------------------|
| (i) An ace | (ii) A card of heart |
| (iii) A red card | (iv) A pictured card |
| (v) A king of heart | (vi) Black card |

- (vii) A card of diamond (viii) A face card

(Faisalabad Board, 2010; Lahore Board 2008, 2009 & 2010)

- 5.47:- (a) If two dice are thrown, what are the various total number of dots that may turn up
 (b) Find the probability of throwing a sum 10 with 2 dice.
 (c) Find the probability of throwing a sum 11 with 2 dice
 (d) Two fair dice are thrown. Show that the probability of throwing more than 7 is equal to that of throwing less than 7.
 (e) Two good dice are rolled. Let A denote the event "The sum shown is 8" and B the event "The two shows the same number". Find $P(A)$ and $P(B)$.
- 5.48:- (a) One student is chosen by lot from a class of 15 boys and 20 girls. What is the probability that a girl is chosen?
 (b) One card is drawn from a deck of 52 cards. What is the probability of drawing a spade?
 (c) One chip is drawn from a bag containing five black chips and four pink chips. What is the probability (i) It is a black chip (ii) It is a pink chip.
- 5.49:- Three fair dice are thrown. Find the probability of throwing (a) 7 (b) 9 (c) 15 (d) 13 (e) 9 or 11.
- 5.50:- In a single throw of two fair dice, find the probability that the product of the numbers on the dice is (i) between 8 and 15 (both inclusive) (ii) divisible by 4.
- 5.51:- From a pack of 52 cards two are drawn at random. Find the probability that one is a king and the other a queen.
- 5.52:- From a bag containing "4 white and 5 black balls" a man draws 3 balls at random. What is the probability of drawing 3 black balls.
- 5.53:- From a bag containing 5 white and 8 black balls, a man draws 4 balls. Find the probability of 4 black balls.
- 5.54:- A set of eight cards contains one joker. A and B are two players and A chooses 5 cards at random, B taking the remaining 3 cards. What is the probability that A had the joker?
- 5.55:- four persons are chosen at random from a group containing 3 men, 2 women and 4 children. What is the probability that exactly 2 of them will be children.
- 5.56:- A bag contains 5 white 6 red and 4 black balls. 3 balls are drawn at random. What is the probability that (i) all the three balls are white (ii) balls are either white or red (iii) one is red and 2 are black (iv) one ball of each colour.
- 5.57:- A bag contains 4 white, 5 red and 6 black balls. 3 balls are drawn at random. Find the probability (i) No ball drawn is black (ii) Exactly 2 are black (iii) All are of the same colour.

- 5.58:-** A batch of 50 lamps is known to have 6% defective. If 5 lamps chosen at random are tested (a) what is the probability that none of them will be defective (b) what is the chance that exactly 2 will be defective.
- 5.59:-** A bag contains 6 white, 4 red and 4 black balls. Find the probability of getting at least two red balls if four balls are drawn.
- 5.60:-** A bag contains 5 white and 7 black balls. If 3 balls are drawn, what is the probability that (i) All are white (ii) 2 white and 1 black (iii) All are of the same colour.
- 5.61:-** A bag contains 14 identical balls, 4 of which are red, 5 black and 5 white. Six balls are drawn at random from the bag. Find the chance that (i) 3 are red (ii) at least two are white.
- 5.62:-** A bag contains 3 white, 2 red, 5 green and 4 yellow balls. If 4 balls are selected at random, find the probability that balls of all colours are selected.
- 5.63:-** One bag contains 4 white and 2 black balls, another bag contains 3 white and 5 black balls. If one ball is drawn from each bag. Find the probability that (i) both are white (ii) both are black (iii) one is white and one is black.
- 5.64:-** A bag contains 5 white, 7 black, B bag contains 4 white, 6 black balls and C bag contains 2 white and 4 black balls. A ball is drawn from a bag selected at random. Find the probability that it is white.
- 5.65:-** Two drawings each of 3 balls are made from a bag containing 5 white and 8 black balls. The balls are not being replaced before the next trial. What are the probabilities that the first drawing will give 3 white balls and the second 3 black balls.
- 5.66:-** Two cards are drawn from a well-shuffled pack of 52 cards. Find the probability that they are both aces; if the first card is (i) replaced (ii) not replaced.
- 5.67:-** In a certain college the students engage in various sports in the following proportions.
Football (F) : 60% of all students.
Basketball (B) : 50% of all students.
Both football and basketball : 30% of all students.
If a student is selected at random what is the probability that he will
(i) play football or basketball (ii) play neither sport
- 5.68:-** A bag contains two red and three white balls. One ball is drawn from the bag and is replaced for the next. Find the probability that two drawings are red balls.

- 5.69:-** Refer to problem 5.68, if the first ball is not returned to the bag for second draw, what is the probability that the two drawings are red balls.
- 5.70:-** If two balanced dice are rolled, find out the conditional probability that the sum of dots will be 7, given that it is odd.
- 5.71:-** In a firm 20 percent of the employees have accounting background, while 5 percent of the employees are executives and having accounting background. If an employee has an accounting background, what is the probability that the employee is an executive.
- 5.72:-** A man tosses two fair dice what is the conditional probability that the sum of the two dice will be 7, given that (i) the sum is odd (ii) the sum is greater than 6 (iii) the two dice had the same outcome.

ANSWERS EXERCISE NO. 5

- 5.1:- (i) 39916800 (ii) 3628800 (iii) 11880 (iv) 73920 (v) 3360
(vi) $\frac{2}{35}$ (vii) $\frac{1}{286}$ (viii) 286
- 5.2:- (i) 720 (ii) 32024 (iii) 151200 (iv) 40320 (v) 6375600
- 5.3:- 120 5.4:- 24
- 5.5:- 360 5.6:- 1680
- 5.7:- 15000 5.8:- 151200
- 5.9:- 60
- 5.10:- (i) 56 (ii) 35 (iii) 10626 (iv) 3876 (v) 1326
(vi) $\frac{2n!}{(r+1)!(2n-r-1)!}$
- 5.11:- 28 5.12:- 330
- 5.13:- 45 5.14:- 120
- 5.15:- 60
- 5.16:- $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$
- 5.17:- $x^4 - 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$
- 5.18:- $x^5 - 15x^4y + 90x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5$
- 5.19:- $32x^5 - 80x^4y + 80x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5$
- 5.20:- $1 - 18a^2 + 134a^4 - 540a^6 + 1214a^8 - 1458a^{10} + 729a^{12}$
- 5.21:- $a^6 - 6a^4 + 15a^2 - 20 + \frac{15}{a^2} - \frac{6}{a^4} + \frac{1}{a^6}$
- 5.22:- $-35750x^{10}$ 5.23:- $38080a^{13}x^{12}$
- 5.24:- 252 5.25:- $320320x^9$
- 5.26:- 1.0356339 5.27:- 4.796
- 5.28:- 0.984886 5.29:- 3.5832
- 5.30:- 1) $\in$ 2) $\notin$ 3) $\in$ 4) $\notin$ 5) $\notin$ 6) $\notin$
- 5.31:- (a) true (b) true (c) false (d) true (e) false (f) false
- 5.32:- (i) finite (ii) finite (iii) infinite (iv) finite (v) infinite (vi) infinite
- 5.33:- 1) $A = \{1, 2, 3, 4, 5, 6, 7\}$ 2) $B = \{7, 8, 9, 10, 11\}$
3) $C = \{2, 4, 6, 8, \dots\}$
- 5.34:- Yes 5.35:- Yes 5.36:- No
- 5.37:- a) $D \subseteq C$ b) $D \subseteq A$ c) $A \subseteq B$ or $B \subseteq A$
- 5.38:- i) Yes ii) No iii) No iv) Yes
- 5.39:- A and B, B and D, C and D
- 5.40:- i) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \dots\}$
ii) $A \cup B = \{a, b, c, d, e, i, o, u\}$
- 5.41:- i) $\{4, 6\}$ ii) $\{a, d, e\}$
- 5.42:- a) $A' = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 20, 21, \dots, 25\}$
b) $B' = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\}$
c) $(A \cup B)' = \{22, 23, 24, 25\}$
d) $A' \cap B' = \{22, 23, 24\}$

- 5.45:- (i) $\frac{1}{2}$ (ii) $\frac{1}{6}$ (iii) $\frac{1}{6}$ (iv) $\frac{6}{36}$ (v) $\frac{1}{2}$
 (vi) $\frac{7}{8}$ (vii) $\frac{8}{26}$ (viii) $\frac{1}{2}$
- 5.46:- (i) $\frac{1}{13}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{2}$ (iv) $\frac{3}{13}$ (v) $\frac{1}{52}$
 (vi) $\frac{1}{26}$ (vii) $\frac{1}{4}$ (viii) $\frac{4}{13}$
- 5.47:- (a) 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
 (b) $\frac{3}{36}$ (c) $\frac{2}{36}$ (d) $\frac{15}{36}$ (e) $P(A) = \frac{4}{13}$ and $P(B) = \frac{1}{6}$
- 5.48:- (a) $\frac{4}{7}$ (b) $\frac{1}{4}$ (c) (i) $\frac{5}{9}$ (ii) $\frac{4}{9}$
- 5.49:- (a) 0.069 (b) $\frac{25}{216}$ (c) $\frac{10}{216}$ (d) $\frac{21}{216}$ (e) $\frac{13}{54}$
- 5.50:- (i) $\frac{1}{3}$ (ii) $\frac{15}{36}$
- 5.51:- $\frac{8}{663}$
- 5.52:- $\frac{10}{84}$
- 5.53:- $\frac{1}{143}$
- 5.54:- $\frac{5}{8}$
- 5.55:- $\frac{10}{21}$
- 5.56:- (i) $\frac{2}{91}$ (ii) $\frac{6}{91}$ (iii) $\frac{36}{455}$ (iv) $\frac{24}{91}$
- 5.57:- (i) $\frac{12}{65}$ (ii) $\frac{27}{91}$ (iii) $\frac{34}{455}$
- 5.58:- (a) 0.724 (b) 0.023
- 5.59:- $\frac{37}{91}$
- 5.60:- (i) $\frac{1}{22}$ (ii) $\frac{7}{22}$ (iii) $\frac{9}{44}$ (iv) $\frac{9}{44}$
- 5.61:- (i) $\frac{480}{3003}$ (ii) $\frac{2289}{3003}$
- 5.62:- $\frac{120}{1001}$
- 5.63:- $\frac{65}{176}$
- 5.64:- (i) $\frac{1}{4}$ (ii) $\frac{5}{24}$ (iii) $\frac{13}{24}$
- 5.65:- $\frac{7}{429}$
- 5.66:- (i) $\frac{1}{169}$ (ii) $\frac{1}{221}$
- 5.67:- (i) 0.8 (ii) 0.2
- 5.68:- $\frac{4}{25}$
- 5.69:- $\frac{1}{10}$
- 5.70:- $\frac{1}{3}$
- 5.71:- 0.25
- 5.72:- (i) $\frac{1}{3}$ (ii) $\frac{2}{7}$ (iii) 0

5

PROBABILITY

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	An experiment of three coins are tossed has sample points	4	8	9	None of these
		(Faisalabad Board, 2008)			
2)	The probability of an event always lies between	-1 and +1	-1 and 0	0 and +1	0 and ∞
		(Lahore Board, 2008 & 2010)			
3)	The probability is the measure of	Un-certainty	Certainty	Chances	None of these
		(Lahore Board, 2008)			
4)	When two coins are tossed, the possible outcomes are	6	4	12	3
		(Gujranwala Board, 2009; Faisalabad Board, 2009 & 2010) (Lahore Board, 2009)			
5)	The probability of sure even is	0	100	1	$\frac{1}{2}$
		(Lahore Board, 2009)			
6)	If $P(A) = 0.70$, then $P(\bar{A})$ is	0.70	0.30	0	1
		(Faisalabad Board, 2010)			
7)	For two mutually exclusive events A and B: $P(A) = 0.70$; $P(B) = 0.3$ then $P(A \cup B)$ is	0	0.40	0.30	1
		(Lahore Board, 2010)			
8)	Four books can be arranged in a shelf is	8 ways	2 ways	16 ways	24 ways
		(Lahore Board, 2010)			

S#	Questions	Answers			
9)	The probability of a black card from pack of 52 cards is	$\frac{1}{52}$	$\frac{1}{2}$	$\frac{13}{52}$	$\frac{3}{4}$
(Lahore Board, 2010)					
10)	The value of probability can not be greater than	Zero	Unity	Sample	Two
(Gujranwala Board, 2010)					
11)	When two coins are tossed simultaneously, the probability of one head is	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
(Rawalpindi Board, 2010)					
12)	If n coins are tossed, the possible outcomes are	n	2n	2^n	n^2
(Rawalpindi Board, 2010)					
13)	The complete sample space is called	Empty	Impossible event	Equally likely event	Sure event
(Sargodha Board, 2010)					

QUESTION NO. 2

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The idea of probability was first applied in gambling games in:	Pakistan	America	France	India
2)	Today the application of the theory of probability is the most important in making decisions in:	Business	Insurance	Management	All above
3)	n-factorial is denoted by:	n!	m!	r!	P!
4)	$(n+2)! =$	$n(n-1)!$	$(n-1)(n-2)!$	$(n+2)(n+1)n!$	$(n \times 2)!$
5)	Three objects can be arranged in:	3 ways	4 ways	5 ways	6 ways

S#	Questions	Answers			
6)	The total number of possible combinations of a set of a different objects, taken r at a time is denoted by:	nP_r	nC_r	nD_r	None of these
7)	${}^nC_r =$ _____	$\frac{n!}{r!}$	$\frac{n!}{(n-r)!}$	$\frac{n!}{r!(n-r)!}$	$\frac{r!}{n!}$
8)	A set having zero as its element is called:	Empty set	Infinite set	Singleton set	Null set
9)	Two books are to be selected at random without replacement out of four books. The number of possible selections are:	6	2	4	3
10)	Three books of different colours are to be arranged in book-shelf. The possible arrangements are:	3	1	6	2
11)	A set consisting of all the elements under consideration is called:	Empty set	Disjoint set	Universal set	Sub set
12)	A set consisting of all the elements under consideration is called:	U	λ	ϕ	μ
13)	Sets having elements in common are called:	Overlapping sets	Disjoint sets	Infinite sets	None of these
14)	The set A is a subset of the set B if every element of A is an element of B. We use the symbol as:	$A \supseteq B$	$A \subseteq B$	$A = B$	$A + B$
15)	The union of two sets is denoted by:	$(A \cup B)$	$(A \cap B)$	$A = B$	$A + B$
16)	The intersection of two sets is denoted by:	$A \cup B$	$A \cap B$	$A \subset B$	$A - B$

S#	Questions	Answers			
17)	An act or the process of obtaining an observation is called:	Experiment	Sample space	Event	Set
18)	If an experiment provides different results under essentially identical conditions, it is called:	Sample space	Simple event	Random experiment	Sub set
19)	A set of all possible outcomes of the random experiment is called:	Sample space	Impossible event	Union of sets	Sub set
20)	Any subset of a sample space is called:	Finite set	Nul set	Event	All of these
21)	An event that contains more than one sample point is called:	Simple event	Compound event	Impossible event	None of these
22)	When a die is rolled, all possible out comes are:	2	4	6	8
23)	When 2 dice are rolled, all possible out comes are:	4	10	22	36
24)	When n dice are rolled, the possible sample points in the sample space are:	6^n	n^6	6^n	None of these
25)	When two coins are tossed, the possible out comes are:	6	4	36	12
26)	If three coins are tossed, the possible outcomes are:	8	3	1	10
27)	If n coins are tossed, the possible outcomes are:	n	2n	2^n	n^2
28)	If two events A and B can not occur together, then they are:	Mutually exclusive	Not mutually exclusive	Independent	All of these

S#	Questions	Answers			
29)	If two events A and B have same chance to occur, then they are:	Mutually exclusive	Equally likely	Exhaustive	None of these
30)	If A and B are mutually exclusive events, $P(A)=0.25$, $P(B)=0.50$, then $P(A \cup B) =$	0.75	0.25	0.50	1
31)	The probability of an event always lies between:	-1 and +1	-1 and 0	0 and 1	None of these
32)	$P(A \cup B) = P(A) + P(B)$ if A and B are:	Mutually exclusive events	Independent events	Non mutually exclusive events	None of these
33)	The probability of appearing a tail when a fair coin is tossed is:	0	$\frac{1}{2}$	$\frac{1}{4}$	1
34)	The probability of drawing a queen from well-shuffled pack of 52 playing cards is:	$\frac{4}{52}$	$\frac{2}{52}$	$\frac{13}{52}$	$\frac{26}{52}$
35)	If two cards are drawn at random from a standard deck of cards, the number of possible sample points is:	52	52!	1326	2000
36)	The probability of drawing a jack from a deck of cards is:	$\frac{1}{13}$	$\frac{1}{4}$	$\frac{1}{52}$	$\frac{5}{52}$
37)	The probability of selecting a jack of club from a deck of cards is:	$\frac{13}{52}$	$\frac{4}{52}$	$\frac{1}{52}$	$\frac{2}{52}$
38)	The probability of getting exactly one head when two coins are tossed is:	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
39)	Five cards are selected at random from a pack of 52	52	${}^{52}C_5$	52^5	5^{52}

S#	Questions	Answers			
	cards. The possible outcomes are:				
40)	A fair coin is tossed 200 times, the expected number of heads is:	100	7	50	20
41)	When two dice are rolled, the maximum total on the two faces of the dice will be:	6	36	12	10
42)	If A and B are independent events then:	$P(A \cup B) = P(A) + P(B)$	$P(A \cap B) = P(A) + P(B)$	$P(A \cap B) = P(A) \cdot P(B)$	None of these
43)	If $P(A/B) = P(A)$ and $P(B/A) = P(B)$, then the events A and B are said to be:	Independent	Dependent	Mutually exclusive	Simple
44)	If $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$, then events A and B are said to be:	Equally likely	Mutually exclusive	Independent	Exhaustive
45)	Probability of an event can never be:	One	Negative	Positive	Zero
46)	The conditional probability $P(A/B)$ is:	$\frac{P(A \cap B)}{P(B)}$	$\frac{P(A \cap B)}{P(A)}$	$\frac{P(A \cap B)}{P(B)}$	$\frac{P(A \cap B)}{P(A)}$

QUESTION NO. 3

Write the short answer to the following questions in the space provided

Q. 1: What is difference between permutation and combination?

Ans: A permutation of a number of objects is an arrangement of these objects in some order. A selection of distinct objects without some order is called a combination. symbolically

$${}^n P_r = \frac{n!}{(n-r)!} \text{ is a formula for permutation and}$$

$${}^n C_r = \frac{n!}{r!(n-r)!} \text{ is a formula for combination}$$

Q. 2: What do you mean by n!?

Ans: The product of first "n" natural numbers is called n factorial denoted by $n! = n(n-1)(n-2) \dots 1$

Q. 3: Define the term experiment.

Ans: An act or the process of obtaining an observation is called experiment.

Q. 4: Define the term trail.

Ans: Performing of an experiment is called a trail.

Q. 5: Define the term random experiment.

(Lahore Board, 2009 & 2010)

(Rawalpindi Boar, 2010)

(Sargodha Board, 2010)

Ans: An experiment is said to be random, if it provides different results, if it is repeated a large number of times under identical conditions.

Q. 6: What is a sample space?

Ans: A set of all possible outcomes of a random experiment is called a sample space. It is denoted by S.

Q. 7: Give two examples of a random experiment.

Ans: The tossing of a fair coin and throwing of a balanced die are examples of random experiment.

Q. 8: What is an event?

(Gujranwala Board, 2010)

Ans: Any subset of the sample space is called an event. It is denoted by A, B, C, D etc.

Q. 9: Make a sample space, when a cubical die is rolled.

(Gujranwala Board, 2010)

(Sargodha Board, 2010)

Ans: $S = \{1, 2, 3, 4, 5, 6\}$

Q. 10: Define sample points.

Ans: The elements of the sample space are called sample points.

Q. 11: What are the properties of a random experiment?

Ans: A random experiment has three properties.

- (i) The experiment can be repeated
- (ii) The experiment always has at least two outcomes
- (iii) The outcome of each trail is unpredictable

Q. 12: Define simple event.

(Lahore Board, 2010)

(Rawalpindi Board, 2010)

Ans: A simple event is one which contains only one sample point of the sample space.

Q. 13: Define compound event.

Ans: Compound event contains two or more sample points of the sample space.

Q. 14: What is impossible event?

Ans: An event consisting of the empty set is called impossible event.

Q. 15: Define sure event or certain event.

Ans: An event that consists of the sample space itself is called sure or certain event.

Q. 16: Define mutually exclusive events with example.

Ans: Two events A and B are said to be mutually exclusive if they can not occur together. Thus occurrence of head or tail in a single throw of a coin are mutually exclusive events.

Q. 17: Define not mutually exclusive events with example.

Ans: If two events can occur together, they are not mutually exclusive. If we draw a card from an ordinary deck of 52 cards, it can be both a queen and a card of spade.

Q. 18: Define equally likely events with example.

Ans: Two or more events are said to be equally likely events if they have same chance to occur. If we toss a coin, the head and tail have the same chance to be occur.

Q. 19: Define independent events with example.

(Lahore Board, 2008 & 2009)

(Sargodha Board, 2010)

Ans: Two events A and B are said to be independents if the events in no way affect each other. For example, suppose event A is the drawing of king from a pack of 52 cards and event B is head on tossing a fair coin. The events are independent because drawing of a card does not affect the head when tossing a coin.

Q. 20: Define dependent events.

Ans: Two events are said to be dependent when the happening of any one event is affected by the happening or non happening of the other.

Q. 21: Define the exhaustive events.

(Lahore Board, 2009)

Ans: Two or more events are said to be collectively exhaustive if their union generates the whole sample space.

Q. 22: Define favourable cases.

Ans: The cases which entail the occurrence of an event are said to be favourable cases to the event.

Q. 23: Are independent events also mutually exclusive?

Ans: No, mutually exclusive events are always independent.

Q. 24: What are two approaches to define probability?

Ans: Two approaches to define probability are:

(a) Subjective approach

(b) Objective approach

Q. 25: Give the classical priori definition of probability.

(Gujranwala Board, 2010)

Ans: If there are "n" equally likely, mutually exclusive and exhaustive sample points in a sample space and "m" out of them are favourable to an event A, then

$$P(A) = \frac{m}{n}$$

Q. 26: Give the relative frequency or posteriori definition of probability.

Ans: If a random experiment is repeated a large number of times say n under uniform conditions and the event A is observed m times, then probability of event A, is

$$P(A) = \lim_{n \rightarrow \infty} \frac{m}{n}$$

Q. 27: Give the axiomatic (mathematical approach) of probability.

Ans: The probability that an event A will occur is the number P(A). The number P(A) satisfy the following axioms:

- (i) $0 \leq P(A) \leq 1$
- (ii) $P(S) = 1$
- (iii) If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Q. 28: When does probability become negative?

Ans: Probability can never be negative.

Q. 29: State the addition law of probability for two mutually exclusive events.

Ans: If A and B are two mutually exclusive events then the probability that anyone of them occur is given by sum of their respective probabilities, that is $P(A \cup B) = P(A) + P(B)$

Q. 30: State the addition law of probability for two not mutually exclusive events.

Ans: If A and B are not mutually exclusive events then the probability that at least one of them occur is given by the sum of their respective probabilities minus the probability of their joint occurrence that is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Q. 31: State the multiplication law of probability for two independent events.

Ans: If A and B are two independent events, then the probability that both occur is given as:

$$P(A \cap B) = P(A).P(B)$$

Q. 32: State multiplication law of probability for two dependent events.

Ans: If A and B are two dependent events, then

$$P(A \cap B) = P(A).P(B/A) \text{ provided } P(A) \neq 0 \text{ OR}$$

$$P(A \cap B) = P(B).P(A/B) \text{ provided } P(B) \neq 0$$

Q. 33: What is conditional probability?

Ans: If A and B are two events, and $P(B) \neq 0$, then conditional probability for event A given that the event B has occurred, denoted by $P(A/B)$, is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Q. 34: Define set.

(Lahore Board, 2010)

Ans: Well defined collection of distinct objects is called set. It is denoted by capital letters A, B, C,

Q. 35: What is empty set?

(Lahore Board, 2010)

Ans: A set consisting no element is called a null or empty set. It is denoted by $\emptyset$ or $\{\}$

ANSWERS

QUESTION NO. 1

- | | | |
|----------------|-------------------|------------------|
| 1) 8 | 2) 0 and +1 | 3) Un-certainty |
| 4) 4 | 5) -1 | 6) 0.30 |
| 7) 1 | 8) 24 ways | 9) $\frac{1}{2}$ |
| 10) Unity | 11) $\frac{1}{2}$ | 12) 2^n |
| 13) Sure event | | |

QUESTION NO. 2

- | | | |
|-------------------------------|--------------------------------|-------------------|
| 1) France | 2) All above | 3) $n!$ |
| 4) $(n+2)(n+1)n!$ | 5) 6 ways | 6) nC_r |
| 7) $\frac{n!}{r!(n-r)!}$ | 8) Singleton set | 9) 6 |
| 10) 6 | 11) Universal set | 12) ϕ |
| 13) Overlapping set | 14) $A \subseteq B$ | 15) $A \cup B$ |
| 16) $A \cap B$ | 17) Experiment | 18) Random |
| 19) Sample space event | 20) Event | 21) Compound |
| 22) 6 | 23) 36 | 24) 6^n |
| 25) 4 | 26) 8 | 27) 2^n |
| 28) Mutually exclusive | 29) Equality likely | 30) 0.75 |
| 31) 0 and 1 | 32) Mutually exclusive events | |
| 33) $\frac{1}{2}$ | 34) $\frac{4}{52}$ | 35) 1326 |
| 36) $\frac{1}{13}$ | 37) $\frac{1}{52}$ | 38) $\frac{2}{4}$ |
| 39) ${}^{52}C_5$ | 40) 100 | 41) 12 |
| 42) $P(A \cap B) = P(A).P(B)$ | 43) Independent | 44) Independent |
| 45) Negative | 46) $\frac{P(A \cap B)}{P(B)}$ | |

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