

3

MEASURES OF LOCATION OR CENTRAL TENDENCY

Sr. No.	Contents	Page
3.1	Introduction	58
3.2	Objects of Measures of central tendency	58
3.3	Characteristics of a good average	59
3.4	Types of averages	59
3.5	Summation notation	60
3.6	The arithmetic mean	60
3.6.1	Arithmetic mean for ungrouped data	61
3.6.2	Frequency Distribution	64
3.7	Properties of arithmetic mean	70
3.8	Weighted arithmetic mean	71
3.9	Advantage and disadvantages of A.M	72
3.10	Median	72
3.10.1	Ungrouped data	72
3.10.2	Grouped data	74
3.10.3	Quartiles, deciles and percentiles	78
3.10.4	A graphic method of location of median, quartiles, deciles and percentiles	85
3.11	Advantages and disadvantages of median	86
3.12	The mode	87
3.12.1	Ungrouped data	87
3.12.2	Grouped data	87
3.12.3	Graphical representation of mode	89
3.13	Advantages and disadvantages of mode	91
3.14	Relation between mean, median and mode	92
	Exercise	94
	Answers	104

3.1 INTRODUCTION

An average is a single value that is typical or representative for a group of numbers. The values included in a group of data usually vary in magnitude: some of them are small and some are large. Obviously, a representative value for a group of numbers is neither the smallest nor the largest value, but is number whose value is some where in the middle of the group. Thus, an average is frequently referred to as a measures of central tendency. As a measure of central tendency indicates the location or general position of the distribution or the set of data in the range of values. It is also known as a measures of location or position.

Dr. Bowley defines an average in the following terms:

"An average is purely mathematical conception such as the average length of life in a varied population which does not correspond to any particular group but is only short way of expressing an arithmetical result."

Croton and Cowden, defines it as "A single value with in the range of the data that is used to represent the values in the series. Since an average is some where with in the range of the data, it is sometimes called a measures of central value."

An average is, therefore, a single simple numerical expression which reflects the result of a complex group, (large numbers reduced to a mean). It is a typical item representing the entire group and, therefore, is also called "Type". It is a representative number of the group and as such throws light on its characteristics better than any other value in the group. Two points should be noted (i) A measure of central tendency should be some where with in the range of the data. (ii) A measure of central tendency should remain unchanged by a rearrangement of the value in a different order.

3.2 OBJECTS OF MEASURES OF CENTRAL TENDENCY

The objects for which average are generally computed and used, may be summarised as follows:

- (i) The most important object of measuring central tendency or average is to find out a value that can represent the whole data.
- (ii) One of the objects of measuring average is to help the comparative study. An average, therefore, affords a basis of comparison with other groups. For example if we are given the individual marks in statistics of total students of two colleges A and B, it is difficult to compare the marks obtained. However, if some short of average marks obtained by the students of the two colleges is computed, the comparison would become easier.
- (iii) The Human mind can not grasp and remember the whole mass of figures and, naturally, have to run to some kind of numerical precise. Statistical average provide a condensed picture of a large group. It is

easy to remember averages but externally difficult to remember the whole data. For instance, it is particularly difficult to remember the individual marks obtained in statistics by 100 students of a college. But, on the other hand, if the individual marks of the students are condensed and put in to a single expression (the averages), it may be remembered. In other words, it is easy to retain in mind the average marks obtained in statistics by 100 students (say 40% instead of 100 individual marks figures).

Averages have many uses. An average is frequently used as a device in summarizing a group of numbers, especially a large group of numbers, for describing the statistical data. Examples are the average age of student in a college, the average weekly wage of manufacturing workers in a state, and the average family income in a nation.

3.3 CHARACTERISTICS OF A GOOD AVERAGE

An average should possess the following characteristics:

- (i) It should be rigidly defined.
- (ii) A good average should base on all the observations of the series. Other wise it can not be a representative value.
- (iii) It should be capable of being computed with reasonable ease and rapidity.
- (iv) A good average should be easily understandable.
- (v) It should be as little affected by fluctuations of sampling as possible.
- (vi) It should be capable of further algebraic treatment.
- (vii) It should be in the form of a mathematical formula.

An average is said to be good, If it possess the all or most of the qualities stated above.

3.4 TYPES OF AVERAGES

The most commonly known averages in statistics are:

- (i) The arithmetic mean.
- (ii) The median.
- (iii) The mode.
- (iv) The geometric mean.
- (v) The harmonic mean.

Each average has its particular characteristics. The arithmetic mean, geometric mean and harmonic mean are mathematical in character while the median and mode are averages of location. In general, the first three averages are used more frequently. In this chapter, we discuss about the first order averages, since I. Com syllabus consists the first order averages. The following averages are called the first order averages.

- (i) The Arithmetic mean
- (ii) The Median
- (iii) The mode

3.5 SUMMATION NOTATION

The symbols $\sum x_i$ is used to denote the sum of all X_i 's from 1 to n i.e.,

$$\sum_{i=1}^n x_i = x_1 + x_2 + x_3 + \dots + x_n$$

$$\sum_{i=1}^6 x_i = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$\sum_{i=1}^n a_i x_i = a_1 x_1 + a_2 x_2 + a_3 x_3 + \dots + a_n x_n$$

$$\sum_{i=1}^n x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots + x_n y_n$$

$$\sum_{i=1}^n x_i^2 = x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2$$

We usually write $\sum x$ instead of $\sum_{i=1}^n x_i$

Similarly $\sum x_i y_i = \sum xy$, $\sum_{i=1}^n x_i^2 = \sum x^2$

Σ is the capital sigma in the Greek alphabet and is customarily used as an observation for sum.

3.6 THE ARITHMETIC MEAN

Arithmetic mean is the most common type of averages. It is a value obtained by dividing sum of all the values by their numbers. If a series is denoted by

$$X_1, X_2, X_3, \dots, X_n$$

then the arithmetic mean of this series denoted by $\bar{x}$ (read as "x bar") will be:

Where
$$\bar{x} = \frac{X_1 + X_2 + X_3 + \dots + X_n}{n} = \frac{\sum x}{n}$$

$\bar{X}$ = Arithmetic mean.

$\sum x$ = Sum of all the value of the variable.

n = Number of values.

There are three methods to calculate arithmetic mean.

- (i) Direct or long method.
- (ii) Indirect or short-cut method.

(iii) Step-deviation or coding method.

3.6.1 Arithmetic Mean For Ungrouped Data (Individual Observations)

(A) Direct Method:

As already explained the arithmetic mean is the quotient of the sum of all values divided by the number of values in the given set of data.

$$\text{Arithmetic Mean} = \bar{X} = \frac{\text{Sum of all the Values}}{\text{Number of Values}}$$

$$\bar{X} = \frac{\sum X}{n}$$

The following examples will show how arithmetic mean is calculated in a given situation.

EXAMPLE 3.1

Calculate the arithmetic mean of the following data.

20, 50, 72, 28, 53, 54, 59, 64, 72, 74, 75, 78, 79

SOLUTION

Computation of arithmetic mean by direct method are.

Size of Items (X)	Size of Items (X)
20	64
50	72
72	74
28	75
53	78
54	79
59	Total : 778

$$\text{Arithmetic Mean} = \bar{X} = \frac{\sum X}{n}$$

Here $\sum X = 778$ $n = 13$

$$\text{Then } \bar{X} = \frac{\sum X}{n} = \frac{778}{13} = 59.85$$

EXAMPLE 3.2

Calculate the arithmetic mean of the following items;

25, 27, 28, 29, 30, 32, 34, 36, 31, 30, 32, 35, 36, 37, 38

SOLUTION

Size of Items (X)	Size of Items (X)	Size of Items (X)
25	34	36
27	36	37
28	31	38
29	30	Total : 480
30	35	
32	32	

$$\bar{X} = \frac{\sum X}{n} \quad \sum x = 480, n = 15; \text{A.M.} = \frac{\sum X}{n} = \frac{480}{15} = 32$$

Arithmetic mean of the series is 32.

EXAMPLE 3.3

The following are salary figures of 30 individuals employed in a firm in Faisalabad; calculate the arithmetic means.

60, 67, 74, 85, 90, 95, 76, 88, 107, 115, 120, 122, 93, 138, 140, 150, 166, 134, 160, 170, 174, 168, 214, 175, 208, 219, 225, 240, 276, 290

SOLUTION

Salaries (X)	Salaries (X)	Salaries (X)
60	122	214
67	93	175
74	138	208
85	140	219
90	150	225
95	166	240
76	134	276
88	160	291
107	170	Total: 4440
115	174	
120	168	

$$\bar{X} = \frac{\sum X}{n} \quad \sum X = 4440, n = 30 \quad \text{A.M.} = \frac{\sum X}{n} = \frac{4440}{30} = 148$$

(B) Short-Cut Method

Direct method can be used only when the size of the figures is small. If it is not so, there would be considerable difficulty in the calculation of the arithmetic mean. To overcome this drawback, short-cut method is used. Calculations can be simplified by using that is known as short-cut method. In this method, any of the variable may be **assumed mean** or **guessed mean** or **provisional mean**. Deviation of values of each item from provisional mean should be found and put down with proper algebraic signs. The algebraic sum of these deviations divided by the number of items when added to the provisional mean gives required arithmetic mean. The short-cut method will enable us to save as considerable amount of time when a large group of data is involved in computing the mean. The short-cut method for computing the arithmetic mean is given below:

$$\bar{X} = \text{P.M.} + \frac{\sum (X - \text{P.M.})}{n} = \text{P.M.} + \frac{\sum D}{n}$$

Where P.M. = Provisional mean; D = Deviation of values from P.M.

N = The total number of values.

EXAMPLE 3.4

Calculate the arithmetic mean from the following data. Monthly income of 12 employees are: Rs. 120, 100, 110, 130, 109, 101, 150, 200, 190, 170, 125, 175

SOLUTION

Income (x)	D
120	-30
100	-50
110	-40
130	-20
109	-41
101	-49
150	0
200	50
190	40
170	20
125	-25
175	25
Total: 1680	-120

(i) **Direct Method**

$$\bar{X} = \frac{\sum X}{n} \quad \sum X = 1680, n = 12 = \frac{1680}{12} = 140$$

(ii) **Short-cut Method**

$$\bar{X} = P.M. + \frac{\sum D}{n} \quad P.M. = 150, \sum D = -120$$

$$\bar{X} = 150 + \frac{-120}{12} = 150 - 10 = 140$$

EXAMPLE 3.5

Calculate the arithmetic mean for the data given in example 3.2 using short cut method.

SOLUTION

Size of Items (x)	D
25	-5
27	-3
28	-2
29	-1
30	0
32	2
34	4
36	6
31	1
30	0
32	2
35	5
36	6
37	7
38	8
.....	30

$$\bar{X} = P.M. + \frac{\sum D}{n} \quad P.M. = 30, \sum D = 30, n = 15$$

$$\bar{X} = 30 + \frac{30}{15} = 30 + 2 = 32$$

3.6.2 Frequency Distribution

When a large group of items is collected, the values of the items should be organised in order to facilitate Statistical analysis. When the value on quantity and number of values within each class indicated, a more compact tabular presentation of the data can be obtained. The table showing grouped data based on quantity is called a frequency distribution,

Class limits and mid point values:

The lower and upper class limits in frequency distribution indicate the boundaries of each class in the distribution.

The mid point to each class is usually used to represent each original value grouped into the class for purposes of further analysis.

$$\text{Class mid point} = X = \frac{\text{Lower Class Limit} + \text{Upper Class Limit}}{2}$$

(A) Direct Method

The arithmetic mean for group data is obtained as follows. The first step is to take mid-point of each class as a representative of that class. The mid points of each class is multiplied by their corresponding frequencies to obtain the total product which is then divided by the number of items. Suppose the value x_1 occurs f_1 times, the value x_2 occurs f_2 times, the value x_3 occurs f_3 times and so on, then

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + \dots + f_nx_n}{f_1 + f_2 + f_3 + \dots + f_n} = \frac{\sum fx}{\sum f}$$

Where x = The mid point of individual class

f = The frequency of individual class

$\sum f$ = The sum of the frequencies

EXAMPLE 3.6

The miles traveled by 20 students in coming to Commerce college Faisalabad from their homes are given below. Calculate the arithmetic mean.

SOLUTION

Miles Traveled	No of Students	Mid Point (x)	fx
0 and under 2	2	1	2
2 and under 4	5	3	15
4 and under 6	4	5	20
6 and under 8	8	7	56
8 and under 10	1	9	9
-----	20	-----	102

$$\sum f = 20, \sum fx = 102$$

$$\text{Arithmetic Mean} = \bar{X} = \frac{\sum fx}{\sum f} = \frac{102}{20} = 5.1 \text{ miles}$$

EXAMPLE 3.7

The frequency distribution below gives the cost of production of sugarcane in different holdings. Obtain the arithmetic mean.

Cost	:	2-6	6-10	10-14	14-18
Frequency	:	1	9	21	47
Cost	:	18-22	22-26	26-30	30-34
Frequency	:	52	36	19	3

SOLUTION

Class Interval	Frequency	x	fx
2-6	1	4	4
6-10	9	8	72
10-14	21	12	252
14-18	47	16	752
18-22	52	20	1040
22-26	36	24	864
26-30	19	28	532
30-34	3	32	96
—	188	—	3612

Applying the method

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{3612}{188} = 19.21$$

EXAMPLE 3.8

From the following data, find the arithmetic mean.

Class Interval	:	2.50-2.99	3.00-3.49	3.50-3.99	4.00-4.49
Frequency	:	1	5	11	16
Class Interval	:	4.50-4.99	5.00-5.49	5.50-5.99	6.00-6.49
Frequency	:	20	16	13	4
Class Interval	:	6.50-6.99	7.00-7.49	7.50-7.99	
Frequency	:	2	0	1	

SOLUTION

Class Interval	F	X	FX
2.50-2.99	1	2.745	2.745
3.00-3.49	5	3.245	16.225
3.50-3.99	11	3.745	41.195
4.00-4.49	16	4.245	67.920
4.50-4.99	20	4.745	94.900
5.00-5.49	16	5.245	83.920
5.50-5.99	13	5.745	74.685
6.00-6.49	4	6.245	24.980
6.50-6.99	2	6.745	13.490
7.00-7.49	0	7.245	0
7.50-7.99	1	7.745	7.745
-----	89	-----	427.805

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{427.805}{89} = 4.807$$

(B) Indirect or Short cut Method

The procedure for arithmetic mean from continuous frequency distribution is given below:

Find out the mid-points of each class. Select the provisional mean. The answer is not affected by the value selected as the assumed mean. Any point, including zero, may be used as the provisional mean. However, in order to simplify the computation, the mid-point of one of the centrally located classes in the given data should be selected as the assumed mean. Find the deviation of each class mid point from the assumed mean in original units of data. The column will be known symbolically as D. Multiply D with the respective frequency. This column is known as fD. Add these products to obtain total deviation of all items included in the data, or $\sum fD$.

$$\bar{X} = P.M. + \frac{\sum fD}{\sum f}$$

Where P.M. = Provisional Mean,

$\sum fD$ = Total product

$\sum f$ = Number of items

The application of this method is shown in the following examples.

EXAMPLE 3.9

Obtain the arithmetic mean for the data given in example 3.7 by short cut method.

SOLUTION

Computation of mean (short cut method)

Class Interval	f	x	D=x-20	fD
2-6	1	4	-16	-16
6-10	9	8	-12	-108
10-14	21	12	-8	-168
14-18	47	16	-4	-188
18-22	52	20	0	0
22-26	36	24	4	144
26-30	19	28	8	152
30-34	3	32	12	36
Total	188			-148

Applying the short cut method

Where P.M. = 20, $\sum fD = -148$, $\sum f = 188$

$$\bar{X} = P.M. + \frac{\sum fD}{n} = 20 + \frac{-148}{188} = 20 - 0.79 = 19.21$$

EXAMPLE 3.10

The following table gives the marks obtained by a class of students on a certain examination. Calculate the average marks per students by (i) Direct method (ii) Short cut method.

Marks	:	10-20	20-30	30-40	40-50	50-60
No of Students	:	1	2	3	5	7
Marks	:	60-70	70-80	80-90	90-100	
No of Students	:	12	16	10	4	

SOLUTION

Marks	No of Students	x	fx	D	fD
10-20	1	15	15	-40	-40
20-30	2	25	50	-30	-60
30-40	3	35	105	-20	-60
40-50	5	45	225	-10	-50
50-60	7	55	385	0	0
60-70	12	65	780	10	120
70-80	16	75	1200	20	320
80-90	10	85	850	30	300
90-100	4	95	380	40	160
Total:	60		3990		690

(i) By direct method

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{3990}{60} = 66.5$$

(ii) By short cut method taking P.M. = 55

$$\bar{X} = P.M. + \frac{\sum fD}{\sum f} = 55 + \frac{690}{60} = 55 + 11.5 = 66.5$$

EXAMPLE 3.11

The following table gives the length of life of 400 radio tubes.

Length of Life	Number of Radio Tubes
300 – 399	12
400 – 499	32
500 – 599	64
600 – 699	76
700 – 799	88
800 – 899	60
900 – 999	32
1000 – 1099	26
1100 – 1199	10
-----	400

Calculate A.M. by short cut method.

SOLUTION

Length of life	F	X	D	fD
300 – 399	12	349.5	-400	-4800
400 – 499	32	449.5	-300	-9600
500 – 599	64	549.5	-200	-12800
600 – 699	76	649.5	-100	-7600
700 – 799	88	749.5	0	0
800 – 899	60	849.5	100	6000
900 – 999	32	949.5	200	6400
1000 – 1099	26	1049.5	300	7800
1100 – 1199	10	1149.5	400	4000
-----	400	---	---	-10600

$$P.M. = 749.5, \sum fD = -10600, \sum f = 400$$

$$\bar{X} = P.M. + \frac{\sum fD}{\sum f} = 749.5 + \frac{-10600}{400} = 749.5 - 26.5 = 723$$

(C) Step Deviation or Coding Method

If class intervals all have equal size 'h', the deviation $D = x - P.M.$ can be expressed $D/h = U$ i.e.

$$\frac{X - P.M.}{h} = U$$

Where U can be positive, negative or zero. The arithmetic mean can be simplified by the formula given below.

$$\bar{X} = P.M. + \frac{\sum fU}{\sum f} \times h$$

Which is equivalent to the formula.

$$\bar{X} = P.M. + \frac{\sum fD}{\sum f}$$

Thus, U is also the number of classes deviated from the provisional mean class, such as -1, -2, -3, (that is, minus one, two, three,, classes smaller than the provisional mean class) and +1, +2, +3, (that is one, two three classes, larger than the provisional mean, respectively). This is called the step deviation method or coding method for computing the mean. It is very short method and should always be used for grouped data where class interval sizes are equal.

EXAMPLE 3.12

Find the arithmetic mean for the data given in example 3.10 by using step deviation method.

SOLUTION

Marks	f	x	D=x-55	U	fU
10-20	1	15	-40	-4	-4
20-30	2	25	-30	-3	-6
30-40	3	35	-20	-2	-6
40-50	5	45	-10	-1	-5
50-60	7	55	0	0	0
60-70	12	65	10	1	12
70-80	16	75	20	2	32
80-90	10	85	30	3	30
90-100	4	95	40	4	16
Total :	60	---	---	---	69

$$P.M. = 55, \Sigma fU = 69, \Sigma f = 60, h = 10$$

$$\bar{X} = P.M. + \frac{\Sigma fU}{\Sigma f} \times h = 55 + \frac{69}{60} \times 10 = 55 + \frac{690}{60} = 55 + 11.5 = 66.5$$

EXAMPLE 3.13

Find arithmetic mean of the following data.

Weights (lbs):	100-110	110-120	120-130	130-140
Frequency :	45	57	66	75
Weights (lbs):	140-150	150-160	160-170	170-180
Frequency :	68	59	49	44

SOLUTION

Weights	f	X	D=X-135	U	fU
100-110	45	105	-30	-3	-135
110-120	57	115	-20	-2	-114
120-130	66	125	-10	-1	-66
130-140	75	135	0	0	0
140-150	68	145	10	1	68
150-160	59	155	20	2	118
160-170	49	165	30	3	147
170-180	44	175	40	4	176
Total :	463	---	---	---	194

$$\bar{X} = P.M. + \frac{\sum fU}{\sum f} \times h = 135 + \frac{194}{463} \times 10 = 135 + 4.19 = 139.19 \text{ pounds.}$$

EXAMPLE 3.14

Find the arithmetic mean for the data given in example 3.11 by using step deviation method.

SOLUTION

Length of Life	F	X	D	U	fU
300-399	12	349.5	-400	-4	-48
400-499	32	449.5	-300	-3	-96
500-599	64	549.5	-200	-2	-128
600-699	76	649.5	-100	-1	-76
700-799	88	749.5	0	0	0
800-899	60	849.5	100	1	60
900-999	32	949.5	200	2	64
1000-1099	26	1049.5	300	3	78
1100-1199	10	1149.5	400	4	40
Total :	400	---	---	---	-106

$$P.M. = 749.5, \sum fU = -106, \sum f = 400, h = 100$$

$$\bar{X} = P.M. + \frac{\sum fU}{\sum f} \times h = 749.5 + \frac{-106}{400} \times 100 = 749.5 - 26.5 = 723$$

3.7 PROPERTIES OF ARITHMETIC MEAN

Following are the properties of arithmetic mean.

- (i) The sum of the deviations of a set of values from their arithmetic mean is always equal to zero.

$$(a) \sum (X - \bar{X}) = 0$$

$$\text{i.e. } \sum X - n\bar{X} = 0 \text{ as } \bar{X} = \frac{\sum X}{n} \text{ or } n\bar{X} = \sum X$$

$$(b) \sum f(X - \bar{X}) = 0$$

$$\text{i.e. } \sum fX - \bar{X} \sum f = 0 \text{ as } \bar{X} = \frac{\sum fX}{\sum f} \text{ or } \bar{X} \sum f = \sum fX$$

- (ii) The sum of the squares of deviations of a set of values from their mean is minimum i.e.

$$\sum (X - \bar{X})^2 \text{ or } \sum f(X - \bar{X})^2 \text{ is always minimum.}$$

If $\sum (X - \bar{X})^2 = \sum d^2$ is sum of squared deviations from mean and $\sum (X - P.M.)^2 = \sum D^2$ is any other sum of squared deviations from other than mean, then

$$\sum d^2 < \sum D^2$$

- (iii) If a constant value is added or subtracted to each value of a set of data, the arithmetic mean will also be added or subtracted by the same constant.

- (iv) If $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_k$ be the means of k distributions with respective frequencies $n_1, n_2, \dots, n_k$, then $\bar{X}$ of the distributions is given by

$$\bar{X} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2 + \dots + n_k \bar{X}_k}{n_1 + n_2 + \dots + n_k} = \frac{\sum n_i \bar{X}_i}{\sum n_i}$$

EXAMPLE 3.15

The mean height of 25 students in section A is 66", the mean height of 35 students in section B is 61" and the mean height of 40 students in section C is 59". Find the over all mean height of 100 students.

SOLUTION

Here $n = n_1 + n_2 + n_3 = 25 + 35 + 40 = 100$

$\bar{X}_1 = 60", \bar{X}_2 = 61", \bar{X}_3 = 59"$

Over all mean height of all students

$$\begin{aligned}\bar{X} &= \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2 + n_3 \bar{X}_3}{n_1 + n_2 + n_3} \\ &= \frac{25(60) + 35(61) + 40(59)}{25 + 35 + 40} \\ &= \frac{1500 + 2135 + 2360}{100} = \frac{5995}{100} = 59.95"\end{aligned}$$

3.8 WEIGHTED ARITHMETIC MEAN

In simple arithmetic mean equal importance or weight is given to all items. When each of the values in a set of data is assigned a weight according to the relative importance in the group, the computed mean is called weighted mean. The weighted mean is obtained by as follows:

First, multiply each value by the weight assigned to the value, second, add these products; and third divide the sum of the products by the sum of the weights.

$$\bar{X}_w = \frac{W_1 X_1 + W_2 X_2 + \dots + W_n X_n}{W_1 + W_2 + \dots + W_n} = \frac{\sum WX}{\sum W}$$

EXAMPLE 3.16

Find the weighted mean from the following data.

Group	Index No.	Weight
Food	125	7
Clothing	133	5
Fuel and Light	141	4
House Rent	173	1
Miscellaneous	182	3

SOLUTION

Group	X	W	WX
Food	125	7	875
Clothing	133	5	665
Fuel and Light	141	4	564
House Rent	173	1	173
Miscellaneous	182	3	546
	---	20	2823

$$\bar{X}_w = \frac{\sum WX}{\sum W} = \frac{2823}{20} = 141.15$$

3.9 ADVANTAGES AND DISADVANTAGES OF THE ARITHMETIC MEAN

Advantages

- It is easy to understand and calculate, and is commonly used.
- It makes use of all the data in the group and can be determined, therefore, with mathematical exactness.
- It provides a good basis for comparison.
- It can be used for further analysis and treatment.
- It is relatively stable statistic.

Disadvantages

- It may give undue weight to extreme abnormal values.
- It is difficult to locate mean by mere inspection.
- It gives satisfactory results when computed from a homogeneous data.
- It can not be computed accurately even when one item is missing.
- It may not lie in the middle of the series, if the series is skewed.

3.10 MEDIAN

Median is the value of the middle item of a series when it is arranged in ascending or descending order of magnitude. Median divides the series in two equal halves in such a way that in one half, the values are less than median and in the other half the values more than median.

The median of a set of values is the value of the middle item when the values are arranged in order of magnitude. To find the median, first arrange the values in an array, either from the smallest to the largest or from the largest to the smallest, second, locate the middle value; that is, the number of values above the median is the same as the number of values below the median. The methods of locating the median for ungrouped data and for grouped data are discussed below:

3.10.1 Ungrouped Data

Suppose the variable x has n values and they are arranged according to magnitude, so that

$$X_1 \leq X_2 \leq X_3 \leq \dots \leq X_n$$

(A) Odd Number

If the number of values i.e. n is an odd number, the median is calculated by the formula given below.

$$\text{Med} = \text{The Value of } \left(\frac{n+1}{2} \right) \text{th observation.}$$

For example, if 15 items are arranged in order the 8th item will be the median.

EXAMPLE 3.17

Find the median of the following items.

5, 7, 9, 12, 10, 8, 7, 15, 21

SOLUTION

Serial No	Size of Items
1	5
2	7
3	7
4	8
5	9
6	10
7	12
8	15
9	21

$$\text{Median} = \text{The Value of } \left(\frac{n+1}{2} \right) \text{th observation.}$$

$$= \text{The value of } \left(\frac{9+1}{2} \right) \text{th item}$$

$$= \text{The value of 5th item} = 9$$

(B) Even Number

If the number of values i.e. n is even, then median is obtained by finding the arithmetic mean of the two middle values i.e.

$$\text{Med} = \frac{1}{2} \left[\text{The Value of } \frac{n}{2} \text{th item} + \text{The value of } \frac{n+2}{2} \text{th item} \right]$$

EXAMPLE 3.18

The following are the marks obtained in a paper of statistics in an examination by 12 students. Find the median.

36, 56, 41, 46, 54, 57, 55, 51, 52, 44, 37, 59

SOLUTION

Marks are arranged in ascending order of magnitude

36, 37, 41, 44, 46, 51, 52, 54, 55, 56, 57, 59

Here $n = 12$ (Even Number)

$$\begin{aligned}\text{Med} &= \frac{1}{2} \left[\text{The Value of } \frac{n}{2} \text{th item} + \text{The value of } \frac{n+2}{2} \text{th item} \right] \\ &= \frac{1}{2} \left[\text{The Value of } \frac{12}{2} \text{th item} + \text{The value of } \frac{12+2}{2} \text{th item} \right] \\ &= \frac{1}{2} \text{ The Value of 6th item} + \text{The value of 7th item} \\ &= \frac{1}{2} 51 + 52 = 51.2\end{aligned}$$

3.10.2 Grouped Data

(A) Continuous Series

When frequency distribution is available in a continuous series, the median is the value of $n/2$ th item from either end. To find the median from frequency distribution, we form a cumulative frequency. The cumulative frequency of each class is the sum of the frequency of the class and the frequencies of the previous classes. For example, the cumulative frequency of the third class in the table is the sum of the frequencies in the first three classes. Find the median class. The median class is the class that contains the median. The median lies in the class which corresponds to the cumulative frequency in which $n/2$ lies. Interpolate the median from the values included in the median class. Median is obtained by formula.

$$\text{Median} = L + \frac{h}{f} \left(\frac{n}{2} - C \right)$$

Where L = Lower class boundary of median class.

n = No of items in the data (i.e. Total frequency)

f = Frequency of median class

h = Size of class interval of median class

C = Cumulative frequency of the class preceding the median class or sum of frequencies of all classes lower than the median.

EXAMPLE 3.19

Find the median of the data given below.

Class Interval	100-200	200-300	300-400	400-500	500-600
Frequency	15	18	30	20	17

SOLUTION

Class Interval	Frequency	Cummulative Frequency
100-200	15	15
200-300	18	33
300-400	30	63
400-500	20	83
500-600	17	100 = n

$$\frac{n}{2} = \frac{100}{2} = 50, L = 300, c = 33, h = 100$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{n}{2} - C \right)$$

$$= 300 + \frac{100}{30} \times (50 - 33) = 300 + \frac{100 \times 17}{30} = 356.67$$

EXAMPLE 3.20

From the figures given below find the median.

Age	20-25	25-30	30-35	35-40
No of Students	50	70	100	180
Age	40-45	45-50	50-55	55-60
No of Students	150	120	70	59

SOLUTION

Age	No of Students	Cummulative Frequency (c.f)
20-25	50	50
25-30	70	120
30-35	100	220
35-40	180	400
40-45	150	550
45-50	120	670
50-55	70	740
55-60	60	800

$$\frac{n}{2} = 400, L = 35, c = 220, h = 5, n = 800, f = 180$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{n}{2} - C \right)$$

$$= 35 + \frac{5}{180} \times (400 - 220) = 35 + \frac{5 \times 180}{180} = 35 + 5 = 40$$

EXAMPLE 3.21

The weights of 40 male students at a university are given in the following frequency table. Calculate median.

Weights in (lbs) :	118-126	127-135	136-144	145-153
Frequency :	3	5	9	12
Weights in (lbs) :	154-162	163-171	172-180	
Frequency :	5	4	2	

SOLUTION

To find the median weight, by the process of linear interpolation, the data are assumed to be continuous. There is the need of class-boundaries. To find the class boundaries, take the gap between the upper limit of the first group and the lower limit of the second group and divide it by two. Deduct

this quotient from the lower limit and add in the upper limit. By this process we get the class boundaries for all groups.

Weights in (lbs)	Class Boundaries	Frequency	Cummulative Frequency
118-126	117.5-126.5	3	3
127-135	126.5-135.5	5	8
136-144	135.5-144.5	9	17
145-153	144.5-153.5	12	29
154-162	153.5-162.5	5	34
163-171	162.5-171.5	4	38
172-180	171.5-180.5	2	40

$$n = 40, \frac{n}{2} = 20, L = 144.5, c = 17, h = 9, f = 12$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{n}{2} - C \right)$$

$$= 144.5 + \frac{9}{12} \times (20 - 17) = 144.5 + \frac{9 \times 3}{12} = 144.5 + 2.25 = 146.75$$

EXAMPLE 3.22

Find the median of the following frequency distribution.

Weekly Income :	35-39	40-49	50-54	55-59
No of Workers :	15	13	17	29
Weekly Income :	60-64	65-69		
No of Workers :	10	5		

SOLUTION

Weekly Income	Class Boundaries	F	C.F.
35-39	34.5-39.5	15	15
40-44	39.5-44.5	13	28
45-49	44.5-49.5	17	45
50-54	49.5-54.5	29	74
55-59	54.5-59.5	11	85
60-64	59.5-64.5	10	95
65-69	64.5-69.5	5	100

$$n = 100, \frac{n}{2} = 50, L = 49.5, c = 45, h = 5, f = 29$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{n}{2} - C \right)$$

$$= 49.5 + \frac{5}{29} \times (50 - 45) = 49.5 + 0.87 = 50.37$$

EXAMPLE 3.23

Following table shows the distribution of a maximum load in short tons supported by certain cables produced by some company.

Max. Load	:	9.3-9.7	9.8-10.2	10.3-10.7	10.8-11.2
No of Cables	:	2	5	12	18
Max. Load	:	11.3-11.7	11.8-12.2	12.3-12.7	12.8-13.2
No of Cables	:	14	6	3	4

Determine median.

SOLUTION

Max. Load	Class boundaries	F	C.F.
9.3-9.7	9.25-9.75	2	2
9.8-10.2	9.75-10.25	5	7
10.3-10.7	10.25-10.75	12	19
10.8-11.2	10.75-11.25	18	37
11.3-11.7	11.25-11.75	14	51
11.8-12.2	11.75-12.25	6	57
12.3-12.7	12.25-12.75	4	61
12.8-13.2	12.75-13.25	1	62

$$n = 62, \frac{n}{2} = 31, L = 10.75, c = 19, h = 0.5, f = 18$$

$$\begin{aligned} \text{Median} &= L + \frac{h}{f} \left(\frac{n}{2} - C \right) \\ &= 10.75 + \frac{0.5}{18} \times (31 - 19) = 10.75 + \frac{0.5 \times 12}{18} \\ &= 10.75 + 0.333 = 11.083 \end{aligned}$$

EXAMPLE 3.24

Find the median of the following data.

Weight	:	4.20-4.24	4.25-4.29	4.30-4.34	4.35-4.39
Frequency	:	10	14	28	30
Weight	:	4.40-4.44	4.45-4.49	4.50-4.54	
Frequency	:	22	12	4	

SOLUTION

Weight	Class Boundaries	F	C.F.
4.20-4.24	4.195-4.245	10	10
4.25-4.29	4.245-4.295	14	24
4.30-4.34	4.295-4.345	28	52
4.35-4.39	4.345-4.395	30	82
4.40-4.44	4.395-4.445	22	104
4.45-4.49	4.445-4.495	12	116
4.50-4.54	4.495-4.545	4	120

$$n = 120, \frac{n}{2} = 60, L = 4.345, c = 52, h = 0.05, f = 30$$

$$\begin{aligned}\text{Median} &= L + \frac{h}{f} \left(\frac{n}{2} - C \right) = 4.345 + \frac{0.05}{30} \times (60 - 52) \\ &= 4.345 + \frac{0.05 \times 8}{30} = 4.345 + 0.013 = 4.358\end{aligned}$$

(B) Discrete Series

In discrete series the size of $\left(\frac{n+1}{2}\right)$ th item is taken as median.

The items are first arrayed in ascending or descending order of magnitude and their respective frequencies are written again then. Then the cumulative frequencies are calculated and the value of middle item is located.

EXAMPLE 3.25

Find the median for the following distribution.

No of Children :	0	1	2	3	4	5	6	7	8	9
No of Families :	2	5	8	12	14	15	13	9	7	3

SOLUTION

No of Children (x)	No of Family (f)	Cummulative Frequency (c-f)
0	2	2
1	5	7
2	8	15
3	12	27
4	14	41
5	15	56
6	13	69
7	9	78
8	7	85
9	3	88

Median = The value of $\left(\frac{n+1}{2}\right)$ th item.

$$= \text{The value of } \left(\frac{88+1}{2}\right)\text{th item} = 44.5\text{th item} = 5$$

3.10.3 Quartiles, Deciles and Percentiles

The method by which the median is determined can be further extended to divide a series into more than two parts.

Quartiles:

The quartiles of a distribution are the values which divide the distribution into four equal parts so that one fourth of the data fall below the

first quartile, Q_1 , half of the data fall below the second quartile, Q_2 , and three fourth of the data fall below the third quartile, Q_3 . Q_1 is called the lower quartile and Q_3 is called upper quartile, by definition Q_2 is equal to median.

Deciles:

The values which divide an arrayed set of data into ten equal parts are called deciles and are denoted by $D_1, D_2, \dots, D_9$ respectively.

Percentiles:

Percentiles are the values in an arranged distribution which divide the distribution into hundred equal parts. Percentiles are denoted by $P_1, P_2, \dots, P_{25}, \dots, P_{50}, \dots, P_{75}, \dots, P_{99}$.

(A) Quartiles, Deciles, Percentiles, From Ungrouped data and discrete Series:

The values of quartiles, deciles and percentiles can be determined, following the same principle as that of determining the median. Thus in the case of individual observation (ungrouped data) and discrete series, the following formulae should be used:

Quartiles: The quartiles are calculated by the following.

$$Q_1 = \text{The value of } \left(\frac{n+1}{4} \right) \text{th item.}$$

$$Q_2 = \text{The value of } 2 \left(\frac{n+1}{4} \right) \text{th item} = \text{Median}$$

$$Q_3 = \text{The value of } 3 \left(\frac{n+1}{4} \right) \text{th item}$$

Deciles: The deciles are calculated by the following.

$$D_1 = \text{The value of } \left(\frac{n+1}{10} \right) \text{th item.}$$

$$D_2 = \text{The value of } 2 \left(\frac{n+1}{10} \right) \text{th item} = \text{Median}$$

$$D_3 = \text{The value of } 3 \left(\frac{n+1}{10} \right) \text{th item} \dots$$

$$D_5 = \text{The value of } 5 \left(\frac{n+1}{10} \right) \text{th item} \dots = \text{Median} \dots$$

$$D_9 = \text{The value of } 9 \left(\frac{n+1}{10} \right) \text{th item}$$

Percentiles: The following formulae are used to calculate the percentiles.

$$P_1 = \text{The value of } \left(\frac{n+1}{100} \right) \text{th item}$$

$$P_2 = \text{The value of } 2 \left(\frac{n+1}{100} \right) \text{th item} \dots$$

$$D_9 = \text{The value of } 9 \left(\frac{n+1}{10} \right) \text{th item}$$

$$= \text{The value of } 9 \left(\frac{21+1}{10} \right) \text{th item}$$

$$= \text{The value of 19.8th item}$$

$$= \text{The value of 19th item} + 0.8[20\text{th item} - 19\text{th item}]$$

$$= 90 + 0.8[99 - 90] = 90 + 0.8(9) = 90 + 7.2 = 97.2$$

$$P_{55} = \text{The value of } 55 \left(\frac{n+1}{100} \right) \text{th item}$$

$$= \text{The value of } 55 \left(\frac{21+1}{100} \right) \text{th item}$$

$$= \text{The value of 12.1th item}$$

$$= \text{The value of 12th item} + 0.1[13\text{th item} - 12\text{th item}]$$

$$= 65 + 0.1[66 - 65] = 65 + 0.1(1) = 65 + 0.1 = 65.1$$

(B) Quartiles, Deciles, Percentiles from Continuous Series.

The values of quartiles, deciles and percentiles for continuous series are determined by following formulae.

Quartiles:

$$Q_1 = L + \frac{h}{f} \left(\frac{n}{2} - C \right)$$

$$Q_2 = L + \frac{h}{f} \left(\frac{2n}{4} - C \right) = L + \frac{h}{f} \left(\frac{n}{2} - C \right) = \text{Med.}$$

$$Q_3 = L + \frac{h}{f} \left(\frac{3n}{4} - C \right)$$

Deciles:

$$D_1 = L + \frac{h}{f} \left(\frac{n}{10} - C \right)$$

$$D_2 = L + \frac{h}{f} \left(\frac{2n}{10} - C \right) \dots\dots\dots$$

$$D_5 = L + \frac{h}{f} \left(\frac{5n}{10} - C \right) = L + \frac{h}{f} \left(\frac{n}{2} - C \right) = \text{Med.} \dots\dots$$

$$D_9 = L + \frac{h}{f} \left(\frac{9n}{10} - C \right)$$

Percentiles:

$$P_1 = L + \frac{h}{f} \left(\frac{n}{100} - C \right)$$

$$P_2 = L + \frac{h}{f} \left(\frac{2n}{100} - C \right) \dots\dots\dots$$

$$P_{25} = L + \frac{h}{f} \left(\frac{25n}{100} - C \right) = L + \frac{h}{f} \left(\frac{n}{4} - C \right) = Q_1 \dots\dots\dots$$

$$P_{50} = L + \frac{h}{f} \left(\frac{50n}{100} - C \right) = L + \frac{h}{f} \left(\frac{n}{2} - C \right) = \text{Med} \dots\dots\dots$$

$$P_{75} = L + \frac{h}{f} \left(\frac{75n}{100} - C \right) = L + \frac{h}{f} \left(\frac{3n}{4} - C \right) = Q_3 \dots\dots\dots$$

$$P_{99} = L + \frac{h}{f} \left(\frac{99n}{100} - C \right)$$

EXAMPLE 3.27

Required Q_1 , Q_3 , D_7 , P_{80} from the following data.

Marks Obtained	0-5	5-10	10-15	15-20	20-25	25-30
No of Students	10	15	20	25	18	14

SOLUTION

Marks Obtained	No of Students	Cummulative Frequency
0-5	10	10
5-10	15	25
10-15	20	45
15-20	25	70
20-25	18	88
25-30	14	102 = n

$$\frac{n}{4} = \frac{102}{4} = 25.5$$

$$Q_1 = L + \frac{h}{f} \left(\frac{n}{4} - C \right) = 10 + \frac{5}{20} (25.5 - 25) = 10 + \frac{5 \times 0.5}{20} = 10 + 0.125 = 10.125$$

$$\frac{3n}{4} = \frac{3 \times 102}{4} = 76.5$$

$$Q_3 = L + \frac{h}{f} \left(\frac{3n}{4} - C \right) = 20 + \frac{5}{18} (76.5 - 70) = 20 + \frac{5 \times 6.5}{18} = 20 + 1.75 = 21.75$$

$$\frac{7n}{10} = \frac{7 \times 102}{10} = 71.4$$

$$D_7 = L + \frac{h}{f} \left(\frac{7n}{10} - C \right) = 20 + \frac{5}{18} (71.4 - 70) = 20 + \frac{5 \times 1.4}{18} = 20 + 0.39 = 20.39$$

$$\frac{80n}{100} = \frac{80 \times 102}{100} = 81.6$$

$$D_{80} = L + \frac{h}{f} \left(\frac{80n}{100} - C \right) = 20 + \frac{5}{18} (81.6 - 70) = 20 + \frac{5 \times 11.6}{18} = 20 + 3.22 = 23.22$$

$$P_{25} = \text{The value of } 25 \left(\frac{n+1}{100} \right) \text{th item} \dots = Q_1 \dots$$

$$P_{50} = \text{The value of } 50 \left(\frac{n+1}{100} \right) \text{th item} \dots = \text{Median} \dots$$

$$P_{75} = \text{The value of } 75 \left(\frac{n+1}{100} \right) \text{th item} = Q_3 \dots$$

$$P_{99} = \text{The value of } 99 \left(\frac{n+1}{100} \right) \text{th item}$$

From above discussion we find the following relation.

$$\text{Median} = Q_2 = D_5 = P_{50}$$

$$Q_1 = P_{25}, Q_3 = P_{75}$$

EXAMPLE 3.26

Find med, Q_1 , Q_3 , D_9 , and P_{55} for the following data. 71, 81, 90, 100, 99, 78, 76, 66, 65, 52, 42, 47, 37, 33, 90, 7, 9, 16, 13, 21, 51

SOLUTION

Arrayed data.

7, 9, 13, 16, 21, 33, 37, 42, 47, 51, 52, 65, 66, 71, 76, 78, 81, 90, 90, 99, 100

Here $n = 21$

$$\text{Median} = \text{The value of } \left(\frac{n+1}{2} \right) \text{th item}$$

$$= \text{The value of } \left(\frac{21+1}{2} \right) \text{th item}$$

$$\text{Median} = \text{The value of 11th item} = 52$$

$$Q_1 = \text{The value of } \left(\frac{n+1}{4} \right) \text{th item}$$

$$= \text{The value of } \left(\frac{21+1}{4} \right) \text{th item}$$

$$= \text{The value of 5.5th item}$$

$$= \text{The value of 5th item} + 0.5[6\text{th item} - 5\text{th item}]$$

$$= 21 + 0.5[33 - 21] = 21 + 0.5(12) = 21 + 6.0 = 27$$

$$Q_3 = \text{The value of } 3 \left(\frac{n+1}{4} \right) \text{th item}$$

$$= \text{The value of } 3 \left(\frac{21+1}{4} \right) \text{th item}$$

$$= \text{The value of 16.5th item}$$

$$= \text{The value of 16th item} + 0.5[17\text{th item} - 16\text{th item}]$$

$$= 78 + 0.5[81 - 78] = 78 + 0.5(3) = 78 + 1.5 = 79.5$$

EXAMPLE 3.28

The height of 100 college students measured the nearest inch are given below.

Height in inches	:	63	64	65	66	67
Number of Students	:	4	6	10	20	30
Height in inches	:	68	69	70	71	
Number of Students	:	13	12	3	2	

Calculate the median, Q_1 , Q_3 , D_7 and P_{65}

SOLUTION

The values are given as mid points. The data relate to heights, we therefore, treat the data as continuous. We find the class boundaries as shown below.

Height in Inches	No of Students	Class boundaries	Cumulative Frequency
63	4	62.5-63.5	4
64	6	63.5-64.5	10
65	10	64.5-65.5	20
66	20	65.5-66.5	40
67	30	66.5-67.5	70
68	13	67.5-68.5	83
69	12	68.5-69.5	92
70	3	69.5-70.5	98
71	2	70.5-71.5	100 = n

$$n = 100$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{\frac{n}{2}}{2} - C \right) = 66.5 + \frac{1}{30} (50 - 40) = 66.5 + 0.33 = 66.83$$

$$\frac{n}{4} = 25$$

$$Q_1 = L + \frac{h}{f} \left(\frac{\frac{n}{4}}{4} - C \right) = 65.5 + \frac{1}{20} (25 - 20) = 65.5 + 0.25 = 65.75$$

$$\frac{3n}{4} = 75$$

$$Q_3 = L + \frac{h}{f} \left(\frac{\frac{3n}{4}}{4} - C \right) = 67.5 + \frac{1}{30} (75 - 70) = 67.5 + 0.17 = 67.67$$

$$\frac{7n}{10} = \frac{7 \times 100}{10} = 70$$

$$D_7 = L + \frac{h}{f} \left(\frac{\frac{7n}{10}}{10} - C \right) = 66.5 + \frac{1}{30} (70 - 40) = 66.5 + 1 = 67.5$$

$$\frac{65n}{100} = \frac{65 \times 100}{100} = 65$$

$$D_{65} = L + \frac{h}{f} \left(\frac{65n}{100} - C \right) = 66.5 + \frac{1}{30} (65 - 40) = 66.5 + \frac{25}{30} = 66.5 + 0.83 = 67.33$$

EXAMPLE 3.29

Find med, Q_1 , and Q_3 for the following data.

Weight (lbs):	3.0	3.2	3.4	3.6	3.8	4.0
Frequency :	4	15	26	25	40	55
Weight (lbs):	4.2	4.4	4.6	4.8	5.0	5.2
Frequency :	69	35	21	10	8	2

SOLUTION

The data relate to weights. It is a continuous variable. The class boundaries are given below.

Weights (lbs) (X)	Frequency (f)	Class Boundaries (c.b)	Cummulative Frequency (c.f)
3.0	4	2.9-3.1	4
3.2	15	3.1-3.3	19
3.4	26	3.3-3.5	45
3.6	25	3.5-3.7	70
3.8	40	3.7-3.9	110
4.0	55	3.9-4.1	165
4.2	69	4.1-4.3	234
4.4	35	4.3-4.5	269
4.6	21	4.5-4.7	290
4.8	10	4.7-4.9	300
5.0	8	4.9-5.1	308
5.2	2	5.1-5.3	310 = n

$$n = 310, \frac{n}{2} = 155, \frac{n}{4} = 77.5, \frac{3n}{4} = 232.5$$

$$\begin{aligned} \text{Median} &= L + \frac{h}{f} \left(\frac{n}{2} - C \right) = 3.9 + \frac{0.2}{55} (155 - 110) \\ &= 3.9 + \frac{0.2 \times 45}{55} = 3.9 + 0.164 = 40.064 \end{aligned}$$

$$\begin{aligned} Q_1 &= L + \frac{h}{f} \left(\frac{n}{4} - C \right) = 3.7 + \frac{0.2}{40} (77.5 - 70) = 3.7 + \frac{0.2 \times 7.5}{40} \\ &= 3.7 + 0.038 = 3.738 \end{aligned}$$

$$Q_3 = L + \frac{h}{f} \left(\frac{3n}{4} - C \right) = 4.1 + \frac{0.2}{69} (232.5 - 165) = 4.1 + \frac{0.2 \times 67.5}{69}$$

$$= 4.1 + 0.196 = 4.296$$

3.10.4 Graphic Method of Locating Median, Quartiles, Deciles and Percentiles

The median, quartiles, deciles and percentiles may be read from a cumulative frequency curve with a high degree of accuracy. For this purpose a cumulative frequency curve is drawn for the given frequency distribution taking the values of class boundaries on the x-axis and cumulative frequency on y-axis. Then the position of median, quartiles, deciles or percentiles is located. For median, a point corresponding to $n/2$ is marked along y-axis. From this point a straight line parallel to x-axis is drawn which meets the cumulative frequency curve at a certain point. From this point on the curve, a perpendicular line is drawn on the x-axis. The value on x-axis will be median. The procedure for finding Q_1 , Q_3 , D_4 , D_9 , P_{57} , P_{80} etc. is the same as in case of median except that a point corresponding to

$$\frac{n}{4}, \frac{3n}{4}, \frac{4n}{10}, \frac{9n}{10}, \frac{57n}{100}, \frac{80n}{100} \text{ is marked along y-axis}$$

EXAMPLE 3.30

Find median, two quartiles, 6th decile and 85th percentile from the following frequency distribution graphically.

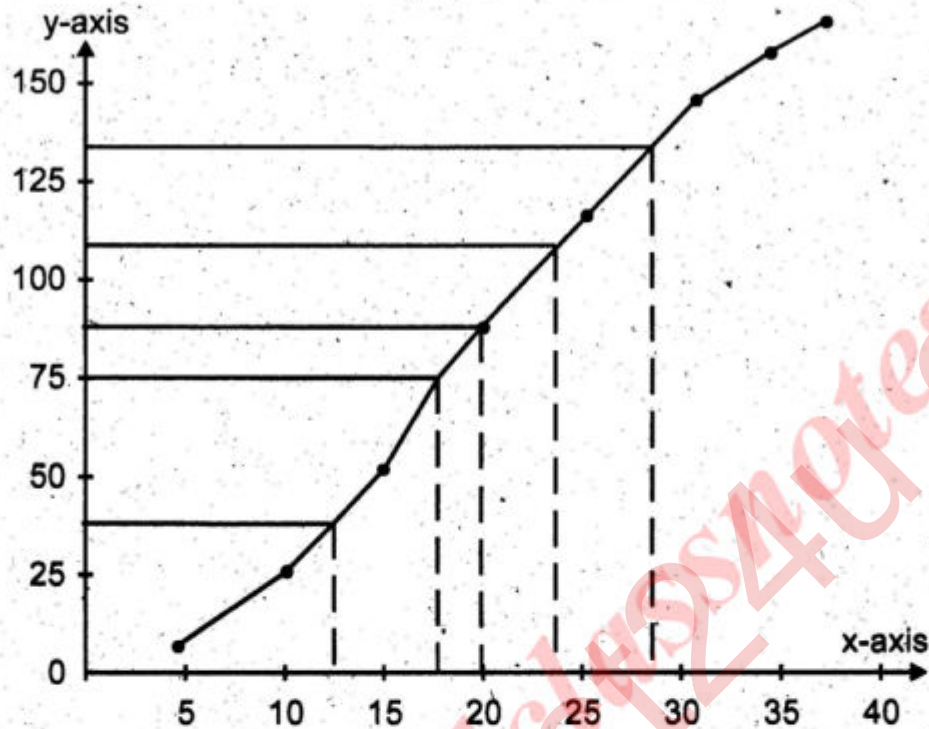
Marks :	0-5	5-10	10-15	15-20
F :	10	15	25	40
Marks :	20-25	25-30	30-35	35-40
F :	30	16	10	4

SOLUTION

Marks	F	C.F
0-5	10	10
5-10	15	25
10-15	25	50
15-20	40	90
20-25	30	120
25-30	16	135
30-35	10	146
35-40	4	150 = n

In plotting the cumulative frequency curve, we use the upper class boundaries of each interval and place the points above these. All are plotted and then a curve is drawn.

CUMMULATIVE FREQUENCY CURVE OR OGIVE



For Median $\frac{n}{2} = 75$ Median = 18.125

For Q_1 , $\frac{n}{4} = 37.5$ $Q_1 = 12.5$

For Q_3 , $\frac{3n}{4} = 112.5$ $Q_3 = 23.75$

For D_6 , $\frac{6n}{10} = 90$ $D_6 = 20$

For P_{85} , $\frac{85n}{100} = 127.5$ $P_{85} = 28$

3.11 ADVANTAGES AND DISADVANTAGES OF MEDIAN

Advantages

- (i) It is easily located in individual and discrete series.
- (ii) It is not affected by extreme values.
- (iii) It is easy to calculate median.
- (iv) It can be found even for distribution with open classes at either end.

Disadvantages

- (i) It is not as familiar average as the arithmetic mean.

- (ii) It does not give proper weight to the important items, it treats all (iii) items equally.
- (iv) It is not capable of further algebraic manipulation.
- (v) Median can not be calculated unless the values are arranged according to size.

3.12 THE MODE

Mode is the most repeated or most common or the most frequent value which occurs in a set of values. The mode is generally regarded as the most typical value in a series of data.

3.12.1 Ungrouped Data

The mode for ungrouped data can be obtained by inspection. A distribution may have one or more than one modes. A distribution having one mode is called a unimodal distribution. When there are two modes in a set of data, the data is called bimodal or multimodal. Following are the examples of ungrouped data.

EXAMPLE 3.31

Find the mode of the values. 1, 4, 10, 8, 5, 11, 20, 10 and 6 representing the miles traveled by 9 students.

SOLUTION

The mode of the 9 values is 10. The value 10 occurs twice, but each of the other values occurs only once, so

$$\text{Mode} = 10$$

EXAMPLE 3.32

Find the mode of the values 2, 3, 3, 8, 10, 10, 12, 18

SOLUTION

There are two modes, 3 and 10 since each of the two values occurs twice

$$\text{Mode} = 3, 10$$

EXAMPLE 3.33

Find the mode of the values 1, 2, 3, 4; 5, 6, 7, 8, 9, 10

SOLUTION

Since none of the values occurs more than once, there is no mode.

No Mode

3.12.2 Grouped Data

(A) Continuous Series

The mode for grouped data can be obtained by various methods. When the data are grouped into a frequency distribution, the mode would lie in the class, that carries the highest frequency. This class is called the modal class. The formula for computing the mode for a frequency distribution is

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

Where L = Lower class boundary of modal class.

f_1 = Frequency of the modal class or maximum frequency
 f_2 = Frequency of the class after the modal class
 h = Size of the class interval of the modal class

EXAMPLE 3.34

Calculate mode of the data given below:

Daily Income :	0-10	10-20	20-30	30-40	40-50	50-60
No of Persons:	10	13	15	20	16	8

SOLUTION

Daily Income	No of Persons
0-10	10
10-20	13
20-30	15
30-40	20
40-50	16
50-60	8

Mode is clearly located in the class 30-40

$$f_m = 20, f_1 = 15, f_2 = 16, h = 10, L = 30$$

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h = 30 + \frac{20 - 15}{(20 - 15) + (20 - 16)} \times 10$$

$$= 30 + \frac{5 \times 10}{5 + 4} = 30 + \frac{50}{9} = 30 + 5.55 = 35.55$$

EXAMPLE 3.35

The frequency distribution of the weights (in grams) of mangoes of a given variety is given below. Calculate the mode.

Weight in grams:	410-419	420-429	430-439	440-449
No of Mangoes:	14	20	42	54
Weight in grams:	450-459	460-469	470-479	
No of Mangoes:	45	18	7	

SOLUTION

Weight (in grams)	Class Boundaries	No of Mangoes
410-419	409.5-419.5	14
420-429	419.5-429.5	20
430-439	429.5-439.5	42
440-449	439.5-449.5	54
450-459	449.5-459.5	45
460-469	459.5-469.5	18
470-479	469.5-479.5	7

$$f_m = 54, f_1 = 42, f_2 = 45, h = 10, L = 439$$

$$\begin{aligned}\text{Mode} &= L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h = 439.5 + \frac{20 - 15}{(20 - 15) + (20 - 16)} \times 10 \\ &= 439.5 + \frac{12 \times 10}{12 + 9} = 439.5 + \frac{120}{21} = 439.5 + 5.761 = 445.26\end{aligned}$$

EXAMPLE 3.36

Estimate the mode for the data given is 3.23.

SOLUTION

Max. Load	Class boundaries	F
9.3-9.7	9.25-9.75	2
9.8-10.2	9.75-10.25	5
10.3-10.7	10.25-10.75	12
10.8-11.2	10.75-11.25	18
11.3-11.7	11.25-11.75	14
11.8-12.2	11.75-12.25	6
12.3-12.7	12.25-12.75	4
12.8-13.2	12.75-13.25	1

$$f_m = 18, f_1 = 12, f_2 = 14, h = 0.5, L = 10.75$$

$$\begin{aligned}\text{Mode} &= L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h \\ &= 10.75 + \frac{18 - 12}{(18 - 12) + (18 - 14)} \times 0.5 \\ &= 10.75 + \frac{6 \times 0.5}{6 + 4} = 10.75 + \frac{3}{10} = 10.75 + 0.3 = 11.05\end{aligned}$$

(B) Discrete Series

In a discrete frequency distribution, the mode is that value which has maximum frequency. It is found by inspection.

EXAMPLE 3.37

Find mode from the following distribution.

No of Children :	1	2	3	4	5	6
No of Couples :	10	15	45	18	15	10

SOLUTION

The data are discrete. The maximum frequency is 45, hence

$$\text{Mode} = 3$$

3.12.3 Graphical Representation of Mode

Mode, like median may be located through graph. The method of finding the mode graphically is given below.

- Construct a histogram of the grouped frequency distribution.

- (ii) Find the modal class i.e. the class with the tallest rectangle.
- (iii) Join the top left hand corner of the modal rectangle to the top left hand corner of the adjacent rectangle and similarly with the top right hand corner to the top right hand corner of the adjacent rectangle. These lines will intersect.
- (iv) The mode is the point where these two lines cross.

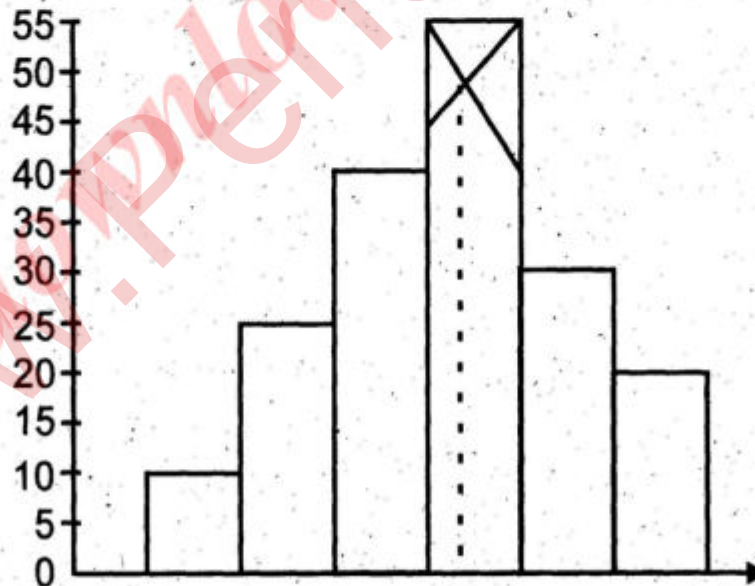
EXAMPLE 3.38

The earnings of 180 bank employees are as follows. Find the modal salary.

Earnings (Rs.)	Number of Employees
1800-2400	10
2400-3000	25
3000-3600	40
3600-4200	55
4200-4800	30
4800-5400	20

SOLUTION

Construct a histogram. Find the modal class, this is represented by the tallest rectangle. Join the top left hand corner of the modal rectangle to the top left hand corner of the adjacent rectangle and similarly with the top right hand corner of the adjacent rectangle. These lines will intersect. The mode occurs at the intersection.



It is evident from the graph that the modal salary is Rs. 3830.

EXAMPLE 3.39

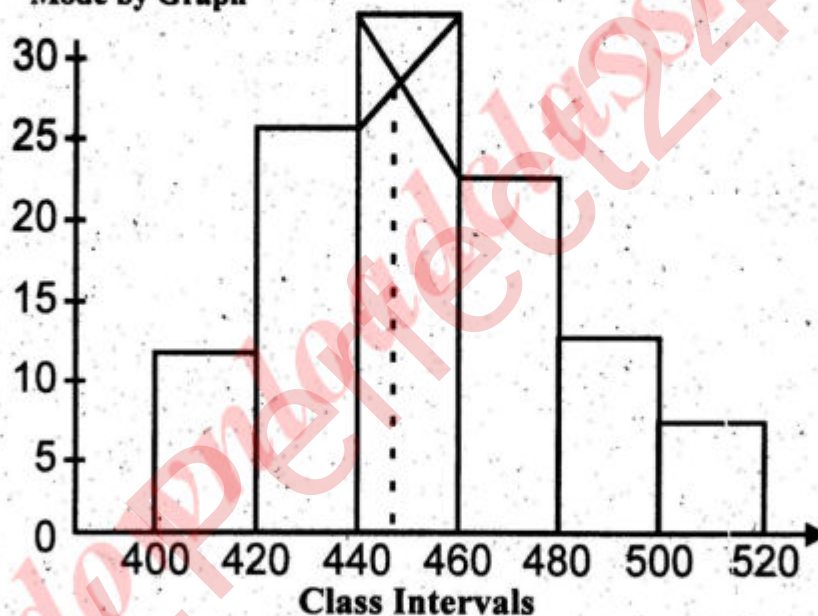
Find mode of the following data. (i) Numerically (ii) Graphically

Class Interval :	400-420	420-440	440-460
Frequency :	12	27	34
Class Interval :	460-480	480-500	500-520
Frequency :	24	15	8

SOLUTION(i) **Mode by Calculation or Numerically**

$$f_m = 34, f_1 = 27, f_2 = 24, h = 20, L = 440$$

$$\begin{aligned} \text{Mode} &= L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h = 440 + \frac{34 - 27}{(34 - 27) + (34 - 24)} \times 20 \\ &= 440 + \frac{7 \times 20}{7 + 10} = 440 + 8.82 = 448.82 \end{aligned}$$

(ii) **Mode by Graph****3.13 ADVANTAGES AND DISADVANTAGES OF MODE****Advantages**

- (i) It is easy to understand.
- (ii) Extreme values do not affect its value.
- (iii) It can be determined from open end distribution.
- (iv) Mode can be found at once by inspection from the ungrouped data.
- (v) It is very useful for meteorological forecasts.

Disadvantages

- i. It is not clearly defined.
- ii. Exact location is often uncertain.

iii. It is not based on all the values.

iv. There will be no mode if there is no common value in the data.

3.14 RELATION BETWEEN MEAN, MEDIAN AND MODE

For a symmetrical distribution the values of mean, median and mode are the same; that is

$$\text{Mean} = \text{Median} = \text{Mode}$$

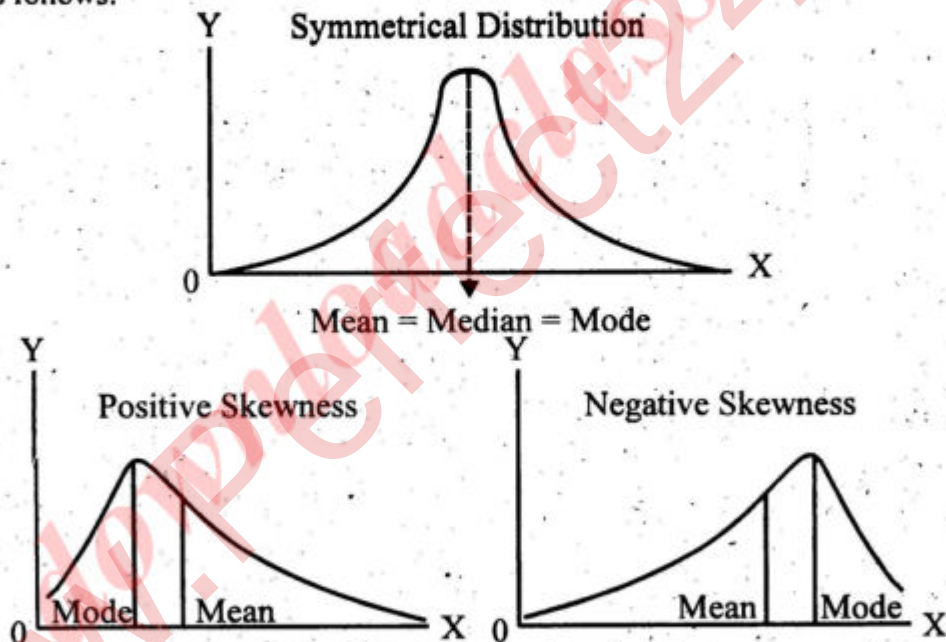
When a distribution is not symmetrical, these values will be different. When a distribution is a symmetrical, it is called skewed distribution. A distribution is said to have positive skewness if

$$\text{Mean} > \text{Median} > \text{Mode}$$

A distribution is said to have negative skewness if

$$\text{Mode} > \text{Median} > \text{Mean}$$

The shapes of the curves of symmetrical and skewed distribution are as follows:



When the distribution is moderately asymmetrical, the relationship is expressed by the formula as:

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

This is also called empirical relation between mean, median and mode.

EXAMPLE 3.40

By using empirical relation, find mode.

Groups :	3-6	6-9	9-12	12-15	15-18	18-21	21-24
F :	3	5	8	11	9	7	4

SOLUTION

Groups	F	X	D	U	FU	C.F.
3-6	3	4.5	-9	-3	-9	3
6-9	5	7.5	-6	-2	-10	8
9-12	8	10.5	-3	-1	-8	16
12-15	11	13.5	0	0	0	27
15-18	9	16.5	3	1	9	36
18-21	7	19.5	6	2	14	43
21-24	4	22.5	9	3	12	47 = n
Total:	47	---	---	---	8	---

$$A.M. = P.M. + \frac{\sum fu}{\sum f} \times h$$

$$= 13.5 + \frac{8}{47} \times 3 = 13.5 + \frac{24}{47} = 13.5 + 0.51 = 14.01$$

$$\text{Median} = L + \frac{h}{f} \left(\frac{n}{2} - C \right) = 12 + \frac{3}{11} (23.5 - 16)$$

$$= 12 + \frac{3}{11} \times 7.5 = 12 + \frac{22.5}{11} = 12 + 2.1 = 14.1$$

Hence Mode = 3 Median - 2 Mean

$$= 3(14.1) - 2(14.01) = 42.3 - 28.02 = 14.28$$

EXERCISE NO. 3

- 3.1:- What is an average? Name the most commonly known averages.
- 3.2:- What do you understand by "Central Tendency"? What are the advantages and disadvantages of arithmetic mean.
- 3.3:- Define (i) Arithmetic Mean, (ii) Weighted Mean (iii) Median (iv) Mode
- 3.4:- Fill in the blanks.
- The sum of the deviation of a set of n values from their mean is
 - The most common or most frequent value of the set of data is called
 - Median - mode = (mean - median)
 - Mode = 3 median
 - In a symmetrical distribution, the mean, median and mode are
 - Median of a set of values lies between mean and
 - A value which divides arranged distribution into two parts is called.....
 - Mode = for grouped data in continuous variable.
 - is not affect by extreme values.
 - $\bar{X} = P.M. + \frac{\sum f}{\sum f}$
- 3.5:- State which of the following statements are true and false.
- A single representative value of a set of data is the average value.
 - The algebraic sum of the deviations of a set of data from their mean is always equal to zero.
 - Cummulative frequency is non decreasing.
 - The most common value in a set of data is median.
 - The median can not be calculated from a frequency distribution with open end distribution.
 - The arithmetic mean is always located in the class which has the greatest number of frequencies.
 - Mean, median and mode are equal in a skewed distribution.
 - The sum of the squared deviations from the mean of that data is always maximum.
 - Median is a calculating average.
 - Mode depends upon all the values of a set of data.
 - Mean remains unchanged in value if a constant is taken away from all observations.
 - Median is always a central value in the data even if the data are not arranged.
 - Median is affected by extreme values.
 - When the distribution consists of a small number of values, the mode may not exist.

- xv. In a symmetrical distribution, mean, median and mode are same.
- xvi. Measure of location of a frequency distribution is called an average.
- xvii. Median is not equal to second quartile.
- xviii. Mean is more affected by changes in the extreme values than mode.
- xix. Sum of the deviation from mean is not zero.
- xx. The third quartile is equal to seventy five percentile.

3.6:- Calculate the arithmetic mean for the following
25, 20, 35, 28, 20, 18, 28, 19, 16, 8

3.7:- Find the mean of the sets.
i. 84, 92, 73, 67, 88, 74, 91, 16, 74
ii. 3.41, 2.86, -1.84, 2.31, -3.80, 2.14, 0.87
iii. 15, 13, 16, 15, 14, 15, 12, 18, 16, 15, 14, 18, 13, 14, 15
iv. 53, 46, 50, 49, 52, 53, 44, 55

3.8:- Find the mean of the following weights of football players.
216, 209, 215, 189, 204, 215, 165, 193, 181, 172, 203, 202, 234, 195,
207, 258, 234, 212, 196, 184, 194, 209, 207, 217, 222, 192, 192, 180,
269, 206, 178, 168, 176, 168, 182, 204, 188, 198, 206, 191, 206, 178,
192, 174

3.9:- The following table gives the marks obtained by a batch of 5 candidates in an examination in History, Statistics and Economics.

Roll No.	History	Statistics	Economics
1	41	46	50
2	35	50	52
3	38	39	41
4	34	50	46
5	30	38	39

In which subject is the level of knowledge highest.

3.10:- (i) Following are the daily wages in rupees of the employees working in a store. Calculate the Arithmetic Mean by using deviation from an assumed mean 2350

2000, 2100, 2200, 2300, 2400, 2500, 2600, 2700

(ii) Given the following set of data:

160, 180, 168, 172, 150, 155

Calculate Arithmetic Mean by using assumed mean 162.

(iii) The deviation from $x = 300$ of 10 different values are:

-140, -113, -71, -55, -55, 0, 5, 25, 75, 654

Calculate Arithmetic Mean by direct method

(iv) The deviation of the values $x = 20$ of 10 different values are:

-5, -8, -11, -5, -4, -2, 0, 5, 0, 10

Calculate Arithmetic Mean by direct method

(v) Sum of the deviation from $x = 110$ for 8 values is 70. Find the Arithmetic Mean of the values.

(vi) If the mean of 10 numbers is 10. If an eleventh number is added in the result the mean again becomes 10. What is the value of the eleventh number.

(vii) The average fine paid by 8 students in a college is Rs. 65. If 7 of the student paid Rs. 30, 80, 39, 100, 27, 80, 64 then find the fine paid by 8th student.

(viii) The mean of 20 values is 15. If one more value is added, the mean becomes 20. Find the added value.

3.11:- Calculate mean, median, Q_1 , Q_3 , D_9 and P_{55} from the data given below:

5, 7, 9, 16, 13, 11, 10, 8, 17, 16, 19, 28, 33, 25, 24, 21, 51, 71, 81, 90, 100, 99, 78, 76, 66, 63, 52, 42, 47, 33, 37, 40

3.12:- Calculate the arithmetic mean of the following data by:

(i) Direct Method (ii) Short Cut Method

Classes :	0-10	10-20	20-30	30-40	40-50
No of Students:	4	8	16	7	5

3.13:- Calculate the arithmetic mean of the following data.

Age (Years):	1-4	5-8	9-12	13-16	17-20
No of Cows :	3	7	10	8	2

3.14:- Find out the arithmetic mean.

Marks:	3-5	5-7	7-9	9-11	11-13
No of Students :	8	12	20	9	1

3.15:- Calculate the arithmetic mean.

Class - interval :	1-5	6-10	11-15	16-20	21-25
Frequency :	7	10	16	32	24
Class - interval :	26-30	31-35	36-40	41-45	
Frequency :	18	10	5	1	

3.16:- Find the arithmetic mean of the following scores.

Scores:	10-24	24-39	40-54	55-69	70-84
F:	10	14	23	29	26
Scores:	85-99	100-114	115-129	130-144	145-159
F:	16	13	7	1	4

3.17:- Find A.M of the following distribution of marks

Marks :	0-5	5-10	10-15	15-20	20-25
F :	4	6	10	16	12

3.18:- Find A.M of the following data.

Class-interval:	0-5	5-15	15-40	40-90
Frequency:	5	15	33	65
Class - interval:	90-100	100-130	130-150	150-200
Frequency:	76	69	49	35

3.19:- (a) Find the median of the following grades obtained by 25 students in statistics examination.

80, 46, 84, 73, 54, 35, 64, 21, 70, 84, 76, 89, 77, 57, 49, 68, 93, 50, 58, 71, 59, 63, 72, 98, 67

(b) Find mean and median of the numbers.

(i) 3, 5, 2, 6, 5, 9, 5, 2, 8, 6

(ii) 51.6, 48.7, 50.3, 48.9

3.20:- Calculate Q_1 , Q_3 , D_6 and P_{42} for the following data.

(i) 19, 22, 18, 20, 49, 52, 50, 48, 41

(ii) 48, 28, 92, 22, 69, 37, 69, 92, 88, 82

(iii) 60, 45, 43, 52, 49, 50, 38, 30, 44, 55

3.21:- Find the median of the following distribution.

Size of items:	4	5	6	7	8	9	10
F	2	5	8	9	12	14	14
Size of items:	11	12	13	14	15	16	17
F	15	11	13	9	7	16	17

3.22:- Find median number of leaves per branch.

No. of leaves :	2	3	4	5	6	7
No. of branches:	10	70	110	300	320	120
No. of leaves :	8	9	10	11	12	
No. of branches:	3	20	8	7	5	

3.23:- Eight coins were tossed together and the number of heads resulting was observed. Calculate mean, median and quartiles of the distribution.

No. of Heads:	0	1	2	3	4	5	6	7	8
Frequency:	1	9	26	59	72	52	29	7	1

(Lahore Board, 2008)

3.24:- For the following data, calculate the value of median

C - I :	35-45	45-55	55-65	65-75	75-85
Frequency :	8	12	20	11	9

3.25:- Calculate the median of the following data.

Marks	10-19	20-29	30-39	40-49	50-59	60-69
No of Students	3	7	20	12	10	8

3.26:- Calculate the median of the following data.

Classes :	10-14	15-19	20-24	25-29	30-34	35-39
Frequency :	10	15	25	18	12	10

3.27:- Calculate the median for the following distribution.

Monthly Wages	No. of Workers
50-55	6
55-60	10
60-65	22
65-70	30
70-75	16
75-80	12
80-100	15

- 3.28:-** Find median, lower upper quartiles, 7th decile and 80th percentile from the data given below:

Groups :	0-8	8-16	16-24	24-32	32-40	40-48
Frequency :	3	17	18	22	6	4

- 3.29:-** The yield of grain (x, lbs) from small points are grouped into classes with a common class interval 0.2 in the table below. Calculate mean, median and Q_1 , Q_3 .

X:	2.8	3.0	3.2	3.4	3.6	3.8	4.0
F:	4	15	20	47	63	78	88
X:	4.2	4.4	4.6	4.8	5.0	5.2	
F:	60	59	35	10	8	4	

- 3.30:-** From the following data, find out the median, the two quartiles and mode.

Wages in Rs. :	20	21	22	23	24
No. of workers :	8	10	11	16	20
Wages in Rs. :	25	26	27	28	
No. of workers :	25	15	9	6	

- 3.31:-** Calculate the median of the distribution of marks obtained by 80 students as given below:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	3	9	15	30	18	5

Draw also the cumulative frequency curve of the above data and read the value of the median.

- 3.32:-** Find out the median, quartiles, 7th decile from the following data.

Monthly Rent	No. of Families Paying the Rent
20-40	6
40-60	9
60-80	11
80-100	14
100-120	20
120-140	15
140-160	10
160-180	8
180-200	7

- 3.33:-** In the following cricket scores. Find out the modal score for each batsman.

Batsman A: 27, 0, 14, 16, 2, 27, 5, 27, 16, 27

Batsman B: 5, 15, 38, 5, 72, 91, 106, 4, 3, 0, 5

Batsman C: 27, 14, 36, 7, 21, 9, 19, 36

- 3.34:-** In the following lists of bowling performance, which is the modal performance for each bowler.

Bowler A: Wickets taken: 4, 3, 1, 4, 4, 5, 3, 1, 2, 4, 5

Bowler B: Wickets taken: 2, 2, 2, 0, 1, 7, 3, 2, 2, 15

Bowler C: Wickets taken: 5, 1, 4, 7, 1, 1, 3, 2, 3, 3, 4

- 3.35:-** Find mean, median and mode of

(i) 1, 3, 4, 0, 4, 2, 3, 1, 2, 3, 0, 1, 1, 1, 5, 1

(ii) 8, 3, 21, 10, 2, 7, 0, 27, 19, 6, 7, 4, 10, 5, 7, 24

- 3.36:-** (a) Prove that $\text{Mode} = 3 \text{ med} - 2 \text{ mean}$ of the following data.

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	10	19	31	32	25	16

- (b) Compute the A.M. by (i) short cut and (ii) coding method of the data present in part (a).

(Faisalabad Board 2008)

- 3.37:-** (a) A variable y is determined from a variable x by the equation $y = 10 - 4x$. Find y when $x = -3, -2, -1, 0, 1, 2, 3, 4, 5$ and show that $y = 10 - 4x$

- (b) Find median and mode for the data. 3, 5, 2, 6, 5, 9, 5, 2, 8, 6

(Faisalabad Board, 2009)

- 3.38:-** (a) Calculate mean of the following values of x and their deviations $D = (x - 60)$, 83, 52, 66, 58, 50, 47, 55, 60

- (b) Compute median and mode using following data.

Groups	100-200	200-300	300-400	400-500	500-600
Frequency	10	28	35	25	7

(Faisalabad Board, 2010)

- 3.39:-** (a) The following table gives daily income of eight operators in a tools factory. Find the arithmetic mean.

No of Operator	A	B	C	D
Income	12	15	18	20
No of Operator	E	F	G	H
Income	25	30	22	35

- (b) Calculate median for the following data.

Marks	10-19	20-29	30-39	40-49	50-59	60-69
F:	5	20	35	20	15	9

(Lahore Board Group II, 2008; Lahore Board Group I, 2010)

- 3.40:-** (a) Compute arithmetic mean from the following data.

Group	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	4	6	8	6	4	2

- (b) Find median from the following data.

Group	30 – 40	40 – 45	45 – 55	55 – 60	60 – 70
Frequency	3	5	8	4	5

(Lahore Board Group I, 2009)

- 3.41:- (a) Compute mode for the following frequency distribution.

Class Limits	3.0–3.9	4.0–4.9	5.0–5.9	6.0–6.9	7.0–7.9	8.0–8.9
Frequency	13	27	40	30	16	4

- (b) A man purchased 120 shares of a firm @ Rs. 20 each. He purchased 80 shares of another firm @ Rs. 25 each. Find his average purchase price per share considering his total deal of 200 shares.

(Lahore Board Group II, 2009)

- 3.42:- Find AM and Mode for the data given below:

x : 40, 45, 48, 50, 55, 57, 55, 59, 65, 70, 75 and 55.

(Lahore Board Group I, 2010)

- 3.43:- (a) Find "Arithmetic Mean" of following distribution of marks.

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	4	6	10	16	12

- (b) Find 'Median' and "Mode" of the following:

10, 13, 15, 17, 13, 12, 19, 18

(Lahore Board Group II, 2010)

- 3.44:- (a) Arithmetic mean of 15 values is 20 and by adding 3 more values, the mean remains unchanged. Find new values if the ratio is

a : b : c

3 : 2 : 1

- (b) Calculate median for given frequency distribution.

X	3	5	7	9	11	13	15
F:	200	600	800	1300	900	700	500

(Gujranwala Board, 2010)

- 3.45:- (a) For the following data, calculate median.

Class	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34
Frequency	5	12	30	25	6

- (b) Find Arithmetic Mean for following data.

1, 3, 4, 0, 4, 2, 3, 1, 2, 3, 0, 1, 1, 1, 5, 1

(Rawalpindi Board, 2010)

- 3.46:- (a) Calculate median and mode from the given data.

2.1, 1.9, 3.5, 0.5, 1.9, 3.1, 6.6, 0.4, 3.7, 3.2, 5.1, 2.5, 1.6, 4.8, 2.8, 1.9, 3.1, 6.7, 8.4, 5.2, 7.7, 3.1, 4.8

- (b) Find arithmetic mean for the following distribution.

Income (Rs.)	35 – 39	40 – 44	45 – 49	50 – 54
No of Workers	3	11	22	19
Income (Rs.)	55 – 59	60 – 64		
No of Workers	4	1		

(Sargodha Board, 2010)

- 3.47:- Find mode of the following data

- 3.48:-** Find mode of the following data
- | | | | | | |
|-----------|-------|-------|-------|-------|-------|
| Weights : | 20-24 | 24-28 | 28-32 | 32-36 | 36-40 |
| F : | 4 | 16 | 27 | 20 | 3 |
- 3.49:-** Calculate mode of the following data
- | | | | | | |
|----------|-------|-------|-------|-------|-------|
| Groups : | 50-54 | 55-59 | 60-64 | 65-69 | 70-74 |
| F : | 5 | 18 | 22 | 15 | 10 |
- 3.50:-** Find mean, median and mode.
- | | | | | | |
|------------------|----|-----|-----|----|----|
| No. of persons : | 1 | 2 | 3 | 4 | 5 |
| No. of houses : | 26 | 113 | 120 | 95 | 60 |
| No. of persons : | 6 | 7 | 8 | 9 | 10 |
| No. of houses : | 42 | 21 | 14 | 5 | 4 |
- 3.51:-** The height of 100 college students measured to the nearest inch are given below. Calculate mean, median and mode.
- | | | | | | |
|------------------|-------|-------|-------|-------|-------|
| Height : | 60-62 | 63-65 | 66-68 | 69-71 | 72-74 |
| No. of students: | 5 | 18 | 42 | 27 | 8 |
- 3.52:-** Find mean, median, mode, Q_1 , Q_3 from the following distribution giving the heights in inches of 100 nine year old boys in primary school.
- | | | | | | | |
|--------------|----|----|----|----|----|----|
| Height : | 47 | 48 | 49 | 50 | 51 | 52 |
| No. of boys: | 2 | 5 | 9 | 10 | 12 | 15 |
| Height : | 53 | 54 | 55 | 56 | 57 | |
| No. of boys: | 20 | 17 | 6 | 3 | 1 | |
- 3.53:-** Find mean, median and mode of the following.
- | | | | | | | |
|---------|-------|-------|-------|-------|-------|--------|
| Marks : | 10-25 | 25-40 | 40-55 | 55-70 | 70-85 | 85-100 |
| F : | 6 | 20 | 44 | 26 | 3 | 1 |
- 3.54:-** The following data indicate daily income of small industrial unit.
- | | | | | | |
|--------------|---------|---------|---------|---------|---------|
| Income (Rs.) | 118-126 | 127-135 | 136-144 | 145-153 | 154-162 |
| Workers | 3 | 5 | 12 | 6 | 4 |
- Calculate arithmetic mean of income.
- (Lahore Board Group I, 2008)**
- 3.55:-** Find mean, median and mode of the following distribution. Give an empirical relation.
- | | | | | |
|----------|-------|-------|-------|-------|
| Groups : | 0-4 | 4-8 | 8-12 | 12-16 |
| F : | 2 | 5 | 8 | 11 |
| Groups : | 16-20 | 20-24 | 24-28 | 28-32 |
| F : | 12 | 9 | 4 | 1 |
- 3.56:-** The heights of 100 college students measured to nearest inch are given in the following table.
- | | | | | | | | | | |
|--------|----|----|----|----|----|----|----|----|----|
| Height | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 |
| F: | 4 | 6 | 10 | 20 | 30 | 13 | 12 | 3 | 2 |

Calculate the arithmetic mean, the median and mode.

3.57:- From the data of Q. 3.45, draw the frequency polygon and ogive to determine the median and the first and third quartiles. Check your results from the formulae.

3.58:- Draw (i) Histogram and (ii) An ogive from the following data.

Height:	53-55	56-58	59-61	62-64	65-67	68-70
F:	15	111	182	105	19	7

Find approximate value of the median from the ogive and check your answer by calculation.

3.59:- Find 1st quartile, third quartile and 5th percentile for the following data.

Class - interval	Frequency
0-4.9	3
5-9.9	4
10-14.9	9
15-19.9	11
20-24.9	15
25-29.9	13
30-34.9	7
35-39.9	5
40-44.9	2
45-49.9	1

3.60:- Compute mean, median, mode, 6th decile and 74 percentile for the data given below:

Classes	Frequency
.7312-.7313	10
.7314-.7315	15
.7316-.7317	20
.7318-.7319	25
.7320-.7321	30
.7322-.7323	8
.7324-.7325	2

3.61:- Find mean, median, mode Q_1 , Q_3 for the data given below:

Groups :	93-97	98-102	103-107	108-112
F :	2	5	12	17
Groups :	113-117	118-122	123-127	128-132
F :	14	6	3	1

3.62:- Calculate mean, median, mode Q_3 , D_7 , P_{65} from the following data.

Classes :	100-104	105-109	110-114	115-119	120-124
F:	10	15	20	25	30
Classes :	125-129	130-134	135-139	140-144	
F :	20	16	14	10	

3.63:- Calculate A.M, Med and mode.

Marks :	10-19	20-29	30-39	40-49	50-59
F :	5	25	40	20	10

3.64:- For the following data obtain the A.M., Median, Mode

Marks :	30-39	40-49	50-59	60-69
Frequency:	6	10	11	12
Marks :	70-79	80-89	90-99	
Frequency:	22	18	8	

3.65:- For the following distribution of weekly incomes (to nearest rupees) of 100 house holds, calculate the arithmetic mean, med and mode.

Weekly income	35-39	40-44	45-49	50-54	55-59	60-64	65-69
No. of h-holds	15	13	17	25	15	10	5

3.66:- Calculate the Arithmetic mean, median, mode and D_7 from the following data.

C.I :	0-5	5-10	10-15	15-20	20-25
F :	25	45	50	70	90
C.I :	25-30	30-35	35-40	40-45	45-50
F :	99	91	75	51	34

Also represent the above data by means of a suitable diagram.

3.67:- (a) Define Arithmetic mean, Median, Mode.

(b) Calculate Arithmetic mean, Median, Mode.

Weekly wages (Rs.)	No. of workers
0-40	6
40-80	15
80-120	22
120-160	30
160-200	45
200-240	27
240-280	13
280-320	6

ANSWERS EXERCISE NO.3

- 3.4:- (i) Zero (ii) Mode (iii) 2 (iv) 2 mean (v) equal
 (vi) Mode (vii) Median
 (viii) $\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$
 (ix) Median (x) $\sum fD$
- 3.5:- (i) True (ii) True (iii) True (iv) False
 (v) False (vi) False (vii) False (viii) False
 (ix) False (x) False (xi) False (xii) False
 (xiii) False (xiv) True (xv) True (xvi) True
 (xvii) False (xviii) True (xix) False (xx) True
- 3.6:- $\bar{X} = 21.7$
- 3.7:- (i) $\bar{X} = 73.22$, (ii) $\bar{X} = 0.85$, (iii) $\bar{X} = 14.87$, (iv) $\bar{X} = 50.25$
- 3.8:- $\bar{X} = 199.5$ Pounds
- 3.9:- Economics
- 3.10:- (i) $\bar{X} = 2350$ (ii) $\bar{X} = 164.17$ (iii) $\bar{X} = 332.5$
 (iv) $\bar{X} = 18$ (v) $\bar{X} = 118.75$ (vi) Eleventh Value = 10
 (vii) Fine of 8th Student = Rs. 100
 (viii) Added Value = 120
- 3.11:- $\bar{X} = 40.25$, Median = 33, $Q_1 = 16$, $Q_3 = 65.25$, $D_9 = 87.3$, $P_{55} = 37.45$
- 3.12:- $\bar{X} = 25.25$ 3.13:- $\bar{X} = 10.37$
- 3.14:- $\bar{X} = 7.32$ 3.15:- $\bar{X} = 20.36$
- 3.16:- $\bar{X} = 69.66$ 3.17:- $\bar{X} = 18$
- 3.18:- $\bar{X} = 96.354$
- 3.19:- (a) Median = 68 (b) (i) $\bar{X} = 5.1$, Median = 5,
 (ii) $\bar{X} = 49.875$, Median = 49.6
- 3.20:- (i) $Q_1 = 19.5$, $Q_3 = 49.5$, $D_6 = 48$, $P_{42} = 25.8$
 (ii) $Q_1 = 34.35$, $Q_3 = 89$, $D_6 = 76.8$, $P_{42} = 61.02$
 (iii) $Q_1 = 41.75$, $Q_3 = 52.75$, $D_6 = 49.6$, $P_{42} = 44.62$
- 3.21:- Median = 10 3.22:- Median = 5
- 3.23:- $\bar{X} = 3.973$, Median = 4, $Q_1 = 3$, $Q_3 = 5$
- 3.24:- Median = 60 3.25:- Median = 39.5
- 3.26:- Median = 23.5 3.27:- Median = 67.92
- 3.28:- Median = 22.67, $Q_1 = 14.82$, $Q_3 = 29.27$, $D_7 = 28$, $P_{80} = 30.55$
- 3.29:- $\bar{X} = 4$ Median = 3.94, $Q_1 = 3.62$, $Q_3 = 4.28$
- 3.30:- Median = 24.25, $Q_1 = 22.567$, $Q_3 = 25.5$, Mode = 24.83
- 3.31:- Median = 34.33
- 3.32:- Median = 110 $Q_1 = 78.2$, $Q_3 = 140$, $D_7 = 133.33$
- 3.33:- For A Mode = 27 For B Mode = 5 For C mode = 36

- 3.34:- For A mode = 4 For B mode = 2 For C mode = 1,3
 3.35:- (i) $\bar{X} = 2$, Median = 1.5, Mode = 1
 (ii) $\bar{X} = 10$, Median = 7, Mode = 7
 3.36:- (a) Mode = 42.41 (b) A.M. = 41.84
 3.37:- (b) Med = 5, Mode = 5
 3.38:- (a) A.M. = 58.875 (b) Med = 341.43, Mode = 341.18
 3.39:- (a) A.M. = 22.125 (b) Med = 37.21
 3.40:- (a) A.M. = 13.5 (b) Med 50.624
 3.41:- (a) Mode = 5.52 (b) A.M. = Rs. 22
 3.42:- A.M. = 56.16
 3.43:- (a) A.M. = 15.21 (b) Med = 14, Mode = 13
 3.44:- (a) Three values are 30, 20, 10; Mode = 9.385
 3.45:- (a) Med = 23.17 (b) A.M. = 2
 3.46:- (A) Med = 3.1, Mode = 1.8, 3.1 (b) A.M. = 48.08
 3.47:- Mode = 30.44
 3.48:- Mode = 61.32
 3.49:- Mode = 29.62
 3.50:- $\bar{X} = 4.24$, Median = 4, Mode = 3
 3.51:- $\bar{X} = 67.45$ ", Median = 67.43", Mode = 67.35"
 3.52:- $\bar{X} = 52$, Median = 52.3 Mode = 53.125,
 $Q_1 = 50.4$, $Q_3 = 53.62$
 3.53:- $\bar{X} = 47.95$, Median = 48.18, Mode = 48.57
 3.54:- Rs. 140.9
 3.55:- $\bar{X} = 15.7$, Median = 16, Mode = 17,
 By relation mode = 16.16
 3.56:- $\bar{X} = 66.8$, Median = 66.83, Mode = 66.87
 3.57:- Median = 66.83, $Q_1 = 65.75$, $Q_3 = 67.88$
 3.58:- Median = 60.04
 3.59:- $Q_1 = 15.63$, $Q_3 = 28.99$, $P_5 = 5.575$
 3.60:- $\bar{X} = 0.731799$, Median = 0.73183, Mode = 0.731987,
 $D_6 = 0.731918$, $P_{74} = 0.732026$
 3.61:- $\bar{X} = 110.92$, Median = 110.74, Mode = 110.625,
 $Q_1 = 105.83$, $Q_3 = 115.714$
 3.62:- $Q_3 = 129.5$, $D_7 = 127.5$, $P_{65} = 125.5$, Mean = 121.5,
 Median = 121.17, Mode = 121.17
 3.63:- A.M = 35, Med = 34.5, Mode = 33.79
 3.64:- A.M = 68.293, Med = 71.545, Mode = 76.643
 3.65:- A.M = 50.1, Med = 50.5, Mode = 51.72
 3.66:- A.M = 26.27, Med = 26.77, Mode = 27.65, $D_7 = 33.407$
 3.67:- A.M = 162.44, Med = 168, Mode = 178.18

3

MEASURES OF LOCATION OR CENTRAL TENDENCY

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The sum of values divided by their numbers is called	Mode	Mean	Median	None of these
(Faisalabad & Lahore Boards, 2008)					
2)	If $Z = X - Y$ then is $\frac{\bar{Z}}{\bar{Z}}$	$\bar{X} + \bar{Y}$	$\bar{X} - \bar{Y}$	0	$\frac{\bar{X}}{\bar{Y}}$
(Faisalabad Board, 2008)					
3)	Sum of deviation from mean is	Zero	Positive	Negative	None of these
(Lahore Board, 2008)					
4)	The most frequent value in the data is called	Mode	Median	G.M	H.M
(Lahore Board, 2008; Faisalabad Board, 2010)					
5)	In symmetrical distribution mean, median and mode are always	Unequal	Equal	Different	Negative
(Lahore Board, 2008)					
6)	The mid point of a class ranging from '40' to '60' is	30	40	50	60
(Gujranwala Board, 2009)					
7)	Mode of the word 'Professor' is	R	S	O	R, S and O
(Faisalabad Board, 2009)					
8)	The mean, median, mode of constant 'a' are	0	$\frac{a}{2}$	a	a^2
(Faisalabad Board, 2009)					
9)	The most frequent (common) value in the data is	Mean	Median	Mode	Both mean and median
(Lahore Board Group I&II, 2009)					

S#	Questions	Answers			
10)	The notation for mean is	X	$\bar{X}$	X	$\bar{X}_w$
		(Lahore Board, 2009)			
11)	For classes, 50 – 54, 55 – 59, 60 – 64, The class interval is	4	5	10	None of these
		(Lahore Board, 2009)			
12)	The size of class is also called	Class interval	Class frequency	Class mark	Class boundary
		(Lahore Board, 2009)			
13)	We must arrange the data, before calculating	Mean	Median	Mode	None of these
		(Lahore Board, 2009)			
14)	The median of 3, 4, 5, 6, 9, 10, 12, is	5	6	9	5.5
		(Faisalabad Board, 2010)			
15)	For a certain distribution, if $\sum(x - 5) = 0$ the value of mean is	0	5	-5	None of these
		(Lahore Board, 2010)			
16)	Mode of the series 2, 3, 3, 3, 4, 4, 5, 0 is	2	3	4	6
		(Lahore Board, 2010)			
17)	To find the average of blue colour pieces of cloth, we use	Median	Mode	Mean	All
		(Lahore Board, 2010)			
18)	The mean of 10 numbers is 9.2, then the sum of numbers is	72	82	92	102
		(Lahore Board, 2010)			
19)	Averages are also called measures of	Variation	Location	Skewness	Median
		(Gujranwala Board, 2010)			
20)	The sum of squared deviations is least from	Median	Mean	Mode	None
		(Gujranwala Board, 2010)			

S#	Questions	Answers			
21)	The number of observations in particular class is known as	Frequency	Class interval	Mid point	Middle value
		(Rawalpindi Board, 2010)			
22)	The mode in word 'STATISTICS' is /are	I	T	S	T and S
		(Rawalpindi Board, 2010)			
23)	A data having single mode is	Uni-model	Bi-model	Tri-model	Multi-model
		(Rawalpindi Board, 2010)			
24)	A value obtained by dividing sum of all the values by their number is called	A.M.	Median	Mode	G.M.
		(Sargodha Board, 2010)			
25)	To find out mid-point of class the formula is	$\frac{\text{Upper Limit} + \text{Lower Limit}}{2}$	$\frac{\text{Upper Limit} - \text{Lower Limit}}{2}$	$\frac{\text{Upper Limit} + \text{Lower Limit}}{2}$	$\frac{\text{Upper Limit} - \text{Lower Limit}}{2}$
		(Sargodha Board, 2010)			
26)	Quartiles divide the values in equal parts	2	3	4	5
		(Sargodha Board, 2010)			
27)	Mid point of class 10 – 20 is	10	20	30	15
		(Sargodha Board, 2010)			

QUESTION NO. 2

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The central value which represents a set of data is called measure of:	Central tendency	Skewness	Dispersion	Symmetry
2)	The value obtained by dividing the sum of all the values by their numbers is called:	Median	Mode	Arithmetic mean	Skewness

S#	Questions	Answers			
3)	The sum of deviations taken from mean is:	Always equal to zero	Some times equal to zero	Never equal to zero	Less than zero
4)	The sum of deviations of observations is zero if deviations are taken from:	Mode	Median	Arithmetic Mean	Provisional Mean
5)	For a data if $\Sigma(x-20) = 14$, $\Sigma(x-25) = 0$, $\Sigma(x-40) = 29$, then the value of A.M. is:	20	25	40	29
6)	Sum of deviations from arithmetic mean is always:	Less than zero	More than zero	Equal to zero	None of these
7)	The smallest and the largest values of any given class of a frequency distribution are called:	Class limits	Class marks	Class interval	Class frequency
8)	The difference between the upper and lower class boundaries of a class are known as:	Class limits	Class interval	Class marks	Class frequency
9)	The number of observations or objects falling into a particular class is called	Class frequency	Relative frequency	Cumulative frequency	None of these
10)	The average value of the lower and upper limits is called:	Class mark	Class width	Class frequency	Class boundaries
11)	Class mark or class mid point is equal to:	Upper limit - lower limit	Upper limit + lower limit	Upper Limit + Lower Limit 2	Upper Limit - Lower Limit 2
12)	The mid point of a class ranging from 40 to 60 is:	30	40	50	60
13)	Given 8 quantities, x_1 to x_8 , the correct	$\sum_{i=1}^8 x_i$	$\sum_{i=2}^8 x_i$	$\sum_{i=3}^8 x_i$	$\sum_{i=4}^8 x_i$

S#	Questions	Answers			
	notation for adding quantities 3 to 6 is:				
14)	Given $x_1 = 10$, $x_2 = 20$, $x_3 = 25$, $x_4 = 11$ $\sum_{i=1}^3 x_i$ equal:	45	55	65	75
15)	We calculate arithmetic mean by:	Direct method	Short cut method	Step deviation	All the above methods
16)	The most common value in a set of data is called:	A.M.	Median	Mode	None of these
17)	The algebraic sum of the observations of a set of data from their mean is always equal to:	Three	Two	One	Zero
18)	The sum of squares of the observations from their mean is always:	Minimum	Maximum	Central	Zero
19)	If the mean of 15 observations is 20, then the sum of these numbers is:	100	200	300	400
20)	The average which is used to find the middle value of data is called:	Arithmetic mean	Median	Mode	None of these
21)	We must arrange the data before calculating:	Mean	Median	Mode	All of these
22)	The combined mean is calculated by the formula:	$\frac{x_1 + x_2}{n_1 + n_2}$	$\frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$	$\frac{n_1 x_1 + n_2 x_2}{n_1 + n_2}$	$\frac{n_1 + n_2}{x_1 + x_2}$
23)	Which of the following is not based upon all the values	A.M.	Mode	Both A.M. & mode	None of these
24)	In a symmetrical distribution $Q_1 = 20$, Med = 30, then the value of Q_3 will be:	80	60	40	10
25)	Upper quartile = Q_3 =	P_{75}	D_4	Median	D_7

S#	Questions	Answers			
26)	The formula of median used for discrete grouped frequency distribution is:	$\frac{L + \frac{1}{2}(f - f_0)}{f - f_0}$	$\frac{1}{2} \left(\frac{n+1}{f} \right) \times h$	$\frac{(n+1)}{2} \times h$	None of these
27)	The most frequent value of data is called:	Mean	Median	Mode	None of these
28)	The mode of the letters in the word 'MATHEMATICS' is:	M	A	T	All M, A, T
29)	The mode of the letters in the word 'STATISTICS' is:	S	T	Both S & T	A
30)	Median is a value which divides an arranged distribution into:	Four parts	Three parts	Two parts	None of these
31)	Cumulative frequency is called:	Increasing	Decreasing	Both Increasing & Decreasing	Constant
32)	Mode will lie in a class which is said to be:	Modal class	Middle class	Class boundaries	Class frequency
33)	If the data are arranged, median is central value:	Always	Never	Some time	None of these
34)	Corresponds to maximum frequency:	Median	AM	Mode	Skewness
35)	The relation between mean, median, mode i.e. Mode = 3 Median - 2 mean is called:	Empirical	Symmetrical	Asymmetrical	All of these
36)	For positively skewed distribution Mean Median Mode:	=	<	>	≠
37)	For negatively skewed distribution Mean Median Mode	=	<	>	≠

S#	Questions	Answers			
38)	For a symmetrical distribution Mean Median Mode:	=	<	>	≠
39)	In a symmetrical distribution, the relation between mean, median and mode is:	Mean = Median = Mode	Mean < Median < Mode	Mean > Median > Mode	Mean > Median < Mode
40)	Mean, Median and Mode are equal in	Skewed distribution	Symmetrical distribution	Both skewed & symmetrical distribution	None of these
41)	For positively skewed distribution:	Mean = Median = Mode	Mean > Median > Mode	Mean < Median < Mode	Mean < Med > Mode
42)	For negatively skewed distribution:	Mean = Median = Mode	Mean > Median > Mode	Mean < Median < Mode	Mean < Median > Mode
43)	The arithmetic mean, median, mode are called:	First order averages	Second order averages	Third order averages	Fourth order average

QUESTION NO. 3

Write the short answer to the following questions in the space provided

Q. 1: What is a statistical average?

(Lahore Board, 2009)

(Rawalpindi Board, 2010)

Ans: An average is a single value that is typical or representative for a group of numbers.

Q. 2: What are other names of average?

Ans: As an average indicates the location or general position of the distribution or an average is some where in the middle of the group, so it is also called measures of central tendency or measures of location or measures of position.

Q. 3: Why are the averages called measures of central tendency?

Ans: As the averages tend to lie in the center of the distribution they are called measures of central tendency.

Q. 4: What is "Measures of central tendency"?

(Lahore Board, 2010)

Ans: Since the averages which mostly fall in the centre of data and used to represent the whole data. So these averages are called measures of central tendency.

Q. 5: What are the objects of measures of central tendency or an average.

Ans: Objects of measures of central tendency are as follows:

- (i) To find out a value that can be represent the whole data.
- (ii) To help the comparative study.
- (iii) To provide a condensed picture of a large group.

Q. 6: What are the important types of averages?

Ans: The most commonly used averages are:

- (i) Arithmetic mean
- (ii) Geometric mean
- (iii) Harmonic mean
- (iv) Median
- (v) Mode

Q. 7: What are characteristics of a good average?

(Lahore Board, 2009 & 2010)

(Sargodha Board, 2010)

Ans: It should be:

- (i) Easy to calculate
- (ii) Easy to understand
- (iii) Rigidly defined
- (iv) Based on the values of data
- (v) Least affected by extreme values

Q. 8: What type of averages are mathematical in character?

Ans: The arithmetic mean, geometric mean and harmonic mean are mathematical in character.

Q. 9: What type of average are averages of location?

Ans: The median and mode are averages of location.

Q. 10: What does Σ denote for?

Ans: Σ is used to denote the sum of all the values of a series.

Q. 11: Define arithmetic mean.

(Lahore Board, 2009)

(Gujranwala & Sargodha Boards, 2010)

Ans: The arithmetic mean is defined as a value obtained by dividing sum of all the values by their numbers.

Q. 12: Give any three algebraic properties of A.M.

(Lahore Board, 2008; Faisalabad Board, 2009; Rawalpindi Board, 2010)

Ans:

- (i) $\Sigma(X - \bar{X}) = 0$
- (ii) $\Sigma(X - \bar{X})^2$ is minimum
- (iii) If $y_i = ax_i + b$ then $\bar{Y} = a\bar{X} + b$

Q. 13: The sum of observations from their mean —?

Ans: The sum of observations from their mean is zero.

Q. 14: What will be the mean of 36, 36, 36, 36?

Ans: As we know that arithmetic mean of a constant is constant itself, so mean is 36.

Q. 15: Discuss the used of weighted mean is statistics.

Ans: When all the observations are not of equal importance, then we used weighted arithmetic mean.

Q. 16: Where arithmetic mean is preferred to measure center of the data.

Ans: Arithmetic mean is preferred when:

- (i) The distribution is approximately symmetrical.
- (ii) A measure is needed that reflects the total.
- (iii) The measure of central tendency having the greatest stability is required.

Q. 17: Write three merits (advantages) of arithmetic mean.

(Faisalabad Board, 2009)

- Ans:**
- (i) Simple to understand and easy to calculate.
 - (ii) Rigidly defined.
 - (iii) Capable of further mathematical treatments.

Q. 18: Define median.

(Lahore Board, 2008, 2009 & 2010)

(Faisalabad & Sargodha Boards, 2010)

Ans: The median is defined as the middle most value of the arranged data.

Q. 19: Give the merits of the median.

(Lahore Board, 2008 & 2009)

(Sargodha Board, 2010)

Ans: Median is:

- (i) Simple to understand and easy to calculate
- (ii) Not affected by extreme observations
- (iii) Possible to locate graphically

Q. 20: What are the demerits of the median?

(Lahore Board, 2008)

(Faisalabad Board, 2009)

(Gujranwala Board, 2010)

Ans: Median is:

- (i) Not rigidly defined
- (ii) Not based on all values
- (iii) Not capable of further mathematical treatment

Q. 21: Is sum of absolute deviations of the observations from mean is minimum.

Ans: No, the sum of absolute deviations taken from median is minimum.

Q. 22: What do you mean by quartiles?

Ans: Quartiles are the values, which divide the arranged data into four equal parts.

Q. 23: Define deciles and percentiles?

Ans: Deciles and percentiles are the values, which divide the arranged data into ten and hundred equal parts.

Q. 24: Define mode with example.

(Lahore Board, 2008 & 2010)

(Faisalabad Board 2008, 2009 & 2010)

(Rawalpindi Board, 2010)

Ans: The most repeated or most common or most frequent value in the set of data is called mode. For example: 20, 10, 18, 10, 18, 7, 10, 30, 15

Here most repeated value is 10, so mode = 10

Q. 25: Which measure of location can measure more than one value for a data.

Ans: The only measure of location that can assume more than one value for a data is mode.

Q. 26: What is the mode of the following data?

10, 20, 30, 40, 50, 60, 70, 80, 90

Ans: No mode, as there is no repeated value.

Q. 27: Define bi-modal distribution.

(Lahore Board, 2009)

Ans: A distribution having two modes is defined bi-modal distribution.

Q. 28: Discuss the relationship between mean, median and mode.

Ans: For a skewed distribution:

$$\text{Mode} \doteq 3 \text{ Med} - 2 \text{ Mean}$$

Which is called empirical relation.

For a symmetrical distribution, the mean, median and mode are equal

i.e. $\text{Mean} = \text{Median} = \text{Mode}$

Q. 29: Give the two merits of the mode.

- Ans:** (i) Mode is simple to understand and easy to calculate
(ii) It is not affected by extreme values.

Q. 30: Give three merits of the mode.

- Ans:** (i) Mode is not rigidly defined.
(ii) It is not based on all the values.
(iii) It is not capable of further mathematical treatment.

Q. 31: What is mode of this data?

10, 20, 30, 10, 40, 50, 10, 60

(Faisalabad Board, 2009)

Ans: Here most common or most repeated number is 10, so mode = 10

Q. 32: Write down the formula to compute mode for grouped data.

(Faisalabad Board, 2010)

Ans: The formula to compute mode for group data is

$$\text{Mode} = L + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

Where f_m = Maximum frequency

f_1 = Preceding value of frequency to f_m

f_2 = Next value of frequency to f_m

L = Lower limit of mode – group

h = Interval size

Q. 33: What is meant by uni-modal distribution?

(Faisalabad & Sargodha Boards, 2010)

Ans: A distribution, having single value of mode is called uni-modal distribution.

For example,

55, 58, 57, 58, 60, 50, 53, 58

is a uni-model distribution have mode = 58 only.

Q. 34: What do you understand by "Change of Origin"?

(Lahore Board, 2008)

Ans: The addition or subtraction of any constant value from each value of a variable is called change of origin.

Q. 35: What is modal class?

(Lahore Board, 2010)

Ans: The class in which maximum frequency lies, is called modal class.

Q. 36: State the formula of arithmetic mean (grouped data / ungrouped data).

(Lahore Board, 2010)

Ans: Ungrouped Data

i) $A.M = \frac{\sum X}{n}$ Direct Method

ii) $A.M = P.M + \frac{\sum D}{n}$ Indirect or Short cut Method

Grouped Data

i) $A.M = \frac{\sum fX}{\sum f}$ Direct Method

ii) $A.M = P.M + \frac{\sum fD}{\sum f}$ Indirect or Short cut Method

iii) $A.M = P.M + \frac{\sum fu}{\sum f} \times h$ Coding Method

Q. 37: In a symmetrical distribution arithmetic mean is 10. Find the value of median and mode. (Gujranwala Board, 2010)

Ans: In a symmetrical distribution mean, med and mode are equal.

So, if mean = 10, then med = 10 and mode = 10

Q. 38: Find median 5, 10, 12, 13, 15, 7, 9

(Lahore Board, 2010)

Ans: For median, arrange the values in ascending and descending order.

5, 7, 9, 10, 12, 13, 15

Here $n = 7$ (odd number), so

Med = The value of $\left(\frac{n+1}{2}\right)$ th item $\Rightarrow$ The value of $\left(\frac{7+1}{2}\right)$ th item

= The value of $\left(\frac{8}{2}\right)$ th item = The value of 4th item = 10

Hence med = 10

Q. 39: Find out arithmetic mean given $\sum X = 308$, $n = 7$

(Lahore Board, 2008)

Ans: Here $\sum X = 308$, $n = 7$

$A.M = \frac{\sum X}{n} = \frac{308}{7} = 44$

Q. 40: For a certain frequency, the value of mean is 15, median is 20. What will be the value of mode?

(Lahore Board, 2008)

Ans: Here Mean = 15, Median = 20

We know that

$$\text{Mode} = 3\text{med} - 2\text{mean} \Rightarrow 3(20) - 2(15) = 60 - 30 = 30$$

Q. 41: What position exist when mean = med = mode?

(Faisalabad Board, 2008)

Ans: A distribution is said to be symmetrical if mean, mode are equal.

Q. 42: Find A.M given that $X = 10 + 5u$, $\Sigma fu = 46$, and $n = 125$

(Lahore Board, 2008)

Ans: • Here $X = 10 + 5u$, $\Sigma fu = 46$, $n = \Sigma f = 125$

We know that

$$\bar{X} = P.M + \frac{\Sigma fu}{\Sigma f} \times h \Rightarrow u = \frac{X - P.M.}{h}$$

We have $X = 10 + 5u$

$$5u = X - 10$$

$$u = \frac{x - 10}{5} = \frac{X - P.M.}{h}$$

Hence P.M. = 10; $h = 5$

Now

$$\bar{X} = 10 + \frac{46}{125} \times 5 = 10 + \frac{46}{h25} = 10 + 1.84 = 11.84$$

ANSWERS

QUESTION NO. 1

- | | | | |
|---|------------------------|---------------|-----------|
| 1) Mean | 2) $\bar{X} - \bar{Y}$ | 3) Zero | 4) Mode |
| 5) Equal | 6) 50 | 7) R, S and 0 | 8) a |
| 9) Mode interval | 10) $\bar{X}$ | 11) 5 | 12) Class |
| 13) Median | 14) 6 | 15) 5 | 16) 3 |
| 17) Mode | 18) 92 | 19) Location | 20) Mean |
| 21) Frequency | 22) T & S | 23) Uni-modal | 24) A.M |
| 25) $\frac{\text{Upper Limit} + \text{Lower Limit}}{2}$ | | 26) 4 | 27) 15 |

QUESTION NO. 2

- | | | |
|--------------------------|---|---|
| 1) Central tendency | 2) Arithmetic mean | 3) Always equal to zero |
| 4) Arithmetic mean | 5) 25 | 6) Equal to zero |
| 7) Class limits | 8) Class interval | 9) Class frequency |
| 10) Class mark | 11) $\frac{\text{Upper limit} + \text{lower limit}}{2}$ | 12) 50 |
| 13) $\sum_{i=3}^6 x_i$ | 14) 55 | 15) All the above methods |
| 16) Mode | 17) Zero | 18) Minimum |
| | | 19) 300 |
| 20) Median | 21) Median | 22) $\frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$ |
| | | 23) Mode |
| 24) 40 | 25) P_{75} | 26) None of these |
| | | 27) Mode |
| 28) All M, A, T | 29) Both S and T | 30) Two parts |
| | | 31) Increasing |
| 32) Modal class | 33) Always | 34) Mode |
| | | 35) Empirical |
| 36) > | 37) < | 38) = |
| 39) Mean = Median = Mode | | 40) Symmetrical distribution |
| 41) Mean > Median > Mode | | 42) Mean < Median < Mode |
| 43) First order averages | | |

www.Perfect24u.com