Unit – 7 : Plane Analytical Geometry
Straight Line.

Definition: Distance between two points $P(x_1, y_1)$ And $Q(x_2, y_2)$ is given by

$$d = |PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\ .$$

Example .1: Find the distance between the two points $P(3, -2)$ and $Q(-1, -5)$

Solution: As we know that the distance between two points is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \ldots\ldots(i)$$

Put $x_1 = 3,\ x_2 = -1,\ y_1 = -2$ and $y_2 = -5$ in (i)

$$\Rightarrow d = \sqrt{(-1-3)^2 + (-5-(-2))^2}$$

$$= \sqrt{(-4)^2 + (-5+2)^2} = \sqrt{16 + (-3)^2}$$

$$\Rightarrow d = \sqrt{16+9} = \sqrt{25} = 5$$

Ratio formula: Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points of a line segment.

Let $R(x, y)$ be the point which divides PQ in ratios $m_1 : m_2$ internally, then the coordinates of R is

$$R(x, y) = R\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

(1) If $R(x, y)$ is the mid point of PQ then

$$R(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

(2) Let $R(x, y)$ be the point which divides PQ in ratios $m_1 : m_2$ externally, then the coordinates of R is

$$R(x, y) = R\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}\right).$$

Example .2: Find the coordinates of the point which divides the line segment PQ joining the two points

(a) $P(1\,,\,2)$ and $Q(3\,,\,4)$ in the ratio $5:7$

(b) $P(3\,,\,4)$ and $Q(-6\,,\,2)$ in the ratio $3:-2$.

Solution: (a) $P(1\,,\,2)$ and $Q(3\,,\,4)$ in the ratio $5:7$

Let $R(x\,,\,y)$ be the point which divides PQ in ratios $m_1:m_2$ internally, then the coordinates of R is

$$R(x\,,y)=R\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}\,,\frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \quad .....(i)$$

Put $x_1=1$, $y_1=2$, $x_2=3$, $y_2=4$

and $m_1=5$, $m_2=7$ put (i)

$$R(x\,,\;y)=R\left(\frac{(5)(3)+(7)(1)}{5+7},\;\frac{(5)(4)+(7)(2)}{5+7}\right)$$

$$=R\left(\frac{15+7}{12},\frac{20+14}{12}\right)=R\left(\frac{22}{12},\frac{34}{12}\right)$$

$$=R\left(\frac{11}{6},\frac{17}{16}\right)$$

(b) $P(3\,,\,4)$ and $Q(-6\,,\,2)$ in the ratio $3:-2$.

Let $R(x\,,\,y)$ be the point which divides PQ in ratios $m_1:m_2$ internally, then the coordinates of R is

$$R(x\,,y)=R\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}\,,\frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \quad .....(i)$$

Put $x_1=3$, $y_1=4$, $x_2=-6$, $y_2=2$

and $m_1=3$, $m_2=-2$ put (i)

$$R(x\,,\;y)=R\left(\frac{(3)(-6)+(-2)(3)}{3-2},\;\frac{(3)(2)+(-2)(4)}{3-2}\right)$$

$$=R\left(\frac{-18-6}{1},\frac{6-8}{1}\right)$$

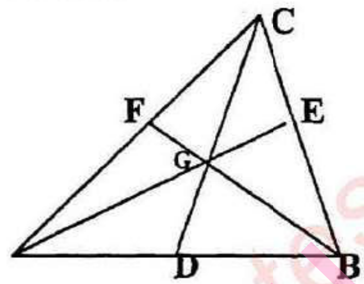
$$=R(-24\,,-2) \qquad Ans.$$

Concurrent lines: Two or more than two lines are concurrent if they are passing through the same point .

(i) **Show that the Medians of triangle are concurrent.**

Solution : The line through the vertex and mid point of a side of a triangle is called median.

Let $A(x_1 , y_1)$, $B(x_2 , y_2)$ and $C(x_3 , y_3)$ are the vertices of a triangle ABC.



Let D , E , F are the Mid points of sides AB , BC , CA respectively.

Then coordinates of the mid points $D\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2}\right)$,

$E\left(\frac{x_2 + x_3}{2} , \frac{y_2 + y_3}{2}\right)$ and $F\left(\frac{x_1 + x_3}{2} , \frac{y_1 + y_3}{2}\right)$.

Now AE , BF , CD are the medians of triangle ABC.

Let $G(x , y)$ be the centroid of triangle ABC which divides each median in the ratio of $2:1$

Let $G(x , y)$ divides the median AE in ratio $m_1 = 2$, $m_2 = 1$

Then by ratio formula the coordinates of $G(x , y)$ is

$$R(x , y) = R\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} , \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right) \text{.....(i)}$$

Put values $x_1 = x_1$, $x_2 = \frac{x_2 + x_3}{2}$, $y_1 = y_1$, $y_2 = \frac{y_2 + y_3}{2}$

in (i)

$$G(x, y) = G\left(\frac{2\left(\frac{x_2 + x_3}{2}\right) + (1)(x_1)}{2+1} , \frac{2\left(\frac{y_2 + y_3}{2}\right) + (1)(y_1)}{2+1}\right)$$

$$\Rightarrow G(x , y) = G\left(\frac{x_1 + x_2 + x_3}{3} , \frac{y_1 + y_2 + y_3}{3}\right) \text{......(ii)}$$

Let $G(x , y)$ divides the median BF in ratio $m_1 = 2$, $m_2 = 1$

Put values

$x_1 = x_2$, $x_2 = \dfrac{x_1 + x_3}{2}$, $y_1 = y_2$, $y_2 = \dfrac{y_1 + y_3}{2}$ in (i)

$$G(x,\ y) = G\left(\frac{2\left(\frac{x_1 + x_3}{2}\right) + (1)(x_2)}{2+1},\ \frac{2\left(\frac{y_1 + y_3}{2}\right) + (1)(y_2)}{2+1}\right)$$

$$\Rightarrow\ G(x\ ,y) = G\left(\frac{x_1 + x_2 + x_3}{3}\ ,\frac{y_1 + y_2 + y_3}{3}\right) \ldots\ldots(\text{iv})$$

Let $G(x\ ,y)$ divides the median CD in ratio $m_1 = 2$, $m_2 = 1$

Put values

$x_1 = x_3$, $x_2 = \dfrac{x_1 + x_2}{2}$, $y_1 = y_3$, $y_2 = \dfrac{y_1 + y_2}{2}$ in (i)

$$G(x,\ y) = G\left(\frac{2\left(\frac{x_1 + x_2}{2}\right) + (1)(x_3)}{2+1},\ \frac{2\left(\frac{y_1 + y_2}{2}\right) + (1)(y_3)}{2+1}\right)$$

$$\Rightarrow\ G(x\ ,y) = G\left(\frac{x_1 + x_2 + x_3}{3},\frac{y_1 + y_2 + y_3}{3}\right) \ldots\ldots(\text{iv})$$

From (ii) , (iii) and (iv) $G(x\ ,y)$ lie on each median .
Thus medians of triangle passes through only one point $G(x\ ,y)$. Hence medians of triangle are concurrent .

<u>Example .3:</u> Find the centroid of the triangle ABC ,whose vertices are $A(3\ ,-5)$, $B(-7,\ 4)$ *and* $C(10\ ,\ -2)$.

<u>Solution:</u> Given $A(3\ ,-5)$, $B(-7,\ 4)$ *and* $C(10\ ,\ -2)$

Since centroid of triangle ABC is

$$G(x\ ,y) = G\left(\frac{x_1 + x_2 + x_3}{3}\ ,\frac{y_1 + y_2 + y_3}{3}\right) \ldots\ldots(\text{i})$$

Put $x_1 = 3$, $x_2 = -7$, $x_3 = 10$, $y_1 = -5$, $y_2 = 4$, $y_3 = -2$ **in(i)**

$$G(x\ ,\ y)=G\left(\frac{3-7+10}{3}\ ,\ \ \frac{-5+4-2}{3}\right)$$

$$\Rightarrow G(x\ ,\ y)=G\left(\frac{6}{3}\ ,-\frac{3}{3}\right)$$

$$=G(2\ ,-1)$$

Exercise 7.1:

Q. 1: The three points $A(-1\ ,3)$, $B(2\ ,1)$ *and* $C(5\ ,-1)$. Show that $|AB|+|BC|=|AC|$.

Solution: Given $A(-1\ ,3)$, $B(2\ ,1)$ *and* $C(5\ ,-1)$.

The distance between two points $A(x_1\ ,\ y_1)$ and $B(x_2\ ,\ y_2)$ is given by

$$d=|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\quad ......(i)$$

We find the lengths of sides AB , BC and AC putting values in (i)

$$|AB|=\sqrt{(2-(-1))^2+(1-3)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$$

$$|BC|=\sqrt{(5-2)^2+(-1-1)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$$

$$|AC|=\sqrt{(5-(-1))^2+(-1-3)^2}=\sqrt{(6)^2+(-4)^2}=\sqrt{36+24}$$

$$=\sqrt{52}=\sqrt{4\times 13}=2\sqrt{13}$$

Now $|AB|+|BC|=\sqrt{13}+\sqrt{13}=2\sqrt{13}=|AC|$

Thus $|AB|+|BC|=|AC|$ **proved .**

Q. 2: In each case , find the mid point of the line segment PQ joining the two points $P(x_1\ ,y_1)$ and $Q(x_2\ ,y_2)$:

(a) $P(10\ ,20)$, $Q(-12\ ,-8)$

Solution: The mid point $C(x,\ y)$ of two points $P(x_1\ ,y_1)$ and $Q(x_2\ ,y_2)$ is given by

$$C(x\,,y)=C\left(\frac{x_1+x_2}{2}\,,\frac{y_1+y_2}{2}\right)\quad .....(i)$$

Put $x_1=10$, $y_1=20$, $x_2=-12$, $y_2=-8$ **in(i)**

$$C(x\,,\ y)=C\left(\frac{10-12}{2}\,,\frac{20-8}{2}\right)=C\left(-\frac{2}{2}\,,\frac{12}{2}\right)$$

$\Rightarrow C(x\,,y)=C(-1\,,6)$ *Ans.*

(b) $P(a\,,-b)$, $Q(-a\,,b)$

Solution : Let mid point $C(x,\ y)$ of two points $P(x_1\,,y_1)$ and $Q(x_2\,,y_2)$ is given by

$$C(x\,,y)=C\left(\frac{x_1+x_2}{2}\,,\frac{y_1+y_2}{2}\right)\quad .....(i)$$

Put $x_1=a$, $y_1=-b$, $x_2=-a$, $y_2=b$ **in(i)**

$$C(x\,,\ y)=C\left(\frac{a-a}{2}\,,\frac{-b+b}{2}\right)=C\left(\frac{0}{2},\frac{0}{2}\right)$$

$\Rightarrow C(x\,,y)=C(0\,,0)$ *Ans.*

(c) $P\left(\frac{1}{2},-\frac{1}{4}\right)$, $Q\left(\frac{3}{5},\frac{4}{7}\right)$

Solution : Let mid point $C(x,\ y)$ of two points $P(x_1\,,y_1)$ and $Q(x_2\,,y_2)$ is given by

$$C(x\,,y)=C\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)\quad .....(i)$$

Put $x_1=\frac{1}{2}$, $y_1=-\frac{1}{4}$, $x_2=\frac{3}{5}$, $y_2=\frac{4}{7}$ **in(i)**

$$C(x\,,\ y)=C\left(\frac{\frac{1}{2}+\frac{3}{5}}{2}\,,\frac{-\frac{1}{4}+\frac{4}{7}}{2}\right)=C\left(\frac{1}{2}\left(\frac{5+6}{10}\right),\frac{1}{2}\left(\frac{-7+16}{28}\right)\right)$$

$$\Rightarrow C(x\,,y)=C\left(\frac{11}{2\times10}\,,\frac{9}{2\times28}\right)=C\left(\frac{11}{20},\frac{9}{56}\right)\quad Ans.$$

Q.3: **In each case, find the coordinates of the point $R(x, y)$ which divides the line segment PQ joining the two points**

(a) $P(1, 2)$, $Q(3, 4)$ **in the ratio of** $5:7$

Solution: $P(1, 2)$, $Q(3, 4)$

Let $R(x, y)$ **be the point which divides** PQ **in ratios** $m_1 : m_2$ **then the coordinates of** R **is**

$$R(x, y) = R\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \quad .....(i)$$

Put $x_1 = 1$, $y_1 = 2$, $x_2 = 3$, $y_2 = 4$

and $m_1 = 5$, $m_2 = 7$ **put (i)**

$$R(x, y) = R\left(\frac{(5)(3)+(7)(1)}{5+7}, \frac{(5)(4)+(7)(2)}{5+7}\right)$$

$$= R\left(\frac{15+7}{12}, \frac{20+14}{12}\right) = R\left(\frac{22}{12}, \frac{34}{12}\right)$$

$$= R\left(\frac{11}{6}, \frac{17}{16}\right)$$

(b) $P(3, 4)$, $Q(-6, 2)$ **in the ratio** $3:-2$.

Solution: **Let** $R(x, y)$ **be the point which divides** PQ **in ratios** $m_1 : m_2$, **then the coordinates of** R **is**

$$R(x, y) = R\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \quad .....(i)$$

Put $x_1 = 3$, $y_1 = 4$, $x_2 = -6$, $y_2 = 2$

and $m_1 = 3$, $m_2 = -2$ **put (i)**

$$R(x, y) = R\left(\frac{(3)(-6)+(-2)(3)}{3-2}, \frac{(3)(2)+(-2)(4)}{3-2}\right)$$

$$= R\left(\frac{-18-6}{1}, \frac{6-8}{1}\right)$$

$$= R(-24, -2) \quad Ans.$$

(c) $P(-6\ ,\ 7)$, $Q(5\ ,\ -4)$ in the ratio $\frac{2}{7}:1$

Solution : $P(-6\ ,\ 7)$, $Q(5\ ,\ -4)$ and

$m_1 : m_2 = \frac{2}{7} : 1$ multiplied the R . H . S. of ratio by 7

Or $m_1 : m_2 = 2 : 7$

Let $R(x\ ,\ y)$ be the point which divides PQ in

ratios $m_1 : m_2$, then the coordinates of R is

$$R(x\ ,y) = R\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}\ ,\frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \ldots\ldots\text{(i)}$$

Put $x_1 = -6$, $y_1 = 7$, $x_2 = 5$, $y_2 = -4$

and $m_1 = 2$, $m_2 = 7$ put (i)

$$R(x\ ,\ y) = R\left(\frac{(2)(5) + (7)(-6)}{2+7}\ ,\ \frac{(2)(-4) + (7)(7)}{2+7}\right)$$

$$= R\left(\frac{10-42}{2+7}\ ,\ \frac{-8+49}{2+7}\right)$$

$$= R\left(-\frac{32}{9}\ ,\ \frac{41}{9}\right) \quad Ans.$$

Q.4: In each case, in what rate is the line segment PQ (joining the points $P_1(x_1\ ,\ y_1)$ *and* $Q(x_2\ ,y_2)$) divided by the point $R(x\ ,\ y)$.

(a) $P(8\ ,\ 10)$, $Q(-12\ ,\ 6)$, $R\left(-\frac{4}{7}\ ,\ \frac{58}{7}\right)$

Solution: Let $R\left(-\frac{4}{7}\ ,\ \frac{58}{7}\right)$ divides the join of

$P(8\ ,\ 10)$, $Q(-12\ ,\ 6)$ in ratio of $m_1 : m_2$

Then by ratios formula

$$R(x\ ,y) = R\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}\ ,\frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \ldots\ldots\text{(i)}$$

Put $x_1 = 8$, $y_1 = 10$, $x_2 = -12$, $y_2 = 6$,

$x = -\frac{4}{7}$, $y = \frac{58}{7}$ in (i)

$$R\left(-\frac{4}{7}, \frac{58}{7}\right) = R\left(\frac{m_1(-12)+m_2(8)}{m_1+m_2}, \frac{m_1(6)+m_2(10)}{m_1+m_2}\right)$$

$$\Rightarrow R\left(-\frac{4}{7}, \frac{58}{7}\right) = R\left(\frac{-12m_1+8m_2}{m_1+m_2}, \frac{6m_1+10m_2}{m_1+m_2}\right)$$

Equating the x and y coordinates

$$\frac{-12m_1+8m_2}{m_1+m_2} = -\frac{4}{7} \quad \text{and} \quad \frac{6m_1+10m_2}{m_1+m_2} = \frac{58}{7}$$

$-84m_1 + 56m_2 = -4m_1 - 4m_2$ **and** $42m_1 + 70m_2 = 58m_1 + 58m_2$

$4m_1 - 84m_1 + 60m_2 = 0$ **and** $42m_1 - 58m_1 + 70m_2 - 58m_2 = 0$

$\Rightarrow -80m_1 + 60m_2 = 0$ **and** $-16m_1 + 12m_2 = 0$

$\Rightarrow 80m_1 = 60m_2$ **and** $16m_1 = 12m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{60}{80} = \frac{3}{4}$ **and** $\frac{m_1}{m_2} = \frac{12}{16} = \frac{3}{4}$

Thus $m_1 = 3$, $m_2 = 4$ **or** $m_1 = 3$, $m_2 = 4$

Thus the required ratios is $m_1 : m_2 = 3:4$

(b) $P(-2, 4)$, $Q(3, 6)$, $R\left(\frac{4}{5}, \frac{3}{5}\right)$

<u>Solution</u>: Let $R\left(\frac{4}{5}, \frac{3}{5}\right)$ **divides the join of**

$P(-2, 4)$, $Q(3, 6)$ **in ratio of** $m_1 : m_2$

Then by ratios formula

$$R(x, y) = R\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \text{.....(i)}$$

Put $x_1 = -2$, $y_1 = 4$, $x_2 = 3$, $y_2 = 6$,

$x = \frac{4}{5}$, $y = \frac{3}{5}$ **in (i)**

$$R\left(\frac{4}{5}, \frac{3}{5}\right) = R\left(\frac{m_1(3)+m_2(-2)}{m_1+m_2}, \frac{m_1(6)+m_2(4)}{m_1+m_2}\right)$$

$$\Rightarrow R\left(\frac{4}{5}, \frac{3}{5}\right) = R\left(\frac{3m_1-2m_2}{m_1+m_2}, \frac{6m_1+4m_2}{m_1+m_2}\right)$$

Equating the x and y coordinates

$$\frac{3m_1-2m_2}{m_1+m_2} = \frac{4}{5} \quad \text{and} \quad \frac{6m_1+4m_2}{m_1+m_2} = \frac{3}{5}$$

$$\Rightarrow 15m_1 - 10m_2 = 4m_1 + 4m_2$$

$$\Rightarrow 15m_1 - 4m_1 = 10m_2 + 4m_2$$

$$\Rightarrow 11m_1 = 14m_2$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{14}{11}$$

Thus the ratios is $m_1 : m_2 = 14 : 11$ Ans.

<u>Q.5</u>: Find the centroid of the triangle ABC, whose vertices are the following.

(a) $A(4, -2)$, $B(-2, 4)$, $C(5, 5)$

<u>Solution</u>: The centroid of triangle ABC is given by

$$G(x, y) = G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) \quad \text{......(i)}$$

Put $x_1 = 4$, $y_1 = -2$, $x_2 = -2$, $y_2 = 4$, $x_3 = 5$, $y_3 = 5$ **in (i)**

$$\Rightarrow G(x, y) = G\left(\frac{4-2+5}{3}, \frac{-2+4+5}{3}\right)$$

$$= G\left(\frac{7}{3}, \frac{7}{3}\right)$$

(b) $A(3, 5)$, $B(4, 6)$, $C(3, -1)$

<u>Solution</u>: The centroid of triangle ABC is given by

$$G(x, y) = G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) \quad \text{......(i)}$$

Put $x_1 = 3$, $y_1 = 5$, $x_2 = -4$, $y_2 = 6$, $x_3 = 3$, $y_3 = -1$ **in (i)**

$$=> G(x\ ,\ y) = G\left(\frac{3-4+3}{3}\ ,\ \frac{5+6-1}{3}\right)$$

$$= G\left(\frac{2}{3}\ ,\ \frac{10}{3}\right).$$

(c) $A(1\ ,1)$, $B(-2\ ,-2)$, $C(4\ ,5)$

<u>Solution</u>: The centroid of triangle ABC is given by

$$G(x\ ,\ y) = G\left(\frac{x_1+x_2+x_3}{3},\quad \frac{y_1+y_2+y_3}{3}\right) \quad ......(i)$$

Put $x_1 = 1$, $y_1 = 1$, $x_2 = -2$, $y_2 = -2$, $x_3 = 4$, $y_3 = 5$ **in (i)**

$$=> G(x\ ,\ y) = G\left(\frac{1-2+4}{3}\ ,\ \frac{1-2+5}{3}\right)$$

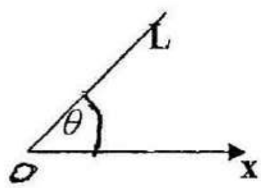
$$= G\left(\frac{3}{3}\ ,\ \frac{4}{3}\right) = G\left(1\ ,\ \frac{4}{3}\right) \quad Ans.$$

<u>Slope of a straight line</u>: The steepness of a straight line is called slope of the line.

The slope of the line is denoted by m.

If θ **is the inclination of the straight line then**

slope = $m = \tan\theta$



<u>Slope in two points form</u>:

Let $A(x_1\ ,y_1)$ *and* $B(x_2\ ,y_2)$ **be any two points on a straight line then the slope of the line is given by**

$$m = \frac{y_2 - y_1}{x_2 - x_2}.$$

<u>Example 5</u>: Find the slope m of the line L through the two points

(a) $E(2\ ,\ 4)$ *and* $F(4\ ,6)$ **(b)** $M(3\ ,1)$ *and* $N(-1\ ,3)$

<u>Solution</u>: (a) $E(2\ ,\ 4)$ *and* $F(4\ ,6)$

The slope of a line through the two points $A(x_1\ ,y_1)$ *and* $B(x_2\ ,y_2)$ **is given by**

$$m = \frac{y_2 - y_1}{x_2 - x_2} \quad .....(i)$$

Put $x_1 = 2$, $y_1 = 4$, $x_2 = 4$ *and* $y_2 = 6$ **in** (i)

$$m = \frac{6-4}{4-2} = \frac{2}{2} = 1 \text{ Ans.}$$

(b) $M(3\ ,1)$ *and* $N(-1\ ,3)$

Put $x_1 = 3$, $y_1 = 1$, $x_2 = -1$ *and* $y_2 = 3$ **in** (i)

$$m = \frac{3-1}{-1-3} = -\frac{2}{4} = -\frac{1}{2} \text{ Ans.}$$

Parallel lines: Two lines L_1 and L_2 having slopes m_1 *and* m_2 respectively. Then the lines L_1 and L_2 are parallel if $m_1 = m_2$

Perpendicular lines: Two lines L_1 and L_2 having slopes m_1 *and* m_2 respectively. Then the lines L_1 and L_2 are perpendicula if $m_1 \times m_2 = -1$

Equation of a straight line parallel to x–axis:

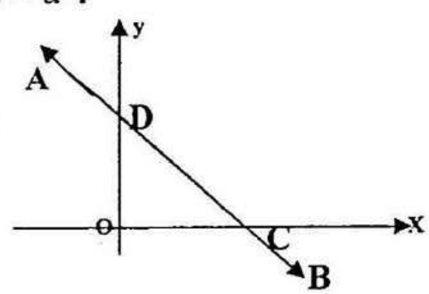
Equation of a straight line parallel to x–axis and passes through the point $(0\ ,\ a)$ is $y = a$.

Equation of a straight line parallel to y–axis:

Equation of a straight line parallel to y–axis and passes through the point $(d\ ,\ 0)$ is $x = d$.

Definition (Intercepts):

If a straight line AB intersects x – axis at point C and y –axis at point D, then OC is called the x – intercept of AB on the x –axis and OD is called the y – intercept on y –axis.



Note: For x intercept put $y = 0$ in the given equation.

For y intercept put $x = 0$ in the given equation.

Example .8: Find the x and y intercepts of a line

$$2x + 4y + 6 = 0$$

Solution: $2x + 4y + 6 = 0$ .....(i)

For x intercept : Put $y = 0$ in (i)

$$2x + 0 + 6 = 0 \quad \Rightarrow \quad 2x = -6$$

$$\Rightarrow \quad x = -3$$

For y intercept : Put $x = 0$ in (i)

$$2(0) + 4y + 6 = 0 \quad \Rightarrow \quad 4y = -6$$

$$\Rightarrow \quad y = -\frac{6}{4} = -\frac{3}{2}$$

<u>Standard form of equation of a straight line</u> :

(1) <u>Slope –Intercept form</u> : If a line L having slope m and y – intercept c on the y – axis .
Then the equation of the line is given by

$$y = mx + c$$

If the line passes through the origin i.e. c = 0
Then the equation of the line is given by

$$y = mx$$

(2) <u>Point - slope form</u> : If a line having slope m and Passing through the point $A(x_1 \ , \ y_1)$ **,**
Then the equation of the line is given by

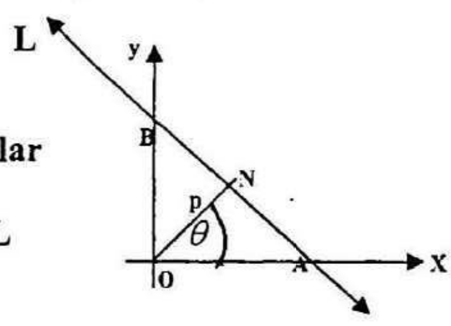
$$y - y_1 = m(x - x_1)$$

(3) <u>Two – points form</u> : If a line passing through the two points $A(x_1 \ , y_1)$ *and* $B(x_2 \ , \ y_2)$ **,**
Then the equation of the line is given by

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

(4) <u>Double intercept form</u> : If a line L intersects x – axis at $x = a$ **and y – axis at** $y = b$ **, where** $a \neq 0 \ , b \neq 0$
Then the equation of the line is given by

$$\frac{x}{a} + \frac{y}{b} = 1$$



(5) <u>Normal form</u> :
If a line L having perpendicular distance p from the origin and the perpendicular from the line L makes an angle θ **with positive**

x –axis ,

Then the equation of the line is given by

$$x\cos\theta + y\sin\theta = p$$

Example .10: Find an equation of the line with slope 4, when the y – intercept is 6.

Solution : Then the equation of the line is given by

$$y = mx + c \quad .....(i)$$

Put $m = 4$, $c = 6$ in(i)

$$y = 4x + 6 \quad \text{Ans.}$$

Example .11: Find an equation of the line with slope 4, and passes through the point $(2 , 4)$.

Solution : Then the equation of the line is given by

$$y - y_1 = m(x - x_1) \quad .....(i)$$

Put $m = 4$, $x_1 = 2$, $y_1 = 4$ in (i)

$$y - 4 = 4(x - 2) \quad \Rightarrow \quad y - 4 = 4x - 8$$

$$\Rightarrow \quad 0 = 4x - y - 8 + 4 \qquad or \quad 4x - y - 4 = 0 \quad \text{Ans.}$$

Example .12: Find an equation of the line that passes through the points $P(-1 , -2)$ and $Q(-5 , 0)$.

Solution: Then the equation of the line through $A(x_1 , y_1)$ and $B(x_2 , y_2)$ is given by

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \quad ....(i)$$

Put $x_1 = -1$, $y_1 = -2$, $x_2 = -5$, $y_2 = 0$ in (i)

$$y - (-2) = \frac{(0 - (-2))}{-5 - (-1)}(x - (-1))$$

$$\Rightarrow \quad y + 2 = \frac{2}{-5 + 1}(x + 1) \quad \Rightarrow \quad y + 2 = -\frac{2}{4}(x + 1)$$

$$\Rightarrow \quad y + 2 = -\frac{1}{2}(x + 1) \quad \Rightarrow \quad 2y + 4 = -x - 1$$

$$\Rightarrow \quad x + 2y + 4 + 1 = 0 \quad \Rightarrow \quad x + 2y + 5 = 0 \qquad Ans.$$

Example .13: Find the equation of a line whose x and y intercepts are $(3, 0)$ and $(0, 4)$ respectively.

Solution: Then the equation of the line whose x intercepts a and y intercepts b is given by

$$\frac{x}{a}+\frac{y}{b}=1 \quad ....(i)$$

Put $a=3$ and $b=4$ in (i)

$$\frac{x}{3}+\frac{y}{4}=1 \qquad \text{Multiplied by } 12$$

=> $4x+3y=12$ or $4x+3y-12=0$ **Ans.**

(6) **Symetric form**: If θ is the inclination of a line L and passing through the point $P(x_1, y_1)$.

Then equation of the line in symmetric form is

$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}$$

Example.13: Find the equation of a straight line with inclination 45^o and passing through the point $(2, \sqrt{2})$

Solution: Then equation of the line in symmetric form is

$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta} \quad .....(i) \quad , \text{put } \theta=45^o, \ x_1=2, \ y_1=\sqrt{2}$$

$$\Rightarrow \quad \frac{x-2}{\cos 45^o}=\frac{y-\sqrt{2}}{\sin 45^o} \Rightarrow \frac{x-2}{\frac{1}{\sqrt{2}}}=\frac{y-\sqrt{2}}{\frac{1}{\sqrt{2}}}$$

=> $x-2=y-\sqrt{2}$ *both sides multiplied by* $\frac{1}{\sqrt{2}}$

=> $x-y-2+\sqrt{2}=0$ **Ans.**

Example .15: Find the corresponding equation of a line, if the length of the perpendicular distance from the origin is 3 units that makes an angle of 120^o.

Solution: The equation of the line in normal form is given by

$$x\cos\theta+y\sin\theta=p \quad .....(i)$$

Put $p=3$ and $\theta=120^o$ in (i)

$$x\cos 120^o + y\sin 120^o = 3 \quad \Rightarrow \quad (x)\left(-\frac{1}{2}\right)+y\left(\frac{\sqrt{3}}{2}\right)=3$$

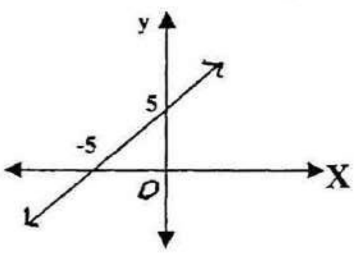
$$\Rightarrow -x+\sqrt{3}y=6 \quad \Rightarrow \quad 0=x-\sqrt{3}\,y+6$$

Or $x-\sqrt{3}y+6=0$ **Ans.**

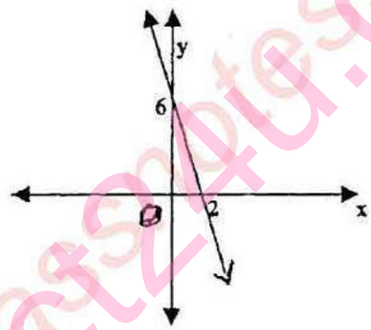
Exercise 7.2:

Q.1: Find the equation of lines that that are represented on the coordinate planes .

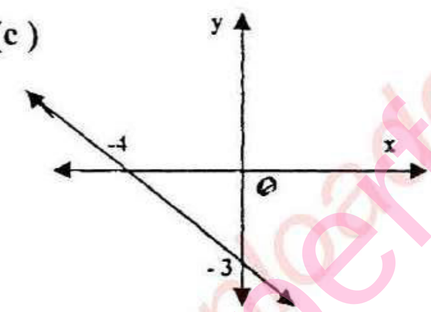
(a)



(b)



(c)



Solution: (a) Since the line intersects x – axis at (-5 , 0) and y –axis at (0 , 5) .

The equation of the line is given by $\frac{x}{a}+\frac{y}{b}=1$ **.......(i)**

Put $a=-5$ **and** $b=5$ **in (i)**

$$\Rightarrow \frac{x}{-5}+\frac{y}{5}=1 \quad \Rightarrow \quad -x+y=5$$

$$\Rightarrow 0=x-y+5 \quad or \quad x-y+5=0$$ **Ans.**

(b) Since the line intersects x – axis at (2 , 0) and y –axis at (0 , 6) .

The equation of the line is given by $\frac{x}{a}+\frac{y}{b}=1$ ....(i)

Put $a=2$ and $b=6$ in (i)

$\frac{x}{2}+\frac{y}{6}=1$ multiplied by 6

=> $3x+y=6$ Or $3x+y-6=0$ Ans.

(c) Since the line intersects x – axis at (– 4 , 0) and y –axis at (0 , -3).

The equation of the line is given by $\frac{x}{a}+\frac{y}{b}=1$ ....(i)

Put $a=-4$ and $b=-3$ in (i)

$\frac{x}{-4}+\frac{y}{-3}=1$, Both sides multiplied by -12

=> $3x+4y=-12$ => $3x+4y+12=0$

=> $4y=-3x-12$ *or* $y=-\frac{3}{4}x-4$ *Ans.*

<u>Q.2</u>: In each case, find the slope, it is defined :

(a) $(-5,4)$ and $(3,6)$ (b) $4x+7y=1$

(c) Parallel to $2y-4x=7$ (d) perpendicular to $6x=y-3$

<u>Solution</u>: The slope of two points $A(x_1,y_1)$ and $B(x_2,y_2)$

is given by $m=\frac{y_2-y_1}{x_2-x_1}$ ....(i)

put $x_1=-5$, $y_1=4$, $x_2=3$, $y_2=6$ in(i)

$$m=\frac{6-4}{3-(-5)}=\frac{2}{3+5}$$

=> $m=\frac{2}{8}=\frac{1}{4}$ Ans.

(b) $4x+7y=1$ => $7y=-4x+1$

=> $y=-\frac{4}{7}x+\frac{1}{7}$

Thus slope of the given line is $m=-\frac{4}{7}$ Ans.

(c) **Parallel to** $2y-4x=7$

$$\Rightarrow 2y=4x+7 \quad \text{or} \quad y=\frac{4}{2}x+\frac{7}{2}$$

$$\Rightarrow y=2x+\frac{7}{2}$$

Thus the slope of the given line is $m=2$

Then the slope of the required line parallel to the given line is also $m=2$.

(d) **perpendicular to** $6x=y-3$

$$\Rightarrow y=6x+3$$

Then the slope of the given line is $m_1=6$

Let m_2 **be the slope of the line perpendicular to the given line. Then** $m_1.m_2=-1$

$$\Rightarrow m_2=-\frac{1}{m_1}=-\frac{1}{6} \text{ Ans.}$$

Q3: What are the x and y – intercepts for each of the following lines ?

(a) $y=2x+6$

Solution: $y=2x+6$ ....(i)

For x – intercept : Put $y=0$ **in (i)**

$0=2x+6 \Rightarrow 2x=-6 \Rightarrow x=-3$

For y – intercept : Put $x=0$ **in (i)**

$y=2(0)+6=6$

Thus x – intercept $=-3$ **and y – intercept** $=6$

(b) $y=-3x+9$

Solution: $y=-3x+9$ ......(i)

For x – intercept : Put $y=0$ **in (i)**

$0=-3x+9 \Rightarrow 3x=9 \Rightarrow x=3$

For y – intercept : Put $x=0$ **in (i)**

$y=-3(0)+9=9$

Thus x – intercept $=3$ **and y – intercept** $=9$.

(c) $y=x+2$

Solution: $y = x + 2$ ......(i)

For x – intercept : Put $y = 0$ in (i)

$0 = x + 2 \quad \Rightarrow \quad x = -2$

For y – intercept : Put $x = 0$ in (i)

$y = 0 + 2 = 2$

Thus x – intercept $= -2$ and y – intercept $= 2$.

Q.4: In each case, show that the pair of lines are parallel or perpendicular or neither.

(a) $x - 2y - 6 = 0 \ldots$(i) , $2x + y - 5 = 0$ .....(ii)

Solution: from (i) => $x - 2y - 6 = 0 \quad \Rightarrow \quad x - 6 = 2y$

$\Rightarrow \quad y = \frac{1}{2}x - 3 \ldots$ (a)

Compare (a) with $y = mx + c$ ...(b) .

Then slope of line (i) is $m_1 = \frac{1}{2}$

From (ii) => $2x + y - 5 = 0$ or $y = -2x + 5$ ....(c)

Compare (c) with (b) .

Then slope of the line (c) is $m_2 = -2$

Now $m_1.m_2 = \left(\frac{1}{2}\right)(-2) = -1$

Hence the lines (i) and (ii) are perpendicular to each other.

(b) $3x + 4y - 8 = 0 \ldots$(i) , $x + 3y - 2 = 0$ .....(ii)

Solution: from (i) => $3x + 4y - 8 = 0 \quad \Rightarrow \quad 4y = -3x + 8$

$\Rightarrow \quad y = -\frac{3}{4}x + 2 \ldots$ (a)

Compare (a) with $y = mx + c$ ...(b) .

Then slope of line (i) is $m_1 = -\frac{3}{4}$

From (ii) => $x + 3y - 2 = 0$ or $3y = -x + 2$

$\Rightarrow \quad y = -\frac{1}{3}x + \frac{2}{3}$ ....(c)

Compare (c) with (b) .

Then slope of the line (c) is $m_2 = -\frac{1}{3}$

Now $m_1.m_2 = \left(-\frac{3}{4}\right)\left(-\frac{1}{3}\right) = \frac{1}{4} \neq -1$

And $m_1 \neq m_2$

The lines (i) and (ii) neither parallel nor perpendicular to each other.

(c) $2x - y - 7 = 0$ ...(i) , $4x - 2y - 5 = 0$ .....(ii)

Solution: from (i) => $2x - y - 7 = 0$ => $2x - 7 = y$

=> $y = 2x - 7$ .... (a)

Compare (a) with $y = mx + c$ ...(b) .

Then slope of line (i) is $m_1 = 2$

From (ii) => $4x - 2y - 5 = 0$ or $4x - 5 = 2y$

=> $y = \frac{4}{2}x - \frac{5}{2}$ or $y = 2x - \frac{5}{2}$ ....(c)

Compare (c) with (b) .

Then slope of the line (c) is $m_2 = 2$

Since $m_1 = .m_2$

Hence the lines (i) and (ii) are parallel to each other.

(d) $2x - 5y - 7 = 0$...(i) , $6x - 15y + 5 = 0$ .....(ii)

Solution: from (i) => $2x - 5y - 7 = 0$ => $2x - 7 = 5y$

=> $y = \frac{2}{5}x - \frac{7}{5}$.... (a)

Compare (a) with $y = mx + c$ ...(b) .

Then slope of line (i) is $m_1 = \frac{2}{5}$

From (ii) => $6x - 15y + 5 = 0$ or $6x + 5 = 15y$

=> $y = \frac{6}{15}x + \frac{5}{15}$ or $y = \frac{2}{5}x + \frac{1}{3}$ ....(c)

Compare (c) with (b) .

Then slope of the line (c) is $m_2 = \frac{2}{5}$

Since $m_1 = m_2$

Hence the lines (i) and (ii) are parallel to each other.

Q.5: In each case, find the equation of a line that passes through the pair of points:

(a) $O(0\ ,\ 0)$ *and* $A(2\ ,\ 6)$ (b) $E(1\ ,\ 0)$ *and* $F(2\ ,\ 5)$

(c) $I(1\ ,\ 1)$ *and* $J(3\ ,\ 3)$

Solution: (a) $O(0\ ,\ 0)$ *and* $A(2\ ,\ 6)$

Then the equation of the line through $A(x_1\ ,\ y_1)$ *and* $B(x_2\ ,\ y_2)$ is given by

$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \text{ ....(i)}$$

Put $x_1 = 0\ ,\ y_1 = 0\ ,\ x_2 = 2\ ,\ y_2 = 6$ in (i)

$$y - 0 = \frac{(6-0)}{(2-0)}(x-0) \Rightarrow y = \frac{6}{2}x$$

$$\Rightarrow y = 3x \quad Ans.$$

(b) $E(1\ ,\ 0)$ *and* $F(2\ ,\ 5)$

Put $x_1 = 1\ ,\ y_1 = 0\ ,\ x_2 = 2\ ,\ y_2 = 5$ in (i)

$$y - 0 = \frac{(5-0)}{(2-1)}(x-1) \Rightarrow y = 5(x-1)$$

$$\Rightarrow y = 5x - 5 \quad or \quad 0 = 5x - y - 5$$

$$\Rightarrow 5x - y - 5 = 0 \quad Ans.$$

(c) $I(1\ ,\ 1)$ *and* $J(3\ ,\ 3)$

Put $x_1 = 1\ ,\ y_1 = 1\ ,\ x_2 = 3\ ,\ y_2 = 3$ in (i)

$$y - 1 = \frac{(3-1)}{(3-1)}(x-1) \Rightarrow y - 1 = \frac{2}{2}(x-1)$$

$$\Rightarrow y - 1 = x - 1 \quad or \quad 0 = x - y - 1 + 1$$

$$\Rightarrow x - y = 0 \quad Ans.$$

Q.6: In each case, the equation of a line that passes through the point $A(x\ ,\ y)$ having slope m:

(a) $A(1\ ,\ 2)\ ,\ m = 4$ (b) $A(-1\ ,-2)\ ,\ m = -\frac{1}{2}$

(c) $A(-3\,,5)$, $m=-3$ (d) $A(7\,,-8)$, $m=5$

Solution: (a) $A(1\,,\,2)$, $m=4$

Then the equation of the line is given by

$$y-y_1=m(x-x_1) \quad .....(i)$$

Put $m=4$, $x_1=1$, $y_1=2$ in (i)

$y-2=4(x-1)$ => $y-2=4x-4$

=> $0=4x-y-4+2$ => $4x-y-2=0$ **Ans.**

(b) $A(-1\,,-2)$, $m=-\frac{1}{2}$

Put $m=-\frac{1}{2}$, $x_1=-1$, $y_1=-2$ in (i)

$y-(-2)=-\frac{1}{2}(x-(-1))$ => $y+2=-\frac{1}{2}(x+1)$

=> $2y+4=-x-1$ => $x+2y+5=0$ *Ans.*

(c) $A(-3\,,5)$, $m=-3$

Put $m=-3$, $x_1=-3$, $y_1=5$ in (i)

$y-5=-3(x-(-3))$ => $y-5=-3(x+3)$

=> $y-5=-3x-9$ *or* $3x+y-5+9=0$

=> $3x+y+4=0$ **Ans.**

(d) $A(7\,,-8)$, $m=5$

Put $m=5$, $x_1=7$, $y_1=-8$ in (i)

$y-(-8)=5(x-7)$ => $y+8=5x-35$

=> $0=5x-y-35-8$ **Or** $5x-y-43=0$ **Ans.**

Q.7: In each case , find the equation of a line that exists the $y-$ intercept c and slope m :

(a) $c=2$, $m=2$ (b) $c=4$, $m=8$

(c) $c=-4$, $m=\frac{1}{2}$

Solution : (a) $c=2$, $m=2$

Then the equation of the line is given by

$$y=mx+c \quad .....(i)$$

Put $m=2$, $c=2$ in(i)

$$y=2x+2 \quad \Rightarrow \quad 0=2x-y+2$$

$\Rightarrow 2x-y+2=0$ **Ans.**

(b) $c=4,\ m=8$

Put $c=4,\ m=8$ **in(i)**

$$y=8x+4 \quad \Rightarrow \quad 0=8x-y+4$$

$\Rightarrow 8x-y+4=0$ **Ans.**

(c) $c=-4$, $m=\dfrac{1}{2}$:

Put $c=-4$, $m=\dfrac{1}{2}$ **in (i)**

$$y=\frac{1}{2}x-4 \quad \Rightarrow \quad 2y=x-8$$

$\Rightarrow 0=x-2y-8$ *or* $x-2y-8=0$ *Ans.*

<u>Q.8</u>: Transform the equation $7x-10y+13=0$ into

(a) slope intercept form (b) symmetric form

(c) Normal form

<u>Solution</u>: $7x-10y+13=0$ **...(i)**

(a) From (i) $\Rightarrow 7x+13=10y$

$\Rightarrow y=\dfrac{7}{10}x+\dfrac{13}{10}$ **is the required slope intercept form**

With slope $m=\dfrac{7}{10}$ **and intercept** $=\dfrac{13}{10}$

(b) symmetric from : (i) $\Rightarrow 7x+13=10y$

$\Rightarrow 10y=7x+13$ *or* $10y=7x+7+6$

$\Rightarrow 10y-6=7x+7 \Rightarrow 10\left(y-\dfrac{6}{10}\right)=7(x+1)$

$\Rightarrow \dfrac{\left(y-\dfrac{3}{5}\right)}{\dfrac{1}{10}}=\dfrac{x-(-1)}{\dfrac{1}{7}}$ *is required symmetric form*

With $\cos\theta = \frac{1}{10}$, $\sin\theta = \frac{1}{7}$ *and point* $\left(-1,\ \frac{3}{5}\right)$

(c) Normal form : From (i) $7x - 10y = -13$ .....(i)

Both sides of eq(i) is divided by $\sqrt{(7)^2 + (-10)^2} = -\sqrt{149}$

$$=> \frac{7x}{-\sqrt{149}} - \frac{10y}{-\sqrt{148}} = \frac{-13}{-\sqrt{149}}$$

$$=> \left(-\frac{7}{\sqrt{149}}\right)x + \left(\frac{10}{\sqrt{148}}\right)y = \frac{13}{\sqrt{149}}$$ **which is the**

requird normal form with $\cos\theta = -\frac{7}{\sqrt{149}}$ **and**

$\sin\theta = \frac{10}{\sqrt{149}}$ **and** $P = \frac{13}{\sqrt{149}}$

Position of a point with respect to a line :

Let $ax + by + c = 0$ **.....(i) is a straight line with** $b > 0$ **.**

Then a point $P(x_1\ , y_1)$ **lie above the line (i) if**

$ax_1 + by_1 + c > 0$

and the point $Q(x_2\ ,\ y_2)$ **lie below the line if**

$ax_2 + by_2 + c < 0$ **.**

Note : (i) $O(0\ ,\ 0)$ **is the origin .If substituted the origin in the expression** $ax + by + c = 0$ **, if** $c > 0$ **then the origin in the positive region and if** $c < 0$ **then the origin is in the negative region .**

(ii) If the two points $P(x_1\ , y_1)$ **and** $Q(x_2\ ,\ y_2)$ **lie above or below the line** $ax + by + c = 0$ **then the points** $P(x_1\ , y_1)$ **and** $Q(x_2\ ,\ y_2)$ **lie on the same side of the line.**

(iii) If point $P(x_1\ , y_1)$ **lie above and the point** $Q(x_2\ ,\ y_2)$ **lie below the line** $ax + by + c = 0$ **,then the points** $P(x_1\ , y_1)$ **and** $Q(x_2\ ,\ y_2)$ **have opposite signs .**

Example .16: Determine whether the point $P(10\ ,\ -6)$ **lies**

above or below the line $9x+10y-3=0$ .Show that the point and the origin lie on the same or on the opposite sides of the given line.

Solution: $9x+10y-3=0$ ....(i)

Put the point $P(10\,,-6)$ in (i)

$$9(10)+10(-6)-3=90-60-3=27>0$$

Thus the point $P(10\,,-6)$ lie above the line.

Now put the origin $O(0\,,\ 0)$ in (i)

$$9(0)+10(0)-3=0+0-3=-3<0$$

Then the origin $O(0\,,\ 0)$ lie below the line.

Thus the origin $O(0\,,\ 0)$ and the point $P(10\,,-6)$ lie on the opposite side of the given line.

Perpendicular Distance of a point from a line:

Let $ax+by+c=0$ .....(i) is a line and $P(x_1\,,y_1)$ is a point out side from the line (i).

Then the perpendicular distance of the point $P(x_1\,,y_1)$ from the line (i) is $d=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Example 17: Find the perpendicular distance from a line $7x+3y-9=0$ to a point $P(2\,,3)$.

Solution: $7x+3y-9=0$ ......(i) and point $P(2\,,3)$.

The perpendicular distance of the point $P(x_1\,,y_1)$ from the line (i) is

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \quad ......(ii)$$

Put $a=7\,,b=3\,,c=-9\,,x_1=2\,,\ y_1=3$ in (ii)

$$d=\frac{|(7)(2)+(3)(3)-9|}{\sqrt{(7)^2+(3)^2}}$$

$$=> d=\frac{|14+9-9|}{\sqrt{49+9}}=\frac{14}{\sqrt{58}} \quad Ans.$$

Angles between two coplanar intersecting lines :

Let $a_1x + b_1y + c_1 = 0$ ....(i) and

$a_2x + b_2y + c_2 = 0$ ....(ii) are two lines.

Let m_1 *and* m_2 are the slopes of lines (i) and (ii) respectively. Let θ is the angle between the two lines.

Then $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1m_2}$.

Note: (i) If $\dfrac{m_1 - m_2}{1 + m_1m_2}$ is positive then the angle is acute.

(ii) If $\dfrac{m_1 - m_2}{1 + m_1m_2}$ is negative then the angle is obtuse.

Example .18: Find the angle from the line $7x + 3y - 9 = 0$ to the line $5x - 2y + 2 = 0$

Solution: $7x + 3y - 9 = 0$ ...(i) and $5x - 2y + 2 = 0$ ....(i)

We change (i) and (ii) to slope intercept form $y = mx + c$ to find slopes of these lines.

From (i) => $3y = -7x + 9$

=> $y = -\dfrac{7}{3}x + \dfrac{9}{3}$ => $y = -\dfrac{7}{3}x + 3$

Then the slope of the line (i) is $m_1 = -\dfrac{7}{3}$

From (ii) => $-2y = -5x - 2$

=> $y = \dfrac{5}{2}x + \dfrac{2}{2}$ => $y = \dfrac{5}{2}x + 1$

Then the slope of the line (ii) is $m_2 = \dfrac{5}{2}$

If θ is the angle from line (i) to line (ii)

Then $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1m_2}$ put values

$$\tan\theta = \frac{-\frac{7}{3}-\frac{5}{2}}{1+\left(-\frac{7}{3}\right)\left(\frac{5}{2}\right)} = \frac{\frac{-14-15}{6}}{\frac{6-35}{6}} = -\frac{29}{6}\times\left(-\frac{6}{29}\right) = 1$$

=> $\theta = \tan^{-1}(1) = 45^{o}$ **Ans.**

<u>Equation of a family of lines passing through the point of Intersection of two lines</u> :

Let the two lines are $a_1x + b_1y + c_1 = 0$ **....(i)**

and $a_2x + b_2y + c_2 = 0$ **.....(ii)**

Then Equation of a family of lines passing through the point of intersection of two lines (i) and (ii) is

$a_1x + b_1y + c_1 + \lambda(a_2x + b_2y + c_2) = 0$ **.....(A) , where** λ **is constant .**

<u>Example 19</u>: Develop the family of lines through the point of intersection of the lines $2x - 3y + 4 = 0$ **and** $2x + y - 1 = 0$.

Find the line from the family of lines which is

(i) Parallel to the line whose slope is $m_1 = -\frac{2}{3}$

(ii) Perpendicular to the line $4x + 3y - 1 = 0$

<u>Solution</u>: Given lines are $2x - 3y + 4 = 0$ **.....(i)**

$2x + y - 1 = 0$ **.....(ii)**

Then Equation of a family of lines passing through the Point of intersection of two lines (i) and (ii) is

$2x - 3y + 4 + \lambda(2x + y - 1) = 0$ **....(A)**

=> $2x - 3y + 4 + 2\lambda x + \lambda y - \lambda = 0$

=> $(2 + 2\lambda)x + (\lambda - 3)y + 4 - \lambda = 0$ ....(B)

Eq (A) or (B) shows the family of lines through the point of intersection of (i) and (ii) .

Now we find the slope of (B) .

From (B) => $(\lambda - 3)y = -(2 + 2\lambda)x + \lambda - 4$

$$\Rightarrow y=-\frac{(2+2\lambda)}{(\lambda-3)}x+\lambda-4 \quad \Rightarrow y=-\frac{(2+2\lambda)}{-(3-\lambda)}x+\lambda-4$$

$$\Rightarrow y=\frac{(2+2\lambda)}{(3-\lambda)}x+\lambda-4 \quad .....(C)$$

Then the slope of family of lines (A) is $m_2=\dfrac{2+2\lambda}{3-\lambda}$

(i)The family of lines is parallel to the line whose slope is

$m_1=-\dfrac{2}{3}$ **. Then** $m_1=m_2$

$$\Rightarrow \quad -\frac{2}{3}=\frac{2+2\lambda}{3-\lambda} \quad \Rightarrow -6+2\lambda=6+6\lambda$$

$$\Rightarrow 2\lambda-6\lambda=6+6 \quad \Rightarrow -4\lambda=12$$

$$\Rightarrow \lambda=-3$$

Put $\lambda=-3$ **in (A) to find the line parallel to the line whose slope is** $m_1=-\dfrac{2}{3}$

$$2x-3y+4-3(2x+y-1)=0$$

$$\Rightarrow 2x-3y+4-6x-3y+3=0$$

$$\Rightarrow -4x-6y+7=0$$

$$\Rightarrow 4x+6y-7=0 \quad Ans.$$

(ii) Perpendicular to the line $4x+3y-1=0$ **....(iii)**

$$\Rightarrow 3y=-4x+1 \quad \Rightarrow y=-\frac{4}{3}x+\frac{1}{3}$$

Then the slope (iii) is $m_3=-\dfrac{4}{3}$

Since the family of lines (A) is perpendicular to the line (iii)
Then $m_2.m_3=-1$ **put values**

$$\left(\frac{2+2\lambda}{3-\lambda}\right)\left(-\frac{4}{3}\right)=-1$$

$$\Rightarrow -4(2+2\lambda)=-3(3-\lambda)$$

$\Rightarrow\ 8+8\lambda=9-3\lambda \quad \Rightarrow\ 8\lambda+3\lambda=9-8$

$\Rightarrow\ 11\lambda=1 \quad \Rightarrow\ \lambda=\dfrac{1}{11}$

Put $\lambda=\dfrac{1}{11}$ in (A) to find the line from the family of (A) which is perpendicular to (iii)

$$2x-3y+4+\frac{1}{11}(2x+y-1)=0$$

$\Rightarrow\ 22x-33y+44+2x+y-1=0$

$\Rightarrow\ 24x-32y+43=0 \qquad Ans.$

<u>Angles of the triangle</u>: If $A(x_1\ ,\ y_1)$, $B(x_2\ ,\ y_2)$ and $C(x_3\ ,y_3)$ are the vertices of triangle ABC .

The slopes of the sides AB , BC and CA are

$$m_1=\frac{y_2-y_1}{x_2-x_1}\ ,\ m_2=\frac{y_3-y_2}{x_3-x_2}\ \ and\ \ m_3=\frac{y_1-y_3}{x_1-x_3}$$

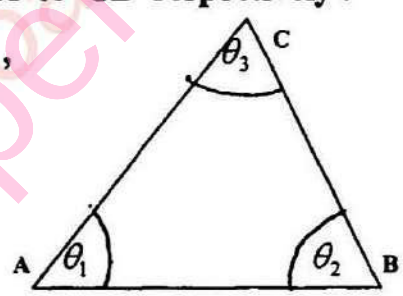
respectively .

Let θ_1 , θ_2 *and* θ_3 are the angles between the sides AB to AC , BC to BA and CA to CB respectively .

Then $\tan\theta_1=\dfrac{m_1-m_3}{1+m_1m_3}$,

$\tan\theta_2=\dfrac{m_2-m_1}{1+m_1m_2}$ **and**

$\tan\theta_3=\dfrac{m_3-m_2}{1+m_2m_3}$ **are**



The angles formulae from the sides AB to AC , BC to BA , and CA to CB respectively .

<u>Example 20</u>: Find the angles of the triangle ABC , whose vertices are $A(-2\ ,-3)$, $B(4\ ,-1)$ and $C(2\ ,\ 3)$.

<u>Solution</u>: The slope of side AB is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-3)}{4-(-2)} = \frac{-1+3}{4+2} = \frac{2}{6} = \frac{1}{3}$$

The slope of side BC is

$$m_2 = \frac{y_3 - y_2}{x_3 - x_2} = \frac{3-(-1)}{2-4} = \frac{3+1}{-2} = -\frac{4}{2} = -2$$

The slope of side CA **is**

$$m_3 = \frac{y_1 - y_3}{x_1 - x_3} = \frac{-3-3}{-2-2} = \frac{-6}{-4} = \frac{3}{2}$$

Let θ_1 **is the angle from the side AB to AC.**

Then $\tan\theta_1 = \dfrac{m_1 - m_3}{1+m_1m_3}$ **put values**

$$\tan\theta_1 = \frac{\frac{1}{3}-\frac{3}{2}}{1+\left(\frac{1}{3}\right)\left(\frac{3}{2}\right)} = \frac{\frac{2-9}{6}}{\frac{6+3}{6}} = -\frac{7}{6}\times\frac{6}{9} = -\frac{7}{9}$$

$$\Rightarrow \quad \theta_1 = \tan^{-1}\left(-\frac{7}{9}\right)$$

Let θ_2 **is the angle from the side BC to BA.**

Then $\tan\theta_2 = \dfrac{m_2 - m_1}{1+m_1m_2}$

put values

$$\tan\theta_2 = \frac{-2-\frac{1}{3}}{1+(-2)\left(\frac{1}{3}\right)} = \frac{\frac{-6-1}{3}}{\frac{3-2}{3}} = -\frac{7}{3}\times\frac{3}{1} = -7$$

$$\Rightarrow \quad \theta_2 = \tan^{-1}(-7)$$

Let θ_3 **is the angle from the side CA to CB.**

Then $\tan\theta_3 = \dfrac{m_3 - m_2}{1+m_2m_3}$ **put values**

$$\tan\theta_3 = \frac{\frac{3}{2}-(-2)}{1+(-2)\left(\frac{3}{2}\right)} = \frac{\frac{3+4}{2}}{1-3} = \frac{7}{2(-2)} = -\frac{7}{4}$$

$$\Rightarrow \theta_3 = \tan^{-1}\left(-\frac{7}{4}\right)$$

Exercise 7.3:

Q.1: Show that the point $P(x_1, y_1)$ lies above or below the line $ax+by+c=0$. Also show that the point $P(x_1, y_1)$ and the origin lie on the same side or on the opposite side of the line $ax+by+c=0$.

(a) $P(4, -5)$, $4x-3y-17-0$

(b) $P(-3, 8)$, $5x+7y+9=0$

Solution: (a) $4x-3y-17-0$ .....(a)

Here $b=-3$ is negative then eq(a) is multiplied by -1.

$\Rightarrow -4x+3y+17=0$ ......(i)

Put the point $P(4, -5)$ in (i) i.e. $x=4$, $y=-5$ in (i)

$-4(4)+3(-5)+17=-16-15+17=-14<0$

Then the point $P(4, -5)$ lie below the line (i).

Now take origin $O(0, 0)$: put $x=0$, $y=0$ in (i)

$4(0)-3(0)-17=0-0-17=-17<0$

Then the origin $O(0, 0)$ is below the line (i).

Thus the point $P(4, -5)$ and the origin $O(0, 0)$ lie on the same side of the line (i).

(b) $5x+7y+9=0$ ....(i) , $P(-3, 8)$

Put $x=-3$, $y=8$ in (i)

$5(-3)+7(8)+9=-15+56+9=50>0$

Then the point $P(-3, 8)$ lie above the line (i).

Now take origin $O(0, 0)$: put $x=0$, $y=0$ in (i)

$5(0)+7(0)+9=0+0+9=9>0$

Then the origin is also lie above the line (i).

Thus the point $P(-3\ ,8)$ and the origin $O(0\ ,\ 0)$ lie on the same side of the line (i).

Q.2: In each case, show that the points $P(x_1\ ,\ y_1)$ and $Q(x_2\ ,\ y_2)$ are on the same side or on the opposite side of the line $ax+by+c=0$

(a) $P(-4\ ,\ 2)$, $Q(11\ ,-3)$; $5x+14y-11=0$

Solution: $5x+14y-11=0$ ....(i)

Take point $P(-4\ ,2)$: Put $x=-4$, $y=2$ in (i)

$5(-4)+14(2)-11=-20+28-11=-3<0$

Then the point $P(-4\ ,\ 2)$ is below the line (i).

Now take the point $Q(11\ ,-3)$:

Put $x=11$, $y=-3$ in (i)

$5(11)+14(-3)-11=55-42-11=2>0$

Then the point $Q(11\ ,-3)$ lie above the line (i)

Thus the points $P(-4\ ,\ 2)$ and $Q(11\ ,-3)$ are lie on the opposite sides of the line (i).

(b) $P(-3\ ,\ 5)$, $Q(1,\ -2)$; $2x-3y-10=0$

Solution: $2x-3y-10=0$ ......(a)

Here $b=-3$ is negative then eq(a) is multiplied by -1.

$-2x+3y+10=0$ .....(i)

Take point $P(-3\ ,\ 5)$:

Put $x=-3$, $y=5$ in (i)

$-2(-3)+3(5)+10=6+15+10=31>0$

Then the point $P(-3\ ,\ 5)$ is above the line (i)

Take $Q(1,\ -2)$: put $x=1$, $y=-2$ in (i)

$-2(1)+3(-2)+10=-2-6+10=2>0$

Then the point $Q(1,\ -2)$ is also lie above the line (i).

Thus the points $P(-3\ ,\ 5)$ and $Q(1,\ -2)$ lie on the same side of the line (i).

Q.3: In each case, find the perpendicular distance from the line $ax+by+c=0$ to a point $P(x_1, y_1)$.

(a) $P(3, -4)$, $4x-3y+6=0$ (b) $P(5, 8)$, $3x-2y+7=0$

(c) $P(3, -1)$, $5x+12y-16=0$

Solution: $4x-3y+6=0$ ......(i) and point $P(3, -4)$.

The perpendicular distance of the point $P(x_1, y_1)$ from the line (i) is

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \quad ......(A)$$

Put $a=4$, $b=-3$, $c=6$, $x_1=3$, $y_1=-4$ in (A)

$$d=\frac{|(4)(3)+(-3)(-4)+6|}{\sqrt{(4)^2+(-3)^2}}$$

$$\Rightarrow d=\frac{|12+12+6|}{\sqrt{16+9}}=\frac{30}{\sqrt{25}}=\frac{30}{5}=6 \quad Ans.$$

(b) $3x-2y+7=0$, $P(5, 8)$

Put $a=3$, $b=-2$, $c=7$, $x_1=5$, $y_1=8$ in (A)

$$d=\frac{|(3)(5)+(-2)(8)+7|}{\sqrt{(3)^2+(-2)^2}}$$

$$\Rightarrow d=\frac{|15-16+7|}{\sqrt{9+4}}=\frac{6}{\sqrt{13}} \quad Ans.$$

(c) $5x+12y-16=0$, $P(3, -1)$

Put $a=5$, $b=12$, $c=-16$, $x_1=3$, $y_1=-1$ in (A)

$$d=\frac{|(5)(3)+(12)(-1)-16|}{\sqrt{(5)^2+(12)^2}}$$

$$\Rightarrow d=\frac{|15-12-16|}{\sqrt{25+144}}=\frac{|-13|}{\sqrt{169}}=\frac{13}{13}=1 \quad Ans.$$

Q.4: In each case, find the angle θ from the line L_1 to line L_2, if the slopes of the lines L_1 *and* L_2 are the following :

(a) $L_1 : m_1 = \frac{1}{2}$, $L_2 : m_2 = 3$ (b) $L_1 : m_1 = 2$, $L_2 : m_2 = 3$

Solution: Let θ is the angle from the line L_1 to line L_2 ,

then $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$ ....(A)

(a) $L_1 : m_1 = \frac{1}{2}$, $L_2 : m_2 = 3$ put values in (A)

$$\tan\theta = \frac{3 - \frac{1}{2}}{1 + \left(\frac{1}{2}\right)(3)} = \frac{\frac{6-1}{2}}{\frac{2+3}{2}} = \frac{5}{2} \times \frac{2}{5} = 1$$

=> $\theta = \tan^{-1}(1) = 45^o$ *Ans.*

(b) $L_1 : m_1 = 2$, $L_2 : m_2 = 3$ put values in (A)

$$\tan\theta = \frac{3-2}{1+(3)(2)} = \frac{1}{1+6} = \frac{1}{7}$$

=> $\theta = \tan^{-1}\left(\frac{1}{7}\right)$ *Ans.*

Q.5: In each case, find the angle θ from the line L_1 to L_2?

(a) L_1 : joins $(1, 2)$ and $(7, -1)$, L_2 : joins $(3, 2)$ and $(5, 6)$

(b) L_1 : joins $(2, 7)$ and $(7, 10)$, L_2: joins $(1, 1)$ and $(-5, 3)$.

Solution:

(a) L_1 : joins $(1, 2)$ and $(7, -1)$, L_2 : joins $(3, 2)$ and $(5, 6)$

Slope of L_1 is $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{7-1} = -\frac{3}{6} = -\frac{1}{2}$

Slope of L_2 is $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{5-3} = \frac{4}{2} = 2$

Let θ is the angle from the line L_1 to line L_2 ,

then $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$ ....(A), put values in (A)

$$\tan\theta = \frac{2-\left(-\frac{1}{2}\right)}{1+2\left(-\frac{1}{2}\right)} = \frac{2+\frac{1}{2}}{1-1} = \frac{\frac{5}{4}}{0} = \infty$$

=> $\theta = \tan^{-1}(\infty) = 90^o$ *Ans.*

(b) L_1 **: joins** $(2\ ,7)$ **and** $(7\ ,10)$ **,** L_2**: joins** $(1,1)$ **and** $(-5,3)$**.**

Slope of L_1 **is** $m_1 = \frac{y_2-y_1}{x_2-x_1} = \frac{10-7}{7-2} = \frac{3}{5}$

Slope of L_2 **is** $m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{3-1}{-5-1} = -\frac{2}{6} = -\frac{1}{3}$

Let θ **is the angle from the line** L_1 **to line** L_2 **,**

Then $\tan\theta = \frac{m_2-m_1}{1+m_1m_2}$ **.......(A) Put values in (A)**

$$\tan\theta = \frac{-\frac{1}{3}-\frac{3}{5}}{1+\left(\frac{3}{5}\right)\left(-\frac{1}{3}\right)} = \frac{-\frac{5+9}{15}}{\frac{15-3}{15}} = -\frac{14}{15}\times\frac{15}{12} = -\frac{7}{6}$$

=> $\theta = \tan^{-1}\left(-\frac{7}{6}\right) = -\tan^{-1}\left(\frac{7}{6}\right)$

Since $\tan\theta$ **is negative in the second quadrant then** θ

lie in the second quadrant then

$\theta = 180^\circ - \tan^{-1}\left(\frac{7}{6}\right)$ **and acute angle**

$\theta = \left|-\tan^{-1}\left(\frac{7}{6}\right)\right| = \tan^{-1}\left(\frac{7}{6}\right)$ **Ans.**

<u>Q.6</u>: In each case, find the angle θ **from the line** L_1 **to** L_2**?**

(a) $L_1:\ x-2y+3=0$, $L_2:\ 3x-y+7=0$

(b) $L_1:\ 2x+4y-10=0$, $L_2:\ 5x-3y+1=0$

Note: If $ax+by+c=0$ .....(i) is a straight line.
To find the slope of (i) we change (i) into slope intercept form $(y=mx+c)$.

from (i) => $by=-ax-c$ => $y=-\frac{a}{b}x-\frac{c}{b}$

Then slope of (i) is $m=-\frac{a}{b}$

Solution: (a) $L_1: x-2y+3=0$ ....(i)
and $L_2: 3x-y+7=0$ ......(ii)

From (i) => $-2y=-x-3$ => $y=\frac{1}{2}x+\frac{3}{2}$

Then the slope of L_1 is $m_1=\frac{1}{2}$.

From (ii) => $-y=-3x-7$ => $y=3x+7$

Then the slope of L_2 is $m_2=3$.

Let θ is the angle from the line L_1 to line L_2,

Then $\tan\theta=\frac{m_2-m_1}{1+m_1m_2}$ ....(A)

Put values in (A)

$$\tan\theta=\frac{3-\frac{1}{2}}{1+\left(\frac{1}{2}\right)(3)}=\frac{\frac{6-1}{2}}{\frac{2+3}{2}}=\frac{5}{2}\times\frac{2}{5}=1$$

=> $\theta=\tan^{-1}(1)=45^o$ *Ans.*

(b) $L_1: 2x+4y-10=0$ .....(i)
$L_2: 5x-3y+1=0$ ....(ii)

From (i) => $4y=-2x+10$ => $y=-\frac{2}{4}x+\frac{10}{4}$

=> $y=-\frac{1}{2}x+\frac{5}{2}$

Then the slope of L_1 is $\quad m_1 = -\frac{1}{2}$

From (ii) $\Rightarrow -3y = -5x - 1 \Rightarrow y = \frac{5}{3}x + \frac{1}{3}$

Then the slope of L_2 is $\quad m_2 = \frac{5}{3}$

Let θ is the angle from the line L_1 to line L_2,

Then $\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$ ....(A)

Put values in (A)

$$\tan\theta = \frac{\frac{5}{3} - \left(-\frac{1}{2}\right)}{1 + \left(\frac{5}{3}\right)\left(-\frac{1}{2}\right)} = \frac{\frac{10+3}{6}}{\frac{6-5}{6}} = \frac{13}{6} \times \frac{6}{1} = 13$$

$\Rightarrow \quad \theta = \tan^{-1}(13) \quad$ *Ans.*

Q.7: In each case, find the angles of the triangle ABC whose vertices are the following:

(a) $A(1, 2)$, $B(4, 2)$, $C(-2, 3)$

Solution: The slope of the side AB is

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{4-1} = \frac{0}{3} = 0$$

The slope of the side BC is

$$m_2 = \frac{y_3 - y_2}{x_3 - x_2} = \frac{3-2}{-2-4} = -\frac{1}{6}$$

The slope of the side CA is

$$m_3 = \frac{y_1 - y_3}{x_1 - x_3} = \frac{2-3}{1-(-2)} = -\frac{1}{3}$$

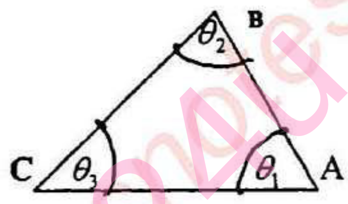
Let θ_1 is the angle from AB to AC, then

$$\tan\theta_1 = \frac{m_1 - m_3}{1 + m_1 m_3} \quad \text{put values}$$

$$\tan\theta_1 = \frac{0-\left(-\frac{1}{3}\right)}{1+(0)\left(\frac{1}{3}\right)} = \frac{1}{3}$$

$$\Rightarrow \quad \theta_1 = \tan^{-1}\left(\frac{1}{3}\right)$$

Let θ_2 is the angle from BC to BA , then



$$\tan\theta_2 = \frac{m_2 - m_1}{1+m_1 m_2} \quad \textbf{put values}$$

$$\tan\theta_2 = \frac{-\frac{1}{6}-0}{1+(0)\left(-\frac{1}{6}\right)} = -\frac{1}{6}$$

$$\Rightarrow \theta_2 = \tan^{-1}\left(-\frac{1}{6}\right) = -\tan^{-1}\left(\frac{1}{6}\right)$$

$$\Rightarrow \theta_2 = 180^o - \tan^{-1}\left(\frac{1}{6}\right)$$

Let θ_3 is the angle from CA to CB , then

$$\tan\theta_3 = \frac{m_3 - m_2}{1+m_2 m_3} \quad \textbf{put values}$$

$$\tan\theta_3 = \frac{-\frac{1}{3}-\left(-\frac{1}{6}\right)}{1+\left(-\frac{1}{3}\right)\left(-\frac{1}{6}\right)} = \frac{\frac{-2+1}{6}}{\frac{18+1}{18}} = -\frac{1}{6}\times\frac{18}{19} = -\frac{3}{19}$$

$$\Rightarrow \theta_3 = \tan^{-1}\left(-\frac{3}{19}\right) = -\tan^{-1}\left(\frac{3}{19}\right) \quad \Rightarrow \theta_3 = 180^o - \tan^{-1}\left(\frac{3}{19}\right)$$

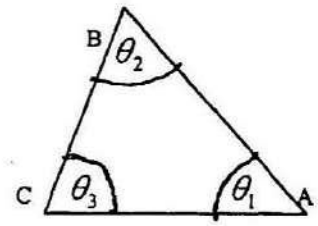
(b) $A(3\ ,-4)$, $B(1\ ,5)$, $C(2\ ,-4)$

Solution: **The slope of the side** AB **is**

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-(-4)}{1-3} = \frac{5+4}{-2} = -\frac{9}{2}$$

The slope of the side BC **is**

$$m_2 = \frac{y_3 - y_2}{x_3 - x_2} = \frac{-4-5}{2-1} = -\frac{9}{1} = -9$$



The slope of the side CA **is**

$$m_3 = \frac{y_1 - y_3}{x_1 - x_3} = \frac{-4-(-4)}{3-2}$$

$$= \frac{-4+4}{1} = 0$$

Let θ_1 **is the angle from CA to AB , then**

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_1 m_3} \quad \textbf{put values}$$

$$\tan\theta_1 = \frac{0-\left(-\frac{9}{2}\right)}{1+\left(-\frac{9}{2}\right)(0)} = \frac{\frac{9}{2}}{1+0} = \frac{9}{2}$$

$$\Rightarrow \quad \theta_1 = \tan^{-1}\left(\frac{9}{2}\right)$$

Let θ_2 **is the angle from BC to BA , then**

$$\tan\theta_2 = \frac{m_1 - m_2}{1 + m_1 m_2} \quad \textbf{put values}$$

$$\tan\theta_2 = \frac{-\frac{9}{2}-(-9)}{1+(-9)\left(-\frac{9}{2}\right)} = \frac{\frac{-9+18}{2}}{\frac{2+81}{2}} = \frac{9}{2} \times \frac{2}{83} = \frac{9}{83}$$

$$\Rightarrow \quad \theta_2 = \tan^{-1}\left(\frac{9}{83}\right)$$

Let θ_3 **is the angle from CA to CB , then**

$$\tan\theta_3 = \frac{m_2 - m_3}{1 + m_2 m_3}$$

put values

$$\tan\theta_3 = \frac{-9-0}{1+(0)(-9)} = \frac{-9}{1} = -9$$

=> $\theta_3 = \tan^{-1}(-9) = -\tan^{-1}(9)$

=> $\theta_3 = 180^o - \tan^{-1}(9)$

<u>Q.8</u>: Find the equation of the straight line from the family of straight lines through the point of intersection of the lines
(a) $2x-3y+4=0$, $3x+4y-5=0$ **and is perpendicular to the line** $6x-7y-18=0$

<u>Solution</u>: $2x-3y+4=0$ **……(i)** , $3x+4y-5=0$ **…..(ii)**

$6x-7y-18=0$ **……..(iii)**

The equation of line through the intersection of (i) and (ii) is

$2x-3y+4+\lambda(3x+4y-5)=0$ **……(A)**

=> $2x-3y+4+3\lambda x+4\lambda y-5\lambda=0$

=> $(2+3\lambda)x+(4\lambda-3)y+4-5\lambda=0$

=> $(4\lambda-3)y=-(2+3\lambda)x+5\lambda-4$

=> $y=-\left(\frac{2+3\lambda}{4\lambda-3}\right)x+\frac{5\lambda-4}{4\lambda-3}$ **……(B)**

Thus the slope of line (A) is $m_1=-\frac{(2+3\lambda)}{4\lambda-3}$

From (iii) => $-7y=-6x+18$

=> $y=\frac{6}{7}x-\frac{18}{7}$

Then the slope of line (iii) is $m_2=\frac{6}{7}$

As line (A) is perpendicular to line (iii) , then

$m_1.m_2=-1$ **put values**

=> $-\left(\frac{2+3\lambda}{4\lambda-3}\right)\left(\frac{6}{7}\right)=-1$ => $6(2+3\lambda)=7(4\lambda-3)$

$$\Rightarrow 12+18\lambda = 28\lambda - 21 \quad \Rightarrow 18\lambda - 28\lambda = -12-21$$

$$\Rightarrow -10\lambda = -33 \quad \Rightarrow \lambda = \frac{33}{10}$$

Put value of λ in (A)

$$2x-3y+4+\frac{33}{10}(3x+4y-5)=0$$

$$\Rightarrow 20x-30y+40+99x+132y-165=0$$

$$\Rightarrow 119x+102y-125=0 \quad Ans.$$

(b) $3x-4y+1=0$, $5x+y-1=0$ **and cuts off equal intercepts from the axes.**

<u>Solution</u>: $3x-4y+1=0$ **....(i)** , $5x+y-1=0$ **......(ii)**

The equation of line through the intersection of (i) and (ii) is

$$3x-4y+1+\lambda(5x+y-1)=0 \quad \text{......(A)}$$

For x – intercepts : Put $y=0$ in (A)

$$3x-0+1+\lambda(5x+0-1)=0 \quad \Rightarrow 3x+5\lambda x+1-\lambda=0$$

$$\Rightarrow x(3+5\lambda)=\lambda-1 \quad \Rightarrow x=\frac{\lambda-1}{3+5\lambda} \quad \text{....(iii)}$$

For y – intercepts : Put $x=0$ in (A)

$$0-4y+1+\lambda(0+y-1)=0 \quad \Rightarrow -4y+1+\lambda y-\lambda=0$$

$$\Rightarrow (\lambda-4)y=\lambda-1 \quad \Rightarrow y=\frac{\lambda-1}{\lambda-4} \quad \text{.....(iv)}$$

Since given that x and y – intercepts are same , then from (iii) and (iv) we have ,

$$\frac{\lambda-1}{3+5\lambda}=\frac{\lambda-1}{\lambda-4} \quad \Rightarrow \quad \frac{1}{3+5\lambda}=\frac{1}{\lambda-4}$$

$$\Rightarrow \lambda-4=3+5\lambda \quad \Rightarrow -4-3=5\lambda-\lambda$$

$$\Rightarrow -7=4\lambda \quad Or \quad \lambda=-\frac{7}{4}$$

Put value of λ in (A)

$$3x-4y+1-\frac{7}{4}(5x+y-1)=0$$

=> $12x-16y+4-35x-7y+7=0$

=> $-23x-23y+11=0$ => $23x+23y-11=0$

=> $23x+23y=11$ *Ans.*

(c) $x-2y=a$, $x+3y=2a$ **and is parallel to the line** $3x+4y=0$

Solution: $x-2y=a$ **.....(i)** , $x+3y=2a$ **......(ii)**

$3x+4y=0$ **......(iii)**

The equation of line through the intersection of (i) and (ii) is

$x-2y-a+\lambda(x+3y-2a)=0$ **......(A)**

=> $x-2y-a+\lambda x+3\lambda y-2a\lambda=0$

=> $(1+\lambda)x+(3\lambda-2)y-a-2a\lambda=0$

=> $(3\lambda-2)y=-(1+\lambda)x+a+2a\lambda$

=> $y=-\frac{(1+\lambda)}{3\lambda-2}x+\frac{a+2a\lambda}{3\lambda-2}$ .....(B)

Then the slope of (A) is $m_1=-\frac{1+\lambda}{3\lambda-2}$

Now from (iii) => $4y=-3x$ *or* $y=-\frac{3}{4}x$

Then the slope of (iii) is $m_2=-\frac{3}{4}$

Since line (A) and (iii) are parallel to each other, then

$m_1=m_2$ **Put values**

=> $-\frac{1+\lambda}{3\lambda-2}=-\frac{3}{4}$ => $4(1+\lambda)=3(3\lambda-2)$

=> $4+4\lambda=9\lambda-6$ => $4+6=9\lambda-4\lambda$

=> $5\lambda=10$ => $\lambda=2$

Put $\lambda=2$ **in (A)**

$$x-2y-a+2(x+3y-2a)=0$$

=> $x-2y-a+2x+6y-4a=0$

=> $3x+4y-5a=0$ OR $3x+4y=5a$ *Ans.*

(d) $2x-y=0$, $3x+2y=0$ **and is perpendicular to the line** $3x+y-6=0$

<u>Solution</u>: $2x-y=0$ **......(i)** , $3x+2y=0$ **......(ii)**

$3x+y-6=0$ **.....(iii)**

The equation of line through the intersection of (i) and (ii) is

$2x-y+\lambda(3x+2y)=0$ **.......(A)**

=> $2x-y+3\lambda x+2\lambda y=0$

=> $(2+3\lambda)x-(1-2\lambda)y=0$ => $(1-2\lambda)y=(2+3\lambda)x$

=> $y=\dfrac{2+3\lambda}{1-2\lambda}x$ .....(B)

Then the slope of (i) is $m_1=\dfrac{2+3\lambda}{1-2\lambda}$

From (iii) => $y=-3x+6$

Then the slope of (iii) is $m_2=-3$

Since the line (A) is perpendicular to line (iii), then

$m_1.m_2=-1$ **put values**

=> $\left(\dfrac{2+3\lambda}{1-2\lambda}\right)(-3)=-1$

=> $3(2+3\lambda)=1-2\lambda$ => $6+9\lambda=1-2\lambda$

=> $9\lambda+2\lambda=1-6$ => $11\lambda=-5$

=> $\lambda=-\dfrac{5}{11}$ *put* $\lambda=-\dfrac{5}{11}$ $in(A)$

$2x-y-\dfrac{5}{11}(3x+2y)=0$ => $22x-11y-15x-10y=0$

=> $7x-21y=0$ => $7(x-3y)=0$

=> $x-3y=0$ *Ans.*

<u>Concurrent lines</u>: **Two or more than two lines are said**

to be concurrent if they are passing through a single point and the point of intersection is called concurrency point.

Condition of concurrency of three straight lines:

Let $L_1 : \; a_1x_1 + b_1y_1 + c_1 = 0$ ......(i)

$L_2 : \; a_2x + b_2y + c_2 = 0$ ....(ii)

$L_3 : \; a_3x + b_3y + c_3 = 0$ ....(iii)

are three straight lines , then the lines (i) , (ii) and (iii) are concurrrnt if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Example .21: Show that the three lines $x + 4y + 3 = 0$, $5x - 4y - 5 = 0$ and $2x + 2y + 1 = 0$ are concurrent . If the lines are concurrent , then find out the point of concurrency .

Solution: $x + 4y + 3 = 0$ .....(i) , $5x - 4y - 5 = 0$ .....(ii)

$2x + 2y + 1 = 0$ .....(iii)

We know that the three lines are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \text{ , Then}$$

$$\begin{vmatrix} 1 & 4 & 3 \\ 5 & -4 & -5 \\ 2 & 2 & 1 \end{vmatrix} = 1(-4 + 10) - 4(5 + 10) + 3(10 + 8)$$

$$= 6 - 60 + 54$$

$$= 0$$

Hence the lines (i) , (ii) and (iii) are concurrent .

Now we find the point of concurrency .

We solve (i) and (ii) , adding (i) and (ii)

$$x+4y+3=0$$
$$\underline{5x-4y-5=0}$$
$$6x-2=0$$

$\Rightarrow \; 6x=2 \quad \Rightarrow \; x=\frac{2}{6}=\frac{1}{3}$

Put $x=\frac{1}{3}$ in (i) $\quad \Rightarrow \; \frac{1}{3}+4y+3=0$

$\Rightarrow \; 4y=-\frac{1}{3}-3 \quad \Rightarrow \; 4y=\frac{-1-9}{3}$

$\Rightarrow \; y=\frac{-10}{4\times 3}=-\frac{5}{6}$

Now put $x=\frac{1}{3}$, $y=-\frac{5}{6}$ in (iii)

$2\left(\frac{1}{3}\right)+2\left(-\frac{5}{6}\right)+1=0 \Rightarrow \frac{2}{3}-\frac{5}{3}+1=0$

$\Rightarrow \; \frac{2-5}{3}+1=0 \quad \Rightarrow \; -\frac{3}{3}+1=0$

$\Rightarrow \; -1+1=0 \quad \Rightarrow \; 0=0$

Thus the point $\left(\frac{1}{3}, -\frac{5}{6}\right)$ satisfied the eq (iii) .

Hence $\left(\frac{1}{3}, -\frac{5}{6}\right)$ is the point of concurrency .

<u>Area of triangle ABC</u>: Let $A(x_1 , y_1)$, $B(x_2 , y_2)$ and $C(x_3 , y_3)$ are the vertices of triangle ABC .

Then the area of triangle ABC is

$$A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

<u>Homogeneous equation</u>: $ax+by=0$ is called first degree homogenous equation , where a , b are called constants .

$ax^2+2hxy+by^2=0$ is called a homogeneous equation

of second degree, where a, h, b are constants and a, h, b are not simultaneously zero.

Note: Every homogenous equation of second degree represent two straight lines passing through the origin. The two lines are

(i) real and distinct if $h^2 - ab > 0$

(ii) real and coincident if $h^2 - ab = 0$

(iii) Imaginary if $h^2 - ab < 0$

The angle between the two lines is

$\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{a+b}$, where a is the coefficient of x^2, $2h$ is the coefficient of xy and b is the coefficient of y^2

Example: 28: Find two first degree straight lines in x and y when the second degree homogeneous equation is $5x^2 + 3xy - 8y^2 = 0$.

Solution: $5x^2 + 3xy - 8y^2 = 0$

$\Rightarrow 5x^2 - 5xy + 8xy - 8y^2 = 0$

$\Rightarrow 5x(x-y) + 8y(x-y) = 0$

$\Rightarrow (x-y)(5x+8y) = 0$

$\Rightarrow x - y = 0$ and $5x + 8y = 0$ are the required two straight lines passing through the origin.

Example .29: Find the angle in between the lines represented by the second degree homogeneous equation $x^2 - xy - 6y^2 = 0$

Solution: $x^2 - xy - 6y^2 = 0$ ....(i)

The angle between the two lines is

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} \quad .....(ii)$$

Here $a = 1$, $2h = -1 \Rightarrow h = -\dfrac{1}{2}$ and $b = -6$ Put in (ii).

$$\tan\theta = \frac{2\sqrt{\left(-\frac{1}{2}\right)^2-(1)(-6)}}{1-6} = \frac{2\sqrt{\frac{1}{4}+6}}{-5} = -\frac{2}{5}\left(\sqrt{\frac{1+24}{4}}\right)$$

$$= -\frac{2}{5}\left(\sqrt{\frac{25}{4}}\right) = -\frac{2}{5}\times\frac{5}{2} = -1$$

=> $\theta = \tan^{-1}(-1) = 135^o$ **Ans.**

Exercise 7.4:

Q.1: In each case, find the point of intersection $P(x, y)$ of the pair of lines :

(a) $2x+4y-10=0 \quad , \quad 5x-3y+1=0$

Solution: $2x+4y-10=0$ **....(i) and** $5x-3y+1=0$ **....(ii)**

Eq (i) is multiplied by 3 **and eq (ii) is multiplied by 4**

$6x+12y-30=0$ **....(iii) and** $20x-12y+4=0$ **.....(ii)**

Adding (iii) and (iv)

$$\begin{array}{r} 6x+12y-30=0 \\ 20x-12y+4=0 \\ \hline 26x-26=0 \end{array}$$

=> $26x = 26$ => $x = \frac{26}{26} = 1$

Put $x=1$ **in (i)**

$2(1)+4y-10=0$ => $4y-8=0$

=> $4y=8$ => $y=2$

Thus the intersection point is $(1, 2)$ **.**

(b) $2x+y-8=0 \quad , \quad 3x+2y-2=0$

Solution: $2x+y-8=0$ **.....(i) ,** $3x+2y-2=0$ **......(ii)**

Eq(i) is multiplied by 2 => $4x+2y-16=0$ **......(iii)**

Now eq(ii) is subtract from (iii)

$$4x+2y-16=0$$
$$\underline{\pm 3x \pm 2y \mp 2=0}$$
$$x-14=0$$

=> $x=14$ put $x=14$ in (i)

=> $2(14)+y-8=0$ => $28+y-8=0$

=> $y+20=0$ => $y=-20$

Thus the intersection point is $(14\ ,\ -20)$ Ans.

<u>Q.2</u>: Show that the following lines are concurrent. If the lines are concurrent, then find out the point at which the given lines can make concurrency.

(a) $x-y-2=0\ ,\ 2x-y-5=0\ ,\ 11x-5y-28=0$

<u>Solution</u>: $x-y-2=0$ ......(i) , $2x-y-5=0$ ....(ii) ,
$11x-5y-28=0$ ....(iii)

We know that the three lines are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0\ ,\ \textbf{Then consider}$$

$$\begin{vmatrix} 1 & -1 & -2 \\ 2 & -1 & -5 \\ 11 & -5 & -28 \end{vmatrix} \quad \textit{Expand by first row}$$

$$=1(28-25)+1(-56+55)-2(-10+11)$$
$$=3-1-2=0$$

Hence the lines are concurrent.

Now we solve (i) and (ii) to find the point of concurrency.

Now (ii) – (i)

$$x-y-2=0$$
$$\underline{\pm 2x \mp y \mp 5=0}$$
$$-x+3=0$$

=> $x=3$

Put $x=3$ **in (i)** => $3-y-2=0$

=> $-y+1=0$ => $y=1$

Put $x = 3$, $y = 1$ in (iii)

$11(3) - 5(1) - 28 = 0 \Rightarrow 33 - 5 - 28 = 0$

$\Rightarrow 0 = 0$

Hence point of concurrency is $(3 , 1)$.

(b) $x + 2y - 3 = 0$, $2x - y + 4 = 0$, $x + 4y - 7 = 0$

Solution: $x + 2y - 3 = 0$ ....(i) , $2x - y + 4 = 0$ ....(ii) ,

$x + 4y - 7 = 0$ ....(iii)

We know that the three lines are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \text{ , Then consider}$$

$$\begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 1 & 4 & -7 \end{vmatrix} \quad \textit{Expand by first row}$$

$$= 1(7 - 16) - 2(-14 - 4) - 3(8 + 1)$$

$$= -9 + 36 - 27 = 0$$

Hence the lines are concurrent.

Now we solve (i) and (ii) to find the point of concurrency.

Eq (ii) is multiplied by $2 \Rightarrow 4x - 2y + 8 = 0$ ....(iv)

Adding (i) and (iv)

$$\begin{array}{r} x + 2y - 3 = 0 \\ \underline{4x - 2y + 8 = 0} \\ 5x + 5 = 0 \end{array}$$

$\Rightarrow 5x = -5 \Rightarrow x = -1$

Put $x = -1$ in (ii) $\Rightarrow 2(-1) - y + 4 = 0$

$\Rightarrow -2 - y + 4 = 0 \Rightarrow -y + 2 = 0$

$\Rightarrow y = 2$

Hence point of concurrency is $(-1 , 2)$.

(c) $3x + 2y - 1 = 0$, $2x - 3y + 4 = 0$, $x + y - 2 = 0$

Solution: $3x + 2y - 1 = 0$ ......(i) , $2x - 3y + 4 = 0$......(ii)

$$x + y - 2 = 0 \qquad \text{.......(iii)}$$

We know that the three lines are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \text{ , Then consider}$$

$$\begin{vmatrix} 3 & 2 & -1 \\ 2 & -3 & 4 \\ 1 & 1 & -2 \end{vmatrix} \quad \textit{Expand by first row}$$

$$= 3(6-4) - 2(-4-4) - 1(2+3)$$
$$= 3(2) - 2(-8) - 5$$
$$= 6 + 16 - 5$$
$$= 17 \neq 0$$

Hence the lines are not concurrent.

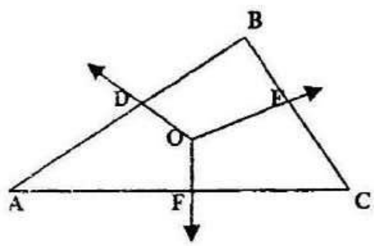
Q.3: If ABC is a triangle with vertices $A(0\ ,\ 0)$, $B(8\ ,\ 6)$ and $C(12\ ,\ 0)$, then show that

(a) The right bisectors of the triangle ABC are concurrent.
(b) The altitudes of the triangle ABC are concurrent.
(c) The medians of the triangle ABC are concurrent.

Solution: Given $A(0\ ,\ 0)$, $B(8\ ,\ 6)$ and $C(12\ ,\ 0)$

(a) **Right bisectors:** The line which bisects its opposite side at right angle is called right bisector.

Let D , E and F are the mid points of sides AB , BC and AC respectively.



D is the mid point of AB , Then the coordinates of

point $D\left(\frac{0+8}{2}\ ,\ \frac{0+6}{2}\right) = D(4\ ,3)$

As E is the mid point of BC . Then

$$E\left(\frac{8+12}{2}\ ,\ \frac{6+0}{2}\right) = E\left(\frac{20}{2}\ ,\ \frac{6}{2}\right)$$
$$= E(10\ ,\ 3)$$

As F is the mid point of AC, then

$$F\left(\frac{0+12}{2}, \frac{0+0}{2}\right) = F(6, 0)$$

Then $\overline{OD}$, $\overline{OE}$ *and* $\overline{OF}$ are the right bisectors of triangle ABC.

Now we find slope of the sides of triangle ABC.

Slope of side $\overline{AB}$ is $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{8-0} = \frac{6}{8} = \frac{3}{4}$

Slope of side $\overline{BC}$ is $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{12-8} = -\frac{6}{4} = -\frac{3}{2}$

Slope of side $\overline{AC}$ is $m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-0}{12-0} = \frac{0}{12} = 0$

Let m_4 be the slope of $\overline{OD}$.

Since $\overline{OD}$ is perpendicular to side $\overline{AB}$, then

$m_1 . m_4 = -1 \quad => m_4 = -\frac{1}{m_1}$ put value

$$m_4 = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$$

Now equation of $\overline{OD}$ passing through the point $D(4, 3)$ having slope $m_4 = -\frac{4}{3}$ is

$y - y_1 = m_4(x - x_1)$ Put values

$=> y - 3 = -\frac{4}{3}(x-4) \quad => 3y - 9 = -4x + 16$

$=> 4x + 3y - 25 = 0 \quad ......(a)$

Let m_5 be the slope of $\overline{OE}$.

Since $\overline{OE}$ is perpendicular to side $\overline{BC}$, then

$m_2.m_5 = -1 \quad => m_5 = -\frac{1}{m_2}$ put value

$$m_5 = -\frac{1}{-\frac{3}{2}} = \frac{2}{3}$$

Now equation of $\overline{OE}$ passing through the point $E(10, 3)$ having slope $m_5 = \frac{2}{3}$ is

$y - y_1 = m_5(x - x_1)$ Put values

$=> y - 3 = \frac{2}{3}(x - 10) \quad => \quad 3y - 9 = 2x - 20$

$=> 0 = 2x - 3y - 20 + 9 \quad => \quad 2x - 3y - 11 = 0 \quad ......(b)$

Let m_6 be the slope of $\overline{OF}$.

Since $\overline{OF}$ is perpendicular to side $\overline{AC}$, then

$m_3.m_6 = -1 \quad => m_6 = -\frac{1}{m_3} = -\frac{1}{0}$

Now equation of $\overline{OF}$ passing through the point $F(6, 0)$ having slope $m_6 = -\frac{1}{0}$ is

$y - y_1 = m_6(x - x_1)$ Put values

$=> y - 0 = -\frac{1}{0}(x - 6) \quad => \quad (0)(y) = -(x - 6)$

$=> \quad x + 0.y - 6 = 0 \quad ......(c)$

We know that the three lines are concurrent if

$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, Then from (a) , (b) and (c)

Now consider

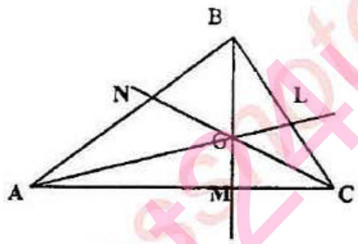
$$\begin{vmatrix} 4 & 3 & -25 \\ 2 & -3 & -11 \\ 1 & 0 & -6 \end{vmatrix} \quad \textbf{Expand by first row}$$

$$= 4(18+0) - 3(-12+11) - 25(0+3)$$
$$= 72 + 3 - 75$$
$$= 0$$

Hence the right bisectors of the triangle are concurrent.

(b) <u>Altitudes of triangle</u>: The line through the vertex of a triangle and perpendicular to its opposite side is called altitude of the triangle.

Let $\overline{AL}$, $\overline{BM}$ and $\overline{CN}$ are the altitudes of the triangle ABC.



Let m_4 be the slope of $\overline{AL}$.

Since $\overline{AL}$ is perpendicular to side $\overline{BC}$, then

$$m_2 . m_4 = -1 \qquad \Rightarrow m_4 = -\frac{1}{m_2} \quad \textbf{put value}$$

$$m_4 = -\frac{1}{-\frac{3}{2}} = \frac{2}{3}$$

Now equation of $\overline{AL}$ passing through the point $A(0\ ,\ 0)$ having slope $m_4 = \frac{2}{3}$ is

$$y - y_1 = m_4(x - x_1) \quad \textbf{Put values}$$

$$\Rightarrow y - 0 = \frac{2}{3}(x - 0) \qquad \Rightarrow 3y = 2x$$

$$\Rightarrow 0 = 2x - 3y \quad \Rightarrow \quad 2x - 3y = 0 \quad ......(a)$$

Let m_5 be the slope of $\overline{BM}$.

Since $\overline{BM}$ is perpendicular to side $\overline{AC}$, then

$m_3.m_5 = -1 \quad => m_5 = -\frac{1}{m_3} = -\frac{1}{0}$

Now equation of $\overline{BM}$ passing through the point $B(6, 8)$

having slope $m_5 = -\frac{1}{0}$ is

$y - y_1 = m_5(x - x_1)$ Put values

$=> y - 6 = -\frac{1}{0}(x-8) \quad => (0)(y-6) = -(x-8)$

$=> x - 8 = 0 \quad => x + 0.y - 8 = 0$ ......(b)

Let m_6 be the slope of $\overline{CN}$.

Since $\overline{CN}$ is perpendicular to side $\overline{AB}$, then

$m_1.m_6 = -1 \quad => m_6 = -\frac{1}{m_1}$ put value

$$m_6 = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$$

Now equation of $\overline{CN}$ passing through the point $C(12, 0)$

having slope $m_6 = -\frac{4}{3}$ is

$y - y_1 = m_6(x - x_1)$ Put values

$=> y - 0 = -\frac{4}{3}(x-12) \quad => 3y = -4x + 48$

$=> 4x + 3y - 48 = 0$ ......(c)

We know that the three lines are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$, Then from (a) , (b) and (c)

Now consider

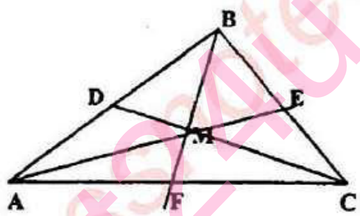
$$\begin{vmatrix} 2 & -3 & 0 \\ 1 & 0 & -8 \\ 4 & 3 & -48 \end{vmatrix} \quad \textbf{Expand by first row}$$

$$= 2(0+24)+3(-48+32)+0$$
$$= 48+3(-16) = 48-48$$
$$= 0$$

Hence the medians of the triangle are concurrent.

(c) Medians of triangle: The line through the vertex which bisect its opposite side is called median of triangle.

Let D, E and F are the mid points of sides AB, BC and AC respectively.



D is the mid point of AB, Then the coordinates of

point $D\left(\frac{0+8}{2}, \frac{0+6}{2}\right) = D(4, 3)$

As E is the mid point of BC. Then

$$E\left(\frac{8+12}{2}, \frac{6+0}{2}\right) = E\left(\frac{20}{2}, \frac{6}{2}\right)$$
$$= E(10, 3)$$

As F is the mid point of AC, then

$$F\left(\frac{0+12}{2}, \frac{0+0}{2}\right) = F(6, 0)$$

Then $\overline{AE}$, $\overline{BF}$ *and* $\overline{CD}$ **are the medians of triangle ABC.**

Equation of $\overline{AE}$: $A(0, 0)$ **and** $E(10, 3)$

Then equation is $y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$, **put values**

$$y - 0 = \frac{(3-0)}{(10-0)}(x-0)$$

$$\Rightarrow y = \frac{3}{10}x \quad \Rightarrow \quad 10y = 3x$$

$$\Rightarrow 0 = 3x - 10y \quad \Rightarrow 3x - 10y = 0 \; ......(a)$$

<u>Equation of</u> $\overline{BF}$: $B(8\,,6)$ **and** $F(6\,,0)$

Then equation is $y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$ **, put values**

$$y - 6 = \frac{(0-6)}{(6-8)}(x-8)$$

$$\Rightarrow y - 6 = \frac{-6}{-2}(x-8) \quad \Rightarrow y - 6 = 3(x-8)$$

$$\Rightarrow y - 6 = 3x - 24 \quad \Rightarrow 0 = 3x - y + 6 - 24 = 0$$

$$\Rightarrow 3x - y - 18 = 0 \; ....(b)$$

<u>Equation of</u> $\overline{CD}$: $C(12\,,0)$ **and** $D(4\,,3)$

Then equation is $y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$ **, put values**

$$\Rightarrow y - 0 = \frac{(3-0)}{(4-12)}(x-12)$$

$$\Rightarrow y = -\frac{3}{8}(x-12) \Rightarrow 8y = -3x + 36$$

$$\Rightarrow 3x + 8y - 36 = 0 \; ......(c)$$

We know that the three lines are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$ **, Then from (a) , (b) and (c)**

Now consider

$$\begin{vmatrix} 3 & -10 & 0 \\ 3 & -1 & -18 \\ 3 & 8 & -36 \end{vmatrix} \quad \textbf{Expand by first row}$$

$$= 3(36+144)+10(-108+54)+0$$
$$= 3(180)+10(-54)$$
$$= 540-540$$
$$= 0$$

Hence the medians of triangle are concurrent.

<u>Q.4</u>: Find the area of the triangular region whose vertices are the following :

(a) $P_1(0\ ,0)\ ,\ P_2(2\ ,4)\ \ and\ \ P_3(-2\ ,\ 2)$

<u>Solution</u> : The area of triangle ABC is

$$A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \textbf{Put values}$$

$$A=\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2 & 4 & 1 \\ -2 & 2 & 1 \end{vmatrix} \quad \textbf{Expand by first row}$$

$$\Rightarrow \quad A=\frac{1}{2}[0-0+1(4+8)]=\frac{12}{2}=6$$

(b) $P_1(-1\ ,-2)\ ,\ P_2(2\ ,5),\ \ P_3(5\ ,\ 2)$

<u>Solution</u> : The area of triangle ABC is

$$A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \quad \textbf{Put values}$$

$$\Rightarrow \quad A=\frac{1}{2}\begin{vmatrix} -1 & -2 & 1 \\ 2 & 5 & 1 \\ 5 & 2 & 1 \end{vmatrix} \quad \textbf{Expand by first row}$$

$$A=\frac{1}{2}[-1(5-2)+2(2-5)+1(4-25)]$$

$$=\frac{1}{2}(-3-6-21)=-\frac{30}{2}=-15$$

But area is always positive then $A=15$

(c) $P_1(4\ ,-5)$, $P_2(5\ ,-6)$, $P_3(3\ ,\ 1)$

Solution : The area of triangle ABC is

$$A=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$ **Put values**

$$\Rightarrow\ A=\frac{1}{2}\begin{vmatrix}4 & -5 & 1\\ 5 & -6 & 1\\ 3 & 1 & 1\end{vmatrix}$$ **Expand by first row**

$$\Rightarrow\ A=\frac{1}{2}[4(-6-1)+5(5-3)+1(5+18)]$$

$$=\frac{1}{2}(-28+10+23)$$

$$=\frac{5}{2}\quad Ans.$$

Q.5: Find the area bounded by the triangle ABC is zero , whose vertices are the following .

(a) $A(-3\ ,6)$, $B(3\ ,\ 2)$, $C(6\ ,0)$.

(b) $A(-2\ ,4)$, $B(3\ ,-6)$, $C(1\ ,-2)$.

(c) Are the vertices in parts (a) and (b) of the triangle ABC are collinear .

Solution: Note: If area of a tringle ABC is zero then the vertices of the triangle are collinear points .

(a) $A(-3\ ,6)$, $B(3\ ,\ 2)$, $C(6\ ,0)$

The area of triangle ABC is

$$A=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix} \quad \textbf{Put values}$$

$$A=\frac{1}{2}\begin{vmatrix}-3 & 6 & 1\\ 3 & 2 & 1\\ 6 & 0 & 1\end{vmatrix} \quad \textbf{Expand by first row}$$

$$\Rightarrow A=\frac{1}{2}[-3(2-0)-6(3-6)+1(0-12)]$$

$$=\frac{1}{2}[-6-6(-3)-12]$$

$$\Rightarrow A=\frac{1}{2}(6+18-12)=0$$

(b) $A(-2,4)$, $B(3,-6)$, $C(1,-2)$.

The area of triangle ABC is

$$A=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix} \quad \textbf{Put values}$$

$$\Rightarrow A=\frac{1}{2}\begin{vmatrix}-2 & 4 & 1\\ 3 & -6 & 1\\ 1 & -2 & 1\end{vmatrix}, \textbf{Expand by first row}$$

$$\Rightarrow A=\frac{1}{2}[-2(-6+2)-4(3-1)+1(-6+6)]$$

$$=\frac{1}{2}[-2(-4)-4(2)+0]$$

$$=\frac{1}{2}(8-8)=0$$

(c) Hence the vertices of part (a) and (b) are collinear points.

Q.6: Find two first degree straiigth lines in x and y, when the second degree homogeneous equations are the

following . Also find the angles in between the lines

(a) $3x^2 - 2xy - 5y^2 = 0$

Solution: $3x^2 - 2xy - 5y^2 = 0$

$3x^2 + 3xy - 5xy - 5y^2 = 0$

=> $3x(x+y) - 5y(x+y) = 0$

=> $(x+y)(3x-5y) = 0$

=> $x + y = 0 \quad and \quad 3x - 5y = 0$

are the required straight lines .

The angle between the two lines is

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} \quad .....(i)$$

Here $a = 3$, $2h = -2$ => $h = -1$ and $b = -5$ Put in (i) .

$$\tan\theta = \frac{2\sqrt{(-1)^2 - (3)(-5)}}{3-5} = \frac{2\sqrt{1+15}}{-2}$$

=> $\tan\theta = -\sqrt{16} = -4$

=> $\theta = \tan^{-1}(-4)$ *Ans.*

(b) $4x^2 - 9xy + 5y^2 = 0$

Solution: $4x^2 - 9xy + 5y^2 = 0$

=> $4x^2 - 4xy - 5xy + 5y^2 = 0$

=> $4x(x-y) - 5y(x-y) = 0$

=> $(x-y)(4x-5y) = 0$

=> $x - y = 0 \quad and \quad 4x - 5y = 0$

are the required straight lines .

The angle between the two lines is

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} \quad .....(i)$$

Here $a = 4$, $2h = -9$ => $h = -\frac{9}{2}$ and $b = 5$ Put in (i) .

$$\tan\theta = \frac{2\sqrt{\left(-\frac{9}{2}\right)^2-(4)(5)}}{4+5} = \frac{2\sqrt{\frac{81}{4}-20}}{9}$$

$$= \frac{2\sqrt{\frac{81-80}{4}}}{9} = \frac{2\sqrt{1}}{2\times 9} = \frac{1}{9}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{9}\right) \quad \textbf{Ans.}$$

Q.7: Show the two first degree straight lines in x and y are coincident, perpendicular or neither, when they are represented by the following second degree homogeneous equations.

(a) $x^2 + 5xy - y^2 = 0$

Solution: **Note: (i) Two lines are coincident if** $h^2 - ab = 0$
(ii) Two lines perpendicular to each other if $a + b = 0$
(iii) Neither if $h^2 - ab \neq 0$ **and** $a + b \neq 0$

Here $a = 1$, $2h = 5 \Rightarrow h = \frac{5}{2}$, $b = -1$

Here $a + b = 1 - 1 = 0$
Then the two lines perpendicular to each other.

Now $-y^2 + 5xy + x^2 = 0$ **both sides divided by** $-x^2$

$$\Rightarrow \frac{-y^2}{-x^2} - \frac{5xy}{x^2} - \frac{x^2}{x^2} = 0$$

$$\Rightarrow \left(\frac{y}{x}\right)^2 - 5\frac{y}{x} - 1 = 0 \quad .....(i)$$

Let $\frac{y}{x} = t$ **in (i)**

$t^2 - 5t - 1 = 0$, **then by quadratic formula**

$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$, put $a=1$, $b=-5$, $c=-1$

$$\frac{y}{x}=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-1)}}{2(1)}=\frac{5\pm\sqrt{25+4}}{2}$$

$\Rightarrow$ $y=\left(\frac{5+\sqrt{29}}{2}\right)x$ *and* $y=\left(\frac{5-\sqrt{29}}{2}\right)x$ **are the requaired straight lines.**

(b) $2x^2-xy-y^2=0$

Solution: **Here** $a=2$, $2h=-1 \Rightarrow h=-\frac{1}{2}$, $b=-1$

Now $h^2-ab=\left(-\frac{1}{2}\right)^2-(2)(-1)=\frac{1}{4}+2=\frac{9}{2}\neq 0$

$a+b=2-1=1\neq 0$

Then straight lines neither concident nor perpendicular.

$2x^2-xy-y^2=0 \quad \Rightarrow \quad 2x^2-2xy+xy-y^2=0$

$\Rightarrow 2x(x-y)+y(x-y)=0$

$\Rightarrow (x-y)(2x+y)=0$

$\Rightarrow x-y=0$ *and* $2x+y=0$

are required straight lines.

Q.8: Find a joint equation of the straight line that passes through the origin and

(a) Perpendicular to the lines represented by

$3x^2-7xy+2y^2=0$

Solution: $3x^2-7xy+2y^2=0$

$\Rightarrow 3x^2-6xy-xy+2y^2=0$

$\Rightarrow 3x(x-2y)-y(x-2y)=0$

$\Rightarrow (x-2y)(3x-y)=0$

$\Rightarrow x-2y=0$ *and* $3x-y=0$

$\Rightarrow x=2y$ *and* $3x=y$

$$=> \quad y=\frac{1}{2}x \;.....(i) \qquad and \quad y=3x\;......(ii)$$

Then the slopes of (i) and (ii) are

$$m_1=\frac{1}{2} \quad and \quad m_2=3$$

Let $y=m_3x$ ....(iii) and $y=m_4x$ ......(iv) are the required lines passing through the origin.

Since (iii) is perpendicular to (i) then

$$m_3.m_1=-1 \qquad => \quad m_3=-\frac{1}{m_1}=-\frac{1}{\frac{1}{2}}=-2$$

Also (iv) is perpendicular to (ii), then

$$m_4.m_2=-1 \qquad => \quad m_4=-\frac{1}{m_2}=-\frac{1}{3}$$

Put $m_3=-2$ and $m_4=-\frac{1}{3}$ in (iii) and (iv)

$$y=-2x \quad and \quad y=-\frac{1}{3}x$$

$$=> 2x+y=0 \quad and \quad 3y+x=0$$

Now joint equation is

$$(2x+y)(x+3y)=0$$

$$=> \quad 2x^2+6xy+xy+3y^2=0$$

$$=> \quad 2x^2+7xy+3y^2=0 \qquad Ans.$$

(b) Perpendicular to the lines represented by

$$x^2-(2\tan\theta)xy-y^2=0$$

<u>Solution</u>: $-y^2-(2\tan\theta)xy+x^2=0$, **divided by** x^2

$$=> \frac{-y^2}{-x^2}-\frac{(2\tan\theta)xy}{-x^2}+\frac{x^2}{-x^2}=0$$

$$=> \quad \left(\frac{y}{x}\right)^2+(2\tan\theta)\frac{y}{x}-1=0$$

Let $\frac{y}{x}=t$ in (i)

$t^2+(2\tan\theta)t-1=0$, then by quadratic formula

$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, put $a=1$, $b=2\tan\theta$, $c=-1$

$\Rightarrow \frac{y}{x}=\frac{-2\tan\theta\pm\sqrt{(2\tan\theta)^2-4(1)(-1)}}{2(1)}$

$\Rightarrow \frac{y}{x}=\frac{-2\tan\theta\pm\sqrt{4\tan^2\theta+4}}{2}=\frac{-2\tan\theta\pm\sqrt{4(\tan^2\theta+1)}}{2}$

$\Rightarrow \frac{y}{x}=\frac{-2\tan\theta\pm\sqrt{4\sec^2\theta}}{2}=\frac{-2\tan\theta\pm 2\sec\theta}{2}$

$\Rightarrow \frac{y}{x}=-\tan\theta\pm\sec\theta$

$\Rightarrow y=(-\tan\theta+\sec\theta)x$ .....(i)

and $y=-(\tan\theta+\sec\theta)x$ ...(ii)

Then the slope of (i) is $m_1=-\tan\theta+\sec\theta$

$\Rightarrow m_1=-\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}=\frac{1-\sin\theta}{\cos\theta}$

The slope of (ii) is $m_2=-\tan\theta-\sec\theta$

$\Rightarrow m_2=-\frac{\sin\theta}{\cos\theta}-\frac{1}{\cos\theta}=-\frac{(1+\sin\theta)}{\cos\theta}$

Let $y=m_3x$ ....(iii) and $y=m_4x$ ......(iv) are the required lines passing through the origin.

Since (iii) is perpendicular to (i) then

$m_3.m_1=-1 \quad \Rightarrow m_3=-\frac{1}{m_1}=-\frac{1}{\frac{1-\sin\theta}{\cos\theta}}=-\frac{\cos\theta}{1-\sin\theta}$

Also (iv) is perpendicular to (ii) , then

$m_4.m_2=-1 \quad \Rightarrow m_4=-\frac{1}{-\left(\frac{1+\sin\theta}{\cos\theta}\right)}=\frac{\cos\theta}{1+\sin\theta}$

Put $m_3 = -\dfrac{\cos\theta}{1-\sin\theta}$ *and* $m_4 = \dfrac{\cos\theta}{1+\sin\theta}$ **in (iii) and (iv)**

$$y = -\left(\frac{\cos\theta}{1-\sin\theta}\right)x \quad and \quad y = \left(\frac{\cos\theta}{1+\sin\theta}\right)x$$

$$\Rightarrow \quad y + \left(\frac{\cos\theta}{1-\sin\theta}\right)x = 0 \quad and \quad y - \left(\frac{\cos\theta}{1+\sin\theta}\right)x = 0$$

$$\Rightarrow (1-\sin\theta)y + (\cos\theta)x = 0 \quad and \quad (1+\sin\theta)y - (\cos\theta)x = 0$$

Now joint equation is

$$[(1-\sin\theta)y + (\cos\theta)x]\,[(1+\sin\theta)y - (\cos\theta)x] = 0$$

$$\Rightarrow (1-\sin\theta)(1+\sin\theta)y^2 - (1-\sin\theta)(\cos\theta)xy + (1+\sin\theta)(\cos\theta)xy - (\cos^2\theta)x^2 = 0$$

$$\Rightarrow (1-\sin^2\theta)y^2 + [-\cos\theta + \sin\theta\cos\theta + \cos\theta + \sin\theta\cos\theta]xy - (\cos^2\theta)x^2 = 0$$

$$\Rightarrow (\cos^2\theta)y^2 + (2\sin\theta\cos\theta)xy - (\cos^2\theta)x^2 = 0$$

Divided by $\cos^2\theta$

$$\Rightarrow \quad y^2 + \left(\frac{2\sin\theta\cos\theta}{\cos^2\theta}\right)xy - x^2 = 0$$

$$\Rightarrow \quad y^2 + 2\frac{\sin\theta}{\cos\theta}xy - x^2 = 0 \quad \textit{multiplied by } -1$$

$$\Rightarrow x^2 - 2\tan\theta\ xy - y^2 = 0 \qquad \textit{Ans.}$$

(c) Perpendicular to the lines represented by

$$ax^2 + 2hxy + by^2 = 0$$

<u>**Solution:**</u> $by^2 + 2hxy + ax^2 = 0$, **divided by** x^2

$$\Rightarrow b\frac{y^2}{x^2} + \frac{2hxy}{x^2} + \frac{ax^2}{x^2} = 1$$

$$\Rightarrow \quad b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0 \quad \ldots\ldots\text{(i)}$$

Let $\dfrac{y}{x} = t$ **in (i)**

$bt^2 + 2ht + a = 0$, **then by quadratic formula**

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{, put} \quad a = b \;,\; b = 2h \;,\; c = a$$

$$\Rightarrow \frac{y}{x} = \frac{-2h \pm \sqrt{(2h)^2 - 4(b)(a)}}{2b} = \frac{-2h \pm \sqrt{4h^2 - 4ab}}{2b}$$

$$\Rightarrow \frac{y}{x} = \frac{-2h \pm 2\sqrt{h^2 - ab}}{2b} = \frac{-h \pm \sqrt{h^2 - ab}}{2b}$$

$$\Rightarrow y = \left(\frac{-h + \sqrt{h^2 - ab}}{b}\right)x \;...(i)$$

$$\text{and} \quad y = \left(\frac{-h - \sqrt{h^2 - ab}}{b}\right)x \;.....(ii)$$

Then the slope of (i) is $m_1 = \dfrac{-h + \sqrt{h^2 - ab}}{b}$

And slope (ii) is $m_2 = \dfrac{-h - \sqrt{h^2 - ab}}{b}$

Let $y = m_3 x$ **....(iii) and** $y = m_4 x$ **......(iv) are the required lines passing through the origin.**

Since (iii) is perpendicular to (i) then

$$m_3 . m_1 = -1 \Rightarrow m_3 = -\frac{1}{m_1} = -\frac{1}{\frac{-h + \sqrt{h^2 - ab}}{b}} = \frac{b}{h - \sqrt{h^2 - ab}}$$

Also (iv) is perpendicular to (ii) , then

$$m_4 . m_2 = -1 \Rightarrow m_4 = -\frac{1}{-\left(\frac{h + \sqrt{h^2 - ab}}{b}\right)} = \frac{b}{h + \sqrt{h^2 - ab}}$$

Put of m_3 *and* m_4 **in (iii) and (iv)**

$$y = \left(\frac{b}{h - \sqrt{h^2 - ab}}\right)x \quad \text{and} \quad y = \left(\frac{b}{h + \sqrt{h^2 - ab}}\right)x$$

$$\Rightarrow y-\left(\frac{b}{h-\sqrt{h^2-ab}}\right)x=0 \quad and \quad y-\left(\frac{b}{h+\sqrt{h^2-ab}}\right)x=0$$

$$\Rightarrow \left(h-\sqrt{h^2-ab}\right)y-bx=0 \quad and \quad \left(h+\sqrt{h^2-ab}\right)y-bx=0$$

Now the joint equation is

$$\Rightarrow \left[\left(h-\sqrt{h^2-ab}\right)y-bx\right]\left[\left(h+\sqrt{h^2-ab}\right)y-bx\right]=0$$

$$\Rightarrow \left(h-\sqrt{h^2-ab}\right)\left(h+\sqrt{h^2-ab}\right)y^2-b\left(h-\sqrt{h^2-ab}\right)xy$$

$$-b\left(h+\sqrt{h^2-a}\right)xy+b^2x^2=0$$

$$\left(h^2-\left(h^2-ab\right)\right)y^2-b\left(h-\sqrt{h^2-ab}+h+\sqrt{h^2-ab}\right)xy+b^2x^2=0$$

$$\Rightarrow aby^2-2bhxy+b^2x^2=0$$ **, divided by b**

$$\Rightarrow ay^2-2hxy+bx^2=0 \quad Or \quad bx^2-2hxy+ay^2=0$$ **Ans.**

Review Exercise. 7:

Q.1: Choose the correct options.

(i) The distance between points $A(7,5)$ *and* $B(-5,-7)$ **is**

(a) $2\sqrt{37}$ **(b)** $12\sqrt{2}$ **(c)** $2\sqrt{2}$ **(d)** 0

(ii) To find the mid point of a line we use the formula

(a) $\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}$ **(b)** $\frac{m_1x_2-m_2x_1}{2}, \frac{m_1y_2-m_2y_1}{2}$

(c) $\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$ **(d)** $\frac{m_1x_2+m_2x_1}{2}, \frac{m_1y_2+m_2y_1}{2}$

(iii) For the points (a, b) *and* $(5, 7)$ **the slope of line is**

(a) $\frac{7-b}{5-a}$ **(b)** $\frac{5}{7}$

(c) $\frac{2}{3}$ **(d)** $-\frac{2}{3}$

(iv) y intercept of the line $2x+4y=-6$ **is**

(a) $\frac{3}{2}$ **(b)** $-\frac{3}{2}$ **(c)** $\frac{2}{3}$ **(d)** $-\frac{2}{3}$

(v) $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$ **is a**

(a) slope intercept form (b) two point form
(c) point slope form (d) double intercept form

(vi) The normal form of an equation is
(a) $x\cos\theta + y\sin\theta = p$ (b) $x\cos\theta - y\sin\theta = p$
(c) $x\sin\theta + y\tan\theta = 0$ (d) $x\cos\theta - y\sin\theta = 0$

(vii) Two given line are perpendicular if their slopes m_1 and m_2 are
(a) $m_1 + m_2 = 1$ (b) $m_1 - m_2 = 0$
(c) $m_1.m_2 = 1$ (d) $m_1.m_2 = -1$

(viii) $5x + 7y = 0$ is a (a) homogeneous linear equation
(b) non-homogeneous linear equation
(c) quadratic equation (d) only linear equation

(ix) The area of the triangular region is $A(4, 5)$, $B(-7, 4)$ and $C(3, 1)$ is : (a) 31 (b) -31 (c) $\frac{43}{2}$ (d) -47

(x) The angle between two pair of lines can be calculated by using the formula ;
(a) $\frac{2\sqrt{h^2+ab}}{a+b}$ (b) $\frac{2-\sqrt{a^2+hb}}{a+b}$
(c) $\frac{2\sqrt{h^2+pq}}{a-b}$ (d) $\frac{2\sqrt{h^2-ab}}{a+b}$

<u>Answers</u>: (i) b (ii) c (iii) a (iv) b (v) b (vi) a
(vii) d (viii) a (ix) c (x) d