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Unit . 6:

Integration .

Definition : The anti of derivative of a function is called anti derivative or integration of the function .
The integration is the inverse process of derivative .
The integration is denoted by the symbol $\int$.

$$if \quad \frac{d}{dx}(F(x)+c)=f(x)$$

$$\Rightarrow d(F(x)+c)=f(x)dx$$

$$\Rightarrow \int d(F(x)+c)=\int f(x)dx$$

$$\Rightarrow F(x)+c=\int f(x)dx$$

Or $\int f(x)dx=F(x)+c$ is called integration of $f(x)$ and c is called constant of integration .

Some important integrals through differentiation :

Integrals	Differentiation
$(i)\ \int k.dx=kx+c$ where c constant of integration	$\frac{d}{dx}(kx+c)=k$
$(ii)\int x^n dx=\frac{x^{n+1}}{n+1}+c, \quad n\neq -1$	$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}+c\right)=x^n$
$(iii)\int \frac{d}{x}=\ln x+c$	$\frac{d}{dx}(\ln x+c)=\frac{1}{x}$
$(iv)\int \sin kx dx=-\frac{\cos kx}{k}+c$	$\frac{d}{dx}\left(-\frac{\cos kx}{k}+c\right)=\sin kx$
$(v)\int \cos kx dx=\frac{\sin kx}{k}+c$	$\frac{d}{dx}\left(\frac{\sin kx}{k}+c\right)=\cos kx$
$(vi)\int \sec^2 kx dx=\frac{\tan kx}{k}+c$	$\frac{d}{dx}\left(\frac{\tan kx}{k}+c\right)=\sec^2 kx$

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$(vii)\int \cos ec^2 kx dx = -\frac{\cot kx}{k} + c$	$\frac{d}{dx}\left(-\frac{\cot kx}{k} + c\right) = \cos ec^2 kx$
$(viii)\int \sec kx.\tan kx dx = \frac{\sec kx}{k} + c$	$\frac{d}{dx}\left(\frac{\sec kx}{k} + c\right) = sec kx \tan kx$
$(ix)\int \cos eckx.\cot kx dx$	$\frac{d}{dx}\left(-\frac{\cos eckx}{k} + c\right)$
$= -\frac{\cos eckx}{k} + c$	$= \cos eckx \cot kx$
$(xiii)\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$	$\frac{d}{dx}\left(\sin^{-1} x + c\right) = \frac{1}{\sqrt{1-x^2}}$
$(xiii)\int -\frac{dx}{\sqrt{1-x^2}} = \cos^{-1} x + c$	$\frac{d}{dx}\left(\cos^{-1} x + c\right) = -\frac{1}{\sqrt{1-x^2}}$
$(xiv)\int \frac{dx}{1+x^2} = \tan^{-1} x + c$	$\frac{d}{dx}\left(\tan^{-1} x + c\right) = \frac{1}{1+x^2}$
$or \ -\cot^{-1} x + c$	$\frac{d}{dx}\left(-\cot^{-1} x + c\right) = \frac{1}{1+x^2}$
$(xv)\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c$	$\frac{d}{dx}\left(\sec^{-1} x + c\right) = \frac{1}{x\sqrt{x^2-1}}$
$or \ -\cos ec^{-1} x + c$	$\frac{d}{dx}\left(-\cos ec^{-1} x + c\right) = \frac{1}{x\sqrt{x^2-1}}$
$(xvi)\int \frac{dx}{\sqrt{x^2-1}} = \cosh^{-1} x + c$	$\frac{d}{dx}\left(\cosh^{-1} x + c\right) = \frac{1}{\sqrt{x^2-1}}$
$(xvii)\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c$	$\frac{d}{dx}\left(\sin^{-1}\frac{x}{a} + c\right) = \frac{1}{\sqrt{a^2-x^2}}$
$or \ -\cos^{-1}\frac{x}{a}$	
$(18)\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\frac{x}{a} + c$	$\frac{d}{dx}\left(\sinh^{-1}\frac{x}{a} + c\right) = \frac{1}{\sqrt{a^2}}$

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$(xix)\ \int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\frac{x}{a}+c$	$\frac{d}{dx}\left(\cosh^{-1}\frac{x}{a}+c\right)=\frac{1}{\sqrt{x^2-a^2}}$
$(xx)\ \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$	$\frac{d}{dx}\left(\frac{1}{a}\tan^{-1}\frac{x}{a}+c\right)=\frac{1}{a^2+x^2}$
$(xxi)\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+c$	$\frac{d}{dx}\left(\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|+c\right)=\frac{1}{a^2-x^2}$
$(xxii)\ \int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+c$	$\frac{d}{dx}\left(\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|+c\right)=\frac{1}{x^2-a^2}$
$(xxiii)\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a}+c$	$\frac{d}{dx}\left(\frac{1}{a}\sec^{-1}\frac{x}{a}\right)=\frac{1}{x\sqrt{x^2-a^2}}$
$(xxiv)\int e^{mx}dx = \frac{e^{mx}}{m}+c$	$\frac{d}{dx}\left(\frac{e^{mx}}{m}+c\right)=e^{mx}$
$(xxv)\int a^x dx = \frac{a^x}{\ln a}+c,\ a\neq 1,\ a>0$	$\frac{d}{dx}\left(\frac{a^x}{\ln a}+c\right)=a^x$
$(xxvi)\ \int \sinh x\,dx = \cosh x+c$	$\frac{d}{dx}(\cosh x+c)=\sinh x$
$(xxvii)\ \int \cosh x\,dx = \sinh x+c$	$\frac{d}{dx}(\sinh x+c)=\cosh x$

Example .2: Integrate the following

(a) $\int x^{\frac{1}{3}}dx$ **(b)** $\int \frac{1}{x^3}dx$

Solution: (a) $\int x^{\frac{1}{3}}dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1}+c = \frac{x^{\frac{4}{3}}}{\frac{4}{3}}+c = \frac{3}{4}x^{\frac{4}{3}}+c$

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(b) $\int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + c = \frac{x^{-2}}{-2} + c = -\frac{1}{2x^2} + c$

Some rules of integration:

(i) **constant rule** $\int k dx = kx + c$, **where k is constant.**

(ii) **constant multiple rule :** $\int kf(x)dx = k\int f(x)dx + c$.

(iii) **sum rule:** $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

(iv) **Power rule :** $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, $n \neq -1$

Important rule I: $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$

Proof : $\frac{d}{dx}\{\ln|f(x)| + c\} = \frac{f'(x)}{f(x)}$

$$\Rightarrow \quad d\{\ln|f(x)| + c\} = \frac{f'(x)}{f(x)} dx$$

$$\int d\{\ln|f(x)| + c\} = \int \frac{f'(x)}{f(x)} dx$$

$$\Rightarrow \quad \{\ln|f(x)| + c\} = \int \frac{f'(x)}{f(x)} dx$$

Or $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$ **proved.**

Rule II: $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$, $n \neq -1$.

Proof : $\frac{d}{dx}\left(\frac{[f(x)]^{n+1}}{n+1} + c\right) = \frac{(n+1)}{(n+1)}[f(x)]^n f'(x)$

$$= [f(x)]^n f'(x)$$

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$$d\left(\frac{[f(x)]^{n+1}}{n+1}+c\right)=\int[f(x)]^n f'(x)dx$$

$$\Rightarrow \int d\left(\frac{[f(x)]^{n+1}}{n+1}+c\right)=\int[f(x)]^n f'(x)dx$$

$$\Rightarrow \frac{[f(x)]^{n+1}}{n+1}+c=\int[f(x)]^n f'(x)dx$$

$$\Rightarrow \int[f(x)]^n f'(x)dx=\frac{[f(x)]^{n+1}}{n+1}+c$$

Example .3: Evaluate the following integrals :

(a) $\int(10+a)dx$ **(b)** $\int(10+a)x^2dx$

(c) $\int(5x^2+4x+2)dx$

Solution: (a) $\int(10+a)dx=(10+a)\int dx=(10+a)x+c$

(b) $\int(10+a)x^2dx=(10+a)\frac{x^3}{3}+c=\frac{(10+a)x^3}{3}$

(c) $\int(5x^2+4x+2)dx=5\int x^2dx+4\int xdx+2\int dx$

$$=5.\frac{x^3}{3}+\frac{x^2}{2}+2x+c$$

$$=\frac{5x^3}{3}+\frac{x^2}{2}+2x+c \quad Ans.$$

Integration by substitution : In this method some complicated functions can be integrated .

(i) if integration of the form $\int[f(x)]^n f'(x)dx$ **then**

Put $f(x)=t \Rightarrow f'(x)dx=dt$

(ii) If integration of the form $\int\frac{f'(x)}{f(x)}dx$ **then**

Put $f(x)=t \Rightarrow f'(x)dx=dt$

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iii) If integration of the form $\int e^{f(x)} f'(x)dx$ then

Put $f(x) = t \Rightarrow f'(x)dx = dt$

Example.4: Evaluate the following iintegrals

(a) $\int (x^3 - 5x + 7)^4 (3x^2 - 5)dx$ **(b)** $\int \frac{dx}{9x+7}$ **(c)** $\int e^{(3x+2)}dx$

Solution: (a) $\int (x^3 - 5x + 7)^4 (3x^2 - 5)dx$

Let $x^3 - 5x + 7 = t$ **.....(i)**

$$\frac{d}{dx}(x^3 - 5x + 7) = \frac{dt}{dx}$$

$$\Rightarrow (3x^2 - 5) = \frac{dt}{dx}$$

$$\Rightarrow (3x^2 - 5)dx = dt \ldots\ldots(ii)$$

Put (i) and (ii) in the given integral

$$\int (x^3 - 5x + 7)^4 (3x^2 - 5)dx = \int t^4 dt$$

$$= \frac{t^5}{5} + c \quad put\ (i)$$

$$= \frac{(x^3 - 5x + 7)^5}{5} + c \quad Ans.$$

(b) $\int \frac{dx}{9x+7}$

Put $9x + 7 = t$ **.....(i)**

$$\frac{d}{dx}(9x + 7) = \frac{dt}{dx}$$

$$\Rightarrow 9 = \frac{dt}{dx}$$

$$\Rightarrow dx = \frac{1}{9}dt \ldots\ldots\ldots(ii)$$

Put (i) and (ii) in the given integral

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$$\int \frac{dx}{9x+7} = \frac{1}{9}\int \frac{dt}{t} = \frac{1}{9}\ln|t| + c \qquad \textit{put value of } t$$

$$= \frac{1}{9}\ln|9x+7| + c$$

(c) $\int e^{(3x+2)}dx$

Let $3x+2=t$ **.....(i)**

$$\Rightarrow \frac{d}{dx}(3x+2) = \frac{dt}{dx}$$

$$\Rightarrow 3 = \frac{dt}{dx} \Rightarrow dx = \frac{1}{3}dt \quad .....(ii)$$

Put (i) and (ii) in the given integral

$$\int e^{(3x+2)}dx = \frac{1}{3}\int e^t dt$$

$$= \frac{1}{3}e^t + c \qquad \textit{put value of}$$

$$= \frac{1}{3}e^{(3x+2)} + c \qquad \textit{Ans.}$$

Example. 5: If the area A of halling wound changes at a rate approximnated by $\frac{dA}{dt} = -4t^{-3}$, $1 \le t \le 10$,

" t" is the time in days and the area A = 2 square centimetres on day t = 1 . What will be the area of wound be in 10 days ? .

Solution: The rate at which the wound area changes is

$$\frac{dA}{dt} = -4t^{-3} \quad .....(i)$$

Integrate (i) w.r.t. to t get wound area $A(t)$.

$$\Rightarrow \int = -\int 4t^{-3}dt$$

$$\Rightarrow A = -4\frac{t^{-3+1}}{-3+1} + c$$

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Or $A(t) = -\frac{4}{-2}t^{-2} + c \quad \Rightarrow A(t) = \frac{2}{t^2} + c$ ....(ii)

To find the value of c . Put $A = 2$, $t = 1$ in (ii)

$$\Rightarrow 2 = \frac{2}{(1)^2} + c \Rightarrow 2 = 2 + c$$

$$\Rightarrow c = 2 - 2 = 0$$

Put $c = 0$ in (ii) $\Rightarrow A(t) = \frac{2}{t^2}$ .....(iii)

The wound area after 10 days : put $t = 10$ in (iii)

$$A(10) = \frac{2}{(10)^2} = \frac{2}{100} = \frac{1}{50} = 0.02 \ cm^2$$

Exercise 6.1:

Q.1: Evaluate the following integral

(a) $\int x^3 dx$

Solution: $\int x^3 dx = \frac{x^4}{4} + c$

(b) $\int (a+15)\, dx$

Solution: $\int (a+15)\, dx = (a+15)\int dx$

$$= (a+15)x + c$$

(c) $\int \frac{1}{5x} dx$

Solution : $\int \frac{1}{5x} dx = \frac{1}{5}\int \frac{1}{x} dx$

$$= \frac{1}{5}\ln x + c$$

(d) $\int (3x^2 + 4x - 5) dx$

Solution: $\int (3x^2 + 4x - 5) dxx = 3\int x^2 dx + 4\int x dx - 5\int dx$

$$= 3.\frac{x^3}{3} + 4.\frac{x^2}{2} - 5x + c$$

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$$= x^3 + 2x^2 - 5x + c.$$

(e) $\int (1+3t)t^3\,dt$

Solution: $\int (1+3t)t^3\,dt = \int (t^3 + 3t^4)\,dt$

$$= \int t^3\,dt + 3\int t^4\,dt$$

$$= \frac{t^4}{4} + \frac{3t^5}{5} + c$$

(f) $\int \frac{1}{x^3}\,dx$

Solution: $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx$

$$= \frac{x^{-3+1}}{-3+1} + c$$

$$= \frac{x^{-2}}{-2} + c = -\frac{1}{x^2} + c \quad \textbf{ans.}$$

(g) $\int e^{5x}\,dx$

Solution: $\int e^{5x}\,dx = \frac{e^{5x}}{5} + c$ **Ans.**

(h) $\int \frac{5}{1+x^2}\,dx$

Solution: $\int \frac{5}{1+x^2}\,dx = 5\int \frac{dx}{1+x^2}$

$$= 5\tan^{-1} x + c$$

(i) $\int t^3\sqrt{t}\,dt$

Solution: $\int t^3\sqrt{t}\,dt = \int t^3 t^{\frac{1}{2}}\,dt$

$$= \int t^{3+\frac{1}{2}}\,dt = \int t^{\frac{6+1}{2}}\,dt$$

$$= \int t^{\frac{7}{2}}\,dt$$

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$$= \frac{t^{\frac{7}{2}+1}}{\frac{7}{2}+1} + c$$

$$= \frac{t^{\frac{7+2}{2}}}{\frac{7+2}{2}} + c = \frac{9t^{\frac{9}{2}}}{2} + c \quad \textbf{Ans.}$$

Q.2: Evaluate the following indefinite integrals by method of substitution.

(a) $\int (3x+4)^8 dx$

Solution: $\int (3x+4)^8 dx$

Let $3x+4=t$ .....(i)

$$\Rightarrow \quad \frac{d}{dx}(3x+4) = \frac{dt}{dx}$$

$$\Rightarrow \quad 3 = \frac{dt}{dx} \quad \Rightarrow \quad dx = \frac{1}{3}dt \quad .....(ii)$$

Put (i) and (ii) in the given integral

$$\int (3x+4)^8 dx = \frac{1}{3}\int t^8 dt$$

$$= \frac{1}{3}.\frac{t^9}{9} + c \quad \textit{put value of } t$$

$$= \frac{(3x+4)^5}{27} + c \quad \textit{Ans.}$$

(b) $\int 3x^2(x^3-4)dx$

Solution: $\int (x^3-4)(3x^2)dx$

Let $x^3-4=t$ .....(i)

$$\frac{d}{dx}(x^3-4) = \frac{dt}{dx}$$

$$\Rightarrow \quad 3x^2 = \frac{dt}{dx} \quad \Rightarrow \quad 3x^2dx = dt \quad .....(ii)$$

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Put (i) and (ii) in the given integral

$$\int (x^3-4)(3x^2)dx = \int t\,dt$$

$$= \frac{t^2}{2} + c \quad \textit{put value of } t$$

$$= \frac{(x^3-4)^2}{2} + c \quad \textit{Ans.}$$

(c) $\int (3x^2+7)(x^3+7x)^8\,dx$

Solution : $\int (x^3+7x)^8(3x^2+7)dx$

Let $x^3+7x = t$ .....(i)

$$\frac{d}{dx}(x^3+7x) = \frac{dt}{dx}$$

$$\Rightarrow \quad 3x^2+7 = \frac{dt}{dx}$$

$$\Rightarrow \quad (3x^2+7)dx = dt \quad .....(ii)$$

Put (i) and (ii) in the given integral

$$\int (x^3+7x)^8(3x^2+7)dx = \int t^8\,dt$$

$$= \frac{t^9}{9} + c \quad \textit{put value of } t$$

$$= \frac{(x^3+7x)^9}{9} + c \quad \textit{Ans.}$$

(d) $\int \frac{3x^2-7}{(x^3-7x)^4}dx$

Solution: $\int \frac{3x^2-7}{(x^3-7x)^4}dx$

Let $x^3-7x = t$ .....(i)

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$$\frac{d}{dx}(x^3-7x)=\frac{dt}{dx}$$

$$\Rightarrow \quad 3x^2-7=\frac{dt}{dx}$$

$$\Rightarrow \quad (3x^2-7)dx=dt \quad \text{.....}(ii)$$

Put (i) and (ii) in the given integral

$$\int \frac{3x^2-7}{(x^3-7x)^4}dx=\int \frac{dt}{t^4}$$

$$=\int t^{-4}dt$$

$$=\frac{t^{-3}}{-3}=-\frac{1}{3t^3} \quad \textit{put value of } t$$

$$\Rightarrow \int \frac{3x^2-7}{(x^3-7x)^4}dx=-\frac{1}{3(x^3-7x)^3}+c \quad \textit{Ans.}$$

(e) $\int \frac{6t}{\sqrt{3t^2-7}}dt$

Solution: $\int \frac{6t}{\sqrt{3t^2-7}}dt=\int \frac{6t}{(3t^2-7)^{\frac{1}{2}}}dt$

$$=\int (3t^2-7)^{-\frac{1}{2}}(6t)dt \quad \text{.....(i)}$$

Let $3t^2-7=x$ **.....(ii)**

$$\frac{d}{dt}(3t^2-7)=\frac{dx}{dt}$$

$$\Rightarrow \quad 6t=\frac{dx}{dt} \quad \textit{or} \quad (6t)dt=dx \quad \text{.....}(iii)$$

Put (ii) and (iii) in (i)

$$\int \frac{6t}{\sqrt{3t^2-7}}dt=\int x^{-\frac{1}{2}}dx$$

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$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{x^{\frac{-1+2}{2}}}{\frac{-1+2}{2}} + c = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{x} + c \quad \text{put vaiue of x}$$

$$= 2\sqrt{3t^2 - 7} + c \quad \text{Ans.}$$

(f) $\int \frac{\sqrt{r^{\frac{1}{3}} + 4}}{r^{\frac{2}{3}}} dr$

Solution: $\int \frac{\sqrt{r^{\frac{1}{3}} + 4}}{r^{\frac{2}{3}}} dr = \int \left(r^{\frac{1}{3}} + 4\right)\left(r^{-\frac{2}{3}}\right) dr$ .....(i)

Let $r^{\frac{1}{3}} + 4 = t$ ....(ii)

$$\Rightarrow \frac{d}{dr}\left(r^{\frac{1}{3}} + 4\right) = \frac{dt}{dr}$$

$$\Rightarrow \frac{1}{3} r^{\frac{1}{3}-1} = \frac{dt}{dr} \quad or \quad r^{-\frac{2}{3}} dr = 3dt \quad ....(iii)$$

Put (ii) and (iii) in (i)

$$\int \frac{\sqrt{r^{\frac{1}{3}} + 4}}{r^{\frac{2}{3}}} dr = 3\int t^{\frac{1}{2}} dt$$

$$= 3\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = 3\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c \quad \text{put value of t.}$$

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$$= 2\left(r^{\frac{1}{3}} + 4\right)^{\frac{2}{3}} + c$$

(g) $\int \frac{x + 3x^2}{\sqrt{x}} dx$

Solution: $\int \frac{x + 3x^2}{\sqrt{x}} dx = \int \left(\frac{x}{\sqrt{x}} + \frac{3x^2}{\sqrt{x}}\right) dx$

$$= \int \left(x^{1-\frac{1}{2}} + 3x^{2-\frac{1}{2}}\right) dx$$

$$= \int x^{\frac{1}{2}} dx + 3\int x^{\frac{3}{2}} dx$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 3\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= \frac{2}{3}x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c \quad Ans.$$

(h) $\int \frac{x+1}{(x^2 + 2x + 2)^2} dx$

Solution: $\int \frac{x+1}{(x^2 + 2x + 2)^2} dx$

Let $x^2 + 2x + 2 = t$ .....(i)

$$\frac{d}{dx}(x^2 = 2x + 2) = \frac{dt}{dx}$$

$$\Rightarrow 2x + 2 = \frac{dt}{dx} \quad \Rightarrow 2(x+1)dx = dt$$

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$$\Rightarrow \quad (x+1)dx = \frac{1}{2}dt \quad .....(ii)$$

Put (i) and (ii) in the given integral

$$\int \frac{x+1}{(x^2+2x+2)^2}dx = \frac{1}{2}\int \frac{dt}{t^2}$$

$$= \frac{1}{2}\int t^{-2}dt$$

$$= \frac{1}{2}.\frac{t^{-1}}{-1} + c$$

$$= -\frac{1}{2t} + c \qquad \textit{put value of } t$$

$$= -\frac{1}{2(x^2+2x+2)} + c \qquad \textit{Ans.}$$

Q.3: Evaluate the following indefinite integrals by method of substitution :

(a) $\int 6e^{6t}dt$

Solution: $\int 6e^{6t}dt$

Let $\quad 6t = u \quad .....(i)$

$$\frac{d}{dt}(6t) = \frac{du}{dt} \quad \Rightarrow \quad 6 = \frac{du}{dt}$$

$$\Rightarrow \quad dt = \frac{1}{6}du \quad .....(ii)$$

Put (i) and (ii) in the given integral

$$\int 6e^{6t}dt = \frac{1}{6}\int e^u du$$

$$= \frac{1}{6}e^u + c \qquad \textit{put value of } u$$

$$= \frac{1}{6}e^{6t} + c \qquad \textit{Ans.}$$

(b) $\int x\,e^{(5x^2+1)}dx$

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Solution: $\int x e^{(5x^2+1)} dx$

Let $5x^2 + 1 = t$ .....(i)

$$\frac{d}{dx}(5x^2+1) = \frac{dt}{dx}$$

$$\Rightarrow 10x = \frac{dt}{dx}$$

$$\Rightarrow \quad xdx = \frac{1}{10} dt \quad .....(ii)$$

Put (i) and (ii) in the given integral

$$\int x e^{(5x^2+1)} dx = \frac{1}{10}\int e^t dt$$

$$= \frac{e^t}{10} + c \qquad \textit{Put value of } t$$

$$= \frac{e^{(5x^2+1)}}{10} \cdot c \qquad \textit{Ans.}$$

(c) $\int (x^2 - 2) e^{(x^3-6x+4)} dx$

Solution: $\int (x^2 - 2) e^{(x^3-6x+4)} dx$

Let $x^3 - 6x + 4 = t$ .....(i)

$$\frac{d}{dx}(x^3 - 6x + 4) = \frac{dt}{dx}$$

$$\Rightarrow 3x^2 - 6 = \frac{dt}{dx} \quad \Rightarrow 3(x^2 - 2)dx = dt$$

$$\Rightarrow (x^2 - 2)dx = \frac{1}{3} dt \quad .....(ii)$$

Put (i) and (ii) in the given integral

$$\int (x^2 - 2) e^{(x^3-6x+4)} dx = \frac{1}{3}\int e^t dt$$

$$= \frac{1}{3} e^t + c \quad \textit{put value of } t$$

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$$= \frac{1}{3} e^{(x^3-6x+4)} + c \quad \textbf{Ans.}$$

(d) $\int 8^{(7-3x^2)}(-6x)dx$

Solution: $\int 8^{(7-3x^2)}(-6x)dx$

Let $7-3x^2 = t$ .....(i)

$$\frac{d}{dx}(7-3x^2) = \frac{dt}{dx}$$

$$\Rightarrow \quad 6x = \frac{dt}{dx}$$

$$\Rightarrow \quad (-6x)dx = dt \quad .....(ii)$$

Put (i) and (ii) in the given integral.

$$\int 8^{(7-3x)}(-6x)dx = \int 8^t dt$$

$$= \frac{8^t}{\ln 8} + c \qquad \textit{put value of } t$$

$$= \frac{8^{(7-3x^2)}}{\ln 8} + c$$

Q.4: Find the equation of the particle curve that has slope $4x^3 + 6x^2$ at a point $(1, 0)$.

Solution: Since the slope of the particle is equal to the the derivative of the function then

$$\frac{dy}{dx} = 4x^3 + 6x^2$$

$$\Rightarrow dy = (4x^3 + 6x^2)dx$$

$$\int dy = \int (4x^3 + 6x^2)dx$$

$$y = 4\int x^3 dx + 6\int x^2 dx$$

$$\Rightarrow \quad y = 4\frac{x^4}{4} + 6\frac{x^3}{3} + c$$

$$y = x^4 + 2x^3 + c \quad .....(i)$$

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To find c put $x=1$, $y=0$ **in (i)**

$$0=(1)^4+2(1)^2+c$$

$$\Rightarrow 0=1+2+c \quad \Rightarrow -3=c$$

$$\Rightarrow c=-3$$

Put $c=-3$ **in (i)**

$$y=x^4+2x^3-3 \quad \textbf{Ans.}$$

Q.5: A certain curve has a slope $x(2x^2-1)^2$ **that passes through the point** $(3\ ,\ 3)$ **. What is the equation of the specific curve ? .**

Solution : Since the slope of the particle is equal to the the derivative of the function then

$$\frac{dy}{dx}=x(2x^2-1)^2$$

$$\Rightarrow dy=(2x^2-1)^2\,xdx$$

$$\int dy=\int(2x^2-1)^2\,xdx \quad \text{.....(i)}$$

Let $2x^2-1=t$ .....(ii)

$$\frac{d}{dx}(2x^2-1)=\frac{dt}{dx}$$

$$\Rightarrow 4x=\frac{dt}{dx} \quad \Rightarrow xdx=\frac{1}{4}dt \quad \text{.....}(iii)$$

Put (ii) and (iii) in (i)

$$\int dy=\frac{1}{4}\int t^2dt$$

$$\Rightarrow y=\frac{1}{4}.\frac{t^3}{3}+c \qquad \textit{put value of } t$$

$$\Rightarrow y=\frac{(2x^2-1)^3}{12}+c \quad \text{.....(iv)}$$

To find c put $x=3$, $y=3$ **in (iv)**

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$$3 = \frac{(2(3)^2 - 1)^3}{12} + c$$

$$\Rightarrow \quad 3 = \frac{(17)^3}{12} + c \quad \Rightarrow \quad 3 - 409.42 = c$$

$$c = -406.42 \quad \text{put in (i)}$$

$$\Rightarrow \quad y = \frac{(2x^2 - 1)^3}{12} - 406.42 \quad \text{Ans.}$$

Q.6: If the rate of excretion of a biochemical compound is given by $f'(t) = 0.01e^{-0.01t}$

(a) Find an expression $f(t)$ **, the total amount excreted by time t (in minutes) .**

(b) If $f(0) = 0$ **units are excreted at a time** $t = 0$ **, how many units are excreted in 10 minutes .**

Solution: (a) $f'(t) = 0.01e^{-0.01t}$

$$\Rightarrow \quad \frac{d}{dt}(f(t)) = 0.01e^{-0.01t}$$

$$\Rightarrow \quad d(f(t)) = 0.01e^{-0.01t}dt \quad \text{integrate}$$

$$\Rightarrow \quad \int d(f(t)) = \int 0.01e^{-0.01t}dt$$

$$\Rightarrow \quad f(t) = \frac{(0.01)e^{-0.01t}}{-0.01} + c$$

$$\Rightarrow \quad f(t) = -e^{-0.01t} + c \quad \ldots\ldots\ldots(i)$$

(b) Put $t = 0$ *and* $f(t) = 0$ **in (i)**

$$0 = e^{-(0.01)(0)} + c \quad \Rightarrow \quad 0 = e^0 + c$$

$$\Rightarrow \quad 0 = 1 + c \quad \text{or} \quad c = -1$$

Put $c = -1$ **in (i)** $\Rightarrow \quad f(t) = e^{-0.01t} - 1 \quad \ldots\ldots(ii)$

Put $t = 10$ **in (ii)** $\Rightarrow \quad f(10) = e^{-(0.01)(10)} - 1$

$$\Rightarrow \quad f(10) = e^{-0.1} - 1 = 0.095 \quad \text{units .}$$

Q.7: For an average person , the rate of change of weight W in (pounds) with respect to height h in (inches) is

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approximately by $\frac{dW}{dh} = 0.0015h^2$.

(a) Find W(h), if the weight is $W = 108$ pounds in response of height $h = 60$ inches.

(b) Find the weight W of a person who is 5 feet 10 inches h tall.

Solution: (a) $\frac{dW}{dh} = 0.0015h^2$

$$dW = 0.0015h^2 dh$$

$$\Rightarrow \int dW = 0.0015\int h^2 dh$$

$$\Rightarrow W = (0.0015)\frac{h^3}{3} + c$$

$$\Rightarrow W = (0.0005)h^3 + c \quad .....(i)$$

Put $W = 108$, $h = 60$ in (i)

$$108 = (0.0005)(60)^3 + c$$

$$\Rightarrow 108 = 108 + c$$

$$\Rightarrow c = 108 - 108 = 0$$

Put $c = 0$ in (i)

$W(h) = 0.0005h^3$ .....(ii) Ans.

(b) put $h = 5\ feet\ 10\ inches = 70\ inch$ in (ii)

$W(70) = (0.0005)(70)^3 = 171.5\ lb$.

Q.8: The rate of growth of the population $N(t)$ of a newly incorporated city t

$\frac{dN}{dt} = 400 + 600\sqrt{t}$, $0 \le t \le 9$.

If the population was 5, 000 at the time incorporation, find the population 9 years later.

Solution: $\frac{dN}{dt} = 400 + 600\sqrt{t}$

$$\Rightarrow dN = \left(400 + 600\sqrt{t}\right)dt$$

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$$\int dN = \int \left(400 + 600\, t^{\frac{1}{2}}\right) dt$$

$$\Rightarrow \quad N = 400t + \frac{(600)t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$\Rightarrow \quad N = 400t + \frac{(600)}{\frac{3}{2}} t^{\frac{3}{2}} + c$$

$$\Rightarrow \quad N = 400t + \frac{1200}{3} t^{\frac{3}{2}} + c$$

$$\Rightarrow \quad N = 400t + 400\, t^{\frac{3}{2}} + c \quad \ldots\ldots\ldots(i)$$

Put $t = 0$, $N = 5000$ **in (i)**

$$5000 = (400)(0) + (400)(0)^{\frac{3}{2}} + c$$

$$\Rightarrow \quad 5000 = c$$

$$\Rightarrow \quad c = 5000 \quad \textbf{put in (i)}$$

$$N(t) = 400t + 400\, t^{\frac{3}{2}} + 5000 \quad \ldots\ldots\textbf{(ii)}$$

Put $t = 9$ **in (ii)**

$$N(9) = (400)(9) + (400)(9)^{\frac{3}{2}} + 5000$$
$$= 3600 + 10800 + 5000$$
$$= 19400$$

6.3.3: Meathod of substitution to evaluate the integral of the following types . $\int \frac{1}{a^2 - x^2} dx$, $\int \sqrt{a^2 - x^2}\, dx$, $\int \frac{dx}{\sqrt{a^2 - x^2}}$

Solution: (a) $\int \frac{1}{a^2 - x^2} dx$

This integration we solve by partial fraction .

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As $\dfrac{1}{a^2-x^2}=\dfrac{1}{(a-x)(a+x)}$

Let $\dfrac{1}{(a-x)(a+x)}=\dfrac{A}{a-x}+\dfrac{B}{a+x}$ .....(i)

Multiplied both sides by $(a-x)(a+x)$

$1= A(a+x)+B(a-x)$ .....(ii)

Put $x=a$ in (ii)

$1=A(a+a)+B(a-a)$

$\Rightarrow 1=2aA+0 \quad \Rightarrow A=\dfrac{1}{2a}$

Put $x=-a$ in (ii)

$1=0+B(a+a) \quad \Rightarrow 1=2aB$

$\Rightarrow B=\dfrac{1}{2a}$

Put values of A and B in (i)

$$\frac{1}{(a-x)(a+x)}=\frac{\frac{1}{2a}}{a-x}+\frac{\frac{1}{2a}}{a+x}$$

$$=\frac{1}{2a}\left(\frac{1}{a-x}+\frac{1}{a+x}\right)$$

$$\Rightarrow \int\frac{1}{(a-x)(a+x)}dx=\frac{1}{2a}\int\frac{1}{a-x}dx+\frac{1}{2a}\int\frac{1}{a+x}dx \quad \text{....(A)}$$

For $\int\dfrac{dx}{a-x}$

Let $a-x=u$

$\dfrac{d}{dx}(a-x)=\dfrac{du}{dx}$

$-1=\dfrac{du}{dx}$

$\Rightarrow dx=-du$

$\int\dfrac{dx}{a-x}=-\int\dfrac{du}{u}$

For $\int\dfrac{dx}{a+x}$

$a+x=u$

$\dfrac{d}{dx}(a+x)=\dfrac{du}{dx}$

$1=\dfrac{du}{dx}$

$dx=du$

$\int\dfrac{dx}{a+x}=\int\dfrac{du}{u}$

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$$\Rightarrow \int \frac{dx}{a-x} = -\ln|u| \qquad \int \frac{dx}{a+x} = \ln|u|$$

$put \quad u = a-x \qquad put \quad u = a+x$

$$\Rightarrow$$

$$\int \frac{dx}{a-x} = -\ln|a-x| \ldots(i) \qquad \int \frac{dx}{a+x} = \ln|a+x| \ldots(ii)$$

Put (i) and (ii) in (A)

$$\Rightarrow \int \frac{1}{(a-x)(a+x)}dx = -\frac{1}{2a}\ln|a-x| + \frac{1}{2a}\ln|a+x|$$

$$\Rightarrow \int \frac{dx}{a^2-x^2} = \frac{\ln|a+x|}{2a} - \frac{\ln|a-x|}{2a} + c$$

(b) $\int \sqrt{a^2-x^2}\,dx$

Let $x = a\sin u \quad \Rightarrow \quad \frac{d}{dx}(x) = \frac{d}{dx}a\sin u$

$$\Rightarrow 1 = a\cos u\frac{du}{dx} \quad \Rightarrow \quad dx = a\cos u\,du$$

$$\int \sqrt{a^2-x^2}\,dx = \int \left(\sqrt{a^2-a^2\sin^2 u}\right)(a\cos u)du$$

$$= \int \sqrt{a^2(1-\sin^2 u)}(a\cos u)du$$

$$= \int \left(\sqrt{a^2\cos^2 u}\right)(a\cos u)du$$

$$= \int (a\cos u)(a\cos u)du$$

$$= \int a^2\cos^2 u\,du \qquad put \ \cos^2 x = \frac{1+\cos 2x}{2}$$

$$= a^2\int \frac{1+\cos 2u}{2}du = \frac{a^2}{2}\int (1+\cos 2u)du$$

$$= \frac{a^2}{2}\left(u + \frac{\sin 2u}{2}\right) + c$$

$$= \frac{a^2}{2}u + \frac{a^2}{2}.\frac{2\sin u.\cos u}{2} + c$$

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$$= \frac{a^2}{2}u + \frac{a^2}{2}.\sin u\sqrt{1-\sin^2 u} + c \quad \text{.....(A)}$$

As $a\sin u = x \Rightarrow \sin u = \frac{x}{a} \Rightarrow u = \sin^{-1}\left(\frac{x}{a}\right)$ **put in (A)**

$$\int \sqrt{a^2 - x^2}\,dx = \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{a^2}{4}.\frac{x}{a}\sqrt{1-\frac{x^2}{a^2}} + c$$

$$= \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{a^2}{4}.\frac{x}{a}\sqrt{\frac{a^2-x^2}{a^2}} + c$$

$$= \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{a^2}{4}.\frac{x}{a}\frac{\sqrt{a^2-x^2}}{a} + c$$

$$= \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + c$$

c. $\int \frac{1}{\sqrt{a^2-x^2}}dx$

Solution: Let $x = a\sin\theta$ ....(i)

$$\frac{d}{dx}(x) = \frac{d}{dx}(a\sin\theta) \Rightarrow 1 = a\cos\theta\frac{d\theta}{dx}$$

$$\Rightarrow dx = a\cos\theta\,d\theta \quad \text{.....(ii)}$$

Put (i) and (ii) in the given integral.

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{a\cos\theta d\theta}{\sqrt{a^2-(a\sin\theta)^2}}$$

$$= \int \frac{a\cos\theta\,d\theta}{\sqrt{a^2(1-\sin^2\theta)}}$$

$$= \int \frac{a\cos\theta d\theta}{\sqrt{a^2\cos^2\theta}} = \int \frac{a\cos\theta d\theta}{a\cos\theta}$$

$$= \int d\theta = \theta + c \quad \text{.....(iii)}$$

As from (i) $\sin\theta = \frac{x}{a} \Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$ **.....(iv)**

Put (iv) in (iii)

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$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{2}\right)+c\,.$$

(ii) $\int \frac{dx}{a^2+x^2}$, $\int \frac{dx}{\sqrt{a^2+x^2}}$

Solution: $\int \frac{dx}{a^2+x^2}$ **Let** $x = a\tan\theta$ **....(i)**

$$\frac{d}{dx}x = a\frac{d}{dx}\tan\theta \Rightarrow 1 = a\sec^2\theta\,\frac{d\theta}{dx}$$

$\Rightarrow$ $dx = a\sec^2\theta d\theta$ **.....(ii) , put (i) and (ii)**

$$\int \frac{dx}{a^2+x^2} = \int \frac{a\sec^2\theta d\theta}{a^2+(a\tan\theta)^2}$$

$$= \int \frac{a\sec^2\theta d\theta}{a^2+a^2\tan^2\theta} = \int \frac{a\sec^2\theta d\theta}{a^2(1+\tan^2\theta)}$$

$$= \frac{1}{a}\int \frac{\sec^2\theta d\theta}{\sec^2\theta} = \frac{1}{a}\int d\theta$$

$$= \theta + c \quad \text{..... (iii)}$$

From (i) $\Rightarrow$ $\tan\theta = \frac{x}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{x}{a}\right)$ **....(iv)**

Put (iv) in (iii) $\Rightarrow$ $\int \frac{dx}{a^2+x^2} = \tan^{-1}\left(\frac{x}{a}\right)+c$ **Ans.**

(b) $\int \frac{dx}{\sqrt{a^2+x^2}}$

Solution: Let $I = \int \frac{dx}{\sqrt{a^2+x^2}}$

Let $x = a\tan\theta \Rightarrow dx = a\sec^2\theta\, d\theta$

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$$Now \quad I = \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2 + a^2\tan^2\theta}}$$

$$\Rightarrow I = \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2(1+\tan^2\theta)}} = \int \frac{a\sec^2\theta\, d\theta}{\sqrt{a^2\sec^2\theta}}$$

$$\Rightarrow I = \int \frac{a\sec^2\theta\, d\theta}{a\sec\theta}$$

$$= \int \sec\theta\, d\theta$$

$$\Rightarrow I = \int \frac{\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta}\, d\theta$$

$$\Rightarrow I = \ln|\sec\theta+\tan\theta|$$

$$\Rightarrow I = \ln\left|\sqrt{\tan^2\theta+1}+\tan\theta\right| \qquad As \quad a\tan\theta = x$$

$$= \ln\left|\sqrt{\frac{x^2}{a^2}+1} + \frac{x}{a}\right| \qquad \tan\theta = \frac{x}{a}$$

$$\Rightarrow I = \ln\left|\sqrt{\frac{x^2+a^2}{a^2}} + \frac{x}{a}\right|$$

$$= \ln\left|\frac{\sqrt{x^2+a^2}+x}{a}\right| = \ln\left|\sqrt{x^2+a^2}+x\right| - \ln a$$

$$\Rightarrow I = \ln\left|x+\sqrt{x^2+a^2}\right| + c \quad Ans \qquad AS \quad c = -\ln a$$

(c) $I = \int \frac{dx}{\sqrt{x^2-a^2}}$

Solution: **Let** $x = a\sec\theta$

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$$\Rightarrow dx = a\sec\theta\tan\theta\, d\theta$$

Now $$\Rightarrow I = \int \frac{a\sec\theta.\tan\theta\, d\theta}{\sqrt{a^2\sec^2\theta - a^2}}$$

$$= \int \frac{a\sec\theta.\tan\theta\, d\theta}{\sqrt{a^2(\sec^2\theta - 1)}} = \int \frac{a\sec\theta.\tan\theta\, d\theta}{\sqrt{a^2\tan^2\theta}}$$

$$= \int \frac{a\sec\theta.\tan\theta}{a.\tan} d\theta = \int \sec\theta\, d\theta$$

$$= \int \frac{\sec\theta(\sec\theta + \tan\theta)}{\sec\theta + \tan\theta} d\theta$$

$$= \ln|\sec\theta + \tan\theta|$$

$$= \ln\left|\sec\theta + \sqrt{\sec^2\theta - 1}\right|$$

$$As \quad a\sec\theta = x \Rightarrow \sec\theta = \frac{x}{a}$$

Now $$I = \ln\left|\frac{x}{a} + \sqrt{\frac{x^2}{a^2} - 1}\right|$$

$$= \ln\left|\frac{x + \sqrt{x^2 - a^2}}{a}\right|$$

$$= \ln\left|x + \sqrt{x^2 - a^2}\right| - \ln a$$

$$\Rightarrow I = \ln\left|x + \sqrt{x^2 - a^2}\right| + c \qquad \boxed{As - \ln a = c}$$

OR Also $$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\frac{x}{a}$$

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Example 6: Evaluate the integral $\int \frac{dx}{x^2+4x+5}$

Solution: $\int \frac{dx}{x^2+4x+5} = \int \frac{dx}{x^2+4x+4+1}$

$$= \int \frac{dx}{(x+2)^2+1} \quad \text{.....(i)}$$

Let $x+2=t \Rightarrow d(x+2)=dt$

$\Rightarrow dx = dt$ put values in (i)

$$\int \frac{dx}{x^2+4x+5} = \int \frac{dt}{t^2+1}$$

$$= \tan^{-1} t + c \quad \text{put value of } t$$

$$= \tan^{-1}(x+2) + c \quad \text{Ans.}$$

Example .7: Evaluate the integral $\int \frac{dx}{\sqrt{x^2+4x+5}}$

Solution: $\int \frac{dx}{\sqrt{x^2+4x+5}} = \int \frac{dx}{\sqrt{x^2+4x+4+1}}$

$$= \int \frac{dx}{\sqrt{(x+2)^2+1}} \quad \text{.....(i)}$$

Let $x+2=t \Rightarrow d(x+2)=dt$

$\Rightarrow dx = dt$ put values in (i)

$$\int \frac{dx}{\sqrt{x^2+4x+5}} = \int \frac{dt}{\sqrt{t^2+1}}$$

$$= \sinh^{-}(t) + c$$

$$= \sinh^{-1}(x+2) + c.$$

use formula

$$\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x$$

Example .9: Evaluate the integral $\int \frac{x+2}{\sqrt{x^2+4x+5}} dx$

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Solution:

$$\int \frac{x+2}{\sqrt{x^2+4x+5}}dx = \int \frac{x+2}{\sqrt{x^2+4x+4+1}}dx$$

$$= \int \frac{(x+2)dx}{\sqrt{(x+2)^2+1}} \quad \text{.....(i)}$$

Let $x+2=t \quad \Rightarrow \quad d(x+2)=dt$

$\Rightarrow \quad dx = dt$ **put values in (i)**

$$\int \frac{dx}{\sqrt{x^2+4x+5}} = \int \frac{tdt}{\sqrt{t^2+1}}$$

$$= \frac{1}{2}\int (t^2+1)^{-\frac{1}{2}}(2t)dt$$

$$= \frac{1}{2}\frac{(t^2+1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= \frac{1}{2}.\frac{(t^2+1)^{\frac{1}{2}}}{\frac{1}{2}} + c = \sqrt{t^2+1} + c$$

$$= \sqrt{(x+2)^2+1} + c \quad Ans.$$

use rule

$$\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

Integration by parts : As we know that

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$$

$$= f(x)g'(x) + g(x)f'(x)$$

$$\Rightarrow \frac{d}{dx}[f(x)g(x)] - g(x)f'(x) = f(x)g'(x)$$

$$or \quad f(x)g'(x) = \frac{d}{dx}[f(x)g(x)] - g(x)f'(x)$$

Take integral

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$$\int f(x)g'(x)dx = \int \frac{d}{dx}[f(x)g(x)]dx - \int g(x)f'(x)dx$$

$$\Rightarrow \int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx \quad .....(A)$$

Eq(A) is the formula of integration by parts.

The standard form of integration by parts is obtained by Putting $u = f(x)$ *and* $v = g(x)$ **then**

$du = f'(x)dx$ *and* $dv = g'(x)dx$ **in (A)**

$$\int u\,vdv = u\,v - \int vdu \quad .....(B)$$

Eq(B) shows the standard form of integration by parts.

Note: Before to attempt a question, choose u **and** $\frac{dv}{dx}$

to obtain du **and** v.

Note:(i) If there is Logarithm or Inverse Trigonometric function .Then take these function is u.

(ii) If there is no logarithm or inverse trigonometric function then take simple function i.e x^n *is* u

(iii) If there is only logarithm or inverse trigonometric function then take "1" is $\frac{dv}{dx}$.

Example:10: Evaluate the integral $\int x\,e^x dx$.

Solution: Here $u = x$ **and** $\frac{dv}{dx} = e^x \Rightarrow dv = e^x dx$

$\Rightarrow du = dx$ **and** $v = e^x$

As we know that integration by parts is given by

$\int u\,dv = u\,v - \int vdu$ **put values**

$$\Rightarrow \int x\,e^x dx = x.e^x - \int e^x dx = xe^x - e^x \quad \text{Ans.}$$

6.4.2: Evaluate the following integrals

(a) $\int \sqrt{a^2 - x^2}\,dx$

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Solution: Let $I = \int 1.\sqrt{a^2 - x^2}\, dx \rightarrow (i)$

Let $u = \sqrt{a^2 - x^2}$ and $\frac{dv}{dx} = 1$

$$\frac{du}{dx} = \frac{d}{dx}\left(a^2 - x^2\right)^{\frac{1}{2}} = \frac{1}{2}\left(a^2 - x^2\right)^{-\frac{1}{2}}(-2x)$$

$$\Rightarrow \quad du = \frac{-x}{\sqrt{a^2 - x^2}}dx \quad \text{and} \quad dv = dx \quad \Rightarrow \quad v = x$$

As we know that integration by parts is given by

$$\int u\, dv = u\, v - \int v du \quad \text{.....(ii)}$$

Put values u , du , dv and v in (ii) .

$$\Rightarrow I = \int \sqrt{a^2 - x^2}\, dx = \sqrt{a^2 - x^2}\,(x) - \int \frac{-x}{\sqrt{a^2 - x^2}}(x)dx$$

$$\Rightarrow \quad I = x\sqrt{a^2 - x^2} - \int \frac{-x^2}{\sqrt{a^2 - x^2}}\, dx$$

Adding and subtract a^2

$$\Rightarrow \quad I = x\sqrt{a^2 - x^2} - \int \frac{a^2 - x^2 - a^2}{\sqrt{a^2 - x^2}}\, dx$$

$$\Rightarrow I = x\sqrt{a^2 - x^2} - \int \left(\frac{a^2 - x^2}{\sqrt{a^2 - x^2}} - \frac{a^2}{\sqrt{a^2 - x^2}} \right) dx$$

$$= x\sqrt{a^2 - x^2} - \int \sqrt{a^2 - x^2}\, dx + a^2 \int \frac{dx}{\sqrt{a^2 - x^2}}$$

Put eq (i)

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$$\Rightarrow I = x\sqrt{a^2 - x^2} - I + a^2 \sin^{-1}\frac{x}{a}$$

$$\Rightarrow I + I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a} + c$$

$$\Rightarrow 2I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\frac{x}{a} + c$$

$$\Rightarrow I = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + \frac{c}{2} \quad Ans$$

$$\boxed{\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}}$$

(ii) $\int \sqrt{a^2 + x^2}\, dx$

Solution: Let $I = \int 1.\sqrt{a^2 + x^2}\ dx \rightarrow (i)$

Let $u = \sqrt{a^2 + x^2}$ **and** $\frac{dv}{dx} = 1$

$$\frac{du}{dx} = \frac{d}{dx}\left(a^2 + x^2\right)^{\frac{1}{2}} = \frac{1}{2}\left(a^2 + x^2\right)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{a^2 + x^2}}$$

$$\Rightarrow \quad du = \frac{x}{\sqrt{a^2 + x^2}}dx$$

And $dv = dx \quad \Rightarrow \quad v = x$

As we know that integration by parts is given by

$$\int u\, dv = u\, v - \int v du \quad \text{.....(ii)}$$

Put values u , du , dv and v in (ii) .

$$\Rightarrow I = \int 1.\sqrt{a^2 - x^2}dx = (\sqrt{a^2 + x^2}).x - \int \frac{x}{\sqrt{a^2 + x^2}}.xdx$$

$$\Rightarrow I = x\sqrt{a^2 + x^2} - \int \frac{x^2}{\sqrt{a^2 + x^2}}dx \quad \textit{Adding and subtracting } a^2$$

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$$= x\sqrt{a^2+x^2} - \int \frac{a^2+x^2-a^2}{\sqrt{a^2+x^2}}dx$$

$$= x\sqrt{a^2+x^2} - \int \left(\frac{a^2+x^2}{\sqrt{a^2+x^2}} - \frac{a^2}{\sqrt{a^2+x^2}}\right)dx$$

$$\Rightarrow I = x\sqrt{a^2+x^2} - \int \sqrt{a^2+x^2}\,dx + a^2\int \frac{dx}{\sqrt{a^2+x^2}}$$

$$\Rightarrow I = x\sqrt{a^2+x^2} - I + a^2 \sinh^{-1}\frac{x}{a}$$

$$\Rightarrow I + I = x\sqrt{a^2+x^2} + a^2 \sinh^{-1}\frac{x}{a} + c$$

$$\Rightarrow I = \frac{x\sqrt{a^2+x^2}}{2} + \frac{a^2 \sinh^{-1}\frac{x}{a}}{2} + \frac{c}{2} \quad Ans$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\frac{x}{a}$$

Example 11: **Evaluate the integral** $\int x \ln x\,dx$

Solution: **Let** $u = \ln x$ **and** $\frac{dv}{dx} = x$

Then $\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x}dx$

And $dv = xdx \Rightarrow v = \frac{x^2}{2}$

As we know that integration by parts is given by

$$\int u\,dv = uv - \int v\,du \quad \text{.....(i)}$$

Put values u , du , dv and v in (i) .

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$$\int x \ln x dx = (\ln x)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$= \left(\frac{x^2}{2}\right)\ln x - \frac{1}{2}\int x dx$$

$$= \left(\frac{x^2}{2}\right)\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + c$$

$$= \left(\frac{x^2}{2}\right)\ln x - \frac{x^2}{4} + c \quad Ans.$$

Example .12: **Evaluate the integral** $\int e^x \sin x dx$.

Solution: **Let** $I = \int e^x \sin x dx$ .....(i)

Here let $u = e^x$ **and** $\frac{dv}{dx} = \sin x$

$$\Rightarrow \frac{du}{dx} = e^x \quad \Rightarrow \quad du = e^x dx$$

Also $dv = \sin x \, dx \quad \Rightarrow \int dv = \int \sin x dx$

Or $v = -\cos x$

As we know that integration by parts is given by

$$\int u \, dv = uv - \int v du \quad .....(i)$$

Put values u , du , dv and v in (i).

$$I = \int e^x \sin x dx = e^x(-\cos x) - \int (-\cos x)e^x dx$$

$$\Rightarrow I = -e^x \cos x + \int e^x \cos x dx \quad .....(ii)$$

Now again take $u = e^x$ **and** $\frac{dv}{dx} = \cos x$

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$$\Rightarrow \frac{du}{dx} = e^x \quad \Rightarrow \quad du = e^x dx$$

Also $dv = \cos x \; dx \quad \Rightarrow \quad \int dv = \int \cos x dx$

Or $v = \sin x$ **put in (i)**

Then $\int e^x \cos x dx = e^x \sin x - \int (\sin x)(e^x) dx$ **.....(iii)**

Put (iii) in (ii)

$$I = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx + c \quad .....(iv)$$

put (i) *in* (iv)

$$\Rightarrow I = -e^x \cos x + e^x \sin x - I + c$$

$$\Rightarrow I + I = -e^x \cos x + e^x \sin x + c$$

$$\Rightarrow 2I = e^x (\sin x - \cos x) + c$$

$$\Rightarrow I = \frac{e^x (\sin x - \cos x)}{2} + \frac{c}{2} \qquad Ans.$$

Exercise 6.2:

Q.1: Evaluate the following indefinite integrals by method of substitution :

(a) $\int \sin^4 x \cos x dx$

Solution: $\int \sin^4 x \cos x dx$

Let $\sin x = t \quad \Rightarrow \frac{d}{dx}(\sin x) = \frac{dt}{dx}$

$$\Rightarrow \cos x = \frac{dt}{dx} \quad \Rightarrow \quad \cos x \, dx = dt$$

put values in the given integral

$$\int \sin^4 x \cos x \, dx = \int t^4 dt$$

$$= \frac{t^5}{5} + c \qquad put \; value \; of \; t$$

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$$= \frac{\sin^5 x}{5} + c \quad Ans.$$

(b) $\int \sqrt[3]{\sin^2 x}\cos x dx$

Solution: $\int \sqrt[3]{\sin^2 x}\cos x dx = \int \sin^{\frac{2}{3}} x \cos x \, dx$ .....(i)

Let $\sin x = t \quad \Rightarrow \frac{d}{dx}(\sin x) = \frac{dt}{dx}$

$\Rightarrow \cos x = \frac{dt}{dx} \quad \Rightarrow \quad \cos x\, dx = dt$

put values in the given integral

$$\int \sqrt[3]{\sin^2 x}\cos x dx = \int t^{\frac{2}{3}} dt$$

$$= \frac{t^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c = \frac{t^{\frac{5}{3}}}{\frac{5}{3}}$$

$$= \frac{3t^{\frac{5}{3}}}{5} + c \quad put\ value\ of\ t$$

$$= \frac{3\sin^{\frac{5}{3}} x}{5} + c \quad Ans.$$

(c) $\int \frac{\sin x \ln(\cos x)}{\cos x} dx$

Solution: **Let** $\ln(\cos x) = t$ ....(i)

$\Rightarrow \frac{d}{dx}\ln(\cos x) = \frac{dt}{dx}$

$\Rightarrow \frac{1}{\cos x}(-\sin x) = \frac{dt}{dx}$

$\Rightarrow \frac{\sin x}{\cos x} dx = -dt$ .....(ii)

Put (i) and (ii) in the given integral .

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$$\int \ln(\cos x)\frac{\sin x}{\cos x}dx = -\int t\,dt$$

$$= -\frac{t^2}{2} + c \quad \textit{put value of } t$$

$$= -\frac{(\ln \sin x)^2}{2} + c \quad \textit{Ans.}$$

(d) $\int e^x \sin e^x dx$

Solution:

$$\textit{Let} \quad e^x = t \ldots\ldots(i) \quad \Rightarrow \quad \frac{d}{dx}e^x = \frac{dt}{dx}$$

$$\Rightarrow \quad e^x = \frac{dt}{dx} \quad \Rightarrow \quad e^x dx = dt \ldots\ldots\ldots(ii)$$

Put (i) and (ii) in the given integral.

$$\int e^x \sin e^x dx = \int \sin t\,dt$$

$$= -\cos t + c \quad \textit{put value of } t$$

$$= -\cos e^x + c \quad \textit{Ans.}$$

(e) $\int \frac{\cot\sqrt{x}}{\sqrt{x}}dx$

Solution: $\int \frac{\cot\sqrt{x}}{\sqrt{x}}dx = \int \frac{\cos\sqrt{x}}{\sin\sqrt{x}}.\frac{dx}{\sqrt{x}}$ .....(i)

Let $\sin\sqrt{x} = t$ .....(ii)

$$\frac{d}{dx}\sin\left(x^{\frac{1}{2}}\right) = \frac{dt}{dx}$$

$$\Rightarrow \quad \cos\left(x^{\frac{1}{2}}\right)\frac{d}{dx}x^{\frac{1}{2}} = \frac{dt}{dx}$$

$$\Rightarrow \quad \left(\cos\sqrt{x}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = \frac{dt}{dx}$$

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$$\Rightarrow \quad \frac{\cos\sqrt{x}}{\sqrt{x}}dx = 2dt \quad .....(iii)$$

Put (ii) and (iii) in (i)

$$\int \frac{\cot\sqrt{x}}{\sqrt{x}}dx = 2\int \frac{dt}{t}$$

$$= 2\ln|t| + c \qquad \textit{put values of } t$$

$$= 2\ln|\sin\sqrt{x}| + c \qquad \textit{Ans.}$$

(f) $\int \frac{\sin x - \cos x}{\sin x + \cos x}dx$

Solution: Let $\sin x + \cos x = t$ .....(i)

$$\frac{d}{dx}(\sin x + \cos x) = \frac{dt}{dx}$$

$$\Rightarrow (\cos x - \sin x) = \frac{dt}{dx} \quad \Rightarrow -(\sin x - \cos x)dx = dt$$

$$(\sin x - \cos x)dx = -dt \quad .....(ii)$$

Put (i) and (ii) in the given integral .

$$\int \frac{\sin x - \cos x}{\sin x + \cos x}dx = -\int \frac{dt}{t}$$

$$= -\ln|t| + c \qquad \textbf{put value of t}$$

$$= -\ln|\sin x + \cos x| + c$$

Q. 2: Use suitable substitutions and tables to evaluate the the following indefinite integral .

(a) $\int \frac{dx}{x^2 + 16}$

Solution: $\int \frac{dx}{x^2 + 16}$

Let $x = 4\tan\theta$ .....(i)

$$\frac{dx}{dx} = 4\frac{d}{dx}\tan\theta \quad \Rightarrow \quad 1 = 4\sec^2\theta\frac{d\theta}{dx}$$

$$\Rightarrow \quad dx = 4\sec^2\theta d\theta \quad .....(ii)$$

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Put (i) and (ii) in the given integral.

$$\int \frac{dx}{x^2+16} = \int \frac{4\sec^2\theta}{16\tan^2\theta+16}\,d\theta$$

$$= \int \frac{4sce^2\theta\,d\theta}{16(\tan^2\theta+1)}$$

$$= \frac{1}{4}\int \frac{\sec^2\theta}{\sec^2\theta}\,d\theta = \frac{1}{4}\int d\theta$$

$$= \frac{1}{4}\theta + c \quad \text{.....(iii)}$$

As from (i) $\tan\theta = \frac{x}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{x}{4}\right)$ **.....(iv)**

Put (iv) in (iii)

$$\int \frac{dx}{x^2+16} = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + c \qquad \textit{Ans.}$$

(b) $\int \frac{\sin x}{\cos^2 x+1}\,dx$

Solution: **Let** $\cos x = t$ **.....(i)**

$$\frac{d}{dx}\cos x = \frac{dt}{dx} \Rightarrow -\sin x = \frac{dt}{dx}$$

$$\Rightarrow \sin x\,dx = -dt \quad \text{.....(ii)}$$

Put (i) and (ii) in the given integral

$$\int \frac{\sin x}{\cos^2 x+1}\,dx = -\int \frac{dt}{t^2+1}$$

$$= -\tan^{-1} t + c \qquad \textit{put value of } t$$

$$= -\tan^{-1}(\cos x) + c \qquad \textit{Ans.}$$

(c) $\int \frac{dx}{\sqrt{e^{2x}-4}}$

Solution : $\int \frac{dx}{\sqrt{e^{2x}-4}} = \int \frac{dx}{\sqrt{(e^x)^2-(2)^2}}$ **.....(i)**

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Let $e^x = t$.....(ii) $\Rightarrow \frac{d}{dx}e^x = \frac{dt}{dx}$

$\Rightarrow e^x = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{e^t}$

$\Rightarrow dx = \frac{dt}{t}$.....(iii)

Put (ii) and (iii) in (i)

$$\int \frac{dx}{\sqrt{e^{2x}-4}} = \int \frac{dt}{t\sqrt{t^2-(2)^2}}$$

$= \sec^{-1}\left(\frac{t}{2}\right) + c$ *put value of t*

$= \sec^{-1}\left(\frac{e^x}{2}\right) + c$ *Ans.*

(d) $\int \frac{2x+5}{x^2+4x+5}dx$

Solution: $\int \frac{2x+5}{x^2+4x+5}dx = \int \frac{2x+5}{x^2+4x+4+1}dx$

$= \int \frac{2x+4+1}{(x+2)^2+1}dx$

$= \int \frac{2(x+2)}{(x+2)^2+1}dx + \int \frac{1}{(x+2)^2+1}dx$ .....(i)

Put $x+2=t$ ....(ii)

$\frac{d}{dx}(x+2) = \frac{dt}{dx} \Rightarrow 1 = \frac{dt}{dx} \Rightarrow dx = dt$ .....(iii)

Put (ii) and (iii) in (i)

$$\int \frac{2x+5}{x^2+4x+5}dx = \int \frac{2tdt}{t^2+1} + \int \frac{1}{t^2+1}dt$$

$= \ln|t^2+1| + \tan^{-1}t + c$

use rule

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|$$

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$$= \ln\left|(x+2)^2+1\right| + \tan^{-1}(x+2) + c$$

$$= \ln\left|x^2+4x+5\right| + \tan^{-1}(x+2) + c$$

(e) $\int \frac{2+x}{\sqrt{4-2x-x^2}}\,dx$

Solution: $\int \frac{2+x}{\sqrt{4-2x-x^2}}\,dx = \int \frac{2+x}{\sqrt{4-(x^2+2x)}}\,dx$

$$= \int \frac{2+x}{\sqrt{4+1-(x^2+2x+1)}}\,dx$$

$$= \int \frac{x+1+1}{\sqrt{5-(x+1)^2}}\,dx$$

$$= \int \frac{x+1}{\sqrt{5-(x+1)^2}}\,dx - \int \frac{1}{\sqrt{5-(x+1)^2}}\,dx \quad \text{....(i)}$$

Put $x+1=t$ .....(ii)

$$\frac{d}{dx}(x+1) = \frac{dt}{dx} \quad \Rightarrow \quad 1 = \frac{dt}{dx}$$

$$\Rightarrow \quad dx = dt \quad \text{.....(iii)}$$

use $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$

Put (ii) and (iii) in (i)

$$\int \frac{2+x}{\sqrt{4-2x-x^2}}\,dx = \int \frac{t}{\sqrt{5-t^2}}\,dt + \int \frac{1}{\sqrt{5-t^2}}\,dt$$

$$= -\frac{1}{2}\int (5-t^2)^{-\frac{1}{2}}(-2t)\,dt + \int \frac{dt}{\sqrt{(\sqrt{5})^2-t^2}}$$

$$= -\frac{1}{2}\cdot\frac{(5-t^2)^{\frac{1}{2}+1}}{-\frac{1}{2}+1} + \sin^{-1}\left(\frac{t}{\sqrt{5}}\right) + c$$

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$$= -\frac{(5-t^2)^{\frac{1}{2}}}{\frac{1}{2}} + \sin^{-1}\left(\frac{t}{\sqrt{5}}\right) + c \quad put\ value\ of\ t$$

$$= -2\sqrt{5-(x+1)^2} + \sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right) + c$$

$$= -2\sqrt{5-x^2-2x-1} + \sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right) + c$$

$$= -2\sqrt{4-2x-x^2} + \sin^{-1}\left(\frac{x+1}{\sqrt{5}}\right) + c \quad Ans.$$

Q.3: Evaluate the following using integration by parts .

(a) $\int x^2 e^{2x} dx$

Solution: Here $u = x^2$ and $\frac{dv}{dx} = e^{2x} \Rightarrow dv = e^{2x}dx$

$\Rightarrow du = 2xdx$ and $v = \frac{e^{2x}}{2}$

As we know that integration by parts is given by

$\int u\,dv = uv - \int v du$ ....(a) put values

$$\Rightarrow \int x^2 e^{2x} dx = x^2 . \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} . 2x dx$$

$$= \frac{x^2 . e^{2x}}{2} - \int x . e^{2x} dx \quad .....(i)$$

Here $u = x$ and $\frac{dv}{dx} = e^{2x} \Rightarrow dv = e^{2x}dx$

$\Rightarrow du = dx$ and $v = \frac{e^{2x}}{2}$ put (a)

$$\Rightarrow \int x\, e^{2x} dx = x . \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + c \quad ,,,,,(ii)$$

Put (i) and (ii) in (a)

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$$\int x^2 e^{2x} dx = \frac{x^2}{2} e^{2x} - \left(\frac{xe^{2x}}{2} - \frac{e^{2x}}{4} \right) + c$$

$$= \frac{x^2}{2} e^{2x} - \frac{xe^{2x}}{2} + \frac{e^{2x}}{4} + c \quad \textbf{Ans.}$$

(b) $\int x \cos x dx$

Solution: Here $u = x$ and $\frac{dv}{dx} = \cos x$

$du = dx$ and $dv = \cos x dx$

$$\Rightarrow \int dv = \int \cos x dx \Rightarrow v = \sin x$$

Since the integration by parts is given by

$$\int u dv = uv - \int v du \quad \text{.....(A)}$$

Put values of u, v, du and dv in (A).

$$\int x \cos x dx = x(\sin x) - \int \sin x dx$$

$$= x \sin x + \cos x + c \quad \textit{Ans.}$$

(c) $\int e^x \sin(x) dx$

Solution: Let $I = \int e^x \sin x dx$ .....(i)

Here let $u = e^x$ and $\frac{dv}{dx} = \sin x$

$$\Rightarrow \frac{du}{dx} = e^x \Rightarrow du = e^x dx$$

Also $dv = \sin x \, dx \Rightarrow \int dv = \int \sin x dx$

Or $v = -\cos x$

As we know that integration by parts is given by

$$\int u \, dv = uv - \int v du \quad \text{.....(A)}$$

Put values u, du, dv and v in (A).

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$$I = \int e^x \sin x dx = e^x(-\cos x) - \int (-\cos x)e^x dx$$

$$\Rightarrow I = -e^x \cos x + \int e^x \cos x dx \quad .....(ii)$$

Now again take $u = e^x$ **and** $\frac{dv}{dx} = \cos x$

$$\Rightarrow \frac{du}{dx} = e^x \quad \Rightarrow \quad du = e^x dx$$

Also $dv = \cos x \, dx \quad \Rightarrow \quad \int dv = \int \cos x dx$

Or $v = \sin x$ **put values of u , v , du and dv in (A)**

Then $\int e^x \cos x dx = e^x(\sin x) - \int (\sin x)(e^x) dx$

$$\Rightarrow \int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx \text{ .....(iii)}$$

Put (iii) in (ii)

$$I = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx + c \ .$$

$$= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx \quad .....(iv)$$

put (i) in (iv)

$$\Rightarrow I = -e^x \cos x + e^x \sin x - I + c$$

$$\Rightarrow I + I = e^x \sin x - e^x \cos x + c$$

$$\Rightarrow 2I = e^x(\sin x - \cos x) + c$$

$$\Rightarrow I = \frac{e^x(\sin x - \cos x)}{2} + c \quad Ans.$$

(d) $\int e^x \cos x dx$

Solution: Let $I = \int e^x \cos x dx$ **.....(i)**

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Here let $u = e^x$ and $\frac{dv}{dx} = \cos x$

$$\Rightarrow \frac{du}{dx} = e^x \quad \Rightarrow \quad du = e^x dx$$

Also $dv = \cos x\ dx \quad \Rightarrow \quad \int dv = \int \cos x dx$

Or $v = \sin x$

As we know that integration by parts is given by

$$\int u\, dv = u\, v - \int v du \quad \text{.....(A)}$$

Put values u , du , dv and v in (A) .

$$I = \int e^x \cos x dx = e^x (\sin x) - \int (\sin x) e^x dx$$

$$\Rightarrow I = e^x \sin x - \int e^x \sin x dx \quad \text{.....(ii)}$$

Now again take $u = e^x$ and $\frac{dv}{dx} = \sin x$

$$\Rightarrow \frac{du}{dx} = e^x \quad \Rightarrow \quad du = e^x dx$$

Also $dv = \sin x\ dx \quad \Rightarrow \quad \int dv = \int \sin x dx$

Or $v = -\cos x$ put values of u , v , du and dv in (A)

Then $\int e^x \sin x dx = e^x(-\cos x) - \int (-\cos x)(e^x) dx$

$$\Rightarrow \int e^x \sin x dx = -e^x \cos x + \int e^x \cos x dx \text{.....(iii)}$$

Put (iii) in (ii)

$$I = e^x \sin x - \left(-e^x \cos x + \int e^x \cos x\, dx\right) + c \ .$$

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$$= e^x \sin x + e^x \cos x - \int e^x \cos x\, dx \quad .....(iv)$$

put (*i*) *in*(*iv*)

$$\Rightarrow I = e^x \sin x + e^x \cos x - I + c$$

$$\Rightarrow I + I = e^x \sin x + e^x \cos x + c$$

$$\Rightarrow 2I = e^x(\sin x + \cos x) + c$$

$$\Rightarrow I = \frac{e^x(\sin x + \cos x)}{2} + C \qquad Ans.$$

(e) $\int x \sin^{-1} x dx$

Solution: Here $u = \sin^{-1} x$ **and** $\frac{dv}{dx} = x$

$$\frac{du}{dx} = \frac{d}{dx}\sin^{-1} x \quad \Rightarrow \quad du = \frac{1}{\sqrt{1-x^2}}dx$$

Also $dv = xdx \quad \Rightarrow \int dv = \int xdx$

$$\Rightarrow v = \frac{x^2}{2}$$

Since the integration by parts is given by

$$\int udv = uv - \int vdu \quad .....(A)$$

Put values of u , v , du and dv in (A).

$$\int x \sin^{-1} x dx = \left(\sin^{-1} x\right)\left(\frac{x^2}{2}\right) - \int \frac{x^2}{2} \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \frac{1-x^2+1}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \left(\frac{1-x^2}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}\right) dx$$

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$$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \frac{1-x^2}{\sqrt{1-x^2}}dx + \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \sqrt{1-x^2}\,dx + \frac{1}{2}\sin^{-1}x$$

use formula $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x$

and $\int \sqrt{a^2-x^2}\,dx = \frac{x\sqrt{a^2-x^2}}{2} - \frac{a^2}{2}\sin^{-1}\frac{x}{a}$

$$= \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\sin^{-1}x + \frac{1}{2}\int \sqrt{(1)^2-x^2}\,dx$$

$$= \left(\frac{x^2+1}{2}\right)\sin^{-1}x + \frac{1}{2}\left[\frac{x\sqrt{1-x^2}}{2} - \frac{1}{2}\sin^{-1}x\right]$$

$$= \left(\frac{x^2+1}{2}\right)\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \left(\frac{x^2+1}{2} - \frac{1}{4}\right)\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \left(\frac{2x^2+2-1}{4}\right)\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + c$$

$$= \left(\frac{2x^2+1}{4}\right)\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + c \quad \text{ans.}$$

(f) $\int e^{3t}\sin\left(e^{3t}\right)dt$

Solution: $\int e^{3t}\sin\left(e^{3t}\right)dt$

Let $e^{3t} = x$ .....(i) $\Rightarrow \frac{d}{dt}e^{3t} = \frac{dx}{dt}$

$\Rightarrow 3e^{3t}dt = dx$ or $e^{3t}dt = \frac{1}{3}dt$ .....(ii) , **put values**

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$$\int e^{3t}\sin\left(e^{3t}\right)dt = \frac{1}{3}\int x\,\sin x\,dx \qquad \text{.....(iii)}$$

Here $u = x$ **and** $\frac{dv}{dx} = \sin x$

$$\frac{du}{dx} = \frac{d}{dx}x \;\Rightarrow\; \frac{du}{dx} = 1 \;\Rightarrow\; du = dx$$

Also $dv = \sin x dx \;\Rightarrow\; \int dv = \int \sin x dx$

$$\Rightarrow\; v = -\cos x$$

Since the integration by parts is given by

$$\int udv = uv - \int vdu \quad \text{.....(A)}$$

Put values of u , v , du and dv in (A) .

$$\int e^{3t}\sin\left(e^{3t}\right)dt = \frac{1}{3}\left[x(-\cos x) - \int(-\cos x)dx\right]$$

$$= \frac{1}{3}\left[x(-\cos x) + \int \cos x dx\right]$$

$$= \frac{1}{3}\left[-x\cos x + \sin x\right] + c \qquad put \;\; x = e^{3t}$$

$$\int e^{3t}\sin\left(e^{3t}\right)dt = -\frac{e^{3t}\cos\left(e^{3t}\right)}{3} + \frac{\sin\left(e^{3t}\right)}{3} + c \qquad Ans.$$

Use partial fraction to find integration:

Example .13: Evaluate the following integrals

(a) $\int \frac{8x-1}{x^2-x-2}dx$

Solution:Since it is the proper fraction then we solve it by partial fraction .

First we factorize the denominator

$$x^2 - x - 2 = x^2 + x - 2x - 2$$

$$= x(x+1) - 2(x+1)$$

$$= (x+1)(x-2)$$

Then $\frac{8x-1}{x^2-x-2} = \frac{8x-1}{(x+1)(x-2)}$

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Let $\frac{8x-1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$ .....(i)

Both sides multiplied by $(x+1)(x-2)$

$8x-1 = A(x-2) + B(x+1)$ .....(ii)

Put $x=-1$ in (ii)

$8(-1)-1 = A(-1-2)+0$

$\Rightarrow -8-1 = -3A \quad \Rightarrow -9 = -3A$

$\Rightarrow A = 3$

Put $x=2$ in (ii)

$8(2)-1 = 0 + B(2+1) \quad \Rightarrow 15 = 3B$

$\Rightarrow B = 5$

Put values of A and B in (i)

$\frac{8x-1}{(x+1)(x-2)} = \frac{3}{x+1} + \frac{5}{x-2}$ integrate

$\int \frac{8x-1}{(x+1)(x-2)}dx = \int \left(\frac{3}{x+1} + \frac{5}{x-2}\right)dx$

$\Rightarrow \int \frac{8x-1}{x^2-x-2}dx = \int \frac{3}{x+1}dx + \int \frac{5}{x-2}dx$

$= 3\ln|x+1| + 5|x-2| + c$ *Ans.*

(b) $\int \frac{x^2-6x+3}{(x-2)^3}dx$

Solution: Let $\frac{x^2-6x+3}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$ ....(i)

Both sides multiplied by $(x-2)^3$

$x^2-6x+3 = A(x-2)^2 + B(x-2) + C$ .....(ii)

Put $x-2=0 \Rightarrow x=2$ in (ii)

$(2)^2 - 6(2) + 3 = 0 + 0 + C \quad \Rightarrow 4-12+3 = C$

$\Rightarrow C = -5$

From (ii)

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$$x^2-6x+3=A(x^2-4x+4)+B(x-2)+C$$

$$\Rightarrow x^2-6x+3=Ax^2+(-4A+B)x+4A-2B+C$$

Now equating the coefficients of x^2 , x

$$A=1$$

And $-4A+B=-6$ **put values of A**

$$\Rightarrow -4(1)+B=-6 \qquad \Rightarrow B=-6+4=-2$$

Put values of A , B and C in (i)

$$\frac{x^2-6x+3}{(x-2)^3}=\frac{1}{x-2}+\frac{-2}{(x-2)^2}+\frac{-5}{(x-2)^3}$$

$$\Rightarrow \int\frac{x^2-6x+3}{(x-2)^3}dx=\int\left(\frac{1}{x-2}+\frac{-2}{(x-2)^2}+\frac{-5}{(x-2)^3}\right)dx$$

$$=\int\frac{dx}{x-2}-2\int(x-2)^{-2}dx-5\int(x-2)^{-3}dx$$

$$=\ln|x-2|-2\cdot\frac{(x-2)^{-1}}{-1}-5\cdot\frac{(x-2)^{-2}}{-2}+c$$

$$=\ln|x-2|+\frac{2}{x-1}+\frac{5}{2(x-2)^2}+c \qquad Ans.$$

(c) $\int\frac{2x^3+x^2+2x+4}{(x^2+1)^2}dx$

Solution: it is proper fraction then

Let $\frac{2x^3+x^2+2x+4}{(x^2+1)^2}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}$ .....(i)

Both sides of eq(i) is multiplied by $(x^2+1)^2$.

$$2x^3+x^2+2x+4=(Ax+B)(x^2+1)+Cx+D$$

$$\Rightarrow 2x^3+x^2+2x+4=Ax^3+Ax+Bx^2+B+Cx+D$$

$$\Rightarrow 2x^3+x^2+2x+4=Ax^3+Bx^2+(A+C)x+B+D$$

Equating the coefficients of x^3 , x^2 , x **and constants .**

$$A=2\ , \quad B=1$$

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$A+C=2 \quad put \quad A=2$

$\Rightarrow 2+C=2 \quad \Rightarrow \quad C=0$

$R+D=4 \quad Put \quad B=1$

$\Rightarrow 1+D=4 \quad \Rightarrow \quad D=3$

Put values of A , B , C and D in (i)

$$\frac{2x^3+x^2+2x+4}{(x^2+1)^2}=\frac{2x+1}{x^2+1}+\frac{3}{(x^2+1)^2}$$

$$\Rightarrow \int\frac{2x^3+x^2+2x+4}{(x^2+1)^2}dx=\int\left(\frac{2x+1}{x^2+1}+\frac{3}{(x^2+1)^2}\right)dx$$

$$=\int\frac{2x}{x^2+1}dx+\int\frac{1}{x^2+1}dx+3\int\frac{1}{(x^2+1)^2}dx \quad ...(A)$$

Now we solve $\int\frac{2x}{x^2+1}dx$ **:**

Let $x^2+1=t \quad \Rightarrow \quad \frac{d}{dx}(x^2+1)=\frac{dt}{dx}$

$\Rightarrow \quad 2x=\frac{dt}{dx} \quad \Rightarrow \quad 2xdx=dt$ **, Then**

$$\int\frac{2x}{x^2+1}dx=\int\frac{dt}{t}=\ln|t|=\ln|x^2+1| \quad .....(ii)$$

And $$\int\frac{dx}{x^2+1}=\tan^{-1}x \quad .....(iii)$$

Now we solve $\int\frac{1}{(x^2+1)^2}dx$ **:**

Let $x=\tan\theta \quad \Rightarrow \quad dx=\sec^2\theta\, d\theta$

$$\int\frac{1}{(x^2+1)^2}dx=\int\frac{\sec^2\theta d\theta}{(\tan^2\theta+1)^2}=\int\frac{\sec^2\theta d\theta}{(\sec^2\theta)^2}$$

$$=\int\frac{1+\cos 2\theta}{2}d\theta$$

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$$= \frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta d\theta$$

$$= \frac{1}{2}\theta + \frac{1}{2}.\frac{\sin 2\theta}{2} + c \qquad put \ \ \theta = \tan^{-1} x$$

$$= \frac{1}{2}\tan^{-1} x + \frac{\sin\left(2\tan^{-1} x\right)}{4} + c \quad \text{.....(iv)}$$

Put (ii) , (iii) and (iv) in (A) .

$$\int \frac{2x^3 + x^2 + 2x + 4}{\left(x^2+1\right)^2} dx = \ln\left|x^2+1\right| + \tan^{-1} x + \frac{3}{2}\tan^{-1} x$$

$$+ \frac{3}{4}\sin\left(2\tan^{-1} x\right) + c$$

$$= \ln\left|x^2+1\right| + \frac{5}{2}\tan^{-1} x + \frac{3}{4}\sin\left(2\tan^{-1} x\right) + c$$

Exercise 6.3:

Q.1: Evaluate the indefinite integrals after decomposing the following rational functions into partial fractions .

(a) $\dfrac{1}{x(x-3)}$

Solution : it is proper fraction then ,

Let $\dfrac{1}{x(x-3)} = \dfrac{A}{x} + \dfrac{B}{x-3}$ .....(i)

Both sides multiplied by $x(x-3)$

$$1 = A(x-3) + Bx \quad \text{.....(ii)}$$

Put $x = 0$ **in (ii)**

$$1 = A(0-3) + 0 \qquad \Rightarrow \quad A = -\frac{1}{3}$$

Put $x - 3 = 0 \quad \Rightarrow x = 3$ **in (ii)**

$$1 = 0 + 3B \qquad \Rightarrow \quad B = \frac{1}{3}$$

Put values of A , B in (i)

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$$\frac{1}{x(x-3)} = \frac{-\frac{1}{3}}{x} + \frac{\frac{1}{3}}{x-3} \qquad \textbf{integrate}$$

$$\Rightarrow \int \frac{1}{x(x-3)}dx = -\frac{1}{3}\int \frac{1}{x}dx + \frac{1}{3}\int \frac{dx}{x-3}$$

$$= \frac{1}{3}\left[-\ln|x| + \ln|x-3|\right] + c$$

$$= \frac{1}{3}\ln\left|\frac{x-3}{x}\right| + c \qquad Ans.$$

(b) $\dfrac{3x^2+2x-1}{x(x+1)}$

Solution: $\dfrac{3x^2+2x-1}{x(x+1)} = \dfrac{3x^2+2x-1}{x^2+x}$

It is improper fraction , then by usual division

$$x^2+x \overline{)\ 3x^2+2x-1\ } \ \ 3$$

$$\pm 3x^2 \pm 3x$$

$$-x-1$$

Then $\dfrac{3x^2+2x-1}{x(x+1)} = 3 - \dfrac{x+1}{x(x+1)}$

$\Rightarrow$ $\dfrac{3x^2+2x-1}{x(x+1)} = 3 - \dfrac{1}{x}$ **integrate**

$$\int \frac{3x^2+2x-1}{x(x+1)}dx = \int\left(3 - \frac{1}{x}\right)dx$$

$$= 3x - \ln|x| + c \qquad Ans.$$

(c) $\dfrac{4x^3+4x^2+x-1}{x^2(x+1)^2}$

Solution: It is proper fraction .

Let $\dfrac{4x^3+4x^2+x-1}{x^2(x+1)^2} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2}$ .....(i)

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Eq(i) is multiplied by $x^2(x+1)^2$

$$4x^3+4x^2+x-1=Ax(x+1)^2+B(x+1)^2+Cx^2(x+1)+Dx^2 \quad \text{...(ii)}$$

Put $x=0$ **in (ii)**

$$0+0+0-1=0+B(0+1)^2+0+0$$

$$-1=B \quad \Rightarrow \quad B=-1$$

Put $x+1=0 \Rightarrow x=-1$ **in (ii)**

$$4(-1)^3+4(-1)^2-1-1=0+0+0+D(-1)^2$$

$$\Rightarrow -4+4-2=D \quad \Rightarrow \quad D=-2$$

From (ii)

$$4x^3+4x^2+x-1=Ax(x^2+2x+1)+B(x^2+2x+1)+Cx^2(x+1)+Dx^2$$

$$=Ax^3+2Ax^2+Ax+Bx^2+2Bx+B+Cx^3+Cx^2+Dx^2$$

$$\Rightarrow 4x^3+4x^2+x-1=(A+C)x^3+(2A+B+C+D)x^2+(A+2B)x+B$$

Equating the coefficients of x^3, x^2, x.

$$A+C=4 \quad \text{.....(ii)}$$

$$2A+B++C+D=4 \quad \text{.....(iii)}$$

$$A+2B=1 \quad \text{.....(iv)}$$

Put $B=-1$ **in (iv)**

$$A+2(-1)=1 \quad \Rightarrow \quad A=1+2=3$$

Put $A=3$ **in (ii)**

$$3+C=4 \quad \Rightarrow \quad C=4-3=1$$

Put values of A , B , C and D in (i)

$$\frac{4x^3+4x^2+x-1}{x^2(x+1)^2}=\frac{3}{x}+\frac{-1}{x^2}+\frac{1}{x+1}+\frac{-2}{(x+1)^2} \quad \text{integrate}$$

$$\int\frac{4x^3+4x^2+x-1}{x^2(x+1)^2}dx=\int\left(\frac{3}{x}-\frac{1}{x^2}+\frac{1}{x+1}-\frac{2}{(x+1)^2}\right)dx$$

$$=3\int\frac{dx}{x}-\int x^{-2}dx+\int\frac{1}{x+1}dx-2\int(x+1)^{-2}dx$$

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$$= 3\ln|x| - \frac{x^{-1}}{-1} + \ln|x+1| - 2.\frac{(x+1)^{-1}}{-1} + c$$

$$= 3\ln|x| + \frac{1}{x} + \ln|x+1| + \frac{2}{x+1} + c \quad Ans.$$

(d) $\frac{1}{x^3-1}$

Solution: $\frac{1}{x^3-1} = \frac{1}{(x-1)(x^2+x+1)}$

Let $\frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$ .....(i)

Eq (i) is multiplied by $(x-1)(x^2+x+1)$

$1 = A(x^2+x+1) + (Bx+C)(x-1)$ .....(ii)

Put $x-1=0 \Rightarrow x=1$ **in (ii)**

$1 = A(1+1+1)+0 \Rightarrow 3A=1 \Rightarrow A=\frac{1}{3}$

From (ii) $\Rightarrow 1 = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C$

$\Rightarrow 0x^2 + 0x + 1 = (A+B)x^2 + (A-B+C)x + A - C$

Equating the coefficients of x^2 , x **and constants.**

$A+B=0$ .....(ii), $A-B+C=0$ .....(iii)

$A-C=1$ .....(iv) **Put** $A=\frac{1}{3}$ **in (ii)**

$\frac{1}{3}+B=0 \Rightarrow B=-\frac{1}{3}$ **Put** $A=\frac{1}{3}$ **in (iv)**

$\frac{1}{3}-C=1 \Rightarrow \frac{1}{3}-1=C \Rightarrow C=\frac{2}{3}$

Put values of A , B , C in(i)

$$\frac{1}{(x-1)(x^2+x+1)} = \frac{\frac{1}{3}}{x-1} + \frac{-\frac{1}{3}x+\frac{2}{3}}{x^2+x+1}$$ **integrate**

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$$\int \frac{dx}{(x-1)(x^2+x+1)} = \frac{1}{3}\int \frac{1}{x}dx - \frac{1}{3}\int \frac{x-2}{x^2+x+1}dx$$

$$= \frac{1}{3}\ln|x| - \frac{1}{3}\int \frac{x+1-3}{x^2+x+1}dx$$

$$= \frac{1}{3}\ln|x| - \frac{1}{3}\int \frac{x+1}{x^2+x+1}dx - \frac{1}{3}\int \frac{3}{x^2+x+1}dx$$

$$= \frac{1}{3}\ln|x| - \frac{1}{3}\int \frac{x+1}{x^2+x+1}dx + \int \frac{dx}{x^2+x+1}$$

Use rule $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)|$

$$= \frac{1}{3}\ln|x| - \frac{1}{3}\ln|x^2+x+1| + \int \frac{dx}{x^2+x+\frac{1}{4}+\frac{3}{4}}$$

$$= \frac{1}{3}\left(\ln|x| - \ln|x^2+x+1|\right) + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{3}\ln\left|\frac{x^2+x+1}{x}\right| + \frac{1}{\frac{1}{2}}\tan^{-1}\left(\frac{x}{\frac{1}{2}}\right)$$

$$= \frac{1}{3}\ln\left|x+1+\frac{1}{x}\right| + 2\tan^{-1}2x + c$$

use $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$

(e) $\int \frac{x^4-x^2+2}{x^2(x-1)}dx$

Solution: $\frac{x^4-x^2+2}{x^2(x-1)} = \frac{x^4-x^2+2}{x^3-x^2}$

It is improper then

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$$x^3 - x^2 \overline{)\; x^4 \quad -x^2 + 2} \;\; x + 1$$

$$\frac{\pm x^4 \mp x^3}{x^3 - x^2 + 2}$$

$$\frac{\pm x^3 \mp x^2}{2}$$

Then $\dfrac{x^4}{x^2(x-1)} = x + 1 + \dfrac{2}{x^2(x-1)}$ .....(i)

Now by partial fraction

Let $\dfrac{2}{x^2(x-1)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x-1}$ .....(ii)

Eq (ii) multiplied by $x^2(x-1)$

$$2 = Ax(x-1) + B(x-1) + Cx^2 \quad \text{.....(iii)}$$

Put $x = 0$ **in (iii)**

$$2 = 0 + B(0-1) + 0 \quad \Rightarrow \quad B = -2$$

Put $x - 1 = 0 \quad \Rightarrow \quad x = 1$ **in (iii)**

$$2 = 0 + 0 + C(1)^2 \quad \Rightarrow \quad C = 2$$

From (iii) $\Rightarrow \quad 2 = Ax^2 - Ax + Bx - B + Cx^2$

$$\Rightarrow 0x^2 + 0x + 2 = (A + C)x^2 + (-A + B)x - B$$

Equating the coefficient of x^2.

$$A + C = 0 \qquad \text{Put} \quad C = 2$$

$$\Rightarrow A + 2 = 0 \quad \Rightarrow \quad A = -2$$

Put values of A , B and C in (iii)

$$\frac{2}{x^2(x-1)} = \frac{-2}{x} + \frac{-2}{x^2} + \frac{2}{x-1} \quad \text{.....(iv)}$$

Put (iv) in (i)

$$\frac{x^4}{x^2(x-1)} = x + 1 - \frac{2}{x} - \frac{2}{x^2} + \frac{2}{x-1}$$ **integrate**

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$$\Rightarrow \int \frac{x^4 dx}{x^2(x-1)} = \int (x+1)dx - 2\int \frac{dx}{x} - 2\int x^{-2}dx + 2\int \frac{dx}{x-1}$$

$$= \frac{x^2}{2} + x - 2\ln|x| - 2\frac{x^{-1}}{-1} + 2\ln|x+1| + c$$

$$= \frac{x^2}{2} + x + 2\ln\left|\frac{x+1}{x}\right| + \frac{2}{x} + c \qquad Ans.$$

(f) $\int \frac{dx}{x^2-1}$

Solution: $\int \frac{dx}{x^2-1}$

Let $\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$ .....(i)

Eq (i) is multiplied by $(x-1)(x+1)$

$$1 = A(x+1) + B(x-1) \quad .....(ii)$$

Put $x = 1$ **in (ii)**

$$1 = A(1+1) + 0 \qquad \Rightarrow \quad A = \frac{1}{2}$$

Put $x = -1$ **in (ii)**

$$1 = 0 + B(-1-1) \qquad \Rightarrow \quad B = -\frac{1}{2}$$

Put values of A and B in (i)

$$\frac{1}{x^2-1} = \frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x+1}$$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\int \left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx$$

$$= \frac{1}{2}\left[\ln|x-1| - \ln|x+1|\right] + c$$

$$= \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + c \qquad Ans.$$

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(g) $\int \frac{-x-3}{2x^2-x-1}dx$

Solution: $\frac{-x-3}{2x^2-x-1} = \frac{-x-3}{2x^2-2x+x-1}$

$$= \frac{-x-3}{2x(x-1)+1(x-1)}$$

$$\Rightarrow \frac{-x-3}{2x^2-x-1} = \frac{-x-3}{(x-1)(2x+1)}$$

Let $\frac{-x-3}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$ .....(i)

Both sides multiplied by $(x-1)(2x+1)$

$-x-3 = A(2x+1)+B(x-1)$ .....(ii)

Put $x=1$ **in (ii)**

$-1-3 = A(2+1)+0 \Rightarrow 3A=-4 \Rightarrow A=-\frac{4}{3}$

Put $x=-\frac{1}{2}$ **in (ii)**

$\frac{1}{2}-3 = 0+B\left(-\frac{1}{2}-1\right) \Rightarrow -\frac{5}{2} = -\frac{3}{2}B \Rightarrow B=\frac{5}{3}$

Put values of A and B in(i)

$$\frac{-x-3}{(x-1)(2x+1)} = \frac{-\frac{4}{3}}{x-1} + \frac{\frac{5}{3}}{2x+1}$$

$$\int \frac{-x-3}{(x-1)(2x+1)}dx = \frac{1}{3}\int \frac{-4}{x-1}dx + \frac{5}{3}\int \frac{dx}{2x+1}$$

$$= -\frac{4}{3}\ln|x-1| + \frac{5}{6}\int \frac{2dx}{2x+1}$$

$$= -\ln\left|(x-1)^{\frac{4}{3}}\right| + \frac{5}{6}\ln|2x+1|$$

use $\int \frac{f'(x)}{f(x)}dx$
$= \ln\lvert f(x)\rvert$

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$$= -\ln\left|(x-1)^{\frac{4}{3}}\right| + \ln\left|(2x+1)^{\frac{5}{6}}\right| + c$$

$$= \ln\left|\frac{(x-1)^{\frac{4}{3}}}{(2x+1)^{\frac{5}{6}}}\right| + c$$

(h) $\int \frac{x^2-1}{x^2-2x-15}dx$

Solution: **It is improper fraction then by usual division**

Then

$$x^2-2x-15\overline{)\;x^2\quad -1}\;\Big|\;1$$

$$\pm x^2 \mp 2x \mp 15$$

$$2x+14$$

Then $\frac{x^2-1}{x^2-2x-15} = 1 + \frac{2x+14}{x^2-2x-15}$ .....(i)

Now $\frac{2x+14}{x^2-2x-15} = \frac{2x+14}{x^2+3x-5x-15}$

$$= \frac{2x+14}{x(x+3)-5(x+3)}$$

$$= \frac{2x+14}{(x+3)(x-5)}$$

Let $\frac{2x+14}{(x-3)(x+5)} = \frac{A}{x+3} + \frac{B}{x-5}$ .....(ii)

Eq(ii) multiplied by $(x+3)(x-5)$.

$$2x+14 = A(x-5) + B(x+3) \quad \text{.....(iii)}$$

Put $x=-3$ **in (iii)**

$$2(-3)+14 = A(-3-5)+0 \quad \Rightarrow \quad -6+14 = -8A$$

$$\Rightarrow \quad -8A = 8 \qquad \Rightarrow \quad A = -1$$

Put $x=5$ **in (iii)**

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$$2(5)+14=0+B(5+3)$$

$$\Rightarrow\ 10+14=8B \quad \Rightarrow\ 8B=24$$

$$\Rightarrow\ B=3$$

Put values of A and B in (ii)

$$\frac{2x+14}{(x+3)(x-5)}=\frac{-1}{x+3}+\frac{3}{x-5} \quad \text{.....(iv)}$$ **Put (iv) in (i)**

$$\frac{x^2-1}{x^2-2x-15}=1-\frac{1}{(x+3)}+\frac{3}{(x-5)}$$ **integrate**

$$\int\frac{x^2-1}{x^2-2x-15}dx=\int\left(1-\frac{1}{x+3}+\frac{3}{x-5}\right)dx$$

$$=\int dx-\int\frac{dx}{x+3}+\int\frac{3dx}{x-5}+c$$

$$=x-\ln|x+3|+3\ln|x-5|+c$$

$$=x-\ln|x+3|+\ln|(x-5)^3|+c$$

$$=x+\ln\left|\frac{(x-5)^3}{x+3}\right|+c \quad Ans.$$

(i) $\int\frac{x^2}{(x+1)^3}dx$

Solution: Let $\frac{x^2}{(x+1)^3}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{(x+1)^3}$ .....(i)

Eq(i) is multiplied by $(x+1)^3$

$$x^2=A(x+1)^2+B(x+1)+C \quad \text{.....(ii)}$$

Put $x+1=0 \Rightarrow x=-1$ **in (ii)**

$$(-1)^2=0+0+C \quad \Rightarrow\ C=1$$

From (ii) $\Rightarrow\ x^2=A(x^2+2x+1)+B(x+1)+C$

$$\Rightarrow\ x^2=Ax^2+2Ax+A+Bx+B+C$$

$$\Rightarrow\ x^2+0x+0=Ax^2+(2A+B)x+A+B+C$$

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Equating the coefficients of x^2 *and* x.

$A=1$

$2A+B=0$ Put $A=1$

$\Rightarrow 2(1)+B=0 \Rightarrow B=-2$

Put values of A, B and C in (i)

$$\frac{x^2}{(x+1)^3}=\frac{1}{x+1}+\frac{-2}{(x+1)^2}+\frac{1}{(x+1)^3}$$ integrate

$$\int\frac{x^2}{(x+1)^3}dx=\int\frac{1}{x+1}dx-2\int(x+1)^{-2}dx+\int(x+1)^{-3}dx$$

$$=\ln|x+1|-2\frac{(x+1)^{-1}}{-1}+\frac{(x+1)^{-2}}{-2}+c$$

$$=\ln|x+1|+\frac{2}{x+1}-\frac{1}{2(x+1)^2}+c \quad Ans.$$

(j) $\int\frac{x^2+2}{(x^2+1)^2}dx$

Solution: Let $\frac{x^2+2}{(x^2+1)^2}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}$ .....(i)

Eq(i) is multiplied by $(x^2+1)^2$.

$x^2+2=(Ax+B)(x^2+1)+Cx+D$

$\Rightarrow x^2+2=Ax^3+Ax+Bx^2+B+Cx+D$

$\Rightarrow 0x^3+x^2+0x+2=Ax^3+Bx^2+(A+C)x+B+D$

Equating the coefficients of x^3, x^2, x and constants.

$\Rightarrow A=0$, $B=1$

$A+C=0$ Put $A=0$

Then $0+C=0 \Rightarrow C=0$

$B+D=2$ Put $B=1$

$\Rightarrow 1+D=2 \Rightarrow D=2-1=1$

Put values of A, B, C and D in (i)

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$$\frac{x^2+2}{(x^2+1)^2}=\frac{0.x+1}{x^2+1}+\frac{0.x+1}{(x^2+1)^2}$$

$$\Rightarrow \frac{x^2+2}{(x^2+1)^2}=\frac{1}{x^2+1}+\frac{1}{(x^2+1)^2} \quad \textbf{integrate}$$

$$\int\frac{x^2+2}{(x^2+1)^2}dx=\int\frac{dx}{x^2+1}+\int\frac{dx}{(x^2+1)^2}$$

$$=\tan^{-1}x+\int\frac{dx}{(x^2+1)^2} \quad .....(A)$$

Now we solve $\int\frac{1}{(x^2+1)^2}dx$**:**

Let $x=\tan\theta \quad \Rightarrow \quad dx=\sec^2\theta\, d\theta$

$$\int\frac{1}{(x^2+1)^2}dx=\int\frac{\sec^2\theta d\theta}{(\tan^2\theta+1)^2}=\int\frac{\sec^2\theta d\theta}{(\sec^2\theta)^2}$$

$$=\int\frac{1}{\sec^2\theta}d\theta=\int\cos^2\theta d\theta$$

$$=\int\frac{1+\cos 2\theta}{2}d\theta$$

$$=\frac{1}{2}\int d\theta+\frac{1}{2}\int\cos 2\theta d\theta$$

$$=\frac{1}{2}\theta+\frac{1}{2}.\frac{\sin 2\theta}{2}+c \qquad put \quad \theta=\tan^{-1}x$$

$$=\frac{1}{2}\tan^{-1}x+\frac{\sin(2\tan^{-1}x)}{4}+c \quad .....(ii)$$

Put (ii) in (i)

$$\int\frac{x^2+2}{(x^2+1)^2}dx=\tan^{-1}x+\frac{1}{2}\tan^{-1}x+\frac{\sin(2\tan^{-1}x)}{4}+c$$

$$=\frac{3}{2}\tan^{-1}x+\frac{\sin(2\tan^{-1}x)}{4}+c \quad Ans.$$

Q.2: The rate at which the body eliminates a drug

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(in millilitres per hour) is given by

$\frac{d}{dt}R(t)=\frac{60t}{(t+1)^2(t+2)}$ .Where t is the number of hours

Since the drug was administered . If $R(0)=0$ is the current drug elimination . How much of the drug is eliminated during the first hour after it was administered ? . The fourth hour it was administered ? .

Solution: Given that $\frac{d}{dt}R(t)=\frac{60t}{(t+1)^2(t+2)}$

$\Rightarrow d\,R(t)=\frac{60t}{(t+1)^2(t+2)}dt$ integrated

$\Rightarrow \int d\,R(t)=\int\frac{60t}{(t+1)^2(t+2)}dt$

$\Rightarrow R(t)=\int\frac{60t}{(t+1)^2(t+2)}dt$ .....(i)

Now by partial fraction

Let $\frac{60t}{(t+1)^2(t+2)}=\frac{A}{t+1}+\frac{B}{(t+1)^2}+\frac{C}{t+2}$ .....(ii)

Eq(ii) is multiplied by $(t+1)^2(t+2)$

$60t=A(t+1)(t+2)+B(t+2)+C(t+1)^2$ ....(iii)

Put $t+1=0 \Rightarrow t=-1$ in (iii)

$60(-1)=0+B(-1+2)+0 \Rightarrow B=-60$

Put $t+2=0 \Rightarrow t=-2$ in (iii)

$60(-2)=0+0+C(-2+1)^2 \Rightarrow -120=C(-1)^2$

$\Rightarrow C=-120$

From (iii) $\Rightarrow 60t=A(t^2+2t+t+2)+B(t+2)+C(t^2+2t+1)$

$\Rightarrow 60t=At^2+3At+2A+Bt+2B+Ct^2+2Ct+C$

$\Rightarrow 0.t^2+60t+0=(A+C)t^2+(3A+B+2C)t+2A+2B+C$

Equating the coefficient of t^2

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$A+C=0 \qquad put \;\; C=120$

$\Rightarrow A-120=0 \qquad \Rightarrow A=120$

Put values of A , B and C in (ii)

$$\frac{60t}{(t+1)^2(t+2)}=\frac{120}{t+1}-\frac{60}{(t+1)^2}-\frac{120}{t+2}$$ **integrate**

$$R(t)=\int\frac{60t}{(t+1)^2(t+2)}dt=120\int\frac{dt}{1+t}-60\int\frac{dt}{(1+t)^2}-120\int\frac{dt}{t+2}$$

$$=120\ln(1+t)-60\int(1+t)^{-2}dt-120\ln(t+2)+c$$

$$=120\ln(1+t)-60.\frac{(1+t)^{-1}}{-1}-120\ln(t+2)+c$$

$$\Rightarrow R(t)=120\ln(1+t)+\frac{60}{1+t}-120\ln(t+2)+c \quad \text{.....(iv)}$$

As $t=0$ then $R=0$ put in (iv)

$$0=120\ln 1+\frac{60}{1+0}-120\ln(2)+c \qquad \textbf{As ln 1= 0}$$

$$\Rightarrow 0=0+60-(120)(0.693)+c$$

$$\Rightarrow 0=60-83.16+c \qquad \Rightarrow 0=-23.16+c$$

$$\Rightarrow c=23.16$$

Put $c=23.16$ in (iv)

$$\Rightarrow R(t)=120\ln(1+t)+\frac{60}{1+t}-120\ln(t+2)+23.16 \quad \text{....(A)}$$

After one hour one hour drug administration :

Put $t=1$ in (A)

$$R(1)=120\ln(1+1)+\frac{60}{1+1}-120\ln(1+2)+23.16$$

$$=120\ln 2+30-120\ln 3+23.16$$

$$=(120)(0.693)+30-(120)(1.098)+23.16$$

$$=83.16+30-131.76+23.16$$

$$\Rightarrow R(1)=4.56\ ml$$

After three hour drug administration :

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Put $t = 3$ in (A)

$$R(3) = 120\ln(3+1) + \frac{60}{3+1} - 120\ln(3+2) + 23.16$$
$$= 120(1.39) + 15 - 120(1.61) + 23.16$$
$$= 166.8 + 15 - 193.2 + 23.16 = 11.76$$

After four hour drug administration :

Put $t = 4$ in (A)

$$R(4) = 120\ln(4+1) + \frac{60}{4+1} - 120\ln(4+2) + 23.16$$
$$= 120\ln 5 + 12 - 120\ln(6) + 23.16$$
$$= (120)(1.61) + 12 - 120(1.79) + 23.16$$
$$= 193.2 + 12 - 214.8 + 23.16$$
$$= 13.56$$

After fourth hour drug administration :

$R(\textit{fourth hour}) = R(4) - R(3) = 13.56 - 11.76 = 1.8$

Q.3: The rate of change of the voting population of a city with respect to time t (in years) is estimated to be $\frac{dN}{dt} = \frac{100t}{(1+t^2)^2}$. Where $N(t)$ is in thousands . If $N(0)$ is the current voting population , then how much will this population increase during the next 3 years ?

Solution: $\frac{dN}{dt} = \frac{100t}{(1+t^2)^2}$

$$\Rightarrow dN = \frac{100t}{(1+t^2)^2}dt \quad \text{integrate}$$

$$\int dN = \int \frac{100t}{(1+t^2)^2}dt \quad \text{.....(A)}$$

Let $t = \tan\theta \quad \Rightarrow \quad \frac{dt}{d\theta} = \frac{d}{d\theta}\tan\theta$

$$\Rightarrow \frac{dt}{d\theta} = \sec^2\theta \quad \Rightarrow \quad dt = \sec^2\theta d\theta$$

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Put these values in (A).

$$N(t) = 100\int \frac{\tan\theta \sec^2\theta d\theta}{(1+\tan^2\theta)^2} = 100\int \frac{\tan\theta \sec^2\theta d\theta}{(\sec^2\theta)^2}$$

$$= 100\int \frac{\tan\theta d\theta}{\sec^2\theta} = 100\int \frac{\sin\theta}{\cos\theta}.\cos^2\theta d\theta$$

$$= 100\int \sin\theta\cos\theta d\theta \quad .....(i)$$

let $\sin\theta = u$ => $\cos\theta d\theta = du$ *put in(i)*

$$N(t) = 100\int udu = 100\frac{u^2}{2} + c \quad put \quad t = \sin\theta$$

$$=> N(t) = 50\sin^2\theta + c$$

$$= 50\frac{\sin^2\theta}{\cos^2\theta}.\cos^2\theta + c$$

$$=> N(t) = 50\frac{\tan^2\theta}{\sec^2\theta} + c$$

$$= \frac{50\tan^2\theta}{1+\tan^2\theta} + c \quad put \quad \tan\theta = t$$

Then $N(t) = \frac{50t^2}{1+t^2} + c$ .....(B)

As given when $t = 0$ then $N = 0$ Put in (B)

$$0 = \frac{0}{1+0} + c \quad => c = 0$$

Put $c = 0$ in (B), Then

$$N(t) = \frac{50t^2}{1+t^2} \quad .....(C)$$

The population increases next three years : Put $t = 3$ in (C).

$$N(3) = \frac{50(3)^2}{1+(3)^2} = \frac{(50)(9)}{10} = 45$$

Since given the population in thousands.
Thus the population increases next three years is
= 45 thousands.

Q.4: A oil tanker aground on a reef is losing oil and producing

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an oil slick that is radiating outward at the rate approximated by $\frac{dr}{dt} = \frac{100}{\sqrt{t^2+9}}$, $t \geq 0$.

where t is the radius (in feet) of the circular slick after t minutes. Find the radius of the slick after 4 minutes if the radius is $r = 0$ **when** $t = 0$.

Solution: $\frac{dr}{dt} = \frac{100}{\sqrt{t^2+9}}$, $t \geq 0$

$$\Rightarrow \quad dr = \frac{100}{\sqrt{t^2+9}}dt \quad \text{integrate}$$

$$\int dr = \int \frac{100}{\sqrt{t^2+9}}dt \quad \ldots.(i)$$

Let $t = 3\tan\theta \quad \Rightarrow \quad dt = 3\sec^2 d\theta$ put in (i)

$$\int dr = \int \frac{100\,(3\sec^2\theta)d\theta}{\sqrt{9\tan^2\theta+9}}$$

$$\Rightarrow \quad r = 100\int \frac{3sce^2\theta d\theta}{\sqrt{9\sec^2\theta}} = 100\int \frac{3\sec^2\theta d\theta}{3\sec\theta}$$

$$\Rightarrow \quad r = 100\int \sec\theta d\theta = 100\int \frac{\sec\theta(\sec\theta+\tan\theta)}{\sec\theta+\tan\theta}d\theta$$

$$\Rightarrow \quad r = 100\ln|\sec\theta+\tan\theta| + c$$

$$= 100\ln\left|\tan\theta+\sqrt{\tan^2\theta+1}\right| + c \qquad put \ \tan\theta = \frac{t}{3}$$

$$\Rightarrow \quad r = 100\ln\left(\frac{t}{3}+\sqrt{\frac{t^2}{9}+1}\right) + c$$

$$\Rightarrow \quad r = 100\ln\left(\frac{t+\sqrt{t^2+9}}{3}\right) + c \quad \ldots\ldots\ldots(ii)$$

Put $t = 0$, $r = 0$ **in (ii)**

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$$0 = 100\ln\left(\frac{0+\sqrt{9}}{3}\right)+c \Rightarrow 0 = 100\ln(1)+c$$

$$\Rightarrow c = 0$$

Put $c = 0$ **in (ii)**

$$r = 100\ln\left(\frac{t+\sqrt{t^2+9}}{3}\right) \quad \text{.....(iii)}$$

Radius after 4 minutes : Put $t = 4$ **in (iii)**

$$r(4) = 100\ln\left(\frac{4+\sqrt{16+9}}{3}\right) = 100\ln\left(\frac{4+5}{3}\right)$$

$$= 100\ln(3) = (100)(1.098) = 109.86$$

Or approximately $r = 110$ **feet.**

Definite integral : If $f(x)$ **is continuous function on the interval** $[a, b]$ **. Then the integration of** $f(x)$ **from** *point* "a" *to point* "b" **is called definet integral . It is denoted by**

$$\int_a^b f(x)dx .$$

If integration of $f(x)$ *is* $F(x)$ **i.e .** $\int f(x)dx = F(x)$ **. Then the definet integral is** $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b)-F(a)$

Example .14 : Evaluate $\int_1^4 x^2 d$

Solution: $\int_1^4 x^2 dx = \frac{x^3}{3}\Big|_1^4 = \frac{1}{3}\left((4)^3-(1)^3\right)$

$$= \frac{1}{3}(64-1) = \frac{63}{3} = 21$$

Definite integral is the limit of a sum:

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If $f(x)$ is continuous function on the interval $[a\ ,\ b]$ and $[a\ ,\ b]$ is divided into n equal sub intervals each of width is $\Delta x = \frac{b-a}{n}$ and the right end points of each subinterval are $x_1\ , x_2\ , \ldots\ldots, x_{n-1}\ , x_n$.

If $n \to \infty$ then definite integral of $f(x)$ from $x = a$ to $x = b$ is given by

$$\int_a^b f(x)dx = \lim_{n\to\infty} \Delta x[f(x_1) + f(x_2) + \ldots\ldots + f(x_n)] \ldots\ldots \text{(A)}$$

Example .15: Find the actual area of the region bounded by by the curve $f(x) = x^2$ and the x-axis in the interval is $[0\ ,\ 2]$

Solution: For n equal subintervals the width of each subinterval is $\Delta x = \frac{2-0}{n} = \frac{2}{n}$

The right end points of each subinterval are

$$x_1 = a + \Delta x = 0 + \frac{2}{n} = \frac{2}{n}$$

$$x_2 = a + 2\Delta x = 0 + 2\left(\frac{2}{n}\right) = \frac{4}{n}$$

$$x_3 = a + 3\Delta x = \frac{6}{n}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$x_n = a + n\Delta x = \frac{2n}{n}$$

As $f(x) = x^2$ .....(i)

$$f(x_1) = f\left(\frac{2}{n}\right) = \left(\frac{2}{n}\right)^2 = \frac{4}{n^2}$$

$$f(x_2) = f\left(\frac{4}{n}\right) = \left(\frac{4}{n}\right)^2 = \frac{4^2}{n^2}$$

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$$f(x_3)=f\left(\frac{6}{n}\right)=\left(\frac{6}{n}\right)^2=\frac{6^2}{n^2}$$

.....

$$f(x_n)=f\left(\frac{2n}{n}\right)=\left(\frac{2n}{n}\right)^2=\frac{2^2n^2}{n^2}$$

As we know that the actual area is given by

$$\mathbf{A}=\int_a^b f(x)dx=\lim_{n\to\infty}\Delta x\left[f(x_1)+f(x_2)+.....+f(x_n)\right] \quad .....\text{(ii)}$$

Put values in (ii)

$$\Rightarrow A=\int_0^2 x^2dx=\lim_{n\to\infty}\frac{2}{n}\left[\frac{2^2}{n^2}+\frac{4^2}{n^2}+\frac{6^2}{n^2}+.....+\frac{(2n)^2}{n^2}\right]$$

$$\Rightarrow A=\lim_{n\to\infty}\frac{2.2^2}{n.n^2}\left[1+2^2+3^2+.....+n^2\right] \quad .....\text{(iii)}$$

As we know that

$$1^2+2^2+3^2+.....+n^2=\frac{n(n+1)(2n+1)}{6} \quad .....\text{(iv)}$$

Put (iv) in (iii)

$$A=\lim_{n\to\infty}\frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$=\lim_{n\to\infty}\frac{4}{3n^3}\left[n\left(2n^2+n+2n+1\right)\right]$$

$$=\lim_{n\to\infty}\frac{4}{3n^3}\left(2n^3+3n^2+n\right)$$

$$\Rightarrow A=\lim_{n\to\infty}\frac{4}{3}\left(\frac{2n^3}{n^3}+\frac{3n^2}{n^3}+\frac{n}{n^3}\right)=\lim_{n\to\infty}\frac{4}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)$$

$$\Rightarrow A=\frac{4}{3}\left(2+\frac{3}{\infty}+\frac{1}{\infty}\right)=\frac{4}{3}(2+0+0)$$

$$\Rightarrow A=\frac{8}{3} \qquad Ans.$$

Fundamental theorem of Integral calculus : If $f(x)$ is

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continuous function on the interval $[a, b]$ and let $f(x)$ has integration $F(x)$ i.e $\int f(x)dx = F(x)$ then

$$\int_a^b f(x)dx = \left|F(x)\right|_{x=a}^{x=b} = F(b) - F(a).$$

Properties of definite integral :

(i) $\int_a^a f(x)dx = 0$

(ii) If $f(x) = g(y)$ then $\int_a^b f(x)dx = \int_a^b g(y)dy$

(iii) if $a < c < b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(iv) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(v) If a function is odd then $\int_{-a}^a f(x)dx = 0$

(vi) If a function is even then

$$\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$$

Example .17: Evaluate the following definite integrals :

(a) $\int_0^1 \frac{2e^{4x} - 3}{e^{2x}}dx$

Solution:

$$\int_0^1 \frac{2e^{4x} - 3}{e^{2x}}dx = \int_0^1 \left(\frac{2e^{4x}}{e^{2x}} - \frac{3}{e^{2x}}\right)dx$$

$$= \int_0^1 2e^{2x}dx - \int_0^1 3e^{-2x}dx$$

$$= \left|2.\frac{e^{2x}}{2}\right|_0^1 - \left|3.\frac{e^{-2x}}{-2}\right|_0^1$$

$$= \left|e^{2x}\right|_0^1 + \frac{3}{2}\left|e^{-2x}\right|_0^1$$

$$= e^2 - e^0 + \frac{3}{2}\left(e^{-2} - e^0\right)$$

$$= 7.39 - 1 + (1.5)(0.135 - 1)$$

$$= 6.09 \quad \textit{Ans.}$$

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(b) $\int_{\pi/3}^{\pi/2} x\sin x^2 dx$

Solution: Let $x^2 = t \Rightarrow \frac{d}{dx}x^2 = \frac{dt}{dx}$

$$\Rightarrow 2x = \frac{dt}{dx} \Rightarrow xdx = \frac{1}{2}dt$$

As $x = \frac{\pi}{3}$ then $t = \frac{\pi^2}{9}$

As $x = \frac{\pi}{2}$ then $t = \frac{\pi^2}{4}$

$$\int_{\pi/3}^{\pi/2} x\sin x^2 dx = \frac{1}{2}\int_{\pi^2/9}^{\pi^2/4} \sin t dt$$

$$= \frac{1}{2}\left|-\cos t\right|_{\pi^2/9}^{\pi^2/4}$$

$$= -\frac{1}{2}\left(\cos\left(\frac{\pi^2}{4}\right) - \cos\left(\frac{\pi^2}{9}\right)\right) \quad put\ \pi = 3.14$$

$$= -\frac{1}{2}(\cos(2.4647) - \cos(1.0966)) \quad \textbf{use radians}$$

$$= -\frac{1}{2}(-0.7812 - 0.4566) = -\frac{1}{2}(-1.2378)$$

$$= 0.6189 \quad Ans.$$

Definite integral through Integration by parts :

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$$

Example .18: Evaluate the following definite integrals .

(a) $\int_0^1 xe^x dx$

Solution: Here $u = x$ and $\frac{dv}{dx} = e^x \Rightarrow dv = e^x dx$

$$\Rightarrow du = dx \quad \text{and} \quad v = e^x$$

As we know that integration by parts is given by

$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$ put values of u , v , du and dv

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$$\Rightarrow \quad \int_0^1 xe^x dx = xe^x\Big|_0^1 - \int_0^1 e^x dx$$

$$= \left(1.e^1 - 0\right) - e^x\Big|_0^1$$

$$= e - \left(e^1 - e^0\right) = e - e + 1$$

$$= 1 \qquad Ans.$$

(b) $\int_0^1 e^x \sin x dx$

Solution: Let $I = \int_0^1 e^x \sin x dx$ .....(i)

Here let $u = e^x$ **and** $\frac{dv}{dx} = \sin x$

$$\Rightarrow \frac{du}{dx} = e^x \quad \Rightarrow \quad du = e^x dx$$

Also $dv = \sin x \, dx \quad \Rightarrow \quad \int dv = \int \sin x dx$

Or $v = -\cos x$

As we know that integration by parts is given by

$$\int_a^b u dv = uv\Big|_a^b - \int_a^b v du \quad .....(A)$$

put values of u , v , du and dv in (A) .

$$I = \int_0^1 e^x \sin x dx = e^x\left(-\cos x\right)\Big|_0^1 - \int_0^1 \left(-\cos x\right)e^x dx$$

$$\Rightarrow I = -\left(e^1 \cos 1 - e^0 \cos 0\right) + \int_0^1 \cos x e^x dx$$

$$\Rightarrow I = -\left[(2.718)(0.54) - 1\right] + + \int_0^1 \cos x e^x dx \quad Use\ radians$$

$$\Rightarrow I = -0.468 + \int_0^1 \cos x e^x dx \quad .....(ii)$$

Now we solve again $\int_0^1 \cos x e^x dx$

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Now again take $u = e^x$ and $\frac{dv}{dx} = \cos x$

$$\Rightarrow \frac{du}{dx} = e^x \quad \Rightarrow \quad du = e^x dx$$

Also $dv = \cos x\, dx \quad \Rightarrow \quad \int dv = \int \cos x dx$

Or $v = \sin x$ put in (A)

$$\int_0^1 \cos x e^x dx = e^x \sin x\Big|_0^1 - \int_0^1 e^x \sin x dx$$

$$= \left(e^1 \sin 1 - e^0 \sin 0\right) - \int_0^1 e^x \sin x dx$$

$$= [(2.718)(0.84) - 0] - \int_0^1 e^x \sin x dx$$

$$= 2.28 - \int_0^1 e^x \sin x dx \quad .....(iii)$$

Put (i) in (iii)

$$\int_0^1 \cos x\, e^x dx = 2.28 - I \quad .....(iv)$$

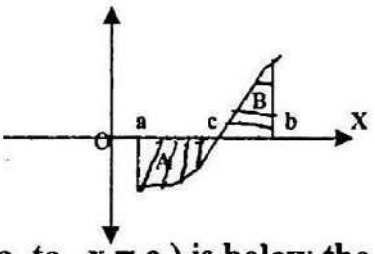
Put (iv) in (ii)

$$\Rightarrow \quad I = -0.468 + 2.28 - I$$

$$\Rightarrow \quad I + I = 1.82 \quad \Rightarrow \quad 2I = 1.82$$

$$\Rightarrow \quad I = 0.91 \qquad Ans.$$

Area between a curve and the x-axis : If $y = f(x)$ is continuous function on the interval $[a\ ,\ b]$, then the area above the x-axis is counted positively and the area below the x- axis is counted negatively .



In the figure the area A (from x = a to x = c) is below the x- axis and the area B (from x = c to x = b) is above the x – axis . Thus the area under the curve from a to b is

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given by

$$\int_a^b f(x)dx = \int_a^c -f(x)dx + \int_c^b f(x)dx \ .$$

As $f(x) \le 0$ in $[a\ ,\ c]$ and $f(x) \ge 0$ in $[c\ \ b]$.

Note: c is the point where the value of $f(x)$ is zero.

Example .19: Find the area between the x-axis and the curve $f(x) = x^2 - 4$ from $x = 0$ to $x = 4$

Solution: Given $f(x) = x^2 - 4$ .....(i)

And interval is $[0\ ,\ 4]$.

First we find intercept of $f(x)$ on the x-axis . i.e. we find those values of x –axis on which $f(x)$ are zero .

put $f(x) = 0$ in (i)

$$x^2 - 4 = 0 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

Thus the subintervals of the interval $[0\ ,\ 4]$ are $[0\ ,\ 2]$ and $[2\ ,\ 4]$

Since $f(x) \le 0$ in $[0\ ,\ 2]$ and $f(x) \ge 0$ in $[2\ ,\ 4]$.

Then the area is given by

$$A = -\int_0^2 f(x)dx + \int_2^4 f(x)dx \quad \text{put value of } f(x)$$

$$\Rightarrow A = -\int_0^2 (x^2 - 4)dx + \int_2^4 (x^2 - 4)dx$$

$$\Rightarrow A = -\left(\frac{x^3}{3} - 4x\right)\Big|_0^2 + \left(\frac{x^3}{3} - 4x\right)\Big|_2^4$$

$$A = -\left[\left(\frac{(2)^3}{3} - 4(2)\right) - \left(\frac{0}{3} - 4(0)\right)\right] + \left[\left(\frac{(4)^3}{3} - 4(4)\right) - \left(\frac{(2)^3}{3} - 4(2)\right)\right]$$

$$\Rightarrow A = -\left(\frac{8}{3} - 8 - 0\right) + \left(\frac{64}{3} - 16 - \frac{8}{3} + 8\right)$$

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$$\Rightarrow A = -\left(\frac{8-24}{3}\right)+\left(\frac{64-48-8+24}{3}\right)$$

$$\Rightarrow A = -\left(-\frac{16}{3}\right)+\left(\frac{32}{3}\right)=\frac{16}{3}+\frac{32}{3}$$

$$\Rightarrow A = \frac{48}{3}=16 \quad \textit{square unit}$$

Example.20: **Find the area between the x- axis and the curve** $f(x)=x^2-2x$ **from** $x=-1$ **to** $x=3$ **.**

Solution: $f(x)=x^2-2x$ ....(i) **and the interval is** $[-1,\ 3]$ **.**

First we find intercept of $f(x)$ **on the x-axis .**

Put $f(x)=0$ **in (i)**

$$x^2-2x=0 \Rightarrow x(x-2)=0$$

$$\Rightarrow x=0 \quad \textit{and} \quad x-2=0$$

$$\Rightarrow x=2$$

Thus the subintervals of the interval $[-1,\ 3]$ **are** $[-1,\ 0]$, $[0\ ,2]$ **and** $[2\ ,3]$

Since $f(x)\geq 0$ **in** $[-1,\ 0]$ **and** $f(x)\leq 0$ **in** $[0\ ,2]$

Also $f(x)\geq 0$ **in** $[2\ ,3]$ **.**

Thus the area is given by

$A=\int_{-1}^{0} f(x)dx+\int_{0}^{2}-f(x)dx+\int_{2}^{3} f(x)dx$ **put values of** $f(x)$

$$\Rightarrow A=\int_{-1}^{0}(x^2-2x)dx-\int_{0}^{2}(x^2-2x)dx+\int_{2}^{3}(x^2-2x)dx$$

$$\Rightarrow A=\left(\frac{x^3}{3}-\frac{2x^2}{2}\right)\Big|_{-1}^{0}-\left(\frac{x^3}{3}-\frac{2x^2}{2}\right)\Big|_{0}^{2}+\left(\frac{x^3}{3}-\frac{2x^2}{2}\right)\Big|_{2}^{3}$$

$$\Rightarrow A=\left(\frac{x^3}{3}-x^2\right)\Big|_{-1}^{0}-\left(\frac{x^3}{3}-x^2\right)\Big|_{0}^{2}+\left(\frac{x^3}{3}-x^2\right)\Big|_{2}^{3}$$

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$$\Rightarrow A=\left[\left(\frac{0}{3}-0\right)-\left(\frac{(-1)^3}{3}-(-1)^2\right)\right]-\left[\left(\frac{(2)^3}{3}-(2)^2\right)-\left(\frac{0}{3}-0\right)\right]$$
$$+\left[\left(\frac{(3)^3}{3}-(3)^2\right)-\left(\frac{(2)^3}{3}-(2)^2\right)\right]$$

$$\Rightarrow A=\left[0-\left(-\frac{1}{3}-1\right)\right]-\left[\frac{8}{3}-4\right]+\left[\frac{27}{3}-9-\frac{8}{3}+4\right]$$

$$=\frac{4}{3}+\frac{4}{3}+\frac{27-27-8+12}{3}$$

$$=\frac{4}{3}+\frac{4}{3}+\frac{4}{3}=\frac{12}{3}$$

$$\Rightarrow A=4 \quad square \quad unit.$$

Example .21: Use MAPLE command "int" to solve .

(a) Indefinite integral of a function

$f(x)=x^4+x^3+x^2+x+1$ **w.r.t. to variable x .**

(b) definite integral of a function $f(x)=x^2$ **in** $[0\ ,1]$

variable x .

(c) definite integral of a function $f(x)=xe^x$

In interval $[0\ ,\ 1]$.

Solution: (a) Open the MAPLE programme . Open the new document and use keybard keys write

int(x^4 + x^3 + x^2 + x + 1, x); **and press enter key**

You get the following result .

Command:

$int(x^4+x^3+x^2+x+1, x);$

$$\frac{1}{5}x^5+\frac{1}{4}x^4+\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

(b)

Open the MAPLE programme . Open the new document and use keybard keys write

int(x^2, x = 0..1); **and press enter key**

You get the following result .

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Command:

$int(x^2, x=0..1);$

$$\frac{1}{3}$$

(c)

Open the MAPLE programme . Open the new document and use keybard keys write

int(x * e^ x , x = 0..1); and press enter key

You get the following result .

Command:

$int(x \cdot e^x, x=0..1);$

$$\frac{e\ln(e) - e + 1}{\ln(e)^2}$$

As ln e =1

$$\frac{e - e + 1}{(1)^2}$$

$$1$$

Exercise 6.4:

Q.1: Evaluate the following definite integrals .

(a) $\int_3^4 5x\,dx$

Solution: $\int_3^4 5x\,dx = 5.\frac{x^2}{2}\Big|_3^4 = \frac{5}{3}\left((4)^2 - (3)^2\right) = \frac{5}{3}(16-9) = \frac{35}{3}$ **Ans.**

(b) $\int_{12}^{20} x^3\,dx$

Solution:

$$\int_{12}^{20} x^3\,dx = \frac{x^4}{4}\Big|_{12}^{20} = \frac{1}{4}\left((20)^4 - (12)^4\right) = \frac{1}{4}(160000 - 20736)$$

$$= \frac{1}{4}(139264) = 34816 \quad \textbf{Ans.}$$

(c) $\int_1^2 \left(2x^{-2} - 3\right)dx$

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Solution: $\int_1^2 (2x^{-2} - 3)dx = \left(2\frac{x^{-1}}{-1} - 3x\right)\Big|_1^2$

$$= -\left(\frac{2}{x} + 3x\right)\Big|_1^2$$

$$= -\left(\frac{2}{2} + 3(2) - \left[\frac{2}{1} + 3(1)\right]\right)$$

$$= -(1 + 6 - 2 - 3)$$

$$= -2 \quad Ans.$$

(d) $\int_1^4 3\sqrt{x}dx$

Solution: $\int_1^4 3\sqrt{x}dx = 3\int_1^4 x^{\frac{1}{2}}dx$

$$= 3\left(\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)\Big|_1^4 = 3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big|_1^4$$

$$= 2x^{\frac{3}{2}}\Big|_1^4 = 2\left[(4)^{\frac{3}{2}} - (1)^{\frac{3}{2}}\right]$$

$$= 2\left[(2^2)^{\frac{3}{2}} - 1\right] = 2(2^3 - 1)$$

$$= 2(8-1) = 14 \quad Ans.$$

(e) $\int_2^3 12(x^2 - 4)^5 x dx$

Solution: $\int_2^3 12(x^2 - 4)^5 x dx = 6\int_2^3 (x^2 - 4)^5 (2x)dx$ .....(A)

Let $x^2 - 4 = t \quad \Rightarrow \quad \frac{d}{dx}(x^2 - 4) = \frac{dt}{dx}$

$\Rightarrow \quad 2x = \frac{dt}{dx} \quad or \quad 2xdx = dt$

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When $x=2$ then $t=(2)^2-4=0$

When $x=3$, then $t=(3)^2-4=5$

Put values in (A).

$$\int_2^3 12\left(x^2-4\right)^5 x\,dx = 6\int_0^5 t^5\,dt$$

$$= 6\left(\frac{t^6}{6}\right)\Big|_0^5 = t^6\Big|_0^5$$

$$= (5)^6-(0)^5$$

$$= 5^6 \qquad Ans.$$

(f) $\int_{-1}^{1}\frac{e^{-x}-e^{x}}{\left(e^{-x}+e^{x}\right)^2}dx$

Solution: $\int_{-1}^{1}\frac{e^{-x}-e^{x}}{\left(e^{-x}+e^{x}\right)^2}dx = \int_{-1}^{1}\left(e^{-x}+e^{x}\right)^{-2}\left(e^{-x}-e^{x}\right)dx$ .....(i)

Let $e^{-x}+e^{x}=t \quad \Rightarrow \quad \frac{d}{dx}\left(e^{-x}+e^{x}\right)=\frac{dt}{dx}$

$\Rightarrow -e^{-x}+e^{x}=\frac{dt}{dx}$ or $-\left(e^{-x}-e^{x}\right)dx=dt$

$\Rightarrow \left(e^{-x}-e^{x}\right)dx=-dt$

When $x=-1$ then $t=e^{-(-1)}+e^{-1}=e+e^{-1}$

When $x=1$ then $t=e^{-1}+e^{1}=e+e^{-1}$

Put values in (i)

$$\int_{-1}^{1}\frac{e^{-x}-e^{x}}{\left(e^{-x}+e^{x}\right)^2}dx = -\int_{e+e^{-1}}^{e+e^{-1}} t^{-2}\,dt = 0$$

Ans. use $\int_a^a f(x)dx = 0$

Other methode:

Since $f(x)=\frac{e^{-x}-e^{x}}{\left(e^{-x}+e^{x}\right)^2}$

Then $f(-x)=\frac{e^{-(-x)}-e^{-x}}{\left(e^{-(-x)}+e^{-x}\right)^2}=\frac{e^{x}-e^{-x}}{\left(e^{x}+e^{-x}\right)^2}$

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$$\Rightarrow \quad f(-x) = -\frac{e^{-x} - e^{x}}{\left(e^{-x} + e^{x}\right)^2} = -f(x)$$

Thus $f(x)$ is odd function then we know that if $f(x)$ odd function then $\int_{-a}^{a} f(x)dx = 0$

Thus $\int_{-1}^{1} \frac{e^{-x} - e^{x}}{\left(e^{-x} + e^{x}\right)^2} dx = 0$ **Ans.**

(g) $\int_{\pi/2}^{\pi} \cos\left(\frac{x}{2} + \pi\right) dx$

Solution: As we know that $\cos(x + \pi) = -\cos x$

Then $\int_{\pi/2}^{\pi} \cos\left(\frac{x}{2} + \pi\right) dx = -\int_{\pi/2}^{\pi} \cos\frac{x}{2} dx$ .....(i)

Let $\frac{x}{2} = t \quad \Rightarrow \quad dx = 2dt$

When $x = \pi$ then $t = \frac{\pi}{2}$

When $x = \frac{\pi}{2}$ then $t = \frac{\pi}{4}$

Put values in the given integral.

$$\int_{\pi/2}^{\pi} \cos\left(\frac{x}{2} + \pi\right) dx = -2\int_{\pi/4}^{\pi/2} \cos t dt$$

$$= -2\sin t\Big|_{\pi/4}^{\pi/2}$$

$$= -2\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right)$$

$$= -2\left(1 - \frac{1}{\sqrt{2}}\right) = -2 + \frac{2}{\sqrt{2}}$$

$$= -2 + \sqrt{2} \quad Ans.$$

(h) $\int_{0}^{\pi/4} \sec^2\theta d\theta$

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Solution: $\int_0^{\pi/4} \sec^2\theta d\theta = \tan\theta\Big|_0^{\pi/4}$

$$= \tan\left(\frac{\pi}{4}\right) - \tan 0 = 1 \quad \textbf{Ans.}$$

Q.2: Evaluate the following definite integrals .

(a) $\int_1^2 \frac{5t^2 - 3t + 18}{t(9 - t^2)} dt$

Solution: we solve it by partial fraction .

Let $\frac{5t^2 - 3t + 18}{t(3-t)(3+t)} = \frac{A}{t} + \frac{B}{3-t} + \frac{C}{3+t}$ .....(i)

Eq(i) is multiplied by $t(3-t)(3+t)$

$$5t^2 - 3t + 18 = A(3-t)(3+t) + Bt(3+t) + Ct(3-t) \quad \text{.....(ii)}$$

Put $t = 0$ **in (ii)**

$$0 - 0 + 18 = A(3-0)(3+0) + 0 + 0$$

$$\Rightarrow \quad 18 = 9A \quad \Rightarrow \quad A = 2$$

Put $t = 3$ **in (ii)**

$$5(3)^2 - 3(3) + 18 = 0 + B(3)(3+3) + 0$$

$$\Rightarrow 45 - 9 + 18 = 18B \quad \Rightarrow 18B = 54$$

$$\Rightarrow \quad B = 3$$

Put $t = -3$ **in (ii)**

$$5(-3)^2 - 3(-3) + 18 = 0 + 0 + C(3)(3+3)$$

$$\Rightarrow 45 + 9 + 18 = 18C \quad \Rightarrow \quad 18C = 72$$

$$\Rightarrow \quad C = 4$$

Put values of A , B and C in (i)

$$\frac{5t^2 - 3t + 18}{t(3-t)(3+t)} = \frac{2}{t} + \frac{3}{3-t} + \frac{4}{3+t} \quad \textbf{integrate}$$

$$\int_1^2 \frac{5t^2 - 3t + 18}{t(9 - t^2)} dt = \int_1^2 \left(\frac{2}{t} + \frac{3}{3-t} + \frac{4}{3+t}\right) dt$$

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$$= \int_1^2 \frac{2}{t}dt + \int_1^2 \frac{3}{3-t}dt + \int_1^2 \frac{4}{3+t}dt$$

$$= 2\ln t\Big|_1^2 - 3\ln(3-t)\Big|_1^2 + 3\ln(3+t)\Big|_1^2$$

$$= 2(\ln 2 - \ln 1) - 3[\ln(3-2) - \ln(3-1)] + 4[\ln(3+2) - \ln(3+1)]$$

$$= 2(0.693 - 0) - 3[\ln 1 - \ln 2] + 4[\ln 5 - \ln 4]$$

$$= 1.386 - 3(0 - 0.693) + 4(1.609 - 1.386)$$

$$= 1.386 + 2.079 + 0.892$$

$$= 4.357 \quad Ans$$

b) $\int_1^2 \frac{4}{t^3+4t}dt$

Solution: $\int_1^2 \frac{4}{t^3+4t}dt = \int_1^2 \frac{4}{t(t^2+4)}dt$

Let $\frac{4}{t(t^2+4)} = \frac{A}{t} + \frac{Bt+C}{t^2+4}$ .....(i)

Eq(i) is multiplied by $t(t^2+4)$.

$$4 = A(t^2+4) + (Bt+C)(t)$$

$$\Rightarrow 0t^2 + 0t + 4 = At^2 + 4A + Bt^2 + Ct$$

$$\Rightarrow 0.t^2 + 0.t + 4 = (A+B)t^2 + Ct + 4A$$

Equating the coefficients of x^2 , x **and constant.**

$$A + B = 0 \quad .....(ii)$$

$$\Rightarrow C = 0 \quad \text{and} \quad 4A = 4$$

$$\Rightarrow A = 1$$

Put $A = 1$ **in(ii)** $\Rightarrow 1 + B = 0 \Rightarrow B = -1$

Put values of A , B and C in (i)

$$\frac{4}{t(t^2+4)} = \frac{1}{t} + \frac{-t+0}{t^2+4} \quad \text{integrate}$$

$$\int_1^2 \frac{4}{t(t^2+4)}dt = \int_1^2 \frac{1}{t}dt - \frac{1}{2}\int_1^2 \frac{2t}{t^2+4}dt$$

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$$= \ln t\Big|_1^2 - \frac{1}{2}\ln\left(t^2+4\right)\Big|_1^2 \qquad use \int \frac{f'(x)}{f(x)}dx = \ln[f(x)]$$

$$= \ln 2 - \ln 1 - \frac{1}{2}[\ln(4+4) - \ln(1+4)]$$

$$= 0.693 - 0 - \frac{1}{2}(2.08 - 1.61)$$

$$= 0.693 - 0.235$$

$$= 0.458 \qquad Ans.$$

Q.3: Use the definite integral to find out the area b etween the curve $f(x)$ and the x-axis over the indicated interval $[a\ ,\ b]$.

(a) $f(x) = 4 - x^2$, $[0,\ 3]$

Solution: $f(x) = 4 - x^2$ .....(i)

First we find intercept of $f(x)$ on the x – axis .

Put $f(x) = 0$ in (i)

$$4 - x^2 = 0 \qquad or \qquad x^2 = 4$$

$$\Rightarrow x = \pm 2$$

Thus the subinterval of the interval $[0,\ 3]$ are $[0\ ,\ 2]$ and $[2\ ,\ 3]$

Since $f(x) \geq 0$ in $[0\ ,\ 2]$ and $f(x) \leq 0$ in $[2\ ,\ 3]$.

Then the area is given by

$$A = \int_0^2 f(x)dx + \int_2^3 -f(x)dx \qquad \textbf{put value of } f(x)$$

$$\Rightarrow A = \int_0^2 \left(4 - x^2\right)dx - \int_2^3 \left(4 - x^2\right)dx$$

$$= \left(4x - \frac{x^3}{3}\right)\Big|_0^2 - \left(4x - \frac{x^3}{3}\right)\Big|_2^3$$

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$$\Rightarrow A = 4(2) - \frac{1}{3}(2)^3 - \left[4(0) - \frac{0}{3}\right] - \left[4(3) - \frac{(3)^3}{3} - \left[4(2) - \frac{(2)^3}{3}\right]\right]$$

$$\Rightarrow A = 8 - \frac{8}{3} - 0 - \left[12 - 9 - \left(8 - \frac{8}{3}\right)\right]$$

$$\Rightarrow A = \frac{24-8}{3} - \left(3 - \frac{24-8}{3}\right) = \frac{16}{3} - \frac{9-16}{3}$$

$$\Rightarrow A = \frac{16+7}{3} = \frac{23}{3} = 7.67 \text{ square units} \quad \text{Ans.}$$

(b) $f(x) = x^2 - 5x + 6$, , $[0, 3]$

Solution: $f(x) = x^2 - 5x + 6$ .....(i)

First we find intercept of $f(x)$ **on the x- axis .**

Put $f(x) = 0$ **in (i)**

$$x^2 - 5x + 6 = 0 \quad \Rightarrow x^2 - 2x - 3x + 6 = 0$$

$$\Rightarrow x(x-2) - 3(x-2) = 0 \Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x - 2 = 0 \quad \text{and} \quad x - 3 = 0$$

$$\Rightarrow x = 2 \quad \text{and} \quad x = 3$$

Thus the subinterval of the interval $[0, 3]$ **are** $[0, 2]$ **and** $[2, 3]$

Since $f(x) \geq 0$ **in** $[0, 2]$ **and** $f(x) \leq 0$ **in** $[2, 3]$.

Then the area is given by

$$A = \int_0^2 f(x)dx + \int_2^3 -f(x)dx \quad \text{put value of } f(x)$$

$$\Rightarrow A = \int_0^2 (x^2 - 5x + 6)dx - \int_2^3 (x^2 - 5x + 6)dx$$

$$= \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right)\Big|_0^2 - \left(\frac{x^3}{3} - \frac{5x^2}{2} + 6x\right)\Big|_2^3$$

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$$\Rightarrow A=\left(\frac{(2)^3}{3}-\frac{5}{2}(2)^2+6(2)-\left[\frac{0}{3}-\frac{0}{2}+0\right]\right)$$

$$-\left[\left(\frac{(3)^3}{3}-\frac{5}{2}(3)^2+6(3)\right)-\left(\frac{(2)^3}{3}-\frac{5}{2}(2)^2+6(2)\right)\right]$$

$$\Rightarrow A=\left(\frac{8}{3}-10+12-0\right)-\left[\left(9-\frac{45}{2}+18\right)-\left(\frac{8}{3}-10+12\right)\right]$$

$$\Rightarrow A=\frac{8}{3}+2-\left[\frac{18-45+18}{2}-\left(\frac{8}{3}+2\right)\right]$$

$$=\frac{14}{3}-\left(\frac{9}{2}-\frac{14}{3}\right)=\frac{14}{3}-\frac{9}{2}+\frac{14}{3}$$

$$\Rightarrow A=\frac{28-27+28}{6}=\frac{29}{6} \quad \textit{Square uniits.}$$

(c) $f(x)=x^2-6x+8$, $[0\ ,\ 4]$.

Solution: $f(x)=x^2-6x+8$ .....(i)

First we find intercept of $f(x)$ **on the x- axis .**

Put $f(x)=0$ **in (i)**

$$x^2-6x+8=0 \quad \Rightarrow x^2-2x-4x+8=0$$

$$\Rightarrow x(x-2)-4(x-2)=0 \quad \Rightarrow (x-2)(x-4)=0$$

$$\Rightarrow x-2=0 \quad \textit{and} \quad x-4=0$$

$$\Rightarrow x=2 \quad \textit{and} \quad x=4$$

Thus the subinterval of the interval $[0,\ 4]$ **are** $[0\ ,\ 2]$ **and** $[2\ ,\ 4]$

Since $f(x)\geq 0$ **in** $[0\ ,\ 2]$ **and** $f(x)\leq 0$ **in** $[2\ ,\ 4]$.

Then the area is given by

$$A=\int_0^2 f(x)dx+\int_2^4 -f(x)dx$$ **put value of** $f(x)$

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$$\Rightarrow A=\int_0^2\left(x^2-6x+8\right)dx-\int_2^4\left(x^2-6x+8\right)dx$$

$$=\left(\frac{x^3}{3}-\frac{6x^2}{2}+8x\right)\Big|_0^2-\left(\frac{x^3}{3}-\frac{6x^2}{2}+8x\right)\Big|_2^4$$

$$\Rightarrow A=\left(\frac{x^3}{3}-3x^2+8x\right)\Big|_0^2-\left(\frac{x^3}{3}-3x^2+8x\right)\Big|_2^4$$

$$\Rightarrow A=\left[\left(\frac{(2)^3}{3}-3(2)^2+8(2)\right)-(0-0+0)\right]$$

$$-\left[\left(\frac{(4)^3}{3}-3(4)^2+8(4)\right)-\left(\frac{(2)^3}{3}-3(2)^2+8(2)\right)\right]$$

$$\Rightarrow A=\left(\frac{8}{3}-12+16\right)-\left[\left(\frac{64}{3}-48+32\right)-\left(\frac{8}{3}-12+16\right)\right]$$

$$=\frac{8}{3}+4-\left(\frac{64}{3}-16-\frac{8}{3}-4\right)=\frac{8+12}{3}-\frac{64-48-8-12}{3}$$

$$\Rightarrow A=\frac{20}{3}-\frac{-4}{3}=\frac{20+4}{3}=\frac{24}{3}=8 \text{ Square units.}$$

(d) $f(x)=5x-x^2$ **on** $[1,\ 3]$

Solution: $f(x)=5x-x^2$ .....(i)

First we find intercept of $f(x)$ **on the x- axis .**

Put $f(x)=0$ **in (i)**

$$5x-x^2=0 \Rightarrow x(5-x)=0$$

$$\Rightarrow x=0 \quad \textit{and} \quad 5-x=0 \Rightarrow x=5$$

Thus $f(x)$ **intercepts x –axis at** $x=0$ **and** $x=5$

Then there is no subintervals of the interval $[1,\ 3]$ **.**

And $f(x)>0$ **in** $[1,\ 3]$ **.**

Then the area is given by

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$$A=\int_1^3 f(x)dx=\int_1^3 (5x-x^2)dx \Rightarrow A=\left(\frac{5x^2}{2}-\frac{x^3}{3}\right)\Big|_1^3$$

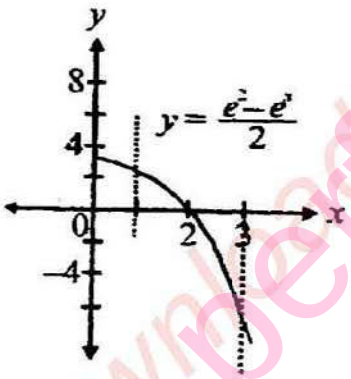
$$\Rightarrow A=\left(\frac{5(3)^2}{2}-\frac{(3)^3}{3}\right)-\left(\frac{5(1)^2}{2}-\frac{(1)^3}{3}\right)$$

$$=\frac{45}{2}-9-\left(\frac{5}{2}-\frac{1}{3}\right)=\frac{45-18}{2}-\frac{15-2}{6}$$

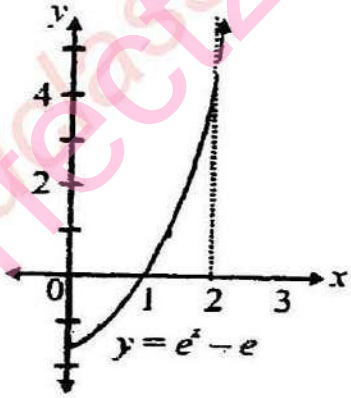
$$\Rightarrow A=\frac{27}{2}-\frac{13}{6}=\frac{81-13}{6} \Rightarrow A=\frac{68}{6}=\frac{34}{3}\ \textit{Square units.}$$

Q.4: Set up definite integrals in problems *a* to *d* that represent the indicated shaded areas.

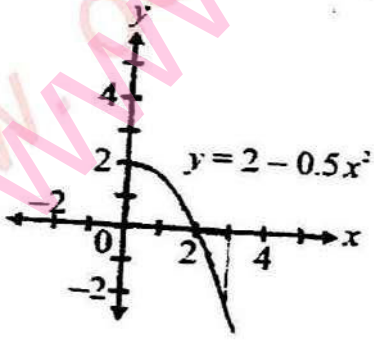
(a)



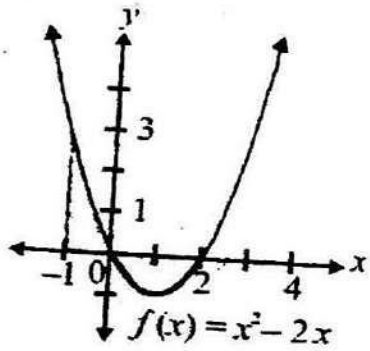
(b)



(c)



(d)



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Solution: (a) Here $f(x)=\dfrac{e^2-e^x}{2}$

From figure (a) we see that $f(x)\geq 0$ in $[1\ ,\ 2]$ and $f(x)\leq 0$ in $[2\ ,3]$.

Then the area is given by

$$A=\int_1^2 f(x)dx+\int_2^3 -f(x)dx \quad \text{put value of } f(x)$$

$$\Rightarrow A=\int_1^2\left(\frac{e^2-e^x}{2}\right)dx-\int_2^3\left(\frac{e^2-e^x}{2}\right)dx$$

$$\Rightarrow A=\frac{1}{2}\left(e^2x-e^x\right)_1^2-\frac{1}{2}\left(e^2x-e^x\right)_2^3$$

$$\Rightarrow A=\frac{1}{2}\left[2e^2-e^2-\left(e^2-e^1\right)\right]-\frac{1}{2}\left[3e^2-e^3-\left(2e^2-e^2\right)\right]$$

$$\Rightarrow A=\frac{1}{2}\left(e^2-e^2+e\right)-\frac{1}{2}\left(3e^2-e^3-e^2\right)$$

$$\Rightarrow A=\frac{e}{2}-\frac{2e^2-e^3}{2}=\frac{2.718}{2}-\frac{2(7.389)-20.086}{2}$$

$$\Rightarrow A=1.359+2.654=4.01 \ \textit{square units.}$$

(b) $f(x)=e^x-e$

From figure (b) we see that $f(x)\geq 0$ in $[1\ ,\ 2]$ and $f(x)\leq 0$ in $[0\ ,1]$.

Then the area is given by

$$A=\int_0^1 -f(x)dx+\int_1^2 f(x)dx \quad \text{put value of } f(x)$$

$$A=-\int_0^1\left(e^x-e\right)dx+\int_1^2\left(e^x-e\right)dx$$

$$\Rightarrow A=-\left(e^x-xe\right)_0^1+\left(e^x-xe\right)_1^2$$

$$\Rightarrow A=-\left[(e-e)-\left(e^0-0\right)\right]+\left[e^2-2e-(e-e)\right]$$

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$$\Rightarrow A = -(0-1)+(e^2-2e) = 1+7.389-2(2.718)$$

$$\Rightarrow A = 2.953 \quad Ans.$$

(c) Here $f(x) = 2-0.5x^2$ **.....(i)**

From figure (ac we see that $f(x) \geq 0$ **in** $[0, 2]$ **and** $f(x) \leq 0$ **in** $[2, 3]$.

Then the area is given by

$$A = \int_0^2 f(x)dx + \int_2^3 -f(x)dx \quad \textbf{put value of } f(x)$$

$$\Rightarrow A = \int_0^2 (2-0.5x^2)dx - \int_2^3 (2-0.5x^2)dx$$

$$= \left(2x - \frac{0.5x^3}{3}\right)\Big|_0^2 - \left(2x - \frac{0.5x^3}{3}\right)\Big|_2^3$$

$$\Rightarrow A = \left(2(2) - \frac{(0.5)(2)^3}{3}\right) - \left(0 - \frac{0}{3}\right) - \left[\left(2(3) - \frac{(0.5)(3)^3}{3}\right) - \left(2(2) - \frac{(0.5)(2)^3}{3}\right)\right]$$

$$\Rightarrow A = \left(4 - \frac{4}{3}\right) - \left(6 - 4.5 - \left(4 - \frac{4}{3}\right)\right)$$

$$\Rightarrow A = \frac{12-4}{3} - \left(1.5 - \frac{12-4}{3}\right)$$

$$\Rightarrow A = \frac{8}{3} - 1.5 + \frac{8}{3} = \frac{8}{3} - \frac{15}{10} + \frac{8}{3}$$

$$\Rightarrow A = \frac{8}{3} - \frac{3}{2} + \frac{8}{3} = \frac{16-9+16}{6}$$

$$\Rightarrow A = \frac{23}{6} \quad square\ units.$$

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(d) Here $f(x) = x^2 - 2x$

From figure (d) we see that $f(x) \geq 0$ **in** $[-1, 0]$ **and** $f(x) \leq 0$ **in** $[0, 2]$.

Then the area is given by

$$A = \int_{-1}^{0} f(x)dx + \int_{0}^{2} -f(x)dx \quad \text{put value of } f(x)$$

$$A = \int_{-1}^{0} (x^2 - 2x)dx - \int_{0}^{2} (x^2 - 2x)dx$$

$$\Rightarrow A = \left(\frac{x^3}{3} - \frac{2x^2}{2}\right)\Big|_{-1}^{0} - \left(\frac{x^3}{3} - \frac{2x^2}{2}\right)\Big|_{0}^{2}$$

$$\Rightarrow A = \left(\frac{x^3}{3} - x^2\right)\Big|_{-1}^{0} - \left(\frac{x^3}{3} - x^2\right)\Big|_{0}^{2}$$

$$\Rightarrow A = \left(\frac{0}{3} - 0\right) - \left(\frac{(-1)^3}{3} - (-1)^2\right) - \left[\left(\frac{(2)^3}{3} - (2)^2\right) - \left(\frac{0}{3} - 0\right)\right]$$

$$\Rightarrow A = 0 - \left(-\frac{1}{3} - 1\right) - \left(\frac{8}{3} - 4 - 0\right)$$

$$\Rightarrow A = -\left(-\frac{4}{3}\right) - \left(\frac{8-12}{3}\right) = \frac{4}{3} + \frac{4}{3} = \frac{8}{3} \quad \textit{square units.}$$

Q.5: An oil tanker is leaking oil at a rate given iin barrels per hour by $\frac{dL}{dt} = \frac{80\ln(t+1)}{t+1}$ **, where t is the time in hour after the tanker hits a hidden rock (when t = 0).**

(a) Find the total number of barrels that the ship will leak on the first day .

(b) Find the total number of barrels that the ship will leak on the second day .

(c) What is happing over the long run to the amount of oil leaked per day ? .

Solution: Given that $\frac{dL}{dt} = \frac{80\ln(t+1)}{t+1}$

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(a) $dL = \dfrac{80\ln(t+1)}{t+1}dt$

The total number of barrels that the ship is leaked on the first day (0 to 24 hours) is

$$B = 80\int_0^{24}\frac{\ln(t+1)}{t+1}dt \quad \text{.....(i)}$$

Let $\ln(t+1) = u \Rightarrow \dfrac{d}{dt}\ln(t+1) = \dfrac{du}{dt}$

$$\Rightarrow \frac{1}{t+1} = \frac{du}{dt} \quad or \quad \frac{dt}{t+1} = du$$

When $t = 0$ *then* $u = \ln(0+1) = 0$

When $t = 24$ *then* $u = \ln(24+1) = \ln 25$

Put values in (i).

$$B = 80\int_0^{\ln 25} u\,du = 80\left(\frac{u^2}{2}\right)\Big|_0^{\ln 25}$$

$\Rightarrow B = 40\left[(\ln 25)^2 - 0\right] = 40(3.22)^2 = 414.44$ **barrels first day.**

(b) **The total number of barrels that the ship is leaked on the second day (24 to 48 hours) is**

$$B = 80\int_{24}^{48}\frac{\ln(t+1)}{t+1}dt \quad \text{.....(ii)}$$

Let $\ln(t+1) = u \Rightarrow \dfrac{d}{dt}\ln(t+1) = \dfrac{du}{dt}$

$$\Rightarrow \frac{1}{t+1} = \frac{du}{dt} \quad or \quad \frac{dt}{t+1} = du$$

When $t = 24$ *then* $u = \ln(24+1) = \ln 25$

When $t = 48$ *then* $u = \ln(48+1) = \ln 49$

Put values in (ii).

$$B = 80\int_{\ln 25}^{\ln 49} u\,du = 80\left(\frac{u^2}{2}\right)\Big|_{\ln 25}^{\ln 49}$$

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$$\Rightarrow B = 40\left[(\ln 49)^2 - (\ln 25)^2\right] = 40\left[(3.89)^2 - (3.22)^2\right]$$

$$\Rightarrow B = 40(15.15 - 10.36) = 191.6 \text{ barrels second day.}$$

(c) We see that the amount of oil leaked per day is decreases when the number of days are increases. Thus over the long run the amount of oil leaked per day is decreased to 0.

Q.6: Use MAPLE command "int" to evaluate

(a) $f(x) = x^2 + 3x + 1$ **, w.r.t. to x.**

(b) $f(x) = e^{2x}\sin x$ **w..r.t. to x.**

Solution: (a) Open the MAPLE program. Then open the new document and use the keyboard keys and write int(x^2+3*x+1,x); **press enter key we will get the following command and result.**

Command:

```
> int(x^2 + 3·x + 1, x);
```

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + x$$

(b) Open the MAPLE program. Then open the new document and use the keyboard keys and write int(e^x*sin(x),x); **press enter key we will get the following command and result.**

Command:

```
int(e^x·sin(x), x);
```

$$\frac{\dfrac{e^{x\ln(e)}\tan\left(\frac{1}{2}x\right)^2}{\ln(e)^2+1} - \dfrac{e^{x\ln(e)}}{\ln(e)^2+1} + \dfrac{2\ln(e)\,e^{x\ln(e)}\tan\left(\frac{1}{2}x\right)}{\ln(e)^2+1}}{1+\tan\left(\frac{1}{2}x\right)^2}$$

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Review Exercise 6:

Chose the corect option.

Q.1: The process of finding antiderivative is called

(a) differentiation (b) integration

(c) probability (d) linear equation

(ii) $\int \tan\theta d\theta =$ .....

(a) $\ln|\sin\theta|+c$ **(b)** $\ln|\cos\theta|+c$

(c) $-\ln|\cos\theta|+c$ **(d)** $-\ln|\sin\theta|+c$

(iii) $\int \frac{dx}{\sqrt{a^2-x^2}} =$ .....

(a) $\ln\left|x+\sqrt{x^2+a^2}\right|$ **(b)** $\sin^{-1}\frac{a}{x}$

(c) $\sin^{-1}\left(\frac{x}{a}\right)+c$ **(d)** $\cos^{-1}\left(\frac{x}{a}\right)+c$

(iv) $\int \frac{dt}{\sqrt{t^2-36}} =$ ....

(a) $\frac{1}{2a}\ln\left|\frac{t-6}{t+6}\right|$ **(b)** $\frac{1}{2}\ln\left|\frac{t-6}{t+6}\right|$

(c) $\ln\left(\frac{t+\sqrt{t^2-36}}{6}\right)$ **(d)** $\frac{1}{6}\left(\ln\left|\frac{t}{6}+1\right|+6\right)+c$

(v) $\int (x^3-4)dx =$ .....

(a) $\frac{x^4}{4}-4x+C$ **(b)** $\frac{x^3}{3}-4x+C$

(c) $-\frac{x^4}{4}+4x+C$ **(d)** $\frac{x^4}{4}+\frac{4x^3}{3}-\frac{4x^2}{2}-4x+C$

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(vi) $\int f(x)g'(x)dx = ...$

(a) $f'(x)g(x)+\int g'(x)f(x)dx$ (b) $f(x)g(x)-\int g(x)f'(x)dx$

(c) $f'(x)g(x)-\int g'(x)f'(x)dx$ (d) $f(x)g(x)-\int g(x)f(x)dx$

(vii) $\int \tan^4 x dx$ = ...

(a) $\frac{2}{3}\tan^2 x + x - \tan x + c$ (b) $\frac{1}{3}\tan^3 x + x - \tan x + c$

(c) $\frac{3}{4}\tan^2 x - x + \tan x + c$ (d) $3\tan^3 x + x + \tan x + c$

(viii) $\int \frac{x+8}{x^2-64}dx$

(a) $\ln|x-8|+c$ (b) $\frac{1}{2}\ln|x^2-64|+\frac{1}{2}\left|\frac{x}{8}+1\right|-\frac{1}{2}\ln\left|\frac{x}{2}-1\right|+c$

(c) $\frac{1}{2}\ln|x^2-64|+\frac{1}{2}\left|\frac{x}{8}+1\right|-\frac{1}{2}\ln\left|\frac{x}{8}-1\right|+c$

(d) $\frac{1}{2}\ln\left|\frac{x}{8}+1\right|+\frac{1}{2}\ln\left|\frac{x}{8}-1\right|+c$

(ix) $\int_0^2 e^{2x}dx$ = ... (a) $\frac{e^4-1}{2}$ (b) $\frac{e^3-1}{2}$ (c) $\frac{e^3-1}{2}$ (d) $\frac{e^4+1}{2}$

(x) $\int_0^2 x^3 dx$ = ... (a) 1 (b) 2 (c) 3 (d) 4

Answers: (i) b (ii) c (iii) c (iv) c (v) a (vi) b
(vii) b (viii) c (ix) a (x) d

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