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Unit- 11 : Partial Differentiation:

Function of two variables : The function $z = f(x, y)$ is called a function of two variables x and y if a unique value of z is obtained from each ordered pair of real numbers (x, y). The real numbers x and y are called independent variables and z is called dependent variable. The set of all real numbers for which z is defined is called domain of the function z.

The set of values $f(x, y)$ is the range.

Partial derivatives : If $z = f(x, y)$ is a function of two variables. Then the partial derivatives of $z = f(x, y)$ with respect to x is denoted by $\frac{\partial f}{\partial x}$ or f_x and given by

$$\frac{\partial f}{\partial x} = f_x = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}.$$

The partial derivatives of $z = f(x, y)$ with respect to y is denoted by $\frac{\partial f}{\partial y}$ or f_y and given by

$$\frac{\partial f}{\partial y} = f_y = \lim_{\Delta y \to 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}.$$

Note: (i) In f_x , y remains constant.

(ii) In f_y , x remains constant.

Example . 2: The function is $f(x, y) = x^3 y + xy^2$. Find partial derivatives f_x and f_y.

Solution: $f(x, y) = x^3 y + xy^2$ ......(i)

Differentiate w.r.t. to x.

$$\frac{\partial f}{\partial x} = f_x = \frac{\partial}{\partial x}\left(x^3 y + xy^2\right)$$

=> $f_x = 3x^2 y + y^2$ Ans.

Differentiate (i) w.r.t. to y.

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$$\frac{\partial f}{\partial y} = f_y = \frac{\partial}{\partial y}\left(x^3 y + xy^2\right)$$

=> $f_y = x^3 + 2xy$ **Ans.**

Exercise 11.1:

Q.1: **If** $f(x,y) = x^2 y + xy^2$ **and** t **is any real number, then find out the following:**

(a) $f(0, 0)$ **(b)** $f(-1, 0)$ **(c)** $f(0, -1)$

(d) $f(t, t)$ **(e)** $f(t, t^2)$ **(f)** $f(1-t, t)$

Solution: $f(x,y) = x^2 y + xy^2$ ......(i)

Put $x = 0$, $y = 0$ **in (i)**

$$f(0, 0) = (0)^2(0) + (0)(0)^2 = 0$$

(b) $f(x,y) = x^2 y + xy^2$ ......(i)

Put $x = -1$, $y = 0$ **in (i)**

$$f(-1, 0) = (-1)^2(0) + (-1)(0)^2 = 0$$

(c) $f(x,y) = x^2 y + xy^2$ ......(i)

Put $x = 0$, $y = -1$ **in (i)**

$$f(0, -1) = (0)^2(-1) + (0)(-1)^2 = 0$$

(d) $f(x,y) = x^2 y + xy^2$ ......(i)

Put $x = t$, $y = t$ **in (i)**

$$f(t, t) = (t)^2(t) + (t)(t)^2 = t^3 + t^3 = 2t^3$$

(e) $f(x,y) = x^2 y + xy^2$ ......(i)

Put $x = t$, $y = t^2$ **in (i)**

$$f(t, t) = (t)^2(t^2) + (t)(t^2)^2 = t^4 + t^5 \quad Ans.$$

(f) $f(x,y) = x^2 y + xy^2$ ......(i)

Put $x = 1-t$, $y = t$ **in (i)**

$$f(1-t, t) = (1-t)^2(t) + (1-t)(t)^2$$

$$= (1 - 2t + t^2)(t) + t^2 - t^3$$

$$= t - 2t^2 + t^3 + t^2 - t^3 = t - t^2 \quad Ans.$$

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Q.2: **The function is** $f(x,y,z)=x^2ye^{2x}+(x+y-z)^2$. **Find the function value at the following points:**

(a) $f(0,0,0)$ **(b)** $f(1,-1,1)$ **(c)** $f(-1,1,-1)$

(d) $\frac{\partial}{\partial x}f(x,x,x)$ **(e)** $\frac{\partial}{\partial y}f(1,y,1)$ **(f)** $\frac{\partial}{\partial z}f(1,1,z^2)$

Solution: **(a)** $f(x,y,z)=x^2ye^{2x}+(x+y-z)^2 \ldots\ldots$(i)

Put $x=0$, $y=0$, $z=0$ **in (i)**

$f(0,0,0)=0+(0+0-0)^2=0$ **Ans.**

(b) $f(1,-1,1)$

Put $x=1$, $y=-1$, $z=1$ **in (i)**

$f(1,-1,1)=(1)^2(-1)e^2+(1-1-1)^2$

$=-e^2+(-1)^2=-e^2+1$ *Ans.*

(c) $f(-1,1,-1)$

Put $x=-1$, $y=1$, $z=-1$ **in (i)**

$f(-1,1,-1)=(-1)^2(1)e^{-2}+(-1+1-(-1))^2$

$=e^{-2}+(1)^2=e^{-2}+1$ *Ans.*

(d) $\frac{\partial}{\partial x}f(x,x,x)$:

Put $x=x$, $y=x$, $z=x$ **in (i)**

$f(x,x,x)=x^2.x.e^{2x}+(x+x-x)^2$

$=x^3e^{2x}+x^2$. **Now**

$\Rightarrow \frac{\partial}{\partial x}f(x,x,x)=\frac{\partial}{\partial x}(x^3e^{2x}+x^2)$

$=x^3.\frac{\partial}{\partial x}e^{2x}+e^{2x}.\frac{\partial}{\partial x}x^3+\frac{\partial}{\partial x}x^2$

$=2x^3e^{2x}+3x^2e^{2x}+2x$ **Ans.**

(e) $\frac{\partial}{\partial y}f(1,y,1)$:

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Put $x=1$, $y=y$, $z=1$ in (i)

$$f(1,\ y,\ 1)=(1)^2 ye^2+(1+y-1)^2$$

$$\Rightarrow\ f(1,y,1)=ye^2+y^2 \quad \text{.Now}$$

$$\frac{\partial}{\partial y}f(1,y,1)=\frac{\partial}{\partial y}(ye^2+y^2)$$

$$=e^2+2y \quad Ans.$$

(f) $\frac{\partial}{\partial z}f(1,1,z^2)$

Put $x=1$, $y=1$, $z=z^2$ in (i)

$$f(1,1,z^2)=(1)^2(1)e^2+(1+1-z^2)^2$$

$$\Rightarrow\ f(1,1,z^2)=e^2+(2-z^2)^2$$

$$\frac{\partial}{\partial z}f(1,1,\ z^2)=\frac{\partial}{\partial z}\left(e^2+(2-z^2)^2\right)$$

$$=0+2(2-z^2)\frac{\partial}{\partial z}(2-z^2)$$

$$=2(2-z^2)(-2z)$$

$$=-4z(2-z^2)=4z(z^2-2) \quad Ans.$$

<u>Q.3</u>: Find the partial derivatives f_x and f_y of each of the following functions.

(a) $f(x,y)=\sin x^2.\cos y$

<u>Solution</u>: $f(x,y)=\sin x^2.\cos y$ .....(i)

Differentiate (i) w.r.t. to x partially

$$\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}(\sin x^2.\cos y)$$

$$\Rightarrow\ f_x=\cos y\frac{\partial}{\partial x}\sin x^2$$

$$=(\cos y)(2x\cos x)$$

$$\Rightarrow\ f_x=2x\cos x\cos y \quad Ans.$$

Differentiate (i) w.r.t. to y partially

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$$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\left(\sin x^2.\cos y\right)$$

$$\Rightarrow f_y=\sin x^2.\frac{\partial}{\partial y}(\cos y)$$

$$=-\sin x^2.\sin y \quad Ans.$$

(b) $f(x,y)=\sqrt{3x^2+y^4}$

Solution: $f(x,y)=\sqrt{3x^2+y^4}$ **.....(i)**

Differentiate (i) w.r.t. to x partially

$$\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\left(3x^2+y^4\right)^{\frac{1}{2}}$$

$$\Rightarrow f_x=\frac{1}{2}\left(3x^2+y^4\right)^{\frac{1}{2}-1}\frac{\partial}{\partial x}\left(3x^2+y^4\right)$$

$$=\frac{1}{2}\left(3x^2+y^4\right)^{-\frac{1}{2}}6x$$

$$=\frac{3x}{\sqrt{3x^2+y^4}} \quad Ans.$$

Differentiate (i) w.r.t. to y partially

$$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\left(3x^2+y^4\right)^{\frac{1}{2}}$$

$$\Rightarrow f_y=\frac{1}{2}\left(3x^2+y^4\right)^{\frac{1}{2}-1}.\frac{\partial}{\partial y}\left(3x^2+y^4\right)$$

$$=\frac{1}{2}\left(3x^2+y^4\right)^{-\frac{1}{2}}\left(4y^3\right)$$

$$=\frac{2y^3}{\sqrt{3x^2+y^4}} \quad Ans.$$

(c) $f(x,y)=xy^3\tan^{-1}y$

Solution: $f(x,y)=xy^3\tan^{-1}y$ **.....(i)**

Differentiate (i) w.r.t. to x partially

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$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(xy^3 \tan^{-1} y\right)$$

$$\Rightarrow f_x = y^3 \tan^{-1} y \frac{\partial}{\partial x}(x) = y^3 \tan^{-1} y \quad Ans.$$

Differentiate (i) w.r.t. to y partially.

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(xy^3 \tan^{-1} y\right)$$

$$\Rightarrow f_y = x\left(y^3 \frac{\partial}{\partial y}\tan^{-1} y + \tan^{-1} y \frac{\partial}{\partial y} y^3\right)$$

$$\Rightarrow f_y = x\left(\frac{y^3}{1+y^2} + 3y^2 \tan^{-1} y\right) \quad \textbf{Ans.}$$

(d) $f(x, y) = x^3 + x^2y + xy^2 + y^3$

Solution: $f(x, y) = x^3 + x^2y + xy^2 + y^3$ .....(i)

Differentiate (i) w.r.t. to x partially.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^3 + x^2y + xy^2 + y^3\right)$$

$$\Rightarrow f_x = 3x^2 + 2xy + y^2 \quad \textbf{Ans.}$$

Differentiate (i) w.r.t. to y partially.

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^3 + x^2y + xy^2 + y^3\right)$$

$$\Rightarrow f_y = x^2 + 2xy + 3y^2 \quad Ans.$$

(e) $f(x, y) = \sin^{-1} xy$

Solution: $f(x, y) = \sin^{-1} xy$ ......(i)

Differentiate (i) w.r.t. to x partially.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\sin^{-1} xy\right)$$

$$\Rightarrow f_x = \frac{1}{\sqrt{1-(xy)^2}} \cdot \frac{\partial}{\partial x}(xy)$$

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$$=> \quad f_x = \frac{y}{\sqrt{1-x^2y^2}} \quad Ans.$$

Differentiate (i) w .r. t. to y partially .

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\sin^{-1} xy\right)$$

$$=> f_y = \frac{1}{\sqrt{1-(xy)^2}} \cdot \frac{\partial}{\partial y}(xy)$$

$$=> \quad f_y = \frac{x}{\sqrt{1-x^2y^2}} \quad Ans.$$

(f) $f(x, y) = x^2 e^{x+y} \cos y$

<u>Solution</u>: $f(x, y) = x^2 e^x . e^y \cos y$ ......**(i)**

Differentiate (i) w .r. t. to x partially .

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^2 e^x e^y \cos y\right)$$

$$=> f_x = e^y \cos y \, \frac{\partial}{\partial x}\left(x^2 e^x\right) \quad \textbf{use product rule}$$

$$=> f_x = e^y \cos y \left(x^2 \frac{\partial}{\partial x} e^x + e^x \frac{\partial}{\partial x} x^2 \right)$$

$$= e^y \cos y \left(x^2 e^x + 2xe^x\right)$$

$$= e^y . e^x \cos y \left(x^2 + 2x\right)$$

$$=> f_x = e^{x+y} \cos y \left(x^2 + 2x\right) \quad Ans.$$

Differentiate (i) w .r. t. to y partially .

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^2 e^x e^y \cos y\right)$$

$$=> f_y = x^2 e^x \frac{\partial}{\partial x}\left(e^y \cos y\right) \quad \textbf{use product rule}$$

$$=> f_x = x^2 e^x \left(e^y \frac{\partial}{\partial x} \cos y + \cos y \frac{\partial}{\partial x} e^y \right)$$

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$$= x^2e^x\left(-e^y \sin y + \cos y e^y\right)$$

$$= x^2e^x.e^y\left(-\sin y + \cos y\right)$$

$$\Rightarrow \; f_x = x^2e^{x+y}\left(\cos y - \sin y\right) \quad Ans.$$

Q.4: **The product function z for the United states was once estimated as** $z = x^{0.7}.y^{0.3}$ **.**

where x stands for the amount of labour and y stands for the amount of capital. Find the marginal productivity of labour $\frac{\partial z}{\partial x}$ **and of capital** $\frac{\partial z}{\partial y}$ **.**

Solution: **Given the production function is**

$$z = x^{0.7}.y^{0.3} \quad \ldots\ldots(i)$$

Differentiate (i) w.r.t. to x partially.

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(x^{0.7}.y^{0.3}\right) \quad \Rightarrow \quad \frac{\partial z}{\partial x} = y^{0.3}\frac{\partial}{\partial x}\left(x^{0.7}\right)$$

$$\Rightarrow \; \frac{\partial z}{\partial x} = (0.7)y^{0.3}x^{0.7-1} \quad \Rightarrow \quad \frac{\partial z}{\partial x} = 0.7\, y^{0.3}.x^{-0.3} \quad Ans.$$

Differentiate (i) w.r.t. to y partially.

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(x^{0.7}.y^{0.3}\right) \quad \Rightarrow \quad \frac{\partial z}{\partial y} = x^{0.7}\frac{\partial}{\partial y}\left(y^{0.3}\right)$$

$$\Rightarrow \; \frac{\partial z}{\partial y} = (0.3)x^{0.7}y^{0.3-1} \quad \Rightarrow \quad \frac{\partial z}{\partial y} = (0.3)\,x^{0.7}y^{-0.7} \quad Ans.$$

Q.5: **A similar production function for Canada is :**

$$z = f(x,y) = x^{0.4}y^{0.6}$$

where x stands for the amount of labour and y stands for the amount of capital. Find the marginal productivity of labour $\frac{\partial z}{\partial x}$ **and of capital** $\frac{\partial z}{\partial y}$ **.**

Solution: $z = x^{0.4}y^{0.6} \quad \ldots\ldots(i)$

Differentiate (i) w.r.t. to x partially.

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(x^{0.4}.y^{0.6}\right) \quad \Rightarrow \quad \frac{\partial z}{\partial x} = y^{0.6}\frac{\partial}{\partial x}\left(x^{0.4}\right)$$

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$=> \dfrac{\partial z}{\partial x} = (0.4)y^{0.6}x^{0.4-1} \quad => \dfrac{\partial z}{\partial x} = 0.4\,y^{0.6}\,x^{-0.6}$ *Ans.*

Differentiate (i) w .r. t. to y partially .

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(x^{0.4}\,y^{0.6}\right) \quad => \frac{\partial z}{\partial y} = x^{0.4}\frac{\partial}{\partial y}\left(y^{0.6}\right)$$

$=> \dfrac{\partial z}{\partial y} = (0.6)x^{0.4}y^{0.6-1} \quad => \dfrac{\partial z}{\partial y} = (0.6)\,x^{0.4}\,y^{-0.4}$ *Ans.*

Q.6: If $f(x, y) = x^2y + xy^2$. Then find f_x and f_y by using definition of partial derivatives .

Solution: $f(x, y) = x^2y + xy^2$ ......(i)

Differentiate w . r. t. to x .

$$\frac{\partial f}{\partial x} = f_x = \frac{\partial}{\partial x}\left(x^2y + xy^2\right)$$

$=> f_x = 2xy + y^2$ **Ans.**

Differentiate (i) w .r. t. to y .

$$\frac{\partial f}{\partial y} = f_y = \frac{\partial}{\partial y}\left(x^3y + xy^2\right)$$

$=> f_y = x^3 + 2xy$ **Ans.**

Homogenous function: A function $f(x, y)$ is called homogenous function of degree n if

$$f(\lambda x, \lambda y) = \lambda^n f(x, y).$$

Homogenous polynomial : A polynomial function $f(x, y)$ is called homogenous function of degree n if the degree of each of its terms in x and y are equal to n .

The homogenous polynomial is of the form

$$f(x, y) = a_o x^n + a_1 x^{n-1}y + a_2 x^{n-2}y^2 + \dots\dots\dots + a_{n-1}xy^{n-1} + a_n y^n$$

$$f(x, y) = x^n\left[a_o + a_1\left(\frac{y}{x}\right) + a_2\left(\frac{y}{x}\right)^2 + \dots\dots + a_{n-1}\left(\frac{y}{x}\right)^{n-1} + a_n\left(\frac{y}{x}\right)^n\right]$$

$$= x^n f\left(\frac{y}{x}\right)$$

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$$\Rightarrow f(x, y) = x^n f\left(\frac{y}{x}\right)$$

Example .5: Show that the function $f(x, y) = 2xy + y^2$ is a homogenous function of degree 2 .

Solution: $f(x, y) = 2xy + y^2$ ....(i)

Now replace x *by* λx **and** y *by* λy **in (i)**

$$f(\lambda x, \lambda y) = 2(\lambda x)(\lambda y) + (\lambda y)^2$$

$$= 2\lambda^2 xy + \lambda^2 y^2$$

$$= \lambda^2(2xy + y^2) = \lambda^2 f(x, y)$$

Thus $f(\lambda x, \lambda y) = \lambda^2 f(x, y)$

Hence eq(i) is homogenous equation of degree 2.

Statement of Euler Theorem :

If $z = f(x,y)$ *is homogeneous function of degree* "n". *Then* $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$ **or** $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$.

Proof: *Let* $f(x,y) = x^n + x^{n-1}y + ... + xy^{n-1} + y^n \rightarrow (i)$

be a homogenous fi:nction of degree n.

$$f(x,y) = x^n\left[1 + \frac{y}{x} + ... + \left(\frac{y}{x}\right)^{n-1} + \left(\frac{y}{x}\right)^n\right]$$

$$= x^n f\left(\frac{y}{x}\right) \rightarrow (ii)$$

$$\Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(x^n f\left(\frac{y}{x}\right)\right)$$

$$\frac{\partial f}{\partial x} = x^n f'\left(\frac{y}{x}\right)\frac{\partial}{\partial x}\left(\frac{y}{x}\right) + f\left(\frac{y}{x}\right)\frac{\partial}{\partial x}(x^n)$$

$$= x^n f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right) + nx^{n-1} f\left(\frac{y}{x}\right)$$

multilied by "x" both sides

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$$=> x\frac{\partial f}{\partial x} = -x^{n-1}y\,f'\left(\frac{y}{x}\right) + nx^n f\left(\frac{y}{x}\right) \to (iii)$$

Now differentiate (ii) w.r.t y

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(x^n f(\frac{y}{x})\right)$$

$$=> \frac{\partial f}{\partial y} = x^n f'\left(\frac{y}{x}\right).\frac{\partial}{\partial y}\left(\frac{y}{x}\right)$$

$$= x^n.\frac{1}{x}.f'\left(\frac{y}{x}\right)$$

Multiiplied by "y" both sides

$$=> y\frac{\partial f}{\partial y} = y\,x^{n-1} f'\left(\frac{y}{x}\right) \to (iv)$$

adding (iii) and (iv)

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -x^{n-1}y\,f'\left(\frac{y}{x}\right) + nx^n f\left(\frac{y}{x}\right)$$
$$+ x^{n-1}y\,f'\left(\frac{y}{x}\right)$$

$$=> x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nx^n f\left(\frac{y}{x}\right) = nf \quad \text{Proved}$$

Hence $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nx^n f\left(\frac{y}{x}\right) = nf(x,y)$ *which is the proof*

Example .8:

If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ *to prove that* $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

Solution: $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right) \quad => \tan u = \frac{x^3+y^3}{x-y}$

Let $f(x,y) = \tan u = \frac{x^3+y^3}{x-y} \to (i)$

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$put\ \ x = \lambda x\ \ \&\ \ y = \lambda y\ \ in\ Eq(i)$

$$f(\lambda x, \lambda y) = \frac{(\lambda x)^3 + (\lambda y)^3}{\lambda x - \lambda y} = \frac{\lambda^3(x^3 + y^3)}{\lambda(x - y)}$$

$$= \lambda^2\left(\frac{x^3 + y^3}{x - y}\right)$$

$$= \lambda^2 f(x, y)$$

Hence the function f is homogeneous of degree "2"

Then by Euler's theorem

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f \rightarrow (ii)$$

As put $f = \tan u$ **in (ii)**

$$=> \ x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2\tan u \rightarrow (iii)$$

Since $f = \tan u$ **.....(iv)**

Differentiate (iv) w. r. t. x and y partially.

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\tan u$$

$$\frac{\partial f}{\partial x} = \sec^2 u.\frac{\partial u}{\partial x} \qquad multiplied\ by\ x$$

$$x\frac{\partial f}{\partial x} = x\sec^2 u\frac{\partial u}{\partial x} \rightarrow (a)$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\tan u$$

$$=> \ \frac{\partial f}{\partial y} = \sec^2 u\frac{\partial u}{x} \qquad multiplied\ both\ sides\ y$$

$$y\frac{\partial f}{\partial y} = y\sec^2 u\frac{\partial u}{\partial y} \rightarrow (b) \qquad Adding\ (a)\ and\ (b)$$

$$=> x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x\sec^2 u\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y}$$

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$$= \sec^2 u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) \ldots\ldots(v)$$

Put (v) in Eq (iii)

$$\sec^2 u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = 2\tan u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2\sin u}{\cos u} \times \frac{1}{\sec^2 u}$$

$$= \frac{2\sin u}{\cos u} \times \cos^2 u$$

$$= 2\sin u.\cos u = \sin 2u$$

Hence $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ *Proved*

Example. 9:Use MAPLE command "diff" to find the partial derivative of

(i) $f(x,y) = x^3 + y^3 + 3xy^2 + 4x^2y$ **w.r.t. variables x and y**

(ii) $f(x,y) = y\sin x + x\cos y + x^2$ **w.r.t. variables x and y**

Solution: (a) Open the MAPLE program . Then open the document and use the keyboards keys and write

$diff(x\text{^}3 + y\text{^}3 + 3 * x * y\text{^}2 + 4 * x\text{^}2 * y, x);$ **then**

Press enter key you get result .

$diff(x\text{^}3 + y\text{^}3 + 3 * x * y\text{^}2 + 4 * x\text{^}2 * y, y);$ **then**

Press enter key you get result .

Command:

```
> diff(x^3 + y^3 + 3·x·y^2 + 4·x^2·y, x);
                    3x^2 + 8xy + 3y^2
> diff(x^3 + y^3 + 3·x·y^2 + 3·x^2·y, y);
                    3x^2 + 6xy + 3y^2
```

(b) Open the MAPLE program . Then open the document and use the keyboards keys and write

$diff(y * \sin(x) + x * \cos(x) + x\text{^}2, x);$ **then**

Press enter key you get result .

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$diff(y*\sin(x)+x*\cos(x)+x^{\wedge}2, y);$ **then**

Press enter key you get result.

Command:

$> diff\left(y\cdot\sin(x)+x\cdot\cos(x)+x^2, x\right);$

$$y\cos(x)+\cos(x)-x\sin(x)+2x$$

$> diff\left(y\cdot\sin(x)+x\cdot\cos(x)+x^2, y\right)$

$$\sin(x)$$

EXERCISE 11.2

Q:1: Are the following functions homogenous ?

(a) $u=f(x, y)=\sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

Solution: $f(x, y)=\sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ ....(i)

Now replace x *by* λx **and** y *by* λy **in (i)**

$$f(\lambda x, \lambda y)=\sin^{-1}\left(\frac{\lambda x+\lambda y}{\sqrt{\lambda x}+\sqrt{\lambda y}}\right)$$

$$=\sin^{-1}\left[\frac{\lambda(x+y)}{\sqrt{\lambda}\left(\sqrt{x}+\sqrt{y}\right)}\right]$$

$$=\sin^{-1}\sqrt{\lambda}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$\neq \lambda^n f(x, y)$$

Hence (i) is not homogenous function.

(b) $u=f(x, y)=\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$

Solution: $f(x, y)=\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ ....(i)

Now replace x *by* λx **and** y *by* λy **in (i)**

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$$f(\lambda x,\ \lambda y)=\frac{\lambda x+\lambda y}{\sqrt{\lambda x}+\sqrt{\lambda y}}$$

$$=\frac{\lambda(x+y)}{\sqrt{\lambda}\left(\sqrt{x}+\sqrt{y}\right)}$$

$$=\sqrt{\lambda}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$$

$$=\lambda^{\frac{1}{2}}f(x,\ y)$$

Hence (i) is homogenous function of degree $\frac{1}{2}$.

(c) $z=f(x,y)=x^3e^{\frac{y}{x}}-3y^2\sqrt{x^2+y^2}$

Solution: $f(x,y)=x^3e^{\frac{y}{x}}-3y^2\sqrt{x^2+y^2}$ ……(i)

Now replace x by λx and y by λy in (i)

$$f(\lambda x,\ \lambda y)=(\lambda x)^3e^{\frac{\lambda y}{\lambda x}}-3(\lambda y)^2\sqrt{(\lambda x)^2+(\lambda y)^2}$$

$$=\lambda^3x^3e^{\frac{y}{x}}-3\lambda^2y^2\sqrt{\lambda^2x^2+\lambda^2y^2}$$

$$=\lambda^3x^3e^{\frac{y}{x}}-3(\lambda^2y^2)(\lambda)\sqrt{x^2+y^2}$$

$$\Rightarrow f(\lambda x,\ \lambda y)=\lambda^3\left(x^3e^{\frac{y}{x}}-3y^2\sqrt{x^2=y^2}\right)\ \text{……..(ii)}$$

Put (i) in (ii)

$$f(\lambda x,\lambda y)=\lambda^3f(x,y)$$

Hence (i) is homogenous function of degree 3.

(d) $z=f(x,y)=\left(x^2+3y^2\right)^{\frac{1}{3}}$

Solution: $f(x,y)=\left(x^2+3y^2\right)^{\frac{1}{3}}$ ……(i)

Now replace x by λx and y by λy in (i)

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$$f(\lambda x\,,\ \lambda y)=\left[(\lambda x)^2+3(\lambda y)^2\right]^{\frac{1}{3}}$$

$$=\left(\lambda^2\right)^{\frac{1}{3}}\left(x^2+3y^2\right)^{\frac{1}{3}}$$

$$=\lambda^{\frac{2}{3}}\left(x^2+3y^2\right)^{\frac{1}{3}}\ldots\ldots\ldots(ii)$$

Put (i) in (ii)

$$f(\lambda x\,,\ \lambda y)=\lambda^{\frac{2}{3}}f(x,\ y)$$

Thus (i) is homogenous function of degree $\frac{2}{3}$.

<u>Q.2</u>: Verify Euler's Theorem for the following homogenous functions .

(a) $z=f(x,y)=ax^2+2hxy+by^2$

<u>Solution</u>: Let $f=ax^2+2hxy+by^2$ **……(i)**

It is homogenous equation of degree $n=2$

As we know that by Euler's theorem

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf \quad \text{.......(ii)}$$

Differentiate (i) w . r . t. x and y partially .

$$\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}\left(ax^2+2hxy+by^2\right)$$

$$\Rightarrow \frac{\partial f}{\partial x}=2ax+2hy \quad \text{multiplied by } x$$

$$\Rightarrow x\frac{\partial f}{\partial x}=2ax^2+2hxy \quad \text{.......(a)}$$

$$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\left(ax^2+2hxy+by^2\right)$$

$$\Rightarrow \frac{\partial f}{\partial y}=2hx+2by \quad \text{multiplied by } y$$

$$\Rightarrow y\frac{\partial f}{\partial y}=2hxy+2by^2 \quad \text{.......(b)}$$

Adding (a) and (b)

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$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=2ax^2+2hxy+2hxy+2by^2$$

$$=2ax^2+4hxy+2by^2$$

$$=2\left(ax^2+2hxy+by^2\right)\ldots.(iii)$$

Put (i) in (iii)

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=2f$$

Hence Euler ,s theorem is verified.

(b) $z=f(x,y)=\left(x^2+xy+y^2\right)^{-1}$

Solution: $f=\left(x^2+xy+y^2\right)^{-1}$ **.....(i)**

Put $x=\lambda x,\ y=\lambda y$

$$f(\lambda x,\lambda y)=\left(\lambda^2x^2+\lambda x\lambda y+\lambda^2y^2\right)^{-1}$$

$$=\lambda^{-2}\left(x^2+xy+y^2\right)^{-1}=\lambda^{-2}f(x,y)$$

The function is homogeneous of degree "−2".

As we know that the Euler ,s theorem

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-2f\rightarrow(A)$$

$$As\quad f=\left(x^2+xy+y^2\right)^{-1}\Rightarrow\frac{\partial u}{\partial x}=\frac{\partial}{\partial x}\left(x^2+xy+y^2\right)^{-1}$$

$$\frac{\partial f}{\partial x}=-\left(x^2+xy+y^2\right)^{-2}(2x+y)$$

$$\frac{\partial f}{\partial x}=\frac{-2x-y}{\left(x^2+xy+y^2\right)^2}\quad \textbf{multiplied by } x$$

$$\Rightarrow x\frac{\partial f}{\partial x}=\frac{-2x^2-yx}{\left(x^2+xy+y^2\right)^2}\rightarrow(ii)\quad \textbf{Now}$$

$$\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}\left(x^2+xy+y^2\right)^{-1}$$

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$$=> \frac{\partial u}{\partial y} = -\left(x^2+xy+y^2\right)^{-2}(x+2y)$$

$$= \frac{-x-2y}{\left(x^2+xy+y^2\right)^2}$$ **Multiplied by y**

$$=> y\frac{\partial f}{\partial y} = \frac{-xy-2y^2}{\left(x^2+xy+y^2\right)^2} \rightarrow (iii)$$

Add (ii) and (iii) we get

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \frac{-2x^2-xy-xy-2y^2}{\left(x^2+xy+y^2\right)^2}$$

$$= \frac{-2x^2-2xy-2y^2}{\left(x^2+xy+y^2\right)^2}$$

$$= -2\frac{\left(x^2+xy+y^2\right)}{\left(x^2+xy+y^2\right)^2}$$

$$= \frac{-2}{\left(x^2+xy+y^2\right)}$$

$$= -2\left(x^2+xy+y^2\right)^{-1} \quad put\ (i)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -2u\ .$$

Hence Euler's Theorem is verified .

Q:3: **If** $u = f\left(\frac{y}{x}\right)$ **show that** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Solution: Let $u(x,y) = f\left(\frac{y}{x}\right)$

$$put\ \ x = \lambda x, y = \lambda y$$

$$=> f(\lambda x, \lambda y) = f\left(\frac{\lambda y}{\lambda x}\right)$$

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$$= \lambda^0 f\left(\frac{y}{x}\right)$$

Thus the given function is homogeneous of degree "0"

Differentiate (i) w.r.t. to x and y partially

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(f\left(\frac{y}{x}\right)\right)$$

$$\Rightarrow \frac{\partial u}{\partial x} = f'\left(\frac{y}{x}\right)\frac{\partial}{\partial x}\left(\frac{y}{x}\right)$$

$$= f'\left(\frac{y}{x}\right)\left(\frac{-y}{x^2}\right)$$

$$\Rightarrow x\frac{\partial u}{\partial x} = \frac{-y}{x} f'\left(\frac{y}{x}\right) \rightarrow (ii)$$

$$Also\ \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(f\left(\frac{y}{x}\right)\right)$$

$$\Rightarrow \frac{\partial u}{\partial y} = f'\left(\frac{y}{x}\right).\frac{1}{x}$$

$$\Rightarrow y\frac{\partial u}{\partial y} = \frac{y}{x} f'\left(\frac{y}{x}\right) \rightarrow (iii)$$

Now adding Eq (ii) and Eq (iii) we get,

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{-y}{x}\phi'\left(\frac{y}{x}\right) + \frac{y}{x}\phi'\left(\frac{y}{x}\right)$$

$$= 0 \quad Hence\ proved$$

Q.4: If $z = xyf\left(\frac{x}{y}\right)$ **, then show that** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2z$

Solution: $z = xyf\left(\frac{x}{y}\right)$ **.....(i)**

Replace x *and* y *by* λx *and* λy **in (i)**

$$z(\lambda x, \lambda y) = \lambda x.\lambda yf\left(\frac{\lambda x}{\lambda y}\right)$$

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$$\Rightarrow \; z(\lambda x,\ \lambda y) = \lambda^2 xyf\left(\frac{x}{y}\right)$$

$$= \lambda^2 z(x, y)$$

It is homogenous equation of degree 2.

Differentiate (i) w.r.t. to x and y partially.

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(xy\ f\left(\frac{x}{y}\right)\right)$$

$$\Rightarrow \; \frac{\partial z}{\partial x} = y\frac{\partial}{\partial x}\left(xf\left(\frac{x}{y}\right)\right)$$

$$= y\left[x\frac{\partial}{\partial x}f\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\frac{\partial}{\partial x}(x)\right]$$

$$\Rightarrow \; \frac{\partial z}{\partial x} = y\left[x\,f'\left(\frac{x}{y}\right)\frac{\partial}{\partial x}\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\right]$$

$$\Rightarrow \; \frac{\partial z}{\partial x} = y\left[\frac{x}{y}f'\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\right]$$

Both sides multiplied by x

$$\Rightarrow \; x\frac{\partial z}{\partial x} = xy\left[\frac{x}{y}f'\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\right] \quad \ldots\ldots\textbf{(a)}$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(xy\ f\left(\frac{x}{y}\right)\right)$$

$$\Rightarrow \; \frac{\partial z}{\partial y} = x\frac{\partial}{\partial y}\left(yf\left(\frac{x}{y}\right)\right)$$

$$= x\left[y\frac{\partial}{\partial y}f\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\frac{\partial}{\partial y}(y)\right]$$

$$\Rightarrow \; \frac{\partial z}{\partial y} = x\left[y\,f'\left(\frac{x}{y}\right)\frac{\partial}{\partial y}\left(\frac{x}{y}\right) + f\left(\frac{x}{y}\right)\right]$$

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$$\Rightarrow \frac{\partial z}{\partial y} = x\left[-\frac{x}{y}f'\left(\frac{x}{y}\right)+f\left(\frac{x}{y}\right)\right]$$

Both sides multiplied by y

$$\Rightarrow y\frac{\partial z}{\partial y} = xy\left[-\frac{x}{y}f'\left(\frac{x}{y}\right)+f\left(\frac{x}{y}\right)\right] \;\ldots\ldots\text{(b)}$$

Adding (a) and (b)

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y} = xy\left[\frac{x}{y}f'\left(\frac{x}{y}\right)+f\left(\frac{x}{y}\right)\right]+xy\left[-\frac{x}{y}f'\left(\frac{x}{y}\right)+f\left(\frac{x}{y}\right)\right]$$

$$= xy\left[\frac{x}{y}f'\left(\frac{x}{y}\right)+f\left(\frac{x}{y}\right)-\frac{x}{y}f'\left(\frac{x}{y}\right)+f\left(\frac{x}{y}\right)\right]$$

$$= 2xyf\left(\frac{x}{y}\right) \quad put(i)$$

$$= 2z$$

Hence $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = 2z$ **proved.**

<u>Q.5</u>:(a) If $u=\tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, **then prove that**

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \sin u.\cos u$$

<u>Solution</u>: *Given* $u=\tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$

$$\Rightarrow \tan u = \frac{x^2+y^2}{x+y} \quad let \;\; \tan u = f(x,y)$$

$$\Rightarrow f(x,y) = \frac{x^2+y^2}{x+y} \rightarrow (i)$$

$Put \;\; x=\lambda x \;\; \& \;\; y=\lambda y \;\; in \;\; Eq(i)$

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$$f(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + \lambda^2 y^2}{\lambda x + \lambda y}$$

$$= \frac{\lambda^2(x^2+y^2)}{\lambda(x+y)} = \lambda\left(\frac{x^2+y^2}{x+y}\right)$$

$$= \lambda f(x,y)$$

Hence given function is homogeneous of degree "1"

Now by Euler's Theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f \quad . \; put \;\; f = \tan u$$

$$\Rightarrow \quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \tan u \rightarrow (i)$$

As $f = \tan u$

$$\Rightarrow \quad \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\tan u$$

$$\frac{\partial f}{\partial x} = \sec^2 u.\frac{\partial u}{\partial x} \quad \textit{multiplied by } x$$

$$\Rightarrow \quad x\frac{\partial f}{\partial x} = x\sec^2 u.\frac{\partial u}{\partial x} \rightarrow (ii) \quad . \text{ Now}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\tan u)$$

$$\Rightarrow \quad \frac{\partial f}{\partial y} = \sec^2 u.\frac{\partial u}{\partial y} \quad \textit{multiplied by } y$$

$$\Rightarrow y\frac{\partial f}{\partial y} = y\sec^2 u.\frac{\partial u}{\partial y} \rightarrow (iii) \quad \textit{adding (ii) and (iii)}$$

$$\Rightarrow x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x\sec^2 u.\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y}$$

$$\Rightarrow \quad x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \sec^2 u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] \;.....\text{(iv)}$$

Put (i) in (iv)

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$$\tan u = \sec^2 u\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right]$$

$$\Rightarrow \frac{\tan u}{\sec^2 u} = x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin u}{\cos u}.\cos^2 u$$

$$\Rightarrow \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin u\cos u \quad \textbf{proved}.$$

(b) If $u = \sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

Solution: *Given* $u = \sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

$$\Rightarrow \quad \sin u = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$$

Let $f(x,\ y) = \sin u = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ .....(i)

put $x = \lambda x, y = \lambda y \quad in(i)$

$$f(\lambda x, \lambda y) = \frac{\sqrt{\lambda x}-\sqrt{\lambda x}}{\sqrt{\lambda x}+\sqrt{\lambda x}} = \frac{\lambda^{\frac{1}{2}}\left(\sqrt{x}-\sqrt{x}\right)}{\lambda^{\frac{1}{2}}\left(\sqrt{x}+\sqrt{x}\right)}$$

$$= \lambda^0 f(x,y)$$

Thus the function is homogeneous of degree "0"

By Euler's Theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 0.f = 0 \rightarrow (ii)$$

As $f = \sin u \qquad \Rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\sin u$

$\frac{\partial f}{\partial x} = \cos u.\frac{\partial u}{\partial x}$ *multiplied by* x

$$\Rightarrow x\frac{\partial f}{\partial x} = x\cos u.\frac{\partial u}{\partial x} \;....(a) \quad Now \quad \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\sin u)$$

$$\frac{\partial f}{\partial y} = \cos u.\frac{\partial u}{\partial y} \quad \textit{multiplied by } y$$

$$\Rightarrow y\frac{\partial f}{\partial y} = y\cos u.\frac{\partial u}{\partial y} \;....(b)$$ **Adding (a) and (b)**

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x\cos u.\frac{\partial u}{\partial x} + y\cos u.\frac{\partial u}{\partial y} \quad .....(iii)$$

Put (ii) in (iii)

$$0 = x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y}$$

$$\Rightarrow \cos u\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = 0 \quad \text{if} \quad \cos u \neq 0$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0 \quad \textit{Hence proved}.$$

Q.6: Use MAPLE command "diff" to find the partial derivation of

(a) $f(x,y) = \dfrac{x^3+y^3}{x^2+y^2}$, $w.r.t.$ "y"

(b) $f(x,y) = -\dfrac{\sin^2(x)}{y} + x\cos(y)$, $w.r.t.$ "y"

(c) $f(x,y) = \dfrac{1}{y}\tan(x) - x\cot^2(y)$, $w.r.t.$ "x"

(d) $f(s,t) = \dfrac{4t^2+3s^2}{s-t}$, $w.r.t.$ "s"

Solution: (a) Open the MAPLE programme . Open the new document and use key board keys and write $diff(x\hat{}3 + y\hat{}3/x\hat{}2 + y\hat{}2\ , y);$ **then press enter key you get result .**

Command:

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> $diff\left(\frac{x^3+y^3}{x^2+y^2}, y\right);$ > $\frac{3y^2}{x^2+y^2} - \frac{2(x^3+y^3)y}{(x^2+y^2)^2}$

(b) Open the MAPLE programme . Open the new document and use key board keys and write $diff(-\sin\text{^}2(x)/y + x*\cos(y), y);$ **then press enter key you get result .**

Command:

> $diff\left(-\frac{\sin^2(x)}{y} + x\cdot\cos(y), y\right);$

$$\frac{\sin(x)^2}{y^2} - x\sin(y)$$

(c) Open the MAPLE programme . Open the new document and use key board keys and write $diff\left(\frac{1}{y}*\tan(x) - x*\cot\text{^}2(y), x\right);$ **then press enter key you get result .**

Command:

> $diff\left(\frac{1}{y}\cdot\tan(x) - x\cdot\cot^2(y), x\right);$

$$\frac{1+\tan(x)^2}{y} - \cot(y)^2$$

(d) Open the MAPLE programme . Open the new document and use key board keys and write $diff(4*t\text{^}3+3*s\text{^}2/s-t, s);$ **then press enter key you get result .**

Command:

> $diff\left(\frac{4\cdot t^3+3\cdot s^2}{s-t}, s\right);$

$$\frac{6s}{s-t} - \frac{4t^3+3s^2}{(s-t)^2}$$

Review Exercise 11.

Q.1: Choose the correct option.

(i) **In the function** $z = f(x, y)$, x *and* y **are :**

(a) alphanumeric variables (b) dependent variables
(c) independent variables (d) dependent constants

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(ii) If $f(x, y)=\sqrt{x^2+y^2-1}$ then $f(1,5)$ is

(a) 25 (b) 5 (c) 125 (d) $3\sqrt{3}$

(iii) $f(x, y)=x^2+y^2$ then f_x or $\frac{\partial f}{\partial x}$ is :

(a) $2y$ (b) $2x$ (c) $2x+2y$ (d) $2x+y^2$

(iv) $diff\left(\frac{1}{x+y}, y\right)$; is

(a) $\frac{1}{x^2}$ (b) $-\frac{1}{y^2}$ (c) $-\frac{1}{(x+y)^2}$ (d) $\frac{1}{(x+y)^2}$

(v) The Euler's Theorem stated that :

(a) $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=nz$ (b) $x\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=nz$

(c) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$ (d) $-x\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=nz$

(vi) The functions of more than independent variables are called (a) monomial functions (b) polynomial functions (c) Multivariate functions (d) non of these

(vii) If $f(x,y,z)=\sqrt{x^2+y^2-z}$, then $f(1, 0, 1)$ is :

(a) 1 (b) -1 (c) 0 (d) 2

(viii) $f(x, y)=\cos y.e^{3x}$ then f_x is

(a) $3\cos ye^{3x}-\sin y$ (b) $3\cos ye^{3x}$

(c) $-3e^{x}.\cos x$ (d) $8\cos ye^{3x}$

(ix) $f(x, y)=3xe^{y}+x^3y^2$, then f_y is

(a) $3xe^{y}+2x^3y$ (b) $3xe^{y}+2y^3x^2$

(c) $3xe^{y}-2x^3y$ (d) $3x^2y^2+3e^{y}$

<u>Answers</u>: (i) c (ii) b (iii) b (iv) c (v) c (vi) c (vii) c (viii) b (ix) a .

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