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Solved Exercise 1.3

Q1 Evaluate each limit by using theorem of limits.

Solution

i. $\lim_{x\to 3}(2x+4)$

$= \lim_{x\to 3}(2x) + \lim_{x\to 3} 4$

$= 2\lim_{x\to 3}(x) + \lim_{x\to 3} 4$

$= 2(3) + 4$

$= 6 + 4 = 10$

ii. $\lim_{x\to 1}(3x^2 - 2x + 4)$

$= \lim_{x\to 1}(3x^2) + \lim_{x\to 1}(2x) + \lim_{x\to 1}(4)$

$= 3\lim_{x\to 1}(x^2) + 2\lim_{x\to 1}(x) + \lim_{x\to 1}(4)$

$= 3(1)^2 - 2 + 4$

$= 3 - 2 + 4$

$= 5$

iii. $\lim_{x\to 3}\sqrt{x^2 + x + 4}$

$= \lim_{x\to 3}(x^2) + \lim_{x\to 3}(x) + \lim_{x\to 3}(4)$

$= (3)^2 + 3 + 4$

$= (16)^{\frac{1}{2}}$

$= 4$

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iv. $\lim_{x\to 2} x\sqrt{x^2-4}$

$= (\lim_{x\to 2} x)(\lim_{x\to 2} \sqrt{x^2-4})$

$= (2)(\sqrt{4-4})$

$= (2)(\sqrt{0})$

$= 0$

v. $\lim_{x\to 2}(\sqrt{x^3+1}-\sqrt{x^2+5})$

$= \lim_{x\to 2}(x^3+1)^{\frac{1}{2}} - \lim_{x\to 2}(x^2+5)^{\frac{1}{2}}$

$= (\sqrt{2^3})+1-(\sqrt{2^2})+5$

$= \sqrt{9}-\sqrt{9} = 0$

vi. $\lim_{x\to -2} \frac{2x^3+5x}{3x-2}$

$= \frac{\lim_{x\to 2} 2x^3+5x}{\lim_{x\to 2} 3x-2} = \frac{2\lim_{x\to 2} x^3+5\lim_{x\to 2} x}{3\lim_{x\to 2} x-\lim_{x\to 2} 2}$

$= \frac{2(-2)^3+(5(-2))}{3(-2)-2}$

$= \frac{-16-10}{-6-2}$

$= \frac{-26}{-8} = \frac{13}{4}$

Q2 Evaluate each limit by using algebraic techniques.

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i. $\lim_{x\to -1} \frac{x^3-x}{x+1}$

$$= \lim_{x\to -1} \frac{x(x^3-x)}{x+1}$$

$$= \lim_{x\to -1} \frac{x(x^2-1)}{x+1}$$

$$= \lim_{x\to -1} \frac{x(x-1)(x+1)}{x+1}$$

$$= \lim_{x\to -1} x(x-1)$$

$$= (-1)(-1-1)$$

$$= 2$$

ii. $\lim_{x\to 1} \frac{3x^3+4x}{x^2+x}$

$$= \frac{3(1)^3+4(1)}{(1)^2+(1)}$$

$$= \frac{3+4}{1+1}$$

$$= \frac{7}{2}$$ (Book answer is 4)

iii. $\lim_{x\to 2} \frac{x^3-8}{x^2-x-6}$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{x^2-3x+2x-6}$$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{x(x-3)+2(x-3)}$$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x+2)(x-3)}$$

$$= \lim_{x\to 2} \frac{(2-2)(2^2+(2)(2)+4)}{(2+2)(2-3)}$$

$$= 0$$

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iv. $\lim_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}$

$$= \lim_{x\to 1}\frac{(x^3-1)-3x^2+3x}{x(x^2-1)}$$

$$= \lim_{x\to 1}\frac{(x-1)(x^2+x+1)-3x(x-1)}{x(x-1)(x+1)}$$

$$= \lim_{x\to 1}\frac{(x-1)(x^2+x+1-3x)}{x(x-1)(x+1)}$$

$$= \lim_{x\to 1}\frac{(x^2-2x+1)}{x(x+1)}$$

$$= \lim_{x\to 1}\frac{(x-1)^2}{x(x+1)}$$

$$= \frac{(1-1)^2}{1(1+1)} = \frac{0}{2} = 0$$

v. $\lim_{x\to -1}(\frac{x^3+x^2}{x^2-1})$

$$= \lim_{x\to -1}(\frac{x^2(x+1)}{(x+1)(x-1)})$$

$$= \lim_{x\to -1}(\frac{x^2}{(x-1)})$$

$$=(\frac{(-1)^2}{(-1-1)})$$

$$=\frac{(-1)^2}{(-2)}$$

$$=\frac{-1}{2}$$

vi. $\lim_{x\to 4}\frac{2x^2-32}{x^3-4x^2}$

$$= \lim_{x\to 4}\frac{2(x^2-16)}{x^2(x-4)}$$

$$= \lim_{x\to 4}\frac{2(x^2-16)}{x^2(x-4)}$$

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$$= \lim_{x\to 4} \frac{2(x-4)(x+4)}{x^2(x-4)}$$

$$= \lim_{x\to 4} \frac{2(x+4)}{x^2}$$

$$= \frac{2(4+4)}{4*4}$$

$$= \frac{16}{16} = 1$$

vii. $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$

$$= \lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{(\sqrt{x})^2-(\sqrt{2})^2}$$

$$= \lim_{x\to 2} \frac{1}{\sqrt{x}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

viii. $\lim_{x\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$$= \lim_{x\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} * \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{x\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{x\to 0} \frac{1}{(\sqrt{x+h})} + \sqrt{x}$$

$$= \frac{1}{(\sqrt{0+h})} + \sqrt{0}$$

$$= \frac{1}{(\sqrt{h})}$$

ix. $\lim_{x\to a} \frac{x^n-a^n}{x^m-a^m}$

Let $x = a + h$

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$$x = a$$

$$a = a + h$$

$$h = 0$$

$$= \lim_{x \to a} \frac{(a+h)^n - a^n}{(a+b)^m - a^m}$$

$$= \lim_{x \to a} \frac{a^n \left[\left(1 + \frac{h}{a}\right)^n - 1\right]}{a^m \left[\left(1 + \frac{b}{a}\right)^m - 1\right]}$$

Using binomial series

$$= \lim_{h \to 0} \frac{a^n \left[1 + \frac{n}{1}\left(\frac{b}{a}\right)^1 + \frac{n(n-1)}{2}\left(\frac{b}{a}\right)^c + \cdots - 1\right]}{a^m \left[1 + \frac{m}{1}\left(\frac{b}{a}\right)^1 + \frac{m(m-1)}{2}\left(\frac{b}{a}\right)^c + \cdots - 1\right]}$$

$$= \lim_{h \to 0} \frac{a^n \left[n\left(\frac{n}{a}\right) + \frac{(n-1)b^2}{2\ a^2} + m\right]}{a^m \left[\left(\frac{n}{a}\right) + \frac{m(m-1)b^2}{2\ a^2} + m\right]}$$

Taking h common

$$= \lim_{h \to 0} \frac{a^n \left[\left(\frac{n}{a}\right) + \frac{n(n-1)\,b}{2\ ac} + \right]}{a^m \left[\left(\frac{m}{a}\right) + \frac{m(m-1)\,b}{2\ a^2} + \right]}$$

$$= \frac{a^n \left[\left(\frac{n}{a}\right) + 0\right]}{a^m \left[\left(\frac{m}{a}\right) + 0\right]}$$

$$= \frac{a^n . \frac{n}{a}}{a^m . \frac{m}{a}}$$

$$= \frac{n}{m} . a^{n-m}$$

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Q3 Evaluate the following limits.

i. $\lim_{h\to 0}\frac{sin7x}{x}$

$= \lim_{h\to 0}\frac{sin7x}{\frac{7x}{7}}$

$= 7\lim_{h\to 0}\frac{sin7x}{7x}$

$= 7(1)$

$= 7$

ii. $\lim_{x\to 0}\frac{sinx^0}{x}$

As $180^0 = \pi rad$

$1^0 = \frac{\pi}{180^0} rad$

$x^0 = x\frac{\pi}{180^0}$ put in above

$= \lim_{x\to 0}\frac{sinx\frac{\pi}{180^0}}{x}.\frac{\frac{\pi}{180^0}}{\frac{\pi}{180^0}}$

$= \lim_{x\to 0}\frac{sin\frac{x\pi}{180^0}}{\frac{x\pi}{180^0}}.\frac{\pi}{180^0}$

$= (1).\frac{\pi}{180^0}$

$= \frac{\pi}{180^0}$

iii. $\lim_{x\to 0}\frac{1-cos\theta}{sin\theta}$

$= \lim_{x\to 0}\frac{1-cos\theta}{sin\theta}.\frac{1+cos\theta}{1+cos\theta}$

$= \lim_{x\to 0}\frac{1-cos^2\theta}{sin\theta(1+cos\theta)}$

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$$= \lim_{x\to 0} \frac{sin^2\theta}{sin\theta(1+cos\theta)}$$

$$= \lim_{x\to 0} \frac{sin\theta}{(1+cos\theta)}$$

$$= \frac{0}{1+1} = \frac{0}{2} = 0$$

iv. $\lim_{x\to\pi} \frac{sin\theta}{\pi - x}$

Let $x = \pi - \theta$

As $x \to \pi, \theta \to 0$

$$= \lim_{\theta\to 0} \frac{\sin(\pi)-\theta}{\pi-(\pi-\theta)}$$

But $\sin(\pi - \theta)$

$$= sin\theta$$

$$= \lim_{\theta\to 0} \frac{sin\theta}{\theta}$$

$$= 1$$

v. $\lim_{x\to 0} \frac{\sin ax}{\sin bx}$

$$= \lim_{x\to 0} \left[\left(\frac{bx}{\sin bx}\right)\left(\frac{\sin ax}{ax}\right)\left(\frac{ax}{bx}\right)\right]$$

$$= \lim_{x\to 0} \left[\left(\frac{\sin ax}{ax}\right)\right] . \lim_{x\to 0} \left[\left(\frac{1}{\frac{\sin bx}{bx}}\right)\left(\frac{a}{b}\right)\right]$$

$$= \left(\frac{a}{b}\right)(1) - \frac{1}{1}$$

$$= \left(\frac{a}{b}\right)$$

vi. $\lim_{x\to 0} \frac{x}{\tan x}$

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$$= \lim_{x\to 0} \frac{cos\ x.x}{sin\ x}$$

$$= \lim_{x\to 0} \frac{cos\ x}{\frac{sin\ x}{x}} = \lim_{x\to 0} \frac{cos\ x.x}{sin\ x}$$

$$= (1)cos\ 0^0$$

$$= (1)(1)$$

$$= 1$$

vii. $\lim_{x\to 0} \frac{1-cos\ 2x}{x^2}$

$$= \lim_{x\to 0} \frac{1-(2cos^2\ x-1)}{x^2}, \qquad cos\ 2x = 2cos^2\ x - 1$$

$$= \lim_{x\to 0} \frac{(2cos^2\ x)}{x^2}$$

$$= \lim_{x\to 0} \frac{(sin^2\ x)}{x^2}$$

$$= 2\lim_{x\to 0} \frac{sin\ x}{x}$$

$$= 2(1)$$

$$= 2$$

viii. $\lim_{x\to 0} \frac{1-cos\ x}{sin^2 x}$

$$= \lim_{x\to 0} \frac{1-cos\ x}{sin^2 x} \cdot \frac{1+cos\ x}{1+cos\ x}$$

$$= \lim_{x\to 0} \frac{sin^2\ x}{sin^2 x(1+cos\ x)}$$

$$= \lim_{x\to 0} \frac{1+cos\ x}{1+cos\ x}$$

$$= \lim_{x\to 0} \frac{1}{1+cos\ x}$$

$$= \frac{1}{1+cos\ 0}$$

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$= \frac{1}{1+1} = \frac{1}{2}$

ix. $\lim_{\theta \to 0} \frac{sin^2\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{sin^2\theta}{\theta} . \theta$

$= \lim_{\theta \to 0} \frac{sin^2\theta}{\theta} . \lim_{\theta \to 0}(0)$

$= (1^2)(0) = 0$

x. $\lim_{x \to 0} \frac{sec\, x - cos\, x}{x}$

$= \lim_{x \to 0} \frac{sec\, x - cos\, x}{x}$

$= \lim_{x \to 0} \frac{1 - cos^2 x}{\frac{cos\, x}{x}}$

$= \lim_{x \to 0} \frac{1 - cos^2 x}{x.cos\, x}$

$= \lim_{x \to 0} \frac{sin^2 x}{x.cos\, x}$

$= \lim_{x \to 0} \frac{sin\, x}{x} . tan\, x$

$= \lim_{x \to 0} \frac{sin\, x}{x} \lim_{x \to 0} tan\, x$

$= (1)(0) = 0$

xi. $\lim_{\theta \to 0} \frac{1 - cosp\theta}{1 - cosq\theta}$

$= \lim_{\theta \to 0} \frac{2sin^2 \frac{p\theta}{2}}{2sin^2 \frac{q\theta}{2}}$

$= \lim_{\theta \to 0} sin\, \frac{2pq}{2} . \frac{1}{sin\theta} . (\frac{q\theta}{2})^2$

$= \lim_{\theta \to 0} (\frac{sin \frac{p\theta}{2}}{\frac{p\theta}{2}}) \lim_{\theta \to 0} (\frac{\frac{1}{sin \frac{q\theta}{2}}}{\frac{q\theta}{2}}) \frac{\frac{p^2}{4}}{\frac{q^2}{4}}$

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$$= 1.1\frac{p^2}{q^2}$$

$$= \frac{p^2}{q^2}$$

xii. $\lim_{\theta\to 0}\frac{tan\theta - sin\theta}{sin^3\theta}$

$$= \lim_{\theta\to 0}\frac{sin\theta - sin\theta cos\theta}{\frac{cos\theta}{sin^3\theta}}$$

$$= \lim_{\theta\to 0}\frac{sin\theta(\frac{1\ cos\theta}{cos\theta})}{sin^3\theta}$$

$$= \lim_{\theta\to 0}\frac{1-cos\theta}{sin^2\theta - cos\theta}$$

$$= \lim_{\theta\to 0}\frac{1-cos\theta}{(1+cos\theta)(1-cos\theta).cos\theta}$$

$$= \lim_{\theta\to 0}\frac{cos\theta}{(1+cos\theta)}$$

$$= \frac{cos\theta}{(1+cos\theta)}$$

$$= \frac{cos0.cos0}{(1+cos\theta)}$$

$$= \frac{1}{1+1}.1$$

$$= \frac{1}{2}$$

Q4 Express each limit in terms of e.

i. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$

$$= \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$$

$$= \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right)^2 \qquad \left(1+\frac{1}{n}\right)^{n} = e$$

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$= e^2$

ii. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$

$= \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{\frac{n}{2}}$

$= \lim_{n\to\infty}(\left(1+\frac{1}{n}\right)^{n})^{\frac{1}{2}}$ $\qquad \left(1+\frac{1}{n}\right)^{n} = e$

$= e^{\frac{1}{2}}$

iii. $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n}$

$= \lim_{n\to\infty}\left(1+(-\frac{1}{n})\right)^{-(-n)}$

$= \lim_{n\to\infty}\left(1+(-\frac{1}{n})^{-n}\right)^{(-1)}$

$= \lim_{n\to\infty} e^{(-1)}$

$= e^{(-1)}$

iv. $\lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{n}$

$= \lim_{n\to\infty}\left(1+\frac{1}{3n}\right)^{3.\frac{n}{3}}$

$= \lim_{n\to\infty}\left((1+\frac{1}{3n})^{3n}\right)^{\frac{1}{3}}$

$= e^{\frac{1}{3}}$

v. $\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{n}$

$= \lim_{n\to\infty}\left(\left(1+\frac{4}{n}\right)^{\frac{n}{4}}\right)^{4}$

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$= e^4$

vi. $\lim_{x\to 0}(1+3x)^{\frac{2}{x}}$

$= \lim_{x\to 0}(1+3x)^{\frac{2}{3x}\cdot 3}$

$= \lim_{x\to 0}((1+3x)^{\frac{1}{3x}})^6$

$= e^6$

vii. $\lim_{x\to 0}(1+2x^2)^{\frac{1}{x^2}}$

$= \lim_{x\to 0}((1+2x^2)^{\frac{1}{2x^2}})^2$

$= e^2$

viii. $\lim_{h\to 0}(1-2h)^{\frac{1}{h}}$

$= \lim_{h\to 0}(1+(-2h))^{\frac{1}{2h}(-2)}$

$= e^{-2}$

ix. $\lim_{x\to 0}(\frac{x}{1+x})^x$

$= \lim_{x\to 0}(\frac{1+x}{x})^{-x}$

$= \lim_{x\to 0}(1+\frac{1+x}{x})^{-x}$

$= \lim_{x\to 0}((1+\frac{1}{x})^x)^{-1}$

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x. $\lim_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, x<0$

Let $y=\frac{1}{x}$

As $x\to 0, y\to\infty$

$$=\lim_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

$$=\lim_{y\to 0}\frac{e^{y}-1}{e^{y}+1}$$

$$=\frac{0-1}{0+1}$$

$$=-1$$

xi. $\lim_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, x>0$

Let $y=\frac{1}{x}$

As $x\to 0, y\to\infty$

$$=\lim_{x\to 0}\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

$$=\lim_{x\to 0}\frac{1-\frac{1}{e^{y}}}{1+\frac{1}{e^{y}}}$$

$$=\frac{1-0}{1+0}$$

$$=1$$

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