CHAPTER

# 15 A.C CIRCUIT

**Q1: Write a note on (a) direct current (b) alternating current**

**Answer: a) Direct Current [D·C]:**

**Definition:** "The current which flows in one direction is known as direct current."

**Explanation:**

In direct current electricity, the electrons flow continuously in one direction from the source of power through the conductor to a load and then back to the source of power. The voltage in direct current remains constant. The D·C power sources include batteries and D·C generators.

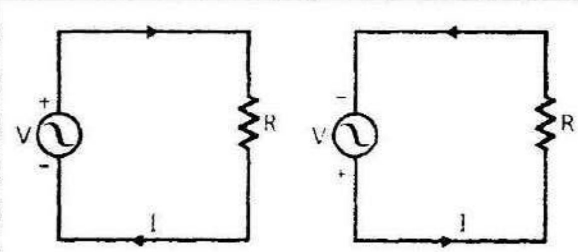
**b) Alternating Current (A·C):**

**Definition:** "The current which changes its direction many times in a second is known as alternating current (A·C)."

**Explanation:**

In alternating current, an A·C generator is used to make the flow of electrons first in one direction and then in the opposite direction. A source which produces potential difference of changing polarity with time is known as alternating source.

A voltage which changes its polarity at regular interval of time is known as alternating voltage. When an alternating voltage is applied in a circuit, the current flows first in one direction and then in the opposite direction. The direction of current at any instant depends upon the




(i) Current in one direction

(ii) Current in opposite direction

polarity of the applied voltage.

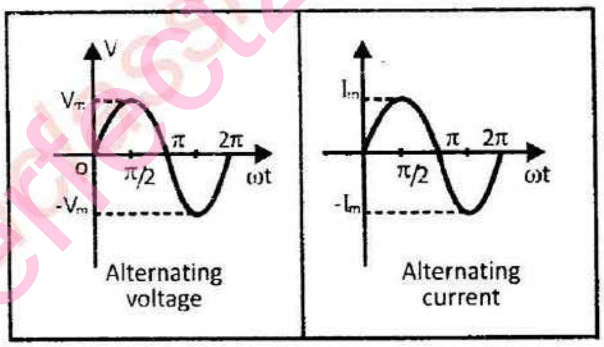
The graph of alternating voltage gives us a distinct shape which is known as sine wave. So the alternating voltage is also known as sinusoidal alternating voltage and is given by,

$$\boxed{V = V_m \sin(\omega t)} \text{ \_\_\_\_\_\_\_\_\_\_ } (1)$$

In equation (1), "$V$" represents the instantaneous value of alternating voltage, "$V_m$" represents the maximum value of alternating voltage and "$\omega$" represents the angular frequency measured in "rad/sec". Similarly, the sinusoidal alternating current at any instant is given by,

$$\boxed{I = I_m \sin(\omega t)} \text{ \_\_\_\_\_\_\_\_\_\_ } (2)$$

The graph of alternating current and voltage shows that they varies both in magnitudes as well as in directions.



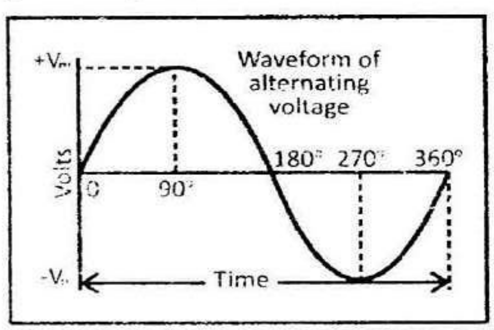

They rises from zero to maximum positive value, then falls to zero again and then increases to a maximum value in the reverse direction and then falls to zero again, as shown in the figure.

**Q2: Define the following terms: Instantaneous value, cycle, time period and frequency**

**Answer: a) Instantaneous Value:**

"The value of an alternating quantity at any instant is known as instantaneous value."

The instantaneous values of alternating voltage and current are represented by "$V$" and "$I$" respectively. For example, in given figure, O, $+V_m$, $-V_m$ represents the instantaneous values of alternating volt-



age of $0^\circ$, $90^\circ$ and $270^\circ$ respectively.

**b) Cycle:**

"One complete set of positive and negative values of an alternating quantity is known as cycle." For example, the above figure represents one cycle of alternating voltage. There are "360 degree" of "$2\pi$ radian" in one cycle.

**c) Cycle:**

"The time during which one cycle is complete is known as time period." It is represented by "T" and is measured in second.

**d) Frequency:**

"The number of cycles in one second is known as frequency of an alternating quantity." (OR)

"The reciprocal of time period is known as frequency." It is represented by "f".

$$f = \frac{1}{T}$$

The unit of frequency is "cycle/sec" or "hertz".

$$\boxed{1\ \text{hertz (Hz)} = 1\text{cycle / sec (c / s)}}$$

**Q3: Define the mean, peak and rms values of sinusoidal current and voltage. Obtain the mathematical expressions for the rms value of current and voltage.**

Answer: **a) Peak Values:**

**Definition:** "The maximum value of sinusoidal voltage or sinusoidal current in one cycle is known as peak value."

**Explanation:** The peak value of a sine wave occurs twice each cycle i.e. once at the positive maximum value and once at the negative maximum value. The peak value plays an important role in testing of materials.

The peak value of an alternating voltage and current are represented by "$V_m$" and "$I_m$" respectively.

**b) Mean Value:**

**Definition:** "The sum of the entire positive and negative values of sinusoidal voltage or current in one time period divided by the time period known as mean value."

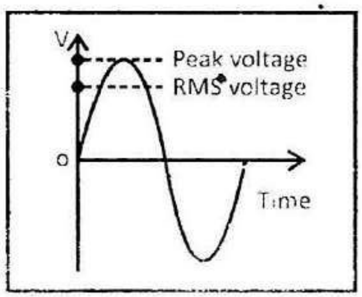
**Explanation:** The sum of all the positive values in one cycle is the area of the upper lobe and the sum of all the negative values is the area of the lower lobe. So the sum taken over one cycle is zero.

### c) <u>Root-Mean-Square (R.M.S) Value (or) Effective Value:</u>

**<u>Definition:</u>** "The square root of mean square values of sinusoidal voltage or current is known as their r.m.s values."

**<u>Explanation:</u>** As the mean value of sinusoidal voltage and current over one cycle is zero, so we cannot use the mean value for power calculation. Therefore, we use another suitable method to measure the effectiveness of an alternating current. The equivalent average value for an alternating current system that provides the same power to the load as a D.C equivalent circuit is known as the "effective value."

This effective power in an alternating current system is equal to "$I^2R$ Average". As power is proportional to the square of current, so the effective current "$I$" will be equal to "$\sqrt{I^2}$ average". Therefore, the effective current in an AC system is known as the root-mean-squared (R.M.S) value.




The effective or r.m.s value of an alternating current is that steady current (D.C) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.

For example, if the effective or r.m.s value of an alternating current is 7A, then the alternating current will produce the same heating effect as that produced by 7A direct current.

**Q4: Derive an expression for r.m.s current and r.m.s voltage. (ETEA 2010)**

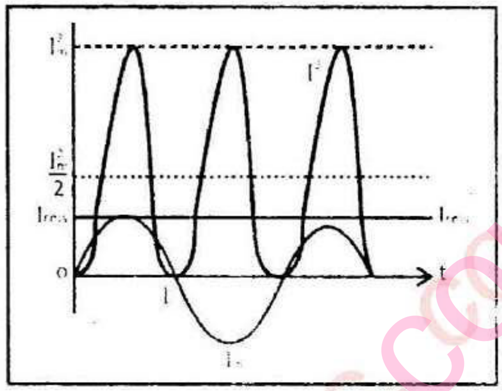
<u>Answer:</u>

The effective value or r.m.s value can be used to calculate the magnitude of alternating current or voltage. The equation for sinusoidal alternating current is given by,

$$I = I_0 \sin(\omega t) \quad \text{\_\_\_\_\_\_} (1)$$

Now if this current is allowed to pass through a resistor "R", then power delivered at any instant is given by,

$$P = I^2R \_\_\_\_\_\_\_\_ (2)$$



Putting equation (1) in equation (2), we get,

$$P = I_m^2 \sin^2(\omega t)R \_\_\_\_\_\_ (3)$$

Now as the value of $\sin^2(\omega t)$ varies between 0 and 1, so its average value is "1/2", thus equation (3) becomes,

$$P = I_m^2 \times \frac{1}{2} \times R$$

$$\Rightarrow P = \frac{1}{2} I_m^2 R \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (4)$$

In terms of r·m·s value, equation (2) can be written as,

$$P = I_{rms}^2 R \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ (5)$$

Comparing equation (4) and equation (5), we get,

$$I_{rms}^2 R = \frac{1}{2} I_m^2 R$$

$$\Rightarrow I_{rms}^2 = \frac{1}{2} I_m^2 \Rightarrow I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\Rightarrow I_{rms} = 0.7071\, I_m$$

$$\Rightarrow \boxed{I_{rms} = 0.707\, I_m}$$

Similarly, the r·m·s value of voltage is given by,

$$\boxed{V_{rms} = 0.707\, V_m}$$

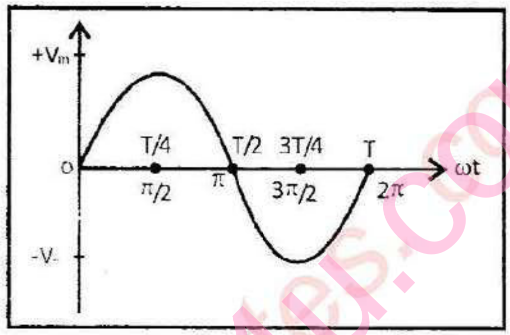
**Note:** In Pakistan, the AC mains supply that is available in the normal wall outlets is 50 hertz and $220V_{rms}$. So the peak value of voltage will be:

$$V_m = \frac{V_{rms}}{0.707}$$

$$\Rightarrow V_m = \frac{220}{0.707} = 310V \Rightarrow V_m = 310 \text{ volts}$$

**Q5: Explain the term "phase" and phase difference in case of A·C circuits·**

**Answer:** **Phase of A·C:** In the department of electrical engineering, we mostly deal with relative phases or phase difference between various alternating quantities, rather than with their absolute values· The word "phasor" or "phase vector" is a way to represent a sine or cosine function graphically·




Now consider an alternating voltage wave of time period "T" as shown in the figure·

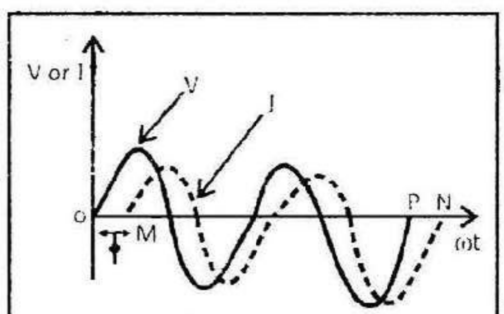
The maximum positive value "$+V_m$" occurs at "$\frac{T}{4}$" second or "$\frac{\pi}{2}$" radians· Thus the phase of maximum positive value is "$\frac{T}{4}$" second or "$\frac{\pi}{2}$" radians· Similarly, the phase of negative peak value "$-V_m$" is "$\frac{3T}{4}$" second or "$\frac{3\pi}{2}$" radians·

From the above discussion, we conclude that, phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference·

**Phase Difference:**

When two alternating quantities of the same frequency have different zero point, then they are said to have a phase difference·

In most of practical circuits, the alternating voltage and current have different phases· Thus the voltage may be passing through its zero point while the current has passed or it is yet to pass through




Phase difference of current and voltage

its zero point in the same direction. So we can say that the voltage and current have a phase difference.

The angle between zero points is called "angle of phase difference". It is represented by "$\phi$" and measured in "degrees" or "radians".

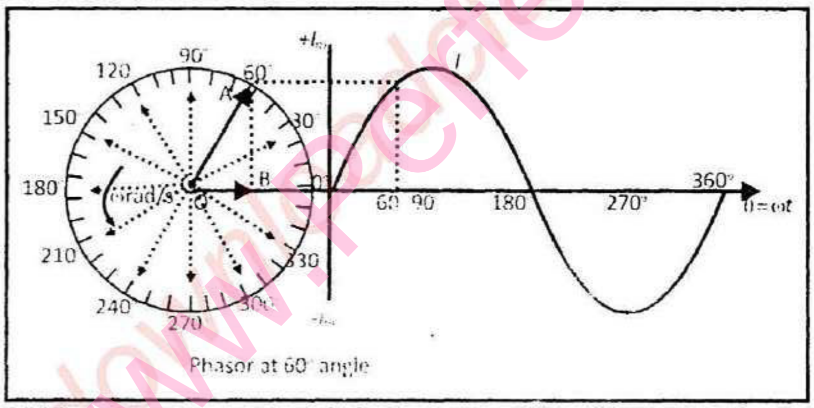
In given figure, the voltage "V" is passing through its zero point "O" and is rising in the positive direction. While the current is passing through its zero point "M" and is rising in the positive direction.

In above case, OM = $\phi$ represents the phase difference between voltage and current. The equation for voltage is given by,

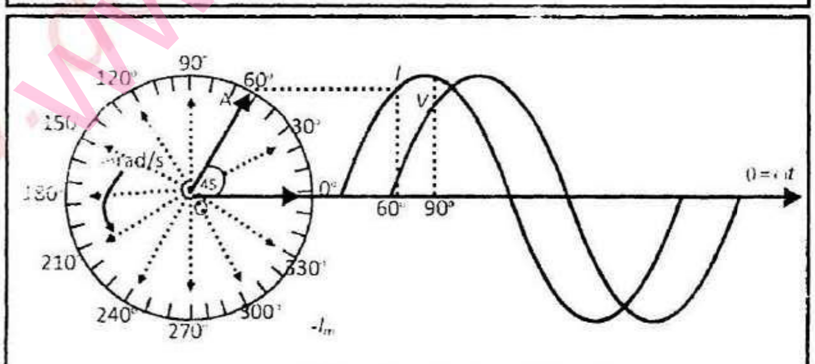
$$\boxed{V = V_m \sin(\omega t)} \qquad (1)$$

As in above case, the current lags behind the voltage by "$\phi$", so the equation for current can be written as,

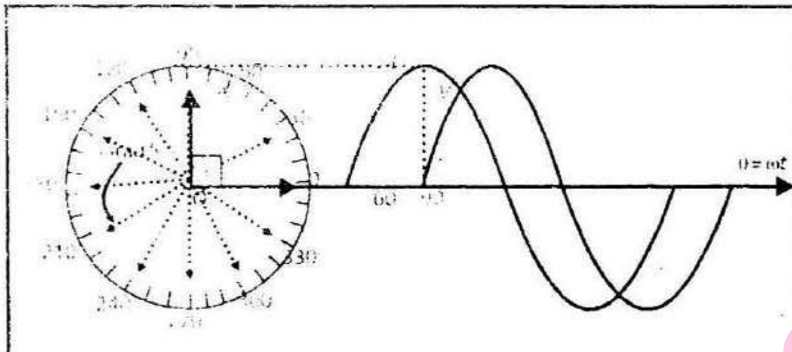
$$\boxed{I = I_m \sin(\omega t - \phi)} \qquad (2)$$



Phasor at 60° angle



(a) Wave "I" leads wave "V" by 45°

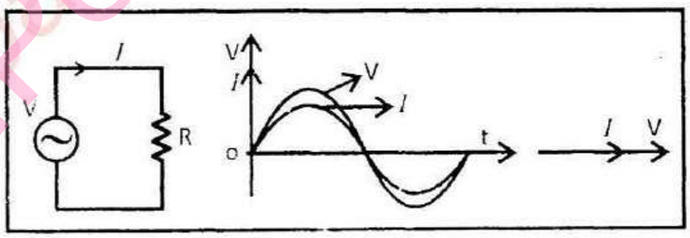



(b) Wave "I" leads wave "V" by 90

**Q6: A sinusoidal alternating voltage of angular frequency "ω" is connected across a resistor "R". Find mathematical expressions for the instantaneous voltage, current and average power dissipated per cycle of the applied voltage. (ETEA 2012)**

Answer:

Consider a circuit in which a pure resistor "R" is connected across a sinusoidal alternating voltage source, as shown in the figure. The free electrons will start flow in one direction for the first half cycle of the supply and then flow in the opposite direction during the next half-cycle. In this way, an alternating current in established in the circuit.




The applied voltage and current pass through their zero values at the same instant and attain their positive and negative peaks at the same instant. Thus the current and applied voltages are in phase, as shown in the figure.

Now the instantaneous voltage across "R" is given by,

$$\boxed{V = V_m \sin(\omega t)} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Now the current due to this voltage is given by,

$$I = \frac{V}{R} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

Putting equation (1) in equation (2), we get,

$$I = \frac{V_m \sin(\omega t)}{R} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

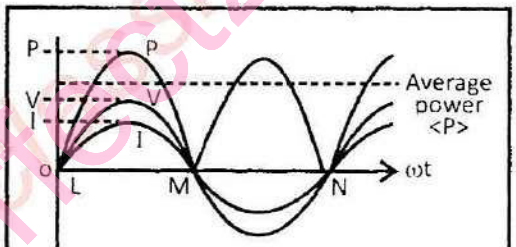
In equation (3), $\frac{V_m}{R}$ = maximum current = $I_m$ so equation (3) becomes,

$$\boxed{I = I_m \sin(\omega t)} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

Equations (1), (4) represents the expressions for instantaneous voltage and current respectively. Where "$V_m$" and "$I_m$" represents the peak value of alternating voltage and current respectively.

**Power Loss in a Resistance Circuit:**

We can obtain the power curve for a pure resistive circuit from the product of corresponding instantaneous values of voltage and current [P = IV]. The maximum and average value of power is shown in the figure.



Power curve

The power becomes zero for a moment at points L, M and N as shown in the figure. In order to get a certain value of power loss in a resistive circuit, we find an expression for average power in terms of r.m.s values. We know that,

$$P = IV \Rightarrow \langle P \rangle = \langle IV \rangle \quad \text{\_\_\_\_\_\_\_\_\_\_(5)}$$

Putting equation (1), (4) in equation (5), we get,

$$\langle P \rangle = \langle I_m \sin(\omega t) \cdot V_m \sin(\omega t) \rangle$$

$$\Rightarrow \langle P \rangle = I_m V_m \langle \sin^2(\omega t) \rangle$$

$$\Rightarrow \langle P \rangle = I_m V_m \times \frac{1}{2}$$

$$\Rightarrow \langle P \rangle = I_m V_m \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$\Rightarrow \langle P \rangle = \frac{I_m}{\sqrt{2}} \times \frac{V_m}{\sqrt{2}}$$

$$\langle \sin^2(\omega t) \rangle = \frac{1}{2}$$

$$\frac{1}{2} = \frac{\sqrt{1}}{\sqrt{2}} \times \frac{\sqrt{1}}{\sqrt{2}}$$

$$\frac{I_m}{\sqrt{2}} = I_{rms}$$

$$\frac{V_m}{\sqrt{2}} = V_{rms}$$

$$\Rightarrow \boxed{\langle P \rangle = I_{rms} V_{rms}}$$

Equation (8) represents the average power dissipated in resistor "R" during one complete cycle.

**Q7: A sinusoidal A.C voltage of frequency "ω" is connected across an inductor of inductance "L". Find the mathematical expressions for the instantaneous voltage, current and average power dissipated per cycle of the applied voltage. (ETEA 2015)**

Answer:

An inductor is a coil shaped conductor which resists changes in electric current passing through it.

Now consider an inductor of inductance "L" which is connected to a source of alternating voltage, as shown in the figure.

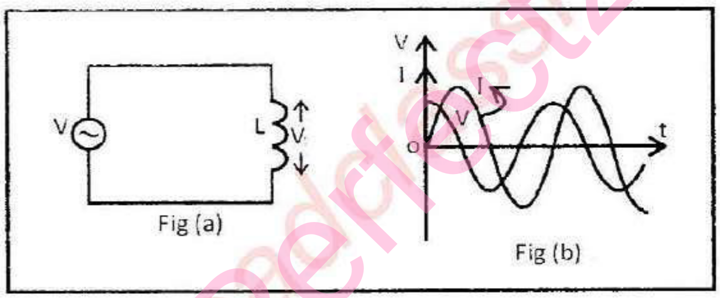


Fig (a) Fig (b)

When the circuit is turned-ON, then alternating current flows in the circuit, which is given by,

$$\boxed{I = I_m \sin(\omega t)} \quad \text{———} (1)$$

The changing current sets up a back emf in the coil whose magnitude is given by,

$$\varepsilon = L\frac{\Delta I}{\Delta t} \quad \text{———} (2)$$

Now to maintain a constant current in the circuit, the applied voltage must be equal to the back emf, so we have,

$$V = \varepsilon = L\frac{\Delta I}{\Delta t}$$

$$\Rightarrow V = L\frac{\Delta I}{\Delta t} \quad \text{———} (3)$$

$$\sin\frac{\pi}{2} = \sin 90^\circ = 1$$

$$\cos\frac{\pi}{2} = \cos 90^\circ = 0$$

Putting equation (1) in equation (3), we get,

$$V = L\frac{\Delta}{\Delta t}\left[I_m \sin(\omega t)\right]$$

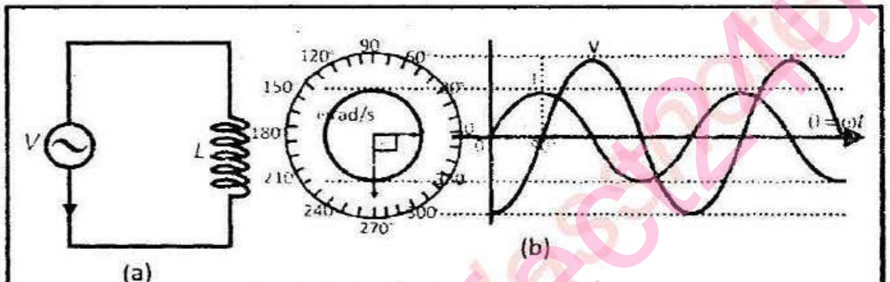
$$\Rightarrow V = LI_m\omega\cos(\omega t) \_\_\_\_\_\_\_\_\_\_ (4)$$

Here $LI_m\omega = V_m$, so equation (4) becomes,

$$\boxed{V = V_m\cos(\omega t)} \_\_\_\_\_\_\_\_\_\_ (5)$$

Equation (5) can be also written as,

$$V = V_m \sin\left(\omega t + \frac{\pi}{2}\right) \_\_\_\_\_\_\_\_\_\_ (6)$$



Inductor in AC circuit

**Formula:**

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

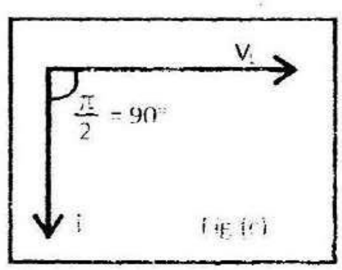
$$\sin\left(\omega t + \frac{\pi}{2}\right) = \sin\omega t\cdot\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cdot\cos\omega t$$

$$\Rightarrow \sin\left(\omega t + \frac{\pi}{2}\right) = \sin\omega t \times 0 + 1 \times \cos\omega t$$

$$\Rightarrow \sin\left(\omega t + \frac{\pi}{2}\right) = 0 + \cos\omega t$$

$$\Rightarrow \sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t$$

The waveform of current and voltage shows that the voltage passes from non zero point while the current passes from zero point at that instant. So the current and voltage are not in phase. Equation (6) shows that voltage leads the current by $\frac{\pi}{2}$. The phasor diagram of



voltage and current in inductive circuit is shown in figure (c).

The inductance of the coil opposes the change in current and serves to delay the increase or decrease of current in the circuit. Thus the current lags behind the voltage by $90^\circ$ or $\frac{\pi}{2}$ radians.

## Inductive Reactance:

"The opposition offered by an inductor to the flow of alternating current is known as inductive reactance of an inductor."

We know that,

$$V_m = I_m R \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} (7)$$

In case of inductor, we replace "R" by "$X_L$", so equation (7)

$$V_m = I_m X_L$$

$$\Rightarrow X_L = \frac{V_m}{I_m} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} (8)$$

We also know that,

$$V_m = I_m \omega L \text{ so equation (8)}$$

$$X_L = \frac{I_m \omega L}{I_m}$$

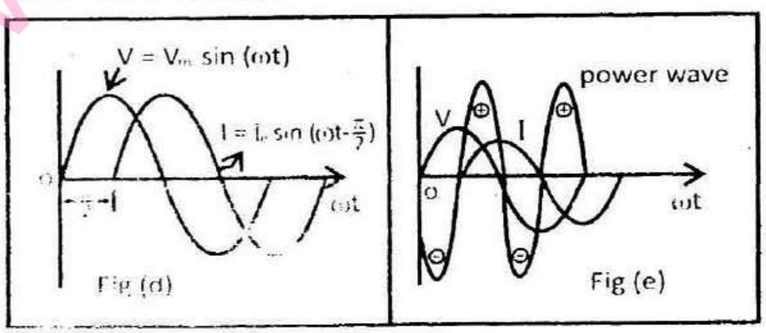
$$\Rightarrow X_L = \omega L \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} (9)$$

As $f = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f$, so equation (9)

$$\boxed{X_L = 2\pi f L} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_} (10)$$

Equation (10) shows that the inductive reactance depends upon the frequency "f" of A.C. In D.C the inductive reactance is zero i.e. $X_L = 0$.

## Power Loss in an Inductor:



We know that,

$$< P > = < IV > \text{ ______________ } (11)$$

Putting equation (7), (5) in equation (11), we get,

$$< P > = < I_m \sin(\omega t) \cdot V_m \cos(\omega t) >$$

$$\Rightarrow < P > = I_m V_m < \sin(\omega t)\cos(\omega t) > \text{ ____________ } (12)$$

Now figure (e) shows that:

1. During first $90^\circ$ of the cycle, the power is negative. It means that the power is flowing from the coil to the source.
2. During next $90^\circ$ of the cycle, the power is positive. It means that the power is flowing from the source to the coil.
3. During next $90^\circ$ of the cycle, the power is negative. It means that the power is flowing from coil to the source.
4. During last $90^\circ$ of the cycle, the power is positive. It means that the power is flowing from source to the coil.

The power curve over one cycle shows that the positive power is equal to the negative power. Thus the resultant power over one cycle is equal to zero. It shows that, a pure inductor consumes no power. In other words, we can say that the electric power merely flows from the source to the coil and then from coil to the source.

Thus equation (12)

| For one cycle, |
|---|
| $\theta = \omega t = 360^\circ$ |
| $\sin(360^\circ) = 0$ |
| $\cos(360^\circ) = 1$ |
| $\Rightarrow \sin(\omega t)\cos(\omega t) = 0$ |

$$\Rightarrow < P > = I_m V_m < \sin(\omega t)\cos(\omega t) >$$

$$\Rightarrow < P > = I_m V_m \times 0$$

$$\Rightarrow \boxed{< P > = 0}$$

**Q8: Define and explain choke coil (or) inductive choke.**

**Answer: Choke Coil:**

A choke is an inductor which is used in A.C circuits. It offers high reactance to frequencies above a certain frequency range without appreciable limiting the flow of current.

A choke can prevent electric signals along undesired paths. It is used as filter in power supply to prevent ripple. It is used as radio frequency choke (RFC) to prevent radio frequency signals from entering audio frequency circuits. In this way, the undesired signals and noise can be attenuated.

> Q9: A sinusoidal voltage of angular frequency "ω" is connected across a capacitor of capacitance "C". Find mathematical expression for the instantaneous voltage, current and average power dissipated per cycle of the applied voltage. (ETEA 2006-2010)

Answer:

Consider a capacitor of capacitance "C" is connected to an alternating voltage source, as shown in the figure.

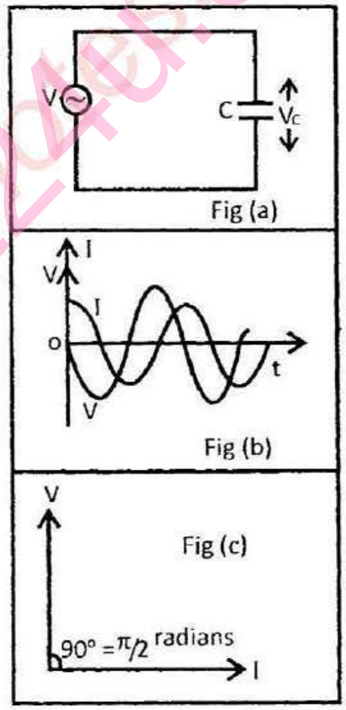


Fig (a)
Fig (b)
Fig (c)

When the circuit is turned-ON, the capacitor is charged in one direction and then in the other direction due to alternating voltage. As a result the electrons move to and fro around the circuit and thus produces alternating current in the circuit.

Now the alternating voltage applied across the circuit is given by,

$$\boxed{V = V_m \sin(\omega t)} \quad \text{\_\_\_\_\_\_\_\_(1)}$$

Now by definition, we have,

$$I = \frac{\Delta q}{\Delta t} \qquad [q = CV]$$

$$\Rightarrow \quad I = \frac{\Delta}{\Delta t}(CV) \quad \text{\_\_\_\_\_\_\_\_(2)}$$

Putting equation (1) in equation (2), we get,

$$I = \frac{\Delta}{\Delta t}[CV_m \sin(\omega t)]$$

$$\Rightarrow \quad I = CV_m\omega \frac{\Delta}{\Delta t} \sin(\omega t)$$

$$\Rightarrow \quad I = CV_m \cos(\omega t) \quad \text{\_\_\_\_\_\_\_\_(3)}$$

Here $CV_m\omega$ = maximum current = $I_m$

So equation (3) becomes,

$$\boxed{I = I_m \cos(\omega t)} \quad \text{\_\_\_\_\_\_\_\_(4)}$$

Equation (1), (4) represents the expressions for instantaneous voltage and current in capacitive circuit. Equation (4) can be also written as,

$$I = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{\_\_\_\_\_\_(5)}$$

**Formula:**

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\Rightarrow \sin\left(\omega t + \frac{\pi}{2}\right) = \sin\omega t\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos\omega t$$

$$\Rightarrow \sin\left(\omega t + \frac{\pi}{2}\right) = \sin\omega t \times 0 + 1 \times \cos\omega t$$

$$\Rightarrow \sin\left(\omega t + \frac{\pi}{2}\right) = \cos\omega t$$

The waveform of current and voltage is shown in figure (b), which shows that the voltage passes from its zero point, while the current at that instant passes from non-zero point. Thus the current and voltage are not in phase. Equation (5) shows that the current leads the voltage by 90° or "π/2" radians. The phasor diagram of current and voltage in capacitance circuit is shown in figure (c).

**Capacitive Reactance: (ETEA 2010)**

**Definition:** "The opposition offered by a capacitor to the flow of alternating current in a circuit is known as capacitance reactance."

**Explanation:** We know that,

$$V_m = I_m R \quad \text{\_\_\_\_\_\_(6)}$$

In case of capacitive reactance, we replace "R" by "$X_C$", so equation (6) becomes,

$$V_m = I_m X_c$$

$$\Rightarrow X_c = \frac{V_m}{I_m} \quad \text{\_\_\_\_\_\_(7)}$$

As we have, $I_m = CV_m\omega$, so equation (7) becomes,

$$X_c = \frac{V_m}{CV_m\omega}$$

$$\Rightarrow X_c = \frac{1}{C\omega}$$

$$\Rightarrow \boxed{X_c = \frac{1}{2\pi f C}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(8)} \qquad \boxed{\begin{array}{l} f = \omega / 2\pi \\ \Rightarrow \omega = 2\pi f \end{array}}$$

Equation (8) shows that the capacitive reactance depends upon the frequency of A·C· In case of D·C, "$X_c$" will be infinite·

## Power Loss in a Capacitive Circuit:

We know that the average power is given by,

$$< P > = < IV > \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(9)}$$

Putting equations (1), (4) in equation (9), we get,

$$< P > = < I_m \cos(\omega t) \sin(\omega t) > \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(10)}$$

Figure (e) shows that the positive power and negative power are equal in one cycle· Thus the power loss in pure capacitive circuit in one cycle is zero· So equation (10) ⇒

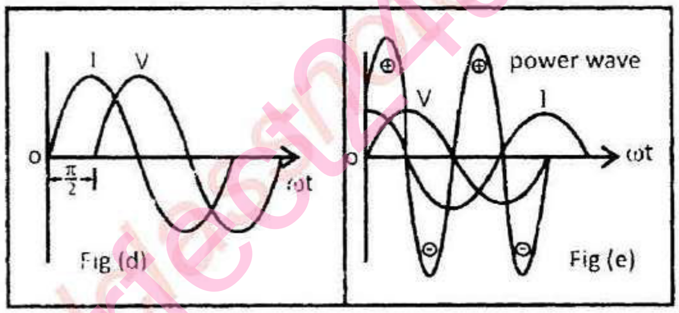



Fig (d) Fig (e)

$$\boxed{< P > = 0}$$

**Q10: Define impedance· What is its unit?**

**Answer: Impedance:**

**Definition:** "The combined opposition to the flow of alternating current by resistance "R", capacitive reactance "$X_c$" and inductive reactance "$X_L$" or any two of them is known as impedance·"

**Explanation:** The impedance is represented by "Z"· In terms of r·m·s values, it can be written as,

$$\boxed{Z = \frac{V_{rms}}{I_{rms}}}$$

Its unit is ohm·

**Q11: Define RL series A·C circuit· Explain with phasor diagram, that in an RL series circuit, the current will lag or lead the applied alternating voltage·**

**Answer:** **RL-Series A·C Circuit:**

**Definition:** "An A·C circuit in which the resistor and inductor are connected in series to an alternating voltage source is known as RL series A·C circuit·"

### Explanation:

Consider an A·C circuit in which an inductor and resistor are connected in series with alternating voltage source, as shown in the figure·

The phasor diagram of the circuit is shown in figure (c), which shows that the voltage drop "$V_R$" is in phase with current "$I$"· Its magnitude and direction is represented by the phasor "OP"· While the voltage drop "$V_L$" leads the current "$I$" by 90°· Its magnitude and direction is represented by the phasor "PM" in the given figure·

### Determination of Impedance:

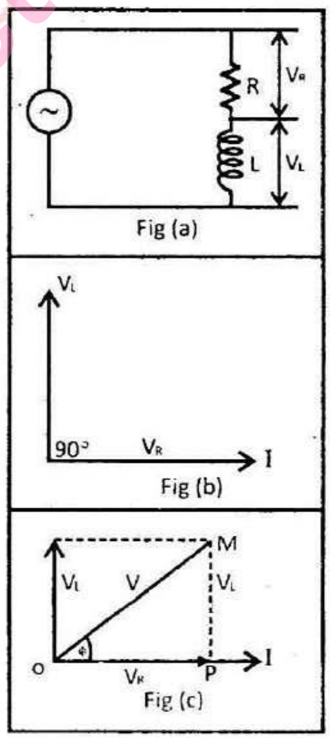


Fig (a)
Fig (b)
Fig (c)

We know that,

$$Z = \frac{V}{I} \quad \text{\_\_\_\_\_\_\_\_(1)}$$

Now according to phasor diagram, the voltage "V" of the source is equal to the phasor sum of "$V_R$" and "$V_L$" i·e·

$$V^2 = V_R^2 + V_L^2$$

$$\Rightarrow V^2 = I^2R^2 + I^2X_L^2$$

$$\Rightarrow V^2 = I^2(R^2 + X_L^2)$$

$$\Rightarrow V = I\sqrt{R^2 + X_L^2} \quad \text{\_\_\_\_\_\_\_\_(2)}$$

Putting equation (2) in equation (1), we get,

$$Z = \frac{I\sqrt{R^2 + X_L^2}}{I}$$

$$\Rightarrow \boxed{Z = \sqrt{R^2 + X_L^2}} \quad \text{\_\_\_\_\_\_\_\_(3)}$$

Equation (3) represents the impedance of RL-series circuit· In such circuit, the current "I" lags behind the voltage "$V_L$" by 90° or $\frac{\pi}{2}$ radians)·

## Determination of Phase Angle $\phi$:

From figure (c), we have,

$$\tan\phi = \frac{V_L}{V_R}$$

$$\Rightarrow \tan\phi = \frac{IX_L}{IR} \Rightarrow \tan\phi = \frac{X_L}{R}$$

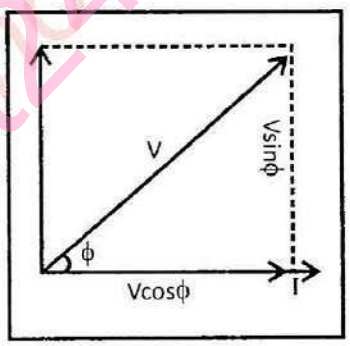
$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)} \quad \text{\_\_\_\_\_\_\_\_}(4)$$

Equation (4) shows that for given values of $X_L$, $R$ we can calculate the phase angle "$\phi$".

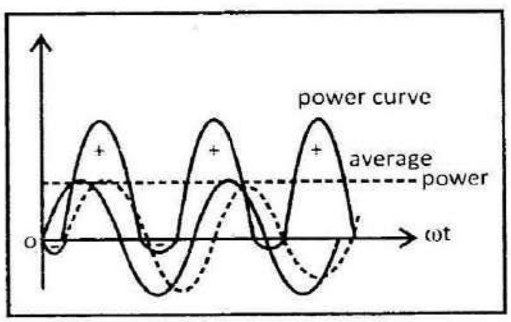
## Power in RL-Series Circuit:

We know that,

$$P = \text{current} \times \text{voltage} \quad \text{\_\_\_\_\_\_}(1)$$




Now from figure, it is clear that the component "$V\cos\phi$" is in phase with current "$I$", so this component is responsible for power consumption in RL circuit. The power curve also shows that the area of positive curve is greater than the area of negative curve. Thus the net power over one cycle is positive. Therefore, we can say that power is consumed in RL series circuit.




Thus equation (1) can be written as,

$$\boxed{P = IV\cos\phi} \quad \text{\_\_\_\_\_\_}(2)$$

Equation (2) represents the expression for power loss in RL series circuit.

**Q12: Define and explain the following:**
(a) Power factor (b) Q-factor
(c) RL-series impedance triangle

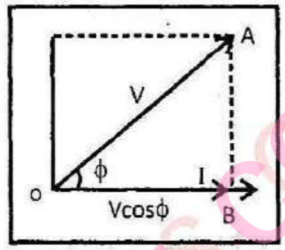
**Answer:** a) **Power Factor:**

**Definition:** In expression, "$P = IV\cos\phi$", the term "$\cos\phi$" is known as power factor.

***Explanation:*** The power factor represents the cosine of angle between "I" and "V".

In case of pure resistive circuit, the current and voltage are in phase, so $\phi = 0^\circ$, so power factor = $\cos\phi = \cos 0^\circ = 1$

This shows that the power factor of pure resistive circuit is equal to one. i.e.




$$\boxed{\text{Power factor} = 1}$$

In case of pure inductive or capacitive circuit, the phase angle is $90^\circ$, so power factor = $\cos\phi = \cos 90^\circ = 0$

$$\Rightarrow \boxed{\text{Power factor} = 0}$$

Thus the power factor of pure inductive or capacitive circuit is zero. For a circuit containing R, L and C in varying proportions, the value of power factor lies between 0 and 1. The value of "$\phi$" in term $\cos\phi$ is called power angle.

## b) Q-Factor:

***Definition:*** "The ability of a component to store energy is known as its quality factor [Q-factor]."

***Explanation:*** The Q-factor of a circuit is a ratio of energy stored in the circuit to the energy dissipated in one cycle.

$$\boxed{Q\text{ - factor} = 2\pi \times \frac{\text{maximum energy stored}}{\text{energy loss per cycle}}}$$

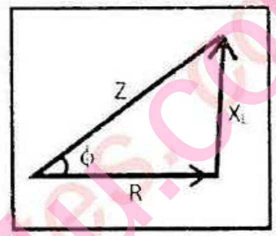
The ratio of inductive reactance "$X_L$" and resistance "R" of a circuit also represents the Q-factor of the coil. i.e.

$$\boxed{Q\text{ - factor} = \frac{X_L}{R}}$$

The Q-factor is used to describe the quality or effectiveness of a coil. The Q-factor of a coil will be greater if its inductive reactance "$X_L$" is greater than "R" and vice versa.

### 3) Impedance Triangle:

**Definition:** The phasor diagram is a triangle whose sides represent R, $X_L$ and Z. Such triangle is known as impedance triangle.




**Explanation:** Using impedance triangle, we can calculate,

1. The power angle "φ" i.e.

$$\cos\phi = \frac{R}{Z} \Rightarrow \boxed{\phi = \cos^{-1}\left(\frac{R}{Z}\right)}$$

2. The phase angle i.e. $\tan\phi = \dfrac{X_L}{R}$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

**Q13: Define RC-series A.C circuit. In RC-series circuit will the current lags or lead the applied voltage. Explain the answer with a phasor diagram.**

**Answer: RC-Series A.C Circuit:**

**Definition:** "An A.C circuit in which a resistor and capacitor are connected in series is known as RC-series A.C circuit."

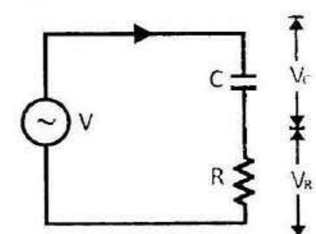


Fig (a)

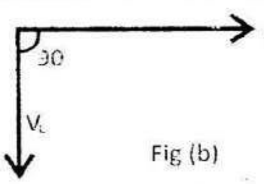


Fig (b)

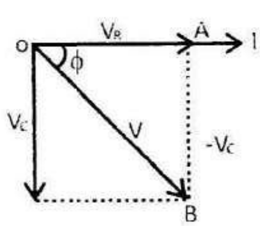


Fig (c)

**Explanation:**

Consider a circuit in which a resistor "R" and capacitor "C" are connected in series with an alternating voltage source as shown in the figure.

The voltage drop "$V_R$" is in phase with current and is represented by the phasor OA. The voltage drop "$V_C$" lags behind the current by 90° and is represented by the phasor AB.

**Determination of Impedance:**

According to phasor diagram, the voltage of the source is equal to the phasor sum of "$V_R$" and "$-V_C$". So we have,

$$V^2 = V_R^2 + (-V_c)^2$$

$$\Rightarrow V^2 = R_R^2 + V_C^2 = I^2R^2 + I^2X_c^2$$

$$\Rightarrow V^2 = I^2(R^2 + X_c^2)$$

$$\Rightarrow V = I\sqrt{R^2 + X_c^2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Now the impedance is given by,

$$Z = \frac{V}{I} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(2)}$$

Putting equation (1) in equation (2), we get,

$$Z = \frac{I\sqrt{R^2 + X_c^2}}{I}$$

$$\Rightarrow \boxed{Z = \sqrt{R^2 + X_c^2}} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

Equation (3) represents the impedance of RC-series circuit. In RC-series circuit, the current leads the voltage "V" by "$\phi$" and voltage "$V_c$" by 90°.

**Phase Angle:**

From $\Delta OAB$ [figure (c)], we have,

$$\tan\phi = \frac{V_c}{V_R}$$

$$\Rightarrow \tan\phi = \frac{IX_c}{IR}$$

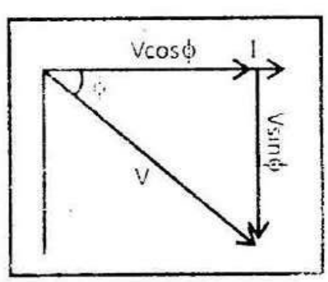
$$\Rightarrow \tan\phi = \frac{X_c}{R} \Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X_c}{R}\right)} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

Equation (4) shows that for given values of "$X_c$" and "R", we can calculate the value of phase angle "$\phi$".

**Power in RC-Circuit:**

We know that,

$$P = \text{current} \times \text{voltage} \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$




Now from the figure, it is clear that the component "$V\cos\phi$" is in phase with current, so this component is responsible for the consumption of power in RC-series A.C circuit. Thus equation (1) can be written as,

$$\boxed{P = IV\cos\phi} \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

Equation (2) represents the expression for power loss in RC-series A·C circuit·

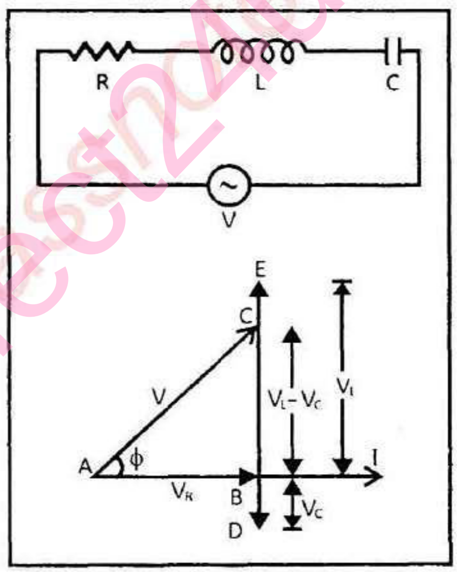
**Q14: What is LRC-series A·C circuit? Find an expression for impedance of RLC-series circuit·**

**Answer: RLC-Series A·C Circuit:**

**Definition:** A circuit in which a resistor, inductor and a capacitor are connected in series with alternating voltage source then such circuit is known as RLC-series A·C circuit·

**Expression for Impedance:**

Consider a circuit in which a resistor, inductor and capacitor are connected in series, as shown in the figure·




In resistive circuit, the current and voltage "$V_R$" are in phase· In inductive circuit, the voltage "$V_L$" leads the current by 90°, while in capacitive circuit, the voltage "$V_C$" lags behind the current by 90°· Thus "$V_L$" and "$V_C$" are out of phase by 180°·

Now according to phasor diagram, the voltage "V" of the source is equal to the phasor sum of "$V_R$" and "$V_L - V_C$" i·e·

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$\Rightarrow V^2 = I^2R^2 + (IX_L - IX_C)^2$$

$$\Rightarrow V^2 = I^2\left[R^2 + (X_L - X_C)^2\right]$$

$$\Rightarrow V = I\sqrt{(R)^2 + (X_L - X_C)^2} \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

Now the impedance of the circuit is given by,

$$Z = \frac{V}{I} \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

Putting equation (1) in equation (2), we get,

$$Z = \frac{I\sqrt{(R)^2 + (X_L - X_C)^2}}{I}$$

$$\Rightarrow \boxed{Z = \sqrt{(R)^2 + (X_L - X_C)^2}}$$

Equation (3) represents the impedance of RLC-series A·C circuit· In equation (3), the quantity $(X_L - X_C)$ is known as reactance of the circuit· It is represented by "X"· So we have,

$$X = X_L - X_C$$

Thus equation (3) can be also written as,

$$\boxed{Z = \sqrt{R^2 + X^2}} \quad \text{———— (4)}$$

**Power Factor of the Circuit:**

In $\Delta ABC$, we have,

$$\cos\phi = \frac{V_R}{V} \qquad [V_R = IR,\ V = IZ]$$

$$\Rightarrow \cos\phi = \frac{IR}{IZ}$$

$$\Rightarrow \cos\phi = \frac{R}{Z} \quad \text{———— (5)} \qquad X^2 = (X_L - X_C)^2]$$

Putting equation (4) in equation (5), we get,

$$\boxed{\cos\phi = \frac{R}{\sqrt{R^2 + X^2}}} \quad \text{———— (6)}$$

Equation (6) represents the power factor of RLC-series circuit·

**Phase Angle:**

In $\Delta ABC$, we have,

$$\tan\phi = \frac{(V_L - V_C)}{V_R} = \left(\frac{IX_L - IX_C}{IR}\right) = \frac{I(X_L - X_C)}{IR}$$

$$\boxed{\begin{aligned} V_R &= IR \\ V_L &= IX_L \\ V_C &= IX_C \\ X_L - X_C &= X \end{aligned}}$$

$$\Rightarrow \tan\phi = \frac{(X_L - X_C)}{R}$$

$$\Rightarrow \tan\phi = \frac{X}{R}$$

$$\Rightarrow \boxed{\phi = \tan^{-1}\left(\frac{X}{R}\right)} \quad \text{———— (7)}$$

Equation (7) represents the phase angle of the circuit.

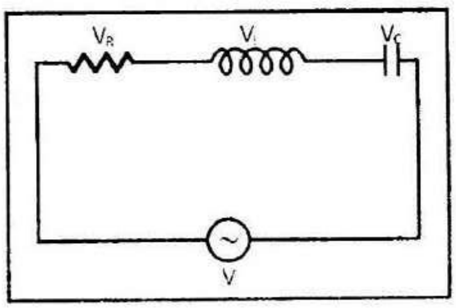
**Note:**

1. If $X_L > X_C$, then the phase angle will be positive and the circuit will be inductive. In such a case, the current will lag behind the applied voltage "V" by $\phi$.
2. If $X_L < X_C$, then the phase angle will be negative and the circuit will be capacitive. In such a case, the current leads the applied voltage "V" by $\phi$.
3. If $X_L = X_C$, then the phase angle will be zero and the circuit will be purely resistive. In such a case, the current and applied voltage "V" will be in phase. i.e. $\phi = 0°$

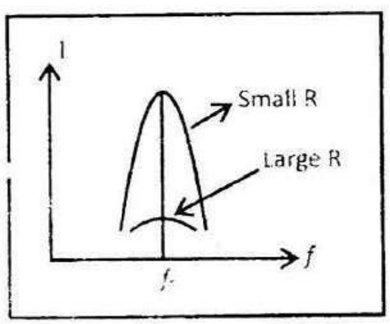
**Q15: Explain the resonance in RLC-series circuit. Show that the resonance occurs at the frequency, $f_r = \frac{1}{2\pi\sqrt{LC}}$. Discuss the properties and uses of RLC-series resonant circuit. (ETEA 2011)**

Answer:

Consider a circuit in which a resistor, inductor and capacitor are connected to a source of alternating voltage as shown in the figure.




Now if the capacitive reactance and inductive reactance balance each other, then the impedance of the circuit becomes equal to the resistance "R" only. Under such conditions, the impedance is minimum and current is maximum. The circuit under such condition is known as resonant circuit and its frequency is known as resonant frequency "$f_r$". The resonance curve shows that at resonance frequency, the impedance is minimum and current is maximum.




Expression for Resonance Frequency

For resonance condition, we have.

$$X_L = X_C \Rightarrow 2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\boxed{\begin{aligned} X_L &= 2\pi f L \\ X_C &= \frac{1}{2\pi f C} \end{aligned}}$$

$$\Rightarrow f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\Rightarrow f_r = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\Rightarrow \boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \quad \text{———————} (1)$$

Equation (1) represents the expression for resonance frequency.

**Properties of RLC-Series Resonant Circuit:**

1. The frequency of RLC-series resonant circuit is equal to, $\frac{1}{2\pi\sqrt{LC}}$
2. The current and voltage "$V_R$" are in phase.
3. "$X_L$" and "$X_C$" are equal and are out of phase by 180°.
4. The impedance is minimum and current is maximum.
5. The radio of "$X_L$" and "R" is known as Q-factor. Q-factor $= \frac{X_L}{R}$.

**Uses of Resonance Circuit:**

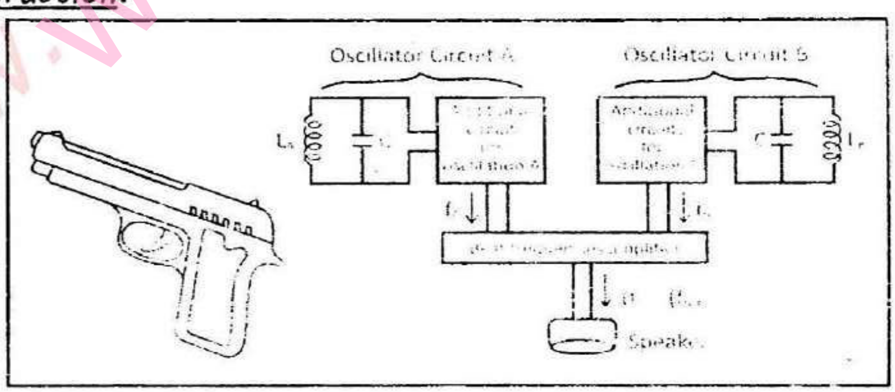
1. The resonant circuit is used in turning a radio or T.V sets.
2. It is used in variable oscillators.
3. It is used in detection instruments.

| Q16: What are metal detectors? Describe their principle and uses. |
| --- |

**Answer: Metal Detectors:**

The metal detectors are the electrical instruments which are used for detection and security purposes.

**Construction:**



It consists of two elements. One element is known as transmission element which generates a magnetic field. The other element is known as receiving element which get the information about certain substance and converts these information into electrical signals. These signals are finally registered on a sonic alarm.

## Principle of Metal Detectors:

The metal detectors consist of a capacitor-inductor circuit as shown in the figure. The combination of capacitor and inductor acts as mass-spring system in which the energy oscillates between the inductor and capacitor. Such circuit is known as electrical oscillator.

## Detection of Metal:

For the detection of metal, two oscillators "A" and "B" are used in operation, as shown in the figure.

When the oscillator "A" is brought near the metal to be detected, the inductance decreases due to which the frequency of oscillator "A" increases. Due to change in frequency, the phenomenon of beats takes place and a loud sound is heard with the help of sonic alarm. In this way, the metal is detected.

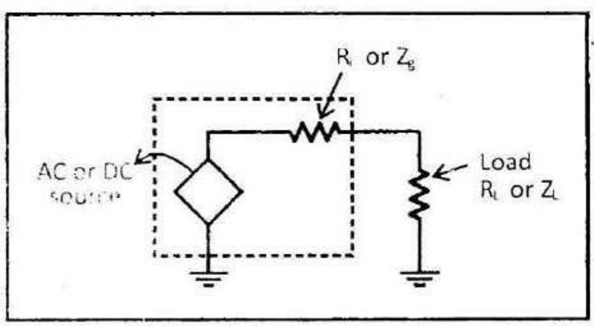
## Uses of Metal Detectors:

The metal detectors are used for security purposes. These detectors can be used to detect hidden objects.

> Q17: Discuss that maximum power is transferred when the impedance of the source and load match to each other.

Answer: **Maximum Power Transfer Theorem:**

**Statement:** The maximum power transfer theorem states that to transfer the maximum amount of power from a source to a load, the load impedance should match the source impedance.

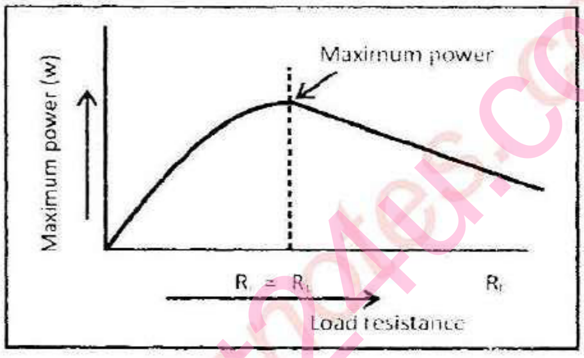



**Explanation:** Let "$R_i$" is the internal resistance of the source and "$R_L$" represents the load resistance, then maximum power will be transferred from source to load, if

$$\boxed{R_i = R_L}$$

In terms of impedance, we can say that, If "$Z_g$" represents the impedance of a generator and (source) "$Z_L$" represents the impedance of the load, then maximum power to the load, if

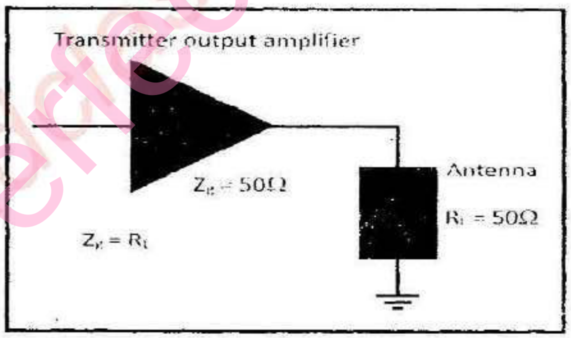
$$\boxed{Z_g = Z_L}$$




The graph of load resistance and load power is shown in the figure, which shows that maximum power will be transferred from source to the load if $R_i$ and $R_L$ match each other.

**Example:**

An antenna is the example of maximum power transferred from source to load. In this case the impedance of antenna is equal to the impedance of transmitter output amplifier as shown in the figure.




**Q18: Use Maxwell's theory to show that, $E = \frac{1}{2\pi r}\frac{\Delta\phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t}$ & $I = \varepsilon_o \frac{\Delta\phi}{\Delta t}$. Describe the statement of four Maxwell's equations.**

**Answer: Maxwell's Equations:**

In 1884 Clark Maxwell formulated Coulomb's law, Ampere's law and Faraday's law of electromagnetic induction into a set of simultaneous equations.

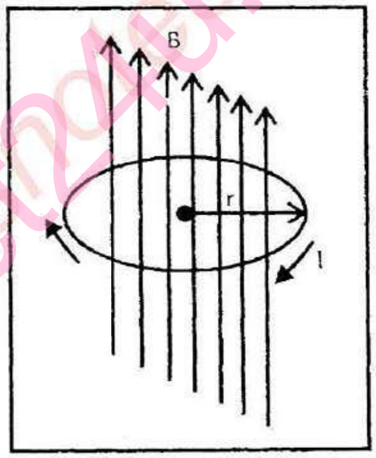
In this way, Maxwell unified the subjects of electricity and magnetism into electromagnetism. The four statements of Maxwell's equations are given below:

1. A changing magnetic field produces an electric field.
2. A changing electric field produces a magnetic field.
3. A changing electric field creates a magnetic field which in turn creates an electric field.
4. Light is a type of electromagnetic waves which travels with a speed of $3 \times 10^8$ m/sec in free space.

The details about above statements are given below:

### 1. A Changing Magnetic Field Produces on Electric Field:

We know that when magnetic flux through a certain region changes, an induced emf is produced in the region. Due to this emf, an induced current can be produced in a conductor placed in that region. Now we know that the induced emf is given by,




$$V = \frac{\Delta}{\Delta t}\phi \text{ ———————— } (1)$$

We also know that the potential difference arises due to electric field. This shows that at each point of the loop, an electric field will be produced due to charging magnetic field. Now we know that,

$$V = E \cdot d \text{ ———————— } (2)$$

Now if a unit positive charge complete one rotation along the circular loop, then its distance covered will be equal to "$2\pi r$", so equation (2) becomes,

$$V = E \times 2\pi r \text{ ———————— } (3)$$

Comparing equation (1), (3), we get,

$$E + 2\pi r = \frac{\Delta}{\Delta t}\phi \Rightarrow \boxed{E = \frac{1}{2\pi r}\frac{\Delta}{\Delta t}\phi} \text{ ———— } (4)$$

As $\phi = BA$, so equation (4)

$$E = \frac{1}{2\pi r}\frac{\Delta}{\Delta t}(BA) \qquad \left[\frac{A}{2\pi r} = \text{constant}\right]$$

$$\Rightarrow \boxed{E = \frac{A}{2\pi r}\frac{\Delta}{\Delta t}\phi} \text{ ———————— } (5)$$

n equation (5) $\frac{A}{2\pi r}$ = constant, so equation (5) becomes

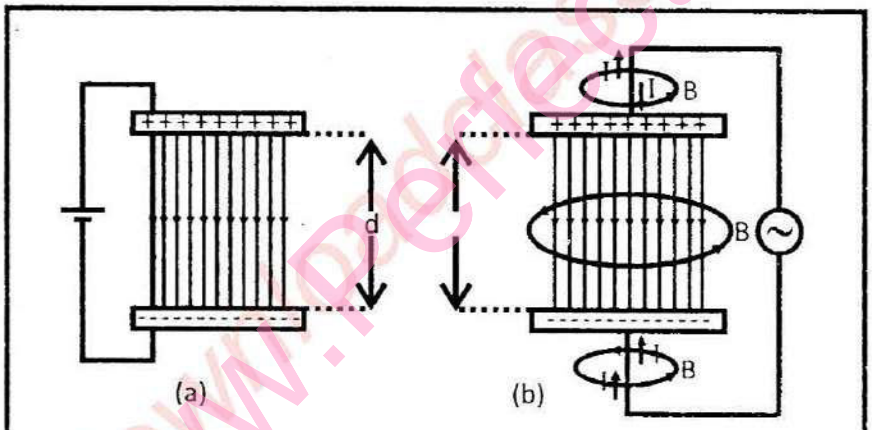
$$E = \text{constant} \cdot \frac{\Delta}{\Delta t} B$$

$$\boxed{E \propto \frac{\Delta}{\Delta t} B} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

Relation (4) shows that the electric field intensity is directly proportional to the rate of change of magnetic flux density. This shows that a changing magnetic field can produce electric field.

## 2. A Changing Electric Field can Produce a Magnetic Field:

Consider a capacitor is connected to an alternating voltage source, as shown in the figure. The current will flow in the circuit continuously and does not stop. This shows that the electric field between the plates of the capacitor varies with time.




Current through the dielectric of the capacitor is due to changing E-field

Now we know that the charge produced on the capacitor is given by,

$$Q = CV \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

Multiplying both sides by "$\frac{\Delta}{\Delta t}$", we get,

$$\frac{\Delta}{\Delta t} Q = \frac{\Delta}{\Delta t}(CV) \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

We also know that,

$$C = \frac{A\varepsilon_o}{d} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(3)}$$

Putting equation (3) in equation (2), we get,

$$\frac{\Delta}{\Delta t} Q = \frac{\Delta}{\Delta t}\left(\frac{A\varepsilon_o V}{d}\right) \text{\_\_\_\_\_\_\_\_} (4)$$

We know that,

$$\frac{\Delta Q}{\Delta t} = \text{current} = I$$

And $\frac{V}{d} = E$, so equation (4) $[V = E \cdot d]$

$$\Rightarrow I = \frac{\Delta}{\Delta t}(\varepsilon_o EA) = \varepsilon_o \frac{\Delta}{\Delta t}(EA)$$

$$\Rightarrow \boxed{I = \varepsilon_o \frac{\Delta}{\Delta t}(\phi_e)} \text{\_\_\_\_\_\_\_\_} (5)$$

EA = electric flux
$\Rightarrow EA = \phi_e$

Equation (5) shows that the changing electric flux is equivalent to a current, such current is known as displacement current. Now according to ampere's law, each type of current produces magnetic field around itself. Thus a changing electric field produces a magnetic field.

- The current produced due to flow of charges is called conduction current.
- While the current due to changing electric flux is called displacement current.

**3.** A changing electric field creates a magnetic field which in turn creates an electric field. In this way, an electromagnetic disturbance or waves are generated. The fundamental requirement for the generation of electromagnetic waves in an electric charge with changing velocity, i.e. an accelerated charge will create changing electric flux. The velocity of an oscillating charge as it moves to and fro along a wire is always changing and as a result, an electromagnetic waves are produced.

**4.** Light is a type of electromagnetic waves which need no material medium for its propagation. It can pass through vacuum. Maxwell predicted theoretically that the velocity of electromagnetic waves in free space is given by,

$$C = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \text{\_\_\_\_\_\_\_\_} (1)$$

$$\Rightarrow C = \frac{1}{\sqrt{(8.85 \times 10^{-12} \times 4\pi \times 10^{-7})}}$$

$$\Rightarrow \boxed{C = 3 \times 10^8 \, m/sec}$$

$\varepsilon_o$ = permittivity of free space = $8.85 \times 10^{-12} C^2/N \cdot m^2$
$\mu_o$ = permeability of free space = $4\pi \times 10^{-7} Wb/A \cdot m$

Q19: (a) What are electromagnetic waves? (b) Discuss the various types of electromagnetic waves.

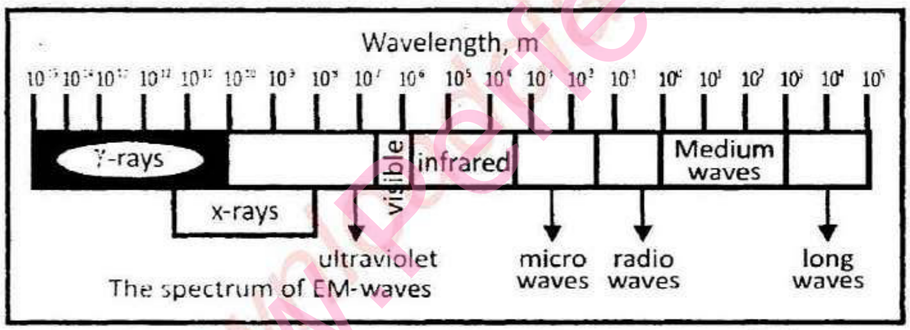
**Answer: a) Electromagnetic Waves:**

**Definition:** "Those waves which can pass through space with rapid speed and need no material medium for their propagation are known as electromagnetic waves."

**Explanation:**

Faraday's law shows that a changing magnetic field gives rise to an electric field. Ampere-Maxwell law shows that a changing electric field gives rise to a magnetic field. It follows that when either electric or magnetic field varies with time, the other field is induced in space. The net effect is that an electromagnetic disturbance is generated due to changing electric and magnetic fields. The disturbance propagates in the form of a electromagnetic wave.

**b) Types of Electromagnetic Waves:**



The spectrum of EM-waves

The various types of electromagnetic waves are given below:

1. **Light Waves:** Light waves are electromagnetic radiations which are emitted by the sun. The wavelength of light waves ranges from 400nm to 700nm. Light is often emitted when the outermost electrons in atoms change their state of motion. The colour of light tells us something about the atoms or the object from which it was emitted. The study of the light emitted from the sun and other stars gives information about their composition.

2. **Infrared Waves:** These are electromagnetic radiations whose wavelength ranges from 0.7 micrometer to about 1mm. These radiations are commonly emitted by atoms or molecules when they

change their rotational or vibrational motion. The warmth which your hand feel near a glowing light bulb is initially a result of the infrared radiations emitted from the bulb.

Objects of temperature ranges from 3K to 3000K emit their most intense thermal radiation in the infrared region of the spectrum.

The infrared radiations are used in hand remote control for operating T.V, DVD set etc in daily life. The signal between a remote control handset and the device it control consists of pulses of infrared light which is invisible to human eye. The infrared radiation is also used for cooking the surface of food.

### 3. ***Microwaves:***

A handset remote control

The microwaves are also called short radio waves whose wavelength ranges from 1mm to 1m. These radiations are commonly produced by electromagnetic oscillators in electric circuits like in microwave ovens. These radiations are used to transmit telephone conversations. These radiations are also emitted by neutral hydrogen atom present between the stars of our galaxy.

### 4. ***Radio Waves:***

These are electromagnetic waves whose wavelengths is longer than 1m. These radiations are produced from terrestrial sources through electrons oscillating in wires of electric circuits. These radiations are also emitted from extraterrestrial sources called radio astronomy. These radiations give us useful information about the universe.

### 5. ***Ultraviolet Rays:***

These are electromagnetic radiations whose wavelength ranges from 1nm to 400nm. These radiations can be produced due to atomic transition of electrons. These radiations are also emitted from the sun. But due to ozone layer, very little amount of these radiations can reach our earth. Brief exposure to ultraviolet radiation can cause common sun burn but long term exposure can cause serious effects like skin cancer. These radiations can be used for destroying bacteria, viruses, molds and other harmful organisms.

6. **X-rays:**

These are electromagnetic radiations whose wavelength ranges from 0.01nm to 10nm. These radiations can be produced due to inner shell transition of electrons in an atom. These radiations are also produced when a high speed charged particle become slowdown due to collision with certain target. These radiations are used in health, industry, agriculture and detective departments for various purposes.

7. **Gamma Rays (γ-rays):**

These are electromagnetic radiations whose wavelength is less than 10 pico meter. These radiations can be produced during the transitions of an atomic nucleus from one state to another. These radiations can also emitted during the decay process of certain radioactive element. These radiations posses high penetrating power and can damage human body.

**Q20: What is the principle of ECG? Sketch the wave curve of heart beats and explain the terms positive deflection and negative deflection.**

**Answer: ECG:**

The ECG stands for "Electro Cardio Gram" which is used for diagnosing the conditions of heart.

**Principle of ECG:**

An ECG is a recording of the small electric waves being generated during heart activity. The electric activity starts at the top of the heart and then spreads down. A normal heart beat is initiated by a small pulse of electric current. This tiny electric "shock" spreads rapidly in the heart and makes the heart muscle to contract. If the whole heart muscle contracted at the same time, then there would be no pumping effect.

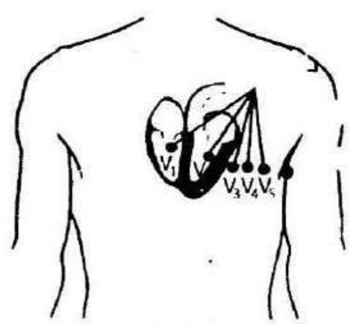


Fig (a)
Sensitive electrodes are placed on main parts of the body.

In this way, the electric activity starts at the top of heart and spreads down and then up again. This causes the heart muscle to contract in an optimal way for pumping blood. The electric waves in the heart are recorded in millivolts by the electro cardiograph.

Our heart produces time-varying voltages as it beats. These heart voltages produce small voltage differences between points on our skin that can be measured and used to diagnose the condition of our heart. The waves are registered by electrodes placed on certain parts of the body, which are then printed on paper in the form of a curve, as shown in figure (b).

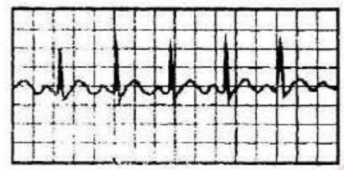


Fig (b)

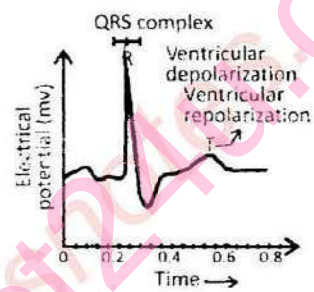



Fig (c). An ECG curve reflects the perspective of the electrode recording it.

## Characteristics of ECG Curve:

1. When the curve falls below the base line, it shows a negative deflection.
2. When the curve rises above the base line, it shows a positive deflection.
3. The negative deflection shows that the recorded wave has travelled away from the electrode.
4. The positive deflection shows that the recorded wave has travelled towards the electrode.
5. The tiny rise and fall in the voltage between the electrodes placed either side of the heart which is displaced as a wavy line either on a screen or on paper.
6. A typical plot of voltage difference between two points on the human body verses time is shown in figure (c).

- The P deflection corresponds to the contraction of the atria at the start of the heart beat.
- The QRS complex corresponds to the contraction of ventricles.
- The T deflection corresponds to a re-polarization or recovery of the ventricular cells in preparation for the next beat.

## SOLUTIONS OF EXAMPLES & ASSIGNMENTS

**EXAMPLE 15·1:**
An A·C circuit consists of a pure resistance of 20Ω and is connected across A·C supply of 220V, 50Hz· Calculate (a) current (b) power consumed and (c) equation for voltage and current·

**Solution:**

**Given Data:**

Resistance = $R = 20\Omega$

Voltage = $V_{rms} = 220V$

Frequency = $f = 50Hz$

**Required:**

a) Current = $I$ = ?

b) Power consumed = $P$ = ?

c) Equation for voltage and current = ?

**Calculation:**

a) We know that,

$$V = IR$$

$$\Rightarrow I = \frac{V}{R} \Rightarrow I_{rms} = \frac{V_{rms}}{R} = \frac{220}{20}$$

$$\Rightarrow \boxed{I_{rms} = 11\ amp}$$

b) We know that, power consumed = $IV \Rightarrow P = IV$

$$\Rightarrow P = I_{rms} \cdot V_{rms} = (11 \times 220)\ watt$$

$$\Rightarrow \boxed{P = 2420\ watt}$$

c) For alternating current, the equation is given by,

$$I = I_m \sin(\omega t)$$

$$\Rightarrow I = \sqrt{2} I_{rms} \sin(2\pi ft)$$

$$\Rightarrow I = \sqrt{2} \times 11 \sin(2 \times 3\cdot14 \times 50t)$$

$$\Rightarrow \boxed{I = 15\cdot55 \sin(314t)}$$

$$I_m = \sqrt{2}\, I_{rms}$$
$$\omega = 2\pi f$$
$$V_m = \sqrt{2}\, V_{rms}$$

The equation for alternating voltage is given by,

$$V = V_m \sin(\omega t)$$

$$\Rightarrow V = \sqrt{2} V_{rms} \sin(2\pi ft)$$

$$\Rightarrow V = \sqrt{2} \times 220 \sin(2 \times 3\cdot14 \times 50t)$$

$$\Rightarrow \boxed{V = 311\cdot1 \sin(314t)}$$

**ASSIGNMENT 15.1:**

A 60Ω resistor is connected across A.C supply of 220V. Calculate the current drawn from the supply and the power consumed by the heating element.

**Solution:**

**Given Data:**

Resistance = $R = 60\Omega$

Voltage = $V_{rms} = 220V$

**Required:**

a) Current = $I$ = ?

b) Power consumed = $P$ = ?

**Calculation:**

a) We know that, $V = IR$

$$\Rightarrow I = \frac{V}{R} = \frac{220}{60}$$

$$\Rightarrow \boxed{I = 3.66A}$$

b) We know that,

$$P = IV = 3.66 \times 220 \text{ watt}$$

$$\Rightarrow \boxed{P = 805 \text{ watt}}$$

**EXAMPLE 15.2:**

A pure inductive coil allows a current of 20A to flow from a 220V, 50Hz supply. Find (1) inductive reactance (2) inductance of the coil (3) power absorbed. Write down the equation for voltage and current.

**Solution:**

**Given Data:**

Current = $I_{rms}$ = 20A, Voltage = $V_{rms}$ = 220V

Frequency = $f$ = 50Hz

**Required:**

a) Inductive reactance = $X_L$ = ?

b) Inductance = $L$ = ?

c) Power absorbed = $P$ = ?

d) Equations for current and voltage = ?

**Calculation:**

a) We know that, $V = IX_L$

$$\Rightarrow X_L = \frac{V}{I} = \frac{220}{20}$$

$$\Rightarrow \boxed{X_L = 11\Omega}$$

b) We know that,

$$X_L = 2\pi f L$$

$$\Rightarrow L = \frac{X_L}{2\pi f} = \frac{11}{(2 \times 3{\cdot}14 \times 50)}$$

$$\Rightarrow \boxed{L = 0{\cdot}035 \text{ Henry}}$$

c) In case of pure inductive circuit, no power is consumed or absorbed, so $\boxed{P = 0}$

d) Equation for voltage is given by,

$$V = V_m \sin(\omega t)$$

$$\Rightarrow V = \sqrt{2}\, V_{rms} \sin(2\pi f t)$$

$$\Rightarrow V = \sqrt{2} \times 220 \sin(2 \times 3{\cdot}14 \times 50t)$$

$$\boxed{V_m = \sqrt{2}\ V_{r{\cdot}m{\cdot}s}}$$

$$\Rightarrow \boxed{V = 311{\cdot}1 \sin(314t)}$$

The equation for current in inductive circuit is given by,

$$I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\Rightarrow I = \sqrt{2}\, I_{rms} \sin\left(2\pi f t - \frac{\pi}{2}\right)$$

$$\Rightarrow I = \sqrt{2} \times 20 \sin\left(2 \times 3{\cdot}14 \times 50t - \frac{\pi}{2}\right)$$

$$\boxed{I_m = \sqrt{2}\ I_{r{\cdot}m{\cdot}s}}$$

$$\Rightarrow \boxed{I = 28{\cdot}28 \sin\left(314t - \frac{\pi}{2}\right)}$$

**ASSIGNMENT 15·2:**

A pure inductor is connected across a 10V, 200Hz supply, and the current flowing through it is measured as 0·4A. Determine the value of its inductance.

**Solution:**

**Given Data:**

Voltage = $V$ = 10V, Frequency = $f$ = 200Hz

Current = 0·4A

**Required:**

Inductance = $L$ = ?

$$\boxed{X_L = 2\pi f L}$$

**Calculation:**

We know that, $V = IX_L$

$$\Rightarrow V = I \times 2\pi fL \Rightarrow L = \frac{V}{2\pi f I}$$

$$\Rightarrow L = \left(\frac{10}{2 \times 3{\cdot}14 \times 200 \times 0{\cdot}4}\right) H = 0{\cdot}0199H$$

$$\Rightarrow L = 19{\cdot}9 \times 10^{-3} H$$

$$\Rightarrow \boxed{L = 19{\cdot}9mH}$$

**EXAMPLE 15·3:**
The current through an 60mH inductor is $0{\cdot}2\sin(377t-25^\circ)A$. Write the mathematical expression for the voltage across it.

**Solution:**

**Given Data:**

Inductance = $L = 60mH = 60 \times 10^{-3}H$

$$\Rightarrow L = 0{\cdot}060H$$

Current through inductor = $I = 0{\cdot}2\sin(377t-25^\circ)A$ ________(1)

From equation (1), we have,

$$I_m = 0{\cdot}2A$$

We know that in case of inductive circuit, the voltage lead the current by 90°, so 90° will be added to the phase angle of voltage, so we get,

$V = IR$
$\Rightarrow V = IX_L$
$\Rightarrow V_m = I_m X_L$

$$V = V_m \sin(377t - 25^\circ + 90^\circ)$$

$$\Rightarrow V = V_m \sin(377t + 65^\circ)$$

$$\Rightarrow V = I_m X_L \sin(377t + 65^\circ)$$

$$\Rightarrow V = I_m(2\pi f L)\sin(377t + 65^\circ)$$

$$\Rightarrow V = I_m L(2fL)\sin(377t + 65^\circ)$$

$$\Rightarrow V = 0{\cdot}2 \times 0{\cdot}060 \times 377\sin(377t + 65^\circ)$$

$$\Rightarrow \boxed{V = 4{\cdot}5\sin(377t + 65^\circ)V}$$

∵ given equation
$\omega t = 377t$
$\Rightarrow \omega = 377$ ___(a)
As $\omega = 2\pi f$ ___(b)
Putting equation (b) in equation (a)
$\boxed{2\pi f = 377}$
Also
$X_L = 2\pi fL$

**EXAMPLE 15·4:**
A 318μF capacitor is connected across a 220V, 50Hz system. De-

termine (a) the capacitive reactance (b) r·m·s value of current and (c) equations for voltage and current

**Solution:**

**Given Data:**

Capacitance = $C = 318\mu F = 318 \times 10^{-6} F$

r·m·s voltage = $V_{rms} = 220V$

Frequency = $f = 50Hz$

**Required:**

a) The capacitive reactance = $X_c$ = ?

b) rms value of current = $I_{rms}$ = ?

c) Equations for voltage and current = ?

**Calculation:** a) We know that,

$$\text{Capacitive reactance} = X_c = \frac{1}{2\pi fC}$$

$$\Rightarrow X_c = \frac{1}{3.14 \times 50 \times 318 \times 10^{-6}} = 10.26\Omega$$

$$\Rightarrow \boxed{X_c = 10.26\Omega}$$

b) We know that,

$$\text{r·m·s value of current} = I_{rms} = \frac{V_{rms}}{X_c}$$

$$\Rightarrow I_{rms} = \frac{220}{10.26} = 21.44A$$

$$\Rightarrow \boxed{I_{rms} = 21.44A}$$

c) The equation for voltage is given by,

$$V = V_m \sin(\omega t)$$

$$\Rightarrow V = \sqrt{2} V_{rms} \sin(2\pi ft)$$

$$\Rightarrow V = \sqrt{2} \times 220 \sin(2 \times 3.14 \times 50t)$$

$$\Rightarrow \boxed{V = 311.1 \sin(314t)}$$

$V_m = \sqrt{2} V_{rms}$
$\omega = 2\pi f$
$I_m = \sqrt{2} I_{rms}$

The equation for current is given by,

$$I = I_m \sin(\omega t + 90^\circ)$$

$$\Rightarrow I = \sqrt{2} I_{rms} \sin(2\pi ft + 90^\circ)$$

$$\Rightarrow I = \sqrt{2} \times 21.44 \sin\left(2 \times 3.14 \times 50t + \frac{\pi}{2}\right)$$

$$\Rightarrow \boxed{I = 30.32 \sin\left(314t + \frac{\pi}{2}\right)}$$

In case of capacitive circuit current leads the voltage by 90, so we replace $\sin(\omega t)$ by $\sin(\omega t + 90^\circ)$

$90^\circ = \frac{\pi}{2}$ rad

**ASSIGNMENT 15·3:**
A perfect capacitor is connected across a 6V, 5KHz supply and the resulting current flow is 88·6mA· Calculate the capacitance value·

**Solution:**

**Given Data:**

Voltage = $V = 6V$, Frequency = $f = 5KHz = 5 \times 10^3 Hz$

Current = $I = 88{\cdot}6mA = 88{\cdot}6 \times 10^{-3} A$

**Required:**

Capacitance = $C = ?$

**Calculation:**

We know that, $V = IR$ ______________(1)

For capacitor of reactance "$X_c$" equation (1) can be written as,

$$V = IX_c = I \times \frac{1}{2\pi fC}$$

$$\Rightarrow V = \frac{I}{2\pi fC} \Rightarrow C = \frac{I}{2\pi fV} \qquad \boxed{X_c = \frac{1}{2\pi fC}}$$

$$\Rightarrow C = \left(\frac{88{\cdot}6 \times 10^{-3}}{2 \times 3{\cdot}14 \times 5 \times 10^3 \times 6}\right) F = 0{\cdot}47 \times 10^{-3-3} F$$

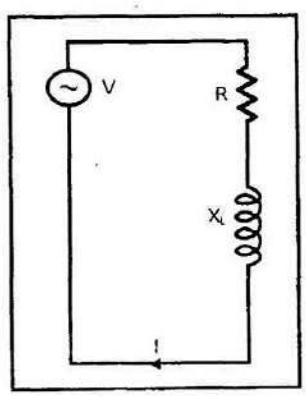
$$\Rightarrow C = 0{\cdot}47 \times 10^{-6} F$$

$$\Rightarrow \boxed{C = 0{\cdot}47\mu F} \qquad \boxed{10^{-6} = \text{micro}}$$

**EXAMPLE 15·5:**
A coil having a resistance of 7Ω and an inductance of 31·8mH is connected to 220V, 50Hz supply· Calculate (a) the circuit current (b) phase angle (c) power factor and (d) power consumed

**Solution:**

**Given Data:**




Inductance = $L = 31{\cdot}8mH$

$= 31{\cdot}8 \times 10^{-3} H = 0{\cdot}0318H$

Voltage = $V = 220V$

Coil resistance = $R = 7\Omega$

Frequency = $f = 50Hz$

**Required:**

a) The circuit current = $I = ?$

b) Phase angle = $\phi = ?$

c) Power factor = ?

d) Power consumed = P = ?

<u>Calculation:</u>

a) We know that, $V = IR$ ____________(1)

For impedance "Z", we replace "R" by "Z" so we get,

$$V = IZ$$

$$\Rightarrow 220 = I\sqrt{R^2 + X_L^2}$$

$$\Rightarrow 220 = I\sqrt{R^2 + (2\pi fL)^2}$$

$$\Rightarrow 220 = I\sqrt{7^2 + (2 \times 3\cdot14 \times 50 \times 0\cdot0318)^2}$$

$$\Rightarrow 220 = I\sqrt{49 + 10^2}$$

$$\Rightarrow 220 = I\sqrt{149} = I \times 12\cdot2$$

$$\Rightarrow 220 = (12\cdot2)I$$

$$\Rightarrow I = \left(\frac{220}{12\cdot2}\right)A$$

$$\Rightarrow \boxed{I = 18\cdot03A}$$

$$\boxed{\begin{array}{l} Z = \sqrt{R^2 + X_L^2} \\ X_L = 2\pi fL \end{array}}$$

$$\boxed{\begin{array}{l} X_c = \dfrac{1}{2\pi fC} \\ X_c = \dfrac{1}{2 \times 3\cdot14 \times 50 \times 79\cdot5 \times 10^{-6}} \\ X_c = 40\Omega \end{array}}$$

b) We know that,

$$\tan\phi = \frac{X_L}{R} \Rightarrow \phi = \tan^{-1}\frac{(X_L)}{R}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{2\pi fL}{R}\right)$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{2 \times 3\cdot14 \times 50 \times 0\cdot0318}{7}\right)$$

$$\boxed{X_L = 2\pi fL}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{9\cdot9852}{7}\right) = \tan^{-1}\left(\frac{10}{7}\right) = \tan^{-1}(1\cdot43)$$

$$\Rightarrow \boxed{\phi = 55^\circ \text{ Lag}}$$

c) We know that, power factor = $\cos\phi$

$\Rightarrow$ Power factor = $\cos(55^\circ) = 0\cdot573$

$\Rightarrow$ $\boxed{\text{Power factor} = 0\cdot573 \text{ Lag}}$

d) We know that,

Power consumed = $IV\cos\phi$

$$\Rightarrow P_{consumed} = 18\cdot03 \times 220 \times \cos(55^\circ)$$

$$\Rightarrow P_{consumed} = (18\cdot03 \times 220 \times 0\cdot573) \text{ watt}$$

$$\Rightarrow \boxed{P_{consumed} = 2272\cdot8 \text{ watt}}$$

**ASSIGNMENT 15·4:**

220V, 50Hz, AC source is connected to an inductance of 0·2H and a resistance of 20Ω in series. What is the current in the circuit?

__Solution:__

__Given Data:__

Voltage = $V = 220V$, Frequency = $f = 50Hz$

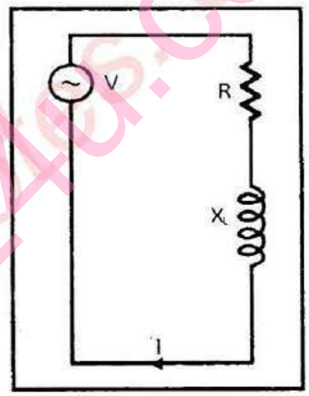
Inductance = $L = 0{\cdot}2H$, Resistance = $R = 20\Omega$

__Required:__

Current in inductive circuit = $I$ = ?

__Calculation:__

For RL circuit, we have,




$$V = IZ = I\sqrt{R^2 + X_L^2}$$

$$\Rightarrow 220 = I\sqrt{(20)^2 + (2\pi fL)^2}$$

$$\Rightarrow 220 = I\sqrt{(20)^2 + (2 \times 3{\cdot}14 \times 50 \times 0{\cdot}2)^2}$$

$$\Rightarrow 220 = I\sqrt{400 + 3943{\cdot}84}$$

$$\Rightarrow 220 = I\sqrt{4343{\cdot}84} = (65{\cdot}9)I$$

$$\Rightarrow 220 = (65{\cdot}9)I$$

$$\Rightarrow I = \left(\frac{220}{65{\cdot}9}\right)A$$

$$\Rightarrow \boxed{I = 3{\cdot}33A}$$

$$\boxed{\begin{aligned} Z &= \sqrt{R^2 + X_L^2} \\ X_L &= 2\pi fL \end{aligned}}$$

**EXAMPLE 15·6:**

A 100V, 50Hz AC supply is applied to a capacitor of capacitance 79·5μF connected in series with a non-inductive resistance of 30Ω. Find (1) impedance (2) current (3) phase angle and (4) equation for the instantaneous value of current.

__Solution:__

__Given Data:__

Voltage = $V = 100V$

Resistance = $R = 30\Omega$

Frequency = $f = 50Hz$

Capacitance = $C = 79{\cdot}5\mu F = 79{\cdot}5 \times 10^{-6}F$

Capacitive reactance = $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2 \times 3{\cdot}14 \times 50 \times 79{\cdot}5 \times 10^{-6}} = 40\Omega$

<u>Required:</u>

1) Impedance = Z = ?
2) Current = I = ?
3) Phase angle = $\phi$ = ? and
4) Equation for the instantaneous value of current = $I_{inst}$ = ?

<u>Calculation:</u>

1) For RC circuit, the impedance is given by,

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{(30)^2 + (40)^2}$$

$$\Rightarrow Z = \sqrt{900 + 1600} \Rightarrow Z = \sqrt{2500}$$

$$\Rightarrow \boxed{Z = 50\Omega}$$

2) We know that,

$$V = IZ \Rightarrow I_{rms} = \frac{V}{Z}$$

$$\Rightarrow I_{rms} = \left(\frac{100}{50}\right)A$$

$$\Rightarrow \boxed{I_{rms} = 2A}$$

3) Phase angle = $\phi = \tan^{-1}\left(\frac{X_c}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right)$

$$\Rightarrow \boxed{\phi = 53^\circ \text{ lead}}$$

4) The equation for instantaneous current is given by,

$$I = I_m \sin(\omega t + \phi)$$

$$\Rightarrow I = \sqrt{2}\, I_{rms} \sin(2\pi f t + \phi)$$

$$\boxed{I_m = \sqrt{2}\, I_{rms}}$$

$$\Rightarrow I = \sqrt{2} \times 2 \sin(2 \times 3.14 \times 50t + 53^\circ)$$

$$\boxed{\omega = 2\pi f}$$

$$\Rightarrow \boxed{I = 2.828 \sin(314t + 53^\circ)}$$

**EXAMPLE 15·7:**
A 220V, 50Hz AC supply is applied to a coil of 0·06H inductance and 2·5Ω resistance connected in series with a 6·8μF capacitor. Calculate (a) impedance (b) current (c) phase angle between current and voltage (d) power factor and (e) power consumed

<u>Solution:</u>

<u>Given Data:</u>

Inductance = L = 0·06H

Resistance = R = 2·5Ω

Voltage = V = 220V

Frequency = $f$ = 50Hz

Capacitance = $C = 6{\cdot}8\mu F = 6{\cdot}8 \times 10^{-6} F$

**Required:**

a) Impedance = $Z$ = ?

b) Current = $I$ = ?

c) Phase angle between current and voltage = $\phi$ = ?

d) Power factor = $\cos\phi$ = ? and

e) Power consumed = $P$ = ?

$X_L = 2\pi fL$

$X_L = 2 \times 3{\cdot}14 \times 50 \times 0{\cdot}06$

$X_L = 18{\cdot}85\Omega$

And $X_c = \dfrac{1}{2\pi fC}$

$X_c = \dfrac{1}{2 \times 3{\cdot}14 \times 50 \times 6{\cdot}8 \times 10^{-6}}$

$X_c = \dfrac{10^6}{2135{\cdot}2}$

$X_c = 468\Omega$

**Calculation:**

a) For RLC series circuit, we have,

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

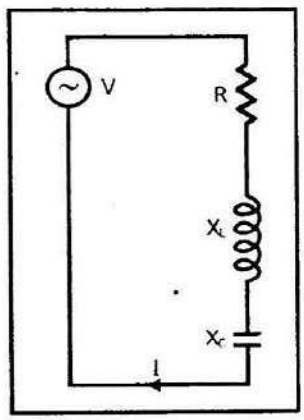
$$\Rightarrow Z = \sqrt{(2{\cdot}5)^2 + (18{\cdot}85 - 468)^2}$$

$$\Rightarrow \boxed{Z = 449{\cdot}2\Omega}$$

b) Current in RLC circuit is given by,

$$I = \frac{V}{Z} = \frac{220}{449{\cdot}2}$$

$$\Rightarrow \boxed{I = 0{\cdot}489A}$$




c) The phase angle is given by,

$$\phi = \tan^{-1}\left(\frac{X_L - X_c}{R}\right)$$

$$\Rightarrow \phi = \tan^{-1}\left[\frac{18{\cdot}85 - 468}{2{\cdot}5}\right]$$

$$\Rightarrow \boxed{\phi = -89{\cdot}7^\circ}$$

The negative sign shows that the current is leading the voltage.

d) Power factor = $\cos\phi = \cos(89{\cdot}7^\circ)$

$$\Rightarrow \boxed{\text{Power factor} = 0{\cdot}0052}$$

e) Power loss = $IV\cos\phi = 0{\cdot}489 \times 220 \times 0{\cdot}0052$

$$\Rightarrow \boxed{\text{Power loss} = 0{\cdot}60 \text{ watt}}$$

## Exercise

**Choose the best possible answer:**

1. The r.m.s value of a sine wave is 100A. Its peak value is:
   a) 70.7A b) 141.4A✓ c) 150A d) 282.8A

**Solution:**

$$I_m = \sqrt{2}\, I_{rms} = \sqrt{2} \times 100 = 141.4A$$

2. A sine wave has a frequency of 50Hz. Its angular frequency is ...... radian/second.
   a) 100π✓ b) 50π c) 25π d) 5π

**Solution:**

$$f = 50Hz$$
$$\omega = 2\pi f = 2\pi \times 50$$
$$\Rightarrow \omega = 100\pi$$

3. The power factor of a purely resistive circuit is:
   a) Zero b) Unity✓ c) Lagging d) Leading

**Solution:**

In purely resistive circuit, the current and voltage are in phase and apparent power and real power remains the same.

4. A capacitor is perfectly insulator for:
   a) Direct current✓ b) Alternating current
   c) Direct as well as alternating current
   d) Induced emf

**Solution:**

A capacitor is perfectly insulator for direct current. It is because when it is used in a direct current circuit, it charges upto its supply voltage but blocks the flow of current through it because the dielectric of a capacitor is non conductive and basically an insulator.

5. The peak value of alternating current is $5\sqrt{2}A$. The mean square value of current will be:
   a) 5A b) 2.5A c) $5\sqrt{2}A$ d) $5^2$✓

**Solution:**

$$I_m = 5\sqrt{2}$$

As $\quad I_m = \sqrt{2}\, I_{rms}$

$\Rightarrow \quad 5\sqrt{2} = \sqrt{2}\, I_{rms}$

$\Rightarrow \quad 5 = I_{rms} \Rightarrow I_{rms} = 5$

Now the square of mean current will be:

$$I^2_{rms} = 5^2$$

6. In choke coil the reactance $X_L$ and resistance $R$ are related as:

a) $X = = R$ b) $X_L << R$ c) $X_L >> R$✓ d) $X_L = \infty$

**Solution:**

As we know that,

$$V^2 = V_R^2 + V_L^2$$

$$V^2 = (IR)^2 + (IX_L)^2$$

$$\Rightarrow V^2 = I^2R^2 + I^2X_L^2$$

$$\Rightarrow I^2X_L^2 = V^2 - I^2R^2$$

$$\Rightarrow X_L^2 = \frac{V^2 - I^2R^2}{I^2}$$

$$\Rightarrow X_L = \sqrt{\frac{V^2 - I^2R^2}{I^2}}$$

$$\Rightarrow X_L >> R$$

7. The phase difference between the current and voltage at resonance is:

a) 0✓ b) $\pi$ c) $-\pi$ d) $\pi/2$

**Solution:**

At resonance, we have $X_L = X_C$ and thus the impedance is purely resistive i.e. $Z = R$. Thus the current and voltage will be in phase (parallel) and thus we have, phase difference = 0.

8. In AC system we generate sine wave form because:

a) It can be easily draw

b) It produces least disturbance in electrical circuits✓

c) It is nature standard

d) Other waves cannot be produced easily

9. An alternating voltage is given by 20sin 157t. The frequency of alternating voltage is:

a) 50Hz b) 25Hz✓ c) 100Hz d) 75Hz

**Solution:**

We have,

$$V = V_m \sin(\omega t) \text{ \_\_\_\_\_\_\_\_ (1)}$$

We have given that,

$$V = 20\sin(157t) \text{ \_\_\_\_\_\_\_\_ (2)}$$

Comparing equation (1) and equation (2), we get,

$$\omega t = 157t \Rightarrow \omega = 157$$

$$\Rightarrow 2\pi f = 157$$

$$\Rightarrow f = \frac{157}{2\pi} = \frac{157}{2 \times 3{\cdot}14} \qquad \boxed{\omega = 2\pi f}$$

$$\Rightarrow f = 25Hz$$

10· In LR circuit which one of the following statements is correct?

a) L and R opposes each other

b) R value increases with frequency

c) The inductive reactance increases with frequency✓

d) The inductive reactance decreases with frequency

**Solution:**

$$X_L = 2\pi fL \text{ \_\_\_\_\_\_\_\_ (1)}$$

Equation (1) shows that "$X_L$" increases when frequency "f" increases·

11· An alternating quantity (voltage or current) is completely known if we know it's:

a) Maximum value b) Frequency and phase

c) Effective value d) Both (a) and (b)✓

**Solution:**

$$I = I_m \sin(2\pi\phi \pm \phi)$$

And $V = V_m \sin(2\pi ft \pm \phi)$

12· For electromagnetic waves, Maxwell generalized:

a) Gausses law for magnetism

b) Gausses law for electricity

c) Faradays law b) Amperes law✓

13· An electromagnetic wave goes from air to glass which of the following does not change?

a) Radio waves b) X-rays

c) Ultraviolet radiation d) Ultrasound waves✓

14. The circuit in which current and voltage are in phase, the power factor is:

a) Zero  b) 1✓  c) -1  d) 2

**Solution:**

Power factor = $\cos\phi$ ____________(1)

When current and voltage are in phase, then we have $\phi = 0^\circ$, so equation (1) becomes,

Power factor = $\cos 0^\circ = 1$

$\Rightarrow$ $\boxed{\text{Power factor} = 1}$

15. With increase in frequency of an AC supply, the impedance of an RLC series circuit:

a) Remains constant  b) Increases✓

c) Decreases

d) Capacitive reactance $X_C$ decreases only

**Solution:**

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow Z = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

As $X_L > X_C$

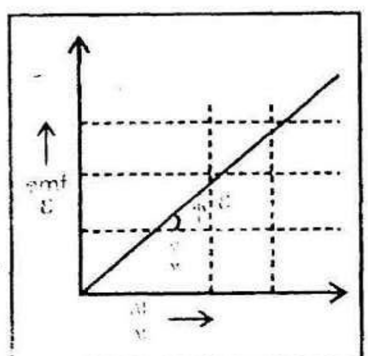
So when "f" increases then "Z" also increases.

## CONCEPTUAL QUESTIONS

**Q1: Sketch a graph of emf induced in an inductive coil against rate of change of current. What is the significance of the gradient?**

Answer:

When current in an inductive coil changes, the magnetic flux linking the coil changes. As a result an emf is induced in the coil whose magnitude is directly proportional to the rate of change of current in the coil.




Thus if we draw a graph between emf "ε" and "ΔI/Δt", we get a straight line as shown in the figure. The gradient (slope) of the graph is given by,

$$\text{Gradient} = \tan\phi = \frac{\varepsilon}{\left(\frac{\Delta I}{\Delta t}\right)}$$

$$\Rightarrow \text{Gradient} = \frac{\varepsilon}{\left(\frac{\Delta I}{\Delta t}\right)} \text{ ———— (1)}$$

<u>*Significance of the Gradient:*</u>

*In case of self induction phenomenon we have,*

$$\varepsilon = L\frac{\Delta I}{\Delta t}$$

$$\Rightarrow L = \frac{\varepsilon}{\left(\frac{\Delta I}{\Delta t}\right)} \text{ ———— (2)}$$

*Comparing equation (1) and equation (2), we get,*

$$\boxed{\text{Gradient} = L} \text{ ———— (3)}$$

*Equation (3) shows that the significance of the gradient is that it gives us the value of the self inductance of the coil.*

**Q2: (a) Current and voltage provided by an AC generator are sometimes negative and sometimes positive. Why for, an AC generator connected to a resistor, power can never be negative?**
**(b) Explain using sketch graphs, why the frequency of variation of power in an AC generator is twice as that of current and voltage?**

*Answer:*

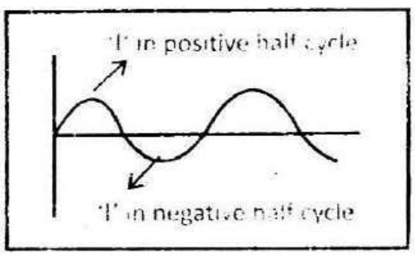
**<u>Part (a)</u>:** *We know that the power in terms of current "I" is given by,*

$$P = I^2R \text{ ———— (1)}$$

*While in terms of voltage, the power is given by,*

$$P = \frac{V^2}{R} \text{ ———— (2)}$$

*Now we know that the alternating current and voltage consists of positive and negative half cycles. But the power of the generator can never be negative. It is because according to equation (1) and equation (2) for any value of "I" and*



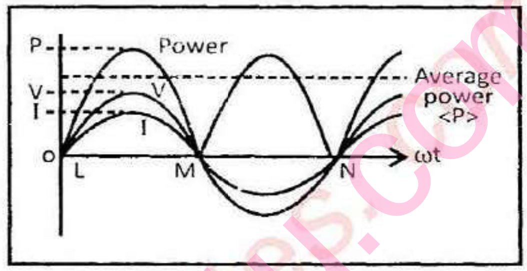
"V", the terms "$I^2$" and "$V^2$" remains positive. So according to equation (1) and equation (2), the power always remains positive. It can never be negative.

**Part (b):** We know that the average power is given by,

$$< P > = I_{rms} V_{rms}$$

$$\Rightarrow < P > = \frac{I_m}{\sqrt{2}} \cdot \frac{V_m}{\sqrt{2}} = \frac{I_m V_m}{2}$$

$$\Rightarrow 2 < P > = I_m V_m \quad \text{\_\_\_\_\_(3)}$$




Equation (3) shows that the frequency of variation of power in an A.C generator is twice as that of the current and voltage.

**Q3: What determines the gradient of a graph of inductive reactance against frequency?**

***Answer:***

The gradient of a graph of inductive reactance against frequency determines the inductance of the inductor.

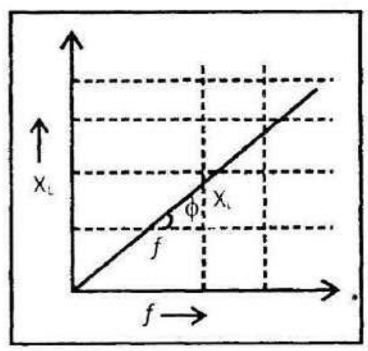
***Reason and Explanation:***

The relation between inductive reactance and frequency is given by,

$$X_L = 2\pi f L \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

Equation (1) shows that the inductive reactance is directly proportional to the frequency "f" i.e. the inductive reactance increases when the frequency increases and vice versa.

Thus if we plot a graph between "$X_L$" and "$f$", we get a straight line as shown in the figure.




Now the slope or gradient of the graph is given by,

$$\text{Gradient} = \tan\phi = \frac{X_L}{f}$$

$$\Rightarrow \text{Gradient} = \frac{X_L}{f} \quad \text{\_\_\_\_\_\_\_\_(2)}$$

From equation (1), we have,

$$X_L = 2\pi f L$$

$$\Rightarrow \frac{X_L}{f} = 2\pi L \quad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

Putting equation (3) in equation (2), we get,

$$\boxed{\text{Gradient} = 2\pi L} \quad \text{\_\_\_\_\_\_\_\_\_\_(4)}$$

Equation (4) shows that from the graph of "$X_L$" and "f", we can determine the value of inductance "L" of the inductor.

**Q4: How does doubling the frequency affect the reactance of (a) an inductor (b) a capacitor?**

Answer:

When we double the frequency, then the reactance of an inductor will also be double. While that of the capacitor will becomes half.

**Reason and Explanation:**

**Part (a):** In case of an inductor, we have,

$$X_L = 2\pi fL \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

When frequency becomes double then equation (1) becomes,

$$X_L' = 2\pi f' L \Rightarrow X_L' = 2\pi(2f)L$$

$$\Rightarrow X_L' = 2\left[2\pi fL\right] \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

Putting equation (1) in equation (2), we get,

$$\boxed{X_L' = 2X_L} \quad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

Equation (3) shows that the reactance of the inductor will becomes double if the frequency increases two times.

**Part (b):** In case of a capacitor, we have,

$$X_c = \frac{1}{2\pi fC} \quad \text{\_\_\_\_\_\_\_\_\_\_(4)}$$

Now when the frequency becomes double, then we have, $f' = 2f$, so equation (4) becomes,

$$X_c' = \frac{1}{2\pi f'C} \Rightarrow X_c' = \frac{1}{2\pi(2f)C}$$

$$X_c' = \frac{1}{2}\left(\frac{1}{2\pi fC}\right) \quad \text{\_\_\_\_\_\_\_\_\_\_(5)}$$

Putting equation (4) in equation (5), we get,

$$\boxed{X_c' = \frac{1}{2}X_c} \quad \text{\_\_\_\_\_\_\_\_\_\_(6)}$$

Equation (6) shows that the reactance of the capacitor becomes half if the frequency becomes double.

**Q5: If the peak value of a sine wave is 1000 volts, what is the effective value?**

*Answer:*

If the peak value of a sine wave is 1000 volts, then the effective value will be equal to 707 volts.

Reason and Explanation:

Peak value voltage = $V_m = 1000V$

Effective value = $V_{rms}$ = ?

We know that the relation between effective value (rmf value) and peak value of voltage is given by,

$$V_{rms} = 0.707V_m \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

Putting the value of "$V_m$" in equation (1), we get,

$$V_{rms} = 0.707 \times 1000 \text{ volts} = 707 \text{ volts}$$

$$\Rightarrow \boxed{\text{Effective value} = V_{rms} = 707 \text{ volts}}$$

**Q6: Show that reactance is measured in ohms for both inductors and capacitors.**

*Answer:*

The reactance for both inductors and capacitors are measured in ohms.

Reason and Explanation:

Part (a): We know that the inductive reactance is given by,

$$X_L = \omega L \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

Dividing and multiplying the R.H.S of equation (1) by "$I_m$" we get,

$$X_L = \frac{I_m \omega L}{I_m} \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

We also know that,

$$I_m \omega L = V_m \quad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

Putting equation (3) in equation (2), we get,

$$X_L = \frac{V_m}{I_m} \Rightarrow X_L = \frac{\text{volt}}{\text{amp}} = \text{ohm}$$

$$\Rightarrow \boxed{X_L = \text{ohm}}$$

Thus "ohm" is the unit of an inductive reactance, which represents the opposition to the flow of current in inductive circuit.

Part (b): For capacitive circuit, we have

$$X_c = \frac{1}{\omega C} \quad \underline{\qquad\qquad} (4)$$

Multiplying and dividing the R·H·S of equation (4) by "$V_m$" we get,

$$X_c = \frac{V_m}{V_m \omega C} \quad \underline{\qquad\qquad} (5)$$

We also know that,

$$V_m \omega C = I_m \quad \underline{\qquad\qquad} (6)$$

Putting equation (6) in equation (5), we get,

$$X_c = \frac{V_m}{I_m} \Rightarrow X_c = \frac{volt}{amp} = ohm$$

$$\Rightarrow \boxed{X_c = ohm}$$

Thus "ohm" is the unit of capacitive reactance which represents the opposition to the flow of current in capacitive circuit.

**Q7: What is the principle of ECG?**

Answer:

ECG stands for "Electro Cardio Gram" which is used for diagnosing the condition of heart. It works in the following steps:

1. It works on the detection and amplification of smallest electrical changes on our skin.
2. The electrical changes on skin takes place due to depolarization of heart muscle during heart beat.
3. During heart beat, a small voltage difference occurs between two points on our skin.
4. The small voltage difference can be recorded with the help of electrodes which are placed on various parts of the body.
5. Finally the information about the heart condition are printed on a paper in the form of wave curve.

## COMPREHENSIVE QUESTIONS

**Q1: Define mean, peak and rms value of sinusoidal current and sinusoidal voltage. Obtain mathematical expression for the rms value of current.**

Answer: See answer of question # 3 in the text.

**Q2: A sinusoidal alternating voltage of angular frequency "$\omega$" is connected across a resistor "R". Find mathematical expression for instantaneous voltage instantaneous current and the average power dissipated per cycle of the applied voltage.**

Answer: See answer of question # 6 in the text.

**Q3: A sinusoidal alternating voltage of angular frequency "$\omega$" is connected across a capacitor C. Find mathematical expression for instantaneous voltage instantaneous current and the average power dissipated per cycle of the applied voltage.**

Answer: See answer of question # 10 in the text.

**Q4: Explain the term impedance of an AC circuit. Find expression for the of the RLC series circuit.**

Answer: See answer of question # 15 in the text.

**Q5: Describe that maximum power is transferred when the impedances of source and load match to each other.**

Answer: See answer of question # 18 in the text.

**Q6: In an RL series circuit will the current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.**

Answer: See answer of question # 12 in the text.

**Q7: In an RC series circuit will be current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.**

Answer: See answer of question # 14 in the text.

**Q8: Describe the statement of four Maxwell's equations. Using Maxwell's theory show that $E = \frac{1}{2\pi r}\frac{\Delta\phi}{\Delta t} = \frac{A}{2\pi r}\frac{\Delta B}{\Delta t}$ & $I = \frac{\varepsilon_o \Delta(\phi)}{\Delta t}$**

Answer: See answer of question # 19 in the text.

Q9: Describe the principle of metal detectors with suitable diagram.

Answer: See answer of question # 17 in the text.

## NUMERICAL QUESTIONS

Q1: The peak voltage of an AC supply is 300V. What is the rms voltage?

Solution:

Given Data:

Peak voltage = $V_m$ = 300V

Required:

rms voltage = $V_{rms}$ = ?

Calculation:

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\Rightarrow V_{rms} = 0.707 V_m$$

We know that,

$$V_{rms} = 0.707\, V_m$$

$$\Rightarrow V_{rms} = 0.707 \times 300V$$

$$\Rightarrow \boxed{V_{rms} = 212.1 \text{ volts}}$$

Q2: The rms value of current in an AC circuit is 10A. What is the peak current?

Solution:

Given Data:

rms current = $I_{rms}$ = 10A

Required:

Peak current = $I_m$ = ?

Calculation:

We know that,

$$I_{rms} = 0.707\, I_m$$

$$\Rightarrow I_m = \left(\frac{I_{rms}}{0.707}\right)A = \left(\frac{10}{0.707}\right)A = 14.1A$$

$$\Rightarrow \boxed{I_m = 14.1A}$$

Q3: The A.C voltage across a 0.5μF capacitor is 16sin $(2 \times 10^3 t)$. Find (a) the capacitive reactance (b) the peak value of current through the capacitor.

Solution:

Given Data:

Capacitance = $C = 0.5\mu F = 0.5 \times 10^{-6}F$

AC voltage = $V = 16\sin(2 \times 10^{3}t)V$

Required:

a) Capacitance reactance = $X_C$ = ?

b) Peak current = $I_m$ = ?

Calculation:

Part (a): $V = 16\sin(2 \times 10^{3}t)$ ______________(1)

Also $V = V_m \sin(\omega t)$ ______________(2)

Comparing equations (1), (2), we get,

$V_m = 16V, \ \omega = 2 \times 10^{3} \Rightarrow 2\pi f = 2 \times 10^{3}$ $\boxed{\omega = 2\pi f}$

Now we know that,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 10^{3} \times 0.5 \times 10^{-6}}$$

$$\Rightarrow \boxed{X_C = 1000\Omega}$$

Part (a): We know that,

$$V_m = I_m X_C \Rightarrow I_m = \frac{V_m}{X_C}$$

$$\Rightarrow I_m = \left(\frac{16}{1000}\right)A \Rightarrow I_m = 0.016A$$

$$\Rightarrow I_m = 16.0 \times 10^{-3}A = 16mA$$

$$\Rightarrow \boxed{I_m = 16mA}$$

**Q4: The voltage across a $0.01\mu F$ capacitor is $240 \sin(1.25 \times 10^{4}t - 30^{\circ})V$. Write the mathematical expression for the current through it.**

Solution:

Given Data:

Capacitance = $C = 0.01\mu F = 0.01 \times 10^{-6}F$

Voltage equation = $V = 240 \sin(1.25 \times 10^{4}t - 30^{\circ})V$

Required:

Mathematical expression for current = ?

Calculation:

We have given that,

$$V = 240\sin(1.25\times10^4 t - 30^\circ) \qquad \boxed{V = IX_c}$$

$$\Rightarrow IX_c = 240\sin(1.25\times10^4 t - 30^\circ)$$

$$\Rightarrow I = \frac{240\sin(1.25\times10^4 t - 30^\circ)}{X_c} \quad \_\_\_\_\_\_(1)$$

In case of capacitive circuit, the current leads the voltage by $\frac{\pi}{2}$ i.e. $90^\circ$, so equation (1) becomes,

$$I = \frac{240\sin(1.25\times10^4 t - 30^\circ + 90^\circ)}{X_c} \quad \_\_\_\_\_\_(2)$$

We also know that,

$$I = I_m\sin(\omega t + \phi) \quad \_\_\_\_\_\_(3)$$

Comparing equation (2) and equation (3), we get,

$$\omega = 1.25\times10^4$$

$$\Rightarrow 2\pi f = 1.25\times10^4 \quad \_\_\_\_\_\_(4) \qquad \boxed{\omega = 2\pi f}$$

$$\text{Now} \quad X_c = \frac{1}{2\pi fC} = \left(\frac{1}{1.25\times10^4\times0.01\times10^{-6}}\right)\Omega$$

$$\Rightarrow X_c = 8000\Omega$$

Putting $X_c = 800\Omega$ in equation (2), we get,

$$I = \frac{240}{8000}\sin(1.25\times10^4 t - 30^\circ + 90^\circ)$$

$$\Rightarrow \boxed{I = 0.03\sin(1.25\times10^4 t + 60^\circ)A}$$

**Q5: An inductor with an inductance of 100μH passes a current of 10mA when its terminal voltage is 6.3V. Calculate the frequency of A.C supply.**

Solution:

Given Data:

Inductance = $L = 100\mu H = 100\times10^{-6}H$

Current = $I = 10mA = 10\times10^{-3}A$

Voltage = $V = 6.3V$

Required:

Frequency of A.C supply = $f$ = ?

Calculation:

We know that,

$$X_L = 2\pi fL \Rightarrow f = \frac{X_L}{2\pi L}$$

$$\Rightarrow f = \frac{\left(\frac{V}{I}\right)}{2\pi L}$$

$$\Rightarrow f = \frac{V}{2\pi LI}$$

$$\Rightarrow f = \frac{6.3}{(2 \times 3.14 \times 100 \times 10^{-6} \times 10 \times 10^{-3})}$$

$$\Rightarrow f = 1 \times 10^6\,Hz \Rightarrow \boxed{f = 10^6\,Hz}$$

$X_L = 2\pi fL$
$V = IX_L$
$\Rightarrow X_L = \frac{V}{I}$

**Q6: (a) Calculate the inductive reactance of a 3·00mH inductor, when 60·0Hz and 10·0kHz AC voltages are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120V?**

<u>Solution:</u>

<u>Given Data:</u>

Inductance = $L = 3mH = 3 \times 10^{-3}H$, $V_{rms} = 120V$

Frequency = $f_1 = 60Hz$, $f_2 = 10kHz = 10 \times 10^3 Hz$

<u>Required:</u>

a) Inductive reactance = $X_L$ = ?

b) $I_{rms}$ = ?

<u>Calculation:</u>

a) We know that,

$$X_L = 2\pi fL$$

$$\Rightarrow X_{1L} = 2\pi f_1 L = 2 \times 3.14 \times 60 \times 3 \times 10^{-3}$$

$$\Rightarrow \boxed{X_{1L} = 1.13\Omega}$$

For $f_2 = 10 \times 10^3\,Hz$, we have,

$$X_{2L} = 2\pi f_2 L = 2 \times 3.14 \times 10 \times 10^3 \times 3 \times 10^{-3}$$

$$\Rightarrow \boxed{X_{2L} = 188\Omega}$$

b) As $V = IX_L$

$$\Rightarrow V_{rms} = I_{rms} X_L$$

$$\Rightarrow V_{rms} = I_{rms} X_{1L}$$

$$\Rightarrow I_{rms} = \frac{V_{rms}}{X_{1L}} = \frac{120}{1.13}$$

$$\Rightarrow \boxed{I_{rms} = 106A}$$

For $X_{2L} = 188\Omega$, we have,

$$I_{rms} = \frac{V_{rms}}{X_{2L}} = \frac{120}{188}$$

$$\Rightarrow \boxed{I_{rms} = 0.637A}$$

**Q7: For the same RLC series circuit having a 40.0Ω resistor, a 3.00mH inductor, and a 5.00μF capacitor. (a) Find the resonant frequency (b) Calculate $I_{rms}$ at resonance if $V_{rms}$ is 120V.**

**Solution:**

**Given Data:**

Resistance = $R = 40\Omega$

Inductance = $L = 3mH = 3 \times 10^{-3} H$

Capacitance = $C = 5\mu F = 5 \times 10^{-6} F$

$V_{rms} = 120V$

**Required:**

a) Resonant frequency = $f_r = ?$

b) rms current = $I_{rms} = ?$

**Calculation:**

a) We know that the resonance frequency is given by,

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad \text{———————} (1)$$

Putting the values in equation (1), we get,

$$f_r = \frac{1}{2 \times 3.14\sqrt{3 \times 10^{-3} \times 5 \times 10^{-6}}}$$

$$\Rightarrow f_r = \frac{1}{2 \times 3.14 \times 1.22 \times 10^{-4}}$$

$$\Rightarrow f_r = 0.130 \times 10^{4} Hz = 1.30 \times 10^{3} Hz$$

$$\Rightarrow \boxed{f_r = 1.30kHz}$$

b) As $V = iR$

$$\Rightarrow V_{rms} = I_{rms} R$$

$$\Rightarrow \quad I_{rms} = \frac{V_{rms}}{R} = \frac{120}{40} = 3A$$

$$\Rightarrow \quad \boxed{I_{rms} = 3A}$$

**Q8: A resistor of resistance 30Ω is connected in series with a capacitor of capacitance 79·5μF across a power supply of 50Hz and 100V. Find (a) impedance (b) current (c) phase angle and (d) equation for the instantaneous value of current.**

Solution:

Given Data:

Resistance = $R = 30\Omega$

Capacitance = $C = 79{\cdot}5\mu F = 79{\cdot}5 \times 10^{-6} F$

Frequency = $f = 50Hz$

Voltage = $V_{rms} = 100V$

Required:

a) Impedance = $Z$ = ?

b) rms current = $I_{rms}$ = ?

c) Phase angle = $\phi$ = ?

d) Equation for instantaneous current = $I_{inst}$ = ?

Calculation:

a) The impedance of RC series circuit is given by,

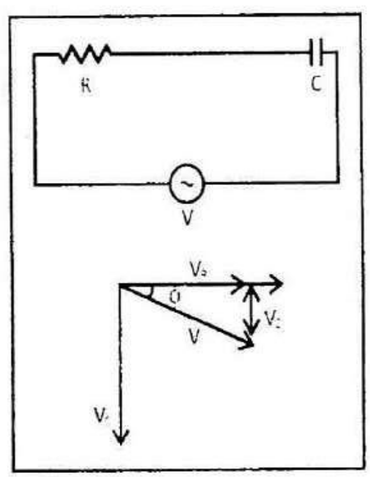
$$Z = \sqrt{R^2 + X_c^2} \quad \text{\_\_\_\_\_\_\_\_\_\_\_\_(1)}$$

We have, $X_c = \frac{1}{2\pi fC}$

$$\Rightarrow \quad X_c = \frac{1}{2 \times 3{\cdot}14 \times 50 \times 79{\cdot}5 \times 10^{-6}}$$

$$= 4{\cdot}00 \times 10^{-5} \times 10^{6}$$

$$\Rightarrow \quad X_c = 4{\cdot}00 \times 10^{1}\,\Omega \quad \Rightarrow \quad X_c = 40\Omega$$




Putting the values of "R" and "$X_C$" in equation (1), we get,

$$Z = \sqrt{(30)^2 + (40)^2}\,\Omega$$

$$= \sqrt{900 + 1600}\,\Omega = \sqrt{2500}$$

$$\Rightarrow \quad \boxed{Z = 50\Omega}$$

b) The rms current is given by,

$$I_{rms} = \frac{V_{rms}}{Z} = \left(\frac{100}{50}\right)A$$

$$\Rightarrow \boxed{I_{rms} = 2A}$$

c) The phase angle is given by,

$$\tan\phi = \frac{V_c}{V_R} \Rightarrow \tan\phi = \left(\frac{IX_c}{IR}\right) = \frac{X_c}{R}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{X_c}{R}\right) = \tan^{-1}\left(\frac{40}{30}\right) \Rightarrow \phi = 53^\circ$$

$$\Rightarrow \boxed{\phi = 53^\circ \text{ lead}}$$

> In capacitance circuit, the current lead the voltage "$V_c$"

d) We know that,

$$I = I_m \sin(\omega t + \phi) \text{ \_\_\_\_\_\_\_\_ } (2)$$

We also know that,

$$I_{rms} = 0.707\, I_{m} \Rightarrow I_m = \frac{I_{rms}}{0.707}$$

$$\Rightarrow I_m = \left(\frac{2}{0.707}\right)A \Rightarrow I_m = 2.828A$$

Also we have,

$$\omega = 2\pi f = 2 \times 3.14 \times 50 \Rightarrow \omega = 314 \text{ rev/sec}$$

Putting the values of "$I_m$" and "$\omega$" and "$\phi$" in equation (2), we get,

$$\boxed{I = 2.828 \sin(314t + 53^\circ)}$$

**Q9: A coil having a resistance of 7Ω and an inductance of 31.8mH is connected to 230V, 50Hz supply. Calculate (a) the circuit current (b) phase angle (c) power factor (d) power consumed.**

Solution:

Given Data:

Resistance = $R = 7\Omega$

Inductance = $L = 31.8mH = 31.8 \times 10^{-3}H$

Voltage = $V = 230V$

Frequency = $f = 50Hz$

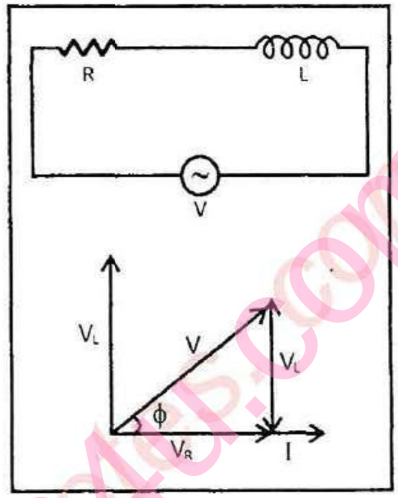
Required:

a) Circuit current = $I$ = ?

b) Phase angle = $\phi$ = ?

c) Power factor = $\cos\phi$ = ?

d) Power consumed = P = ?




In inductance resistive circuit, the current lag behind the voltage "$V_L$"

<u>Calculation:</u>

a) We know that,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$\Rightarrow I = \frac{V}{\sqrt{R^2 + (2\pi f L)^2}}$$

$$\Rightarrow I = \frac{230}{\sqrt{7^2 + (2 \times 3.14 \times 50 \times 31.8 \times 10^{-3})^2}}$$

$$\Rightarrow I = \frac{230}{\sqrt{49 + 99.7}} = \frac{230}{\sqrt{148.8}}$$

$$\Rightarrow I = \frac{230}{12.2} \Rightarrow \boxed{I = 18.85A}$$

b) $$\tan\phi = \left(\frac{V_L}{V_R}\right) = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{2\pi f L}{R}\right)$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{2 \times 3.14 \times 50 \times 31.8 \times 10^{-3}}{7}\right)$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{9.99}{7}\right) = \tan^{-1}(1.43)$$

$$\Rightarrow \boxed{\phi = 55^\circ \text{ lag}}$$

c) Power factor = $\cos\phi$

$$\Rightarrow P.F = \cos 55^\circ$$

$$\Rightarrow \boxed{P.F = 0.573}$$

d) Power consumed = $IV\cos\phi = 18.85 \times 230 \times \cos 55^\circ$

$$\Rightarrow P = 18.85 \times 230 \times 0.573$$

$$\Rightarrow \boxed{P = 2484.24 \text{ watt}}$$

**Q10: A capacitor which has a resistance of $10\Omega$ and a capacitance value of $100\mu F$ is connected to a supply voltage given as $V_{(t)} = 100 \sin(314t)$. Calculate the capacitive reactance, circuit impedance and peak current flowing into the capacitor.**

__Solution:__

__Given Data:__

Resistance $= R = 10\Omega$

Capacitance $= C = 100\mu F = 100 \times 10^{-6} F$

Voltage $= V = 100\sin(314t)$

__Required:__

a) Capacitance reactance $= X_c = ?$

b) Circuit impedance $= Z = ?$

c) Peak current $= I_m = ?$

__Calculation:__

a) Voltage equation is given by,

$$V = 100\sin(314t) \quad \text{\_\_\_\_\_\_\_\_(1)}$$

Also $$V = V_m \sin(\omega t) \quad \text{\_\_\_\_\_\_\_\_(2)}$$

Comparing equation (1) and equation (2), we get,

$$V_m = 100V, \; \omega = 314$$

$$\Rightarrow \boxed{2\pi f = 314}$$

$$\Rightarrow f = \frac{314}{2\pi} = \frac{314}{2 \times 3.14} \qquad \boxed{\omega = 2\pi f}$$

$$\Rightarrow f = 50Hz$$

For RC-series circuit, we have,

$$Z = \sqrt{R^2 + X_c^2} \quad \text{\_\_\_\_\_\_\_\_(1)}$$

Here $$X_c = \frac{1}{2\pi fC} = \frac{1}{314 \times 100 \times 10^{-6}} = \frac{1}{314 \times 10^{-4}}$$

$$\Rightarrow X_c = 3.185 \times 10^{1}\,\Omega$$

$$\Rightarrow \boxed{X_c = 31.8\Omega}$$

Putting the value of "$X_c$" and "R" in equation (1), we get,

$$Z = \sqrt{(10)^2 + (31.8)^2}$$

$$\Rightarrow Z = \sqrt{100 + 1011.24}$$

$$\Rightarrow Z = \sqrt{1111.24}$$

$$\Rightarrow \boxed{Z = 33.4\Omega}$$

c) $V_m = I_m Z$

$$\Rightarrow I_m = \frac{V_m}{Z} = \frac{100}{33.4} A$$

$$\Rightarrow I_m = 2.99A$$

$$\Rightarrow \boxed{I_m = 3A}$$