

Numerical Problems

1. The peak voltage of an ac supply is 300 V. What is the rms voltage?

Give:

Maximum voltage = $V_{\max} = 300\text{V}$

To find:

Root mean square voltage (V_{rms}) = ?

Solution:

The rms voltage is defined as

$$V_{\text{rms}} = V_{\text{rms}} = \frac{V_{\max}}{\sqrt{2}} = 0.707 V_{\max}$$

Putting values:

$$V_{\text{rms}} = 0.707 \times 300\text{V}$$

$$\boxed{V_{\text{rms}} = 212.1\text{V}}$$

2. The rms value of current in an ac circuit is 10 A. What is the peak current?

Given:

Root mean square current $I_{\text{rms}} = 10\text{A}$

To find:

Maximum current $I_{\max} = ?$

Solution:

The rms current is defined as

$$I_{\text{rms}} = \frac{I_{\max}}{\sqrt{2}}$$

Putting values:

$$I_{\max} = \sqrt{2} \times 10\text{A}$$

$$I_{\max} = 1.41 \times 10\text{A}$$

Or $I_{\max} = 14.1\text{ A}$

3. The a.c. voltage across a $0.5\text{ }\mu\text{F}$ capacitor is $16 \sin (2 \times 10^3 t)\text{ V}$. Find (a) the capacitive reactance (b) the peak value of current through the capacitor.

Given:

Capacitance $C = 0.5\text{ }\mu\text{F} = 0.5 \times 10^{-6}\text{ F}$

Instantaneous voltage $V_c = 16 \sin (2 \times 10^3 t)\text{ v}$

Required:

Capacitive reactance $X_c = ?$

Peak current $I_{\max} = ?$

Solution:

Few more data points can be calculated by comparing the given instantaneous voltage equation with the general instantaneous voltage equation for capacitor.

The given equation is $V_c = 16 \sin (2 \times 10^3 t)$ (i)

The general equation is $V_c = V_{\max} \sin \omega t$ (ii)

Comparing equation (i) and (ii), we get

The peak voltage is $V_{\max} = 16\text{V}$

The angular frequency = $\omega = 2 \times 10^3\text{ rad / s}$

(a) The capacitive reactance x_c is given by the relation

$$X_c = \frac{1}{\omega_c}$$

$$X_c = \frac{1}{2 \times 10^3 \text{ rad/s} \times 0.5 \times 10^{-6} \text{ C/v}} = X_c = 1 \times 10^3 \Omega = 1 \text{ k}\Omega$$

(b) By Ohm's law the peak current be written as

$$I_{\max} = \frac{V_{\max}}{X_c}$$

Putting values: $I_{\max} = \frac{16}{10^3} \Rightarrow 16 \times 10^{-3} \text{ A} \Rightarrow 16 \text{ mA}$

$$I_{\max} = 16 \times 10^{-3} \text{ A} = 16 \text{ mA}$$

4. The voltage across a $0.01 \mu\text{F}$ capacitor is $240 \sin (1.25 \times 10^4 t - 30^\circ)$, write the mathematical expression for current through it.

Given: Capacitance $C = 0.01 \mu\text{F} = 0.01 \times 10^{-6} \text{ F}$

Instantaneous voltage $V_c = 240 \sin (1.25 \times 10^4 t - 30^\circ)$

To find: Instantaneous current $I_c = ?$

Solution:

Few more data points can be calculated by comparing the given instantaneous voltage equation with the general instantaneous.

The given equation is $V_c = 240 \sin (1.25 \times 10^4 t - 30^\circ)$ (i)

The general equation is $V_c = V_{\max} \sin \omega t$ (ii)

Comparing equation (i) and equation (ii), we get

the peak voltage is $V_{\max} = 240 \text{ V}$

The angular frequency is $\omega t = 1.25 \times 10^4 \text{ rad/s}$

Instantaneous current for capacitor i_c can be written as $I_c = I_{\max} \sin (\omega t - \phi + 90^\circ)$ (iii)

By Ohm's law the peak current can be written as

$$I_{\max} = \frac{V_{\max}}{X_c}$$

Putting equation (iv) in equation (iii), we get

$$I_c = V_{\max} \times \omega_c \times \sin (\omega - \Phi + 90^\circ)$$

Putting

$$I_c = 240\text{V} \times 1.25 \times 10^4 \text{ rad / s} \times 0.01 \times 10^{-6} \text{F} \times \sin (1.25 \times 10^4 t - 30^\circ + 90^\circ)$$

$$\text{Or } I_c = 3 \times 10^{-2} \text{ V/s} \times \gamma \times \sin (1.25 \times 10^4 t + 60^\circ)$$

$$\text{Hence } I_c = 3 \times 10^{-2} \text{ A} \times \sin (1.25 \times 10^4 t + 60^\circ)$$

$$I_c = 3 \times 10^{-2} \sin (1.25 \times 10^4 t + 60^\circ)$$

5. An inductor with an inductance of $100\mu\text{H}$ passes a current of 10mA when its terminal voltage is 6.3V . Calculate the frequency of A.C supply.

Given:

$$\text{Inductance } L = 100\mu\text{H} = 100 \times 10^{-6} \text{ H}$$

$$\text{Current } I = 10\text{mA} = 10 \times 10^{-3} \text{ A}$$

$$\text{Voltage } v = 6.3\text{V}$$

Required:

$$\text{Ac frequency } f = ?$$

Solution:

The inductive reactance X_L is

$$X_L = \omega L = 2\pi f \times L$$

$$\text{Or } f = \frac{X_L}{2\pi L} \quad (i)$$

The inductive reactance X_L can also be written as

$$X_L = \frac{V}{I} = \frac{6.3\text{V}}{10 \times 10^{-3} \text{ A}}$$

$$\text{or } X_L = 0.63 \times 10^3 \Omega \quad (ii)$$

$$f = 1.003 \times 10^6 \text{ Hz}$$

6. (a) Calculate the inductive reactance of a 3.00 mH inductor, when 60.0 Hz and 10.0 kHz AC voltage are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120 V?

Given:

Inductance $L = 3.00 \text{ mH} = 3.00 \times 10^{-3} \text{ H}$

AC frequency $f_1 = 60 \text{ Hz}$

AC frequency $f_2 = 10 \text{ kHz} = 10 \times 10^3 \text{ Hz}$

Root mean square voltage $V_{\text{rms}} = 120 \text{ V}$

Required:

(a) Inductive reactance $X_{L1} = ?$

(b) Inductive reactance $X_{L2} = ?$

(c) $I_{\text{rms}1}$ at frequency $f_1 = ?$

(d) $I_{\text{rms}2}$ at frequency $f_2 = ?$

Solution:

The inductive reactance at frequency f_1 is

$$X_{L1} = 2\pi f_1 \times L$$

Putting values: $X_{L1} = 2 \times 3.14 \times 60 \text{ Hz} \times 3.00 \times 10^{-3} \text{ H}$

or $X_{L1} = 1.13041 \text{ } \Omega \text{ s} \times \text{V s/ A}$

or $X_{L1} = 1.1304 \text{ ohm}$

The inductive reactance at frequency f_1 is

$$X_{L2} = 2\pi f_2 \times L$$

Putting values: $X_{L2} = 2 \times 3.14 \times 10 \times 10^3 \text{ Hz} \times 3.00 \times 10^{-3} \text{ H}$

The rms current at frequency f_1 is $I_{rms1} = \frac{V_{rms}}{X_{L1}}$

Putting values: $I_{rms1} = \frac{120}{1.1304\Omega}$

or $I_{rms1} = 106.16A$

The rms current at frequency f_2 is $I_{rms2} = \frac{V_{rms}}{X_{L2}}$

Putting values: $I_{rms2} = \frac{120}{188.4\Omega}$

or $I_{rms2} = 0.637A$

7. For the same RLC series circuit having a 40.0Ω resistor, a 3.00 mH inductor, and a $5.00\text{ }\mu\text{F}$ capacitor: (a) Find the resonant frequency, (b) Calculate I_{rms} at resonance if V_{rms} is 120 V .

Given:

Resistance $R = 40.0\Omega$

Inductance $L = 3.00\text{mH} = 3.00 \times 10^{-3}\text{H}$

Capacitance $C = 5.00\mu\text{F} = 5.00 \times 10^{-6}\text{F}$

Root mean square voltage $V_{rms} = 120\text{V}$

Required:

(a) Resonant frequency $f_0 = ?$

(b) I_{rms} at frequency $f_0 = ?$

Solution:

The resonant frequency is $f_0 = \frac{1}{2\pi\sqrt{L \times C}}$

Putting values: $f_0 = \frac{1}{6.28\sqrt{15.00 \times 10^{-9}\text{Vs/A} \times \text{C/v}}}$

Hence, $f_0 = \frac{1}{6.28\sqrt{1.500 \times 10^{-9}\text{Vs/A} \times \text{As/v}}}$

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Since at resonance the series RLC circuit behaves as purely resistive circuit and the impedance $Z=R$, therefore the rms current at resonance is

$$I_{rms} = V_{rms} / R$$

putting values:

$$I_{rms} = \frac{120V}{40\Omega}$$

or $I_{rms} = 3.00A$

8. A coil of pure inductance 318 mH is connected in series with a pure resistance of 75Ω . The voltage across resistor is 150V and the frequency of power supply is 50Hz. Calculate the voltage of power supply and the phase angle.

Given:

Inductance $L = 318mH = 318 \times 10^{-3}H$

Resistance $R = 75\Omega$

AC frequency $f = 50Hz$

Voltage across resistor $V_R = 150V$

Required:

Supply voltage $v = ?$, Phase angle $\theta = ?$

Solution:

The inductive reactance X_L is $X_L = \omega L = 2\pi f \times L$

or $X_L = 2 \times 3.14 \times 50Hz \times 318 \times 10^{-3}H$

Or $X_L = 99852 \times 10^{-3}Vs/A \times V/s$

Hence, $X_L = 99.852V / A = 99.852\Omega$

The supply voltage relation is $V = I \times Z$ (i)

For series AC circuit the current is same through all components therefore

$$I = \frac{V_R}{R} = \frac{150V}{75\Omega} = 2A \quad (ii)$$

For series R_L circuit the impedance Z is given by the relation

$$Z = \sqrt{R^2 + X_L^2}$$

Putting values: $Z = \sqrt{(75\Omega)^2 + (99.85\Omega)^2}$

$$Z = 124.88\Omega \quad (iii)$$

Putting equation (ii) and equation (iii) in equation (i), we get

$$V = 2A \times 124.88\Omega$$

$$V = 249.76V = 250V$$

The Phase angle ' θ ' is $\theta = \tan^{-1} \frac{X_L}{R}$

Putting values $= \tan^{-1} \frac{99.852\Omega}{75\Omega}$

$$\theta = 53.06^\circ$$

9. A resistor of resistance 30Ω is connected in series with a capacitor of capacitance $79.5 \mu F$ across a power supply of 50Hz and $100V$. Find (a) impedance (b) current (c) phase angle and (d) equation for the instantaneous value of current.

Given:

Capacitance $C = 79.5 \mu F = 79.5 \times 10^{-6}F$

Resistance $R = 30\Omega$

AC frequency $f = 50\text{Hz}$

Root mean square voltage $V_{rms} = 100V$

Required: (a) The impedance $Z = ?$

(b) The current $I = ?$

(c) Phase angle $\theta = ?$

Solution: The capacitive reactance X_c is

$$X_c = \frac{1}{\omega_c} = \frac{1}{2\pi f \times C}$$

$$X_c = \frac{1}{2.314 \times 50 \text{ Hz} \times 79.5 \times 10^{-6} \text{ F}}$$

$$X_c = \frac{1}{24.963 \times 10^{-3} \text{ I/s} \times \text{C/V}}$$

$$X_c = \frac{1}{24.963 \times 10^{-3} \text{ A/V}} = 0.0400 \times 10^3 \text{ V/A} = 40 \Omega$$

(a) The impedance Z is $Z = \sqrt{R^2 + X_c^2}$

$$Z = \sqrt{(30 \Omega)^2 + (40 \Omega)^2}$$

$$\boxed{Z = 50 \Omega}$$

(b) The current 'I' is given as $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$

Putting values: $I_{\text{rms}} = \frac{100 \text{ V}}{50 \Omega}$

$$\boxed{I_{\text{rms}} = 2 \text{ A}}$$

(c) The Phase angle θ is $\theta = \tan^{-1} \frac{X_c}{R}$

Putting values: $\theta = \tan^{-1} \frac{40 \Omega}{30 \Omega}$

$$\boxed{\theta = 53.12^\circ}$$

(d) The instantaneous current i is

$$i_{\text{inst}} = I_{\text{max}} \sin(2\pi ft + \theta) \text{ since}$$

$$I_{\text{max}} = \sqrt{2} I_{\text{rms}}, \text{ therefore}$$

$$i_{\text{inst}} = \sqrt{2} I_{\text{rms}} \sin(2\pi ft + \theta)$$

Putting values: $i_{\text{inst}} = \sqrt{2} \times 2 \text{ A} \times \sin\{2 \times 3.14 \times 50 + 53.12^\circ\}$

$$\boxed{i_{\text{inst}} = 2.8284 \times \sin(314t + 53.12^\circ)}$$

Given:

Inductance $L = 3.18\text{mH} = 31.8 \times 10^{-3} \text{ A}$

Resistance $R = 7\Omega$

AC frequency $f = 50\text{Hz}$

Root mean voltage $V_{\text{rms}} = 230\text{V}$

Required:

(a) Current $I = ?$

(b) Phase angle $\theta = ?$

(c) Power factor $\cos \theta = ?$

(d) Power consumed $< p > = ?$

Solution:

The inductive reactance X_L is $X_L = \omega_L = 2\pi f \times L$

Putting values $X_L = 2 \times 3.14 \times 50\text{Hz} \times 31.8 \times 10^{-3}\text{H}$

Or $X_L = 9985.2 \times 10^{-3} \text{ V s / A x/1 s}$ hence, $X_L = 9.9852\text{V / A} = 9.9852\Omega$

The impedance is $Z = \sqrt{R^2 + X_L^2}$

Putting values: $Z = \sqrt{(7\Omega)^2 + (9.9852\Omega)^2}$

Hence, $Z = 12.194\Omega$

(a)

The current I is given as $I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$

Putting values: $I_{\text{max}} = \frac{230\text{V}}{12.194\Omega}$

$$I_{\text{max}} = 18.86\text{A}$$

(b) The phase angle θ is $\theta = \tan^{-1} \frac{X_L}{R}$

Putting values: $\theta = \tan^{-1} \frac{9.9852\Omega}{7\Omega}$

The power factor $\cos \theta$ is

$$\cos \theta = \cos 50^\circ = 0.5731$$

The power consumed is

$$\langle p \rangle = I_{\text{rms}} V_{\text{rms}} \cos \theta$$

Putting values $\langle p \rangle = 18.86\text{A} \times 230\text{V} \times 0.573$

Or

$$\langle p \rangle = 2484.24 \text{ W}$$

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