Chap 8 TRIGONOMETRY

MULTIPLE CHOICE (MCQ's)

(i) Trignometeric ratio of $\sin 30^0$ is ----------

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

(ii) Trignometeric ratio of $\text{Cos } 30^0$ is ----------

(a) $\frac{1}{2}$ (b) $\sqrt{3}$ **(c)** $\frac{\sqrt{3}}{2}$ (d) 1

(iii) Trignometeric ratio of $\text{Cos } 30^0$ is ----------

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{2}}$

(iv) Trignometeric ratio of $\text{Sin } 45^0$ is ----------

(a) $\sqrt{3}$ (b) 1 (c) $\frac{1}{2}$ **(d)** $\frac{1}{\sqrt{2}}$

(v) Trignometeric ratio of $\text{Cos } 45^0$ is ----------

(a) $\frac{1}{\sqrt{3}}$ **(b)** $\frac{1}{2}$ c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

(vi) Trignometeric ratio of $\text{Tan } 45^0$ is ----------

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

(vii) Trignometeric ratio of $\text{Sin } 60^0$ is ----------

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{2}}$

(viii) Trignometeric ratio of Cos 60° is ----------

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

(ix) Trignometeric ratio of Tan 60° is ----------

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

(x) Cosec 30° is equal to ----------

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 2 (d) 1

(xi) sec 60° is equal to ----------

(a) 2 b) $\sqrt{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{\sqrt{2}}$

(xii) Cot 30° is equal to ----------

(a) 1 b) $\sqrt{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

(xiii) Cosec 60° is equal to ----------

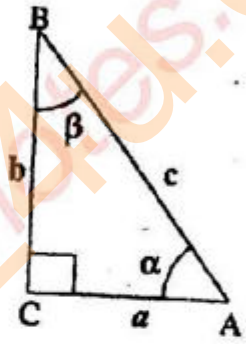
(a) $\frac{\sqrt{3}}{2}$ b) $\sqrt{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{1}{2}$

(xiv) Cot 30° is equal to ----------

(a) $\sqrt{2}$ b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

EXERCISE 8.1

Q.1. For the adjacent figure state whether the following statements are true or false.



(i) $\sin\beta = \frac{a}{c}$ (ii) $\cos\beta = \frac{b}{c}$,

(iii) $\tan\beta = \frac{b}{a}$ (iv) $\cot\alpha = \frac{a}{b}$

(v) $\sec\alpha = \frac{c}{b}$ (vi) $\text{cosec}\,\alpha = \frac{c}{a}$

(vii) $\text{cosec}\,\alpha = \frac{c}{b}$ (viii) $\sin\alpha = \frac{1}{\cos\alpha}$

(ix) $\tan\beta = \frac{1}{\cot\beta}$ (x) $\sec\alpha = \frac{1}{\cos\beta}$

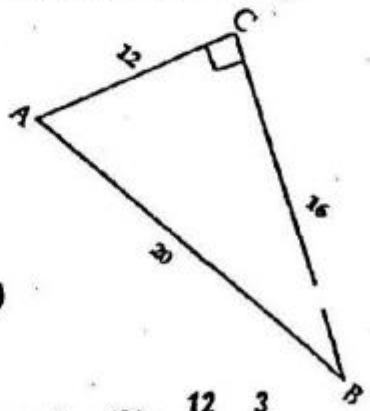
SOLUTION.

(i) True (ii) False (iii) False (iv) True

(v) False (vi) False (vii) False (viii) False

(ix) True (ii) False

Q.2. Look at the adjacent triangle and find the following.



(i) sin (m∠A) (ii) cos (m∠A)

(iii) tan (m∠A) (iv) sin (m∠A)

(v) cos (m∠A) (vi) tan (m∠B)

(vii) cot (m∠A) (viii) sec (m∠A)

(ix) cosec (m∠A) (x) cot (m∠B)

(xi) sec (m∠B) (xii) cosec (m∠B)

SOLUTION.

(i) $\sin(m\angle A) = \frac{16}{20} = \frac{4}{5}$ (ii) $\cos(m\angle A) = \frac{12}{20} = \frac{3}{5}$

(iii) $\tan(m\angle A) = \frac{16}{20} = \frac{4}{3}$ (iv) $\sin(m\angle B) = \frac{12}{20} = \frac{3}{5}$

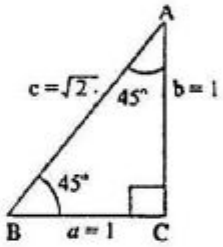
(v) $\sin(m\angle B) = \frac{16}{20} = \frac{4}{5}$ (vi) $\tan(m\angle B) = \frac{12}{16} = \frac{3}{4}$

(vii) $\cot(m\angle A) = \frac{12}{16} = \frac{5}{4}$ (viii) $\cot(m\angle B) = \frac{20}{12} = \frac{5}{3}$

(ix) $\operatorname{cosec}(m\angle A) = \frac{20}{16} = \frac{5}{4}$ (x) $\cot(m\angle B) = \frac{16}{12} = \frac{4}{3}$

(xi) $\sec(m\angle B) = \frac{20}{16} = \frac{5}{4}$ (xii) $\operatorname{cosec}(m\angle B) = \frac{20}{12} = \frac{5}{3}$

Q.3. Use the adjacent figure to find following.



(i) $\sin 45°$ (ii) $\cos 45°$

(iii) $\tan 45°$ (iv) $\cot 45°$

(v) $\sec 45°$ (iv) $\operatorname{cosec} 45°$

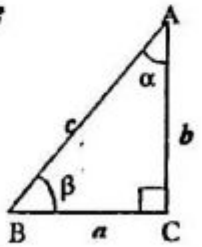
SOLUTION.

(i) $\sin 45° = \frac{a}{c} = \frac{1}{\sqrt{2}}$ (i) $\cos 45° = \frac{b}{c} = \frac{1}{\sqrt{2}}$

(iii) $\sin 45° = \frac{a}{b} = \frac{1}{1} = 1$ (iv) $\cot 45° = \frac{b}{a} = \frac{1}{1} = 1$

(v) $\sin 45° = \frac{c}{b} = \frac{\sqrt{2}}{1} = \sqrt{2}$ (iv) $\cot 45° = \frac{c}{a} = \frac{\sqrt{2}}{1} = \sqrt{2}$

Q.4. With the help of adjacent figure prove that:



(i) $\sin\alpha = \cos\beta$ (ii) $\tan\alpha = \cot\beta$

(iii) $\sec\alpha = \operatorname{cosec}\beta$ (iv) $\tan\beta = \cot\alpha$

(iv) $\tan\beta = \cot\alpha$ (vi) $\sec\beta = \operatorname{cosec}\alpha$

SOLUTION.

(i) $\because \sin\alpha = \frac{a}{c}$ (ii) $\because \tan\alpha = \frac{a}{b}$ and

and $\cos\beta = \frac{a}{c}$ $\cot\beta = \frac{a}{b}$

$\therefore \quad \sin\alpha = \cos\beta$

$\therefore \quad \tan\alpha = \cot\beta$

(iii) $\because \quad \sec\alpha = \frac{c}{b}$

and $\quad \operatorname{cosec}\beta = \frac{c}{b}$

$\therefore \quad \sec\alpha = \operatorname{cosec}\beta$

(iv) $\because \quad \sin\beta = \frac{b}{c}$ and

$\cos\alpha = \frac{b}{c}$

$\therefore \quad \sin\beta = \cos\alpha$

(v) $\because \quad \tan\beta = \frac{b}{a}$ and

$\cot\alpha = \frac{b}{a}$

$\therefore \quad \tan\beta = \cot\alpha$

(vi) $\because \quad \sec\beta = \frac{c}{a}$ and

$\operatorname{cosec}\alpha = \frac{c}{a}$

$\therefore \quad \sec\beta = \operatorname{cosec}\alpha$

Q5. Use the figure of Question 4 to prove that.

(i) $\cot\alpha = \frac{\cos\alpha}{\sin\alpha}$ **(ii)** $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

(iii) $\cot\beta = \frac{\cos\beta}{\sin\beta}$ **(iv)** $\tan\beta = \frac{\sin\beta}{\cos\beta}$

(v) $\tan\alpha\cot\alpha = 1$ **(vi)** $\sin\alpha\operatorname{cosec}\alpha = 1$

(vii) $\sin\beta\operatorname{cosec}\beta = 1$ **(viii)** $\sec\beta\cos\beta = 1$

(ix) $\sec\alpha\cos\alpha = 1$ **(x)** $\tan\beta\cot\beta = 1$

SOLUTION:

(i) $\cot\alpha = \frac{b}{a}$

$\cos\alpha = \frac{b}{c}$ and $\sin\alpha = \frac{a}{c}$

(ii) $\tan\alpha = \frac{a}{b}$,

$\sin\alpha = \frac{a}{c}$ and $\cos\alpha = \frac{b}{c}$

$$\frac{\cos\alpha}{\sin\alpha} = \frac{\frac{b}{c}}{\frac{a}{c}}$$

Now

$$= \frac{b}{a} = \cot\alpha$$

(iii) $\cot\beta = \frac{a}{b}$,

$\text{Cos}\,\beta = \frac{a}{c}$ and $\sin\beta = \frac{b}{c}$

$$\frac{\cos\beta}{\sin\beta} = \frac{\frac{a}{c}}{\frac{b}{c}}$$

Now

$$= \frac{a}{b} = \cot\beta$$

(v) $\tan\alpha \cot\alpha$

$$= \frac{a}{b} \times \frac{b}{a} = 1$$

(vii) $\sin\beta \operatorname{cosec}\beta$

$$= \frac{b}{c} \times \frac{c}{b}$$

$$= 1$$

(ix) $\sec\alpha \cos\alpha$

$$= \frac{c}{a} \times \frac{a}{c}$$

$$= 1$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\frac{a}{c}}{\frac{b}{c}}$$

Now

$$= \frac{a}{b} = \tan\alpha$$

(iv) $\tan\beta = \frac{b}{a}$,

$\sin\beta = \frac{b}{c}$ and $\cos\beta = \frac{a}{c}$

$$\frac{\sin\beta}{\cos\beta} = \frac{\frac{b}{c}}{\frac{a}{c}}$$

Now

$$= \frac{b}{a} = \tan\beta$$

(vi) $\sin\alpha \operatorname{cosec}\alpha$

$$= \frac{a}{c} \times \frac{c}{a} = 1$$

(viii) $\sec\beta \cos\beta$

$$= \frac{c}{a} \times \frac{a}{c}$$

$$= 1$$

(x) $\tan\beta \cot\beta$

$$= \frac{b}{a} \times \frac{a}{b}$$

$$= 1$$

Q.6. In right triangle KLM, name ratio represented for the given angle.

(i) $\frac{17}{15}, \theta$ (ii) $\frac{17}{8}, \theta$ (iii) $\frac{15}{8}, \theta$ (iv) $\frac{17}{15}, \alpha$ (v) $\frac{8}{15}, \alpha$ (vi) $\frac{17}{8}, \alpha$

Where $\theta = m\angle M$ and $\alpha = m\angle K$

SOLUTION.

(i) $\sec\theta = \frac{17}{15}$ (ii) $\operatorname{cosec}\theta = \frac{17}{8}$

(iii) $\cot\theta = \frac{15}{18}$ (iv) $\operatorname{cosec}\alpha = \frac{17}{15}$

(v) $\cot\alpha = \frac{8}{15}$ (vi) $\sec\alpha = \frac{17}{8}$

Q7. Refer to ΔDEF to show that each of the following is true.

(i) $(\sec\theta)^2 - (\tan\theta)^2 = 1$ (ii) $(\operatorname{cosec}\theta)^2 - (\cot\theta)^2 = 1$

(iii) $\sec\theta \cos\theta = 1$

(iv) $\operatorname{cosec}\theta \sin\theta = 1$

(v) $\tan\theta \cot\theta = 1$

(vi) $\frac{\cos\theta}{\sin\theta} = \cot\theta$

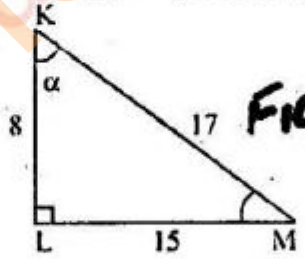


Fig. Q.6

SOLUTION.

(i) $(\sec\theta)^2 - (\tan\theta)^2$

$= \left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2$

$= \frac{25}{16} - \frac{9}{16}$

$= \frac{25-9}{16}$

$= \frac{16}{16} = 1$

(ii) $(\operatorname{cosec}\theta)^2 - (\cot\theta)^2 = 1$

$= \left(\frac{5}{4}\right)^2 - \left(\frac{3}{4}\right)^2$

$= \frac{25}{9} - \frac{16}{9}$

$= \frac{25-16}{9}$

$= \frac{9}{9} = 1$

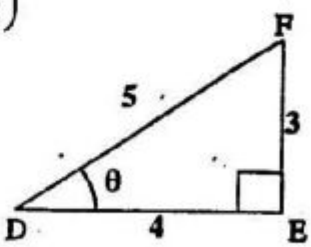


Fig. Q.7

(iii) $\sec \cos \theta$

$= \frac{5}{4} \times \frac{4}{5}$

$= 1$

(iv) $\operatorname{cosec} \theta \sin \theta$

$= \frac{5}{3} \times \frac{3}{5}$

$= 1$

(v) $\tan \theta \cot \theta$

$= \frac{3}{\cancel{4}} \times \frac{\cancel{4}}{3}$

$= 1$

(vi) $\frac{\cos \theta}{\sin \theta}$

$= \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3} = \cot \theta$

EXERCISE 8.2

Q.1. Fill in the blanks.

(i) $\operatorname{cosec} 60° = \frac{1}{\ldots\ldots}$, (ii) $\sec 30° = \frac{1}{\ldots\ldots\ldots}$,

(iii) $\sec 60° = \ldots\ldots\ldots$, (iv) $\sin 20° = \sin (90° - 70°) = \cos \ldots\ldots$

(v) $\cos 10° = \cos (90° - \ldots\ldots) = \sin \ldots\ldots$

(vi) $\text{Tan } 40° = \tan (\ldots\ldots - 50°) = \cot \ldots\ldots$

(vii) $\text{Sec } \ldots\ldots = \sec (90° - 10°) = \ldots\ldots 10°$

(viii) $\text{Cosec } 10° = \sec \ldots\ldots\ldots$

(ix) $\tan 20° = \ldots\ldots 70°$

SOLUTION.

(i) $\operatorname{cosec} 60° = \frac{1}{\sin 60°}$ (ii) $\sec 30° = \frac{1}{\ldots\ldots\ldots}$,

(iii) $\sec 60° = \ldots\ldots$, (iv) $\sin 20° = \sin (90° - 70°) = \cos 70°$

(v) $\cos 10° = \cos (90° - 80°) = \sin 80°$ (vi) $\tan 90° = \tan (90° - 50°) = \cot 50$

(vii) $\sec 80° = \sec (90° - 10°) = \operatorname{cosec} 10°$ (viii) $\operatorname{cosec} 10° = \sec 10° = \sec \ldots\ldots$

(ix) $\tan 20° = \cot 70°$

Q.2.(a) Find the value of α when the value of sin α is:

(i) $\frac{1}{2}$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $\frac{1}{\sqrt{2}}$

SOLUTION.

(a) (i) $\because \sin 30° = \frac{1}{2} \Rightarrow \alpha = 30°$ (ii) $\because \sin 60° = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60°$

(iii) $\because \sin 45° = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45°$

(b) (i) $\because \cos 30° = \frac{\sqrt{3}}{2} \Rightarrow \beta = 30°$ (ii) $\because \cos 45° = \frac{1}{\sqrt{2}} \Rightarrow \beta = 45°$

(iii) $\because \cos 60° = \frac{1}{2} \Rightarrow \beta = 60°$

(c) (i) $\because \tan 45° = 1 \Rightarrow \gamma = 45°$ (ii) $\because \tan 60° = \sqrt{3} \Rightarrow \gamma = 60°$

(iii) $\because \tan 30° = \frac{1}{\sqrt{3}} \Rightarrow \gamma = 30°$

(d) (i) $\because \sec 60° = 2 \Rightarrow \theta = 60°$ (ii) $\because \sec 45° = \sqrt{2} \Rightarrow \theta = 45°$

(iii) $\because \sec 30° = \frac{2}{\sqrt{3}} \Rightarrow \theta = 60°$

b. **Find the value of β when the cos β is:**

(i) 1 (ii) $\frac{1}{\sqrt{2}}$ (iii) $\frac{1}{2}$

c. **Find the value of γ when the value of the tan γ is.**

(i) 1 (ii) $\sqrt{3}$ (iii) $\frac{1}{\sqrt{3}}$

d. **Find the value of θ when the value of sec θ is:**

(i) 2 (ii) $\sqrt{2}$ (iii) $\frac{2}{\sqrt{3}}$

Q3. Find the value of following (Using table and / or formula for complementary angles and reciprocal relations).

(i) cot 45° (ii) sec 45° (iii) cosec 45°
(iv) cot 30° (v) sec 30° (vi) cosec 30°
(vii) cot 60° (viii) sec 60° (ix) cosec 60°
(x) sin30° (xi) cos45° (xii) tan 60°

SOLUTION.

(i) $\cot 45° = 1$

(ii) $\sec 45° = \sqrt{2}$

(iii) $\operatorname{cosec} 45° = \sqrt{2}$

(iv) $\cot 30° = \sqrt{3}$

(v) $\sec 30° = \frac{2}{\sqrt{3}}$

(vi) $\operatorname{cosec} 30° = 2$

(vii) $\cot 60° = \frac{1}{\sqrt{3}}$

(viii) $\sec 60° = 2$

(ix) $\operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$

(x) $\sin 30° = \frac{1}{2}$

(xi) $\cos 45° = \frac{1}{\sqrt{2}}$

(xii) $\tan 60° = \sqrt{3}$

Q.4 Find the value of the following.

(i) $\sin 30° \operatorname{cosec} 30° + \cos 30° \sec 30°$
(ii) $\cos 60° \sin 60° + \operatorname{cosec} 60° \sec 60°$
(iii) $\sin 45° \cos 45° - \tan 45° - \cot 45°$
(iv) $\sin 30° \cos 45° - \sin 30° \sin 45°$
(v) $\frac{\tan 30^0 + \tan 45^0}{1 - \tan 30^0 \tan 45^0}$ (vi) $\frac{\cot 45^0 + \cot 30^0 + 1}{\cot 30^0 - \cot 45^0}$

SOLUTION.

(i) $\sin 30° \operatorname{cosec} 30° + \cos 30° \sec 30° = \frac{1}{2} \times 2 + \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} = 1 \times 2 = 2$

(ii) $\cos 60° \sin 60° + \operatorname{cosec} 60° \sec 60° = \frac{1}{2} \times 2 + \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} \times 2$

$= \frac{\sqrt{3}}{4} + \frac{4}{\sqrt{3}} = \frac{3+16}{4\sqrt{3}}$

(iii) $\sin 45° + \cos 45° - \tan 45° - \cot 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - 1$

$= \frac{1+1}{\sqrt{2}} - 2 = \frac{2}{\sqrt{2}} - 2 = \sqrt{2} - 2$

(iv) $\sin 30° \cos 450° - \sin 30° \sin 45° = \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{1-1}{2\sqrt{2}} = \frac{0}{2\sqrt{2}} = 0$

(v) $\frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \tan 45^\circ} = \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}} \times 1} = \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2}$

$= \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2 + \sqrt{3}$

(vi) $\frac{\cot 45^\circ \cot 30^\circ + 1}{\cot 30^\circ \tan 45^\circ} = \frac{1 \times \sqrt{3} + 1}{\sqrt{3} - 1} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - (1)^2}$

$= \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2}$

$= \frac{2(2+\sqrt{3})}{2} = 2 + \sqrt{3}$

Q.5. Verify that:

(i) $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

(ii) $\tan 30^\circ + \cot 30^\circ = \sec 30^\circ \operatorname{cosec} 30^\circ$

(iii) $(\cos 60^\circ - \sin 60^\circ)^2 + 2 \sin 60^\circ \cos 60^\circ = 1$

(iv) $2\sin 45^\circ + \frac{1}{2}\operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$ (v) $1 - 2\sin^2 30^\circ = 2\cos^2 30^\circ - 1$

(vi) $\sin^2 60^\circ + \cos^2 60^\circ = 1$ (vii) $\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = 1 - 2\sin^2 30^\circ$

(viii) $1 + \tan^2 60^\circ = \sec^2 60^\circ$ (ix) $\cos^2 45^\circ - \sin^2 45^\circ = \frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$

(x) $1 + \cot^2 30^\circ = \operatorname{cosec}^2 30^\circ$

SOLUTION.

(i) $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ \Rightarrow \frac{\sqrt{3}}{2} = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

(ii) $\tan 30^\circ + \cot 30^\circ = \sec 30^\circ \operatorname{cosec} 30^\circ \Rightarrow \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{2}{\sqrt{3}} \times 2$

$\Rightarrow \frac{1+3}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\therefore \quad \frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}}$ Verified

(iii) $(\cos 60^\circ - \sin 60^\circ)^2 + 2\sin 60^\circ \cos 60 = 1 \Rightarrow \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 + 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = 1$

$\Rightarrow \quad \frac{1 - 2\sqrt{3} + 3}{4} + \frac{\sqrt{3}}{2} = 1$

$\Rightarrow \quad \frac{4 - 2\sqrt{3} + 2\sqrt{3}}{4} = 1$

$\Rightarrow \quad \frac{4}{4} = 1$

$\Rightarrow \quad 1 = 1$ Verified

(iv) $2 \sin 45° + \frac{1}{2} \operatorname{cosec} 45° = \frac{3}{\sqrt{2}}$

$2 \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \sqrt{2} = \frac{3}{\sqrt{2}} \Rightarrow \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$\frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} \Rightarrow \therefore \frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}}$ Verified

(v) $1 - 2 \sin^2 30° = 2 \cos^2 30° - 1$

$\Rightarrow 1 - 2\left(\frac{1}{2}\right)^2 = 2\left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow 1 - 2 \times \frac{1}{4} = 2 \times \frac{3}{4} - 1$

$\Rightarrow 1 - \frac{1}{2} = \frac{3}{2} - 1 \Rightarrow 1 - \frac{1}{2} = \frac{3}{2} - 1$

$\Rightarrow \frac{1}{2} = \frac{1}{2}$ Verified

(vi) $\sin^2 60° + \cos^2 60° = 1$

$\Rightarrow \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1 \Rightarrow \frac{3}{4} + \frac{1}{4} = 1$

$\Rightarrow \frac{3+1}{4} = 1 \Rightarrow \frac{4}{4} = 1$

$1 = 1$ Verified

(vii) $\frac{1 - \tan^2 30°}{1 + \tan^2 30°} = 1 - 2 \sin^2 30°$

$\Rightarrow \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = 1 - 2\left(\frac{1}{2}\right)^2 \Rightarrow \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = 1 - 2 \times \frac{1}{4}$

$$\Rightarrow \quad \frac{\frac{3-1}{3}}{\frac{3+1}{3}} = 1 - \frac{1}{2} \qquad \Rightarrow \qquad \frac{2}{4} = \frac{2-1}{2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2} \qquad \therefore \qquad 4 = 4 \quad \text{Verified}$$

(viii) $1 + \tan^2 60° = \sec^2 60°$

$$\Rightarrow \quad 1 + \left(\sqrt{3}\right)^2 = (2) \qquad \Rightarrow \qquad 1 + 3 = 4 \qquad \therefore \quad 4 = 4 \text{ Verified}$$

(ix) $\cos^2 45° - \sin^2 45° = \frac{1 - \tan^2 45^0}{1 + \tan^2 45^0}$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 \quad \Rightarrow \quad \frac{1}{2} - \frac{1}{2} = \frac{1-1}{2} \qquad \therefore \quad 0 = 0$$

(x) $1 + \cot^2 30° = \operatorname{cosec}^2 30°$

$$\Rightarrow \quad 1 + \left(\sqrt{3}\right)^2 = (2)^2 \qquad \Rightarrow \qquad 1 + 3 = 4 \qquad \therefore \quad 4 = 4 \quad \text{Verified}$$

EX.8.3

Q.2

(viii) $\dfrac{\cot^2 \alpha - 1}{1 + \cot^2 \alpha} = \dfrac{\frac{\cos^2 \alpha}{\sin^2 \alpha} - 1}{1 + \frac{\cos^2 \alpha}{\sin^2 \alpha}}$

$$= \frac{\frac{\cos^2 \alpha - \sin^2 \alpha}{\sin^2 \alpha}}{\frac{\sin^2 \alpha - \cos^2 \alpha}{\sin^2 \alpha}} = \frac{\cos^2 \alpha - (1 - \cos^2 \alpha)}{1}$$

$$= \cos^2 \alpha - 1 + \cos^2$$

$$= 2\cos^2 \alpha - 1 \qquad \text{Verified}$$

(ix) $\frac{1}{1+\sin\alpha}+\frac{1}{1-\sin\alpha}$

$= \frac{1-\sin\alpha+1+\sin\alpha}{(1+\sin\alpha)(1-\sin\alpha)}$

$= \frac{2}{1-\sin^2\alpha}$

$= \frac{2}{\cos^2\alpha}$

$= 2\sec^2\alpha$ Verified

(x) $\frac{\cot\beta-1}{\cot\beta+1}$

$= \frac{\frac{\cos\beta}{\sinh\beta}-1}{\frac{\cos\beta}{\sin\beta}+1}$

$= \frac{\frac{\cos\beta-1}{\sinh\beta}}{\frac{\cos\beta+1}{\sin\beta}}$

$= \frac{\cos\beta-1}{\cos\beta+1}$ Verified

(xii) $\frac{1+\operatorname{cosec}\gamma}{\operatorname{cosec}\gamma-1}$

$= \frac{1+\frac{1}{\sin\gamma}}{\frac{1}{\sin\gamma}-1}$

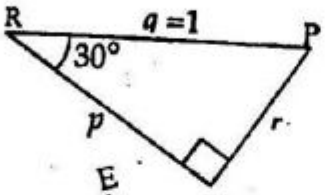
$= \frac{\frac{\sin\gamma+1}{\sin\gamma}}{\frac{1-\sin\gamma}{\sin\gamma}}$

$= \frac{1+\sin\gamma}{1-\sin\gamma}$ Verified

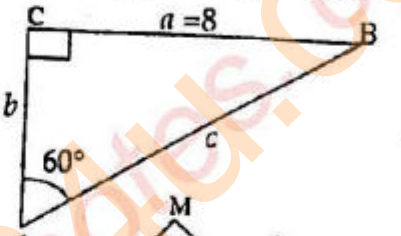
EXERCISE 8.4

Q.1. Find the unknown elements of the following right triangles.

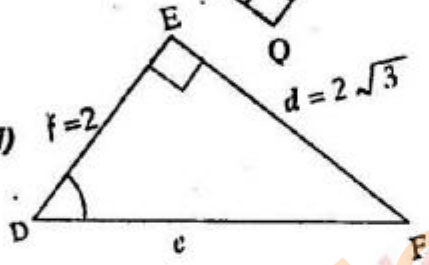
(i)



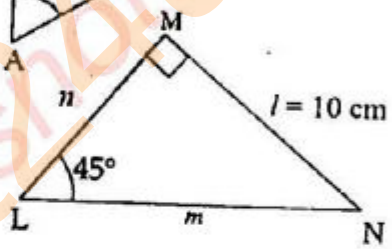
(ii)



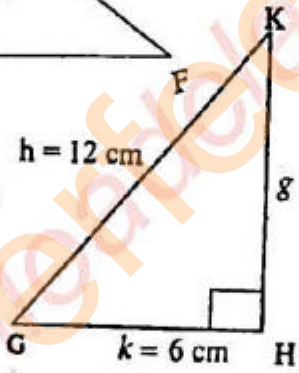
(iii)



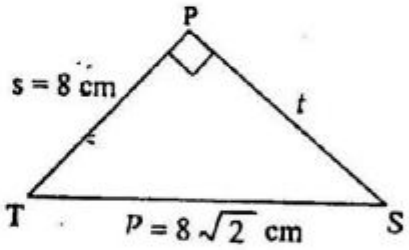
(iv)



(v)



(vi)



SOLUTION

i) $\therefore m\angle P = 90° - m\angle P$ $\quad\therefore m\angle P = 90° - 30° = 60°$

$\therefore \sin 30° = \frac{r}{q} \Rightarrow \frac{1}{2} = \frac{r}{q}$

$\therefore \quad r = \frac{1}{2}$

(ii) $\therefore m\angle B = 90° - m\angle A$ $\quad\therefore m\angle B = 90° - 60° = 30°$

$\therefore \sin 60° = \frac{a}{c} \Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{c}$

$\therefore \quad c = 8 \times \frac{2}{\sqrt{3}} = \frac{16}{\sqrt{3}}$

Using Pythagoras theorem, $q^2 = r^2 + p^2$

$\Rightarrow (1)^2 = \left(\frac{1}{2}\right)^2 + p^2 \quad \Rightarrow \quad p^2 = 1 - \frac{1}{4}$

$\Rightarrow p^2 = \frac{3}{4} \quad \Rightarrow \quad p = \frac{\sqrt{3}}{2}$

(iii) $\therefore \tan(m\angle F) = \frac{f}{d} \quad \Rightarrow \quad \tan(m\angle F) = \frac{2}{2\sqrt{3}}$

$= \frac{1}{\sqrt{3}} \qquad \therefore m\angle F = 30°$

$m\angle D = 90° - m\angle F = 90° - 30° = 60°$

Using Pythagoras Theorem,

$e^2 = f^2 + d^2 = (2)^2 + \left(2\sqrt{3}\right)^2$

$= 4 + 4 \times 3 = 16$

$\therefore \quad e = 4$

(iv) $\therefore m\angle N = 90° - m\angle L \qquad \therefore m\angle N = 90 - 45° = 45°$

$\therefore m\angle N = m\angle L \qquad \therefore l = n = 10$

Using Pythagoras Theorem,

$m^2 = l^2 + n^2 = (10)^2 + (10)^2 = 2 \times 100$

$\therefore m = 10\sqrt{2}$

(v) Using Pythagoras' Theorem,

$h^2 = g^2 + k^2 \quad \Rightarrow \quad g^2 = h^2 - k^2$

$= (12)^2 - (6)^2 \quad \Rightarrow \quad = 144 - 36$

$= 180$

$\therefore \quad g = \sqrt{108} = \sqrt{3 \times 36} = 6\sqrt{3}$

$\therefore \quad \cos(m\angle G) = \frac{k}{h} = \frac{6}{12} = \frac{1}{2}$

$\therefore \quad m\angle G = 60°$

Using Pythagoras Theorem,

$c^2 = a^2 + b^2 \quad \Rightarrow \quad b^2 = c^2 - a^2$

$= \left(\frac{16}{\sqrt{3}}\right)^2 - (8)^2 \quad = \frac{16 \times 4 \times 4}{3} - 64$

$= \frac{64 \times 4 - 64 \times 3}{3} \quad = \frac{64}{3}(4 - 3)$

$= \frac{64}{3} \quad \therefore \quad b = \frac{8}{\sqrt{3}}$

(vi) $\therefore \sin(m\angle s) = \frac{s}{p} = \frac{8}{8\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\therefore m\angle s = 45°$

$\therefore m\angle T = 90° - m\angle s = 90° - 45$

$= 45°$

$\therefore m\angle s = m\angle T \quad \therefore t = s = 8$

Q2. Solve triangle ACB when $m\angle C = 90°$ and

(i) $m\angle B = 60°$, $a = 2$ cm (ii) $m\angle B = 30°$, $a = 4$ cm

(iii) $m\angle A = 30°$, $c = 6$ cm (iv) $m\angle A = 45°$, $c = 3\sqrt{2}$

(v) $m\angle B = 30°$, $b = 3$ cm (vi) $m\angle B = 45°$ $b = 8$ cm

(vii) $m\angle A = 60°$, $c = 4$ cm (viii) $m\angle A = 45°$, $a = 10$cm

SOLUTION.

(i) $m\angle A = ?$, $b =$

$\therefore m\angle A = 90° - m\angle B = 90° - 60°$

$\therefore m\angle A = 30°$

$\therefore \tan(m\angle A) = \frac{a}{b} \Rightarrow \tan 30° = \frac{2}{b}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2}{b} \quad \therefore b = 2\sqrt{3}$ cm

Using Pythagoras Theorem

$e^2 = a^2 + b^2 = (2)^2 + (2\sqrt{3})^2$

$= 4 + 44 \times 3 = 16$

$\therefore c = 4$ cm

(ii) $m\angle B = ?, b = ?, c = ?$

$\therefore m\angle B = 90° - m\angle A = 90° - 30°$

$\therefore m\angle B = 60°$

$\therefore \tan(m\angle A) = \frac{a}{b} \Rightarrow \tan 30°$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{4}{b} \quad \therefore b = \frac{4}{\sqrt{3}}$ cm

Using Pythagoras Theorem,

$c^2 = a^2 + b^2$

$= (4)^2 + \left(\frac{4}{\sqrt{3}}\right)^2 = 16 + \frac{16}{3}$

$= \frac{48+16}{3} = \frac{64}{3}$

$\therefore c = \frac{8}{\sqrt{3}}$ cm

(iii) $m\angle A = ?,\ a = ?,\ b = ?$

$m\angle A = 90° - m\angle B = 90° - 30° = 60°$

$\sin(m\angle B) = \frac{a}{c} \Rightarrow \sin 30° = \frac{a}{6}$

$\Rightarrow \frac{1}{2} = \frac{a}{6} \therefore a = 3\ cm$

Using Pythagoras Theorem

$c^2 = a^2 + b^2 \Rightarrow b^2 = c^2 - a^2$

$= (6)^2 - (3)^2 \Rightarrow = 36 - 9$

$= 27 \therefore b = 3\sqrt{3}\ cm$

(iv) $m\angle B = ?,\ a = ?,\ b = ?$

$m\angle B = 90° - m\angle A = 90° - 45° = 45°$

$\therefore m\angle A = m\angle B$

$\therefore a = b$

Using Pythagoras Theorem

$c^2 = a^2 + b^2$

$= 2a^2 \quad (\because a = b)$

$\Rightarrow a^2 = \frac{c^2}{2} = \left(\frac{3\sqrt{2}}{2}\right)^2$

$= \frac{9\times 2}{2}$

$\Rightarrow a^2 = 9 \therefore a = 3\ cm \Rightarrow b = a = 3\ cm$

(v) $m\angle A = ?,\ a = ?,\ c = ?$

$\therefore m\angle A = 90° - m\angle B = 90° - 30°$

$\therefore m\angle A = 60°$

$\therefore \sin(m\angle B) = \frac{b}{c} \Rightarrow \sin 30° = \frac{3}{c}$

$\Rightarrow \frac{1}{2} = \frac{3}{c} \quad \therefore c = 6\ cm$

Using Pythagoras Theorem

$c^2 = a^2 + b^2$

$\Rightarrow a^2 = c^2 - b^2$

$= (6)^2 - (3)^2$

$= 36 - 9$

$\therefore a = 3\sqrt{3}\ cm$

(vii) $m\angle B = ?,\ a = ?,\ b = ?$

$\therefore m\angle B = 90° - m\angle A \quad \therefore m\angle B = 90° - 60° = 30°$

$\therefore \sin(m\angle A) = \frac{a}{c} \Rightarrow \sin 60° = \frac{a}{4}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a}{4} \quad \therefore a = 2\sqrt{3}\ cm$

Using Pythagoras Theorem

$c^2 = a^2 + b^2 \Rightarrow b^2 = c^2 - a^2$

$= (4)^2 - \left(2\sqrt{3}\right)^2 \Rightarrow = 16 - 4 \times 3$

$= 16 - 12 = 4$

$\therefore b = 2cm$

(viii) $m\angle B = ?,\ b = ?,\ c = ?$

$\therefore m\angle B = 90° - m\angle A$

$\therefore m\angle B = 90° - 45° = 45°$

$\therefore m\angle A = m\angle B$

$\therefore b = a = 10$ cm

Using Pythagoras Theorem

$c^2 = a^2 + b^2 \qquad = (10)^2 + (10)^2$

$= 100 + 100 \qquad = 2 \times 100$

$\therefore c = 10\sqrt{2}$ cm

Q3. Solve triangle ACB when $m\angle C = 90°$ and

(i) $c = 10\sqrt{2}$ cm, $a = 10$ cm

(ii) $b = 2\sqrt{3}$ cm, $a = 2$cm

(iii) $a = b = 5$cm

(iv) $c = 18$cm, $b = 9$ cm

(v) $c = 4$ cm, $a = 2$cm

(vi) $b = 3$cm, $a = 3\sqrt{3}$ cm

(vii) $c = 8$cm, $b = 4\sqrt{3}$ cm

SOLUTION.

(i) *Using Pythagoras Theorem,*

$c^2 = a^2 + b^2$

$\Rightarrow \quad b^2 = c^2 - a^2$

$= \left(10\sqrt{2}\right)^2 - (10)^2$

$= 200 - 100$

$= 100$

$\therefore b = 10$ cm

$\therefore a = b$

$\therefore m\angle A = m\angle B$

$\because m\angle A + m\angle B = 90°$

$\therefore 2m\angle A = 90°$

$\Rightarrow \quad m\angle A = 45$

$\therefore \quad m\angle B = 45°$

(ii) Using Pythagoras Theorem,

$c^2 = a^2 + b^2$

$= (2)^2 + \left(2\sqrt{3}\right)^2$

$= 4 + 4 \times 3$

$= 16$

$\therefore c = 4cm$

$\because \tan(m\angle A) = \frac{a}{b}$

$\Rightarrow \tan(m\angle A) = \frac{2}{2\sqrt{3}}$

$\Rightarrow \tan(m\angle A) = \frac{1}{\sqrt{3}}$

$\therefore m\angle A = 30°$

$\because m\angle B = 90° - m\angle A$

$\therefore m\angle B = 90° - 30° = 60°$

(iii) $\because a = b$

$\therefore m\angle A = m\angle B$

$\because m\angle A = m\angle B$

$\because m\angle A = m\angle B = 90°$

$\Rightarrow 2\, m\angle A = 90°$

$\Rightarrow m\angle A = 45°$

$\therefore m\angle B = 45^c$

Using Pythagoras Theorem.

$c^2 = a^2 + b^2$

$= (5)^2 + (5)^2$

$= 25 + 25$

$= 2 \times 25$

$\therefore c = 5\sqrt{2}$ cm

(iv) Using Pythagoras theorem

$c^2 = a^2 + b^2$

$\Rightarrow b^2 = c^2 - a^2$

$= (18)^2 - (a)^2$

$= 4 \times 81 - 81$

$= 3 \times 81$

$\therefore a = 9\sqrt{3}$ cm

$\because \sin(m\angle A) = \frac{a}{c}$

$\Rightarrow \sin(m\angle A) = \frac{9\sqrt{3}}{18}$

$\Rightarrow \sin(m\angle A) = \frac{\sqrt{3}}{2}$

$\therefore m\angle A = 60°$

$\because m\angle B = 90° - m\angle A$

$\therefore m\angle B = 90° - 60° = 30°$

(v) Using Pythagoras theorem

$c^2 = a^2 + b^2$

$\Rightarrow b^2 = c^2 - a^2$

$= (4)^2 - (2)^2$

$= 16 - 4$

$= 12$

$\therefore b = 2\sqrt{3}$ cm

$\because \sin(m\angle A) = \frac{2}{4} = \frac{1}{2}$

$\therefore m\angle A = 30°$

$\because m\angle B = 90° - m\angle A$

$\therefore m\angle B = 90° - 30° = 60°$

(vi) $\because \tan(m\angle A) = \frac{a}{b}$

$\Rightarrow \tan(m\angle A) = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$\therefore m\angle A = 60°$

$\because m\angle B = 90° - m\angle A$

$\because m\angle B = 90° - 60° = 30°$

Using Pythagoras theorem

$c^2 = a^2 + b^2$

$= 9 \times 3 + 9$

$= 4 \times 9$

$\therefore c = 2 \times 3 = 6$cm

(vii) Using Pythagoras theorem

$c^2 = a^2 + b^2$

$\Rightarrow a^2 = c^2 - b^2 = (8)^2 - (4\sqrt{3})^2 = 64 - 48 = 16$

$\therefore a = 4$cm

$\because \tan(m\angle A) = \frac{a}{b}$

$$\Rightarrow \quad \tan(m\angle A) = \frac{a}{b}$$

$$\Rightarrow \quad \tan(m\angle A) = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$\therefore m\angle A = 30°$

$\therefore m\angle B = 90° - m\angle A = 90° - 30° = 60°$

Q4. The diagonal of a rectangular field is (x + 8) metre and the sides are x metre and (x + 4) mere. Find the value of x.

SOLUTION

Using Pythagoras theorem

$$(x+8)^2 = (x)^2 + (x+4)^2$$

$$\Rightarrow \quad x^2 + 16x + 64 = x^2 + x^2\,8x + 16$$

$$\Rightarrow \quad x^2 - 8x - 48 = 0$$

$$\therefore \quad x = \frac{(-8) \pm \sqrt{(-8)^2 - 4(1)(-48)}}{2 \times 1}$$

$$= \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-48)}}{2 \times 1}$$

$$= \frac{8 \pm \sqrt{64 + 192}}{2}$$

$$= \frac{8 \pm \sqrt{64 + 192}}{2}$$

$$= \frac{8 \pm 16}{2}$$

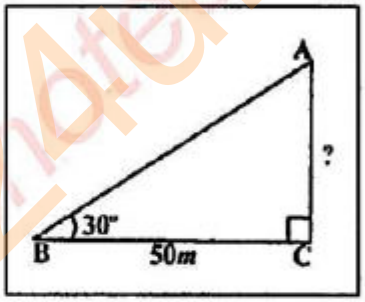
$$= \frac{24}{2}$$

$$= 12$$

EXERCISE 8.5

Q1. A tree is at a distance of 50m form a point on the ground. The angle of elevation of the top of the tree form this point is of 30°. Find the height of the tree.

SOLUTION



Let $\overline{AC}$ represent the tree.

$$\therefore \quad \tan(m\angle B) = \frac{m\overline{AC}}{m\overline{BC}}$$

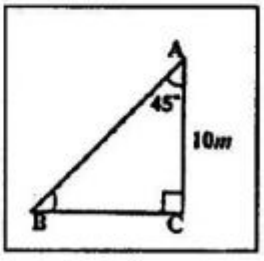
$$\Rightarrow \quad \tan 30° = \frac{m\overline{AC}}{50}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{m\overline{AC}}{50}$$

$$\therefore \quad m\overline{AC} = 50 \times \frac{1}{\sqrt{3}} = \frac{50}{1.732} = 28.87m$$

Q2. On the bank of a sea, there is a light house, 100m high. The angle of depression of a ship form the top of the light house is of 45°. Find the distance between the foot of the light house and the ship.

SOLUTION.



Let $\overline{AC}$ represent the tree and let $m\overline{BC}$ be the distance between the foot of the light house and the sky?

$$\therefore \quad m\angle B = 90° - m\angle A = 90° - 45° = 45°$$

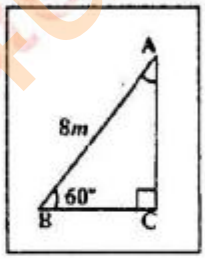
$$\therefore \quad m\angle A = m\angle B.$$

$$\therefore \quad m\overline{BC} = m\overline{AC} = 10m.$$

Q3. A ladder 8 meters long reaches a point on a wall such that it makes an angle of 60° with the ground. Find the height of the point on the wall where the top end of the ladder touches it.

SOLUTION.

Let $\overline{AB}$ represent the tree and let $m\overline{AC}$ be the height of the point on the wall where the top end of the ladder touches it.



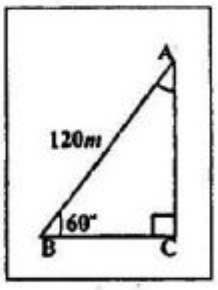
$$\therefore \quad \sin(m\angle B) = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\sin 60° = \frac{m\overline{AC}}{8}$$

$$\therefore m\overline{AC} = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3} = 4 \times 1.732 = 6.93m$$

Q4. A kite has 120m of string attached to it, when at an elevation of 60°. How far it is above the hand holding it?

SOLUTION



Let $\overline{AB}$ represent the string and let $m\overline{AC}$ be the height above the hand holding the kite.

$$\therefore \quad \sin(m\angle B) = \frac{m\overline{AC}}{m\overline{AB}}$$

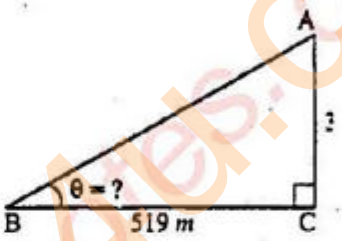
$$\Rightarrow \quad \sin 60° = \frac{m\overline{AC}}{120}$$

$$\therefore m\overline{AC} = 120 \times \frac{\sqrt{3}}{2} = 60\sqrt{3} \quad 60 \times 1.7332 = 103.92$$

Q5. Find the measure of an angle of elevation of the sun, if a tower 300m height casts a shadow 519m long.

SOLUTION.

Let $\overline{AB}$ represent the string and let $m\overline{AC}$ be the height above the hand holding the kite.



$$\therefore \quad \sin(m\angle B) = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\Rightarrow \quad \tan\theta = \frac{300}{519} = \frac{100}{173}$$

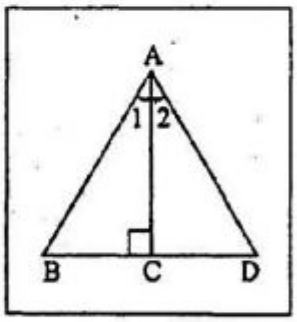
$$\frac{1}{173} = \frac{1}{1.73} = \frac{1}{\sqrt{3}}$$

$$= \frac{}{100}$$

$$\therefore \quad \theta = 30°$$

Q6. An isosceles triangle has a vertical angle of 120°-and a base 40cm long. Find the length of its altitude.

SOLUTION.



Let $\overline{AC}$ is the altitude.

$\therefore m\angle A = 120°; m\overline{BD} = 40cm$

$$\therefore m\overline{BC} = \frac{1}{2} m\angle A = \frac{1}{2} \times 40 = 20cm$$

$$m\angle 1 = \frac{1}{2} m\angle A = \frac{1}{2} \times 120° = 60°$$

$$\therefore \tan(m\angle 1) = \frac{m\overline{BC}}{m\overline{AC}}$$

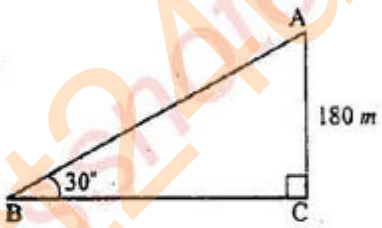
$$\Rightarrow \quad \tan 60° = \frac{20}{m\overline{AC}}$$

$$\therefore m\overline{AC} = \frac{20}{\sqrt{3}} = \frac{20}{1.73} = 11.55cm$$

Q7. A tree of 180 dm height, on one bank of the river, makes angle of 30° directly on the opposite side of the river. Find the width of the river.

SOLUTION

Let $\overline{AC}$ represent a tree and let $m\overline{BC}$ is the width of river.



$$\therefore \quad \tan(m\angle B) = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\Rightarrow \quad \tan 30° = \frac{180}{m\overline{BC}}$$

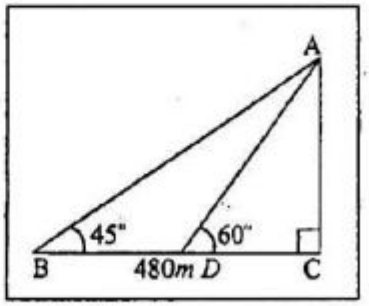
$$\frac{180}{1} = 180 \times \sqrt{3} \times 1.73$$

$$\therefore \quad m\overline{BC} = \sqrt{3}$$

$$= 180 = 311.76cm$$

Q8. The angle of elevation of the top hill form a place is of 45°. In going towards the hill 480m form the place, the angle of elevation of the top becomes of 60°. Find the height of the hill.

SOLUTION. Let $m\overline{AC}$ be the height of the hill.



From ΔABC,

$$m\overline{AC} = m\overline{BC}$$

$$= m\overline{BD} + m\overline{DC}$$

$$= 480 + m\overline{DC} \quad \ldots\ldots\ldots (1)$$

From ΔADC,

$$Tan\ 60° = \frac{m\overline{AC}}{m\overline{DC}}$$

$$\Rightarrow \quad \sqrt{3} = \frac{480 + m\overline{DC}}{m\overline{DC}} \quad [\text{from}(1)]$$

$$\Rightarrow \quad \sqrt{3}\, m\overline{DC} = 480 + m\overline{DC}$$

$$\Rightarrow \quad (\sqrt{3}-1)\, m\overline{DC} = 480$$

$$\therefore \quad m\overline{DC} = \frac{480}{\sqrt{3}-1} = \frac{480}{1.73-1} = \frac{480}{0.73} = 655.48m$$

Putting this in (1), we get

$$m\overline{AC} = 480 + 655.48$$
$$= 135.48m$$

Q9. Ali standing in a stream finds that the measure of the angles of elevation of two trees, of heights 6m and 8m, on opposite banks in the line with him are of 30° and 45°, respectively. Find the width of the stream.

SOLUTION. Let $m\overline{CE}$ be the width of the stream.

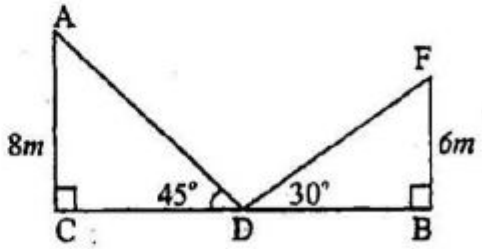
Than $m\overline{CE} = m\overline{CD} + m\overline{DE}$ ..... (1)

$$\therefore \quad \tan(m\angle FDE) = \frac{mFE}{m\overline{DE}}$$

$$\Rightarrow \quad \tan 30° = \frac{6}{m\overline{DE}}$$

$$\frac{6}{\frac{1}{\sqrt{3}}} = 6 \times \sqrt{3} = 10.39m$$

$$\therefore \quad m\overline{DE} = \frac{6}{\frac{1}{\sqrt{3}}}$$

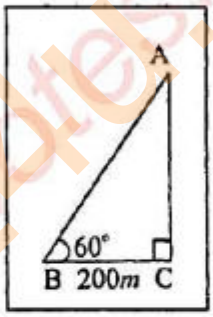


For ΔACD, $m\overline{CD} = m\overline{AC} = 8m$

Putting $m\overline{DE} = 10.39 = 18.39m$

Q10. The foot of tower is at a distance of 20m form a point on the ground. The angle of elevation of the top of the tower form his point is of 60°. Find the height of the tower.

SOLUTION



Let $m\overline{AC}$ be the height of the tower.

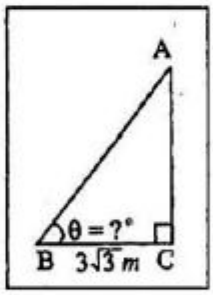
$$\therefore \quad \tan(m\angle B) = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\Rightarrow \quad \tan 60° = \frac{m\overline{AC}}{20}$$

$$\therefore \quad m\overline{AC} = 20 \times \sqrt{3} = 20 \times 1.73 = 34.64m$$

Q11. Find the measure of angle of elevation of the sun such that the shadow of a bamboo 9m long is $3\sqrt{3}m$

SOLUTION.



Let θ be the measure of angle of elevation of the sun such that shadow of a bamboo $\overline{AC}$ is $\overline{BC}$

$$\therefore \quad \tan(m\angle B) = \frac{m\overline{AC}}{m\overline{BC}}$$

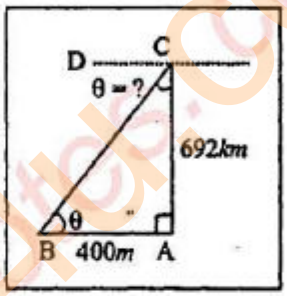
$$\Rightarrow \quad \tan\theta = \frac{9}{3\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\therefore \quad \theta = 60°$$

Q12. An artificial satellite, directly above a city A, is at distance of 692 km (approximately). The distance between the city A and another city B is 400km. find the measure of angle of depression of the city B from the artificial satellite.

SOLUTION.

Let θ be the measure of angle of depression of the city B form an artificial satellite C.



$\therefore$ $(m\angle CBA) = \angle BCD\ \theta$ (Alternate angle)

$\Rightarrow$ $\tan(m\angle CBA = \frac{m\overline{AC}}{m\overline{BC}}$

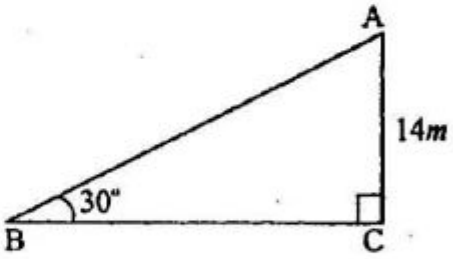
$\Rightarrow$ $\tan\theta = \frac{962}{400} = 1.73 = \sqrt{3}$

$\therefore$ $\theta = 60°$

Q13. A pole 14m high of the bank of a stream makes an angle of 30° with a place on the opposite bank. Find the breadth of the stream.

SOLUTION

Let $\overline{AB}$ represent the bamboo.

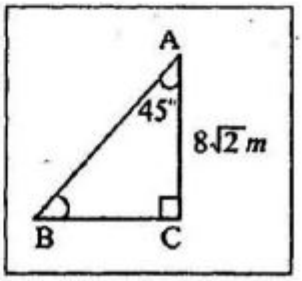


$\therefore$ $\cos(m\angle A = \frac{m\overline{AC}}{m\overline{BC}}$

$\Rightarrow$ $\tan 30° = \frac{14}{m\overline{BC}}$

$\therefore m\overline{BC} = \frac{\frac{14}{1}}{\sqrt{3}} = 14 \times \sqrt{3} = 14 \times 1.73 = 24.25m$

Q14. A bamboo makes an angle of 45° with a wall and reaches a height of $8\sqrt{2}$ m on the wall. Find the length of the bamboo.



SOLUTION.

Let $\overline{AB}$ represent the bamboo.

$\therefore$ $\cos(m\angle A = \frac{m\overline{AC}}{m\overline{BC}}$

$$\Rightarrow \quad m\overline{AB} = \frac{m\overline{AC}}{\cos(m\angle A)1}$$

$$= \frac{8\sqrt{2}}{\cos 45^0} = \frac{\frac{8\sqrt{2}}{1}}{\sqrt{2}} = 8 \times \sqrt{2} \times \sqrt{2} = 16m$$

Q15. A light house is 750 dm high the surface of a sea. Form the top of the light house the angle of depression of two ships are of 30° and of 45° respectively. If the line joining the ships passes through the foot of the light house, then find the distance between the ships are on opposite sides of the light house.

SOLUTION

Let $\overline{AC}$ represent a light house and let B and D are two ships and let $m\overline{BD}$ be the distance between them.

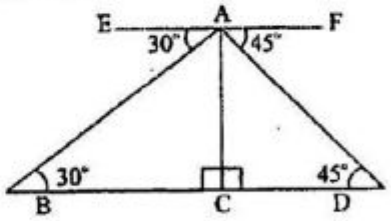
From $\triangle ABD$,

$$m\overline{BD} = m\overline{BC} + m\overline{CD} \quad \ldots\ldots (1)$$

$\therefore$ $m\angle ABC = m\angle EAB = 30°$ (Alternate angles)

$\because$ $m\angle ADC = m\angle FAD = 45°$ (Alternate angles)

$\therefore$ $m\angle D = \angle CAD = 45°$



$\therefore$ $m\overline{CD} = m\overline{AC} = 750$ dm

$\therefore$ $\tan(m\angle ABC) = \frac{m\overline{AC}}{m\overline{BC}}$

$$\Rightarrow \quad m\overline{BC} = \frac{750}{\tan 30^0} = \frac{750}{\frac{1}{\sqrt{3}}} = 750 \times \sqrt{3} = 129dm$$

Eq (1) becomes.

$$m\overline{BD} = 1299 + 750 = 2049 \text{ dm}$$

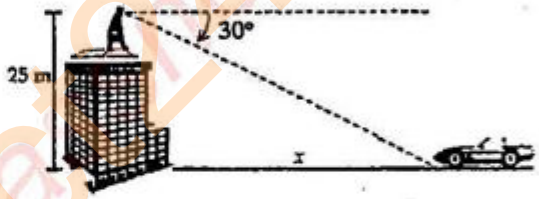
Q16. An observer stands on the top of a building 25m above the ground and the angle of depression to a car on the ground has measure of 30°. About how far is the car from the building?

SOLUTION.

From the figure,

$$\tan 30° = \frac{\text{Height of building}}{x}$$

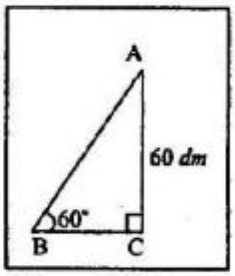
$$\Rightarrow \quad x = \frac{25}{\frac{1}{\sqrt{3}}} = 25 \times 1.73 = 77.85m$$



Q.17. A ladder is placed against a wall making an angle of 60° with the flower and reaching a height of 60 dm. find the length of the ladder.

SOLUTION.

Let $\overline{AB}$ represent a ladder reaching height of 60m at a wall.



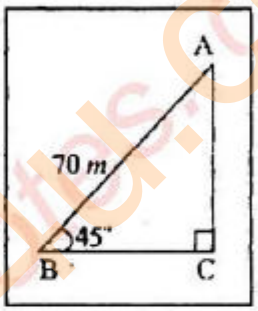
$$\because \quad \sin(m\angle B) = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\Rightarrow \quad m\overline{AB} = \frac{60}{\sin 60°} = \frac{60}{\frac{\sqrt{3}}{2}} = \frac{120}{\sqrt{3}} = 69.28dm$$

Q18. A kite if flying at an angle of 45° with the around. About 70 metres of string have been let out ignoring the sag in the string. Find the height of the kite above the ground.

Solution.

Let $m\overline{AC}$ be the height of the kite above the ground and let $m\overline{AB}$ be the length of the string.



$\because \quad m\overline{AB} = 70m$

$\because \quad \sin(m\angle B) = \frac{m\overline{AC}}{m\overline{AB}}$

$\Rightarrow \quad m\overline{AC} = m\overline{AB}\ (m\angle B)$

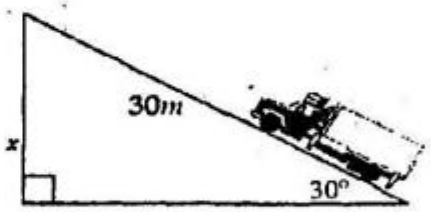
$= 70 \times \sin 45°$

$= 70 \times \frac{1}{\sqrt{2}} = \frac{70}{1.41} = 49.5m$

Q19. A truck drives up a 30 metre incline. The angle that the truck drives up is of 30°. To the nearest metre, how high (vertically) did the truck rise?

Solution.

From the figure.



$\sin 30° = \frac{x}{30}$

$\Rightarrow \quad x = 30 \sin 30°$

$\Rightarrow \quad = 30 \times \frac{1}{2} = 15m$

MISCELLANEOUS EXERCISE VI

Q1. Show that:

(i) $2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

(ii) $\cot 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = 0$

(iii) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = 0$

(iv) $\cos^2 30 - \sin^2 30 = \cos 60^\circ$

(v) $1 - 2\sin^2 30^\circ = 2\cos^2 30^\circ - 1$

(vi) $4 \tan 60^\circ \tan 30^\circ \tan 45^\circ \sin 30^\circ \cos 60^\circ = 1$

(vii) $\sin^2 45^\circ + \cos^2 45^\circ = 1$

SOLUTION.

(i) $2 \sin 45^\circ + \frac{1}{2} \operatorname{cosec} 45^\circ = \frac{3}{\sqrt{2}}$

L.H.S. $= 2 \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \sqrt{2} = \frac{4+2}{2\sqrt{2}} = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}}$

L.H.S. = R.H.S. Proved

(ii) $\cot 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = 0$

L.H.S. $= \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

L.H.S. = R.H.S. Proved

(iii) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$

L.H.S. $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{1}{2}$

$= \sin 30^\circ$

L.H.S. = R.H.S. Proved

(iv) $\cos^2 30 - \sin^2 30 = \cos 60°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$

$= \cos 60°$

(v) $1 - 2\sin^2 30° = 2\cos^2 30° - 1$

$\Rightarrow \quad 1 - 2\left(\frac{1}{2}\right)^2 = 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$

$\Rightarrow \quad 1 - 2 \times \frac{1}{4} = 2 \times \frac{3}{4} - 1$

$\Rightarrow \quad 1 - \frac{1}{2} = \frac{3}{2} - 1$

$\therefore \quad \frac{1}{2} = \frac{1}{2}$ L.H.S. = R.H.S. Proved

(vi) $4 \tan 60° \tan 30° \tan 45° \sin 30° \cos 60° = 1$

$= 4 \times \sqrt{3} \times \frac{1}{\sqrt{3}} \times 1 \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{4 \times \sqrt{3}}{\sqrt{3} \times 4} = 1$ L.H.S. = R.H.S. Proved

(vii) $\sin^2 45° + \cos^2 45° = 1$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2} = 1$ L.H.S. = R.H.S. Proved

Q2. Prove the following identities.

(i) $\cos\theta \sec\theta - \tan\cos\theta = 1$

(ii) $\sec\theta - \tan\theta \sin\theta = \cos\theta$

(iii) $\cos\theta\sqrt{1+\cot^2\theta} = \cot\theta$

(iv) $\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta} = \cot\theta$

(v) $\text{cosec}\,\theta \sin\theta + \dfrac{1}{\cot^2\theta} = \sec^2\theta$

(vi) $\dfrac{\text{cosec}\,\theta - \sec\theta}{\text{cosec}\,\theta + \sec\theta} = \dfrac{1-\tan\theta}{1+\tan\theta}$

SOLUTION.

(i) $2\cos\theta \sec\theta - \tan\cos\theta = 1$

L.H.S.

$= 2\cos\theta \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\cdot\dfrac{\cos\theta}{\sin\theta}$

$= 2(1) - 1 = 2 - 1 = 1$

L.H.S. = R.H.S. Proved

(iii) $\cos\theta\sqrt{1+\cot^2\theta} = \cot\theta$

L.H.S. $= \cos\theta\sqrt{\text{cosec}^2\theta}$

$= \cos\theta\,\text{cosec}\,\theta$

$= \cos\theta \dfrac{1}{\sin\theta}$

$= \cos\theta$

L.H.S. = R.H.S. Proved

(iv) $\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta} = \cot\theta$

L.H.S. $= \dfrac{\sqrt{\cos^2\theta}}{\sin\theta}$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \cos\theta$

L.H.S. = R.H.S. Proved

(v) $\text{cosec}\,\theta \sin\theta + \dfrac{1}{\cot^2\theta}$

L.H.S. $= \dfrac{1}{\sin\theta}\cdot\sin\theta + \dfrac{1}{\dfrac{\cos^2\theta}{\sin^2\theta}}$

$= 1 + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

	$= \frac{1}{\cos^2\theta}$ $= \sec^2\theta$ L.H.S. = R.H.S. Proved
(vi) $\frac{\cos ec\theta - \sec\theta}{\cos ec\theta + \sec\theta} = \frac{1-\tan\theta}{1+\tan\theta}$ L.H.S. $= \frac{\cos ec\theta + \sec\theta}{\cos ec\theta + \sec\theta}$ $= \frac{\frac{1}{\sin\theta} - \frac{1}{\cos\theta}}{\frac{1}{\sin\theta} + \frac{1}{\cos\theta}}$ $= \frac{\frac{1}{\sin\theta}\left[1 - \frac{\sin\theta}{\cos\theta}\right]}{\frac{1}{\sin\theta}\left[1 + \frac{\sin\theta}{\cos\theta}\right]}$ $= \frac{1-\tan\theta}{1+\tan\theta}$ L.H.S. = R.H.S. Proved	

Q3. A ladder makes an angle of 60° with horizontal and reaches upto the height of 5 metres on a wall. Find the length of the ladder.

SOLUTION.

Let $\overline{AB}$ represent a ladder.

$\because \sin(m\angle B) = \frac{m\overline{AC}}{m\overline{AB}}$

$$\Rightarrow \quad m\overline{AB} = \frac{m\overline{AC}}{\sin(m\angle B)} = \frac{5}{\sin 60^\circ}$$

$$= \frac{5}{\frac{\sqrt{3}}{2}} = \frac{10}{\sqrt{3}} = \frac{10}{1.73} = 5.77\,m$$

Q.4. A tree of 90 dm height, on one bank of a river makes an angle of 30° directly on the opposite side of the river. Find the width of the river.

SOLUTION.

Let $\overline{AC}$ represent a tree and let $\overline{BC}$ be the width of the river.

$$\because \quad \tan(m\angle B) = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\Rightarrow \quad m\overline{BC} = \frac{m\overline{AC}}{\tan(m\angle B)} = \frac{90}{\tan 30^\circ} = \frac{90}{\frac{1}{\sqrt{3}}} = 90 \times \sqrt{3}$$

$$= 45.70dm$$

Q.5. Find the angle of elevation of the sun when a 6 meters height pole makes a shadow of length $2\sqrt{3}$ meters on the horizontal plane.

SOLUTION.

Let $\overline{AC}$ represent a pole and $m\overline{BC}$ be the length of its shadow.

Let θ be angle of elevation of the sun.

$$\therefore \quad \tan(m\angle B) = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\Rightarrow \quad \tan\theta = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\therefore \quad \theta = 60^\circ$$

Q6. From the top of a clif 300 m high, measures of the angle of depression of the top and bottom of a tower are 30° and 60° respectively. Find the height of the tower.

SOLUTION.

Let $\overline{AC}$ represent the tower.

From ΔDEA,

$$\tan m\angle ADE = \frac{m\overline{AE}}{m\overline{ED}}$$

$$\Rightarrow \quad \tan 30° = \frac{m\overline{AE}}{m\overline{ED}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} \quad \frac{m\overline{AE}}{m\overline{ED}}$$

$$\Rightarrow \quad m\overline{ED} = m\overline{AE}\ \sqrt{3} \qquad \ldots\ldots\ldots\ldots (1)$$

From ΔDEB.

$$\tan m\angle BDE = \frac{m\overline{EB}}{m\overline{ED}}$$

$$\Rightarrow \quad \tan 60° = \frac{m\overline{DC}}{m\overline{ED}} \qquad (\because m\overline{EB} = m\overline{DC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{300}{m\overline{ED}} \quad \Rightarrow \quad m\overline{ED} = \frac{300}{\sqrt{3}} \qquad \ldots\ldots\ldots (2)$$

From (1)and (2), we get $\quad m\overline{AE} = \frac{300}{\sqrt{3}}$

$$\Rightarrow \quad m\overline{AE} = \frac{300}{3} = 100m$$

Now $\quad m\overline{EB} = m\overline{EA} + m\overline{AB} \quad \Rightarrow \quad m\overline{AB} = m\overline{EB} - m\overline{EA}$

$= 300 - 100$

$= 200m$ **Ans.**