More Notes, Books Visit Now: www.Perfect24u.com

Chap 6 DEMONSTRATIVE GEOMETRY

MULTIPLE CHOICE (MCQ's)

(i) The line segment AB consists of ------------------

S(a) A, B and the points between A,B

(b) Finite number of points

(c) all points member A,B

(d) Point AB

(ii) Sum of all the three angles of triangles is ---------------

(a) 90^0 **(b) 180^0** © 270^0 (d) 360^0

(iii) Sum of measures of any two sides of a triangle is----------

(a) greater than the measure of third side

(b) equal to the measure of third side

(c) smaller than the measure of third side

(d) above of these

(iv) How many mid points has a line segment?

(a) only four (b) only three © only two **(d) only one**

(v) How many non–collinear points are in a plane?

(a) three (b) four © two (d) one

(vi) Set of all the points which line outside a triangle is called,

(a) base of triangle (b) diagonal of the triangle

(c) Interior of triangle (d) exterior of triangle

(vii) Number of end point of a ray,

Perfect24u.com

www.Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

(a) 0 **(b) 1** (c) 2 (d) infinite

(viii) An angle of 90^0 is called-----------

(a) Right angle (b) Acute angle

(c) Obtuse angle (d) None of above

(ix) Two intersecting lines divide the plane into ----------.

(a) 4 parts (b) 2 parts (c) 1 parts (d) 0 parts

(x) The outer sides of two adjacent supplementary angles----------.

(a) From a line (b) make a right angle

(c) Intersect with each other (d) make a line segment

(xi) Which one of the following pairs constitute complementary angles?

(a) 1^0 , 90^0 **(b) 1^0 , 89^0**

© 100^0 , 80^0 (d) 90^0 , 90^0

(xii) Each of two complementary angles can be ------------

(a) Right angle **(b) Acute angles**

(c) Obtuse angle (d) None of above

(xiii) If supplementary angle are congruent to each other, then the measurement of each angle is,

(a) 60^0 **(b) 90^0** © 180^0 (d) 45^0

(xiv) If complementary angles are congruent, the measure of each angle is---------------.

(a) 60^0 (b) 90^0 © 180^0 **(d) 45^0**

(xv) One and only one line can pass through--------

(a) 1 point **(b) 2 points**

(c) 3 points (d) 4 points

(xvi) Three non–collinear points determine a

(a) Plane (b) space (c) square (d) Rectangle

(xvii) One and only one place passes through ------------

(a) 1 point (b) 2 points

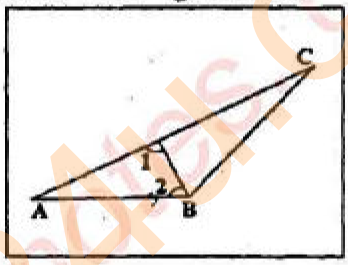
(c) 3 points (d) 4 points

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

EXERCISE 6.1

Q.1. The greatest side of a triangle subtends the greatest angle.



SOLUTION. Given: In ΔABC, $\overline{AC}$ is greatest side.

To Prove that: $m\angle B > m\angle A$ and $m\angle B > m\angle C$.

Construction: From $\overline{AC}$ cut of $\overline{AD} \cong \overline{AB}$. join B and D.

Proof:

STATEMENTS	REASONS
1. In ΔABD, $\overline{AB} \cong \overline{AD}$	1. Construction
2. $\therefore$ $m\angle 1 > m\angle 2$	2. Opposite
3. But $\angle 1$ is interior angle of Δ ABC.	3. By definition of exterior angle.
4. $\therefore$ $m\angle 1 > m\angle C$	4. Theorem 2 (Maths Part1) exterior angle is greater than non-adjacent interior angle.
5. or $m\angle 2 > m\angle C$	5. By (2)
6. $\therefore$ $m\angle B = m\angle ABD + m\angle CBD$ $= m\angle 2 + m\angle CBD$	6. Angle Addition postulate
7. So, $m\angle B > m\angle C$	7. Transitive property.
8. Similarly, $m\angle B > m\angle A$.	8. As proved above.

Q.E.D.

Q.2. If two sides of a triangle are unequal, the angle opposite to the smaller side must be acute.

SOLUTION:

Given: In ΔABC, $m\overline{BC} > m\overline{AB}$, $m\overline{AC} > m\overline{AB}$

To prove that: $\angle C$ is acute, i.e., $m\angle C > 90°$.

More Notes, Books Visit Now: www.Perfect24u.com

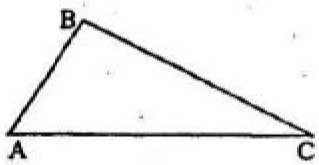
PROOF:

STATEMENTS	REASONS
1. $\because\ m\overline{BC} > m\overline{AB}$	1. Given
2. $\therefore\ m\angle A > m\angle C$	2. Theorem 1
3. or $m\angle C > m\angle A$	3. Property of Inequality
4. $\because\ m\overline{AC} > m\overline{AB}$	4. Given
5. $\therefore\ m\angle B > m\angle C$	5. Theorem 1
6. $m\angle A + m\angle B > m\angle C$	6. By (2) and (5)
7. or $180^\circ > -m\angle C > m\angle C$	7. $m\angle A + m\angle B > m\angle C = 180^\circ$
8. or $180^\circ > 2m\angle C$	8. By (7)
9. or $m\angle C < 90^\circ$ i.e. $\angle C$ is acute	9. By (8)

Q.E.D.

Q.3. The greatest angle of a triangle has the greatest side opposite to it.

SOLUTION.



Given: In ΔABC, $\angle A$ is the greatest angle.

To prove that: $m\overline{BC} > \overline{AB}$ and $m\overline{BC} > m\overline{AC}$

PROOF:

STATEMENTS	REASONS
1. $\because\ m\angle A > m\angle B$	1. Given
2. $\therefore\ m\overline{BC} > m\overline{AC}$	2. Theorem 1 (a)
3. $\because\ m\angle A > m\angle C$	3. Given
4. $\therefore\ m\overline{BC} > m\overline{AB}$	4. Theorem 1 (a)

Q.E.D.

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

Q.4. In a right-angle triangle the hypotenuse is the greatest side.

SOLUTION:

Given: *In a right-angled triangle ABC, $\angle C$ is a right angle and $\overline{AB}$ is the hypotenuse.*

To prove that: *$\overline{AB}$ is the greatest side, i.e.*

$m\overline{AB} > m\overline{BC}$ and $m\overline{AB} > m\overline{AC}$

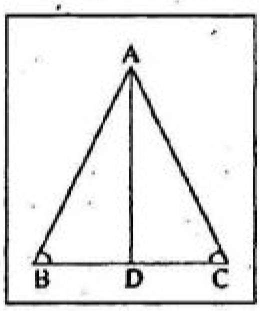
Proof:

STATEMENTS	REASONS
1. $\because m\angle C = 90°$	1. Given
2. $\therefore m\angle C > m\angle A$ and $m\angle C > m\angle B$	2. $m\angle A + m\angle B + m\angle C = 180°$
3. $\because m\angle C > m\angle A$	3. By (2)
4. $\therefore m\overline{AB} > m\overline{BC}$	4. Theorem 1 (a)
5. $\because m\angle C > m\angle B$	5. By (2)
6. $\therefore m\overline{AB} > m\overline{AC}$	6. Theorem (a)
7. $\overline{AB}$ is the greatest side.	7. By (4) and (6)

Q.E.D.

Q.5. Give an alternate proof of Theorem 1(a) assuming that if $m\overline{AC} > m\overline{AB}$, then by trichotomy property of order relation either

(i) $m\overline{AC} = m\overline{AB}$ or (ii) $m\overline{AC}$ $m\overline{AB}$ and refute the assumption.



SOLUTION:

Given: *In ΔABC, $m\angle B = m\angle C$ with $m\overline{AC} > m\overline{AB}$*

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

To prove that: $m\overline{AC} = m\overline{AB}$

Construction: Join A to the mid point D of $\overline{BC}$.

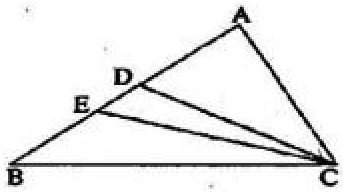
Proof:

STATEMENTS	REASONS
1. In $\Delta ADB \leftrightarrow \Delta ADC$	1.
(i) $\overline{AD} \cong \overline{AD}$	(i) Common
(ii) $m\angle B = m\angle C$	(ii) Given
iii) BD L DC	(iii) Construction
2. $\therefore \Delta ADB \cong \Delta ADC$	2. S.A.S E S.A.S
3. $\overline{AC} \cong \overline{AB}$	3. By (2)
4. Or $m\overline{AC} = m\overline{AB}$	4. By (3)

Q.E.D.

(ii) Given: In ΔABC, $m\angle B < m\angle C$ with $m\overline{AC} \not> m\overline{AB}$

To prove that: $m\overline{AC} < m\overline{AB}$



Construction Make $\angle ACD \cong \angle B$. Draw $\overline{AE}$ the bisector of $\angle DCB$, i.e. $m\angle 1 = m\angle 2$

Proof:

STATEMENTS	REASONS
1. $\angle AEC$ is the exterior angle of ΔEBC.	1. By definition of exterior angle
2. $\therefore m\angle AEC = m\angle B + m\angle 2$	2. By Theorem 5 corollary 5.
$= m\angle B + m\angle 1$	Construction ($m\angle 1 = m\angle 2$)
$= m\angle ACD + m\angle 1$	$m\angle B = \angle ACD$ (Construction)
$= m\angle ACE$	By angle addition postulate
3. $\therefore \overline{AE} \cong \overline{AE}$	3. By (2)
4. $\therefore m\overline{AB} > m\overline{CB}$	4. $\therefore m\overline{AB} > m\overline{AE}$
5. Or $m\overline{AC} < m\overline{AB}$	5. By (4)

Q.E.D.

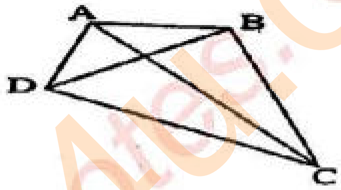
Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

EXERCISE 6.2

Q1. Sum of sides of a quadrilateral is greater than the sum of its diagonals.

SOLUTION.



Given: ABCD is a Quadrilateral. $\overline{AC}$ and $\overline{BD}$ are the its diagonal.

To prove that: $|\overline{AB}|+|\overline{BC}|+|\overline{CD}|+|\overline{AD}|>|\overline{AC}|+|\overline{BD}|$

Proof:

STATEMENTS	REASONS
1. In ΔABD, $|\overline{AD}|+|\overline{AB}|>|\overline{AC}|$	1. Theorem 2
2. In ΔABC, $|\overline{AB}|+|\overline{BC}|>|\overline{AC}|$	2. Theorem 2
3. In ΔADC, $|\overline{DB}|+|\overline{CD}|>|\overline{AC}|$	3. Theorem 2
4. In ΔBCD, $|\overline{BC}|+|\overline{CD}|>|\overline{BD}|$	4. Theorem 2
5. $|\overline{AD}|+|\overline{AB}|+|\overline{AB}|+|\overline{BC}|+|\overline{AD}|+|\overline{CD}|>|\overline{BD}|+|\overline{AC}|+|\overline{AC}|+|\overline{BD}|$	5. By (1), (2), (3) and (4)
6. $2|\overline{AB}|+2|\overline{BC}|+2|\overline{CD}|+2|\overline{AD}|>2|\overline{AC}|+2|\overline{BD}|$	6. Adding the same quantity
7. $\therefore |\overline{AB}|+|\overline{BC}|+|\overline{CD}|+|\overline{AD}|>|\overline{AC}|+|\overline{BD}|$	7. Dividing by 2

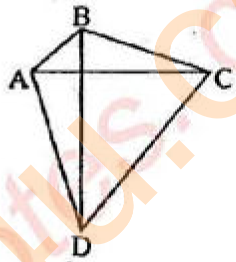
Q.E.D.

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

2Q. Any there sides of a quadrilateral are together greater than the fourth.

SOLUTION.



Given: $\triangle BCD$ is a quadrilateral. $\overline{AC}$ and $\overline{BD}$ are its A diagonals.

To prove that: $|\overline{AB}| + |\overline{BC}| + |\overline{CD}| > |\overline{AD}|$

Proof:

STATEMENTS	REASONS
1. In $\triangle ABC$, $|\overline{AB}| + |\overline{BC}| > |\overline{AC}|$	1. Theorem 2
2. In $\triangle ACD$, $|\overline{AC}| + |\overline{CD}| > |\overline{AD}|$	2. Theorem 2
3. $\therefore$ $|\overline{AB}| + |\overline{BC}| + |\overline{AC}| + |\overline{CD}| > |\overline{AC}| + |\overline{DC}|$	3. By (1) and (2)
4. Or $|\overline{AB}| + |\overline{BC}| + |\overline{CD}| > |\overline{AD}|$	4. Dividing by $|\overline{AC}|$

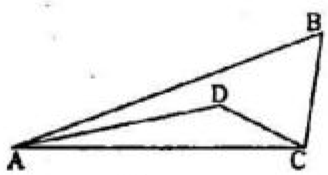
Q.E.D.

Q.3. The sum of the line segments drawn from the ends of the base of a triangle to a point in its interior is less than the sum of its two sides.

SOLUTION.

Given: D is the point in the interior of $\triangle ABC$.

$\overline{AD}$ and $\overline{CD}$ are line segments drawn from the ends of the base $\overline{BC}$ of $\triangle ABC$ to D.



To prove that: $|\overline{AD}| + |\overline{CD}| < |\overline{AB}| + |\overline{BC}|$

More Notes, Books Visit Now: www.Perfect24u.com

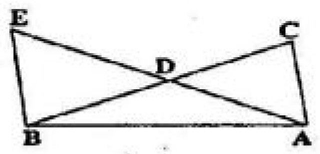
Proof:

STATEMENTS	REASONS
1. In ΔABC, $\lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert > \lvert\overline{AC}\rvert$	1. Theorem 2
2. In ΔADC, $\lvert\overline{AD}\rvert + \lvert\overline{DC}\rvert > \lvert\overline{AC}\rvert$	2. Theorem 2
3. $\lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert > \lvert\overline{DC}\rvert$	3. D is in the interior of ΔABC
4. $\therefore \lvert\overline{AD}\rvert + \lvert\overline{DC}\rvert < \lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert$	4. By (3)

Q.E.D.

Q.4. Prove that any two sides of a triangle are together greater than twice the median of the third side.

SOLUTION:



Given: In ΔABC, D is the mid-point of BC,

i.e. $\lvert\overline{BD}\rvert + \lvert\overline{DC}\rvert = \lvert\overline{BC}\rvert$

Construction: Join A to D $\overline{AD}$ is the median to $\overline{BC}$.

Produce $\overline{AD}$ such that $\overline{DE} \cong \overline{AD}$. Join E to B.

To prove that: $\lvert\overline{AB}\rvert + \lvert\overline{AC}\rvert > 2\lvert\overline{AD}\rvert$

Proof:

STATEMENTS	REASONS
1. In $\Delta BDE \leftrightarrow \Delta ADC$	1.
(i) $\overline{DE} \cong \overline{AD}$	(i) Construction
(ii) $\overline{BD} \cong \overline{DC}$	(ii) Given
(iii) $m\angle BDE = m\angle ADC$	(iii) Theorem 1 (Maths for class IX)
2. $\therefore \Delta BDE \cong \Delta ADC$.	2.
3. $\overline{BE} \cong \overline{AC}$	3. By congruence of triangles

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

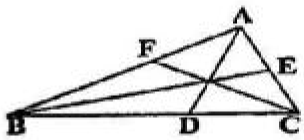
4. Now in ΔABE, $|\overline{AB}| + BE > |\overline{AE}|$	4. Theorem 2
5. Or $|\overline{AB}| + |\overline{AC}| > |\overline{AE}|$	5. By (3)
6. Or $|\overline{AB}| + |\overline{AC}| > |\overline{AD}| + |\overline{DE}|$	6. Construction
7. Or $|\overline{AB}| + |\overline{AC}| > |\overline{AD}| + |\overline{AD}|$	7. Construction
8. $\therefore |\overline{AB}| + |\overline{AC}| > |\overline{AD}|$	8. By (7)

Q.E.D.

Q5. Prove that the sum of the three medians of triangle is less than the sum of its sides.

[Hints: Use the result of Question 4 above]

SOLUTION.



Given: D. E and F are mid-points of $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ respectively. $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ are medians to the sides $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ respectively.

To prove that: $\overline{AD} + \overline{BE} + \overline{CF} < \overline{AB} + \overline{BC} + \overline{AC}$

Proof:

STATEMENTS	REASONS
1. $2|\overline{AD}| < |\overline{AB}| + |\overline{AC}|$	1. The result of Q No. 4
2. $2|\overline{BE}| < |\overline{AB}| + |\overline{BC}|$	2. The result of Q. No. 4
3. $2|\overline{CF}| < |\overline{BC}| + |\overline{AC}|$	3. The result of Q. No. 4

Perfect24u.com

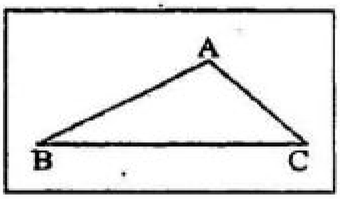
More Notes, Books Visit Now: www.Perfect24u.com

4.	$2\lvert\overline{AD}\rvert + 2\lvert\overline{BE}\rvert + 2\lvert\overline{CF}\rvert + \lvert\overline{AB}\rvert +$ $\lvert\overline{AC}\rvert + \lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert + \lvert\overline{BC}\rvert + \lvert\overline{AC}\rvert$	4.	BY (1), (2), (3)
5.	$2\{\lvert\overline{AD}\rvert + \lvert\overline{BE}\rvert + \lvert\overline{CF}\rvert\} < 2\{\lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert + \lvert\overline{AC}\rvert\}$	5.	By (4)
6.	$\therefore \lvert\overline{AD}\rvert + \lvert\overline{BE}\rvert + \lvert\overline{CF}\rvert < \lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert + \lvert\overline{AC}\rvert$	6.	Dividing by 2

Q.E.D.

Q6 Difference of any two sides of a triangle is less than the third side.

SOLUTION.



Given: ΔABC

To prove that: $\lvert\overline{AB}\rvert - \lvert\overline{AC}\rvert < \lvert\overline{BC}\rvert$

Proof:

Statements	Reasons
1. $\lvert\overline{AC}\rvert + \lvert\overline{BC}\rvert > \lvert\overline{AB}\rvert$	1. Theorem 2
2. Or $\lvert\overline{AC}\rvert - \lvert\overline{AB}\rvert > -\lvert\overline{BC}\rvert$	2. By (1)
3. Or $-\lvert\overline{AC}\rvert + \lvert\overline{AB}\rvert < \lvert\overline{BC}\rvert$	3. Multiplying by -1
4. $\therefore \lvert\overline{AB}\rvert - \lvert\overline{AC}\rvert < \lvert\overline{BC}\rvert$	4. By (3)

Q.E.D.

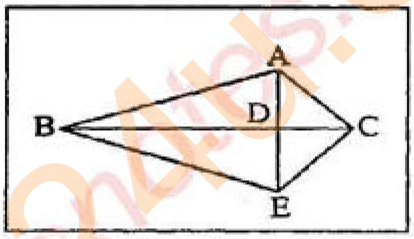
Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

EXERCISE 6.3

Q1. Prove that two sides of a triangle are together greater than twice the perpendicular drawn to the opposite side from the vertex in which they meet.

SOLUTION:



Given: In ΔABC, $\overline{AD}$ is the perpendicular.

To prove that: $|\overline{AB}| + |\overline{AC}| > |\overline{AD}|$

Construction: Produce $\overline{AD}$ such that

$\overline{AD} \cong |\overline{AD}|$. Join E to B and C.

Proof:

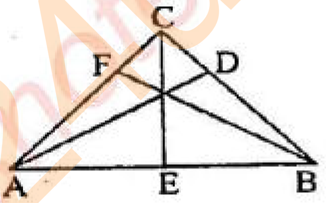
STATEMENTS	REASONS
1. In $\Delta ABD \leftrightarrow \Delta EBD$	1.
(i) $\overline{BD} \cong \overline{BD}$	(i) Common
(ii) $m\angle ADB = m\angle EDB$	(ii) $\overline{AE} \perp \overline{BD}$
(iii) $\overline{AD} \cong \overline{DE}$	(iii) Construction
2. $\therefore \Delta ABD \cong \Delta EBD$	2. S.A.S. Postulate
3. $\therefore |\overline{AB}| = |\overline{BE}|$	3. By congruence of triangles
4. In ΔABE, $|\overline{AB}| + |\overline{BE}| > |\overline{AE}|$	4. Theorem 2
5. Or $|\overline{AB}| + |\overline{AB}| > |\overline{AD}| + |\overline{DE}|$	5. $|\overline{AB}| = |\overline{BE}|$; $|\overline{AE}| = |\overline{AD}| + |\overline{DE}|$
6. $\therefore 2|\overline{AB}| > 2|\overline{AD}|$	6. $|\overline{AD}| = |\overline{DE}|$
7. $|\overline{AB}| > |\overline{AD}|$	7. Dividing by 2
8. Similarly, $|\overline{AC}| > |\overline{AD}|$	8. As proved above

More Notes, Books Visit Now: www.Perfect24u.com

9. Now $\|\overline{AB}\| + \|\overline{AC}\| > \|\overline{AD}\| + \|\overline{AD}\|$	9. By (7) and (8)
10. $\therefore \|\overline{AB}\| + \|\overline{AC}\| > \|\overline{AD}\|$	10. By (9)

Q.E.D.

Q.2. Perimeter of a triangle is greater than the sum of its three perpendiculars.



SOLUTION.

Given: ΔABC is a triangle. $\overline{AD} \perp \overline{BC}$, $\overline{BF} \perp \overline{AC}$ and $\overline{CE} \perp \overline{AC}$

To prove that: $|\overline{AB}| + |\overline{BC}| + |\overline{AC}| > |\overline{AD}| + |\overline{BF}| + |\overline{CE}|$

Proof.

STATEMENTS	REASONS
1. $\|\overline{AB}\| + \|\overline{AC}\| > 2\|\overline{AD}\|$	1. The result of Q. No. 1
2. $\|\overline{AB}\| + \|\overline{BC}\| > 2\|\overline{BF}\|$	2. The result of Q. No. 1
3. $\|\overline{AC}\| + \|\overline{BC}\| > 2\|\overline{CE}\|$	3. The result of Q. No. 1
4. $\therefore \|\overline{AB}\| + \|\overline{AC}\| + \|\overline{AB}\| + \|\overline{BC}\| + \|\overline{AC}\| + \|\overline{BC}\|$ $> 2\|\overline{AD}\| + 2\|\overline{BF}\| + 2\|\overline{CE}\|$	4. By (1), (2) and (3)
5. Or $2\|\overline{AB}\| + 2\|\overline{BC}\| + 2\|\overline{AC}\| >$ $2\|\overline{AD}\| + 2\|\overline{BF}\| + 2\|\overline{CE}\|$	5. By (4)
6. $\therefore \|\overline{AB}\| + \|\overline{BC}\| \|\overline{AC}\| > \|\overline{AD}\| + \|\overline{BF}\| + \|\overline{CE}\|$	6. Dividing by 2

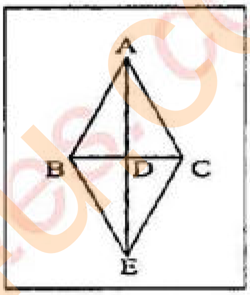
Q.E.D.

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

Q3 Congruent sides of an isosceles triangle are together greater than twice the median to the base.

SOLUTION.



Given: In an isosceles $\triangle ABC$, $\overline{AD}$ is the median to $\overline{BC}$.

Construction: Produce $\overline{AD}$ such that $\overline{AD} \cong \overline{DE}$.

Join E to B and C.

To prove that: $|AB| + |AC| > 2|AD|$

Proof:

STATEMENTS	REASONS
1. In $\triangle ABD \leftrightarrow \triangle EBD$	1.
(i) $\overline{AD} \cong \overline{DE}$	(i) Construction
(ii) $\overline{BD} \cong \overline{BD}$	(ii) D is the mid-point of $\overline{BC}$
(iii) $m\angle ADB = m\angle EDB$	(iii) $\overline{AD} \perp \overline{BC}$
2. $\therefore \triangle ABD = m\angle EBD$	2. S.A.S Postulate
3. $\therefore |AB| = |BE|$	3. Congruence of triangles
4. In $\triangle ABE$, $|AB| + |BE| > |AE|$	4. Theorem 2
5. $|AC| + |AB| > |AD| + |DE|$	5. $|AB| = |AC|$ (Given) $|AB| = |BE|$ (By 3) $|AE| = |AD| + |DE|$ (Construction)
6. $\therefore |AB| + |AC| > 2|AD|$	6. $|AD| = |DE|$ (Construction)

Q.E.D.

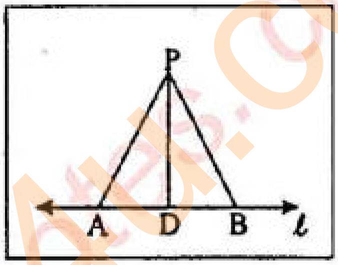
www.Perfect24u.com

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

Q.4. To a line, there can be drawn at most two congruent segments from a given point outside it.

SOLUTION.



Given: *Two segments $\overline{PA}$ and $\overline{PB}$ are drawn to the line l from the a given point P outside it.*

To prove that: $\overline{PA} \cong \overline{PB}$

Construction Draw $\overline{PD} \perp l$

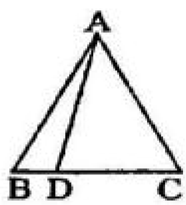
Proof:

STATEMENTS	REASONS
1. In $\Delta ADP \leftrightarrow \Delta BDP$	1.
(i) $\overline{PD} \cong \overline{PD}$	(i) Common
(ii) $m\angle ADP = m\angle BDP$	(ii) $\overline{PD} \perp l$
(iii) $\overline{AD} \cong \overline{DB}$	(iii) Construction
2. $\therefore \Delta ADP \cong \Delta BDP$	2. S.A.S Postulate
3. $\therefore \overline{PA} \cong \overline{PB}$	3. Congruence of triangles

Q.E.D.

Q.5. A line segment drawn from the vertex of an isosceles triangle to any point on the base is less than either of the congruent sides.

SOLUTION.



Given: *In an isosceles ΔABC. $\overline{AD}$ is the line segment A from A to any point D on the base $\overline{BC}$ and $\overline{AB} \cong \overline{AC}$.*

To prove that: $|\overline{AD}| < |\overline{AB}|$ or $|\overline{AD}| < |\overline{AC}|$

More Notes, Books Visit Now: www.Perfect24u.com

Proof:

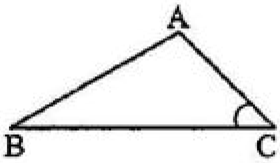
STATEMENTS	REASONS
1. $\because \lvert\overline{AB}\rvert = \lvert\overline{AC}\rvert$	1. Given
2. $\therefore \angle C = \angle B$	2. Angles opposite to congruent sides
3. $\angle ADB$ is the exterior angle of ΔADC	3. By definition of exterior angle
4. $\therefore m\angle ADC > m\angle C$	4. Exterior angle > interior angle
5. Or $m\angle ADC > m\angle B$	5. By (2)
6. $\therefore \lvert\overline{AB}\rvert > \lvert\overline{AD}\rvert$	6. Theorem 1 (b)
7. Similarly, $\lvert\overline{AC}\rvert > \lvert\overline{AD}\rvert$	7. By (6)

Q.E.D.

Q6 Any side of a triangle is less than half the sum of its three sides.

SOLUTION: **Given:** ΔABC.

To prove that: $\lvert\overline{AB}\rvert < \frac{1}{2}\left[\lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert + \lvert\overline{AC}\rvert\right]$



Proof:

STATEMENTS .	REASONS
1. In ΔABC, $\lvert\overline{AC}\rvert + \lvert\overline{BC}\rvert > \lvert\overline{AB}\rvert$	1. Theorem 2
2. Or $\lvert\overline{AB}\rvert < \lvert\overline{AC}\rvert + \lvert\overline{BC}\rvert$	2. By (1)
3. $\therefore \lvert\overline{AB}\rvert + \lvert\overline{AB}\rvert < \lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert + \lvert\overline{AC}\rvert$	3. Adding the same quantity
4. Or $2\lvert\overline{AB}\rvert < \lvert\overline{AB}\rvert + \lvert\overline{BC}\rvert + \lvert\overline{AC}\rvert$	4.

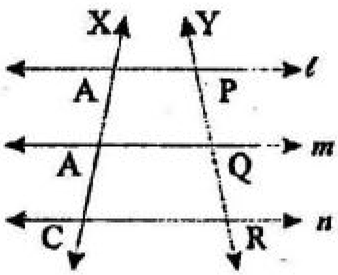
More Notes, Books Visit Now: www.Perfect24u.com

5. $\overline{AB} < 2 - [\overline{AB} + \overline{BC} + \overline{AC}]$	5. Dividing by 2
6. Similarly, $\overline{AC} < \frac{1}{2}[\overline{AB} + \overline{BC} + \overline{AC}]$ $\overline{BC} < \frac{1}{2}[\overline{AB} + \overline{BC} + \overline{AC}]$	6. As proved above

Q.E.D.

EXERCISE 6.4

Q.1. Three parallel lines l, m, n cut two other lines X and Y in points A, B, C and P, Q, R respectively. Prove that:
$\frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{PQ}}{m\overline{QR}}$



SOLUTION.

Given: Three parallel lines l, m and n cut two other lines X and Y in points A, B, C and P, Q, R respectively.

To prove that: $\frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{PQ}}{m\overline{QR}}$

Proof:

STATEMENTS	REASONS
1. $\because l \parallel m$ and $m \parallel n$	1. Given
2. $\therefore m\overline{AB} = m\overline{BC}$	2. Theorem 15 (Maths Class IX)
3. Also $m\overline{PQ} = m\overline{QR}$	3. Theorem 15 (Maths Class Do
4. Now $\frac{m\overline{AB}}{m\overline{BC}} = 1$	4. Dividing the same quantity

More Notes, Books Visit Now: www.Perfect24u.com

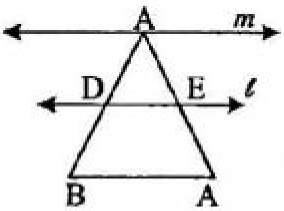
5. Also $\frac{m\overline{PQ}}{m\overline{QR}} = 1$	5. By reason (4)
6. $\frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{PQ}}{m\overline{QR}}$	6. Transitive property

Q.E.D.

Q2 Prove that the line drawn from the mid-point of a side of a triangle parallel to the base bisects the other side.

SOLUTION.

Given: In ΔABC, D is the mid-point of $\overline{AB}$ and the A line l is drawn from D and parallel to the base $\overline{BC}$ and cuts $\overline{AC}$ at E



To prove that: $m\overline{AE} = m\overline{CE}$.

Construction Draw a line m through A parallel to $\overline{BC}$.

Proof:

STATEMENTS	REASONS
1. $\therefore l \parallel \overline{BC}$	1. Given
2. and $m \parallel \overline{BC}$	2. Construction
3. $\therefore l \parallel m$	3. By (1) and (2)
4. $\therefore m\overline{AD} = m\overline{DB}$	4. Given
5. $\therefore \overline{AB}$ is a transversal	5. By (1), (2), (3) and (4)
6. Also $\overline{AC}$ is a transversal	6. By (4)
7. $\therefore m\overline{AE} = m\overline{BC}$	7. Theorem 15 (Maths Class IX)

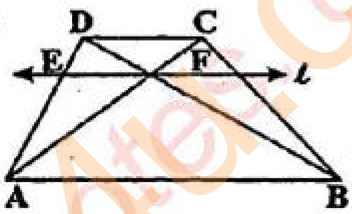
Q.E.D.

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

Q.3. The diagonals $\overline{AC}$ and $\overline{BD}$ of a trapezium ABCD intersect in Q. Prove that $m\overline{OA} : m\overline{OC} = m\overline{OB} : m\overline{OD}$

SOLUTION:



Given: In a trapezium ABCD $\overline{AC}$ and $\overline{BD}$ intersect in O and $\overline{AB} \parallel \overline{DC}$

To prove that: $m\overline{OA} : m\overline{OC} = m\overline{OB} : m\overline{OD}$

Construction: Draw a line l through O parallel to $\overline{DC}$. The line "l" cuts $\overline{DA}$ at E and $\overline{CB}$ at F.

Proof:

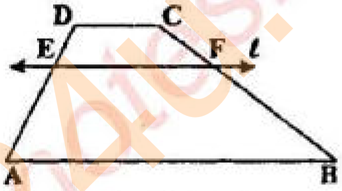
STATEMENTS	REASONS
1. $\overline{DA}$ is transversal to $\overline{DC}$, l and $\overline{AB}$ and $\overline{CB}$ cuts $\overline{DC}$, l and $\overline{AB}$.	1. Construction
2. $\therefore \dfrac{m\overline{AB}}{m\overline{BC}} = \dfrac{m\overline{PQ}}{m\overline{QR}}$	2. Theorem 15 (Maths Class IX)
3. In ΔCAB, $\dfrac{m\overline{OC}}{m\overline{OA}} = \dfrac{m\overline{CF}}{m\overline{FB}}$	3. Theorem 4
4. In ΔCAB, $\dfrac{m\overline{DE}}{m\overline{EA}} = \dfrac{m\overline{OD}}{m\overline{OB}}$	4. Theorem 4
5. $\therefore \dfrac{m\overline{OC}}{m\overline{OA}} = \dfrac{m\overline{OD}}{m\overline{OB}}$	5. Transitive property of equation.
6. $\therefore \dfrac{m\overline{OA}}{m\overline{OC}} = \dfrac{m\overline{OB}}{m\overline{OD}}$	6. Invertendo property of proportion.

Q.E.D.

More Notes, Books Visit Now: www.Perfect24u.com

Q4 Prove that the line drawn parallel to the sides of a trapezium divides the non- parallel sides proportionally.

SOLUTION.



Given: In a trapezium ABCD, $\overline{AB} \parallel \overline{DC}$ and a line is parallel to $\overline{AB}$ and $\overline{DC}$ and cuts $\overline{DA}$ at E and $\overline{CB}$ at F.

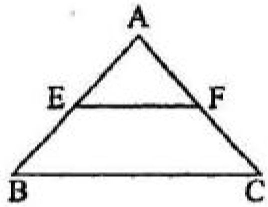
To prove that: $\dfrac{m\overline{DE}}{m\overline{EA}} = \dfrac{m\overline{CF}}{m\overline{FB}}$

Proof:

STATEMENTS	REASONS
1. $\overline{DA}$ is a transversal to $\overline{DC}$, l and $\overline{AB}$ and $\overline{CB}$ cuts $\overline{DC}$, 1 and $\overline{AB}$.	1. Given
2. $\therefore$ $\dfrac{m\overline{DE}}{m\overline{EA}} = \dfrac{m\overline{CF}}{m\overline{FB}}$	2. Theorem 15 (Maths Class IX)

Q.5. Prove that the line segment joining the mid-points of two sides of a triangle is parallel to the third side.

SOLUTION.



Given : In ΔABCD, $\overline{EF}$ is the line segment that cuts $\overline{AB}$ and $\overline{AC}$ at their mid-points E and F respectively

such that: $\dfrac{m\overline{AE}}{m\overline{EB}} = \dfrac{m\overline{AF}}{m\overline{FC}}$

To prove that: $\overline{EF} = \overline{BC}$

Proof:

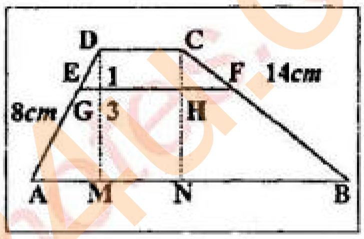
Statements	Reasons
1. $\dfrac{m\overline{AE}}{m\overline{EB}} = \dfrac{m\overline{AF}}{m\overline{FC}}$	1. Given
2. $\overline{EF} = \overline{BC}$	2. Theorem 4 (a)

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

Q.6. Prove that the line which divides the non-parallel sides of a trapezium proportionally is parallel t' the third side.

SOLUTION.



Given: In a trapezium ABCD, the line 1 which divides $\overline{AD}$ and $\overline{BC}$ at E and F respectively

such that: $\frac{m\overline{AE}}{m\overline{ED}} = \frac{m\overline{BF}}{m\overline{FC}}$ and $\overline{AB} \parallel \overline{DC}$

To prove that: $l \parallel \overline{BC}$ and $l \parallel \overline{DC}$

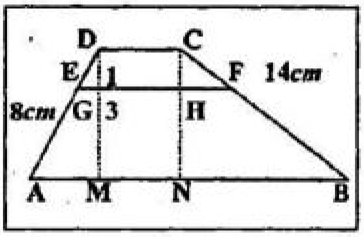
Proof:

Statements	Reasons
1. $\therefore \frac{m\overline{AE}}{m\overline{ED}} = \frac{m\overline{BF}}{m\overline{FC}}$	1. Given
2. $l \parallel \overline{AB}$	2. Theorem 4 (a) corollary 1
3. and $l \parallel \overline{DC}$	3. $\overline{AB} \parallel \overline{AB}$

Q.E.D.

Q.7. The measure of non-parallel sides of a trapezoid are 8cm and 14cm. A line parallel to the parallel sides divides the altitude of the trapezoid in the ratio 1:3. Find the measures of the line segments made by this line on the nonparallel sides of the trapezoid.

SOLUTION.



Given: $m\overline{DA} = 8cm$ $m\overline{CB} = 14$ cm

$$\frac{m\overline{DG}}{m\overline{GM}} = \frac{m\overline{CH}}{m\overline{HN}} = \frac{1}{3}$$

To Find: $m\overline{DE} = ?$; $m\overline{EA} = ?$; $m\overline{CF} = ?$; $m\overline{FB} = ?$

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

In ΔADM, $\dfrac{m\overline{DE}}{m\overline{EA}} = \dfrac{m\overline{DG}}{m\overline{GM}}$

$\Rightarrow \dfrac{m\overline{EA}}{m\overline{DE}} = \dfrac{m\overline{DG}}{m\overline{DG}}$

(Invertendo property of proportion)

$\Rightarrow \dfrac{m\overline{DE}+m\overline{EA}}{m\overline{DE}} = \dfrac{m\overline{DG}+m\overline{GM}}{m\overline{DG}}$

(Componendo property of proportion)

$\Rightarrow \dfrac{m\overline{DA}}{m\overline{DE}} = \dfrac{m\overline{DG}+m\overline{GM}}{m\overline{DG}}$ $\quad (\because m\overline{DE}+m\overline{EA}=m\overline{DA})$

$\Rightarrow \dfrac{m\overline{DE}}{m\overline{DA}} = \dfrac{m\overline{DG}}{m\overline{DG}+m\overline{GM}}$

(Invertendo property of proportion)

$\Rightarrow \dfrac{m\overline{DE}}{8} = \dfrac{1}{1+32}$ (Given)

$\therefore m\overline{DE} = \dfrac{1}{4}\times 8 = 2cm$

$\therefore m\overline{DE} + m\overline{EA} = m\overline{DA}$

$\therefore m\overline{EA} = m\overline{DA} - m\overline{DE} = 8 - 2 = 6cm$

Similarly, $\dfrac{m\overline{CF}}{m\overline{CB}} = \dfrac{m\overline{CH}}{m\overline{CH}+m\overline{HN}}$

$\Rightarrow \dfrac{m\overline{CF}}{14} = \dfrac{1}{1+3}$

$\therefore m\overline{CF} = 14 \times \dfrac{1}{4} = \dfrac{7}{2} = 3.5\ cm$

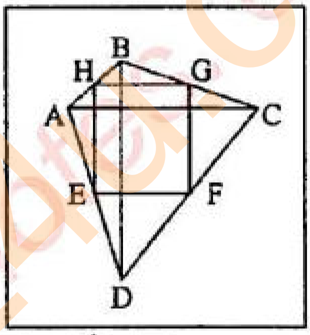
$\because m\overline{CF} + m\overline{FB} = m\overline{CB}$

$\therefore m\overline{FB} = m\overline{CB} - m\overline{CF} = 14 - 3.5 = 10.5cm$

More Notes, Books Visit Now: www.Perfect24u.com

Q.8. Prove that the line segments joining the mid-points of consecutive sides of a quadrilateral form ||m.

SOLUTION:



Given: In a quadrilateral ABCD. E. F. G and H are the mid-points of $\overline{AD}$, $\overline{DC}$, $\overline{CB}$ and $\overline{BA}$ respe ively.

To prove that: EFGH is a parallelogram i.e.

$EF \parallel \overline{HG}$ and $FG \parallel EH$.

Construction: Join B to D and A to C.

Proof

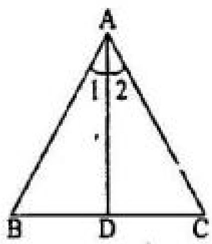
STATEMENTS	REASONS
1. In ΔADC, $\overline{EF} \parallel \overline{AC}$	1. The result of Q. No. 5
2. In ΔABC, $\overline{HG} \parallel \overline{AC}$	2. The result of Q. No. 5
3. $\therefore$ $\overline{EF} \parallel \overline{HG}$	3. By (1) and (2)
4. Similarly, $\overline{FG} \parallel \overline{EH}$	4. By (3)

Q.E.D

EXERCISE 6.5

Q.1. If the bisector of the vertical angle of a triangle bisects the base, the triangle is isosceles.

SOLUTION.



Given: In ΔABC, $\overline{AD}$ is the bisector of the vertical angle $\angle A$.

It also bisects the base $\overline{BC}$ at D, i.e. $m\overline{BD} = m\overline{DC}$

To prove that: $m\overline{AB} = m\overline{AC}$

Proof:

Perfect24u.com

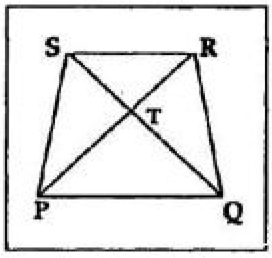
More Notes, Books Visit Now: www.Perfect24u.com

STATEMENTS	REASONS
1. $\frac{m\overline{BD}}{m\overline{DC}} = \frac{m\overline{AB}}{m\overline{AC}}$	1. Theorem 5
2 $1 = \frac{m\overline{AB}}{m\overline{AC}}$	2. $m\overline{BD} = m\overline{DC}$
3. $\therefore m\overline{AB} = m\overline{AC}$	3. by (2)
4. i.e. ΔABC is an isosceles triangle	4. By (3)

Q.E.D.

Q.2. PQRS is a quadrilateral and the bisector of angles Q and S meet the diagonal PR at T. Prove that: $\frac{m\overline{PQ}}{m\overline{QR}} = \frac{m\overline{PS}}{m\overline{RS}}$

SOLUTION.



Given: PQRS is quadrilateral and the bisector of angles Q and S meet the diagonal $\overline{PR}$ at T.

To prove that : $\frac{m\overline{PQ}}{m\overline{QR}} = \frac{m\overline{PS}}{m\overline{RS}}$

Proof:

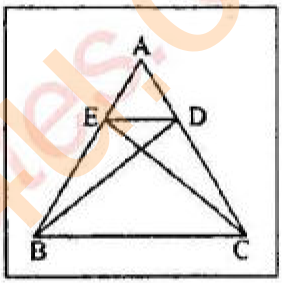
STATEMENTS	REASONS
1. In Δ PQR, $\frac{m\overline{PQ}}{m\overline{QR}} = \frac{m\overline{PT}}{m\overline{TR}}$, where $\overline{TS}$ is the bisector of $\angle Q$	1. Theorem 5
2. In ΔPSR, $\frac{m\overline{PS}}{m\overline{RS}} = \frac{m\overline{PT}}{m\overline{TR}}$, Where the bisector of $\angle S$	2. Theorem 5
3. $\therefore \frac{m\overline{PQ}}{m\overline{QR}} = \frac{m\overline{PS}}{m\overline{RS}}$	By (1) and (2)

Q.E.D.

More Notes, Books Visit Now: www.Perfect24u.com

Q3 A line segment bisecting the base angle B of an isosceles ΔABC meets the opposite side $\overline{AC}$ in D and through D, $\overline{DE} \parallel \overline{BC}$ is drawn cutting $\overline{AB}$ in E. Prove that $\overline{CE}$ bisects angle ACB.

SOLUTION.



Given: In an isosceles ΔABC, $m\overline{AB} = m\overline{AC}$. $\overline{BD}$ is bisector of $\angle B$ such that $\frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{AD}}{m\overline{DC}}$

$\because$ $\overline{DE} \parallel \overline{BC}$

$\therefore$ $\frac{m\overline{AE}}{m\overline{EB}} = \frac{m\overline{AD}}{m\overline{DC}}$ (Theorem 4)

To prove that: $\overline{CE}$ bisects angle ACB, i.e. $\frac{m\overline{BC}}{m\overline{AC}} = \frac{m\overline{AE}}{m\overline{EB}}$

Construction Join C to E.

STATEMENTS	REASONS
1. $\because$ $\frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{AD}}{m\overline{DC}}$	1. Given
2. and $\frac{m\overline{AE}}{m\overline{EB}} = \frac{m\overline{AD}}{m\overline{DC}}$	2. Given
3. $\therefore$ $\frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{AE}}{m\overline{EB}}$	3. Transitive property of equation
4. or $\frac{m\overline{PC}}{m\overline{BC}} = \frac{m\overline{AE}}{m\overline{EB}}$	4. Given
5. $\therefore$ $\overline{CE}$ is the bisector of $\angle ACB$	5. Theorem 5

Q.E.D.

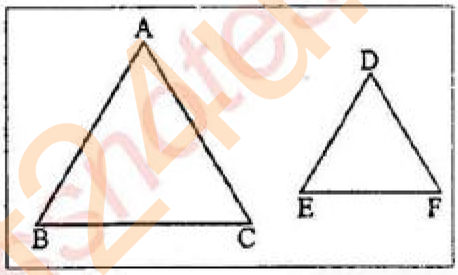
Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

EXERCISE 6.6

Q.1. If two triangle have three sides of the one parallel to the corresponding three sides of the other, prove that their sides are proportional.

SOLUTION.



Given: In two triangles ABC and DEF,

$\overline{AB} \parallel \overline{DE}$, $\overline{AC} \parallel \overline{DF}$ and $\overline{BC} \parallel \overline{EF}$

To prove that: $\dfrac{m\overline{AB}}{m\overline{ED}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$

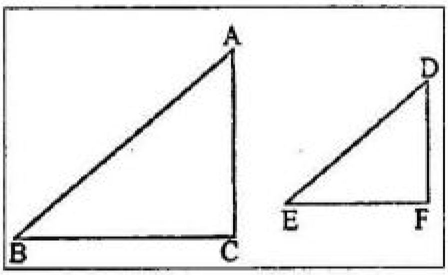
Proof:

Statements	Reasons
1. $\because$ $\overline{EF} \parallel \overline{AC}$ and $\overline{AC} \parallel \overline{DF}$	1. Given
2. $\therefore \angle A \cong \angle D$	2. Theorem 4 corollary 4 (Math Class IX)
3. Similarly, $\angle B \cong \angle E$ And $\angle C \cong \angle F$	3. As proved above
4. $\therefore \dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$	4. Theorem 6

Q.E.D.

Q.2. Two right-triangles have their sides proportional if an acute angle of the one is congruent to an acute angle of the other.

SOLUTION.



Given: In two right triangle ABC and DEF , $\angle A \cong \angle O$ and $m\angle C = m\angle F = 90°$

To prove that: $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{BC}}{m\overline{EF}} = \dfrac{m\overline{AC}}{m\overline{DF}}$

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

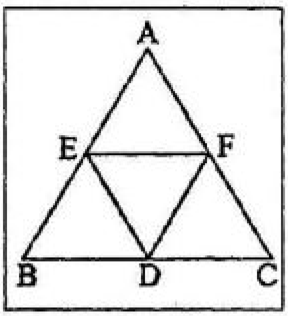
Proof:

STATEMENTS	REASONS
1. $\because \angle A \cong \angle D$ and $\angle C \cong \angle F$	1. Given
2. $\therefore \angle B \cong \angle E$	2. Theorem 5 corollary 6 (Math Class IX)
4. $\therefore \dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{BC}}{m\overline{EF}} = \dfrac{m\overline{AC}}{m\overline{DF}}$	3. Theorem 6

Q.E.D.

Q.3. Segments which join the mid-points of the sides of a triangle form a triangle similar to the original triangle.

SOLUTION.



Given: In ΔABC, D. E and F are the mid-points of $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ respectingly. Join D to E, E to F and F to D.

To prove that: $\Delta DEF \sim \Delta ABC$, i.e.

(i) $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}}$

(ii) $\angle D \cong \angle A$, $\angle E \cong \angle B$ and $\angle C \cong \angle F$

Proof:

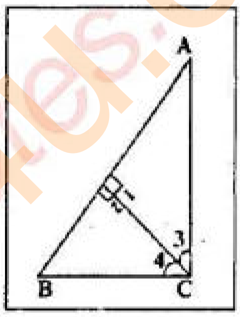
STATEMENTS	REASONS
1. $\because \overline{AB} \parallel \overline{DE}$ and $\overline{AC} \parallel \overline{DF}$	1. From figure
2. $\therefore \angle A \cong \angle D$	2. Theorem 4 corollary 4 (Math Class IX)
3. Similarly, $\angle B \cong \angle E$ And $\angle C \cong \angle F$	3. As proved above
4. $\therefore \dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{FE}}$	4. Theorem 6
5. $\therefore \Delta DEF \cong \Delta ABC$	5. By definitive of similar triangles

More Notes, Books Visit Now: www.Perfect24u.com

Q.E.D.

Q.4. In a right-triangle, the perpendicular drawn from the right angle to the hypotenuse divides the triangle into two parts each similar to the original triangle.

SOLUTION.



Given: In a right triangle ABC. $\overline{CD}$ is the perpendicular draw from right angle $\angle C$ to AB.

To prove that: (i) $\Delta CDB \sim \Delta ACB$

(ii) $\Delta CDA \sim \Delta ACB$

Proof:

STATEMENTS	REASONS
1. Right triangles CDB and ACB have $\angle B$ as common.	1. From figure
2. $\therefore \frac{m\overline{AB}}{m\overline{BC}} = \frac{m\overline{AC}}{m\overline{CD}} = \frac{m\overline{BC}}{m\overline{BD}}$	2. The result of Q. No. 2
3. $\therefore \Delta CDB \cong \Delta ACB$	3. Theorem 6
4. Similarly, $\Delta CDA - \Delta ACB$	4. As proved above

Q.E.D.

EXERCISE 6.7

Q1 Measures of the sides of a triangle are given. Decide which of these are right As and why?

(i) **3 cm, 4cm, 5 cm** **(ii)** **6cm, 8cm, 10cm**

(iii) **5cm, 12cm, 13cm**

(iv) **$(x^2 + y^2)$ units, $(2xy)$ units $(x^2 - y^2)$ units**

(v) **6, 7, 8 units each.**

More Notes, Books Visit Now: www.Perfect24u.com

SOLUTION.

(i) Using Pythagoras Theorem,

$$(Hyp.)^2 = (Perp)^2 + (Base)^2$$
$$(5)^2 = (3)^2 + (4)^2$$
$$\Rightarrow \quad 25 = 9 + 16$$
$$\therefore \quad 25 = 25$$

So, the given measures of the sides of a triangle are the measures of the sides of a right triangle.

(ii) Using Pythagoras Theorem,

$$(Hyp.)^2 = (Perp)^2 + (Base)^2$$
$$(10)^2 = (6)^2 + (8)^2$$
$$\Rightarrow \quad 100 = 36 + 64$$
$$\therefore \quad 100 = 100$$

So, the given measures of the sides of a triangle are the measures of the sides of a right triangle.

(iii) By Pythagoras Theorem,

$$(Hyp.)^2 = (Perp)^2 + (Base)^2$$
$$(13)^2 = (5)^2 + (12)^2$$
$$169 = 25 + 144$$
$$169 = 169$$

So, the given measures of the sides of a triangle are the measures of the sides of a right triangle.

(iv) Using Pythagoras Theorem,

$$(Hyp.)^2 = (Perp)^2 + (Base)^2$$
$$(x^2 + y^2)^2 = (2xy)^2 + (x^2 - y)^2$$
$$= 4x^2 y^2 + x \;\; 2x^2 y^2 + y^4$$
$$= (x^2 + y^2)^2$$

So, the given measures of the sides of a triangle are the measures of the sides of a right triangle.

(v) Using Pythagoras Theorem,

$$(8)^2 = (6)^2 + (7)^2$$
$$\Rightarrow \quad 64 \neq 35 + 49$$

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

So, the given measures of the sides of a triangle are the measures of the sides of a right triangle.

Q2. The top of a ladder, 65ft long, is resting with a wall at a height of 60 ft. Find how far is the foot of ladder from the foot of the wall.

SOLUTION.

Let $\overline{AC}$ show the wall and $\overline{AB}$ show a ladder.

Suppose that $m\overline{BC}$ is the distance between the foot of ladder and the foot of the wall. Using Pythagoras Theorem,

$$(m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2$$

$$\begin{aligned}(m\overline{BC})^2 &= (m\overline{AB})^2 - (m\overline{AC})^2\\ &= (65)^2 - (60)^2\\ &= 4225 - 3600\\ &= 625\end{aligned}$$

$$\therefore m\overline{BC} = 25 \text{ ft.}$$

Q3.(a) Length of each side of an equilateral triangle is 6 units, find the length of any one altitude of the triangle.

(b) Length of each side of an equilateral triangle is 2x units, find the length of each altitude of the triangle.

SOLUTION.

(a) Let ΔABC be an equilateral triangle. Then $m\overline{AB} = m\overline{BC} = m\overline{AC} = 6$ units.

Let $m\overline{AD}$ be the length of an attitude of ΔABC.

Then $m\overline{BD} = m\overline{DC}$ i.e. $m\overline{BC} = 2\,m\overline{BD}$ $\Rightarrow m\overline{BD} = \frac{6}{2} = 3$ units.

Using Pythagoras Theorem for ΔADB,

$$(m\overline{AB})^2 = (m\overline{BD})^2 + (m\overline{AD})^2$$

$$\begin{aligned}\Rightarrow \quad (m\overline{AD})^2 &= (m\overline{AB})^2 - (m\overline{BD})^2\\ &= (6)^2 - (3)^2\end{aligned}$$

Perfect24u.com

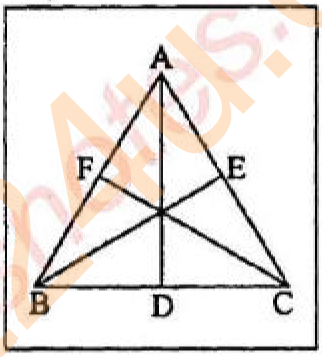
More Notes, Books Visit Now: www.Perfect24u.com

$= 36 - 9 = 27$

$\therefore \quad m\overline{AD} = 3\sqrt{3}$ units

(b) Let ΔABC be an equilateral triangle.

Then $m\overline{AB} = m\overline{BC} = m\overline{AC} = 2x$ units.



Let $m\overline{AD}$, $m\overline{BE}$ and $m\overline{CF}$ be the lengths of attitudes to $\overline{BC}$, $\overline{AC}$ and $\overline{CF}$ respectively.

Than $m\overline{BD} = m\overline{DC}$, i.e. $m\overline{BC} = 2m\overline{BD}$

$\Rightarrow \frac{2x}{2} = x$ units.

Using Pythagoras Theorem for AADB,

$(m\overline{AB})^2 =. (m\overline{AD})^2 + (mBD)^2$

$(m\overline{AD})^2 = (m\overline{AB})^2 (m\overline{BD})^2$

$= (2x)^2$

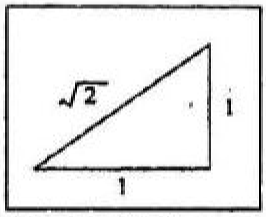
$= 4x^2 - x^2 = 3x^2$

$m\overline{AD} = \sqrt{3}x$ units

Similarly, $m\overline{BE} = \sqrt{3}x$ units

$m\overline{CF} = \sqrt{3}x$ units

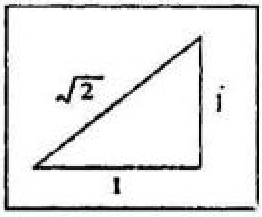
Q4 Draw the following line segments using the Pythagoras Theorem:



[Hints: Break up each number into two parts such that each part is a perfect square, e.g., 2 = 12 + 12, 13 = 22 + 32 etc. Then construct right Δ, with these sides containing the right angle. The hypotenuse will be $\sqrt{2}, \sqrt{13}$ etc.viz]

SOLUTION. Using Pythagoras Theorem,

$(\sqrt{2})^2 = (1)^2 + (1)^2$

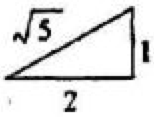


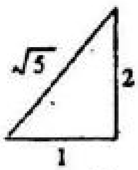
The triangle with these measures of their sides is drawn in the fig.

More Notes, Books Visit Now: www.Perfect24u.com

Using Pythagoras Theorem, $= (\sqrt{5})^2 + (1)^2$

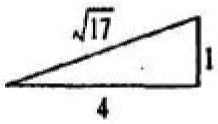
The triangle with these measures of their sides is drawn in the figures (2) and (3).

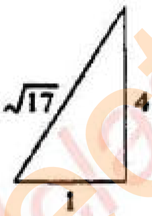




Using Pythagoras Theorem, $\left(\sqrt{17}\right)^2 = (4)^2 + (1)^2$

The triangle with these measures of their sides is drawn in the figures (4) and (5).

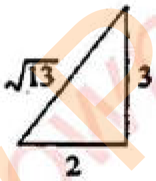




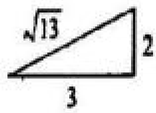
Using Pythagoras Theorem, $(\sqrt{13})^2 = (3)^2 + (2)^2$

The triangle with these measures of their sides is drawn in the figures (6) and (7).

(Figure 6)

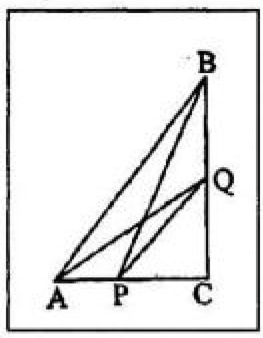


(Figure 7)



Q.5. P and Q are points respectively on the sides $\overline{AC}$ and $\overline{BC}$ of ΔABC, right angled at C. Prove that:

$$\left(m\overline{PQ}\right)^2 + \left(m\overline{AB}\right)^2 = \left(m\overline{PB}\right)^2 + \left(m\overline{QA}\right)^2.$$



SOLUTION.

Using Pythagoras Theorem,

$\left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2$ ---------- *(1)*

$\left(m\overline{CD}\right)^2 = \left(m\overline{CE}\right)^2 + \left(m\overline{DE}\right)^2$ ---------- *(2)*

www.Perfect24u.com

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

$(m\overline{PB})^2 = (m\overline{PC})^2 + (m\overline{BC})^2$ ---------- (3)

$(m\overline{QA})^2 = (m\overline{AC})^2 + (m\overline{QC})^2$ ---------- (4)

Now, $(m\overline{PQ})^2 + (m\overline{QA})^2 = (m\overline{PC})^2 + (m\overline{QC})^2 + (m\overline{AC})^2 + (m\overline{BC})^2$

$= (m\overline{PB})^2 + (m\overline{BC})^2 + (m\overline{BC})^2 + (m\overline{QA})^2 + (m\overline{AC})^2 + (m\overline{AC})^2 + (m\overline{BC})^2$

---------- [From (3) and (4)]

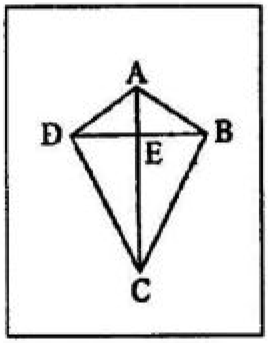
$\therefore \quad (m\overline{QC})^2 + (m\overline{AB})^2 = (m\overline{BC})^2 + (m\overline{QA})^2$

Q6. Diagonals of a quadrilateral ABCD, cut at right angles. Show that $(m\overline{AB})^2 + (m\overline{CD})^2 = (m\overline{BC})^2 + (m\overline{AD})^2$

SOLUTION.

Let $\overline{AC}$ and $\overline{BD}$ be the diagonals of a quadrilateral ΔBCD which are cutting each other at right angles.

Using Pythagoras Theorem for rights ΔAEB, ΔCED, ΔB_iEC and ΔAED,



$(m\overline{AB})^2 = (m\overline{AC})^2 + (m\overline{BC})^2$ ---------- (1)

$(m\overline{PQ})^2 = (m\overline{CE})^2 + (m\overline{DE})^2$ ---------- (2)

$(m\overline{PB})^2 = (m\overline{PC})^2 + (m\overline{BC})^2$ ---------- (3)

$(m\overline{AD})^2 = (m\overline{AE})^2 + (m\overline{DE})^2$ ---------- (4)

Now, $(m\overline{AB})^2 + (m\overline{CD})^2 = (m\overline{AE})^2 + (m\overline{BE})^2 + (m\overline{CE})^2 + (m\overline{DE})^2$

$= [(m\overline{AE})^2 + (m\overline{CE})^2 + (m\overline{AE})^2 + (m\overline{DE})^2]$

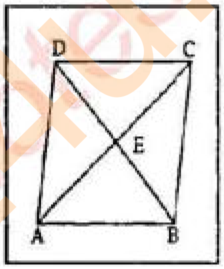
$(m\overline{AB})^2 + (m\overline{CD})^{2^2} = (m\overline{BC})^2 + (m\overline{AD})^2$ ---------- [From (3) and (4)]

More Notes, Books Visit Now: www.Perfect24u.com

Q.7. Prove that sum of squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

SOLUTION.

Let ABCD be a rhombus and let $\overline{AC}$ and $\overline{BD}$ be its diagonals. Since the diagonals of a rhombus bisects each other right angles, therefore



$$|\overline{AC}| = |\overline{AE}| + |\overline{EC}|$$

$$|\overline{BD}| = |\overline{BE}| + |\overline{ED}|$$

$\because$ $|\overline{AE}| = |\overline{EC}|$ and $|\overline{BE}| = |\overline{ED}|$

$\therefore$ $|\overline{AC}| = 2|\overline{AE}|$ ---------- (1)

$BE = 2|\overline{BE}|$ ---------- (2)

Using Pythagoras Theorem for right triangles ΔAEB, ΔBEC, ΔCED and ΔAED,

$$|\overline{AB}|^2 = |\overline{AE}|^2 + |\overline{BR}|^2 \quad \text{---------- (3)}$$

$$|\overline{BC}|^2 = |\overline{BE}|^2 + |\overline{EC}|^2 = |\overline{BE}|^2 + |\overline{AE}|^2 \quad \text{---------- (4)}$$

$$|\overline{CD}|^2 = |\overline{CE}|^2 + |\overline{ED}|^2 = |\overline{AE}|^2 + |\overline{BE}|^2 \quad \text{---------- (5)}$$

$$|\overline{AD}|^2 = |\overline{AE}|^2 + |\overline{ED}|^2 = |\overline{AE}|^2 + |\overline{BE}|^2 \quad \text{---------- (6)}$$

From (1), (2), (3) and (4)

$$|\overline{AB}|^2 + |\overline{BC}|^2 + |\overline{CD}|^2 + |\overline{AD}|^2$$

$$= |\overline{AE}|^2 = |\overline{BE}|^2 + |\overline{BE}|^2 + |\overline{AE}|^2 + |\overline{AE}|^2 = |\overline{BE}|^2 + |\overline{AE}|^2 + |\overline{BE}|^2$$

$$= 4|\overline{AE}|^2 + 4|\overline{BE}|^2$$

$$= \{2|\overline{AE}|^2\} + \{2|\overline{BE}|^2\}$$

$$= |\overline{AC}|^2 + |\overline{BD}|^2 \quad \text{---------- [From (1) and (2)}$$

Hence the result.

Perfect24u.com

More Notes, Books Visit Now: www.Perfect24u.com

www.Perfect24u.com

Perfect24u.com