Chap 5 FUNDAMENTAL CONCEPTS OF GEOMETRY

INTRODUCTION CHAPTER

See the Introduction Chapter in Ex. 5.1 Question No. 1

MULTIPLE CHOICE (MCQ's)

(i) The line segment AB consists of ------------------

S(a) A, B and the points between A,B

(b) Finite number of points

(c) all points member A,B

(d) Point AB

(ii) Sum of all the three angles of triangles is -------------

(a) 90^0 **(b) 180^0** © 270^0 (d) 360^0

(iii) Sum of measures of any two sides of a triangle is---------

(a) greater than the measure of third side

(b) equal to the measure of third side

(c) smaller than the measure of third side

(d) above of these

(iv) How many mid points has a line segment?

(a) only four (b) only three © only two **(d) only one**

(v) How many non–collinear points are in a plane?

(a) three (b) four © two (d) one

(vi) Set of all the points which line outside a triangle is called,

(a) base of triangle (b) diagonal of the triangle

(c) Interior of triangle (d) exterior of triangle

(vii) Number of end point of a ray,

(a) 0 **(b) 1** (c) 2 (d) infinite

(viii) An angle of 90^0 is called-----------

(a) Right angle (b) Acute angle

(c) Obtuse angle (d) None of above

(ix) Two intersecting lines divide the plane into ---------.

(a) 4 parts (b) 2 parts (c) 1 parts (d) 0 parts

(x) The outer sides of two adjacent supplementary angles-----------.

(a) From a line (b) make a right angle

(c) Intersect with each other (d) make a line segment

(xi) Which one of the following pairs constitute complementary angles?

(a) 1^0 , 90^0 **(b) 1^0 , 89^0**

© 100^0 , 80^0 (d) 90^0 , 90^0

(xii) Each of two complementary angles can be -----------

(a) Right angle **(b) Acute angles**

(c) Obtuse angle (d) None of above

(xiii) If supplementary angle are congruent to each other, then the measurement of each angle is,

(a) 60^0 **(b) 90^0** © 180^0 (d) 45^0

(xiv) If complementary angles are congruent, the measure of each angle is---------------.

(a) 60^0 (b) 90^0 © 180^0 **(d) 45^0**

(xv) One and only one line can pass through---------

(a) 1 point **(b) 2 points** (c) 3 points (d) 4 points

(xvi) Three non–collinear points determine a

(a) Plane (b) space (c) square (d) Rectangle

(xvii) One and only one place passes through -------------

(a) 1 point (b) 2 points

(c) 3 points (d) 4 points

EXERCISE 5.1

Q.1. Illustrate the following by drawing figures:

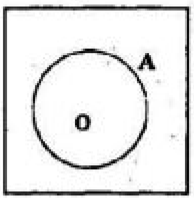
(i)	**Circle**	**(ii)**	**Semi-Circle**
(iii)	**Arc**	**(iv)**	**Minor and major arcs**
(v)	**Radial segment**	**(vi)**	**Chord and diameter**
(vii)	**Tangent**		
(viii)	**Interior and exterior of a circle**		
(ix)	**Secant**	**(x)**	**Congruent arcs**
(xi)	**Congruent circles**		
(xii)	**Sector and segment of a circle**		

SOLUTION:

(i) <u>CIRCLE</u>

A circle is a set of all the points in a plane which are equidistant from a fixed point of the plane.

The fixed point is called the centre of the circle. The distance between any point on the circle and its centre is called the radius of the circle.

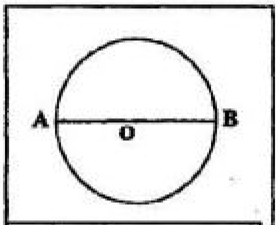


In the adjacent figure, 0 is the centre of the circle and m $\overline{OA}$ = 1cm.

(2) <u>SEMI CIRCLE</u>

The position of a circle intercepted by a diameter is called a "Semi-circle".

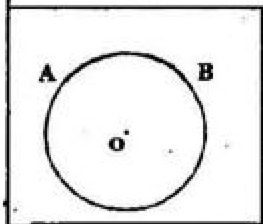
In this figure, diameter $\overline{AB}$ divides the circle in two equal parts. Each part is a semi-circle.



(3) <u>ARC</u>

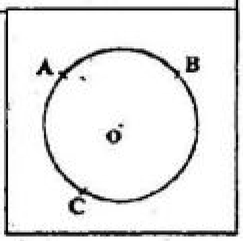
Any position or part of a circle is called an arc of the circle.

In this figure, the position AB of the circle is an arc. It is denoted by $\overset{\frown}{AB}$.



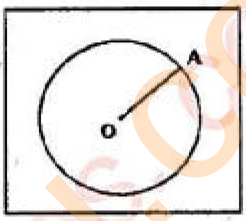
(iv) <u>MINOR AND MAJOR ARCS</u>

An arc which is less than a semi-circle is called a minor arc. An arc which is greater than a semi-circle is called a major arc.



In this figure, $\widehat{AB}$ is a minor arc. $\widehat{ABC}$ is a major arc

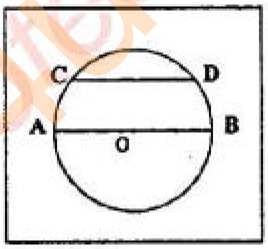
(v) RADIAL SEGMENT



The segment joining the centre of a circle to any point on the circle is called a radial segment.

In this figure, $\overline{OA}$ is a radial segment.

(vi) CHORD AND DIAMETER

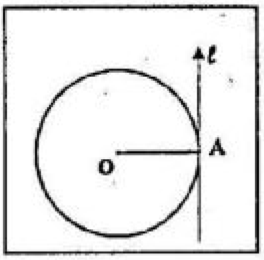


The line segment whose end-points are any two points of a circle is called a chord of the circle.

A chord which passes through the centre of the circle is called a diameter.

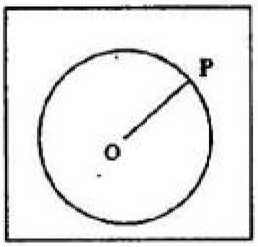
In this figure, $\overline{CD}$ is a chord and $\overline{AB}$ is the diameter.

(vii) TANGENT



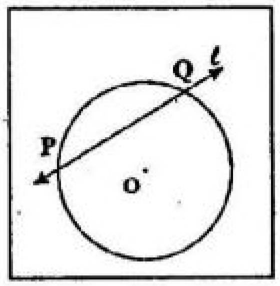
If a circle and a line "l" lie in a plane and the line "l" intersects the circle at only one point A, then the line "l" is called tangent to the circle and the common point A is called the point of tangency or the point of contact.

(viii) INTERIOR AND EXTERIOR OF A CIRCLE



In a plane, if a circle is drawn with any centre O, then the set of points whose distance from centre O is less than the radius of the circle, is called the interior of the circle and the set of points whose distance from the centre O is greater the radius of the circle, is called the exterior of the circle.

(ix) SECANT



If a circle and a line "l" lie in a plane and the line "l" intersects the circle in two distinct point P and Q, then the line "l" is called the secant to the circle.

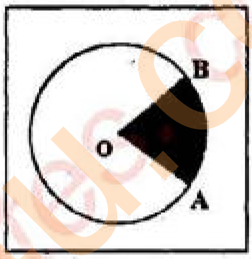
(x) CONGRUENT ARCS TWO ARCS:

(1) Which lie on the same circle (or congruent circle), and

(2) their central angles are congruent, are called congruent arcs.

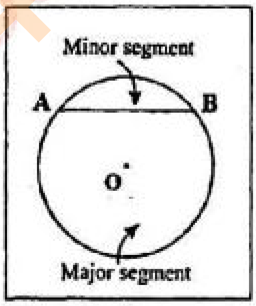
(XI) SECTOR AND SEGMENT OF A CIRCLE

Two circles are said to be congruent if their radii are equal.



(XII) SECTOR AND SEGMENT IF A CIRCLE

The portion of a circular region bounded by an arc "$\overset{\frown}{AB}$" and two radial segments $\overline{OA}$ and $\overline{OB}$ is called sector of the circle. In this figure OAB is the sector.



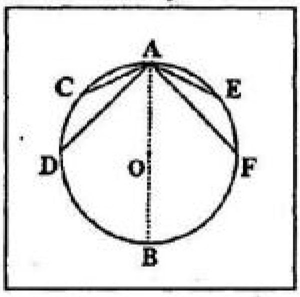
A chord of a circle which divides the circular region into two parts. Each part is called a segment of the circle.

The region bounded by the chord and the minor arc is called minor segment and the region bounded by the chord and the major arc is called major segment as shown in the figure.

2(a) From a point of the circle how .many diameters, chords and radial segments can be drawn?

(b) From the centre of the circle how many diameters, chords and radial segments can be drawn?

SOLUTION:.



(a) From a point A of a circle only one diameter "$\overline{AB}$ " can be drawn.

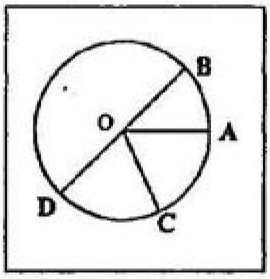
From a point a point A of a circle many chords $\overline{AB}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, etc. can be drawn.

From a point A of a circle only one radial segment $\overline{AO}$ can be drawn.

(b) From the centre O of the circle no one diameter can be drawn.

From the centre of O of the circle no one chord can be drawn.

From the centre O of the circle many radial segments $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$, etc can be drawn.



Q3 . Differentiate between:

(i) Radius and radial segment

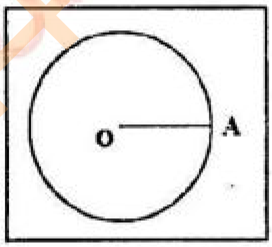
(ii) Interior and exterior of a circle

(iii) Chord and diameter

(iv) Major arc and minor arc

SOLUTION:

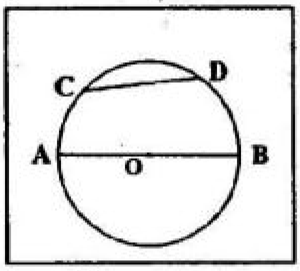
(i) The segment joining the centre O of a circle to a point A on the circle is called radial segment "$\overline{OA}$".



The length of the radial segment "$\overline{OA}$" (i.e., m $\overline{OA}$ or $|\overline{OA}|$) is called radius of the circle.

(ii) In a plane, a set of points whose distance from the centre of a circle is less than the radius of the circle, is called the interior of the circle and a set of points whose distance from the centre of the circle is greater then the radius of the circle, is called the exterior of the circle.

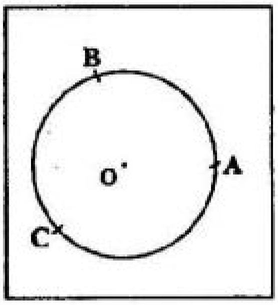
(iii) The line segment whose end-points C and D are any two points of the circle is called a chord $\overline{CD}$ of the circle.



The chord $\overline{AB}$ which passes through the centre "O" of the circle is called a diameter.

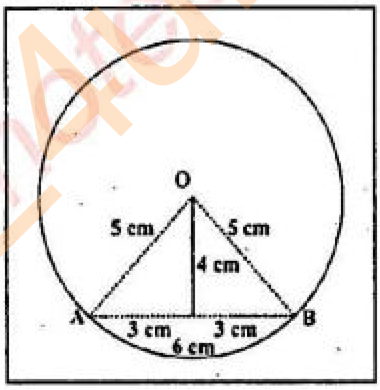
(iv) A minor arc is an arc which is less than a semi-circle In this figure arc AB is a minor arc.

A major arc is an arc which is greater than a semi-circle. In this figure are ACB is a major arc.



Q.4. Take a segment $\overline{AB}$, 6cm long. Draw its right bisector at P. Take O a point on the right bisector such that $m\overline{OP}$ = 4cm. Draw a circle with O as centre and $\overline{OA}$ as radial segment. Does this circle pass through point B. If yes, why? Measure $\overline{OA}$ and verify it through calculation.

SOLUTION. $\therefore \quad |\overline{OA}| = 5cm$



In ΔAPO,

$|\overline{OA}|^2 = |\overline{AP}|^2 + |\overline{OP}|^2$

$= |\overline{BP}|^2 + |\overline{PO}|^2 \qquad \because |\overline{AP}| = |\overline{BP}|$

$= |\overline{OB}|^2 \quad \Rightarrow \quad |\overline{OA}| = |\overline{OB}| = 5cm$

So, the circle passes through B.

Q5. In a circle, there are three congruent arcs. If the measure of the central angle of one arc is 30°, find the measure of the central angle of the other arcs.

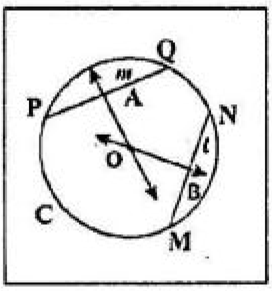
SOLUTION.

Since the measures of the central angles of the congruent arcs are equal, therefore the measure of the central angles of the other two arcs are 30° and 30° because the measure of central angle of one arc is 30°.

EXERCISE 5.2

Q1 The right bisectors of two chords of a circle intersect at the centre of the circle.

[Hint: Apply Theorem 1(b))].



SOLUTION. __**Given:**__ $\overline{PQ}$ and $\overline{MN}$ are two chords of a circle with centre O.

__**Construction:**__ Draw two right bisectors *l* and *m* to chords $\overline{PQ}$ and $\overline{MN}$ at A and B respectively.

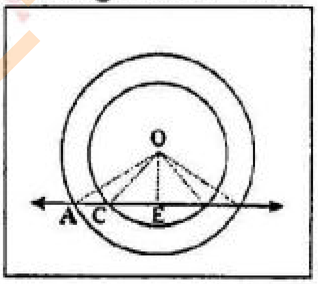
__**To prove that:**__ *l* and m in pass through O.

Proof:

STATEMENTS	REASONS
1. *l* Passes through O.	1. By theorem 1(b) (second converse of theorem 1)
2. m passes through O.	2. By Theorem 1(b)
3. *l* and m in passes through O	3. By (1) and (2)

Q.E.D.

Q2 Two concentric circles intercept congruent segments on any line that cuts circles.



SOLUTION:

Given : Two concentric circles are with the centre 0.

Construction: Draw a perpendicular from O to E on the line.

To prove that: $\overline{AC} \cong \overline{DB}$ and $\overline{CE} \cong \overline{ED}$

Proof:

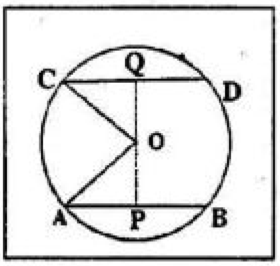
STATEMENTS	REASONS
1. In right Δs, $\Delta OEC \leftrightarrow$ AGED	1.
(i) $\overline{OE} \cong \overline{OE}$	(i) common
(ii) $\overline{OE} \cong \overline{OD}$	(ii) Radial segments of same circle.
2. $\therefore \Delta OEC \cong OED$	2. In right Δs, H.S. $\cong$ H.S.
3. $\therefore \overline{CE} \cong \overline{ED}$	3. By congruence of Δs.
4. In right $\Delta OEA \leftrightarrow \Delta OEB$	4.
(i) $\overline{OE} \cong \overline{OE}$	(i) common
(ii) $\overline{OA} \cong \overline{OD}$	(ii) Radial segments of same circle.
5. $\therefore \Delta OEA \cong OEB$	5. In right Δs, H.S. $\cong$ H.S.
6. $\therefore \overline{AE} \cong \overline{EB}$	6. By congruence of Δs.

7. $m\overline{AE} = m\overline{EB}$	7. By (6).
8. $m\overline{AC} + m\overline{CE} = m\overline{ED} + m\overline{DB}$	8. $m\overline{AE} = m\overline{AC} + m\overline{CE}$ and $m\overline{EB} = m\overline{ED} + m\overline{DB}$
9. $m\overline{AC} = m\overline{DB}$	9. By (3)
10. $\therefore \overline{AC} \cong \overline{DB}$	10. By (9)

Q.E.D.

Q.3. Prove that congruent chords of a circle are equidistant from the centre.

SOLUTION.



Given: $\overline{AB}$ and $\overline{CD}$ are two congruent chords in a circle with centre O.

$\overline{OP} \perp \overline{AB}$ and $\overline{OQ} \perp \overline{CD}$

To prove that: $\overline{OP} \cong \overline{OQ}$

Construction: Draw $\overline{OA}$ and $\overline{OC}$.

Proof:

Statements	Reasons
1. $m\overline{AP} = \frac{1}{2} m\overline{AB}$	1. $\overline{OP} \perp \overline{AB}$ (Theorem 1)
2. $m\overline{CQ} = \frac{1}{2} m\overline{CD}$	2. $\overline{OQ} \perp \overline{CD}$ (Theorem 1)
3. But $m\overline{AB} = m\overline{CD}$	3. Given
4. $\therefore m\overline{AP} = m\overline{CQ}$	4. Transitive property of equation.
5. In right Δs, $APO \leftrightarrow \Delta CQO$	5.
(i) $\overline{AO} \cong \overline{CO}$	(i) Radial segments of same circle
(ii) $\overline{AP} \cong \overline{CQ}$	(ii) By (4)
6. $\therefore \Delta APO \leftrightarrow \Delta CQO$	6. In right Δs, H.S. $\cong$ H.S.

7. $\therefore \overline{OP} \cong \overline{OQ}$	7. By congruence of Δs.

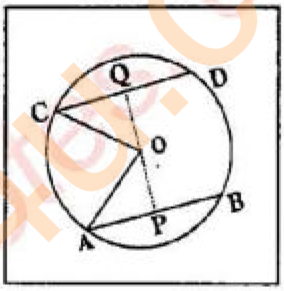
Q.E.D

Q4 Write the converse of question 3 and prove it.

SOLUTION.

The converse of question 3 is:

"Two chords of a circle which are equidistant from the centre are congruent".



Given: $\overline{AB}$ and $\overline{CD}$ are two chords in a circle with centre O'. In this circle $\overline{OP} \perp \overline{AB}$ and $\overline{OP} \perp \overline{CD}$.

To prove that: $\overline{AB} \cong \overline{CD}$ (i.e., the two chords are congruent).

Construction: Draw $\overline{OA}$ and $\overline{OC}$.

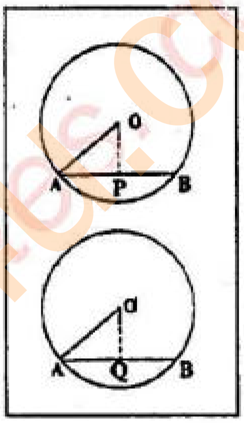
Proof:

Statements	Reasons
1. In right Δs, $\Delta AOP \leftrightarrow \Delta COQ$	1.
(i) $\overline{OP} \cong \overline{OQ}$	(i) Given
(ii) $\overline{AO} \cong \overline{CO}$	(ii) Radial segments of same circle.
2. $\therefore \Delta AOP \cong \Delta COQ$	2. In right Δs, H.S $\cong$ H.S.
3. $\therefore \overline{AP} = m\overline{CQ}$	3. By congruence of Δs.
4. But P and Q are mid-points of $\overline{AB}$ and $\overline{CD}$	4. By Theorem 1.
5. $\therefore 2m\ \overline{AP} = 2m\ \overline{CQ}$	5. Multiplication property of equation.
6. or $m\overline{AB} = m\overline{CD}$	6. $2m\ \overline{AP} = m\overline{AB}$, $2m\overline{CQ} = m\overline{CD}$
7. $\therefore \overline{AB} \cong \overline{CD}$	7. By (6)

Q.E.D.

Q.5. Chords which are equidistant from the centers in two congruent circles are congruent.

SOLUTION.



Given: $\overline{AB}$ and $\overline{CD}$ are two chords with centers O and O' respectively. In these circles, $\overline{OP} \perp \overline{AB}$ and $O'Q \perp CD$.

To prove that: $\overline{AB} \cong \overline{CD}$

(i.e., two chords are congruent).

Construction: Draw $\therefore$ $\overline{OA}$ and $\overline{O'C}$.

Proof.

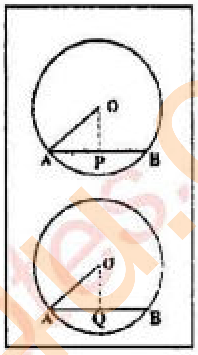
STATEMENTS	REASONS
1. In right Δs, $\Delta AOP \leftrightarrow \Delta CO'Q$	1.
(i) $\overline{OP} \cong \overline{O'Q}$	(i) Given
(ii) $\overline{AO} \cong \overline{CO'}$	(ii) Radial segments of same circle.
2. $\therefore$ $\Delta AOP \cong \Delta CO'Q$	2. In right Δs, H.S $\cong$ H.S.
3. $\therefore$ $\overline{AP} = m\overline{CQ}$	3. By congruence of Δs.
4. But P and Q are mid-points of $\overline{AB}$ and $\overline{CD}$	4. By Theorem 1.
5. $\therefore$ $2m\,\overline{AP} = 2m\,\overline{CQ}$	5. Multiplication property of equation.
6. or $m\overline{AB} = m\overline{CD}$	6. $2m\,\overline{AP} = m\overline{AB}$, $2m\overline{CQ} = m\overline{CD}$
7. $\therefore$ $\overline{AB} \cong \overline{CD}$	7. By (6)

Q.E.D.

Q.6. State the converse if the question 5 and prove it.

SOLUTION.

The converse of the question is congruent chords of congruent circles are equidistant from their centers.



Given: $\overline{AB}$ and $\overline{CD}$ are two congruent chords in two congruent circles with centers O and O'. In these circles $\overline{OP} \perp \overline{AB}$ and $O'Q \perp \overline{CD}$

To prove that: $\overline{OP} \cong \overline{O'Q}$

Construction: Draw ∴ $\overline{OA}$ and $\overline{O'C}$

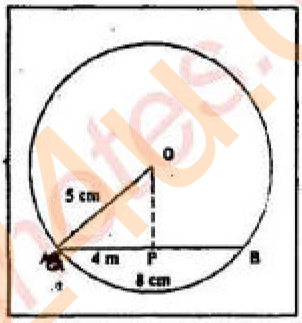
Proof:

STATEMENTS	REASONS
1. $m\overline{AP} = \frac{1}{2}m\overline{AB}$	1. $\overline{OP} \perp \overline{AB}$ (Theorem 1)
2. $m\overline{CQ} = \frac{1}{2}m\overline{CD}$	2. $\overline{OQ} \perp \overline{CD}$ (Theorem 1)
3. But $m\overline{AB} = m\overline{CD}$	3. Given
4. $\therefore m\overline{AP} \neq m\overline{CQ}$	4. Transitive property of equation.
5. In right Δs, APO ↔ΔCQO	5.
(i) $\overline{AO} \cong \overline{O'C}$	(i) Radial segments of same circle
(ii) $\overline{AP} \cong \overline{CQ}$	(ii) By (4)
6. ∴ ΔAPO ↔ ΔCQO'	6. In right Δs, H.S. ≅ H.S.
7. ∴ $\overline{OP} \cong \overline{O'Q}$	7. By congruence of Δs.

Q.E.D.

Q.7. In a circle of radius 5 cm, a chord measuring 8cm has been drawn, find its distance from the centre of the circle.

SOLUTION:



Given: $m\overline{OA} = 5\text{ cm}$, $m\overline{AB} = 8\text{ cm}$

To find: $m\overline{OP}$

Calculation: By Theorem 1, $\overline{OP} \perp \overline{AB}$ and $\overline{OP}$ bisects the chord $\overline{AB}$.

$\therefore \quad m\overline{AP} = m\overline{BP}$

$\Rightarrow \quad m\overline{AP} = \frac{1}{2} m\overline{AB} = \frac{1}{2} \times 8 = 4cm$

Let $m\overline{OP}$ be the distance from the centre O to the chord AB.

According Pythagoras theorem,

$$(m\overline{OA}) = (m\overline{AP})^2 + (m\overline{OP})^2$$

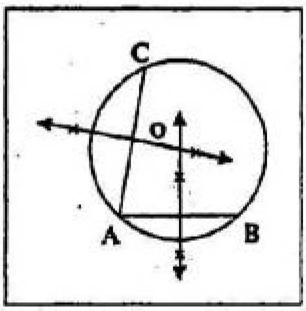
$$(5)^2 = (4)^2 + (m\overline{OP})^2$$

$$(m\overline{OP})^2 = 25 - 16$$

$$(m\overline{OP})^2 = 9 \quad \Rightarrow \quad m\overline{OP} = 3\text{ cm}$$

Q8. Take any three non-collinear points. Draw a circle through these points.

SOLUTION.



GIVEN: A, B, C are three non-collinear points.

To draw: A circle through A, B and C.

Steps of Construction:

1. Join A to B and to C.
2. Draw right bisectors of $\overline{AB}$ and $\overline{AC}$ which cut each other at O.
3. With centre O and with radius congruent $\overline{OA}$ or $\overline{OB}$ or $\overline{OC}$ draw a circle.
4. The obtained circle is the required circle.

EXERCISE 5.3

Q1. Three villages are situated in such a way that B is east of A at a distance of 6 kms and C is north of B at a distance of 8 kms. Residents of these villages plan to construct a mosque at a place so that they have to walk same distance from each village:

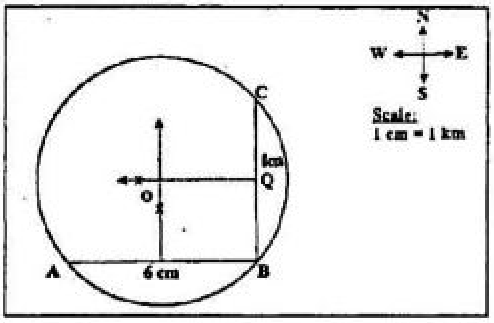
(a) Determine the location of mosque by drawing a neat figure using ruler and compass.

(b) How much distance would each villager have to walk down?

(c) If two villages P and Q emerge later on, one mid-way between A and B and the other mid-way between B and C respectively. How much distance would P and Q villagers have to walk to reach the mosque?

SOLUTION:

(a) Steps of drawing the required fig:



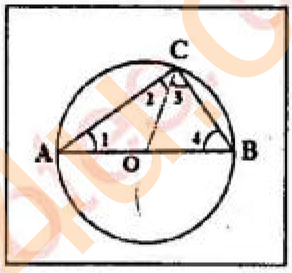
1. Draw the point B from A at a distance of 6 kms in east.
2. Draw the point C from B at a distance of 8 kms in north.
3. Draw the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ at P and Q respectively.
4. Draw a circle of centre O which passes through A, B and C.
5. Thus, at O three villages residents can construct a mosque has the same distance from each village.

(b) Required distance = in $\overline{AB}$ = m$\overline{BO}$ = m$\overline{CO}$ = 5 kms.

(c) Distance between mosque and village P = m$\overline{OP}$ = 4 kms.

Distance between mosque and village Q = m$\overline{OQ}$ = 3 kms.

Q.2. Prove that angle subtended by a semi-circle is a right angle.



SOLUTION. Given: $\widehat{ACB}$ is a semi-circle with centre O and $\angle ACB$ is the angle subtended by it.

To prove that : $m\angle ACB = 90°$, i.e., ACB is a right angle.

Construction: Draw $\overline{OC}$

Proof:

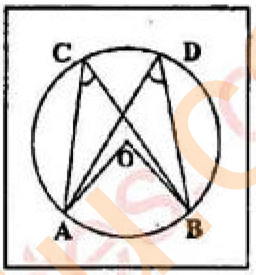
STATEMENTS	REASONS
1. In ΔAOC	1.
(i) $\overline{OA} \cong \overline{OC}$	(i) Radial segments
	(ii) Angles opposite to congruent sides.
2. In ΔBOC.	2.
(i) $\overline{OB} \cong \overline{OC}$	(i) Radial segments
(ii) $m\angle 3 = m\angle 4$	(ii) Angles opposite to congruent sides.
3. $\therefore m\angle 2 + m\angle 3 = m\angle 1 + m\angle 4$	3. Addition property of equation
4. $m\angle ACB = m\angle BAC + m\angle ABC = 180°$	4. Transitive property of equation.
5. $\because m\angle ACB + m\angle BAC + m\angle ABC$	5. Theorem 5 (Maths IX)
6. $\therefore m\angle ACB + m\angle ACB = 180°$	6. By (4)
7. or $2\, m\angle ACB = 180°$	
8. $\therefore m\angle ACB = 90°$	

Q.E.D.

Q3 Prove that the angles subtended by the same major arc are congruent.

SOLUTION.

Given: ∠ACB and ∠ADB are the angles subtended by the same major arc ACB or ADB of circle with centre O.



To prove that: ∠ACB ∠ADB

Construction: Draw AO and BO.

∠AOB is the central angle of $\widehat{AB}$.

Proof:

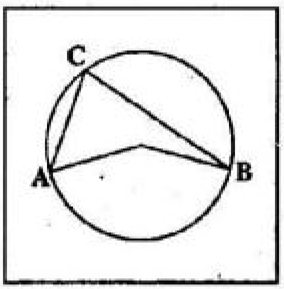
STATEMENTS	REASONS
1. m ∠AOB = 2m∠ACB	1. Theorem 4 (construction)
2. m∠AOB = 2 ∠ADB	2. Theorem 4 (construction)
3. 2m ∠ACB = 2 m ∠ADB	3. Equal to the same quantity.
4. m∠ ACB = m∠ADB	4. Dividing by 2.
5. ∠ACB ≅ ∠ADB	5. By (4)

Q.E.D.

Q4 Angle inscribed in a major arc is acute.

SOLUTION:

Given: ∠ACB is the angle inscribed in a major arc ACB of the circle with centre 0.



To prove that: m ∠ACB<90°.

Construction: Draw AO and OB.

∠AOB is the central angle of $\widehat{AB}$.

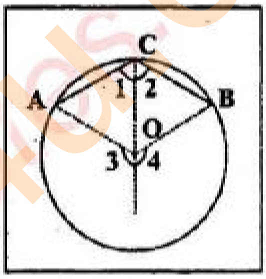
Proof:

STATEMENTS	REASONS
1. m∠AOB = 2 m∠ACB	1. Theorem 4 (construction)
2. m ∠AOB < 180°	2. Measure of semi-circle is 180°.

3. or $2m\angle ACB < 180°$	3. By (1)
4. or $m\angle ACB < 90°$	4. Dividing by 2.

Q.E.D.

Q5 Angle inscribed in a minor arc is obtuse.



SOLUTION.

Given: $\angle ACB$ is the angle inscribed in a minor arc AB of the circle with centre O.

Construction: Draw $\overline{OA}$ and $\overline{OB}$ and $\overline{OC}$ and extent $\overline{OC}$ toward O.

Proof:

STATEMENTS	REASONS
1. In ΔAOC, $\overline{OA} \cong \overline{OC}$	1. Radial segments of the same circle.
2. in $\angle 1 = m\angle CAO$	2. Angles opposite to congruent sides.
3. $m\angle 3 = m\angle 1 + m\angle CAO$	3. Exterior angle is the sum of opposite interior angles.
4. or $m\angle 3 = m\angle 1 + m\angle 1$	4. By (2)
5. or $m\angle 3 = 2m\angle 1$	5. Addition of same quantity
6. Similarly, $m\angle 4 = 2m\angle 2$	6. By (5)
7. $m\angle 3 + m\angle 4 = 2m\angle 1 + 2m\angle 2$	7. By (5) and (6)
8. or $m\angle 3 + m\angle 4 = 2(m\angle 1 + m\angle 2)$	8. Taking 2 as common.
9. $m\angle AOB < 180°$	9. Measure of semi-circle is 180°
10. $\therefore m\angle 3 + m\angle 4 > 180°$	10. By (9)
11. or $2(m\angle 3 + m\angle 2) > 180°$	11. By (8)
12. or $m\angle 1 + m\angle 2 > 90°$	12. Dividing by 2
13. $\therefore m\angle ACB > 90°$	13. Angle Addition postulate.

Q.E.D.

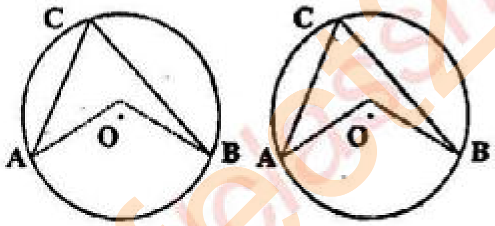
Q6 The inscribed angles of two congruent major arcs of congruent circles congruent.

SOLUTION.

Given: Circles with centers O and O' are congruent circles. $\overset{\frown}{ACB}$ and $\overset{\frown}{A'C'B}$ are congruent. $\angle ACB$ and $\angle A'C'B'$ are their inscribed angles.

To prove that: $\angle ACB \cong \angle A'C'B'$

Construction: Draw $\overline{OA}$, $\overline{OB}$, $\overline{OA}'$ and $\overline{OB}'$.



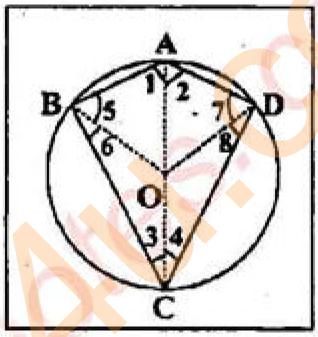
Proof:

STATEMENTS	REASONS
1. ∵ Major arcs ACB and A'C'B' are congruent.	1. Given
2. ∴ Minor arcs AB and A'B' are congruent.	2. Circles are congruent.
3. $m\angle AOB = m\angle A'O'B'$	3. By (2) [central angles are also equal]
4. $m\angle AOB = m\angle ACB$	4. Theorem 4.
5. $m\angle A'O'B' = m\angle A'C'B'$	5. Theorem 4.
6. $2m\angle ACB = m\angle A'CB'$	6. By (4) and (5)
7. $m\angle ACB = m\angle A'CB'$	7. Dividing by 2
8. $m\angle ACB \cong m\angle A'C'B'$	8. By (7)

Q.E.D.

Q.7. Opposite angles of cyclic quadrilateral are supplementary.

SOLUTION:



Given: ABCD is a cyclic quadrilateral to a circle with centre O.

To prove that: $m\angle A + m\angle C = 180°$ and

$m\angle B + m\angle D = 180°$

Construction: Draw $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ and $\overline{OD}$.

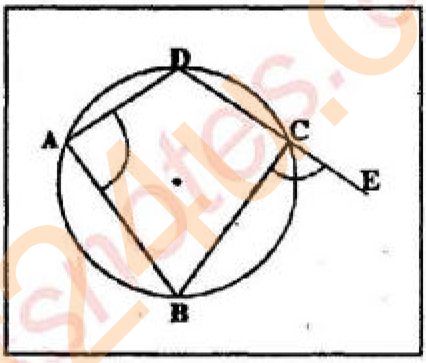
Proof.

STATEMENTS	REASONS
1. In ΔAOB, $\overline{OA} \cong \overline{OB}$	1. Radial segments of the same circle.
2. $\therefore$ $m\angle 1 = m\angle 5$	2. Angles opposite to congruent sides.
3. Similarly, $m\angle 2 = m\angle 7$ $m\angle 3 = m\angle 6$ and $m\angle 4 = m\angle 8$	3. Proved in (2).
4. In ΔABC, $m\angle 1 \times m\angle 5 + m\angle 6 + m\angle 3 = 180°$	4. Sum of measures of all angles of a triangle is 180°.
5. or $m\angle 1 + m\angle 1 + m\angle 3 + m\angle 3 = 180°$	5. By (2) and (3).
6. or $2(m\angle 1 + m\angle 3) = 180°$	6. By addition property.
7. or $m\angle 1 + m\angle 3 = 90°$ ...(i)	7. Dividing by 2.
8. Similarly, $m\angle 2 + m\angle 4 = 90°$ ... (ii)	8. Proved above.
9. $m\angle 1 + m\angle 3 + m\angle 2 + m\angle 4 = 90° + 90°$	9. Adding equations (i) and (ii)
10. or $(m\angle 1 + m\angle 2) + (m\angle 3 + m\angle 4) = 180°$	10. Arranging L.H.S.
11. or $mA + m\angle C = 180°$	11. By construction.
12. Similarly, $m\angle B + m\angle D = 180°$	12. By (11)

Q.E.D.

Q.8. If a side of a cyclic quadrilateral is produced, the exterior angle so formed is congruent to the opposite interior of the quadrilateral.

SOLUTION.



Given: AB CD is a cyclic quadrilateral to the circle with centre. $\overline{DC}$ is produced to E. $\angle 1$ is the exterior angle and $\angle 3$ is the interior angle opposite to the adjacent interior angle $\angle 2$ of $\angle 1$.

To prove that: $m\angle 1 = m\angle 3$

Proof.

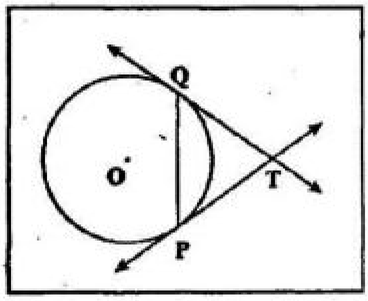
Statements	Reasons
1. $m\angle 1 + m\angle 2 = 180°$	1. Supplementary angles postulate
2. $m\angle 2 + m\angle 3 = 180°$	2. Opposite angles of cyclic quadrilateral
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$	3. By (1) and (2)
4. So, $m\angle 1 = m\angle 3$	4. Subtracting $m\angle 2$ from each side.

Q.E.D.

EXERCISE 5.4

Q.1. If two tangents to a circle meet each other, they make congruent angles with the chord passing through the points of tangency.

SOLUTION.



Given: $\overline{TP}$ and $\overline{TQ}$ are two tangents to a circle with centre O, at the points P and Q respectively. $\overline{PQ}$ is the chord

To prove that: $m\angle P = m\angle Q$

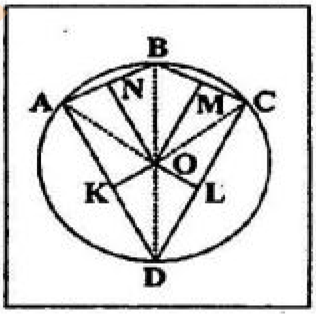
Proof:

STATEMENTS	REASONS
1. In ΔTPQ, $\overline{TP} \cong \overline{TQ}$	1. By theorem 6
2. $m\angle P = m\angle Q$	2. Angles opposite to congruent sides.

Q.E.D.

Q.2. If a quadrilateral circumscribe a circle, the sum of one pair of opposite sides is equal to the sum of the other pair.

SOLUTION.



Given: ABCD is a quadrilateral circumscribed by a circle with centre "O".

To prove that: $|\overline{AB}| + |\overline{CD}| = |\overline{AD}| + |\overline{BC}|$

Construction: Draw $\overline{AC}$ and $\overline{BD}$.

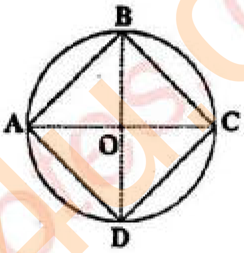
Proof:

STATEMENTS	REASONS
1. In $\Delta BEC \leftrightarrow \Delta AEB$ (i) $BE \cong BE$ (ii) $m\angle E = \angle E$ (iii) $AE \cong EC$	1. (i) Common (ii) $\overline{BO} \perp \overline{AC}$ (Theorem 1) (iii) Theorem 1
2. $\therefore \Delta BEC = \Delta AEB$	2. S.A.S. Δ S.A.S.
3. $\therefore |\overline{AB}| = |\overline{BC}|$	3. Congruence of triangles.
4. Similarly $|\overline{AD}| = |\overline{CD}|$	4. As proved above
5. $\therefore |\overline{AB}| + |\overline{CD}| = |\overline{BC}| + |\overline{AD}|$	5. By (3) and (4)

Q.E.D.

Q.3. If a||ᵐ circumscribe a circle, it must be a rhombus.

SOLUTION.



Given: ABCD is a parallelogram circumscribe by a circle with centre 0.

To prove that: $|\overline{AB}| + |\overline{BC}| = |\overline{CD}| = |\overline{AD}|$

Construction: Join O to A, B, C and D.

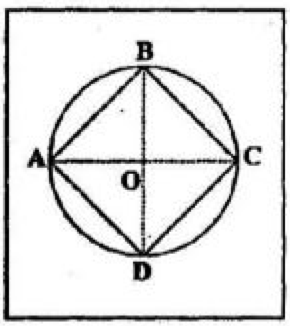
Proof:

STATEMENTS	REASONS
1. In $\Delta ADB \leftrightarrow \Delta BOC$	1.
(i) $\overline{BO} \cong \overline{BO}$	(i) Common
(ii) $\therefore$ $m\angle AOB = \angle BOC$	(ii) $\overline{BO} \perp \overline{AC}$ (Theorem 1)
(iii) $\overline{AO} \cong \overline{OC}$	(iii) Theorem 1
2. $\therefore \Delta AOB \cong \Delta BOC$	2. S.A.S. ≅ S.A.S.
3. $\therefore$ $|\overline{AB}| = |\overline{BC}|$	3. Congruence of triangles
4. Similrarly, $\therefore$ $|\overline{BC}| = |\overline{CD}|$ and $|\overline{CD}| + |\overline{AD}|$	4. As proved above
5. $|\overline{AB}| = |\overline{BC}| = |\overline{CD}| = |\overline{AD}|$	5. By (3) and (4)

Q.E.D.

Q.4. If a rectangle is described about a circle, it must be a square.

SOLUTION.



Given: ABCD is a rectangle which is described about a circle with centre 0,

i.e., $\overline{AB} = \overline{BC} = \overline{CD} = \overline{AD}$

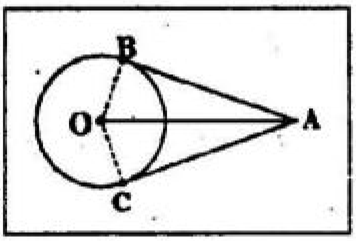
To prove that: Draw $\overline{OA}$, $\overline{OB}$, $\overline{OC}$ and $\overline{OD}$.

Proof:

STATEMENTS	REASONS
1. In $\Delta AOB \leftrightarrow \Delta AOB$	1.
(i) $\overline{AO} \cong \overline{AO}$	(i) Common
(ii) ∴ $m\angle AOD = \angle AOB$	(ii) $\overline{BO} \perp \overline{AC}$ (Theorem 1)
(iii) $\overline{DO} \cong \overline{OB}$	(iii) Theorem 1
2. ∴ $\Delta AOB \cong \Delta BOC$	2. S.A.S. ≅ S.A.S.
3. ∴ $\lvert\overline{AD}\rvert + \lvert\overline{AB}\rvert$	3. Congruence of triangles
4. Similrarly, $\lvert\overline{BC}\rvert = \lvert\overline{CD}\rvert$ and $= \lvert\overline{CD}\rvert = \lvert\overline{AD}\rvert$	4. As proved above
5. $\lvert\overline{AB}\rvert = \lvert\overline{BC}\rvert$ and $= \lvert\overline{CD}\rvert = \lvert\overline{AD}\rvert$	5. By (3) and (4)

Q.E.D.

Q.5. In this figure, is a circular wheel with radius 6cm. There is a small pulley at a distance of 10cm from the centre of the wheel. A belt is moving round the wheel and the pulley as shown. Find the length of the belt which is not touching the circular wheel.



SOLUTION.

Given: In the circle with centre O $\lvert\overline{OB}\rvert + \lvert\overline{OC}\rvert$ = 6 cm

$\lvert\overline{OA}\rvert$ is a distance from the centre of the wheel to a small pulley, i.e., $\lvert\overline{OA}\rvert$ = 10cm.

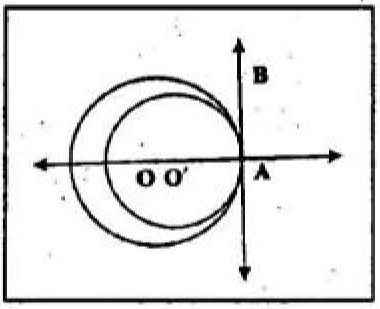
Proof:

STATEMENTS	REASONS
1. $\overline{\|AB\|}+\overline{\|BC\|}$	1. By Theorem (6)
2. $\overline{\|AB\|}+\overline{\|BC\|} = \overline{\|AB\|}+\overline{\|AB\|}$	2. By (1)
3. $\overline{\|AB\|}+\overline{\|BC\|} = 2\ \overline{\|AB\|}$	3. By (2)
4. ∴ In ΔABO, $\overline{\|OA\|}^2 = \overline{\|OB\|}^2 + \overline{\|AB\|}^2$	4. By Pythagoras theorem
5. $\|10\|^2 = \|6\|^2 + \overline{\|AB\|}^2$	5. $\overline{\|OA\|} = 10\ cm,\ \overline{\|OB\|} = 6\ cm$
6. or $\overline{\|AB\|}^2 = 100-36$	
7. or $\overline{\|AB\|}^2 = 64$	7. By subtracting
8. ∴ $\overline{\|AB\|} = 8$ cm	8. By taking square root
9. $\overline{\|AB\|} = \overline{\|BC\|} = 2\times 8 = 16\ cm$	9.By (3) and (8

Q.E.D.

Q.6. If two circles are internally tangent, then the line of centers passes through the common point of tangency.

SOLUTION.



Given: Two circles with centers O and O' touch each other at the point A internally. $\overleftrightarrow{OO'}$ is the line of centers.

To proves that: $\overleftrightarrow{OO'}$ passes through the commonpoint of tangency A.

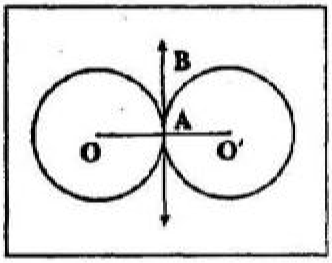
Construction: Draw a tangent $\overleftrightarrow{AB}$ at A.

Proof:

STATEMENTS	REASONS
1. The point A is at both circles.	1. Give
2. $\therefore$ $\overline{OA}$ and $\overline{O'A}$ are radical segment of the circles.	2. S.A.S. $\cong$ S.A.S.
3. But $\overline{AB}$ is the common tangent at A.	3. Congruence of triangles
4. $\therefore$ $\overline{OA} \perp \overline{AB}$ and $\overline{O'A} \perp \overline{AB}$	4. Theorem 5(b)
5. $m\angle OAB = 90°$ and	5. By (4)
6. $\therefore m\angle OAB + m\angle O'AB = 180°$	6. By (5)
7. Or $m\angle OAB + m\angle O'AB = 90° + 90°$	
8. $\therefore$ OO'A is a straight line.	8. Supplement Postulate.
9. But through O and O' only one straight	9. Postulate 1 (maths for class IX) (i.e.. two points determine a line)
10. $\therefore$ $\overleftrightarrow{OO'A}$ and $\overleftrightarrow{OO'}$ are the same line.	
11. So, $\overleftrightarrow{OO'}$ passes through the point of tangency A.	

Q.E.D.

Q.7. If the distance between the centers of two circles is the sum of their radii, prove that the circles are tangent externally.



SOLUTION.

Given: O and O' are centers of two circles, $|\overline{OO'}|$ is the distance between O and O'.

$|\overline{OO'}| = |\overline{OA'}| + |\overline{O'+A}|$, where A is the point on both the circles.

To prove that: The circles are tangent externally.

Construction: Draw a perpendicular $\overleftrightarrow{AB}$ to $\overline{OO'}$ at A.

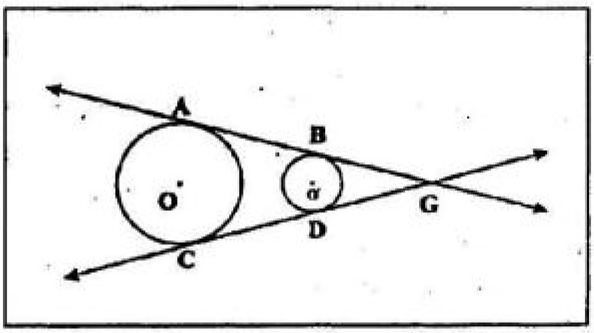
Proof:

STATEMENTS	REASONS
1. $\because |\overline{OO'}| = |\overline{OA'}| + |\overline{O'+A}|$	1. Given.
2. $\therefore$ A is between O and O'.	2. By definition of between.
3. $\therefore$ Points O, A and O' lie on line.	3. By (2)
4. $\because \overleftrightarrow{AB} \perp \overline{OA}$ and $\overleftrightarrow{AB} \perp \overline{O'A}$	4. Construction.
5. $\overline{AB}$ is tangent on circles with centers O and O' externally.	5. Theorem 5

Q.E.D.

Q8 Prove that the common external tangent segments of two circles are congruent. [Hint: Produce the common external tangents to meet at some point G (say) or prove otherwise].

SOLUTION.



Given: O and O' are centres of two circles. $\overline{AB}$ and $\overline{CD}$ are the common external tangent segments of these circles.

To prove that: $\overline{AB} \cong \overline{CD}$

Construction:

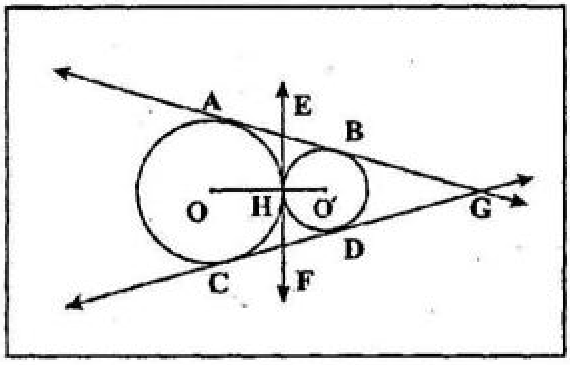
Produce the common external tangents to meet at a point G.

STATEMENTS	REASONS
1. $\overline{AG} \cong \overline{CG}$	1. Theorem 6.
2. $\overline{BG} \cong \overline{DG}$	2. Theorem 6.
3. $\therefore$ A, B and G lie on the same line.	3. Given
4. $m\overline{AB} + m\overline{BG} = m\overline{AG}$	4. By (3)
5. Similarly, $m\overline{CD} + m\overline{DG} = m\overline{CG}$	5. As proved above.
6. $m\overline{AB} + m\overline{BG} = m\overline{CD} + m\overline{DG}$	6. By (1), (4) and (5)
7. $m\overline{AB} = m\overline{CD}$	7. By (2)
8. $\therefore \overline{AB} \cong \overline{CD}$	8. By (7)

Q.E.D.

Q9 Prove that the common internal tangent segments of two circles are congruent.

SOLUTION:



Given: O and O' are centers of two circles. AB and CD are the common internal tangent segments of these circles. AB and CD cut OO' at G.

To prove that: $\overline{AB} \cong \overline{CD}$

Proof

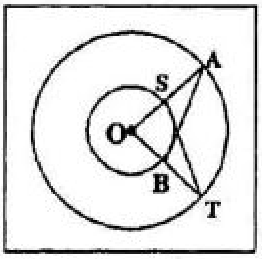
STATEMENTS	REASONS
1. $\overline{AG} \cong \overline{CG}$	1. Theorem 6.
2. $\overline{GB} \cong \overline{GD}$	2. Theorem 6.

3. $m\overline{AG} + m\overline{GB} = m\overline{GD}$	3. Given
4. $m\overline{AG} + m\overline{GD} = m\overline{CD}$	4. Given
5. $m\overline{AG} + m\overline{AB} = m\overline{GB}$	5. By (3)
6. $m\overline{AG} + m\overline{CD} = m\overline{GD}$	6. By (4)
7. $\therefore\ m\overline{AB} - m\overline{GB} = m\overline{CD} - m\overline{GD}$	7. By (1), (5) and (6)
8. or $m\overline{AB} = m\overline{CD}$	8. By (2)
9. $\therefore m\overline{AB} \cong \overline{CD}$	9. By (8)

Q.E.D.

Q10. A point A is in the exterior of a circle with centre O. $\overline{OT}$ meets the circle at S. $\overline{ST}$ is a tangent to this circle at S which meets another circle with centre O and radial segment $\overline{OA}$ at T. $\overline{OA}$ is drawn to cut the first circle at Be Prove that $\overline{BA}$ is tangent to 1st circle at B. [Hint: Prove $\Delta OBA \cong \Delta OST$

SOLUTION:



Given: $\overline{OS} \cong \overline{OB}$; $\overline{OA}$; $\overline{OT}$; $\overline{ST}$ is a tangent to circle at S.

To prove that: BA is tangent to the first circle at B, i.e., $m\angle ABO = 90°$.

Proof:

STATEMENTS	REASON
1. In $\Delta OBA \leftrightarrow \Delta OST$	1.
(i) $\overline{OA} \cong \overline{OT}$	(i) Given
(ii) $\overline{OB} \cong \overline{OS}$	(ii) Given
(iii) $m\angle AOB = m\angle SOB$	(iii) Same central angles.

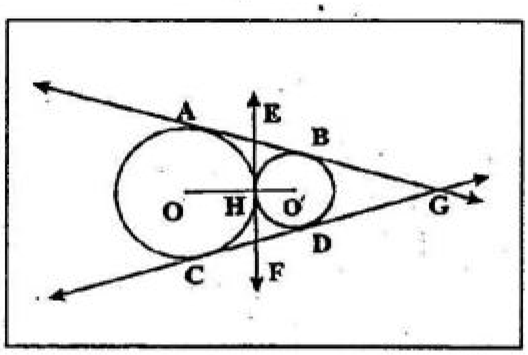
2. $\because$ $\Delta OBA \cong \Delta OST$	2. S.A.S. postulate
3. $\therefore$ $\overline{AB} \cong \overline{ST}$	3. By (2)
4. $m\angle TSO = 90°$	4. $\overline{ST}$ is a tangent to circle at S
5. $\therefore m\angle ABO = 90°$	5. By (2)
6. $\therefore \overline{BA}$ is a tangent to the first circle at B.	6. By (3)

Q.E.D.

Q.11. Two circles are externally tangent. Prove that the internal common tangent bisects both the common external tangent segments. Also prove that the segments of common internal tangent which lie between the two external tangents is congruent to each of the segment.

SOLUTION.

Given: O and O′ are the centers of two circles. At H, these circles touch externally. At the point of tangency H, $\overline{EF}$ is the internal common tangent. $\overline{AB}$ and $\overline{CD}$ are the common external tangent segments. $\overline{EF}$ cuts $\overline{AB}$ and $\overline{CD}$ at the points E and F respectively.



Construction: Produce $\overline{AB}$ and $\overline{CD}$ to meet at a point G.

To prove that:

(i) $\overline{EA} \cong \overline{EB}$; $\overline{FC} \cong \overline{FD}$ B and (ii) $\overline{EF} \cong \overline{AB}$; $\overline{EF} \cong \overline{CD}$

Proof:

Statements	Reason
1. From E, $\overline{EA}$ and $\overline{EH}$ are tangent segments to the circle with centre O.	1. Given

2. $\therefore$ $\lvert\overline{EA}\rvert = \lvert\overline{EH}\rvert$	2. Theorem 6
3. Similarly, $\lvert\overline{EB}\rvert = \lvert\overline{EH}\rvert$	3. By (1) and (2)
4. $\therefore$ $\lvert\overline{EA}\rvert = \lvert\overline{EB}\rvert$	4. Transitive property
5. or $\overline{EA} \cong \overline{EB}$	5. By (4)
6. i.e., E is the midpoint of $\overline{AB}$	6. By (5)
7. Also from F, $\overline{FH}$ are tangent segments to the circle with centre O.	7. Given
8. $\therefore$ $\lvert\overline{FC}\rvert = \lvert\overline{FH}\rvert$	8. Theorem
9. Similarly, $\lvert\overline{FD}\rvert = \lvert\overline{FH}\rvert$	9. By (7) and (8)
10. $\therefore$ $\lvert\overline{FC}\rvert = \lvert\overline{FD}\rvert$	10. Transitive property.
11. or $\overline{FC} \cong \overline{FD}$	11. By (10)
12. $\lvert\overline{EA}\rvert + \lvert\overline{EB}\rvert = 2\lvert\overline{EH}\rvert$	12. By (2) and (3)
13. or $\lvert\overline{EB}\rvert = 2\lvert\overline{EH}\rvert$	13. By (6)
14. Similarly, $\lvert\overline{CD}\rvert = 2\lvert\overline{FH}\rvert$	14. By (12) and (13)
15. $\overline{AB} \cong \overline{CD}$	15. Theorem 6
16. $\overline{AB} + \overline{CD} = 2\lfloor\lvert\overline{EH}\rvert = 2\lvert\overline{FH}\rvert\rfloor$	16. By (13) and (14)
17. $2\lvert\overline{AB}\rvert = 2\lvert\overline{CD}\rvert = 2\lvert\overline{EF}\rvert$	17. By (15)
18. $\therefore \lvert\overline{AB}\rvert = \lvert\overline{CD}\rvert = \lvert\overline{EF}\rvert$	18. Dividing by 2
19. or $\overline{AB} \cong \overline{EF}$; $\overline{CD} \cong \overline{EF}$	19. By (18)