

MATHEMATICS NOTES FOR CLASS 10TH (FOR SINDH)

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Chap 4 INFORMATION HANDLING

INTRODUCTION CHAPTER

INFORMATIONS HANDLING

1. How many primary or secondary schools will be required before year 2010 A.D. ?
2. How many college or university will be required before year 2015 ?
3. How many hospital and Doctor will have to be required in the next five years to meet the demands of population ?
4. How many house, flats and city will be needed during the next 10 years.

To know about something is known as "information" and to present that in a manageable way so that useful conclusions can be drawn, is called "information Handling."

DATA

The numerical figures obtained from any field of study are known as data. Or the information given in quantitative or qualitative form regarding particular characteristics is called "data". Data can be obtained from any sources i.e. office records, publishing papers can be obtained directly from the field according to the needs.

DATA SET

The data collected for a particular purpose is called "data set".

VARIABLE

Its is a characteristics that can take different values of the elements in the data set. Variables are of two types depending upon the form of the characteristics.

- (1) Quantitative Variable (2) Continuous Variable

Qualitative variable

It is that variable whose characteristics of interest is measurable or can be expressed numerically, e.g. weight , height, income, length, volume, number of children, etc. Qualitative variables are of two types.

- (1) Discrete Variable (2) Continuous variable

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MULTIPLE CHOICE (MCQ's)

- (i) In (70– 79) the mid point is,
 (a) 120 (b) 25.5 (c) **74.5** (d) 10
- (ii) In (50–59) the size of class interval is,
 (a) **10** (b) 28 (c) **58** (d) 08
- (iii) In 2,3,4,5,6,7,9 the mode is
 (a) 3 (b) 3.5 (c) **4** (d) 7
- (iv) In 25,30,10,48,100 the mode is
 (a) 30 (b) 50 (c) **80** (d) **90**
- (v) The sum of 5 observations is 125, its mean is,
 (a) 90 (b) 50 (c) **10** (d) **25**
- (vi) The mean of 20 observations is 100, its sum is,
 (a) **2000** (b) 1000 (c) **900** (d) 500
- (vii) A series contains values 5,9,3,1,4,6 its median is,
 (a) 2 (b) 3.5 (c) **4.5** (d) 5.5
- (viii) If a standard deviation of a series is 6, then its variance is,
 (a) 30 (b) **36** (c) 25 (d) 10
- (ix) Present the information in a manageable any so that useful conclusions can be drawn is called-----
 (a) **Information Handling** (b) Primary data
 (c) Secondary data (d) Grouped data
- (x) The numerical data obtained from any field of study is known as,
 (a) **Data** (b) Primary data
 (c) Secondary data (d) Grouped data
- (xi) The person who collects the data is known as,
 (a) Inspector (b) Clerk
 (c) **Investigator** (d) Cashier

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- (xii) How many types of data?
(a) one (b) **Two** (c) Three (d) Four
- (xiii) The data directory collected from its source is called -----
(a) **Secondary Data** (b) **Primary data**
(c) Grouped data (d) ungrouped data
- (xiv) The raw data when put in some order change into,
(a) **Secondary Data** (b) **Primary data**
(c) Grouped data (d) ungrouped data
- (xv) Any real number can be taken
(a) **Constant** (b) **Variable**
(c) **Discrete Variable** (d) **Continuous Variable**
- (xvi) The mid point of any class is called -----
(a) **Class Mark** (b) **Class interval**
(c) **Lower limit** (d) **Upper Limit**
- (xvii) Age of an individual is a
(a) **Constant** (b) **Class frequency**
(c) **Discrete Variable** (d) **Continuous Variable**
- (xviii) Any fixed quantity that has a single value is known as
(a) **Constant** (b) **Class frequency**
(c) **Discrete Variable** (d) **Continuous Variable**
- (xix) Process of changing the data into certain groups or classes having similar characteristics is called -----,
(a) **Classification** (b) **Tabulation**
(c) **Frequency distribution** (d) **Dispersion**
- (xx) The data which have been passed through some statistical treatments at least once is called-----
(a) **Secondary Data** (b) **Primary data**
(c) **Grouped data** (d) **ungrouped data**

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EXERCISE 4.1

Q1. Discuss the advantages of tabulation and classification.

SOLUTION : TABULATION

After the collected of data need to be presented data in a systematic way and manageable from that provides a clear picture about it. The procedure employed to reduce and simplify the raw data is called classification and tabulation. These two procedures work together. Classification is preliminary to tabulation. Which can be easily handled and analysed.

Classification is the process of sorting the data into classes or groups having similar properties according to their observed characteristics.

The classified data must be tabulated in manner that gives the best use of the given information.

Q2. Differentiate between the primary and secondary data, statistical population and sample.

SOLUTION : PRIMARY DATA

The data directly collected by some other agency source. When an original enquiry is conducted for the collection of information, the collected data is called primary data.

SECONDARY DATA

The data which has been passed through some statistical treatments atleast once is called secondary data. For example, the raw data when put in some order, change into a secondary data.

STATISTICAL POPULATION

The collection of all observation (elements) relating to a characteristics is called statistical population or simply population.

Sample : It is a subset of the statistical population.

Q.3. Prepare the frequency distribution using tally method as well as listing method form the data of rainfall (In cms) for 25 years : 5, 25, 16, 8, 12, 9, 1, 14, 21, 23, 4, 11, 22, 3, 2, 5, 11, 7, 25, 13, 17, 2, 6, 7, 9.

(i) Find the mid-points and Class frequencies for each case.

(ii) Construct a frequency polygon.

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SOLUTION : Tally Method

Class intervals	Tally marks	Frequency
1 – 5		7
6 – 10		7
11 – 15		5
16 – 20		2
21 – 25		5
Total		25

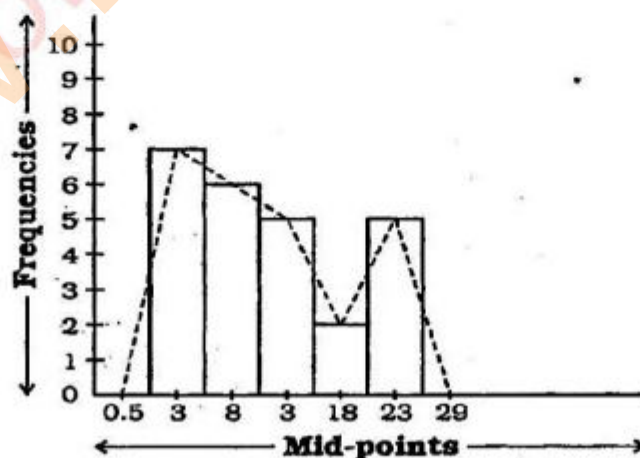
Listing Method (or Direct Method)

Class Intervals	Entries	Frequency	Mid-points
1 – 5	5, 4, 3, 2, 5, 2	7	3
6 – 10	8, 9, 7, 6, 7, 9	6	8
11 – 15	12, 14, 11, 11, 13	5	13
16 – 20	16, 17	2	18
21 – 25	25, 21, 23, 22, 25	5	23
Total		25	

(i) Mid points 3, 8, 13, 18, 23

Class Frequencies : 7, 6, 5, 2, 5

(ii) Frequency Polygon



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Q.4. The following distribution represents the scores achieved by a group of science students in physics Laboratory. Answer the following questions.

Scores	No. of students
24 – 28	03
29 – 33	16
34 – 38	12
39 – 43	23
44 – 48	15
49 – 53	26
Total	95

- (i) What is the upper limit of the last class?
- (ii) What is the lower limit of the class interval (49–53)?
- (iii) What is the mid value in the class interval (44–48)?
- (iv) What is the class frequencies of the classes (29–33) and (44–48)?
- (v) What is the lower limit of the class having 16 as its class frequency?

SOLUTION :

- (i) The upper limit of the last class is 53.
- (ii) The lower limit of the class interval (49 – 53) is 49.
- (iii) The mid value in the class interval (44 – 48) is 46.
- (iv) The class frequencies of the classes (29–33) and (44–48) are 16 and 15.
- (v) The lower limit of the class having 16 as its class frequency is 29.

Q5. Following are the number of mistakes made by a group of students of class-X in a test of Essay Writing. Using an appropriate size of class interval make a frequency distribution and also indicate the number of class intervals:

14, 7, 12, 9, 21, 16, 16, 3, 19, 17, 24, 14, 15, 8, 11, 16, 15, 6, 5, 8, 13, 11, 20, 18, 22, 16, 2, 1, 3.

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SOLUTION : Frequency Distribution

Class Intervals	Entries	Frequency
1 – 4	1,2,3,3	4
5 – 8	7,8,6,5,8	5
9 – 12	12, 9, 11, 11	4
13 – 16	14, 16, 14, 15,13,16, 15, 18	8
17 – 20	17, 19, 20, 18	4
21 – 24	21, 24, 22	3
Total		28

In this distribution, size of class intervals is 4.

Q.6. A group of 10th class students obtained the following marks (out of 100 marks) in English test:

38, 59, 58, 33, 40, 58, 45, 46, 43, 45, 45, 50, 52, 49, 50, 57, 63, 55, 49, 50, 65, 49, 48, 44, 42, 47, 46, 47, 46, 53, 40, 20, 12.

Classify the data into a frequency distribution by Direct Method taking 5 as the size of the class interval. Also find the class interval with least class frequency.

SOLUTION : Frequency Distribution

Class	Entries	Frequency
10 — 15	12	1
16 — 20	20	1
21 — 25		
26 — 30		
31 — 35	33	1
36 — 40	38,40,40	3
41 — 45	45,43,45,45,44,42,44	7
46 — 50	46,50,49,50,49,50,49,48,	12
51 — 55	52,55,53	3

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56 — 60	59,58,58,57	4
61 — 65	63,65	2

Here least class frequency = 1.

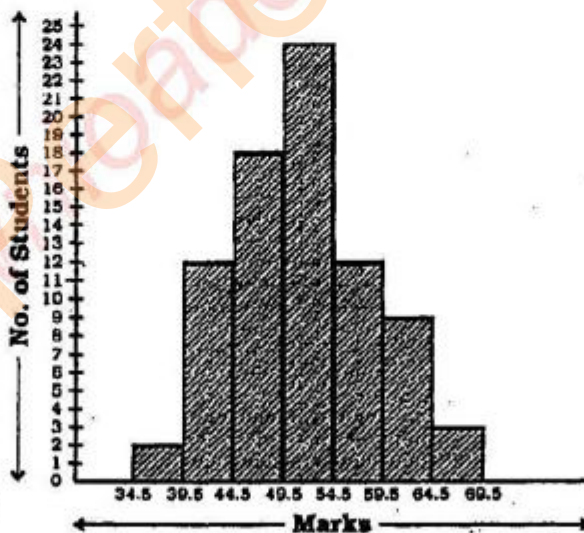
The class intervals with least class frequency are (10-15), (16-20) and (31-35).

Q.7. The marks obtained by 80 students of 10th class in Chemistry test were grouped into the following frequency distribution.

Marks	35-39	40-44	45-49	50-54	55-59	60-64	65-69
No. of Students	2	12	18	24	12	9	3

SOLUTION :

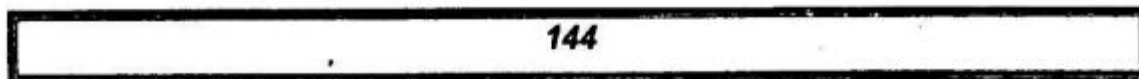
Draw a histogram for the above distribution.



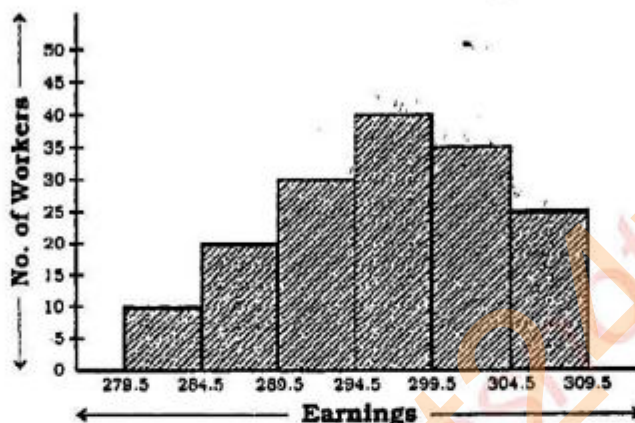
Q.8. Draw a Histogram for the distribution of daily earnings (Rs) of workers given below:

Earnings	280-284	285-289	290-294	295-299	300-304	305-309
No. of Workers	10	20	30	40	35	25

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SOLUTION :



Q.9. Represent the Information given below in tabular form.

"The number of students in a school in the year 1970 was 500, of these 450 students were boys and rest girls. In 1980, the number of boys increased by 100 percent and that of girls by 200 percent compared to their strengths in 1970. In 1990, the total number of students in the school was 1500, the number of boys being triple the numbers of girls".

SOLUTION :

Year	No. of Students	No. of Boys	No. of Girls
1970	500	450	50
1980	1050	900	150
1990	1500	1125	375

Q10. Make a frequency polygon for the following grouped data stating the main steps involved in making the said graph:

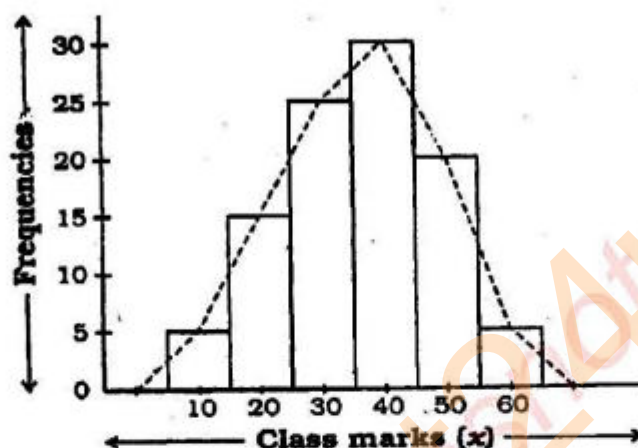
Class mark (x)	10	20	30	40	50	60
Frequency	5	15	25	30	20	5

SOLUTION :

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Diagrams Representation

Visual representation of data in the form of points, lines, areas and other geometrical forms is known as graphical representation. Such visual representation can be divided into main types, graphs and diagram. The graph is a representation of data by continuous curves, while a diagram is any other form of visual representation.

Simple Bar Diagram

It consists of horizontal or vertical bars of equal widths or lengths proportional to the value they represent.

Multiple Bar Diagram

It is used for the comparison of two (or more) variable characteristics, simultaneously. For example, we may represent the marks obtained by students in three tests, quarterly half-yearly, and yearly, through a multiple bar diagram, this is done by making three separate simple bar diagrams.

Pie Diagram

It is a graphic device consisting of dividing a circle into sectors, looking like pie shaped pieces, whose areas are proportional to the various parts into which the whole quality is divided. It is also known as circle diagram or sector diagram. It is used for the comparison of values of different items by the making the corresponding sectors of a circle.

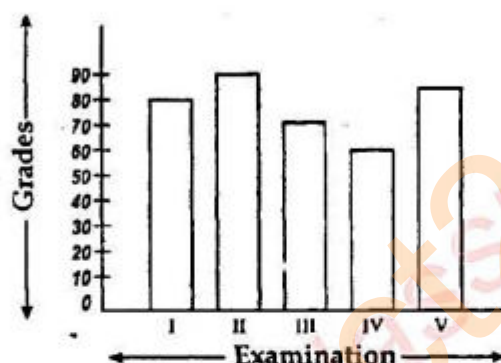
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EXERCISE 4.2

Q.1. The grades of a student Nadia in five examinations were 80, 90, 70, 60, and 85. show the above grades by making a simple bar diagram.

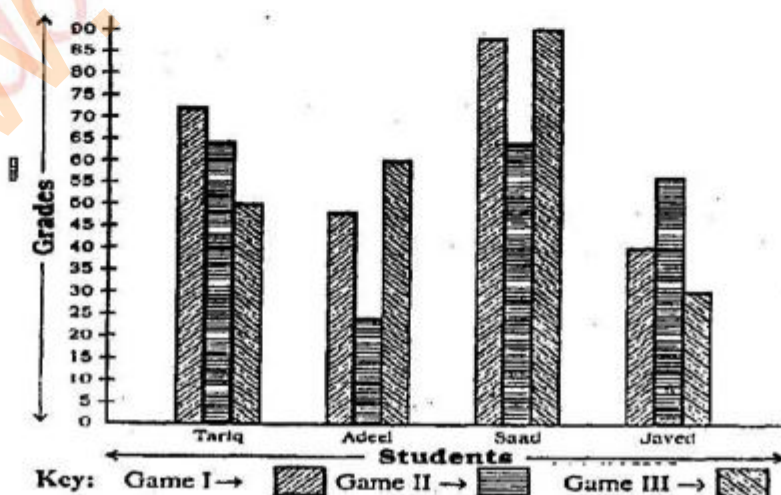
SOLUTION :



Q.2. Prepare a multiple bar diagram to show the points out of (100) made by four students in three games.

Students	Points in Game-I	Points in Game -II	Points in Game-III
Tariq	72	64	50
Adeel	48	24	60
Saad	88	64	90
Javed	40	56	30

SOLUTION :



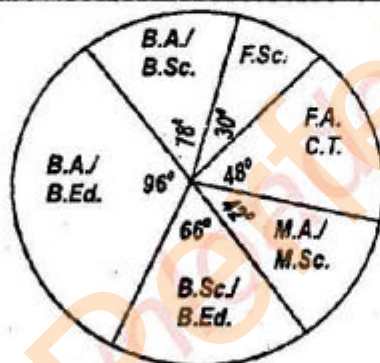
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Q3. The data given below show the number of teachers according to their academic qualifications in an English medium school. Construct a Pie Chart.

Students	Points in Game-I
F.A.C.T.	8
F.Sc.	5
B.A./B.Sc.	13
B.A., B.Ed.	16
B.Sc., B.Ed.	11
M.A./M.Sc.	7

SOLUTION :



Q4. The following are the runs made in a two-day cricket match by the students in a High School.

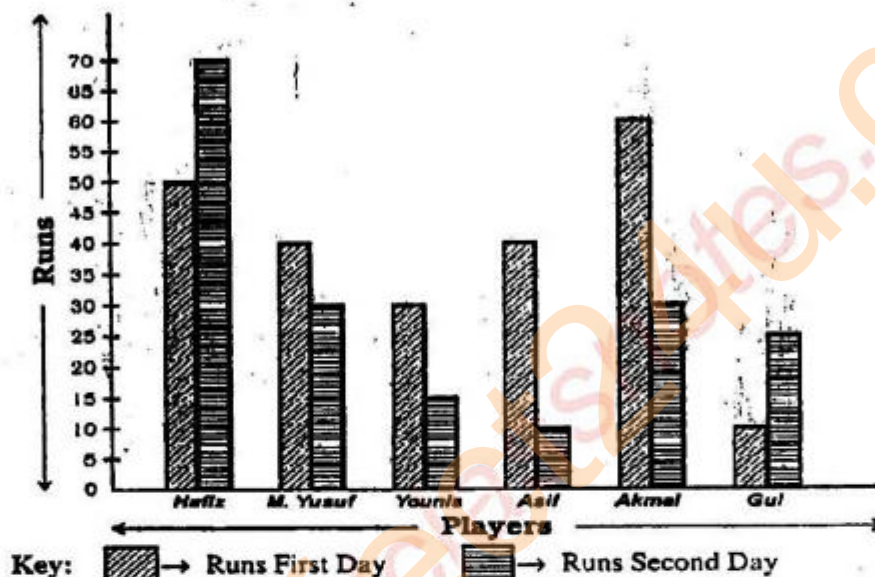
Players	Runs First day	Runs Second day
Hafiz	50	70
M. Yousuf	40	30
Younis	30	15
Asif	40	10
Akmal	60	30
Gul	10	25

Draw a suitable diagram for the runs made in the match.

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SOLUTION :



EXERCISE 4.3

Q1. What is an average? Explain the method of calculation of Arithmetic mean for ungrouped data and as well as for grouped data.

SOLUTION : AVERAGE

An average is simple value is intended to represent a set of data. It is a value around which the elements in the set of data tend to cluster.

Method of Finding Arithmetic Mean for Ungrouped Data.

If $x_1, x_2, x_3, x_4, \dots, x_n$ are the values of different observations and $f_1, f_2, f_3, f_4, \dots, f_n$ are their frequencies. Then

$$\text{A.M.} = \bar{x} = \frac{f_1 x_1 + f_2 x_2 + f_3 x_3 + f_4 x_4 + \dots + f_n x_n}{f_1 + f_2 + \dots + f_n}$$

$$\text{i.e., } \bar{x} = \frac{\sum fx}{\sum f}$$

Method of Finding Arithmetic Mean for Grouped Data

Let $x_1, x_2, x_3, x_4, \dots, x_n$ be the mid points of the class intervals with corresponding frequencies $f_1, f_2, f_3, f_4, \dots, f_n$. Then the Arithmetic Mean is

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obtained by dividing the sum of the products of "f" and "x" by the total of all frequencies. Thus.

$$A.M. = \bar{x} = \frac{\sum fx}{\sum f}$$

Q2. Find the Arithmetic Mean in each of the following.

(i) 3.2, 6, 10, 12, 15, -20, 25, 28, 30.8

(ii) 14, 12, 18, 19, 0, -19, -18, 12, 14

(iii) 6.5, 11, 12.3, 9, 8.1, 16, 18, 20.5, 25

SOLUTION : (i) $A.M. = \frac{3.2 + 6 + 10 + 12 + 15 + (-20) + 25 + 28 + 30.8}{9} = \frac{110}{9} = 12.22$

(ii) $A.M. = \frac{3.2 + 6 + 10 + 12 + 15 + (-19) + (-18) + (-12) + (-14)}{9} = \frac{0}{9} = 0$

(iii) $A.M. = \frac{6.5 + 11 + 12.3 + 9 + 8.1 + 16 + 18 + 20.5 + 25}{9} = \frac{126.4}{9} = 14.04$

Q3. Define method and give its merits and demerits. How do we calculate median for grouped data.

SOLUTION : MEDIAN

Median is the value which divides a set of data into two parts, i.e., 50% of the data are greater than the median value and 50% are smaller than it.

Thus it is necessary to arrange the data in ascending or descending or descending order.

Merits of Median

It is easily calculated and understood

It is useful in case of quantitative data.

It is not affected by extreme values.

Demerits of Median

It is not rigorously defined.

It is necessary to arrange the data into an array.

It is not capable of lending itself to mathematical treatment.

It does not use value of all the observations.

Median for Grouped Data.

It is obtained by the following formula.

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$$\text{Median} = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

Where l = Lower class boundary of median class

h = length of median class

f = Frequency of the median class

n = Total frequency, i.e., Σf

c = Cumulative frequency preceding the median class.

Q.4. Following are the heights (in inches) of 12 students. Height : 51, 55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 64. Find the Median.

SOLUTION : Writing the given heights in ascending order,

51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 56, 57, 52, 64

$$\begin{aligned} \text{Median} &= \frac{1}{2} \left[\frac{n}{2} \text{th} + \left(\frac{n+2}{2} \right) \text{th} \right] \\ &= \frac{1}{2} \left[\frac{12}{2} \text{th} + \left(\frac{12+2}{2} \right) \text{th} \right] \\ &= \frac{1}{2} [6\text{th} + 7\text{th}] = \frac{1}{2} [56 + 57] \\ &= \frac{113}{2} = 56.5 \end{aligned}$$

Q5. Define mode and explain the methods of its calculation.

SOLUTION : MODE

In a set of data, mode is a value of the variable which appears or occurs most often. It is the most common value.

A set of data may have one mode, more than one mode or no mode at all.

Q7. Marks obtained by the students in chemistry are given below. Find.

(i) Arithmetic Mean of their marks by direct and shortcut formula

(ii) Median of their marks

(iii) Mode

Marks obtained	25-29	30-34	35-39	40-44	45-49
No. of students	9	18	35	17	5

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SOLUTION : (i) Direct Method

Marks obtained	Mix point (x)	(No. of Students) Frequency (f)	f x
25-29	27	9	243
30-34	32	18	576
35-39	37	35	1295
40-44	42	17	714
45-49	47	5	235
Total	-	$\Sigma f x = 84$	$\Sigma f x = 3063$

$$A.M. = \frac{\Sigma f x}{\Sigma f} = \frac{3063}{84} = 36.46$$

Shortcut Formula

Marks obtained	Mix point (x)	(No. of Students) Frequency (f)	f x	f D
25-29	27	9	-10	-90
30-34	32	18	-5	-90
35-39	37	35	0	0
40-44	42	17	5	85
45-49	47	5	10	50
Total	-	$\Sigma f x = 84$	-	$\Sigma f D = -45$

$$\text{Now, } A.M. = A + \frac{\Sigma f D}{\Sigma f} = 37 + \frac{-45}{84} = 37 - 0.54 = 36.46$$

(ii)

Marks Obtained	Class Boundaries	(No. of Students) Frequency (f)	C.F.
25-29	24.5-29.5	9	9
30-34	29.5-34.5	18	9+18 = 27
35-39	34.5-39.5	35	27+35 = 62

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40-44	39.5-44.5	17	62 + 17 = 79
45-49	44.5-49.5	5	79 + 5 = 84
Total		$\Sigma f = 84$	

$$\text{Median} = \frac{n}{2} \text{th item} = \frac{84}{2} = 42\text{th item.}$$

As 42th item lies in (35-39), it is the median class with $l = 35$, $h = 5$,
 $f = 35$, $c = 27$, $n = 84$.

$$\begin{aligned} \text{Median} &= l + \frac{h}{f} \left(\frac{n}{2} - c \right) = 35 + \frac{5}{35} \left(\frac{84}{2} - 27 \right) \\ &= 35 + \frac{1}{7} \times 15 = 37.14 \end{aligned}$$

(iii) The modal class is (35-39) because it has maximum frequency 35.

Here, $l = 35$, $f_m = 35$, $f_1 = 18$, $f_2 = 17$, $h = 5$.

$$\begin{aligned} \text{S, Mode} &= l + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)} = 35 + \frac{(35 - 18) \times 5}{(35 - 18) + (35 - 17)} \\ &= 35 + \frac{17 \times 5}{17 + 18} = 35 + \frac{85}{35} \\ &= 35 + 2.43 = 37.43 \end{aligned}$$

8. A set of data contains the values as 148, 145, 160, 157, 156, 160, 160, 165. Show that mode > median > mean.

$$\begin{aligned} \text{Solution : Mean} &= \frac{148 + 145 + 160 + 157 + 156 + 160 + 160 + 165}{8} \\ &= \frac{1095}{8} = 136.875 \end{aligned}$$

Writing the given values in ascending order,

145, 148, 156, 157, 160, 160, 160, 165

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$$\begin{aligned}\text{Median} &= \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} + \left(\frac{n+2}{2} \right)^{\text{th}} \right] = \frac{1}{2} \left[\left(\frac{8}{2} \right)^{\text{th}} + \left(\frac{8+2}{2} \right)^{\text{th}} \right] \\ &= \frac{1}{2} [4^{\text{th}} + 5^{\text{th}}] = \frac{1}{2} [157 + 160] = 158.5\end{aligned}$$

$$\text{Mode} = 160$$

$$\therefore 160 > 158.5 > 136.875$$

So, mode > median > mean.

Q9. Frequency distribution for number of deaths, due to various diseases, during January, in Hyderabad, is given below.

Cause of death	T.B.	Diabetes	Malaria	Cholera	Cancer	Heart	B.P
Frequency	3	2	2	1	3	5	1

Find the modal cause of death.

Can you find the arithmetic mean and median?

SOLUTION : The modal cause of death is heart because number of frequencies is 5 which is maximum number of death by heart diseases.

No, we cannot find the arithmetic mean and median because the value of different observations are not given.

Q.10 Calculate the arithmetic mean from the following information:

(i) $D = x - 140$, $\Sigma D = 600$ and $n = 10$

(ii) $u = \frac{x-130}{6}$, $\Sigma f D = -150$ and $n = 25$

(iii) $D = x - 20$, $\Sigma f D = 300$ and $\Sigma f = 20$

(iv) $u = \frac{x-120}{5}$, $\Sigma f D = 60$ and $\Sigma f = 200$

SOLUTION : (i) $\therefore D = x - A = x - 140 \Rightarrow A = 140$

$$\therefore \text{A.M.} = A + \frac{\Sigma D}{n} = 140 + \frac{600}{10}$$

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$$= 140 + 60 = 200$$

$$(ii) \quad \therefore u = \frac{x-A}{h} = \frac{x-130}{6} \Rightarrow A = 130, h = 6$$

$$\therefore A.M. = A + \frac{\sum u \times h}{n} = 130 + \frac{(-150) \times 6}{25} = 130 - 36 = 94$$

$$(iii) \quad \therefore D = x - A = x - 20 \Rightarrow A = 20$$

$$\therefore A.M. = A + \frac{\sum D}{n} = 20 + \frac{300}{20} \\ = 20 + 15 = 35$$

$$(iv) \quad \therefore u = \frac{x-A}{h} = \frac{x-120}{5} \Rightarrow A = 120, h = 5$$

$$\therefore A.M. = A + \frac{\sum u \times h}{n} = 120 + \frac{60}{200} \times 5 = 120 + \frac{3}{2} \\ = 121.5$$

Q.11. Given below is a frequency distribution derived by making a substitution as $D = x - 20$, Calculate the arithmetic mean.

D	-6	-4	-2	0	2	4	6
f	2	3	6	20	24	12	3

SOLUTION :

D	-6	-4	-2	0	2	4	6	-
f	2	3	6	20	24	12	3	$\Sigma f = 70$
fD	-12	-12	-12	0	48	48	18	$\Sigma fD = 78$

$$\therefore A.M. = A + \frac{\sum D}{n} = 20 + \frac{78}{70} \\ = 20 + 1.11 = 21.11$$

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EXERCISE 4.4.

Q1. What is range? How is it calculated?

SOLUTION : RANGE

Range is defined as the difference between the largest and the smallest observations in the data set. It is denoted by R.

Calculation of Range for Raw or Ungrouped Data

$$R = x_{\max} - x_{\min}$$

where $x_{\min}$ denotes the maximum or largest value of x, and $x_{\max}$ denotes the minimum or smallest value of x.

Calculation of Range for Grouped Data

$R = \text{Upper boundary of the highest class} - \text{lower boundary of the lowest class}$

Q2. Find the range of the following set of measurement:

21.25, 3.12, 25, 14.28, 9.1, 16.24, 4.45, 1.13, 9, 26.28.

SOLUTION : Here, $x_{\max} = 26.28$ and $x_{\min} = 1.13$

$$\text{So, } R = x_{\max} - x_{\min} = 26.28 - 1.13 = 25.15$$

Q3. Define variance. Give a method of its calculation.

SOLUTION : The variance is a single value obtained by dividing the sum of squares of the deviations taken from the Arithmetic Mean by the number of observations in the data set, i.e.,

$$\text{Var}(x) = \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n} = \frac{\sum d_i^2}{n} \quad (\text{Deviation Method})$$

$$\text{or} \quad \sigma^2 = \frac{\sum D_i^2}{n} - \left(\frac{\sum D_i}{n} \right)^2 \quad (\text{shortcut method})$$

$$\text{or} \quad \sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2 \quad (\text{Direct Method})$$

where $D_i = x_i - A$ and $A = \text{provisional mean}$

As the variance is based upon the square of the deviations, so it is always positive and does not pose the problem of being zero.

Note: σ is read as sigma. It is a Greek letter.

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Q4. Find the variance of the following sets of observations:

X = 11, 13, 25, 15, 12, 18, 17, 23, 20, 16

Y = 115, 108, 95, 118, 130, 114, 116, 120

SOLUTION :

x_i	11	13	25	15	12	18	17	23	20	16	$\Sigma x_i = 170$
x_i^2	121	169	625	225	144	324	289	529	400	256	$\Sigma x_i^2 = 3082$

$$\begin{aligned}
 \therefore \sigma^2 &= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n} \right)^2 \text{ (Direct Method)} \\
 &= \frac{3082}{10} - \left(\frac{170}{10} \right)^2 \quad (\because n = 10) \\
 &= 308.2 - 289 = 19.2
 \end{aligned}$$

x_i	115	108	95	118	130	114	116	120	$\Sigma x_i = 916$
x_i^2	13225	11664	9025	13924	16900	12996	13456	14400	$\Sigma x_i^2 = 105590$

$$\begin{aligned}
 \therefore \sigma^2 &= \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n} \right)^2 \text{ (Direct Method)} \\
 &= \frac{105590}{8} - \left(\frac{916}{8} \right)^2 \quad (\because n = 8) \\
 &= 13198.75 - (114.5)^2 \\
 &= 13198.75 - 13110.25 = 88.5
 \end{aligned}$$

Q5. What is standard deviation? Describe a method of its calculation.

SOLUTION : STANDARD DEVIATION

It is defined as the positive square root of the variance. It is denoted by σ or s . It is a measure of the average spread around the mean.

Calculation of Standard Deviation

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$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} = \sqrt{\frac{\sum d_i^2}{n}} \quad (\text{Deviation Method})$$

$$\text{or } \sigma = \sqrt{\frac{\sum D_i^2}{n} \left(\frac{\sum D_i}{n} \right)^2}, \quad (\text{Shortcut Method})$$

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} \left(\frac{\sum x_i}{n} \right)^2}, \quad (\text{Direct Method})$$

where $D_i = x_i - A$ and $A = \text{provisional mean.}$

Q.6. The following are the marks obtained (out of 200) by eight students in two subjects:

Students	1	2	3	4	5	6	7	8
Marks (Urdu)	145	142	133	138	147	151	135	160
Marks (Civics)	148	150	141	135	150	161	142	162

SOLUTION : Compare the S.D. of marks obtained in each subject.

No. of Students	Marks (Urdu) x_{1i}	Marks (civics) x_{2i}	x_{1i}^2	x_{2i}^2
1	145	148	21025	21904
2	142	150	20164	22500
3	133	141	17689	19881
4	138	135	19044	18225
5	147	150	21609	22500
6	151	161	22801	25921
7	135	142	18225	20164
8	160	162	25600	26244
$n = 8$	$\sum x_{1i} = 1151$	$\sum x_{2i} = 1189$	$\sum x_{1i}^2$	$\sum x_{2i}^2$

S.D. of marks obtained in Urdu is:

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$$\begin{aligned}\sigma &= \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{\frac{166157}{8} - \left(\frac{1151}{8}\right)^2} \\ &= \sqrt{20769.625 - (143.875)^2} \\ &= \sqrt{69.609} = 8.343\end{aligned}$$

S.D. of marks obtained in Civics is:

$$\begin{aligned}\sigma &= \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{\frac{177339}{8} - \left(\frac{1189}{8}\right)^2} \\ &= \sqrt{22167.375 - 22089.391} \\ &= \sqrt{22167.375 - 22089.391} \\ &= \sqrt{77.984} = \sqrt{8.831}\end{aligned}$$

$$\therefore \sigma_2 > \sigma_1$$

It shows that eight students are better in Urdu than Civics.

Q.7. Find the range :

$$X = 10, 30 ; Y = 10, 22, 30 ; Z = 120, 122, 130, 135, 140$$

Is there any effect of number of item on the range?

Is there any effect of starting and end values of item?

$$\text{SOLUTION : Range of } X = x_{\max} - x_{\min} = 30 - 10 = 20$$

$$\text{Range of } Y = x_{\max} - x_{\min} = 30 - 10 = 20$$

$$\text{Range of } Z = x_{\max} - x_{\min} = 140 - 120 = 20$$

No, there is no effect of number of items on the range. Yes, there is an effect of starting and end values of items.

Q8. Find the two numbers if:

(i) Their mean is 23 and S.D. is 5

(ii) Their variance is 4 and mean is 7

(iii) Their S.D. is 2 and mean is 7

(iv) Their range is 2 and mean is 7

$$\text{SOLUTION : (i) A.M.} = \bar{x} = 23 \text{ and S.D.} = \sigma = 5.$$

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Let x and y be the required numbers, then

$$\bar{x} = \frac{x+y}{2}$$

i.e., $23 = \frac{x+y}{2} \Rightarrow y = 46 - x$ (1)

$$\sigma = \sqrt{\frac{(x-\bar{x})^2 + (y-\bar{x})^2}{2}}$$

$$5 = \sqrt{\frac{(x-23)^2 + (46-x-23)^2}{2}}$$

$$5 = \sqrt{\frac{2(x-23)^2}{2}} \Rightarrow 5 = x - 23$$

$$x = 28 \Rightarrow y = 46 - 28 = 18$$

Thus, the required numbers are 28 and 18.

(ii) **Variance** = $\sigma^2 = 4$ and **A.M.** = $\bar{x} = 7$

Let x and y be the required numbers. Then

$$\bar{x} = \frac{x+y}{2}$$

i.e. $7 = \frac{x+y}{2} \Rightarrow y = 14 - x$ (1)

$$\sigma^2 = \frac{(x-\bar{x})^2 + (y-\bar{x})^2}{2}$$

i.e. $4 = \frac{(x-7)^2 + (7-x)^2}{2}$

$$\Rightarrow 4 = \frac{2(x-7)^2}{2} \Rightarrow 4 = x^2 - 14x + 49$$

$$\Rightarrow x^2 - 14x + 45 = 0$$

$$\therefore x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(45)}}{2(1)}$$

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$$x = \frac{14 \pm \sqrt{196 - 180}}{2}$$

$$= \frac{14 \pm 4}{2} \Rightarrow x = 9 ; 5$$

Subtracting $x = 9 ; 5$ in (1), we get $\therefore y = 5 ; 9$
Thus the required numbers are 9 and 5.

(III) S.D. = $\sigma = 2$ and A.M. = $\bar{x} = 7$

Let x and y be the required numbers. Then

$$\bar{x} = \frac{x+y}{2}$$

i.e., $7 = \frac{x+y}{2} \Rightarrow y = 14 - x \quad \dots\dots (1)$

$$\sigma = \sqrt{\frac{(x-\bar{x})^2 + (y-\bar{x})^2}{2}}$$

i.e. $2 = \sqrt{\frac{(x-7)^2 + (14-x-7)^2}{2}}$

$$\Rightarrow 2 = \sqrt{\frac{(x-7)^2 + (7-x)^2}{2}} \Rightarrow 2 = \sqrt{\frac{2(x-7)^2}{2}}$$

$$\Rightarrow 4 = x - 7 \Rightarrow x = 11 ; y = 3$$

Thus, the required numbers such that $x > y$. then

$$R = x - y$$

i.e., $2 = x - y \Rightarrow y = x - 2 \quad \dots\dots (1)$

$$\bar{x} = \frac{x+y}{2}$$

i.e. $7 = \frac{x+y}{2} \Rightarrow y = 14 - x \quad \dots\dots (2)$

Comparing (1) and (2), we get $x - 2 = 14 - x$

$$\Rightarrow 2x = 16 \Rightarrow x = 8 \quad \therefore y = 8 - 2 = 6$$

Thus, the required numbers are 8 and 6.

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Q9. Find the variance and standard deviation from the following information: $\bar{x} = 19.5$, $n = 10$, $\Sigma x^2 = 5555$.

SOLUTION : $\therefore \bar{x} = \frac{\Sigma x}{n} = 19.5$

$$\begin{aligned} \therefore \sigma^2 &= \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n} \right)^2 \\ &= \frac{5555}{10} - (19.5)^2 \quad (\because \Sigma x^2 = 5555 ; n = 10) \\ &= 555.5 - 380.25 \\ &= 175.25 \end{aligned}$$

$$S.D. = \sqrt{\sigma^2} = \sqrt{175.25} = 13.24$$

Q10. The heights (In cms) of 16 children are given below:

4, 67, 62, 66, 63, 64, 63, 69, 63, 65, 67, 71, 65, 64, 72, 66. Calculate the Mean and Standard Deviation. Find the percentage of observations lying within the limits $(\bar{x} + 1s)$, $(\bar{x} + 2s)$, $(\bar{x} + 3s)$.

SOLUTION :

No. of Student	Heights x_i	x_i^2	No. of	Heights x_i	x_i^2
1	64	4096	9	63	3969
2	67	4489	10	65	4225
3	62	3844	11	67	4489
4	66	4356	12	71	5041
5	63	3969	13	65	4225
6	64	4096	14	64	4096
7	63	3969	15	72	5184
8.	69	4761	16	66	4356

Here $n = 16$, $\Sigma x_i = 1051$, $\Sigma x_i^2 = 69165$

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$$\text{Mean} = \frac{\sum x_i}{n} = \frac{1051}{16} = 65.69$$

$$\begin{aligned} \text{S.D. } \sigma &= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} \\ &= \sqrt{\frac{69165}{16} - \left(\frac{1051}{16}\right)^2} \\ &= \sqrt{4322.8125 - 4314.8476} \\ &= \sqrt{7.9649} = 2.82 \end{aligned}$$

$$\therefore \bar{x} \pm 1s = 65.69 \pm 2.82 = 62.87 ; 68.51$$

Between 62.87 and 68.51, values are: 62, 63, 63, 63, 64, 64, 64, 65, 65, 66, 66, 67, 67

There are 13 values between 62.87 and 68.51.

$$\begin{aligned} \therefore \text{Percentage of observations lying within the limit } (\bar{x} \pm 1s) &= \frac{13}{16} \times 100 \\ &= 81.25\% \end{aligned}$$

$$\begin{aligned} \bar{x} \pm 2s &= 65.69 \pm 2(2.82) = 65.69 \pm 5.64 \\ &= 60.05 ; 71.33 \end{aligned}$$

Between 60.05 and 71.33, values are: 64, 67, 62, 66, 63, 64, 63, 69, 63, 65, 67, 71, 65, 64, 66, 67, 67

There are 15 values between 60.05 and 71.33.

$$\begin{aligned} \text{Percentage of observations lying within the limit } (\bar{x} \pm 2s) &= \frac{15}{16} \times 100 = 93.75\% \end{aligned}$$

$$\begin{aligned} \therefore \bar{x} \pm 3s &= 65.69 \pm 3(2.82) = 65.69 \pm 8.46 \\ &= 57.23 ; 74.15 \end{aligned}$$

All the values are between 57.23 and 74.15.

$$\therefore \text{Percentage of observations lying within the limit } (\bar{x} \pm 3s) = \frac{16}{16} \times 100$$

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= 100%

Q.11. Variance of consecutive even numbers from 1 to 16 is 21.

Find the Variance of consecutive odd numbers from 1 to 16.

SOLUTION : Variance of consecutive odd numbers from 1 to 16 is:

$$\begin{aligned}\sigma &= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} \\&= \sqrt{\frac{(1)^2 + (3)^2 + (7)^2 + (11)^2 + (13)^2 + (15)^2}{8} - \left(\frac{1+3+7+9+11+13+15}{8}\right)^2} \\&= \sqrt{\frac{1+9+25+49+81+121+169+225}{8} - \left(\frac{64}{8}\right)^2} \\&= \sqrt{\frac{680}{8} - (8)^2} = \sqrt{85 - 64} = \sqrt{21}\end{aligned}$$

Q.12: X = 10, 20, 30 ;

Y = 10, 20, 20, 30 ;

Z = 10, 10, 20, 20, 30, 30 ; W = 40, 10, 15, 25, 30, 30 ;

(a) Find mean, median, mode and range in each case.

(b) Find A.M., median, mode, range and S.D. for W after :

(i) adding 5 to each item

(ii) subtracting 5 from each item

(iii) multiplying each item by 4

(c) Find the effect on median, mode, range and S.D. for each case
Inserting two

(I) relatively high values i.e., 50 and 60.

(II) relatively low. values i.e., 0 and 2..

SOLUTION :

(a) For X :

$$\text{Mean} = \frac{10+20+30}{3} = \frac{60}{3} = 20$$

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$$\text{Median} = \left(\frac{n+1}{2}\right) \text{th item} = \text{th item} = 2\text{nd item} = 20$$

Mode = 0 because no repetition of numbers.

$$\text{Range} = x_{\max} - x_{\min} = 30 - 10 = 20$$

For Y:

$$\text{Mean} = \frac{10 + 20 + 20 + 30}{4} = \frac{80}{4} = 20$$

$$\begin{aligned} \text{Median} &= \frac{1}{2} \left[\left(\frac{n}{2}\right) \text{th} + \left(\frac{n+2}{2}\right) \text{th} \right] \\ &= \frac{1}{2} (2^{\text{nd}} \text{ item} + 3^{\text{rd}} \text{ item}) (\because n = 4) \\ &= \frac{1}{2} (20 + 20) = \frac{40}{2} = 20 \end{aligned}$$

Mode = 20 becomes 20 is repeated twice.

$$\text{Range} = x_{\max} - x_{\min} = 30 - 10 = 20$$

For Z:

$$\begin{aligned} \text{Mean} &= \frac{10 + 10 + 20 + 20 + 30 + 30}{6} \\ &= \frac{120}{6} = 20 \end{aligned}$$

$$\begin{aligned} \text{Median} &= \frac{1}{2} \left[\left(\frac{n}{2}\right) \text{th} + \left(\frac{n+2}{2}\right) \text{th} \right] \\ &= \frac{1}{2} (3^{\text{rd}} \text{ item} + 4^{\text{th}} \text{ item}) (\because n = 6) \\ &= \frac{1}{2} (20 + 20) = \frac{40}{2} = 20 \end{aligned}$$

Mode = 10, 20 and 30 because they are repeated twice

$$\text{Range} = x_{\max} - x_{\min} = 30 - 10 = 20$$

For W:

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$$\text{Mean} = \frac{10 + 10 + 15 + 25 + 30 + 30}{6} = \frac{120}{6} = 20$$

$$\begin{aligned} \text{Median} &= \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} + \left(\frac{n+2}{2} \right)^{\text{th}} \right] \\ &= \frac{1}{2} (3^{\text{rd}} \text{ item} + 4^{\text{th}} \text{ item}) \quad (\because n = 6) \\ &= \frac{1}{2} (15 + 25) = \frac{1}{2} (40) = 20 \end{aligned}$$

Mode = 10 and 30 because they are repeated twice.

$$\text{Range} = x_{\max} - x_{\min} = 30 - 10 = 20$$

(b)(i) Adding 5 to each item of W, we have 15, 15, 20, 30, 35, 35

SOLUTION:

$$\text{A.M.} = \frac{15 + 15 + 20 + 30 + 35 + 35}{6} = \frac{150}{6} = 25$$

$$\begin{aligned} \text{Median} &= \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} + \left(\frac{n+2}{2} \right)^{\text{th}} \right] \\ &= \frac{1}{2} (3^{\text{rd}} \text{ item} + 4^{\text{th}} \text{ item}) \quad (\because n = 6) \\ &= \frac{1}{2} (20 + 30) = \frac{50}{2} = 25 \end{aligned}$$

Mode = 15 and 35 because they are repeated twice.

$$\text{Range} = x_{\max} - x_{\min} = 35 - 15 = 20$$

$$\begin{aligned} \text{S.D.} &= \sqrt{\frac{2x_i^2 \left(\frac{\sum x_i}{n} \right)^2}{n}} \\ &= \sqrt{\frac{(15)^2 + (15)^2 + (20)^2 + (30)^2 + (35)^2}{6} - \left(\frac{15 + 15 + 20 + 30 + 35 + 35}{6} \right)^2} \\ &= \sqrt{\frac{225 + 225 + 400 + 900 + 1225 + 1225}{6} - \left(\frac{150}{6} \right)^2} \end{aligned}$$

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$$= \sqrt{\frac{4200}{6} - (25)^2}$$

$$= \sqrt{700 - 625} = \sqrt{75} = 8.66$$

(II) Subtracting 5 from each item of W, we have 5, 5, 10, 20, 25, 25

SOLUTION:

A.M. $= \frac{5+5+10+20+25+25}{6} = \frac{90}{6} = 15$

Median $= \frac{1}{2} \left[\left(\frac{n}{2} \right)^{th} + \left(\frac{n+1}{2} \right)^{th} \right]$

$$= \frac{1}{2} (3^{rd} \text{ item} + 4^{th} \text{ item}) \quad (\because n = 6)$$

$$= \frac{1}{2} (10 + 20) = \frac{30}{2} = 15$$

Mode = 5 and 25 because they are repeated.

Range $= x_{max} - x_{min} = 25 - 5 = 20$

S.D $= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2}$

$$= \sqrt{\frac{(5)^2 + (5)^2 + (10)^2 + (20)^2 + (25)^2 + (25)^2}{6} - \left(\frac{5+5+10+20+25+25}{6} \right)^2}$$

$$= \sqrt{\frac{25+25+100+400+625+625}{6} - \left(\frac{90}{6} \right)^2}$$

$$= \sqrt{\frac{1800}{6} - (15)^2}$$

$$= \sqrt{300 - 225} = \sqrt{75} = 8.66$$

(iii) Multiplying each item of W by 4, we have 40, 40, 60, 100, 120, 120

SOLUTION:

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$$\text{A.M.} = \frac{40 + 40 + 60 + 100 + 120 + 120}{6} = \frac{480}{6} = 80$$

$$\begin{aligned} \text{Median} &= \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} + \left(\frac{n+2}{2} \right)^{\text{th}} \right] \\ &= \frac{1}{2} (3^{\text{rd}} \text{ item} + 4^{\text{th}} \text{ item}) \quad (\because n = 6) \\ &= \frac{1}{2} (60 + 100) = \frac{160}{2} = 80 \end{aligned}$$

$$\text{Mode} = 40 \text{ and } 120 \text{ because they are repeated.}$$

$$\text{Range} = x_{\max} - x_{\min} = 120 - 40 = 80$$

$$\begin{aligned} \text{S.D} &= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2} \\ &= \sqrt{\frac{(40)^2 + (40)^2 + (60)^2 + (100)^2 + (120)^2 + (120)^2}{6} - \left(\frac{40+40+60+100+120+120}{6} \right)^2} \\ &= \sqrt{\frac{1600 + 1600 + 3600 + 10000 + 14400 + 14400}{6} - \left(\frac{480}{6} \right)^2} \\ &= \sqrt{\frac{45600}{6} - (80)^2} = \sqrt{7600 - 6400} = \sqrt{1200} = 34.64 \end{aligned}$$

(c)(i) Inserting two relatively high values, i.e., 50 and 60 in the values of X, we have 10, 20, 30, 50, 60

$$\text{SOLUTION: A.M.} = \frac{10 + 20 + 30 + 50 + 60}{5} = \frac{170}{5} = 34$$

$$\text{Median} = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ item} = 3^{\text{rd}} \text{ item} = 30$$

Mode = 0 because no repetition of numbers.

$$\text{Range} = x_{\max} - x_{\min} = 60 - 10 = 50$$

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$$\begin{aligned}
 S.D &= \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} \\
 &= \sqrt{\frac{(10)^2 + (20)^2 + (30)^2 + (50)^2 + (60)^2}{5} - \left(\frac{10 + 20 + 30 + 50 + 60}{5}\right)^2} \\
 &= \sqrt{\frac{100 + 400 + 900 + 2500 + 3600}{5} - \left(\frac{170}{5}\right)^2} \\
 &= \sqrt{\frac{7500}{5} - (34)^2} \\
 &= \sqrt{1500 - 1156} = \sqrt{344} = 18.54
 \end{aligned}$$

Except mode there is a change in A.M., median, range and S.D. as increase in them.

(ii) Inserting two relatively low values, i.e., 0 and 2 in the values of X, we have 0, 2, 10, 20, 30

SOLUTION: A.M. = $\frac{0+2+10+20+30}{5} = \frac{62}{5} = 12.4$

Median = $\left(\frac{n+1}{2}\right)^{\text{th}}$ item = 3rd item = 10

Mode = 0 because no repetition of numbers.

Range = $x_{\max} - x_{\min} = 30 - 0 = 30$

$$\begin{aligned}
 S.D &= \sqrt{\frac{0^2 + 2^2 + 10^2 + 20^2 + 30^2}{5} - \left(\frac{0 + 2 + 10 + 20 + 30}{5}\right)^2} \\
 &= \sqrt{\frac{0 + 4 + 100 + 400 + 900}{5} - \left(\frac{62}{5}\right)^2} = \sqrt{\frac{1404}{5} - (12.4)^2} \\
 &= \sqrt{280.8 - 153.76} = \sqrt{127.04} = 11.27
 \end{aligned}$$

Except mode there is a change in A.M., median, range and S.D. as increase in them.

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MISCELLANEOUS EXERCISE III

Q.1. Briefly describe, why do we summarize the data?

SOLUTION : We know that the data forms the basis to draw conclusion and make decisions. Teachers guess the ability of their students on the basis of their marks, families plan to spend on health, education, clothing, rent, transport, food, etc. on the basis of their income.

Before any program is prepared, the relevant data is collected. After the collection of data, the next step is to present data in a way that provides a clear picture about it. To procedure employed to reduce and simplify the raw data is called classification and tabulation. These two procedures work together.

Q.2. Following are the weights of 30 children recorded to the nearest Kgs : 38, 64, 50, 32, 44, 25, 49, 57, 46, 58, 40, 47, 36, 48, 52, 44, 68, 26, 38, 76, 63, 19, 54, 65, 46, 73, 42, 47, 35, 53.

Make a frequency table taking size of class intervals as 6. Also draw the frequency polygon.

SOLUTION : FREQUENCY TABLE

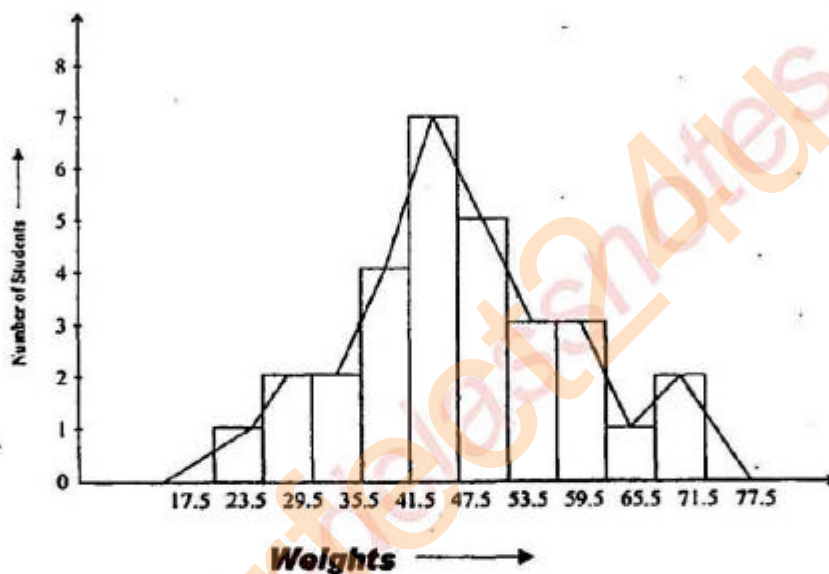
Class Intervals (Weights)	Entries	No. of Children Frequency (<i>f</i>)	Class boundaries	Mid-points
18 — 23	19	1	17.5 — 23.5	20.5
24 — 29	25, 26	2	23.5 — 29.5	26.5
30 — 35	32, 35	2	29.5 — 35.5	32.5
36 — 41	38, 40, 36, 38	4	35.5 — 41.5	38.5
42 — 47	44, 46, 47, 44, 46, 42, 47	7	41.5 — 47.5	44.5
48 — 53	50, 49, 48, 52, 53	5	47.5 — 53.5	50.5
54 — 59	57, 58, 54	3	53.5 — 59.5	56.5
60 — 65	64, 63, 65	3	59.5 — 65.5	62.5
66 — 71	68	1	65.5 — 71.5	68.5

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72 — 77	76, 73	2	71.5 — 77.5	74.5
Total		$\Sigma f = 30$		

Required Frequency Polygon



Marks	(20-24)	(25-29)	(30-34)	(35-39)	(40-44)	(45-49)
Students	25	28	32	25	13	12

With reference to the above table find the following:

1. **Upper class boundary of the 5th class.**
2. **Lower class boundary of the last class.**
3. **Mid-point of the 2nd class.**
4. **Size of the class interval.**
5. **Frequency of the 4th class.**
6. **The class interval with least frequency.**

SOLUTION :

(i) Upper class boundary of the 5th class = $44 + \frac{1}{2} = 44.5$

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(ii) Lower class boundary of the last class = $44 + \frac{1}{2} = 44.5$

(iii) Mid-point of the 2nd = $\frac{25+29}{2} = \frac{54}{2} = 27$

(iv) Size of the class interval = $29 - 24 = 5$

(v) Frequency of the 4th class = 25

(vi) The class interval with least frequency = (45 — 49)

Q4 Following distribution shows maximum load (in Kg.) supported by certain ropes. Find the mean of maximum load using "short method" and "coding method".

Max Load (Kg)	Number of ropes
193 — 197	2
198 — 202	5
203 — 207	8
208 — 212	12
213 — 217	6
218 — 222	2

SOLUTION : Shortcut Method for finding mean of maximum load

∴ Mean = $\bar{x} = A + \frac{\sum fd}{\sum f}$ where A is assumed mean and $D = x - A$

Max Load (Kg)	Mid-points (x)	Number of ropes	$D = x - A$	fD
193 — 197	195	2	10	20
198 — 202	200	5	5	25
203 — 207	205	8	0	0
208 — 212	210	12	5	60
213 — 217	215	6	10	60
218 — 222	220	2	15	30

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Total		$\Sigma f = 35$		$\Sigma f D = 105$
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Let $A = 205$

$$\therefore \bar{x} = 205 + \frac{105}{35} = 205 + 3 = 208$$

Coding Method for finding mean of maximum load

$$\therefore \text{Mean} = \bar{x} = A + \frac{\Sigma f}{\Sigma f} \times h,$$

Where $u = \frac{x-A}{h}$, h is size of class interval

Max. load (kgs)	Mid points x	No. of ropes Frequency f	$x - A$	$u = \frac{x-A}{h}$	$f u$
193 — 197	195	2	-10	-2	-4
198 — 202	200	5	-5	-1	-5
203 — 207	205	8	0	0	0
208 — 212	210	12	5	1	12
213 — 217	215	6	10	2	12
218 — 222	220	2	15	3	6
Total		$\Sigma f = 35$	—		$\Sigma f u = 21$

Let $A = 205, h = 5$

$$\therefore \bar{x} = 205 + \frac{21}{35} \times 5 = 205 + 3 = 208$$

Q5 Ahmed rolled two dice eight times. Each time the sum of dots was recorded as 8, 5, 11, 3, 6, 6, 9, 4. Find the median and the mode of the sums of dots.

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$$\begin{aligned}
 \text{SOLUTION : Median} &= \left[\left(\frac{n}{2} \right)^{\text{th}} + \left(\frac{n+2}{2} \right)^{\text{th}} \right] \\
 &= \frac{1}{2} [4^{\text{th}} + 5^{\text{th}}] \quad (\because n = 8) \\
 &= \frac{1}{2} (3 + 6) = \frac{9}{2} = 4.5
 \end{aligned}$$

Mode = 6 because 6 is repeated twice.

Q.6. For a set of 10 observations the mean is 20. On checking it is discovered that the observation 19 was incorrectly recorded where as the correct value was 23. Calculate the correct mean from the information.

SOLUTION :

$$\Rightarrow \text{Wrong mean} = \frac{\Sigma f}{n} = 20$$

$$\text{Wrong sum} = \Sigma f = 20 \times n = 20 \times 10 = 200$$

$$\text{Sum without 10} = \Sigma f - 19 = 200 - 19 = 181$$

$$\text{Sum without 19 but with 23} = 181 + 23 = 204$$

Q.7. Differentiate between discrete and continuous variables. Classify the following variables as discrete continuous:

- The number of students in a class.
- The number of tails in tossing two coins.
- The number of fours in rolling 3 dice.
- Milk produced daily by a goat.
- The life time of bulbs.

SOLUTION : Discrete variable is that variable whose value arises through counting, e.g., length, volume and weight.

- | | |
|-------------------------|-------------------------|
| (a) Discrete variable | (b) Discrete variable |
| (c) Discrete variable | (d) Continuous variable |
| (e) Continuous variable | |

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Q.8.(a) Can we find the A.M. for each of the following:

(i) $A = \{\text{Intelligent, Hardworking, Rich, Honest, Lazy}\}$

(ii) $B = \{\text{Rs.30, Palsa 50, Pounds 70, Dollars 80}\}$

(iii) $C = \{\text{2 months, 5 days, 6 years, 10 hours, 20 minutes}\}$

(iv) $D = \{\text{North, South, East, West}\}$

(b) Can we find median or mode of the above data?

SOLUTION :

(a) (i) No because A has no numbers.

(ii) Yes, after changing pounds and dollars into rupees and paisas.

(iii) Yes, after changing years, months, days and hours into minutes.

(iv) No because D has no numbers.

(b) For Median: (i) No. (ii) Yes, $\frac{1}{2} [2^{\text{nd}} \text{ item} + 3^{\text{rd}} \text{ item}]$

(iii) Yes, $\left(\frac{n+1}{2}\right)^{\text{th}} \text{ item} = 3^{\text{rd}} \text{ item} = 6 \text{ years.}$

For Mode. (i), (ii), (iii), (iv) Yes, mode = 0

Q.9.Explain the meaning of term "Dispersion". What are the types of dispersion? Describe one method of measuring dispersion.

SOLUTION :

Dispersion is an important concept which reflects the process of control tendency as a single value representative for the whole set of data.

The main of dispersion are:

1 Range

2 Variance.

3. Standard Deviation

1. Range

It is defined as the difference between the largest and the smallest observations in the data set.

2. Variance

It is a single value obtained by dividing the sum of squares of deviations taken from the arithmetic mean by the number of observations in the data set.

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3. Standard Deviation

It is defined as the positive square root of the variance.

Q10A college is six storied building. The number of classes as in each story is: 15, 12, 11, 14, 13.

(I) Find the Standard Deviation in the data.

(II) Subtract 3 from each number in the above data; then find S.D. of the new data thus obtained.

SOLUTION : (I) Standard Deviation =
$$\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{(15)^2 + (12)^2 + (11)^2 + (14)^2 + (13)^2}{5} - \left(\frac{15 + 12 + 11 + 14 + 13}{5}\right)^2}$$

$$= \sqrt{\frac{225 + 144 + 121 + 196 + 169}{5} - \left(\frac{65}{5}\right)^2}$$

$$= \sqrt{\frac{855}{5} - (13)^2}$$

$$= \sqrt{171 - 169}$$

$$= \sqrt{2} = 1.423 \text{ Ans.}$$

(II) Subtracting 3 from each number in the given data, we have the following 12, 9, 8, 11, 10

SOLUTION : S.D. =
$$\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{(12)^2 + (9)^2 + (8)^2 + (11)^2 + (10)^2}{5} - \left(\frac{12 + 9 + 8 + 11 + 10}{5}\right)^2}$$

$$= \sqrt{\frac{144 + 81 + 64 + 121 + 100}{5} - \left(\frac{50}{5}\right)^2}$$

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$$= \sqrt{\frac{510}{5} - (10)^2} = \sqrt{102 - 100} = \sqrt{2} = 1.42$$

Q11. Fiaz spends 40% of his income on food, 25% on electricity and clothing 15% on his friends and the rest on miscellaneous items. Represent the expenditure by Pie-diagram.

SOLUTION : Let a circle represent the expenditure.

Then 40% of Fiaz's income on food = 40% of expenditure circle

$$= \frac{40}{100} \times 360^\circ = 90^\circ$$

25% of Fiaz's income on electricity = 25% of expenditure circle

$$= \frac{25}{100} \times 360^\circ = 90^\circ$$

15% of Fiaz's income on his friends = 15% of expenditure circle

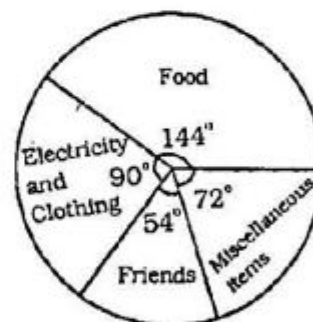
=

$$\frac{15}{100} \times 360^\circ = 54^\circ$$

The rest of Fiaz's income on miscellaneous items

$$= 360^\circ - (144^\circ + 90^\circ + 54^\circ) = 360^\circ - 288^\circ = 72^\circ$$

Required Pie Diagram



Q12. The two partners Mr. Mustafa and Mrs. Mustafa run a company. The weekly wages (In Rs.) of employees who work in the company are:

Wages (Rs)	400-600	600-800	800-1000	1000-1200	1200-1400
Employees	5	7	11	21	6

Calculate A.M. If:

(I) $D = x - 900$

(II) $D = x - 700$

(III) $u = \frac{x - 1100}{200}$

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SOLUTION : (I)

Wages (Rs)	Mid-points x	Employees Frequency (f)	$D = x - 900$	fD
400-600	500	5	-400	-2000
600-800	700	7	-200	-1400
800-1000	900	11	0	0
1000-1200	1100	21	200	4200
1200-1400	1300	6	400	2400
Total		$\Sigma f = 50$		$\Sigma fD = 3200$

$$\therefore \text{A.M.} = \bar{x} = A + \frac{\Sigma fD}{\Sigma f},$$

Where A = assumed mean

$$\text{Let } A = 900, \text{ then } \bar{x} = 900 + \frac{3200}{50} = 900 + 64 = 964$$

$$\therefore \text{A.M.} = \bar{x} = A + \frac{\Sigma fD}{\Sigma f},$$

where A = assumed mean

$$\text{Let } A = 900. \text{ Then } \bar{x} = 900 + \frac{3200}{50} = 900 + 64 = 964$$

(II)

Wages (Rs)	Mid-points x	Employees Frequency (f)	$D = x - 900$	fD
400-600	500	5	-200	-1000
600-800	700	7	0	0
800-1000	900	11	200	2200
1000-1200	1100	21	400	8400
1200-1400	1300	6	600	3600
Total		$\Sigma f = 50$		$\Sigma fD = 3200$

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$$\therefore \text{A.M.} = \bar{x} = A + \frac{\sum fd}{\sum f}$$

where A = assumed mean

$$\text{Let } A = 900. \text{ Then } \bar{x} = 700 + \frac{13200}{50} = 700 + 264 = 964$$

$$(III) \therefore u = \frac{x - A}{h} = \frac{x - 1100}{200}$$

$$\therefore A = 1100, h = 200$$

Wages (Rs)	Mid points x	Employees Frequency (f)	x-1100	$u = \frac{x-1100}{200}$	fu
400-600	500	5	-600	-3	-15
600-800	700	7	-400	-2	-14
800-1000	900	11	-200	-1	-11
1000-1200	1100	21	0	0	0
1200-1400	1300	6	200	1	6
Total	—	$\sum f = 50$			$\sum fu = -34$

$$\text{A.M.} = \bar{x} = A + \frac{\sum fu}{\sum f} \times h,$$

$$= 1100 + \frac{-34}{50} \times 200 = 1100 - 136 = 964$$

Q.13. Four friends Alia, Nazia, Rani and Saba rolled two dice independently six times. Each time the product of their numbers was recorded as given below:

Alia: 1, 3, 9, 36, 16, 36 ;

Nazia: 4, 6, 16, 25, 3, 9

Rani: 4, 2, 6, 16, 4, 5;

Saba: 1, 1, 3, 36, 2, 9

On the Average who got larger product?

SOLUTION :

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$$\text{Average of product for Alia} = \frac{1+3+9+36+16+36}{6} = \frac{101}{6} = 16.83$$

$$\text{Average of product of Nazia} = \frac{4+6+16+25+3+9}{6} = \frac{63}{6} = 10.5$$

$$\text{Average of product for Rani} = \frac{4+2+6+16+4+5}{6} = \frac{37}{6} = 6.16$$

$$\text{Average of product for Saba} = \frac{1+1+3+36+2+9}{6} = \frac{52}{6} = 8.66$$

On the average Alia got larger product.

Q.14. Find the median of a distribution with $\bar{x} = 34.6$ and mode = 30.8. verify that the median is between A.M. and mode.

(Hint: Mode = 3 Median 2 Mean)

SOLUTION : $\therefore$ A.M. = $\bar{x} = 34.6$; Mode = 30.8

We have

$$\text{Mode} = 3 \text{ median} - 2 \text{ Mean}$$

$$30.8 = 3 \text{ Median} - 2 \times 34.6$$

$$\text{Median} = \frac{30.8 + 69.2}{3} = \frac{100}{3} = 33.33$$

$$\therefore 34.6 > 33.33 > 30.8$$

$$\text{i.e., Mean} > \text{Median} > \text{mode}$$

Q.15. Find x_1 and x_2 if $n = 2$ and :

(i) Variance = 2, $\bar{x} = 7$; (ii) Range = 2, $\bar{x} = 7$;

(iii) $= 5\sqrt{2}$, $\bar{x} = 23$

SOLUTION : (i) $\therefore \bar{x} = 7 \Rightarrow \frac{x_1 + x_2}{2} = 7 \Rightarrow x_2 = 14 - x_1$

$$\text{We have Variance} = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\text{i.e. } 2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2}{2}$$

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$$\Rightarrow 2 = \frac{(x_1 - 7)^2 + (14 - x_1 - 7)^2}{2}$$

$$\Rightarrow 2 = \frac{(x_1 - 7)^2 + (7 - x_1)^2}{2}$$

$$\Rightarrow 2 = \frac{2(x_1 - 7)^2}{2}$$

$$\Rightarrow 2 = x_1^2 - 14x_1 + 49$$

$$\Rightarrow x_1^2 - 14x_1 + 47 = 0$$

$$\therefore x_1 = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(47)}}{2(1)} = \frac{14 \pm \sqrt{196 - 188}}{2}$$

$$\therefore = \frac{14 \pm \sqrt{8}}{2} = \frac{14 \pm 2\sqrt{2}}{2} = 7 \pm \sqrt{2}$$

Equation (1) becomes $x_2 = 14 - (\pm \sqrt{2}) = 7 \mp \sqrt{2}$

Thus, $x_1 = 7 - \sqrt{2}$; $x_2 = 7 + \sqrt{2}$ or $x_1 = 7 + \sqrt{2}$; $x_2 = 7 - \sqrt{2}$

(ii) Range = 2

If $x_1 > x_2$, then $x_1 = x_2 \Rightarrow x_1 = 2 + x_2$ (1)

$$\therefore \bar{x} = 7 \Rightarrow \frac{x_1 + x_2}{2} = 7 \Rightarrow x_1 = 14 - x_2$$
 (2)

Comparing (1) and (2), we have $2 + x_2 = 14 - x_2$

$$\Rightarrow 2x_2 = 12 \quad \therefore x_2 = 6 \quad ; \quad x_1 = 14 - 6 = 8$$

$$(iii) \therefore \bar{x} = 23 \Rightarrow \frac{x_1 - x_2}{2} = 23 \Rightarrow x_2 = 46 - x_1$$

$$\therefore S.D. = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2}{2}}$$

$$5\sqrt{2} = \sqrt{\frac{(x_1 - 23)^2 + (46 - x_1 - 23)^2}{2}}$$

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$$5\sqrt{2} = \sqrt{\frac{(x_1 - 23)^2 + (23 - x_1)^2}{2}}$$

$$5\sqrt{2} = \sqrt{\frac{2(x_1 - 23)^2}{2}}$$

$$x_1 = 5\sqrt{2} + 23$$

$$x_2 = 46 - (5\sqrt{2} + 23) = 23 - 5\sqrt{2}$$

$$\text{Thus, } x_1 = 26 + 5\sqrt{2} ; x_2 = 23 - 5\sqrt{2}$$

Q16. Calculate the Variance and S.D. of natural numbers from:

(i) 1 to 11

(ii) 56 to 62

(iii) 1 to 22

SOLUTION :

$$\begin{aligned} \text{(i) } \therefore \text{ Variance} &= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2 \\ &= \frac{(1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2 + (6)^2 + (7)^2 + (8)^2 + (9)^2 + (10)^2 + (11)^2}{11} \\ &= \left(\frac{1+2+3+4+5+6+7+8+9+10+11}{11} \right)^2 \\ &= \frac{506}{11} - \left(\frac{66}{11} \right)^2 = 46 - 36 = 10 \end{aligned}$$

$$\therefore \text{ S.D.} = \sqrt{10}$$

$$\begin{aligned} \text{(ii) } \text{ Variance} &= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2 \\ &= \frac{(56)^2 + (57)^2 + (58)^2 + (59)^2 + (60)^2 + (61)^2 + (62)^2}{7} \\ &= \left(\frac{56+57+58+59+60+61+62}{7} \right)^2 \end{aligned}$$

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$$= \frac{24395}{7} - \left(\frac{413}{7}\right)^2 = 3485 - 3481 = 4$$

$$\text{S.D.} = \sqrt{4} = 2$$

$$\begin{aligned} \text{(III) } \therefore \text{ Variance} &= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \\ &= \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 + 11^2}{22} \\ &\quad + \frac{12^2 + 13^2 + 14^2 + 15^2 + 16^2 + 17^2 + 18^2 + 19^2 + 20^2 + 21^2 + 22^2}{22} \\ &= \left(\frac{1+2+3+4+5+6+7+8+9+10+11+12+13}{22} \right. \\ &\quad \left. + \frac{14+15+16+17+18+19+20+21+22}{22} \right)^2 \\ &= \frac{3784}{22} - \left(\frac{253}{22}\right)^2 = 172.45 - 132.25 = 40.5 \end{aligned}$$

$$\therefore \text{S.D.} = \sqrt{40.5} = 6.34$$

Q.17. What can be said about a sample of observations whose.

- (i) S.D. = 0 (ii) Range = 0 (iii) Mode = 0
(iv) Median = 0 (v) A.M. = 0?

Solution :

- (i) Sample of same observations.
(ii) Sample of only one observation
(iii) Sample of observations without repetition.
(iv) Sample of odd observation with zero in middle.
(v) Sample of zeros as observation or
Sample of observation with opposite signs but equal magnitude.

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