Chap 3 VARIATION

INTRODUCTION CHAPTER

1. RATIO:

Ratio is the relation of one quantity to another of the same kind. The ratio of a to b is usually written as a : b. The quantities a and b are called "terms" of the ratio. The first term is called the "antecedent", the second term the "consequent".

The ratio between two quantities a and b is denoted as:

$$a : b \quad \text{and} \quad a : b. = \frac{a}{b} = a \div b$$

In a : b , a and b are respectively the first and second terms of the ratio

Similarly $a^3 : b^3$ is called the "triplicate ratio" of a: b. Also $a^{\frac{1}{2}} : b^{\frac{1}{2}}$ is called the sub-duplicate ratio of a : b.

2. PROPORTION:

We know that the relation of quantity between two ratios a : b and c : d is called a proportion four the four quantities a, b , c and d.

$$\frac{a}{b} = \frac{c}{d}$$

or $a : b = c : d$ is a proportion and in general, it is written as

Continued proportion

If a : b = b : c, then the quantities a, b, c are said to be in continued proportion. if a : b :: b: c then $b^2 = ac$.

3. VARIATIONS

In our daily life we often come across quantities which keep their values changing i.e., they do not have a fixed or constant value but are different at different times or locations. For example, temperature and population of a country. Such changes in values of these quantities are called variations.

Variation are two types. (i) Direct variation (ii) Inverse variation

I. Direct Variation

The quantities x and y are said to be in direct variation if increase (or decrease) in the value of x leads to corresponding increase (or decrease) in the value of y.

The expression "y" varies directly as "x" is written as $y \alpha x$.

So, $y = kx$, where k is constant.

We often see the examples of direct variation in our daily life. For example following quantities vary directly:

(i) Work and days
(ii) Temperature and volume
(iii) Weight and mass of a body
(iv) Circumference and radius of a circle

II. Inverse variation:

If two variables are related to each other in such a way that increase in one causes decrease in the other and vice versa, then the variables are inverse variation to each other.

The expression "y" varies inversely as " x" is written as $y \alpha \frac{1}{x}$.

So, $y = \frac{k}{x}$, where k is constant.

Properties of proportion

1. Invertendo property of proportion. If a : b :: c: d then b : a :: d: c
2. Alternado property of proportion. If a : b :: c: d then a: c :: b : d
3. Componendo property of proportion. If a: b ::c: d then a +b : b =c +d:d
4. Dividendo property of proportion. If a: b:: c: d, then a–b: b = c –d: d
5. Companendo and dividendo property of proportion.

If a : b :: c : d then a + b : a–b :: c +d : c–d

4. FINDING THE FOURTH PROPORTIONAL.

a : b :: c : d then d is called the fourth proportional i.e. $\frac{a}{b} = \frac{c}{d}$

ad = bc (by cross multiplication). in the fourth proportion = $d = {bc}/{a}$

5. FINDING THE FOURTH PROPORTIONAL.

a : b :: b : c then c Is called the third proportional i.e. $\frac{a}{b} = \frac{b}{c}$

$ac = b^2$ (by cross multiplication). in the third proportion = $c = {b^2}/{a}$

MULTIPLE CHOICE (MCQ's)

(i) If a : b :: c : d , then which one statement is true?

(a) $a \div b$ (b) $\frac{1}{a} = c$ © $a = c$ (d) $a = \frac{1}{c}$

(ii) If $\frac{x}{y} = \frac{y}{z} = k$, then $x = ?$

(a) zk^2 b) yk^2 © z^2k (d) y^2k

(iii) If $x : 5 = 4 : 2$, then

(a) $x = 5$ (b) $x = 2$ © $x = 10$ (d) $x = 7$

(iv) $y^2 k$, then

(a) $y^2 = \frac{1}{x}$ (b) $y \propto \frac{1}{y^2}$ © $x = \frac{1}{y^2}$ (d) $x \propto \frac{1}{y^2}$

(v) If $x \propto \frac{1}{y}$, then

(a) $x = \frac{y}{k}$ (b) $x = \frac{k}{y}$ © $x = \frac{1}{y}$ (d) $x = \frac{1}{k}$

(vi) If © $p \propto q^3$, then $p = ?$

(a) $p = kq^3$ (b) $p = \frac{1}{kq^3}$ © $p = \frac{q^3}{k}$ (d) $p = \frac{k}{q^3}$

(vii) If $x : y :: u : w$, then

(a) $\frac{w}{u}$ (b) w.u (c) $\frac{u}{w}$ (d) $w^2.u$

(viii) What is mean proportional of 4 and 16? ------------

(a) 1/8 (b) ± 8 (c) ± 4 (d) 4

(ix) If $\propto$ t, then,

(a) $S = kt$ (b) $St = k$ (c) $St = 1$ (d) $S = t$

(x) In $6 : 4 :: 3 : x$, What is the value of x ?

(a) 2 (b) 3 (c) 4 (d) 5

(xi) If $y = \frac{1}{p^3}$, then $yp^3 = ?$

(a) 5 (b) 8 (c) k (d) $\frac{1}{k}$

(xii) If $\frac{x}{y} = \frac{y}{z} = k$, then $x = ?$

(a) k^2 (b) $2k^2$ (c) $\frac{1}{k^2}$ (d) $\frac{1}{z^2 k}$

(xiii) If $\frac{a}{b} = \frac{c}{d}$, then $ac = ?$

(a) $\frac{1}{c^2}$ (b) a^2 (c) b^2 (d) c^2

(xiv) If [illegible], then $x = ?$

(a) ± 4 (b) ± 8 (c) ± 5 (d) 9

(xv) In $16 : x :: x : 9$, then $x = ?$

(a) ± 12 (b) ± 25 (c) 25 (d) 12

(xvi) Variation are of ________ types.

(a) two (b) three (c) four (d) five

EXERCISE 3.1

Q.1. Which ratio is greater:

(i) 5 : 6 or 11: 12

SOLUTION:

$$5:6=\frac{5}{6}=\frac{5\times2}{6\times2}=\frac{10}{12} \quad \text{or} \quad 11:12=\frac{11}{12}$$

$$\therefore \quad \frac{10}{12}<\frac{11}{12} \quad \Rightarrow \quad \frac{5}{6}<\frac{11}{12}$$

Thus, 11 : 12 is greater ratio.

(ii) 22 : 27 or 32 : 45

SOLUTION:

$$22:27=\frac{22}{27}=\frac{22\times5}{27\times5}=\frac{110}{135} \quad \text{or} \quad 32:45=\frac{32}{45}=\frac{32\times3}{45\times3}=\frac{96}{135}$$

$$\therefore \quad \frac{110}{135}>\frac{96}{135} \quad \Rightarrow \quad \frac{22}{27}>\frac{32}{45}$$

Thus, 22 : 27 is greater ratio.

Q.2. Find the ratio compounded of:

(i) 3: 5, 7 :9 and 15 : 28

SOLUTION: Required ratio $=\frac{3}{5}\times\frac{7}{9}\times\frac{15}{28}=\frac{3\times7\times15}{5\times9\times28}$

$=\frac{1}{4}=1:4$ Ans.

(ii) $(a+x):(a-x)$, $(a^2+x^2):(a+x)^2$ and $(a^2-x^2)^2:a^4-x^4$

SOLUTION: Required ratio is:

$$\frac{a+x}{a-x}\times\frac{a^2+x^2}{(a+x)^2}\times\frac{(a^2-x^2)^2}{a^4-x^4}=\frac{(a+x)(a^2+x^2)(a^2-x^2)^2}{(a-x)(a+x)^2(a^2-x^2)(a^2+x^2)}$$

$$=\frac{a^2-x^2}{(a-x)(a+x)} \qquad =\frac{a^2-x^2}{a^2-x^2}$$

$=1:1$ Ans.

(iii) 2a : 3b and the duplicate ratio of $9b^2 : ab$

SOLUTION: Duplicate ratio of $9b^2 : ab$ is $(9b^2)^2 : (ab)^2$

$$\text{Required ratio} = \frac{2a}{3b} \times \frac{(9b^2)^2}{(ab)^2} = \frac{2a \times 81b^4}{3b \times a^2 b^2}$$

$$= \frac{2 \times 27b}{a} = 54\ b : a \quad \text{Ans.}$$

Q.3. If x +7 :2 (x+14) is the duplicate ratio of 5 : 8, find x.

SOLUTION: Duplicate ratio of 5 : 8 $= 5^2 : 8^2$

According to the given condition,

$(x+7) : 2(x+14) = 5^2 : 8^2$

$$\Rightarrow \frac{x+7}{2(x+14)} = \frac{25}{64} \Rightarrow 64x + 448 = 50x + 700$$

$$\Rightarrow 14x = 252 \quad \therefore \quad x = 18 \quad \text{Ans.}$$

Q.4. Find two numbers in the ratio of 7 : 12 so that the greater number exceeds the smaller by 275.

SOLUTION: Suppose x be the smaller number.

Then x+275 is the greater number.

According to the given condition,

$$\Rightarrow \frac{x}{x+275} = \frac{7}{12} \Rightarrow 12x = 7x + 1925$$

$$\Rightarrow 5x = 1925 \Rightarrow x = 385$$

$$\therefore x + 275 = 385 + 275 = 660$$

Thus, required numbers are 385 and 660. Ans.

Q.5. What number must be added to each term of the ratio 5 : 27 to make it equal to 1 : 3?

SOLUTION: Suppose x be the required number.

Then $\frac{5+x}{27+x} = \frac{1}{3} \Rightarrow 15 + 3x = 27 + x$

$\Rightarrow 2x = 12$

$\therefore x = 6$ Ans.

Q.6. Two numbers are in the same ratio 7 : 8 and their sum is 105. Find the numbers.

SOLUTION: Suppose x and y be the required numbers.

Then $x : y = 7 : 8$ and $x + y = 105$

$\Rightarrow \quad y = \frac{8}{7}x$ ---------- (1) $\quad y = 105 - x$ ---------- (2)

Comparing (1) and (2), we get

$\frac{8}{7}x = 105 - x \quad \Rightarrow \quad 8x = 7(105 - x)$

$\Rightarrow \quad 8x = 735 - 7x \quad \Rightarrow \quad 15x = 735$

$\therefore \quad x = 49$

Substituting $x = 49$ in equation (2), we have $y = 105 - 49 = 56$

Thus, the required numbers are 49 and 56. **Ans.**

Q.7. Two number are in the ratio 13 : 11 and their difference is 12, find the numbers.

SOLUTION: Suppose x and y be the required numbers such that x > y.

Then $x : y = 13 : 11$ and $x - y = 12$

$\Rightarrow \quad y = \frac{11}{13}x$ ----------(1) $\quad \Rightarrow \quad y = x - 12$ ----------(2)

Comparing equation (1) and equation (2), we get

$\frac{11}{13}x = x - 12$

$\Rightarrow \quad 11x = 13x - 156 \quad \therefore \quad x = 78$

Substituting $x = 78$ in equation (2),

$y = 78 - 12 = 66$

Thus, the required numbers are 78 and 66.

Q.8. Two numbers are in the ratio 5:3. If 11 be added to each, then the sums are in the ratio is 18:13, find the numbers.

SOLUTION: Suppose x and y be the required numbers.

Then $x : y = 5 : 3$ and $x + 11 : y + 11 = 18 : 13$

$\Rightarrow \quad y = \frac{3}{5}x$ ---------- (1) , $\quad \frac{x+11}{y+11} = \frac{18}{13}$ ---------- (2)

Substituting $y = \frac{3}{5}x$ in (2), we get $\quad \frac{x+11}{\frac{5}{3}x+11} = \frac{18}{13}$

$\Rightarrow \quad 13(x+11) = 18\left(\frac{3}{5}x+11\right) \quad \Rightarrow \quad 65x + 715 = 54 + 990$

$\Rightarrow \quad 11x = 275 \quad \therefore \quad x = 25$

Substituting $x = 25$ in (1), we get $y = \frac{3}{5} \times 25 = 3 \times 5 = 15$

Thus, the required numbers are 25 and 15.

Q.9. For what value of a will the ratio 23 +a: 19+a be equal to 54

SOLUTION: Suppose $\quad 23 + a : 19 + a = 5 : 4$

$\Rightarrow \quad \frac{23+a}{19+a} = \frac{5}{4} \quad \Rightarrow \quad 92 + 4a = 95 + 51$

$\therefore \quad a = -3$ **Ans.**

Q.10. Find the fourth proportional to: (i) 3, 5, 12

SOLUTION: Suppose x be fourth proportional.

Then $\quad 3:5::12:x \quad \Rightarrow \quad 3x = 60$

$\therefore \quad x = 20$ **Ans.**

(ii) 8, 4, 2

SOLUTION: Suppose x be the fourth proportional.

Then $\quad 8x = 8 \quad \therefore \quad x = 1$ **Ans.**

Q.11. Find the third proportional to: (i) 6, 18

SOLUTION: Suppose x be third proportional.

Then, $6 : 8 :: 18 : x \quad \Rightarrow \quad 6x = 18 \times 8$

$\therefore \quad x = 54$ **Ans.**

(ii) 3, 6

SOLUTION: Suppose x be the third proportional.

Then $3:6::6:x \Rightarrow 3x = 6\times 6$

$\therefore \quad x = 12$ **Ans.**

Q.12. Find the mean proportional between: (i) $\frac{1}{3}, 27$

SOLUTION: Suppose x be the mean proportional.

Then $\frac{1}{3} : x :: x : 27 \Rightarrow x^2 = \frac{1}{3}\times 27$

$\Rightarrow x^2 = 9 \quad \therefore \quad x = 3$

(ii) 14, 56

SOLUTION: Let x be the mean proportional.

Then $14 : x :: x : 56 \Rightarrow x^2 = 14\times 56$

$\Rightarrow x^2 = 784 \quad \therefore \quad x = 28$ **Ans.**

Q.13. Find four numbers which are proportional such that their sum is 40, the ratio of the third to the fourth is $3:5$ and the difference of the second and fourth is 5.

SOLUTION: Suppose a, b, c and d be the required four numbers.

Then $a : b = c : d$

$\Rightarrow \frac{a}{c} = \frac{c}{d} \Rightarrow a = \frac{bc}{d}$ --------- (1)

and $a + b + c + d = 40$ --------- (2)

$c : d = 3 : 5 \Rightarrow c = \frac{3}{5}d$ ---------(3)

$b - d = 5 \Rightarrow d = b - 5$ ---------(4)

Substituting $d = b - 5$ in (2), we get

$$c = \frac{3}{5}(b-5) \quad \text{--------- (5)}$$

Substituting $d = b - 5$, $c = \frac{3}{5}(b-5)$ in (1), we get

$$a = \frac{b\frac{3}{5}(b-5)}{(b-5)} \Rightarrow a = \frac{3}{5}b \quad \text{--------- (6)}$$

subtracting $a=\frac{3}{5}b, c=\frac{3}{5}(b-5), d=(b-5)$ in equation (2).

$\frac{3}{5}b+b+\frac{3}{5}(b-5)+(b-5)=40 \quad \Rightarrow \quad \frac{3b+5b+3b-15+5b-25}{5}=40$

$\Rightarrow \quad 16b-40=200 \quad \therefore \quad b=\frac{240}{16}=15$

Subtracting b = 15 in equations (3), (4) and (5), we get

$d=15-5=10$

$c=\frac{3}{5}(15-5)=\frac{3}{5}\times 10=6$

$a=\frac{3}{5}\times 15=3\times 3=9$

Thus, the required numbers are 9,15,6 and 10.

Q.14. Find four numbers which are proportional such that the product of the extremes is 200 and the ratio of the second to the first is 2:1 and of the second to the fourth is 1:4.

SOLUTION: Suppose a, b, c, and d be the required four numbers.

Then $a:b::c:d \Rightarrow a=\frac{bc}{d}$ ---------(1)

$ad+200 \Rightarrow d=\frac{200}{a}$ --------- (2)

$b:a=2:1 \Rightarrow b=2a$ --------- (3)

$b:d=1:4 \Rightarrow d=4b$ --------- (4)

Substituting b =2a in (4), we get

$d=4\times 2a \quad \Rightarrow \quad d=8a$ --------- (5)

comparing equations (2) and (3), we get

$\frac{200}{a}=8a \quad \Rightarrow \quad a^2=25 \quad \therefore \quad a=5$

substituting a = 5 in equations (3) and (5), we get

$b=2\times 5=10$ and $d=8\times 5=40$

Substituting a =5, b =10, d = 40 in equation (1), we get

$5 = \frac{10c}{40} \quad \Rightarrow \quad c = 20$ **Ans.**

Thus, the required four proportional are 5,10,20 and 40.

Q.15. Find the numbers in continued proportion such that their sum is 13 and product is 27.

SOLUTION: Suppose a, b and c be three numbers in continued proportion. Then $a:b::b:c \Rightarrow b^2 = ac$ --------- (1)

$a+b+c=13$ ---------(2)

$abc = 27 \Rightarrow ac = \frac{27}{b}$ --------- (3)

substituting $ac = \frac{27}{b}$ in (1), we get

$$b^2 = \frac{27}{b} \Rightarrow b^3 = 27 \Rightarrow b = 3$$

Substituting b =3 in (1), we get

$$9 = ac \Rightarrow c = \frac{9}{a} \quad \text{---------} \quad (4)$$

substituting b= 3 and $c = \frac{9}{a}$ in (2) we get

$$a+3+\frac{9}{a} = 13 \quad \Rightarrow \quad a^2+3a+9 = 13a$$

$$\Rightarrow \quad a^2-10a+9=13a \quad \Rightarrow \quad a^2-10a+9=0$$

$$\Rightarrow \quad (a-9)(a-1)=0 \quad \therefore \quad a = 9, 1$$

Substituting a =9, 1 in (4), we get $c = \frac{9}{9} = 1, \quad c = \frac{9}{1} = 9$

Thus, the required numbers are: 9, 3,1 or 1, 3, 9

Q.16. If x varies directly as y and x = 9 y when y =15, find x when y =10.

SOLUTION: If x varies directly as y, then $a \alpha y$

$\Rightarrow \quad x = ky$ --------- (1) $\quad \therefore \quad x = 9, \quad y = 15$

$\therefore \quad 9 = 15k$

$\Rightarrow \quad k = \frac{9}{15} = \frac{3}{5}$

substituting $k = \frac{3}{5}$ and $y = 10$ in (1), we get $\quad x = \frac{3}{5} \times 10 = 6$

Q.17. If y varies inversely as x and x =7 when y =3, find x when $y = 2\frac{1}{2}$.

SOLUTION: If y varies inversely as x, then $y \;\alpha\; \frac{1}{x}$

$\Rightarrow \quad y = \frac{k}{x}$ ---------- (1) $\qquad \therefore \quad x = 7, \;\; y = 3$

$3 = \frac{k}{7} \quad \Rightarrow \quad k = 21$

Substituting $k = 21, y = 2\frac{1}{3} = \frac{7}{3}$ in equation (1),

we get $\quad \frac{7}{3} = \frac{21}{x} \quad \Rightarrow \quad x = 21 \times \frac{3}{7} = 9$

18. A α B show that B α A:

(i) If A α B and B α C, then show that A α C.

(ii) If A α B and B α C, show that $A+B \,\alpha\, C$, $A-B \alpha C$, and $\sqrt{AB} \alpha C$.

SOLUTION: If A α B, then A= kb, where $\frac{1}{k}$ is a constant. B α A

(i) If A α B and B α C, then, $A = k_1 B$ and $B = k_2 C$

$A = k_1 k_2 C$,

Where $k_1 k_2$ is also a constant. $A \,\alpha\, C$

(ii) $A + B = K_1B + K_2S = K_1K_2C + K_2C$

$= (K_1K_2 + K_2)C$,

Where $K_1K_2 + K_2$ is a constant.

$A + B \,\alpha\, C$

$A - B = K_1B - K_2C = K_1K_2 = (K_1K_2 - K_2)C$

Where $K_1 K_2 -$ is a constant. $\quad \therefore \quad \sqrt{AB} \alpha C$

111

Q.19. $Y = A + B$, when $A \propto x$ and $B \propto x^2$ and when $x = 1$, $y=4$; when $x = 2$, $y = 10$, determine the relation between x and y, find when $= 3$.

SOLUTION: If $A \propto x$ and $B \propto x^2$, then $A = k_1x$; Bk_2x^2

Now $y = A + B = k_1x + k_2x^2$ $\quad \because \quad x = 1, \quad y = 4$

$\therefore \quad 4 = k_1 + k_2$

$\Rightarrow \quad k_2 = 4 - k_1 \qquad$ ---------(1) $\quad \because \quad x = 2, \quad y = 10$

$10 = 2k_1 + 4k^2 \qquad$ --------- (2)

subtracting $k_2 = 4 - k_1$ in (2), we get

$10 = 2k_1 + 4(k - k_1)$

$10 = 2k_1 + 16 - 4k_1$

$2k_1 = 6$

$\therefore \quad k_1 = 3$

Equation (1) becomes

$k_2 = 4 - 3 = 1$

Substituting $k_1 = 3$, $k_2 = 1$ in $y = k_1x + k_2x_2$, we get

$y = 3x + x^2$

which is the required relation.

If $x = 3$, then

$y = 3(3) + (3)^2 = 9 + 9 = 18$

EXERCISE 3.2

Q.1. If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$ **and** $a+b+c\neq 0$ **prove that** $a=b=c$.

SOLUTION: *By the corollary of Theorem 3.1,*

$$\text{Each fraction} = \frac{a+b+c}{(b+c)+(c+a)+(a+b)}$$

$$=\frac{a+b+c}{2(a+b+c)}, a+b+c\neq 0$$

$$=\frac{1}{2}$$

Now $\quad \frac{a}{b+c}=\frac{1}{2};\quad \frac{b}{c+a}=\frac{1}{2};\quad \frac{c}{a+b}=\frac{1}{2}$

$\Rightarrow \quad 2a=b+c;\quad 2b=c+a;\quad 2c=a+b$

$\Rightarrow \quad 3a=a+b+c;\quad 3b=a+b+c;\quad 3c=a+b+c$

$\Rightarrow \quad a=\frac{a+b+c}{3};\quad b=\frac{a+b+c}{3};\quad c=\frac{a+b+c}{3}$

$\therefore \quad a=b=c$

Q.2. If $\frac{a+b}{x+y}=\frac{b+c}{y+z}=\frac{c+a}{z+x}$ **prove that each of them is equal** $\frac{a+b+c}{x+y+z}$.

SOLUTION: *By the corollary of Theorem 3.1,*

$$\text{Each fraction} = \frac{(a+b)+(b+c)+(c+a)}{(x+y)+(y+z)+(z+x)}$$

$$=\frac{2(a+b+c)}{2(x+y+z)}$$

$$=\frac{a+b+c}{x+y+z}$$

Q.3. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, show that $(a^2+c^2+e^2)(b^2+d^2+f^2)=(ab+cd+ef)^2$

SOLUTION: Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ Then $a=bk$, $c=dk$, $e=fk$

Now L.H.S $=(a^2+c^2+e^2)(b^2+d^2+f^2)$

$=(b^2k^2+d^2k^2+f^2k^2)(b^2+d^2+f^2)$

$=k^2(b^2+d^2+f^2)(b^2+d^2+f^2)$

$=k^2(b^2+d^2+f^2)^2$

$=(b^2k+d^2k+f^2k)^2$

$=(ab+cd+ef)^2$ $=$ R.H.S.

Q.4. If $\frac{x}{3x-y-z}=\frac{y}{3y-z-x}=\frac{z}{3y-x-y}$, then each of these ratio is equal to 1, if $x+y+z\neq 0$.

SOLUTION: By the corollary of Theorem 3.1,

Each fraction $=\frac{x+y+z}{(3x-y-z)+(3y-z-x)+(3z-x-y)}$

$=\frac{x+y+z}{x+y+z}$

$=1,$ $\Rightarrow$ $x+y+z\neq 0.$

Q.5. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, prove that $\frac{a^4b^2+a^2e^2-e^4f}{b^6+b^2f^2-f^5}=\frac{a^4}{b^4}$

SOLUTION: Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$ Then $a=bk$, $c=dk$, $e=fk$

Now L.H.S $=\frac{a^4b^2+a^2e^2-e^4f}{b^6+b^2f^2-f^5}=\frac{b^4k^4b^2+b^2k^2f^2k^2-f^4k^4f}{b^6+b^2f^2-f^5}$

$$=\frac{k^4(b^6+b^2f^2+f^5)}{b^6+b^2f^2-f^5}=k^4$$

$$=\left(\frac{a}{b}\right)^4=\frac{a^4}{b^4}= R.H.S.$$

Q.6. If $x:y=a:b-a$, prove that: $x^2+xy+y^2:a^2-ab+b^2=x^2:a^2$

SOLUTION: Let $\frac{x}{y}=\frac{a}{b-a}=k \Rightarrow$ Then $\frac{x}{y}=k; \frac{a}{b-a}=k$

$$\Rightarrow \quad y=\frac{x}{k}; \quad b=\frac{(1+k)}{k}a$$

Now L.H.S. $=\dfrac{x^2+xy+y^2}{a^2+ab+b^2}$

$$=\frac{x^2+x.\frac{x}{k}+\frac{x^2}{k^2}}{a^2-a.\frac{(1+k)}{k}.a+\frac{(1+k)^2}{k^2}.a^2}$$

$$=\frac{x^2\left(1+\frac{1}{k}+\frac{1}{k^2}\right)}{a^2\left(1-\frac{1+k}{k}+\frac{(1+k)^2}{k^2}\right)}$$

$$=\frac{x^2}{a^2}.\frac{\frac{k^2+k+1}{k^2}}{\frac{k^2-k(1+k)+(1+k)^2}{k^2}}$$

$$=\frac{x^2}{a^2}.\frac{k^2+x+1}{k^2-k-k^2+1+2k+k^2}=\frac{x^2}{a^2}.\frac{k^2+k+1}{k^2+k+1}$$

$$=\frac{x^2}{a^2}=R.H.S.$$

Q.7. $\frac{a}{y+z}=\frac{b}{z+x}=\frac{c}{x+y}$, **prove that:** $\frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$.

SOLUTION: Let $\frac{a}{y+z}=\frac{b}{z+x}=\frac{c}{x+y}=k.$

Then $\frac{a}{y+z}=k\ ;\ \frac{b}{z+x}=k\ ;\ \frac{c}{x+y}=k$

$\Rightarrow\quad y+z=\frac{a}{k}\ ;\ z+x=\frac{b}{k}\ ;\quad x+y=\frac{c}{k}$

$\Rightarrow\quad y=\frac{a}{k}-z;\ z=\frac{b}{k}-x;\quad x=\frac{c}{k}-y$

$\Rightarrow\quad y=\frac{a}{k}-\left(\frac{b}{k}-x\right);z=\frac{b}{x}-\left(\frac{c}{k}-y\right);x=\frac{c}{k}-\left(\frac{a}{k}-z\right)$

$\Rightarrow\quad y=\frac{a}{k}-\left\{\frac{b}{k}-\left(\frac{c}{k}-y\right)\right\};\ z=\frac{b}{k}-\left\{\frac{c}{k}-\left(\frac{a}{k}-z\right)\right\};\ x=\frac{c}{k}-\left\{\frac{a}{k}-\left(\frac{b}{k}-x\right)\right\}$

$\Rightarrow\quad y=\frac{a}{k}-\frac{b}{k}+\frac{c}{k}-y\ ;\ z=\frac{b}{k}-\frac{c}{k}+\frac{a}{k}-z\ ;\ x=\frac{c}{k}-\frac{a}{k}+\frac{b}{k}-x$

$\Rightarrow\quad 2y=\frac{a-b+c}{k}\ ;\ 2z=\frac{b-c+a}{k}\ ;\ 2x=\frac{c-a+b}{k}$

$\Rightarrow\quad \frac{y}{c+a-b}=\frac{1}{2k};\ \frac{z}{a+b-c}=\frac{1}{2k};\ \frac{x}{b+c-a}=\frac{1}{2k}$

$\therefore\quad \frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}=\frac{1}{2k}$

$$\Rightarrow \quad \frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$$

EXERCISE 3.3

If $a:b::c:d$, show that

Q.1. $c^2:d^2=(a^2+c^2):(b^2+d^2)$

Solution: $\because \quad \frac{a}{b}=\frac{c}{d} \quad \Rightarrow \quad \frac{a}{c}=\frac{b}{d}$ *(By alternendo)*

$\Rightarrow \quad \frac{a^2}{c^2}=\frac{b^2}{d^2} \quad \Rightarrow \quad \frac{a^2+c^2}{c^2}=\frac{b^2+d^2}{d^2}$ *(By componendo)*

$\Rightarrow \quad \frac{c^2}{a^2+c^2}=\frac{d^2}{b^2+d^2}$ *(By invertendo)*

$\therefore \quad \frac{c^2}{d^2}=\frac{a^2+c^2}{b^2+d^2}$ *(By alternendo)*

$$c^2:d^2=a^2+c^2:b^2+d^2$$

Q.2. $ac:bd=(a^2+c^2):(b^2+d^2)$

Solution: $\because \quad \frac{a}{b}=\frac{c}{d} \quad \Rightarrow \quad \frac{a}{c}=\frac{b}{d}$ *(By alternendo)*

$\Rightarrow \quad \frac{ac}{c^2}=\frac{bd}{d^2} \Rightarrow \quad \frac{ac}{bd}=\frac{c^2}{d^2}$ *(By alternendo)*

$\Rightarrow \quad \frac{ac}{bd}=\frac{a^2+c^2}{b^2+d^2}$ *(From Q. No.1)*

$\therefore \quad ac:bd=(a^2+c^2):(b^2+d^2)$

Q.3. $\frac{a^2-c^2}{ac}=\frac{b^2-d^2}{bd}$

SOLUTION: $\because \quad \frac{a}{b}=\frac{c}{d} \quad \Rightarrow \quad \frac{a}{c}=\frac{b}{d}$ ------------(1)

$\Rightarrow \quad \frac{ac}{c^2}=\frac{bd}{d^2} \quad \Rightarrow \quad \frac{ac}{bd}=\frac{c^2}{d^2}$ ------------ (2)

From equation(1), $\frac{a^2}{c^2}=\frac{b^2}{d^2} \Rightarrow \frac{a^2-c^2}{c^2}=\frac{b^2-d^2}{d^2}$ (By dividendo)

$\Rightarrow \frac{a^2-c^2}{b^2-d^2}=\frac{c^2}{d^2}$ ------------ (3)

From equations (2) and (3), we get

$=\frac{a^2-c^2}{b^2-d^2}=\frac{ac}{bd}$, $\therefore \frac{a^2-c^2}{ac}=\frac{b^2-d^2}{bd}$

Q.4. $a^2:b^2 = 3a^2+5c^2 : 3b^2+5d^2$

SOLUTION: $\because \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a^2}{b^2}=\frac{c^2}{d^2}$ ------------ (1)

$\Rightarrow \frac{3a^2}{3b^2}=\frac{5c^2}{5d^2} \Rightarrow \frac{3a^2}{5c^2}=\frac{3b^2}{5d^2}$

$\Rightarrow \frac{3a^2+5c^2}{5c^2}=\frac{3b^2+5d^2}{5d^2}$

$\Rightarrow \frac{3a^2+5c^2}{3b^2+5d^2}=\frac{5c^2}{5d^2}$

$\Rightarrow \frac{3a^2+5c^2}{3b^2+5d^2}=\frac{c^2}{d^2}$

$=\frac{a^2}{b^2}$ $\therefore a^2:b^2=3a^2+5c^2:3b^2+5d^2$

Q.5. $\frac{a}{c},\frac{ma+nb}{mc+nd}=\frac{b^2}{d^2}$

SOLUTION: $\because \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a}{c}=\frac{b}{d}$

$\Rightarrow \frac{ma}{mc}=\frac{nb}{nd} \Rightarrow \frac{ma}{nb}=\frac{mc}{nd}$

$\Rightarrow \frac{ma+nb}{nb}=\frac{mc+nd}{nd} \Rightarrow \frac{ma+nb}{mc+nd}=\frac{nb}{nd}$

$$\Rightarrow \quad \frac{ma+nb}{mc+nd}=\frac{b}{d}=\frac{a}{c} \qquad \therefore \qquad \frac{a}{c}\cdot\frac{ma+nb}{mc+nd}=\frac{a^2}{c^2}$$

Q.6. $\dfrac{a}{(a+b)(a+c)}=\dfrac{1}{a+b+c+d}$

SOLUTION: $\because \quad \dfrac{a}{b}=\dfrac{c}{d}(\therefore bc=ad)$

$$\Rightarrow \quad \frac{a+b}{b}=\frac{c+d}{d} \Rightarrow \quad a+b=\frac{b}{d}(c+d) \text{ ------------ } (1)$$

Now $\dfrac{a}{c}=\dfrac{b}{d}$

$$\Rightarrow \quad \frac{a+c}{c}=\frac{b+d}{d} \Rightarrow \quad a+c=\frac{c}{d}(b+d) \text{ ------------ } (2)$$

Multiplying (1) and (2), we get

$$(a+b)(a+c)=\frac{b}{d}(c+d).\frac{c}{d}(b+d)$$

$$=\frac{bc}{d^2}(bc+cd+bd+d^2)$$

$$=\frac{ad}{d^2}(ad+cd+bd+d^2)$$

$$=\frac{ad^2}{d^2}(a+b+c+d)$$

$$\therefore \quad \frac{a}{(a+b)(a+c)}=\frac{1}{a+b+c+d}$$

Q.7. $a+\dfrac{1}{b}:b+\dfrac{1}{a}=c+\dfrac{1}{d}:d+\dfrac{1}{c}$

SOLUTION: $\because \quad \dfrac{a}{b}=\dfrac{c}{d}$

$$\Rightarrow \quad \frac{a(ab+1)}{b(ab+1)}=\frac{c(cd+1)}{d(cd+1)}$$

$$\Rightarrow \quad \frac{\frac{ab+1}{b}}{\frac{ab+1}{a}}=\frac{\frac{cd+1}{d}}{\frac{cd+1}{c}}$$

$$\Rightarrow \quad \frac{\left(a+\frac{1}{b}\right)}{\left(b+\frac{1}{a}\right)}=\frac{\left(c+\frac{1}{d}\right)}{\left(d+\frac{1}{c}\right)}$$

$\therefore \quad a+\frac{1}{b}:b+\frac{1}{a}=c+\frac{1}{d}:d+\frac{1}{c}$ **proved**

Q.8. If a, b, c are in continued proportion, prove that:

$$\frac{a^2+b^2+c^2}{a+b+c}=a-b+c$$

and hence find three numbers in continued proportion so that their sum 14 and the sum of their squares is 84.

SOLUTION: *If a, b, c are in continued proportion, then* $a:b=b:c$

$\Rightarrow \quad b^2=ac$ ------------ (1)

L.H.S $= \dfrac{a^2+b^2+c^2}{a+b+c}$

$$=\frac{a^2+ac+c^2}{a+b+c}=\frac{(a+c)^2-ac}{a+b+c}=\frac{(a+c)^2-b^2}{a+b+c}$$

$$=\frac{(a+b+c)(a+c-b)}{a+b+c}=\frac{(a+b+c)(a-b+c)}{a+b+c}=$$

$= a-b+c=R.H.S$

If the sum of a, b, and c is 14 and the sum of their square is 84, then

$a+b+c=14$

$a^2+b^2+c^2=84 \qquad \Rightarrow \qquad (a-b+c)(a+b+c)=84$

$\Rightarrow \quad (a-b+c)(14)=84 \qquad \therefore \qquad a-b+c=\frac{84}{14}=6$ ---------- (3)

Adding equation (2) and (3), we get

$2a+2c=20 \Rightarrow a+c=10$ ------------ (4)

Subtracting a + c =10 in equation (2), we get

$b+10=14 \Rightarrow b=4$

Equation (1) becomes ac = 16

$\Rightarrow \quad c=\frac{16}{a}$ ------------ (5)

Equation (4) becomes $a+\frac{16}{a}=10$

$\Rightarrow \quad a^2-10a+16=0$

$$a=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(16)}}{2(1)}=\frac{10\pm\sqrt{100-64}}{2}$$

$$=\frac{10\pm 6}{2}=8,2$$

From equation (5), c =2 ,8.

Thus, the required three numbers are 2,4,8.

Solve the following for x = 2,4,8 Ans.

Q.9. $\frac{(x+3)^2+(x-1)^2}{(x+3)^2-(x-1)^2}=\frac{5}{4}$

SOLUTION: By componendo and dividendo property.

$$\frac{(x+3)^2+(x-1)^2+(x+3)^2-(x-1)^2}{(x+3)^2+(x-1)^2-(x+3)^2+(x-1)^2}=\frac{5+4}{5-4}$$

$\Rightarrow \quad \frac{2(x+3)^2}{2(x-1)^2}=\frac{9}{1}$

$\Rightarrow \quad \left(\frac{x+3}{x-1}\right)^2=3^2$

$\Rightarrow \quad \frac{x+3}{x-1}=3$

$\Rightarrow \quad x+3=3x-3 \Rightarrow x=3$ **Ans.**

Q.10. $\frac{x^2+16x+63}{x^2+6x+8}=\frac{x^2+16+60}{x^2+6x+5}$

SOLUTION: By componendo and dividendo property.

$$\frac{x^2+16x+63}{x^2+6x+8}=\frac{x^2+16x+60}{x^2+6x+5}$$

$$\Rightarrow \frac{x^2+16x+63}{x^2+16x+60}=\frac{x^2+6x+8}{x^2+6x+5}$$

$$\Rightarrow \frac{x^2+16x+63+x^2+16x+60}{x^2+16x+63-x^2-16x-60}$$

$$=\frac{x^2+6x+8+x^2+6x+5}{x^2+6x+8-x^2-6x-5}$$

$$\Rightarrow \frac{2x^2+32x+123}{3}=\frac{2x^2+12x+13}{3}$$

$$\Rightarrow 2x^2+32x+123=2x^2+12x+13$$

$$\Rightarrow 20x=-110 \quad \therefore \quad x=-\frac{11}{2} \quad \textbf{Ans.}$$

Q.11. $\frac{(x-1)(x-5)}{(x-6)(x-10)}=\frac{(x-2)(x-4)}{(x-7)(x-9)}$

SOLUTION:

$$\frac{x^2-6x+5}{x^2-16x+60}=\frac{x^2-6x+8}{x^2-16x+63}$$

By componendo and dividendo property.

$$\Rightarrow \frac{x^2-6x+5}{x^2-6x+8}=\frac{x^2-16x+60}{x^2-16x+63}$$

$$\Rightarrow \frac{x^2-6x+5+x^2-6x+8}{x^2-6x+5-x^2+6x-8}=0$$

$$\frac{x^2-16x+60+x^2-16x+63}{x^2-16x+60-x^2+16x-63}$$

$\Rightarrow \quad \dfrac{2x^2 12x+13}{-3}=\dfrac{2x^2-32x+123}{-3}$

$\Rightarrow \quad 2x^2-12x+13=2x^2-32+123$

$\Rightarrow \quad -12x+32x=123-13$

$\Rightarrow \quad 20x=110 \qquad \therefore \qquad x=\dfrac{11}{2}$ **Ans.**

Q.12. $\dfrac{x^2-3x+5}{3x-5}=\dfrac{x^2-2x+3}{2x-3}$

SOLUTION: By componendo property,

$$\frac{x^2-3x+5+3x-5}{3x-5}=\frac{x^2-2x+3+2x+-3}{2x-3}$$

$\Rightarrow \quad \dfrac{x^2}{3x-5}=\dfrac{x^2}{2x-3}$

$\Rightarrow \quad 2x^3-3x^2=3x^3-5x^2$

$\Rightarrow \quad x^3-2x^2=0$

$\Rightarrow \quad x^2(x-2)=0$

$\Rightarrow \quad x^2=0, \quad x-2=0$

$\therefore \quad x=0, \quad x=2$ **Ans.**

Q.13. $\dfrac{\sqrt{x+10}-\sqrt{x-10}}{\sqrt{x+10}+\sqrt{x-10}}=\dfrac{1}{5}$

SOLUTION:

$$\frac{\sqrt{x+10}-\sqrt{x-10}}{\sqrt{x+10}+\sqrt{x-10}}=\frac{1}{5}$$

By componendo and dividendo property.

$$\frac{\sqrt{x+10}-\sqrt{x-10}+\sqrt{x+10}+\sqrt{x-10}}{\sqrt{x+10}-\sqrt{x-10}-\sqrt{x+10}-\sqrt{x-10}}=\frac{1+5}{1-5}$$

$\Rightarrow$ $$\frac{2\sqrt{x+10}}{-2\sqrt{x-10}}=\frac{6}{-4}$$

$\Rightarrow$ $\sqrt{\frac{x+10}{x-10}}=\frac{3}{2}$ $\Rightarrow$ $\frac{x+10}{x-10}=\frac{9}{4}$

$\Rightarrow$ $4x+40=9x-90$

$\Rightarrow$ $5x=130$ $\therefore$ $x=26$ **Ans.**

Q.14. $$\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}=\frac{4x-1}{2}$$

Solution: By componendo and dividendo property.

$$\frac{\sqrt{x+1}+\sqrt{x-1}+\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}-\sqrt{x+1}+\sqrt{x-1}}=\frac{4x-1+2}{4x-1-2}$$

$\Rightarrow$ $\frac{2\sqrt{x+1}}{2\sqrt{x-1}}=\frac{4x+1}{4x-3}$ $\Rightarrow$ $\sqrt{\frac{x+1}{x-1}}=\frac{4x+1}{4x-3}$

$\Rightarrow$ $$\frac{x+1}{x-1}=\frac{16x^2+8x+1}{16x^2-24x+9}$$

$\Rightarrow$ $16x^3-24x^2+9x+16x^2-24x+9=$

$16x^3+8x^2+x-16x^2-8x-1$

$\Rightarrow$ $16x^3-8x^2-15x+9-16x^3+8x^2+7x+1=0$

$\Rightarrow$ $-8x+10=0$

$\therefore$ $x=\frac{5}{4}$ **Ans.**

EXERCISE 3.4

Q.1. The area of a circle varies as the square of its radius. Find the radius of a circle whose area is equal to the sum of the areas of two circles whose radii are 5 and 12 cms. respectively.

SOLUTION: Let A and r be the area and radius of the circle.

Then $A \propto r^2$

$\Rightarrow \quad A = kr^2 \quad$ ------------(1)

$\Rightarrow \quad \pi r^2 = kr^2$

$\therefore \quad k = \pi$

According to the given condition,

$A = \pi 5^2 + \pi(12)^2$

$\Rightarrow \quad \pi r^2 = \pi 25 + \pi 144$

$\Rightarrow \quad r^2 = 169$

$\therefore \quad r = 13$ cm **Ans.**

Q.2. When a body falls from rest its distance from the starting point varies as the square of the times.

If body falls through $402\frac{1}{2}$ feet in 5 seconds, how far does it fall in 10 seconds? Also how far does it fall the 10^{th} second?

SOLUTION: Let d be the distance covered by the body in time t.

Then $d \propto t^2 \quad \Rightarrow \quad d = kt^2$ ---------- (1)

$$\Rightarrow \quad k = \frac{402\frac{1}{2}}{25} = \frac{805}{50} \qquad (\because d = 402\tfrac{1}{2}\text{feet}, t = 5\text{ sec})$$

$\because \quad d = \frac{805}{50} \times (10)^2 = 1610 \text{ feet} \qquad \left(\because k = \frac{805}{50}, t = 10 \text{ sec}\right)$

$\Rightarrow \quad d = \frac{805}{50} \times (9)^2 = 1304.1 \text{feet} \qquad (\because t = 9 \text{ sec})$

In 10^{th} second,

$d = 1610 - 134.1 = 305.9 \text{feet}$

Thus, in 10 seconds the body covers a distance of 1610 feet and in 10^{th} second it covers a distance of 305.9 feet.

Q.3. Given that the volume of a sphere varies as the cube of its radius, and when the radius is $3\frac{1}{2}$ feet, the volume is $179\frac{2}{3}$ cubic feet, find the volume when the radius is 1 foot 9 inches.

SOLUTION: Let V be volume of a sphere of radius r.

Then $v \propto r^3$

$\Rightarrow \quad V = kr^3.$

$\Rightarrow \quad 179\frac{2}{3} = k\left(3\frac{1}{2}\right)^3 \qquad \left(\because r = 3\frac{1}{2} \text{feet}, v = 179\frac{2}{3} \text{feet}\right)$

$k = \frac{539}{3} \div \frac{343}{8} = \frac{88}{21}$

$V = \frac{88}{21} \times (1.75)^3 \qquad (\because r = 1 \text{ feet } 9 \text{ inches } = 1.75 \text{ feet})$

$= 4.191 \times 5.359$

$= 22.46$ cubic feet **Ans.**

Q.4. If S is the distance traveled in t seconds by a body starting from rest and moving with a uniform acceleration a, it has been found that S varies jointly as a square of t. If the body

traveled a meter in 3 second moving with an acceleration of 2 meter per second^2; find a when t= 5 seconds and S = 75 meter.

SOLUTION: As $S \propto at^2$

$$S = k\,at^2$$

If $S = 9m$, $a = 2m/s^2$, $t = 3sec$,

then $9 = k \times 2 \times 9 \Rightarrow k = \frac{1}{2}$

Equation (1) becomes $S = \frac{1}{2}at^2$

If $S = 75$ m, $t = 5s$, then $75 = \frac{1}{2}\alpha\,(5)^2$

$\alpha = \frac{75 \times 2}{25} = 6m/s^2$. **Ans.**

Q.5. Two photographs of different sizes, show exactly the same picture. The smaller photograph measures 10 cm by 14 cm and the larger photograph measures 15 cm by 21 cm. The height of tree given in the larger photograph is 12 cm. Find the height of the same tree in the smaller photograph.

SOLUTION: As the height h_1 of the smaller photograph varies the height h_2 of the larger photograph, i.e.

$$h_1 \propto h_2 \Rightarrow h_1 = kh_2$$

$k = \frac{h_1}{h_2} = \frac{14}{21} = \frac{2}{3}$ $(\because h_1 = 14\text{ cm}, h_2 = 21\text{ cm})$

So, the sizes of tree in smaller photograph = k × height of tree in larger photograph.

$$= \frac{2}{3} \times 12 = 8\text{ cm}$$

Q.6. If the ratio a : b is unaltered when x and y are added to its items in order. Show that x : y = a:b.

SOLUTION: $\because \quad a : b :: (a + x) : (b + y)$

$$\Rightarrow \quad \frac{a}{b}=\frac{a+x}{b+y} \Rightarrow ab+ay=ab+bx$$

$$\Rightarrow \quad ay=bx$$

$$\Rightarrow \quad bx=ay \quad \Rightarrow \quad \frac{x}{y}=\frac{b}{a} \Rightarrow x:b=b:a$$

Q.7. Measure of the sides of a triangle are 6, 11 and 15 cms. A triangle similar to it, has perimeter equals to 160 cm. Find the measure of the sides.

SOLUTION: *Let a,b,c be the measures of the sides of the required angle.*

Then $\quad \frac{6}{a}=\frac{11}{b}=\frac{15}{c}=k$

$$\Rightarrow \quad \frac{6}{a}=k\ ;\ \frac{11}{b}=k\ ;\ \frac{15}{c}=k$$

$$\Rightarrow \quad a=\frac{6}{k}\ ;\ b=\frac{11}{k}\ ;\ c=\frac{15}{k}$$

$\because$ *Perimeter of the required angle =160*

$$\Rightarrow \quad a+b+c=160$$

$$\Rightarrow \quad \frac{6}{k}+\frac{11}{k}+\frac{15}{k}=160$$

$$\Rightarrow \quad \frac{6+11+15}{k}=160$$

$$\Rightarrow \quad k=\frac{32}{160}=\frac{1}{5}$$

$$\therefore \quad a=\frac{6}{\frac{1}{5}}=6cm\ ;\ b=\frac{11}{\frac{1}{5}}=55cm\ ;\ c=\frac{15}{\frac{1}{5}}=75cm.$$

Q.8. The perimeter of a triangle is 176 dm. Its sides are proportional to 2 : 4 : 5. Find the lengths of its sides.

SOLUTION: *Let a,b and c be the required lengths. Then*

$$\frac{2}{a}=\frac{4}{b}=\frac{5}{c}=k$$

$\Rightarrow \quad a=\frac{2}{k}; \quad b=\frac{4}{k}; \quad c=\frac{5}{k}$

$\because$ Perimeter of the triangle = 176

$\Rightarrow \quad a+b+c=176$

$\Rightarrow \quad \frac{2}{k}+\frac{4}{k}+\frac{5}{k}=176$

$\Rightarrow \quad k=\frac{11}{176}=\frac{1}{16}$

$\therefore a=\frac{2}{\frac{1}{16}}=32$ dm ; $b=\frac{4}{\frac{1}{16}}=64dm$; $c=\frac{5}{\frac{1}{16}}=80dm$

MISCELLANEOUS EXERCISE

Q.1. Show that the ratio of the duplicate of 4 : 5 ; sub duplicate of 25 : 16, and the reciprocal of 4 : 5 is unity.

SOLUTION: Duplicate of 4 : 5 = $4^2 : 5^2 = 16 : 25$

Sub duplicate of 25 : 16 $=(5^2)^{\frac{1}{2}} : (4^2)^{\frac{1}{2}} = 54$

Inverse of 4 : 5 = 5: 4

$\therefore$ Ratio compounded = $\frac{16}{25}\times\frac{5}{4}\times\frac{5}{4}=1$

Q.2. What is the least integer which when subtracted from both the terms of 5 : 6 will give a ratio less than 13 : 18?

SOLUTION: Let x be the required integer.

Then $\quad \frac{5-x}{6-x}<\frac{13}{18}$

$\Rightarrow \quad 90-18x<78-13x \qquad \Rightarrow \quad 90-78<-13x+18x$

$\Rightarrow \quad 12 < 5x \quad \Rightarrow \quad x > \frac{12}{5} \quad \therefore x = 3$ **Ans.**

Q.3. If 1 : 2 is duplicated subtracting x from its terms, find x.

SOLUTION:

$\therefore \quad 1^2 : 2^2 = (1-x) : (2-x)$

$\Rightarrow \quad \frac{1}{4} = \frac{1-x}{2-x}$

$\Rightarrow \quad 2-x = 4-4x \quad \Rightarrow \quad 3x = 2 \quad \therefore \quad x = \frac{2}{3}$

Q.4. Find four proportional such that the sum of the extremes is 21, the sum of the means is 19, and the sum of the squares of all the four is 442.

SOLUTION: Let a ,b , c, d be the required four proportional.

Then $a : b :: c : d$

$\Rightarrow \quad ad = bc$ ---------- (1)

$a + d = 21$ ---------- (2)

$b + c = 19$ ---------- (3)

$a^2 + b^2 + c^2 + d^2 = 442$ ---------- (4)

Squaring both the sides of equation (2) and (3), we get

$a^2 + 2ad + d^2 = 441 ; \quad b^2 + 2bc + c^2 = 361$

$\Rightarrow \quad a^2 + d^2 = 441 - 2ad ; \quad b^2 + c^2 = 361 - 2bc$

Subtracting $a^2 + d^2 = 441 - 2ad$ and $b^2 + c^2 = 361 - 2bc$ in equation (4),

$441 - 2ad + 361 - 2bc = 442$

$\Rightarrow \quad -(ad + bc) = -360$

$\Rightarrow \quad 2bc = 180 \quad (\because ad = bc)$

$\Rightarrow \quad b = \frac{90}{c}$ ------------ (5)

Equation (3) becomes $\frac{90}{c} + c = 19$

$\Rightarrow \quad c^2 - 19 + 90 = 0$

$\therefore \quad c = \frac{-(-19) \pm \sqrt{(-19)^2 - 4(1)(90)}}{2(1)} = \frac{19 \pm \sqrt{361 - 360}}{2} = 10, 9$

Equation (5) becomes b = 9,10

Equation (1) becomes ad = 90

$\Rightarrow \quad d = \dfrac{90}{a}$

Equation (2) becomes $a + \dfrac{90}{a} = 21$

$\Rightarrow \quad a^2 - 21a + 90 = 0$

$$a = \frac{-(-21) \pm \sqrt{(-21) - 4(1)(90)}}{2(1)} = \frac{21 \pm \sqrt{441 - 360}}{2} = \frac{21 \pm 9}{2} = 15{,}6$$

Equation (6) becomes d = 6 , 15

Thus, the required four proportionals are:15 , 10 ,9 ,6 or 6, 9 , 10 ,15

Q.5. If a : b is the duplicate ratio of a – c : b – c, a and b being unequal, then c is a mean proportional between a and b.

SOLUTION: $\because \quad a : b = (a-c)^2 : (b-c)^2$

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{a^2 - 2ac + c^2}{b^2 - 2bc + c^2}$

$\Rightarrow \quad ab^2 - 2ab + ac^2 = a^2b - 2abc + bc^2$

$\Rightarrow \quad ab^2 + ac^2 = a^2 = a^2b + bc^2$

$\Rightarrow \quad a^2b - ab^2 = ac^2 - bc^2$

$\Rightarrow \quad ab(a-b) = c^2(a-4)$

$\Rightarrow \quad ab = c^2, \qquad (\because a - b \neq 0 \text{ or } a \neq b.$

Q.6.(i) If 7x +5y = 4x+3y, show that $x \propto y$.

SOLUTION: 7x + 5y =4x +3y

$\Rightarrow \quad 3x = -2y \quad \Rightarrow \quad x = -\dfrac{2}{3}y \qquad \therefore \ x \propto y.$

(ii) If $x^2y^2 + 1 = 2xy$, show that x varies inversely as y.

SOLUTION: $x^2y^2 + 1 = 2xy$

$\Rightarrow \quad x^2y^2 - 2xy + 1 = 0 \quad \Rightarrow \quad (xy-1)^2$

$\Rightarrow \quad xy - 1 = 0 \quad \Rightarrow \quad xy = 1$

$\Rightarrow \quad x = 1.\frac{1}{y} \qquad \therefore \quad x \alpha \frac{1}{y}$

Q.7.(I) If a : b = 6 : 7, find 7a +10b : 18b – 7a.

SOLUTION: $\because \quad a : b = 6 : 7 \Rightarrow 7a = 7a.$

$$\therefore \quad 7a + 10b : 18b - 7a = \frac{4a+10b}{18b-7a}$$

$$= \frac{6b+10b}{18b-6b} \qquad (\because 7a = 6b)$$

$$= \frac{16b}{12b}$$

$$= \frac{4}{3} = 4 : 3$$

(II) If x : y =3 :4, find $4x - 3y : 12x - 3y.$

SOLUTION: $\because \quad x : y = 3 : 4 \Rightarrow 4x = 3y$

$$\therefore \quad 4x - 3y : 12x - 3y = \frac{4x-3y}{12x-3y}$$

$$= \frac{4x-4x}{12x-4x} = \frac{0}{8x} = 0$$

(III) If $\frac{5a^2+7b^2}{3a^2-b^2} = 1\frac{40}{47}$, **find a: b.**

SOLUTION: $\because \quad \frac{5a^2+7b^2}{3a^2-b^2} = 1\frac{40}{47}$

$\Rightarrow \quad \frac{5a^2+7b^2}{3a^2-b^2} = \frac{87}{47} \qquad \Rightarrow \quad 235a^2 + 329b^2 = 261a^2 - 87b^2$

$\Rightarrow \quad 26a^2 = 416b^2 \qquad \Rightarrow \quad \left(\frac{a}{b}\right)^2 = \frac{16}{1}$

$\Rightarrow \quad \frac{a}{b} = \frac{4}{1} \qquad \therefore \quad a : b = 4 : 1$

Q.8. If $\frac{a-b}{a+b}=\frac{2b-c}{2b+c}=\frac{c-2a}{c+2a}$, then prove that each of the ratio = 0, unless $2a+2b\neq 0$.

SOLUTION: By theorem 3.1 with $l=2, m=1, n=1$

$$\text{Each fraction} = \frac{2(a-b)+1(2b-c)+1(c-2a)}{2(a+b)+1(2b+c)+1(c+2a)}$$

$$= \frac{2a-2b+2b-c+c-2a}{2a+2b+2b+c+c+2a}$$

$$= \frac{0}{4a+4b+2c}$$

$$= 0, \qquad 2a+2b+c\neq 0$$

Q.9. If $x = p + q$ when $q \; \alpha \; y^2$ and $q \; \alpha \frac{1}{y}$, find the equation between x and y if $y = 1$ When $x = 7$ and $y = 2$ when $x = 14$.

SOLUTION: $\because \quad p \, \alpha \; y^2 \quad \Rightarrow \quad p = k_1 y^2$

$$q \alpha \frac{1}{y} \quad \Rightarrow \quad q = k_2 . \frac{1}{y}$$

$$\therefore \quad p+q = k_1 y^2 + k_2 . \frac{1}{y}$$

$$\Rightarrow \quad x = k_1 y^2 + k_2 . \frac{1}{y} \quad \text{---------- (1)} \qquad (\because = p+q)$$

If $y = 1$ when $x = 7$, then $7 = k_1 + k_2 \Rightarrow k_2 = 7 - k_1$

If $y = 2$ when $x = 14$, then $14 = 4k_1 + \frac{k_2}{2}$

$$14 = 4k_1 + \frac{7-k_1}{2} \quad \Rightarrow \quad 7k_1 = 21$$

$$\therefore \quad k_1 = 3 \;\; ; \;\; k_2 = 4$$

Equation (1) becomes $x = 3y^2 + 4.\frac{1}{y}$

$\therefore \quad 3y^3 - xy + 4 = 0$, Which is the required equation.

Q.10. If the volume of a sphere varies as the cube of its radius, find the radius of a sphere whose volume equals the sum of the volumes of three spheres whose radii are 3 ft, 4ft and 5 ft respectively.

SOLUTION: Let V be the volume of a sphere of radius r.

Then $V \alpha r^3 \quad \Rightarrow \quad V = kr^3$ --------- (1)

Let V_1, V_2 and V_3 be the volumes of three spheres whose radii are 3 ft, 4ft and 5 ft respectively.

Then $V_1 = 27k$; $V_2 = 64k$; $V_3 = 125k$

$\because \quad V = V_1 + V_2 + V_3$

$\Rightarrow \quad V = 27k + 64k + 215k \quad \Rightarrow \quad V = 216k$

$\Rightarrow \quad k = \frac{V}{216}$

Equation (1) becomes $V = \frac{V}{216} . r^3$

$\Rightarrow \quad r^3 = V . \frac{216}{V} \quad \Rightarrow \quad r^3 = 216$

$\Rightarrow \quad r^3 = (6)^3 \qquad r = 6$ ft **Ans.**

Q.11. If 10 men take 118 days to mow 60 acres of grass, how long will 15 men take to mow 80 acres.

SOLUTION: Let x be the required numbers of days. Then

Men	Days	Areas (acres)
10	1 8	60
5	X	80

There is inverse proportion in men and days. So, 15 : 10 :: 18 : x

There is direct proportion in areas and days. So, 60 : 80 :: 18 : x

Combining both the proportions, $\left.\begin{matrix} 15 : 10 \\ 60 : 80 \end{matrix}\right] :: 118 : x$

We know the product of extremes = product of means

$$\therefore \quad 15 \times 50 \times x = 10 \times 80 \times 18$$

$$\Rightarrow \quad x = \frac{10 \times 80 \times 18}{15 \times 60} \quad \therefore \quad x = 16$$

Thus, 15 men will take 16 days to mow 80 acres.

Q.12. If 18 companies of equal ability can Set up 24 sheets in 8 days, how many sheets can 45 compositors of the same ability set up in 14 days?

SOLUTION: Let x be the required numbers of sheets. Then

Sheets	Min	Days
24	18	8
x	45	14

There are direct proportions in sheets and men; in sheets and days.

So, 24:x :: 18:45

8:45

We know the product of extremes = product of means

$$\Rightarrow \quad x \times 18 \times 8 = 24 \times 45 \times 14$$

$$\Rightarrow \quad x = \frac{24 \times 45 \times 14}{18 \times 8}$$

$$= 15 \times 7$$

$$= 105$$

Thus, 45 compositors of the same ability can set up 105 sheets in 14 days.

Q.13. Ages of a father and his sons are proportional to 10:3. eight years later the ages will be in ratio of 12:5. find their present ages.

SOLUTION: Let x years be present age of the Father and let y years be the present age of the son.

$$\text{Then} \quad x : y = 10 : 3 \Rightarrow y = \frac{3}{10}x \text{ ---------- } (1)$$

Now $(x+8):(y+8)=12:5$

$\Rightarrow \quad 5(x+8)=12(y+8) \quad \Rightarrow \quad 5x+40=12y+96$

$\Rightarrow \quad 5x-12y=56 \quad \left(\because y=\frac{3}{10}x\right)$

$\Rightarrow \quad 50x-36x=560 \quad \Rightarrow \quad 14x=560$

$\therefore \quad x=40 \quad , \quad y=\frac{3}{10}\times 40=12$

Thus, the present age of father is 40 years and the present age of son is 12 years.

Q.14. A father is twice old as his son 8 years back their ages were in the ratio of 8:3 find their present ages.

SOLUTION: *Let x year and y years be the present ages of the father and the son respectively.*

Then $\quad x=2y \Rightarrow = \frac{1}{2}x$

Since 8 years back their ages were in the ratio of 8:3 therefore,

$$\frac{x-8}{y-8}=\frac{8}{3}$$

$\Rightarrow \quad 3(x-8)=8(y-8)$

$\Rightarrow \quad 3x-24=8y-64$

$\Rightarrow \quad 3x-8y=-40$

$\Rightarrow \quad 3x-8\frac{1}{2}x=-40 \quad \left(\because y=\frac{1}{2}.x\right)$

$\Rightarrow \quad 3x-4x=-40$

$\Rightarrow \quad -x=-40$

$\Rightarrow \quad y=\frac{1}{2}\times 40=20$

Thus, the present age of father is 40 years and the present age of son is 20 years.