



Radiant Publishers

Bank Road, Mardan Tel: 0937-866462

MATH

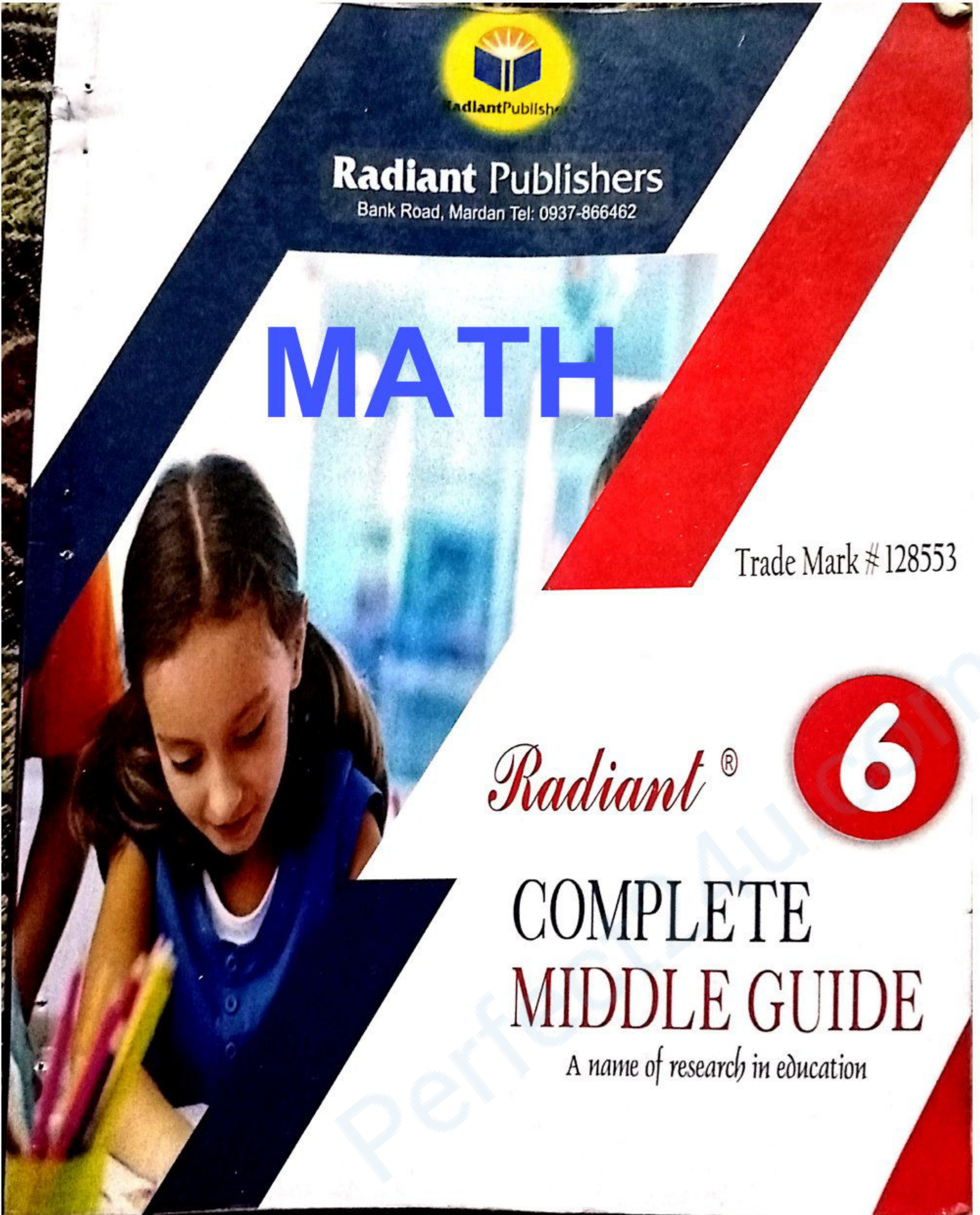
Trade Mark # 128553

Radiant®

6

COMPLETE MIDDLE GUIDE

A name of research in education



Maths

Unit 1

Multiples and Factors

Exercise 1.1

1. Find all the factors of the following numbers.

a. 26

Solution:

We can find the factors of 26 as,

$$26 = 1 \times 26$$

$$26 = 2 \times 13$$

So factors of 26 are 1, 2, 13 and 26

b. 112

Solution:

We can find the factors of 112 as

$$112 = 1 \times 112$$

$$112 = 2 \times 56$$

$$112 = 4 \times 28$$

$$112 = 7 \times 16$$

$$112 = 8 \times 14$$

So factors of 112 are 1, 2, 4, 7, 8, 14, 16, 28, 56 and 112.

c. 150

Solution:

We can find the factors of 150 as

$$150 = 1 \times 150$$

$$150 = 2 \times 75$$

$$150 = 3 \times 50$$

$$150 = 5 \times 30$$

$$150 = 6 \times 25$$

$$150 = 10 \times 15$$

So factors of 150 are 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75 and 150.

d. 175

Solution:

We can find the factors of 175 as:

$$175 = 1 \times 175$$

$$175 = 5 \times 35$$

$$175 = 7 \times 25$$

So factors of 175 are 1, 5, 7, 25, 35 and 175.

e. 68

Solution:

We can find the factors of 68 as:

$$68 = 1 \times 68$$

$$68 = 2 \times 34$$

$$38 = 4 \times 17$$

So factors of 68 are 1, 2, 4, 17, 34 and 68.

f. 145

Solution:

We can find the factor of 145 as:

$$145 = 1 \times 145$$

$$145 = 5 \times 29$$

So factors of 145 are 1, 5, 29 and 145.

g. 80

Solution:

We can find the factor of 80 as:

$$80 = 1 \times 80$$

$$80 = 2 \times 40$$

$$80 = 4 \times 20$$

$$80 = 5 \times 16$$

$$80 = 8 \times 10$$

So factors of 80 are 1, 2, 4, 5, 8, 10, 16, 20, 40 and 80.

h. 100

Solution:

We can find the factors of 100 as

$$100 = 1 \times 100$$

$$100 = 2 \times 50$$

$$100 = 4 \times 25$$

$$100 = 5 \times 20$$

$$100 = 10 \times 10$$

So factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50 and 100.

i. 23

Solution:

We can find the factors of 23 as

$$23 = 1 \times 23$$

So factors of 23 are 1, 23.

j. 47

Solution:

We can find the factors of 47 as

$$47 = 1 \times 47$$

So factors of 47 are 1 and 47.

k. 58

Solution:

We can find factors of 58 as:

$$58 = 1 \times 58$$

$$58 = 2 \times 29$$

So factors of 58 are 1, 2, 29 and

58.

l. 64

Solution:

We can find factors 64 as:

$$64 = 1 \times 64$$

$$64 = 2 \times 32$$

$$64 = 4 \times 16$$

$$64 = 8 \times 8$$

So factors of 64 are 1, 2, 4, 8, 16, 32, and 64.

m. 42

Solution:

We can find factors 42 as:

$$42 = 1 \times 42$$

$$42 = 2 \times 21$$

$$42 = 3 \times 14$$

$$42 = 6 \times 7$$

So factors of 42 are 1, 2, 3, 6, 7, 14, 21 and 42.

n. 36

Solution:

$$36 = 1 \times 36$$

$$36 = 2 \times 18$$

$$36 = 3 \times 12$$

$$36 = 4 \times 9$$

$$36 = 6 \times 6$$

So factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

o. 70

Solution:

We can find factors of 70 as:

$$70 = 1 \times 70$$

$$70 = 2 \times 35$$

$$70 = 5 \times 14$$

$$70 = 7 \times 10$$

So factors of 70 are 1, 2, 5, 7, 10, 14, 35 and 70.

p. 90

Solution:

We can find factors of 90 as:

$$90 = 1 \times 90$$

$$90 = 2 \times 45$$

$$90 = 3 \times 30$$

$$90 = 5 \times 18$$

$$90 = 6 \times 15$$

$$90 = 9 \times 10$$

So factors of 90 are 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45 and 90.

Q2: Find the multiples of the given number less than 250.

a. 12

Solution:

As $1 \times 12 = 12$, $2 \times 12 = 24$, $3 \times 12 = 36$,

$4 \times 12 = 48$, $5 \times 12 = 60$, $6 \times 12 = 72$,

$7 \times 12 = 84$, $8 \times 12 = 96$, $9 \times 12 = 108$,

$10 \times 12 = 120$, $11 \times 12 = 132$, $12 \times 12 = 144$,

$13 \times 12 = 156$, $14 \times 12 = 168$, $15 \times 12 = 180$,

$16 \times 12 = 192$, $17 \times 12 = 204$, $18 \times 12 = 216$,

$19 \times 12 = 228$, $20 \times 12 = 240$

So Multiples of 12 less than 250 are

12, 24, 36, 48, 60, 72, 84, 96, 108, 120,

132, 144, 156, 168, 180, 192, 204, 216,

228, 240.

b. 25

Solution:

As $1 \times 25 = 25$, $2 \times 25 = 50$, $3 \times 25 = 75$,

$4 \times 25 = 100, 5 \times 25 = 125, 6 \times 25 = 150,$
 $7 \times 25 = 175, 8 \times 25 = 200, 9 \times 25 = 225$
 So Multiples of 25 less than 250 are
 25, 50, 75, 100, 125, 150, 175, 200, 225.

c. 37

Solution:

As $1 \times 37 = 37, 2 \times 37 = 74,$
 $3 \times 37 = 111, 4 \times 37 = 148,$
 $5 \times 37 = 185, 6 \times 37 = 222.$

So Multiples of 37 less than 250 are
 37, 74, 111, 148, 185 and 222.

d. 48

Solution:

As $1 \times 48 = 48, 2 \times 48 = 96,$
 $3 \times 48 = 144, 4 \times 48 = 192,$
 $5 \times 48 = 240.$

So Multiples of 48 are 48, 96, 144,
 192, 240.

e. 60

Solution:

As $1 \times 60 = 60, 2 \times 60 = 120,$
 $3 \times 60 = 180, 4 \times 60 = 240$

So Multiples of 60 less than 60 are
 60, 120, 180, 240

f. 75

Solution:

As $1 \times 75 = 75, 2 \times 75 = 150,$
 $3 \times 75 = 225$

So Multiples of 75 are 75, 150 and
 225.

g. 45

Solution:

As $1 \times 45 = 45, 2 \times 45 = 90,$
 $3 \times 45 = 135, 4 \times 45 = 180$
 $5 \times 45 = 225$

So Multiples of 45 less than 250 are
 45, 90, 135, 180, 225.

h. 57

Solution:

As $1 \times 57 = 57, 2 \times 57 = 114$

$$3 \times 27 = 171, 4 \times 57 = 228$$

So Multiples of 57 less than 250 are
 57, 114, 171, 228.

Q3: Find the multiples of 14 less than 500.

Solution:

As $1 \times 14 = 14, 2 \times 14 = 28, 3 \times 14 = 42,$
 $4 \times 14 = 56, 5 \times 14 = 70, 6 \times 14 = 84,$
 $7 \times 14 = 98, 8 \times 14 = 112, 9 \times 14 = 126,$
 $10 \times 14 = 140, 11 \times 14 = 154, 12 \times 14 = 168,$
 $13 \times 14 = 182, 14 \times 14 = 196, 15 \times 14 = 210,$
 $16 \times 14 = 224, 17 \times 14 = 238,$
 $18 \times 14 = 252, 19 \times 14 = 266, 20 \times 14 = 280,$
 $21 \times 14 = 294, 22 \times 14 = 308, 23 \times 14 = 322,$
 $24 \times 14 = 336, 25 \times 14 = 350, 26 \times 14 = 364,$
 $27 \times 14 = 378, 28 \times 14 = 392, 29 \times 14 = 406,$
 $30 \times 14 = 420, 31 \times 14 = 434, 32 \times 14 = 448,$
 $33 \times 14 = 462, 34 \times 14 = 476, 35 \times 14 = 490$

So Multiples of 14 less than 500 are
 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154,
 168, 182, 196, 210, 224, 238, 252, 266, 280,
 294, 308, 322, 336, 350, 364, 378, 392, 406,
 420, 434, 448, 462, 476, 490.

Q4: Is 18 a factor of 256?

Solution:

To find 18 is a factor of 256 or
 not, we divide 256 by 18.

$$\begin{array}{r} 14 \\ 18 \overline{) 256} \\ \underline{18} \\ 76 \\ \underline{72} \\ 4 \end{array}$$

As 18 does not divide 256
 completely, so 18 is not a factor of
 256.

Exercise 1.2

Q1. Find the prime factors of the following numbers and write it in the form of index notation.

a. 144

Solution:

$$\begin{array}{r} 2 \overline{) 144} \\ \underline{2} \\ 72 \\ \underline{2} \\ 36 \\ \underline{2} \\ 18 \\ \underline{2} \\ 9 \\ \underline{3} \\ 3 \end{array}$$

The prime factorization of 144
 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Index form $= 2^4 \times 3^2$

b. 314

Solution:

$$\begin{array}{r} 2 \overline{) 314} \\ \underline{2} \\ 157 \end{array}$$

The prime factorization of
 $314 = 2 \times 157$

Index form $= 2^1 \times 157^1$

c. 5624

Solution:

$$\begin{array}{r} 2 \overline{) 5624} \\ \underline{2} \\ 2812 \\ \underline{2} \\ 1406 \\ \underline{2} \\ 703 \\ \underline{19} \\ 37 \end{array}$$

The prime factorization of
 $5624 = 2 \times 2 \times 2 \times 19 \times 37$

Index form $= 2^3 \times 19^1 \times 37^1$

d. 3084

Solution:

$$\begin{array}{r} 2 \overline{) 3084} \\ \underline{2} \\ 1542 \\ \underline{2} \\ 771 \\ \underline{3} \\ 257 \end{array}$$

The prime factorization of
 $3084 = 2 \times 2 \times 3 \times 257$

Index form $= 2^3 \times 3^1 \times 257^1$

e. 7880

Solution:

$$\begin{array}{r} 2 \overline{) 7880} \\ \underline{2} \\ 3940 \\ \underline{2} \\ 1970 \\ \underline{2} \\ 985 \\ \underline{5} \\ 197 \end{array}$$

The prime factorization of
 $7880 = 2 \times 2 \times 2 \times 5 \times 197$

Index form $= 2^3 \times 5^1 \times 197^1$

f. 6024

Solution:

$$\begin{array}{r} 2 \overline{) 6024} \\ \underline{2} \\ 3012 \\ \underline{2} \\ 1506 \\ \underline{2} \\ 753 \\ \underline{3} \\ 251 \end{array}$$

The prime factorization of
 $6024 = 2 \times 2 \times 2 \times 3 \times 251$

Index form $= 2^3 \times 3^1 \times 251^1$

g. 8460

Solution:

$$\begin{array}{r} 2 \overline{) 8460} \\ \underline{2} \\ 4230 \\ \underline{2} \\ 2115 \\ \underline{3} \\ 705 \\ \underline{5} \\ 141 \\ \underline{3} \\ 47 \end{array}$$

The prime factorization of
 $8460 = 2 \times 2 \times 3 \times 3 \times 5 \times 47$

Index form $= 2^2 \times 3^2 \times 5^1 \times 47^1$

h. 2986

Solution:

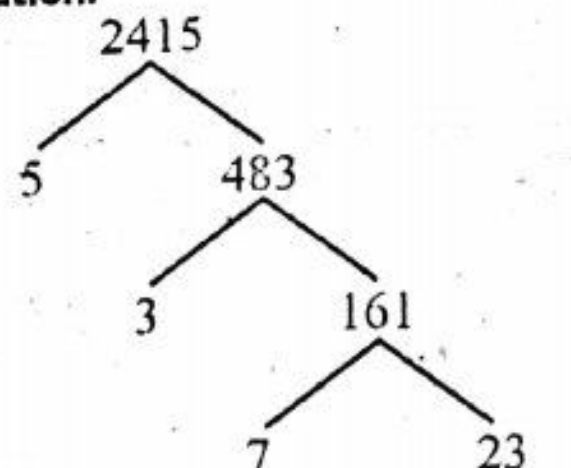
$$\begin{array}{r} 2 \overline{) 2986} \\ 1493 \end{array}$$

The prime factorization of
 $2986 = 2 \times 1493$
 Index form = $2^1 \times 1493^1$

Q2: Find the prime factors of the following numbers by using factor tree method and write it in the form of index notation.

a. 2415

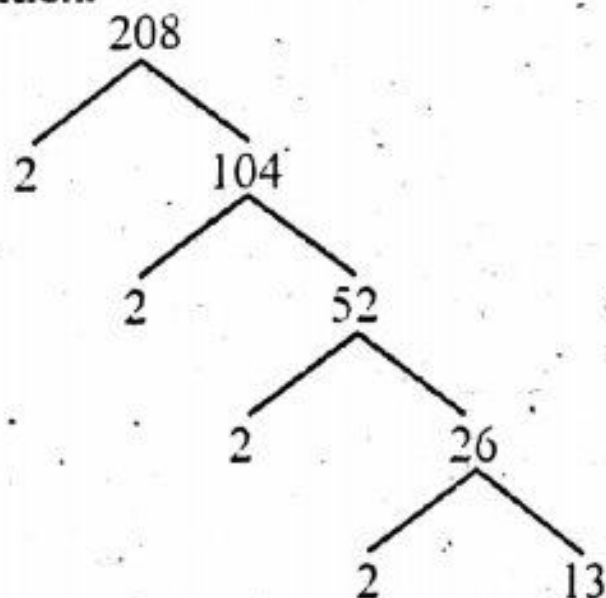
Solution:



The prime factorization of
 $2415 = 3 \times 5 \times 7 \times 23$
 Index form = $3^1 \times 5^1 \times 7^1 \times 23^1$

b. 208

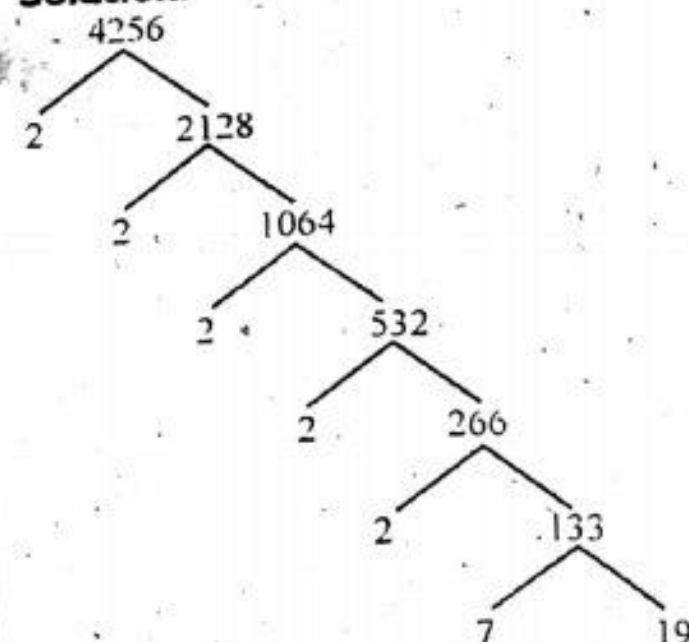
Solution:



The prime factorization of
 $208 = 2 \times 2 \times 2 \times 2 \times 13$
 Index form = $2^4 \times 13^1$

c. 4256

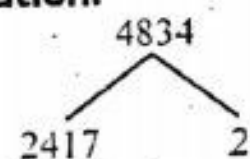
Solution:



The prime factorization of
 $4256 = 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 19$
 Index form = $2^5 \times 7^1 \times 19^1$

d. 4834

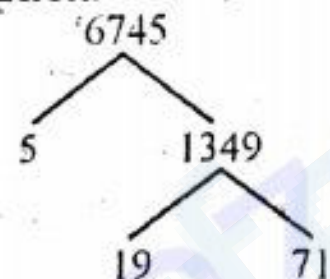
Solution:



The prime factorization of
 $4834 = 2 \times 2417$
 Index form = $2^1 \times 2417^1$

e. 6745

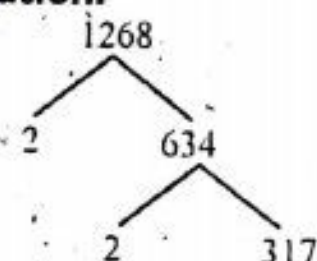
Solution:



The prime factorization of
 $6745 = 5 \times 19 \times 71$
 Index form = $5^1 \times 19^1 \times 71^1$

f. 1268

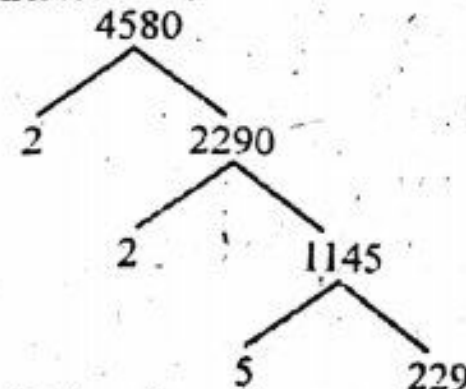
Solution:



The prime factorization of
 $1268 = 2 \times 2 \times 317$
 Index form = $2^2 \times 317^1$

g. 4580

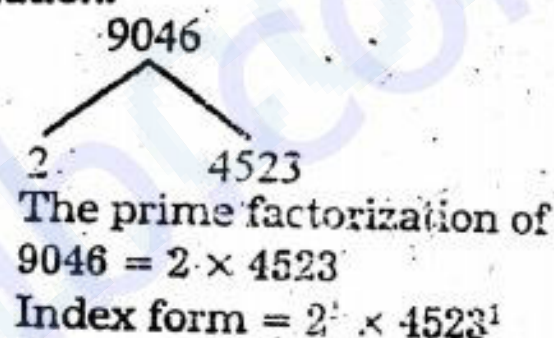
Solution:



The prime factorization of
 $4580 = 2 \times 2 \times 5 \times 229$
 Index form = $2^2 \times 5^1 \times 229^1$

h. 9046

Solution:



The prime factorization of
 $9046 = 2 \times 4523$
 Index form = $2^1 \times 4523^1$

Q3: Write the following in expanded form.

a. 2^6

Solution:

$$2^6$$

Expanded form = $2 \times 2 \times 2 \times 2 \times 2 \times 2$

b. 4×7^5

Solution:

$$4 \times 7^5$$

Expanded form = $4 \times 7 \times 7 \times 7 \times 7 \times 7$

c. $3 \times 5^2 \times 11^4$

Solution:

$$3 \times 5^2 \times 11^4$$

Expanded form = $3 \times 5 \times 5 \times 11 \times 11 \times 11 \times 11$

d. $2^2 \times 3^3 \times 5^6$

Solution:

Expanded form = $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

e. $13^4 \times 17^3$

Solution:

$$13^4 \times 17^3$$

Expanded form = $13 \times 13 \times 13 \times 13 \times 13 \times 17 \times 17 \times 17$

f. 4×7^5

Solution:

$$4 \times 7^5$$

Expanded form = $4 \times 7 \times 7 \times 7 \times 7 \times 7$

g. $3^2 \times 5^4 \times 17^2$

Solution:

$$3^2 \times 5^4 \times 17^2$$

Expanded form = $3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 17 \times 17$

h. $7^2 \times 11^3 \times 29$

Solution:

$$7^2 \times 11^3 \times 29$$

Expanded form = $7 \times 7 \times 11 \times 11 \times 11 \times 29$

Q4: Express the following in index notation.

a. $5 \times 5 \times 5$

Solution:

$$5 \times 5 \times 5 = 5^3$$

b. $2 \times 2 \times 5 \times 5 \times 5 \times 5$

Solution:

$$2 \times 2 \times 5 \times 5 \times 5 \times 5 = 2^2 \times 5^4$$

c. $1 \times 2 \times 2 \times 7 \times 7 \times 7$

Solution:

$$1 \times 2 \times 2 \times 7 \times 7 \times 7 = 1^1 \times 2^2 \times 7^3$$

d. $11 \times 11 \times 13 \times 13 \times 13 \times 19 \times 19$

Solution:

$$11 \times 11 \times 13 \times 13 \times 13 \times 19 \times 19 = 11^2 \times 13^3 \times 19^2$$

e. $7 \times 7 \times 7 \times 11 \times 11$

Solution:

$$7 \times 7 \times 7 \times 11 \times 11 = 7^3 \times 11^2$$

Exercise 1.3

1. Find the HCF of the following numbers using prime factorization method.

a. 224, 142

Solution:

First we find prime factorization of 224, 142.

$$\begin{array}{r|l} 2 & 224 \\ 2 & 112 \\ 2 & 56 \\ 2 & 28 \\ 2 & 14 \\ & 7 \end{array} \quad \begin{array}{r|l} 2 & 142 \\ & 71 \end{array}$$

Prime factors of 224 = $2 \times 2 \times 2 \times 2 \times 2 \times 7$

Prime factor of 142 = 2×71

Common factors are 2

So H.C.F = 2.

b. 520, 420, 124

Solution:

First we find prime factorization of 520, 420 and 124

$$\begin{array}{r|l} 2 & 520 \\ 2 & 260 \\ 2 & 130 \\ 5 & 65 \\ & 13 \end{array} \quad \begin{array}{r|l} 2 & 420 \\ 2 & 210 \\ 2 & 105 \\ 5 & 35 \\ & 7 \end{array} \quad \begin{array}{r|l} 2 & 124 \\ 2 & 62 \\ & 31 \end{array}$$

Prime factors of 520 = $2 \times 2 \times 2 \times 5 \times 13$

Prime factors of 420 = $2 \times 2 \times 3 \times 5 \times 7$

Prime factors of 124 = $2 \times 2 \times 31$

Common factors are 2, 2

So H.C.F = 2×2

= 4

c. 168, 400, 540

Solution:

First we find prime factorizations 168, 400 and 540

$$\begin{array}{r|l} 2 & 168 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array} \quad \begin{array}{r|l} 2 & 400 \\ 2 & 200 \\ 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ & 5 \end{array} \quad \begin{array}{r|l} 2 & 540 \\ 2 & 270 \\ 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

Prime factors of 168 = $2 \times 2 \times 2 \times 3 \times 7$

Prime factors of 400 = $2 \times 2 \times 2 \times 2 \times 5 \times 5$

Prime factors of 540 = $2 \times 2 \times 3 \times 3 \times 3 \times 5$

Common factors are 2, 2

So H.C.F = 2×2

= 4

d. 366, 145, 780

Solution:

First we find prime factorization of 366, 145, and 780.

$$\begin{array}{r|l} 3 & 366 \\ 3 & 183 \\ & 61 \end{array} \quad \begin{array}{r|l} 5 & 145 \\ & 29 \end{array} \quad \begin{array}{r|l} 5 & 780 \\ 2 & 156 \\ 2 & 78 \\ 3 & 39 \\ & 13 \end{array}$$

Prime factors of 366 = $2 \times 3 \times 61$

Prime factors of 145 = 5×29

Prime factors of 780 = $2 \times 2 \times 3 \times 5 \times 13$

As there are no common factors, So

H.C.F = 1

e. 678, 424, 784

Solution:

First we find prime factorizations

$$\begin{array}{r|l} 2 & 678 \\ 3 & 339 \\ & 113 \end{array} \quad \begin{array}{r|l} 2 & 424 \\ 2 & 212 \\ 2 & 106 \\ & 53 \end{array} \quad \begin{array}{r|l} 2 & 784 \\ 2 & 392 \\ 2 & 196 \\ 2 & 98 \\ 7 & 49 \\ & 7 \end{array}$$

Prime factors of 678 = $2 \times 3 \times 113$

Prime factors of 424 = $2 \times 2 \times 2 \times 53$

Prime factors of 784 = $2 \times 2 \times 2 \times 2 \times 7 \times 7$

Common factors are 2

So H.C.F = 2.

f. 905, 560

Solution:

First we find prime factorizations of 905, 560

$$\begin{array}{r|l} 5 & 905 \\ & 181 \end{array} \quad \begin{array}{r|l} 5 & 560 \\ 2 & 112 \\ 2 & 56 \\ 2 & 28 \\ 2 & 14 \\ & 7 \end{array}$$

Prime factors of 905 = 5×181

Prime factors of 560 = $2 \times 2 \times 2 \times 2 \times 5 \times 7$

Common factors are 5

So H.C.F = 5.

g. 248, 104, 565

Solution:

First we find prime factorizations of 248, 104 and 565

$$\begin{array}{r|l} 2 & 248 \\ 2 & 124 \\ 2 & 62 \\ & 31 \end{array} \quad \begin{array}{r|l} 2 & 104 \\ 2 & 52 \\ 2 & 26 \\ & 13 \end{array} \quad \begin{array}{r|l} 53 & 565 \\ & 113 \end{array}$$

Prime factors of 248 = $2 \times 2 \times 2 \times 31$

Prime factors of 104 = $2 \times 2 \times 2 \times 13$

Prime factors of 565 = 5×113

As there are no common factors

So H.C.F = 1

h. 578, 452

Solution:

First we find prime factorizations

$$\begin{array}{r|l} 2 & 578 \\ 17 & 289 \\ & 17 \end{array} \quad \begin{array}{r|l} 2 & 452 \\ 2 & 226 \\ & 113 \end{array}$$

Prime factors of 578 = $2 \times 17 \times 17$

Prime factors of 452 = $2 \times 2 \times 113$

Common factors are 2,

So H.C.F = 2.

i. 385, 855, 620

Solution:

First we find prime factorizations, of 385, 855 and 620

$$\begin{array}{r|l} 5 & 385 \\ 7 & 77 \\ & 11 \end{array} \quad \begin{array}{r|l} 5 & 855 \\ 5 & 175 \\ 5 & 35 \\ & 7 \end{array} \quad \begin{array}{r|l} 5 & 620 \\ 2 & 124 \\ 2 & 62 \\ 2 & 31 \\ & 2 \end{array}$$

Prime factors of 385 = $5 \times 7 \times 11$

Prime factors of 855 = $5 \times 5 \times 5 \times 7$

Prime factors of 620 = $5 \times 2 \times 2 \times 2 \times 2 \times 2$

Common factors are 5

So H.C.F = 5.

Q2: Find the HCF of the following numbers using division method.

a. 100, 350, 480

Solution:

$$\begin{array}{r} 3 \\ 100 \overline{) 350} \\ \underline{300} \quad 2 \\ 50 \overline{) 100} \\ \underline{50} \quad 50 \\ 50 \overline{) 50} \\ \underline{50} \quad 0 \end{array}$$

So, the H.C.F of 100 and 350 is 50.

Now, find the H.C.F of 50 and 480.

$$\begin{array}{r} 9 \\ 50 \overline{) 480} \\ \underline{450} \quad 30 \\ 30 \overline{) 30} \\ \underline{30} \quad 0 \end{array}$$

So the H.C.F of 100, 350 and 480 is 10.

b. 678, 456

Solution:

$$\begin{array}{r}
 1 \\
 456 \overline{) 678} \\
 \underline{456} 2 \\
 222 18 \\
 \underline{222} 0 \\
 216 2 \\
 \underline{216} 0 \\
 12 2 \\
 \underline{12} 0 \\
 \times
 \end{array}$$

So the H.C.F of 678 and 456 is 6.

c. 400, 350, 250

Solution:

$$\begin{array}{r}
 1 \\
 350 \overline{) 400} \\
 \underline{350} 7 \\
 50 350 \\
 \underline{350} 0 \\
 \times
 \end{array}$$

The H.C.F of 400 and 350 is 50.

Now we find H.C.F of 50 and 250.

$$\begin{array}{r}
 5 \\
 50 \overline{) 250} \\
 \underline{250} 0 \\
 \times
 \end{array}$$

So, H.C.F of 400, 350 and 250 is 50.

d. 100, 245, 168

Solution:

$$\begin{array}{r}
 2 \\
 100 \overline{) 245} \\
 \underline{200} 2 \\
 45 100 \\
 \underline{90} 4 \\
 10 45 \\
 \underline{40} 2 \\
 5 10 \\
 \underline{10} 0 \\
 \times
 \end{array}$$

The H.C.F of 100 and 245 is 5.

Now we find H.C.F of 5 and 168.

$$\begin{array}{r}
 33 \\
 5 \overline{) 168} \\
 \underline{165} 1 \\
 3 5 \\
 \underline{3} 1 \\
 2 3 \\
 \underline{2} 2 \\
 1 2 \\
 \underline{2} 0 \\
 \times
 \end{array}$$

So, H.C.F of 100, 245, 168 is 1.

e. 332, 480, 290

Solution:

$$\begin{array}{r}
 1 \\
 332 \overline{) 480} \\
 \underline{332} 2 \\
 158 332 \\
 \underline{316} 9 \\
 16 158 \\
 \underline{144} 1 \\
 14 16 \\
 \underline{14} 7 \\
 2 14 \\
 \underline{14} 0 \\
 \times
 \end{array}$$

The H.C.F of 332 and 480 is 2.

Now we find H.C.F of 2 and 290.

$$\begin{array}{r}
 145 \\
 2 \overline{) 290} \\
 \underline{290} 0 \\
 9 \\
 \underline{8} 10 \\
 10 \\
 \underline{10} 0 \\
 \times
 \end{array}$$

So H.C.F of 332, 480, 290 = 2

f. 734, 202

Solution:

$$\begin{array}{r}
 202 \overline{) 734} \\
 \underline{606} 1 \\
 128 202 \\
 \underline{128} 1 \\
 74 128 \\
 \underline{74} 1 \\
 54 74 \\
 \underline{54} 2 \\
 20 54 \\
 \underline{40} 1 \\
 14 20 \\
 \underline{14} 2 \\
 6 14 \\
 \underline{6} 12 \\
 3 12 \\
 \underline{6} 6 \\
 2 6 \\
 \underline{4} 2 \\
 \times
 \end{array}$$

The H.C.F of 734 and 202 is 2.

$$\begin{array}{r}
 145 \\
 2 \overline{) 290} \\
 \underline{290} 0 \\
 \times
 \end{array}$$

So H.C.F of 332, 480 and 290 is 2.

Q3: Find the LCM of the given numbers using prime factorization method.

a. 120, 150, 145

Solution:

First we find prime factorizations

$$\begin{array}{r|l}
 2 & 120 \\
 2 & 60 \\
 2 & 30 \\
 3 & 15 \\
 & 5
 \end{array}
 \quad
 \begin{array}{r|l}
 2 & 150 \\
 3 & 75 \\
 5 & 25 \\
 & 5
 \end{array}
 \quad
 \begin{array}{r|l}
 5 & 145 \\
 & 29
 \end{array}$$

The prime factors of 120 = $2 \times 2 \times 2 \times 3 \times 5$ The prime factors of 150 = $2 \times 3 \times 5 \times 5$ The prime factors of 145 = 5×29

Common factors = 2, 5, 3

Non common factors = 2, 2, 5, 29

L.C.M = Common factors $\times$ Non common factors

$$= 2 \times 5 \times 3 \times 2 \times 2 \times 5 \times 29$$

$$= 17400$$

b. 108, 116, 128

Solution:

First we find prime factorizations

$$\begin{array}{r|l}
 2 & 180 \\
 2 & 90 \\
 3 & 45 \\
 3 & 15 \\
 & 5
 \end{array}
 \quad
 \begin{array}{r|l}
 2 & 116 \\
 2 & 58 \\
 & 29
 \end{array}
 \quad
 \begin{array}{r|l}
 2 & 128 \\
 2 & 64 \\
 2 & 32 \\
 2 & 16 \\
 2 & 8 \\
 2 & 4 \\
 & 2
 \end{array}$$

The prime factors of 108 = $2 \times 2 \times 3 \times 3 \times 3$ The prime factors of 116 = $2 \times 2 \times 29$ The prime factors of 128 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Common factors = 2, 2

Now common factors = 3, 3, 3, 29, 2, 2, 2, 2, 2

L.C.M = Common factors $\times$ Non common factors

$$= 2 \times 2 \times 3 \times 3 \times 3 \times 29 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$= 100,224$$

c. 240, 412

Solution:

$$\begin{array}{r|l}
 2 & 240 \\
 2 & 120 \\
 2 & 60 \\
 2 & 30 \\
 3 & 15 \\
 & 5
 \end{array}
 \quad
 \begin{array}{r|l}
 2 & 412 \\
 2 & 206 \\
 & 103
 \end{array}$$

The prime factors of 240 = $2 \times 2 \times 2 \times 2 \times 3 \times 5$ The prime factors of 412 = $2 \times 2 \times 103$

Common factors = 2, 2

Non common factors = 2,2,3,5,103
 L.C.M = Common factors $\times$ Non
 Common factors

$$= 2 \times 2 \times 2 \times 3 \times 5 \times 103$$

$$= 24720$$

d. 158, 250, 240

Solution:

First we find prime factorizations,

$\begin{array}{r} 2 \overline{)158} \\ 79 \end{array}$	$\begin{array}{r} 2 \overline{)250} \\ 5 \overline{)125} \\ 5 \overline{)25} \\ 5 \end{array}$	$\begin{array}{r} 2 \overline{)240} \\ 2 \overline{)120} \\ 5 \overline{)60} \\ 2 \overline{)30} \\ 3 \overline{)15} \\ 5 \end{array}$
--	--	--

The prime factors of 158 = 2×79

The prime factors of 250 = $2 \times 5 \times 5 \times 5$

The prime factors of 240 = $2 \times 2 \times 2 \times 2 \times 3 \times 5$

Common factors = 2, 5

Non Common factors = 79, 5, 2, 2, 2, 3

L.C.M = Common factors $\times$ Non
 common factors

$$= 2 \times 5 \times 79 \times 5 \times 5 \times 2 \times 2 \times 2 \times 3$$

$$= 474000$$

e. 296, 564, 444

Solution:

First we find prime factorizations

$\begin{array}{r} 2 \overline{)296} \\ 2 \overline{)148} \\ 2 \overline{)74} \\ 37 \end{array}$	$\begin{array}{r} 2 \overline{)564} \\ 2 \overline{)282} \\ 3 \overline{)141} \\ 47 \end{array}$	$\begin{array}{r} 2 \overline{)444} \\ 2 \overline{)222} \\ 3 \overline{)111} \\ 37 \end{array}$
---	--	--

The prime factors of 296 = $2 \times 2 \times 2 \times 37$

The prime factors of 564 = $2 \times 2 \times 47 \times 3$

The prime factors of 444 = $2 \times 2 \times 3 \times 37$

Common factors = 2, 2, 3, 37

Non common factors = 2, 47

L.C.M = Common factors $\times$ Non
 common factors

$$= 2 \times 2 \times 3 \times 37 \times 2 \times 47$$

$$= 41,736$$

f. 246, 378, 984

Solution:

First we find prime factorizations,

$\begin{array}{r} 2 \overline{)246} \\ 3 \overline{)123} \\ 41 \end{array}$	$\begin{array}{r} 2 \overline{)378} \\ 3 \overline{)189} \\ 3 \overline{)63} \\ 3 \overline{)21} \\ 7 \end{array}$	$\begin{array}{r} 2 \overline{)984} \\ 2 \overline{)492} \\ 2 \overline{)246} \\ 3 \overline{)123} \\ 41 \end{array}$
---	--	---

The prime factors of 246 = $2 \times 3 \times 41$

The prime factors of 378 = $2 \times 3 \times 3 \times 3 \times 7$

The prime factors of 984 = $2 \times 2 \times 2 \times 3 \times 41$

Common factors = 2, 3, 41

Non common factors = 3, 3, 7, 2, 2

L.C.M = Common factors $\times$ Non
 common factors

$$= 2 \times 3 \times 41 \times 3 \times 3 \times 7 \times 2 \times 2$$

$$= 61992$$

g. 112, 142

Solution:

First we find prime factorizations,

$\begin{array}{r} 2 \overline{)112} \\ 2 \overline{)56} \\ 2 \overline{)28} \\ 2 \overline{)14} \\ 7 \end{array}$	$\begin{array}{r} 2 \overline{)142} \\ 71 \end{array}$
---	--

The prime factors of 112 = $2 \times 2 \times 2 \times 2 \times 7$

The prime factors of 142 = 2×71

Common factors = 2

Non Common factors = 2, 2, 2, 7, 71

L.C.M = Common factors $\times$ Non
 common factors

$$= 2 \times 2 \times 2 \times 2 \times 7 \times 71$$

$$= 7,952$$

h. 764, 986, 130

Solution:

First we find prime factorizations,

$\begin{array}{r} 2 \overline{)764} \\ 2 \overline{)382} \\ 191 \end{array}$	$\begin{array}{r} 2 \overline{)986} \\ 493 \end{array}$	$\begin{array}{r} 2 \overline{)130} \\ 5 \overline{)65} \\ 13 \end{array}$
--	---	--

The prime factors of 764 = $2 \times 2 \times 191$

The prime factors of 986 = 2×493

The prime factors of 130 = $2 \times 5 \times 13$

Common factors = 2

Non Common factors = 2, 191, 493, 5, 13

L.C.M = Common factors $\times$ Non
 common factors

$$= 2 \times 2 \times 191 \times 493 \times 5 \times 13$$

$$= 24,482,380$$

i. 864, 789, 556

Solution:

First we find prime factorizations

$\begin{array}{r} 2 \overline{)864} \\ 2 \overline{)432} \\ 2 \overline{)216} \\ 2 \overline{)108} \\ 2 \overline{)54} \\ 3 \overline{)27} \\ 3 \overline{)9} \\ 3 \end{array}$	$\begin{array}{r} 3 \overline{)789} \\ 263 \end{array}$	$\begin{array}{r} 2 \overline{)556} \\ 2 \overline{)278} \\ 139 \end{array}$
---	---	--

The prime factors of 864 = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

The prime factors of 789 = 3×263

The prime factors of 556 = $2 \times 2 \times 139$

Common factors = 2, 2, 3

Non common factors = 2, 2, 2, 3, 3, 263, 139

L.C.M = Common factors $\times$ Non
 common factors

$$= 2 \times 2 \times 3 \times 2 \times 2 \times 2 \times 3 \times 3 \times 263 \times 139$$

$$= 31,585,248$$

Q4: Find the LCM of the given
 number using division method.

a. 280, 250, 290

Solution:

$\begin{array}{r} 2 \overline{)280, 250, 290} \\ 5 \overline{)140, 125, 145} \\ 2 \overline{)28, 25, 29} \\ 7 \overline{)7, 25, 29} \\ 5 \overline{)1, 25, 29} \\ 5 \overline{)1, 5, 29} \\ 29 \overline{)1, 1, 29} \\ 1, 1, 1 \end{array}$

L.C.M = Product of prime factors
 $= 2 \times 5 \times 2 \times 2 \times 7 \times 5 \times 5 \times 29$
 $= 203,000$

b. 456, 230, 900

Solution:

$\begin{array}{r} 2 \overline{)456, 230, 900} \\ 5 \overline{)228, 115, 450} \\ 2 \overline{)228, 23, 90} \\ 2 \overline{)114, 23, 45} \\ 3 \overline{)57, 23, 45} \\ 3 \overline{)19, 23, 5} \\ 19 \overline{)19, 23, 1} \\ 23 \overline{)1, 23, 1} \\ 1, 1, 1 \end{array}$
--

L.C.M = Product of Prime factors
 $= 2 \times 5 \times 2 \times 2 \times 3 \times 3 \times 5 \times 19 \times 23$
 $= 786,600$

c. 150, 560, 450

Solution:

$\begin{array}{r} 2 \overline{)150, 560, 450} \\ 5 \overline{)75, 280, 225} \\ 3 \overline{)15, 56, 45} \\ 5 \overline{)5, 56, 15} \\ 3 \overline{)1, 56, 3} \\ 2 \overline{)1, 56, 1} \\ 2 \overline{)1, 28, 1} \\ 2 \overline{)1, 14, 1} \\ 7 \overline{)1, 7, 1} \\ 1, 1, 1 \end{array}$

L.C.M = Product of prime factors
 $= 2 \times 5 \times 3 \times 5 \times 3 \times 2 \times 2 \times 7$
 $= 25,200$

d. 789, 648, 832

Solution:

2	789, 648, 832
2	789, 324, 416
2	789, 162, 208
2	789, 81, 104
2	789, 81, 52
2	789, 81, 26
3	789, 81, 13
3	263, 27, 13
3	263, 9, 13
3	263, 3, 13
263	263, 1, 13
13	1, 1, 13
	1, 1, 1

L.C.M = Product of prime factors
= 17,724,096

e. 898, 986, 786

Solution:

2	898, 986, 786
3	449, 493, 393
17	449, 493, 131
	449, 29, 131

L.C.M = Product of prime factors
= $2 \times 3 \times 17 \times 449 \times 29 \times 131$
= 173,986,602

f. 446, 340, 850

Solution:

2	446, 340, 850
3	223, 170, 425
5	223, 85, 425
5	223, 17, 85
17	223, 17, 17
	223, 1, 1

L.C.M = Product of prime factors
= $2 \times 2 \times 5 \times 5 \times 17 \times 223$
= 379,100

g. 908, 682, 672

Solution:

2	908, 682, 672
2	454, 341, 336
2	227, 341, 168
2	227, 341, 84
2	227, 341, 42
3	227, 341, 21
7	227, 341, 7
11	227, 341, 1
31	227, 31, 1
227	227, 1, 1
	1, 1, 1

L.C.M = Product of prime factors
= $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7 \times 11 \times 31 \times 227$
= 52,017,504

h. 408, 126, 522

Solution:

2	408, 126, 522
2	204, 63, 261
2	102, 63, 261
3	51, 63, 261
3	17, 21, 87
7	17, 7, 29
17	17, 1, 29
29	1, 1, 29
	1, 1, 1

L.C.M = Product of prime factors
= $2 \times 2 \times 2 \times 3 \times 7 \times 17 \times 29$
= 248,472

i. 765, 635, 235

Solution:

5	765, 635, 235
3	153, 127, 47
3	51, 127, 47
17	17, 127, 47
47	1, 127, 47
127	1, 127, 1
	1, 1, 1

L.C.M = Product of prime factors
= $5 \times 3 \times 3 \times 17 \times 47 \times 127$
= 4,566,285

Q5: The LCM of two numbers is 456 and HCF is 6. If one number is 24 then find the other number.

Solution:

Given that L.C.M = 456

And H.C.F = 6

a = 24

b = ?

As we know that

H.C.F $\times$ L.C.M = a $\times$ b

By putting values

 $6 \times 456 = 24 \times b$ $2736 = 24b$ $\Rightarrow b = \frac{2736}{24}$

b = 114 Ans.

Q6: The HCF of two numbers is 7 and LCM is 588. Find the product of two numbers.

Solution:

Given that HCF = 7

LCM = 588

ab = ?

As we know that

a $\times$ b = H.C.F $\times$ L.C.M

By putting values

ab = 7 $\times$ 588

ab = 4116 Ans.

Q7: The HCF of two numbers is 4 and the product of the two numbers is 512. Find the LCM of the numbers.

Solution:

Given that HCF = 4

a $\times$ b = 512

L.C.M = ?

As we know that

a $\times$ b = H.C.F $\times$ L.C.M

By putting values

 $512 = 4 \times \text{L.C.M}$ $\Rightarrow \text{L.C.M} = \frac{512}{4}$

L.C.M = 128

Exercise 1.4

Q1: Find the minimum capacity of a water tank that can completely measure the amounts of 460 l, 245 l and 525 l.

Solution:

To find the minimum capacity of a water tank, we have to find L.C.M.

5	460, 245, 525
7	92, 49, 105
2	92, 7, 15
2	46, 7, 15
3	23, 7, 15
7	23, 7, 5
5	23, 1, 5
23	23, 1, 1
	1, 1, 1

So L.C.M = Product of prime factors
= $5 \times 7 \times 2 \times 2 \times 3 \times 7 \times 5 \times 23$
= 338,100 l.

Q2: Find the greatest number that can divide 460, 648 and 984.

Solution:

To find greatest number we have to find H.C.F.

2	460	2	648	2	984
2	230	2	324	2	492
5	115	2	162	2	246
23	23	3	81	3	123
	1	3	27		41
		3	9		
		3	3		
			1		

Prime factors of 460 = $2 \times 2 \times 5 \times 23$ Prime factors of 648 = $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$ Prime factors of 984 = $2 \times 2 \times 2 \times 3 \times 41$

Common factors = 2, 2

H.C.F = $2 \times 2 = 4$ **Q3: Find the smallest number which exactly divide 406, 348 and 256.****Solution:**

To find smallest number which exactly divided 406, 348, 256. we can find H.C.F.

2	406
7	203
29	29
	1

2	348
2	174
3	87
29	29
	1

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factors of 406 = $2 \times 7 \times 29$ Prime factors of 348 = $2 \times 2 \times 3 \times 29$ Prime factors of 256 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

H.C.F = 2

So required number = 2

Q4: The LCM of two numbers is 96 and HCF is 24. If one number is 32 then find the other number.**Solution:**

Given L.C.M = 96

H.C.F = 24

 $a = 32, b = ?$

As we know that

 $a \times b = H.C.F \times L.C.M$ $32 \times b = 24 \times 96$

$$b = \frac{24 \times 96}{32}$$

 $b = 72$ Ans.**Q5: The HCF of two numbers is 18 and LCM is 75. Find the product of two numbers.****Solution:**

Given H.C.F = 18

L.C.M = 75

 $a \times b = ?$

As we know that

 $a \times b = H.C.F \times L.C.M$ $= 18 \times 75$ $a \times b = 1350$ **Q6: LCM of two numbers is 678 and the two numbers is 4746. Find the HCF of the numbers.****Solution:**

Given L.C.M = 678

 $a \times b = 4746$

H.C.F = ?

As we know that

 $H.C.F \times L.C.M = a \times b$ $H.C.F \times 678 = 4746$ $H.C.F = \frac{4746}{678}$

H.C.F = 7 Ans.

Q7: Umar has two pieces of ropes. One is 18 metres long and the other is 112 metres long. He wants to cut it into pieces that are all of the same length, without any remainder. What is the greatest length that he can cut them into?**Solution:**

To find the greatest length that he can cut them into equal parts, we can find H.C.F of 18, 112.

2	18
2	9
3	3
	1

2	112
2	56
2	28
2	14
7	7
	1

Prime factors of = $2 \times 3 \times 3$ Prime factors of 112 = $2 \times 2 \times 2 \times 2 \times 7$

Common factors = 2

So H.C.F = 2

Length of each piece = 2m

Now as (length of each piece) $\times$ number of piece $=$ Total length of rope $\Rightarrow$ Number of pieces = $\frac{\text{Total length of rope}}{\text{length of each piece}}$

When total length of rope = 18m, then

$$\text{Number of pieces} = \frac{18}{2}$$

$$= 9 \text{ pieces}$$

When total length of rope = 112m

Then

$$\text{Number of pieces} = \frac{112}{2}$$

$$= 56 \text{ pieces}$$

Q8: For an Iftar party, Aiza wants to serve chicken bread and nuggets to the guests invited. She wants to do it in such a way that each serving contains the same combination of chicken bread and nuggets. If there are 24 slices of chicken bread and 120 nuggets, find the maximum number of servings she can prepare.**Solution:**

To find maximum number of servings that she can prepare.

We can find H.C.F of 24, 120.

2	24
2	12
2	6
3	3
	1

2	120
2	60
2	30
3	15
5	5
	1

Prime factors of 24 = $2 \times 2 \times 2 \times 3$ Prime factors of 120 = $2 \times 2 \times 2 \times 3 \times 5$

Common factors = 2, 2, 2, 3

So H.C.F = $2 \times 2 \times 2 \times 3$
 $= 24$ **Q9: Nida goes for a walk every 10 days and Sehrish goes for a walk every 25 days to the same park. If they both went for a walk today, then how many days from now will they next be in the park on the same day?****Solution:**

To find walk in the park on the same day, we find H.C.F.

2	10
5	5
	1

5	25
5	5
	1

Prime factors of 10 = 2×5 Prime factors of 25 = 5×5

Common factor = 5

So H.C.F = 5

So Next walk in the park on the same day after 5 days.

Q10: Find the smallest number which is exactly divisible by 40, 224 and 400 without leaving any remainder.**Solution:**

2	40, 224, 400
2	20, 112, 200
2	10, 56, 100
2	5, 28, 50
2	5, 14, 25
5	5, 7, 25
7	1, 7, 5
5	1, 1, 5
	1, 1, 1

L.C.M = Product of prime factors
 $= 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 7 \times 5$
 $= 5600$

Exercise 1.5**Q1: Find the square of the following numbers.****a. 25****Solution:**

$$\text{Square of } 25 = 25^2 = 25 \times 25 = 625$$

b. 77**Solution:**

$$\text{Square of } 77 = 77^2 = 77 \times 77 = 5929$$

c. 98**Solution:**

$$\text{Square of } 98 = 98^2 = 98 \times 98 = 9604$$

d. 11**Solution:**

$$\text{Square of } 11 = 11^2 = 11 \times 11 = 121$$

e. 46**Solution:**

$$\text{Square of } 46 = 46^2 = 46 \times 46 = 2116$$

f. 69**Solution:**

$$\text{Square of } 69 = 69^2 = 69 \times 69 = 4761$$

g. 13**Solution:**

$$\text{Square of } 13 = 13^2 = 13 \times 13 = 169$$

h. 37**Solution:**

$$\text{Square of } 37 = 37^2 = 37 \times 37 = 1369$$

i. 45**Solution:**

$$\text{Square of } 45 = 45^2 = 45 \times 45 = 2025$$

j. 79**Solution:**

$$\text{Square of } 79 = 79^2 = 79 \times 79 = 6241$$

k. 67**Solution:**

$$\text{Square of } 67 = 67^2 = 67 \times 67 = 4489$$

l. 50**Solution:**

$$\text{Square of } 50 = 50^2 = 50 \times 50 = 2500$$

Q2: A glass is cut into square shape. If the length of the glass is 98 cm, what is the area of the surface of the glass? (Hint: Area of Square = Length $\times$ Length)

Solution:

$$\text{Length of the glass} = 98 \text{ cm}$$

As we know that

$$\text{Area} = \text{length} \times \text{length}$$

By putting values,

$$\text{Area} = 98 \times 98$$

$$= 9604 \text{ cm}^2$$

Review Exercise**Q1: Circle the correct option.****a. The prime factors of 24 are ____.****i. $2 \times 2 \times 2 \times 3$ ✓****ii. $2 \times 2 \times 3 \times 3$** **iii. $2 \times 2 \times 2 \times 2$** **iv. $3 \times 3 \times 2 \times 3$** **b. The HCF to two or more prime numbers is equal to their ____****i. sum****ii. product****iii. difference****iv. one ✓****c. The LCM of two or more prime numbers is equal to their ____****i. sum****ii. product ✓****iii. difference****iv. quotient****d. The $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$ in exponent notation is ____****i. $2^5 \times 3^2 \times 5^2$** **ii. $2^4 \times 3^2 \times 5$** **iii. $2^5 \times 3 \times 5$** **iv. $2^5 \times 3^2 \times 5$ ✓****e. The product of two numbers is equal to the product of their LCM and ____****i. first number****ii. second number****iii. common multiple****iv. HCF ✓****Q2: Define the following terms:****a. Factors****b. Multiples****c. Prime Factorization****d. Highest Common Factors****e. Least Common Factors****Solution:****a. Factors:**

Factors is a number that divides another number without leaving any remainder.

b. Multiples:

Multiples are the numbers obtained by multiplying a certain number by another number.

c. Prime Factorization:

Prime Factorization is a process in which a number can be written as a product of its prime factors.

d. Highest Common Factors:

Highest common factor of two or more numbers is the greatest factor that divides the given numbers exactly without any remainder.

e. Least Common Multiple:

Least common multiple of two or more numbers is the smallest common multiple of given numbers.

Q3: Find the factors of the following numbers.**a. 78****Solution:**

We can find factors of 78 as:

$$78 = 1 \times 78$$

$$78 = 2 \times 39$$

$$78 = 3 \times 26$$

$$78 = 6 \times 13$$

So factors of 78 are 1, 2, 3, 6, 13, 26, 39, 78

b. 840**Solution:**

We can find factors of 840 as

$$840 = 1 \times 840$$

$$840 = 2 \times 420$$

$$840 = 3 \times 280$$

$$840 = 4 \times 210$$

$$840 = 5 \times 168$$

$$840 = 6 \times 140$$

$$840 = 7 \times 120$$

$$840 = 8 \times 105$$

$$840 = 10 \times 84$$

$$840 = 12 \times 70$$

$$840 = 14 \times 60$$

$$840 = 15 \times 56$$

$$840 = 20 \times 42$$

$$840 = 21 \times 40$$

$$840 = 24 \times 35$$

$$840 = 28 \times 30$$

So factors of 840 are 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 14, 15, 20, 21, 24, 28, 30, 35, 40, 42, 56, 60, 70, 84, 105, 120, 140, 168, 210, 280, 420, 840

c. 144**Solution:**

We can find factors of 144 as:

$$144 = 1 \times 144$$

$$144 = 2 \times 72$$

$$144 = 3 \times 48$$

$$144 = 4 \times 36$$

$$144 = 6 \times 24$$

$$144 = 8 \times 18$$

$$144 = 9 \times 16$$

$$144 = 12 \times 12$$

So factors of 144 are 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

d. 705

Solution:

We can find factors of 705 as:

We can find factors of 705 as

$$705 = 1 \times 705$$

$$705 = 3 \times 235$$

$$705 = 5 \times 141$$

$$705 = 15 \times 47$$

So factors of 705 are 1, 3, 5, 15, 47, 141, 235, 705.

Q4: Find the multiples of the given number less than 250.

a. 18

Solution:

As $1 \times 18 = 18$, $2 \times 18 = 36$, $3 \times 18 = 54$,
 $4 \times 18 = 72$, $5 \times 18 = 90$, $6 \times 18 = 108$,
 $7 \times 18 = 126$, $8 \times 18 = 144$, $9 \times 18 = 162$,
 $10 \times 18 = 180$, $11 \times 18 = 198$,
 $12 \times 18 = 216$, $13 \times 18 = 234$

So Multiples of 18 less 250 are 18, 36, 54, 72, 90, 108, 126, 144, 162, 180, 198, 216, 234.

b. 30

Solution:

As $1 \times 30 = 30$, $2 \times 30 = 60$,
 $3 \times 30 = 90$, $4 \times 30 = 120$,
 $5 \times 30 = 150$, $6 \times 30 = 180$,
 $7 \times 30 = 210$, $8 \times 30 = 240$

So Multiple of 30 less than 250 are 30, 60, 90, 120, 150, 180, 210, 240.

c. 55

Solution:

As $1 \times 55 = 55$, $2 \times 55 = 110$,
 $3 \times 55 = 165$, $4 \times 55 = 220$,

So, Multiple of 55 less than 250 are 55, 110, 165, 220.

d. 82

Solution:

$$\text{As } 1 \times 82 = 82 \quad 2 \times 82 = 164$$

$$3 \times 82 = 246$$

So, Multiples of 82 are 82, 164, 246.

Q5: Find the prime factors of the following numbers and write it in the form of index notation.

a. 566

Solution:

$$\begin{array}{r|l} 2 & 566 \\ & 283 \end{array}$$

Prime factors of 566 = 2×283

Index Notation = $2^1 \times 283^1$

b. 788

Solution:

$$\begin{array}{r|l} 2 & 788 \\ 2 & 394 \\ & 197 \end{array}$$

Prime factors of 788 = $2 \times 2 \times 197$

Index Notation = $2^2 \times 197^1$

c. 1000

Solution:

$$\begin{array}{r|l} 2 & 1000 \\ 2 & 500 \\ 2 & 250 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array}$$

Prime factors of 1000 = $2 \times 2 \times 2 \times 5 \times 5 \times 5$

Index Notation = $2^3 \times 5^3$

d. 3088

Solution:

$$\begin{array}{r|l} 2 & 3088 \\ 2 & 1544 \\ 2 & 772 \\ 2 & 386 \\ & 193 \end{array}$$

Prime factors of 3088 = $2 \times 2 \times 2 \times 2 \times 193$

Index Notation = $2^4 \times 193^1$

e. 8777

Solution:

$$\begin{array}{r|l} 8777 & 8777 \\ & 1 \end{array}$$

Prime factors of 8777 = 1×8777

Index Notation = $1^1 \times 8777^1$

f. 4026

Solution:

$$\begin{array}{r|l} 2 & 4026 \\ 3 & 2013 \\ 11 & 671 \\ 61 & 61 \\ & 1 \end{array}$$

Prime factors 4026 = $2 \times 3 \times 11 \times 61$

Index Notation = $2^1 \times 3^1 \times 11^1 \times 61^1$

g. 6054

Solution:

$$\begin{array}{r|l} 2 & 6054 \\ 3 & 3027 \\ 1009 & 1009 \\ & 1 \end{array}$$

Prime factors 6054 = $2 \times 3 \times 1009$

Index Notation = $2^1 \times 3^1 \times 2009^1$

h. 5478

Solution:

$$\begin{array}{r|l} 2 & 5478 \\ 3 & 2739 \\ 11 & 913 \\ 83 & 83 \\ & 1 \end{array}$$

Prime factors of 5478 = $2 \times 3 \times 11 \times 83$

Index Notation = $2^1 \times 3^1 \times 11^1 \times 83^1$

Q6: Write the following in expanded form.

a. 5^6

Solution:

Expanded form of $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

b. $2^3 \times 5^5$

Solution:

Expanded form of $2^3 \times 5^5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5$

c. $3^7 \times 7^2 \times 11^4$

Solution:

Expanded form $3^7 \times 7^2 \times 11^4 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 11 \times 11 \times 11 \times 11$

d. $1^2 \times 5^3 \times 3^6$

Solution:

Expanded form of $1^2 \times 5^3 \times 3^6 = 1 \times 1 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Q7: Find the HCF of the following numbers using prime factorization method.

a. 455, 254

Solution:

$$\begin{array}{r|l} 5 & 455 \\ 7 & 91 \\ 13 & 13 \\ & 1 \end{array} \quad \begin{array}{r|l} 5 & 254 \\ 127 & 127 \\ & 1 \end{array}$$

Prime factors of 455 = $5 \times 7 \times 13$

Prime factors of 254 = 2×127

As there is no common factors,

So H.C.F = '1'.

b. 569, 369, 545

Solution:

$$\begin{array}{r|l} 569 & 569 \\ & 1 \end{array} \quad \begin{array}{r|l} 3 & 369 \\ 3 & 123 \\ 41 & 41 \\ & 1 \end{array} \quad \begin{array}{r|l} 8 & 545 \\ 109 & 109 \\ & 1 \end{array}$$

Prime factors of 569 = 1×569

Prime factors of 369 = $3 \times 3 \times 41$

Prime factors of 545 = 5×109

As there is no common factors,

So H.C.F = 1

c. 690, 468

Solution:

$$\begin{array}{r|l} 2 & 690 \\ 3 & 345 \\ 5 & 115 \\ 23 & 23 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 468 \\ 2 & 234 \\ 3 & 117 \\ 3 & 39 \\ 13 & 13 \\ & 1 \end{array}$$

Prime factors of 690 = $2 \times 3 \times 5 \times 23$

Prime factors of 468 = $2 \times 2 \times 3 \times 3 \times 13$

Common factors = 2, 3

So H.C.F = 2×3
= 6

d. 255, 751, 155

Solution:

$$\begin{array}{r|l} 5 & 255 \\ 3 & 51 \\ 17 & 17 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 7 & 751 \\ & 1 \\ \hline & \end{array} \quad \begin{array}{r|l} 5 & 155 \\ 31 & 31 \\ \hline & 1 \end{array}$$

Prime factors of 255 = $3 \times 5 \times 17$

Prime factors of 751 = 1×751

Prime factors of 155 = 5×31

As there is no common factors,

So H.C.F = 1.

Q8. Find the HCF of the following numbers by division method.

a. 256, 164, 144

Solution:

$$\begin{array}{r} 1 \\ 164 \overline{) 256} \\ \underline{164} \quad 1 \\ 112 \overline{) 164} \\ \underline{112} \quad 2 \\ 52 \overline{) 112} \\ \underline{52} \quad 2 \\ 104 \overline{) 112} \\ \underline{8} \quad 6 \\ 8 \overline{) 52} \\ \underline{48} \quad 2 \\ 4 \overline{) 8} \\ \underline{4} \quad 8 \\ 8 \overline{) 8} \\ \underline{8} \quad 0 \end{array}$$

Now we find H.C.F of 4 and 144.

$$\begin{array}{r} 36 \\ 4 \overline{) 144} \\ \underline{12} \quad 24 \\ 24 \overline{) 24} \\ \underline{24} \quad 0 \end{array}$$

So H.C.F of 256, 164 and 144 = 4

b. 178, 278, 240

Solution:

$$\begin{array}{r} 1 \\ 178 \overline{) 278} \\ \underline{178} \quad 1 \\ 100 \overline{) 178} \\ \underline{100} \quad 78 \\ 78 \overline{) 100} \\ \underline{78} \quad 22 \\ 18 \overline{) 78} \\ \underline{18} \quad 66 \\ 22 \overline{) 66} \\ \underline{22} \quad 1 \\ 10 \overline{) 22} \\ \underline{10} \quad 12 \\ 10 \overline{) 12} \\ \underline{10} \quad 2 \\ 2 \overline{) 10} \\ \underline{10} \quad 0 \end{array}$$

Now we find H.C.F of 2 and 240

So,

$$\begin{array}{r} 120 \\ 2 \overline{) 240} \\ \underline{2} \quad 0 \\ 4 \overline{) 0} \\ \underline{4} \quad 0 \end{array}$$

So H.C.F of 178, 278, 240 = 2

c. 448, 385, 245

Solution:

$$\begin{array}{r} 1 \\ 385 \overline{) 448} \\ \underline{385} \quad 63 \\ 63 \overline{) 385} \\ \underline{63} \quad 378 \\ 378 \overline{) 385} \\ \underline{378} \quad 9 \\ 7 \overline{) 63} \\ \underline{7} \quad 63 \\ 63 \overline{) 63} \\ \underline{63} \quad 0 \end{array}$$

Now we find H.C.F of 7, 245

$$\begin{array}{r} 35 \\ 7 \overline{) 245} \\ \underline{21} \quad 35 \\ 35 \overline{) 35} \\ \underline{35} \quad 0 \end{array}$$

So H.C.F of 448, 385, 245 = 7

d. 212, 380, 180

Solution:

$$\begin{array}{r} 1 \\ 212 \overline{) 380} \\ \underline{212} \quad 168 \\ 168 \overline{) 212} \\ \underline{168} \quad 44 \\ 44 \overline{) 168} \\ \underline{44} \quad 132 \\ 36 \overline{) 44} \\ \underline{36} \quad 8 \\ 8 \overline{) 36} \\ \underline{8} \quad 32 \\ 32 \overline{) 36} \\ \underline{32} \quad 4 \\ 4 \overline{) 8} \\ \underline{4} \quad 8 \\ 8 \overline{) 8} \\ \underline{8} \quad 0 \end{array}$$

Now divide 180 by 4

$$\begin{array}{r} 45 \\ 4 \overline{) 180} \\ \underline{16} \quad 20 \\ 20 \overline{) 20} \\ \underline{20} \quad 0 \end{array}$$

So H.C.F of 448, 385 and 245 = 4

Q9: Find the LCM of the following numbers using prime factorization method.

a. 600, 345, 476

Solution:

$$\begin{array}{r|l} 2 & 600 \\ 2 & 300 \\ 2 & 150 \\ 5 & 75 \\ 3 & 15 \\ 5 & 5 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 345 \\ 5 & 115 \\ 23 & 23 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 2 & 476 \\ 2 & 238 \\ 7 & 119 \\ 17 & 17 \\ \hline & 1 \end{array}$$

Prime factors of 600 = $2 \times 2 \times 2 \times 3 \times 5 \times 5$

Prime factors of 345 = $3 \times 5 \times 23$

Prime factors of 476 = $2 \times 2 \times 7 \times 17$

Common factors = 2, 2, 3, 5

Non common factors = 2, 5, 23, 7, 17

L.C.M = Common factors $\times$ Non common factors

$$= 2 \times 2 \times 3 \times 5 \times 2 \times 5 \times 23 \times 7 \times 17$$

$$= 1642,200$$

b. 620, 434, 545

Solution:

$$\begin{array}{r|l} 2 & 620 \\ 2 & 310 \\ 5 & 155 \\ 31 & 31 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 2 & 434 \\ 7 & 217 \\ 31 & 31 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 5 & 545 \\ 19 & 19 \\ \hline & 1 \end{array}$$

Prime factors of 620 = $2 \times 2 \times 5 \times 31$

Prime factors of 434 = $2 \times 7 \times 31$

Prime factors of 545 = 5×19

Common factors = 2, 5, 31

Non common factors = 2, 7, 19

L.C.M = common factor $\times$ Non common factor

$$= 2 \times 5 \times 31 \times 2 \times 7 \times 19$$

$$= 82,460$$

c. 156, 135, 215

Solution:

$$\begin{array}{r|l} 2 & 156 \\ 2 & 78 \\ 3 & 39 \\ 13 & 13 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 3 & 135 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ \hline & 1 \end{array} \quad \begin{array}{r|l} 5 & 215 \\ 43 & 43 \\ \hline & 1 \end{array}$$

Prime factors of 156 = $2 \times 2 \times 3 \times 13$

Prime factors of 135 = $3 \times 3 \times 3 \times 5$

Prime factors of 215 = 5×43

Common factors = 3, 3, 5

Non common factors = 2, 2, 3, 43

L.C.M = Common factors $\times$ Non Common factor

$$= 3 \times 3 \times 5 \times 2 \times 2 \times 3 \times 43$$

$$= 23,220$$

d. 165, 425, 725

Solution:

$$\begin{array}{r|l} 3 & 165 \\ 5 & 55 \\ \hline & 11 \end{array} \quad \begin{array}{r|l} 5 & 425 \\ 5 & 85 \\ \hline & 17 \end{array} \quad \begin{array}{r|l} 5 & 725 \\ 5 & 145 \\ \hline & 29 \end{array}$$

Prime factors of 165 = $5 \times 3 \times 11$

Prime factors of 425 = $5 \times 5 \times 17$

Prime factors of 725 = $5 \times 5 \times 29$

Common factors = 5, 5

Non common factors = 3, 11, 14, 29

L.C.M = Common factors $\times$ Non common factors

= $5 \times 5 \times 3 \times 11 \times 17 \times 29$

= 406,725

Q10: Find the LCM of the following numbers using prime factorization method.

a. 212, 404, 68

Solution:

$$\begin{array}{r|l} 2 & 212, 404, 68 \\ 2 & 106, 202, 34 \\ 2 & 53, 101, 17 \end{array}$$

L.C.M = Product of prime factors

= $2 \times 2 \times 53 \times 101 \times 17$

= 364004

b. 306, 428, 845

Solution:

$$\begin{array}{r|l} 2 & 306, 428, 845 \\ 2 & 153, 211, 845 \\ 3 & 153, 107, 845 \\ 3 & 51, 107, 845 \\ 5 & 17, 107, 845 \\ 17 & 17, 107, 169 \\ 107 & 1, 107, 169 \\ 13 & 1, 1, 169 \\ 13 & 1, 1, 13 \\ & 1, 1, 1 \end{array}$$

L.C.M = Product of prime factors

= $2 \times 2 \times 3 \times 3 \times 5 \times 17 \times 107 \times 13 \times 13$

= 55,333,980

c. 340, 146, 456

Solution:

$$\begin{array}{r|l} 2 & 340, 146, 456 \\ 2 & 170, 76, 228 \\ 2 & 85, 73, 114 \\ 5 & 85, 73, 57 \\ 3 & 17, 73, 57 \\ 17 & 17, 73, 19 \\ 73 & 1, 73, 19 \\ 19 & 1, 1, 19 \\ & 1, 1, 1 \end{array}$$

L.C.M = Product of prime factors.

= $2 \times 2 \times 2 \times 5 \times 3 \times 17 \times 73 \times 19$

= 2,829,480

d. 560, 956, 890

Solution:

$$\begin{array}{r|l} 2 & 560, 956, 890 \\ 2 & 280, 478, 445 \\ 5 & 140, 239, 445 \\ 2 & 28, 239, 89 \\ 2 & 14, 239, 89 \\ 7 & 7, 239, 89 \\ 89 & 1, 239, 89 \\ 239 & 1, 239, 1 \\ & 1, 1, 1 \end{array}$$

L.C.M = Product of prime factors

= $2 \times 2 \times 5 \times 2 \times 2 \times 7 \times 78 \times 239$

= 11,911,760.

Q11: Find square of the following numbers.

a. 16

Solution:

Square of 16 = $16^2 = 16 \times 16$
= 256

b. 12

Solution:

Square of 12 = $12^2 = 12 \times 12$
= 144

c. 44

Solution:

Square of 44 = $44^2 = 44 \times 44$
= 1936

d. 62

Solution:

Square of 62 = $62^2 = 62 \times 62$
= 3844

Q12: The length, breadth and height of a room is 680 cm, 510 cm and 340 cm respectively. Find the longest tape which can measure the dimensions of the room exactly.

Solution:

To find the longest tape, we can find H.C.F of 680, 510, 340.

$$\begin{array}{r|l} 2 & 680 \\ 2 & 340 \\ 2 & 170 \\ 5 & 85 \\ 17 & 17 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 510 \\ 3 & 255 \\ 5 & 85 \\ 17 & 17 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 340 \\ 2 & 170 \\ 5 & 85 \\ 17 & 17 \\ & 1 \end{array}$$

Prime factors of 680 = $2 \times 2 \times 2 \times 5 \times 17$

Prime factors of 510 = $2 \times 3 \times 5 \times 17$

Prime factors of 340 = $2 \times 2 \times 5 \times 17$

Common factors = 2, 5, 7

H.C.F = Product of common factors

H.C.F = $2 \times 5 \times 17$

= 170cm

So the longest tap = 170cm.

Q13: A light flashes every 12 seconds, another every 18 seconds and a third every single minute. At 2:45 am, the three flashed simultaneously. What time the three lights will again flash simultaneously?

Solution:

To find the time that three lights will again flash simultaneously, we can find H.C.F of 12 sec, 18 sec and 1 min = 60 sec.

$$\begin{array}{r|l} 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 18 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array}$$

Prime factors of 12 = $2 \times 2 \times 3$

Prime factors of 18 = $2 \times 3 \times 3$

Prime factors of 60 = $2 \times 2 \times 3 \times 5$

Common factors = 2, 3

H.C.F of 12, 18, 60 = $2 \times 3 = 6$ sec

So after the time 2:45 + 6sec = 2:51a.m the three lights will flash again simultaneously.

Q14: Find the greatest number which exactly divides 145, 360 and 578.

Solution:

To find greatest number use can find H.C.F of 145, 360, 578.

$$\begin{array}{r|l} 5 & 145 \\ 29 & 29 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 360 \\ 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ 5 & 5 \\ & 1 \end{array} \quad \begin{array}{r|l} 2 & 578 \\ 17 & 289 \\ 17 & 17 \\ & 1 \end{array}$$

Prime factors of 145 = 5×29

Prime factors of 360 = $2 \times 2 \times 2 \times 3 \times 3 \times 5$

Prime factors of 578 = $2 \times 17 \times 17$

As there is no common factors, So H.C.F = 1.

Q15: Find the smallest number which is exactly divisible by 200, 400 and 800.

Solution:

To find the smallest number which is exactly divisible by 200, 400, 800, we can find L.C.M.

2	200, 400, 800
2	100, 200, 400
2	50, 100, 200
2	25, 50, 100
2	25, 25, 50
5	25, 25, 25
5	5, 5, 5
	1, 1, 1

L.C.M = Product of Prime Factors
 $= 2 \times 2 \times 2 \times 2 \times 5 \times 5$
 $= 800$

Q16: Find the minimum length of ribbon that can be exactly cut into pieces of length 450cm, 750cm and 800cm.

Solution:

To find minimum length of ribbon, we can find L.C.M of 450, 750, and 800 cm.

2	450, 750, 800
2	225, 375, 400
2	225, 375, 200
2	225, 375, 100
2	225, 375, 50
5	225, 375, 25
5	45, 75, 5
3	9, 15, 1
3	3, 5, 1
5	1, 5, 1
	1, 1, 1

L.C.M = Product of prime factors
 $= 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 3 \times 3 \times 5$
 $= 3600$

Q17: If LCM of two numbers is 678 and product of two number is 4986. Find HCF of the numbers.

Solution:

Given LCM = 678
 $a \times b = 4986$
H.C.F = ?

As we know that
 $H.C.F \times L.C.M = a \times b$

$$H.C.F = \frac{a \times b}{L.C.M}$$

$$= \frac{4986}{678}$$

$$H.C.F = 7.35$$

Q18: If HCF of two numbers is 64 and LCM is 1024. Find the product of the two numbers.

Solution:

Given HCF = 64, LCM = 1024
 $a \times b = ?$

As we know that
 $a \times b = L.C.M \times H.C.F$

By putting value
 $a \times b = 1024 \times 64$
 $a \times b = 65536$

Q19: The HCF of two numbers is 20 and LCM is 5604. If one number is 467 then find the other number.

Solution:

Given HCF = 20
LCM = 5604

$a = 467, b = ?$

As we know that
 $a \times b = HCF \times LCM$

$$b = \frac{HCF \times LCM}{a}$$

By putting values

$$b = \frac{20 \times 5604}{467}$$

$$b = \frac{112080}{467}$$

$$b = 240 \text{ Ans}$$

Unit 2

Integers

Exercise 2.1

Q1: Circle the positive integers from the following.

+2, +10, -9, +11, -3, -5, +8, +9, -3, -23, 7, 10

Solution:

(+2), (+10), (+11), (+8), (+9), (7), (10)

Q2: Write the opposite integers of each of the following.

a. -5 Sol: +5	b. +11 Sol: -11
c. 0 Sol: 0	d. -9 Sol: +9
e. +14 Sol: -14	f. -23 Sol: +23
g. -19 Sol: +19	h. -8 Sol: +8

Q3: Enlist all the integers which come between the following.

a. -4 and +6

Solution:

-3, -2, -1, 0, +1, +2, +3, +4, +5

b. -7 and -1

Solution:

-6, -5, -4, -3, -2

c. -14 and +11

Solution:

-13, -12, -11, -10, -9, -8, -7, -6,

-5, -4, -3, -2, -1, 0, +1, -2, +3,

+4, +5, +6, +7, +8, +9, +10

d. -8 and -18

Solution:

-9, -10, -11, -12, -13, -14, -15,

-16, -17

e. -5 and +5

Solution:

-4, -3, -2, -1, 0, +1, +2, +3, +4
f. +6 and +8

Solution:

+7

g. -11 and -18

Solution:

-10, -9, -8, -7, -6, -5, -4, -3, -2,
-1, 0, +1, +2, +3, +4, +5, +6, +7,
+8, +9, +10, +11, +12, +13, +14,
+15, +16

h. -19 and +20

Solution:

-18, -17, -16, -15, -14, -13, -12,
-11, -10, -9, -8, -7, -6, -5, -4,
-3, -2, -1, 0, +1, +2, +3, +4, +5,
+6, +7, +8, +9, +10, +11, +12, +13,
+14, +15, +16, +17, +18, +19

Q4: Write the six integers which are:

a. $< +5$

Solution:

-1, 0, +1, +2, +3, +4

b. > -6

Solution:

0, -1, -2, -3, -4, -5

c. < -12

Solution:

-18, -17, -16, -15, -14, -13

d. > -16

Solution:

-10, -11, -12, -13, -14, -15

e. ≥ 10

Solution:

10, 11, 12, 13, 14, 15

f. ≤ 50

Solution:

45, 46, 47, 48, 49, 50

Q5: Write an integer for each situation.

a. an earning of Rs 12000

Solution:

+12000

b. a loan of Rs 700

Solution:

-700

c. 30° below 0°.

Solution:

-30°

d. a loss of Rs 720

Solution:

-720

e. a profit of Rs 100

Solution:

+100

f. 5695 metres below sea level

Solution:

-5695

g. Two degrees above zero

Solution:

+2

h. fifteen degrees below zero

Solution:

-15

i. 3 levels below ground level

Solution:

-3

j. 55 metres above sea level

Solution:

+55

Q6: Akram bought an air conditioner for Rs 65,000 and sells it for Rs 62,000. Express his loss or profit as an integer.

Solution:

Bought price of air conditioner = Rs. 65000

And sell price = Rs. 62,000

As sell price < Bought price, So there is loss

Loss = Bought price - sell price
= 65,000 - 62,000

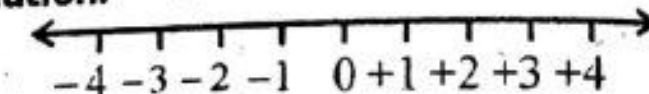
= Rs. 3000
Loss = -3000

Exercise 2.2

Q1: Represent the following sets of integers on a number line.

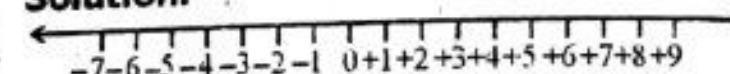
a. -4, to +4

Solution:



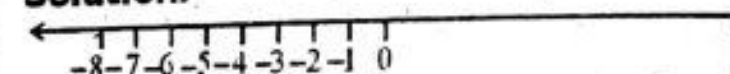
b. -7 to +9

Solution:



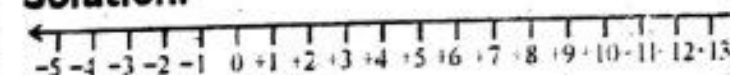
c. -8 to 0

Solution:



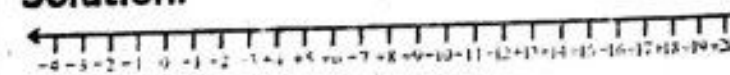
d. -5 to +13

Solution:



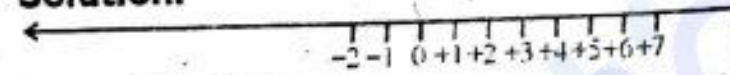
e. -4 to +20

Solution:



f. -2 to +7

Solution:



g. -4 to +5

Solution:



h. -6 to 0

Solution:



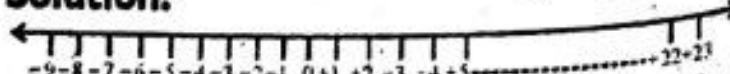
i. -3 to 20

Solution:



j. -9 to +23

Solution:



Q2: Which is greater? Give the reason.

a. -666 or -6666

Solution:

On the number line -6666 lies to the left of -666, so -666 is greater than -6666.

b. -2340 or -234

Solution:

On the number line -2340 lies to the left of -234, so -234 is greater than -2340.

c. -10000 or -1000

Solution:

On the number line -10000 lies to the left of -1000, so -1000 is greater than -10000.

d. -55 or -555

Solution:

On the number line -555 lies to the left of -55, so -55 is greater than -555.

Q3: Arrange the following integers in ascending order.

a. -5, +4, -13, -1, 0, +3, +7

Solution:

Arrangement in ascending order,

-13, -5, -1, 0, +3, +4, +7

b. -5, -3, +3, +2, +11, -12

Solution:

Arrangement in ascending order,

-12, -5, -3, +2, +3, +11

c. -33, -12, 0, +11, +9, +5, -16

Solution:

Arrangement in ascending order,

-33, -16, -12, 0, +5, +9, +11

d. -12, 0, +17, +11, +6, -8, -5

Solution:

Arrangement in ascending order,

-12, -8, -5, 0, +6, +11, +17

e. -3, 0, +1, +8, +7, -5, -4, +9

Solution:

Arrangement in ascending order,

-5, -4, -3, 0, +1, +7, +8, +9

f. -15, -11, 0, +23, +24, +27, -14

Solution:

Arrangement in ascending order,

-15, -14, -11, 0, +23, +24, +27

Q4: Arrange the following integers in descending order.

a. -5, -1, 0, 7, 9, -3, -2

Solution:

Arrangement in descending order,

+9, +7, 0, -1, -2, -3, -5

b. -11, -13, -8, +9, +12, +23

Solution:

Arrangement in descending order,

+23, +12, +9, -8, -11, -13

c. +24, +45, -17, 0, +10, -12, -20

Solution:

Arrangement in descending order,

+45, +24, +10, 0, -12, -17, -20

d. -9, -7, 0, +56, +78, +90, -45, -11

Solution:

Arrangement in descending order,

+90, +78, +56, 0, -7, -8, -9

e. -8, -1, +4, +1, +10, -10, -9

Solution:

Arrangement in descending order,

+10, +4, +1, -1, -8, -9, -10

f. -34, -78, 0, +56, +78, +90, -45, -11

Solution:

The arrangement in descending order

+90, +78, +56, 0, -11, -34, -45, -78

Exercise 2.3

Q1: Write the absolute value of the following.

a. +5 Solution: $ 5 = 5$	b. +67 Solution: $ +67 = 67$
c. -99 Solution: $ -99 = 99$	d. +90 Solution: $ +90 = 90$
e. -80 Solution: $ -80 = 80$	f. -40 Solution: $ -40 = 40$

Q2: Arrange the absolute or numerical values of the following integers in ascending order and descending order.

a. +22, +33, 0, +50, +7, +23

Solution:

Ascending order

0, 7, 22, 23, 33, 50

Descending order

50, 33, 23, 22, 7, 0

b. +6, +11, -3, +5, 0, +77

Solution:

Ascending order

0, 3, 5, 6, 11, 77

Descending order

77, 11, 6, 5, 3, 0

c. -14, 0, -11, -7, -8, +9

Solution:

Ascending order

0, 7, 8, 9, 11, 14

Descending order

14, 11, 9, 8, 7, 0

d. -11, -3, +5, +18, +13, -7

Solution:

Ascending order

3, 5, 7, 13, 11, 18

Descending order

18, 11, 13, 7, 5, 3

e. -16, +4, -7, -9, +2, +10

Solution:

Ascending order

2, 4, 7, 9, 10, 16

Descending order

16, 10, 9, 7, 4, 2

f. -6, +7, -8, -22, +25, -31, -56

Solution:

Ascending order

6, 7, 8, 22, 25, 31, 56

Descending order

56, 31, 25, 22, 8, 7, 6

g. +23, +12, +11, -14, -6, +8, -9

Solution:

Ascending order

6, 8, 9, 11, 12, 14, 23

Descending order

23, 14, 12, 11, 9, 8, 6

h. +23, +13, -15, +6, -5, +7, +9, +3, -10, -8

Solution:

Ascending order

3, 5, 6, 7, 8, 9, 10, 13, 15, 23

Descending order

23, 15, 13, 10, 9, 8, 7, 6, 5, 3

Q3: Aliya and Harris start at same point. Aliya takes 11 steps upstairs and Harris comes 5 steps down the stairs. Write the integers for the two situations. Also write the absolute value of the integers.

Solution:

Aliya upstairs steps = +11

Harris down stairs steps = -5

Absolute value of +11 = $|+11|$
= 11

And Absolute value of $-5 = |-5|$
= 5

Review Exercise 2

Q1: Choose the correct option

a. The smallest positive integer is

(i) 0 (ii) +1✓ (iii) -1

(iv) not determined

b. The greatest negative integer is

(i) 0 (ii) +1 (iii) -1✓

(iv) not determined

c. The absolute value of -16 is

(i) -16 (ii) +16✓ (iii) -60 (iv) +60

Q2: Circle the negative integers from the following.

+5, +15, -78, +100, -67, -12, +90

+5, +15, (-78), +100, (-67), (-12), +90

Q3: Write the opposite integers for each of the following.

a. -7

Solution:

Opposite of -7 = +7

b. +101

Solution:

Opposite of +101 = -101

c. -987

Solution:

Opposite of -987 = +987

d. -50

Solution:

Opposite of -50 = +50

e. +13

Solution:

Opposite of +13 = -13

f. -56

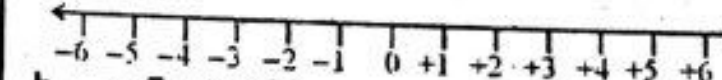
Solution:

Opposite of -56 = +56

Q4: Represent the following sets of integers on a number line.

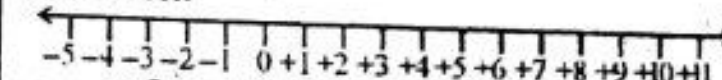
a. -6 to +6

Solution:



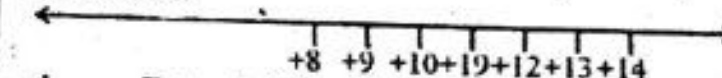
b. -5 to +11

Solution:



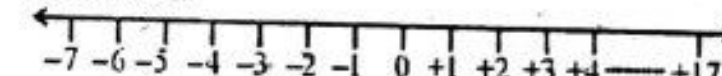
c. +8 to +14

Solution:



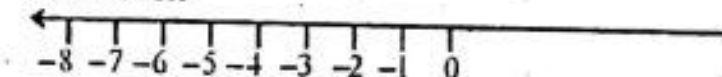
d. -7 to +17

Solution:



e. -8 to -12

Solution:



Q5: Write the integers between the following.

a. -5 and +5

Solution:

-4, -3, -2, -1, 0, +1, +2, +3, +4

b. -6 and +8

Solution:

-5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7

c. -11 and 17

Solution:

-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +10, +11, +12, +13, +14, +15, +16

d. -19 and +20

Solution:

-18, -17, -16, -15, -14, -13, -12, -11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5, +6, +7, +8, +9, +10, +11, +12, +13, +14, +15, +16, +17, +18, +19

Q6: Arrange the following integers in ascending and descending order.

a. -6, -3, -1, 0, 5, 7

Solution:

Ascending order

- 6, -3, -1, 0, +5, +7
Descending order
+7, +5, +0, -1, -3, -6

b. -12, -5, -3, +2, +11

Solution:

Ascending order

-12, -5, -3, -5, -12

Descending order

+11, +3, +2, -3, -5, -12

c. -23, -56, -13, 0, +14, +15, +23

Solution:

Ascending order

-56, -23, -13, 0, +14, +15, +23

Descending order

+23, +15, +14, 0, -13, -23, -56

d. -12, -16, -11, +10, +25, -34

Solution:

Ascending order

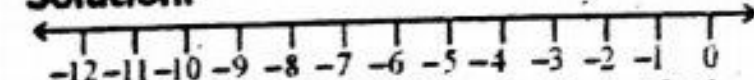
-16, -12, -11, +10, +25, +34

Descending order

+34, +25, +10, -11, -12, -16

Q7: Represent the integers greater than -13 to 0 on a number line.

Solution:



Q8: Write the absolute value of the following.

Solution:

a. -11

Solution:

Absolute value of -11 = $|-11| = 11$

b. +45

Solution:

Absolute value of +45 = $|+45| = 45$

c. -99

Solution:

Absolute value of -99 = $|-99| = 99$

d. +123

Solution:

Absolute value of +123 = $|+123| = 123$

e. -88

Solution:

Absolute value of -88 = $|-88| = 88$

f. -100

Solution:

Absolute value of -100 = $|-100| = 100$

Q9: Arrange the absolute or numerical values of the following integers in ascending order and descending order.

a. +16, +8, 0, +2, +7, +19

Solution:

Ascending order

0, 2, 7, 8, 16, 19

Descending order

19, 16, 8, 7, 2, 0

b. +17, +11, -13, +8, 0, +23

Solution:

Ascending order

0, 3, 8, 11, 17, 23

Descending order

23, 17, 11, 8, 3, 0

c. -17, 0, -16, -4, -1, +4

Solution:

Ascending order

0, 1, 4, 16, 17

Descending order

17, 16, 4, 1, 0

d. -18, -3, +4, +18, +19, -9

Solution:

Ascending order

3, 4, 9, 18, 19

Descending order

19, 18, 9, 4, 3

e. -21, +5, +16, -5, -8, +5, +77

Solution:

Ascending order

5, 8, 15, 21, 27

Descending order

77, 21, 15, 8, 5

Unit 3

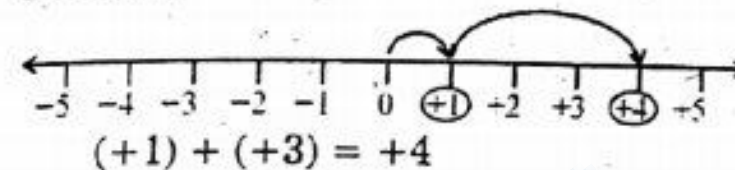
Laws of Integers and Order of Operations

Exercise 3.1

Q1: Represent the sum of the integers on a number line.

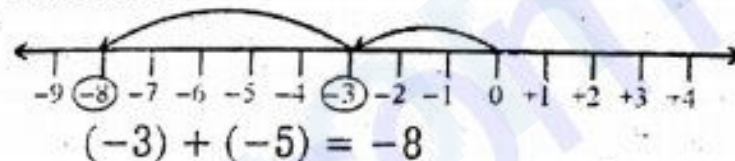
a. (+1) + (+3)

Solution:



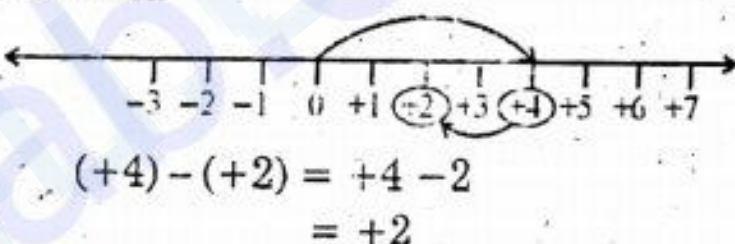
b. (-3) + (-5)

Solution:



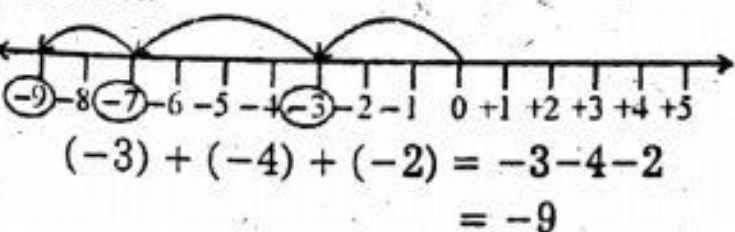
c. (+4) - (+2)

Solution:



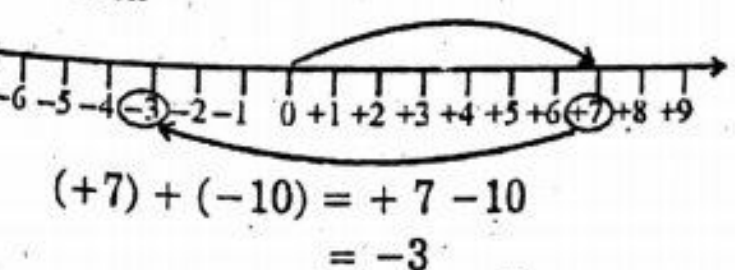
d. (-3) + (-4) + (-2)

Solution:



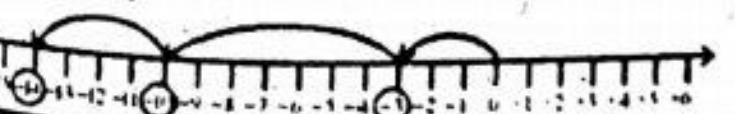
e. (+7) + (-10)

Solution:



f. (-3) + (-7) + (-4)

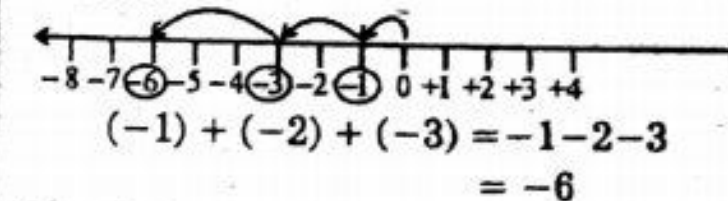
Solution:



$(-3) + (-7) + (-4) = -3 - 7 - 4 = -14$

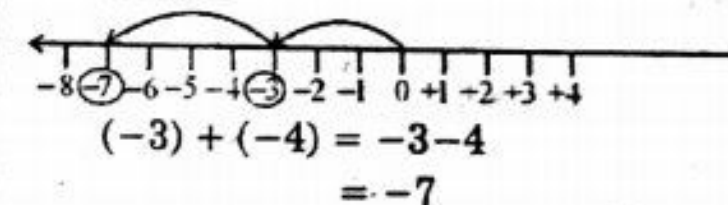
g. (-1) + (-2) + (-3)

Solution:



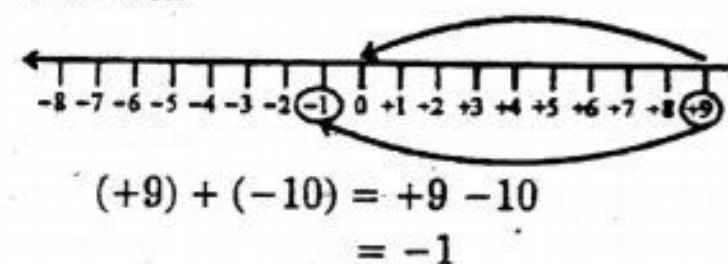
h. (-3) + (-4)

Solution:



i. (+9) + (-10)

Solution:



Q2: Solve the following.

a. (+9) - (+5)

Solution:

$(+9) - (+5) = +9 - 5 = +4$

b. (-4) + (-7)

Solution:

$(-4) + (-7) = -4 - 7 = -11$

c. (+4) - (+6)

Solution:

$(+4) - (+6) = +4 - 6 = -2$

d. (-9) + (-8) + (-2)

Solution:

$(-9) + (-8) + (-2) = -9 - 8 - 2 = -17 - 2 = -19$

e. (+10) - (-2)

Solution:

$$(+10) - (-2) = +10 + 2$$

$$= +12$$

f. $(-2) + (-5) + (+3)$

Solution:

$$(-2) + (-5) + (+3) = -2 - 5 + 3$$

$$= -7 + 3$$

$$= -4$$

g. $(-5) + (-2) + (-7)$

Solution:

$$(-5) + (-2) + (-7) = -5 - 2 - 7$$

$$= -7 - 7$$

$$= -14$$

h. $(+3) - (+2)$

Solution:

$$(+3) - (+2) = +3 - 2$$

$$= +1$$

i. $(+6) - (+4)$

Solution:

$$(+6) - (+4) = +6 - 4$$

$$= +2$$

j. $(-4) + (-6) + (-8)$

Solution:

$$(-4) + (-6) + (-8) = -4 - 6 - 8$$

$$= -10 - 8$$

$$= -18$$

k. $(+23) + (-2) + (-34)$

Solution:

$$(+23) + (-2) + (-34) = 23 - 2 - 34$$

$$= +23 - 36$$

$$= -13$$

l. $(-7) + (+6)$

Solution:

$$(-7) + (+6) = -7 + 6$$

$$= -1$$

Q3: Write the missing integers in the given.

a. $(+11) + (+3) = +14$

b. $(-4) + (-3) = (-7)$

c. $(-3) - (+4) = (-7)$

d. $(-9) + (-5) + (-2) = -16$

e. $(+2) - (-3) = +5$

f. $(-1) + (-3) + (-7) = (-11)$

g. $(-5) + (-7) + (+2) = (-10)$

h. $(-5) + (-2) + (-9) = -16$

Q4: The difference of two integers is +34. If minuend is -23. Find the subtrahend.**Solution:**

Given Minuend = -23

Suppose subtrahend = y

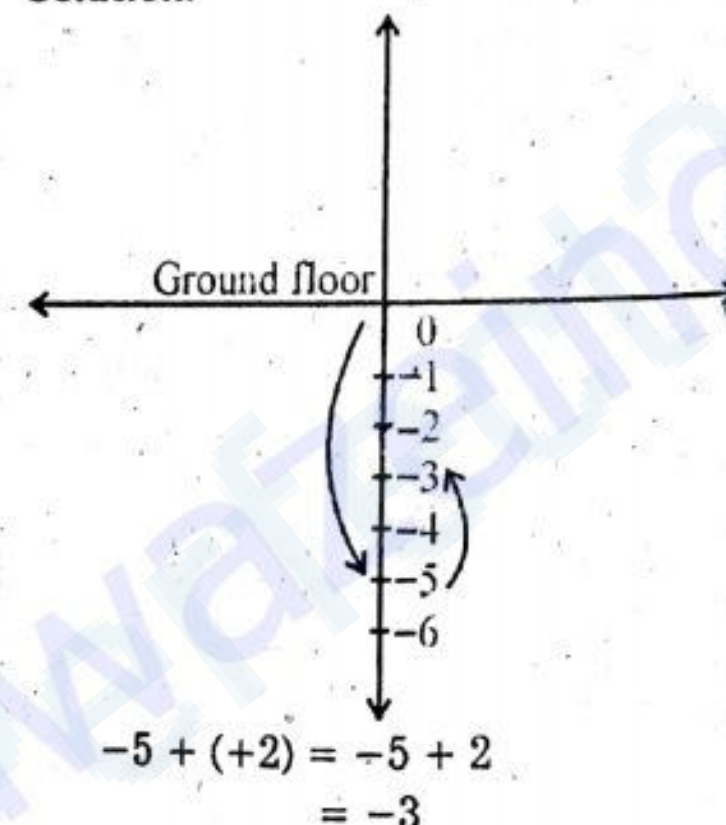
According to given condition

$$-23 - (+y) = +34$$

$$-23 - (+34) = +y$$

$$-57 = y$$

So subtrahend = -57

Q5: An elevator goes down 5 floors from the ground floor. Then it goes up 2 floors. What floor is the elevator on now?**Solution:****Q6: The temperature in the night was -10°C . If the temperature rises 4°C in the morning. What is the new temperature?****Solution:**

Night temperature = -10°C

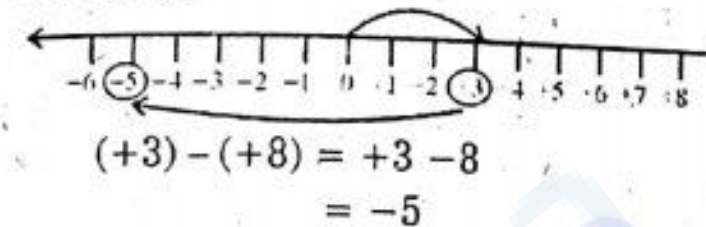
Morning temperature = 4°C

So New temperature = $-10 + (+4)$

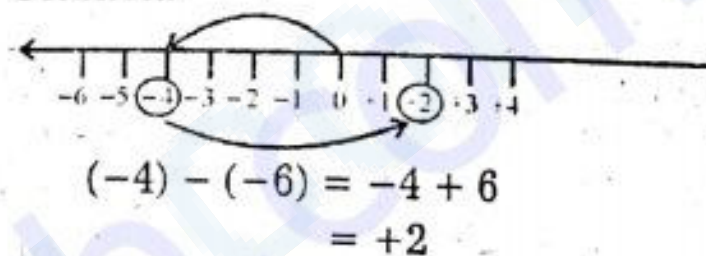
$$= -6^{\circ}\text{C}$$

Exercise 3.2**Q1: Represent the difference of the integers on the number line.**

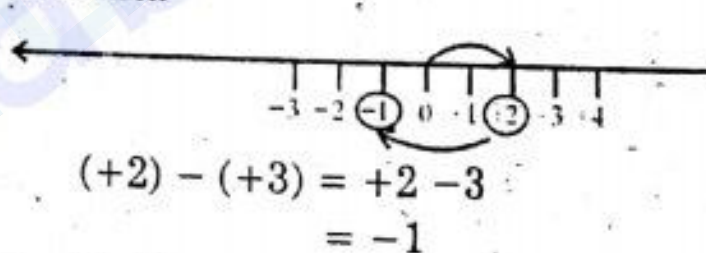
a. $(+3) - (+8)$

Solution:

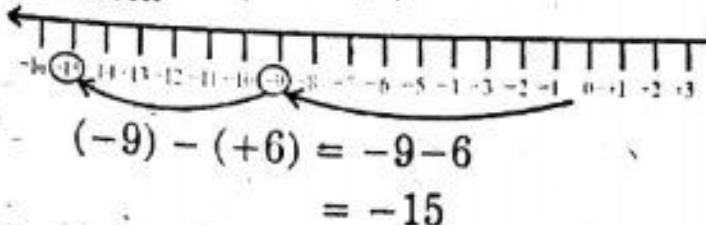
b. $(-4) - (-6)$

Solution:

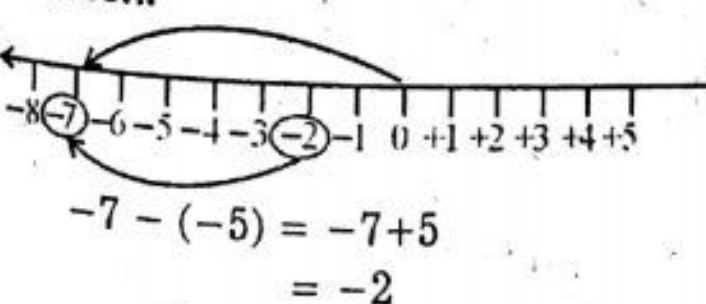
c. $(+2) - (+3)$

Solution:

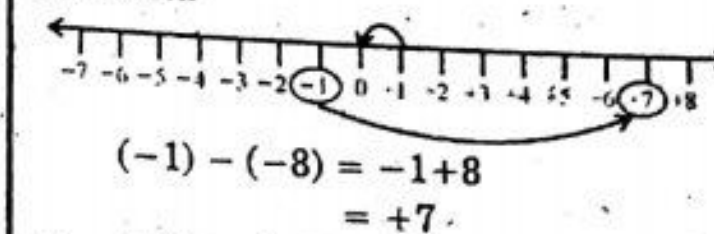
d. $(-9) - (+6)$

Solution:

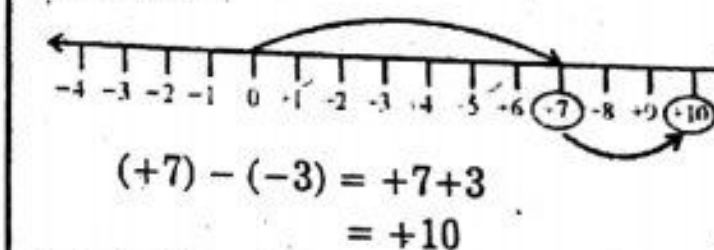
e. $(-7) - (-5)$

Solution:

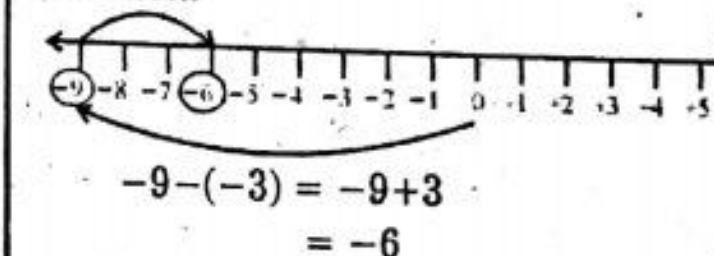
f. $(-1) - (-8)$

Solution:

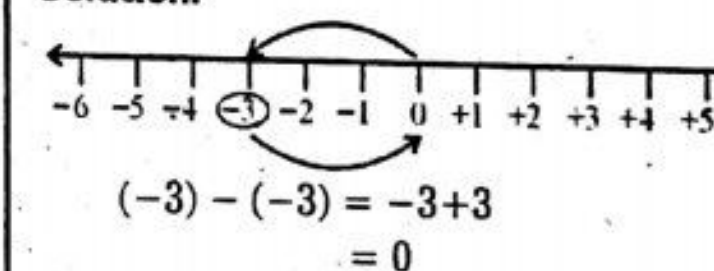
g. $(+7) - (-3)$

Solution:

h. $(-9) - (-3)$

Solution:

i. $(-3) - (-3)$

Solution:**Q2: Solve the following without using a number line.**

a. $(+4) - (+18)$

Solution:

$$(+4) - (+18) = +4 - 18$$

$$= -14$$

b. $(-8) - (-3)$

Solution:

$$(-8) - (-3) = -8 + 3$$

$$= -5$$

c. $(+5) - (+7)$

Solution:

$$(+5) - (+7) = +5 - 7$$

$$= -2$$

d. $(-11) - (-5)$

Solution:

$$(-11) - (-5) = -11 + 5$$

$$= -6$$

e. $(-3) - (-2)$

Solution:

$$(-3) - (-2) = -3 + 2$$

$$= -1$$

f. $(-5) - (-4)$

Solution:

$$(-5) - (-4) = -5 + 4$$

$$= -1$$

g. $(-6) - (-4)$

Solution:

$$(-6) - (-4) = -6 + 4$$

$$= -2$$

h. $(-7) - (-3)$

Solution:

$$(-7) - (-3) = -7 + 3$$

$$= -4$$

i. $(-4) - (+2)$

Solution:

$$(-4) - (+2) = -4 - 2$$

$$= -6$$

j. $(+7) - (-8)$

Solution:

$$(+7) - (-8) = +7 + 8$$

$$= 15$$

k. $(-9) - (+2)$

Solution:

$$(-9) - (+2) = -9 - 2$$

$$= -11$$

l. $(+6) - (+8)$

Solution:

$$(+6) - (+8) = +6 - 8$$

$$= -2$$

Q3: Write the missing integers in the given.

a. $(+3) - (+8) = (-5)$

b. $(-3) - (-1) = (-2)$

c. $(-4) - (+5) = (-9)$

d. $(-8) - (-5) = (-3)$

e. $(-1) - (-8) = +7$

f. $(-9) - (-5) - (-2) = (-2)$

Q4: The sum of two integers is +54. One of them is -11. Find the other.

Solution:

Suppose other number = x

So According to given condition

$$-11 + (x) = +54$$

$$x = (54 + (+11))$$

$$= 54 + 11$$

$$= 65$$

So other number = +65

Q5: A place is 213 m above sea level and another is 114 m below sea level. What is the distance between the two places?

Solution:

213m above sea level = +213m

114m below sea level = -114m

The distance between the two places

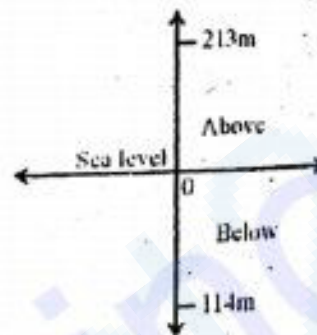
= The difference in level between two places

$$= 213m - (-114m)$$

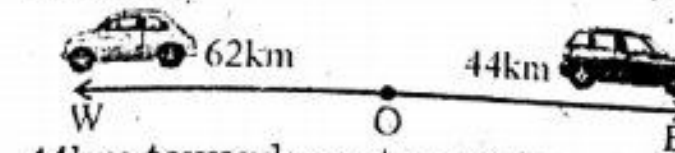
$$= 213m + 114m$$

$$= 327m$$

Q6: Two cars started from the same point. First car went towards the east and covered 44 km in an hour. The second car went towards the west and covered 62 km in 1 hour. Find the distance between the two cars after an hour.



Solution:



44km towards east = +44km

62 km towards west = -62km

The distance between two cars

$$= (+44) - (-62)$$

$$= 44 + 62$$

$$= 106km$$

Q7: Sifwa borrowed Rs 56 from her brother. She returned Rs 45. How much does she owe?

Solution:

Sofa borrowed = Rs 56

Returned Rs = Rs 45

So

$$\text{Sofa owe} = 56 - 45$$

$$= \text{Rs } 11$$

Exercise 3.3

Q1: Use laws of addition of integers and complete the following.

Solution:

a. $(-3) + 5 = \underline{5} + (-3)$

b. $7 + (-4) = -4 + \underline{7}$

c. $(-38) + (-17) = \underline{(-17)} + (-38)$

d. $\underline{(-16)} + (-8) = -8 + (-16)$

e. $\{(4 + (-25))\} + 5 = 4 + \{(\underline{-25}) + 5\}$

f. $(84 + 43) + (-9) = \underline{84} + \{(43 + (-9))\}$

g. $48 + (\underline{2} + 25) = (48 + 2) + 25$

h. $38 + (48 + \underline{12}) = (38 + 48) + 12$

Q2: Which of the following are true?

a. $(-32) + 6 = 6 + (-32)$ True

b. $18 + (-14) = -4 + (-18)$ False

c. $(-38) + 47 = 47 \times (-38)$ False

d. $86 + (-8) = (-8) + 86$ True

e. $(-7 + 4) + 6 = 7 + (4 + 6)$ False

f. $(44 + 93) + (-96) = 44 + \{93 + (-96)\}$ False

g. $38 + [98 + (-86)] = (38 + 98) + (-86)$ True

h. $79 + [(-63) + (-86)] = (79 - 63) - (-86)$ False

Q3: Verify the Commutative Law for the following.

a. -6, +8

Solution:

To verify commutative law for -6, +8.

We prove that $(-6) + 8 = 8 + (-6)$

$$\text{L.H.S} = (-6) + 8 = -6 + 8 = +2$$

$$\text{R.H.S} = +8(-6) = 8 - 6 = +2$$

As L.H.S = R.H.S

So commutative law verified

b. +9, +4

Solution:

To verify commutative law for +9, +4

We prove that $(+9) + (+4) = (+4) + (+9)$

$$\text{L.H.S} = (+9) + (+4) = 9 + 4 = 13$$

$$\text{R.H.S} = (+4) + (+9) = 4 + 9 = 13$$

As L.H.S = R.H.S

So commutative law for +9, +4 is Verified.

c. -7, -3

Solution:

To verify commutative law for -7, -3

We prove that $(-7) + (-3) = (-3) + (-7)$

$$\text{L.H.S} = (-7) + (-3) = -7 - 3 = -10$$

$$\text{R.H.S} = (-3) + (-7) = -3 - 7 = -10$$

As L.H.S = R.H.S

So commutative law for (-7), (-3) is verified.

d. -4, +16

Solution:

To verify commutative law for -4, +16

We prove that $(-4) + (+16) = (+16) + (-4)$

$$\text{L.H.S} = (-4) + (+16) = 16 + 4 = 12$$

$$\text{R.H.S} = (+16) + (-4) = 16 - 4 = 12$$

As L.H.S = R.H.S

So commutative law for -4, +6 is verified.

e. 3, -43

Solution:

To verify commutative law for 3, -43

We prove that $3 + (-43) = (-43) + 3$

$$\text{L.H.S} = 3 + (-43) + 3 = -3 - 43 = -40$$

$$\text{R.H.S} = (-43) + 3 = -43 + 3 = -40$$

As L.H.S = R.H.S

So commutative law for 3, -43 is verified.

f. -8, +9

Solution:

To verify commutative law for -8, +9.

We prove that $(-8) + (+9) = (+9) + (-8)$

$$\text{L.H.S} = (-8) + (+9) = -8 + 9 = +1$$

$$\text{R.H.S} = (+9) + (-8) = +9 - 8 = +1$$

As L.H.S = R.H.S

So commutative law for -8, +9 is verified.

Q4: Verify the Associative Law for the following.

a. 8, -6, -9

Solution:

To verify Associative law for 8, -6, -9

We prove that

$$(8 + (-6)) + (-9) = 8 + ((-6) + (-9))$$

$$\text{L.H.S} = (8 + (-6)) + (-9) = (8 - 6) + (-9)$$

$$= 2 - 9$$

$$= -7$$

$$\text{R.H.S} = 8 + ((-6) + (-9)) = 8 + (-6 - 9)$$

$$= 8 + (-15)$$

$$= 8 - 15$$

$$= -7$$

As L.H.S = R.H.S

So Associative property for 8, -6, -9 is verified.

b. 15, +5, -38

Solution:

To verify associative law for 15, +5, -38

$$\text{We prove that } (15 + 5) + (-38) = 15 + (5 + (-38))$$

$$\text{L.H.S} = (15 + 5) + (-38) = 20 + (-38)$$

$$= 20 - 38 = -18$$

$$\text{R.H.S} = 15 + (5 + (-38)) = 15 + (5 - 38)$$

$$= 15 + (-33) = 15 - 33 = -18$$

As L.H.S = R.H.S

So Associative law for 15, +5, -38 is verified.

c. -34, 37, -48

Solution:

To verify associative law for -34, 37, -48.

$$\text{We prove that } ((-34) + 37) + (-48) = (-34) + (37 + (-48))$$

$$\text{L.H.S} = ((-34) + 37) + (-48)$$

$$= (-34 + 37) + (-48)$$

$$= 3 - 48$$

$$= -45$$

$$\text{R.H.S} = (-34) + (37 + (-48))$$

$$= (-34) + (37 - 48)$$

$$= -34 + (-11) = -34 - 11 = -45$$

As L.H.S = R.H.S

So Associative law for 15, +5, -38 is verified.

d. -39, 18, 50

Solution:

To verify associative law for -39, 18, 50.

We Prove that

$$((-39) + 18) + 50 = (-39) + (18 + 50)$$

$$\text{L.H.S} = ((-39) + 18) + 50 = (-39 + 18) + 50$$

$$= (-21) + 50 = -21 + 50 = 29$$

$$\text{R.H.S} = (-39) + (18 + 50) = (-39) + 68$$

$$= -39 + 68 = 29$$

As L.H.S = R.H.S

So Associative law is verified.

e. -16, -12, 98

Solution:

To verify associative law for -16, 21, 98

We prove that

$$((-16) + (-21)) + 98 = (-16) + ((-21) + 98)$$

$$\text{L.H.S} = ((-16) + (-21)) + 98 = (-16 - 21) + 98$$

$$= (-37) + 98 = -37 + 98 = 61$$

$$\text{R.H.S} = (-16) + ((-21) + 98) = (-16) + (-21 + 98)$$

$$= (-16) + 77 = -16 + 77 = 61$$

As L.H.S = R.H.S

So associative law is verified.

f. -48, -96, -83

Solution:

To verify associative law for -48, -96, -83.

We prove that

$$((-48) + (-96)) + (-83) = (-48) + ((-96) + (-83))$$

$$\text{L.H.S} = ((-48) + (-96)) + (-83)$$

$$= (-48 - 96) + (-83)$$

$$= (-144) + (-83) = -144 - 83$$

$$= -227$$

$$\text{R.H.S} = (-48) + ((-96) + (-83))$$

$$= (-48) + (-96 - 83)$$

$$= (-48) + (-179) = -48 - 179$$

$$= -227$$

As L.H.S = R.H.S

So associative law is verified.

Q5: Verify the following.

a. $(-2) + 5 = 5 + (-2)$

Solution:

$$\text{L.H.S} = (-2) + 5 = -2 + 5 = 3$$

$$\text{R.H.S} = 5 + (-2) = 5 - 2 = 3$$

As L.H.S = R.H.S

So $(-2) + 5 = 5 + (-2)$ is verified

b. $11 + (-3) = -3 + 11$

Solution:

$$\text{L.H.S} = 11 + (-3) = 11 - 3 = 8$$

$$\text{R.H.S} = -3 + 11 = 8$$

As L.H.S = R.H.S

So $11 + (-3) = -3 + 11$ is verified.

c. $(-8) + (-7) = (-7) + (-8)$

Solution:

$$\text{L.H.S} = (-8) + (-7) = -8 - 7 = -15$$

$$\text{R.H.S} = (-7) + (-8) = -7 - 8 = -15$$

As L.H.S = R.H.S

So $(-8) + (-7) = (-7) + (-8)$ is verified

d. $7 + (-8) = -8 + 7$

Solution:

$$\text{L.H.S} = 7 + (-8) = 7 - 8 = -1$$

$$\text{R.H.S} = -8 + 7 = -1$$

As L.H.S = R.H.S

So $7 + (-8) = -8 + 7$ is verified

e. $\{4 + (-7)\} + 3 = 4 + \{-7 + 3\}$

Solution:

$$\text{L.H.S} = \{4 + (-7)\} + 3$$

$$= \{4 - 7\} + 3 = -3 + 3 = 0$$

$$\text{R.H.S} = 4 + \{-7 + 3\} = 4 + (-4) = 4 - 4 = 0$$

As L.H.S = R.H.S

So $\{4 + (-7)\} + 3 = 4 + \{-7 + 3\}$ is verified.

f. $\{(-14) + 9\} + (-6) = -14 + \{9 + (-6)\}$

Solution:

$$\text{L.H.S} = \{(-14) + 9\} + (-6) = \{-14 + 9\} - 6$$

$$= -5 - 6 = -11$$

$$\text{R.H.S} = -14 + \{9 + (-6)\} = -14 + \{9 - 6\}$$

$$= -14 + 3 = -11$$

As L.H.S = R.H.S

So $\{(-14) + 9\} + (-6) = -14 + \{9 + (-6)\}$ is verified.

g. $(-18) + \{8 + (-4)\} = \{(-18) + 8\} + (-4)$

Solution:

$$\text{L.H.S} = (-18) + \{8 + (-4)\} = -18 + \{8 - 4\} \\ = -18 + 4 = -14$$

$$\text{R.H.S} = \{(-18) + 8\} + (-4) = \{-18 + 8\} - 4 \\ = -10 - 4 = -14$$

As L.H.S = R.H.S

So $(-18) + \{8 + (-4)\} = \{(-18) + 8\} + (-4)$ is verified.

$$\text{h. } 50 + \{(-13) + (-16)\} = \{50 + (-13)\} + (-16)$$

Solution:

$$\text{L.H.S} = 50 + \{(-13) + (-16)\} \\ = 50 + \{-3 - 16\} = 50 - 29 = 21$$

$$\text{R.H.S} = \{50 + (-13)\} + (-16) = \{50 - 13\} - 16 \\ = 37 - 16 = 21$$

As L.H.S = R.H.S

So, $50 + \{(-13) + (-16)\} = \{50 + (-13)\} + (-16)$ is verified.

Exercise 3.4

Q1: Find the product of the following without using a number line.

a. $+2, +5$

Solution:

$$(+2) \times (+5) = 10$$

b. $-4, +7$

Solution:

$$(-4) \times (+7) = -4 \times 7 = \\ = -28 \text{ (as } (-1) \times (+1) = -1)$$

c. $+9, -2$

Solution:

$$(+9) \times (-2) = (+) \times (-) \times 9 \times 2 \\ = (-) \times 18 \text{ (as } (+) \times (-) = -) \\ = -18$$

d. $+6, +5$

Solution:

$$(+6) \times (+5) = 6 \times 5 \\ = 30$$

e. $+13, -10$

Solution:

$$(+13) \times (-10) = (+) \times (-) \times 13 \times 10 \\ = (-) \times 130 \text{ (as } (+) \times (-) = -) \\ = -130$$

f. $+14, -20$

Solution:

$$(+14) \times (-20) = (+) \times (-) \times 14 \times 20 \\ = -280 \text{ (as } (+) \times (-) = -)$$

g. $-3, +22$

Solution:

$$(-3) \times (+22) = (-) \times (+) \times 3 \times 22 \\ = -66 \text{ (as } (-) \times (+) = -)$$

h. $-70, +21$

Solution:

$$(-70) \times (21) = (-) \times (+) \times 70 \times 21 \\ = -1470 \text{ (as } (-) \times (+) = -)$$

i. $-11, -22$

Solution:

$$(-11) \times (-22) = (-) \times (-) \times 11 \times 22 \\ = (+) \times 242 \text{ (as } (-) \times (-) = +) \\ = 242$$

j. $+40, -29$

Solution:

$$(+40) \times (-29) = (+) \times (-) \times 40 \times 29 \\ = -1160 \text{ (as } (+) \times (-) = -)$$

k. $-4, +5$

Solution:

$$(-40) \times (+5) = (-) \times (+) \times 4 \times 5 \\ = -20 \text{ (as } (-) \times (+) = -)$$

l. $-10, +10$

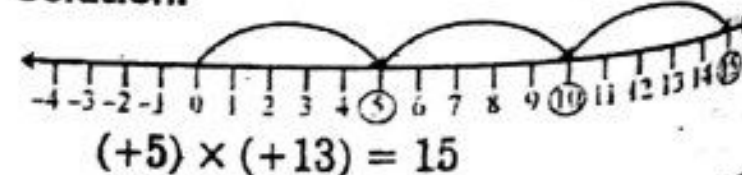
Solution:

$$(-10) \times (10) = (-) \times (+) \times 10 \times 10 \\ = -100 \text{ (as } (-) \times (+) = -)$$

Q2: Find the product of the following using a number line.

a. $+5, +3$

Solution:



b. $-3, -3$

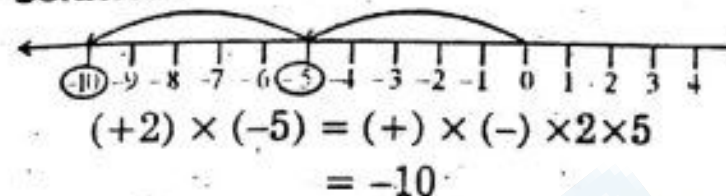
Solution:

As both number (integer) are negative, so it is difficult on the number line.

$$(-3) \times (-3) = (-) \times (-) \times 3 \times 3 \\ = +9 \text{ (as } (-) \times (-) = +)$$

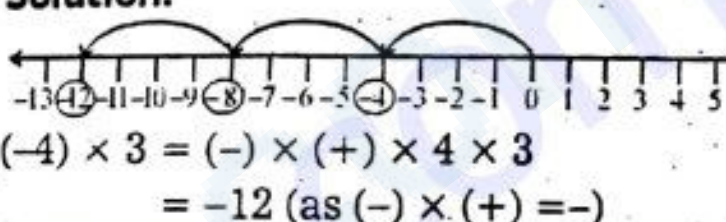
c. $+2, -5$

Solution:



d. $-4, +3$

Solution:



e. $-1, -7$

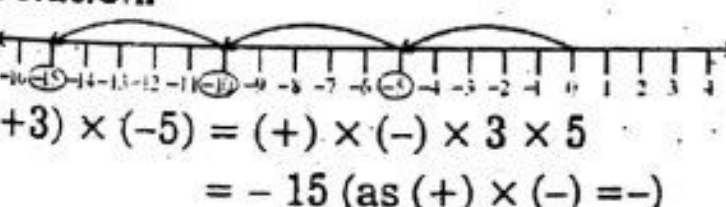
Solution:

As both integers are negative, so it is difficult one the number line.

$$(-1) \times (-7) = (-) \times (-) \times 1 \times 7 \\ = 7 \text{ (as } (-) \times (-) = +)$$

f. $+3, -5$

Solution:



g. $-4, -2$

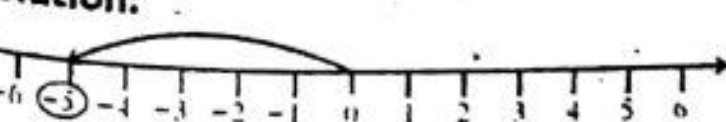
Solution:

As both integers are negative, so it is difficult on the number line.

$$(-4) \times (-2) = (-) \times (-) \times 4 \times 2 \\ = 8 \text{ (as } (-) \times (-) = +)$$

h. $+1, -5$

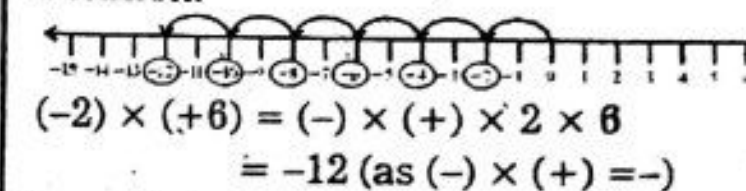
Solution:



$$(+1) \times (-5) = (+) \times (-) \times 1 \times 5 \\ = -5 \text{ (as } (-) \times (+) = -)$$

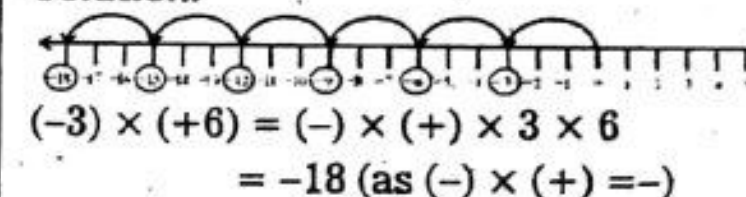
i. $-2, +6$

Solution:



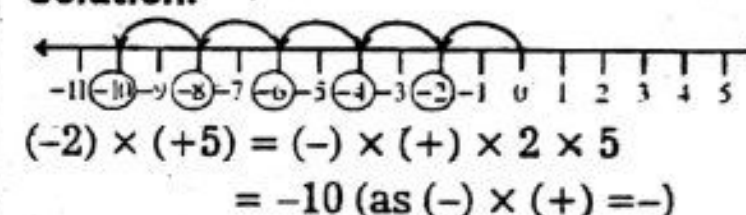
j. $-3, +6$

Solution:



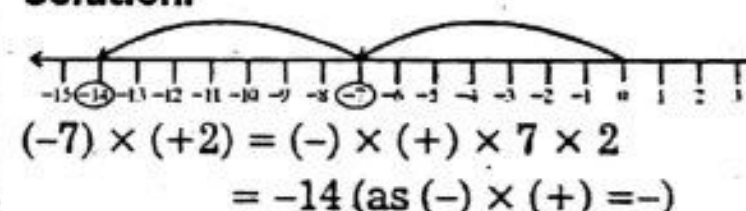
k. $-4, +5$

Solution:



l. $-7, +2$

Solution:



Q3: Find the product of the greatest negative integer and the smallest positive integer.

Solution:

Greatest negative integer = -1

And

Smallest positive integer = 1

$$\text{Product} = (-1) \times (1) \\ = (-) \times (+) \times 1 \times 1 \\ = -1 \text{ (as } (-) \times (+) = -)$$

Q4: Find the product of -100 and +100.

Solution:

$$\text{Product} = (-100) \times (100) \\ = (-) \times (+) \times 100 \times 100 \\ = -10000 \text{ (as } (-) \times (+) = -)$$

Exercise 3.5

Q1: Verify the Commutative Law under multiplication for the following.

a. -2, +6

Solution:

To verify commutative law under multiplication for -2, +6 we prove that $(-2) \times (+6) = (6) \times (-2)$

$$\text{L.H.S} = (-2) \times (+6) = -12$$

$$\text{R.H.S} = (+6) \times (-2) = -12$$

As L.H.S = R.H.S

So $(-2) \times (+6) = (6) \times (-2)$ is verified.

b. 8, -4

Solution:

To verify commutative law under multiplication for 8, -4. We prove that $(8) \times (-4) = (-4) \times 8$

$$\text{L.H.S} = (8) \times (-4) = -32$$

$$\text{R.H.S} = (-4) \times (8) = -32$$

As L.H.S = R.H.S

So $8 \times (-4) = (-4) \times 8$ is verified.

c. -6, 3

Solution:

To verify commutative law under multiplication for -6, 3 we prove that $(-6) \times 3 = 3 \times (-6)$

$$\text{L.H.S} = (-6) \times 3 = -18$$

$$\text{R.H.S} = (3) \times (-6) = -18$$

As L.H.S = R.H.S

So $(-6) \times 3 = 3 \times (-6)$ is verified.

d. -38, -94

Solution:

To verify commutative law under multiplication for -38, -94, we prove that $(-38) \times (-94) = (-94) \times (-38)$

$$\text{L.H.S} = (-38) \times (-94) = 3572$$

$$\text{R.H.S} = (-94) \times (-38) = 3572$$

As L.H.S = R.H.S

So commutative law is verified.

e. -95, 34

Solution:

To verify commutative law under multiplication for -95, 34 we prove that $(-95) \times (34) = (34) \times (-95)$

$$\text{L.H.S} = (-95) \times (34) = -3230$$

$$\text{R.H.S} = (34) \times (-95) = -3230$$

As L.H.S = R.H.S

So commutative law is verified.

f. 48, -82

Solution:

To verify commutative law under multiplication for 48, -82 we prove that $(48) \times (-82) = (-82) \times 48$

$$\text{L.H.S} = (48) \times (-82) = -3936$$

$$\text{R.H.S} = (-82) \times (48) = -3936$$

As L.H.S = R.H.S

So commutative law under multiplication is verified.

Q2: Verify the Associative Law under multiplication for the following.

a. -4, +7, -8

Solution:

To verify associative law under multiplication, we prove that $\{(-4) \times 7\} \times (-8) = (-4) \times \{7 \times (-8)\}$

$$\text{L.H.S} = \{(-4) \times 7\} \times (-8) = (-28) \times (-8)$$

$$= (-) \times (-) \times (28 \times 8) = 224$$

$$\text{R.H.S} = (-4) \times \{7 \times (-8)\} = (-4) \times (-56)$$

$$= 224$$

As L.H.S = R.H.S,

So Associative law under multiplication is verified.

c. -13, 52, -68

Solution:

To verify associative law under multiplication, we prove that $\{(-13) \times 52\} \times (-68) = (-13) \times \{52 \times (-68)\}$

$$\text{L.H.S} = \{(-13) \times 52\} \times 68$$

$$= (-676) \times 68$$

$$= -45,968$$

$$\text{R.H.S} = (-13) \times \{52 \times (-68)\}$$

$$= (-13) \times (-3536)$$

$$= -45,968$$

As L.H.S = R.H.S,

So Associative law under multiplication is verified.

d. -85, -44, 76

Solution:

To verify associative law under multiplication, we prove that $\{(-85) \times (-44)\} \times 76 = (-85) \times \{(-44) \times 76\}$

$$\text{L.H.S} = \{(-85) \times (-44)\} \times 76$$

$$= 3740 \times 76 = 284,240$$

$$\text{R.H.S} = (-85) \times \{(-44) \times 76\}$$

$$= (-85) \times (-3344)$$

$$= 284,240$$

As L.H.S = R.H.S,

So Associative law under multiplication is verified.

e. -63, -93, -81

Solution:

To verify associative law under multiplication, we prove that $\{(-63) \times (-93)\} \times (-81) = (-63) \times \{(-93) \times (-81)\}$

$$\text{L.H.S} = \{(-63) \times (-93)\} \times (-81) = (5859) \times (-81)$$

$$= -474,579$$

$$\text{R.H.S} = (-63) \times \{(-93) \times (-81)\}$$

$$= (-63) \times (753)$$

$$= -474,579$$

As L.H.S = R.H.S,

So Associative law under multiplication is verified.

f. 15, -38, -39

Solution:

To verify associative law under multiplication, we prove that $\{15 \times (-38)\} \times (-39) = 15 \times \{(-38) \times (-39)\}$

$$\text{L.H.S} = \{15 \times (-38)\} \times (-39) = (-570) \times (-39)$$

$$= 22,230$$

$$\text{R.H.S} = 15 \times \{(-38) \times (-39)\}$$

$$= 15 \times (1482)$$

$$= 22,230$$

As L.H.S = R.H.S,

So Associative law under multiplication is verified.

Q3: Write the multiplicative inverse of each of the following.

a. -3

Solution:

The multiplicative inverse of

$$-3 = -\frac{1}{3}$$

b. -2

Solution:

The multiplicative inverse of

$$-2 = -\frac{1}{2}$$

c. +7

Solution:

The multiplicative inverse of

$$+7 = \frac{1}{7}$$

d. +100

Solution:

The multiplicative inverse of

$$+100 = \frac{1}{100}$$

e. -77

Solution:

The multiplicative inverse of

$$-77 = -\frac{1}{77}$$

f. -243

Solution:

The multiplicative inverse of

$$-243 = -\frac{1}{243}$$

g. -48

Solution:

The multiplicative inverse of

$$-48 = -\frac{1}{48}$$

h. -55

Solution:

The multiplicative inverse of

$$-55 = -\frac{1}{55}$$

i. +60

Solution:

The multiplicative inverse of

$$+60 = +\frac{1}{60}$$

$$j. \frac{1}{-2}$$

Solution:

The multiplicative inverse of

$$\frac{1}{-2} = -2$$

$$k. \frac{3}{2}$$

Solution:

The multiplicative inverse of

$$\frac{3}{2} = \frac{2}{3}$$

$$l. \frac{-2}{7}$$

The multiplicative inverse of

$$\frac{-2}{7} = \frac{-7}{2}$$

Q4: Verify the Distributive Law.

$$a. -4 \times [(1 + (-3))] = (-4 \times 1) + [(-4 \times (-3))]$$

Solution:

$$\begin{aligned} \text{L.H.S} &= -4 \times [(1 + (-3))] = -4 \times [1 - 3] \\ &= -4 \times (-2) = 8 \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= (-4 \times 1) + [(-4 \times (-3))] \\ &= -4 + 12 = 8 \end{aligned}$$

As L.H.S = R.H.S

So

$$-4 \times [(1 \times (-3))] = (-4 \times 1) + [(-4 \times (-3))]$$

Here verified.

$$b. -4 \times [(-5) + 1] = [(-4 \times (-5))] + [(-4 \times 1)]$$

Solution:

$$\text{L.H.S} = -4 \times [(-5) + 1] = -4 \times (-4) = 16$$

$$\begin{aligned} \text{R.H.S} &= [(-4 \times (-5))] + [(-4 \times 1)] \\ &= 20 - 4 = 16 \end{aligned}$$

As L.H.S = R.H.S

So

$$-4 \times [(-5) + 1] = [(-4 \times (-5))] + [(-4 \times 1)]$$

Here verified.

$$c. [(-5) - 2] \times (-2) = [(-5 \times (-2))] - [2 \times (-2)]$$

Solution:

$$\begin{aligned} \text{L.H.S} &= [(-5) - 2] \times (-2) = [-5 - 2] \times (-2) \\ &= (-7) \times (-2) = 14 \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= [(-5 \times (-2))] - [2 \times (-2)] \\ &= 10 - (-4) = 10 + 4 = 14 \end{aligned}$$

As L.H.S = R.H.S

So

$$[(-5) - 2] \times (-2) = [(-5 \times (-2))] - [2 \times (-2)]$$

Here verified.

$$d. -8 \times [(1 + (-3))] = (-8 \times 1) + [(-8 \times (-3))]$$

Solution:

$$\begin{aligned} \text{L.H.S} &= -8 \times [(1 + (-3))] = -8 \times [1 - 3] \\ &= -8 \times (-2) = 16 \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= (-8 \times 1) + [(-8 \times (-3))] \\ &= -8 + 24 = 16 \end{aligned}$$

As L.H.S = R.H.S

So

$$-8 \times [(1 + (-3))] = (-8 \times 1) + [(-8 \times (-3))]$$

Hence verified.

$$e. [(-6) - 1] \times (-4) = [(-6 \times (-4))] - [1 \times (-4)]$$

Solution:

$$\begin{aligned} \text{L.H.S} &= [(-6) - 1] \times (-4) = [-6 - 1] \times (-4) \\ &= (-7) \times (-4) = 28 \end{aligned}$$

$$\text{R.H.S} = [(-6 \times (-4))] - [1 \times (-4)]$$

$$= 24 - (-4) = 24 + 4 = 28$$

As L.H.S = R.H.S

So

$$[(-6) - 1] \times (-4) = [(-6 \times (-4))] - [1 \times (-4)]$$

Hence verified.

$$f. -2 \times [(1 + (-3))] = (-2 \times 1) + [(-2 \times (-3))]$$

Solution:

$$\text{L.H.S} = -2 \times [(1 + (-3))] = -2 \times [1 - 3]$$

$$= -2 \times (-2) = 4$$

$$\text{R.H.S} = (-2 \times 1) + [(-2 \times (-3))]$$

$$= (-2) + 6 = -2 + 6 = 4$$

$$\text{So } -2 \times [(1 + (-3))] = (-2 \times 1) + [(-2 \times (-3))]$$

Hence verified.

$$g. [(-7) - 1] \times (-3) = [(-7 \times (-3))] - [1 \times (-3)]$$

Solution:

$$\text{L.H.S} = [(-7) - 1] \times (-3) = [-7 - 1] \times (-3)$$

$$= (-8) \times (-3) = 24$$

$$\text{R.H.S} = [(-7 \times (-3))] - [1 \times (-3)]$$

$$= 21 - (-3) = 21 + 3 = 24$$

As

$$\text{L.H.S} = \text{R.H.S}$$

So

$$[(-7) - 1] \times (-3) = [(-7 \times (-3))] - [1 \times (-3)]$$

Hence verified.

$$h. -4 \times [(7 + (-8))] = (-4 \times 7) + [(-4 \times (-8))]$$

Solution:

$$\text{L.H.S} = -4 \times [(7 + (-8))] = -4 \times [7 - 8]$$

$$= -4 \times (-1) = 4$$

$$\text{R.H.S} = (-4 \times 7) + [(-4 \times (-8))]$$

$$= -28 + 32 = 4$$

$$\text{As L.H.S} = \text{R.H.S}$$

So

$$-4 \times [(7 + (-8))] = (-4 \times 7) + [(-4 \times (-8))]$$

Hence verified

Exercise 3.6**Q1: Solve the following.**

$$a. (+45) \div (-5)$$

Solution:

Take absolute value and divide

$$\begin{aligned} (+45) \div (-5) &= |45| \div |-5| = 45 \div 5 \\ &= 9 \end{aligned}$$

As both numbers have different signs, so the answer will be negative.

$$\text{So } (+45) \div (-5) = -9$$

$$b. (-63) \div (-9)$$

Solution:

Take absolute value and divide

$$\begin{aligned} (-63) \div (-9) &= |-63| \div |-9| = 63 \div 9 \\ &= 7 \end{aligned}$$

As both numbers are negative, so the answer will be positive.

$$\text{So } (-63) \div (-9) = 7$$

$$c. (-80) \div (+4)$$

Solution:

Take absolute value and divide

$$\begin{aligned} (-80) \div (+4) &= |-80| \div |4| = 80 \div 4 \\ &= 20 \end{aligned}$$

As both numbers have different signs, so the answer will be negative.

$$\text{So } (-80) \div (+4) = -20$$

$$d. (+44) \div (-11)$$

Solution:

Take absolute value and divide

$$\begin{aligned} (+44) \div (-11) &= |44| \div |-11| = 44 \div 11 \\ &= 4 \end{aligned}$$

As both numbers have different signs, so the answer will be negative.

$$\text{So } (+44) \div (-11) = -4$$

$$e. (-60) \div (+10)$$

Solution:

Take absolute value and divide

$$(-60) \div (+10) = |-60| \div |+10| = 60 \div 10 = 6$$

As both the numbers have different signs, so the answer will be negative.

$$\text{So } (-60) \div (+10) = -6$$

f. $(-144) \div (+12)$

Solution:

Take absolute value and divide

$$(-144) \div (+12) = |-144| \div |+12| = 144 \div 12 = 12$$

As both numbers have different signs, so the answer will be negative.

$$\text{So } (-144) \div (+12) = -12$$

g. $(+168) \div (-8)$

Solution:

Take absolute value and divide,

$$(+168) \div (-8) = |+168| \div |-8| = 168 \div 8 = 21$$

As both numbers have different signs, so the answer will be negative.

$$\text{So } (+168) \div (-8) = -21$$

h. $(+390) \div (-39)$

Solution:

Take absolute value and divide

$$(+390) \div (-39) = |+390| \div |-39| = 390 \div 39 = 10$$

As both numbers have different signs, so answer will be negative.

$$\text{So } (+390) \div (-39) = -10$$

i. $(-600) \div (-25)$

Solution:

Take absolute value and divide

$$(-600) \div (-25) = |-600| \div |-25| = 600 \div 25 = 24$$

As both numbers have same signs, So answer will be positive.

$$\text{So } (-600) \div (-25) = +24$$

j. $(-85) \div (+17)$

Solution:

Take absolute value and divide

$$(-85) \div (+17) = |-85| \div |+17| = 85 \div 17 = 5$$

As both numbers have different signs, so answer will be negative.

$$\text{So } (-85) \div (+17) = -5$$

Q2: Solve the following.

a. $0 \div 4 = 0$

b. $0 \div 10 = 0$

c. $0 \div (-90) = 0$

d. $0 \div (-5) = 0$

e. $0 \div (-130) = 0$

f. $0 \div (+300) = 0$

Exercise 3.7

Q1. Simplify the following.

a. $1.6 + 3 \times 7 - 5$

$$1.6 + 3 \times 7 - 5 = 1.6 + 21 - 5 = 22.6 - 5 = 17.6$$

b. $42 \div 2 + 45 - 22$

$$42 \div 2 + 45 - 22 = 21 + 45 - 22 = 66 - 22 = 44$$

c. $\frac{7}{9} \times 1\frac{1}{5} \div \frac{8}{15}$

$$\begin{aligned} \frac{7}{9} \times 1\frac{1}{5} \div \frac{8}{15} &= \frac{7}{9} \times \frac{6}{5} \div \frac{8}{15} \\ &= \frac{7}{9} \times \frac{6}{5} \times \frac{15}{8} \\ &= \frac{7}{9} \times \frac{18}{8} = \frac{7}{9} \times \frac{9}{4} = \frac{7}{4} \end{aligned}$$

d. $\frac{15}{21} \times \frac{45}{3} - \frac{1}{15} \div \frac{3}{7}$

$$\begin{aligned} \frac{15}{21} \times \frac{45}{3} - \frac{1}{15} \div \frac{3}{7} &= \frac{15}{21} \times \frac{45}{3} - \frac{1}{15} \times \frac{7}{3} \\ &= \frac{75}{7} - \frac{7}{15} = \frac{75 \times 15 - 7 \times 7}{105} \\ &= \frac{1125 - 49}{105} = \frac{1076}{105} \end{aligned}$$

e. $725 - [20 \times \{10 + 7 + (7 \times 5 - 3)\}]$

$$\begin{aligned} &= 725 - [20 \times \{10 + 7 + (7 \times 2)\}] \\ &= 725 - [20 \times \{10 + 7 + 14\}] \\ &= 725 - [20 \times 31] = 725 - 640 \\ &= 85 \end{aligned}$$

f. $7 + \left\{ \frac{1}{3} + \frac{2}{12} \times \left(\frac{7}{4} - \frac{5}{12} \right) \right\}$

$$= 7 + \left\{ \frac{1}{3} + \frac{2}{12} \times \left(\frac{3 \times 7 - 5}{12} \right) \right\}$$

$$= 7 + \left\{ \frac{1}{3} + \frac{2}{12} \times \left(\frac{21 - 5}{12} \right) \right\}$$

$$= 7 + \left\{ \frac{1}{3} + \frac{2}{12} \times \frac{16}{12} \right\}$$

$$= 7 + \left\{ \frac{1}{3} + \frac{2}{9} \right\}$$

$$= 7 + \frac{1 \times 3 + 2 \times 1}{9} = 7 + \frac{3 + 2}{9}$$

$$= 7 + \frac{5}{9} = \frac{7 \times 9 + 5 \times 1}{9} = \frac{63 + 5}{9}$$

$$= \frac{68}{9} = 7\frac{5}{9}$$

g. $\left\{ \left(10\frac{1}{3} - \frac{2}{9} \div \frac{5}{16} \right) \times \frac{5}{20} + \frac{1}{4} \right\}$

$$= \left\{ \left(\frac{30}{3} - \frac{2}{9} \div \frac{5}{16} \right) \times \frac{5}{20} + \frac{1}{4} \right\}$$

$$= \left\{ \left(\frac{3 \times 30 - 2 \times 1}{9} \times \frac{5}{16} \right) \times \frac{5}{20} + \frac{1}{4} \right\}$$

$$= \left\{ \left(\frac{90 - 2}{9} \div \frac{5}{16} \right) \times \frac{5}{20} + \frac{1}{4} \right\}$$

$$= \left\{ \frac{88}{9} \div \frac{5}{16} \times \frac{5}{20} + \frac{1}{4} \right\}$$

$$= \left\{ \frac{44}{9} \times \frac{16}{5} \times \frac{1}{20} + \frac{1}{4} \right\}$$

$$= \left\{ \frac{352}{9} \times \frac{1}{5} + \frac{1}{4} \right\} = \left\{ \frac{352}{45} + \frac{1}{4} \right\}$$

$$= \frac{352 \times 4 + 1 \times 45}{180} = \frac{1408 + 45}{180}$$

$$= \frac{1453}{180} = 8\frac{13}{180}$$

$$\begin{aligned}
 h. \quad & \frac{1}{3} - \left\{ \frac{17}{4} - \left(5\frac{1}{4} \times 2\frac{1}{2} \right) \right\} \\
 &= \frac{1}{3} - \left\{ \frac{17}{4} - \left(\frac{21}{4} \times \frac{5}{2} \right) \right\} \\
 &= \frac{1}{3} - \left\{ \frac{17}{4} - \frac{105}{8} \right\} \\
 &= \frac{1}{3} - \left\{ \frac{17 \times 2 - 105 \times 1}{8} \right\} = \frac{1}{3} - \left\{ \frac{34 - 105}{8} \right\} \\
 &= \frac{1}{3} - \left\{ \frac{-71}{8} \right\} = \frac{1}{3} + \frac{71}{8} \\
 &= \frac{1 \times 8 + 71 \times 3}{24} = \frac{8 + 213}{24}
 \end{aligned}$$

$$\begin{aligned}
 i. \quad & \frac{71}{8} - \frac{21}{4} \left(\frac{1}{5} \times \frac{3}{4} \right) \\
 &= \frac{71}{8} - \frac{21}{4} \times \frac{2}{21} = \frac{71}{8} - \frac{93}{80} \\
 &= \frac{71 \times 80 - 93 \times 3}{240} = \frac{5680 - 279}{240} \\
 &= \frac{5401}{240} = 22\frac{211}{240}
 \end{aligned}$$

$$\begin{aligned}
 j. \quad & \left(\frac{6}{17} \times \frac{34}{15} \right) - \frac{6}{35} \times \frac{2}{3} \\
 &= \left(\frac{6}{17} \times \frac{34}{15} \right) - \frac{6}{35} \times \frac{2}{3} \\
 &= \frac{2}{1} \times \frac{2}{5} - \frac{2}{35} \times \frac{2}{3} \\
 &= \frac{4}{5} - \frac{4}{35} = \frac{4 \times 7 - 4 \times 1}{35} \\
 &= \frac{28 - 4}{35} = \frac{24}{35}
 \end{aligned}$$

$$\begin{aligned}
 k. \quad & \left\{ \left(6\frac{1}{3} \div \frac{3}{5} \right) \times \frac{18}{9} - \frac{6}{3} \right\} \\
 &= \left\{ \left(\frac{19}{3} \div \frac{3}{5} \right) \times \frac{18}{9} - \frac{6}{3} \right\} \\
 &= \left\{ \left(\frac{19}{3} \times \frac{5}{3} \right) \times \frac{18}{9} - \frac{6}{3} \right\} \\
 &= \left\{ \frac{95}{9} \times \frac{18}{9} - \frac{6}{3} \right\} \\
 &= \frac{190}{9} - \frac{6}{3} \\
 &= \frac{190 - 6 \times 3}{9} = \frac{190 - 18}{9} \\
 &= \frac{172}{9} = 19\frac{1}{9}
 \end{aligned}$$

$$\begin{aligned}
 l. \quad & 22.46 \times 3 \div 2(7.82 - 2.42) + 9.5 \times 3.4 \\
 &= 22.46 \times 3 \div 2(7.82 - 2.42) + 9.5 \times 3.4 \\
 &= 22.46 \times 3 \div 2 \times 5.4 + 9.5 \times 3.4 \\
 &= 22.46 \times 1.5 \times 5.4 + 9.5 \times 3.4 \\
 &= 181.926 + 32.3 \\
 &= 214.226
 \end{aligned}$$

$$\begin{aligned}
 m. \quad & 12.35 + [5.25 + 3(18.43 - 3.66 \times 5.75)] \\
 &= 12.35 + [5.25 + 3(18.43 - 21.045)] \\
 &= 12.35 + [5.25 + 3(-2.615)] \\
 &= 12.35 + [5.25 - 7.745] \\
 &= 12.35 + (-2.595) \\
 &= 12.35 - 2.595 \\
 &= 9.755
 \end{aligned}$$

Exercise 3.8

Q1: Afnan's monthly income is Rs 35,300. If he spent $\frac{2}{10}$ of his income on house rent, $\frac{3}{10}$ on food $\frac{4}{10}$ on other expenditures, find the total amount that he saved.

Solution:

Monthly income = Rs 35,300

$$\begin{aligned}
 \frac{2}{10} \text{ of his income} &= \frac{2}{10} \times 35,300 \\
 &= \text{Rs. } 7060
 \end{aligned}$$

$$\begin{aligned}
 \frac{3}{10} \text{ of his income} &= \frac{3}{10} \times 35,300 \\
 &= 10,590
 \end{aligned}$$

$$\begin{aligned}
 \frac{4}{10} \text{ of his income} &= \frac{4}{10} \times 35,300 \\
 &= 14,120
 \end{aligned}$$

$$\begin{aligned}
 \text{Total remaining amount} &= 35,300 - 7060 - 10,590 \\
 &= 35,300 - 31,770 = 3530
 \end{aligned}$$

So total amount that he saved = Rs. 3530.

Q2: The length of a piece of rope is 111.9 metres. If it is divided into 6 equal pieces and 10 metres of each rope is used, find the length of remaining rope.

Solution:

$$\begin{aligned}
 \text{Total length of piece of rope} &= 111.9\text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Divide into 6 equal piece} &= \frac{111.9}{6} \\
 &= 18.65\text{m}
 \end{aligned}$$

$$\begin{aligned}
 \text{Length of each rope} &= 18.65\text{m}
 \end{aligned}$$

If we used 10 from each rope, then remaining length of each = 18.65 - 10

$$\begin{aligned}
 \text{So total remaining length of a rope} &= 6(8.65) \\
 &= 51.9\text{m}
 \end{aligned}$$

Q3: Dania bought 6 boxes of cupcakes. Each box has 12 cupcakes in it. She distributed $\frac{2}{3}$ of the cupcakes among needy children and put the remaining in a jar equally. How many cupcakes did she put in each jar?

Solution:

Total boxes of cupcakes = 6

As each box has 12 cupcakes, so

$$\begin{aligned}
 \text{Total cupcakes} &= 12 \times 6 = 72
 \end{aligned}$$

$$\begin{aligned}
 \frac{2}{3} \text{ of cupckes} &= \frac{2}{3} \times 72 \\
 &= 48
 \end{aligned}$$

$$\begin{aligned}
 \text{Remainins cupcakes} &= 72 - 48 = 24
 \end{aligned}$$

To put 24 cupcakes in 8 jar equally

$$\begin{aligned}
 &= \frac{24}{8} = 3
 \end{aligned}$$

So '3' cupckes she put in each jar.

Q4: Marwa ordered 3 scarves for Rs 415 each 2 abaya's for Rs 920 each online. There was a discount on these items and she paid half price for each item. She paid additional Rs 140 as courier charges. How much did she spend in total?

Solution:

Price of each scarves = Rs. 415

$$\begin{aligned}
 \text{Price of 3 scararues} &= 3 \times 415 \\
 &= \text{Rs. } 1245
 \end{aligned}$$

Price of each obaya = Rs 920

$$\begin{aligned}
 \text{Primce of 2 obaya} &= 2 \times 920 \\
 &= \text{Rs. } 1840
 \end{aligned}$$

$$\begin{aligned}
 \text{Total} &= 1245 + 1840 = \text{Rs. } 3085
 \end{aligned}$$

$$\begin{aligned}
 \text{Half of total price} &= \frac{1}{2} \times 3085 \\
 &= \text{Rs. } 1542.5
 \end{aligned}$$

Courier charges = Rs. 140

$$\begin{aligned}
 \text{So total spend money} &= 1542.5 + 140 \\
 &= \text{Rs. } 1682.5
 \end{aligned}$$

Q5: Ali has number blocks in three colours. He has 8 blue blocks. There are $\frac{3}{4}$ times as many green blocks as blue blocks, and there are 4 fewer red blocks than green blocks. How many blocks does Ali have?

Solution:

Number of blue blocks = 8

Number of green blocks = $\frac{3}{4}$ of blue blocks

$$= \frac{3}{4} \times 8 = 6$$

Number of Red blocks = 4 less by green

$$= 6 - 4 = 2$$

Q6: Madiha needs 7.5 kg of apples and 5.75 kg of mangoes for an Iftar party. She already has 3.25 kg of mangoes and 2.25 kg apples at home. How many more apples and mangoes should she buy?

Solution:

Required apples for Iftar party = 7.5kg

Required mangoes for Iftar party = 5.75kg

Present apples at home = 2.25kg

Present mangoes at home = 3.25kg

So weight of apples to buy = 7.5 - 2.25

$$= 5.25\text{kg}$$

And

So weight of mangoes to buy = 5.75 - 3.25

$$= 2.5\text{kg}$$

Review Exercise

Q1. Encircle the correct option.

a. The smallest positive integer is

- i. 0 ii. +1✓
iii. -1 iv. not determined

b. The greatest negative integer is

- i. 0 ii. +1
iii. -1✓ iv. not determined

c. The greatest positive integer is

- i. 0 ii. +100✓
iii. +1 iv. not determined

d. The absolute value of -16 is

- i. -16 ii. +16✓
iii. -60 iv. +60

e. The product of two integers with opposite signs is

- i. always positive
ii. always negative✓
iii. sometimes positive sometimes negative
iv. both positive and negative

f. What must be added to -135 to get -142?

- i. -7✓ ii. 7
iii. 277 iv. -227

g. Additive inverse of 6 is

- i. -6✓ ii. 6
iii. $\frac{1}{6}$ iv. $\frac{1}{6}$

h. The value of $4 \times (-5) \times (-2)$ is

- i. -16 ii. +20
iii. +40✓ iv. -40

i. Multiplicative inverse of -100 is

- i. -100 ii. +100
iii. $\frac{1}{100}$ iv. $-\frac{1}{100}$ ✓

j. _____ is a phrase that includes numbers, brackets and operators together to show the value of something in mathematical form.

- i. A set ii. An expression✓
iii. An operation
iv. A rule

k. We solve _____ bracket first in BODMAS rule

- i. [] ii. { }
iii. ()✓ iv. _____

l. In BODMAS rule we solve _____ operation first.

- i. + ii. $\times$ iii. - iv. $\div$ ✓

m. _____ is called Vinculum.

- i. [] ii. { }
iii. () iv. _____✓

n. The correct order of operations is _____.

- i. +, -, $\div$, $\times$ ii. $\div$, +, $\times$, -
iii. $\div$, $\times$, +, -✓ iv. $\div$, $\times$, -, +

Q2: Solve the following.

- a. $85 + (-96) = (-11)$
b. $80 + 57 = 137$
c. $86 + (-38) = 48$
d. $22 + (-41) + (-8) = 27$
e. $(-18) + (-45) + (-89) = (-152)$
f. $(-11) + (-5) + 6 = (-10)$
g. $(-4) - (6) = -10$
h. $80 - (-7) = 87$
i. $6 - (-8) = 14$
j. $(-34) - (-40) = 6$
k. $(-18) - (-45) - (56) = 29$
l. $(-101) - (-55) - 43 = (-89)$
m. $(+6) \times (-9) = (-54)$
n. $(+8) \times (-9) = -72$
o. $(-2) \times (-34) = 68$
p. $(+134) \times (-123) = -16482$
q. $(+6) \div (-2) = (-3)$

r. $(+340) \div (-34) = -10$

s. $(-12) \div (-4) = 3$

t. $(+224) \div (-22) = -10.18$

Q3: The product of two integers is -75. If one of them is -15, then find the other one.

Solution:

Product of two integer = -75

One integer = -15

Let 2nd integer = y

According to given condition

$$(-15) \times y = -75$$

$$y = \frac{-75}{-15} = 5$$

So 2nd integer = 5

Q4: Find an integer which divides -90 to give -45.

Solution:

Suppose required integer = x

So according to given condition

$$\frac{-90}{x} = -45$$

$$\frac{-90}{-45} = x$$

$$x = 2$$

Q5. Verify the following.

a. $(-5) \times 3 = 3 \times (-5)$

Solution:

$$\text{L.H.S} = (-5) \times 3 = (-15)$$

$$\text{R.H.S} = 3 \times (-5) = (-15)$$

As L.H.S = R.H.S

So $(-5) \times 3 = 3 \times (-5)$ is verified.

b. $(-27) + (-97) = (-97) + (-27)$

Solution:

$$\text{L.H.S} = (-27) + (-97) = (-124)$$

$$\text{R.H.S} = (-97) + (-27) = (-124)$$

As L.H.S = R.H.S

So $(-27) + (-97) = (-97) + (-27)$

is verified.

c. $17 + (-9) = -9 + 17$

Solution:

$$\text{L.H.S} = 17 + (-9) = 17 - 9 = 8$$

$$\text{R.H.S} = -9 + 17 = 8$$

As L.H.S = R.H.S

So $17 + (-9) = -9 + 17$ is verified.

d. $-101 + (-77) = -77 + (-101)$

Solution:

$$\text{L.H.S} = -101 + (-77)$$

$$= -101 - 77 = -178$$

$$\text{R.H.S} = -77 + (-101) = -77 - 101 = -178$$

As L.H.S = R.H.S

So $17 + (-9) = -9 + 17$ is verified.

e. $(-6) + [(-3) + (-4)] = [(-6) + (-3)] + (-4)$

Solution:

$$\text{L.H.S} = (-6) + [(-3) + (-4)] = -6 + [-3 - 4]$$

$$= -6 + (-7) = -6 - 7 = -13$$

$$\text{R.H.S} = [(-6) + (-3)] + (-4)$$

$$= [-6 - 3] + (-4) = -9 - 4 = -13$$

As L.H.S = R.H.S

So $(-6) + [(-3) + (-4)] = [(-6) + (-3)] + (-4)$ is verified

f. $(-12) \times [5 \times (-4)] = [(-12) \times 5] \times (-4)$

Solution:

$$\text{L.H.S} = (-12) \times [5 \times (-4)] = (-12) \times (-20)$$

$$= 240$$

$$\text{R.H.S} = [(-12) \times 5] \times (-4)$$

$$= (-60) \times (-4) = 240$$

As L.H.S = R.H.S

So $(-12) \times [5 \times (-4)] = [(-12) \times 5] \times (-4)$ is verified.

g. $[(-6) - 2] (-5) = [(-6) (-5)] - [2 (-5)]$

Solution:

$$\text{L.H.S} = [(-6) - 2] (-5) = [-6 - 2] (-5)$$

$$= (-8) (-5) = 40$$

$$\text{R.H.S} = [(-6) (-5)] - [2 (-5)]$$

$$= 30 + 10 = 40$$

As L.H.S = R.H.S

So $[(-6) - 2] (-5) = [(-6) (-5)] - [2 (-5)]$ is verified.

h. $(-11) \times [(5 + (-7))] = (-11 \times 5) + [(-11 \times (-7))]$

Solution:

$$\text{L.H.S} = (-11) \times [(5 + (-7))]$$

$$= (-11) \times [5 - 7] = (-11) \times (-2)$$

$$\text{R.H.S} = (-11 \times 5) + [(-11) \times (-7)]$$

$$= -55 + (77) = -55 + 77 = 22$$

As L.H.S = R.H.S

So $(-11) \times [(5 + (-7))] = (-11 \times 5) + [(-11) (-7)]$ is verified.**Q6: Simplify the following expressions.**

a. $(2 \times 7) + (12 \div 4)$

Solution:

$$(2 \times 7) + (12 \div 4) = 14 + 3 = 17$$

b. $3 \times (7 + 9) - 6$

Solution:

$$3 \times (7 + 9) - 6 = 3 \times 16 - 6$$

$$= 48 - 6 = 42$$

c. $(8 \times 5) + 5 - 3$

Solution:

$$(8 \times 5) + 5 - 3 = 40 + 5 - 3 = 45 - 3$$

$$= 42$$

d. $204 - [30 \times \{20 + (8 \times 32 \div 4) - (9 \div 63)\}]$

Solution:

$$= 204 - [30 \times \{20 + (8 \times 8) - (7)\}]$$

$$= 204 - [30 \times \{20 \times 64 - 7\}]$$

$$= 204 - [30 \times \{1280 - 7\}]$$

$$= 204 - [30 \times 1270]$$

$$= 204 - 37,190$$

$$= -37,986$$

e. $\left\{ \left(9 \times \frac{1}{3} - \frac{3}{9} \right) \div \frac{5}{16} \right\} \times \left(\frac{5}{12} + \frac{3}{4} \right)$

Solution:

$$\left\{ \left(9 \times \frac{1}{3} - \frac{3}{9} \right) \div \frac{5}{16} \right\} \times \left(\frac{5}{12} + \frac{3}{4} \right)$$

$$= \left\{ \left(3 - \frac{3}{9} \right) \div \frac{5}{16} \right\} \times \frac{5}{16}$$

$$= \left\{ \left(\frac{3 \times 9 - 3}{9} \right) \div \frac{5}{16} \right\} \times \frac{5}{16}$$

$$= \left\{ \left(\frac{27 - 3}{9} \right) \div \frac{5}{16} \right\} \times \frac{5}{16}$$

$$= \left\{ \frac{24}{9} \times \frac{16}{5} \right\} \times \frac{5}{16}$$

$$= \frac{24}{9} \times \frac{16}{5} \times \frac{5}{16}$$

$$= \frac{24}{9}$$

Q7: While travelling Rohaan Paid Rs 36.45 for a rickshaw, Rs 34.50 for a metro bus and Rs 56.56 for a metro train. If he had Rs 1300, find the total amount he was left with.

Solution:

Rohaan paid for a rickshaw travelling = Rs. 36.45

Rohaan paid for a metro bus travelling = Rs 34.50

Rohaan paid for a metro train travelling = Rs. 56.56

Total paid payment = 36.45 + 34.50 + 56.56

$$= \text{Rs. } 127.51$$

Total amount with Rohaan = Rs. 1300

Remaining amount with Rohaan = 1300 - 127.51

$$= \text{Rs. } 1172.5$$

Q8: A shopkeeper has $42\frac{3}{4}$ m of cloths in his stock, if he sells $6\frac{2}{6}$ m of cloth, then how much cloth he has left?

Solution:

Cloths with shopkeeper is his stock

$$= 42\frac{3}{4} \text{ m}$$

$$\text{Sold cloth} = 6\frac{2}{6} \text{ m}$$

$$\text{Remaining (left) cloth} = 42\frac{3}{4} - 6\frac{2}{6}$$

$$= \frac{171}{4} - \frac{38}{6}$$

$$= \frac{3 \times 171 - 2 \times 38}{12}$$

$$= \frac{513 - 76}{12}$$

$$= \frac{437}{12} = 36\frac{5}{12} \text{ m}$$

9. Look at the following price list of the different fruits to answer the questions.

Price list

 Rs 150 per 5g

 Rs 200 per kg

 Rs 100 per dozen

 Rs 150 per dozen

- a. If Rabia bought 3.45 kg of apples and 2.34 dozen of orange. How much did she pay?

Solution:

Rabia bought apples = 3.45kg
Price of per kg of apple = Rs 150
So Price of 3.45 kg = 150×3.45
= Rs. 517.5

Rabia bought oranges = 2.34 dozen
Per dozen price of oranges = Rs.100
So price of 2.34 dozen oranges
= 2.34×100
= Rs. 234

Total paid payment of Rabia = 517.5 + 234
= Rs. 751.5

- b. If the shopkeeper sold 34.45 kg of mangoes, 23.89 kg of apples, 7.67 dozen of oranges and 6.5 dozen banana, find his total sales.

Solution:

Shopkeeper sold mangoes = 34.45kg
Price of per kg mangoes = Rs. 200
Amount of mangoes = 34.45×200
= Rs. 6890

Shopkeeper sold apples = 23.89kg
Price of per kg apples = Rs. 150
Amount for apples = 150×23.89
= Rs. 3583.5

Shopkeeper sold oranges = 7.67 dozen

Price of per dozen oranges = Rs.100
Amount of oranges = 7.67×100
= Rs. 767

Shopkeeper sold banana = Rs. 6.5
Price of per dozen banana = Rs. 150
Amount of banana = 6.5×150
= Rs. 975

So total sale of shopkeeper
= $3583.5 + 6890 + 767 + 975$

= Rs. 12,215.5

- Q10: The pack of large erasers has 8 pieces; the pack of medium erasers has 15 pieces each, and the pack of small erasers has 20 erasers each. Ibrahim has 3 small, 2 medium and 2 large packs of erasers. How many erasers does he have altogether?

Solution:

In large pack of erasers = 8 pieces
In medium pack of erasers = 15 pieces

In small pack of erasers = 20 pieces
Number of small pack = 3.

So total number of erasers in small pack = 3×20
= 60 pieces

Number of medium pack = 2.
So total number of erasers in medium pack = 2×15
= 30 pieces

Number of large pack = 2
So total number of erasers in large pack = 2×8
= 16 pieces

Thus

Number of erasers of All types = $60 + 30 + 16$
= 106 pieces

- Q11: Usman and his family use $13\frac{3}{9}$ liter for water $10\frac{5}{15}$ liter for cleaning and $6\frac{2}{9}$ liter for cooking respectively. How much water do they use?

Solution:

Use of water for bathing = $13\frac{3}{9}$ liters

Use of water for cleaning = $10\frac{5}{15}$ liters

Use of water for cooking = $6\frac{2}{9}$ liters

So total use water = $13\frac{3}{9} + 10\frac{5}{15} + 6\frac{2}{9}$ liters

$$= \frac{120}{3} + \frac{155}{3} + \frac{56}{9}$$

$$= \frac{40}{3} + \frac{31}{3} + \frac{56}{9}$$

$$= \frac{30 \times 40 + 31 \times 3 + 56}{9}$$

$$= \frac{120 + 93 + 56}{9} = \frac{269}{9}$$

$$= 29\frac{8}{9} \text{ liters}$$

- Q12. In a restaurant, a family ordered 3 sandwiches for Rs 250 each, 2 glasses of cold drinks for Rs 180 each and one cup of coffee for Rs 210. They paid additional tax of Rs 112. If they gave a Rs 5000 note to the cashier, how much was their total bill? How much amount did they get as change?

Solution:

Price of Sandwiches = Rs. 250

Number of Sandwiches = 3

Total price of sandwiches = 3×250
= Rs. 750

Price of cold drinks = Rs. 180

Number of cold drinks = 2

Total price of cold drinks = 2×180
= Rs. 360

Price of one cup coffee = Rs 210

Additional tax paid payment = Rs. 112

Total paid payment = $750 + 360 + 210 + 112$

Total Bill = $5000 - 1432$
= Rs. 3568

- Q13: Ali and Fahad have Rs 564.32 and Rs 876.89 respectively. They want to buy a Tafseer book for Rs 2234.23. How many more rupees do they need to buy the Tafseer book?

Solution:

Amount with Ali = Rs. 564.32

Amount with Fahad = Rs. 876.89

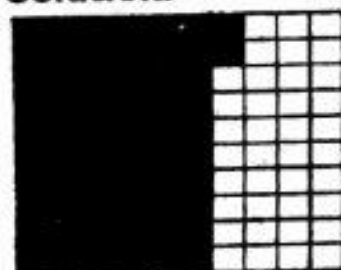
Required amount for Tafseer book = Rs. 2234.23

To buy Tafseer book required more rupees = $2234.23 - 564.32 - 876.89$
= Rs. 793.02

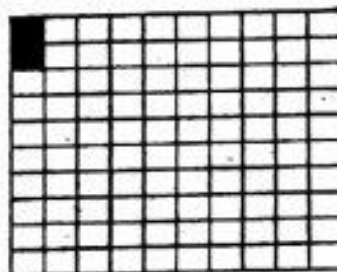
Unit 4 Rate, Ratio and Percentage Exercise 4.1

Q1: The following squares are divided into 100 equal parts. Express the coloured parts as a percentage.

Solution:



62%



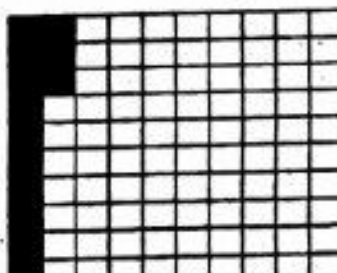
2%

Q2: Colour the squares to show the given percentage.

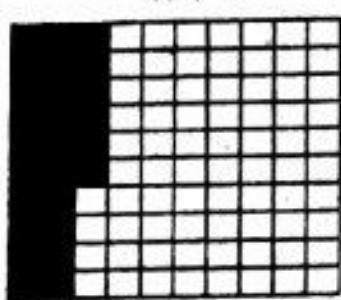
Solution:



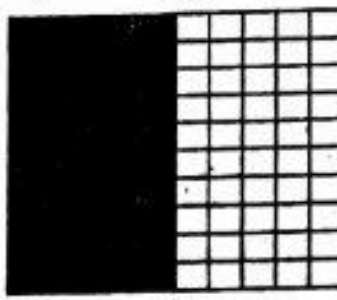
78%



13%



26%



50%

Q3: Mahad got 92 marks out of 100 in a Mathematics test. Express the marks as a percentage.

Solution:

Write:- 92%

Read:- 92 Percent

Q4: Find the value of the following.

a. 15% of 35

Solution:

$$15\% \text{ of } 35 = \frac{15}{100} \times 35 \\ = 5.25$$

b. 90% of 1000

Solution:

$$90\% \text{ of } 1000 = \frac{90}{100} \times 1000 \\ = 900$$

c. 5.2% of 500

Solution:

$$5.2\% \text{ of } 500 = \frac{5.2}{100} \times 500 \\ = 26$$

d. 25% of 175

Solution:

$$25\% \text{ of } 175 = \frac{25}{100} \times 175 \\ = 43.75$$

e. $67\frac{1}{7}\%$ of 780

Solution:

$$67\frac{1}{7}\% \text{ of } 780 = \frac{470}{7}\% \text{ of } 780 \\ = \frac{470}{7} \times \frac{1}{100} \times 780 \\ = 523.7$$

f. 20% of 1000

Solution:

$$20\% \text{ of } 1000 = \frac{20}{100} \times 1000 \\ = 200$$

g. 87% of 200 litres

Solution:

$$87\% \text{ of } 200 \text{ litres} = \frac{87}{100} \times 200 \\ = 174 \text{ litres}$$

h. 2.5% of 3200 grams

Solution:

$$2.5\% \text{ of } 3200 \text{ grams} = \frac{2.5}{100} \times 3200 \\ = 80 \text{ gram}$$

i. 9.25% of Rs 300

Solution:

$$9.25\% \text{ of Rs. } 300 = \frac{9.25}{100} \times 300 \\ = \text{Rs. } 27.75 \text{ gram}$$

j. 45% of 75 marks

Solution:

$$45\% \text{ of } 75 \text{ marks} = \frac{45}{100} \times 75 \\ = 33.75$$

k. 0.5% of 50mm

Solution:

$$0.5\% \text{ of } 50 \text{ mm} = \frac{0.5}{100} \times 50 \\ = 0.25 \text{ mm}$$

l. 4.50% of 100

Solution:

$$4.5\% \text{ of } 100 = \frac{4.5}{100} \times 100 \\ = 4.5$$

Q5: Find the percentage.

a. 50 out of 600

Solution:

$$50 \text{ out of } 600 = \frac{50}{600} \\ = \frac{50}{600} \times 100\% \\ = 8.3\%$$

b. 175 out 900

Solution:

$$175 \text{ out of } 900 = \frac{175}{900} \\ = \frac{175}{900} \times 100\% \\ = 19.44\%$$

c. 7.82 out of 40

Solution:

$$7.82 \text{ out of } 40 = \frac{7.82}{40} \\ = \frac{7.82}{40} \times 100\% \\ = 19.55\%$$

d. 90 out of 100

Solution:

$$90 \text{ out of } 100 = \frac{90}{100} \\ = \frac{90}{100} \times 100\% \\ = 90\%$$

e. 66 out of 190

Solution:

$$66 \text{ out of } 190 = \frac{66}{190} \\ = \frac{66}{190} \times 100\% \\ = 34.73\%$$

f. 89 out of 200

Solution:

$$89 \text{ out of } 200 = \frac{89}{200} \\ = \frac{89}{200} \times 100\% \\ = 44.5\%$$

g. 40 out of 75 marks

Solution:

$$\begin{aligned}
 40 \text{ out of } 75 \text{ marks} &= \frac{40}{75} \\
 &= \frac{40}{75} \times 100\% \\
 &= 53.33\%
 \end{aligned}$$

h. 2 out of 10 children

Solution:

$$\begin{aligned}
 2 \text{ out of } 10 \text{ children} &= \frac{2}{10} \\
 &= \frac{2}{10} \times 100\% \\
 &= 20\%
 \end{aligned}$$

i. Rs 30 out of Rs 500

Solution:

$$\begin{aligned}
 \text{Rs } 30 \text{ out of Rs. } 500 &= \frac{30}{500} \\
 &= \frac{30}{500} \times 100\% \\
 &= 6\%
 \end{aligned}$$

j. 78 m out of 560 m

Solution:

$$\begin{aligned}
 78 \text{ m out of } 560 \text{ m} &= \frac{78}{560} \\
 &= \frac{78}{560} \times 100\% \\
 &= 13.92\%
 \end{aligned}$$

k. 3 months out of 5 years

Solution:

$$\begin{aligned}
 3 \text{ months out of } 5 \text{ years} &= \frac{3}{5} \\
 &= \frac{3}{5} \times 100\% \\
 &= 60\%
 \end{aligned}$$

l. 1 day out of 1 week

Solution:

$$\begin{aligned}
 1 \text{ day out of } 1 \text{ week} \\
 &= 1 \text{ day out of } 7 \text{ days} = \frac{1}{7} \\
 &= \frac{1}{7} \times 100\% = 14.3\%
 \end{aligned}$$

Q6. 18 out of 90 boxes in a shop are blue. Express the number of blue boxes as percentage.

Solution:

$$\begin{aligned}
 18 \text{ out of } 90 \text{ boxes} &= \frac{18}{90} \\
 \text{In percentage } 18 \text{ out of } 90 \text{ boxes} \\
 &= \frac{18}{90} \times 100\% = 20\%
 \end{aligned}$$

Q7: Huma used 12 metres of cloth of stitch four shirts. If she had 30-meter cloth altogether, find the percentage of cloth she used.

Solution:

$$\begin{aligned}
 \text{Huma used cloth} &= 12 \text{ metres} \\
 \text{Total cloth with Human} &= 30 \text{ metres} \\
 \text{So } 12 \text{ m out of } 30 \text{ m} &= \frac{12}{30} \\
 \text{In percentage } 12 \text{ m out of } 30 \text{ m} \\
 &= \frac{12}{30} \times 100\% \\
 &= 40\%
 \end{aligned}$$

Q8: 11 students out of 25 students of a school participated in a marathon. Express this as percentage.

Solution:

$$\begin{aligned}
 11 \text{ students out of } 25 \text{ students} \\
 \text{In percentage} &= \frac{11}{25} \\
 11 \text{ students out of } 25 \text{ students} \\
 &= \frac{11}{25} \times 100\% \\
 &= 44\%
 \end{aligned}$$

Q9: Fahad saved Rs 50 out of Rs 200 from his pocket money. What percentage of money did he save?

Solution:

$$\begin{aligned}
 \text{Rs. } 50 \text{ out of Rs } 200 &= \frac{50}{200} \\
 \text{In percentage Rs. } 50 \text{ out of Rs. } 200 \\
 &= \frac{50}{200} \times 100\% \\
 &= 25\%
 \end{aligned}$$

Q10: Out of 180 eggs, 25% are broken. How many eggs are broken?

Solution:

To find number of broken eggs, we can find,

$$\begin{aligned}
 25\% \text{ of } 180 &= \frac{25}{100} \times 180 \\
 &= 45 \text{ eggs}
 \end{aligned}$$

So, number of broken eggs = 45 eggs

Q11: In a match Imran scored 35% of the total 350 runs. How much runs he scored and how much runs the remaining team scored?

Solution:

$$\begin{aligned}
 \text{Total runs} &= 350 \\
 \text{Imran scored} &= 35\% \\
 \text{Imran runs} &= 35\% \text{ of } 350 \\
 &= \frac{35}{100} \times 350 \\
 &= 122.5 \text{ runs.}
 \end{aligned}$$

$$\begin{aligned}
 \text{Remaining team scored runs} \\
 &= 350 - 122.5 \\
 &= 227.5
 \end{aligned}$$

Q12: Sara donated 12% of her salary as a donation. If her salary is Rs 45000, find the amount she donated.

Solution:

$$\text{Total salary of Sara} = \text{Rs. } 45000$$

Donation = 12%

So, Donation amount of Sara = 12% of 45000

$$\begin{aligned}
 &= \frac{12}{100} \times 45000 \\
 &= \text{Rs. } 5400
 \end{aligned}$$

Exercise 4.2

Q1: Compare the following and tell which one is greater as percentage.

a. 22 out of 60 and 54 out of 60.

Solution:

Percentage of 22 out of 60

$$\begin{aligned}
 &= \frac{22}{60} \times 100\% \\
 &= 36.67\%
 \end{aligned}$$

Percentage of 54 out of 60

$$\begin{aligned}
 &= \frac{54}{60} \times 100\% \\
 &= 90\%
 \end{aligned}$$

As 90% > 36.67%

So, 54 out of 60 is greater

b. 18 out of 20 and 16 out of 20.

Solution:

$$\begin{aligned}
 \text{Percentage of } 18 \text{ out of } 20 &= \frac{18}{20} \times 100\% \\
 &= 90\%
 \end{aligned}$$

$$\begin{aligned}
 \text{Percentage of } 16 \text{ out of } 20 &= \frac{16}{20} \times 100\% \\
 &= 80\%
 \end{aligned}$$

As 90% > 80%

So, 18 out of 20 is greater.

c. 68 out of 120 and 66 out of 130

Solution:

$$\begin{aligned}
 \text{Percentage of } 68 \text{ out of } 120 &= \frac{68}{120} \times 100\% \\
 &= 56.67\%
 \end{aligned}$$

$$\begin{aligned}
 \text{Percentage of } 66 \text{ out of } 130 &= \frac{66}{130} \times 100\% \\
 &= 50.77
 \end{aligned}$$

As $56.67\% > 50.77\%$

So, 68 out of 120 is greater.

d. 440 out of 550 and 520 out of 710

Solution:

Percentage of 440 out of 550

$$= \frac{440}{550} \times 100\% \\ = 80\%$$

Percentage of 520 out of 710

$$= \frac{520}{710} \times 100\% \\ = 73.23\%$$

As $80\% > 73.23\%$

So, 440 out of 550 is greater.

e. 10 out 60 and 15 out of 70

Solution:

Percentage of 10 out of 60

$$= \frac{10}{60} \times 100\% \\ = 16.67\%$$

Percentage of 15 out of 70

$$= \frac{15}{70} \times 100\% \\ = 21.42\%$$

As $21.42\% > 16.67\%$

So, 15 out of 70 is greater.

f. 200 out of 400 and 150 out of 350

Solution:

Percentage of 200 out of 400

$$= \frac{200}{400} \times 100\% \\ = 50\%$$

Percentage of 150 of 350

$$= \frac{150}{350} \times 100\% \\ = 42.86\%$$

As $50 > 42.86\%$,

So 200 out of 400 is greater.

Q2: In town A, 200 out 500 houses are under construction and in town B 250 out of 600 houses are under construction. Compare and tell in which town more percentage of houses are under construction?

Solution:

In town A, under construction house = 200 out 500

$$= \frac{200}{500}$$

In percentage, $\frac{200}{500} \times 100\% = 40\%$

In town B, under construction House,

= 250 out of 600

$$= \frac{250}{600}$$

In percentage, $\frac{250}{600} \times 100\% = 41.67\%$

As $41.67\% > 40\%$,

So, in town B more percentage of houses are under construction.

Q3: In math test Anum got 40 marks out of 50. And in English test she got 35 marks out of 60. In which subject she got less percentage marks.

Solution:

Marks of Anum in maths test

= 40 out of 50

$$= \frac{40}{50}$$

Percentage of $\frac{40}{50} = \frac{40}{50} \times 100\%$

Marks of Anum in English = 35 out of 60

$$= \frac{35}{60}$$

Percentage of $\frac{35}{60} = \frac{35}{60} \times 100\% \\ = 58.33\%$

As $58.33\% < 80\%$

So, In English she got less percentage marks.

Q4: Mahad got 12 out of 15 marks, Hadia got 14 out of 15 marks and Samiha got 11 out of 15 marks in math weekly test. Compare their marks by finding percentage.

Solution:

Mahad marks in Maths test

$$= \frac{12}{15}$$

Percentage of $\frac{12}{15} = \frac{12}{15} \times 100\% \\ = 80\%$

Hadia marks in Maths test

$$= \frac{14}{15}$$

Percentage of $\frac{14}{15} = \frac{14}{15} \times 100\% \\ = 93.33\%$

Samiha marks in Maths test

$$= \frac{11}{15}$$

Percentage of $\frac{11}{15} = \frac{11}{15} \times 100\% \\ = 73.33\%$

From result, Hadia got highest percentage and Samiha got lowest percentage marks.

Q5: Omair's notebook has 82 empty pages out of 120 pages. Sara's notebook has 42 empty pages out of 54 pages. Compare by finding percentage to tell who has more empty pages in his/her notebook.

Solution:

Omair's notebook empty page = 82 out of 120

$$= \frac{82}{120}$$

Percentage of $\frac{82}{120} = \frac{82}{120} \times 100\% \\ = 68.33\%$

Sara's notebook empty page = 42 out of 54

$$= \frac{42}{54}$$

Percentage of $\frac{42}{54} = \frac{42}{54} \times 100\% \\ = 77.77\%$

As $77.77\% > 68.33\%$

So Sara notebook has more empty pages.

Exercise 4.3

Q1. Find increase and decrease in quantity as given in percentage.

a. 500 decreased by 17%

Solution:

Quantity = 500

Percentage = 17%

Decrease in Quantity = 17% of 500

$$= \frac{17}{100} \times 500 \\ = 85$$

So 500 is decreased by 85

So $500 - 85 = 415$.

b. 1400 increased by 9%**Solution:**

Quantity = 1400

Percentage increase = 9%

Increase in Quantity = 9% of 1400

$$= \frac{9}{100} \times 1400$$

$$= 126$$

So 1400 increased by 126.

So $1400 + 126 = 1526$.**c. 18% increase in 1350****Solution:**

Quantity = 1350

Percentage increase = 18% of 1350

$$= \frac{18}{100} \times 1350$$

$$= 243$$

So 1350 increase by 243.

So $1350 + 243 = 1593$ **d. 22% decrease in 760****Solution:**

Quantity = 760

Percentage decrease = 22%

Decrease in Quantity = 22% of 760

$$= \frac{22}{100} \times 760$$

$$= 167.2$$

So 760 Decreased by 167.2

So $760 - 167.2 = 592.8$ **e. 55% decrease in 2500****Solution:**

Quantity = 2500

Percentage decrease = 55%

Decreased in Quantity = 55% of 2500

$$= \frac{55}{100} \times 2500$$

$$= 1365$$

So 2500 decreased by 1365.

So $2500 - 1365 = 1125$ **f. 2% increase in 270****Solution:**

Quantity = 270

Percentage increase = 2%

Increase in Quantity = 2% of 270

$$= \frac{2}{100} \times 270$$

$$= 5.4$$

So 270 increase by 5.4

So $270 + 5.4 = 275.4$ **Q2. Arhams's salary is Rs 45900. If his salary increased by 12%. How much salary increased? Find his new salary?****Solution:**

Arhams's Salary = Rs. 45900

Percentage Salary increase = 12%

Increase in Salary = 12% of 45900

$$= \frac{12}{100} \times 45900$$

$$= \text{Rs. } 5508$$

And New Salary = $45900 + 5508$ $= \text{Rs. } 51,408$ **Q3. On Monday 300 children visited the zoo. On Tuesday 15% less children visited the zoo. How many children visited the zoo on Tuesday?****Solution:**

Children visited to zoo on Monday = 300

On Tuesday percentage decrease = 15%

Decrease in children = 15% of 300

$$= \frac{15}{100} \times 300$$

$$= 45$$

So 300 children decreased by 45 on Tuesday.

So children visited to zoo on Tuesday.

$$= 300 - 45$$

$$= 255$$

Q4: In 2021, Ahads weight was 27 kg. His weight is decreased by 9% in 2022. Find:**a. How much weight did he loose?****b. What is his weight in 2022?****Solution:**

In 2021m Ahad's weight = 27kg

In 2022, weight decrease = 9%

Decrease in weight = 9% of 27

$$= \frac{9}{100} \times 27$$

$$= 2.43 \text{ kg}$$

a. Loose in weight = 2.43 kg**b. Weight in 2022 = $27 - 2.43 \text{ kg}$**

$$= 24.57 \text{ kg}$$

Q5: Marwa's marks in monthly test in March were 56. In April, her marks increased by 24%. What are her marks in April?**Solution:**

Marwa's marks in monthly test = 56

Increase marks = 24%

Increase in marks = 24% of 56

$$= \frac{24}{100} \times 56$$

$$= 13.44 \text{ kg}$$

So marks in April = $56 + 13.44$

$$= 69.44$$

Exercise 4.4**Q1: Find the ratio of:****a. 35 to 55****Solution:**

$$35 \text{ to } 55 = \frac{35}{55} = \frac{7}{11} = 7:11$$

b. 15 min to 3 hours**Solution:**15min to 3hours = 15min to 3×60 min

$$= 15 \text{ min to } 180 \text{ min} = \frac{15}{180} = \frac{1}{12}$$

 $= 1:12$ **c. 8km to 600m****Solution:**8km to 600m = $8 \times 1000\text{m}$ to 600m
 $= 8000\text{m}$ to 600m

$$= \frac{8000}{600} = 40:3$$

d. 500 gram to 4 kg**Solution:**500gram to 4kg = 500gram to $4 \times 1000\text{gram}$

$$= 500\text{gram to } 4000\text{gram} = \frac{500}{4000} = 1:8$$

e. Rs 240 to Rs 300**Solution:**

$$\text{Rs. } 240 \text{ to Rs. } 300 = \frac{240}{300} = 4:5$$

f. 10m to 100cm**Solution:**10m to 100cm = $10 \times 100\text{cm}$ to 100cm

$$= 1000\text{cm to } 100\text{cm} = \frac{1000}{100} = \frac{10}{1}$$

$$= 10:1$$

Q2: Write each of the following ratios in fraction form.

a. $4:5 = \frac{4}{5}$	b. $12:13 = \frac{12}{13}$
c. $19:25 = \frac{19}{25}$	d. $34:63 = \frac{34}{63}$
e. $55:78 = \frac{55}{78}$	

$$f. 3.1:6.2 = \frac{3.1}{6.2} = \frac{31 \times 10}{62 \times 10} = \frac{310}{620} = \frac{1}{2}$$

$$g. 18:69 = \frac{18}{69} = \frac{6}{23}$$

$$h. 72:81 = \frac{72}{81} = \frac{8}{9}$$

Q3: Simplify the following ratios.

- $4:8 = 1:2$ (Dividing by 4)
- $15:10 = 3:2$ (Dividing by 5)
- $30:45 = 2:3$ (Dividing by 15)
- $81:99 = 9:11$ (Dividing by 9)
- $16:39 = 16:39$
- $42:56 = 3:4$ (Dividing by 14)
- $35:100 = 7:20$ (Dividing by 5)
- $25:75 = 5:15$ (Dividing by 5)
 $= 1:3$ (Dividing by 5)
- $8:12 = 2:3$ (Dividing by 4)
- $56:72 = 7:9$ (Dividing by 8)
- $18:36 = 6:12$ (Dividing by 3)
 $= 1:2$ (Dividing by 6)
- $64:90 = 32:45$ (Dividing by 2)

Q4: Write each of the following quantities into ratios and reduce into the simplest form (where possible).

a. Rs 500 and Rs 750

Solution:

$$\text{Rs 500 and Rs 750} = 500:750$$

$$= 100:150 \text{ (Dividing by 5)}$$

$$= 20:30 \text{ (Dividing by 5)}$$

$$= 4:6 \text{ (Dividing by 2)}$$

$$= 2:3 \text{ (Dividing by 2)}$$

b. 45m and 350cm

Solution:

$$45\text{m and } 350 = 45\text{m} : 350\text{cm}$$

$$= 45 \times 100\text{cm} : 350\text{cm}$$

$$= 4500:350$$

$$= 900:70 \text{ (Dividing by 5)}$$

$$= 180:14 \text{ (Dividing by 5)}$$

$$= 90:7 \text{ (Dividing by 2)}$$

$$= 90:7$$

c. 100 days and 35 weeks

Solution:

$$100 \text{ day and } 35 \text{ weeks}$$

$$= 100 \text{ day} : 35 \text{ weeks}$$

$$= 100 \text{ day} : 35 \times 7 \text{ day}$$

$$= 100:245$$

$$= 20:49 \text{ (Dividing by 5)}$$

d. 4 kg and 12 grams

Solution:

$$4 \text{ kg and } 12 \text{ grams}$$

$$= 4 \text{ kg} : 12 \text{ grams}$$

$$= 4 \times 100 \text{ grams} : 12 \text{ grams}$$

$$= 400:12$$

$$= 2000:6 \text{ (Dividing by 2)}$$

$$= 1000:3 \text{ (Dividing by 2)}$$

e. 8 weeks and 64 days

Solution:

$$8 \text{ weeks and } 64 \text{ days}$$

$$= 8 \times 7 \text{ days} : 64 \text{ days}$$

$$= 56:64$$

$$= 7:8 \text{ (Dividing by 8)}$$

f. 40 km and 90 km

Solution:

$$40 \text{ km and } 90 \text{ km}$$

$$= 40:90$$

$$= 8:18 \text{ (Dividing by 2)}$$

$$= 4:9 \text{ (Dividing by 2)}$$

Q5: Write each of the following quantities into ratios and reduce into the simplest form (where possible).

Solution:

$$\text{Total bulb in box} = 80$$

$$\text{Defective bulb} = 15$$

$$\text{So Non-defective} = 80 - 15$$

$$= 65$$

$$\text{So ratio of defective to non-defective}$$

$$= 15:65$$

$$= 3:13 \text{ (Dividing by 5)}$$

Q6: Ahmed and Saad bought a bat for Rs. 450. Ahmed paid Rs. 250 and Saad paid Rs. 200. Find the ratio of the amount Ahmed paid to the amount that Saad paid.

Solution:

$$\text{Total price of bat} = \text{Rs. 450}$$

$$\text{Ahmad paid} = \text{Rs. 250}$$

$$\text{Saad Paid} = \text{Rs. 200}$$

$$\text{Ratio between amount of Ahmad and Saad}$$

$$= 250:200$$

$$= 50:40 \text{ (Dividing by 5)}$$

$$= 10:8 \text{ (Dividing by 5)}$$

$$= 5:4 \text{ (Dividing by 2)}$$

Q7: In a school there are 800 students and 80 teachers. Find the student to teacher ratio in the school.

Solution:

$$\text{Number of Students} = 800$$

$$\text{Number of Teachers} = 80$$

$$\text{Ratio between students and Teachers}$$

$$= 800:80$$

$$= 160:16 \text{ (Dividing by 5)}$$

$$= 10:1 \text{ (Dividing by 16)}$$

Q8: The price of a pack of bread increased to Rs 50 from Rs 48.

Find the ratio of the increase in price to the original price.

Solution:

$$\text{Original price} = \text{Rs. 48}$$

$$\text{Increase price} = \text{Rs. 50}$$

$$\text{Ratio between increase price and original price} = 50:48$$

$$= 25:24 \text{ (Dividing by 2)}$$

Q9: If Rs 340 is divided between Sara and Sonia according to the ratio 3:7, find the amount each gets.

Solution:

$$\text{Total amount} = \text{Rs. 340}$$

$$\text{Ratio} = 3:7$$

$$\text{Sum of ratio} = 3+7=10$$

$$\text{Amount of Sara} = \frac{3}{10} \times 340$$

$$= 3 \times 34 = \text{Rs. 102}$$

$$\text{Amount of Sonia} = \frac{7}{10} \times 340$$

$$= 7 \times 34$$

$$= \text{Rs. 238}$$

Exercise 4.5

Q1: Solve the following.

a. If $a:b = 4:5$ and $b:c = 5:6$, then find $a:b:c$

Solution:

$$\text{The given ratios are}$$

$$a:b = 4:5$$

$$\text{and } b:c = 5:6$$

$$a : b : c$$

$$4 : 5$$

$$5 : 6$$

$$4 : 5 : 6$$

$$\text{So required ratio } a:b:c = 4:5:6$$

b. If $d:e = 2:6$ and $e:f = 7:8$, then find $d:e:f$.

Solution:

The given ratios are

$$d : e = 2 : 6$$

and $e : f = 7 : 8$

$$\begin{array}{ccc} d & e & f \\ 2 & 6 & 7 \\ \hline 14 & 42 & 48 \end{array}$$

$$14 : 42 : 48$$

$$7 : 21 : 24 \text{ (Dividing by 2)}$$

So required ratio are

$$d : e : f = 7 : 21 : 24$$

c. If $g : h = 8 : 9$ and $h : i = 9 : 12$, then find $g : h : i$.

Solution:

Given ratios are

$$g : h = 8 : 9$$

and $h : i = 9 : 12$

$$\begin{array}{ccc} g & h & i \\ 8 & 9 & 12 \\ \hline 8 & 9 & 12 \end{array}$$

$$8 : 9 : 12$$

So required ratio are

$$g : h : i = 8 : 9 : 12$$

d. If $j : k = 6 : 7$ and $k : l = 10 : 1$, then find $j : k : l$.

Solution:

Give ratios are

$$j : k = 6 : 7$$

and $k : l = 10 : 1$

$$\begin{array}{ccc} j & k & l \\ 6 & 7 & 10 \\ \hline 60 & 70 & 7 \end{array}$$

$$60 : 70 : 7$$

So required ratio are

$$j : k : l = 60 : 70 : 7$$

e. If $m : n = 2 : 8$ and $n : o = 5 : 6$, then find $m : n : o$

Solution:

Given ratios are

$$m : n = 2 : 8$$

and $n : o = 5 : 6$

$$\begin{array}{ccc} m & n & o \\ 2 & 8 & 5 \\ \hline 10 & 40 & 24 \end{array}$$

$$10 : 40 : 24$$

$$5 : 20 : 12 \text{ (Divide by 2)}$$

So required ratio are

$$m : n : o = 5 : 20 : 24$$

Q2: The measurement of three angles of a triangle are in the ratio 2 : 2 : 5. Find the measurement of each angle.

Solution:

Given ratio of angles of a triangle = 2 : 2 : 5

$$\text{Sum of Ratios} = 2 + 2 + 5 = 9$$

Summation of angles of a triangle = 180

$$\text{Measurement of 1st angle} = \frac{2}{9} \times 180 = 40$$

$$\text{Measurement of 2nd angle} = \frac{2}{9} \times 180 = 40$$

$$\text{Measurement of 3rd angle} = \frac{5}{9} \times 180 = 100$$

$$\text{So 1st angle} = 40, \text{ 2nd angle} = 40, \text{ 3rd angle} = 100$$

Q3: The ratio of a to b is 8 : 9 and b to c is 4 : 6. Find the continued ratio a : b : c.

Solution:

Given ratios are

$$a : b = 8 : 9$$

and $b : c = 4 : 6$

So the continued ratio are

$$\begin{array}{ccc} a & b & c \\ 8 & 9 & 4 \\ \hline 32 & 36 & 16 \end{array}$$

$$32 : 36 : 16$$

$$16 : 18 : 8 \text{ (Dividing by 2)}$$

So continue ratio = a : b : c = 16 : 18 : 27

Q4: The two ratios of three quantities a, b and c are a : b = 2 : 3 and b : c = 4 : 5. Find their continued ratio.

Solution:

Given ratios are

$$a : b = 2 : 3$$

and $b : c = 4 : 5$

$$\begin{array}{ccc} a & b & c \\ 2 & 3 & 4 \\ \hline 8 & 12 & 15 \end{array}$$

$$8 : 12 : 15$$

So required continued ratio are

$$a : b : c = 8 : 12 : 15$$

Q5: The ratio of Ibrahim's marks to Ahad's marks is 5 : 3 and the ratio of Ibrahim's marks to Marwa's marks is 4 : 7. Find the continued ratio among their marks.

Solution:

Ratio of Ibrahim's marks to Ahad's marks = 5 : 3

and

Ratio of Ibrahim's mark to Marwa's marks = 4 : 7

$$\begin{array}{ccc} \text{Ahad} & \text{Ibrahim} & \text{Marwa} \\ 3 & 5 & 4 \\ \hline 12 & 20 & 35 \end{array}$$

$$12 : 20 : 35$$

So required Ratio among Ahad, Ibrahim and Marwa = 12 : 20 : 35

Q6: Divide Rs 7500 into the ratio 4 : 6 : 5.

Solution:

To amount = Rs. 7500

$$\text{Ratio} = 4 : 6 : 5$$

$$\text{Sum of ratio} = 4 + 6 + 5 = 15$$

$$\text{1st share} = \frac{4}{15} \times 7500 = 4 \times 500 = \text{Rs. 2000}$$

$$\text{2nd share} = \frac{6}{15} \times 7500 = 6 \times 500 = \text{Rs. 3000}$$

$$\text{3rd share} = \frac{5}{15} \times 7500 = 5 \times 500$$

$$= \text{Rs. 2500}$$

Q7: Divide Rs 5376 between Ayesha, Sadia and Madiha in the ratio 8 : 7 : 6.

Solution:

Total amount = Rs. 5376

$$\text{Ratio} = 8 : 7 : 6$$

$$\text{Sum of ratio} = 8 + 7 + 6 = 21$$

$$\text{Ayesha share} = \frac{8}{21} \times 5376 = \text{Rs. 2048}$$

$$\text{Sadia share} = \frac{7}{21} \times 5376 = \text{Rs. 1792}$$

$$\text{Madiha share} = \frac{6}{21} \times 5376 = \text{Rs. 1536}$$

Q8: Divide 3416 among three persons a, b and c, such that the ratio between their shares is:

$$a : b = 4 : 5$$

$$b : c = 8 : 10$$

Solution:

Total shares = 3416

Given ratios are

$$a : b = 4 : 5$$

and $b : c = 8 : 10$

$$\begin{array}{ccc} a & b & c \\ 4 & 5 & 8 \\ & \swarrow & \searrow \\ & 8 & 10 \end{array}$$

$$32 : 40 : 50$$

$$a : b : c = 32 : 40 : 50$$

$$= 16 : 20 : 25 \text{ (Dividing by 2)}$$

$$a : b : c = 16 : 20 : 25$$

$$\text{Sum of ratios} = 16 + 20 + 25$$

$$= 61$$

$$\text{1st person share} = \frac{16}{61} \times 3416$$

$$= 896$$

$$\text{2nd person share} = \frac{20}{61} \times 3416$$

$$= 1120$$

$$\text{3rd person share} = \frac{25}{61} \times 3416$$

$$= 1400$$

Exercise 4.6

Q1: Find the rate for each of the following.

a. 28 kilometers covered in 4 hours

Solution:

Given 28 kilometers covered in 4 hours

$$= \frac{28 \text{ km}}{4 \text{ hr}}$$

$$= 28 : 4 \text{ (Divide by 4)}$$

$$= 7 : 1$$

$$= 7 \text{ km/hr}$$

b. 16 metre rope for Rs. 1376

Solution:

16 metre rope for Rs. 1376

$$= \text{Rs. } 1376 : 16 \text{ m}$$

$$= 688 : 8 \text{ (Divide by 2)}$$

$$= 344 : 4 \text{ (Divide by 2)}$$

$$= 172 : 2 \text{ (Divide by 2)}$$

$$= 86 : 1 \text{ (Divide by 2)}$$

$$= \text{Rs. } 86/\text{m}$$

c. 108 words typed in 6 minutes

Solution:

108 words typed in 6 minutes

$$= 108 \text{ words} : 6 \text{ min}$$

$$= 108 : 6$$

$$= 108 : 1 \text{ (Dividing by 6)}$$

$$= 18 \text{ word} : 1 \text{ min}$$

$$= 18 \text{ word/min}$$

d. 8 kg of mangoes in Rs 400

Solution:

8 kg of mangoes in Rs 400

$$= 400 : 8$$

$$= 50 : 1 \text{ (Dividing by 8)}$$

$$= \text{Rs. } 50/\text{kg}$$

e. Rs 6000 for fencing 50 metre boundary

Solution:

Rs 6000 for fencing 50 metre boundary

$$= 6000 : 50$$

$$= 1200 : 10 \text{ (Dividing by 5)}$$

$$= 240 : 2 \text{ (Dividing by 5)}$$

$$= 120 : 1 \text{ (Dividing by 2)}$$

$$= \text{Rs } 120/\text{m}$$

Q2: The entry ticket for 8 persons in the museum costs Rs 1760. What is the rate per ticket?

Solution:

Total cost = Rs. 1760

Number of ticket = 8

Required Rate = Rs. 1760/8 Ticket

$$= 1760 : 8$$

$$= 880 : 4 \text{ (Dividing by 2)}$$

$$= 220 : 1 \text{ (Dividing by 4)}$$

$$= 220 : 1$$

$$= \text{Rs. } 220/\text{Ticket}$$

Q3: The cost of tiling a floor having an area of 234 meter square is Rs 205, 920. What is the per square meter rate of tiling?

Solution:

Area = 234 m²

Cost = Rs. 205, 920

Required rate = Rs. 205920/234m²

$$= 205920 : 234$$

$$= 102960 : 117 \text{ (Divide by 2)}$$

$$= 34,230 : 39 \text{ (Divide by 3)}$$

$$= 11,440 : 13 \text{ (Divide by 3)}$$

$$= 880 : 1 \text{ (Divide by 13)}$$

$$= \text{Rs. } 880/\text{m}^2$$

Q4: Dania bought 12 kg of apples in Rs 720. Find the rate of 1 kg of apples.

Solution:

Here we consider weight of apples with its price.

We can write it as:

$$\text{Rs. } 720/12\text{kg} = 720 : 12$$

To find per kg rate, we simplify the ratio.

$$720 : 12$$

$$60 : 1 \text{ (Dividing by 12)}$$

$$= \frac{60}{1}$$

So rate of 1 kg of apple

$$= \text{Rs. } 60/\text{kg}$$

Q5: Aleem's income is Rs 3000 after working for 19 hours. What is his per hour pay?

Solution:

To find per hour pay, we can write as,

$$\text{Rs. } 3000/19 = 3000 : 19$$

To find per hour rate, we simplify the ratio.

$$3000 : 19$$

$$= 157.89 : 1 \text{ (Dividing by 19)}$$

$$= 157.89/1$$

$$\text{So per hour pay} = \text{Rs. } 157.89/\text{h}$$

Review Exercise

Q1: Encircle the correct option.

a. When we compare two or more than two quantities of same kind then that comparison is called ____

(i) Ratio✓

(ii) Percentage

(iii) Rate

(iv) Fraction

b. The value of one quantity in unit rate is always:

(i) zero

(ii) two

(iii) three

(iv) one✓

c. There are 14 cars and 12 bikes on the road. The ratio of cars to bike is ____.

(i) 12 : 7

(ii) 6 : 7

(iii) 7 : 6✓

(iv) 7 : 12

d. A ____ is a comparison of three or more quantities in a certain order

(i) ratio

(ii) continued ratio✓

(iii) rate

(iv) percentage

e. 50 out of 500 in percentage form is ____

(i) 1.2%

(ii) 10%✓

(iii) 15%

(iv) 5%

f. The ratio 20 : 50 in simplified form is ____

(i) 1 : 2

(ii) 2 : 5✓

(iii) 10 : 25

(iv) 4 : 10

g. The rate of one week wages of Rs 4200. The rate of a one day wages:

(i) 600 : 1✓

(ii) 7 : 4200

(iii) 1 : 4200

(iv) 1 : 2800

h. If the cost of 12 notebooks is Rs 204, then the cost of 5 notebooks is

- (i) Rs.100 (ii) Rs.102
(iii) Rs 85✓ (iv) Rs. 120

i. The equivalent form of the ratio 3 : 4 is _____

- (i) 6 : 4 (ii) 6 : 8✓
(iii) 9 : 8 (iv) 12 : 12

Q2: What is meant by ratio?

Ans: A term that is used to compare two or more quantities of the same kind is called "Ratio". Symbol ":" is used to represent a ratio.

Q3: What is rate? Give examples.

Ans: A rate is a ratio that compares two quantities with different units of measure.

Rates are used almost every day. For example, unit price of items, speed, service charges etc are often given as rates.

Q4: Define percentage with examples.

Ans: Percentage:

A relative value indicating hundredth parts of any quantity i.e. To express a quantity out of hundred symbol for percent is %.

For example Salman obtained 70 marks out 100, So it expressed as 70%.

Q5: Write the following in the form of ratio and simplify is possible.

a. 600 days, and 120 days

Solution:

$$\begin{aligned} 600 \text{ days and } 120 \text{ days} \\ &= 600 : 120 \\ &= 50 : 10 \text{ (Dividing by 12)} \end{aligned}$$

$$= 5 : 1 \text{ (Dividing by 10)}$$

b. 40g and 120g

Solution:

$$\begin{aligned} 40\text{g and } 120\text{g} &= 40 : 120 \\ &= 4 : 12 \\ &\quad \text{(Dividing by 10)} \\ &= 1 : 3 \text{ (Dividing by 4)} \end{aligned}$$

c. 65 ml and 13 ml

Solution:

$$\begin{aligned} 65\text{ml and } 13\text{ml} &= 65 : 13 \\ &= 5 : 1 \\ &\quad \text{(Dividing by 13)} \end{aligned}$$

d. 14 men and 16 women

Solution:

$$\begin{aligned} 14 \text{ men and } 16 \text{ women} &= 14 : 16 \\ &= 7 : 8 \\ &\quad \text{(Dividing by 2)} \end{aligned}$$

Q6: Aleena covers a distance of 40 m in 25 sec. How much time will she take to cover the distance of:

a. 50m

Solution:

$$\begin{aligned} \text{Distance covered} &= 40\text{m} \\ \text{Time} &= 25 \text{ sec} \end{aligned}$$

As 40m covered in 25 sec

$$\text{So } 40\text{m} = 25\text{sec}$$

$$1\text{m} = \frac{25}{40}$$

$$\text{So } 50\text{m} = \frac{25}{40} \times 50$$

$$= \frac{125}{4}$$

$$= 31\frac{1}{4} \text{ sec}$$

$$\text{So } 50\text{m distance covered in } 31\frac{1}{4} \text{ sec.}$$

b. 120m

Solution:

As 40m distance covered in 25sec.

$$\text{So } 40\text{m} = 25\text{sec}$$

$$1\text{m} = \frac{25}{40} \text{ sec}$$

As we have to find time for 120m.

So

$$\begin{aligned} 120\text{m} &= \frac{25}{40} \times 120 \text{ sec} \\ &= 75 \text{ sec} \end{aligned}$$

Q7: There are 800 eggs. If 120 are rotten, find:

a. What percentage of eggs are fresh?

Solution:

$$\text{Total eggs} = 800$$

$$\text{Rotten eggs} = 120$$

$$\text{Fresh eggs} = 800 - 120 = 680$$

So percentage of 680 fresh eggs out of 800

$$= \frac{680}{800} \times 100\%$$

$$= 85\%$$

b. What is the ratio between the number of rotten and fresh eggs.

Solution:

$$\text{As fresh eggs} = 680$$

$$\text{Rotten eggs} = 120$$

$$\text{Ratio} = 120 : 680$$

$$= 12 : 68 \text{ (Dividing by 10)}$$

$$= 6 : 34 \text{ (Dividing by 2)}$$

$$= 3 : 17 \text{ (Dividing by 2)}$$

c. What is the ratio between the fresh and total number of eggs?

Solution:

$$\text{As fresh eggs} = 680$$

$$\text{Total eggs} = 800$$

$$\text{Ratio among them} = 680 : 800$$

$$= 68 : 80 \text{ (Dividing by 10)}$$

$$= 34 : 40 \text{ (Divide by 2)}$$

$$= 17 : 20 \text{ (Divide by 2)}$$

Q8: The ratio of Saad's salary to Sania's salary is 2 : 3 and the ratio of Sania's salary to Mauz's Salary is 4 : 3. Find the continued ratio among their salary.

Solution:

Ratio of Saad's salary to Sania's salary = 2 : 3 and ratio of Sania's salary to Mauz's salary = 4 : 3

To find continued ratio

Saad	Sania	Mauz
2	3	
	4	3
8	12	9

So continued ratio among their salary

$$= 8 : 12 : 9$$

Q9: An author charges Rs 82,500 for writing 55 pages. What is his per page rate for?

Solution:

$$\text{Total charges of 55 pages} = \text{Rs. } 82,500$$

$$\begin{aligned} \text{So per page rate} &= \frac{\text{Rs. } 82,500}{55} \\ &= \text{Rs. } 1500 \end{aligned}$$

Q10: Umar got 15 marks out of 20 marks in Islamic study and 22 marks out 25 marks in Urdu. Compare and tell in which subject he got more percentage marks?

Solution:

$$\text{Marks in Islamic study} = 15 \text{ out of } 20$$

$$= \frac{1500}{20}$$

$$\text{Percentage} = \frac{15}{20} \times 100\%$$

$$= 75\%$$

Marks in Urdu = 22 out of 25

$$= \frac{22}{25}$$

$$\text{Percentage} = \frac{22}{25} \times 100\%$$

$$= 55\%$$

As $88\% > 75\%$

So in Urdu he got more percentage marks.

Q11: Madeeha bought a table in Rs 1500. If its price decreased by 4% during sale, find the decreased price of table.

Solution:

Price of table = Rs. 1500

Percentage decrease = 4%

Decrease in price = 4% of 1500

$$= \frac{4}{100} \times 1500$$

$$= 60$$

So, Price of the table decreased by 60.

So, Decrease price of table

$$= 1500 - 60$$

$$= \text{Rs } 1440$$

Unit 5

Set

Exercise 5.1

Q1: Identify well-defined sets among these.

a) **A** = the set of grades in your school.

Solution:

Well-defined

b) **B** = the set of interesting games.

Solution:

Not well-defined

c) **C** = the set of the provincial capitals of Pakistan

Solution:

Well-defined

d) **D** = the set of intelligent students in the class

Solution:

Well-defined

e) **E** = the set of easy questions in the exams.

Solution:

Not well-defined

f) **H** = the set of big animals

Solution:

Not well-defined

g) **J** = the set of the Prime Ministers of Pakistan

Solution:

Well-defined

h) Set of small birds in the sky

Solution:

Not well-defined

i) Set of even numbers between 1 and 5

Solution:

Well-defined

j) $\{+, -, +, -\}$

Solution:

Well-defined

k) $\{1, 3, 5, 7\}$

Solution:

Well-defined

l) Set of durable tables

Solution:

Not well-defined

m) Set of nice people in the neighbourhood

Solution:

Not well-defined

n) Set of beautiful people in a town

Solution:

Not well-defined

Q2: List each element of these sets in symbolic form (e.g. $a \in A$)

a. $A = \{1, 3, 5, 7, 9\}$

Solution:

$$1 \in A, 3 \in A, 5 \in A, 7 \in A, 9 \in A$$

b. $B = \{a, b, c, d, e\}$

Solution:

$$a \in B, b \in B, c \in B, d \in B, e \in B$$

c. $C = \{s, t, a, r\}$

Solution:

$$s \in C, t \in C, a \in C, r \in C$$

d. $D = \{\text{Monday, Wednesday, Friday}\}$

Solution:

$$\text{Monday} \in D, \text{Wednesday} \in D, \text{Friday} \in D$$

e. $E = \{\text{chips, cookies, burgers, fries}\}$

Solution:

$$\text{chips} \in E, \text{cookies} \in E, \text{burgers} \in E, \text{fries} \in E$$

Q3: Fill in the blanks using the symbols $\in$ or $\notin$

a. $5 \in \{1, 4, 5\}$

b. $10 \in \{\text{set of even numbers}\}$

c. $\{u\} \notin \{a, i, o, e, u\}$

- d. tiger $\notin$ {set of birds}
 e. January $\in$ {set of months of a year}

Q4: Write the following statements in symbolic form.

- a. 4 is an element of set W

Solution:

$$4 \in W$$

- b. 4 is an element of set P

Solution:

$$4 \in P$$

- c. 0 is not the element of natural number.

Solution:

$$0 \notin N$$

Q5: If $A = \{1, 3, 5, 7, 11\}$ and $B = \{2, 4, 6, 8, 10\}$ then write true and false for the following statements.

- a. $3 \in A$ (T) b. $5 \notin B$ (T) c. $11 \in A$ (T)
 d. $0 \in A$ (T) e. $0 \notin A$ (T) f. $4 \notin B$ (F)
 g. $11 \in B$ (F) h. $2 \notin A$ (T) i. $12 \notin B$ (T) j. $\{7\} \in A$ (F)

Q6: Write the cardinality of the following sets.

- a. $B = \{1, 2, 3, 4, 5, 6, 7\}$

Solution:

$$\text{Cardinality of } B = n(B) = 7$$

- b. $Q = \{a, e, i, o, u\}$

Solution:

$$\text{Cardinality of } Q = n(Q) = 5$$

- c. $N = \{\}$

Solution:

$$\text{Cardinality of } N = n(N) = 0$$

- d. $K = \{\text{maths, science, Urdu, English}\}$

Solution:

$$\text{Cardinality of } K = n(K) = 4$$

- e. $R = \{2, 4, 6\}$

Solution:

$$\text{Cardinality of } R = n(R) = 3$$

- f. $S = \{5, 10, 15, 20\}$

Solution:

$$\text{Cardinality of } S = n(S) = 4$$

- g. $P = \{k, l, m, n, o\}$

Solution:

$$\text{Cardinality of } P = n(P) = 5$$

- h. $E = \{10, 20, 30, 40, 50\}$

Solution:

$$\text{Cardinality of } E = n(E) = 5$$

Exercise 5.2

Q1: Write the following sets in descriptive form.

- a. $A = \{a, b, c, d, e, f\}$

Solution:

$A =$ set of first 6 small letters.

- b. $B = \{1, 3, 5\}$

Solution:

$B =$ set of first 3 odd numbers

- c. $D = \{4, 8, 12, 16, \dots, 32\}$

Solution:

$D =$ set of first '8' multiples of 4.

- d. $E = \{10, 20, 30, \dots\}$

Solution:

$E =$ set of multiples of 10.

- e. $F = \{2, 4, 6, 8, 10, 12\}$

Solution:

$F =$ set of first 6 multiples of 2.

- f. $G = \{\text{Sunday, Saturday}\}$

Solution:

$G =$ set of weekend days of a week.

Q2: Identify the well-defined sets and write them in tabular form.

- a. A set of months of the Islamic year.

Solution:

$\{\text{Muharram, Safar, Rabi-ul-Awwal, Rabi-us Sani, Jamadi-ul-Awwal, Jamadi-ul-Sani, Rajab,}$

Shaban, Ramadan, Shawal, Zil-Qadah, Zul-Hijah}

- b. A set of all odd numbers.

Solution:

$$\{1, 3, 5, 7, 9, \dots\}$$

- c. A set of all provinces of Pakistan

Solution:

$\{\text{Punjab, Sindh, Balochistan, Khyber Pakhtunkhwa}\}$

- d. A set of famous Muslim countries.

Solution:

$\{\text{Pakistan, Saudi Arabia, Bangladesh, Turkey, Iran, Malaysia, Indonesia, ...}\}$

- e. A set of prime numbers less than 20

Solution:

$$\{2, 3, 5, 7, 11, 13, 17, 19\}$$

- f. A set the last seven English alphabets.

Solution:

$$\{t, u, v, w, x, y, z\}$$

- g. A set of even numbers between 55 and 77.

Solution:

$$\{56, 58, 60, 62, 64, 66, 68, 70, 72, 74, 76\}$$

- h. A set of composite numbers between 110 and 119.

Solution:

$$\{111, 112, 114, 115, 116, 117, 118\}$$

Exercise 5.3

Q1: Separate the finite and infinite sets from the following.

- a. $A = \{10, 15, 20, 25, 30\}$. (Finite set)

- b. $B =$ set of numbers less than 20. (Infinite set)

- c. $C =$ The set of all persons of Pakistan. (Infinite set)

- d. $W = \{0, 1, 2, 3, \dots\}$ (Infinite set)

- e. The set of all animals in Lahore zoo. (Finite Set)

- f. Set of all positive integers which are multiple of 6. (Infinite set)

- g. $N = \{1, 2, 3, \dots\}$ (Infinite set)

Q2: Identify empty and non-empty sets. Also give the reason.

- a. The set of quadrilaterals with 7 sides.

Solution:

Empty set because a quadrilateral is a figure of 4 - sides

- b. The set of numbers between 34 and 98.

Solution:

Non-empty set

- c. The set of bicycles having 3 wheels.

Solution:

Non-empty set

- d. The set of prime numbers.

Solution:

Non-empty set

- e. The set of odd multiples of 4

Solution:

Empty set, because multiples of 4 are not odd.

- f. The set of numbers divisible by 2.

Solution:

Non-empty set

- g. The set of prime numbers which have at least 5 factors.

Solution:

Empty set, because prime numbers have only two factors.

Q3: Identify the singleton sets.

- a. The set of composite numbers less than 5.

Solution:

Singleton Sets

- b. The set of prime numbers between 8 and 12.

Solution:

Singleton Set

- c. The set of the provinces of Pakistan.

Solution:

Not Singleton Set

- d. The set of days of the week starting with the letter F.

Solution:

Singleton Set

- e. The set of common factors of 12, 40, 55.

Solution:

Not Singleton Set

- f. The set of numbers which divide both 8 and 15.

Solution:

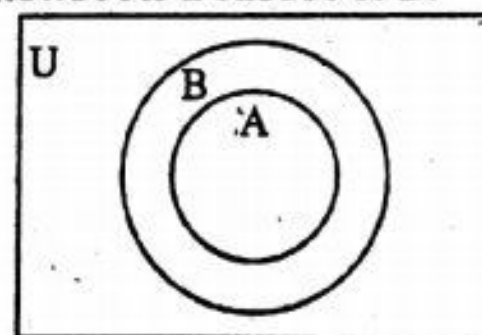
Not Singleton Set

- g. The set of multiples of 9 between 10 and 20.

Solution:

Singleton Set

- Q4: Draw a Venn diagram which shows A is a subset of B.



A is a Subset of B.

Review Exercise

- Q1. Choose the correction option.

- a. A well-defined collection of similar but distinct objects is called a ____

- (i) Set ✓ (ii) Subset

- (iii) null set (iv) infinite set

- b. Empty set is denoted by ____

- (i) $\{\emptyset\}$ or $\{\}$ (ii) $\{\}$ or $\emptyset$ ✓

- (iii) $\{\emptyset\}$ or 0 (iv) $\{0\}$

- c. A set that consists of a limited number of elements is called a/an ____ set.

- (i) equivalent (ii) finite ✓

- (iii) empty (iv) an infinite

- d. The cardinality of set $M = \{5, 10, 15, 20, 25\}$ is ____

- (i) 2 (ii) 3 (iii) 4 (iv) 5 ✓

- e. Elements of a set are denoted by the symbol.

- (i) $\emptyset$ (ii) $\in$ ✓ (iii) $\neq$ (iv) $\supset$

Q2: Define the following.

- a. Set

Definition:

Set is a well-defined collection of similar but distinct objects is called a set.

- b. Element of a set

Definition:

Every element of a set belongs to that set.

The symbol " $\in$ " is used to show that an element "belongs to" the set. For example, for the set $A = \{10, 15, 20, 35\}$. We can say that $10 \in A$.

- c. Tabular form of a set

Definition:

In tabular form, elements of the set are placed within a curly bracket $\{\}$ separated by commas.

- d. Descriptive form of a set

Definition:

In a descriptive form a set is represented in the form of statements using well-defined words.

Example 1:

- a) A = set of prime numbers
b) R = set of 4 provinces of Pakistan

- e. Finite set

Definition:

A set that consists of a limited number of elements is called a finite set.

- g. Empty set

Definition:

When a set contains no elements, it is known as an empty set. An empty set is denoted by curly brackets without any element. It can also be represented by using the symbol $\emptyset$ (read as phi). So, an empty set A can be denoted as: $A = \{\}$ or $A = \emptyset$

- h. Singleton set

Definition:

When a set contains only one element (neither less nor more), it is known as a singleton set.

Examples:

- (i) F = Set of the names of the month having 3 letters

There is only one month which has 3 letters i.e. May. So, it is a singleton set.

- (ii) C = $\{9\}$

Set C contains only 1 element. So, it is a singleton set.

Q3: Which of the following sets are well defined?

- a. The set of 5 triangles with 3 sides.
b. The set of even numbers. (well defined)
c. The set of cars in a showroom. (well defined)
d. The set of prime numbers.

(well defined)

- e.. The set of grains in the sand.
f. The set of numbers which are divisible by 8. (well defined)

Q4: List the elements of the following sets.

- a. The set of 6 countries of world.

Solution:

{Pakistan, India, Iran, Iraq, Turkey, Italy}

- b. The set of vowels

Solution:

{a, e, i, o, u}

- c. Set of the first six months of the year.

Solution:

{January, February, March, April, May, Jun}

- d. Set of colours in a rainbow.

Solution:

{Red, Orange, Yellow, Green, Blue, Violet, Indigo}

- e. Set of 5 pet animals

Solution:

{cat, dog, parrot, Horse, Hen}

Q5: Write the cardinality of the following sets.

- a. The set of all factors of 45 which are even.

Solution:

{ }

- b. The set of prime numbers less than 30.

Solution:

{2, 3, 5, 7, 11, 13, 17, 19, 23}

- c. The set of odd numbers divisible by 4.

Solution:

{ }

Q6. For $P = \{10, 20, 30\}$ and $Q = \{5, 10, 15, 20, \dots\}$, which of the following are correct?

- a. $5 \in P$ incorrect b. $20 \in Q$ correct
 c. $30 \in Q$ incorrect d. $50 \in P$ correct
 e. $60 \in Q$ incorrect f. $0 \in Q$ incorrect

Q7: Identify the correct form of the following sets as tabular or descriptive.

- a. A set of odd numbers between 5 and 15.

Solution:

{7, 9, 11, 13}

- b. A set of prime numbers.

Solution:

{2, 3, 5, 7, 11, ...}

- c. $A = \{1, 2, 3, 4, 5, 6\}$

Solution:

Set of first six Natural number.

- d. $B = \{2, 4, 6, 8, 10\}$

Solution:

Set of first 5 multiples of 2.

- e. A set of days of the week.

Solution:

{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

Q8: Write the following sets into tabular form.

- a. The set of the first five natural numbers.

Solution:

{1, 2, 3, 4, 5}

- b. The set of all vowels of the English alphabet.

Solution:

{a, e, i, o, u}

- c. The set of all odd numbers less than 9.

Solution:

{1, 3, 5, 7}

- d. The set of all numbers which divide 12.

Solution:

{1, 2, 3, 4, 6, 12}

- e. The set of all letters in the word MATHEMATICS.

Solution:

{M, A, T, H, E, I, C, S}

- f. The set of the last four months of the year.

Solution:

{September, October, November, December}

Q9: Write the following sets into descriptive form.

- a. $A = \{7, 8, 9, 10\}$

Solution:

Set of natural numbers between 6 and 11.

- b. $B = \{6, 12, 18, 24, 30\}$

Solution:

Set of first 5 multiples of 6

- c. $C = \{3, 5, 7, 11\}$

Solution:

Set of odd not between 1 and 13

- d. $D = \{r, e, s, t\}$

Solution:

Set of letters to be used in word rest

- e. $E = \{5, 10, 15, 20\}$

Solution:

Set of first four multiples of 5

Q10: Classify the following sets as finite, infinite, empty or singleton set.

- a. The set of stars in the sky. (Infinite)
 b. The set of the sun. (Singleton)
 c. The set of odd numbers. (Infinite)
 d. The set of your friends. (Finite)
 e. The set of all women in the world. (Infinite)
 f. The set of whole numbers between 88 and 89. (Empty)

Unit 6

Patterns and Algebraic Expressions

Exercise 6.1

Q1: Identify the term-to-term rules for the following patterns and write the next three terms.

- a. 4, 9, 14, 19, __, __, __

The term-to-term rule of this pattern is: add 5 in previous term.

The next three terms are: 24, 29, 34.

- b. 8, 16, 32, 64, __, __, __

The term-to-term rule of this pattern is: Multiply previous term with 2.

The next three terms are: 128, 256, 512

- c. 78125, 15625, 3125, __, __, __

The term-to-term rule of this pattern is: Divide previous term with 5.

The next three terms are: 625, 125, 25

- d. 122, 111, 100, 89, __, __, __

The term-to-term rule of this pattern is: Subtract 2 from previous term.

The next three terms are: 78, 67, 56

- e. 5, 10, 20, 40, __, __, __

The term-to-term rule of this pattern is: Multiply previous term with 2.

The next three terms are: 80, 160, 320

- f. 26, 42, 58, 74, __, __, __

The term-to-term rule of this pattern is: Odd 16 in previous term.

The next three terms are: 90, 106, 122

Q2: Identify the position-to-term rules for the following patterns and find:

- a. 8th term b. 30th term

- c. 100th term

- (i) 10, 11, 12, 13, 14, 15, 16, ...

Solution:

Rule = Add Previous term with 1.

- (a) 8th term = 17

- (b) 30th term = 39

- (c) 100th term = 109.

- (ii) 10, 20, 30, 40, 50, 60, 70, ...

Solution:

Rule = Add Previous term with 10.

- (a) 8th term = 80

- (b) 30th term = 300

- (c) 100th term = 1000.

- (iii) 28, 29, 30, 31, 32, 33, 34, ...

Solution:

Rule = Add Previous term with 1.

- (a) 8th term = 35

- (b) 30th term = 27

- (c) 100th term = 127.

- (iv) 5, 10, 15, 20, 25, 30, 35, ...

Solution:

Rule = Add Previous term with 5.

- (a) 8th term = 40

- (b) 30th term = 150

- (c) 100th term = 500.

- (v) 15, 30, 45, 60, 75, 90, 105, ...

Solution:

Rule = Add Previous term with 1.

- (a) 8th term = 120

- (b) 30th term = 450

- (c) 100th term = 1500.

Q3: Ahad daily read the storybook.

On Monday, he read 3 pages. The

next day, he read 6 pages. On

Wednesday, he read 9 pages. If

he keeps increasing the number of

pages daily, how many pages will

he read on Thursday and Friday?

Solution:

Ahad read book on Monday = 3 pages

Ahad read book on Monday = 6 pages

Ahad read book on Wednesday = 9 pages.

To find number of pages on Thursday and Friday.

We first find Rule,

Rule = Add previous term with 3.

So Ahad read book on Thursday = 12 pages

and Ahad read book on Friday = 15 pages

Q4: Rida goes for a walk in the nearby park. On Monday, she completes 2 rounds of the walking track. The next day, she completes 4 rounds of the walking track. On Wednesday, she completes 6 rounds. If she keeps increasing the number of rounds daily, how many rounds does she complete over the next three days of the week?

Solution:

On Monday Rida Complete 2 round of the walking track

Next day = 4 round

On Wednesday = 6 round

To find number of round walk, we first find Rule,

Rule = Add previous term will 2.

So

Completion of round on the next three days are 8, 10, 12

Exercise 6.2

Q1: Underline the variables in each of the following.

- a. $3 - \underline{y} = 4$ b. $\underline{a} + 3 = 30$
 c. $1 + \underline{b} = 0$ d. $10\underline{c} - 4 = 15$
 e. $3 \times 8 = \underline{z}$ f. $4 + 5 = \underline{s}$
 g. $18 + \underline{x} = 23$ h. $\underline{m} \times 6 = 18$

Q2: Use letters instead of symbols in each of these statements.

a. $2 - \square = 3$

Solution:

$$2 - (-1) = 3$$

b. $\Delta + 5 = 20$

Solution:

$$15 + 5 = 20$$

c. $4 + \square = 6$

Solution:

$$4 + 2 = 6$$

d. $\square - 7 = 14$

Solution:

$$21 - 7 = 14$$

e. $3 \times \Delta = 6$

Solution:

$$3 \times 2 = 6$$

f. $9 + \square = 1$

Solution:

$$9 + (-8) = 1$$

Exercise 6.3

Q1: Write the terms of the following algebraic expressions.

a. $4a + 5b$

Solution:

Terms of algebraic expression are $+4a + 5b$.

b. $4y - 3x - 4$

Solution:

The terms of algebraic expression are $+4y, -3x, -4$

c. $6x - y + 7$

Solution:

The terms of algebraic expression are $+6x, -y, +7$

d. $3x^2 + y^2$

Solution:

The terms of algebraic expression are $+3x^2, y^2$

e. $abc - d$

Solution:

The terms of algebraic expression are $abc, -d$

f. $11x^3 + xyz^2 + 4$

Solution:

The terms of algebraic expression are $11x^3, xyz^2, 4$.

g. $4x^3 + 5y^3 + 4$

Solution:

The terms of algebraic expression are $4x^3, 5y^3, 4$

h. $a^2 - b^2 + 3$

Solution:

The terms of algebraic expression are $a^2, -b^2, +3$.

Q2: Identify variable, coefficient and constant in each algebraic expressions.

a. $x^2 + 3$

Solution:

Variable = x

Constant = 3

Coefficient = 1

b. $7y^3 + z - 1$

Solution:

Variable = y, z

Constant = -1

Coefficient = 7, 1

c. $2a + 7b^2$

Solution:

Variable = a, b

Coefficient = 2, 7

Constant = 0.

d. $\frac{1}{2}y^2 + 6$

Solution:

Variable = y

Coefficient = $\frac{1}{2}$

Constant = 6

e. $7a - 4b + c^2$

Solution:

Variable = a, b, c

Coefficient = 7, -4, 1

Constant = 0

Q3: Write the algebraic expressions whose terms are given below.

a. $3a, -2b$

Solution:

Algebraic expressions = $3a - 2b$

b. $3y, -4z, 1$

Solution:

Algebraic expression = $3y - 4z + 1$

c. $9x^2, -y^2, 5$

Solution:

Algebraic expression = $9x^2 - y^2 + 5$

d. $x^2, -y^2$

Solution:

Algebraic expression = $x^2 - y^2$

e. $6abc, -bcd, 2de$

Solution:

Algebraic expression = $6abc - bcd + 2de$

f. $13x^4, xy^2, 7$

Solution:

Algebraic expression = $13x^4 + xy^2 + 7$

Q4: List out the like terms in each of the following sets.

a. $7a - 5a + 8b - a + \frac{a}{3}$

Solution:

Like terms = $7a - 5a - a + \frac{a}{3}$

b. $2p^3q^2 + 4p^3q^2 + 7q^2p^3 - 2p^3q^2$

Solution:

Like terms = $2p^3q^2, -2p^3q^2$

c. $-xy + 3y + 5xy - x - xy - 11$

Solution:

Like terms = $-xy, 5xy, -xy$

d. $2x^2y + 3x^2y + 2xy^2 + 4yx^2 - 2x^2y - 3yx^2$

Solution:

Like terms = $2x^2y, -2x^2y, -3yx^2, 4yx^2$

e. $a^2b^3 - 5a^2b^3 + 7a^2b^3 + 11a^2b^3 - 3b^2a^4$

Solution:

Like terms = $-5a^2b^3, 7a^2b^3, -3b^2a^4$

Exercise 6.4

Q1: Substitute the given values for the variables to evaluate each expression.

a. $3y + 2y$ when $y = 5$

Solution:

Put $y = 5$ in the given expression

So

$$\begin{aligned} 3y + 2y &= 3 \times 5 + 2 \times 5 \\ &= 15 + 10 \\ &= 25 \end{aligned}$$

b. $2a - 2b + c$ when $a = 3, b = -2$ and $c = 4$

Solution:

Put $a = 3, b = -2, c = 4$ in the given expression

So

$$\begin{aligned} 2a - 2b + c &= 2 \times 3 - 2(-2) + 4 \\ &= 6 + 4 + 4 \\ &= 14 \end{aligned}$$

c. $3(x^2 + 4x) - 3y$ when $x = -5$ and $y = 3$

Solution:

Put $x = -5, y = 3$ in the given expression,

So

$$\begin{aligned} 3(x^2 + 4x) - 3y &= 3((-5)^2 + 4(-5)) - 3 \times 3 \\ &= 3(25 - 20) - 9 \\ &= 3(5) - 9 \\ &= 15 - 9 \\ &= 6 \end{aligned}$$

d. x^2y when $x = -3$ and $y = -4$

Solution:

Put $x = -3, y = -4$ in the given expression,

So

$$x^2y = (-3)^2(-4) = 9(-4) = -36$$

e. $(b - d)^2$ when $b = 9$ and $d = -3$

Solution:

Put $b = 9, d = -3$ in the given expression

So

$$(b - d)^2 = (9 - (-3))^2 = (9 + 3)^2 = (12)^2 = 144$$

f. $m^4 + 2m^3 - m^2 + 8$ when $m = -2$

Solution:

Put $m = -2$, in the given expression,

So

$$\begin{aligned} m^4 + 2m^3 - m^2 + 8 &= (-2)^4 + 2(-2)^3 - (-2)^2 + 8 \\ &= 16 - 16 - 4 + 8 \\ &= 0 + 8 - 4 = 4 \end{aligned}$$

g. $\frac{3x}{2} + 1$ when $x = -5$

Solution:

Put $x = -5$ in the given expression,

So

$$\begin{aligned} \frac{3x}{2} + 1 &= \frac{3 \times (-5)}{2} + 1 \\ &= \frac{-15}{2} + 1 = \frac{-15 + 2}{2} \\ &= \frac{-13}{2} \\ &= -6\frac{1}{2} \end{aligned}$$

h. $5y^2 - 2(x^2 - 4x) - 5y$ when $x = 2$ and $y = -2$

Solution:

Put $x = 2$ and $y = -2$ in the given expression, So

$$\begin{aligned} 5y^2 - 2(x^2 - 4x) - 5y &= 5(-2)^2 - 2(2^2 - 4 \times 2) - 5(-2) \\ &= 5 \times 4 - 2(4 - 8) + 10 \\ &= 20 - 2(-4) + 10 \\ &= 30 + 8 = 38 \end{aligned}$$

Q2: Verify the following when; $p = 2, s = 5$.

a. $p + (q + s) = (p + q) + s$

Solution:

$$\text{L.H.S} = p + (q + s)$$

Put $p = 2, q = -3, s = 5$

$$\begin{aligned} \text{So L.H.S} &= 2 + (-3 + 5) \\ &= 2 + 2 = 4 \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= (p + q) + s = (2 - 3) + 5 \\ &= -1 + 5 = 4 \end{aligned}$$

$$\text{As L.H.S} = \text{R.H.S}$$

Hence $p + (q + s) = (p + q) + s$ is verified.

b. $p(q + s) = pq + ps$

Solution:

$$\text{L.H.S} = p(q + s)$$

Put $p = 2, q = -3, s = 5$

$$\text{L.H.S} = (2)(-3 + 5) = 2(2) = 4$$

$$\begin{aligned} \text{R.H.S} &= pq + ps = 2 \times (-3) + 2 \times 5 \\ &= -6 + 10 \\ &= 4 \end{aligned}$$

$$\text{As L.H.S} = \text{R.H.S}$$

Hence $p(q + s) = pq + ps$ is verified.

c. $(p + s)q = pq + sq$

Solution:

$$\text{L.H.S} = (p + s)q$$

Put $p = 2, q = -3, s = +5$

$$\text{L.H.S} = (2 + 5)(-3) = 7(-3) = -21$$

$$\text{R.H.S} = 2(-3) + 5(-3) = -6 - 15 = -21$$

$$\text{As L.H.S} = \text{R.H.S}$$

So $(p + s)q = pq + sq$ is verified.

d. $q(p - s) = qp - qs$

Solution:

$$\text{L.H.S} = q(p - s) = qp - qs$$

Put $p = 2, q = -3, s = 5$

$$\text{L.H.S} = (-3)(2 - 5) = -3(-3) = 9$$

$$\begin{aligned} \text{R.H.S} &= (-3)(2) - (-3)(5) \\ &= -6 + 15 = 9 \end{aligned}$$

$$\text{As L.H.S} = \text{R.H.S}$$

So $q(p - s) = qp - qs$ is verified.

Q3. If $x = 3$ and $y = 2$, then prove that $(x + y)^2 = x^2 + 2xy + y^2$.

Solution:

$$\text{L.H.S} = (x + y)^2$$

Put $x = 3, y = 2$

$$\text{L.H.S} = (3 + 2)^2 = (5)^2 = 25$$

$$\begin{aligned} \text{R.H.S} &= x^2 + 2xy + y^2 \\ &= (3)^2 + 2(3)(2) + 2^2 \\ &= 9 + 12 + 4 = 25 \end{aligned}$$

$$\text{As L.H.S} = \text{R.H.S}$$

Hence $(x + y)^2 = x^2 + 2xy + y^2$ proved

Q4: If $a = 1, b = 3$ and $c = 1$, then evaluate $4b^2 - 2ac$.

Solution:

Put $a = 1, b = 3, c = 1$ in the given expression, So

$$\begin{aligned} 4b^2 - 2ac &= 4(3)^2 - 2 \times 1 \times 1 \\ &= 4 \times 9 - 2 = 36 - 2 = 34 \end{aligned}$$

Exercise 6.5

Q1: Add the following algebraic expressions horizontally.

a. ab, ab

Solution:

$$ab + ab = 2ab$$

b. $7x^2, 9x^2$

Solution:

$$7x^2 + 9x^2 = 16x^2$$

c. $14x, 3x, -7x$

Solution:

$$\begin{aligned} 14x + 3x - 7x &= 17x - 7x \\ &= 10x \end{aligned}$$

d. $12y, 7y, y$

Solution:

$$12y + 7y + y = 20y$$

e. $3yz, -4yz, yz$

Solution:

$$\begin{aligned} 3yz - 4yz + yz &= 3yz + yz - 4yz \\ &= 4yz - 4yz \\ &= 0 \end{aligned}$$

f. $3x + 2y, x + y$

Solution:

$$\begin{aligned} & (3x + 2y) + (x + y) \\ &= 3x + x + 2y + y \\ &= 4x + 3y \end{aligned}$$

g. $4x + 3y + 3, 3x + 5y + 5$

Solution:

$$\begin{aligned} & 4x + 3y + 3 + 3x + 5y + 5 \\ &= 4x + 3x + 3y + 5y + 3 + 5 \\ &= 7x + 8y + 8 \end{aligned}$$

h. $-9x^2 + 5x - 4x^2 + 2x + 1$

Solution:

$$\begin{aligned} & -9x^2 + 5x - 4x^2 + 2x + 1 \\ &= -9x^2 + x^2 - 5x + 2x - 4 + 1 \\ &= -8x^2 + 7x - 3 \end{aligned}$$

i. $6x + 3y - 4z, 6x + 8y + 9z$

Solution:

$$\begin{aligned} & 6x + 3y - 4z, 6x + 8y + 9z \\ &= 6x + 6x + 3y + 8y - 4z + 9z \\ &= 12x + 11y + 5z \end{aligned}$$

j. $7x^2 + 8y, 7 - 2x^2, 6 + 4x^2$

Solution:

$$\begin{aligned} & 7x^2 + 8y + 7 - 2x^2 + 6 + 4x^2 \\ &= 7x^2 - 2x^2 + 4x^2 + 8y + 7 + 6 \\ &= 11x^2 - 2x^2 + 8y + 13 \\ &= 9x^2 + 8y + 13 \end{aligned}$$

Q2: Add the following expressions vertically.

a. $5x + y, x - 9y$

Solution:

$$\begin{array}{r} 5x + y \\ + x - 9y \\ \hline 6x - 8y \end{array}$$

b. $5x^3 - 2y^3, 7x^3 - 3y^3$

Solution:

$$\begin{array}{r} 5x^3 - 2y^3 \\ + 7x^3 - 3y^3 \\ \hline 12x^3 - 5y^3 \end{array}$$

c. $x + 3y - 2z, x - y + z$

Solution:

$$\begin{array}{r} x + 3y - 2z \\ + x - y + z \\ \hline 2x + 2y - z \end{array}$$

d. $2x + 9y - 7, x - y + 2$

Solution:

$$\begin{array}{r} 2x + 9y - 7 \\ + x - y + 2 \\ \hline 3x + 8y - 5 \end{array}$$

e. $5x^2 + 4y^2, -7x^2 - 5$

Solution:

$$\begin{array}{r} 5x^2 + 4y^2 \\ + -7x^2 - 5 \\ \hline -2x^2 + 4y^2 - 5 \end{array}$$

f. $-3x^2 + 3, x^2 + 5$

Solution:

$$\begin{array}{r} -3x^2 + 3 \\ + x^2 + 5 \\ \hline -2x^2 + 8 \end{array}$$

g. $a^2 + b^2 + c^2 - 3abc, a^2 - b^2 + c^2 + abc$

Solution:

$$\begin{array}{r} a^2 + b^2 + c^2 - 3abc \\ + a^2 - b^2 + c^2 + abc \\ \hline 2a^2 + 0 + 2c^2 - 2abc \end{array}$$

h. $x^2 + 5x + 6, 3x^2 + 4x, 2x^2 - x - 3$

Solution:

$$\begin{array}{r} x^2 + 5x + 6 \\ 3x^2 + 4x \\ 2x^2 - x - 3 \\ \hline 6x^2 + 8x + 3 \end{array}$$

i. $(6a + 8b - 7c) + (2b + c - 4a) + (a - 3b - 2c)$

Solution:

$$\begin{array}{r} 6a + 8b - 7c \\ - 4a + 2b + c \\ + a - 3b - 2c \\ \hline 3a + 7b - 8c \end{array}$$

j. $8x^2 - 5xy + 3y^2, 2xy - 6y^2 + 3x^2, y^2 + xy - 6x^2$

Solution:

$$\begin{array}{r} 8x^2 - 5xy + 3y^2 \\ 3x^2 + 2xy - 6y^2 \\ + -6x^2 + xy + y^2 \\ \hline 5x^2 - 2xy - 2y^2 \end{array}$$

Q3: Subtract the following algebraic terms horizontally.

a. $-ab$ from $3ab$

Solution:

$$\begin{aligned} & 3ab - (-ab) = 3ab + ab \\ &= 4ab \end{aligned}$$

b. $4x^3$ from $-9x^3$

Solution:

$$-9x^3 - 4x^3 = -13x^3$$

c. $8y$ from $-7y$

Solution:

$$-7y - 8y = -15y$$

d. $-15y$ from y

Solution:

$$y - (-15y) = y + 15y = 16y$$

e. $3xz$ from $-4xz$

Solution:

$$-4xz - 3xz = -7xz$$

f. $7x + 4y$ from $-x - y$

Solution:

$$\begin{aligned} &= -x - y - (7x + 4y) \\ &= -x - 7x - 4y \\ &= -8x - 4y \end{aligned}$$

g. $-6x + 2y^2 + 3$ from $3x - 5y^2 - 5$

Solution:

$$\begin{aligned} & 3x - 5y^2 - 5 - (-6x + 2y^2 + 3) \\ &= 3x - 5y^2 - 5 - (-6x) - (2y^2) - (+3) \\ &= 3x - 5y^2 - 5 + 6x - 2y^2 - 3 \\ &= 3x + 6x - 5y^2 - 2y^2 - 5 - 3 \\ &= 9x - 7y^2 - 8 \end{aligned}$$

h. $-10x^2 + 3x - 2$ from $x^2 - 5x + 9$

Solution:

$$\begin{aligned} & x^2 - 5x + 9 - (-10x^2 + 3x - 2) \\ &= x^2 - 5x + 9 - (-10x^2) - (+3x) - (-2) \\ &= x^2 - 5x + 9 + 10x^2 - 3x + 2 \\ &= x^2 + 10x^2 - 5x - 3x + 9 + 2 \\ &= 11x^2 - 8x + 11 \end{aligned}$$

i. $5x - 2y - z$ from $6x + 8y - z$

Solution:

$$\begin{aligned} & 6x + 8y - z - (5x - 2y - z) \\ &= 6x - 8y - z - 5x - (-2y) - (-z) \\ &= 6x + 8y - z - 5x + 2y + z \\ &= 6x - 5x + 8y + 2y - z + z \\ &= x + 10y + 0 \\ &= x + 10y \end{aligned}$$

j. $8x^2 - 8y$ from $7 - 2x^2$

Solution:

$$\begin{aligned}
 & 7 - 2x^2 - (8x^2 - 8y) \\
 &= 7 - 2x^2 - 8x^2 - (-8y) \\
 &= 7 - 2x^2 - 8x^2 + 8y \\
 &= 7 - 10x^2 + 8y
 \end{aligned}$$

Q4: Subtract the following expressions vertically.

a. $4a + 5b - 3c$ from $6a - 3b + c$

Solution:

$$\begin{array}{r}
 6a - 3b + c \\
 \pm 4a + 5b - 3c \\
 \hline
 2a - 8b + 4c
 \end{array}$$

b. $3x^2 - 6x - 4$ from $5 + x - 2x^2$

Solution:

$$\begin{array}{r}
 5 + x - 2x^2 \\
 \mp 4 \mp 6x \pm 3x^2 \\
 \hline
 9 + 7x - 5x^2
 \end{array}$$

c. $3x + y - 3z$ from $9x - 5y - z$

Solution:

$$\begin{array}{r}
 9x - 5y + z \\
 \pm 3x \pm y \mp 3z \\
 \hline
 6x - 6y + 4z
 \end{array}$$

d. $2x + 9y - 7$ from $x - y + 2$

Solution:

$$\begin{array}{r}
 x - y + 2 \\
 \pm 2x \pm 9y \mp 7 \\
 \hline
 -x - 10y + y
 \end{array}$$

e. $-7x^2 - 5$ from $4x^2 - 5$

Solution:

$$\begin{array}{r}
 4x^2 - 5 \\
 \mp 7x^2 \mp 5 \\
 \hline
 11x^2
 \end{array}$$

f. $-7x^2 - 5y - 4$ from $x^2 - 5y + 2$

Solution:

$$\begin{array}{r}
 x^2 - 5y + 2 \\
 \mp 7x^2 \pm 5y \mp 4 \\
 \hline
 8x^2 - 10y + 6
 \end{array}$$

g. $9x^2 - 5x + 6$ from $-3x^2 - 4x - 3$

Solution:

$$\begin{array}{r}
 -3x^2 - 4x - 3 \\
 \pm 9x^2 \mp 5x \pm 6 \\
 \hline
 -12x^2 + x - 9
 \end{array}$$

h. $4a + 7b - 7c$ from $9b - c - 3a$

Solution:

$$\begin{array}{r}
 x^2 - 5y + 2 \\
 \pm 7x^2 \pm 5y \mp 4 \\
 \hline
 8x^2 - 10y + 6
 \end{array}$$

i. $10x^2 - xy + z$ from $z^2 + xy - 7x^2$

Solution:

$$\begin{array}{r}
 z^2 + xy - 7x^2 \\
 \pm z^2 \mp xy \pm 10x^2 \\
 \hline
 0 + 2xy - 17x^2 \\
 = 2xy - 17x^2
 \end{array}$$

j. $7a^2 + 5b^2 - c^2 - 3abc$ from $4a^2 - b^2 + 6c^2 - 7abc$

Solution:

$$\begin{array}{r}
 4a^2 - b^2 + 6c^2 - 7abc \\
 \pm 7a^2 \pm 5b^2 \mp c^2 \mp 3abc \\
 \hline
 -3a^2 - 6b^2 + 7c^2 - 4abc
 \end{array}$$

Q5. Subtract $x^2 + y^2 + 3xy$ from $4x^2 + 2xy - 3y^2$

Solution:

$$\begin{array}{r}
 4x^2 + 2xy - 3y^2 \\
 \pm x^2 \pm 3xy \pm y^2 \\
 \hline
 3x^2 - xy - 4y^2
 \end{array}$$

Q6. What should be subtracted from $a^3 - 4a^2 + 5a - 6$ to obtain $a^2 - 2a + 1$?

Solution:

To obtain required result we subtract $a^2 - 2a + 1$ from $a^3 - 4a^2 + 5a - 6$.

$$\begin{array}{r}
 a^3 - 4a^2 + 5a - 6 \\
 \pm a^2 \mp 2a \pm 1 \\
 \hline
 a^3 - 5a^2 + 7a - 7
 \end{array}$$

Q7. Subtract $5a^3 - 2a^2 + a - 6$ from the sum of $6a^3 + a^2 + 1$ and $a^2 - 4$.

Solution:

First sum $6a^3 + a^2 + 1$ and $a^2 - 4$

$$\begin{array}{r}
 6a^3 + a^2 + 1 \\
 \hline
 a^2 - 4
 \end{array}$$

$$6a^3 + 2a^2 - 3$$

Now subtract $5a^3 - 2a^2 + a - 6$ from $6a^3 + 2a^2 - 3$

$$\begin{array}{r}
 6a^3 + 2a^2 - 3 \\
 \pm 5a^3 \mp 2a^2 \mp 6 \pm a \\
 \hline
 a^3 + 4a^2 + 3 - a \\
 = a^3 + 4a^2 - a + 3
 \end{array}$$

Q8. If $X = a - b + c$, $Y = a + b + c$, $Z = a - b - c$ then find;

a. $X + Y$

Solution:

$$\begin{aligned}
 X + Y &= a - b + c + a + b + c \\
 &= 2a + 2c
 \end{aligned}$$

b. $X - Z$

Solution:

$$\begin{aligned}
 X - Z &= a - b + c - (a - b - c) \\
 &= a - b + c - a + b + c \\
 &= 2c
 \end{aligned}$$

c. $Y + Z$

Solution:

$$\begin{aligned}
 Y + Z &= a + b + c + a - b - c \\
 &= 2a
 \end{aligned}$$

d. $Z - X$

Solution:

$$\begin{aligned}
 Z - X &= a - b - c - (a - b + c) \\
 &= a - b - c - a + b + c \\
 &= -2c
 \end{aligned}$$

e. $X + Y - Z$

Solution:

$$\begin{aligned}
 X + Y - Z &= a - b + c + a + b + c - (a - b - c) \\
 &= a - b + c + a + b + c - a + b + c \\
 &= 2a + b + 3c
 \end{aligned}$$

f. $X - Y - Z$

Solution:

$$\begin{aligned}
 X - Y - Z &= a - b + c - (a + b + c) - (a - b - c) \\
 &= a - b + c - a - b - c - a + b + c \\
 &= -a - b + c
 \end{aligned}$$

g. $-X - Y + Z$

Solution:

$$\begin{aligned}
 -X - Y + Z &= -(a - b + c) - (a + b + c) + a - b - c \\
 &= -a + b - c - a - b - c + a - b - c \\
 &= -a - b - 3c
 \end{aligned}$$

Exercise 6.6**Q1. Simplify the following.**

(a) $2(4x+2)$

Solution:

$$2(4x+2) = 2(4x) + 2 \times 2 \\ = 8x + 4$$

(b) $2 + 2x\{2(3x+2)+2\}$

Solution:

$$2 + 2x\{2(3x+2)+2\} \\ = 2 + 2x\{6x+4+2\} \\ = 2 + 2x\{6x+6\} \\ = 2 + 12x^2 + 12x \\ = 2 + 12x^2 + 12x$$

(c) $[6x - \{3x + (2x - x)\}]$

Solution:

$$[6x - \{3x + (2x - x)\}] \\ = [6x - \{3x + x\}] \\ = [6x - 4x] \\ = 2x$$

(d) $[7\ell - \{4m - (4m - 2m)\}]$

Solution:

$$[7\ell - \{4m - (4m - 2m)\}] \\ = [7\ell - \{4m - 2m\}] \\ = [7\ell - 2m] \\ = 7\ell - 2m$$

(e) $3u + \{v - (2u - t + v)\}$

Solution:

$$3u + \{v - (2u - t + v)\} \\ = 3u + \{v - 2u + t + v\} \\ = 3u + \{v - 2u - t + v\} \\ = 3u + \{-2u + t + 2v\} \\ = 3u - 2u + t + 2v \\ = u + t + 2v$$

(f) $2 + 3x\{4(x+2)+2\}$

Solution:

$$2 + 3x\{4(x+2)+2\} \\ = 2 + 3x\{4x+8+2\} \\ = 2 + 3x\{4x+10\} \\ = 2 + 3x\{4x+10\} \\ = 2 + 12x^2 + 30x$$

(g) $11a - \{5b - 3(2a+b)\}$

Solution:

$$11a - \{5b - 3(2a+b)\} \\ = 11a - \{5b - 6a - 3b\} \\ = 11a - \{2b - 6a\} \\ = 11a - 2b + 6a \\ = 17a - 2b$$

(h) $8\{3(4a+5b) - 2(6a-5b)\}$

Solution:

$$8\{3(4a+5b) - 2(6a-5b)\} \\ = 8\{3(4a)+3(5b) - 2(6a) - 2(-5b)\} \\ = 8\{12a+15b-12a+10b\} \\ = 8\{15b\} = 120b$$

Review Exercise**Q1: Choose the correct option.**

a. In algebra, a fixed our unchanged value is called is

- (i) constant ✓ (ii) variable
(iii) term (iv) coefficient

b. Unlike terms are those terms whose _____ are different with different exponents.

- (i) signs (ii) variables ✓
(iii) coefficients (iv) constants

c. In $3x^2$, 2 is the _____.

- (i) variable (ii) constant
(iii) coefficient (iv) exponent ✓

d. If $a = 1$, $b = 2$, the value of the expression $a^2 + 2a - b$ is _____.

- (i) 1 ✓ (ii) 0 (iii) -1 (iv) 2

e. If $x = 3$, $y = 2$, then $3x^2y =$ _____.

- (i) -54 (ii) 54 ✓ (iii) 36 (iv) -36

Q2: Identify the position-to-term rules for the pattern and find 20th term:

11, 22, 33, 44, 55, ...

Solution:

11, 22, 33, 44, 55, ...
Rule = Multiple of 11
20th term = 220.

Q3: Identify the position-to-term rules for the pattern and find 55th term:

16, 17, 18, 19, 20, 21, ...

Solution:

16, 17, 18, 19, 20, 21, ...
Rule = Add '1' in previous term
55th term = 70.

Q4: Identify the term-to-term rules for the following patterns and write the next three terms.

a. 15, 21, 27, 33, _____, _____, _____

The term-to-term rule of this pattern is: Add '6' with previous term.

The next three terms are: 39, 45, 51

b. 5, 10, 20, 40, 80, _____

The term-to-term rule of this pattern is: 2 is multiplied with previous

The next three terms are: 160, 320, 640.

Q5. Subtract the following.

a. $-3x - 5y - z$ from $3x - 6y - 6z$

Solution:

$$\begin{array}{r} 3x - 6y - 6z \\ + 3x + 5y + z \\ \hline 6x - y - 5z \end{array}$$

b. $19p - q + r$ from $8p - 3q - 4r$

Solution:

$$\begin{array}{r} 8p - 3q - 4r \\ + 19p + q + r \\ \hline -11p - 2q - 5r \end{array}$$

Q6. Simplify.

a. $[9x^2 - \{x^2 - 5y(5x - 2y)\}]$

Solution:

$$\begin{aligned} &= [9x^2 - \{x^2 - 25xy + 10y^2\}] \\ &= [9x^2 - x^2 - 25xy - 10y^2] \\ &= 9x^2 - x^2 - 25xy - 10y^2 \\ &= 8x^2 - 25xy - 10y^2 \end{aligned}$$

b. $x - [2y - \{3x - (2y + 3z)\}]$

Solution:

$$\begin{aligned} &= x - [2y - \{3x - (2y + 3z)\}] \\ &= x - [2y - \{3x + 2y + 3z\}] \\ &= x - [2y - 3x + 2y + 3z] \\ &= x - [4y - 3x + 3z] \\ &= x - 4y + 3x - 3z \\ &= 4x - 4y - 3z \end{aligned}$$

Q7. If $x = -2$, $y = 4$ and $z = 5$ then find the value of the following.

a. $9x - 5y + z$

Solution:

Put $x = -2$, $y = 4$, $z = 5$ in the given expression,

$$\begin{aligned} 9x - 5y + z &= 9(-2) - 5(4) + 4 \\ &= -18 - 20 + 4 \\ &= -38 + 4 \\ &= -34 \end{aligned}$$

b. $5x^2 + y - z$

Solution:

Put $x = -2$, $y = 4$, $z = 5$ in the given expression,

$$\begin{aligned} 5x^2 + y - z &= 5 \times (-2)^2 + 4 - 5 \\ &= 5 \times 4 + 4 - 5 = 20 + 4 - 5 \\ &= 19 \end{aligned}$$

c. $-2x^2 - x - 5$

Solution:

Put $x = -2$, $y = 4$, $z = 5$ in the given expression,

$$\begin{aligned} -2x^2 - x - 5 &= -2(-2)^2 + (-2) - 5 \\ &= -2 \times 4 + 2 - 5 \\ &= -8 - 5 + 2 \\ &= -13 + 2 \\ &= -11 \end{aligned}$$

d. $6xy - yz + z$

Solution:

Put $x = -2$, $y = 4$, $z = 5$ in the given expression,

$$\begin{aligned} 6xy - yz + z &= 6(-2)(4) - (2)(5) + 5 \\ &= -48 - 20 + 5 \\ &= -68 + 5 \\ &= -63 \end{aligned}$$

Unit 7 Linear Expressions and Equations

Exercise 7.1

Q1. Identify the following as algebraic equations or expressions.

(a) $2x + 3y = 13$ (Equation)

(b) $5x = 3y$ (Equation)

(c) $n = mc^2$ (Equation)

(d) $6x - 3y$ (Expression)

(e) $10b + 4 = 14$ (Equation)

(f) $8x + 9y$ (Expression)

(g) $3s + 4t$ (Expression)

(h) $x^2 + 3x = 45$ (Equation)

(i) $x^2 - 32x + 23 = 0$ (Equation)

(j) $x^3 + y^3 - 3xy = 42$ (Equation)

Exercise 7.2

Q1. Identify the linear equations from the following.

(a) $y = 2x + 1$ (Linear Equations)

(b) $5x^3 = 6 + 3y$

(c) $\frac{y}{2} = 3 - x$ (Linear Equations)

(d) $y^3 - 2 = 0$

(e) $\frac{x}{2} = 16$ (Linear Equations)

(f) $3 - y^3 = 6$

(g) $y = 3 + z$ (Linear Equations)

(h) $2.2x + 5.7 = 22.5$ (Linear Equations)

Q2. Write at least 5 linear equations in one variable.

Solution:

(i) $2 + x = 5$

(ii) $2x + 1 = 3$

(iii) $5x + 3 = 4x$

(iv) $x + 3 = 2x$

(v) $9x + 11 = 0$

Exercise 7.3

Q1. Write algebraic expressions for the following phrases.

a. 5 added to a number

Solution:

$$x + 5$$

b. A number subtracted from 8

Solution:

$$8 - x$$

c. 20 decreased by a number

Solution:

$$x - 20$$

d. Twice a number decreased by 2

Solution:

$$2x - 2$$

e. Half of a number added to 7

Solution:

$$\frac{1}{2}x + 7$$

f. 12 subtracted from the sum of 6 and a number

Solution:

$$6 + x - 12$$

Q2. Write algebraic equations for the following statements.

a. 6 added to a number is 8.

Solution:

$$x + 6 = 8$$

b. 4 subtracted from a number is 13

Solution:

$$x - 4 = 13$$

c. 6 times a number decreased by 3 is 5

Solution:

$$6x - 3 = 5$$

- d. 3 times the sum of a number and 8 is 14.

Solution:

$$3(x + 8) = 14$$

- e. A number decreased by 6 is 12

Solution:

$$x - 6 = 12$$

- f. Twice the number decreased by 33 is 66

Solution:

$$2x - 33 = 66$$

- g. 2 times the number is 16 less than 10 times the number

Solution:

$$2x = 10 \times 16$$

- h. 6 subtracted from 5 times a number is 14 more than the number

Solution:

$$5x - 6 = 14 + x$$

- i. The product of a number and 2 is 26.

Solution:

$$2x = 26$$

Q3: Write the statements for the following algebraic equations.

- a. $8 - x = 1$

Solution:

A number subtracted from 8 is one.

- b. $9x + 4 = 22$

Solution:

4 added to nine times the number is 22.

- c. $3x - 7 = 23$

Solution:

7 subtracted from three times the number is 23.

- d. $x + 5 = 2x$

Solution:

5 added to the number is twice the number.

- e. $(6 - x) + 2x = 8$

Solution:

Sum of number subtracted from 6 and twice the number and is 8.

- f. $48 - x = 7x$

Solution:

Number subtracted from 48 is 7 times the number.

- g. $4x - 8 = 2x$

Solution:

8 subtracted from 4 times the number is twice the number.

- h. $\frac{36}{x} = 4x$

Solution:

Number divided by 36 is four times the number.

Exercise 7.4

Q1: Solve the following linear equations. Also verify the solution.

- a. $6x + 2 = -19$

Solution:

$$6x + 2 = -19$$

$$6x = -19 - 2$$

$$6x = -21$$

Dividing by '6'

$$\frac{6x}{6} = \frac{-21}{6}$$

$$x = \frac{-21}{6}$$

$$x = \frac{-7}{2}$$

$$x = -3\frac{1}{2}$$

- b. $4x - \frac{1}{4} = 5$

Solution:

$$4x - \frac{1}{4} = 5$$

$$4x = 5 + \frac{1}{4}$$

$$4x = \frac{5 \times 4 + 1 \times 1}{4}$$

$$4x = \frac{20 + 1}{4}$$

$$4x = \frac{21}{4}$$

Dividing by '4'

$$\frac{4x}{4} = \frac{21}{4 \times 4}$$

$$x = \frac{21}{16}$$

$$x = 1\frac{5}{16}$$

- c. $7x - 4 = 23$

Solution:

$$7x - 4 = 23$$

$$7x - 23 + 4$$

$$7x = 27$$

Dividing by '7'

$$\frac{7x}{7} = \frac{27}{7}$$

$$x = \frac{27}{7}$$

$$x = 3\frac{6}{7}$$

- (d) $-6x = 21$

Solution:

$$-6x = 21$$

Dividing by "-6"

$$\frac{-6x}{-6} = \frac{21}{-6}$$

$$x = \frac{-7}{2}$$

$$x = -3\frac{1}{2}$$

- (e) $18 - 7x = -3$

Solution:

$$18 - 7x = -3$$

$$-7x = -3 - 18$$

$$-7x = -21$$

Dividing by "-7"

$$\frac{-7x}{-7} = \frac{-21}{-7}$$

$$x = 3$$

- f. $5(x + 4) = 25$

Solution:

$$5(x + 4) = 25$$

$$5x + 20 = 25$$

$$5x = 25 - 20$$

$$5x = 5$$

Dividing by "5"

$$\frac{5x}{5} = \frac{5}{5}$$

$$x = 1$$

- g. $\frac{4x}{6} - \frac{3x}{5} = 12$

Solution:

$$\frac{4x}{6} - \frac{3x}{5} = 12$$

$$\frac{4x \times 5 - 3x \times 6}{30} = 12$$

$$\frac{20x - 18x}{30} = 12$$

$$\frac{2x}{30} = 12$$

$$\frac{2x}{30} = 12$$

$$\frac{x}{15} = 12$$

$$x = 12 \times 15$$

$$x = 180$$

h. $3x - 5 = 5x - 4$

Solution:

$$3x - 5 = 5x - 4$$

$$3x - 5x = -4 + 5$$

$$-2x = 1$$

$$x = \frac{1}{-2}$$

$$x = -\frac{1}{2}$$

i. $4(x - 3) = 4(3x + 1)$

Solution:

$$4(x - 3) = 4(3x + 1)$$

$$4x - 12 = 12x + 4$$

$$4x - 12x = 4 + 12$$

$$-8x = 16$$

Dividing by "-8"

$$\frac{-8x}{-8} = \frac{16}{-8}$$

$$x = -2$$

j. $\frac{3x}{12} - 10 = \frac{1}{2}$

Solution:

$$\frac{3x}{12} - 10 = \frac{1}{2}$$

$$\frac{3x}{12} = \frac{1 + 10 \times 2}{2}$$

$$\frac{3x}{12} = \frac{1 + 20}{2}$$

$$\frac{x}{4} = \frac{21}{2}$$

$$x = \frac{21 \times 4}{2}$$

$$x = 42$$

k. $\frac{x}{2} - \frac{x}{3} = 8$

Solution:

$$\frac{x}{2} - \frac{x}{3} = 8$$

$$\frac{3 \times (x) - 2 \times (x)}{6} = 8$$

$$\frac{3x - 2x}{6} = 8$$

$$\frac{x}{6} = 8$$

$$x = 6 \times 8$$

$$x = 48$$

l. $6x - 9 - 2(1 - x) = x + 9$

Solution:

$$6x - 9 - 2(1 - x) = x + 9$$

$$6x - 9 - 2 + 2x = x + 9$$

$$6x - 11 + 2x = x + 9$$

$$8x - 11 = x + 9$$

$$8x - x = 9 + 11$$

$$7x = 20$$

$$x = \frac{20}{7}$$

$$x = 2\frac{6}{7}$$

m. $0.4x + 2.4 = 6.5$

Solution:

$$0.4x + 2.4 = 6.5$$

$$0.4x = 6.5 - 2.4$$

$$0.4x = 4.1$$

$$x = \frac{4.1}{0.4} = \frac{41 \times 10}{10 \times 4} = \frac{410}{40}$$

$$x = \frac{41}{4} = 10\frac{1}{4}$$

n. $0.8x - 4 = 12$

Solution:

$$0.8x - 4 = 12$$

$$0.8x = 12 + 4$$

$$0.8x = 16$$

$$x = \frac{16}{0.8}$$

$$x = \frac{16 \times 10}{8}$$

$$x = 20$$

o. $0.3x + 6.9 = 12$

Solution:

$$0.3x + 6.9 = 12$$

$$0.3x = 12 - 6.9$$

$$0.3x = 5.1$$

$$x = \frac{5.1}{0.3}$$

$$x = \frac{51}{10} \times \frac{10}{3}$$

$$x = \frac{57}{8}$$

$$x = 17$$

p. $\frac{0.3x}{3.1} = 3$

Solution:

$$0.3x = 3 \times 3.1$$

$$0.3x = 9.3$$

$$x = \frac{9.3}{0.3} \times \frac{10}{10} = \frac{93}{3}$$

$$x = 31$$

q. $0.5x + 1.1 = 2.3$

Solution:

$$0.5x + 1.1 = 2.3$$

$$0.5x = 2.3 - 1.1$$

$$0.5x = 1.2$$

$$x = \frac{1.2}{0.5}$$

$$x = \frac{12}{5} \times \frac{10}{10}$$

$$x = 2\frac{2}{5}$$

r. $2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$

Solution:

$$2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$$

$$2x - 2 \times 2 - 5x + 5 \times 5 = 4x - 4 \times 8 - 2x + 2 \times 2$$

$$2x - 4 - 5x + 25 = 4x - 32 - 2x + 4$$

$$-3x + 21 = 2x - 28$$

$$-3x - 2x = -28 - 21$$

$$-5x = -49$$

$$\Rightarrow x = \frac{-49}{-5} = \frac{49}{5}$$

$$x = \frac{49}{5}$$

$$x = 9\frac{4}{5}$$

Q2: Dania is 6 years older than her younger sister. After 10 years, the sum of their ages will be 48 years. Find their present ages.

Solution:

Suppose sister age = x Dania age = $x + 6$

After 10 years

Sister age = $x + 10$ and Dania age = $x + 6 + 10 = x + 16$

According to given condition

$$x + 10 + x + 16 = 48$$

$$2x + 26 = 48$$

$$2x = 48 - 26$$

$$2x = 22$$

$$x = \frac{22}{2}$$

$$x = 11$$

So age of Dania sister = 11 year
and Dania age = 17 years

Q3: The price of a shirt has decreased after a sale by Rs 215. Find the original price if the new price is Rs 345.

Solution:

Suppose price of shirt = x

Price of shirt has decrease after sale = $x - 215$

According to given condition,

$$x - 215 = 345$$

$$\Rightarrow x = 345 + 215$$

$$x = 560$$

So original price of shirt = Rs. 560

Q4: The total price of a paintbrush and a pen is Rs 156. Find the price of both the items if the price of the paintbrush is half the price of the pen.

Solution:

Suppose price of the pen = x

and price of the paint brush = $\frac{x}{2}$

According to given condition

$$x + \frac{x}{2} = \text{Rs. } 156$$

$$\frac{2x + 26}{2} = 156$$

$$\frac{3x}{2} = 156$$

$$3x = 2 \times 156$$

$$3x = 312$$

$$\frac{3x}{3} = \frac{312}{3}$$

$$x = 104$$

So price of pen = Rs 101

and price of paint brush = $\frac{x}{2}$
= Rs. 52

Q5: Ali's weight is 13 kg less than Umar's weight. Find the weight of Umar if the sum of their weights is 59 kg.

Solution:

Suppose Umar's weight = x

and Ali's weight = $x - 13$

According to given condition

$$x + x - 13 = 59$$

$$2x = 59 + 13$$

$$2x = 72$$

$$\frac{2x}{2} = \frac{72}{2}$$

$$x = 36$$

Umar weight = 36kg

Ali's weight = $36 - 13$

$$= 23 \text{ kg}$$

Q6: The difference between 45 and 5 times a number is 15. Find the number.

Solution:

Suppose number = x

According to given condition

$$5x - 45 = 15$$

$$5x = 15 + 45$$

$$5x = 60$$

$$\frac{5x}{5} = \frac{60}{5}$$

$$x = 12$$

So required number = 12

Q7: Madiha things of a number. She multiplied it by 6.2 and added 8 to the product, she got the answer 39. Find the number.

Solution:

Suppose thinking number = x

Multiply by 6.2 = $6.2x$

Q8: Add to product = $6.2x + 8$

According to given condition

$$6.2x + 8 = 39$$

$$6.2x = 39 - 8$$

$$6.2x = 31$$

$$\frac{6.2x}{6.2} = \frac{31}{6.2}$$

$$x = \frac{31}{6.2}$$

$$x = \frac{31}{6.2}$$

$$x = \frac{10}{2}$$

$$x = 5$$

So thinking number = 5.

Q8: Ayesha was asked to divide a number by 3 but by mistake she multiplied it by 3. If the correct number is 72 less than the wrong number, find the number.

Solution:

Suppose number = x

So According to given condition

$$3x = \frac{x}{3} + 72$$

$$\Rightarrow 3x - \frac{x}{3} = 72$$

$$\frac{3(3x) - x}{3} = 72$$

$$\frac{9x - x}{3} = 72$$

$$\frac{8x}{3} = 72$$

$$8x = 3 \times 72$$

$$x = \frac{3 \times 72}{8}$$

$$x = 3 \times 9$$

$$x = 27$$

So required number = 27

Review Exercise

Q1: Encircle the correct option.

- A mathematical sentence which has equal sides separated by an equal sign is called a/an _____.
(i) equation✓ (ii) number
(iii) variable (iv) power
- An algebraic equation is an open mathematical statement that shows the equality of two or more _____.
(i) number (ii) expressions✓
(iii) variables (iv) powers
- The equals sign between two expressions means that both sides of the equations are _____.
(i) greater (ii) smaller
(iii) equal✓ (iv) nothing
- A linear equation is an algebraic equation where the highest power of the variables involved is _____.
(i) 0 (ii) 1✓ (iii) 3 (iv) 2
- The solution of $2x - 3 = 1$ is _____.
(i) 5 (ii) 4 (iii) 3 (iv) 2✓
- If $\frac{2x}{5} = 3$, $x =$ _____.
(i) $\frac{15}{2}$ ✓ (ii) $\frac{11}{2}$
(iii) $\frac{13}{5}$ (iv) $\frac{16}{2}$
- $2x + 0.06 = 0.4 + 1.22 =$ _____.
(i) 1 (ii) 0.78✓ (iii) 2.3 (iv) 11
- The solution of $5x - 20 = 3x + 2$ is _____.
(i) 15 (ii) 3 (iii) 10 (iv) 11✓
- "12 added to half of a number" means
(i) $\frac{x}{2} + 12$ ✓ (ii) $12 + \frac{1}{2x}$
(iii) $12x + \frac{1}{2}$ (iv) $12x + \frac{1}{2x}$

j. "5 decreased by a number is 10" means:

- (i) $5 - 10 = x$ (ii) $10x - 5 = x$
 (iii) $10x - 5 = x$ (iv) $x - 5 = 10$ ✓

Q2: Answer the following questions.

a. What is an algebraic equation?

Ans: An algebraic equation is an open mathematical statement that shows the equality of two or more expressions (with at least one variable). The equal sign between two expressions means that both sides of the equation are equal.

b. What is the difference between an equation and an expression?

Ans:

Equations	Expressions
(i) An equation has equals sign.	(i) An expression does not have an equals sign.
(ii) An equation has left and right sides.	(ii) An expression has only one side.
(iii) Equations are true for some values of the variables.	(iii) Expressions are true for all values of the variables.

c. Define linear equation and linear equation in one variable.

Ans: A linear equation is an algebraic equation where the highest power of the variables involved is 1. In other words, we can say that a linear equation contains the expressions having variables with highest power of 1.

A linear equation that has only one variable is said to be a linear equation in one variable.

d. What is meant by solving an equation?

Ans: To find the value of unknown in an equation is known as solving of the equation.

Q3: Identify the following as expressions or equations.

- (a) $ab + 4$ (Expression)
 (b) $3x + 3y = 45$ (Equation)
 (c) $pq = 6$ (Equation)
 (d) $x - 5$ (Expression)
 (e) $3x = 9$ (Equation)
 (f) $x + 5$ (Expression)

Q4: Solve the following linear equations. Also verify the solution.

a. $\frac{(4x+3)}{(2x-3)} = -\frac{4}{2}$

Solution:

$$\begin{aligned}\frac{(4x+3)}{(2x-3)} &= -\frac{4}{2} \\ \Rightarrow 2(4x+3) &= -4(2x-3) \\ 2 \times 4x + 2 \times 3 &= -4(2x) - 4(-3) \\ 8x + 6 &= -8x + 12 \\ 8x + 8x &= 12 - 6 \\ 16x &= 6\end{aligned}$$

$$x = \frac{6}{16} = \frac{3}{8}$$

b. $\frac{(x-9)}{3} = -\frac{(x-3)}{7}$

Solution:

$$\begin{aligned}\frac{(x-9)}{3} &= -\frac{(x-3)}{7} \\ 7(x-9) &= -3(x-3) \\ 7x - 63 &= -3x + 9 \\ 7x + 3x &= 9 + 63 \\ 10x &= 72\end{aligned}$$

$$x = \frac{72}{10} = \frac{36}{5}$$

$$x = 7\frac{1}{5}$$

c. $2.5x + 3 = 7$

Solution:

$$\begin{aligned}2.5x + 3 &= 7 \\ 2.5x &= 7 - 3 \\ 2.5x &= 4 \\ x &= \frac{4}{2.5} = \frac{4}{25} \times 10 \\ \Rightarrow x &= \frac{40}{25} = \frac{8}{5} \\ x &= 1.6\end{aligned}$$

d. $4x + \frac{4}{8} = 3 - x$

Solution:

$$\begin{aligned}4x + x &= 3 - \frac{4}{8} \\ 5x &= \frac{3 \times 8 + 4 \times 1}{8} \\ 5x &= \frac{24 + 4}{8} \\ 5x &= \frac{28}{8} \\ x &= \frac{28}{8} \times \frac{1}{5} \\ x &= \frac{28}{40} \Rightarrow x = \frac{7}{10}\end{aligned}$$

e. $6x = \frac{3}{9}$

Solution:

$$\begin{aligned}6x &= \frac{3}{9} \\ x &= \frac{3}{9} \times \frac{1}{6} \\ x &= \frac{1}{18}\end{aligned}$$

f. $5(a-6) - 3(4a+4) = 7(a+6)$

Solution:

$$5(a-6) - 3(4a+4) = 7(a+6)$$

$$5a - 30 - 12a - 12 = 7a + 42$$

$$-7a - 42 = 7a + 42$$

$$-7a - 7a = 42 + 42$$

$$-14a = 84$$

$$a = \frac{-84}{14}$$

$$a = -6$$

g. $3(y-2) + 4(y-3) = 5y + 14 - 4(y-8)$

Solution:

$$\begin{aligned}3(y-2) + 4(y-3) &= 5y + 14 - 4(y-8) \\ 3y - 6 + 4y - 12 &= 5y + 14 - 4y + 32 \\ 3y + 4y - 6 - 12 &= 5y - 4y + 14 + 32 \\ 7y - 18 &= y + 46 \\ 7y - y &= 46 + 18\end{aligned}$$

$$6y = 64 \Rightarrow y = \frac{64}{6} = \frac{32}{3}$$

$$y = 10\frac{2}{3}$$

i. $\frac{3(2x-4)}{15(x-2)} = 4$

Solution:

$$\begin{aligned}\frac{3(2x-4)}{15(x-2)} &= 4 \\ 3(2x-4) &= 4 \times 15(x-2) \\ 6x - 12 &= 60(x-2) \\ 6x - 12 &= 60x - 120 \\ 6x - 60x &= -120 + 12 \\ -54x &= -108 \\ x &= \frac{-108}{-54} \\ x &= 2\end{aligned}$$

$$j/ \frac{3}{5}(4x+5) = \frac{2}{7}(3x+5)$$

Solution:

$$\frac{3}{5}(4x+5) = \frac{2}{7}(3x+5)$$

$$3 \times 7(4x+5) = 5 \times 2(3x+5)$$

$$21(4x+5) = 10(3x+5)$$

$$84x+105 = 10 \times 3x + 10 \times 5$$

$$84x+105 = 30x+50$$

$$54x = -55$$

$$x = \frac{-55}{54}$$

$$x = -1\frac{1}{54}$$

k. $0.4x + 2.4 = 2.0x + 0.12$

Solution:

$$0.4x + 2.4 = 2.0 + 0.12$$

$$0.4 - 2.0x = 0.12 - 2.4$$

$$-1.6x = -2.28$$

$$x = \frac{-2.28}{-1.6} = \frac{2.28}{1.6}$$

$$= 1.425$$

l. $0.5(0.2p + 3.2) = 0.5p - 1$

Solution:

$$0.5(0.2p + 3.2) = 0.5p - 1$$

$$0.1p + 1.6 = 0.5p - 1$$

$$(0.1)p - (0.5)p = -1 - 1.6$$

$$-0.4 = -2.6$$

$$p = \frac{-2.6}{-0.4} = \frac{2.6}{0.4}$$

$$p = \frac{26}{4} \times \frac{10}{10}$$

$$p = \frac{26}{4}$$

$$p = 6.5$$

Q5. Mr Ali purchased a fridge and an air conditioner for Rs 95,230. If the cost of the air conditioner is Rs 23,000 more than the fridge, find the cost of both items.

Solution:

Suppose cost of fridge = x

Cost air conditioner = $x + 23,000$

According the given condition

$$x + x + 23,000 = 95,230$$

$$2x + 23,000 = 95,230$$

$$2x = 95,230 - 23,000$$

$$2x = 72,230$$

$$x = \frac{72,230}{2}$$

$$x = 36,115$$

Cost of fridge = Rs. 36,115

Cost of Air conditioner = Rs. 59,115

Q6. After sixteen years, Asma will be 28 years old. Find her present age.

Solution:

Suppose present age = x

After sixteen years age 06

Asma = $x + 16$

According to given condition,

$$x + 16 = 28$$

$$x = 28 - 16$$

$$x = 12$$

So present age of Asma = 12 years.

Q7. When five times a number is added to three times itself, the sum obtained is 48. Find the number.

Solution:

Suppose number = x

5 time of a number = $5x$

According to given condition,

$$5x + 3x = 48$$

$$8x = 48$$

$$\Rightarrow x = \frac{48}{8}$$

$$x = 6$$

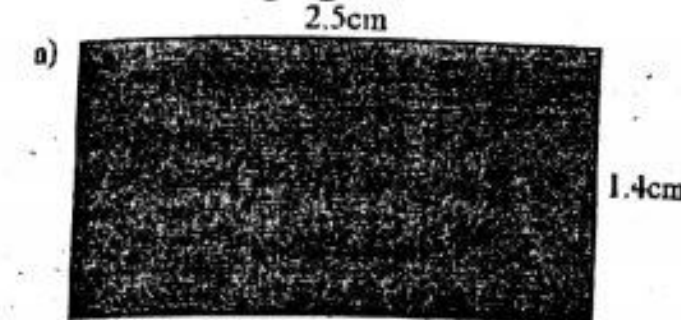
So number = 6

Unit 8

Surface Area and Volume

Exercise 8.1

Q1: Find the perimeter of the following figures.



Solution:

As given figure is rectangle, so

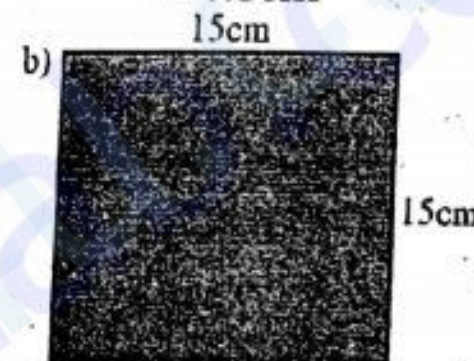
Perimeter of rectangle

$$= 2(\text{length} + \text{width})$$

$$= 2(2.5\text{cm} + 1.4\text{cm})$$

$$= 2(3.9)$$

$$= 7.8\text{cm}$$



Solution:

As given figure is square, so

Perimeter of rectangle = $4 \times \text{length}$

$$= 4 \times 1.5\text{cm}$$

$$= 60\text{cm}$$



Solution:

As given figure is rectangle, so

Perimeter of rectangle

$$= 2(\text{length} + \text{width})$$

$$= 2(45 + 39)$$

$$= 84 \times 2$$

$$= 168\text{cm}$$

Q2: Use the formula to find the area and perimeter of the squares whose lengths for given below:

Solution:

a. 4cm

Solution:

As we know that

Area of square = length $\times$ length

$$= 4 \times 4$$

$$= 16\text{cm}^2$$

And perimeter of square = $4 \times \text{length}$

$$= 4 \times 4$$

$$= 16\text{cm}$$

b. 7.2cm

Solution:

As we know that,

Area of square = length $\times$ length

$$= 7.2\text{cm} \times 7.2\text{cm}$$

$$= 51.84\text{cm}^2$$

And perimeter of square = $4 \times \text{length}$

$$= 4 \times 7.2$$

$$= 28.8\text{cm}$$

c. 10.5cm

Solution:

As we know that

Area of square = length $\times$ length

$$= 10.5 \times 10.5$$

$$= 110.25\text{cm}^2$$

And perimeter of square = $4 \times \text{length}$

$$= 4 \times 10.5$$

$$= 42\text{cm}$$

d. 6.2cm

Solution:

As we know that

Area of square = length $\times$ length

$$= 6.2 \times 6.2$$

$$= 38.44\text{cm}^2$$

$$\begin{aligned}\text{And perimeter} &= 4 \times \text{length} \\ &= 4 \times 6.2 \\ &= 24.8\text{cm}\end{aligned}$$

Q3: Use the formula to find the area and perimeter of the rectangles having the following lengths and widths.

a. Length = 4.7cm and Width = 2cm

Solution:

As we know that

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width} \\ &= 4.7 \times 2 \\ &= 9.4\text{cm}^2\end{aligned}$$

And perimeter of rectangle

$$\begin{aligned}&= 2(\text{length} \times \text{width}) \\ &= 2(4.7 + 2) \\ &= 2(6.7) \\ &= 13.4\text{cm}\end{aligned}$$

b. Length = 16cm and Width = 15.5 cm

Solution:

As we know that

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width} \\ &= 16\text{m} \times 15.5\text{cm} \\ &= 248\text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{And perimeter} &= 2(\text{length} \times \text{width}) \\ &= 2(16\text{m} + 15.5\text{m}) \\ &= 4(31.5) \\ &= 63\text{cm}\end{aligned}$$

c. Length = 23cm and Width = 4cm

Solution:

As we know that

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width} \\ &= 23\text{cm} \times 4\text{cm} \\ &= 92\text{cm}^2\end{aligned}$$

And perimeter of Rectangle

$$\begin{aligned}&= 2(\text{length} + \text{width}) \\ &= 2(23 + 4) \\ &= 2(37) \\ &= 74\text{cm}\end{aligned}$$

d. Length = 20m and Width = 12.2cm

Solution:

As we know that

$$\begin{aligned}\text{Area of Rectangle} &= \text{length} \times \text{width} \\ &= 20 \times 12.2 \\ &= 244\text{cm}^2\end{aligned}$$

And perimeter of Rectangle

$$\begin{aligned}&= 2(\text{length} + \text{width}) \\ &= 2(20 + 12.2) \\ &= 2(32.2) \\ &= 64.4\text{cm}\end{aligned}$$

Q4: Find the length of the rectangle if its area is 325 m² and width is 13m.

Solution:

$$\begin{aligned}\text{Area of rectangle} &= 325\text{m}^2 \\ \text{Width} &= 13\text{m} \\ \text{Length} &= ?\end{aligned}$$

As we know that

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width} \\ 325 &= \text{length} \times 13\text{m}\end{aligned}$$

$$\text{Length} = \frac{325}{13}$$

$$\text{Length} = 25\text{m}.$$

Q5: Find the length of a square if its perimeter is 48 m.

Solution:

$$\text{Perimeter of square} = 48\text{m}$$

$$\text{Length of square} = ?$$

As we know that

$$\begin{aligned}\text{Perimeter of square} &= 4 \times \text{length} \\ 48 &= 4 \times \text{length}\end{aligned}$$

$$\text{Length} \frac{48}{4} = 12$$

$$\text{Length of square} = 12\text{m}.$$

Q6: A rectangular-shaped room is 24 meters long and 20 meters wide. Find the area of the room and the cost to cement the room

at the rate of Rs 200 per metre square.

Solution:

$$\text{Length} = 24\text{m}$$

$$\text{Width} = 20\text{m}$$

$$\begin{aligned}\text{Area of rectangular room} &= \text{length} \times \text{width} \\ &= 24 \times 20 \\ &= 480\text{m}^2\end{aligned}$$

Now we have to find cost of cement,

$$\text{As rate per meter square} = \text{Rs. } 200$$

$$\begin{aligned}\text{So rate for } 480\text{m}^2 &= 200 \times 480 \\ &= \text{Rs. } 96,000\end{aligned}$$

Q7: Find the area of a square shaped swimming pool if its length is 300 m.

Solution:

$$\text{Length} = 300\text{m}$$

As we know that

$$\begin{aligned}\text{Area of square shaped} &= \text{length} \times \text{length} \\ &= 300 \times 300 \\ &= 90000\text{m}^2\end{aligned}$$

Q8: The perimeter of a rectangular flowerbed is 244cm. Find the length of the flowerbed if its width is 44cm.

Solution:

$$\begin{aligned}\text{Perimeter of a rectangular flowerbed} &= 244\text{cm}\end{aligned}$$

$$\text{Length} = ?$$

$$\text{Width} = 44\text{cm}$$

As we know that

$$\text{Perimeter} = 2(\text{Length} + \text{width})$$

$$244 = 2(\text{Length} + 44)$$

$$244 = 2 \times \text{Length} + 2 \times 44$$

$$2 \text{ length} = 244 - 88$$

$$2 \text{ length} = 156$$

$$\Rightarrow \text{length} = \frac{156}{2} = 78\text{cm}$$

Q9: Find the length of a rectangular room if its area is 2048 cm² and width is 32cm. Find the cost of flooring the room at the rate of Rs 250 per cm².

Solution:

$$\text{Length of Rectangular room} = ?$$

$$\text{Area} = 2048\text{cm}^2$$

$$\text{Width} = 32\text{cm}.$$

$$\text{Cost of flooring the room} = ?$$

As we know that

$$\begin{aligned}\text{Area of rectangle} &= \text{length} \times \text{width} \\ 2048 &= \text{Length} \times 32\end{aligned}$$

$$\text{Length} = \frac{2048}{32} = 64$$

$$\text{Length} = 64\text{cm}$$

$$\text{Rate of flooring per cm}^2 = \text{Rs. } 250$$

$$\begin{aligned}\text{So cost of flooring the room for } 2048\text{cm}^2 &= 250 \times 2048 \\ &= \text{Rs. } 512,000\end{aligned}$$

Q10: The cost of fencing a square garden at the rate of Rs. 25 per meter square is Rs. 5625. Find the area of the garden.

Solution:

$$\text{Cost of fencing garden} = \text{Rs. } 5625$$

$$\text{Rate per meter square} = \text{Rs. } 25$$

As we know that

$$\begin{aligned}\text{Total cost of fencing garden} &= \text{Rate m}^2 \times \text{Area}\end{aligned}$$

$$\text{So Area of the garden} = \frac{\text{Total cost}}{\text{Rate m}^2}$$

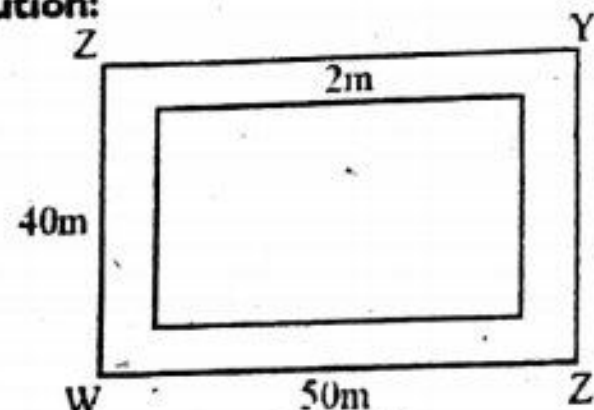
$$= \frac{5625}{25}$$

$$= 225\text{m}^2$$

Exercise 8.2

Q1: A rectangle WXYZ is 50m long and 40 m wide. A path is constructed towards the inner side of the rectangle that is 2 m wide. Find the area of the path.

Solution:



Length of rectangle = 50m

Width of rectangle = 40m

Area = length $\times$ width

$$= 50 \times 40$$

$$= 2000\text{m}^2$$

Now length without of path

$$= 50 - 2 - 2$$

$$= 46\text{m}$$

Width without of pat = $40 - 2 - 2$

$$= 36\text{m}$$

Area of rectangle without path

$$= 45 \times 36$$

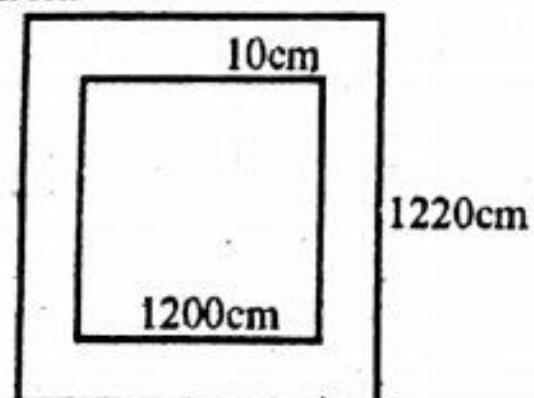
$$= 1656\text{m}^2$$

So Area of the path = $2000 - 1656$

$$= 344\text{m}^2$$

Q2: A square is 1200 cm long. There is a 10 cm wide strip around the square. Find the area of the strip.

Solution:



Length of side of square = 1200cm

Area of square = $1200 \times 1200 = 1440000\text{cm}^2$

Now

Length of the square with strip

$$= 1200 + 20$$

$$= 1220\text{cm}$$

Area of the square with strip

$$= 1220 \times 1220$$

$$= 1,448,400\text{cm}^2$$

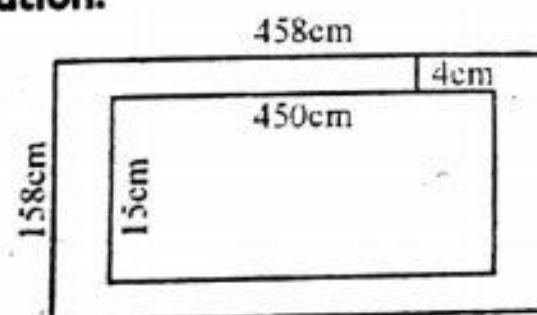
So Area of the strip = Area of the square with strip - Area of the square

$$= 1448,400 - 1440000$$

$$= 8400\text{cm}^2$$

Q3: A 450 cm long and 150cm wide rectangle is enclosed by a strip outside it that is 4 cm wide. Find the area of the strip.

Solution:



Length of rectangle = 450cm

Width of rectangle = 150cm

Area of Rectangle = 450×150

$$= 67,500\text{cm}^2$$

Now length of rectangle with strip = 458cm

Width of rectangle with strip = 158cm

Area of rectangle with strip

$$= 158 \times 450$$

$$= 71,100\text{cm}^2$$

So

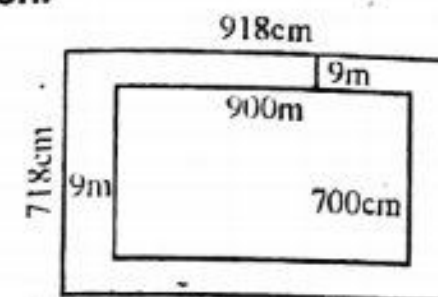
Area of the strip = Area of rectangle with strip - Area of rectangle without strip.

$$= 72,364 - 67,500$$

$$= 4864\text{cm}^2$$

Q4: A rectangle is 900m long and 700m wide. A path is constructed outside the rectangle that is 9m wide. Find the area of the path.

Solution:



Width of Rectangle = 700m

Length of Rectangle = 900m

Area of rectangle = 700×900

$$= 630000\text{m}^2$$

Now

Width of rectangle with path = 718m

Length of rectangle with path = 918m

Area rectangle with path = 918×718

Area of rectangle with path = 659124m^2

Now

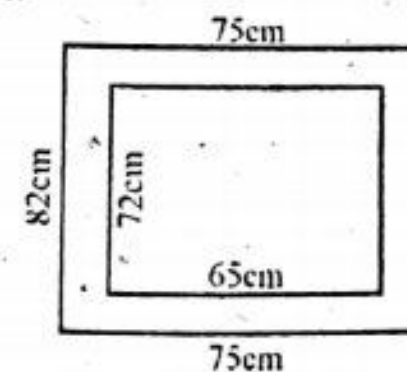
Area of the path = Area of the rectangle with path - Area of the rectangle, without path.

$$= 659124 - 630000$$

$$= 29,124\text{m}^2$$

Q5: A garden is 75cm long and 82 cm wide. A 5 cm wide road is made all around the inside of the garden. Find the area of the road.

Solution:



Length of garden = 75cm

Width of garden = 82cm

Area of garden = 75×82

$$= 6150\text{cm}^2$$

Length of garden without road = 65cm

Width of garden without road = 65×72

Area of garden without round

$$= 65 \times 72$$

$$= 4680\text{cm}^2$$

So

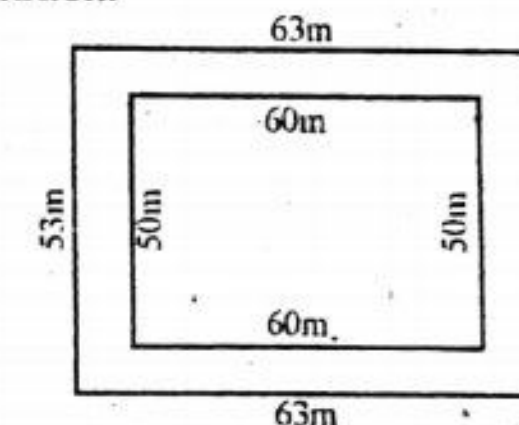
Area of the road = Area of the garden - Area of the garden without road.

$$= 6150 - 4680$$

$$= 1470\text{cm}^2$$

Q6: A playground is 60m long and 50 m wide. A 1.5 m wide concrete path is made all around it towards the outer side of the playground. Find the area of the concrete path.

Solution:



Length of playground = 60m

Width of playground = 50m

Area of playground = 60×50

$$= 3000\text{m}^2$$

Now

Length of playground with path = 63m

Width of playground with path = 53m

Area playground with path

$$= 63 \times 53$$

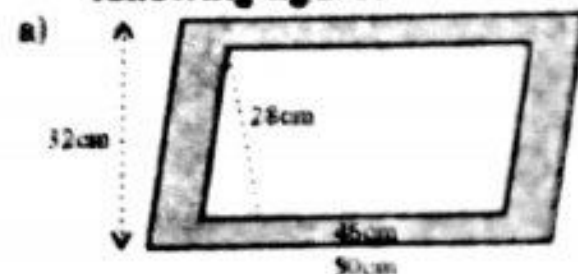
$$= 3339\text{m}^2$$

So

$$\text{Area of concrete path} = 3339 - 3000 \\ = 339\text{m}^2$$

Exercise 8.3

Q1: Find the shaded area of the following figures.



Solution:

$$\text{Altitude} = 32\text{cm}$$

$$\text{Base} = 50\text{cm}$$

$$\text{Area of outer parallelogram} = \text{Altitude} \times \text{Base}$$

$$= 32 \times 50$$

$$= 1600\text{cm}^2$$

Now

$$\text{Altitude} = 28\text{cm}$$

$$\text{Base} = 46\text{cm}$$

$$\text{Area of inner parallelogram} = \text{Base} \times \text{Altitude}$$

$$= 46 \times 28$$

$$= 1288\text{cm}^2$$

Now

Area of the shaded path of parallelogram

$$= 1600 - 1288$$

$$= 312\text{cm}^2$$



Solution:

$$\text{Base of outer triangle} = 9\text{m}$$

$$\text{Altitude of outer triangle} = 6\text{m}$$

$$\text{Area of outer triangle} = \frac{1}{2} (\text{Altitude} \times \text{Base})$$

$$= \frac{1}{2} (6 \times 9)$$

$$= \frac{1}{2} \times 54$$

$$\text{Area of outer triangle} = 27\text{m}^2$$

$$\text{Now Base of inner triangle} = 7\text{m}$$

$$\text{Altitude of inner triangle} = 4\text{m}$$

$$\text{Area of inner triangle} = \frac{1}{2} (\text{Altitude} \times \text{Base})$$

$$= \frac{1}{2} (7 \times 4)$$

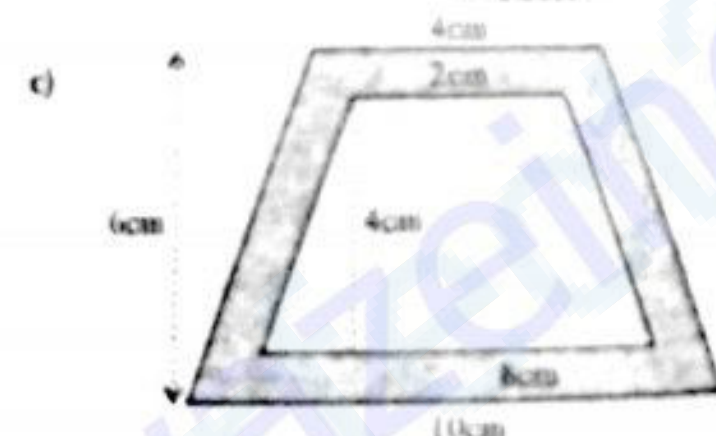
$$= \frac{1}{2} (28)$$

$$= 14\text{m}^2$$

Now

$$\text{Area of the shaded path of triangle} = 27 - 14$$

$$= 13\text{m}^2$$



Solution:

$$\text{Length of outer side of trapezium}$$

$$= b_1 = 10\text{cm}$$

$$\text{Length of outer side, of trapezium}$$

$$= b_2 = 4\text{cm}$$

$$\text{Altitude} = 6\text{cm}$$

$$\text{Area of out trapezium} = \text{Altitude} \times \left(\frac{b_1 + b_2}{2} \right)$$

$$= 6 \times \left(\frac{10 + 4}{2} \right)$$

$$\text{Area of outer trapezium} = 6 \times \frac{14}{2}$$

$$= 6 \times 7 = 42\text{cm}^2$$

$$\text{Length of side 1 of inner trapezium} = b_1 = 8\text{cm}$$

$$\text{Length of side 2 of inner trapezium} = b_2 = 2\text{cm}$$

$$\text{Altitude} = 4\text{cm}$$

$$\text{Area of inner trapezium} = \text{Altitude} \times \left(\frac{b_1 + b_2}{2} \right)$$

$$= 4 \times \left(\frac{8 + 2}{2} \right)$$

$$= 4 \times \frac{10}{2}$$

$$= 4 \times 5 = 20\text{cm}^2$$

$$\text{Area of shaded trapezium} = 42 - 20$$

$$= 22\text{cm}^2$$

Q2: Find the altitude of a trapezium if the length of two bases are 5.6cm and 2.9 cm respectively. The area of the trapezium is 42cm²

Solution:

$$\text{Altitude of trapezium} = ?$$

$$\text{Length of side}_1 = b_1 = 5.6\text{cm}$$

$$\text{Length of side}_2 = b_2 = 2.9\text{cm}$$

$$\text{Area of trapezium} = 42\text{cm}^2$$

As we know that

$$\text{Area of trapezium} = \text{Altitude} \times \left(\frac{b_1 + b_2}{2} \right)$$

$$42\text{cm}^2 = \text{Altitude} \times \left(\frac{5.6 + 2.9}{2} \right)$$

$$42\text{cm}^2 = \text{Altitude} \times \frac{8.5}{2}$$

$$\text{Altitude} = \frac{42 \times 2}{8.5} = \frac{84}{8.5}$$

$$\text{Altitude} = 9.89\text{cm}$$

Q3: Find the area of a trapezium if the length of two bases are 8.8 m and 12.9 m respectively. The perpendicular distance between two bases are 12 m.

Solution:

$$\text{Length of side}_1 \text{ of trapezium} = b_1 = 8.8\text{m}$$

$$\text{Length of side}_2 \text{ of trapezium} = b_2 = 12.9\text{m}$$

$$\text{Altitude} = 12\text{m}$$

As we know that

$$\text{Area of trapezium} = \text{Altitude} \times \left(\frac{b_1 + b_2}{2} \right)$$

$$= 12 \times \left(\frac{8.8 + 12.9}{2} \right)$$

$$= 12 \times \frac{21.7}{2}$$

$$= 12 \times 10.85$$

$$= 130.2\text{m}^2$$

Q4: Find the altitude of the triangular field. If its area is 946cm² and base is 104cm.

Solution:

$$\text{Altitude of the triangular field} = ?$$

$$\text{Area} = 946\text{cm}^2, \text{Base} = 104\text{cm}$$

$$\text{Area of trapezium} = \frac{1}{2} \times (\text{Altitude} \times \text{Base})$$

$$946 = \frac{1}{2} \times \text{Altitude} \times 104$$

$$\Rightarrow \text{Altitude} = \frac{2 \times 946}{104} = \frac{1892}{104}$$

$$\text{Altitude} = 18.19\text{cm}$$

Q5: Find the area of the triangle if the length of its base is 2.5 m and the length of its altitude is 1.2. m

Solution:

Area of triangle = ?

Length of Base = 2.5m

Altitude = 1.2m

$$\begin{aligned}\text{Area of triangle} &= \frac{1}{2} \times (\text{Altitude} \times \text{Base}) \\ &= \frac{1}{2} (1.2 \times 2.5) \\ &= 1.5\text{m}^2\end{aligned}$$

Q6: Find the base of the triangular piece of carpet. If its area is 1568 cm² and altitude is 9.04m

Solution:Base = ?, Area = 1568cm²

Altitude = 9.04cm = 904cm

As

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times (\text{Altitude} \times \text{Base}) \\ 1568 &= \frac{1}{2} (904 \times \text{Base}) \\ \Rightarrow \text{Base} &= \frac{2 \times 1568}{904} = \frac{3136}{904} \\ &= 3.47\text{cm}\end{aligned}$$

Q7: The altitude of the parallelogram is 56 m and its area is 6424 cm². Find the base of parallelogram.

Solution:

Altitude of the parallelogram

= 56cm

= 5600cm

Area = 6424cm²

Base = ?

As

Area of parallelogram = Altitude × Base

$$6424 = 5600 \times \text{Base}$$

$$\text{Base} = \frac{6424}{5600} = 1.147$$

$$\text{Base} = 1.147\text{cm}$$

Q8: The area of the parallelogram is 900 m² and base is 30 m. Find the altitude of parallelogram.

Solution:Given Area of parallelogram = 900m²

Base = 30m, Altitude = ?

As we know that Area of Parallelogram = Altitude × Base

$$900 = \text{Altitude} \times 30$$

$$\text{Altitude} = \frac{900}{30} = 30$$

$$\text{Altitude} = 30\text{m}$$

Exercise 8.4

Q1: Name the following shapes and write their number of faces, edges and vertices.

Solution:

Q2: Label the characteristics/properties of solids.

Solution:**Exercise 8.5**

Q1: Calculate the volume and surface area of the following cubes whose length of the edge are given below

a. 5cm**Solution:**

Length = l = 5cm

As

Volume of cube = l³

$$= 5^3$$

$$= 125\text{cm}^3$$

And surface area of cube = 6(length)²

$$= 6 \times 5^2$$

$$= 150\text{cm}^2$$

b. 8cm**Solution:**

Length = l = 8cm

As

Volume of cube = l³

$$= 8^3$$

$$= 512\text{cm}^3$$

And

Surface area of cube = 6 (length)²

$$= 6 \times 8^2$$

$$= 6 \times 64$$

$$= 384\text{cm}^2$$

c. 4.3cm**Solution:**

Length = l = 4.3cm

As

Volume of cube = l³

$$= (4.3)^3$$

$$= 79.507\text{cm}^3$$

And

$$\begin{aligned}\text{Surface area of cube} &= 6 (\text{length})^2 \\ &= 6 \times (4.3)^2 \\ &= 110.94\text{cm}^2\end{aligned}$$

d. 6.4m**Solution:**

Length = l = 6.4m

As

Volume of cube = (length)³

$$= (6.4)^3$$

$$= 262.144\text{m}^3$$

And

Surface area of cube = 6 × (length)²

$$= 6 \times (6.4)^2$$

$$= 245.76\text{cm}^2$$

e. 2m**Solution:**

Length = l = 2m

As

Volume of cube = (length)³

$$= (2)^3$$

$$= 8\text{m}^3$$

And

Surface area of cube = 6 (length)²

$$= 6 \times (2)^2$$

$$= 24\text{m}^2$$

f. 3m**Solution:**

Length = l = 3m

As

Volume of cube = (length)³

$$= 3^3$$

$$= 27\text{m}^3$$

And

Surface area of cube = 6 (length)²

$$= 6 \times (3)^2$$

$$= 54\text{m}^2$$

g. 12cm**Solution:**

Length = 12cm

As

$$\begin{aligned}\text{Volume of cub} &= (\text{length})^3 \\ &= (12)^3 \\ &= 1728\text{cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cub} &= 6(\text{length})^2 \\ &= 6(12)^2 \\ &= 864\text{cm}^2\end{aligned}$$

h. 90cm

Solution:

$$\text{Length} = 90\text{cm}$$

As

$$\begin{aligned}\text{Volume of cub} &= (\text{length})^3 \\ &= (90)^3 \\ &= 72,900\text{cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cub} &= 6(\text{length})^2 \\ &= 6(90)^2 \\ &= 4860\text{cm}^2\end{aligned}$$

i. 45.6m

Solution:

$$\text{Length} = 45.6\text{m}$$

As

$$\begin{aligned}\text{Volume of cub} &= (\text{length})^3 \\ &= (45.6)^3 \\ &= 94818.816\text{m}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cub} &= 6(\text{length})^2 \\ &= 6 \times (45.6)^2 \\ &= 12476\text{m}^2\end{aligned}$$

j. 100m

Solution:

$$\text{Length} = 100\text{m}$$

As

$$\begin{aligned}\text{Volume of cub} &= (\text{length})^3 \\ &= (100)^3 \\ &= 1000000\text{m}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cub} &= 6(\text{length})^2 \\ &= 6 \times (100)^2 \\ &= 60000\text{m}^2\end{aligned}$$

k. 10 cm

Solution:

$$\text{Length} = 10\text{cm}$$

As

$$\begin{aligned}\text{Volume of cub} &= (\text{length})^3 \\ &= (10)^3 \\ &= 1000\text{m}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cub} &= 6 \times (\text{length})^2 \\ &= 6 \times (10)^2 \\ &= 600\text{cm}^2\end{aligned}$$

l. 55m

Solution:

$$\text{Length} = l = 55\text{m}$$

As

$$\begin{aligned}\text{Volume of cub} &= (\text{length})^3 \\ &= (55)^3 \\ &= 166375\text{m}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cub} &= 6 \times (\text{length})^2 \\ &= 6 \times (55)^2 \\ &= 18150\text{m}^2\end{aligned}$$

Q2: Calculate the volume and surface area of each of the following cuboids.

a. $l=21\text{cm}, w=11\text{cm}, h=6\text{cm}$ **Solution:**

$$l = 21\text{cm}, w = 11\text{cm}, h = 6\text{cm}$$

As

$$\begin{aligned}\text{Volume of cuboids} &= (\text{length}) \times (\text{width}) \times (\text{height}) \\ &= (21) \times (11) \times (6) \\ &= 1386\text{cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cuboids} &= 2 \times [(l \times w) + l \times h + (w \times h)] \\ &= 2 \times [(21 \times 11) + (21 \times 6) + (11 \times 6)] \\ &= 2 \times [231 + 126 + 66] \\ &= 2 \times 423 \\ &= 846\text{m}^2\end{aligned}$$

b. $l = 43\text{cm}, w = 31\text{cm}, h = 9\text{cm}$ **Solution:**

$$l = 43\text{cm}, w = 31\text{cm}, h = 9\text{cm}$$

As we know that

$$\begin{aligned}\text{Volume of cuboid} &= l \times w \times h \\ &= 43 \times 31 \times 9 \\ &= 1386\text{cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cuboid} &= 2 \times [(l \times w) + l \times h + (w \times h)] \\ &= 2 \times [(43 \times 31) + (43 \times 9) + (31 \times 9)] \\ &= 2 \times [1333 + 387 + 279] \\ &= 2 \times (1999) \\ &= 3998\text{cm}^2\end{aligned}$$

c. $l = 34.5\text{m}, w = 15.1\text{m}, h = 800\text{m}$ **Solution:**

$$l = 34.5\text{m}, w = 15.1\text{m}, h = 800\text{m}$$

As we know that

$$\begin{aligned}\text{Volume of cuboids} &= l \times w \times h \\ &= 34.5 \times 15.1 \times 800 \\ &= 416,760\text{cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cuboids} &= 2 \times [(l \times w) + l \times h + (w \times h)] \\ &= 2 \times [(34.5 \times 15.1) + (34.5 \times 800) + (15.1 \times 800)] \\ &= 2 \times [520.95 + 27,600 + 12,080] \\ &= 2 \times (40,200.95) \\ &= 80,401.9\text{m}^2\end{aligned}$$

d. $l = 19\text{cm}, w = 20\text{cm}, h = 21\text{cm}$ **Solution:**

$$l = 19\text{cm}, w = 20\text{cm}, h = 21\text{cm}$$

As we know that

$$\begin{aligned}\text{Volume of cuboids} &= l \times w \times h \\ &= 19 \times 20 \times 21 \\ &= 7980\text{cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area of cuboid} &= 2 \times [(l \times w) + l \times h + (w \times h)] \\ &= 2 \times [(19 \times 20) + (19 \times 21) + (20 \times 21)]\end{aligned}$$

$$= 2 \times [380 + 399 + 420]$$

$$= 2 \times (1199)$$

$$= 2398\text{m}^2$$

Q3: Look at the following measurements of a cuboids and fill the missing values.

	Length (cm)	Width (cm)	Height (cm)	Volume (cm)	Surface area (cm ²)
a	23		4		452
b	21	14	7		
c		5	9	321	
d	45	3		453	

a.

Solution:

$$\text{Given } l = 23\text{cm}, w = ?$$

$$h = 4\text{cm}, v = ?$$

$$\text{Surface Area} = 452\text{cm}^2$$

First we find 'w' from given data,

$$\text{As surface area of cuboid} = 2 \times [(l \times w) + l \times h + (w \times h)]$$

$$452 = 2 \times [(23 \times w) + (23 \times 4) + (w \times 4)]$$

$$452 = 2 \times [23w + 92 + 4w]$$

$$452 = 2 \times [27w + 92]$$

$$27w + 92 = \frac{452}{2}$$

$$27w + 92 = 226$$

$$27w = 226 - 92$$

$$27w = 134$$

$$w = \frac{134}{27} = 4.96$$

$$\Rightarrow w = 4.96$$

Now we find volume,

As

$$v = l \times w \times h$$

$$v = 23 \times 4.96 \times 4 = 456.32$$

$$\text{Volume of cuboid} = 456.32\text{cm}^3$$

b.

Solution:

$$\text{Given } l = 21\text{cm}, w = 14\text{cm}$$

$$h = 7\text{cm}$$

As

$$\begin{aligned}\text{Volume of cuboid} &= l \times w \times h \\ &= 21 \times 14 \times 7 \\ &= 2058 \text{ cm}^3\end{aligned}$$

Now we find surface area,

$$\begin{aligned}\text{Surface area} &= 2 \times [(l \times w) + (l \times h) + (w \times h)] \\ &= 2 \times [(12 \times 14) + (21 \times 7) + (14 \times 7)] \\ &= 2 \times [294 + 147 + 98] \\ &= 2 \times (539) = 1078 \text{ cm}^2\end{aligned}$$

c.

Solution:

$$\begin{aligned}\text{Given } w &= 5 \text{ m, } h = 9 \text{ m, } v = 321 \text{ cm}^3 \\ l &= ?, \text{ Surface area} = ?\end{aligned}$$

As

$$\begin{aligned}\text{Volume of cuboids} &= l \times w \times h \\ 321 &= l \times 5 \times 9 \\ 321 &= 45l \\ \Rightarrow l &= \frac{321}{45} = 7.13 \text{ cm}\end{aligned}$$

Now we find surface area,

$$\begin{aligned}\text{Surface area} &= 2 \times [(l \times w) + (l \times h) + (w \times h)] \\ &= 2 \times [(7.13 \times 5) + (7.13 \times 9) + (5 \times 9)] \\ &= 2 \times [35.67 + 64.2 + 45] \\ &= 2 \times [144.87] \\ &= 289.74 \text{ cm}^2\end{aligned}$$

d.

Solution:

$$\begin{aligned}\text{Given } l &= 45 \text{ cm, } w = 3, h = ? \\ \text{Volume} &= v = 453, \text{ Surface area} = ? \\ \text{As}\end{aligned}$$

$$\begin{aligned}\text{Volume of cuboid} &= l \times w \times h \\ 453 &= 45 \times 3 \times h \\ 453 &= 135 \times h \\ \Rightarrow h &= \frac{453}{135} = 3.36 \text{ cm}\end{aligned}$$

Now we find Surface area,

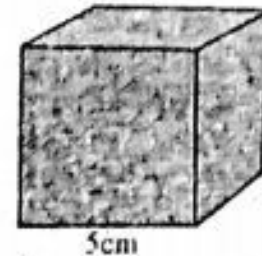
As

$$\begin{aligned}\text{Surface of cuboids} &= 2 \times [(l \times w) + (l \times h) + (w \times h)] \\ &= 2 \times [(45 \times 3) + (45 \times 3.36) + (3 \times 3.36)]\end{aligned}$$

$$\begin{aligned}&= 2 \times [135 + 151 + 10.08] \\ &= 2 \times [296.08] \\ &= 592.16 \text{ cm}^2\end{aligned}$$

Q4: Find the surface area and volume of the given cubes and cuboids.

a)



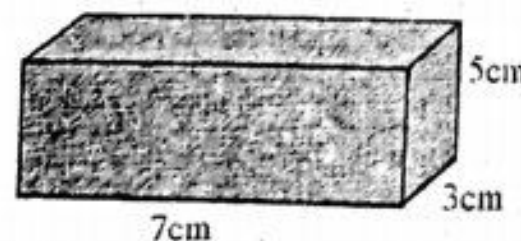
Solution:

$$\begin{aligned}\text{Given length of cub} &= l = 5 \text{ cm,} \\ \text{As volume cub} &= (\text{length})^3 \\ &= 5^3 \\ &= 125 \text{ cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area} &= 6 \times (\text{length})^2 \\ &= 150 \text{ cm}^2\end{aligned}$$

b)



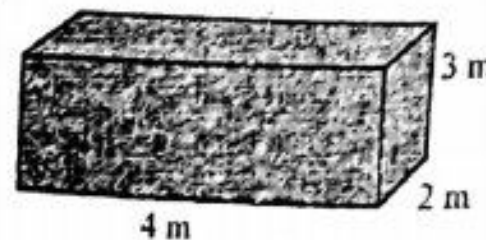
Solution:

$$\begin{aligned}\text{Given } l &= 7 \text{ cm, } w = 3 \text{ cm, } h = 5 \text{ cm} \\ \text{As volume of cuboid} &= l \times w \times h \\ &= 7 \times 3 \times 5 \\ &= 105 \text{ cm}^3\end{aligned}$$

Now we find surface area of cuboids
Surface area = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

$$\begin{aligned}&= 2 \times [(7 \times 3) + (7 \times 5) + (3 \times 5)] \\ &= 2 \times [21 + 35 + 15] \\ &= 2 \times (71) \\ &= 142 \text{ cm}^2\end{aligned}$$

c)



Solution:

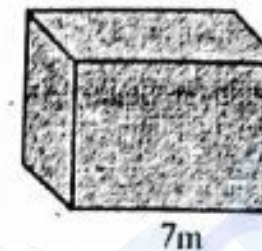
$$\begin{aligned}\text{Given } l &= 4 \text{ cm, } w = 2 \text{ m, } h = 3 \text{ m} \\ \text{Surface area} &= ? \text{ Volume} = ?\end{aligned}$$

$$\begin{aligned}\text{As volume of cuboid} &= l \times w \times h \\ &= 4 \times 2 \times 3 \\ &= 24 \text{ m}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface area} &= 2 \times [(l \times w) + (l \times h) + (w \times h)] \\ &= 2 \times [(4 \times 2) + (4 \times 3) + (2 \times 3)] \\ &= 2 \times [8 + 12 + 6] \\ &= 2 \times (26) \\ &= 52 \text{ m}^2\end{aligned}$$

d)



Solution:

$$\begin{aligned}\text{Given } l &= 7 \text{ m, Surface area} = ? \\ \text{Volume} &= ?\end{aligned}$$

$$\begin{aligned}\text{Surface area} &= 6 \times (l)^2 \\ &= 6 \times 7^2 = 6 \times 49 = 294 \text{ m}^2\end{aligned}$$

Now

$$\begin{aligned}\text{Volume} &= (l)^3 \\ &= (7)^3 \\ &= 343 \text{ m}^3\end{aligned}$$

Exercise 8.6

Q1. A cuboid glass box has dimensions 3m x 2m x 1m. Find the volume of water that it can hold.

Solution:

Length of glass box = 3m

Width of glass box = 2m

Height of glass box = 1m

Volume of water = length x width x height

$$\begin{aligned}&= 3 \times 2 \times 1 \\ &= 6 \text{ m}^3\end{aligned}$$

Q2: A box of cereal is 22 cm high, 12 cm long and 6 cm wide. What is the capacity of the cereal box?

Solution:

$$h = 22 \text{ cm, } l = 12 \text{ cm, } w = 6 \text{ cm}$$

As

$$\begin{aligned}\text{Capacity of cereal box} &= \text{volume} \\ &= h \times l \times w \\ &= 22 \times 12 \times 6 \\ &= 1584 \text{ cm}^3\end{aligned}$$

Q3: A cube shaped container has a length of 72cm. if it is filled with blocks up to a height of 42 cm, find the remaining capacity of the container.

Solution:

$$\begin{aligned}\text{Length of cube shape container} &= 72 \text{ cm} \\ l &= 72 \text{ cm}\end{aligned}$$

As

$$\text{Volume} = (l)^3 = 72^3 = 373,248 \text{ cm}^3$$

Now we find volume of cube filled part.

As $l = w = h$ for cube

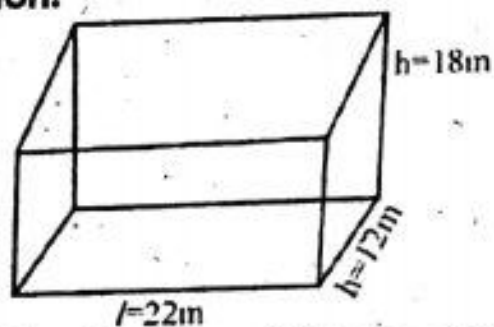
So $h = l = 42 \text{ cm}$

$$\begin{aligned}\text{Volume of filled part} &= l^3 \\ &= (42)^3\end{aligned}$$

$= 42 \times 42 \times 42$
 Volume of filled part = $74,088 \text{ cm}^3$
 So remaining capacity of container
 $= 373,248 - 74,088$
 $= 299,160 \text{ cm}^3$

Q4: A Masjid is 22 m long, 12m wide and 18m high. Find the cost of painting its four walls at the rate of Rs 450 per square meter.

Solution:



Given $l = 22\text{m}$, $w = 12\text{m}$, $h = 18\text{m}$
 Area of four wall = $l \times h + w \times h + l \times h + w \times h$
 $= 2l \times h + 2w \times h$
 $= 2 \times h (l + w)$
 Area of four walls = $2 \times 18 \times (22 + 12)$
 $= 2 \times 18 \times (34) = 1224 \text{ m}^2$
 Now rate of cost of wall per square meter = Rs. 450
 So
 Total cost of painting = 450×1224
 $= \text{Rs. } 550,800$

Q5: Find the cost of painting a cuboids container of length 4 m, width 3.5 m and height of 6 m at the rate of Rs 43.78 per square meter.

Solution:

Length = $l = 4\text{m}$, width = $w = 3.5\text{m}$ and height = $h = 6\text{m}$.
 As
 Surface Area of cuboid = $2 \times [(l \times w) + (l \times h) + (w \times h)]$
 $= 2 \times [(4 \times 3.5) + (4 \times 6) + (3.5 \times 6)]$

$= 2 \times (14 + 24 + 21)$
 $= 2 \times (59)$
 $= 118 \text{ m}^2$
 Now as
 Rate of per square meter = Rs. 43.78
 So Total cost = 118×43.78
 $= \text{Rs. } 5166.04$

Q6: A cube shaped gift box has a length of 3 cm. find whether the surface area of this box is greater or smaller than that of a cuboids box of dimension 5 cm $\times$ 6 cm $\times$ 3 cm.

Solution:

Given length of cube = $l = 3\text{cm}$
 As surface area of cube
 $= 6 \times (\text{length})^2$
 $= 6 \times 3^2$
 $= 6 \times 9 = 54 \text{ cm}^2$
 Now For cuboids box, $l = 5\text{cm}$, $w = 6\text{cm}$ and $h = 3\text{cm}$
 As
 Surface of cuboids box = $2 \times [(l \times w) + (l \times h) + (w \times h)]$
 $= 2 \times [(5 \times 6) + (5 \times 3) + (6 \times 3)]$
 $= 2 \times [30 + 15 + 18]$
 $= 2 \times 63$
 $= 126 \text{ cm}^2$
 As $54 \text{ cm}^2 < 126 \text{ cm}^2$
 So surface area of cube is smaller.

Q7: The total surface area of a cuboids shoebox is 1296 cm^2 . If its length and width is 18 cm and 9 cm respectively, find the height of the shoebox.

Solution:

Surface area of cuboid = 1296 cm^2
 $l = 18\text{cm}$, $w = 9\text{cm}$, $h = ?$
 As
 Surface area = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

$$1296 = 2 \times [(18 \times 9) + (18 \times h) + (9 \times h)]$$

$$1296 = 2 \times [162 + 18h + 9h]$$

$$= 2 \times (162 + 27h)$$

$$162 + 27h = \frac{1296}{2}$$

$$162 + 27h = 648$$

$$27h = 648 - 162$$

$$27h = 486$$

$$h = \frac{486}{27}$$

So height of shoebox = 18cm.

Q8: A wooden box has a length of 3 m, width of 2 m and height of 4 m. Find:

a. The volume of the box.

Solution:

$$l = 3\text{m}, w = 2\text{m}, h = 4\text{m}$$

(a) We have to find volume of the box.

$$\text{As volume of the box} = l \times w \times h$$

$$= 3 \times 2 \times 4$$

$$= 24 \text{ m}^3$$

b. The cost of polishing its surface if the rate of polishing is Rs. 220 per square meter

Solution:

We have to find cost of polishing, for this we first find surface area.

$$\text{Surface area} = 2 \times [(l \times w) + (l \times h) + (w \times h)]$$

$$= 2 \times [(3 \times 2) + (3 \times 4) + (2 \times 4)]$$

$$= 2 \times [6 + 12 + 8]$$

$$= 2 \times (26) = 52 \text{ m}^2$$

As Rate of per square meter = Rs. 220

$$\text{So Total cost} = 220 \times 52$$

$$= \text{Rs. } 11440$$

Q9: A box is 0.56m high and 21 cm wide. Its volume is $51744 \text{ cubic centimeters}$. Find:

a. The length of the box in cm.

Solution:

$$\text{height} = 0.56\text{m}$$

$$= 0.56 \times 100\text{cm}$$

$$= 56\text{cm}$$

$$\text{Volume} = 51744 \text{ cm}^3, w = 21\text{cm}$$

We have to find length

As

$$\text{Volume} = l \times h \times w$$

$$51744 = l \times 56 \times 21$$

$$51744 = 1176 \times l$$

$$\text{So length} = \frac{51744}{1176}$$

$$\text{Length of the box} = 44\text{cm.}$$

b. Total surface area of the box in cm^2

We have to find surface area,

$$\text{Surface area} = 2 \times [(l \times w) + (l \times h) + (w \times h)]$$

$$= 2 \times [(44 \times 21) + (44 \times 56) + (21 \times 56)]$$

$$= 2 \times [924 + 2464 + 1176]$$

$$= 2 \times (4564)$$

$$= 9128 \text{ cm}^2$$

c. Cost of painting its complete outer surface at the rate of Rs 145 per square centimeters. (Hint: First make the units of its dimensions the same)

Solution:

We have to find cost of painting.
 As rate of painting per square meter = Rs. 145

So

$$\text{Total cost of painting} = 145 \times 9128$$

$$= \text{Rs. } 1323560$$

Review Exercise**Q1: Choose the correct option.**

a. The ___ of any shape is the space occupied by it.

- (i) Perimeter (ii) space
(iii) width (iv) area✓

b. The sum of all the sides of a shape is called its ___.

- (i) perimeter✓ (ii) space
(iii) width (iv) area

c. The formula to find the area of a square is ___.

- (i) $4L$ (ii) $L + B$
(iii) $2(L + B)$ (iv) $L \times L$ ✓

d. ___ is the shortest distance between the top and bottom of a geometrical figure.

- (i) base (ii) altitude✓
(iii) top (iv) vertex

e. The formula to find the area of a trapezium is ___.

- (i) $a + \left(\frac{b_1 + b_2}{2}\right)$ (ii) $a \times \left(\frac{b_1 - b_2}{2}\right)$
(iii) $a - \left(\frac{b_1 + b_2}{2}\right)$ (iv) $a \times \left(\frac{b_1 + b_2}{2}\right)$ ✓

f. The formula for finding the area of a triangle is ___.

- (i) $\frac{1}{2}(b + h)$ (ii) $\frac{1}{2}(b \times h)$
(iii) $\frac{1}{2}(b \times h)$ ✓ (iv) $\frac{1}{2}(b - h)$

g. Surface area of cube = ___

- i. 6 length^2 ✓ ii. length^3
iii. 4 length^3 iv. 8 length^3

h. Surface area of cuboid is ___

- i. $2[(l \times w) \times (l + h) \times (b + h)]$
ii. $4[(l \times b) + (l \times h) + (b \times h)]$
iii. $5[(l \times b) + (l \times h) + (b \times h)]$
iv. $2[l \times w + (l \times h) + (w \times h)]$ ✓

- i. Volume of a cuboids = ___
i. $L + W + h$ ii. $l \times W \times h$ ✓
iii. $l - w - h$ iv. $l + W + h$

Q2: Define the following terms:

a. **Perimeter of square and rectangle**

A square is a four sided 2-D shape having all sides equal in length. Its perimeter can be measure by adding all 4 sides. If L is the length of a side of the square, we can write the perimeter as:

$$\text{Perimeter of square} = L + L + L + L \\ = 4 \times L = 4L$$

Perimeter of Rectangle:

We know that length of opposite sides of a rectangle are the same. So, its perimeter can be measured by adding all 4 sides. If L is the length of a side of the rectangle, and W is the width, we can write the perimeter as:

$$\begin{aligned} \text{Perimeter of a rectangle} \\ &= \text{length} + \text{length} + \text{width} + \text{width} \\ &= 2 \text{ length} + 2 \text{ width} \\ &= 2(\text{length} + \text{width}) \\ &= 2(L + W) \end{aligned}$$

b. **Area of Square:**

We can find the area of the square by multiplying its length and width.

$$\begin{aligned} \text{Area of square} &= \text{length} \times \text{length} \\ &= L \times L \end{aligned}$$

Area of Rectangle:

The total surface covered by a rectangle is called its area. The area of the rectangle is the product of their length and width.

$$\text{Area of rectangle} = \text{length} \times \text{width}$$

Altitude:

The altitude of any shape is the shortest distance between the top or vertex of the shape to the opposite side or base.

Q3: Find the perimeter and area of the squared shaped room whose length is 34 cm.

Solution:

$$\text{Given length} = l = 34 \text{ cm}$$

$$\text{Perimeter of square} = ?$$

$$\text{Area of square} = ?$$

$$\begin{aligned} \text{As perimeter of square} &= 4 \times \text{length} \\ &= 4 \times 34 \\ &= 136 \text{ cm} \end{aligned}$$

And

$$\begin{aligned} \text{Area of square} &= \text{length} \times \text{length} \\ &= 34 \times 34 \\ &= 1156 \text{ cm}^2 \end{aligned}$$

Q4: Find the perimeter and area of the rectangular pond whose length is 98 m and width is 88 m.

Solution:

$$\text{Given length} = l = 98 \text{ m}$$

$$\text{width} = w = 88 \text{ m}$$

As

$$\begin{aligned} \text{Area of rectangle} &= l \times w \\ &= 98 \times 88 \\ &= 8624 \text{ cm}^2 \end{aligned}$$

Now

$$\begin{aligned} \text{Perimeter of Rectangle} &= 2(l + w) \\ &= 2(98 + 88) \\ &= 372 \text{ cm} \end{aligned}$$

Q5: Find the length of the playground if its perimeter is 1250 cm and cost to construct a boundary around the play ground at the rate of Rs 90 per meter.

Solution:

$$\text{Given perimeter} = 1250 \text{ cm}$$

$$\text{length} = ?$$

$$\text{As perimeter} = 4 \times L$$

$$1250 = 4 \times L$$

$$\Rightarrow L = \frac{1250}{4} = 312.5 \text{ cm}$$

$$\begin{aligned} \text{As perimeter} &= 1250 \text{ cm} = \frac{1250}{100} \text{ m} \\ &= 12.50 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{As Rate of construction per meter} &= \\ \text{Rs. 90} \end{aligned}$$

$$\begin{aligned} \text{Total cost of } 12.50 \text{ m} &= 90 \times 12.50 \\ &= \text{Rs. 1125} \end{aligned}$$

Q6: Find the length of squared shaped bed sheet if its perimeter is 14 cm.

Solution:

$$\text{Length of square shaped bed} = ?$$

$$\text{Perimeter} = 14 \text{ cm}$$

$$\text{As perimeter of square} = 4 \times \text{length}$$

$$\Rightarrow 14 = 4 \times \text{length}$$

$$\begin{aligned} \Rightarrow \text{length} &= \frac{14}{4} \\ &= 3.5 \text{ cm} \end{aligned}$$

Q7: Find the length of rectangular hall if its area is 980 m² if width is 20 m. Find the cost of cementing the room at the rate of Rs. 420 per cm².

Solution:

$$\text{Length of rectangle} = ?$$

$$\text{Area} = 980 \text{ m}^2$$

$$\text{Width} = w = 20 \text{ m}$$

As

$$\text{Area} = \text{length} \times \text{width}$$

$$980 = \text{length} \times (20)$$

$$\text{Length} = \frac{980}{20} = 49 \text{ m}$$

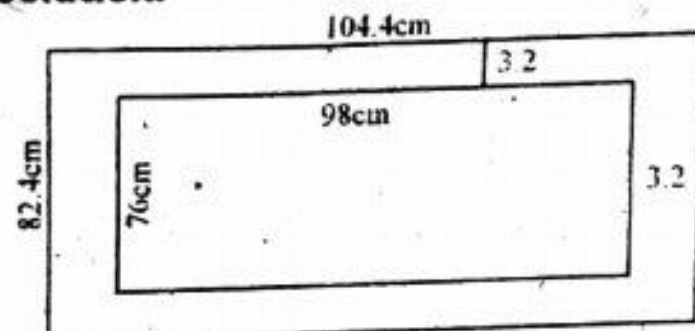
$$\text{As Rate per cm}^2 = \text{Rs. 420}$$

$$\text{So cost} = \text{Rs. } 420 \times 98 \text{ m}^2$$

Total cost = Rs. 41,160

Q8: A pond is 98 cm long and 78cm wide. A road 3.2cm wide is made all around outside the pool. Find the area of the road.

Solution:



Length of pond = 98cm,

Width = 76cm

Area of pond = 98×76
= 7448 cm^2

Length of pond with road = 104.4cm

Width of pond with road = 82.4cm

Area of pond with road = 104.4×82.4
= 8602.56 cm^2

Now area of the road

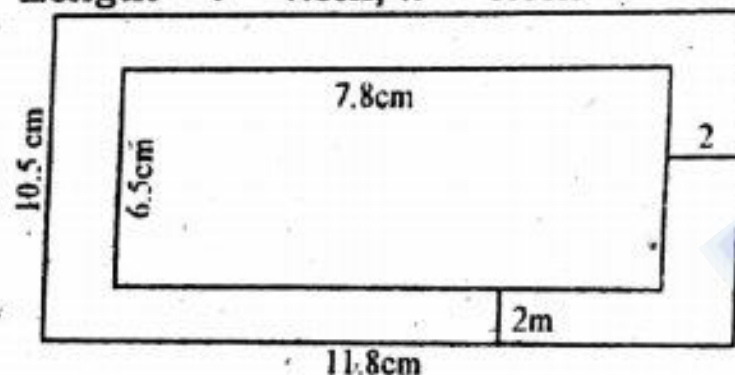
= $8602.56 - 7448$

= 1154.56 cm^2

Q9: A room is 7.8cm long and 6.5cm wide. Aliza wants to carpet 2 cm wide all round inside the room. Find the area of the carpeting part of the room.

Solution:

Length = $l = 7.8\text{m}$, $w = 6.5\text{m}$



Area of room = length $\times$ width

= 7.8×6.5

= 50.7 cm^2

Length of room with carpet = 11.8×10.5

= 123.9 cm^2

So

Area of carpeting part of the room

= $123.9 - 50.7$

= 73.2 cm^2

Q10: Find the altitude of a trapezium if the length of two bases are 34.56 and 32.12 cm respectively. The area of the trapezium is 482 cm^2 .

Solution:

Given

Length of base 1 = $b_1 = 34.56 \text{ cm}$

Length of base 2 = $b_2 = 32.12 \text{ cm}$

Area of trapezium = 482 cm^2

Altitude = ?

As

Area of trapezium = Altitude $\times \frac{b_1 + b_2}{2}$

$482 = \text{Altitude} \times \left(\frac{66.68}{2} \right)$

$482 = \text{Altitude} \times (33.34)$

Altitude = $\frac{482}{33.34} = 14.46 \text{ cm}$

Q11: Find the area of a trapezium if the length of two bases are 44cm and 66cm respectively. The perpendicular distance between two bases are 52cm.

Solution:

Given length of base 1 = $b_1 = 44 \text{ cm}$

Length of base 2 = $b_2 = 66 \text{ cm}$

Altitude = 52cm

Area of trapezium = ?

Area of trapezium = Altitude $\times \frac{b_1 + b_2}{2}$

= $52 \times \frac{44 + 66}{2}$

= $52 \times \frac{110}{2}$

= $52 \times 55 = 2860 \text{ cm}^2$

Q12: Write the names and write the properties of each of the given shape.

Shape	Circular edges	Straight edges	Vertices	Circular faces	Flat faces
Pyramid	1	0	1	1	1
Cuboid	0	12	8	0	06
Cube	0	12	8	0	06
Cylinder	02	0	0	1	02
Sphere	0	0	0	1	0
Triangular prism	1	0	0	1	0

Q13: Find the altitude of the triangular field. If its area is 99 cm^2 and base is 9cm.

Solution:

Altitude = ? , Area = 99 cm^2

Base = 9cm

As Area of triangular region = $\frac{1}{2}$
(altitude $\times$ Base)

Area of triangular region = $\frac{1}{2}$ (Altitude $\times$ Base)

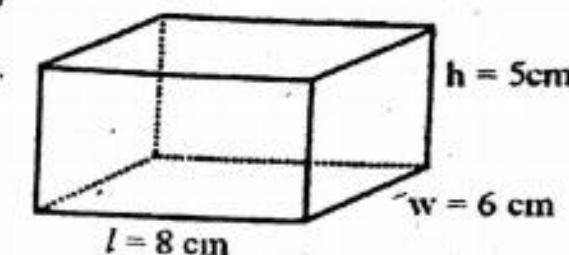
= $99 = \frac{1}{2}$ (Altitude) $\times 9$

Altitude = $\frac{2 \times 99}{9} = 2 \times 11$

Altitude = 22 cm

Q14: Find the volume and surface area of the following cube.

a)



Solution:

$l = 8 \text{ cm}$, $w = 6 \text{ cm}$, $h = 5 \text{ cm}$

As volume = $l \times w \times h$

= 240 cm^3

And

Surface Area = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

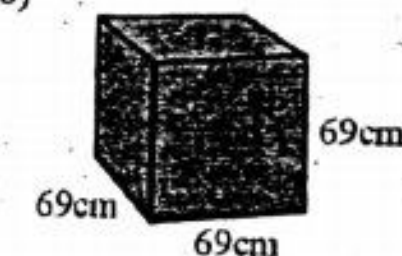
= $2 \times [(8 \times 6) + (8 \times 5) + (6 \times 5)]$

= $2 \times (48 + 40 + 30)$

Surface area = 2×118

= 236 cm^2

b)



Solution:

Length = $l = 69 \text{ cm}$

As

Volume of cube = $l^3 = (69)^3$

= $69 \times 69 \times 69$

= $328,509 \text{ cm}^3$

As

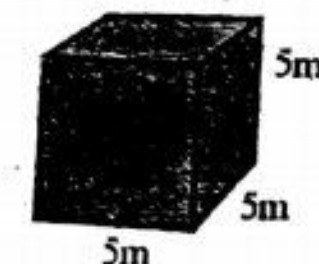
Surface Area = $6 \times (l)^2$

= $6 \times (69)^2$

= $6 \times 69 \times 69$

= $28,566 \text{ cm}^2$

c)



Solution:

Length = $l = 5 \text{ m}$

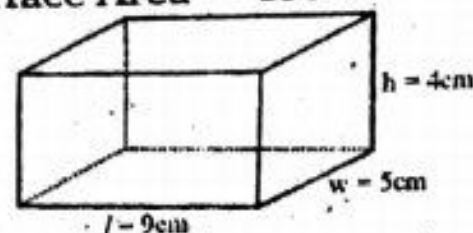
As volume of cube = l^3
 $= 5^3 = 125\text{m}^3$

And

$$\begin{aligned}\text{Surface Area} &= 6 \times (\text{length})^2 \\ &= 6 \times 5^2 \\ &= 6 \times 25\end{aligned}$$

$$\text{Surface Area} = 150\text{m}^2$$

d)



Solution:

$$\begin{aligned}l &= 9\text{cm}, w = 5\text{cm}, h = 4\text{cm} \\ \text{Volume of cuboid} &= 9 \times 5 \times 4 \\ &= 180\text{cm}^3\end{aligned}$$

And

$$\begin{aligned}\text{Surface Area} &= 2 \times [(l \times w) + (l \times h) + (w \times h)] \\ &= 2 \times [(9 \times 5) + (9 \times 4) + (5 \times 4)] \\ &= 2 \times [45 + 36 + 20] \\ &= 2 \times 101\end{aligned}$$

$$\text{Surface Area} = 202\text{cm}^2$$

Q15: Find the total surface area of the block whose length is 9cm, width 2.3cm and height is 3.3cm.

Solution:

$$\begin{aligned}\text{Length} = l &= 9\text{cm}, w = 2.3\text{cm}, \\ h &= 3.3\text{cm}\end{aligned}$$

As

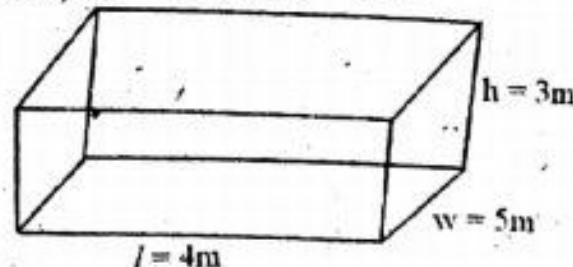
$$\begin{aligned}\text{Surface area} &= 2 \times [(l \times w) + (l \times h) + (w \times h)] \\ &= 2 \times [(9 \times 2.3) + (9 \times 3.3) + (2.3 \times 3.3)] \\ &= 2 \times (20.7 + 29.7 + 7.59) \\ &= 2 \times (57.99)\end{aligned}$$

$$\text{Surface Area} = 115.98\text{cm}^2$$

Q16: A classroom is 4m long, 5m wide and 3m high. Find the cost of whitewash the all four walls and ceiling the room at the rate of Rs 43 per m^2 .

Solution:

$$l = 4\text{m}, w = 5\text{m}, h = 3\text{m}$$



$$\begin{aligned}\text{Now area of four walls} &= l \times h + w \times h \\ &+ l \times h + w \times h \\ &= 2 \times (l \times h) + 2 \times (w \times h) \\ &= 2 [l \times h + w \times h]\end{aligned}$$

By putting values, so

$$\begin{aligned}\text{Area of four walls} &= 2 [4 \times 3 + 5 \times 3] \\ &= 2 \times 9 \times 3 \\ &= 54\text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Now area of ceiling} &= l \times w \\ &= 4 \times 5 \\ &= 20\text{m}^2\end{aligned}$$

$$\begin{aligned}\text{Total area of four walls and area of ceiling} &= 54 + 20 = 74\text{m}^2\end{aligned}$$

$$\text{Rate of per m}^2 = \text{Rs. } 43$$

So

$$\begin{aligned}\text{Total cost} &= 74 \times 43 \\ &= \text{Rs. } 3182\end{aligned}$$

Q17: A water tank is cuboids in shape. The length, width and height of the tank are 3.5m, 4.2m and 5.3m respectively. If we completely fill it with water, find the volume of water in the tank in liter?

Solution:

$$\begin{aligned}l &= 3.5\text{m}, w = 4.2\text{m}, h = 5.3\text{m}, \\ \text{Volume of cuboid tank} &= l \times w \times h \\ &= 3.5 \times 4.2 \times 5.3 \\ &= 77.9\text{cm}^3\end{aligned}$$

Q18: Find the capacity of a can whose length, width and height are 54cm, 35cm and 56cm respectively.

Solution:

$$l = 54\text{cm}, w = 35\text{cm}, h = 56\text{cm}$$

To find capacity, we find the volume

$$\begin{aligned}\text{Volume} &= l \times w \times h \\ &= 54 \times 35 \times 56 \\ &= 105840\text{cm}^3\end{aligned}$$

Q19: The altitude of the parallelogram is 2.4 cm and its area is 84.4 cm^2 . Find the base of parallelogram.

Solution:

$$\text{Altitude} = 2.4\text{cm}, \text{Area} = 84.4\text{cm}^2$$

Base = ?

$$\text{As Area of parallelogram} = \text{Altitude} \times \text{Base}$$

$$84.4 = 2.4 \times \text{Base}$$

$$\text{Base} = \frac{84.4}{2.4}$$

$$\text{Base of parallelogram} = 35.16\text{cm}$$

Q20: Find the area of the triangle if the length of its base is 67.2cm and the length of its altitude is 56.2 cm.

Solution:

$$\text{Area of triangle} = ?$$

$$\text{Base} = 67.2\text{cm}, \text{Altitude} = 56.2\text{cm}$$

As

$$\text{Area of triangle} = \frac{1}{2} (\text{Altitude} \times \text{Base})$$

$$= \frac{1}{2} (67.2 \times 56.2)$$

$$= \frac{1}{2} (3776.64)$$

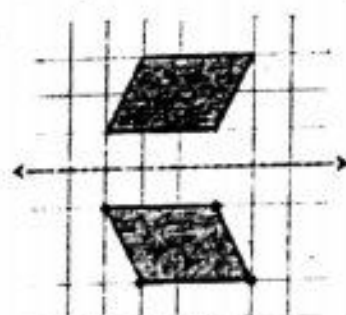
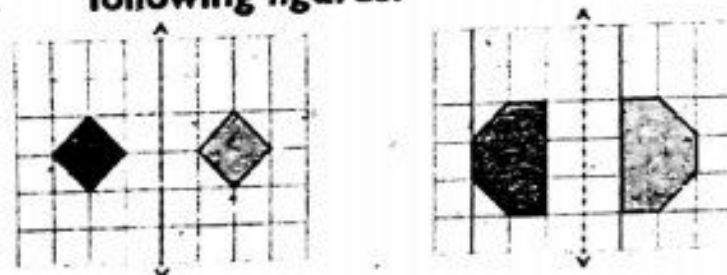
$$\text{Area of triangle} = 1888.32\text{cm}^2$$

Unit 9

Lines, Angles and Symmetry

Exercise 9.1

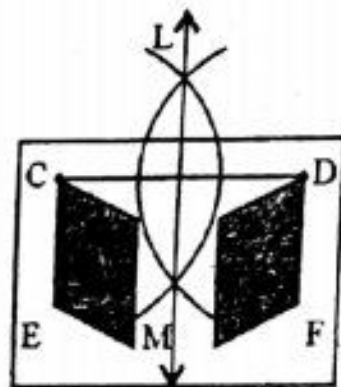
Q1: Find the perimeter of the following figures.



Q2: Find and draw the line of reflection for these images using compass.

Solution:

a.



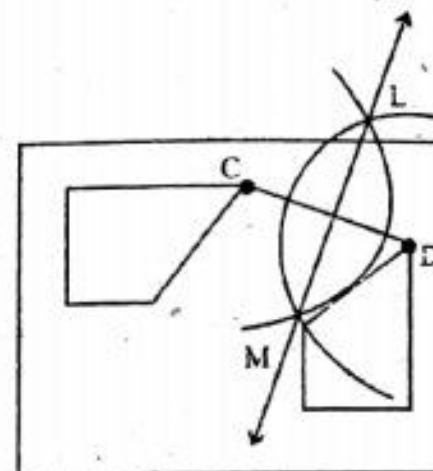
Step of construction:

- Join vertex C and D.
- Place the pointer of the compass at point C.
- Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.
- Similarly using the same opening of the compass, place

the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.

- Join the point L and M by a line using a ruler.

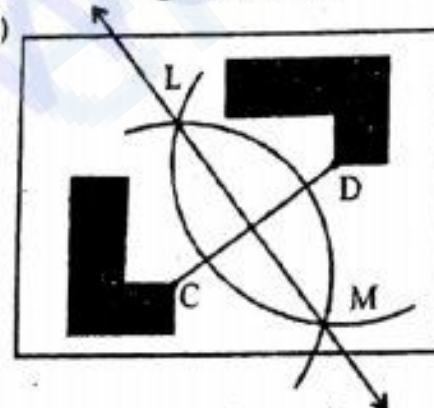
b.



Step of construction:

- Join vertex C and D.
- Place the pointer of the compass at point C.
- Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.
- Similarly using the same opening of the compass, place the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.
- Join the point L and M by a line using a ruler.

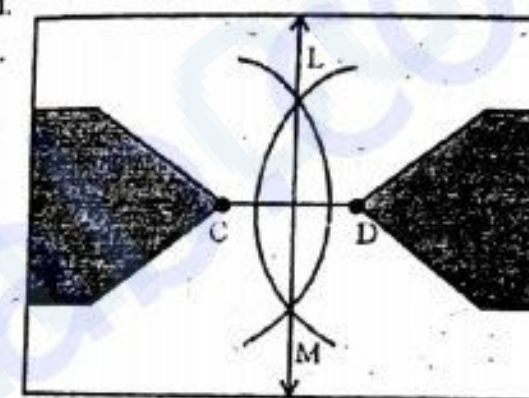
c)



Step of construction:

- Join vertex C and D.
- Place the pointer of the compass at point C.
- Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.
- Similarly using the same opening of the compass, place the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.
- Join the point L and M by a line using a ruler.

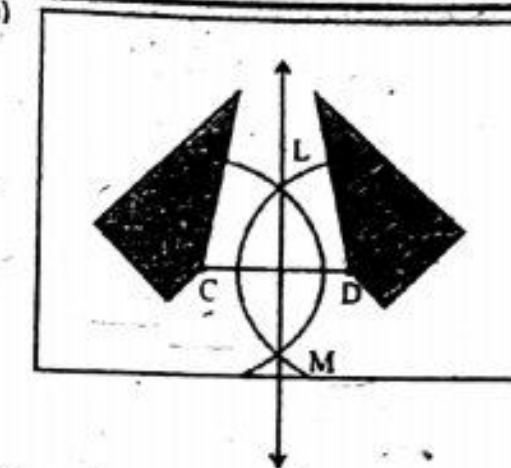
d.



Step of construction:

- Join vertex C and D.
- Place the pointer of the compass at point C.
- Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.
- Similarly using the same opening of the compass, place the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.
- Join the point L and M by a line using a ruler.

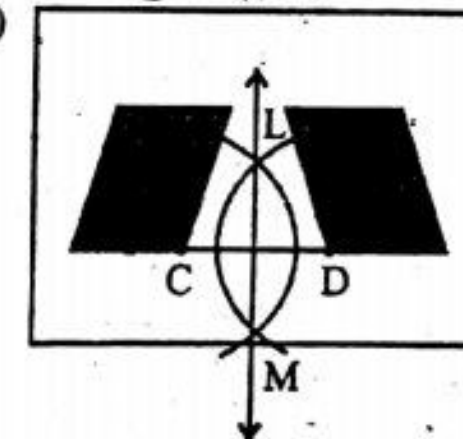
e)



Step of construction:

- Join vertex C and D.
- Place the pointer of the compass at point C.
- Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.
- Similarly using the same opening of the compass, place the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.
- Join the point L and M by a line using a ruler.

f)



Step of construction:

- Join vertex C and D.
- Place the pointer of the compass at point C.
- Open the compass with more than half of the measure of the

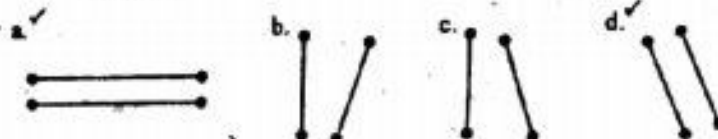
line segment CD and draw an arc on the top and bottom of CD.

- (iv) Similarly using the same opening of the compass, place the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.

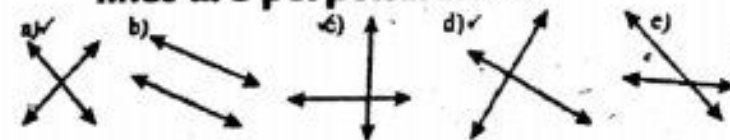
- (v) Join the point L and M by a line using a ruler.

Exercise 9.2

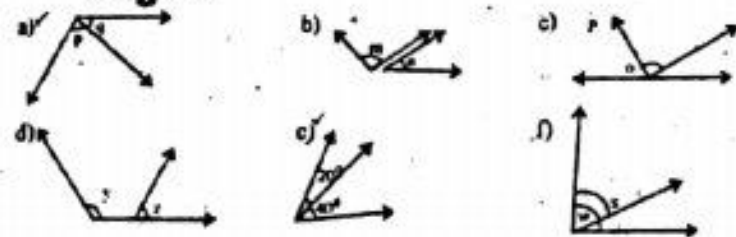
Q1: Identify and tick (✓) the parallel lines.



Q2: Determine which of these pairs of lines are perpendicular.



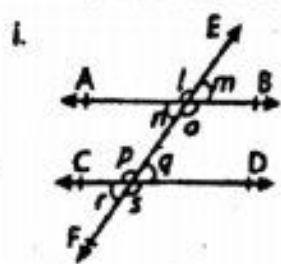
Q3: Tick (✓) the pair of adjacent angles.



Exercise 9.3

Q1: Two parallel lines are intersected by a transversal, as shown in the given figures: observe these figures and write answers of the following questions:

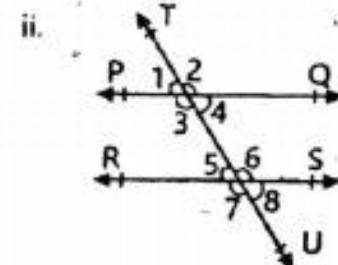
- a. Write names of parallel lines and a transversal.



Solution:

Parallel lines: - $\overline{AB}$ and $\overline{CD}$

Transversal: - $\overline{EF}$



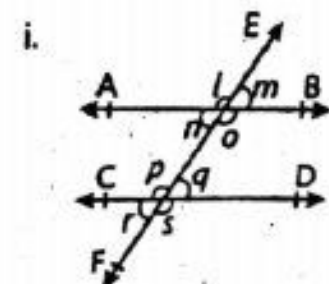
Solution:

Parallel lines: - $\overline{PQ}$ and $\overline{RS}$

Transversal: - $\overline{TU}$

b. Write names of pairs of:

- Corresponding angles.
- Alternate interior angles.
- Alternate exterior angles.

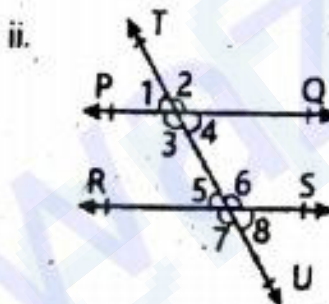


Solution:

Corresponding angles: (l,p), (n,r), (m,q) and (o,s)

Alternate interior angles: (n,e), (o,p)

Alternate exterior angles: (l,s), (m,r)



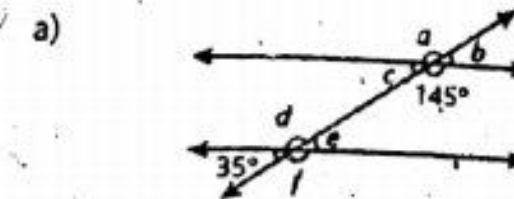
Solution:

Corresponding angles: (1,5), (3,7), (2,6), (4,8)

Alternate interior angles: (3,6), (4,5)

Alternate exterior angles: (1,8), (2,7)

Q2: Using properties of parallel lines, and a transversal, find unknown angles of the following figures:



Solution:

As (a, 145°) are vertical angles, so $a = 145^\circ$

As $a + b = 180^\circ \Rightarrow 145^\circ + b = 180^\circ$

$b = 180^\circ - 145^\circ = 35^\circ$

As (b, c) are vertical angles so $c = 35^\circ$

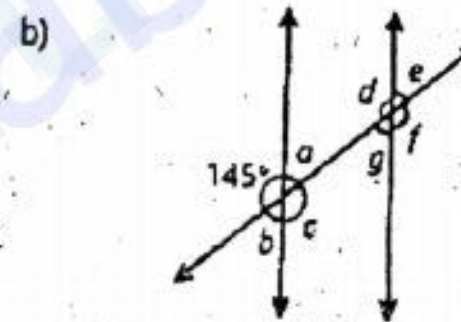
As $c = 35^\circ$ so $e = 35^\circ$ (because c and e are alternating angles)

$d = 145^\circ$ (because is alternate angle of 145°)

As $d = 145^\circ \Rightarrow f = 145^\circ$ (because d and f are vertical angles)

So $b = c = e = 35^\circ$

and $a = f = d = 145^\circ$



Solution:

As (c, 145°) are vertical angles, so $a = 145^\circ$

As (c, d) are vertical angles, so $f = 145^\circ$

As $\angle f, \angle e = 180^\circ \Rightarrow \angle e = 180^\circ - 145^\circ$ so $e = 35^\circ$

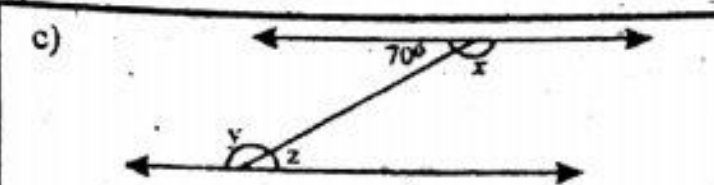
As (e, g) are vertical angles, so $g = 35^\circ$

As (g, a) are alternate angles, so $a = 35^\circ$

As (a, b) are vertical angles, so $b = 35^\circ$

So $c = d = f = 145^\circ$

and $g = a = b = e = 35^\circ$



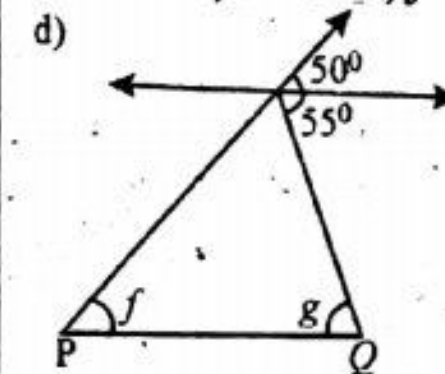
Solution:

As (70°, z) are alternate angles, so $z = 70^\circ$

As $x + 70^\circ = 180^\circ \Rightarrow x = 180^\circ - 70^\circ$ $x = 110^\circ$

As (x, y) are alternate angles so $y = 110^\circ$

So $z = 70^\circ, x = 110^\circ, y = 110^\circ$



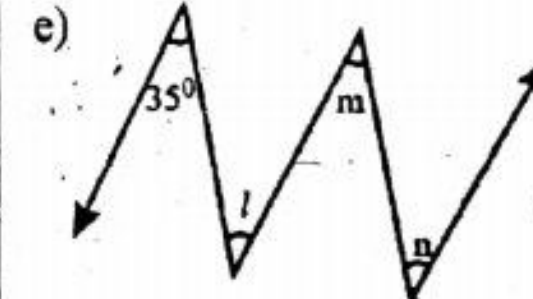
Solution:

As (50°, f) are corresponding angles so $f = 50^\circ$

and

(55°, g) are alternate angles, So $g = 55^\circ$

So $f = 50^\circ, g = 55^\circ$



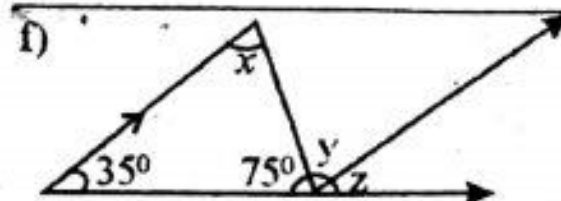
Solution:

As (35°, l) are alternate angles, so $l = 35^\circ$

As (l, m) are alternate angles, So $m = 35^\circ$

As (m, n) are alternate angles, So $n = 35^\circ$

So $l = m = n = 35^\circ$

**Solution:**

$$\text{As } 35^\circ + 75^\circ + x = 180^\circ$$

$$110^\circ + x = 180^\circ$$

$$x = 180^\circ - 110^\circ = 70^\circ$$

As (x, y) are alternate angles, so $y = 70^\circ$

A $75^\circ + y + z = 180^\circ$ (supplementary angles)

and $y = 70^\circ$, So

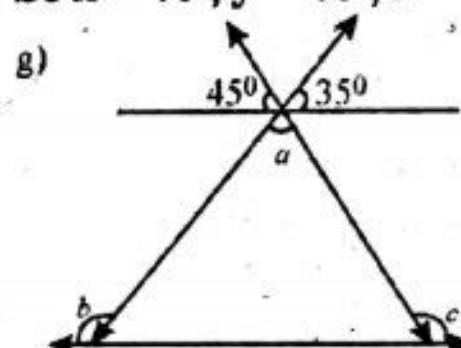
$$75^\circ + 70^\circ + z = 180^\circ$$

$$145^\circ + z = 180^\circ$$

$$\Rightarrow z = 180^\circ - 145^\circ$$

$$= 35$$

$$\text{So } x = 70^\circ, y = 70^\circ, z = 35^\circ$$

**Solution:**

As from figure

$$35^\circ + 45^\circ + a = 180^\circ$$

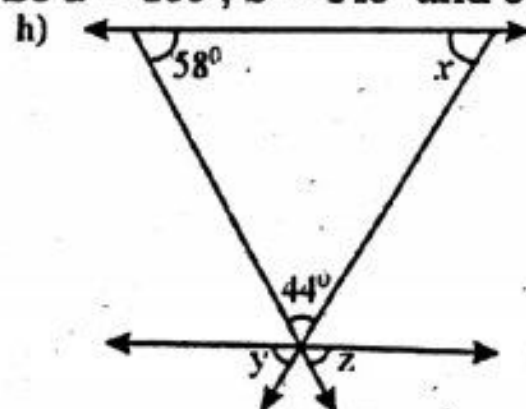
$$\Rightarrow 80^\circ + a = 180^\circ$$

$$\Rightarrow a = 180^\circ - 80^\circ$$

$$a = 100^\circ$$

and $b = 145^\circ, c = 135^\circ$.

So $a = 100^\circ, b = 145^\circ$ and $c = 135^\circ$

**Solution:**

As $58^\circ + 44^\circ + x = 180^\circ$ (Sum of angles of triangle)

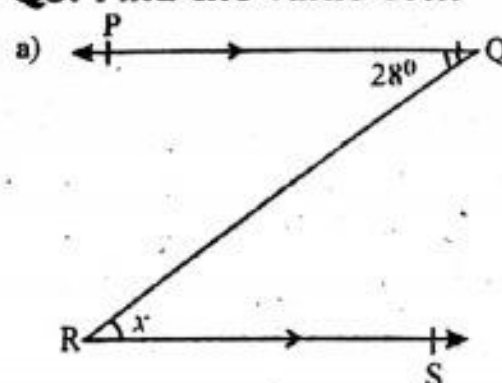
$$102^\circ + x = 180^\circ$$

$$x = 180^\circ - 102^\circ = 78^\circ$$

any $y = 78^\circ$ (because x and y are corresponding)

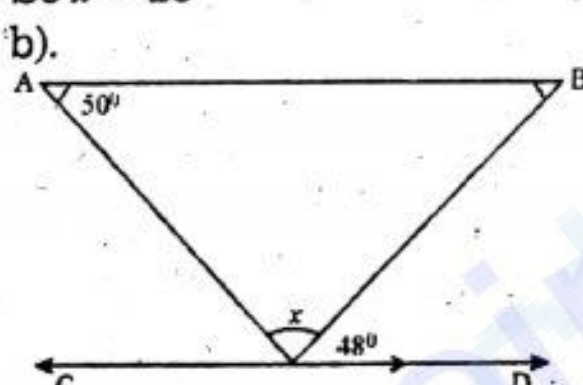
$z = 58^\circ$ (because 58° and z are compounding angles)

So, $x = 78^\circ, y = 78^\circ, z = 58^\circ$

Q3: Find the value of x .**Solution:**

As x and 28° are alternating interior angles.

$$\text{So } x = 28^\circ$$

**Solution:**

$$\text{As } 50^\circ + x + 48^\circ = 180^\circ$$

(Supplementary angles)

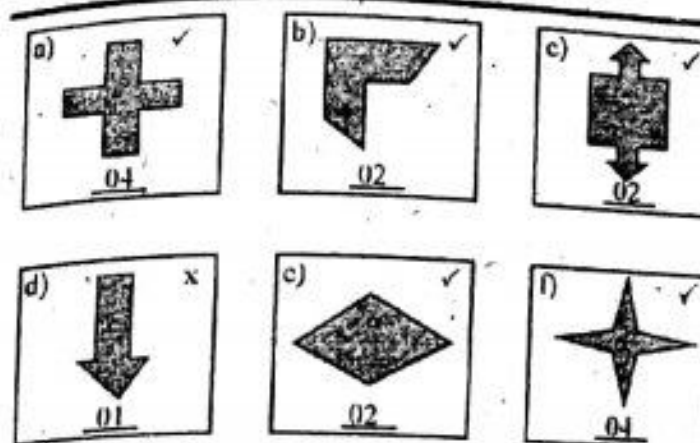
$$98^\circ + x = 180^\circ$$

$$\Rightarrow x = 180^\circ - 98^\circ$$

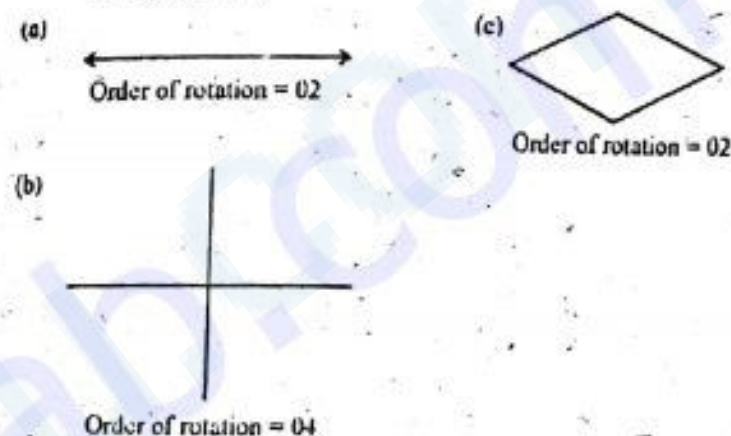
$$x = 82^\circ$$

Exercise 9.4

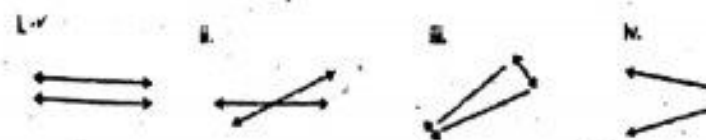
Q1: Tick (✓) the figures that have rotational symmetry. Write the order of rotation and mark the point of rotation for the symmetric figures.



Q2: Draw three shapes having rotational symmetry and mention the order of their rotational symmetry as well as point of rotation.

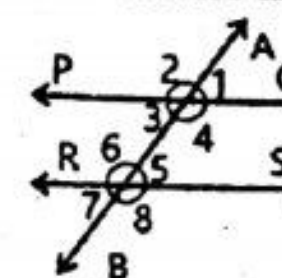
**Review Exercise****Q1: Choose the correct option.**

- A point has _____ dimensions.
i. 1 ii. 2 iii. 3 iv. 0✓
- A cuboids has _____ faces.
i. 8 ii. 6✓ iii. 5 iv. 3
- The pairs of lines that are cannot intersecting each other.

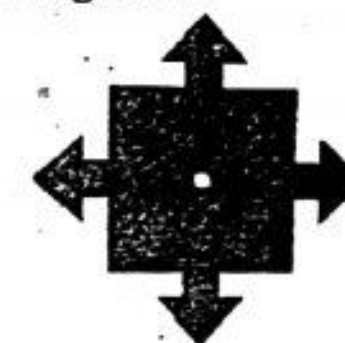


- If a line passes through two or more given lines at different points, it is called:
i. Alternate
ii. non-parallel
iii. parallel
iv. transversal ✓

e. The interior angles are:



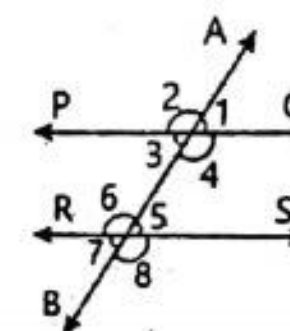
- $\angle 1, \angle 4, \angle 5, \angle 7$
- $\angle 3, \angle 4, \angle 5, \angle 6$ ✓
- $\angle 1, \angle 4, \angle 2, \angle 6$
- $\angle 3, \angle 4, \angle 5, \angle 8$
- The order of rotation of the given figure is:



- 3 ii. 1 iii. 0 iv. 4✓

Q2: Define the following.**a) Transversal:**

If a line passes through two or more given lines at different points, it is called a transversal.

**b) Corresponding angles:**

Any pair of angles each of which is on the same side of one of two lines cut by a transversal and on the same side of the transversal.

c) Alternate angles:

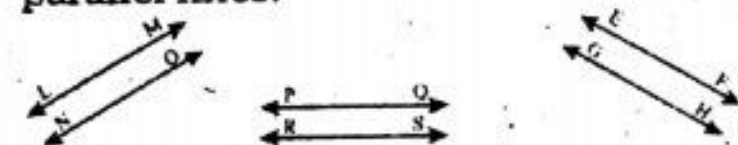
Two angles, formed when a line crosses two other lines, that lie on opposite sides of the transversal line

and on opposite relative sides of the other lines. If the two lines crossed are parallel, the alternate angles are equal.

d) Parallel lines:

The lines which do not intersect each other at a point, when they are extended on either side in the same direction, are called parallel lines.

In real life, we see many parallel lines. e.g. lines in our note-books, railway lines, etc. See more pairs of parallel lines.

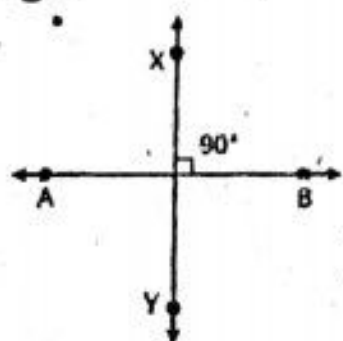


e) Perpendicular lines

The lines that intersect each other at 90° are called perpendicular lines.

Look at the hand of the clock, they are perpendicular to each other at 3 o'clock.

Similarly, the line XY and AB are perpendicular to each other as they are intersecting each other at right angle.

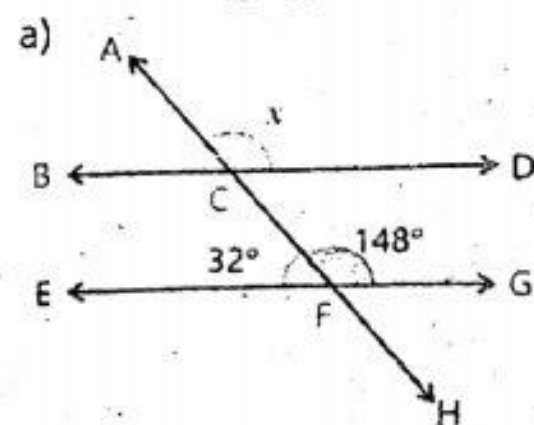


Q3: Differentiate between parallel lines, perpendicular lines and transversals.

Solution:

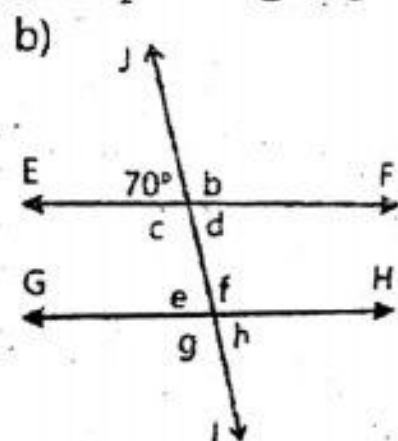
Given in Q2

Q4: Find unknown angles of the following figures.



Solution:

As $(x, 148)$ are pair of corresponding angles, so $x = 148^\circ$



Solution:

As 'd' is vertical angle of 70° , so $d = 70^\circ$.

As $70^\circ + b = 180^\circ$ (supplementary angles)

So $b = 180^\circ - 70^\circ = 110^\circ$

As C is vertical angle of $b = 110^\circ$,

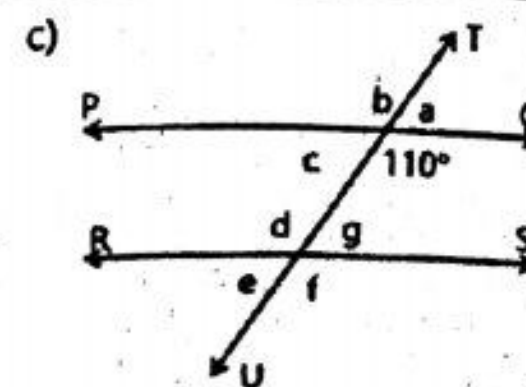
So $C = 110^\circ$

$f = 110^\circ$ (because f is corresponding angle of b).

$g = 110^\circ$ (because g and f are vertical angle)

$e = 70^\circ$ (because e is corresponding angle of 70°)

$h = 70^\circ$ (as h and e are vertical angles)



Solution:

As 'b' is vertical angle of 110° so $b = 110^\circ$.

As $a + b = 180^\circ$ (because a, b are supplementary angles so $a = 70^\circ$)

$c = 70^\circ$ (because a and c are vertical)

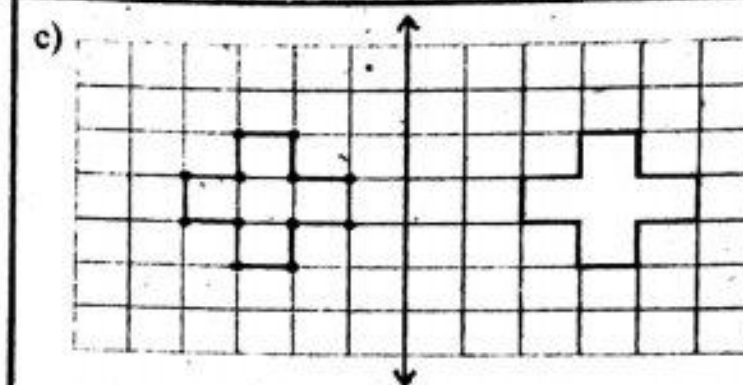
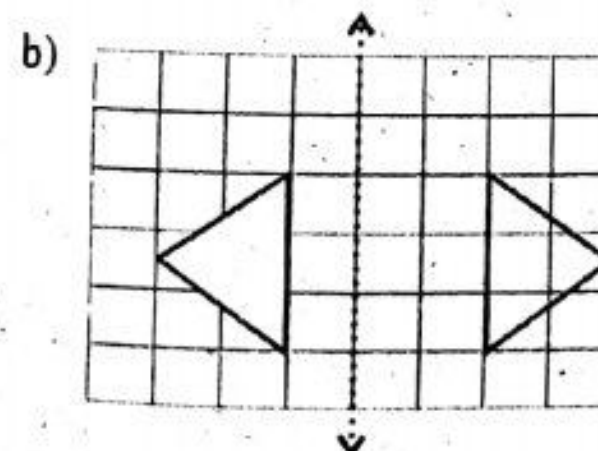
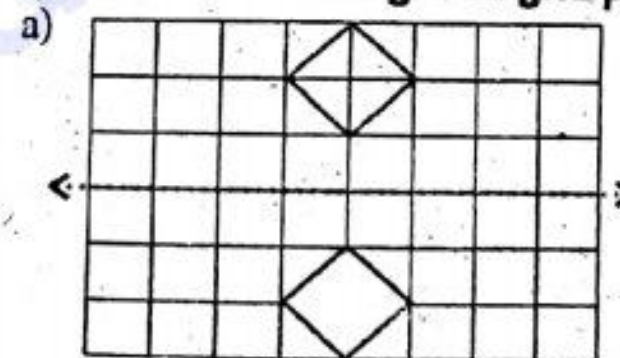
$d = 110^\circ$ (because d is corresponding angle of b)

$f = 110^\circ$ (because f is vertical angle of d).

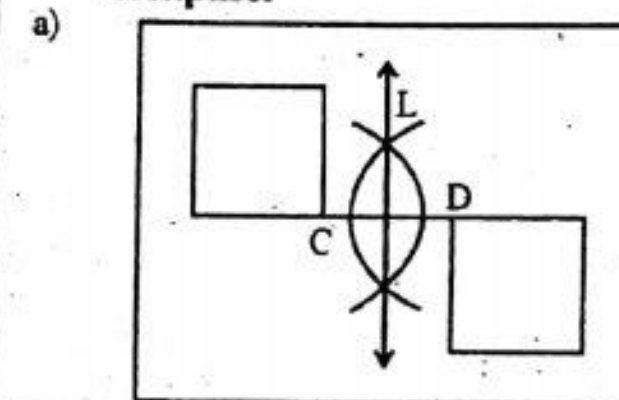
$e = 70^\circ$ (because e is alternate angle of a).

$g = 70^\circ$ (because 'g' is vertical angle of e).

Q5: Reflect the images on grid paper.

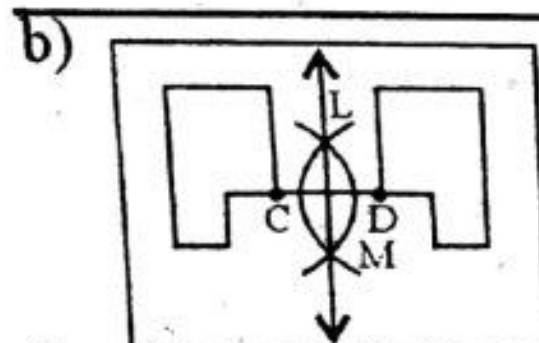


Q6: Find and draw the line of reflection for these images using compass.



Step of construction:

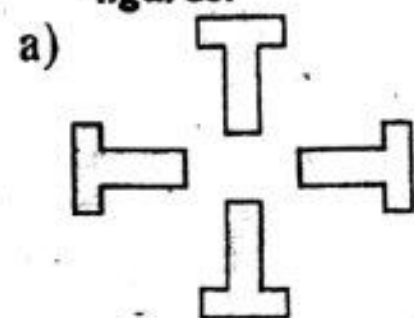
- (i) Join vertex C and D.
- (ii) Place the pointer of the compass at point C.
- (iii) Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.
- (iv) Similarly using the same opening of the compass, place the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.
- (v) Join the point L and M by a line using a ruler.



Step of construction:

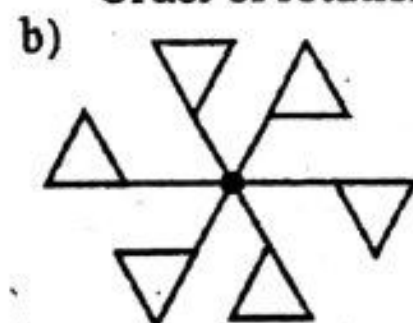
- Join vertex C and D.
- Place the pointer of the compass at point C.
- Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.
- Similarly using the same opening of the compass, place the pointer of the compass at point D and draw an arc on top and bottom of CD, which intersect the previous arc at point L and M.
- Join the point L and M by a line using a ruler.

Q7: Write the order of rotation of each figure and mark the point of rotation for the symmetric figures.



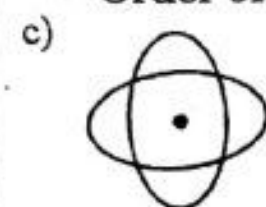
Solution:

Order of rotation = 04



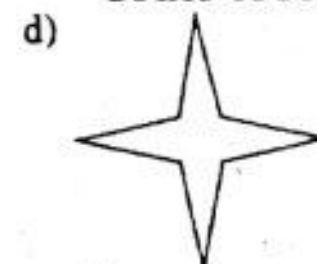
Solution:

Order of rotation = 04



Solution:

Order of rotation = 04



Solution:

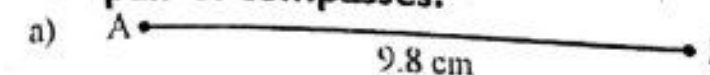
Order of rotation = 04

Unit 10

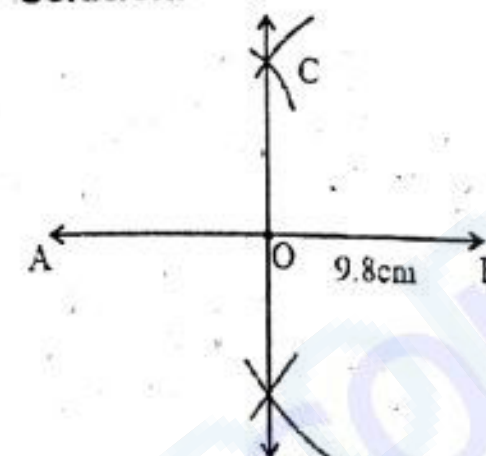
Geometrical Constructions

Exercise 10.1

Q1: Draw the right bisectors of the following line segments using a pair of compasses.



Solution:



Step of construction:

A line segment $\overline{AB}$ is given.

Step I:

Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment and draw $\overline{AB}$ an arc on the top and bottom of $\overline{AB}$.

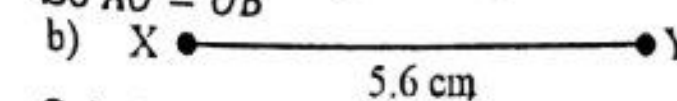
Step II:

Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arc on the top and bottom of $\overline{AB}$ which intersects the previous arcs at point C and D respectively.

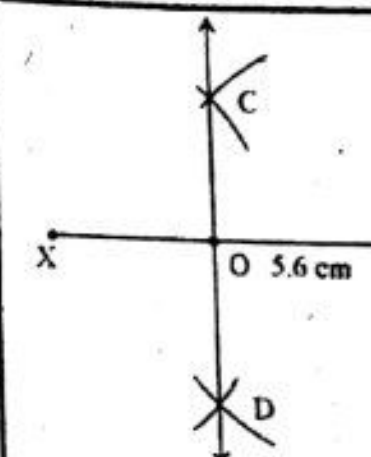
Step III:

Join the point C and D by a line using a ruler that will cut the line segment $\overline{AB}$ at point O. $\overline{CD}$ is the right bisector of $\overline{AB}$ and O is the midpoint.

So $\overline{AO} = \overline{OB}$



Solution:



Step of construction:

A line segment $\overline{XY}$ is given.

Step I:

Place the pointer of the compass at point X. Open the compass with more than half of the measure of the line segment and draw $\overline{XY}$ an arc on the top and bottom of $\overline{XY}$.

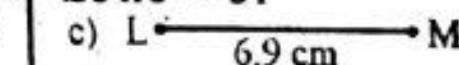
Step II:

Similarly, using the same opening of the compass, place the pointer of the compass at point Y and draw two arc on the top and bottom of $\overline{XY}$ which intersects the previous arcs at point C and D respectively.

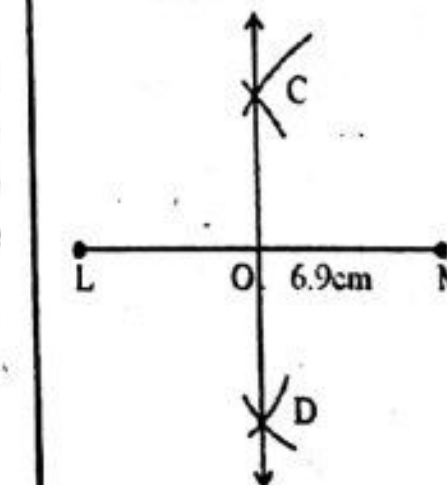
Step III:

Join the point C and D by a line using a ruler that will cut the line segment $\overline{XY}$ at point O. $\overline{CD}$ is the right bisector of $\overline{XY}$ and O is the midpoint.

So $\overline{XO} = \overline{OY}$



Solution:



Step of construction:

A line segment $\overline{LM}$ is given.

Step I:

Place the pointer of the compass at point L. Open the compass with more than half of the measure of the line segment and draw $\overline{LM}$ an arc on the top and bottom of $\overline{LM}$.

Step II:

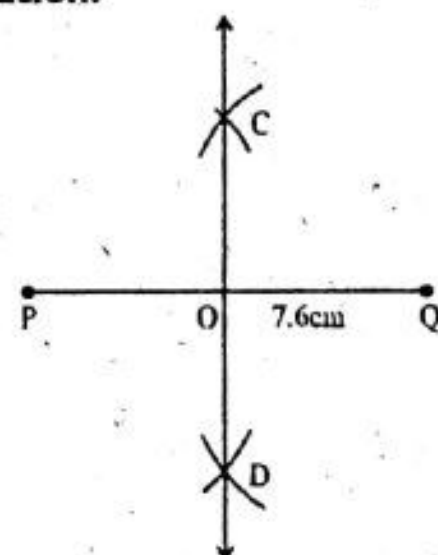
Similarly, using the same opening of the compass, place the pointer of the compass at point M and draw two arc on the top and bottom of $\overline{LM}$ which intersects the previous arcs at point C and D respectively.

Step III:

Join the point C and D by a line using a ruler that will cut the line segment $\overline{LM}$ at point O. $\overline{CD}$ is the right bisector of $\overline{LM}$ and O is the midpoint.

$$\text{So, } \overline{LO} = \overline{OM}$$

d) $\overline{PQ}$ 7.6cm

Solution:**Step of construction:**

A line segment $\overline{PQ}$ is given.

Step I:

Place the pointer of the compass at point P. Open the compass with more than half of the measure of the line segment and draw $\overline{PQ}$ an arc on the top and bottom of $\overline{PQ}$.

Step II:

Similarly, using the same opening of the compass, place the pointer of the compass at point Q and draw two arc on the top and bottom of $\overline{PQ}$ which intersects the previous arcs at point C and D respectively.

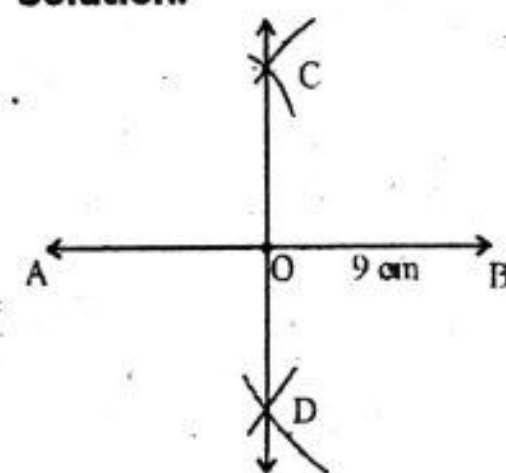
Step III:

Join the point C and D by a line using a ruler that will cut the line segment $\overline{PQ}$ at point O. $\overline{CD}$ is the right bisector of $\overline{PQ}$ and O is midpoint.

$$\text{So } \overline{PO} = \overline{OQ}$$

Q2: Draw the following line segments and bisect them.

a. 9cm

Solution:**Step of construction:**

A line segment $\overline{AB}$ is given.

Step I:

Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment and draw $\overline{AB}$ an arc on the top and bottom of $\overline{AB}$.

Step II:

Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arc on the top and bottom of $\overline{AB}$ which intersects the

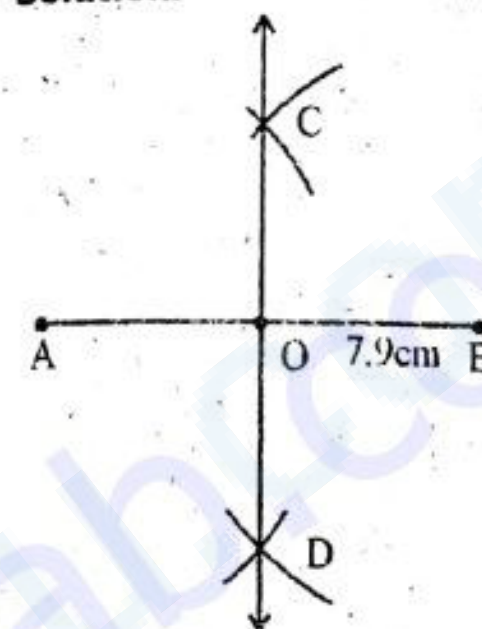
previous arcs at point C and D respectively.

Step III:

Join the point C and D by a line using a ruler that will cut the line segment $\overline{AB}$ at point O. $\overline{CD}$ is the right bisector of $\overline{AB}$ and O is midpoint.

$$\text{So } \overline{AO} = \overline{OB}$$

b. 7.9 cm

Solution:**Step of construction:**

A line segment $\overline{AB}$ is given.

Step I:

Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment and draw $\overline{AB}$ an arc on the top and bottom of $\overline{AB}$.

Step II:

Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arc on the top and bottom of $\overline{AB}$ which intersects the previous arcs at point C and D respectively.

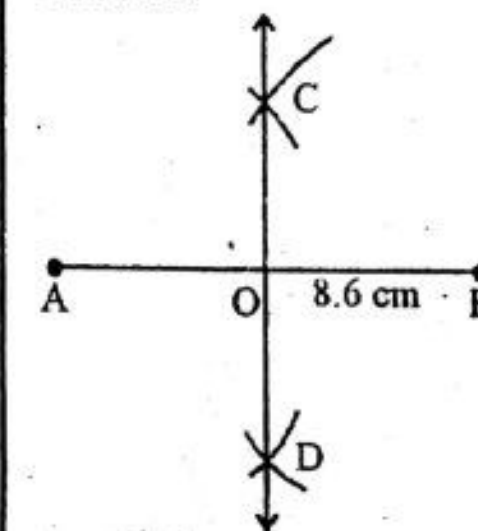
Step III:

Join the point C and D by a line using a ruler that will cut the line

segment $\overline{AB}$ at point O. $\overline{CD}$ is the right bisector of $\overline{AB}$ and O is midpoint.

$$\text{So } \overline{AO} = \overline{BO}$$

c. 8.6 cm

Solution:**Step of construction:**

A line segment $\overline{AB}$ is given.

Step I:

Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment and draw $\overline{AB}$ an arc on the top and bottom of $\overline{AB}$.

Step II:

Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arc on the top and bottom of $\overline{AB}$ which intersects the previous arcs at point C and D respectively.

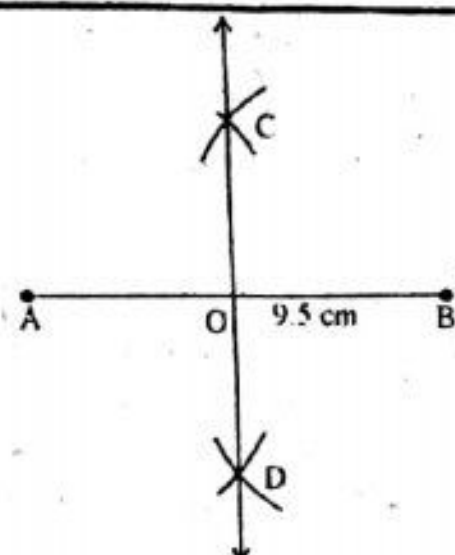
Step III:

Join the point C and D by a line using a ruler that will cut the line segment $\overline{AB}$ at point O. $\overline{CD}$ is the right bisector of $\overline{AB}$ and O is midpoint.

$$\text{So } \overline{AO} = \overline{BO}$$

d. 9.5 cm

Solution:

**Step of construction:**

A line segment $\overline{AB}$ is given.

Step I:

Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment and draw $\overline{AB}$ an arc on the top and bottom of $\overline{AB}$.

Step II:

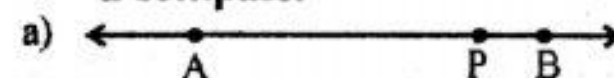
Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arc on the top and bottom of $\overline{AB}$ which intersects the previous arcs at point C and D respectively.

Step III:

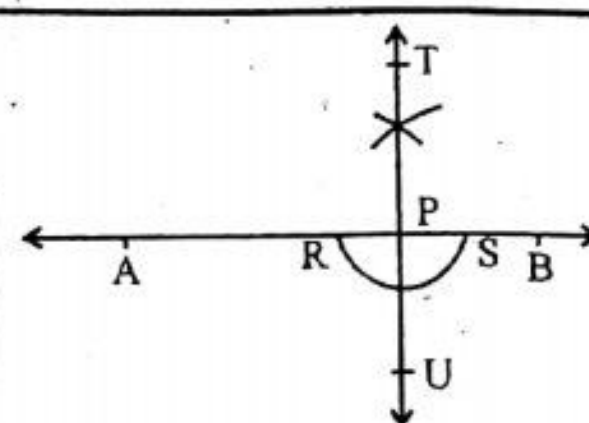
Join the point C and D by a line using a ruler that will cut the line segment $\overline{AB}$ at point O. $\overline{CD}$ is the right bisector of $\overline{AB}$ and O is midpoint.

So $\overline{AO} = \overline{BO}$

Q3: Draw perpendiculars to the lines from the given points on it using a compass.



Solution:

**Step of construction:**

$\overline{AB}$ is the given line and P is a point on it.

Step I:

Place the pointer of the compass at point P and draw an arc of suitable radius that will cut the line at $\overline{AB}$ at two points R and S respectively, such that $\overline{PR} = \overline{PS}$.

Step II:

Place the pointer of the compass at point R and draw an arc of radius greater than $\overline{PS}$ as shown in the figure.

Step III:

With the same opening of the compass, place the pointer of the compass at point S and draw an arc which cuts the previous arc at point T as shown in the figure.

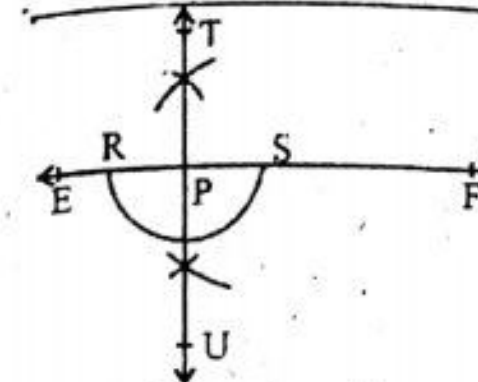
Step IV:

Join the point T to P and extend it to U. So, $\overline{TU}$ is the required perpendicular to the given line $\overline{AB}$ point P.

$\therefore \overline{TU} \perp \overline{PQ}$



Solution:

**Step of construction:**

$\overline{EF}$ is the given line and P is a point on it.

Step I:

Place the pointer of the compass at point P and draw an arc of suitable radius that will cut the line at $\overline{EF}$ at two points R and S respectively, such that $\overline{PR} = \overline{PS}$.

Step II:

Place the pointer of the compass at point R and draw an arc of radius greater than $\overline{PS}$ as shown in the figure.

Step III:

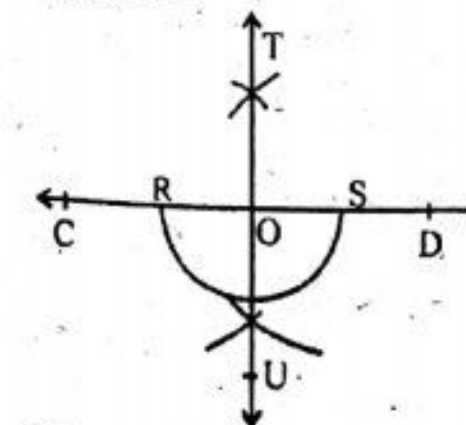
With the same opening of the compass, place the pointer of the compass at point S and draw an arc which cuts the previous arc at point T as shown in the figure.

Step IV:

Join the point T to P and extend it to U. So, $\overline{TU}$ is the required perpendicular to the given line $\overline{EF}$ at point P.



Solution:



Step of construction:

$\overline{CD}$ is the given line and O is a point on it.

Step I:

Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the line at $\overline{CD}$ at two points R and S respectively, such that $\overline{OR} = \overline{OS}$.

Step II:

Place the pointer of the compass at point R and draw an arc of radius greater than $\overline{OR}$ as shown in the figure.

Step III:

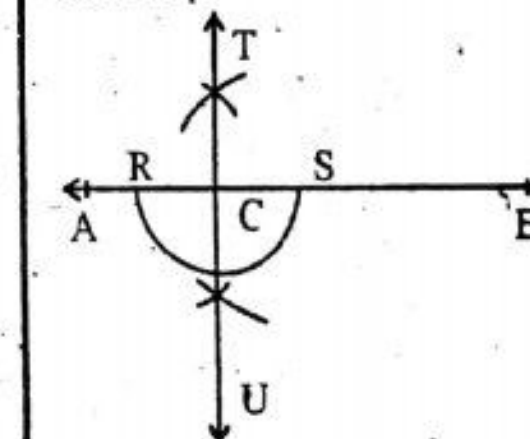
With the same opening of the compass, place the pointer of the compass at point S and draw an arc which cuts the previous arc at point T as shown in the figure.

Step IV:

Join the point T to O and extend it to U. So, $\overline{TU}$ is the required perpendicular to the given line $\overline{CD}$ at point O.



Solution:

**Step of construction:**

$\overline{AB}$ is the given line and C is a point on it.

Step I:

Place the pointer of the compass at point C and draw an arc of suitable

radius that will cut the line at $\overline{AB}$ at two points R and S respectively, such that $\overline{CR} = \overline{CS}$.

Step II:

Place the pointer of the compass at point R and draw an arc of radius greater than $\overline{CR}$ as shown in the figure.

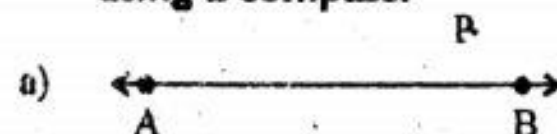
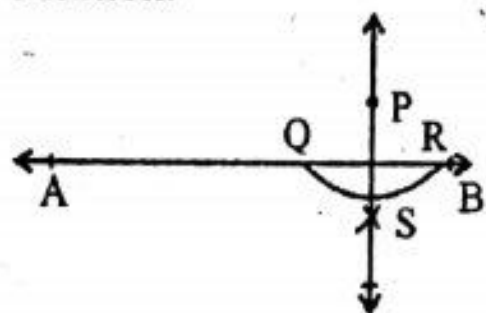
Step III:

With the same opening of the compass, place the pointer of the compass at point S and draw an arc which cuts the previous arc at point T as shown in the figure.

Step IV:

Join the point T to C and extend it to U. So, $\overline{TU}$ is the required perpendicular to the given line $\overline{AB}$ at point C.

Q4: Draw perpendiculars to the given lines from the point P not on it using a compass.

**Solution:****Step of construction:**

$\overline{AB}$ is the given line. A point P (not on it) is also given.

Step I:

Place the pointer of the compass at point P and draw an arc that will cut the line $\overline{AB}$ at two points Q and R respectively.

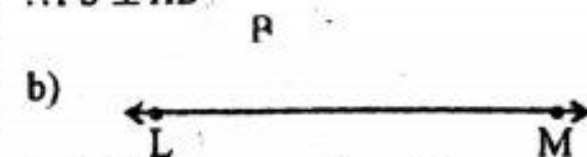
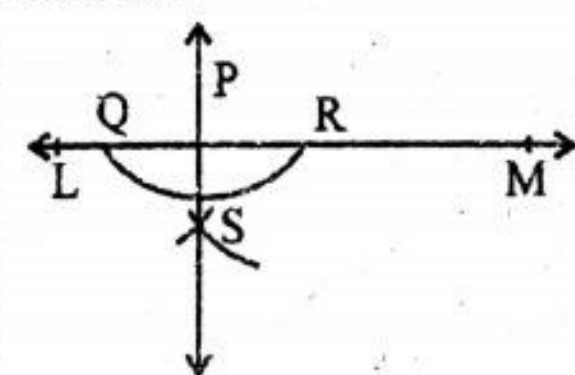
Step II:

Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overline{AB}$ that cut each other at point S.

Step III:

Join P to S. $\overline{PS}$ is the required perpendicular to the given line $\overline{AB}$.

$$\therefore \overline{PS} \perp \overline{AB}$$

**Solution:****Step of construction:**

$\overline{AB}$ is the given line. A point P (not on it) is also given.

Step I:

Place the pointer of the compass at point P and draw an arc that will cut the line $\overline{AB}$ at two points Q and R respectively.

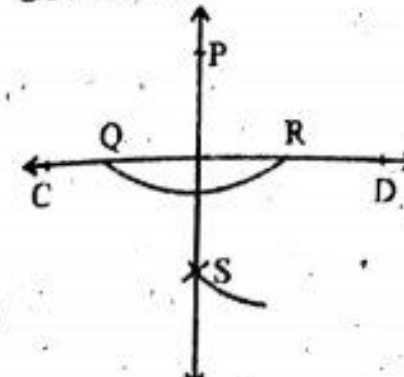
Step II:

Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overline{AB}$ that cut each other at point S.

Step III:

Join P to S. $\overline{PS}$ is the required perpendicular to the given line $\overline{AB}$.

$$\therefore \overline{PS} \perp \overline{AB}$$

**Solution:****Step of construction:**

$\overline{CD}$ is the given line. A point P (not on it) is also given.

Step I:

Place the pointer of the compass at point P and draw an arc that will cut the line $\overline{CD}$ at two points Q and R respectively.

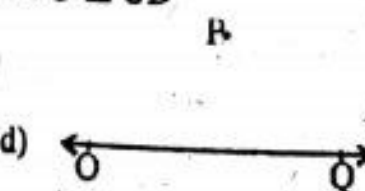
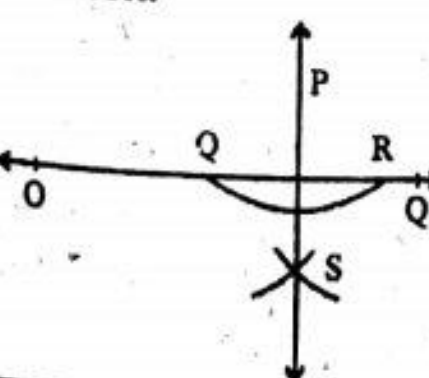
Step II:

Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overline{CD}$ that cut each other at point S.

Step III:

Join P to S. $\overline{PS}$ is the required perpendicular to the given line $\overline{CD}$.

$$\therefore \overline{PS} \perp \overline{CD}$$

**Solution:****Step of construction:**

$\overline{OQ}$ is the given line. A point P (not on it) is also given.

Step I:

Place the pointer of the compass at point P and draw an arc that will cut the line $\overline{OQ}$ at two points Q and R respectively.

Step II:

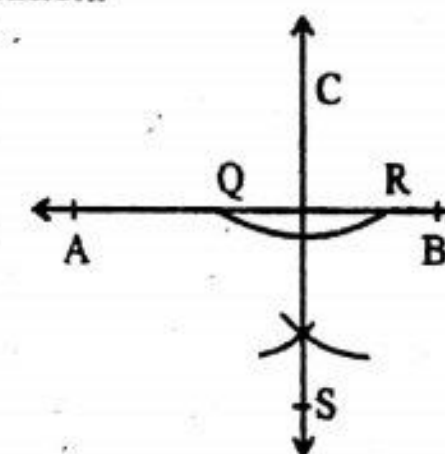
Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overline{OQ}$ that cut each other at point S.

Step III:

Join P to S. $\overline{PS}$ is the required perpendicular to the given line $\overline{OQ}$.

$$\therefore \overline{PS} \perp \overline{OQ}$$

Q5: Draw a line $\overline{AB}$ and draw a perpendicular on it from a point C not on it.

Solution:**Step of construction:**

$\overline{AB}$ is the given line. A point C (not on it) is also given.

Step I:

Place the pointer of the compass at point C and draw an arc that will cut the line $\overline{AB}$ at two points Q and R respectively.

Step II:

Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overline{AB}$ that cut each other at point S.

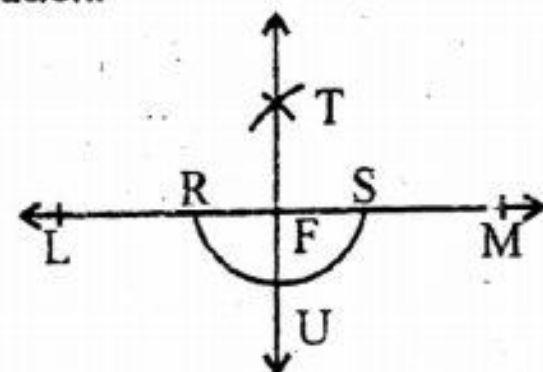
Step III:

Join C to S. $\overline{CS}$ is the required perpendicular to the given line $\overline{AB}$.

$\therefore \overline{CS} \perp \overline{AB}$

Q6: Draw a line $\overline{LM}$ and draw a perpendicular on it from point F on it.

Solution:



$\overline{LM}$ is the given line and F is a point on it.

Step I:

Place the pointer of the compass at point F and draw an arc of suitable radius that will cut the line $\overline{LM}$ at two points R and S respectively, such that $\overline{FR} = \overline{FS}$.

Step II:

Place the pointer of the compass at point R and draw an arc of radius greater than $\overline{FR}$ as shown in the figure.

Step III:

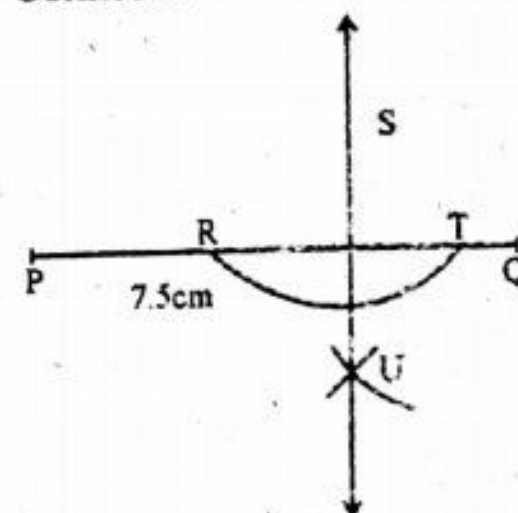
With the same opening of the compass, place the pointer of the compass at point S and draw an arc which cuts the previous arc at point T as shown in the figure.

Step IV:

Join the point T to F and extend it to U. So, $\overline{TU}$ is the required perpendicular to the given line $\overline{LM}$ at point F.

Q7: Draw a line segment $m\overline{PQ} = 7.5$ cm. Draw a perpendicular from point S not on it.

Solution:



Steps of Construction:

$\overline{PQ} = 7.5$ cm is the given line segment. A point S (not on it) is also given.

Step I:

Place the pointer of the compass at point P and draw an arc that will cut the line segment at P two points R and T respectively.

Step II:

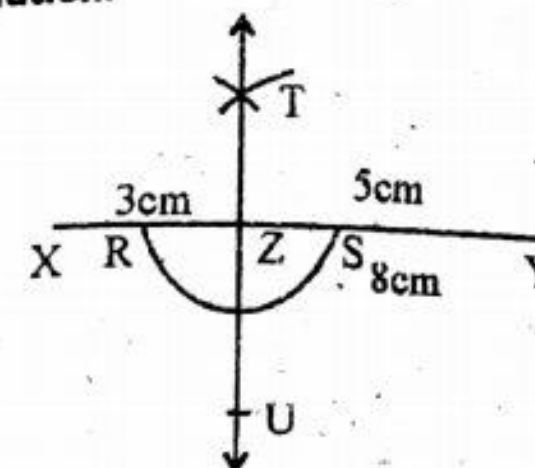
Using the same opening of the compass, place the pointer of the compass at point Q and draw two arcs on the other side of $\overline{PQ}$ that cut each other at point U.

Step III:

Join S to U. $\overline{SU}$ is the required perpendicular to given line segment $\overline{PQ}$.

Q8: Draw a line segment $m\overline{XY} = 8$ cm. Construct a perpendicular at point Z on it such that $\overline{XZ} = 3$.

Solution:



$\overline{XY}$ is the given line of measurement 8 cm and Z is a point on it.

Step I:

Place the pointer of the compass at point Z and draw an arc of suitable radius that will cut the line at line $\overline{XY}$ at two points R and S respectively, such that $\overline{ZR} = \overline{ZS}$.

Step II:

Place the pointer of the compass at point R and draw an arc of radius greater than $\overline{ZR}$ as shown in the figure.

Step III:

With the same opening of the compass, place the pointer of the compass at point S and draw an arc which cuts the previous arc of point T as shown in the figure.

Step IV:

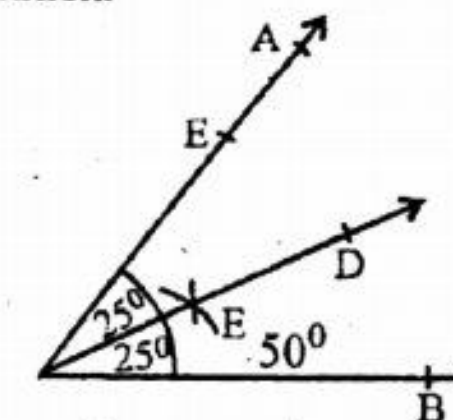
Join the point T to Z and extended it to U. So $\overline{TU}$ is the required perpendicular to the given line $\overline{XY}$ at point Z.

Exercise 10.2

Q1: Construct the following angles by using a protractor and bisect them by using a pair of compasses.

a. 50°

Solution:

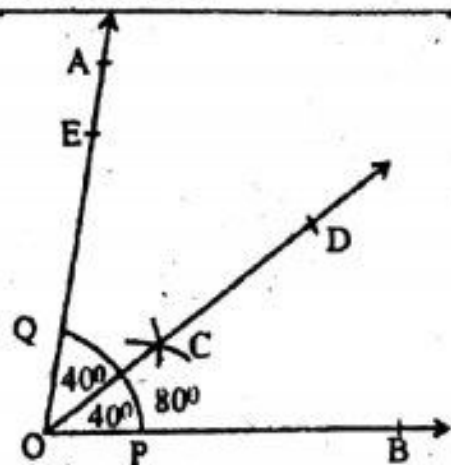


Steps of construction:

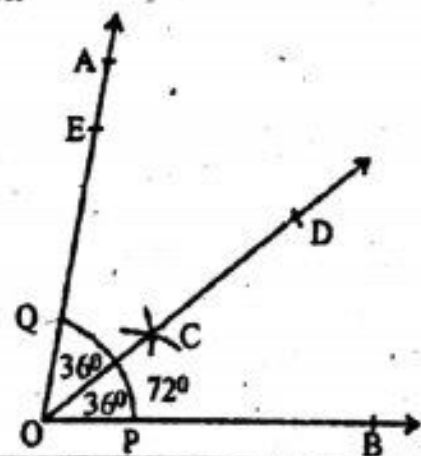
- Draw a ray $\overline{OB}$ using a ruler.
- Place protractor on point O and take a point 'E' which is coincide on 50° .
- Using a ruler draw a ray $\overline{OA}$ passing through the point E. So $\angle AOB = 50^\circ$.
- Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overline{OB}$ at P and cuts the ray $\overline{OA}$ at any point Q.
- Place the pointer of the compass at point P and Q and draw two arcs of suitable radius that will cut each other at point C.
- Using a ruler draw a ray $\overline{OD}$ passing through the point C, this line bisecting the angle $\angle AOB$ into two equal parts. So $m\angle AOC = m\angle BOC$.

b. 80°

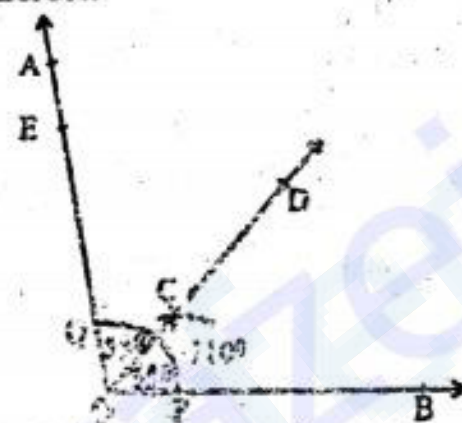
Solution:

**Steps of construction:**

- (i) Draw a ray $\overrightarrow{OB}$ using a ruler.
- (ii) Place protractor on point O and take a point 'E' which is coincide on 80° .
- (iii) Using a ruler draw a ray $\overrightarrow{OA}$ passing through the point E. So $\angle AOB = 80^\circ$.
- (iv) Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at P and cuts the ray $\overrightarrow{OA}$ at any point Q.
- (v) Place the pointer of the compass at point P and Q and draw two arcs of suitable rading that will cut each of point C.
- (vi) Using a ruler draw a ray $\overrightarrow{OD}$ passing through the point C, this line bisecting the angle $\angle AOB$ into two equal parts. So $m\angle AOC = m\angle BOC$.

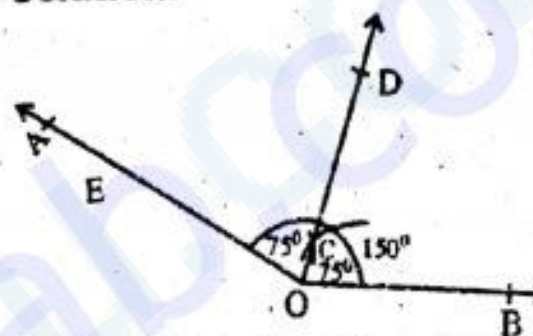
c. 72° **Solution:****Steps of construction:**

- (i) Draw a ray $\overrightarrow{OB}$ using a ruler.
- (ii) Place protractor on point O and take a point 'E' which is coincide on 72° .
- (iii) Using a ruler draw a ray $\overrightarrow{OA}$ passing through the point E. So $\angle AOB = 72^\circ$.
- (iv) Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at P and cuts the ray $\overrightarrow{OA}$ at any point Q.
- (v) Place the pointer of the compass at point P and Q and draw two arcs of suitable rading that will cut each of point C.
- (vi) Using a ruler draw a ray $\overrightarrow{OD}$ passing through the point C, this line bisecting the angle $\angle AOB$ into two equal parts. So $m\angle AOC = m\angle BOC$.

d. 110° **Solution:****Steps of construction:**

- (i) Draw a ray $\overrightarrow{OB}$ using a ruler.
- (ii) Place protractor on point O and take a point 'E' which is coincide on 110° .
- (iii) Using a ruler draw a ray $\overrightarrow{OA}$ passing through the point E. So $\angle AOB = 110^\circ$.

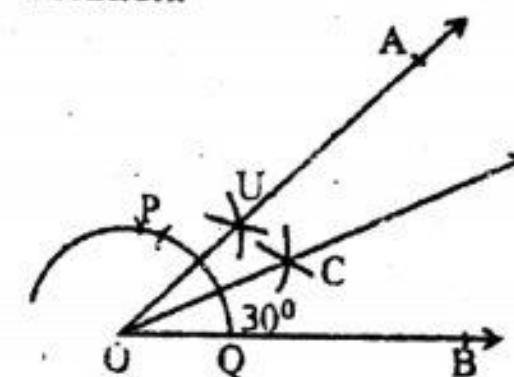
- (iv) Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at P and cuts the ray $\overrightarrow{OA}$ at any point Q.
- (v) Place the pointer of the compass at point P and Q and draw two arcs of suitable rading that will cut each of point C.
- (vi) Using a ruler draw a ray $\overrightarrow{OD}$ passing through the point C, this line bisecting the angle $\angle AOB$ into two equal parts. So $m\angle AOC = m\angle BOC$.

e. 150° **Solution:****Steps of construction:**

- (i) Draw a ray $\overrightarrow{OB}$ using a ruler.
- (ii) Place protractor on point O and take a point 'E' which is coincide on 150° .
- (iii) Using a ruler draw a ray $\overrightarrow{OA}$ passing through the point E. So $\angle AOB = 150^\circ$.
- (iv) Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at P and cuts the ray $\overrightarrow{OA}$ at any point Q.
- (v) Place the pointer of the compass at point P and Q and draw two arcs of suitable rading that will cut each of point C.
- (vi) Using a ruler draw a ray $\overrightarrow{OD}$ passing through the point C,

this line bisecting the angle $\angle AOB$ into two equal parts.
So $m\angle AOC = m\angle BOC$.

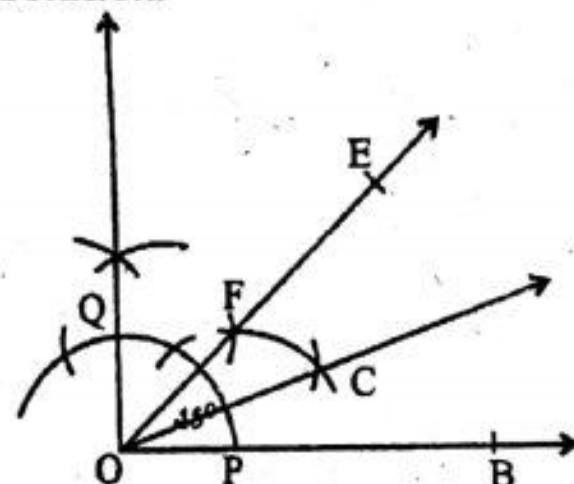
Q2: Construct the following angles by using a pair of compasses. Then bisect them.

a. 30° **Solution:****Steps of construction:**

- (i) Draw a ray $\overrightarrow{OB}$ using a ruler.
- (ii) Place the pointer of the compass of point P and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at a point Q.
- (iii) With the same opening of the compass place of the compass at point Q and draw an arc that will cut the previous arc at point P.
- (iv) Again with the same opening of the compass, place the pointer of the compass at Q and P will cut each other at U.
- (v) Using a ruler draw a ray $\overrightarrow{OA}$ passing the U. $\angle BOA = 30^\circ$ is the required angle.
- (vi) Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.
- (vii) Using a ruler draw a ray $\overrightarrow{OC}$ passing through the point C and bisect the angle $\angle BOA = 30^\circ$ into two equal parts.

b. 45°

Solution:



Steps of construction:

Step I:

Draw $\angle AOB = 90^\circ$

Step II:

Place the pointer of the compass at point P and Q and draw two arcs of same radius that will intersect each other at point F.

Step III:

Using a ruler draw a ray $\overline{OE}$ passing through the point F. $\angle BOE = 45^\circ$ is the required angle.

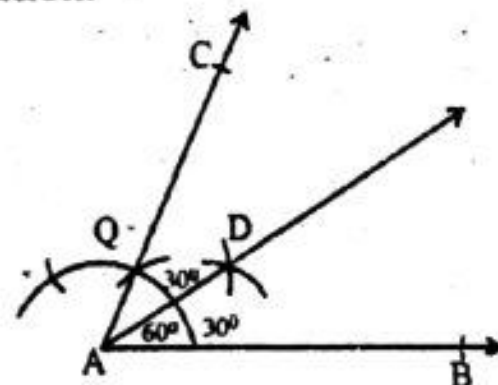
Step IV:

Draw bisector of angle $\angle BOE$, which divide angle into two equal angles.

So $\angle BOC = \angle COE$.

c. 60°

Solution:



Step of construction:

Step I:

Draw a ray $\overline{AB}$ using a ruler.

Step II:

Place the pointer of the compass at point A and draw an arc of suitable radius that will cut the ray $\overline{AB}$ at a point P.

Step III:

With the same opening of the compass, place the pointer of the compass at point P and draw an arc that will cut the previous arc at point Q.

Step IV:

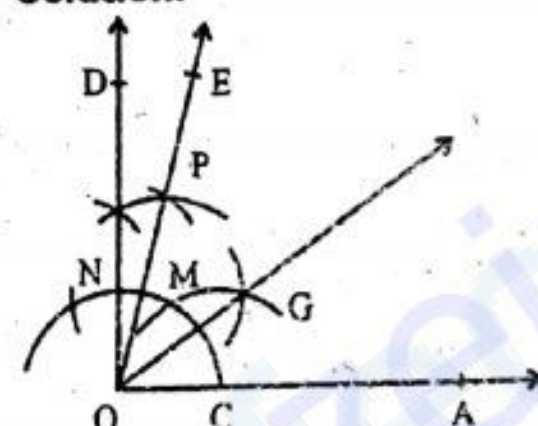
Using a ruler draw a ray $\overline{AC}$ passing through the point Q. $\angle CAB 60^\circ$ is the required angle.

Step V:

Draw bisector of angle $\angle BAC$, which divide the angle into two equal angles i.e. $\angle BAD = \angle DAC = 30^\circ$

d. 75°

Solution:



Steps of construction:

Step I:

Draw an $\angle AOD$ equal to 90° .

Step II:

Mark a point N, where the arm $\overline{OD}$ of $\angle AOD$ is cut by the arc $\overline{CL}$.

Step III:

Place the pointer of the compass at point M and N and draw two arcs of same radius that will cut each other at point P.

Step IV:

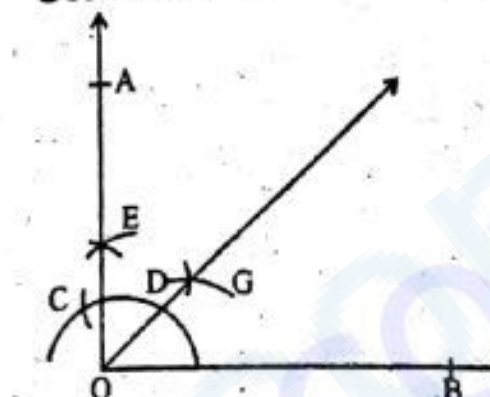
Using a ruler draw a ray $\overline{OE}$ passing through the point P. $\angle AOE = 75^\circ$.

Step V:

Bisect the angle $\angle AOE$, which divide the angle into two equal angles, So $\angle AOG = \angle GOE$.

e. 90°

Solution:



Step of construction:

Step I:

Draw a ray $\overline{OB}$ using a ruler.

Step II:

Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overline{OB}$ at a point X.

Step III:

With the same opening of the compass, place the pointer of the compass at point X and draw an arc that will cut the previous arc at point D.

Step IV:

Again using the same opening of the compass, place the pointer of the compass at point D and draw an arc that will cut the previous arc at point C.

Step V:

Place the pointer of the compass at point D and C and draw two arcs

of same radius that will cut each other at point E.

Step VI:

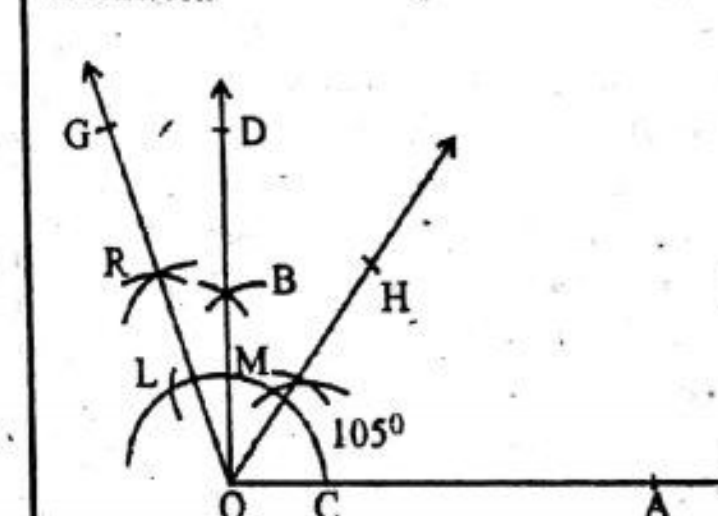
Using a ruler draw a ray $\overline{OA}$ passing through the point E. $\angle AOB = 90^\circ$ is the required angle.

Step VII:

Draw the bisector of angle $\angle BOA$ which divide the angle i.e. $\angle BOG = \angle GOA$.

f. 105°

Solution:



Steps of Construction:

Step I:

Draw an $\angle AOD = 90^\circ$

Step II:

Mark a point N, where the arm $\overline{OD}$ of $\angle AOD$ is cut by the arc $\overline{CL}$.

Step III:

Place the point of the compass at point N and L and draw two arcs of same radius that will cut each other at point R.

Step IV:

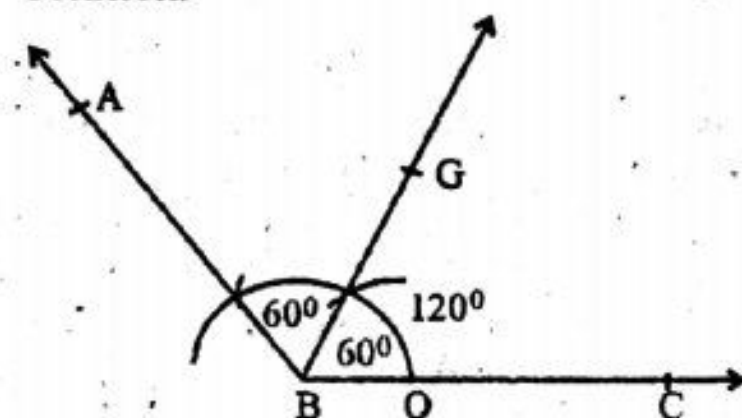
Using a ruler draw a ray $\overline{OR}$ passing through the point R. So, $\angle AOG = 105^\circ$

Step V:

Bisect the angle $\angle AOG$, which divide the angle into two equal angles, So $\angle AOH = \angle HOG$.

g. 120°

Solution:

**Steps of construction:****Step I:**Draw a ray $\overrightarrow{BC}$ using a ruler.**Step II:**Place the pointer of the compass at point B and draw an arc of suitable radius that will cut the ray $\overrightarrow{BC}$ at any point Q.**Step III:**

With the same opening of the compass, place the pointer of the compass at point Q and draw an arc that will cut the previous arc at point R.

Step IV:

With the same opening of the compass, place the pointer of the compass at point R and draw an arc that will cut the initial arc at point P.

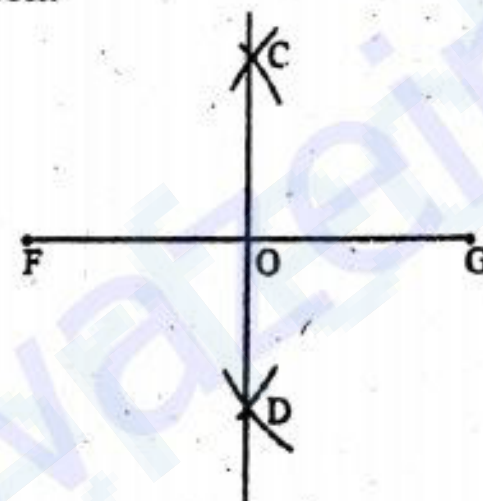
Step V:Using a ruler draw a ray $\overrightarrow{BA}$ passing through the point P. So, the $\angle ABC = 120^\circ$.**Step VI:**Bisect the angle $\angle CBA$, which divide the angle into two equal angles i.e. $\angle CBG = \angle GBA$.**Review Exercise****Q1: Choose the correct option.**

- a. The word geometry comes from two ____ words.
 i. Latin ii. Greek ✓
 iii. German iv. Arabic
- b. Bisection means to divide the line into ____ equal parts.
 i. one ii. two ✓
 iii. three iv. four
- c. Two lines or rays are said to be perpendicular to each other if the angle formed between them is ____.
 i. 60° ii. 45°
 iii. 30° iv. 90° ✓
- d. If we bisect a 90° angle, we get two ____ angles.
 i. 30° ii. 15°
 iii. 40° iv. 45° ✓

Q2: Draw the right bisectors of the following line segments by using a pair of compasses.

- a)
-

Solution:

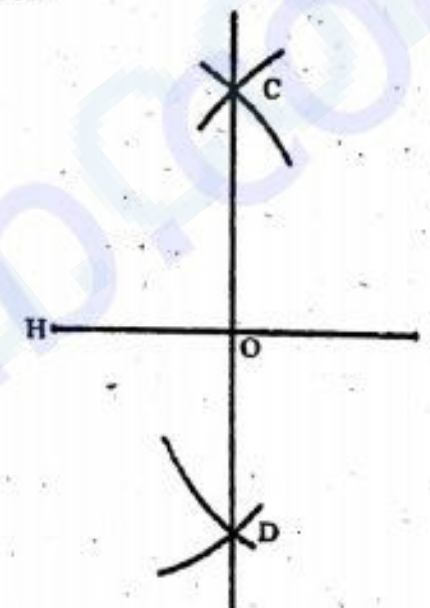
**Step of construction:**A line segment $\overline{FG}$ is given.**Step I:**

Place the pointer of the compass at point F. Open the compass with more than half of the measure of the

line segment and draw $\overline{FG}$ an arc on the top and bottom of $\overline{FG}$.**Step II:**Similarly, using the same opening of the compass, place the pointer of the compass at point G and draw two arc on the top and bottom of $\overline{FG}$ which intersects the previous arcs at point C and D respectively.**Step III:**Join the point C and D by a line using a ruler that will cut the line segment $\overline{FG}$ at point O.

- b)
-

Solution:

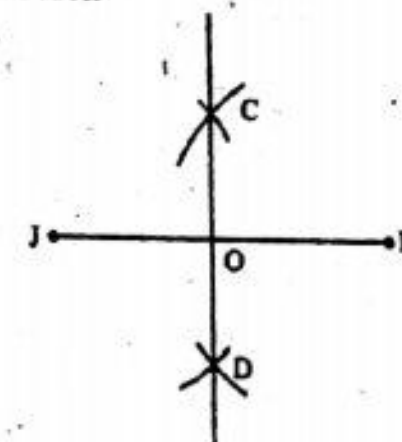
**Step of construction:**A line segment $\overline{HI}$ is given.**Step I:**Place the pointer of the compass at point H. Open the compass with more than half of the measure of the line segment and draw $\overline{HI}$ an arc on the top and bottom of $\overline{HI}$.**Step II:**Similarly, using the same opening of the compass, place the pointer of the compass at point I and draw two arc on the top and bottom of $\overline{HI}$ which intersects the

previous arcs at point C and D respectively.

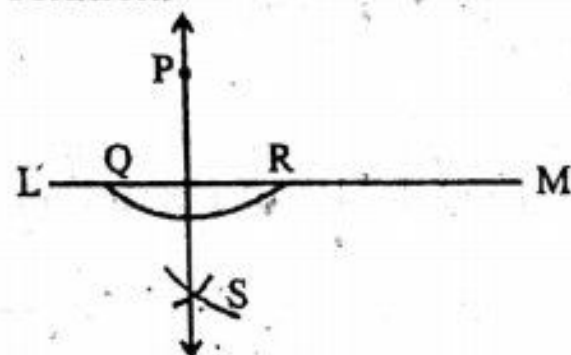
Step III:Join the point C and D by a line using a ruler that will cut the line segment $\overline{HI}$ at point O.

- c)
-

Solution:

**Step of construction:**A line segment $\overline{JK}$ is given.**Step I:**Place the pointer of the compass at point J. Open the compass with more than half of the measure of the line segment and draw $\overline{JK}$ an arc on the top and bottom of $\overline{JK}$.**Step II:**Similarly, using the same opening of the compass, place the pointer of the compass at point K and draw two arc on the top and bottom of $\overline{JK}$ which intersects the previous arcs at point C and D respectively.**Step III:**Join the point C and D by a line using a ruler that will cut the line segment $\overline{JK}$ at point O.**Q3: Draw perpendiculars from the point P to the line segment LM.**

- a)
-

Solution:**Step of construction:**

$\overline{LM}$ is the given line. A point P (not on it) is also given.

Step I:

Place the pointer of the compass at point P and draw an arc that will cut the line $\overline{LM}$ at two points Q and R respectively.

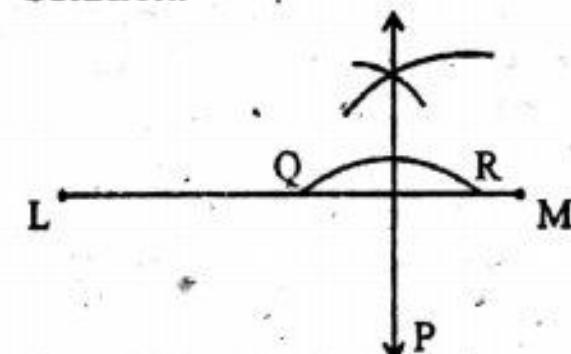
Step II:

Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overline{LM}$ that cut each other at point S.

Step III:

Join P to S. $\overline{PS}$ is the required perpendicular to the given line $\overline{MN}$.

$\therefore \overline{PS} \perp \overline{LM}$

**Solution:****Step of construction:****Step of construction:**

$\overline{LM}$ is the given line. A point P (not on it) is also given.

Step I:

Place the pointer of the compass at point P and draw an arc that will cut the line $\overline{LM}$ at two points Q and R respectively.

Step II:

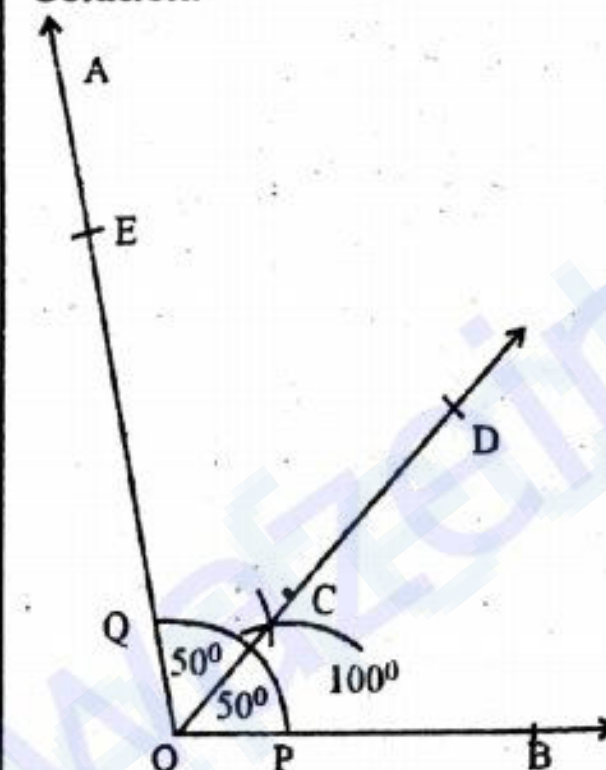
Using the same opening of the compass, place the pointer of the compass at point Q and R and draw two arcs on the other side of $\overline{LM}$ that cut each other at point S.

Step III:

Join P to S. $\overline{PS}$ is the required perpendicular to the given line $\overline{MN}$.

$\therefore \overline{PS} \perp \overline{LM}$

Q4: Construct the following angles by using a protractor and bisect them by using a pair of compass.

a. 100° **Solution:****Steps of construction:**

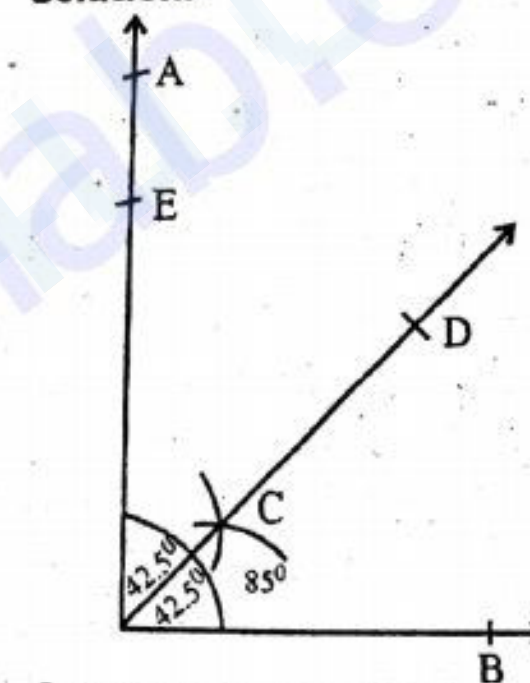
- Draw a ray $\overline{OB}$ using a ruler.
- Place protractor on point O and take a point 'E' which is coincide on 100° .

(iii) Using a ruler draw a ray $\overline{OA}$ passing through the point E
So $\angle AOB = 100^\circ$

(iv) Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overline{OB}$ at P and cuts the ray $\overline{OA}$ at any point Q.

(v) Place the pointer of the compass at point P and Q and draw two arcs of suitable rading that will cut each of point C.

(vi) Using a ruler draw a ray $\overline{OD}$ passing through the point C, this line bisecting the angle $\angle AOB$ into two equal parts.
So $m\angle AOC = m\angle BOC$.

b. 85° **Solution:****Steps of construction:**

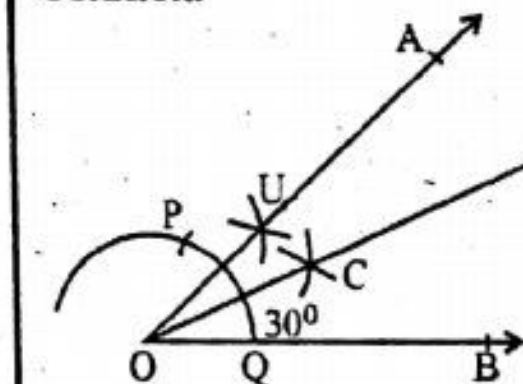
- Draw a ray $\overline{OB}$ using a ruler.
- Place protractor on point O and take a point 'E' which is coincide on 85° .
- Using a ruler draw a ray $\overline{OA}$ passing through the point E
So $\angle AOB = 85^\circ$

(iv) Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overline{OB}$ at P and cuts the ray $\overline{OA}$ at any point Q.

(v) Place the pointer of the compass at point P and Q and draw two arcs of suitable rading that will cut each of point C.

(vi) Using a ruler draw a ray $\overline{OD}$ passing through the point C, this line bisecting the angle $\angle AOB$ into two equal parts.
So $m\angle AOC = m\angle BOC$.

Q5: Construct the following angles by using a pair of compasses. Then bisect them.

a. 30° **Solution:****Steps of construction:**

- Draw a ray $\overline{OB}$ using a ruler.
- Place the pointer of the compass of point P and draw an arc of suitable radius that will cut the ray $\overline{OB}$ at a point Q.
- With the same opening of the compass place of the compass at point Q and draw an arc that will cut the previous arc at point P.
- Again with the same opening of the compass, place the pointer

of the compass at Q and P will cut each other at U.

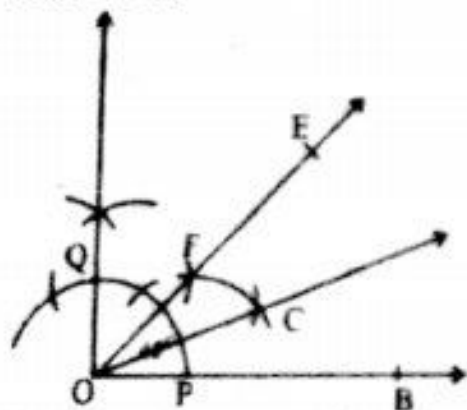
(v) Using a ruler draw a ray $\overrightarrow{OA}$ passing the U. $\angle BOA = 30^\circ$ is the required angle.

(vi) Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.

(vii) Using a ruler draw a ray $\overrightarrow{OC}$ passing through the point C and bisect the angle $\angle BOA = 30^\circ$ into two equal parts.

b. 45°

Solution:



Steps of construction:

Step I:

Draw $\angle AOB = 90^\circ$

Step II:

Place the pointer of the compass at point P and Q and draw two arcs of same radius that will intersect each other at point F.

Step III:

Using a ruler draw a ray $\overrightarrow{OE}$ passing through the point F. $\angle BOE = 45^\circ$ is the required angle.

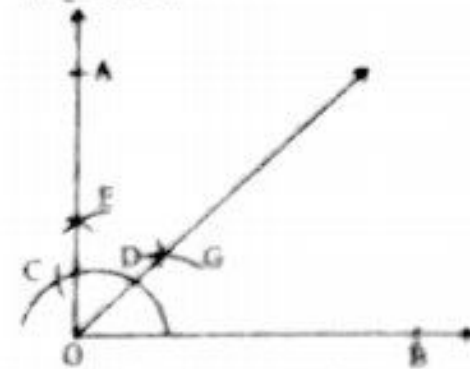
Step IV:

Draw bisector of angle $\angle BOE$, which divide angle into two equal angles.

So $\angle BOC = \angle COE$.

c. 90°

Solution:



Step of construction:

Step I:

Draw a ray $\overrightarrow{OB}$ using a ruler.

Step II:

Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at a point X.

Step III:

With the same opening of the compass, place the pointer of the compass at point X and draw an arc that will cut the previous arc at point D.

Step IV:

Again using the same opening of the compass, place the pointer of the compass at point D and draw an arc that will cut the previous arc at point C.

Step V:

Place the pointer of the compass at point D and C and draw two arcs of same radius that will cut each other at point E.

Step VI:

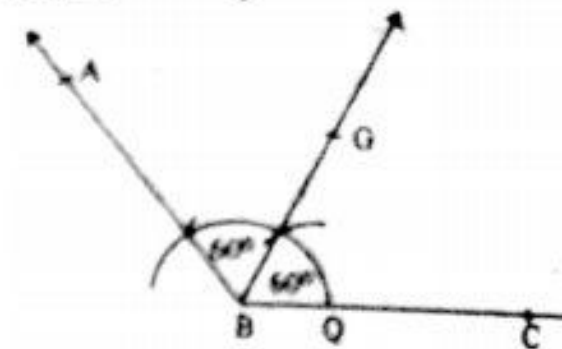
Using a ruler draw a ray $\overrightarrow{OA}$ passing through the point E. $\angle AOB = 90^\circ$ is the required angle.

Step VII:

Draw the bisector of angle $\angle BOA$ which divide the angle i.e. $\angle BOG = \angle GOA$.

d. 120°

Solution:



Steps of construction:

Step I:

Draw a ray $\overrightarrow{BC}$ using a ruler.

Step II:

Place the pointer of the compass at point B and draw an arc of suitable radius that will cut the ray $\overrightarrow{BC}$ at any point Q.

Step III:

With the same opening of the compass, place the pointer of the compass at point Q and draw an arc that will cut the previous arc at point R.

Step IV:

With the same opening of the compass, place the pointer of the compass at point R and draw an arc that will cut the initial arc at point P.

Step V:

Using a ruler draw a ray $\overrightarrow{BA}$ passing through the point P. So, the $\angle ABC = 120^\circ$.

Step VI:

Bisect the angle $\angle CBA$, which divide the angle into two equal angles i.e. $\angle CBG = \angle GBA$.

Unit 11

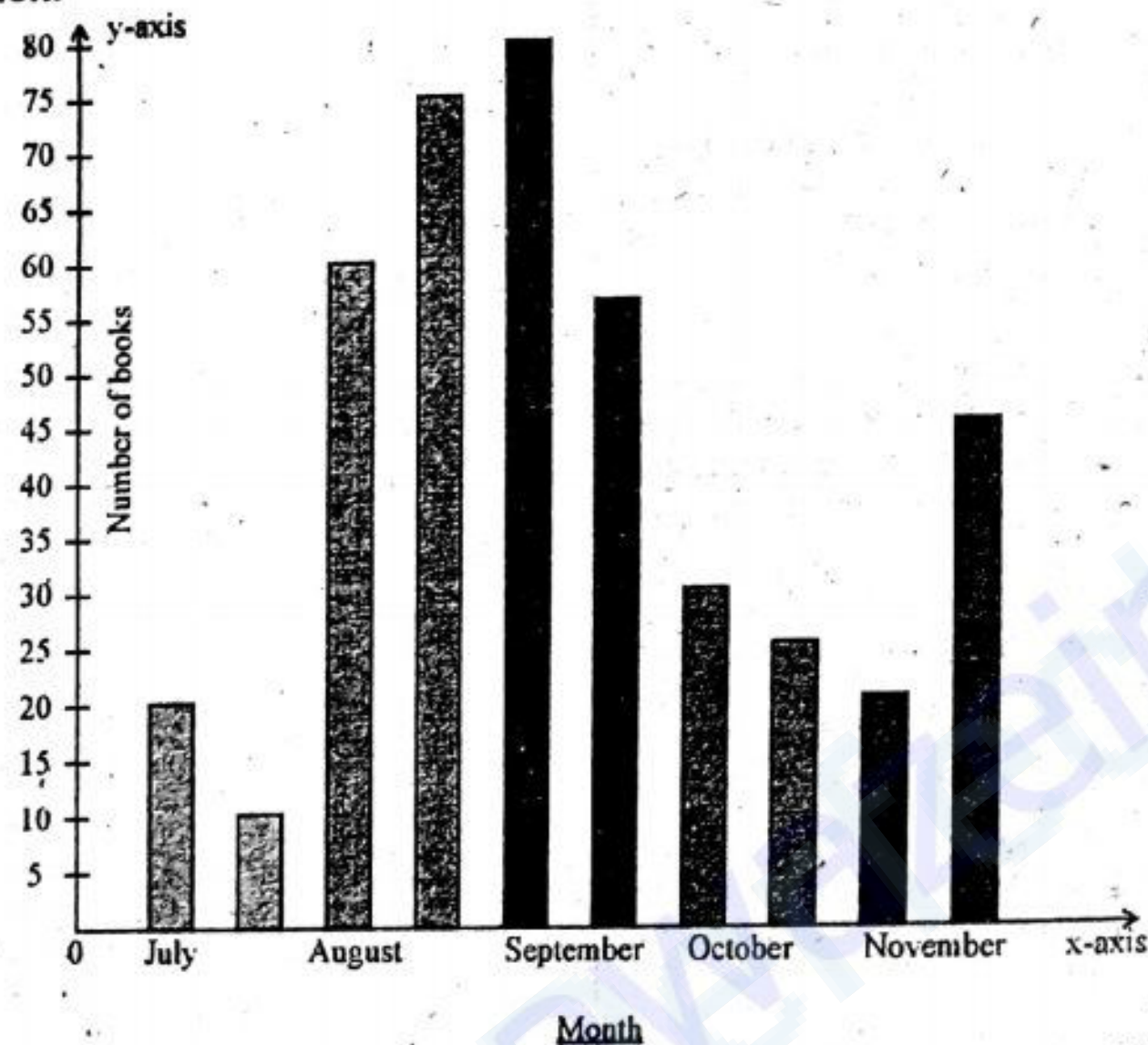
Data Management

Exercise 11.1

Q1: The following data shows the number of new books added to a library during 5 months. Draw a vertical multiple bar graph for this data using appropriate scale.

Language	July	August	September	October	November
English	20	60	80	30	20
Urdu	10	75	55	25	45

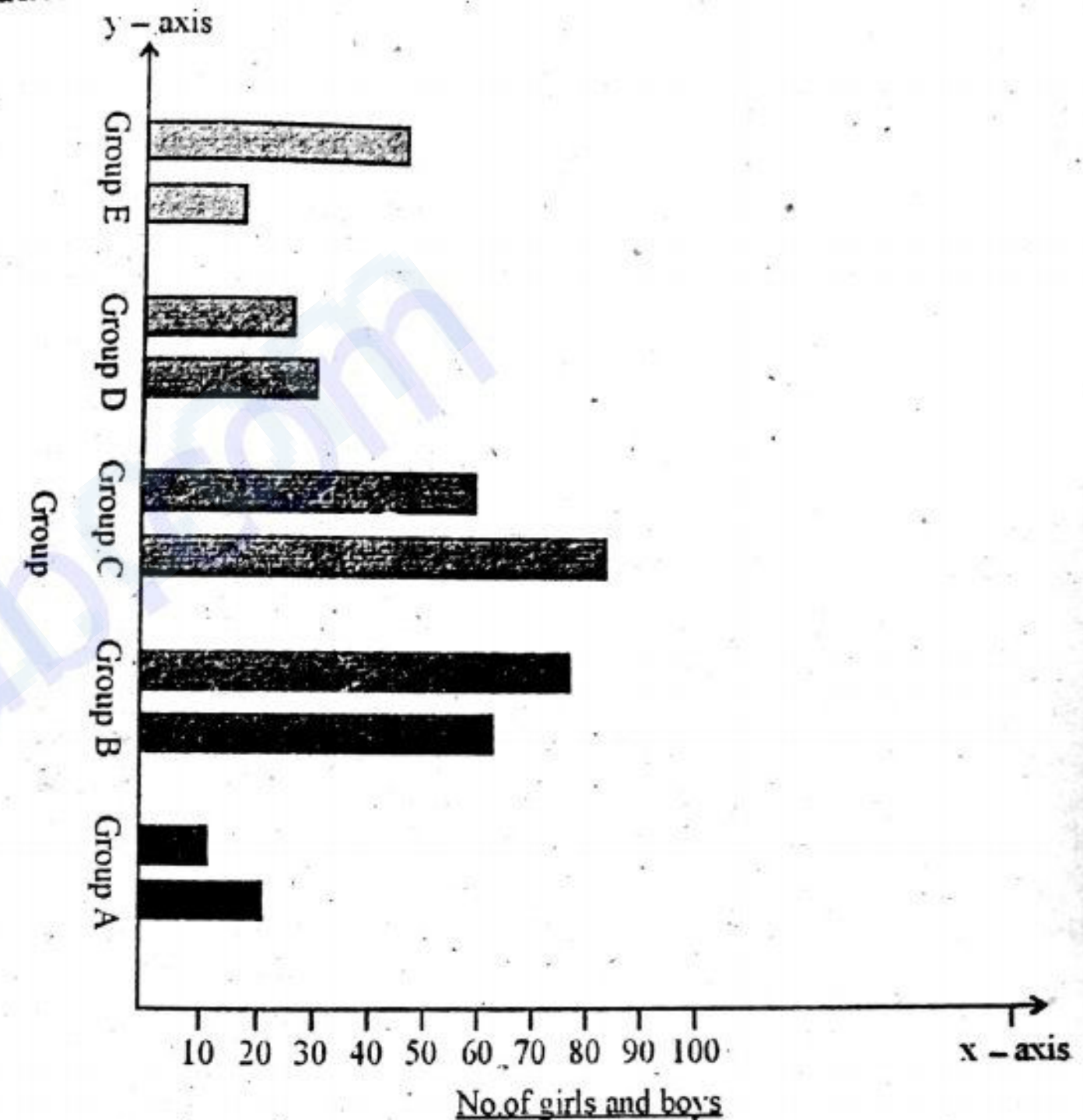
Solution:



Q2: The following data shows the number of girls and boys in different groups made for a mathematics competition. Draw a horizontal multiple bar graph for this data using appropriate scale.

Students	Group A	Group B	Group C	Group D	Group E
Boys	20	60	80	30	20
Girls	10	72	55	25	45

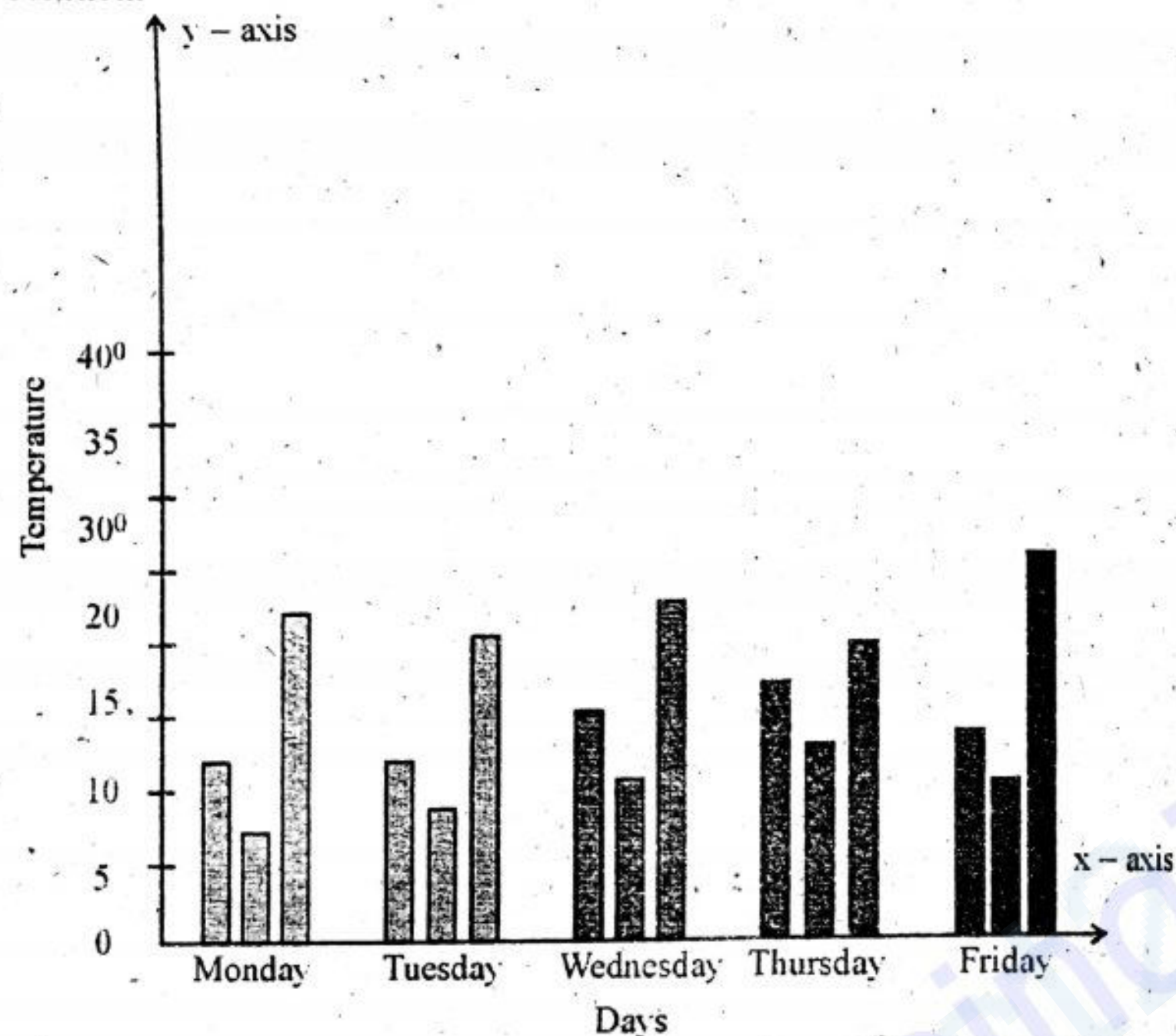
Solution:



Q3: The following data shows the average daily temperature of three cities. Draw a vertical multiple bar graph for this appropriate scale.

Cities	Monday	Tuesday	Wednesday	Thursday	Friday
City A	12°	11°	14°	16°	13°
City B	7°	8°	10°	12°	10°
City C	22°	20°	22°	18°	24°

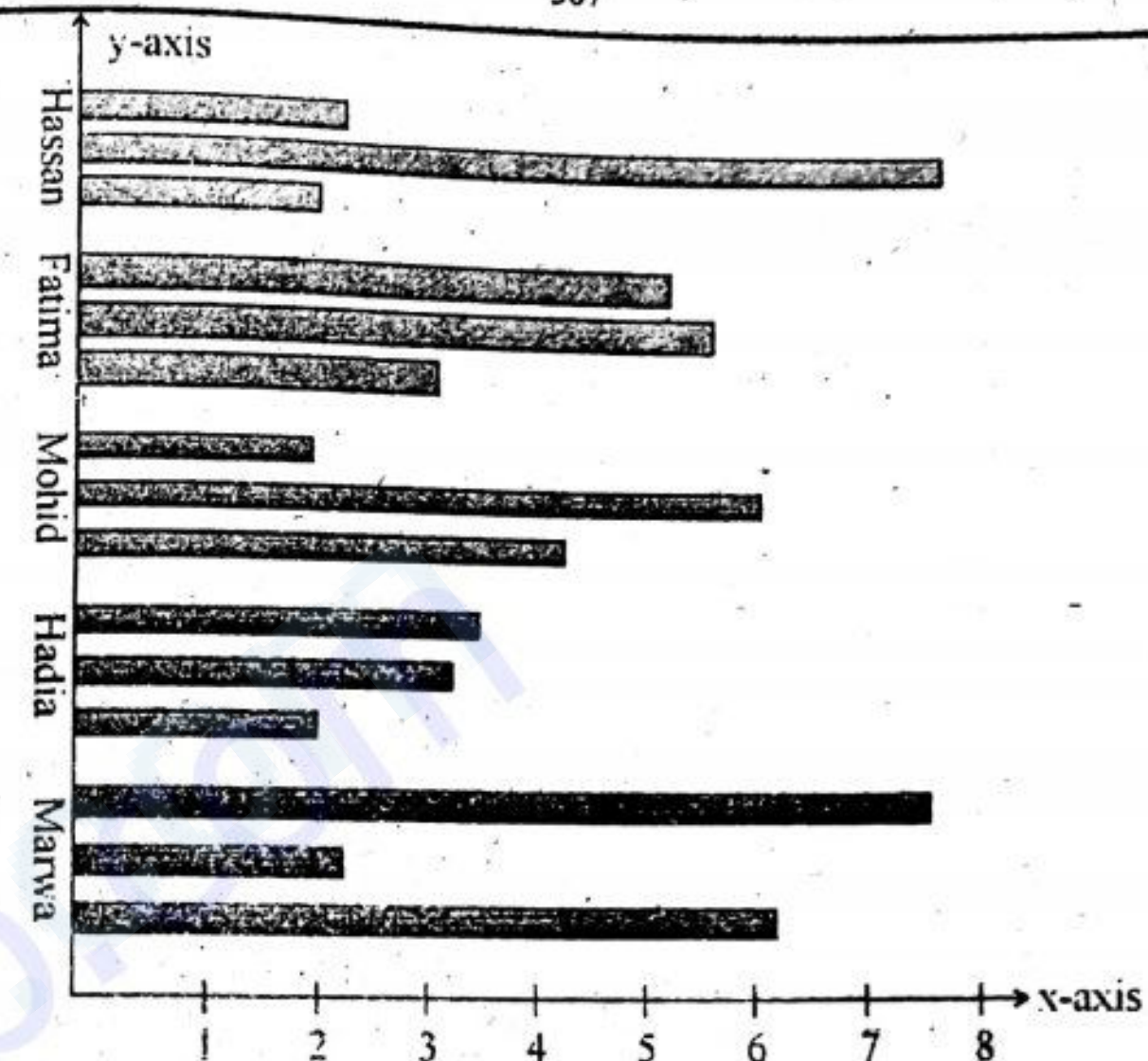
Solution:



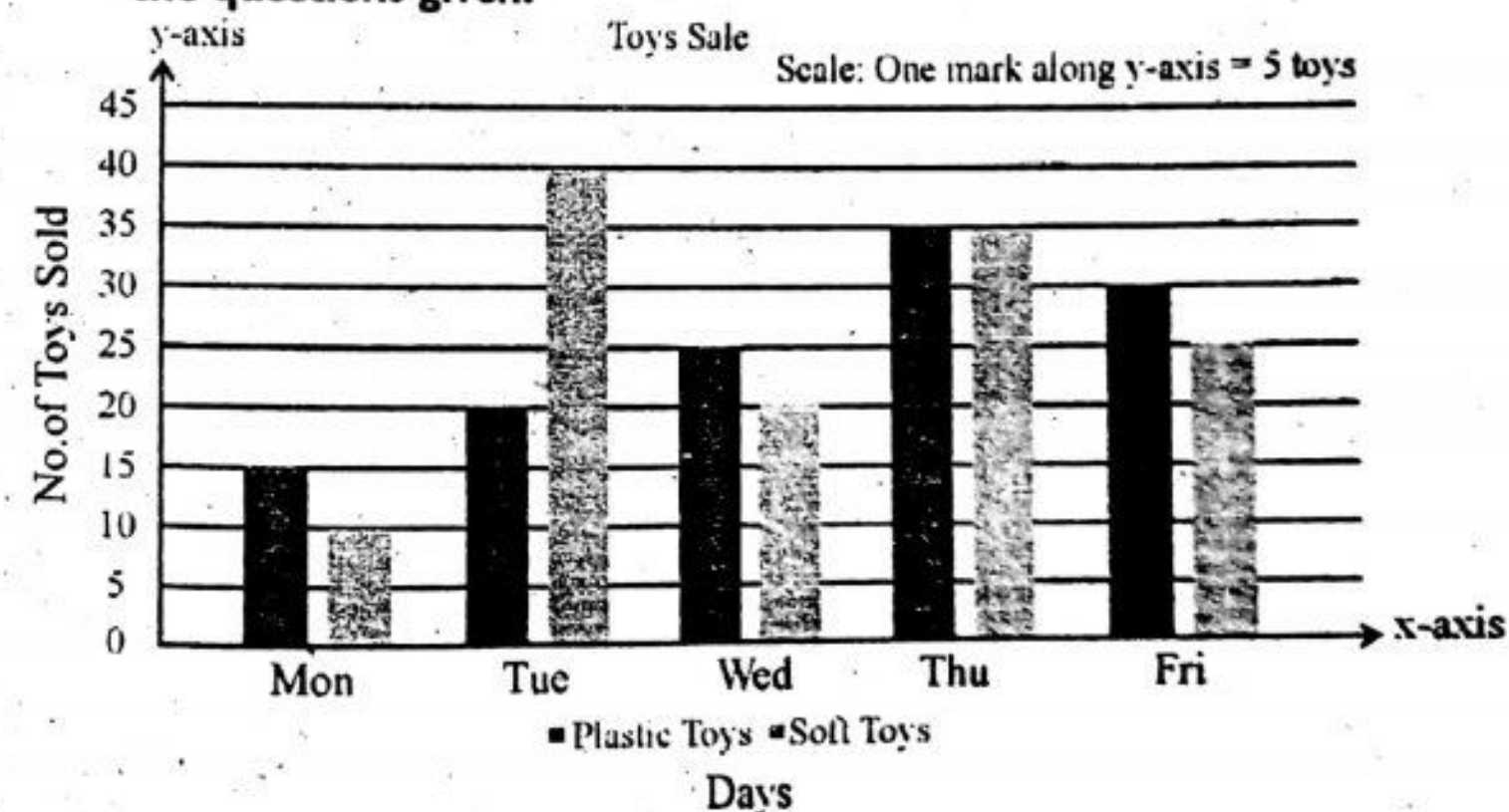
Q4: The following data shows the amount collected by Marwa, Hadia, Mohid, Fatima and Hassan during three weeks to donate to the construction of a local Masjid. Draw a Horizontal multiple bar graph for this data using appropriate scale.

Weeks	Marwa	Hadia	Mohid	Fatima	Hassan
Week 1	Rs 3200	Rs 1000	Rs 2000	Rs 1400	Rs 1000
Week 2	Rs 1200	Rs 1500	Rs 3000	Rs 2200	Rs 4000
Week 3	Rs 4000	Rs 1600	Rs 900	Rs 200	Rs 1200

Solution:



Q5: Observe the given multiple bar graph in vertical form and answer the questions given.



a) What is the title of the graph?

Ans: Multiple bar graph in vertical form

b) What information is shown on the X-axis?

Ans: Days

c) What information is shown on the Y-axis?

Ans: No. of toys sold

d) How many plastic toys were sold on Monday?

Ans: 15

e) How many soft toys were sold on Wednesday?

Ans: 20

f) On which day the least number of plastic toys were sold?

Ans: Monday

g) On which day the greatest number of soft toys were sold?

Ans: Tuesday

h) What is the total number of soft toys sold on Tuesday and Wednesday altogether?

Ans: $40 + 20 = 60$

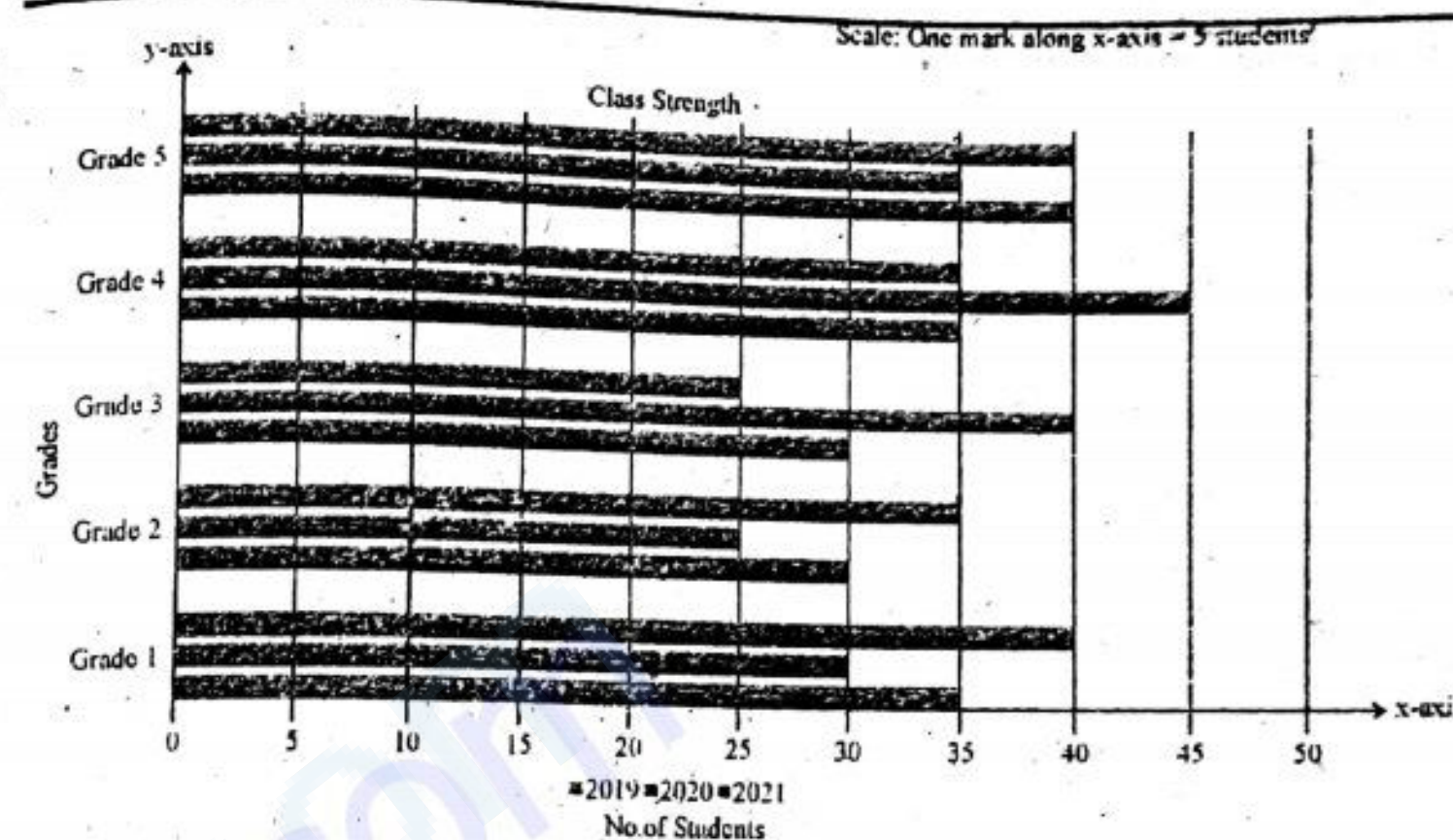
i) On which day the greatest number of plastic toys were sold?

Ans: Tuesday

j) What is the difference between the number of soft toys sold on Tuesday and Thursday?

Ans: 5

Q6: Observe the given multiple bar graph in horizontal form and answer the questions given.



a) What is the title of the graph?

Ans: Multiple bar graph in horizontal form.

b) What information is shown on the X-axis?

Ans: No. of students

c) What information is shown on the Y-axis?

Ans: Grades

d) What is the class strength of grade 5 students in 2021?

Ans: 40

e) What is the class strength of grade 3 students in 2019?

Ans: 25

f) What is the difference in class strength of grade 4 and Grade 5 in 2021?

Ans: 5

g) What is the difference in class strength of grade 1 and Grade 2 in 2020?

Ans: 5

h) Which year has the greatest number of grade 3 students?

Ans: 2020

i) Which year has the least number of grade 2 students?

Ans: 2020

j) In which year the greatest number of students were there in Grade 4?

Ans: 2020

k) In which year the least number of students were there in Grade 5?

Ans: 2020

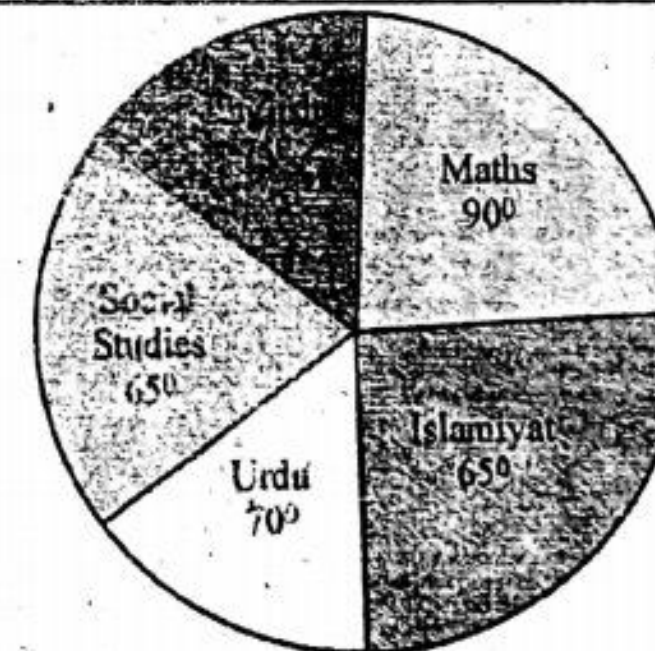
Exercise 11.2

Q1: Razia obtained the following marks in the different subjects:

Subject	Marks
Maths	90
English	75
Social Studies	60
Urdu	70
Islamiyat	65

Solution:

Subject	Marks	Mesurment of central angle
Maths	90	$\frac{90}{360} \times 360 = 90^\circ$
English	75	$\frac{75}{360} \times 360 = 75^\circ$
Social studies	60	$\frac{60}{360} \times 360 = 60^\circ$
Urdu	70	$\frac{70}{360} \times 360 = 70^\circ$
Islamiyat	65	$\frac{65}{360} \times 360 = 65^\circ$
Total	360	sum of central angles = 360°



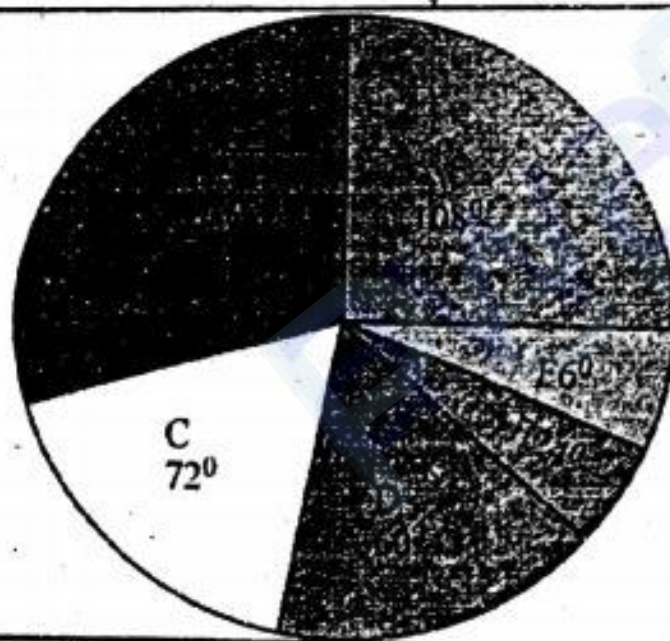
Q2: Draw pie charts for the following data:

(i) The population of 6 cities of Pakistan is given below.

City	Population (in Million)
A	18
B	15
C	12
D	10
E	4
F	1

Solution:

City	Population (in Million)	Mesurment of central angle
A	18	$\frac{18}{60} \times 360^\circ = 108^\circ$
B	15	$\frac{15}{60} \times 360^\circ = 90^\circ$
C	12	$\frac{12}{60} \times 360^\circ = 72^\circ$
D	10	$\frac{10}{60} \times 360^\circ = 60^\circ$
E	4	$\frac{4}{60} \times 360^\circ = 24^\circ$
F	1	$\frac{1}{60} \times 360^\circ = 6^\circ$
Total	60	360°

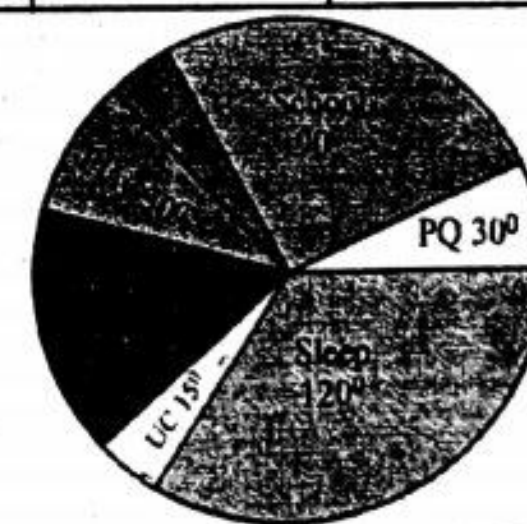


(ii) The following table show Amna's Activities in a day:

Activities	Time (Hours)
Reciting Quran	2
School	6
Meal	1
Play	2
Study	4
Using Computer	1
Sleep	8

Solution:

Activities	Time (Hours)	Mesurment of central angle
Reliting Quran	2	$\frac{2}{24} \times 360^\circ = 30^\circ$
School	6	$\frac{6}{24} \times 360^\circ = 90^\circ$
Meal	1	$\frac{1}{24} \times 360^\circ = 15^\circ$
Play	2	$\frac{2}{24} \times 360^\circ = 30^\circ$
Study	4	$\frac{4}{24} \times 360^\circ = 60^\circ$
Using computer	1	$\frac{1}{24} \times 360^\circ = 15^\circ$
Sleep	8	$\frac{8}{24} \times 360^\circ = 120^\circ$
Total	24	360°

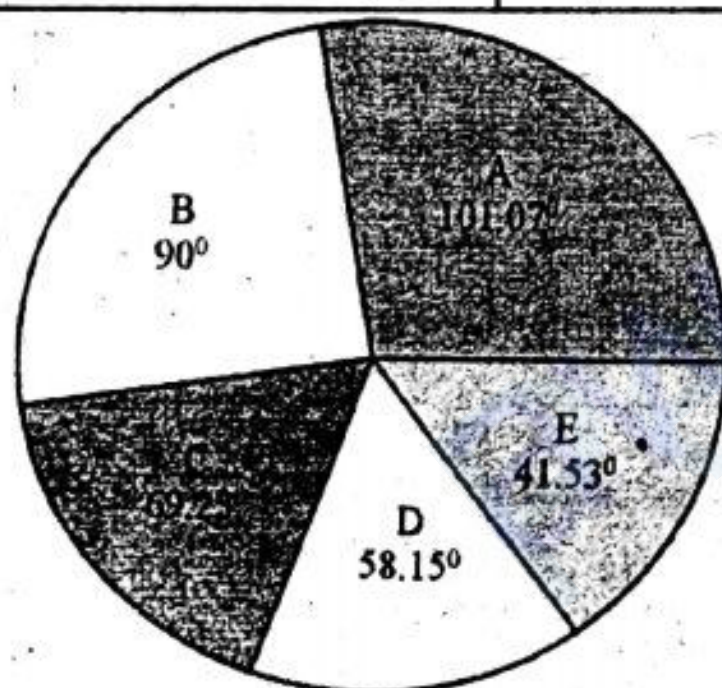


Q3: The number of students in different sections of class VII are given below:

Sections	Number of Students
A	73
B	65
C	50
D	42
E	30

Draw pie chart for the given data.

Sections	Number of students	Measurement of central angles
A	73	$\frac{73}{260} \times 360^\circ = 101.07^\circ$
B	65	$\frac{65}{260} \times 360^\circ = 90^\circ$
C	50	$\frac{50}{260} \times 360^\circ = 69.23^\circ$
D	42	$\frac{42}{260} \times 360^\circ = 58.15^\circ$
E	30	$\frac{30}{260} \times 360^\circ = 41.53^\circ$
Total	260	360°



Q4: The pie graph shows the sale of 180 litres of milk by a milkman in a day.

Read this pie graph and answer the following questions:

(i) How many litres does he supply to homes?

Solution:

Total milk = 180 litres

$$\text{Number of litre supply to home} = \frac{180^\circ}{360} \times 180 \\ = 90^\circ \text{litre}$$

(ii) How many litres does he sell to the customers?

Solution:

Total milk = 180 litres

$$\text{Number of litres sell to the customer} = \frac{120^\circ}{360^\circ} \times 180 \\ = 60 \text{ litre}$$

(iii) How any liters does he use for yogurt?

Solution:

Total milk = 180 litres

$$\text{Number of litres use for yogurt} = \frac{60^\circ}{360} \times 180 \\ = 30 \text{ litre}$$

Q5: The monthly expenditure of Rauf is shown by a pie graph. If the monthly income of Rauf is Rs. 7200, find the answers of the following questions:

(i) What is the largest expenditure of Rauf and how much?

Solution:

Largest expenditure of Rauf = House Rent

Total income = Rs. 7200

Now House Rent expenditure of Rauf

$$= \frac{150^\circ}{360^\circ} \times 7200 \\ = \text{Rs. } 3000$$

(ii) What amount does he spend on education and food?

Solution:

Total income = Rs. 7200

$$\begin{aligned}\text{Amount spend on Education} &= \frac{75^0}{360^0} \times 7200 \\ &= \text{Rs. 1500}\end{aligned}$$

$$\begin{aligned}\text{Amount spend on food} &= \frac{60^0}{36^0} \times 7200 \\ &= \text{Rs. 1200}\end{aligned}$$

(iii) How much does he pay as utility bills?

Solution:

Total income = Rs. 7200

$$\begin{aligned}\text{Utility bills payment} &= \frac{50^0}{360^0} \times 7200 \\ &= \text{Rs. 1000}\end{aligned}$$

(iv) What amount does he save?

Solution:

Total income = Rs. 7200

$$\begin{aligned}\text{Amount that he save} &= \frac{25^0}{360^0} \times 7200 \\ &= \text{Rs. 500}\end{aligned}$$

Q6: Sana's activities in a day are shown by a pie graph. Read the graph and answer the following questions: (Hint: 1 day = 24 hours)

(i) How much time does she spend in school?

Solution:

Total time = 24 hours

$$\begin{aligned}\text{Time that she spend in school} &= \frac{90}{360^0} \times 24 \\ &= 6 \text{ hours}\end{aligned}$$

(ii) How many hours does she sleep?

Solution:

Total time = 24 hours

$$\begin{aligned}\text{Time that she sleep} &= \frac{120^0}{360^0} \times 24 \\ &= 8 \text{ hours}\end{aligned}$$

(iii) How many hours does she study?

Solution:

Total time = 24 hours

$$\begin{aligned}\text{Time that she study} &= \frac{45^0}{360^0} \times 24 \\ &= 3 \text{ hours}\end{aligned}$$

(iv) In which activity does she spend the least time? And how much?

Solution:

Total time = 24 hours

The activity that she spend the least time = play

So the time that she spend in play

$$\begin{aligned}&= \frac{15^0}{360} \times 24 \\ &= 1 \text{ hour}\end{aligned}$$

(v) How much time does she spend in using computer and meal?

Solution:

Total time = 24 hours

$$\begin{aligned}\text{Time that she spend in using computer and meal} &= \frac{30^0}{36^0} \times 24 \\ &= 2 \text{ hours}\end{aligned}$$

(vi) How many hours does she spend on recitation of the Holy Quran and Namaz?

Solution:

Total time = 24 hours

Time that she spend on recitation of the

$$\begin{aligned}\text{holy Quran and Namaz} &= \frac{60^0}{360} \times 24 \\ &= 4 \text{ hours}\end{aligned}$$

Exercise 11.3

Q1: Azan makes a list of students about their favourite subjects. Show this in the form of grouped data and then answer the given questions.

English	Science	Math	Math	Urdu	Urdu	English
Urdu	Math	Math	Science	English	English	Science
Science	Science	Science	Math	Math	English	Urdu

Solution:

Subjects	Tally Marks	No. of students
English	III	05
Science	III 1	06
Maths	III 1	06
Urdu	II	04
Total		21

a) Which subject is the most favourite?

Solution:

The most favourite subjects are science and Maths.

b) Which subject is the least favourite?

Solution:

The least favourite subject is Urdu.

Q2: The given data is the weight of class 6 students:

20kg	24kg	21kg	18kg	20kg	24kg	21kg	18kg
19kg	24kg	19kg	21kg	24kg	20kg	18kg	18kg
20kg	24kg	18kg	24kg	26kg	21kg	19kg	26kg

Show it in the form of grouped data. Also.

Solution:

Weight	Tally Marks	No. of students
17 - 19	III III	08
20 - 20	III III	08
23 - 25	III I	06
29 - 28	II	02
Total		24

a. Tell the number of students that have a weight less than 20 kg.

Solution:

The number of students that have weight less than 20kg = 08

b. Tell the number of students that have a weight greater than 21 kg.

Solution:

The number of students that have weight greater than 21kg = 08

Exercise 11.4**Q1:** Calculate the mean of the following data.

(i) 46, 52, 39, 50, 38

Solution:

46, 50, 39, 50, 38

As we know that

$$\bar{X} = \frac{\sum X}{n}$$

$$= \frac{46 + 50 + 39 + 50 + 38}{5}$$

$$= 44.6$$

(ii) 108, 115, 138, 190

Solution:

108, 115, 138, 190

As we know that

$$\bar{X} = \frac{\sum X}{n}$$

$$= \frac{108 + 115 + 138 + 190}{4}$$

$$= 137.75$$

(iii) 210, 237, 340

Solution:

210, 237, 340

As we know that

$$\bar{X} = \frac{\sum X}{n} = \frac{210 + 237 + 340}{3}$$

$$\bar{X} = \frac{787}{3} = 262.33$$

Q2: The monthly income of 10 employees of a factory is:

Rs. 6000, Rs. 65230, Rs. 78600, Rs. 8000, Rs. 9350

Rs. 9680, Rs. 1030, Rs. 1236, Rs. 14340, Rs. 15000.

Calculate the average of their monthly income.

Solution:

As we know that

$$\text{Average} = \frac{\sum X}{n}$$

$$= \frac{6000 + 65230 + 78600 + 8000 + 9350 + 9680 + 1030 + 1236 + 14340 + 15000}{10}$$

$$= \frac{208,466}{10} = \text{Rs. } 20,846.6$$

Q3: Find the median of the following sets of observations.

(i) 15, 12, 13, 10, 8

Solution:

15, 12, 13, 10, 8

First we write data in ascending order

8, 10, 12, 13, 15

Here $n = 5$.

$$\text{The Median} = \left(\frac{n+1}{2} \right)^{\text{th item}}$$

$$= \left(\frac{5+1}{2} \right)^{\text{th item}}$$

$$= 3^{\text{rd item}}$$

So median = 12

(ii) 100, 106, 101, 108, 98, 93

Solution:

100, 106, 101, 108, 98, 93

First write the data in ascending order

93, 98, 100, 101, 106, 108

Here

$$n = 6$$

$$\begin{aligned}\text{So Median} &= \left(\frac{n+1}{2}\right)\text{th item} \\ &= \left(\frac{6+1}{2}\right)\text{th item}\end{aligned}$$

= 3.5th item in the given observations.

3.5th item is half way between the 3rd and 4th items

3rd value = 100

4th value = 101

$$\begin{aligned}\text{So Median} &= \frac{100+101}{2} \\ &= 100.5\end{aligned}$$

(iii) 84, 72, 95, 90, 65, 68, 69, 70

Solution:

84, 72, 95, 90, 65, 68, 69, 70

First write in ascending order.

65, 68, 69, 70, 72, 84, 90, 95

Here $n = 8$, So

$$\begin{aligned}\text{Median} &= \left(\frac{n+1}{2}\right)\text{th item} \\ &= \left(\frac{8+1}{2}\right)\text{th item} \\ &= \frac{9}{2}\text{th item} = 4.5\text{th item}\end{aligned}$$

4.5th item is the half way between the 4th and 5th items.

4th value = 70

5th value = 72

So

$$\begin{aligned}\text{So Median} &= \frac{70+72}{2}\text{th item} \\ &= 71\end{aligned}$$

So Median = 71

Q4: Find out the mode for each of the following:

(i) 7, 8, 8, 13, 15, 15, 16

Solution:

7, 8, 8, 13, 15, 15, 16

Frequency of 8 is '2' and frequency of 15 is '2' which is greatest frequency.

Hence Mode = $\hat{x} = 8, 15$

(ii) 100, 120, 120, 120, 120, 150

Solution:

100, 120, 120, 120, 120, 150

Frequency of 120 is '4' which is greatest frequency

Hence Mode = $\hat{x} = 120$

(iii) 357, 402, 427, 500

Solution:

As frequency of terms are '1'

So Mode = No Mode

Exercise 11.5**Q1: Describe the following with examples:****a) Experiments:**

An experiment is a process which gives some results/outcomes.

b) Random Experiments:

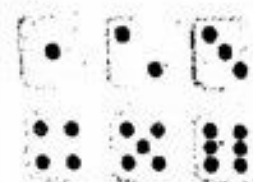
An experiment will be called a random experiment if it has two or more possible results.

c) Outcomes:

The possible result of a random experiment is called an outcome. For example, when rolling a dice, there are total six possible outcomes can be 1 or 2 or 3 or 4 or 5 or 6.

**d) Sample space:**

The sample space is the set of all possible outcomes of an experiment.

**e) Events:**

An event is an outcome or a collection of outcomes.

An event is subset of a sample space.

For example, tossing a coin and getting a head is an event, rolling a die and getting a 5 is an event.

f) Equally Likely events:

The outcomes in a simple space will be called equally likely if each outcome has the same chance of occurring.

Q2: Saad has a bag with 14 blocks. Six are red, two are green, four are blue and two are yellow. What is the probability that if he chooses one block, randomly without seeing, it will be red?

Solution:

Sample space or total number of possible outcomes = Red, Red, Red, Red, Red, Green, Green, Blue, Blue, Blue, Blue, Yellow, Yellow.

So number of possible outcomes = 14

$n(\text{Red}) = 6$, $P(\text{Red}) = ?$

As probability of an event

$$= \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

$$\text{So } P(\text{Red}) = \frac{6}{14} = \frac{3}{7}$$

Also find:**a) Sample Space:****Solution:**

Sample Space = 14

b) P (green)**Solution:**

$$n(\text{green}) = 2, n(s) = 14$$

c) P (yellow)**Solution:**

$$\begin{aligned} \text{So } P(\text{green}) &= \frac{n(\text{Green})}{n(S)} \\ &= \frac{2}{14} = \frac{1}{7} \end{aligned}$$

d) P (Not blue)**Solution:**

$$n(\text{Not Blue}) = 10, n(s) = 14$$

$$\text{So } P(\text{Not Blue}) = \frac{n(\text{Not Blue})}{n(S)}$$

$$\begin{aligned} &= \frac{10}{14} \\ &= \frac{5}{7} \end{aligned}$$

Q3: There are 14 candies in a jar. Hamza takes one candy randomly without looking at the candies.

Find:**(a) The sample space.****Solution:**

Sample space = Orange, Orange, Orange, Orange, Orange, Orange, Orange, Pink, Pink, Pink, Red, Red, Red, Red.

So sample space = Number of possible outcomes event.

Exercise 11.5

Q1: Describe the following with examples:

a) Experiments:

An experiment is a process which gives some results/outcomes.

b) Random Experiments:

An experiment will be called a random experiment if it has two or more possible results.

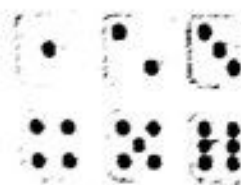
c) Outcomes:

The possible result of a random experiment is called an outcome. For example, when rolling a dice, there are total six possible outcomes can be 1 or 2 or 3 or 4 or 5 or 6.



d) Sample space:

The sample space is the set of all possible outcomes of an experiment.



e) Events:

An event is an outcome or a collection of outcomes.

An event is subset of a sample space.

For example, tossing a coin and getting a head is an event, rolling a die and getting a 5 is an event.

f) Equally Likely events:

The outcomes in a simple space will be called equally likely if each outcome has the same chance of occurring.

Q2: Saad has a bag with 14 blocks. Six are red, two are green, four are blue and two are yellow. What is the probability that if he chooses one block, randomly without seeing, it will be red?

Solution:

Sample space or total number of possible outcomes = Red, Red, Red, Red, Red, Green, Green, Blue, Blue, Blue, Blue, Yellow, Yellow.

So number of possible outcomes = 14

$n(\text{Red}) = 6$, $P(\text{Red}) = ?$

As probability of an event

$$= \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

$$\text{So } P(\text{Red}) = \frac{6}{14} = \frac{3}{7}$$

Also find:

a) Sample Space:

Solution:

Sample Space = 14

b) P (green)

Solution:

$$n(\text{green}) = 2, n(s) = 14$$

c) P (yellow)

Solution:

$$\begin{aligned} \text{So } P(\text{green}) &= \frac{n(\text{Green})}{n(S)} \\ &= \frac{2}{14} = \frac{1}{7} \end{aligned}$$

d) P (Not blue)

Solution:

$$n(\text{Not Blue}) = 10, n(s) = 14$$

$$\text{So } P(\text{Not Blue}) = \frac{n(\text{Not Blue})}{n(S)}$$

$$\begin{aligned} &= \frac{10}{14} \\ &= \frac{5}{7} \end{aligned}$$

Q3: There are 14 candies in a jar. Hamza takes one candy randomly without looking at the candies.

Find:

(a) The sample space.

Solution:

Sample space = Orange, Orange, Orange, Orange, Orange, Orange, Orange, Pink, Pink, Pink, Red, Red, Red, Red.

So sample space = Number of possible outcomes event.

$$= 14$$

b. The probability of picking a chocolate candy.

Solution:

Let 'B' be the event which represent chocolate candy.

So $n(B) = 10$, $n(s) = 14$

$$\text{So } P(B) = \frac{n(B)}{n(S)} = \frac{10}{14} = \frac{5}{7}$$

c. The probability of picking a chocolate candy.

Solution:

Let 'A' be the event which represent strawberry candy.

So $n(A) = 4$, $n(s) = 14$

$$\text{So } P(A) = \frac{n(A)}{n(S)} = \frac{4}{14} = \frac{2}{7}$$

d. The probability of NOT picking a chocolate candy.

Solution:

Let 'C' be the event which represent NOT Picking a chocolate candy.

So $n(c) = 4$, $n(s) = 14$

$$\text{So } P(c) = \frac{n(c)}{n(s)} = \frac{4}{14} = \frac{2}{7}$$

e. The probability of picking an orange candy.

Solution:

Let 'D' be the event which represent orange candy.

$n = (d) = 7$, $n(s) = 14$,

$$\text{So } P(D) = \frac{n(D)}{n(S)} = \frac{7}{14} = \frac{1}{2}$$

f. The probability of NOT picking an orange candy.

Solution:

Let 'E' be the event which represent Not picking the orange candy.

So $n(E) = 7$, $n(S) = 14$

$$\text{So } P(E) = \frac{n(E)}{n(S)} = \frac{7}{14} = \frac{1}{2}$$

g. What is more likely to be picked; an orange candy or a strawberry candy?

Solution:

As $P(\text{orange}) > P(\text{strawberry})$

So orange candy is more likely to be picked.

Q4: Draw 2D shapes (as sample space) for each of these according to the below mentioned probabilities:

a. $P(\text{squares}) = \frac{2}{4}$

Solution:

2D shapes for $P(\text{Squares}) = \frac{2}{4}$

is 

b. $P(\text{circles}) = \frac{3}{8}$

Solution:

2D shapes for $P(\text{circles}) = \frac{3}{8}$

is 

c. $P(\text{triangle}) = \frac{1}{3}$

Solution:

2D shapes for $P(\text{triangles}) = \frac{1}{3}$

is 

Q5: Look at the spinner and find:

a) The sample space

Solution:

Sample space = green, green, green, red, yellow, yellow, red, red, blue.

So sample space = Number of possible out comes
= 8

(b) The probability:

i. $P(\text{red})$

Solution:

Let 'R' be the event which represent Red.

So $n(R) = 3$, $n(S) = 8$

$$\text{So } P(\text{Red}) = \frac{n(R)}{n(S)} = \frac{3}{8}$$

(ii) **P (Green)**

Solution:

Let 'G' be the event which represent Green.

So $n(G) = 3$, $n(S) = 8$

$$\text{So } P(\text{Green}) = \frac{n(G)}{n(S)} = \frac{3}{8}$$

(iii) **P (Blue)**

Solution:

Let 'B' be the event which represent, Blue.

So $n(B) = 1$, $n(S) = 8$

$$\text{So } P(B) = \frac{n(B)}{n(S)} = \frac{1}{8}$$

(iv) **P (yellow)**

Let 'y' be the event which represent yellow.

So $n(y) = 1$, $n(s) = 8$

$$\text{So } P(\text{yellow}) = \frac{n(y)}{n(s)} = \frac{1}{8}$$

Q6: The pond has 10 blue, 8 red, 7 green and 4 yellow fish. What is the probability that a randomly caught fish will be a:

a. Red fish

Solution:

Let 'A' is the event for red fish.

So $n(A) = 8$, $n(S) = 29$

$$\text{So } P(\text{Red fish}) = \frac{n(A)}{n(S)} = \frac{8}{29}$$

b. Blue fish

Solution:

Let 'B' be the event for Blue fish

So $n(B) = 10$, $n(S) = 29$

$$\text{So } P(\text{Blue fish}) = \frac{n(B)}{n(S)} = \frac{10}{29}$$

c. Green fish

Solution:

Let 'c' be the event for Green fish

So $n(c) = 7$, $n(s) = 29$.

$$\text{So } P(\text{Green fish}) = \frac{7}{29}$$

Q7: A box contains 8 buttons out of which 5 are transparent. What is the probability that a randomly selected button will not be a transparent one?

Solution:

Let 'A' be the event which represent Not Transparent,

So $n(A) = 8 - 5 = 3$, $n(S) = 8$

$$\text{So } P(\text{Not transparent}) = P(A) = \frac{3}{8}$$

Q8: In a pack of randomly packed 14 pencils, 11 are blue ones.

a. What is the probability that a randomly selected pencil will not be blue?

Solution:

Let 'A' is the event which represent blue pencil.

So $n(A) = 11$, $n(S) = 14$

$$\text{So } P(\text{Blue pencil}) = P(A) = \frac{n(A)}{n(S)}$$

$$P(A) = \frac{11}{14}$$

b. What is the probability that a randomly selected pencil will not be blue?

Solution:

Let 'B' is the event which represent Not blue pencil.

So $n(B) = 3$, $n(S) = 14$

$$\text{So } P(\text{Not Blue}) = P(B) = \frac{n(B)}{n(S)}$$

$$P(B) = \frac{3}{4}$$

Q9: Ali picks a card randomly.



(a) What is the sample space?

Solution:

Sample space = $n(S) = 6$

(b) What are the possible outcomes?

Solution:

The possible outcomes = 4, 7, 0, 3, 8

(c) What is the probability that a randomly selected card will have a 7?

Solution:

Let 'A' is the event which represent card 7.

So $n(A) = 2$, $n(S) = 6$

$$\text{So } P(A) = P(\text{card } 7) = \frac{n(A)}{n(S)} = \frac{2}{6} \\ = \frac{1}{3}$$

(d) What is the probability that a randomly selected card will have a 3?

Solution:

Let 'B' is the event which represent card 3.

So $n(B) = 1$, $n(S) = 6$

$$\text{So } P(B) = P(\text{card } 3) = \frac{n(B)}{n(S)} = \frac{1}{6}$$

Review Exercise

Q1: Encircle the correct option.

- The information on the basis of which these graphs are drawn is called:
 - Knowledge
 - Table
 - Data ✓
 - Product
- Two or more related categories of data are represented in a:
 - single bar graph
 - multiple bar graph ✓
 - pie bar graph
 - Line bar graph
- The sum of central angles of a Pie graph must be equal to:
 - 160°
 - 380°
 - 360° ✓
 - 180°
- Which of these shows discrete data?
 - The mass of an animal
 - The number of rooms in a hotel ✓
 - The length of a chain
 - The capacity of a water container
- The mode of the data 9, 10, 12, 13, 13, 13, 15, 15, 16, 16 is:
 - 9
 - 15
 - 16
 - 13 ✓
- The median of the data 23, 23, 23, 45, 45, 56, 67, 73, 89 is:
 - 20
 - 23
 - 45 ✓
 - 46

Q2: Define the following with examples:

a) Discrete data:

The values or quantities that are in whole number form are fall under discrete data. The values in discrete data are countable. They cannot be subdivided into further parts. For example:

- The Number of students in Grade 6.
- The population of Islamabad.

b) Continuous data:

The values or quantities that can be presented in fraction or decimal fall under continuous data. The values in continuous data are not countable. They can take any value within a range.

- The weight of grade 6 students.
- The volume of liquid
- The height of person

- The temperature of a city

c) **Ungrouped data:**

When collected data from any source it does not convey to us any meaningful information to understand. This data is called ungrouped data or raw data.

d) **Grouped data:**

When collected data from any source is organized in groups/categories to show some meaning, this data is called grouped data.

e) **Mean:**

The sum of values divided by the number of values is called arithmetic mean.

$$\text{i.e., Arithmetic mean} = \frac{\text{sum of values}}{\text{number of values}}$$

f) **Median:**

The median is the value which divides the values into two equal halves arranged either in ascending or descending order.

g) **Mode:**

The frequently occurring value (or values) in the data denoted by λ is called mode. To find the mode.

Q3: Fahad made a list of students about their favourite games. Show this in the form of grouped data:

Games	Tally	No. of
Cricket	HH I	06
Hockey	IIII	04
Football	IIII	04
Basketball	IIII	04
Badminton	III	03
Total		21

a) Which game is more favourite?

Ans: Cricket

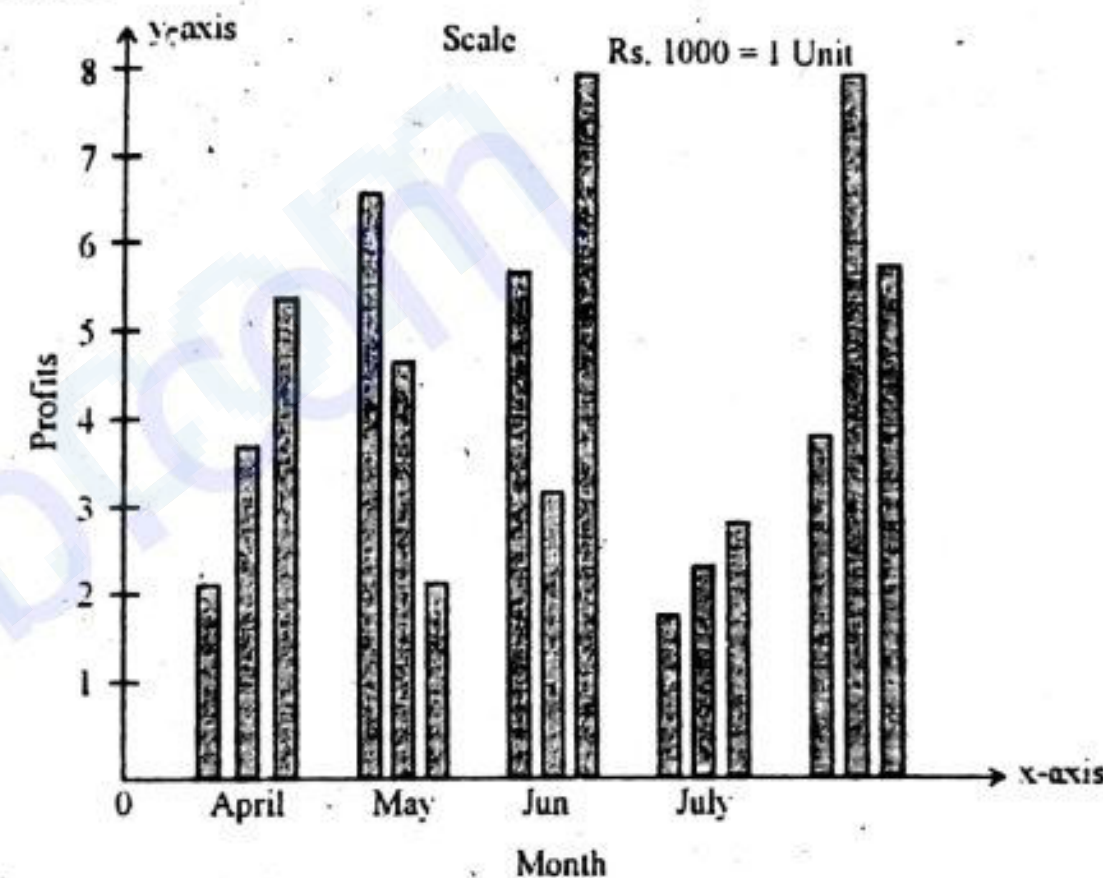
b) Which game is least favourite?

Ans: Badminton

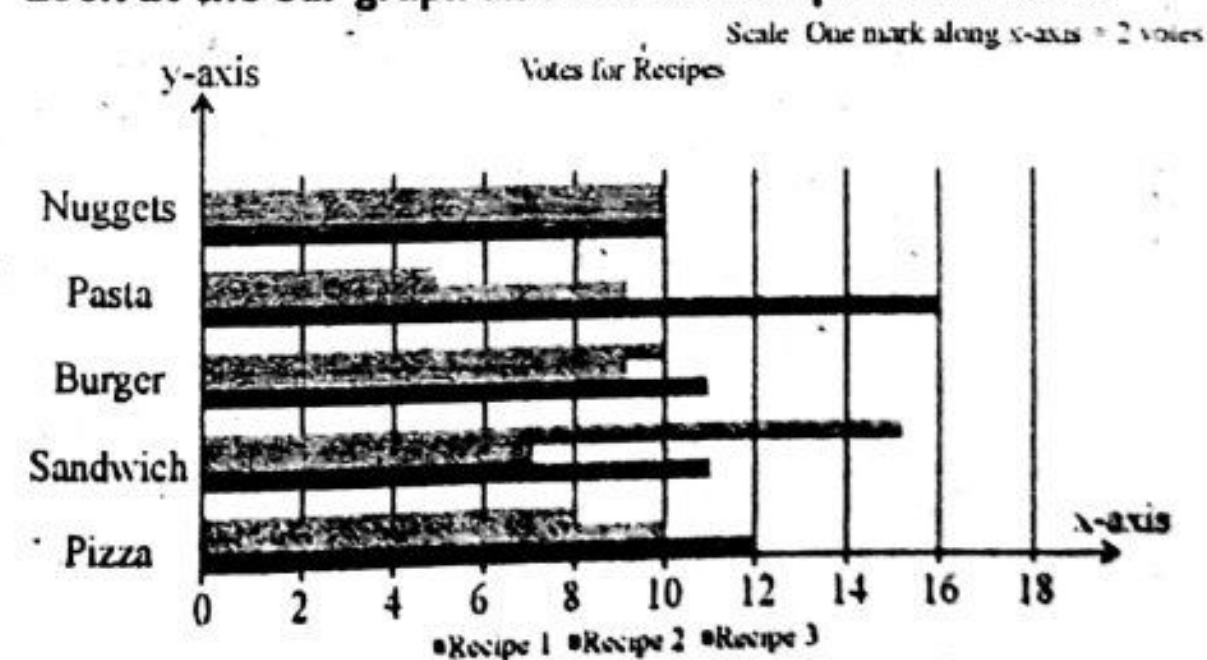
Q4: The following table shows the profit gained by 3 shopkeeper during five months. Draw a multiple bar graph for this data.

	April	May	June	July	August
Umar	Rs 20000	Rs 65000	Rs 55000	Rs 15000	Rs 35000
Ali	Rs 35000	Rs 45000	Rs 30000	Rs 20000	Rs 76000
Kahlid	Rs 52000	Rs 19000	Rs 78000	Rs 25000	Rs 54000

Solution:



Q5: Look at the bar graph and answer the question below.



a) Which items recipe got the greatest vote?

Ans: Greatest vote = $16 \times 2 = 32$

b) How many people vote for the recipe 3 of the pasta?

Ans: Number of vote = $2 \times 5 = 10$

c) How many people vote for three recipes of sandwich?

Ans: Number of vote = $8 \times 2 + 7 \times 2 + 15 \times 2 = 16 + 14 + 30 = 60$

d) How many people vote for the recipe 2 of the pizza?

Ans: Number of vote = $10 \times 2 = 20$

e) How many people vote for recipe 1 of the burger?

Ans: Number of vote = $11 \times 2 = 22$

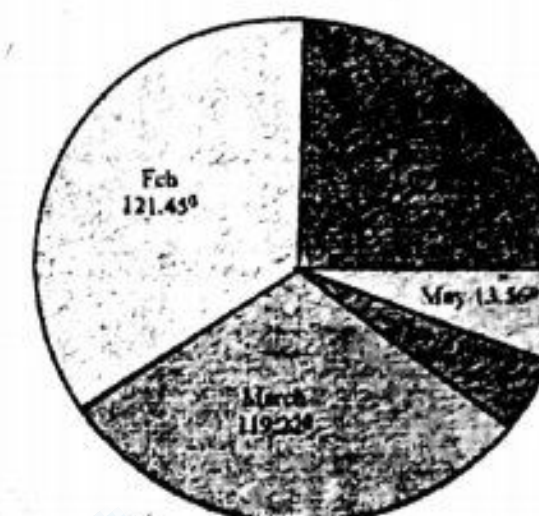
Q6: The following table shows the money Ahmad spends on food in the first 5 months of the year.

Months	January	February	March	April	May
Spent on food	9000	12000	11780	1450	1340

Draw a pie chart for this data.

Solution:

Month	Spent on food	Measurement of central angle
Jan	9000	$\frac{9000}{35,570} \times 360^\circ = 91.09^\circ$
Feb	12000	$\frac{12000}{35,570} \times 360^\circ = 121.45^\circ$
March	11780	$\frac{11780}{35570} \times 360^\circ = 119.22^\circ$
April	1450	$\frac{1450}{35,510} \times 360^\circ = 14.67^\circ$
May	1340	$\frac{1340}{35,570} \times 360^\circ = 13.56$
Total	35,570	360°



Q7: Find mean, median and mode for the following observations,

(i) 52, 48, 50, 49, 47

Solution:

52, 48, 50, 49, 47

$$\text{As } \bar{X} = \frac{\sum X}{n} = \frac{52 + 48 + 50 + 49 + 47}{5}$$

$$= \frac{246}{5}$$

$$= 49.2$$

So Mean = 49.2

For Median: We first write data in ascending order

47, 48, 49, 50, 52

Here

$$n = 5$$

$$\text{So Median} = \left(\frac{n+1}{2} \right)^{\text{th item}}$$

$$= \left(\frac{5+1}{2} \right)^{\text{th item}}$$

$$= \left(\frac{6}{2} \right)^{\text{th item}}$$

$$= 3^{\text{rd item}}$$

So Median = 49

Mode:

As frequency of all terms are '1'

So Mode = No mode

(ii) 207, 301, 205, 350, 322, 205

Solution:

$$\begin{aligned}\text{As Mean} &= \frac{\Sigma X}{n} \\ &= \frac{207 + 301 + 205 + 350 + 322 + 205}{6} \\ &= \frac{1590}{6} \\ &= 265\end{aligned}$$

For Median:

First write in ascending order

205, 205, 207, 301, 322, 350

Here $n = 6$

$$\begin{aligned}\text{Median} &= \left(\frac{n+1}{2} \right) \text{th item} \\ &= \left(\frac{6+1}{2} \right) \text{th item} \\ &= \left(\frac{7}{2} \right) \text{th item} \\ &= 3.5 \text{th item}\end{aligned}$$

As 3.5 is the half way between the 3rd and 4th items.

$$\begin{aligned}\text{So Median} &= \frac{3\text{rd} + 4\text{th}}{2} \\ &= \frac{207 + 301}{2} \\ &= 254\end{aligned}$$

Mode:

As frequency of 205 is '2'

So mode = 205