More Notes, Books Fbise Visit Now: www.perfect24u.com

Exercise 3.6

Evaluate the following definite integrals

Q1. $= \int_1^2 (x^2 + 1)dx$

Solution

$$= \int_1^2 (x^2 + 1)dx$$

$$= \int_1^2 x^2 dx + \int_1^2 1dx = \frac{x^3}{3}\Big|_1^2 + x\Big|_1^2$$

$$= \left(\frac{(2)^3-(1)^3}{3}\right) + \{(2) - (1)\}$$

$$= \left(\frac{8-1}{3}\right) + 1 = \frac{7}{3} + 1$$

$$= \frac{7+3}{3} = \frac{10}{3}$$

Q2. $= \int_1^1 \left(x^{1/3} + 1\right) dx$

Solution

$$= \int_1^1 \left(x^{1/3} + 1\right) dx$$

$$= \int_1^1 x^{1/3} dx + \int_1^1 1\, dx$$

$$= \frac{x^{1/3}}{1/3}\Big|_{-1}^1 + x\Big|_{-1}^1$$

$$= \frac{3}{4} x^{4/3}\Big|_{-1}^1 + \{(1)-(-1)\}$$

$$= \frac{3}{4}[(1)^{4/3} - (-1)^{3/4}] + 1 + 1$$

$$= \frac{3}{4}[1-(-1)^{2/3}] + 2$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

Q3. $= \int_{-2}^{0} \frac{1}{(2x-1)^2} dx$

$= \int_{-2}^{0} (2x-1)^{-2} dx$

$= \frac{1}{2} \int_{-2}^{0} (2x-1)^{-2} dx$

$= \frac{1}{2} \frac{(2x-1)^{-2+1}}{-2+1} \Big|_{-2}^{0} = \frac{-1}{2} (2x-1)^{-1} \Big|_{-2}^{0}$

$= \frac{-1}{2} \frac{1}{2x-1} \Big|_{-2}^{0} = \frac{-1}{2} \left[\frac{1}{2(0)-1} - \frac{1}{2(-2)-1}\right]$

$= \frac{-1}{2}\left[-1 - \frac{1}{5}\right] = \frac{-1}{2}\left[\frac{-5+1}{5}\right]$

$= \left(\frac{-1}{2}x\right)\left(\frac{-4}{5}\right) = \frac{2}{5}$

Q4. $= \int_{-6}^{2} \sqrt{3-x}\, dx$

Solution

$= (-1) \int_{-6}^{2} (3-x)^{1/2} (-1) dx$

$= (-1) \frac{(3-x)^{1/2+1}}{1/2+1} \Big|_{-6}^{2}$

$= (-1)\, {}^{2}/_{3} \; (3-x)^{3/2} \Big|_{-6}^{2}$

$= \frac{-2}{3}[(3-2)^{3/2} - (3-(-6))^{3/2}]$

$= \frac{-2}{3}[(1)^{3/2} - (9)^{3/2}]$

$= \frac{-2}{3}[1 - (3)^{2\times 3/2}]$

$= \frac{-2}{3}[1 - (3)^3]$

$= \frac{-2}{3}[1-27] = \frac{-2}{3}(-26)$

$= \frac{52}{3}$

More Notes, Books Fbise Visit Now: www.perfect24u.com

Solution

$$= \int_1^{\sqrt{5}} \sqrt{(2t-1)^3}\,dx$$

$$= \frac{1}{2}\int_1^{\sqrt{5}} (2t-1)^{3/2} \times (2)dx$$

$$= \frac{1}{2}\frac{(2t-1)^{3/2+1}}{3/2+1}\Big|_1^{\sqrt{5}}$$

$$= \frac{1}{2}\ \frac{2}{5}|(2t-1)^{5/2}|\ _1^{\sqrt{5}}$$

$$= \frac{1}{5}\ (2t-1)^{5/2}\ _1^{\sqrt{5}} = \frac{1}{5}[(2\sqrt{5})-1]^{5/2} - \{2(1)-1\}^{5/2}$$

$$= \frac{1}{5}[(2\sqrt{5}-1)^{5/2} - (1)^{5/2}]$$

$$= \frac{1}{5}[(2\sqrt{5}-1)^{5/2} - 1]$$

Q6. $\int_2^{\sqrt{5}} x\sqrt{x^2-1}dx$

Solution

$$= \int_2^{\sqrt{5}} (x^2-1)^{\frac{1}{2}}\,x\,dx$$

$$= \frac{1}{2}\int_2^{\sqrt{5}} (x^2-1)^{\frac{1}{2}}\,x\,dx$$

$$= \frac{1}{2}\ \frac{(x^2-1)^{1/2+1}}{1/2+1}\Big|_2^{\sqrt{5}}$$

$$= \frac{1}{2}\times\frac{2}{3}\ (x^2-1)^{3/2}\ _2^{\sqrt{5}}$$

$$= \frac{1}{3}\left[[(\sqrt{5})^2-1]^{\frac{3}{2}} - [(2)^2-1]^{\frac{3}{2}}\right]$$

$$= \frac{1}{3}\left[[5-1]^{\frac{3}{2}} - [4-1]^{\frac{3}{2}}\right]$$

$$= \frac{1}{3}\left[[4]^{\frac{3}{2}} - [3]^{\frac{3}{2}}\right]$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

Q7. $\int_1^2 \frac{x}{x^2+2}\,dx$

Solution

$$\int_1^2 \frac{x}{x^2+2}\,dx = \frac{1}{2}\int_1^2 \frac{2x}{x^2+2}\,dx$$

$$= \frac{1}{2}|ln\ x^2+2\ |_1^2 = \frac{1}{2}[ln\{(2)^2+2\} - ln\{(1)^2+2\}]$$

$$= \frac{1}{2}[ln(4+2) - ln(1+2)] = \frac{1}{2}[ln6 - ln3]$$

$$= \frac{1}{2}ln\frac{6}{3} = \frac{1}{2}ln\,2$$

Q8. $\int_2^3 (x - \frac{1}{x})^2\,dx$

Solution

$$\int_2^3 (x-\frac{1}{x})^2\,dx = \int_2^3 [x^2 - 2x(\frac{1}{x}) + \frac{1}{x^2}]\,dx$$

$$= \int_2^3 [x^2 - 2 + x^{-2}]\,dx = \int_2^3 x^2\,dx - 2\int_2^3 1\,dx + \int_2^3 x^{-2}\,dx$$

$$= \frac{x^3}{3}\ \frac{3}{2} - 2\ x\ \frac{3}{2} + \frac{x^{2+1}}{-2+1}\ \frac{3}{2} = \frac{(3)^3-(2)^3}{3} - 2[(3)-(2)] - [\frac{1}{x}]_2^3$$

$$= \frac{27-8}{3} - 2(1) - [\frac{1}{3} - \frac{1}{2}] = \frac{19}{3} - 2 - [\frac{2-3}{3}]$$

$$= \frac{19}{3} - 2 - \frac{1}{6}$$

$$= \frac{38-12+1}{6} = \frac{27}{6} = \frac{9}{2}$$

Q9. $\int_{-1}^1 (x+\frac{1}{2})\sqrt{x^2+x+1}\,dx$

Solution

$$\int_{-1}^1 (x+\frac{1}{2})\sqrt{x^2+x+1}\,dx$$

$$= \frac{1}{2}\int_{-1}^1 (2x+1)(x^2+x+1)^{\frac{1}{2}}\,dx$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

$$= \frac{1}{2} \frac{(x^2+x+1)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \Big|_{-1}^{1} = \frac{1}{2} \times \frac{2}{3} (x^2+x+1)^{\frac{3}{2}} \Big|_{-1}^{1}$$

$$= \frac{1}{3}[\{(1)^2 + (1) + 1)^{\frac{3}{2}} - \{(-1)^2 - 1 + 1)^{\frac{3}{2}}]$$

$$= \frac{1}{3}[(3)^{\frac{3}{2}} - (1)^{\frac{3}{2}}] = \sqrt{3} - \frac{1}{3}$$

Q10. $\int_0^3 \frac{dx}{x^2+9}$

Solution

$$\int_0^3 \frac{dx}{x^2+9} = \int_0^3 \frac{1}{(x)^2+(3)^2} dx$$

$$= \frac{1}{3} \tan^{-1} \frac{1}{3} \Big|_0^3 = \frac{1}{3}[\tan^{-1}(\frac{3}{3}) - \tan^{-1} 0]$$

$$= \frac{1}{3}[\tan^{-1}(1) - \tan^{-1} 0] = \frac{1}{3}(\frac{\pi}{4} - 0) = \frac{\pi}{12}$$

Q11 $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{Cos t dt}$

Solution

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{Cos t dt} = Sin\ t \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} \quad [\because \frac{d}{dt}(Sin\ t) = Cos\ t]$$

$$= \text{Sin}\ \frac{\pi}{3} - Sin\ \frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$\Rightarrow \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \text{Cos t dt} = \frac{\sqrt{3}-1}{2}$$

Q12. $\int_1^2 (x + \frac{1}{x})^{\frac{1}{2}}(1 - \frac{1}{x^2})$ dx

Solution

$$\int_1^2 (x + \frac{1}{x})^{\frac{1}{2}}(1 - \frac{1}{x^2})\ dx$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

$dy = (1 - \frac{1}{x^2})dx$

when $x = 2, y = \frac{5}{2}$

when $x = 1, y = 2$

$= \int_2^{\frac{1}{2}} y^{\frac{1}{2}}\ dy = \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\ {}_2^{\frac{1}{2}} = \frac{2}{3}\ y^{\frac{1}{2}+1}\ {}_2^{\frac{1}{2}} = \frac{2}{3}[\ (5/2)^{\frac{3}{2}} - (2)^{\frac{3}{2}}] = \frac{2}{3}[\frac{5^{3/2}}{2^{3/2}} - 2^{3/2}]$

$= \frac{2}{3}[\frac{5\sqrt{5} - 2^3}{2\sqrt{2}} = \frac{2}{3\sqrt{2}}[5\sqrt{5} - 8]$

Q13. $\int_1^2 \ln x\, dx$

Solution

$\int_1^2 \ln x\, dx = x \ln x\ {}_1^2 - \int_1^2 x \frac{d}{dx}(\ln x)\, dx$

$= x \ln x\ {}_1^2 - \int_1^2 x \times \frac{1}{x}\, dx = 2\ln(2) - 1\ln(1) - \int_1^2 1\, dx$

$= 2 \ln 2 - (0) - x\ {}_1^2 \quad (\because \ln 1 = 0)$

$= 2\ln 2 - (2-1)$

$= 2 \ln 2 - 1$

Q14. $\int_0^2 (e^{x/2} - e^{-x/2})dx$

Solution

$\int_0^2 (e^{x/2} - e^{-x/2})dx = \int_0^2 e^{x/2}\, dx - \int_0^2 e^{-x/2} dx$

$= 2\int_0^2 e^{x/2} \times \frac{1}{2} dx + 2\int_0^2 e^{-x/2} \times (-\frac{1}{2})dx$

$= 2\ e^{x/2}\ {}_0^2 + 2\ e^{-x/2}\ {}_0^2 = 2[e^{2/2} - e^0] + 2[e^{-2/2} - e^0]$

$= 2[e-1] + [e^{-1} - 1] = 2[e - 1 + \frac{1}{e} - 1]$

$= 2[e^{-1} - 2] = 2[\ldots - 2]$

More Notes, Books Fbise Visit Now: www.perfect24u.com

Q15. $\int_0^{\frac{\pi}{4}} \frac{\text{Cos}\,\theta+\text{Sin}\theta}{\text{Cos}\,2\theta} d\theta$

Solution

$$\int_0^{\frac{\pi}{4}} \frac{\text{Cos}\,\theta+\text{Sin}\theta}{2Cos^2\theta} d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\text{Cos}\,\theta}{Cos^2\theta} d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}} \frac{\text{Sin}\,\theta}{\text{Cos}\theta} \times \frac{1}{Cos\theta} d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \text{Sec}\,\theta d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}} Sec\,\theta\, tan\theta\, d\theta$$

$$= \frac{1}{2} \ln(Sec\,\theta\, tan\theta)\Big|_0^{\frac{\pi}{4}} + \frac{1}{2} Sec\,\theta\Big|_0^{\frac{\pi}{4}}$$

$$= \frac{1}{2}[\ln(Sec\frac{\pi}{4} + tan\frac{\pi}{4}) - \ln(Sec\,0 + tan\,0)]$$

$$+ \frac{1}{2}[Sec\frac{\pi}{4} - Sec\,0]$$

$$= \frac{1}{2}[ln|\sqrt{2}+1| - ln|1+0| + \frac{1}{2}[(\sqrt{2}) - (1)]$$

$$= \frac{1}{2}[ln|\sqrt{2}+1| - 0 + \frac{1}{2}[(\sqrt{2}) - (1)]$$

$$= \frac{1}{2}[ln|\sqrt{2}+1| - 0 + \sqrt{2} - 1]$$

Q16. $\int_0^{\frac{\pi}{6}} \text{Cos}^3\theta d\theta$

Solution

$$\int_0^{\frac{\pi}{6}} \text{Cos}^3\theta d\theta = \int_0^{\frac{\pi}{6}} \text{Cos}^2\theta \times \text{Cos}\theta d\theta$$

$$= \int_0^{\frac{\pi}{6}} (1 - \text{Sin}^2\theta)\text{Cos}\theta d\theta = Sin\,\theta\Big|_0^{\frac{\pi}{6}} - \frac{\text{Sin}^3\theta}{3}\Big|_0^{\frac{\pi}{6}}$$

$$= (\text{Sin}\frac{\pi}{6} - Sin\,0) - [\frac{\text{Sin}^3(\frac{\pi}{6}) - \text{Sin}^3(0)}{3}]$$

$$= (\frac{1}{2} - 0) - \frac{1}{3}[(\frac{1}{2})^3 - (0)^3$$

$$= \frac{1}{2} - \frac{1}{3} \times \frac{1}{8} = \frac{1}{2} - \frac{1}{24}$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

Q17. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \mathbf{Cos^2\theta Cot^2\theta d\theta}$

Solution

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{Cos}^2\theta(\text{Cosec}^2\theta - 1)\text{d}\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{Cos}^2\theta \times \text{Cosec}^2\theta \, \text{d}\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{Cos}^2\theta \, \text{d}\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{Cot}^2\theta \, \text{d}\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \left(\frac{1+\text{Cos}^2\theta}{2}\right) \text{d}\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\text{Cot}^2\theta - 1) \, \text{d}\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1 \, \text{d}\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{Cos } 2\theta \, \text{d}\theta$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\text{Cosec}^2\theta \, \text{d}\theta - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1 \, \text{d}\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1 \, \text{d}\theta - \frac{1}{2} \times \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{Cos } 2\theta \times 2\text{d}\theta$$

$$= -Cot\,\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} - \frac{3}{2}\,\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} - \frac{1}{4}\,Sin2\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= -[(\text{Cot}\frac{\pi}{4} - Cot\,\frac{\pi}{6})] - \frac{3}{2}(\frac{\pi}{4} - \frac{\pi}{6}) - \frac{1}{4}(Sin\,\frac{\pi}{2} - Sin\,\frac{\pi}{3})$$

$$= (1-\sqrt{3}) - \frac{3}{2}(\frac{3-2}{12})\,\pi - \frac{1}{4}(-\frac{\sqrt{3}}{2})$$

$$= \sqrt{3} - 1 - \frac{\pi}{8} - \frac{1}{4} + \frac{\sqrt{3}}{8} = \frac{3\sqrt{3}-8-\pi-2+\sqrt{3}}{8}$$

$$= \frac{9\sqrt{3}-10-\pi}{8}$$

Q18. $\int_0^{\frac{\pi}{4}} \mathbf{Cos^4 t dt}$

Solution

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} (2\text{Cos}^2\text{t})(2\text{Cos}^2\text{t})\text{dt} = \frac{1}{4}\int_0^{\frac{\pi}{4}} (1 + \text{Cos } 2\text{t})^2\text{dt}$$

$$= \frac{1}{4}[\int_0^{\frac{\pi}{4}} 1 + \text{Cos}^2 2\text{t} + 2\text{Cos } 2\text{t})\text{dt}\,]$$

$$= \frac{1}{4}[\int_0^{\frac{\pi}{4}} 1 + \frac{1}{2}\int_0^{\frac{\pi}{4}}(1 + Cos\,4t)\,dt + \int_0^{\frac{\pi}{4}}(Cos2t)2\,dt\,]$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

$$= \frac{1}{4}[(\frac{\pi}{4} - 0) + \frac{1}{2}(\frac{\pi}{4} - 0) + \frac{1}{8}(\text{Sin } 4\frac{\pi}{4} - Sin\ 0) + (\text{Sin } 2\frac{\pi}{4} - Sin\ 0)]$$

$$= \frac{1}{4}[\frac{\pi}{4} + \frac{\pi}{8} + \frac{1}{8}(Sin\ \pi - Sin\ 0) + (Sin\ \frac{\pi}{4} - Sin\ 0)]$$

$$= \frac{1}{4}[\frac{\pi}{4} + \frac{\pi}{8} + \frac{1}{8}(0 - 0) + 1 - 0]$$

$$= \frac{1}{4}[\frac{2\pi + \pi}{8} + 1] = \frac{3\pi+8}{32}$$

Q19. $\int_0^{\frac{\pi}{3}} \mathbf{Cos^2\ Sin\theta d\theta}$

Solution

$$= -\int_0^{\frac{\pi}{3}} \text{Cos}^2\theta \times (-\text{Sin}\theta)d\theta = -\left[\frac{\text{Cos}^3\theta}{3}\right]_0^{\frac{\pi}{3}}$$

$$= -[\frac{\text{Cos}^3\frac{\pi}{3} - \text{Cos}^3\theta}{3}] = -[\frac{(1/2)^3 - (1)^3}{3}]$$

$$= -\frac{1}{3}[\frac{1}{8} - 1] = -\frac{1}{3}[\frac{1-8}{3}]$$

$$= -\frac{1}{3} \times \frac{-7}{8} = \frac{7}{24}$$

Q20. $\int_0^{\frac{\pi}{4}} \mathbf{(1 + Cos^2\theta)tan^2\theta d\theta}$

Solution

$$= \int_0^{\frac{\pi}{4}} \text{tan}^2\theta d\theta + \int_0^{\frac{\pi}{4}} \text{Cos}^2\theta \times \frac{\text{Sin}^2\theta}{\text{Cos}^2\theta} d\theta$$

$$= \int_0^{\frac{\pi}{4}} (\text{Sec}^2\theta - 1)d\theta + \int_0^{\frac{\pi}{4}} \text{Sin}^2\theta d\theta$$

$$= \int_0^{\frac{\pi}{4}} \text{Sec}^2\theta\, d\theta - \int_0^{\frac{\pi}{4}} 1\, d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}} 2\text{Sin}^2\theta d\theta$$

$$= \text{tan}\,\theta\ {}_0^{\frac{\pi}{4}} - \theta\ {}_0^{\frac{\pi}{4}} + \frac{1}{2}\int_0^{\frac{\pi}{4}} (1 - \text{Cos}\ 2\theta)d\theta$$

$$= [(\text{tan}\frac{\pi}{4} - tan\ 0)] - (\frac{\pi}{4} - 0) + \frac{1}{2}\int_0^{\frac{\pi}{4}} 1\, d\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} 2Cos\ 2\theta d\theta$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

$$= 1\frac{\pi}{4} + \frac{1}{2}(\frac{\pi}{4} - 0) - \frac{1}{4}(Sin\ 2 \times \frac{\pi}{4} - Sin\ 0)$$

$$= 1 - \frac{\pi}{4} + \frac{\pi}{8} - \frac{\pi}{4} = \frac{3}{4} + \frac{\pi - 2\pi}{4} = \frac{3}{4} - \frac{\pi}{8}$$

$$= \frac{3}{4} - \frac{\pi}{8} = \frac{6-\pi}{8}$$

Q21. $\int_0^{\frac{\pi}{4}} \frac{Sec\theta}{Sin\theta + Cos\,\theta} d\theta$

Solution

$$= \int_0^{\frac{\pi}{4}} \frac{Sec\theta}{Cos\,\theta(\frac{Sin\,\theta}{Cos\,\theta}+1)} d\theta = \int_0^{\frac{\pi}{4}} \frac{Sec^2\theta}{(\tan\theta+1)} d\theta$$

$$= \int_0^{\frac{\pi}{4}} (1 + \tan\theta)^{-1} Sec^2\theta\, d\theta = \ln(1 + tan\,\theta)\Big|_0^{\frac{\pi}{4}}$$

$$= \ln|1+\tan\frac{\pi}{4}| - \ln|1+\tan 0|$$

$$= \ln|1+1| - \ln|1+0| = \ln 2 - \ln 1 = \ln\frac{2}{1}\ \ln 2$$

Q22. $\int_{-1}^{5} |x-3|$

Solution

$$= \int_{-1}^{3} |x-3|\, dx + \int_{3}^{5} |x-3|\, dx$$

$$= \int_{-1}^{3} (x-3)\, dx + \int_{3}^{5} (x-3)\, dx$$

$$= \frac{(x-3)^2}{2}\Big|_{-1}^{3} + \frac{(x-3)^2}{2}\Big|_{3}^{5}$$

$$= \frac{1}{2}[(3-3)^2 - (-1-3)^2] + \frac{1}{2}[(5-3)^2 - (3-3)^2]$$

$$= -\frac{1}{2}[0-16] + \frac{1}{2}[4-0] = 8+2 = 10$$

Q23. $\int_{1/8}^{1} \frac{(x^{1/3}+2)^2}{x^{2/3}}$ dx

More Notes, Books Fbise Visit Now: www.perfect24u.com

$$= \int_{1/8}^{1} (x^{1/3} + 2)^2 \frac{1}{3} \times \frac{-2}{3}\, dx$$

$$= 3 \left.\frac{(x^{1/3}+2)^{2+1}}{2+1}\right|_{1/8}^{1} = \left.(x^{1/3} + 2)^3\right|_{1/8}^{1}$$

$$= ((1)^{1/3} + 2)^3 - ((\tfrac{1}{8})^{1/3} + 2)^3 = (1 + 2)^3 - ((\tfrac{1}{8})^{1/3} + 2)^3$$

$$= 27 - (\tfrac{1}{2} + 2)^{1/3} = 27 - \frac{125}{8} = \frac{91}{8}$$

Q24. $\int_1^3 \frac{x^2-2}{x+1}\, dx$

Solution

$$= \int_1^3 \frac{x^2-1-1}{x+1}\, dx = \int_1^3 (x - 1)\, dx - \int_1^3 \frac{1}{x+1}\, dx$$

$$= \left.\frac{(x-1)}{2}\right|_1^3 - \left. ln(x + 1)\right|_1^3$$

$$= \frac{(3-1)^2-(1-1)^2}{2} - [\ln(3+1) - \ln(1+1)]$$

$$= \frac{2^2-0}{2} - [\ln 4 - \ln 2]$$

$$= \frac{4}{2} \ln(\tfrac{4}{2}) = 2 - \ln 2$$

Q25. $\int_2^3 \frac{3x^2-2x+1}{(x-1)(x^2+1)}\, dx$

Solution

Consider $\frac{3x^2-2x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)}$ .....(i)

Multiplying on both sides by $(x - 1)(x^2 + 1)$ we have

$$3x^2 - 2x + 1 = A(x^2 + 1) + (Bx + C)(x - 1) \text{.....(ii)}$$

Put $x = 1$ in(ii) we have

$$(1)^2 - 2(1) + 1 = A((1)^2 + 1)$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

Comparing Co-efficients of $(x)^2$,x and constant terms of (ii) , we have

$A+B = 3 \quad \Rightarrow 1+B = 3 \Rightarrow B = 2$

$-B + C = -2$

$-2+C = -2 \quad \Rightarrow C = -2+2 = 0 \Rightarrow C = 0$

Hence (1) can be written as:

$$= \int_2^3 \frac{3x^2-2x+1}{(x-1)(x^2+1)}\,dx \int_2^3 \frac{1}{(x-1)}\,dx + \int_2^3 \frac{2x}{(x^2+1)}\,dx$$

$$= \ln(x-1)\Big|_2^3 + \ln(x^2+1)\Big|_2^3$$

$$= \ln|3-1| - \ln|2-1| + \ln|(3)^2+1| - \ln|(2)^2+1|$$

$$= \ln 2 - \ln 1 + \ln 10 - \ln 5 = \ln 2 - \ln 5 + \ln 10$$

$$= \ln\left(\frac{2}{5}\right) + \ln 10 = \ln\left(\frac{2}{5} \times 10\right)$$

$$= \ln 4$$

Q26. $\int_0^{\pi/4} \frac{1}{1+\sin x}\,dx$

Solution

$$= \int_0^{\pi/4} \frac{1}{1+\sin x} \times \frac{(1-\sin x)}{(1-\sin x)}\,dx = \int_0^{\pi/4} \frac{(1-\sin x)}{(1-\sin^2 x)}\,dx$$

$$= \int_0^{\pi/4} \frac{(1-\sin x)}{\cos^2 x}\,dx = \int_0^{\pi/4} \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}]\,dx$$

$$= \int_0^{\pi/4} \sec^2 x\,dx - \int_0^{\pi/4} \sec x \tan x\,dx$$

$$= \tan x\Big|_0^{\pi/4} - \sec x\Big|_0^{\pi/4}$$

$$= \tan \pi/4 - \tan 0 - [\sec \pi/4 - \sec(0)] = 1 - 0 - [\sqrt{2} - 1]$$

$$= 1 - \sqrt{2} + 1 = 2 - \sqrt{2}$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

$$= -\int_0^1 \frac{-3x}{\sqrt{4-3x}}\,dx = -\int_0^1 \frac{4-3x-4}{\sqrt{4-3x}}\,dx$$

$$= -\int_0^1 \frac{4-3x}{\sqrt{4-3x}}\,dx + 4 - \int_0^1 \frac{1}{\sqrt{4-3x}}\,dx$$

$$= -\int_0^1 \sqrt{4-3x}\,dx + 4 \times \left(\frac{-1}{3}\right) \int_0^1 (4-3x)^{\frac{1}{2}}\,(-3)\,dx$$

$$= \frac{1}{3}\int_0^1 (4-3x)^{\frac{1}{2}}\,(-3)dx - \frac{4}{3}\int_0^1 (4-3x)^{\frac{1}{2}}\,(-3)\,dx$$

$$= \frac{1}{3}\ \frac{(4-3x)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\ \Big|_0^1 - \frac{4}{3}\ \frac{(4-3x)^{\frac{1}{2}+1}}{-\frac{1}{2}+1}\ \Big|_0^1$$

$$= \frac{2}{9}[(4-3(1))^{\frac{3}{2}} - (4-0)^{\frac{3}{2}}]\ \frac{8}{3}[(4-3(1))^{\frac{1}{2}} - (4-0)^{\frac{1}{2}}]$$

$$= \frac{2}{9}[(1))^{\frac{3}{2}} - (4)^{\frac{3}{2}}]\ \frac{8}{3}[(1))^{\frac{1}{2}} - \sqrt{4}]$$

$$= \frac{2}{9}[1-(2)^3]\ \frac{8}{3}[1-2] = \frac{2}{9}[1-8]\ \frac{8}{3}[-1]$$

$$= \frac{2}{9}[-7] + \frac{8}{3} = \frac{-14}{9} + \frac{8}{3} = \frac{-14+24}{9} = \frac{10}{9}$$

Q 29. $\int_{\pi/6}^{\pi/2} \frac{\text{Cos x}}{\text{Sin x(2+Sin x)}}\,\text{dx}$

Solution

$$= \int_{\pi/6}^{\pi/2} \left(\frac{2}{Sin\ x} + 1\right)^{-1}(-2Cosec\ x\ Cot\ x)\,dx$$

$$= \int_{\pi/6}^{\pi/2} (2Cosec\ x + 1)^{-1}(-2Cosec\ x\ Cot\ x)\,dx$$

$$= -\frac{1}{6}\int_{\pi/6}^{\pi/2} (2Cosec\ x + 1)^{-1}(-2Cosec\ x\ Cot\ x)\,dx$$

$$= -\frac{1}{2}\ \ln(2\text{Cosec x} + 1)\ \Big|_{\pi/6}^{\pi/2} = -\frac{1}{2}\ \ln\left(\frac{2}{Sin\ x} + 1\right)\ \Big|_{\pi/6}^{\pi/2}$$

$$= -\frac{1}{2}\left[ln\left(\frac{2}{1} + 1\right) - ln\left(\frac{2}{1/2+1}\right)\right] = -\frac{1}{2}[\ln(2+1) - \ln(4+1)]$$

$$= \frac{1}{2}[\ln 3 - \ln 5] = \frac{1}{2}[\ln 5 - \ln 3]$$

More Notes, Books Fbise Visit Now: www.perfect24u.com

Q30. $\int_0^2 \frac{\text{Sin x}}{(1+\text{Cosx})(2+\text{Cosx})}\,\text{dx}$

Solution

Let u = 1+Cos x

When x = 0, u = 2

When $x = \frac{\pi}{2}$, u = 1

du = - Sin x dx

-du = Sin x dx

Now $\int_0^{\pi/2} \frac{\text{Sin x}}{(1+\text{Cosx})(2+\text{Cosx})}\,\text{dx} = \int_2^1 [\frac{-\text{du}}{u(1+u+1)}]\text{du}$

$= -\int_2^1 [\frac{1}{u} - \frac{1}{u+1}]\text{du} = \int_2^1 \frac{1}{u}\,\text{du} + \int_2^1 \frac{1}{u+1}\,\text{du}$

$= -\ \ln u\ {}_1^2 + \ln(u+1)\ {}_1^2$

= -(ln 2 – ln 1) + (ln (2+1) – ln (1+1))

= ln 2+ 0 + ln 3 – ln 2 = -2 ln 2 + ln 3

= -ln 4 + ln 3 = ln 3 – ln 4 = $\ln \frac{3}{4}$

More Notes, Books Fbise Visit Now: www.perfect24u.com