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Exercise 3.4

Q1. Evaluate the following integral by parts add a word representing all the functions are defined

i. $\int x \sin x dx$

Solution

$$I = \int x \sin x \, dx$$

Integral by parts taking x as first function

$$= x(-\cos x) - \int \cos x \, \frac{d}{dx} x(dx)$$

$$= -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + C$$

$$= \sin x - x \cos x + C$$

ii. $\int \ln x \, dx$

Solution

$$I = \int \ln x . 1 dx$$

Integral by parts taking lnx as first function

$$= \ln x \, . \, x \int x \, \frac{d}{dx}(lnx)dx$$

$$= x \ln x - \int x \, \frac{1}{x} dx$$

$$= x \ln - \int dx$$

$$= x \ln x - x + C$$

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Solution

$$I = \int \ln x \, .x dx$$

Integral by parts taking lnx as first function

$$= \ln x \frac{x^2}{2} - \int \frac{x^2}{2} \frac{d}{dx}(\ln x) dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x^2 . \frac{1}{x} dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} \left(\frac{x^{1+1}}{1+1}\right) + C$$

$$= \frac{x^2}{2} \ln x - \frac{1}{2} . \frac{x^2}{2} + C$$

$$= \frac{x^2}{2} \ln x - \frac{1}{4} x^2 + C$$

$$= \frac{x^2}{2} \left[\ln x - \frac{1}{2}\right] + C$$

iv. $\int x^2 \ln x dx$

Solution

$$I = \int x^2 ln \, x \, dx$$

Integral by parts taking lnx as first function

$$= \ln x \frac{x^3}{2} - \int \frac{x^3}{2} \frac{d}{dx}(\ln x) dx$$

$$= \frac{x^3}{2} \ln x - \frac{1}{3} \int x^3 . \frac{1}{x} dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 \, dx$$

$$= \frac{x^3}{3} \ln x - \frac{1}{3} . \frac{x^3}{3} + C$$

$$= \frac{x^3}{2} \ln x - \frac{1}{9} x^3 + C$$

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v. $\int x^3 \ln x \, dx$

Solution

$$I = \int x^3 ln\, x \, dx$$

Integral by parts taking lnx as first function

$$= \frac{x^4}{4} \ln x - \int \frac{x^4}{4} . \frac{1}{x} dx$$

$$= \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx$$

$$= \frac{x^4}{4} \ln x - \frac{1}{4} . \frac{x^4}{4} + C$$

$$= \frac{x^4}{4} \ln x - \frac{1}{16} x^4 + C$$

$$= \frac{x^4}{4} [\ln x - \frac{1}{4}] + C$$

vi. $\int x^4 \ln x \, dx$

Solution

$$I = \int x^4 ln\, x \, dx$$

Integral by parts taking lnx as first function

$$= \frac{x^5}{5} \ln x - \int \frac{x^5}{5} . \frac{1}{x} dx$$

$$= \frac{x^5}{5} \ln x - \frac{1}{5} \int x^4 \, dx$$

$$= \frac{x^5}{5} \ln x - \frac{1}{5} . \frac{x^5}{5} + C$$

$$= \frac{x^5}{5} \ln x - \frac{1}{25} x^5 + C$$

$$= \frac{x^5}{5} [\ln x - \frac{1}{5}] + C$$

vii $\int \tan^{-1} x \, dx$

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$$I = \int \tan^{-1} x \, dx$$

Integral by parts taking $\tan^{-1} x$ as first function

$$= \tan^{-1} x \, . x - \int x \frac{d}{dx}(\tan^{-1} x) dx$$

$$= x \tan^{-1} x - \int \frac{x}{x^2+1} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{x^2+1} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \ln|x^2 + 1| + C$$

viii. $\int x^2 \text{ Sin } x \, dx$

Solution

$$I = \int x^2 \text{ Sin } x \, dx$$

Integral by parts taking x^2 as first function

$$= x^2 \text{ (-Cos x) } - \int - \text{ Cos x } \frac{d}{dx}(x^2) dx$$

$$= -x^2 \text{ Cos x } + \int \text{ Cos x. 2x dx}$$

$$= -x^2 \text{ Cos x} + 2 \int \text{ x Cos x dx}$$

Again integrating by parts x as first function

$$= -x^2 \text{ Cos x} + 2[x \, Sin \, x - \int \text{ Sin x dx}$$

$$= -x^2 \text{ Cos x} + 2[x \, Sin \, x - (-\text{Cos x})] + \text{C}$$

$$= -x^2 \text{ Cos x} + 2x \text{ sin x} + 2 \text{ Cos x} + \text{ C}$$

ix. $\int x^2 \tan^{-1} x \, dx$

Solution

$$I = \int x^2 \tan^{-1} x \, dx$$

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$$= \tan^{-1} x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \frac{d}{dx}(\tan^{-1} x)dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3}\int \frac{x^3}{x^2+1}dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{2}\int (x - \frac{x}{x^2+1})dx \text{ by actual division}$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3}\int x\,dx + \frac{1}{3}\int \frac{x}{x^2+1}dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3}\int x\,dx + \frac{1}{3.2}\int \frac{2x}{x^2+1}dx$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3}\frac{x^2}{2} + \frac{1}{6}\ln|x^2+1| + C$$

$$= \frac{x^3}{3} \tan^{-1} x - \frac{x^2}{6} + \frac{1}{6}\ln|x^2+1| + C$$

x. $\int x \tan^{-1} x\,dx$

Solution

$$I = \int x \tan^{-1} x\,dx$$

Integral by parts taking $\tan^{-1} x$ as first function

$$= \tan^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \frac{d}{dx}(\tan^{-1} x)dx$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}\int \frac{x^2}{x^2+1}dx$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}\int (\frac{x^2+1-1}{x^2+1})dx \text{ by actual division}$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}\int (\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1})dx$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}\int 1\,dx + \frac{1}{32}\int \frac{1}{x^2+1}dx$$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + C$$

$$= \frac{1}{2} \tan^{-1} x\,[x^2+1] - \frac{1}{2}x + C$$

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Solution

$$I = \int x^3 \tan^{-1} x \, dx$$

Integral by parts taking $\tan^{-1} x$ as first function

$$= \tan^{-1} x . \frac{x^4}{4} - \int \frac{x^4}{4} \frac{d}{dx}(\tan^{-1} x) dx$$

$$= \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \int \frac{x^4}{x^2+1} dx$$

$$= \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \int x^2 dx + \frac{1}{4} \int \left(\frac{x^2+1-1}{x^2+1}\right) dx$$

$$= \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \int x^2 dx + \frac{1}{4} \int 1 \, dx - \frac{1}{4} \int \frac{1}{x^2+1} dx$$

$$= \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \frac{x^3}{3} + \frac{1}{4} x - \frac{1}{4} \tan^{-1} x + C$$

$$= \frac{1}{4} [x^4 \tan^{-1} x - \tan^{-1} x - \frac{x^3}{3} + x] + C$$

xii. $\int x^3$ **Cos x dx**

Solution

$$I = \int x^3 \text{ Cos x } dx$$

Integral by parts taking x as first function

$$= x^3 \text{ Sin x } - \int \text{Sin x } 3x^2 dx$$

$$= x^3 \text{ sin x } - 3 \int x^2 \text{Sin x } dx$$

Again integrating by parts

$$= x^3 \text{ sin x} - 3[x^2(-Cosx) - \int (\text{-Cos x}). 2x dx]$$

$$= x^3 \text{ sin } 3x^2 Cos\, x - 6 \int \text{x Cos x } dx$$

$$= x^3 \text{ sin } 3x^2 Cos\, x - 6[x\, Sin\, x - \int \text{Sin x } .1 \, dx]$$

$$= x^3 \text{ sin } 3x^2 Cos\, x - 6x\, Sin\, x + 6 \int \text{Sin x } dx$$

$$= x^3 \text{ sin } 3x^2 Cos\, x - 6x\, Sin\, x - 6\text{Cos x} + C$$

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xiii. $\int \sin^{-1} x \, dx$

Solution

$$I = \int \sin^{-1} x \; dx \, dx$$

Integral by parts taking $\sin^{-1} x$ as first function

$$= \sin^{-1} x . x - \int x \frac{d}{dx}(\sin^{-1} x) dx$$

$$= x\sin^{-1} x - \int x . \frac{1}{\sqrt{1-x^2}} dx$$

$$= x\sin^{-1} x - \int x(1-x^2)^{\frac{-1}{2}} dx$$

$$= x\sin^{-1} x + \frac{1}{2}\int -2x(1-x^2)^{\frac{-1}{2}} dx$$

$$= x\sin^{-1} x + \frac{1}{2}\frac{(1-x^2)^{\frac{-1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= x\sin^{-1} x + \frac{1}{2}\frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= x\sin^{-1} x + \sqrt{1-x^2} + C$$

xiv. $\int x \sin^{-1} x \, dx$

Solution

$$I = \int x \sin^{-1} x \; dx \, dx$$

Integral by parts taking $\sin^{-1} x$ as first function

$$= \sin^{-1} x . \frac{x^2}{2} - \int \frac{x^2}{2} \frac{d}{dx}(\sin^{-1} x) dx$$

$$= \frac{x^2}{2}\sin^{-1} x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \frac{-x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}} dx$$

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$$=\frac{x^2}{2}\sin^{-1}x+\frac{1}{2}\left[\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\sin^{-1}x\right]-\frac{1}{2}\sin^{-1}x+C$$

$$=\frac{x^2}{2}\sin^{-1}x+\frac{x\sqrt{1-x^2}}{4}+\frac{1}{4}\sin^{-1}x-\frac{1}{2}\sin^{-1}x+C$$

$$=\frac{x^2}{2}\sin^{-1}x+\frac{1}{4}\sin^{-1}x-\frac{1}{2}\sin^{-1}x+\frac{x\sqrt{1-x^2}}{4}+C$$

$$=\frac{x^2}{2}\sin^{-1}x-\frac{1}{4}\sin^{-1}x+\frac{x\sqrt{1-x^2}}{4}+C$$

xv. $\int e^x$ **Sin x Cos x dx**

Solution

$$=\int e^x \text{ Sin x Cos x dx}$$

Integrating by parts taking e^x as first function

$$=e^x.\frac{sin^2x}{2}-\int\frac{sin^2x}{2}.e^x\,dx$$

$$=e^x.\frac{sin^2x}{2}-\frac{1}{2}.\frac{1}{2}\int(2sin^2x).e^x\,dx$$

$$=e^x.\frac{sin^2x}{2}-\frac{1}{4}\int(1-Cos2x).e^x\,dx$$

$$=e^x.\frac{sin^2x}{2}-\frac{1}{4}\int e^x\,dx+\frac{1}{4}\int Cos2x.e^x\,dx$$

$$=e^x.\frac{sin^2x}{2}-\frac{1}{4}e^x+\frac{1}{4}\left[e^x.\frac{Sin\,2x}{2}-\int\frac{sin\,2x}{2}e^x\,dx\right.$$

$$=e^x.\frac{sin^2x}{2}-\frac{1}{4}e^x+\frac{e^xSin\,2x}{8}-\frac{1}{8}\int 2Sin\,x\,Cos\,xe^x\,dx$$

$$=e^x.\frac{sin^2x}{2}-\frac{1}{4}e^x+\frac{e^xSin\,2x}{8}-\frac{1}{4}\int e^xSin\,x\,Cos\,x\,dx$$

$$=e^x.\frac{sin^2x}{2}-\frac{1}{4}e^x+\frac{e^xSin\,2x}{8}-\frac{1}{4}\quad(I)$$

$$I+\frac{1}{4}I=e^x.\frac{sin^2x}{2}-\frac{1}{4}e^x+\frac{e^xSin\,2x}{8}$$

$$\frac{5}{4}I=e^x.\frac{sin^2x}{2}-\frac{1}{4}e^x+\frac{e^xSin\,2x}{8}$$

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$$I = \frac{2}{5} e^x . \frac{sin^2 x}{2} - \frac{1}{5} e^x + \frac{1}{10} e^x sin\ 2x + c$$

xvi. $\int x$ **Sin x Cos x dx**

Solution

$$I = \int x \text{ Sin x Cos x dx}$$

$$= \frac{1}{2} \int x \text{ 2 Sin x Cos x dx}$$

$$= \frac{1}{2} \int x \text{ Sin 2x dx}$$

Integrating by parts taking x as first function

$$= \frac{1}{2} [x . \frac{-Cos\ 2x}{2} - \int \frac{-Cos\ 2x}{2} \text{ dx}]$$

$$= \frac{1}{2} [-\frac{xCos\ 2x}{2} + \frac{1}{2} \int Cos\ 2x \text{ dx}]$$

$$= [-\frac{xCos\ 2x}{4} + \frac{1}{4} \frac{Sin\ 2x}{2}] + C$$

$$= \frac{1}{4} [-x\ Cos\ 2x + \frac{2Sin\ xCos\ x}{2}] + C$$

$$= \frac{1}{4} [-x\ Cos\ 2x + Sin\ x\ Cos\ x] + C$$

xvii. $\int$ **x Cos** ^{2}**x dx**

Solution

$$I = \int x \text{ Cos 2x dx}$$

$$= \int x \left(\frac{1+Cos\ 2x}{2}\right) \text{dx}$$

$$= \frac{1}{2} \int (x + x\text{Cos2x}) \text{dx}$$

$$= \frac{1}{2} \int x \text{dx} + \frac{1}{2} \int x\text{Cos2x dx}$$

Integrating by parts

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$$= \frac{x^2}{4} + \frac{1}{2}\left[x.\frac{\sin 2x}{2} - \frac{1}{2}\int \sin 2x.dx\right]$$

$$= \frac{x^2}{4} + \frac{x\sin 2x}{4} - \frac{1}{4}\int \sin 2x.\,dx$$

$$= \frac{x^2}{4} + \frac{x\sin 2x}{4} - \frac{1}{4}\left(-\frac{Cos\,2x}{2}\right) + C$$

$$= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{1}{8}Cos\,2x + C$$

$$= \frac{1}{4}(x^2 + x\sin 2x + \frac{1}{2}Cos\,2x) + C$$

xviii. $\int x\,Sin^2 x\,dx$

Solution

$$I = \int x\,Sin^2 x\,dx$$

$$= \int x\left(\frac{1-Cos\,2x}{2}\right)dx$$

$$= \frac{1}{2}\int x(x - x\cos 2x)\,dx$$

$$= \int \left(\frac{1}{2}x - \frac{1}{2}x\cos 2x\right)dx$$

$$= \frac{1}{2}\int x\,dx - \frac{1}{2}\int x\cos 2x\,dx$$

Integrating by parts

$$= \frac{1}{2}.\frac{x^2}{2} - \frac{1}{2}\left[x.\frac{\sin 2x}{2} - \int \frac{\sin 2x}{2}.dx\right]$$

$$= \frac{x^2}{4} - \frac{1}{4}x\,Sin\,2x + \frac{1}{4}\int \sin 2x.\,dx$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{1}{4}.\frac{Cos2x}{2} + C$$

$$= \frac{1}{4}(x^2 - x\sin 2x - \frac{\cos 2x}{2}) + C$$

xix. $\int (\ln x)^2 dx$

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Integrating by parts taking $(\ln x)^2$ as first function

$$=(\ln x)^2.x - \int x \frac{d}{dx}(lnx)\frac{1}{x}\,dx$$

$$=x(\ln x)^2 - 2\int lnx.1\,dx$$

Again integrating by parts

$$=x(\ln x)^2 - 2[lnx.x - \int x.\frac{d}{dx}(lnx)dx]$$

$$=x(\ln x)^2 - 2[lnx.x - \int x.\frac{1}{x}dx]$$

$$=x(\ln x)^2 - 2[xlnx - \int dx]$$

$$=x(\ln x)^2 - 2[xlnx - x]+C$$

$$=x\ln x(\ln x -2)+2x+C$$

xx. $\int \ln(\tan x)\mathrm{Sec}^2 x\,dx$

Solution

$$I = \int \ln(\tan x)\mathrm{Sec}^2 x\,dx$$

Integrating by parts taking ln(tan x) as first function

$$=\ln(\tan x).\tan x - \int \tan x \frac{d}{dx}(ln\,tan)dx$$

$$= \tan x \ln \tan x - \int \tan x.\frac{1}{tan\,x}.\mathrm{Sec}^2 x\,dx$$

$$= \tan x \ln \tan x - \int \mathrm{Sec}^2 x\,dx$$

$$= \tan x \ln \tan x - \tan x + C$$

xxi. $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\,dx$

Solution

$$I = \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\,dx$$

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$$=-[\sqrt{1-x^2}\sin^{-1}x-\int\sqrt{1-x^2}\frac{d}{dx}(\sin^{-1}x)dx]$$

$$=-[\sqrt{1-x^2}\sin^{-1}x-\int\sqrt{1-x^2}.\frac{1}{\sqrt{1-x^2}}dx]$$

$$=x-\sqrt{1-x^2}\sin^{-1}x+\int 1\,dx$$

$$=x-\sqrt{1-x^2}\sin^{-1}x+C$$

Q2. Evaluate the following integrals

i. $\int \tan^4 x\,dx$

Solution

$$I=\int \tan^4 x\,dx$$

$$=\int \tan^2 x\,tan^2 x\,dx$$

$$=\int tan^2 x.\sec^2 x-tan^2 x)\,dx$$

$$=\int (tan^2 x\sec^2 x)dx-\int tan^2 x\,dx$$

$$=\frac{tan^3 x}{3}-\int(\sec^2 x-1)dx$$

$$=\frac{tan^3 x}{3}-\int\sec^2 x dx+\int dx$$

$$=\frac{tan^3 x}{3}-tan\,x+x+C$$

ii. $\int \mathbf{Sec^4 x\,dx}$

Solution

$$I=\int \sec^4 x\,dx$$

$$=\int \sec^2 x\,sec^2 x\,dx$$

$$=\int(1+tan^2 x).\sec^2 x\,dx$$

$$=\int (tan^2 x\sec^2 x)dx-\int sec^2 x\,dx$$

$$=\frac{tan^3 x}{3}+tan\,x+C$$

iii $\int e^x \mathbf{Sin\,2x\,Cos\,2x\,dx}$

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$$=\frac{1}{2}\int [e^x\ 2\text{Sin } 2x \text{ Cos } 2x]\ dx$$

$$=\frac{1}{2}\int e^x[\ \text{Sin}(\ 2x+x)+\text{Sin}(2x-x)\ ]\ dx$$

$$=\frac{1}{2}\int e^x \text{ Sin } 3x dx+\frac{1}{2}\int e^x \text{ Sin} x\ dx$$

$$=\frac{1}{2}I_1+\frac{1}{2}I_2 \ldots\ldots\ldots\ldots (A)$$

$$I_1 = \int e^x \text{ Sin } 3x dx$$

$$= \frac{-Cos\ 3x}{3}e^x+\frac{1}{3}\int e^x \text{ Cos } 3x\ dx$$

$$= \frac{-Cos\ 3x}{3}e^x+\frac{1}{3}[\frac{Sin\ 3x}{3}e^x-\frac{1}{3}\ \text{Sin } 3xe^x\ dx$$

$$=\frac{-Cos\ 3x}{3}e^x+\frac{1}{3}\text{Sin } 3xe^x-\frac{1}{9}I_1+C_1$$

$$(\ 1+\frac{1}{9})\ I_1 = \frac{-Cos\ 3x}{3}e^x+\frac{1}{9}\text{Sin } 3xe^x\ +C_1$$

$$\frac{10}{9}I_1 = \frac{-Cos\ 3x}{3}e^x+\frac{1}{9}\text{Sin } 3xe^x\ +C_1$$

$$I_1 = \frac{9}{10}[\ \frac{-Cos\ 3x}{3}e^x+\frac{1}{9}\text{Sin } 3xe^x\ ]+\frac{9}{10}C_1$$

$$I_1 = \frac{1}{10}[-3\text{Cos } 3xe^x+\text{Sin } 3xe^x\ ]+\frac{9}{10}C_1$$

$$I_1 = \frac{e^x}{10}[-3\text{Cos } 3x+\text{Sin } 3x\ ]+\frac{9}{10}C_1$$

$I_2 = \int e^x \text{ Sin } x dx$ integrating by parts

$$= -\text{ Cos } xe^x-\int(-\mathbf{Cos}x)\frac{d}{dx}(e^x)dx$$

$$=-\text{ Cos } xe^x+\int \mathbf{Cos}x\ e^x dx$$

$$= -\text{ Cos } xe^x+e^x\ Sin\ x\ -+\int \mathbf{Sin\ x}\ e^x dx+C_2$$

$$I_2 = -\text{ Cos } xe^x+e^x\ Sin\ x\ -I_2+C_2$$

$$2I_2 = e^x[-\ Cos\ x+\ Sin\ x]+C_2$$

$$I_2 = \frac{1}{2}e^x[\ Sin\ x-Cos\ x]+\frac{1}{2}C_2$$

By putting values I_1 and I_2 in (A) we get

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Hence $\int e^x \text{Sin } 2x \text{ Cos } 2x \, dx = \frac{e^x}{4}[\frac{1}{5}Sin\ 3x - \frac{3}{5}\ Cos3x\ + Sin\ x\ -\ Cos\ x] + C$

iv. $\int \tan^3 x \text{ Sec } x \, dx$

Solution

$$I = \int \tan^2 x(\tan x \text{ Sec } x)\, dx$$

$$= \text{Sec } x \tan^2 x - \int \sec x \, 2\tan x \sec^2 x \, dx$$

$$= \text{Sec } x \tan^2 x - 2[\sec x \tan x \, (1 + \tan^2 x)\, dx]$$

$$= \text{Sec } x \tan^2 x - 2\int \sec x \tan x dx - 2\int \tan^3 x \, sec\ x \, dx$$

$$I = \text{Sec } x \tan^2 x - 2\sec x\ - 2]$$

$$I+2I = \text{Sec } x \tan^2 x - 2\sec x$$

$$3I = \text{Sec } x \tan^2 x - 2\sec x$$

$$I = \frac{1}{3}[\text{ Sec } x \tan^2 x - 2\sec x] + C$$

Hence

$$\int \tan^3 x \text{ Sec } x \, dx = \frac{1}{3}[\text{ Sec } x \, (Sec2x\ - 1) - 2\sec x] + C$$

$$= \frac{1}{3}[\text{ Sec } x \tan^2 x - 2\sec x] + C$$

v. $\int x^3 e^{5x} \, dx$

Solution

$$\int x^3 e^{5x} \, dx$$

$$= \frac{e^{5x}}{5} x^3 - \frac{1}{5}\int e^{5x} \frac{d}{dx}(x^3) dx$$

$$= \frac{1}{5} e^{5x} x^3 - \frac{1}{5}\int e^{5x} 3x^2 dx$$

$$= \frac{1}{5}[e^{5x} x^3 - 3\int e^{5x} x^2 dx]$$

$$= \frac{1}{5}[e^{5x} x^3 - 3(\frac{e^{5x} x^2}{5} - \frac{1}{5}\int e^{5x} 2x dx)]$$

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$$=\frac{1}{5}[e^{5x}x^3-\frac{3e^{5x}x^2}{5}+\frac{6}{5}(\frac{e^{5x}x}{5}-\frac{1}{5}\int e^{5x}dx)]$$

$$=\frac{1}{5}[e^{5x}x^3-\frac{3e^{5x}x^2}{5}+\frac{6}{25}e^{5x}x-\frac{6}{52}\times\frac{e^{5x}}{5}]+C$$

$$=\frac{1}{5}[e^{5x}x^3-\frac{3e^{5x}x^2}{5}+\frac{6}{25}e^{5x}x-\frac{6}{125}e^{5x}]+C$$

$$=\frac{1}{5}e^{5x}[x^3-\frac{3x^2}{5}+\frac{6}{25}-\frac{6}{125}]+C$$

vi. $\int e^{-x}\,\text{Sin}\,2x\,dx$

Solution

$$I=\int e^{-x}\text{Sin}^2x\,dx=-\frac{Cos^2x}{2}e^{-x}+\frac{1}{2}\int \text{Cos}\,2x(-e^{-x})dx]$$

$$=-\frac{Cos^2x}{2}e^{-x}-\frac{1}{2}[\frac{Sin\,x\,2x}{2}e^{-x}\int\frac{Sin\,2x}{2}(-e^{-x})dx]$$

$$=-\frac{Cos^2x}{2}e^{-x}-\frac{Sin\ 2x}{4}e^{-x}-\frac{1}{4}\int Sin\,2x(-e^{-x})dx]$$

$$I=\frac{Cos^2x}{2}e^{-x}-\frac{Sin\ 2x}{4}e^{-x}+C_1+\frac{1}{4}I$$

$$I+\frac{1}{4}I=\frac{Cos^2x}{2}e^{-x}-\frac{Sin\ 2x}{4}e^{-x}+C_1$$

$$\frac{1}{5}I=-\frac{1}{2}(Cos2x)e^{-x}-\frac{1}{4}(Sin\,2x)e^{-x}+C_1$$

$$I=\frac{4}{5}\times(-\frac{1}{2}e^{-x})[Cos\,2x-\frac{1}{2}Sin\,2x]+C$$

$$=-\frac{2}{5}e^{-x}[Cos\,2x-\frac{1}{2}\,Sin\,2x\,]+C$$

vii. $\int e^{2x}\,\text{Cos}\,3x\,dx$

Solution

$$I=\int e^{2x}\,\text{Cos}\,3x\,dx$$

$$=\frac{Sin\,3x}{3}e^{2x}-\frac{1}{3}\int \text{Sin}\,3x\,\frac{d}{dx}(e^{2x})dx$$

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$$= \frac{Sin\ 3xe^{2x}}{3} - \frac{2}{3}[- \frac{Cos\ 3xe^{2x}}{3} + \frac{1}{3} \int \text{Cos } 3x \times (e^{2x} \times 2)dx$$

$$,= \frac{Sin\ 3xe^{2x}}{3} + \frac{2Cos\ 3xe^{2x}}{9} \frac{4}{9} I + C$$

$$I + \frac{4}{9} I = \frac{e^{2x}}{3}[Sin\ 3x + \frac{2}{3} Cos\ 3x] + C$$

$$\frac{13}{9} I = \frac{e^{2x}}{3}[Sin\ 3x + \frac{2}{3} Cos\ 3x] + C$$

$$I = \frac{9}{13} \times \frac{e^{2x}}{3}[Sin\ 3x + \frac{2}{3} Cos\ 3x] + C$$

$$\int e^{2x} \text{ Cos } 3x\ dx = \frac{3\ e^{2x}}{13}[Sin\ 3x + \frac{2}{3} Cos\ 3x] + C$$

viii. $\int \text{Cosec}^3 x\ dx$

Solution

$$I = \int \text{Cosec}^3 x\ dx = \int \text{Cosec}^2 x \times Cosec\ x\ dx$$

$$= - \text{Cot x Cosec x} + \int \text{Cot x} \frac{d}{dx} (Cosec\ x)\ dx$$

$$= - \text{Cot x Cosec x} + \int \text{Cot x} \times Cosec\ x\ Cot\ x\ dx$$

$$= - \text{Cot x Cosec x} - \int Cot^2 x\ Cosec\ x\ dx$$

$$= - \text{Cot x Cosec x} - \int (\text{Cosec}^2 x - 1)(Cosec\ x)\ dx$$

$$= - \text{Cot x Cosec x} - \int \text{Cosec}^3 x\ dx + \int Cosec\ x\ dx$$

$$I = - \text{Cot x Cosec x} - I + \ln |\text{Cosec x} - \text{Cot x}| + C_1$$

$$2I = - \text{Cot x Cosec x} - 1 + \ln |\text{Cosec x} - \text{Cot x}| + C_1$$

$$I = - \frac{1}{2}[\text{ Cot x Cosec x} - \ln |\text{Cosec x Cot x}| + C$$

$$\int \text{Cosec}^3 x\ dx = - \frac{1}{2}[\text{ Cot x Cosec x} - \ln |\text{Cosec x Cot x}| + C$$

Q3. Show that

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Solution

$$I = \int e^{ax} \operatorname{Sin} bx\, dx$$

$$= \frac{-\operatorname{Cos} bx}{b} e^{ax} - \int \frac{-\operatorname{Cos} bx}{b} e^{ax} \times a dx$$

$$= \frac{-\operatorname{Cos} bx}{b} e^{ax} + \frac{a}{b}\int \operatorname{Cos} bx\, e^{ax} dx$$

$$= \frac{-\operatorname{Cos} bx}{b} e^{ax} + \frac{a}{b}\left[\frac{\operatorname{Sin} bx\, e^{ax}}{b} - \int \frac{\operatorname{Sin} bx}{b} e^{ax} \times a dx\right]$$

$$= \frac{-\operatorname{Cos} bx}{b} e^{ax} + \frac{a}{b^2} \operatorname{Sin} bx\, e^{ax} - \frac{a}{b^2}\int \operatorname{Sin} bx\, e^{ax}\, dx$$

$$I = \frac{-\operatorname{Cos} bx}{b} e^{ax} + \frac{a}{b^2} \operatorname{Sin} bx\, e^{ax} - \frac{a}{b^2} I + C$$

$$I + \frac{a^2}{b^2} I = \frac{a}{b^2} \operatorname{Sin} bx\, e^{ax} - \frac{\operatorname{Cos} bx}{b} e^{ax} + C$$

$$\left(1 + \frac{a^2}{b^2}\right) I = \frac{1}{b^2}[a \operatorname{Sin} bx - b \operatorname{Cos} b x] e^{ax} + C$$

$$\left(\frac{a^2+b^2}{b^2}\right) I = \frac{1}{b^2}[a \operatorname{Sin} bx - b \operatorname{Cos} b x] e^{ax} + C$$

$$I = \frac{b^2}{a^2+b^2} \times \frac{1}{b^2}[a \operatorname{Sin} bx - b \operatorname{Cos} b x] e^{ax} + C$$

$$= \frac{1}{a^2+b^2}[a \operatorname{Sin} bx - b \operatorname{Cos} b x] e^{ax} + C$$

$$= \frac{1}{a^2+b^2} e^{ax} \sin\left(bx - \tan^{-1}\left(\frac{b}{a}\right)\right) + C$$

Q4. Evaluate the following integrals.

i. $\int \sqrt{a^2 - x^2}\, dx$

Solution

$$I = \int \sqrt{a^2 - x^2}\, dx \quad = \int (1).\sqrt{a^2 - x^2}\, dx$$

$$= \sqrt{a^2 - x^2}\, x - \int x \times \frac{1}{2}(a^2 - x^2)^{\frac{1}{2}}(-2x)\, dx$$

$$= x\sqrt{a^2 - x^2} + \int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx$$

$$= x\sqrt{a^2 - x^2} + \int \frac{a^2 - (a^2 - x^2)}{\quad} dx$$

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$$I = x\sqrt{a^2 - x^2} + a^2 \int \frac{1}{\sqrt{a^2 - x^2}}\, dx - I$$

$$2I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right) + C_1$$

$$I = \frac{1}{2}\left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\left(\frac{x}{a}\right)\right] + C$$

Thus $\int \sqrt{a^2 - x^2}\ dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$

ii. $\int \sqrt{a^2 - x^2}\ dx$

Solution

$$I = \int (1)\sqrt{a^2 - x^2}\ dx$$

$$= x\sqrt{a^2 - x^2} - \int x \frac{d}{dx}\left[(x^2 - a^2)^{\frac{1}{2}}\right] dx$$

$$= x\sqrt{a^2 - x^2} - \int x \times \frac{1}{2}(x^2 - a^2)^{\frac{1}{2}}(2x)\ dx$$

$$= x\sqrt{a^2 - x^2} - \int \frac{x^2}{\sqrt{x^2 - a^2}}\, dx$$

$$= x\sqrt{a^2 - x^2} - \int \frac{x^2 - a^2 + a^2)}{\sqrt{x^2 - a^2}}\, dx$$

$$= x\sqrt{a^2 - x^2} - \int \frac{x^2 - a^2}{\sqrt{x^2 - a^2}}\, dx - \int \frac{a^2}{\sqrt{x^2 - a^2}}\, dx$$

$$= x\sqrt{a^2 - x^2} - I - a^2 \int \frac{1}{\sqrt{x^2 - a^2}}\, dx$$

Let $I_1 = \int \frac{1}{\sqrt{x^2 - a^2}}\, dx$

Put $x = a \operatorname{Sec}\theta, then\ dx = a \operatorname{Sec}\theta \tan\theta\, d\theta$

$$I_1 = \int \frac{1}{\sqrt{(a \operatorname{Sec}\theta)^2 - a^2}} \times a \operatorname{Sec}\theta \tan\theta\, d\theta$$

$$= \int \frac{1}{\sqrt{a^2(\operatorname{Sec}^2\theta - 1)}} \times a \operatorname{Sec}\theta \tan\theta\, d\theta$$

$$= \int \frac{1}{a\sqrt{\tan^2\theta}} \times a \operatorname{Sec}\theta \tan\theta\, d\theta = \int \frac{\operatorname{Sec}\theta \tan\theta}{\tan\theta}\, d\theta$$

$$= \int \operatorname{Sec}\theta\, d\theta$$

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But $x = a\,Sec\,\theta \quad \Rightarrow Sec\theta \frac{x}{a}$

$\tan\theta = \sqrt{(\mathrm{Sec}^2\theta - 1)}$

$= \sqrt{(\frac{x}{a})^2 - 1} \quad = \frac{1}{a}\sqrt{x^2 - a^2}$

$= \ln |\mathrm{Sec}\,\theta + tan\theta| + C_1$

$= \ln |\frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a}| + C_1$

$= \ln |x + \sqrt{x^2 - a^2}| - ln\,a + C_1$

$= \ln |x + \sqrt{x^2 - a^2}| + C_2$

(a)Becomes

$2I = x\sqrt{a^2 - x^2} - a^2 ln|x + \sqrt{x^2 - a^2}| + C_2$

$I = \frac{1}{2}x\sqrt{a^2 - x^2} - \frac{a^2}{2}\, ln|x + \sqrt{x^2 - a^2}| + C$

Hence $= \int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} - \frac{a^2}{2}\, ln|x + \frac{1}{2}\sqrt{x^2 - a^2}| + C$

iii. $\int \sqrt{4 - 5x^2}\, dx$

Solution

$I = \sqrt{5} \int \sqrt{(\frac{4}{5} - x^2)}\, dx$

$= \sqrt{5} \int \sqrt{(\frac{2}{\sqrt{5}})^2 - x^2)}\, dx$

$= \sqrt{5}\, [\frac{x}{2}\sqrt{(\frac{2}{\sqrt{5}})^2 - x^2)}\, dx + \frac{1}{2}(\frac{2}{\sqrt{5}})^2 \sin^{-1}(\frac{x}{\frac{2}{\sqrt{5}}})] + C$

$= \sqrt{5}\frac{x}{2}\sqrt{\frac{4}{5} - x^2} + \sqrt{5}\,(\frac{1}{2} \times \frac{4}{5}\sin^{-1}(\frac{\sqrt{5}\,x}{2}) + C$

$= \frac{\sqrt{5}}{2} \times \frac{\sqrt{(4 - x^2)}}{\sqrt{5}} + \frac{2}{\sqrt{5}}\sin^{-1}(\frac{\sqrt{5}\,x}{2}) + C$

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iv. $\int \sqrt{3-4x^2}\, dx$

Solution

$= \int \sqrt{(\sqrt{3})^2-(2x)^2}$, let $a = \sqrt{3} \Rightarrow a^2 = 3$

$= \int \sqrt{a^2-y^2}\frac{1}{2} dy$ $y = 2x \Rightarrow dy = 2dx$

$= \frac{1}{2}\sqrt{a^2-y^2}\, dy$ $\Rightarrow dx = \frac{1}{2} dy$

$= \frac{1}{2}[\frac{y}{2}(2x)\sqrt{(\sqrt{3})^2-(2x)^2} + \frac{(\sqrt{3})^2}{2}\sin^{-1}(\frac{2x}{\sqrt{3}})] + C$

$= \frac{1}{2}[x\sqrt{3-4x^2} + \frac{3}{4}\sin^{-1}(\frac{2x}{\sqrt{3}})] + C$

v. $\int \sqrt{x^2+4}\, dx$

Solution

Let $I = \int (1)\sqrt{x^2+4}\, dx$

$= x\sqrt{x^2+4} - \int x \times \frac{1}{\sqrt{x^2+4}}(2x)\, dx$

$= x\sqrt{x^2+4} - \int \frac{x^2}{\sqrt{x^2+4}}\, dx$

$= x\sqrt{x^2+4} - \int \frac{x^2+4-4)}{\sqrt{x^2+4}}\, dx$

$= x\sqrt{x^2+4} - \int \frac{x^2+4}{\sqrt{x^2+4}}\, dx + \int \frac{4}{\sqrt{x^2+4}}\, dx$

$= x\sqrt{x^2+4} - I + 4\int \frac{1}{\sqrt{x^2+4}}\, dx$

Let $I_1 = \int \frac{1}{\sqrt{x^2+4}}\, dx$

Put $x = 2\tan\theta, then\ dx = 2\,Sec^2\theta\, d\theta$

$I_1 = \int \frac{1}{\sqrt{(2\tan\theta)^2+4}} \times 2\,Sec^2\theta\, d\theta$

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$$=\int \frac{1}{2\sqrt{\text{Sec}^2\theta}} \times 2\,Sec^2\theta\, d\theta = \int \frac{Sec^2\theta}{\text{Sec}\,\theta}\, d\theta$$

$$=\int \text{Sec}\,\theta\, d\theta$$

$$= \ln |\text{Sec}\,\theta + tan\theta| + C_1$$

$$= \sqrt{(\frac{x}{2})^2 + 1} + \frac{x}{2}) + C_1$$

$$=\ln |\frac{x^2+4}{2} + \frac{x}{2}| + C_1$$

$$=\ln |\frac{\sqrt{x^2+4}+x}{2}| + C_1$$

$$= \ln |\sqrt{x^2+4} + x| - \ln 2 + C_1$$

$$= \ln |x + \sqrt{x^2+4}| + C_2$$

(a)Becomes

$$2I = x\sqrt{a^2+4} + 4ln|x + \sqrt{x^2+4}| + C_2$$

$$I = \frac{1}{2}x\sqrt{a^2+4} + \frac{4}{2} ln|x + \sqrt{x^2+4}| + C$$

$$\text{Hence} = \int \sqrt{x^2+4}\, dx = \frac{x}{2}\sqrt{a^2+4} + 2\, ln|x + \sqrt{x^2+4}| +$$

vi. $\int x^2 e^{ax}\, dx$

Solution

$$=\frac{e^{ax}}{a}.x^2 - \int \frac{e^{ax}}{a}.(2x)\, dx$$

$$=\frac{e^{ax}}{a}.x^2 - \frac{2}{a}(\frac{e^{ax}}{a} - x\int \frac{e^{ax}}{a}.1\, dx)$$

$$=\frac{e^{ax}}{a}.x^2 - \frac{2}{a^2}e^{ax}.x + \frac{2}{a^2}.\frac{e^{ax}}{a} + C$$

$$= \frac{e^{ax}}{a}[x^2 - \frac{2}{a}x + \frac{2}{a^2}] + C$$

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Q5. Evaluate the following integrals

i. $\int e^x(\frac{1}{x} + \ln x)\, dx$

Solution

Let $\int (x) = \ln x$, then $\int (x)\, x = \frac{1}{x}$

$$\int e^x(\frac{1}{x} + \ln x)\, dx = \int e^x\, [f(x)+f'(x)]dx$$

$$= \int \frac{d}{dx}(e^x f(x))dx$$

$$= e^x f(x)+C$$

$$= e^x ln\, x + C$$

ii. $\int e^x(\cos x + \sin x)dx = \int e^x(\sin + \cos x)dx$

Solution

$f(x) = \sin x$ then $f'(x) = \cos x$. So

$$= \int e^x(Cos\, x + Sin\, x)dx = \int e^x[f(x)+f'(x)]dx$$

$$= \int \frac{d}{dx}(e^x f(x))dx$$

$$= e^x f(x)+C$$

$$= e^x Sin\, x + C$$

iii. $\int e^{ax}[a \sec^{-1} x + \frac{1}{x\sqrt{x^2-1}}]dx$

Solution

Let $\sec^{-1} x = f(x)$ then $f'(x) = \frac{1}{x\sqrt{x^2-1}}$

Thus $\int e^{ax}[a \sec^{-1} x + \frac{1}{x\sqrt{x^2-1}}]dx$

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$$=\int \frac{d}{dx}(e^{ax}f(x)+f'(x))dx$$

$$= e^{ax}\sec^{-1}x + C$$

iv. $\int e^{3x}\left(\frac{3\sin x - \cos x}{\sin^2 x}\right)dx$

Solution

$$= \int e^{3x}\left(\frac{3\sin x}{\sin^2 x} - \frac{\cos x}{\sin^2 x}\right)dx$$

$$=\int e^{3x}[3\,Cosec\,x - Cot\,x\,Cosec\,x]$$

$$=\int e^{3x}Cosec\,x - \int e^{3x}\,Cot\,x\,Cosec$$

Integrate by first integral by taking Cosec x as first function

$$=3\,\text{Cosec}\,x\left(\frac{1}{3}e^{3x}\right) - 3\int\left(\frac{1}{3}e^{3x}\right)(-Cot\,x\,Cosec\,x) - \int e^{3x}\,Cot\,x\,Cosec\,x$$

$$=e^{3x}\,Cosec\,x + \int e^{3x}\,Cot\,x\,Cosec\,x - \int e^{3x}\,Cot\,x\,Cosec\,x$$

$$= e^{3x}\,Cot\,x\,Cosec\,x + C$$

v. $\int e^{2x}(-\sin x + 2\cos x)\,dx$

Solution

$$= \int e^{2x}(-\sin x + 2\cos x)\,dx$$

$$= \int e^{2x}(2\cos x - \sin x))\,dx$$

$$= 2\int e^{2x}\,Cos\,x\,dx - \int e^{2x}\,Sin\,x\,dx$$

Integrate by parts first integral by taking Cos x as first function

$$= 2[\text{Cos}\left(\frac{1}{2}e^{2x}\right) - \int\left(\frac{1}{2}e^{2x}\right)(-Sin\,x)dx] - \int e^{2x}\,Sin\,x\,dx$$

$$=e^{2x}\,Cos\,x + \int e^{2x}\,Sin\,x\,dx - \int e^{2x}\,Sin\,x\,dx$$

$$=e^{2x}\,Cos\,x + C$$

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vi. $\int \frac{xe^x}{(1+x)^2}\,dx$

Solution

$$= \int e^x\left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right]dx$$

Let $f(x) = \frac{1}{1+x}$ then $f'(x) = -\frac{1}{(1+x)^2}$

So $\int \frac{xe^x}{(1+x)^2}\,dx$ $\int e^x[f(x)+f'(x)]$

$$= \int \frac{d}{dx}(e^x f(x))dx$$

$$= e^x f(x) + C$$

$$= e^x \frac{1}{1+x} + C$$

vii. $\int e^{-x}[\cos x - \sin x]\,dx$

Solution

$$= \int e^{-x}[\cos x - \sin x]\,dx$$

$$= \int e^{-x}\cos x\,dx - \int e^{-x}\sin x\,dx$$

Integrate by parts first integral by taking e^{-x} *as first function*

$$= e^{-x}(\sin x) - \int \sin x\, e^{-x}(-1)dx - \int e^{-x}\sin x\,dx$$

$$= e^{-x}\sin x + \int e^{-x}\sin x\,dx - \int e^{-x}\sin x\,dx$$

$$= e^{-x}\sin x + C$$

viii. $\int \frac{e^{m\tan^{-1}x}}{1+x^2}\,dx$

Solution

Let $y = \tan^{-1}x$ then

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$$dy = \left(\frac{1}{1+x^2}\right)dx$$

thus $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$

$$= \int e^{m \tan^{-1} x}\left(\frac{1}{1+x^2}\right) dx$$

$$= \int e^{m}.dy$$

$$= \frac{1}{m}.e^{m \tan^{-1} x} + C$$

ix $\int \frac{2x}{1-Sin\, x}\, dx$

Solution

$$= \int \frac{2x}{1-Cos\left(\frac{\pi}{2}-x\right)} dx = \int \frac{2x}{1-sin^2\left(\frac{\pi}{2}-\frac{x}{2}\right)} dx$$

$$= \int x\, Cosec\, 2\left(\frac{\pi}{2}-\frac{x}{2}\right) dx$$

$$= \left(\frac{\pi}{2}-\frac{x}{2}\right) = u$$

$$\frac{1}{2}\, dx = du \Rightarrow dx = -du$$

Thus $\int x\, Cosec^2\left(\frac{\pi}{2}-\frac{x}{2}\right) dx$

$$= \int \left(\frac{\pi}{2} - 2u\right)\, Cosec^2\, ux - 2du$$

$$= \int \left(2u - \frac{\pi}{2}\right)\, Cosec^2\, u\, dx$$

$$= \int (-Cot\, u) \times \left(2u - \frac{\pi}{2}\right)$$

$$= 2[\,(-\text{Cot } u)\left(2u - \frac{\pi}{2}\right) + 2\, ln\, |sin\, u| + C$$

$$= 2x\ \text{Cot}\left(\frac{\pi}{4}-\frac{x}{2}\right) + 4\ \text{ln}\ |\text{Sin}\left(\frac{\pi}{4}-\frac{x}{2}\right)| + C$$

x $\int \frac{e^x(1+x)}{}\, dx$

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$$= \int \frac{e^x(1+x-1)}{(2+x)^2}\, dx$$

$$= \int e^x[\frac{2+x}{(2+x)^2} - \frac{1}{(2+x)^2}]\, dx$$

$$= \int e^x[\frac{1}{(2+x)} - \frac{1}{(2+x)^2}]\, dx$$

$f(x) = \frac{1}{2+x}$

thus $f'(x) = \frac{d}{dx}\left[\frac{1}{2+x}\right] = \frac{d}{dx}[(2+x)^{-1}]$

$= (-1)\,(2+x)^{-2} \times 1$

$= -\frac{1}{(2+x)^2}$

Thus $\int e^x[\frac{1}{(2+x)} - \frac{1}{(2+x)^2}]\, dx$

$= \int e^x[\, f(x) + f'(x)]dx$

$= e^x\, f(x) + C$

$= e^x \frac{1}{(2+x)} + C = \frac{e^x}{(2+x)} + C$

xi. $\int(\frac{1-Sin\,x}{1-Cos\,x})\ e^x$ **dx**

Solution

$$= \int(\frac{1-Sin\,x}{2\,Sin^2\frac{x}{2}})\ e^x\ dx$$

$$= \int e^x[\frac{1}{2\,Sin^2\frac{x}{2}} - \frac{sin\,x}{2\,Sin^2\frac{x}{2}}]\, dx$$

$$= \int[\frac{1}{2}\,Cosec^2\frac{x}{2} - Cot\ \frac{x}{2}]e^x\, dx$$

$$= \int e^x[-Cot\ \frac{x}{2} - Cosec^2\ \frac{x}{2}]\ e^x\, dx$$

$f(x) = -\,Cot\ \frac{x}{2}$

Thus we have

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thus $f'(x) = \frac{1}{2} Cosec^2 \frac{x}{2}$

$$= \int e^x [-Cot \frac{x}{2} - Cosec^2 \frac{x}{2}] dx$$

$$= - \int e^x [f(x) + f'(x)] dx$$

$$= -e^x f(x) + C$$

$$= -e^x \text{ Cot } \frac{x}{2} + C$$

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