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Exercise 3.2

Q1. Evaluate the following indefinite integrals.

i. $\int(3x^2 - 2x + 1)dx$

Solution

$$= \int(3x^2 - 2x + 1)dx$$

$$= \int(3x^2)\text{dx} - \int(2xdx + \int 1dx$$

$$= 3\int(x^2)dx - 2\int x\,dx + \int dx$$

$$= 3\left(\frac{x^{2+1}}{2+1}\right) - 2\left(\frac{x^{1+1}}{1+1}\right) + x + c$$

$$= 3\left(\frac{x^3}{2}\right) - 2\left(\frac{x^2}{2}\right) + x + c$$

$$= x^3 - x^2 + x + c$$

ii. $\int(\sqrt{x} + \frac{1}{\sqrt{x}})dx, x > 0$

Solution

$$\int(\sqrt{x} + \frac{1}{\sqrt{x}})\text{dx}$$

$$= \int(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}})\text{dx}$$

$$= \int(x^{\frac{1}{2}} + x^{-\frac{1}{2}})\text{dx}$$

$$= \int(x^{\frac{1}{2}})\text{dx} + \int(x^{-\frac{1}{2}})\text{dx} \qquad \therefore \int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \text{C}$$

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$$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

iii. $\int x(\sqrt{x+1})\,dx, x > 0$

Solution

$$\int x(\sqrt{x+1})\,dx$$

$$= \int x(x+1)^{\frac{1}{2}}\,dx$$

$$= \int (x^{\frac{3}{2}} + x)dx$$

$$= \int x^{\frac{3}{2}}\,dx + \int x\,dx$$

$$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{x^{1+1}}{\frac{1}{2}+1} + C$$

$$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{x^2}{2} + c$$

$$= \frac{2}{5}x^{\frac{3}{2}} + \frac{1}{2}x^2 + c$$

iv. $\int (2x+3)^{\frac{1}{2}}\,dx$

Solution

$$\int (2x+3)^{\frac{1}{2}}\,dx$$

$$= \frac{1}{2}\int (2x+3)^{\frac{1}{2}}\,2\,dx$$

$$= \frac{1}{2}\frac{(2x+3)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

As $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}$

$$= \frac{1}{3}\frac{(2x+3)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

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$$= \frac{1}{3}(2x+1)^{\frac{3}{2}} + C$$

v. $\int(\sqrt{x}+1)^2\,dx$

Solution

$$\int(\sqrt{x}+1)^2\,dx$$

$$= \int((\sqrt{x})^2 + 2(\sqrt{x})(1) + (1)^2)dx$$

$$= \int(x + 2\sqrt{x}+1)dx$$

$$= \int(x + 2x^{\frac{1}{2}}+1)dx$$

$$= \int(x)dx + \int 2x^{\frac{1}{2}}dx + \int dx$$

$$= \frac{x^{1+1}}{1+1} + 2\int \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + x + C$$

$$= \frac{x^2}{2} + 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + x + C$$

$$= \frac{x^2}{2} + \frac{2.2}{3}x^{\frac{3}{2}} + x + C$$

$$= \frac{x^2}{2} + \frac{4}{3}x^{\frac{3}{2}} + x + C$$

i. $\int(\sqrt{x} - \frac{1}{\sqrt{x}})^2\,dx$

Solution

$$\int(\sqrt{x} - \frac{1}{\sqrt{x}})^2\,dx \qquad \therefore (a-b)^2 = a^2 - 2ab + b^2$$

$$= \int((x)^2 - 2(\sqrt{x})(\frac{1}{\sqrt{x}}) + (\frac{1}{\sqrt{x}})^2)dx$$

$$= \int(x - 2 + \frac{1}{x})\,dx$$

$$= \int x\,dx - 2\int dx + \int \frac{1}{x}dx$$

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$$= \frac{x^2}{2} - 2x + \ln x + C$$

vii. $\int \frac{3x+2}{\sqrt{x}} dx, x > 0$

Solution

$$\int \frac{3x+2}{\sqrt{x}} dx$$

$$= \int \left(\frac{3x}{\sqrt{x}} + \frac{2}{\sqrt{x}}\right) dx$$

$$= \int \left(3\sqrt{x} + \frac{2}{\sqrt{x}}\right) dx$$

$$= \int (3x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}) dx$$

$$= \int 3x^{\frac{1}{2}} dx + \int 2x^{-\frac{1}{2}} dx$$

$$= 3\int x^{\frac{1}{2}} dx + 2\int x^{-\frac{1}{2}} dx$$

$$= 3\left(\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right) + 2\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + C$$

$$= 3\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{6}{3}x^{\frac{3}{2}} + 2.2x^{\frac{1}{2}} + C$$

$$= 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + C$$

viii. $\int \frac{\sqrt{y}(y+1)}{y} dy, y > 0$

Solution

$$\int \frac{\sqrt{y}(y+1)}{y} dy$$

$$= \int \frac{y^{\frac{1}{2}}(y+1)}{y} dy$$

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$$= \int (y^{\frac{1}{2}} + y^{\frac{1}{2}})dy$$

$$= \int y^{\frac{1}{2}}dy + \int y^{\frac{1}{2}}dy$$

$$= \frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{y^{\frac{3}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{y^{\frac{3}{2}}}{\frac{3}{2}} + \frac{y^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2}{3}y^{\frac{3}{2}} + 2y^{\frac{1}{2}} + C$$

ix $\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}} d\theta, \theta > 0$

Solution

$$\int \frac{(\sqrt{\theta}-1)^2}{\sqrt{\theta}} d\theta$$

$$= \int (\frac{\theta - 2\sqrt{\theta}+1}{\sqrt{\theta}})d\theta$$

$$= \int (\frac{\theta}{\sqrt{\theta}} - \frac{2\sqrt{\theta}}{\sqrt{\theta}} + \frac{1}{\sqrt{\theta}})d\theta$$

$$= \int (\sqrt{\theta} - 2 + \frac{1}{\sqrt{\theta}})d\theta$$

$$= \int (\theta^{\frac{1}{2}} - 2 + \theta^{-\frac{1}{2}})d\theta$$

$$= \int (\theta^{\frac{1}{2}})d\theta - \int 2d\theta + \int (\theta^{-\frac{1}{2}})d\theta$$

$$= \int (\theta^{\frac{1}{2}})d\theta - 2\int d\theta + \int (\theta^{-\frac{1}{2}})d\theta$$

$$= \frac{\theta^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 2\theta + \frac{\theta^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{\theta^{\frac{3}{2}}}{\frac{3}{2}} - 2\theta + \frac{\theta^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= \frac{2}{3}\theta^{\frac{3}{2}} - 2\theta + 2\theta^{\frac{1}{2}} + C$$

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x. $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx, x > 0$

Solution

$$= \int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$$

$$= \int (\frac{1+x-2\sqrt{x}}{\sqrt{x}}) dx$$

$$= \int (\frac{dx}{\sqrt{x}} + \int \frac{x}{\sqrt{x}} dx - 2\int \frac{\sqrt{x}}{\sqrt{x}} dx$$

$$= \int x^{\frac{-1}{2}} dx + \int x^{\frac{1-1}{2}} dx - 2\int dx$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \int x^{\frac{1}{2}} dx - 2.x + C$$

$$= 2x^{\frac{1}{2}} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2x + C$$

$$= 2x^{\frac{1}{2}} + 2x^{\frac{3}{2}} - 2x + C$$

xi. $\int \frac{e^{2x}+e^x}{e^x} dx$

Solution

$$= \int \frac{e^{2x}}{e^x} + \frac{e^x}{e^x} dx$$

$$= \int (e^x + 1) dx$$

$$= \int e^x dx \int 1 dx$$

$$= e^x + x + C$$

Q2 Evaluate

i $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}}$ $\{x + a > 0\}$ **{x+ b>0}**

Solution

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Rationalize

$$= \int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} \times \frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})dx}{(\sqrt{x+a})^2-(\sqrt{x+b})^2}$$

$(a+b)(a-b) = a^2 - b^2$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})}{(x+a)-(x-b)} dx$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x+b})}{x+a-x-b} dx$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x})}{a-b} dx$$

$$= \frac{1}{a-b}\int (\sqrt{x+a} - \sqrt{x+b})dx$$

$$= \frac{1}{a-b}[\int (x+a)^{\frac{1}{2}} - (x+b)^{\frac{1}{2}}]\, dx$$

$$= \frac{1}{a-b}\left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{(x+b)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$$

$$= \frac{1}{a-b}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$$

$$= \frac{1}{a-b}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}(x+b)^{\frac{3}{2}}\right] + C$$

$$= \frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}} - (x+b)^{\frac{3}{2}}\right] + C$$

ii. $\int \frac{1-x^2}{1+x^2} dx$

Solution

$$\int \frac{1-x^2}{1+x^2} dx$$

It can be written as

$$= \int \frac{-1-x^2+2}{} dx$$

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$$= \int \left(\frac{2}{1+x^2} - \frac{1+x^2}{1+x^2}\right) dx$$

$$= 2\int \frac{1}{1+x^2} dx - \int 1\, dx$$

$$= 2\tan^{-1} x - x + C$$

iii $\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}$ $\mathbf{x > 0, a > 0}$

$$\int \frac{dx}{\sqrt{x+a}+\sqrt{x}}$$

Rationalize it

$$= \int \frac{dx}{\sqrt{x+a}+\sqrt{x}} \times \frac{\sqrt{x+a}-\sqrt{x}}{\sqrt{x+a}-\sqrt{x}}$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x})dx}{(\sqrt{x+a})^2-(\sqrt{x})^2}$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x})}{x+a-a} dx$$

$$= \int \frac{(\sqrt{x+a}-\sqrt{x})}{x} dx$$

$$= \frac{1}{a}\int (\sqrt{x+a} - \sqrt{x})dx$$

$$= \frac{1}{a}[\int (x+a)^{\frac{1}{2}} - x^{\frac{1}{2}}]\, dx$$

$$= \frac{1}{a}\left[\frac{(x+a)^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]$$

$$= \frac{1}{a}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$$

$$= \frac{1}{a}\left[\frac{2}{3}(x+a)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}\right] + C$$

$$= \frac{1}{3a}\left[(x+a)^{\frac{3}{2}} - x^{\frac{3}{2}}\right] + C$$

iv $\int (a-2x)^{\frac{3}{2}}\, dx$

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$$\int (a-2x)^{\frac{3}{2}}\,dx$$

Multiply and divide by -2

$$= -\frac{1}{2}\int (a-2x)^{\frac{3}{2}}(-2)\,dx$$

$$= -\frac{1}{2}\int (a-2x)^{\frac{3}{2}}(-2\,dx)$$

$$= \frac{1}{2}\frac{(a-2x)^{\frac{3}{2}+1}}{\frac{3}{2}+1}+C$$

$$= -\frac{1}{2}\frac{(a-2x)^{\frac{5}{2}}}{\frac{5}{2}}+C$$

$$= -\frac{1}{5}(a-2x)^{\frac{5}{2}}+C$$

v $\int \frac{(1+e^x)^3}{e^x}\,dx$

Solution

$$\int \frac{(1+e^x)^3}{e^x}\,dx$$

$$=\int \frac{1+e^{3x}+3e^x(1+e^x)}{e^x}\,dx$$

$$(a+b)^3 = a^3+b^3+3ab(a+b)$$

$$=\int \frac{1+3e^x+3e^{2x}+e^{3x}}{e^x}\,dx$$

$$=\int \left(\frac{1}{e^x}+\frac{3e^x}{e^x}+\frac{3e^{2x}}{e^x}+\frac{e^{3x}}{e^x}\right)dx$$

$$= \int (e^{-x}+3+3e^x+e^{2x})dx$$

$$= \int e^{-x}\,dx + 3\int dx + 3\int e^x dx + \int e^{2x}\,dx$$

$$= \frac{e^{-x}}{-1}+3x+3e^x+\frac{e^{2x}}{2}+C \qquad \therefore \int e^{2x}\,dx = \frac{e^{ax}}{a}$$

$$= -e^{-x}+3x+3e^x+\frac{1}{2}e^{2x}+C$$

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vi. $\int \sin(a+b)x\,dx$

Solution

$$\int \sin(a+b)x\,dx$$

Multiply divide by a +b

$$= \frac{1}{a+b}\int \sin(a+b)x\,(a+b)\,dx$$

$$= -\frac{1}{a+b}\int -\sin(a+b)x\,(a+b)\,dx$$

$$= -\frac{1}{a+b}\cos(a+b)\,x + C$$

vii $\int \sqrt{1-\cos 2x}\;dx \quad (1-\cos 2x > 0)$

Solution

$$\int \sqrt{1-\cos 2x}\;dx$$

$$= \int \sqrt{2\sin^2 x}\;dx \qquad 1-\cos 2x = 2\sin^2 x$$

$$= \sqrt{2}\int \sqrt{\sin^2 x}\;dx$$

$$= \sqrt{2}\int \sin x\;dx$$

$$= -\sqrt{2}\cos x + C \qquad \int \sin x\;dx = -\cos x$$

viii. $\int (\ln x)\times\frac{1}{x}dx$

Solution

$$\int (\ln x)\times\frac{1}{x}dx$$

$$= \frac{(\ln x)^{1+1}}{1+1} + C$$

$$= \frac{(\ln x)^2}{2} + C$$

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Solution

$$\int sin^2x\ dx \qquad \therefore\ sin^2x = \frac{1-Cos\ 2x}{2}$$

$$= \int (\frac{1-Cos\ 2x}{2})dx$$

$$= \int (\frac{1}{2} - \frac{1}{2}Cos\ 2x)dx$$

$$= \int \frac{1}{2}\ dx - \int \frac{1}{2}Cos\ 2xdx$$

$$= \frac{1}{2}\int\ dx - \frac{1}{2}\int Cos\ 2xdx$$

$$= \frac{1}{2}x - \frac{1}{2}\frac{sin\ 2x}{2} + C$$

$$\int \text{Cos ax dx} = \frac{sin\ ax}{a}$$

$$= \frac{1}{2}x - \frac{1}{4}sin\ 2x + C$$

$$= \frac{1}{2}(x - \frac{sin\ 2x}{2} + C$$

x. $\int \frac{1}{1+Cos\ x}\ \mathbf{dx}\ (\frac{\pi}{2} < x < \frac{\pi}{2})$

Solution

$$\int \frac{1}{1+Cos\ x}\ dx$$

$$\Rightarrow\ 1 + \text{Cos x} = 2Cos^2\frac{x}{2}$$

$$= \int \frac{1}{\frac{2Cos\ x^2}{2}}\ dx$$

$$= \frac{1}{2}\int \frac{1}{2Cos^2\frac{x}{2}}\ dx$$

$$= \frac{1}{2}\int sec^2\frac{x}{2}\ dx \qquad \therefore \int sec^2x\ dx = \tan x$$

$$= \frac{1}{2}\frac{tan\ \frac{x}{2}}{\frac{1}{2}} + C$$

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xi. $\int \frac{ax+b}{ax^2+2bx+C}\,dx$

Solution

$$\int \frac{ax+b}{ax^2+2bx+C}\,dx$$

$$= \int (ax^2+2bx+C)^{-1}(ax+b)dx$$

Multiply and divide by 2

$$= \frac{1}{2}\int (ax^2+2bx+C)^{-1}\,2(ax+b)dx$$

$$= \frac{1}{2}\int (ax^2+2bx+C)^{-1}\,(2ax+2b)dx$$

$$= \frac{1}{2}\int \left(\frac{2ax+2b}{ax^2+2bx+C}\right) dx$$

$$= \frac{1}{2}\ln\ ax^2+2bx+C+C_1$$

xii. $\int$ **Cos3xsin2x dx**

Solution

$\int$ Cos3xsin2x dx

Multiply and divide by 2

$= \frac{1}{2}\int$ 2Cos3xsin2x dx $\quad \because$ 2Cos ax sin bx = sin(a+b)+sin+(a-b)

$$= \frac{1}{2}\int [\sin(3x+2x)+\sin(3x-2x)]dx$$

$$= \frac{1}{2}\int [\sin 5x+\sin x]\,dx$$

$$= \frac{1}{2}\int \sin 5x\,dx + \frac{1}{2}\int \sin x\,dx \quad \therefore \quad \int \sin ax\,dx = \frac{-Cos\ ax}{a}$$

$$= \frac{1}{2}\left(\frac{-Cos\ 5x}{5}\right) + \frac{1}{2}(-Cos\ x) + C$$

$$= -\frac{1}{2}\left[\frac{Cos\ 5x}{5} + Cos\ x\right] + C$$

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Solution

$$\int \frac{Cos2x-1}{1+Cos2x}\,dx$$

$$= -\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$$

$$= -\int \frac{2+sin^2 x}{2\,Cos^2 x}\,dx$$

$$= -\int \tan^2 x\,dx$$

$$= -\int (\sec^2 x - 1)\,dx$$

$$= -\int \sec^2 x\,dx + \int dx$$

$$= -\tan x + x + C$$

xiv. $\int \tan^2 x\,dx$

Solution

$$\int \tan^2 x\,dx$$

$$= \int (\sec^2 x - 1)\,dx$$

$$= \int \sec^2 x\,dx - \int dx$$

$$= \tan x - x + C$$

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