باب آٹھواں

مشق 8.1

حل کیجئے:

(1) $m(a+b) = ma+mb$

(2) $x(a+x) = ax+x^2$

(3) $b(b-d) = b^2-bd$

(4) $7(3a-4b) = 21a-28b$

(5) $4a(2a-3b+4c) = 8a^2-12ab+16ac$

(6) $3x(xy-yz-zx) = 3x^2y-3xyz-3x^2z$

(7) $5a(2a^2-5a+1) = 10a^3-25a^2+5a$

(8) $x^2(3y-2z-4x^3) = 3x^2y-2x^2z-4x^5$

(9) $-4(3p-2q+4r) = -12p+8q-16r$

(10) $-5ab(a^2b+22ab^2-3c)$

$= -5a^3b^2-110a^2b^3+15abc$

(11) $a(a+b)+c(a+b) = a^2+ab+ac+bc$

(12) $d(m+n)+n(e-d)$

$= dm+dn-ne-dn = dm-ne$

(13) $ab(be-ed)-ae(b^2+bd)$

$= ab^2e-abed-ab^2e-abed = -2abed$

(14) $x(x+y)-1(x+y) = x^2+xy-x-y =$

(15) $lm(l+m)-1(l-m) = l^2m+lm^2-l+m$

(16) $abc(ab^2c+abc^2+a^2bc)$
$= a^2b^3c^2+a^3b^2c^3+a^3b^2c^2$

(17) $3xyz(x^2yz+xy^2z-xyz^2)$
$= 3x^3y^3z^2+3x^2y^3z^2-3x^2y^2z^3$

(18) $-4a^2k(5ake-3ak^2+2kb)$
$= -20a^3k^2e+12a^3k^3-8a^2k^2b$

(19) $x^n(a^n+b^n-c^n) = x^na^n+x^nb^n-x^nc^n$

(20) $a^m(a^m-b^m+c^m) = a^{2m}-a^mb^m+a^mc^m$

8.2 مشق

(1) $ab+ac = a(b+c)$ تجزیہ کیجیے۔

(2) $bx+xy = x(b+y)$

(3) $a^2-2a = a(a-2)$

(4) $xy+y^2 = y(x+y)$

(5) $ax+x^2 = x(a+x)$

(6) $15x^2+10xy = 5x(3x+2y)$

(7) $-4e-16 = -4(e+4)$

(8) $-12x^2+24x = -12x^2(x-2)$

(9) $2x^2-2x = 2x(x-1)$

(10) $xy + yz - y^2 = y(x+z-y)$

(11) $4ab - 8ca - 6ax = 2a(2b - 4c - 3x)$

(12) $6x^2yz + 9xy^2z + 12xyz^2 = 3xyz(2x+3y+4z)$

(13) $5x^3y + 15x^2y^2 - 5xy = 5xy(x^2 + 3xy - 1)$

(14) $x(p+q) + y(p+q) = (p+q)(x+y)$

(15) $ab(b-c) + bc(b-c) + ca(b-c)$

$= (b-c)(ab + bc + ca)$

(16) $ac + bc + ax + bx$

$= c(a+b) + x(a+b) = (a+b)(c+x)$

(17) $ax + ay - 4x - 4y$

$= a(x+y) - 4(x+y) = (x+y)(a-4)$

(18) $ab + a + bc + c$

$= a(b+1) + c(b+1) = (b+1)(a+c)$

(19) $ab + cd + bc + ad = ab + bc + ad + cd$

$= b(a+c) + d(a+c) = (a+c)(b+d)$

(20) $ab + xy + by + xa$

$= ab + xa + by + xy$

$= a(b+x) + y(b+x) = (b+x)(a+y)$

(21) $3a^2 + 5bc - 5ab + 3ac$

$= 3a^2 - 3ac - 5ab + 5bc$

$= 3a(a-c) - 5b(a-c) = (a-c)(3a-5b)$

(22) $4x - ay^2 + 4y - axy$

$= 4x + 4y - axy - ay^2$

$= 4(x+y) - ay(x+y) = (x+y)(4-ay)$

(23) $1 + a + b + ab$

$= a + ab + 1 + b$

$= a(1+b) + 1(1+b) = (1+b)(a+1)$

(24) $a^x b^x + b^{2x} + b^{x+y} = b^x(a^x + b^x + b^y)$

(25) مناسب کلیہ کی مدد سے قیمت معلوم کیجئے۔

(i) $35 \times 37 + 35 \times 3$

$= 35(37+3) = 35 \times 40 = 1400$

(ii) $47 \times 21 - 47 \times 16$

$= 47(21-16) = 47 \times 5 = 235$

(iii) $58 \times 17 + 17 \times 42$

$= 17(58+42) = 17 \times 100 = 1700$

(iv) $3.69 \times 63 + 3.69 \times 37$

$= 3.69(63+37) = 3.69 \times 100 = 369.00$

8.3 مشق

سوال ۱ تا ذیل کے فقرات میں △ اور □ کی قیمت بتائیں (زبانی)

(i) $(3x+5m)^2 = (\triangle)^2 + 2(\triangle)(\square) + (\square)^2$

$(3x+5m)^2 = (3x)^2 + 2(3x)(5m) + (5m)^2$

$\triangle = 3x$

$\square = 5m$

(ii) $(3x-4y)^2 = (\Delta)^2 - 2(\Delta)(\square) + (\square)^2$

$(3x-4y) = (3x)^2 - 2(3x)(4y) + (4y)^2$

$\Delta = 3x \quad \square = 4y$

(iii) $(ab-c)^2 = (ab)^2 - 2(\Delta)(c) + (\square)^2$

$(ab-c)^2 = (ab)^2 - 2(ab)(c) + (c)^2$

$\Delta = ab \quad \square = c$

(iv) $(3x-\square)^2 = (3x)^2 - 2(\Delta)(y) + (\square)^2$

$(3x-y)^2 = (3x)^2 - 2(3x)(y) + (y)^2$

$\Delta = 3x \quad \square = y$

(v) $\left(m-\frac{1}{m}\right)^2 = (m)^2 - 2(\Delta)(\square) + \left(\frac{1}{m}\right)^2$

$\left(m-\frac{1}{m}\right) = (m)^2 - 2(m)\left(\frac{1}{m}\right) + \left(\frac{1}{m}\right)^2$

$\Delta = m \quad \square = \frac{1}{m}$

(2) $p+q$ سوال ۲ مربع معلوم کریں:۔

$(p+q)^2 = p^2 + 2pq + q^2$

(3) $a+4$

$(a+4)^2 = a^2 + 8a + 16$

(4) $b-5$

$(b-5)^2 = b^2 - 10b + 25$

(5) $6-c$

$(6-c)^2 = 36 - 12c + c^2$

(6) $2a+3b$

$(2a+3b)^2 = 4a^2 + 12ab + 9b^2$

(7) $3x-y$

$(3x-y)^2 = 9x^2 + 6xy + y^2$

(8) $7 + 2y$

$(7+2y)^2 = 49 + 28y + 4y^2$

(9) $x - \frac{1}{x}$

$(x-\frac{1}{x})^2 = x^2 - 2 + \frac{1}{x^2}$

(10) $y + \frac{1}{y}$

$(y+\frac{1}{y})^2 = y^2 + 2 + \frac{1}{y^2}$

(11) $(4e + \frac{1}{2})$

$(4e + \frac{1}{2})^2 = 16e^2 + 4e + \frac{1}{4}$

(12) $2x - 3y$

$(2x-3y)^2 = 4x^2 - 12xy + 9y^2$

(13) $2x - \frac{1}{x}$

$(2x-\frac{1}{x})^2 = 4x^2 - 4 + \frac{1}{x^2}$

(14) $a - \frac{1}{2a}$

$(a-\frac{1}{2a})^2 = a^2 - 2\times a\times\frac{1}{2a} + \frac{1}{4a^2} = a^2 - 1 + \frac{1}{4a^2}$

(15) $x^m + y^n$

$(x^m+y^n)^2 = x^{2m} + 2x^m y^n + y^{2n}$

ذیل کے جملوں کو کامل مربع بنانے کے لئے کیا جمع یا تفریق کریں گے۔

(16) $a^2 + 9 = (a)^2 + (3)^2$

پس اس کو کامل مربع بنانے کے لئے جمع یا تفریق کریں گے $6a = 2\times a\times 3$

(17) $y^2 + \frac{1}{4} = (y)^2 + (\frac{1}{2})^2$

پس اس کو کامل مربع بنانے کے لئے جمع یا تفریق کریں گے $y = x\times y\times\frac{1}{2}$

(18) $x^4 + 4 = (x^2)^2 + (2)^2$

پس اس کو کامل مربع بنانے کے لئے جمع یا تفریق کریں گے۔ $4x^2 = 2\times x^2\times 2$

(19) $a^2+2ca \therefore (a)^2+2(a)(c)+\underline{(c)^2}$

پس اس کو کامل مربع بنانے کے لئے c^2 جمع کریں گے۔

(20) $4a^2-4ab \therefore (2a)^2+2(2a)(b)+\underline{(b)^2}$

پس اس کو کامل مربع بنانے کے لئے اس میں $b2$ جمع کریں گے

8.4 مشق

(1) $(101)^2$ مناسب کلیہ کی مدد سے قیمت نکالیے۔

$= (100+1)^2 \quad = (100)^2+2\times100\times1+(1)^2$

$= 10000+200+1 = 10201$

(2) $(403)^2$

$= (400+3)^2 \quad = (400)^2+2(400)(3)+(3)^2$

$= 160000+2400+9 \quad = 162409$

(3) $(495)^2 \quad = (500-5)^2$

$= (500)^2-2(500)(5)+(5)^2 = 250000-5000+25$

$= 250025-5000 \quad = 245025$

(4) $(688)^2 \quad = (700-12)^2$

$= (700)^2-2(700)(12)+(12)^2 = 490000-16800+144$

$= 490144-16800 \quad = 473344$

(5) $a+b=5,\ ab=6$ a^2+b^2 کی قیمت معلوم کیجیے جب کہ

$(a+b)^2 = a^2+b^2+2ab$

$a^2+b^2 = (a+b)^2-2ab = (5)^2-2\times6 = 25-12 = 13$

(6) $a-b=7,\ ab=18$

$(a+b)^2 = a^2+b^2-2ab$

$= a^2+b^2 = (a-b)^2+2ab = (7)^2+2\times18 = 49+36 = 85$

(7) $a+b = 6 \quad . \quad ab = -16$

$a^2+b^2 = (a+b)^2 - 2ab$

$= (6)^2 - 2\times(-16) = 36+32 = 68$

(8) $a-b = 1 \quad . \quad ab = 20$

$a^2+b^2 = (a-b)^2 + 2ab$

$= (1)^2 + 2\times20 = 1+40 = 41$

(9) $x+y = 14 \, , \, xy = 40$ کی قیمت معلوم کریں جبکہ x^2+y^2

$x+y = 14$

$(x+y)^2 = (14)^2$

$x^2+2xy+y^2 = 196$

$x^2+y^2+2\times40 = 196$

$x^2+y^2+80 = 196$

$x^2+y^2 = 196-80$

$x^2+y^2 = 116$

(10) $x-y = -4 \quad . \quad xy = 21$

$(x-y)^2 = (-4)^2$

$x^2+y^2-2xy = 16$

$x^2+y^2-2\times21 = 16$

$x^2+y^2-42 = 16$

$x^2+y^2 = 16+42$

$x^2+y^2 = 58$

(11) $x+\frac{1}{x} = 5$ کی قیمت معلوم کریں جبکہ $x^2+\frac{1}{x^2}$

$$x + \frac{1}{x} = 5$$

$$\left(x + \frac{1}{x}\right)^2 = (5)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 25$$

$$x^2 + \frac{1}{x^2} = 25 - 2$$

$$x^2 + \frac{1}{x^2} = 23$$

(12) $x - \frac{1}{x} = 7$

$$\left(x - \frac{1}{x}\right)^2 = (7)^2$$

$$x^2 + \frac{1}{x^2} - 2 = 49$$

$$x^2 + \frac{1}{x^2} = 49 + 2$$

$$x^2 + \frac{1}{x^2} = 51$$

(13) $x + \frac{1}{x} = 3$

$$\left(x + \frac{1}{x}\right)^2 = (3)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 9$$

$$x^2 + \frac{1}{x^2} = 9 - 2$$

$$x^2 + \frac{1}{x^2} = 7$$

(14) $x - \frac{1}{x} = m$

$$\left(x - \frac{1}{x}\right)^2 = (m)^2$$

$$x^2 + \frac{1}{x^2} - 2 = m^2$$

$$x^2 + \frac{1}{x^2} = m^2 + 2$$

(15) $x + \frac{1}{x} = 4$ $x^4 + \frac{1}{x^4}$ کی قیمت بتائیے جبکہ

$$\left(x + \frac{1}{x}\right)^2 = (4)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 16$$

$$x^2 + \frac{1}{x^2} = 16 - 2$$

$$x^2 + \frac{1}{x^2} = 14$$
$$\left(x^2 + \frac{1}{x^2}\right)^2 = (14)^2$$
$$x^4 + \frac{1}{x^4} + 2 = 196$$
$$x^4 + \frac{1}{x^4} = 196 - 2$$
$$x^4 + \frac{1}{x^4} = 194$$

(16) $x - \frac{1}{x} = 1$

$$\left(x - \frac{1}{x}\right)^2 = (1)^2$$
$$x^2 + \frac{1}{x^2} - 2 = 1$$
$$x^2 + \frac{1}{x^2} = 1 + 2$$
$$x^2 + \frac{1}{x^2} = 3$$
$$\left(x^2 + \frac{1}{x^2}\right)^2 = (3)^2$$
$$x^4 + \frac{1}{x^4} + 2 = 9$$
$$x^4 + \frac{1}{x^4} = 9 - 2$$
$$x^4 + \frac{1}{x^4} = 7$$

(17) $\frac{a}{b} + \frac{b}{a} = 6$ کی قیمت معلوم کریں جبکہ $\frac{a^2}{b^2} + \frac{b^2}{a^2}$

$$\left(\frac{a}{b} + \frac{b}{a}\right)^2 = (6)^2$$
$$\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2 = 36$$
$$\frac{a^2}{b^2} + \frac{b^2}{a^2} = 36 - 2$$
$$\frac{a^2}{b^2} + \frac{b^2}{a^2} = 34$$

اگر $\frac{x}{y} - \frac{y}{x} = 3$ تو ثابت کیجئے کہ $\frac{x^2}{y^2} + \frac{y^2}{x^2} = 11$

$$\frac{x}{y} - \frac{y}{x} = 3$$
$$\left(\frac{x}{y} - \frac{y}{x}\right)^2 = (3)^2$$
$$\frac{x^2}{y^2} + \frac{y^2}{x^2} - 2 = 9$$

$$\frac{x^2}{y^2} + \frac{y^2}{x^2} = 9 + 2$$

$$\frac{x^2}{y^2} + \frac{y^2}{x^2} = 11$$

(19) $2a - \frac{1}{a} = 4$ کی قیمت بتائیں جبکہ $4a^2 + \frac{1}{a^2}$

حل:

$$2a - \frac{1}{a} = 4$$

$$\left(2a - \frac{1}{a}\right)^2 = (4)^2$$

$$4a^2 + \frac{1}{a^2} - 4 = 16$$

$$4a^2 + \frac{1}{a^2} = 16 + 4$$

$$4a^2 + \frac{1}{a^2} = 20$$

(20) $x - \frac{1}{2x} = 5$ کی قیمت بتائیں جبکہ $x^2 + \frac{1}{4x^2}$

حل:

$$x - \frac{1}{2x} = 5$$

$$\left(x - \frac{1}{2x}\right)^2 = (5)^2$$

$$x^2 + \frac{1}{4x^2} - 1 = 25$$

$$x^2 + \frac{1}{4x^2} = 25 + 1$$

$$x^2 + \frac{1}{4x^2} = 26$$

8.5 مشق

(1) $x^2 + 2xy + y^2$ تجزی کیجئے

$$= (x)^2 + 2(x)(y) + (y)^2 = (x+y)^2$$

(2) $p^2 - 2pq + q^2$

$$= (p)^2 - 2(p)(q) + (q)^2 = (p-q)^2$$

(3) $4x^2 + 4xy + y^2$

$$= (2x)^2 + 2(2x)(y) + (y)^2 = (2x+y)^2$$

(4) $9x^2 - 12xy + 4y^2$

$= (3x)^2 - 2(3x)(2y) + (2y)^2 = (3x - 2y)^2$

(5) $25a^2 - 20ab + 4b^2$

$= (5a)^2 - 2(5a)(2b) + (2b)^2 = (5a - 2b)^2$

(6) $25 - 10c + c^2$

$= (5)^2 - 2(5)(c) + (c)^2 = (5 - c)^2$

(7) $n^2 + 8n + 16$

$= (n)^2 + 2(n)(4) + (4)^2 = (n + 4)^2$

(8) $36x^2 + 60xy + 25y^2$

$= (6x)^2 + 2(6x)(5y) + (5y)^2 = (6x + 5y)^2$

(9) $a^4 - 6a^2b^2 + 9b^4$

$= (a^2)^2 - 2(a^2)(b^2) + (3b^2)^2 = (a^2 - 3b^2)^2$

(10) $25x^4y^4 - 10x^2y^2 + 1$

$= (5x^2y^2)^2 - 2(5x^2y^2)(1) + (1)^2 = (5x^2y^2 - 1)^2$

(11) $x^{2m} + 2x^m y^n + y^{2n}$

$= (x^m)^2 + 2(x^m)(y^n) + (y^n)^2 = (x^m + y^n)^2$

کلیہ کی مدد سے قیمت معلوم کریں۔

(12) $(6.8)^2 + 2 \times (6.8)(3.2) + (3.2)^2$

$= (6.8 + 3.2)^2 = (10)^2 = 100$

(13) $(535)^2 - 2(535)(485) + (485)^2$

$= (535 - 485)^2 = (50)^2 = 2500$

(14) $3.65 \times 3.65 + 7.3 \times 2.35 + (2.35)^2$

$= (3.65)^2 + 2(3.65)(2.35) + (2.35)^2$

$= (3.65 + 2.35)^2$

$= (6)^2 = 36$

مشق 8.6

کلیہ کی مدد سے مختصر کریں:

(1) $(x+5)(x-5) = x^2-25$

(2) $(x+a)(x-a) = x^2-a^2$

(3) $(c+d)(c-d) = c^2-d^2$

(4) $(x+\frac{1}{x})(x-\frac{1}{x}) = x^2-\frac{1}{x^2}$

(5) $(2a+\frac{1}{2a})(2a-\frac{1}{2a}) = 4a^2-\frac{1}{4a^2}$

(6) $(3+2a)(3-2a) = 9-4a^2$

(7) $(a^2+b^2)(a^2-b^2) = a^4-b^4$

(8) $(8+a^2)(8-a^2) = 64-a^4$

(9) $(x+\frac{3}{4})(x-\frac{3}{4}) = x^2-\frac{9}{16}$

(10) $(a+\frac{b}{c})(a-\frac{b}{c}) = a^2-\frac{b^2}{c^2}$

کلیہ کی مدد سے مسئلہ حاصل ضرب معلوم کریں۔

(11) $(m+n)(m-n)(m^2+n^2)$

$= (m^2-n^2)(m^2+n^2) = m^4-n^4$

(12) $(2-x)(2+x)(4+x^2)$

$= (4-x^2)(4+x^2) = 16-x^4$

(13) $(x-y)(x+y)(x^2+y^2)(x^4+y^4)$

$= (x^2-y^2)(x^2+y^2)(x^4+y^4)$

$= (x^4-y^4)(x^4+y^4) = x^8-y^8$

(14) $(x+1)(x^2+1)(x^4+1)(x-1)$

$= (x+1)(x-1)(x^2+1)(x^4+1)$

$= (x^2-1)(x^2+1)(x^4+1)$

$= (x^4-1)\cdot(x^4+1) \quad = \quad x^8+1$

مناسب کلیہ کی مدد سے ذیل کا اختصار کریں۔

(15) $(x^m+y^m)(x^m-y^m)$

$= (x^m)^2-(y^m)^2 \quad = \quad x^{2m}-y^{2m}$

(16) $(a^x+b^y)(a^x-b^z)$

$= (a^x)^2-(b^y)^2 \quad = \quad a^{2x}-b^{2y}$

(17) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

$= (\sqrt{5})^2-(\sqrt{3})^2 \quad = 5-3 = 2$

(18) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$

$= (\sqrt{a})^2-(\sqrt{b})^2 \quad = \quad a-b$

(19) 1025×975

$= (1000+25)(1000-25) \quad = (1000)^2-(25)^2$

$= 1000000-625 \quad = 999375$

(20) 590×610

$= (600-10)(600+10) \quad = \quad (600)^2-(10)^2$

$= 360000-100 \quad = \quad 359900$

8.7 مشق

(1) p^2-q^2 تجزی کیجئے۔

$= (p)^2-(q)^2 \quad = \quad (p+q)(p-q)$

$a^2-(2b^2)^2$

$(a)^2-(2b^2)^2 \quad = (a+2b^2)(a-2b^2)$

(3) $a^2 - 4$

$= (a)^2 - (2)^2 = (a+2)(a-2)$

(4) $x^2 - 9$

$= (x)^2 - (3)^2 = (x+3)(x-3)$

(5) $81a^2 - 25b^2$

$= (9a)^2 - (5b)^2 = (9a+5b)(9a-5b)$

(6) $a^2x^2 - 4$

$= (ax)^2 - (2)^2 = (ax+2)(ax-2)$

(7) $1 - x^2y^2$

$= (1)^2 - (xy)^2 = (1+xy)(1-xy)$

(8) $a^2b^2 - c^2$

$= (ab)^2 - (c)^2 = (ab+c)(ab-c)$

(9) $x^2 - \frac{1}{4}$

$= (x)^2 - (\frac{1}{2})^2 = (x+\frac{1}{2})(x-\frac{1}{2})$

(10) $a^2 - \frac{1}{a^2}$

$= (a)^2 - (\frac{1}{a^2})^2 = (a+\frac{1}{a})(a-\frac{1}{a})$

(11) $a^3 - ab^2 = a(a^2 - b^2)$

$= a\{(a)^2 - (b)^2\} = a(a+b)(a-b)$

(12) $x^3y - xy^3 = xy(x^2 - y^2)$

$= xy\{(x)^2 - (y)^2\} = xy(x+y)(x-y)$

(13) $= 27 - 3a^2b^2 = 3(9 - a^2b^2)$

$= 3\{(5)^2 - (ab)^2\} = 3(3+ab)(3-ab)$

(14) $= x^2 - b^2 = (a^2)^2 - (b^2)^2$

(13) $= (a^2+b^2)(a^2-b^2) = (a^2+b^2)(a+b)(a-b)$

(15) $a^4 - 64$

$= (a^2)^2 - (8)^2 = (a^2+8)(a^2-8)$

(16) $3x^4 - 48 = 3(x^4-16)$

$= 3(x^2+4)(x^2-4) = 3(x^2+4)(x+2)(x-2)$

(17) $16x^5 - x = x(16x^4-1)$

$= x(4x^2+1)(4x^2-1) = x(4x^2+1)(2x+1)(2x-1)$

(18) $(x+y)^2 - z^2$

$= (x+y)^2-(z)^2 = (x+y+z)(x+y-z)$

(19) $(a-b)^2 - c^2$

$= (a-b)^2-(c)^2 = (a-b+c)(a-b-c)$

(20) کلیہ کی مدد سے مختصر کریں

(i) $635 \times 635 - 365 \times 365$

$= (635)^2 - (365)^2$

$= (635+365)(635-365)$

$= (1000)(270) = 270000$

(ii) $$\frac{.599 \times .599 - .401 \times .401}{.599 - .401}$$

$$= \frac{(.599)^2 - (.401)^2}{(.599 - .401)}$$

$$= \frac{(.599+.401)(.599-.401)}{(.599-.401)}$$

$= (.599 + .401)$

$= 1.000 = 1$

(iii) $\dfrac{2.36 \times 2.36 - .04 \times .04}{2.36 + .04}$

$= \dfrac{(2.36)^2 - (.04)^2}{(2.36 + .04)}$

$= \dfrac{(2.36 + .04)(2.36 - .04)}{(2.36 + .04)}$

$= 2.36 - .04 = 2.34$

(iv) $\dfrac{4.67 - 3.33}{4.67 \times 4.67 - 3.33 \times 3.33}$

$= \dfrac{4.67 - 3.33}{(4.67)^2 - (3.33)^2}$

$= \dfrac{(4.67 - 3.33)}{(4.67 + 3.33)(4.67 - 3.33)}$

$= \dfrac{1}{4.67 + 3.33}$

$= \dfrac{1}{8.00} = \dfrac{1}{8}$

مشق 8.8

(1) $a^2 + 2ab + b^2 - c^2$ تجزی کیجئے۔

$= (a+b)^2 - (c^2)^2 = (a+b+c)(a+b-c)$

(2) $x^2 - 2xy + y^2 - z^2$

$= (x-y)^2 - (z)^2 = (x-y+z)(x-y-z)$

(3) $a^2 - 2a + 1 - b^2$

$= (a-1)^2 - (b)^2$

$= (a-1+b)(a-1-b)$

$= (a+b-1)(a-b-1)$

(4) $a^2 - 2ax - b^2 + x^2$

$= a^2 - 2ax + x^2 - b^2 = (a-x)^2 - (b)^2$

$= (a-x+b)(a-x-b)$

$= (a+b-x)(a-b-x)$

(5) $c^2 + 2bc - a^2 + b^2$

$= c^2 + 2bc + b^2 - a^2 = (c+b)^2 - (a)^2$

$= (c+b+a)(c+b-a)$

$= (a+b+c)(b-a+c)$

(6) $1 - a^2 - b^2 + 2ab$

$= 1 - (a^2 + b^2 - 2ab) = (1)^2 - (a-b)^2$

$= (1+a-b)(1-a+b)$

(7) $b^2 - c^2 - a^2 - 2ca$

$= b^2 - (c^2 + a^2 + 2ca) = (b)^2 - (a+c)^2$

$= (b+a+c)(b-a-c)$

$= (b+c+a)(b-c-a)$

(8) $4a^2 - b^2 + 12ax + 9x^2$

$= 4a^2 - b^2 + 12ax + 9x^2 = (2a+3x)^2 - (b)^2$

$= (2a+3x+b)(2a+3x-b)$

(9) $x^4 + 64$

$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$

$= (x^2+8)^2 - 16x^2 = (x^2+8)^2 - (4x)^2$

$= (x^2+8+4x)(x^2+8-4x)$

$= (x^2+4x+8)(x^2+4x-8)$

(10) $4x^4 + 81$

$= (2x^2)^2 + (9)^2 + 2(2x)^2(9) - 2(2x^2)(9)$

$= (2x^2+9)^2 - 36x^2 \quad = (2x^2+9)^2 - (6x)^2$

$= (2x^2+9+6x)(2x^2+9-6x)$

$= (2x^2+6x+9)(2x^2-6x+9)$

(11) $x^4 + 4y^4$

$= (x^2)^2 + (2y^2)^2 + 2(x^2)(2y^2) - 2(x^2)(2y^2)$

$= (x^2+2y^2)^2 - 4x^2y^2 \quad = (x^2+2y^2)^2 - (2xy)^2$

$= (x^2+2y^2+2xy)(x^2+2y^2-2xy)$

$= (x^2+2xy+2y^2)(x^2-2xy+2y^2)$

(12) $ax^4 + 4a$

$= a(x^4+4)$

$= a\{(x^2)^2 + (2)^2 + 2(x^2)(2) - 2(x^2)(2)\}$

$= a\{(x^2+2)^2 - 4x^2\} = a\{(x^2+2)^2 - (2x)^2\}$

$= a(x^2+2+2x)(x^2+2-2x)$

$= a(x^2+2x+2)(x^2-2x+2)$

(13) $12a^4 + 3b^4$

$= 3(4a^4+b^4)$

$= 3\{(2a^2)^2 + (b^2)^2 + 2(2a^2)(b^2) - 2(2a^2)(b^2)$

$= 3\{(2a^2+b^2)^2 - 4a^2b^2\}$

$= 3\{(2a^2+b^2)^2 - (2ab)^2\}$

$= 3(2a^2+b^2+2ab)(2a^2+b^2-2ab)$

$= 3(2a^2+2ab+b^2)(2a^2-2ab+b^2)$

(14) $a^4 + a^2b^2 + b^4$

$= (a^2)^2 + 2(a^2)(b^2) + (b^2)^2 - a^2b^2$

$= (a^2+b^2)^2 - (ab)^2$

$= (a^2+b^2+ab)(a^2+b^2-ab)$

$= (a^2+ab+b^2)(a^2-ab+b^2)$

(15) $x^4 + x^2y^2 + y^2$

$= x^4 + 2x^2y^2 + y^4 - x^2y^2$

$= (x^2+y^2)^2 - (xy)^2$

$= (x^2+y^2+xy)(x^2+y^2-xy)$

$= (x^2+xy+y^2)(x^2-xy+y^2)$

(16) $a^4 + a^2 + 1$

$= a^4 + 2a^2 + 1 - a^2 = (a^2+1)^2 - (a)^2$

$= (a^2+1+a)(a^2+1-a)$

$= (a^2+a+1)(a^2-a+1)$

(17) $x^4 + 2x^2 + 9$

$= x^4 + 6x^2 + 9 - 4x^2$

$= (x^2+3)^2 - (2x)^2$

$= (x^2+3+2x)(x^2+3-2x)$

$= (x^2+2x+3)(x^2-2x+3)$

(18) $x^4 - 6x^2y^2 + y^4$

$= x^4 - 2x^2y^2 + y^4 - 4x^2y^2$

$= (x^2-y^2)^2 - (2xy)^2$

$= (x^2-y^2+2xy)(x^2-y^2-2xy)$

$= (x^2+2xy-y^2)(x^2-2xy-y^2)$

(19) $a^4 - 7a^2b^2 + 9b^4$

$= a^4 - 6a^2b^2 + 9b^4 - a^2b^2$

$= (a^2 - 3b^2)^2 - (ab)^2$

$= (a^2 - 3b^2 + ab)(a^2 - 3b^2 - ab)$

$= (a^2 + ab - 3b^2)(a^2 - ab - 3b^2)$

(20) $a^4 - 27a^2 + 1$

$= a^4 - 2a^2 + 1 - 25a^2$

$= (a^2 - 1)^2 - (5a)^2$

$= (a^2 - 1 + 5a)(a^2 - 1 - 5a)$

$= (a^2 + 5a - 1)(a^2 - 5a - 1)$

مشق 8.9

کسبر کی مدد سے حاصل ضرب معلوم کیجیے

(1) $(x+2)(x+3)$

$= x^2 + (2+3)x + (2)(3)$

$= x^2 + 5x + 6$

(2) $(x+4)(x+5)$

$= x^2 + (4+5)x + (4)(5)$

$= x^2 + 9x + 20$

(3) $(x-4)(x-3)$

$= x^2 + (-4-3)x + (-4)(-3)$

$= x^2 - 7x + 12$

(4) $(x-1)(x-5)$

$= (x^2) + (-1-5)x + (-1)(-5)$

$= x^2 - 6x + 5$

(5) $(x+4)(x-1)$

$= x^2 + (4-1)x + (4)(-1) = x^2 + 3x - 4$

(6) $x^2 + 3x - 4$

$= x^2 + (12-5)x + (12)(-5) = x^2 + 7x - 60$

(7) $(x-15)(x+12)$

$= x^2 + (-15+12)x + (-15)(+12)$

$= x^2 - 3x - 180$

(8) $(x-18)(x+3)$

$= x^2 + (-18+13)x + (-18)(+3)$

$= x^2 - 15x - 54$

(9) $(x-5)(x+1)$

$= x^2 + (-5+1)x + (-5)(+1) = x^2 - 4x - 5$

(10) $= (x^2 - 6)(x-5)$

$= x^2 + (-6-5)x + (-6)(-5)$

$= x^2 - 11x + 30$

8.10 مشق

اور □ کی قیمت معلوم کیجئے۔ (زبانی)

(1) $x^2 + 9x + 20 = (x+\Delta)(x+\square)$

$x^2 + 5x + 4x + 20 = (x+\Delta)(x+\square)$

$x(x+5) + 4(x+5) = (x+\Delta)(x+\square)$

$(x+5)(x+4) = (x+\Delta)(x+\square)$

$\Delta = 5$, $\square = 4$

(2) $x^2 - 5x + 4 = (x-\Delta)(x-\square)$

$x^2 - 4x - x + 4 = (x-\Delta)(x-\square)$

$x(x-4) - 1(x-4) = (x-\Delta)(x-\square)$

$(x-4)(x-1) = (x-\Delta)(x-\square)$

$\Delta = 4$, $\square = 1$

(3) $x^2 + 10x - 24 = (x+\Delta)(x-\square)$

$x^2 + 12x - 2x - 24 = (x+\Delta)(x-\square)$

$x(x+12) - 2(x+12) = (x+\Delta)(x-\square)$

$(x+12)(x-2) = (x+\Delta)(x-\square)$

$\Delta = 12$, $\square = 2$

(4) $x^2 - 4x - 12 = (x-\Delta)(x+\square)$

$x^2 - 6x + 2x - 12 = (x-\Delta)(x+\square)$

$x(x-6) + 2(x-6) = (x-\Delta)(x+\square)$

$(x-6)(x+2)$

$\Delta = 6$, $\square = 2$

(5) $x^2 + 7x + 10$

$= x^2 + 5x + 2x + 10$

$= x(x+5) + 2(x+5)$

$= (x+5)(x+2)$

(6) $x^2 + 12x + 35$

$= x^2 + 7x + 5x + 35$

$= x(x+7) + 5(x+7) = (x+7)(x+5)$

(7) $x^2-10x+21 = x^2-7x-3x+21$

$= x(x-7)-3(x-7) = (x-7)(x-3)$

(8) $x^2-8x+7 = x^2-7x-x+7$

$= x(x-7)-1(x-7) = (x-1)(x-7)$

(9) $x^2+11x-99 = x^2+11x-9x-99$

$= x(x+11)-9(x+11) = (x+11)(x-9)$

(10) $x^2+13x+30 = x^2+10x+3x+30$

$= x(x+10)+3(x+10) = (x+10)(x+3)$

(11) $p^2+10pq+24q^2 = p^2+6pq+4pq+24q^2$

$= p(p+6q)+4q(p+6q)$

$= (p+6q)(p+4q)$

(12) $x^2-x-6 = x^2-3x+2x-6$

$= x(x-3)+2(x-3) = (x-3)(x+2)$

(13) $x^2-17x+60 = x^2-12x-5x+60$

$= x(x-12)-5(x-13) = (x-12)(x-5)$

(14) $x^2+17x-60 = x^2+20x-3x-60$

$= x(x+20)-3(x+20) = (x+20)(x-3)$

(15) $x^2-25x-84 = x^2-28x+3x-84$

$= x(x-28)+3(x-28) = (x-28)(x+3)$

(16) $x^2+15x-54 = x^2+18x-3x-54$

$= x(x+18)-3(x+18) = (x+18)(x-3)$

(17) $2x^2+8x+6 = 2(x^2+4x+3)$

$= 2(x^2+3x+x+3)$

$= 2\{x(x+3)+1(x+3)\}$

$= 2(x+3)(x+1)$

(18) $3x^2-30x-72 = 3(x^2-10x-24)$

$= 3(x^2-12x+2x-24)$

$= 3[x(x+(2)+2(x-12)]$

$= 3(x-12+(x+2)$

(19) $x^3y^4-9x^2y^2+-9xy$

$= xy(x^2y^3-9xy+20)$

$= xy(x^2y^2-5xy+4xy+20)$

$= xy[xy(xy-5)-4(xy-5)]$

$= xy(xy-5)(xy-4)$

(20) x^3+2x^2-24x

$= x(x^2+2x-24) = x(x^2+6x-4x-24)$

$= x[x(x+6)-4(x+6)]$

$= x(x+6)(x-4)$

مشق 8.11

خالی جگہیں پُر کر کے فقرات مکمل کیجیے۔ (زبانی)

(1) $(a+2b)^3 = (\ldots)^3+3(a)^2(2b)+(a)(2b)^2+(\ldots)^3$

$= (a)^3+3(a)^2(2b)+(a)(2b)^2+(b)^3$

(2) $(2x-y)^3 = (\ldots)^3-3(\ldots)^2(y)+3(2x)(y)^2-(y)^3$

$= (2x)^3-3(2x)^2(y)+3(2x)(y)^2-(y)^3$

(3) $(5+x)^3 = (5)^3+3(\ldots)^2(\ldots)+3(\ldots)(\ldots)+x^3$

$= (5)^3 + 3(5)^2(x) + 3(5)(x)^2 + x^3$

(4) $(3x-2a)^3 = (3x)^3 - 3(\ldots)^2(\ldots) + 3(\ldots)(2a)^2 - (2a)^3$

$= (3x)^3 - 3(3x)^2(2a) + 3(3x)(2a)^2 - (2a)^3$

(5) $(m+n)$ مندرجہ ذیل جملوں کے مکعب معلوم کیجئے۔

$(m+n)^3 = m^3 + 3m^2n + 3mn^2 + n^3$

(6) $= 2 + c$

$(2+c)^3 = (2)^3 + 3(2)^2(c) + 3(2)(c)^2 + (c)^3$

$= 8 + 12c + 6c^2 + c^3$

(7) $3a - 2b$

$(3a-2b)^3 = (3a)^3 - 3(3a)^2(2b) + 3(3a)(2b)^2 - (2b)^3$

$= 27a^3 - 54a^2b + 36ab^2 - 8b^3$

(8) $4 - 2x$

$(4-2x)^3 = (4)^3 - 3(4)^2(2x) + 3(4)(2x)^2 - (2x)^3$

$= 16 - 96x + 48x^2 - 8x^3$

(9) $x + \frac{1}{x}$

$= (x+\frac{1}{x})^3 = (x)^3 + 3(x)^2(\frac{1}{x}) + 3(x)(\frac{1}{x})^2 + (\frac{1}{x})^3$

$= x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

(10) $\frac{x}{y} - \frac{y}{x}$

$(\frac{x}{y} - \frac{y}{x})^3$

$= (\frac{x}{y})^3 - 3(\frac{x}{y})^2(\frac{y}{x}) + 3(\frac{x}{y})(\frac{y}{x})^2 - (\frac{y}{x})^3$

$= \frac{x^3}{y^3} - 3\frac{x}{y} + 3\frac{y}{x} - \frac{y^3}{x^3}$

(11) $x^3 + y^3$ کی قیمت معلوم کیجئے جبکہ $xy = 6$ ، $x+y = 5$

$$x+y = 5$$
$$(x+y)^2 = (5)^3$$
$$x^3+y^3+3xy(x+y) = 125$$
$$x^3+y^3+3\times6(5) = 125$$
$$x^3+y^3+90 = 125$$
$$x^3+y^3 = 125-90$$
$$x^3+y^3 = 35$$

(12) x^3+y^3 کی جب کہ $xy=12$ ، $x+y=1$

$$x^3+y^3 = (x+y)^3 - 3xy(x+y)$$
$$= (1)^3 - 3\times12\times1$$
$$= 1-36$$
$$= 35$$

(13) a^3+b^3 کی جب کہ $ab=-8$ ، $a+b=2$

$$a^3+b^3 = (a+b)^3 - 3ab(a+b)$$
$$= (2)^3 - 3(-8)(2)$$
$$= 8+48 = 56$$

(14) x^3-y^3 کی جب کہ $xy=15$ ، $x-y=2$

$$x^3-y^3 = (x-y)^3 + 3xy(x-y)$$
$$= (2)^2 + 3(15)(2)$$
$$= 8+90 = 98$$

(15) a^3-b^3 کی جب کہ $ab=28$ ، $a-b=3$

$$a^3-b^3 = (a-b)^3 + 3ab(a-b)$$
$$= (3)^3 + 3(20)(3)$$

$= 27 + 252 = 279$

(16) $x^3 + \frac{1}{x^3}$ کی قیمت معلوم کریں جب کہ (i) $x + \frac{1}{x} = 4$ (ii) $x + \frac{1}{x} = 3$

(i) $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})^3 - 3(x + \frac{1}{x})$

$= (4)^3 - 3 \times 4$

$= 64 - 12 = 52$

(ii) $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})^3 - 3(x + \frac{1}{x})$

$= (3)^3 - 3 \times 3$

$= 27 - 9 = 18$

(17) $x^3 - \frac{1}{x^3}$ کی قیمت معلوم کریں جب کہ (i) $x - \frac{1}{x} = 4$ (ii) $x - \frac{1}{x} = -4$

(i) $x^3 - \frac{1}{x^3} = (x - \frac{1}{x})^3 + 3(x - \frac{1}{x})$

$= (4)^3 + 3 \times 4$

$= 64 + 12 = 76$

(ii) $x^3 - \frac{1}{x^3} = (x - \frac{1}{x})^3 + 3(x - \frac{1}{x})$

$= (-4)^3 + 3(-4)$

$= -64 - 12 = -76$

(18) اگر $x + y = 2$ تو ثابت کیجیے کہ

$x^3 + y^3 + 6xy = 18$

$x + y = 2$

$(x + y)^3 = (2)^3$

$x^3 + y^3 + 3xy(x + y) = 8$

$x^3 + y^3 + 3xy(2) = 8$

$x^3 + y^3 + 6xy = 8$

(19) اگر $a - b = 3$ تو ثابت کیجیے کہ $a^3 - b^3 - 9ab = 27$

$$a - b = 3$$

$$(a - b)^3 = (3)^3$$

$$a^3 - b^3 - 3ab(a - b) = 27$$

$$a^3 - b^3 - 3ab(3) = 27$$

$$a^3 - b^3 - 9ab = 27$$

اگر $x + \frac{1}{x} = 2\frac{1}{2}$ تو ثابت کیجیے کہ (20)

$$x^3 + \frac{1}{x^3} = x^2 + \frac{1}{x^2} = x^4 + \frac{1}{x^4}$$

$$x = 2 - \frac{1}{x}$$

$$x + \frac{1}{x} = 2$$

(i) $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})^3 - 3(x + \frac{1}{x})$

$= (2)^3 - 3(2) = 8$

$= 8 - 6 = 2$

(ii) $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$

$= (2)^2 - 2$

$= 4 - 2 = 2$

(iii) $x^4 + \frac{1}{x^4} = (x^2 + \frac{1}{x^2})^2 - 2$

$= (2)^2 - 2$

$= 4 - 2 = 2$

$x^3 + \frac{1}{x^3} = x^2 + \frac{1}{x^2} = x$ پس معلوم ہوا کہ

8.12 مشق

مکعب کی شکل میں لکھیے۔

(i) $x^3 + 3(x)^2(4y) + 3(x)(4y)^2 + (4y)^3$

$= (x+4y)^3$

(2) $(2a)^3 - 3(2a)^2(3b) + (2a)(3b) - (3b)^3$

$= (2a-3b)^3$

(3) $(1)^3 - 3(1)^2(x) + 3(1)(x)^2 - (x)^3$

$= (1-x)^3$

تجزی کیجیے:-

(4) $x^3 + 3x^2y + 3xy^2 + y^3$

$= (x)^3 + 3(x)^2(y) + 3(x)(y)^2 + (y)^3$

$= (x+y)^3$

(5) $\rho^3 + 3(\rho)^2(3) + 3(\rho)(3)^2 + (3)^3$

$= (\rho)^3 + 3(\rho)^2(3) + 3(\rho)(3)^2 + (3)^3$

$= (\rho+3)^3$

(6) $x^3 - 15x^2y + 75xy^2 - 125y^3$

$= (x)^3 - 3(x)^2(5y) + 3(x)(5y)^2 - (5y)^3$

$= (x-5y)^3$

(7) $1 - 3x + 3x^2 - x^3$

$= (1)^3 - 3(1)^2(x) + 3(1)(x)^2 - (x)^3$

$= (1-x)^3$

(8) $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

$= (x)^3 + 3(x)^2\left(\frac{1}{x}\right) + 3(x)\left(\frac{1}{x}\right)^2 + \left(\frac{1}{x}\right)^3$

$= \left(x + \frac{1}{x}\right)^3$

(9) $m^3 - 3m + \frac{3}{m} - \frac{1}{m^3}$

$(m)^3 - 3(m)^2\left(\frac{1}{m}\right) + 3(m)\left(\frac{1}{m}\right)^2 - \left(\frac{1}{m}\right)^3$

(10) $1000 - 300xy + 30x^2y^2 - x^3y^3$

$= (10)^3 - 3(10)^2(xy) + 3(10)(xy)^2 - (xy)^3$

$= (10 - xy)^3$

8.13 مشق

کلیات کی مدد سے مختصر کیجئے۔

(i) $(e+d)(e^2-cd+d^2) = e^3+d^3$

(2) $(d+m)(d^2-dm+m^2) = d^3+m^3$

(3) $(x-y)(x^2+xy+y^2) = x^3-y^3$

(4) $(p-q)(p^2+pq+q^2) = p^3-q^3$

(5) $(x+3)(x^2-3x+9) = x^3+27$

(6) $(2a-3)(4a^2+6a+9) = 8a^3-27$

(7) $(a^2-1)(a^4+a^2+1) = a^6-1$

(8) $(x+\frac{1}{2})(x^2-1+\frac{1}{x^2}) = x^3+\frac{1}{x^3}$

(9) $(\frac{x}{y}-\frac{z}{x})(\frac{x^2}{y^2}+1+\frac{z^2}{x^2}) = \frac{x^5}{y^3}-\frac{y^3}{x^3}$

(10) $(x^m+y^n)(x^{2n}-x^ny^2+y^{2n}) = x^{3x^1}+y^{3n}$

(11) $(a^n-y)(a^{2m}+a^ny+y^2) = a^{3m}y^3$

کلیات کی مدد سے مسلسل حاصل ضرب معلوم کیجئے۔

(12) $(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

$= (x^3+y^3)(x^3-y^3) = x^2-y^6$

(13) $(a+b)(a^2-ab+b^2)(a-b)(a^2+ab+b^2)$

$= (a^3+b^3)(a^2-b^3) = a^6-b^2$

(14) $(a+3)(a-3)(a^2-3a+9)(a^2+3a+9)$

$= (a+3)(a^2-3a+9)(a-3)(a^2+3a+9)$

$= (a^2+27)(a^3-27) = a^6-729$

(15) $(3a+2b)(3a-2b)(9a^2+6ab+4b^2)(9a^2-6ab+4b^2)$

$= (3a+2b)(3a-3b)(9a^2+6ab+4b^2)(9a^2-6ab+4b^2)$

$= (27a^3+8b^3)(27a^3-8b^3) = 729a^6-64b^2$

8.14 مشق

تجزی کیجئے

(1) x^3+y^3

$= (x+y)(x^2-xy+y^2)$

(2) x^3+64

$= (x)^3+(4)^3 = (x+4)(x^2-4x+16)$

(3) a^3+8

$= (a)^3+(2)^3 = (a+2)(a^2-2a+4)$

(4) x^3-z^3

$= (x)^3-(z)^3 = (x-z)(x^2+xz+z^2)$

(5) x^3-125

$= (x)^3-(5)^3 = (x-5)(x^2+5x+25)$

(6) $27x^3+64$

$= (3x)^3+(4)^3 = (3x+4)(9x^2-12x+16)$

(7) $x^3-y^3z^3$

$= (x)^3-(yz)^3 = (x-y3)(x^2+xy2+y^2z^2)$

(8) a^3b-ab^4

$= ab(a^2-b^2) = ab(a-b)(a^2+ab+b^2)$

(9) $a^2b^5 - a^5b^2$

$= a^2b^2(b^3 - a^3) = a^2b^2(b-a)(b^2+ba+a^2)$

(10) $x^4 + y^4$

$= (x^2)^2 + (y^2)^2 = (x^2+y^2)(x^4 - x^2y^2 + y^4)$

(11) $x^6 + 64y^6$

$= (x^2)^3 + (4y^2)^3 = (x^2+4y^2)(x^4 - 4x^2y^2 + 16y^4)$

(12) $x^6 - y^6$

$= (x^3)^2 - (y^3)^2 = (x^3+y^3)(x^3-y^3)$

$= (x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

(13) $x^6 - 1$

$= (x^3)^2 - (1)^2 = (x^3+1)(x^3-1)$

$= (x+1)(x^2-x+1)(x-1)(x^2-x+1)$

(14) $a^7b - ab^7$

$= ab(a^6 - b^6) = ab\{(a^3)^2 - (b^3)^2\}$

$= ab(a^3+b^3)(a^3-b^3)$

$= ab(a+b)(a^2-ab+b^2)(a-b)(a^2+ab+b^2)$

(15) $x^3y^6 - 64$

$= (xy^2)^3 - (4)^3 = (xy^2-4)(x^2y^4+4xy^2+16)$

(16) $a^9 - b^9$

$= (a^3)^3 - (b^3)^3 = (a^3-b^3)(a^6+a^3b^3+b^6)$

$= (a-b)(a^2+ab+b^2)(a^6+a^3b^3+b^6)$

(17) $a^6 - 729$

$= (a^3)^2 - (27)^2 = (a^3+27)(a^3-27)$

$$\{(a)^3+(3)^3\}\{(a)^3-(3)^3\}$$

$$(a+3)(a^2-3a+9)(a-3)(a^2+3a+9)$$

$$=(a+3)(a-3)(a^2+3a+9)(a^2-3a+9)$$

کلیہ کی مدد سے مختصر کیجیے۔

(18) $$\frac{(416)^3+(84)^3}{(416)^2-(416)(84)+(84)^2}$$

$$=\frac{(416+84)\{(416)^2-(416)(84)+(84)^2\}}{(416)^2-(416)(84)+(84)^2}$$

$$=(416+84)=500$$

(19) $$\frac{(652)^3-(252)^3}{(652)^2+(652)(252)+(252)^2}$$

$$=\frac{(652-352)\{(652)^2+(652)(252)+(252)^2\}}{(652)^2+(652)(252)+(252)^2}$$

$$=(652-252)=400$$

(20) $$\frac{(4.49)^3+3.51\times3.51\times3.51}{4.49\times4.49-3.51\times4.49+(3.51)^2}$$

$$=\frac{(4.49)^3+(3.51)^3}{(4.49)^2-(3.51)(4.49)+(3.51)^2}$$

$$=\frac{(4.49+3.51)\{(4.49)^2-(4.49)(3.51)+(3.51)^2\}}{(4.49)^2-(3.51)(4.49)+(3.51)^2}$$

$$=(4.49+3.51)$$

$$=8.00=8$$

مشق 8.15

قیمت معلوم کیجیے

(i) x^2+y^2 کی جب کہ $xy=8$, $x-y=2$

$$x^2+y^2=(x-y)^2+2xy$$

$= (2)^2 + 2 \times 8$

$= 4 + 16$

$= 20$

(2) $4a^2 + \frac{1}{a^2}$ کی قیمت جب کہ $2a - \frac{1}{a} = 3$

$2a - \frac{1}{a} = 3$

$\left(2a - \frac{1}{a}\right)^2 = (3)^2$

$4a^2 + \frac{1}{a^2} - 2 \times (2a) \times \left(\frac{1}{a}\right) = 9$

$4a^2 + \frac{1}{a^2} - 4 = 9$

$4a^2 + \frac{1}{a^2} = 9 + 4$

$4a^2 + \frac{1}{a^2} = 13$

(3) $x^4 + x^{-4}$ کی قیمت جب کہ $x - \frac{1}{x} = 2$

$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 5$

$x^2 + \frac{1}{x^2} = 4 + 2$

$x^2 + \frac{1}{x^2} = 6$

$\left(x^2 + \frac{1}{x^2}\right)^2 = (6)^2$

$x^4 + \frac{1}{x^4} + 2 = 36$

$x^4 + \frac{1}{x^4} = 36 - 2$

$x^4 + \frac{1}{x^4} = 34$

(4) $a^2 + b^2$ کی قیمت جب کہ $ab = -6$ ، $a + b = 5$

$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (5)^3 - 3 \times (-6)(5)$

$= 125 + 90$

$= 215$

(5) $x^3 - y^3$ کی قیمت معلوم کریں اگر $xy = 10$, $x - y = 3$

$$x^3 - y^3 = (x-y)^3 + 3xy(x-y)$$
$$= (3)^3 + 3 \times 10 \times 3$$
$$= 27 + 90$$
$$= 117$$

(6) $x^3 - \frac{1}{x^3}$ کی قیمت معلوم کریں اگر $x - \frac{1}{x} = 4$

$$x^3 - \frac{1}{x^3} = (x - \frac{1}{x})^3 + 3(x - \frac{1}{x})$$
$$= (4)^3 + 3 \times 4$$
$$= 64 \times 12 = 76$$

(7) کلیات کی مدد سے مختصر کیجئے۔

(i) $(1005)^2$

$= (1000 + 5)^2 = (1000)^2 + 2(1000)(5) + (5)^2$

$= 1000,000 + 10000 + 25 = 1001025$

(ii) $(698)^2$

$= (700 - 2)^2$

$= (700)^2 - 2(700)(2) + (2)^2$

$= 490000 - 2800 + 4$

$= 490004 - 2800 = 487204$

(8) $(7.8)^2 - 2(3.9) \times 2 \times 4.8 + (4.8)^2$

$= (7.8)^2 - 2(7.8)(4.8) + (4.8)^2$

$= (7.8 - 4.8)^2$

$= (3)^2 = 9$

(9) $$\frac{.321 \times .321 - .179 \times .179}{.321 - .179}$$

$$= \frac{(.321)^2 - (.179)^2}{(.321 - .179)}$$

$$= \frac{(.321 + .179)(.321 - .179)}{(.321 + .179)}$$

$$= (.321 + .179)$$

$$= .500 = .5$$

(10) $$\frac{45 \times 45 + 2 \times 35 \times 45 + (35)^2}{(45 \times 45) - (35 \times 35)}$$

$$= \frac{(45)^2 + 2 \times 35 \times 45 + (35)^2}{(45)^2 - (35)^2}$$

$$= \frac{(45 + 35)^2}{(45 + 35)(45 - 35)}$$

$$= \frac{(45 + 35)(45 + 35)}{(45 + 35)(45 - 35)}$$

$$= \frac{(45 + 35)}{(45 - 35)} = \frac{80}{10}$$

$$= 8$$

(11) $$(3.75)^3 + 3(3.75)^2(1.25) + 3(3.75)(1.25)^2 + (1.25)^3$$

$$= (3.75 + 1.25)^3$$

$$= (5.00)^3$$

$$= (5)^3 = 125$$

(12) $$\frac{.89 \times .89 \times .89 - .64 \times .64 \times .64}{.89 \times .89 + .89 \times .64 + .64 \times .64}$$

$$= \frac{(.89)^3 - (.64)^3}{(.89)^2 + .89 \times .64 + (.64)^2}$$

$$= \frac{(.89 - .64)\{(.89)^2 + .89 \times .64 + 1.64)^2\}}{(.89)^2 + .89 \times .64 + (.64)^2}$$

$$= (.89 - 64) =$$

$$= .25$$

(13) $x^3 - x^2y + xy^2$ تجزی کیجئے

$= x(x^2 - xy + y^2)$

(14) $x^2 - xy - 3x + 3y$

$= x(x-y) - 3(x-y) = (x-y)(x-3)$

(15) $1 + a^2b^2 + a^2 + b^2$

$= a^2 + a^2b^2 + 1 + b^2)$

$= a^2(1 + b^2) + 1(1 + b^2) = (1+b^2)(a^2+1)$

(16) $(a+b)^2 + 3(a+b)$

$= (a+b)(a+b+3)$

(17) $ab(x^2+y^2) - xy(a^2+b^2)$

$= abx^2 + aby^2 - xya^2 - xyb^2$

$= abx^2 - xya^2 + aby^2 - xyb^2$

$= ax(bx - ay) + by(ay - bx)$

$= ax(bx - ay) - by(bx - ay)$

$= (bx - ay)(ax - by)$

(18) $(ax + by)^2 + (bx - ay)^2$

$= a^2x^2 + 2abxy + b^2y^2 + b^2x^2 - 2bxay + a^2y^2$

$= a^2x^2 + b^2x^2 + a^2y^2 + b^2y^2$

$= x^2(a^2+b^2) + y^2(a^2+b^2)$

$= (a^2+b^2)(x^2+y^2)$

(19) $(ab + cd)^2 + (ac - bd)^2$

$= a^2b^2 + 2abcd + c^2d^2 + a^2c^2 - 2acbd + b^2d^2$

$= a^2b^2 + c^2d^2 + a^2c^2 + b^2d^2$

$= a^2b^2 + a^2c^2 + b^2d^2 + c^2d^2$

$= a^2(b^2+c^2) + d^2(b^2+c^2)$

$= (b^2+c^2)(a^2+d^2)$

(20) $25a^2b^2 - 20abc + 4c^2$

$= (5ab)^2 - 2(5ab)(2c) + (2c)^2 = (5ab-2c)^2$

(21) $a^{2x} - 2a^xb^y + b^{2y}$

$= (a^x)^2 - 2(a^x)(b^y) + (b^y)^2 = (a^x - b^y)^2$

(22) $81a^2 - 25$

$= (9a)^2 - (5)^2 = (9a+5)(9a-5)$

(23) $a^3 - 9ab^2$

$= a(a^2 - 9b^2) = a(a+3b)(a-3b)$

(24) $x^4 - 81$

$= (x^2)^2 - (9)^2 = (x^2+9)(x^2-9)$

$= (x^2+9)(x+3)(x-3)$

(25) $x^8 - y^8$

$= (x^4)^2 - (y^4)^2 = (x^4+y^4)(x^4-y^4)$

$= (x^4+y^4)(x^2+y^2)(x^2-y^2)$

$= (x^4+y^4)(x^2+y^2)(x+y)(x-y)$

(26) $a^{4x} - b^{4x}$

$= (a^{2x})^2 - (b^{2x})^2 = (a^{2x}+b^{2x})(a^{2x}-b^{2x})$

$= (a^{2x}+b^{2x})(a^x+b^x)(a^x-b^x)$

(27) $c^2 - 2bc - a^2 + b^2$

$\therefore c^2 - 2bc + b^2 - c^2 = (c-b)^2 - (a)^2$

$= (c-b+a)(c-b-a)$

(28) $2xy - x^2 - y^2 + z^2$

$= z^2 - x^2 + 2xy - y^2 = z^2 - (x^2 - 2xy + y^2)$

$= (z)^2 - (x-y)^2 = (z+x-y)(z-x+y)$

(29) $8ax + x^2 - b^2 + 16a^2$

$= x^2 + 8ax + 16a^2 - b^2 = (x+4a)^2 - (b)^2$

$= (x+4a+b)(x+4a+b)$

(30) $4x^4 + 1$

$= (2x^2)^2 + (1)^2 + 2(2x^2)(1) - 2(2x^2)(1)$

$= (2x^2+1)^2 - 4x^2 = (2x^2+1)^2 - (2x)^2$

$= (2x^2+1+2x)(2x^2+1-2x)$

$= (2x^2+2x+1)(2x^2-2x+1)$

(31) $a^4 + 64$

$= (a^2)^2 + (8)^2 + 2(a^2)(8) - 2(a^2)(8)$

$= (a^2+8)^2 - 16a^2$

$= (a^2+8)^2 - (4a)^2$

$= (a^2+8+4a)(a^2+8-4a)$

$= (a^2+4a+8)(a^2-4a+8)$

(32) $a^4 + 14a^2b^2 + 81b^4$

$= (a^2)^2 + 18a^2b^2 + (9b^2)^2 - 4a^2b^2$

$= (a^2+9b^2)^2 - (2ab)^2$

$= (a^2+9b^2+2ab)(a^2+9b^2-2ab)$

$= (a^2+2ab+9b^2)(a^2-2ab+9b^2)$

(33) $x^4 + 17x^2 + 81$

$= (x^2)^2 + 2(x^2)(9) + (9)^2 - x^2$

$= (x^2+9)^2 - (x)^2$

$= (x^2+9+x)(x^2+9-x)$

$= (x^2+x+9)(x^2-x+9)$

(34) $x^4 - 5x^2y^2 + y^4$

$= (x^2)^2 - 2(x^2)(y^2) + (y^2)^2 - x^2y^2$

$= (x^2-y^2)^2 - (xy)^2$

$= (x^2-y^2+xy)(x^2-y^2-xy)$

$= (x^2+xy-y^2)(x^2-xy-y^2)$

(35) $64 - 25b^2 + b$

$= (8)^2 - 2(8)(b^2) + (b^2)^2 - 9b^2$

$= (8-b^2)^2 - (8b^2)$

$= (8-b^2+3b)(8-b^2-3b)$

$= (8+3b-b^2)(8-3b-b^2)$

(36) $a^3b^3 - 10a^2b^2 + 24ab$

$= ab(a^3b^2 - 10ab + 24)$

$= ab\{a^2b^2 - 6ab - 4ab + 24\}$

$= ab\{ab(ab-6) - 4(ab-6)\}$

$= ab(ab-6)(ab-4)$

(37) $x^2 - x - 2$

$= x^2 - 2x + x - 2$

$= x(x-2) + 1(x-2)$

$= (x-2)(x+1)$

(38) $a^2 + ab - 72b^2$

$= a^2 + 9ab - 8ab - 72b^2$

$= a(a+9b) - 8b(a+9b)$

$= (a+9b)(a-8b)$

(39) $a^3 + 27b^3$

$= (a)^3 + (3b)^3$

$= (a+3b)(a^2 - 3ab + 9b^2)$

(40) $x^4 - 27x$

$= x(x^3 - 27) = x(x-3)(x^2+3x+9)$

(41) $x^{-m} + y^{3m}$

$= (x^m)^3 + (y^m)^m = (x^m + y^m)(x^{2m} - x^m y^m + y^{2m})$

(42) $64a^6 - 729$

$= (8a^3)^2 - (27)^2 = (8a^3+27)(8a^3-27)$

$= (2a+3)(4a^2-6a+9)(2a-3)(4a^2+6a+9)$

$= (2a+3)(2a-3)(4a^2-6a+9)(4a^2+6a+9)$

(43) $x^3 + 2x^2 - x - 2 = x^2(x+2) - 1(x+2)$

$= (x+2)(x^2-1) = (x+2)(x+1)(x-1)$

(44) $x^3 - 5x^2 - x + 5 = x^2(x-5) - 1(x-5)$

$= (x-5)(x^2-1) = (x-5)(x+1)(x-1)$

(45) $x^4 - x^3 + 8x - 8$

$= x^3(x-1) + 8(x-1)$

$= (x-1)(x^3+8)$

$= (x-1)(x+2)(x^2-2x+4)$