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Unit 6 MATRICES AND DETERMINANTS

Matrix: The rectangular array of objects enclosed in square bracket "[ ]" is called matrix. It is usually denoted by A, B, C etc.

Order of a matrix:- The number of rows by column is called its order i.e

Order = R by C = R × C. e.g

Order of $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ is 2×2.

Equal matrices:- Two matrices A and B will be equal if (i) There order is same (ii) Corresponding elements are equal.

Ex: 6.1

1. Write the number of rows and columns of each.

(i) $A = \begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ No of rows = 3, No of Column = 3

(ii) $B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ Rows = 3, Columns = 2

(iii) $C = \begin{bmatrix} 1 & -2 & 3 \\ 5 & 6 & 7 \end{bmatrix}$ Rows = 2, Columns = 3

(iv) $D = \begin{bmatrix} 6 & 7 \\ 0 & -2 \end{bmatrix}$ Row = 2, Columns = 2

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2. Determin the order of each matrix in Q1.

Sol:- (i) $A = \begin{bmatrix} 0 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ order $A = 3 \times 3$

(ii) $B = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$ order $B = 3 \times 2$

(iii) $C = \begin{bmatrix} 1 & -2 & 3 \\ 5 & 6 & 7 \end{bmatrix}$ order $C = 2 \times 3$

(iv) $D = \begin{bmatrix} 6 & 7 \\ 0 & -2 \end{bmatrix}$ order $D = 2 \times 2$

Q3 Which of the following pairs of matrices are equal:

(i) equal (ii) Not equal (iii) equal (iv) Not equal (v) Equal.

Types of matrices:-

(i) Row matrix :- A matrix having only one row e.g $[1\ 2\ 3]$

(ii) Column matrix:- A matrix having only one column e.g $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ etc:

(iii) Square matrix:- A matrix in which rows and columns are equal i.e No of rows = No of colmn.

(iv) Rectangular matrix:- A matrix in which Rows ≠ Colmn

(v) Null matrix :- A matrix in which all entries are zero

(vi) Diagonal matrix:- A square matrix in which all the elements are zero but at least one of the diagonal element is non zero

e.g $A = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(vii) Scalar matrix:- A square matrix in which all the elements are zero and the diagonal elt are same.

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e.g $A=\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$, $B=\begin{bmatrix}a&0&0\\0&a&0\\0&0&a\end{bmatrix}$ are scalers.

(VII) Identity matrix:- A square matrix in which all elements are zero and the diagonal elements are 1.

e.g $I=[1]$, $I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ etc.

(VIII) Symmetric : A matrix for which $A^t=A$ is called Symmetric

(IX) Skew Symmetric :- A matrix for which $A^t=-A$.

Ex#6.2

1. Identify row matrix in the following matrices:

(i) $\begin{bmatrix}1&2\\0&2\end{bmatrix}$ (ii) $\begin{bmatrix}1&2&4\end{bmatrix}$ (iii) $\begin{bmatrix}3\\0\\2\end{bmatrix}$ (iv) $\begin{bmatrix}1&2&8\\0&1&3\end{bmatrix}$

(v) $\begin{bmatrix}0&4&9\end{bmatrix}$

Soln:- Only (ii) and (v) are row matrices.

2. Identify column matrix in the following:

(i) $\begin{bmatrix}1&3&0\end{bmatrix}$ (ii) $\begin{bmatrix}1&0\\2&1\\3&4\end{bmatrix}$, (iii) $\begin{bmatrix}1\\4\\9\end{bmatrix}$ (iv) $\begin{bmatrix}1&0\\2&3\\8&7\end{bmatrix}$

(v) $\begin{bmatrix}0\\7\\1\end{bmatrix}$

Soln:- Only (iii) and (v) are column matrix.

3. Identify rectangular and square matrices:

Soln:- (ii), (v), (VI) and (VII) are Square matrice.

(i), (iii), (iv), (VI), (VIII), X are rectangular matrices.

4. Which of the following are null/zero matrices

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Sol:- (i), (ii), (iii), (v), (vi) are null matrices

5. Separate the scalar, Unit and diagonal matrices from the following matrices:

Solu:- (i) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ diagonal

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ diagonal matrix

(iii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = Identity (iv) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ = Scaler

(v) $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ = Scaler, (vi) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ = diagonal

(vii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = Identity (viii) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ = Scaler

6. Find the transpose of the following matrices:

(i) $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$

Solu:- Let $A = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow A^t = \begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}^t = \begin{bmatrix} 1 & 5 \\ 2 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & 1 & 5 \\ 3 & 3 & 0 \\ 4 & 5 & 0 \end{bmatrix}$

Let $A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & 3 & 0 \\ 4 & 5 & 0 \end{bmatrix} \Rightarrow A^t = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 3 & 5 \\ 5 & 0 & 0 \end{bmatrix}$

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(iii) Let $A=\begin{bmatrix}1&0&2\\2&1&0\\0&2&1\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}1&2&0\\0&1&2\\2&0&1\end{bmatrix}$

(iv) Let $A=\begin{bmatrix}0&1&4\\1&7&9\\4&9&2\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}0&1&4\\1&7&9\\4&9&2\end{bmatrix}$.

7. Which of the following are Symmetric matrices:

Soln: (i) Let $A=\begin{bmatrix}1&0&2\\2&1&0\\0&2&1\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}1&2&0\\0&1&2\\2&0&1\end{bmatrix}=A$

As $A^t=A \Rightarrow A$ is symmetric matrix

(ii) Let $A=\begin{bmatrix}1&3&7\\3&0&2\\7&2&4\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}1&3&7\\3&0&2\\7&2&4\end{bmatrix}=A$

As $A^t=A \Rightarrow A$ is symmetric matrix.

(iii) Let $A=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=A$

$\Rightarrow A^t=A$ So A is symmetric.

(iv) Let $A=\begin{bmatrix}0&1&4\\1&7&9\\4&9&2\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}0&1&4\\1&7&9\\4&9&2\end{bmatrix}=A$

As $A^t=A$ So A is symmetric matrix.

8. Which of the following are Skew-Symmetric matrices.

Soln: (i) Let $A=\begin{bmatrix}0&1&7\\-1&0&3\\-7&-3&0\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}0&-1&-7\\1&0&-3\\7&3&0\end{bmatrix}$

$\Rightarrow A^t=-\begin{bmatrix}0&1&7\\-1&0&3\\-7&-3&0\end{bmatrix}=-A$

As $A^t=-A$ So A is Skew Symmetric.

(ii) Let $A=\begin{bmatrix}3&1&0\\-1&2&4\\7&-4&-7\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}3&-1&7\\1&2&-4\\0&4&-7\end{bmatrix}=-\begin{bmatrix}-3&1&-7\\-1&-2&4\\0&-4&7\end{bmatrix}$

$\Rightarrow A^t \neq -A$ So A is not Skew Symmetric

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(iii) Let $A=\begin{bmatrix}0 & 9 & 6\\ -9 & 0 & 3\\ -7 & -3 & 0\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}0 & -9 & -7\\ 9 & 0 & -3\\ 6 & 3 & 0\end{bmatrix}=-\begin{bmatrix}0 & 9 & 7\\ -9 & 0 & 3\\ -6 & 3 & 0\end{bmatrix}$.

As $A^t \neq -A \Rightarrow A$ is not skew symmetric

(iv) Let $A=\begin{bmatrix}0 & 3 & 5\\ -3 & 0 & 1\\ -5 & -1 & 0\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}0 & -3 & -5\\ 3 & 0 & -1\\ 5 & 1 & 0\end{bmatrix}$

$\Rightarrow A^t=-\begin{bmatrix}0 & 3 & 5\\ -3 & 0 & 1\\ -5 & -1 & 0\end{bmatrix}=-A$

As $A^t=-A \Rightarrow A$ is Skew Symmetric

(v) Let $A=\begin{bmatrix}0 & 2 & 7\\ -2 & 0 & 3\\ -7 & -3 & 0\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}0 & -2 & -7\\ 2 & 0 & -3\\ 7 & 3 & 0\end{bmatrix}$

$\Rightarrow A^t=-\begin{bmatrix}0 & 2 & 7\\ -2 & 0 & 3\\ -7 & -3 & 0\end{bmatrix}=-A$.

As $A^t=-A$ so A is Skew Symmetric.

(vi) Let $A=\begin{bmatrix}0 & 1 & 2\\ -1 & 0 & 9\\ -2 & -9 & 0\end{bmatrix} \Rightarrow A^t=\begin{bmatrix}0 & -1 & -2\\ 1 & 0 & -9\\ 2 & 9 & 0\end{bmatrix}$

$\Rightarrow A^t=-\begin{bmatrix}0 & 1 & 2\\ -1 & 0 & 9\\ -2 & -9 & 0\end{bmatrix}=-A$

As $A^t=-A \Rightarrow A$ is Skew Symmetric.

Addition and Subtraction of matrices

If the order of two matrices A and B are same then they are said to be comfortable for addition. For addition/subtraction the corresponding elements are subtracted.

The addition of matrices satisfy the following properties.

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1. Commutative property i.e $A+B=B+A$
2. Associative property i.e $(A+B)+C=A+(B+C)$
3. Additive identity i.e $A+O=O+A=A$
4. Additive inverse: If A is a matrix, then $-A$ is called its inverse and $A+(-A)=O$

Ex: 6.3

1. Add the following matrices

(i) $\begin{bmatrix} 5 & 2 & 0 \\ -1 & 1 & -2 \end{bmatrix}, \begin{bmatrix} 0 & -2 & 4 \\ 2 & 1 & 1 \end{bmatrix}$

Soln:- Let $A=\begin{bmatrix} 5 & 2 & 0 \\ -1 & 1 & -2 \end{bmatrix}$, $B=\begin{bmatrix} 0 & -2 & 4 \\ 2 & 1 & 1 \end{bmatrix}$

$$A+B=\begin{bmatrix} 5 & 2 & 0 \\ -1 & 1 & -2 \end{bmatrix}+\begin{bmatrix} 0 & -2 & 4 \\ 2 & 1 & 1 \end{bmatrix}=\begin{bmatrix} 5+0 & 2-2 & 0+4 \\ -1+2 & 1+1 & -2+1 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 0 & 4 \\ 1 & 2 & -1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, B=\begin{bmatrix} 7 \\ 9 \\ 2 \end{bmatrix}$

Soln:- Let $A=\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$, $B=\begin{bmatrix} 7 \\ 9 \\ 2 \end{bmatrix}$

$$A+B=\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}+\begin{bmatrix} 7 \\ 9 \\ 2 \end{bmatrix}=\begin{bmatrix} 1+7 \\ 0+9 \\ 5+2 \end{bmatrix}=\begin{bmatrix} 8 \\ 9 \\ 7 \end{bmatrix}$$

(iii) $[2], [5]$

Soln: Let $A=[2]$, $B=[5]$

$$A+B=[2]+[5]=[2+5]=[7]$$

(iv) $[2 \quad 2 \quad 2], [3 \quad 1 \quad 4]$

Soln:- Let $A=[2 \quad 2 \quad 2]$, $B=[3 \quad 1 \quad 4]$

$$A+B=[2 \quad 2 \quad 2]+[3 \quad 1 \quad 4]=[2+3 \quad 2+1 \quad 2+4]=[5 \quad 3 \quad 6]$$

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(v) $\begin{bmatrix} 0 & 1 & 4 \\ 4 & 0 & 2 \\ 3 & 5 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 2 & 3 \\ 3 & 1 & 7 \\ 6 & 4 & 1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 0 & 1 & 4 \\ 4 & 0 & 2 \\ 3 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & 3 \\ 3 & 1 & 7 \\ 6 & 4 & 1 \end{bmatrix}$

$\Rightarrow A+B = \begin{bmatrix} 0 & 1 & 4 \\ 4 & 0 & 2 \\ 3 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 3 \\ 3 & 1 & 7 \\ 6 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 1+2 & 4+3 \\ 4+3 & 0+1 & 2+7 \\ 3+6 & 5+4 & 0+1 \end{bmatrix} = \begin{bmatrix} 0 & 3 & 7 \\ 7 & 1 & 9 \\ 9 & 9 & 1 \end{bmatrix}$

(vi) $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$, $\begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$

Then $A+B = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2+5 & 4+3 \\ 6+1 & 8+1 \end{bmatrix}$

$= \begin{bmatrix} 7 & 7 \\ 7 & 9 \end{bmatrix}$

2. Subtract 2nd matrix from 1st matrix.

(i) $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

Soln:- $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3-3 \\ 4-(-2) \\ 5-1 \end{bmatrix} = \begin{bmatrix} 3-3 \\ 4+2 \\ 5-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 4 \end{bmatrix}$

(ii) $[1 \quad 0 \quad 3]$, $[-4 \quad -5 \quad -6]$

Soln:- $[1 \quad 0 \quad 3] - [-4 \quad -5 \quad -6]$

$= [1-(-4) \quad 0-(-5) \quad 3-(-6)]$

$= [1+4 \quad 0+5 \quad 3+6]$

$= [5 \quad 5 \quad 9]$

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(iii) $\begin{bmatrix} 5 & 4 & 3 \\ 2 & -7 & 6 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

Soln:- $\begin{bmatrix} 5 & 4 & 3 \\ 2 & -7 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

$= \begin{bmatrix} 5-1 & 4-3 & 3-5 \\ 2-2 & -7-4 & 6-6 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -2 \\ 0 & -11 & 0 \end{bmatrix}$

(iv) $\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 7 \\ 6 & 8 & 7 \end{bmatrix}$, $\begin{bmatrix} 3 & 0 & 2 \\ 4 & 1 & 5 \\ 7 & 8 & 6 \end{bmatrix}$

Soln:- $\begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 7 \\ 6 & 8 & 7 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 2 \\ 4 & 1 & 5 \\ 7 & 8 & 6 \end{bmatrix}$

$= \begin{bmatrix} 0-3 & 1-0 & 2-2 \\ 4-4 & 5-1 & 7-5 \\ 6-7 & 8-8 & 7-6 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 \\ 0 & 4 & 2 \\ -1 & 0 & 1 \end{bmatrix}$.

3. If $A = \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ -3 & 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 6 & 1 & 1 \\ 4 & 3 & 2 \\ 0 & 1 & -5 \end{bmatrix}$

are matrices Then show that

(i) $A+B = B+A$

Soln:- $A+B = \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ -3 & 2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3+5 & 2+0 & 6+4 \\ 4-2 & 5+6 & 0+1 \\ 1-3 & 3+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 8 & 2 & 10 \\ 2 & 11 & 1 \\ -2 & 5 & 5 \end{bmatrix}$ —(i)

$B+A = \begin{bmatrix} 5 & 0 & 4 \\ -2 & 6 & 1 \\ -3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 2 & 10 \\ 2 & 11 & 1 \\ -2 & 5 & 5 \end{bmatrix}$ —(ii)

From (i) and (ii) we have $A+B = B+A$

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(ii) $A+(B+C)=(A+B)+C$

Soln:- $A+(B+C)=\begin{bmatrix}3&2&6\\4&5&0\\1&3&5\end{bmatrix}+\left(\begin{bmatrix}5&0&4\\-2&6&1\\-3&2&1\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&-1&-5\end{bmatrix}\right)$

$=\begin{bmatrix}3&2&6\\4&5&0\\1&3&5\end{bmatrix}+\begin{bmatrix}5+6&0+1&4+1\\-2+4&6+3&1+2\\-3+0&2-1&1-5\end{bmatrix}$

$=\begin{bmatrix}3&2&6\\4&5&0\\1&3&5\end{bmatrix}+\begin{bmatrix}11&1&5\\2&9&3\\-3&1&-4\end{bmatrix}$

$=\begin{bmatrix}3+11&2+1&6+5\\4+2&5+9&0+3\\1-3&3+1&5-4\end{bmatrix}=\begin{bmatrix}14&3&11\\6&14&3\\-2&4&1\end{bmatrix}$ —(i)

$(A+B)+C=\left(\begin{bmatrix}3&2&6\\4&5&0\\1&3&5\end{bmatrix}+\begin{bmatrix}5&0&4\\-2&6&1\\-3&2&1\end{bmatrix}\right)+\begin{bmatrix}6&1&1\\4&3&2\\0&-1&-5\end{bmatrix}$

$=\begin{bmatrix}3+5&2+0&6+4\\4-2&5+6&0+1\\1-3&3+2&5+1\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&-1&-5\end{bmatrix}$

$=\begin{bmatrix}8&2&10\\2&11&1\\-2&5&6\end{bmatrix}+\begin{bmatrix}6&1&1\\4&3&2\\0&-1&-5\end{bmatrix}$

$=\begin{bmatrix}8+6&2+1&10+1\\2+4&11+3&1+2\\-2+0&5-1&6-5\end{bmatrix}=\begin{bmatrix}14&3&11\\6&14&3\\-2&4&1\end{bmatrix}$ —(ii)

From Equation (i) and (ii) we get that

$A+(B+C)=(A+B)+C$

(iii) $A+O=O+A=A$

Soln:- $A+O=\begin{bmatrix}3&2&6\\4&5&0\\1&3&5\end{bmatrix}+\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$

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(150)

$$= \begin{bmatrix} 3+0 & 2+0 & 6+0 \\ 4+0 & 5+0 & 0+0 \\ 1+0 & 3+0 & 5+0 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 5 \end{bmatrix} = A \quad \text{---(i)}$$

$$O + A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3 & 0+2 & 0+6 \\ 0+4 & 0+5 & 0+0 \\ 0+1 & 0+3 & 0+5 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 5 \end{bmatrix} = A \quad \text{---(ii)}$$

From (i) and (ii) we get;

$$O + A = A + O = A$$

(iv) $A + (-A) = -A + A = O$

Soln: $A + (-A) = \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix} + (-1)\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} -3 & -2 & -6 \\ -4 & -5 & 0 \\ -1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3 & 2-2 & 6-6 \\ 4-4 & 5-5 & 0-0 \\ 1-1 & 3-3 & 4-4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \quad \text{---(i)}$$

$$-A + A = -1\begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 & -6 \\ -4 & -5 & 0 \\ -1 & -3 & -4 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 6 \\ 4 & 5 & 0 \\ 1 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3+3 & -2+2 & -6+6 \\ -4+4 & -5+5 & 0+0 \\ -1+1 & -3+3 & -4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \quad \text{---(ii)}$$

From (i) and (ii) we get $A + (-A) = -A + A = O$

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4. Find the additive inverse of the following matrices:

(i) $[8]$

Let $A=[8]$. Then additive inverse is $-A$

i.e $-A=-1\cdot A=-1[8]=[-8]$

(ii) $\begin{bmatrix}2 & 1\\ 3 & 3\end{bmatrix}$

Soln:- Let $A=\begin{bmatrix}2 & 1\\ 3 & 3\end{bmatrix}$. Then its additive inverse is $-A$

i.e $-A=-1\cdot A=-1\begin{bmatrix}2 & 1\\ 3 & 3\end{bmatrix}=\begin{bmatrix}-2 & -1\\ -3 & -3\end{bmatrix}$

(iii) $\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$

Let $A=\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$. Then additive inverse $=-A$

i.e $-A=-1\cdot A=-1\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}=\begin{bmatrix}-9 & 0 & 0\\ 0 & -9 & 0\\ 0 & 0 & -9\end{bmatrix}$

(iv) $\begin{bmatrix}1 & 3 & 2\\ 5 & 8 & 9\end{bmatrix}$

Let $A=\begin{bmatrix}1 & 3 & 2\\ 5 & 8 & 9\end{bmatrix}$

additive inverse $=-A=-1\cdot A=-1\begin{bmatrix}1 & 3 & 2\\ 5 & 8 & 9\end{bmatrix}$

$=\begin{bmatrix}-1 & -3 & -2\\ -5 & -8 & -9\end{bmatrix}$.

5. Verify the additive identity property for the following

(i) $[2 \quad 4]$

Soln:- Let $A=[2 \quad 4]$, $0=[0 \quad 0]$ then

$A+0=[2 \quad 4]+[0 \quad 0]=[2+0 \quad 4+0]=[2 \quad 4]$

$=A$ —①

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$O+A=\begin{bmatrix}0 & 0\end{bmatrix}+\begin{bmatrix}2 & 4\end{bmatrix}=\begin{bmatrix}0+2 & 0+4\end{bmatrix}=\begin{bmatrix}2 & 4\end{bmatrix}=A$ ——(ii)

From (i) and (ii) we get

$$A+O=O+A=A.$$

(ii) $\begin{bmatrix}1\\0\end{bmatrix}$

Soln:- Let $A=\begin{bmatrix}1\\0\end{bmatrix}$ and $O=\begin{bmatrix}0\\0\end{bmatrix}$ Then;

$$A+O=\begin{bmatrix}1\\0\end{bmatrix}+\begin{bmatrix}0\\0\end{bmatrix}=\begin{bmatrix}1+0\\0+0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}=A \quad \text{——(i)}$$

$$O+A=\begin{bmatrix}0\\0\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}0+1\\0+0\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}=A \quad \text{——(ii)}$$

From (i) and (ii) we get

$$A+O=O+A=A$$

(iii) $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Soln:- Let $A=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$, $O=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$

$$A+O=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=A \quad \text{——(i)}$$

$$O+A=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=A \quad \text{——(ii)}$$

From (i) and (ii) $A+O=O+A=A$

(iv) let $A=\begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}$, $O=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$$A+O=\begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}+\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}=\begin{bmatrix}7 & 0\\0 & 7\end{bmatrix}=A.$$

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Multiplication of matrices:- Suppose A and B are two matrices. Then AB is possible if Columns of matrix A = Rows of B

The multiplication have the following properties.

(i) $AB \neq BA$ in general

(ii) $(AB)C = A(BC)$ i.e Associative property

(iii) $A(B+C) = AB + AC$ i.e Distributive property.

Ex: 6.4

1. Multiply the following matrices:

(i) $\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

Soln:- $\begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} (2\times4) + (3\times1) \end{bmatrix} = \begin{bmatrix} 8+3 \end{bmatrix} = \begin{bmatrix} 11 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$

Soln:- $\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0\times2 + 2\times0 & 0\times2 + 2\times0 \\ 3\times2 + 1\times0 & 3\times2 + 1\times0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 6+0 & 6+0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 6 & 6 \end{bmatrix}$

(iii) $\begin{bmatrix} 4 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix}$

Soln:- $\begin{bmatrix} 4 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4\times5 + 0\times0 & 4\times3 + 0\times1 \\ 1\times5 + 1\times0 & 1\times3 + 1\times1 \end{bmatrix}$

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$= \begin{bmatrix} 20+0 & 12+0 \\ 5+0 & 3+1 \end{bmatrix} = \begin{bmatrix} 20 & 12 \\ 5 & 4 \end{bmatrix}$

(iv) $\begin{bmatrix} 3 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}$

Soln:- $\begin{bmatrix} 3 & 2 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3\times4+2\times1 & 3\times4+2\times1 \\ 3\times4+1\times1 & 3\times4\times1\times1 \end{bmatrix} = \begin{bmatrix} 12+2 & 12+2 \\ 12+1 & 12+1 \end{bmatrix} = \begin{bmatrix} 14 & 14 \\ 13 & 13 \end{bmatrix}$

(v) $\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Soln:- $\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3\times0+3\times1 & 3\times1+3\times0 \\ 2\times0+2\times1 & 2\times1+2\times0 \end{bmatrix}$

$= \begin{bmatrix} 0+3 & 3+0 \\ 0+2 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}$

(vi) $\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Soln:- $\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1\times0+(-2)\times1 \\ (-1)\times0+(-1)\times1 \end{bmatrix} = \begin{bmatrix} 0-2 \\ 0-1 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

2. For the matrices $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$,

$C = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}$ show that

(i) $(A+B)^t = A^t + B^t$

Soln:- $A+B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2+0 & 1+3 \\ 0+4 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$

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$\Rightarrow (A+B)^t = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ ——— (i)

$A^t = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, $B^t = \begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix}$

$A^t + B^t = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2+0 & 0+4 \\ 1+3 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ ——— (ii)

From (i) and (ii) we get that $(A+B)^t = A^t + B^t$.

(ii) $(AB)^t = B^t \cdot A^t$

Soln $AB = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2\times0+1\times4 & 2\times3+1\times1 \\ 0\times0+1\times4 & 0\times3+1\times1 \end{bmatrix} = \begin{bmatrix} 0+4 & 6+1 \\ 0+4 & 0+1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 7 \\ 4 & 1 \end{bmatrix}$

$\Rightarrow (AB)^t = \begin{bmatrix} 4 & 4 \\ 7 & 1 \end{bmatrix}$ ——— (i)

$A^t = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, $B^t = \begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix}$

Now; $B^t \cdot A^t = \begin{bmatrix} 0 & 4 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0\times2+4\times1 & 0\times0+4\times1 \\ 3\times2+1\times1 & 3\times0+1\times1 \end{bmatrix}$

$= \begin{bmatrix} 0+4 & 0+4 \\ 6+1 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 7 & 1 \end{bmatrix}$ ——— (ii)

From (i) and (ii) we get $(AB)^t = B^t \cdot A^t$.

3. If $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 3 \\ 4 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}$

are matrices then show that :

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(i) $(A+B)C = AB + AC$

Soln: $A+B = \begin{bmatrix} 3 & 0 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 2 & -5 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3+2 & 0-5 \\ 2+3 & -1+0 \end{bmatrix}$

$= \begin{bmatrix} 5 & -5 \\ 5 & -1 \end{bmatrix}$

$(A+B)C = \begin{bmatrix} 5 & -5 \\ 5 & -1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 5\times1+(-5)\times(-1) & 5\times(-2)+(-5)\times(-1) \\ 5\times1+(-1)\times(-1) & 5\times(-2)+(-1)\times(-1) \end{bmatrix}$

$= \begin{bmatrix} 5+5 & -10+5 \\ 5+1 & -10+1 \end{bmatrix} = \begin{bmatrix} 10 & -5 \\ 6 & -9 \end{bmatrix}$ ——— (i)

$AC = \begin{bmatrix} 3 & 0 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 3\times1+0\times(-1) & 3\times(-2)+0\times(-1) \\ 2\times1+(-1)\times(-1) & 2\times(-2)+(-1)\times(-1) \end{bmatrix}$

$= \begin{bmatrix} 3+0 & -6+0 \\ 2+1 & -4+1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 3 & -3 \end{bmatrix}$

$BC = \begin{bmatrix} 2 & -5 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2\times1+(-5)\times(-1) & 2\times(-2)+(-5)\times(-1) \\ 3\times1+0\times(-1) & 3(-2)+0(-1) \end{bmatrix}$

$= \begin{bmatrix} 2+5 & -4+5 \\ 3+0 & -6+0 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 3 & -6 \end{bmatrix}$

$AC+BC = \begin{bmatrix} 3 & -6 \\ 3 & -3 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ 3 & -6 \end{bmatrix} = \begin{bmatrix} 3+7 & -6+1 \\ 3+3 & -3-6 \end{bmatrix}$

$= \begin{bmatrix} 10 & -5 \\ 6 & -9 \end{bmatrix}$ ——— (ii)

From (i) and (ii) we get

$(A+B)C = AC + BC$

(ii) $A(B+C) = AB + AC$

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(157)

$$B+C=\begin{bmatrix}2 & -5\\ 3 & 0\end{bmatrix}+\begin{bmatrix}1 & -2\\ -1 & -1\end{bmatrix}=\begin{bmatrix}2+1 & -5-2\\ 3-1 & 0-1\end{bmatrix}$$

$$=\begin{bmatrix}3 & -7\\ 2 & -1\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}3 & 0\\ 2 & -1\end{bmatrix}\begin{bmatrix}3 & -7\\ 2 & -1\end{bmatrix}$$

$$=\begin{bmatrix}3\times3+0\times2 & 3\times(-7)+0\times(-1)\\ 2\times(3)+(-1)\times2 & 2\times(-7)+(-1)(-1)\end{bmatrix}$$

$$=\begin{bmatrix}9+0 & -21+0\\ 6-2 & -14+1\end{bmatrix}=\begin{bmatrix}9 & -21\\ 4 & -13\end{bmatrix}\quad\text{---(i)}$$

$$AB=\begin{bmatrix}3 & 0\\ 2 & -1\end{bmatrix}\begin{bmatrix}2 & -5\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3\times2+0\times3 & 3\times(-5)+0\times0\\ 2\times2+(-1)\times3 & 2\times(-5)+(-1)\times0\end{bmatrix}=\begin{bmatrix}6+0 & -15+0\\ 4-3 & -10+0\end{bmatrix}$$

$$=\begin{bmatrix}6 & -15\\ 1 & -10\end{bmatrix}$$

$$AC=\begin{bmatrix}3 & 0\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & -2\\ -1 & 1\end{bmatrix}=\begin{bmatrix}3\times1+0\times(-1) & 3\times(-2)+0\times1\\ 2\times1+(-1)\times(-1) & 2\times(-2)+(-1)\times1\end{bmatrix}$$

$$=\begin{bmatrix}3+0 & -6+0\\ 2+1 & -4+1\end{bmatrix}=\begin{bmatrix}3 & -6\\ 3 & -3\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}6 & -15\\ 1 & -10\end{bmatrix}+\begin{bmatrix}3 & -6\\ 3 & -3\end{bmatrix}$$

$$=\begin{bmatrix}6+3 & -15-6\\ 1+3 & -10-3\end{bmatrix}=\begin{bmatrix}9 & -21\\ 4 & -13\end{bmatrix}\quad\text{---(ii)}$$

From (i) and (ii) we get

$$A(B+C)=AB+AC$$

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4. If $A=\begin{bmatrix}-5 & 0\\ 0 & -4\end{bmatrix}$, $B=\begin{bmatrix}-2 & -3\\ -1 & 5\end{bmatrix}$ and $C=\begin{bmatrix}0 & -2\\ -1 & -1\end{bmatrix}$, find $2A+3B-4C$.

Soln:-

$$2A+3B-4C=2\begin{bmatrix}-5 & 0\\ 0 & -4\end{bmatrix}+3\begin{bmatrix}-2 & -3\\ -1 & 5\end{bmatrix}-4\begin{bmatrix}0 & -2\\ -1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-10 & 0\\ 0 & -8\end{bmatrix}+\begin{bmatrix}-6 & -9\\ -3 & 15\end{bmatrix}-\begin{bmatrix}0 & -8\\ -4 & -4\end{bmatrix}$$

$$=\begin{bmatrix}-10-6-0 & 0-9+8\\ 0-3+4 & -8+15+4\end{bmatrix}=\begin{bmatrix}-16 & -1\\ 1 & 11\end{bmatrix}$$ Ans.

5. If $A=\begin{bmatrix}2 & 0\\ -1 & 4\end{bmatrix}$. Then prove that:

(i) $(A^t)^t = A$

Soln:- $A=\begin{bmatrix}2 & 0\\ -1 & 4\end{bmatrix}$

$$A^t=\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}$$

$$\Rightarrow (A^t)^t=\begin{bmatrix}2 & 0\\ -1 & 4\end{bmatrix}=A.$$

So $(A^t)^t = A$

(ii) $AA^t \neq A^tA$.

Soln:- $A^t=\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}$

$$AA^t=\begin{bmatrix}2 & 0\\ -1 & 4\end{bmatrix}\begin{bmatrix}2 & -1\\ 0 & 4\end{bmatrix}$$

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$$= \begin{bmatrix} 2(2)+0(0) & 2(-1)+0(4) \\ -1(2)+4(0) & -1(-1)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0 & -2+0 \\ -2+0 & 1+16 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -2 & 17 \end{bmatrix} \quad \text{---(i)}$$

$$A^t \cdot A = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+(-1)(-1) & 2(0)+(-1)(4) \\ 0(2)+4(-1) & 0(0)+4(4) \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & 0-4 \\ 0-4 & 0+16 \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 16 \end{bmatrix} \quad \text{---(ii)}$$

From Eq (i) and (ii) we get $AA^t \neq A^t \cdot A$.

Determinant of a Square matrix:- Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a matrix. Then determinant of A denoted by $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a(d) - b(c) = ad - bc$

→ If $|A| = 0$ then A is called singular matrix

→ If $|A| \neq 0$ then A is called non-singular.

→ If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adjoint of A is

$$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

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Ex: 6.5

1. Find the determinant of the following matrices if possible.

(i) $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & 6 \\ 1 & 3 \end{vmatrix}$

$= 2(3) - 1(6) = 6 - 6 = 0$

(ii) $\begin{bmatrix} 3 & 0 \\ 1 & 4 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 3 & 0 \\ 1 & 4 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix}$

$= 3(4) - 1(0) = 12 - 0 = 12$

(iii) $\begin{bmatrix} 5/2 & 6 \\ 5/6 & 2 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 5/2 & 6 \\ 5/6 & 2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 5/2 & 6 \\ 5/6 & 2 \end{vmatrix} = 5/2(2) - 5/6(6)$

$= 5 - 5 = 0$

(iv) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2(1) - 1(1)$

$= 2 - 1 = 1$

(v) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Soln:- $|A| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$= 0(0) - 1(1) = 0 - 1 = -1$

(vi) $\begin{bmatrix} 7 & 0 \\ 40 & 1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 7 & 0 \\ 40 & 1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 7 & 0 \\ 40 & 1 \end{vmatrix}$

$= 7(1) - 40(0) = 7 - 0 = 7$

(vii) $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = 2(5) - 1(3)$

$= 10 - 3 = 7$

(viii) $\begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$

$= 3(-1) - 2(4)$

$= -3 - 8 = -11$

(ix) $\begin{bmatrix} -2 & 1 \\ -5 & -3 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} -2 & 1 \\ -5 & -3 \end{bmatrix}$

$|A| = \begin{vmatrix} -2 & 1 \\ -5 & -3 \end{vmatrix}$

$= -2(-3) - 1(-5)$

$= 6 + 5 = 11.$

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2. Find which matrices are singular and which are non-sing

(i) $\begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$

Then $|A| = \begin{vmatrix} -3 & 2 \\ -1 & 0 \end{vmatrix}$

$= -3(0) - (-1)(2)$

$= 0 + 2 = 2 \neq 0$

As $|A| \neq 0$ so A is non-Singular

(ii) $\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -4 \\ 0 & 2 \end{vmatrix} = 1(2) - 0(-4)$

$= 2 + 0 = 2 \neq 0$

As $|A| \neq 0$ So A is non-Singular.

(iii) $\begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 0 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & 2 \\ 1 & 0 \end{vmatrix}$

$= 2(0) - 1(2)$

$= 0 - 2 = -2 \neq 0$

$\Rightarrow$ A is non-Singular

(iv) $\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 1 & -1 \\ 1 & 3 \end{vmatrix}$

$= 1(3) - (1)(-1)$

$= 3 + 1 = 4 \neq 0$

So A is non-Singular

(v) $\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 3 & 0 \\ -1 & 0 \end{vmatrix}$

$= 3(0) - (-1)(0)$

$= 0 + 0 = 0$

As $|A| = 0 \Rightarrow$ A is Singular

(vi) $\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}$

Soln: Let $A = \begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}$

$|A| = 1(3) - (-1)(0)$

$= 3 + 0 = 3 \neq 0$

As $|A| \neq 0$ So

A is non-Singular

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3. Find the inverse of matrix: $A = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$

Soln:- We know that $A^{-1} = \frac{1}{|A|} \text{adj } A$ ——(i)

$$|A| = \begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix} = 2(3) - 0(-1) = 6 + 0 = 6 \neq 0$$

As $|A| \neq 0$ so, A^{-1} is possible.

$$\text{adj } A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

Putting the values of adj A and A^{-1} in Eq (i) we get

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{6}\times 3 & \frac{1}{6}\times 0 \\ \frac{1}{6}\times(-1) & \frac{1}{6}\times 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{6} & \frac{1}{3} \end{bmatrix}$$

4. Verify that $(AB)^{-1} = B^{-1}.A^{-1}$ for matrices

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix}$$

Soln:- $$AB = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -4+2 & 3+1 \\ 8+6 & -6+3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 \\ 14 & -3 \end{bmatrix}$$

As $(AB)^{-1} = \frac{1}{|AB|} \text{adj } (AB)$ ——(i)

and $$|AB| = \begin{vmatrix} -2 & 4 \\ 14 & -3 \end{vmatrix} = -2(-3) - 4(14) = 6 - 56 = -50 \neq 0$$

Putting $|AB|$ and adj (AB) in Eq (i)

$$(AB)^{-1} = -\frac{1}{50}\begin{bmatrix} -3 & -4 \\ -14 & -2 \end{bmatrix} \quad \text{——(a)}$$

As $A^{-1} = \frac{1}{|A|} \text{adj } A$ ——(ii)

and $$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} = -1(3) - 2(1) = -3 - 2 = -5 \neq 0$$

$\Rightarrow A^{-1}$ exists.

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$\text{adj } A = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

Putting adj A and $|A|$ in Eq (ii) we get

$A^{-1} = \frac{1}{-5}\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}$

As; $B^{-1} = \frac{1}{|B|}\text{adj } B$ ——— (iii)

and $|B| = \begin{vmatrix} 4 & -3 \\ 2 & 1 \end{vmatrix} = 4(1) - 2(-3) = 4+6 = 10 \neq 0$

where $\text{adj } B = \begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$

Putting adj B and $|B|$ in Eq (iii)

$B^{-1} = \frac{1}{10}\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}$

Now $B^{-1}A^{-1} = \frac{1}{10}\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}\frac{1}{-5}\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}$

$= \frac{1}{10}\left(\frac{1}{-5}\right)\begin{bmatrix} 1 & 3 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ -2 & -1 \end{bmatrix}$

$= \frac{1}{-50}\begin{bmatrix} 3-6 & -1-3 \\ -6-8 & 2-4 \end{bmatrix} = \frac{1}{-50}\begin{bmatrix} -3 & -4 \\ -14 & -2 \end{bmatrix}$ —— (b)

From Eq (i) and Eq (ii) we get that

$(AB)^{-1} = B^{-1}\cdot A^{-1}$

5. Find the inverse matrix of the following matrices.

(i) $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

As $A^{-1} = \frac{1}{|A|}\text{adj } A$ ——— (i)

and $|A| = \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2(1) - 1(0) = 2 - 0 = 2 \neq 0$

As $|A| \neq 0$ so A^{-1} exists.

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$\because$ Adj $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

Putting the values of $|A|$ and adj A in Eq (i) we get

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1 \end{bmatrix}$$

(ii) $\begin{bmatrix} 6 & 3 \\ 1 & 1/2 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 6 & 3 \\ 1 & 1/2 \end{bmatrix}$

As we know that $A^{-1} = \frac{1}{|A|}$ adj A ——— (i)

and $|A| = \begin{vmatrix} 6 & 3 \\ 1 & 1/2 \end{vmatrix} = 3(1/2) - 3(1) = 3 - 3 = 0$

Since $|A| = 0 \Rightarrow A^{-1}$ is not possible.

(iii) $\begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}$

As we know that $A^{-1} = \frac{1}{|A|}$ adj A ——— (i)

and $|A| = \begin{vmatrix} 4 & 1 \\ 1 & 0 \end{vmatrix} = 4(0) - 1(1) = 0 - 1 = -1 \neq 0$

adj $A = \begin{bmatrix} 0 & -1 \\ -1 & 4 \end{bmatrix}$

Putting $|A|$ and adj A in Eq (i) we get

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -1(0) & -1(-1) \\ -1(-1) & -1(4) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix}$$

(iv) $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

As $A^{-1} = \frac{1}{|A|}$ adj A ——— (i)

and $|A| = \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1(1) - 0(3) = 1 - 0 = 1 \neq 0$

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$\text{adj } A = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

Putting $|A|$ and adj A in Eq(i) we get

$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$

and $|A| = \begin{vmatrix} 2 & 4 \\ 2 & 4 \end{vmatrix} = 2(4) - 2(4) = 8 - 8 = 0$

As $|A| = 0$ so A^{-1} is not possible.

(vi) $\begin{bmatrix} 5 & 0 \\ 1 & 2 \end{bmatrix}$

Soln:- Let $A = \begin{bmatrix} 5 & 0 \\ 1 & 2 \end{bmatrix}$

As we know that $A^{-1} = \frac{1}{|A|}\text{adj } A$ ——(i)

and $|A| = \begin{vmatrix} 5 & 0 \\ 1 & 2 \end{vmatrix} = 5(2) - 1(0) = 10 - 0 = 10 \neq 0$

As $|A| \neq 0 \Rightarrow A^{-1}$ exists.

Now $\text{adj } A = \begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix}$

Putting the values of $|A|$ and adj A in Eq(i) we get

$A^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 0 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{10}\times 2 & \frac{1}{10}\times 0 \\ \frac{1}{10}\times -1 & \frac{1}{10}\times 5 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{5} & 0 \\ -\frac{1}{10} & \frac{1}{2} \end{bmatrix}$

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Solution of Simultaneous Linear Equation:

Suppose we have a system of linear Equation:

$$\left.\begin{aligned} ax+by &= n \\ cx+dy &= m \end{aligned}\right\} \quad \text{---------- (i)}$$

There are two methods for the solution of System (i) in matrices.

(i) Matrix inversion method (ii) Cramer's Rule.

Matrix inversion Method

System (i) can be writtes as:

$$\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix} = \begin{bmatrix} n \\ m \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} n \\ m \end{bmatrix}$$

$$\Rightarrow AX = B \quad \text{where } A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}, X=\begin{bmatrix} x \\ y \end{bmatrix}, B=\begin{bmatrix} n \\ m \end{bmatrix}$$

$$\Rightarrow \bar{A}^1(AX) = \bar{A}^1 B$$

$$\Rightarrow (\bar{A}^1 A)X = \bar{A}^1 B \Rightarrow IX = \bar{A}^1 B \Rightarrow X = \bar{A}^1 B$$

Cramer's Rule :- The working rules of Cramer's rule are below.

(i) Find A, A_x, A_y where A_x, A_y Can be obtained by replacing the x and y columns of A by the matrix B respectively

(ii) Calculate $|A|, |A_x|, |A_y|$

(iii) The following formulas will be used for x and y values

$$x = \frac{|A_x|}{|A|} \quad \text{and} \quad y = \frac{|A_y|}{|A|}$$

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Ex#6.6

1. Write each of the following system of Eqs in matrix form

(i) $x+2y=0$, $2x-3y=1$

Soln:- The matrix form is:

$$\begin{bmatrix} x+2y \\ 2x-3y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

(ii) $\frac{1}{2}x-3y=4$, $x-\frac{3}{2}y=\frac{1}{2}$

Soln:- Writting the system in matrix form:

$$\begin{bmatrix} \frac{1}{2}x-3y \\ x-\frac{3}{2}y \end{bmatrix}\begin{bmatrix} 4 \\ \frac{1}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{2} & -3 \\ 1 & -\frac{3}{2} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{1}{2} \end{bmatrix}$$

(iii) $\sqrt{2}x+y=\sqrt{3}$, $x-\sqrt{3}y=1$

Soln:- The matrix form of given system is:

$$\begin{bmatrix} \sqrt{2}x+y \\ x-\sqrt{3}y \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$$

(iv) $\frac{4}{5}x-y=6$, $\frac{1}{3}x+2y=0$

Soln:- The matrix form is

$$\begin{bmatrix} \frac{4}{5}x-y \\ \frac{1}{3}x+2y \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{4}{5} & -1 \\ \frac{1}{3} & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

2. Solve each of the following system of linear Equations by matrix method.

(i) $x-y=2$, $x+2y=1$

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Soln:- The matrix form is:

$$\begin{bmatrix} x-y \\ x+2y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Here $A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$

As $X = A^{-1}B$ ——(i)

and $A^{-1} = \frac{1}{|A|}\,adj\,A$ ——(ii)

Now $|A| = \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 1(2) - 1(-1) = 2+1 = 3 \neq 0$

also $adj\,A = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$

Putting the values of $|A|$ and $adj\,A$ in Eq(i) we get:

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$$

Now putting A^{-1}, B in Eq (i) we have.

$$X = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 4+1 \\ -2+1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/3 \times 5 \\ 1/3 \times -1 \end{bmatrix} = \begin{bmatrix} 5/3 \\ -1/3 \end{bmatrix} \Rightarrow x = 5/3,\ y = -1/3$$

$\therefore$ S.S $= \{(5/3, -1/3)\}$

(ii) $2x + 3y = 0$, $x - 4y = 1$

Soln:- Matrix form of system is:

$$\begin{bmatrix} 2x+3y \\ x-4y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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Here; $A=\begin{bmatrix}2 & 3\\ 1 & -4\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}0\\ 1\end{bmatrix}$

$\because X=A^{-1}B$ ——(i) and $A^{-1}=\frac{1}{|A|}\text{adj}A$ ——(ii)

Now $|A|=\begin{vmatrix}2 & 3\\ 1 & -4\end{vmatrix}=2(-4)-1(3)=-8-3=-11\neq 0$

and $\text{adj}A=\begin{bmatrix}-4 & -3\\ -1 & 2\end{bmatrix}$

Putting $|A|$ and adjA in Eq (ii) we get

$$A^{-1}=\frac{1}{-11}\begin{bmatrix}-4 & -3\\ -1 & 2\end{bmatrix}$$

again putting A^{-1} and B in Eq (i) we get

$$X=\frac{1}{-11}\begin{bmatrix}-4 & -3\\ -1 & 2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=\frac{1}{-11}\begin{bmatrix}0-3\\ 0+2\end{bmatrix}=\frac{1}{-11}\begin{bmatrix}-3\\ 2\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-\frac{1}{11}\times -3\\ -\frac{1}{11}\times 2\end{bmatrix}=\begin{bmatrix}3/11\\ -2/11\end{bmatrix}.$$

$\Rightarrow x=3/11,\ y=-2/11$. So S.S$=\{(3/11, -2/11)\}$

(iii) $4x+5y=\frac{1}{2}$ ——(i), $x-\frac{1}{2}y=3$ ——(ii)

Soln:- Multiplying (i) and (ii) by 2 we get

$8x+10y=1$, $2x-y=6$

Now matrix form is

$$\begin{bmatrix}8x+10y\\ 2x-y\end{bmatrix}=\begin{bmatrix}1\\ 6\end{bmatrix}\Rightarrow\begin{bmatrix}8 & 10\\ 2 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 6\end{bmatrix}$$

Here $A=\begin{bmatrix}8 & 10\\ 2 & -1\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}1\\ 6\end{bmatrix}$

Since $X=A^{-1}B$ ——(iii) and $A^{-1}=\frac{1}{|A|}\text{adj}A$ ——(iv)

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Now $|A| = \begin{vmatrix} 8 & 10 \\ 2 & -1 \end{vmatrix} = 8(-1) - 2(10) = -8 - 20 = -28 \neq 0$

and $adj\,A = \begin{bmatrix} -1 & -10 \\ -2 & 8 \end{bmatrix}$

Putting $|A|$ and $adj\,A$ in Eq (iv) we get

$$A^{-1} = \frac{1}{-28}\begin{bmatrix} -1 & -10 \\ -2 & 8 \end{bmatrix}$$

Again putting A^{-1} and B in Eq (iii) we have.

$$X = \frac{1}{-28}\begin{bmatrix} -1 & -10 \\ -2 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ 6 \end{bmatrix} = \frac{1}{-28}\begin{bmatrix} -1-60 \\ -2+48 \end{bmatrix} = \frac{1}{-28}\begin{bmatrix} -61 \\ 46 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1/28 \times -61 \\ -1/28 \times 46 \end{bmatrix} = \begin{bmatrix} 61/28 \\ -23/14 \end{bmatrix} \Rightarrow x = 61/28,\ y = -\frac{23}{14}$$

$$\therefore S.S = \left\{\left(61/28, -\frac{23}{14}\right)\right\}$$

(iv) $x - y = 2$, $x + y = 4$

Soln:- Matrix form of the system is

$$\begin{bmatrix} x-y \\ x+y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Here $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

As we know that $X = A^{-1}B$ —— (i)

where; $A^{-1} = \frac{1}{|A|}adj\,A$ —— (ii)

But $|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1(1) - (1)(-1) = 1+1 = 2 \neq 0$

and $adj\,A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$.

Putting the values of $|A|$ and $adj\,A$ in Eq (ii)

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$\Rightarrow \bar{A}^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

Putting the values of $\bar{A}^{-1}$ and B in Equation (i).

$$X = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2+4 \\ -2+4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\times 6 \\ \frac{1}{2}\times 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow x = 3,\ y = 1$$

So $S.S = \{(3, 1)\}$

3. Solve Q 2 by cramer's rule.

(i) $x - y = 2$, $x + 2y = 1$

Soln : Matrix form of the system is

$$\begin{bmatrix} x-y \\ x+2y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Here $A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$, $A_x = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, $A_y = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 1 & -1 \\ 1 & 2 \end{vmatrix} = 1(2) - (-1)(1) = 2 + 1 = 3$

$|A_x| = \begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 2(2) - (-1)(1) = 4 + 1 = 5$

$|A_y| = \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = 1(1) - 1(2) = 1 - 2 = -1.$

But $x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{5}{3}$

and $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{-1}{3}$

$\therefore S.S = \{(5/3, -1/3)\}$

(ii) $4x + 5y = \frac{1}{2}$, $x - \frac{1}{2}y = 3$

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Soln: Multiplying these equations by 2 we get.

$8x+10y=1$, $2x-y=6$

Matrix form of the system is

$$\begin{bmatrix} 8x+10y \\ 2x-y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 & 10 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

Here; $A = \begin{bmatrix} 8 & 10 \\ 2 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 1 & 10 \\ 6 & -1 \end{bmatrix}$, $A_y = \begin{bmatrix} 8 & 1 \\ 2 & 6 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 8 & 10 \\ 2 & -1 \end{vmatrix} = 8(-1) - 2(10) = -8-20 = -28 \neq 0$

$|A_x| = \begin{vmatrix} 1 & 10 \\ 6 & -1 \end{vmatrix} = 1(-1) - 6(10) = -1-60 = -61$

$|A_y| = \begin{vmatrix} 8 & 1 \\ 2 & 6 \end{vmatrix} = 8(6) - 2(1) = 48-2 = 46$

So $x = \frac{|A_x|}{|A|} = \frac{-61}{-28} = \frac{61}{28}$

$y = \frac{|A_y|}{|A|} = \frac{46}{-28} = \frac{23}{-14}$

$\therefore$ S.S $= \left\{\left(\frac{61}{28}, \frac{23}{-14}\right)\right\}$.

(iii) $2x+3y=0$, $x-4y=1$

Soln:- Matrix form is

$$\begin{bmatrix} 2x+3y \\ x-4y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Here $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $A_x = \begin{bmatrix} 0 & 3 \\ 1 & -4 \end{bmatrix}$, $A_y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Now; $|A| = \begin{vmatrix} 2 & 3 \\ 1 & -4 \end{vmatrix} = 2(-4) - 1(3) = -8-3 = -11 \neq 0$

$|A_x| = \begin{vmatrix} 0 & 3 \\ 1 & -4 \end{vmatrix} = 0(-4) - 1(3) = 0-3 = -3$

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$|A_y| = \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2(1) - 1(0) = 2 - 0 = 2$

Now $x = \frac{|A_x|}{|A|} = \frac{-3}{-11} = \frac{3}{11}$

$y = \frac{|A_y|}{|A|} = \frac{2}{-11} = -\frac{2}{11}$

$\therefore S.S = \{(3/11, -2/11)\}$

(iv) $x - y = 2, \quad x + y = 4$

Soln:- Matrix form of the system is

$$\begin{bmatrix} x-y \\ x+y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Here, $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $A_x = \begin{bmatrix} 2 & -1 \\ 4 & 1 \end{bmatrix}$, $A_y = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$

Now $|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1(1) - (-1)(1) = 1 + 1 = 2 \neq 0$

$|A_x| = \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 2(1) - (-1)(4) = 2 + 4 = 6$

$|A_y| = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 1(4) - 1(2) = 4 - 2 = 2.$

So $x = \frac{|A_x|}{|A|} = \frac{6}{2} = 3$

$y = \frac{|A_y|}{|A|} = \frac{2}{2} = 1$

$\therefore S.S = \{(3,1)\}.$

4. Cost of 4 dozen eggs and 2 dozen oranges is Rs: 200 and cost 3 dozen eggs and 2 dozen oranges is Rs: 180. Find the cost per dozen of each item?

Soln:- Let price of 1 dozen eggs = x

" " " " oranges = y

According to condition 1: $4x + 2y = 200$ —— (i)

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According to 2nd condition : $3x+2y=180$ —(ii)

Writting (i) and (ii) in matrix form

$$\begin{bmatrix}4x+2y\\3x+2y\end{bmatrix}=\begin{bmatrix}200\\180\end{bmatrix}\Rightarrow\begin{bmatrix}4&2\\3&2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}200\\180\end{bmatrix}$$

Here, $A=\begin{bmatrix}4&2\\3&2\end{bmatrix}$, $A_x=\begin{bmatrix}200&2\\180&2\end{bmatrix}$, $A_y=\begin{bmatrix}4&200\\3&180\end{bmatrix}$

$$|A|=\begin{vmatrix}4&2\\3&2\end{vmatrix}=4(2)-3(2)=8-6=2\neq0$$

$$|A_x|=\begin{vmatrix}200&2\\180&2\end{vmatrix}=200(2)-2(180)=400-360=40$$

$$|A_y|=\begin{vmatrix}4&200\\3&180\end{vmatrix}=4(180)-3(200)=720-600=120$$

Now $x=\dfrac{|A_x|}{|A|}=\dfrac{40}{2}=20$

$y=\dfrac{|A_y|}{|A|}=\dfrac{120}{2}=60$

So price per dozen of eggs $=x=$ Rs : 20

" " " " Orange $=y=$ Rs : 60

5. The sum of ages of sister and her brother is 26 years. Three years ago sisters age was 4 times the age of brother. Find their present ages.

Soln: Let age of brother $=x$ years

and " " Sister $=y$ years

According to condition First : $x+y=26$ —(i)

Three years before, Their ages are: $x-3$, $y-3$ years.

According to condition 2nd : $y-3=4(x-3)$

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$\Rightarrow y-3 = 4x-12$

$\Rightarrow 4x-y = -3+12 \Rightarrow 4x-y=9$ —(ii)

Now we Solving (i) and (ii) by Cramer's rule:

$$\begin{bmatrix} x+y \\ 4x-y \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 26 \\ 9 \end{bmatrix}$$

Here $A = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 26 & 1 \\ 9 & -1 \end{bmatrix}$, $A_y = \begin{bmatrix} 1 & 26 \\ 4 & 9 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = 1(-1) - 4(1) = -1-4 = -5$

$|A_x| = \begin{vmatrix} 26 & 1 \\ 9 & -1 \end{vmatrix} = 26(-1) - 9(1) = -26-9 = -35$

$|A_y| = \begin{vmatrix} 1 & 26 \\ 4 & 9 \end{vmatrix} = 1(9) - 4(26) = 9-104 = -95$

According Cramer's rule

$x = \frac{|A_x|}{|A|} = \frac{-35}{-5} = 7$

$y = \frac{|A_y|}{|A|} = \frac{-95}{-5} = 19$

$\therefore$ Age of brother $= x = 7$ years

" " Sister $= y = 19$ years

6. The sum of two digit number is 13. If the digits are interchanged, the new number so formed is increased by 27. Find the number.

Soln: Let digit at unit place $= x$

and " " tenth " $= y$

According to condition first: $x+y = 13$ —(i)

number is $= x+10y$

Number after changing the position $= y+10x$

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According to 2nd condition: $x+10y = y+10x-27$

$\Rightarrow x+10y-y-10x = -27$

$\Rightarrow -9x+9y = -27$

$x-y = 3$ —(iii) dividing by -9

Now Solving (i) and (ii) by Cramer's rule.

$$\begin{bmatrix} x+y \\ x-y \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \end{bmatrix}$$

Here $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 3 & 1 \\ 13 & -1 \end{bmatrix}$, $A_y = \begin{bmatrix} 1 & -3 \\ 1 & 13 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 1(+1)+1(1) = -1-1 = +2 \neq 0$

$|A_x| = \begin{vmatrix} -3 & 1 \\ 13 & -1 \end{vmatrix} = 3(-1)-13(1) = -3+13 = +10$

$|A_y| = \begin{vmatrix} 1 & -3 \\ 1 & 13 \end{vmatrix} = 1(13)+1(3) = 13+3 = 16$

Now; $x = \frac{|A_x|}{|A|} = \frac{+16}{+2} = 8$

$y = \frac{|A_y|}{|A|} = \frac{10}{2} = 5$

So the required number is 58

Ex : 6.7

(Objective Type Questions)

1. Encircle "T" for True and "F" for false.

(i) For addition of matrices the order must not be same. (T)

(ii) A square matrix "B" is called singular if $|B|=0$ (T)

(iii) If order of matrix A is m-by-n then order of A^{-1} is also m-by-n (F)

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(iv) In scaler multiplication order a matrix changes (F)

2. Fill in the blanks with correct Word / Number

(i) If A is non-Singular then A^{-1} exists.

(ii) A squar matrix "C" is symmetric if $C^t = C$

(iii) Determinant of a square matrix is a real number

(iv) The product matrix $IX = X$

3. Choose the correct option in each case.

(i) b (ii) a (iii) c (iv) a

End of Chapter

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