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Exercise 6.6

Q1. Find an equation of the hyperbola with the given date. Sketch the graph of each.

i. Centre (0,0) , Focus (6,0), vertex (4,0) As: c=ae

Solution

Here $a = 4$ and $c = 6$

From this information we can find b^2.

$$c^2 = a^2 + b^2 \qquad \text{(in hyperbola)}$$

$$b^2 = c^2 - a^2$$

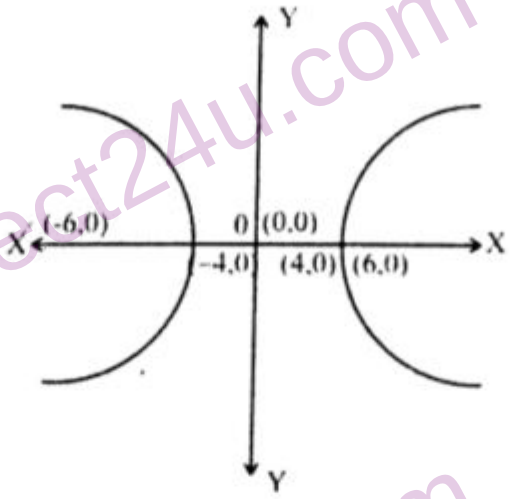
$$\Rightarrow \quad b^2 = 36 - 16$$

$$\Rightarrow \quad b^2 = 20$$

From the location of the vertex and focus, we can also determine that the transverse axis is horizontal (the x term is positive).

$$\frac{x^2}{16} - \frac{y^2}{20} = 1$$

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ii. Foci $(\pm 5, 0)$, vertex (3,0)

Solution

Foci (5,0),(-5,0) As c = ae

⇒ centre is at (0,0) and $c = 5$

Also $a = 3$

From this information we can find b^2.

$$c^2 = a^2 + b^2 \qquad \text{(in hyperbola)}$$

$$b^2 = c^2 - a^2$$

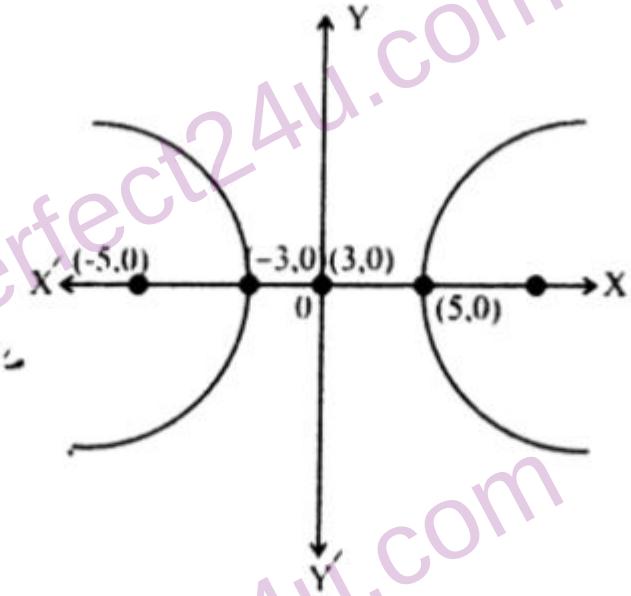
$$\Rightarrow \quad b^2 = 25 - 9$$

$$\Rightarrow \quad b^2 = 16$$

From the location of the vertex and focus, we can also determine that the transverse axis is horizontal (the x term is positive). The equation is:

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

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iii. **F $(2+5\sqrt{2}, 7)$, F$'(2-5\sqrt{2}, -7)$ $2a = 10$**

Solution

From the centre and foci, we have

$a = 5$ and $c = 5\sqrt{2}$

As $c = ae$

As $c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

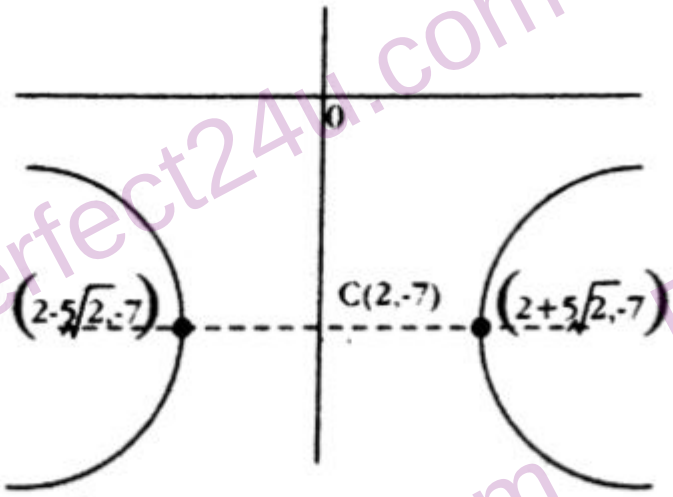
$\Rightarrow \quad b^2 = (5\sqrt{2})^2 - 25 = 50 - 25$

$\Rightarrow \quad b^2 = 25$

Thus the equation of hyperbola is

$$\frac{(x-2)^2}{25} - \frac{(y+7)^2}{25} = 1$$

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iv. Foci $(0, \pm 9)$, drictrices $y = \pm 4$

Solution

Here $c = 9$ $y = \pm\frac{a}{e} = 4$

i.e. $ae = 9$ $a = 4e$

i.e. $a = \frac{9}{4} = 9$

i.e. $a^2 = 36$

$c^2 = a^2 + b^2$ (in hyperbola)

$b^2 = c^2 - a^2$

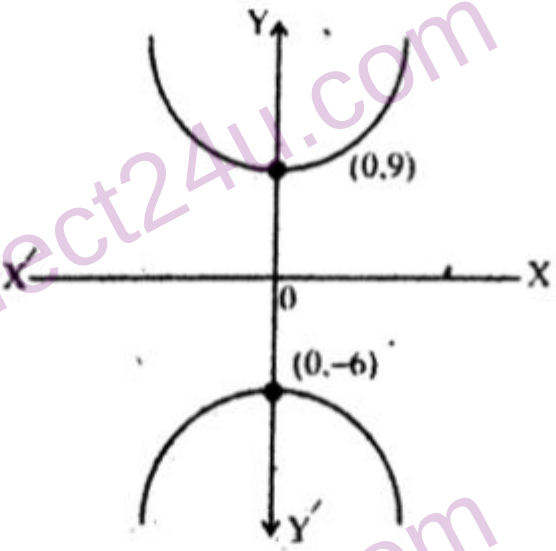
$\Rightarrow$ $b^2 = 81 - 36$

$\Rightarrow$ $b^2 = 45$

Required equation is

$\frac{y^2}{45} - \frac{x^2}{36} = 1$ As foci lies on y-axis

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v. Centre (2,2), horizontal. Transverse axis of length 6 and eccentricity e = 2.

Solution

Here $2a = 6$ As: $c = ae$

$\Rightarrow a = 3$

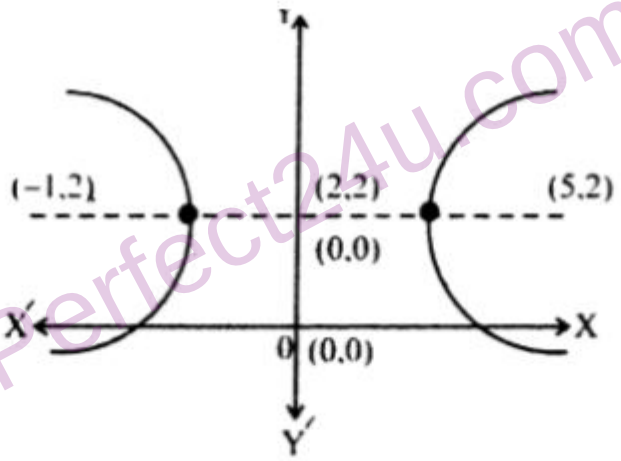
$c = ae = 3.2 = 6$

$b^2 = c^2 - a^2$

$\Rightarrow \quad b^2 = 36 - 9 = 27$

Required equation is $\frac{(x-2)^2}{9} - \frac{(y-2)^2}{27} = 1$ with centre at (2,2).

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vi. Vertices $(2, \pm 3)$, (0,5) lies on the curve.

Solution

$$2a = 6 \quad \Rightarrow \quad a = 3$$

Clear from the vertices, equation of hyperbola is

$$\frac{y^2}{a^2} - \frac{(x-2)^2}{b^2} = 1 \qquad \text{---------- (1)}$$

As (0,5) lies on the equation (1) , so

$$\frac{(5)^2}{a^2} + \frac{(0-2)^2}{b^2} = 1 \qquad \text{---------- (2)}$$

0r $\quad \frac{25}{a^2} - \frac{4}{b^2} = 1$

Also from vertices $(\mathbf{2, \pm 3})$, $a = 3$ i. e. $a^2 = 9$

Put the value of a^2 in eq. (2), we have

$$\frac{25}{9} - \frac{4}{b^2} = 1 \quad \text{or} \quad \frac{4}{b^2} = 1 - \frac{25}{a^2}$$

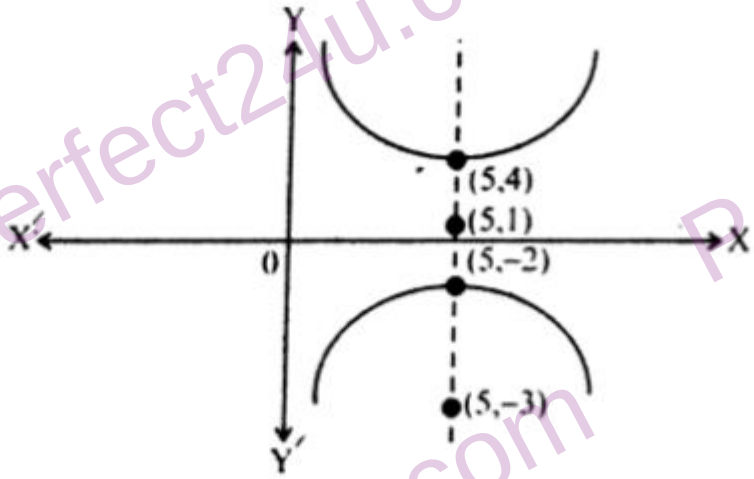
Or $\frac{4}{b^2} = \frac{25-9}{9}$ 0r $\frac{4}{b^2} = \frac{16}{9}$

Or $\frac{1}{b^2} = \frac{4}{9}$ Or $b^2 = \frac{9}{4}$

Substituting the values of a^2 and b^2in eq. (1), we have

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$\frac{y^2}{9} - \frac{(x-2)^2}{(\frac{9}{4})} = 1$



vii. Vertices (5,-2),(5,4) and one focus (5,-3)

Solution

Centre is the midpoint of vertices, so

$C\left(\frac{5+5}{2}, \frac{-2+4}{2}\right) = (5,1)$ as the centre of the hyperbola.

From the vertices and centre, we have $a = 3$ i.e. $a^2 = 9$

From focus and centre, we have $c = 4$

As $c^2 = a^2 + b^2$

$\Rightarrow \quad b^2 = c^2 - a^2$

$\Rightarrow \quad b^2 = (4)^2 - 9$

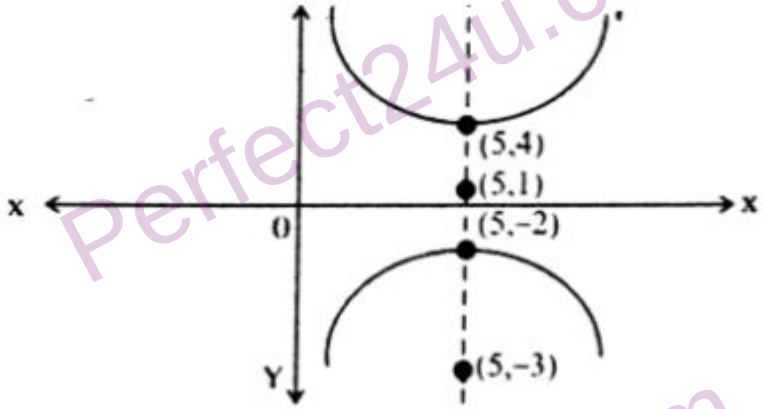
$\Rightarrow \quad b^2 = 16 - 9$

$\Rightarrow \quad b^2 = 7$

Equation of hyperbola is $\frac{(y-1)^2}{a^2} - \frac{(x-5)^2}{b^2} = 1$

Or $\frac{(y-1)^2}{9} - \frac{(x-5)^2}{7} = 1$

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Q2. Find the centre, foci, eccentricity, vertices and equation of directrices.

i. $x^2 - y^2 = 9$

Solution

Given equation can be written as

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \qquad \text{---------- (1)}$$

The transverse axis of (1) lies along the x-axis.

Coordinates of centre are (0,0)

Coordinates of vertices are $(\pm 3, 0)$

Here $a = 3 \qquad b = 3 \qquad c = ae$

$c^2 = a^2 + b^2$, we get

$c^2 = 9 + 9 = 18 \qquad$ or $\qquad c = 3\sqrt{2}$

Foci are $(\pm\sqrt{2}, 0)$.

Equation of directories are $x = \pm\frac{a}{e} = x = \pm\frac{3}{\sqrt{2}}$

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ii. $\frac{x^2}{4} - \frac{y^2}{9} = 1$ ---------- **(1)**

solution

The transverse axis of (1) lies along the x-axis.

Coordinates of centre are (0,0)

Coordinates of vertices are $(\pm 3,0)$

Here $a = 2$ $b = 3$ $c = ae$

$c^2 = a^2 + b^2$, we get

$\Rightarrow$ $c^2 = 4 + 9 = 13$ or $c = \sqrt{13}$

Foci of hyperbola are $(\pm\sqrt{13}, 0)$.

Equation of directories are $x = \pm\frac{a}{e}$

$$x = \pm\frac{2}{\frac{\sqrt{13}}{2}} = x = \pm\frac{4}{\sqrt{13}}$$

so, c(0,0)

$$F(\pm ae, 0) = F(\pm\sqrt{13}, 0)$$

Vertices $V(\pm a, 0) = V(\pm 2,0)$

iii. $\frac{y^2}{16} - \frac{x^2}{9} = 1$

Solution

The transerve axis of (1) lies along the y-axis.

Coordinates of centre are (0,0)

Here $a = 4$ $b = 3$ $c = ae$

From $c^2 = a^2 + b^2$, we get

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$\Rightarrow \quad c^2 = 25 \quad$ or $\quad c = 5$

Foci are $(0, \pm 5)$.

vertices are $(0, \pm 4)$

Equation of directories are $y = \pm\frac{a}{e} = \pm\frac{16}{5}$

iv. $\frac{y^2}{4} - \frac{x^2}{1} = 1$

Solution

The transerve axis of (1) lies along the y-axis.

Coordinates of centre are (0,0)

Here $a = 2 \quad b = 1 \quad c = ae$

From $c^2 = a^2 + b^2$, we get

$c^2 = 4 + 1 = 5 \quad$ or $\quad c = \sqrt{5}$

Foci are $(0, \pm\sqrt{5})$.

vertices are $(0, \pm 2)$

Equation of directories are $y = \pm\frac{a}{e} = \pm\frac{2}{\sqrt{5}}$

v. $\frac{(x-1)^2}{2} - \frac{(y-1)^2}{9} = 1$ ---------- **(1)**

Solution

We write x – 1 = X, y – 1 = Y in (1), to have,

$\frac{x^2}{2} - \frac{y^2}{9} = 1$ ---------- (2)

So that it is a hyperbola with centre at x = 0,

y = 0 i.e. the centre of (1) is (1,1)

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the transerve axis of 2 is Y=0

i.e. Y-1 = 0 is transerve axis of (1).

Here $a = \sqrt{2}$ $b = 3$ so that, we have

$$c = \sqrt{a^2 + b^2} = \sqrt{2+9} = \pm\sqrt{11},$$

foci of (2) are $x = \pm\sqrt{11}$, $y = 0$

i.e. $x - 2 = 0$ $y + 2 = \pm 5$

i.e. $(1+\sqrt{11}, 1)$ and $(1-\sqrt{11}, 1)$

vertices of (2) are $x = \pm\sqrt{2}$, $y = 0$

i.e. $x - 1 = \pm\sqrt{2}$ $y - 1 = 0$

i.e. $x = 1 \pm \sqrt{2}$ $y = 1$

i.e. $(1+\sqrt{2}, 1)$ and $(1-\sqrt{2}, 1)$

Equation of directories are

$$x = \pm\frac{a}{e}$$

$$x - 1 = \pm\frac{\sqrt{2}}{\frac{\sqrt{11}}{\sqrt{2}}}$$

$$x = 1 \pm \frac{2}{\sqrt{11}}$$

vi. $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$ ---------- **(1)**

Solution

If we write x – 2 = X, y +2 = Y in (1), to have,

$\frac{y^2}{9} - \frac{x^2}{16} = 1$ ---------- (2)

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Its centre is at $x = 0$, $y = 0$

i.e. $x - 2 = 0$ $y + 2 = 0$

$x = 2$ $y = -2$ i.e. (2,-2)

The transerve axis is vertical and has equation

$x = 0$ $x - 2 = 0$

i.e. $x = 2$ is the transerve axis of (1). Joint equation of asymptotes of (2) is:

$\frac{y^2}{9} - \frac{x^2}{16} = 1$ i.e. $\frac{y^2}{9} = \frac{x^2}{16}$

i.e. $y^2 = \frac{9}{16}x^2$ i.e. $y = \pm\frac{3}{4}x$

Here $a = 3$ $b = 4$ so that, we have

$c = \sqrt{a^2 + b^2}$

$= \sqrt{2 + 9}$

$= \sqrt{11}$

$= 5$

foci of (2) are $x = 0$ $y = \pm 5$

i.e. $x - 2 = 0$ $y + 2 = \pm 5$

i.e. $x = 2$

$y = -2 \pm 5$

i.e. (2,3) and $(2, -7)$ are foci of (1)

vertices of (2) are $x = 0$, $y = \pm 3$

i.e. $x - 2 = 0$ $y + 2 = \pm 3$

i.e. $x = 2$ $y = -2 \pm 3$

i.e. (2,1) and (2,-5)

Equation of directories are

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$$y = \pm\frac{a}{e}$$

$$\Rightarrow y + 2 = \pm\frac{9}{5}$$

$$\Rightarrow y = -2 \pm \frac{9}{5}$$

vii. $\mathbf{9x^2 - 12x - y^2 - 2y + 2 = 0}$

Solution

$$9x^2 - 12x - y^2 - 2y + 2 = 0$$

$$9x^2 - 12x - (y^2 + 2y) = -2$$

By completing the square

$$[(3x)^2 - 2(3x)(2) + (2)^2] - [y^2 - 2(y)(1) + (1)^2] = -2 + 4 - 1$$

$$(3x - 2)^2 - (y + 1)^2 = 1$$

$$9\left(x - \frac{2}{3}\right)^2 - (y + 1)^2 = 1$$

$$\frac{\left(x-\frac{2}{3}\right)^2}{1/9} - \frac{(y+1)^2}{1} = 1$$

$\frac{x^2}{1/9} - \frac{y^2}{1} = 1$ where $x = x - \frac{2}{3}$ $y = y + 1$

$a^2 = \frac{1}{9}$ $a = \pm\frac{1}{3}$, $b^2 = 1$ $b = \pm 1$

$e^2 = \frac{a^2+b^2}{a^2} = \frac{\frac{1}{9}+1}{1/9} = \frac{\frac{10}{9}}{1/9} = 10$ $e = \sqrt{10}$

$ae = \frac{1}{3}(\sqrt{10}) = \frac{\sqrt{10}}{3} = ac = \frac{\frac{1}{3}}{\sqrt{10}} = \frac{1}{3\sqrt{10}}$

coordinates of centre (0,0)

x=0 y = 0

$x - \frac{2}{3} = 0$ $y + 1 = 0$

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$c(\frac{2}{3}, 1)$

coordinates of foci $(\pm ae, 0)$

$x = \pm a, \quad y = 0$

i.e. $x - \frac{2}{3} = \pm\frac{1}{3} \qquad y + 1 = 0$

i.e. $x = \frac{2}{3} \pm \frac{1}{3} \qquad y = -1$

$x = \frac{2\pm1}{3}$

$x = 1, \frac{1}{3}$

A(1,-1) A'(1/3 , -1)

viii. $\mathbf{4y^2 + 12y - x^2 + 4x + 1 = 0}$

Solution

$4(y^2 + 3y) - (x^2 - 4x) = -1$

$4\left(y^2 + 3y + \frac{9}{4}\right) - (x^2 - 4x + 4) = -1 + \frac{9}{4} - 4$

$4\left(y + \frac{3}{2}\right)^2 - (x - 2)^2 = 4$

$\frac{\left(y+\frac{3}{2}\right)^2}{(1)^2} - \frac{(x-2)^2}{(2)^2} = 1$

$\frac{y^2}{1} - \frac{x^2}{4} = 1$ where $x = x - 2 \qquad y = y + \frac{3}{2}$

Its centre is at x=0 y = 0

$x - 2 = 0 \qquad y + \frac{3}{2} = 0$

$x = 2 \qquad y = -\frac{3}{2}$

$(2, -\frac{3}{2})$

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The transverse axis id vertical and has equation x =0

i.e. $x - 2=0$

i.e. x = 2 is the transerve axis of (1). Joint equation of asymptotes of (2) is:

$\frac{y^2}{1} - \frac{x^2}{4} = 1$ i.e. $\frac{y^2}{1} = \frac{x^2}{4}$

i.e. $y^2 = \frac{1}{4}x^2$ i.e. $y = \pm\frac{1}{2}x$

i.e $y + \frac{3}{2} = \pm\frac{1}{2}(x - 2)$

i.e x-2y-5=0 and x+2y+1=0

also $a = 1$ $b = 2$ so that, we have

$c = \sqrt{a^2 + b^2} = \sqrt{1 + 4} = \sqrt{5}$

foci of (2) are $x = 0$ $y = \pm\sqrt{5}$

i.e. $x - 2 = 0$ $y + \frac{3}{2} = \pm\sqrt{5}$

i.e. $x = 2$ $y = -\frac{3}{2} \pm \sqrt{5}$

i.e. $\left(2, \frac{-3+2\sqrt{5}}{2}\right), \left(2, \frac{-3-2\sqrt{5}}{2}\right)$ are foci of (1)

vertices of (2) are $x = 0$, $y = \pm 1$

i.e. $x - 2 = 0$ $y + \frac{3}{2} = \pm 1$

i.e. $x = 2$ $y = \frac{-3\pm 2}{2}$

i.e. (2, -1/2), (2, -5/2) are vertices of (1)

Equation of directories are

$$y = \pm\frac{a}{e}$$

$$\Rightarrow y + \frac{3}{2} = \pm\frac{1}{\sqrt{5}}$$

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$$\Rightarrow y = -\frac{3}{2} \pm \frac{1}{\sqrt{5}}$$

ix. $x^2 - y^2 + 8x - 2y - 10 = 0$

Solution

$$x^2 - y^2 + 8x - 2y - 10 = 0$$

$$x^2 + 8x - y^2 - 2y = 10$$

$$(x^2 + 8x + 16) - (y^2 + 2y + 1) = 10 + 16 - 1$$

$$(x + 4)^2 - (y + 1)^2 = 25$$

$\frac{(x+4)^2}{25} - \frac{(y+1)^2}{25} = 1$ is the equation of hyperbola

For centre

$x + 4 = 0 \quad y + 1 = 0$

$x = -4 \quad y = -1$

$(-4, -1)$ as the centre

To find eccentricity

Here $a^2 = 25 \quad b^2 = 25$

$$e^2 = \frac{a^2+b^2}{a^2} = \frac{25+25}{25} = \frac{50}{25} = 2$$

$e = \sqrt{2}$

vertices $x + 4 = 5 \quad y + 1 = 0$

i.e. $x = -4 + 5 \quad y = -1$

$x = -4 - 5 \quad y = -1$

thus vertices are $(1, -1), (-9, -1)$.

Foci are given by $(\pm ae, 0)$

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$x+4=\pm 5\sqrt{2}, \qquad y+1=0$

$x=-4\pm 5\sqrt{2}, \qquad y=-1$

thus foci are

$(-4+5\sqrt{2},1), \qquad (-4-5\sqrt{2},1),$

Equation of directories are

$$x+4=\pm\frac{a}{e}$$

$$\Rightarrow x+4=\pm\frac{5}{\sqrt{2}}$$

$$\Rightarrow x=-4\pm\frac{5}{\sqrt{2}}$$

x. $\mathbf{9x^2-y^2-36x-6y+18=0}$

Solution

$9x^2-y^2-36x-6y+18=0$

$9x^2-36x-y^2-6y=-18$

$9(x^2-4x)-(y^2+6y)=-18$

$9(x^2-4x+4)-(y^2+6y+9)=-18+36-9$

$9(x-2)^2-(y+3)^2=9$

$\frac{(x-2)^2}{1}-\frac{(y+3)^2}{9}=1$ is the equation of hyperbola

For centre

$x-2=0 \qquad y+3=0$

$x=2, \qquad y=-3$

$(2,-3)$ as the centre

To find eccentricity

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Here $a^2 = 1$ $b^2 = 9$

$$e^2 = \frac{a^2+b^2}{a^2} = \frac{1+9}{1} = 10$$

$$e = \sqrt{10}$$

vertices

$x - 2 = \pm 1$ $y + 3 = 0$

i.e. $x = 2 \pm 1$ $y = -3$

$x = 2 + 1$ $y = -3$

$x = 2 - 1$ $y = -3$

$x = 3$ $y = -3$

$x = 1$ $y = -3$

thus vertices are $(3,-3), (1,-3)$.

Foci are given by $(\pm ae, 0)$

$x - 2 = \pm(1)\sqrt{10},$ $y + 3 = 0$

$x = 2 \pm \sqrt{10},$ $y = -3$

thus foci are $(2 + \sqrt{10},-3), (2 - \sqrt{10},-3)$,

Equation of directories are

$$x - 2 = \pm\frac{a}{e}$$

$$\Rightarrow x - 2 = \pm\frac{1}{\sqrt{10}}$$

$$\Rightarrow x = 2 \pm \frac{1}{\sqrt{10}}$$

Q3. Let $0 < a < c$, $F(-c, 0)$ and $F'(c, 0)$ be two fixed points. Show that the set of points $P(x, y)$ such that $|PF| - |PF'| = \pm 2a$, is the hyperbola.

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$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **(where $b^2 = c^2 - a^2$)**

Solution

$$|PF| - |PF'| = \pm 2a$$

$$\sqrt{(x-c)^2 + (y-0)^2} - \sqrt{(x+c)^2 + (y-0)^2} = \pm 2a$$

$$\sqrt{(x-c)^2 + y^2} - \sqrt{(x+c)^2 + y^2} = \pm 2a$$

Square both sides

$$(x-c)^2 + y^2 + (x+c)^2 + y^2 - 2\sqrt{(x-c)^2 + y^2}\sqrt{(x+c)^2 + y^2} = 4a^2$$

$$\Rightarrow \quad x^2 + c^2 - 2xc + y^2 + x^2 + c^2 + 2xc + y^2 - 2\sqrt{\{(x-c)^2 + y^2\}\{(x+c)^2 + y^2\}} = 4a^2$$

$$\Rightarrow \quad 2x^2 + 2y^2 + 2c^2 - 2\sqrt{(x^2 + y^2 + c^2)^2 - (2xc)^2} = 4a^2$$

$$\Rightarrow \quad (x^2 + y^2 + c^2) - 2a^2 = \sqrt{(x^2 + y^2 + c^2)^2 - (2xc)^2}$$

Squaring on both sides

$$x^2 + y^2 + c^2 + 4a^4 - 4a^2(x^2 + y^2 + c^2) = (x^2 + y^2 + c^2) - 4x^2c^2$$

$$a^4 - a^2(x^2 + y^2 + c^2) = -x^2c^2$$

$$x^2c^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$$

$$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2)$$

$$\frac{(c^2-a^2)x^2}{a^2(c^2-a^2)} - \frac{a^2y^2}{a^2(c^2-a^2)} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{(c^2-a^2)} = 1$$

Hence proved

Q4. Using problem 3. Find an equation of hyperbola with foci (-5,-5) and (5,5) vertices $(-3\sqrt{3}, -3\sqrt{2})$ and $(3\sqrt{3}, 3\sqrt{2})$.

Solution

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F(-5,-5),F(5,5),P(x,y)

$$|PF| - |PF'| = \pm 2a$$

Distance between the vertices

$$2a = \sqrt{(3\sqrt{2} + 3\sqrt{2})^2 + (3\sqrt{2} + 3\sqrt{2})^2}$$

$$2a = \sqrt{(6\sqrt{2})^2 + (6\sqrt{2})^2}$$

$$= \sqrt{72 + 72} = \sqrt{144} = 12$$

$$|PF| = \sqrt{(x+5)^2 + (y+5)^2} \qquad |PF'| = \sqrt{(x-5)^2 + (y-5)^2}$$

Putting the values in (1)

$$\sqrt{(x+5)^2 + (y+5)^2} - \sqrt{(x-5)^2 + (y-5)^2} = 12$$

Square both sides

$$x^2 + 10x + 25 + y^2 + 10y + 25 + x^2 - 10x + 25 + y^2 - 10y + 25 -$$
$$2\sqrt{x^2 + 10x + 25 + y^2 + 10y + 25}\sqrt{x^2 - 10x + 25 + y^2 - 10y + 25} = 144$$

$$2x^2 + 2y^2 + 100 - 2\sqrt{(x^2 + y^2 + 10x + 10y + 50)(x^2 + y^2 - 10x - 10y + 50)} = 144$$

$$x^2 + y^2 + 50 - \sqrt{(x^2 + y^2 + 10x + 10y + 50)(x^2 + y^2 - 10x - 10y + 50)} = 72$$

Let $x^2 + y^2 + 50 = t$

$$t - \sqrt{(t + 10x + 10y)(t - 10x - 10y)} = 72$$

$$\sqrt{(t + 10x + 10y)(t - 10x - 10y)} = t - 72$$

Square both sides

$$(t + 10x + 10y)(t - 10x - 10y) = t^2 - 144t + 5184$$

$$t^2 - 10tx - 10ty + 10tx - 100x^2 - 100xy - 100xy + 10ty - 100y^2 = t^2 - 144t + 5184$$

$$-100x^2 - 100y^2 - 200xy = -144t + 5184$$

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By replacing the value of t

$$-100x^2 - 100y^2 - 200xy = -144(x^2 + y^2 + 50) + 5184$$

$$-100x^2 - 100y^2 - 200xy = -144x^2 - 144y^2 - 7200 + 5184$$

$$44x^2 + 44y^2 - 200xy + 2016 = 0$$

$$4(11x^2 + 11y^2 - 50xy + 5041) = 0$$

$$11x^2 + 11y^2 - 50xy + 5041 = 0$$

Q5. For any point on the hyperbola the difference of its distance from the point (2,2) and (10,2) is 6. Find an equation of hyperbola.

Solution

F(2,2), F'(10,2)

Let P(x,y) be any point on the hyperbola

Then $\sqrt{(x-10)^2 + (y-2)^2} - \sqrt{(x-2)^2 + (y-2)^2} = 6$

Square both sides

$$x^2 - 20x + 100 + y^2 - 4y + 4 + x^2 - 4x + 4 + y^2 - 4y + 4 -$$

$$2\sqrt{(x-10)^2 + (y-2)^2}\sqrt{(x-2)^2 + (y-2)^2} = 36$$

$$2x^2 + 2y^2 - 24x - 8y + 112 - 2\sqrt{(x^2 - 20x + 104 + y^2 - 4y)(x^2 - 4x + 8 + y^2 - 4y)} = 36$$

$$x^2 + y^2 - 12x - 4y + 56 - \sqrt{(x^2 - 20x + 104 + y^2 - 4y)(x^2 - 4x + 8 + y^2 - 4y)} = 18$$

$$\sqrt{(x^2 - 20x + 104 + y^2 - 4y)(x^2 - 4x + 8 + y^2 - 4y)} = x^2 + y^2 - 12x - 4y + 38$$

$$\sqrt{\{(x^2 - 4x + 8 + y^2 - 4y) - 16x + 96\}(x^2 - 4x + 8 + y^2 - 4y)} = (x^2 + y^2 - 4x - 4y + 8) - 8x + 30$$

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Let $x^2 + y^2 - 4x - 4y + 8 = t$

$$\sqrt{(t - 16x + 96)t} = t - 8x + 30$$

Again squaring both sides we have

$$(t - 16x + 96)(t) = t^2 + 64x^2 - 16xt + 60t + 900$$

$$t^2 - 16xt + 96t = t^2 + 64x^2 - 16xt + 60t + 900 - 480x$$

$$-96t + 64x^2 + 60t + 900 - 480x = 0$$

Dividing by '4`, we get

$$16x^2 - 120x - 9t + 225 = 0$$

Replace the value of t, we have

$$16x^2 - 120x - 9(x^2 + y^2 - 4x - 4y + 8) + 225 = 0$$

$$16x^2 - 120x - 9x^2 - 9y^2 + 36x + 36y - 72 + 225 = 0$$

$$7x^2 - 84x - 9y^2 + 36y + 153 = 0$$

Q6. Two listening posts hear the sound of any enemy gun. The difference in time is one second. If the listening posts are 1400 feet apart, write an equation of the hyperbola passing through the position of enemy gun. (velocity of sound is travel at 1080t/sec.

Solution

Let two listening post S and hear the sound of an enemy gun after t and t-1 seconds respectively.

Since listening posts are 1400 feet apart.

$$\therefore \quad 2C = 1400$$

$$\Rightarrow \quad C = 700$$

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If P is the position of enemy gun, we have

$$|PS'| - |PS| = 2a$$

$$(1080)t - (1080)(t-1) = 2a$$

$$(1080)(t - t + 1) = 2a$$

$$1080 = 2a$$

$$\Rightarrow a = \frac{1080}{2} = 540$$

using $c^2 = a^2 + b^2$ $\quad c = ae$

$$(700)^2 = (540)^2 + b^2$$

$$b^2 = (700)^2 + (540)^2$$

$$= 490000 - 291600$$

$$= 198400$$

Thus equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\frac{x^2}{291600} - \frac{y^2}{198400} = 1$$

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