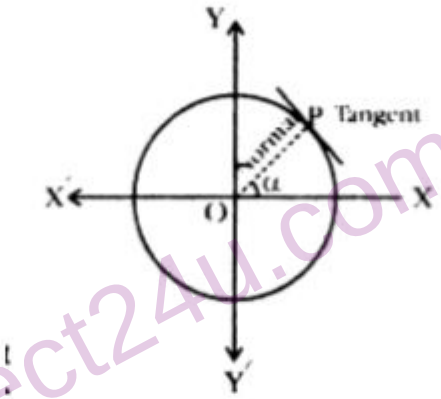
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Exercise 6.3

Q1 Prove that normal lines of a circle pass through the center of the circle.

Solution:



Consider the equation of the circle $x^2 + y^2 = a^2$.....(1)

Any tangent line to the circle (1) at (x_1, y_1) is

$$xx_1 + yy_1 - a^2 = 0$$

Slope of tangent line $= -\frac{x_1}{y_1}$

Slope of normal line $= \frac{x_1}{y_1}$

Equation of the normal line passing through (x_1, y_1) is

$$y - y_1 = \frac{y_1}{x_1}(x - x_1)$$

$$yx_1 - y_1x_1 = y_1(x - x_1)$$

$$yx_1 = y_1x$$

$$y = \frac{y_1}{x_1}x$$

The above equation is satisfied by $(0,0)$. This implies that equation of normal to circle (1) passing through the centre of circle (1) which is $(0,0)$

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Hence the result

Q2 Prove that the straight line drawn from centre of a circle perpendicular to a tangent pass through the point of tangency.

Solution:

Consider the circle

$x^2 + y^2 = r^2$

$xy_1 + yy_1 = a^2$ be an equation of tangent to the circle.

$x^2 + y^2 = r^2$ at point (x_1, y_1)

Slope of perpendicular to tangent is $\frac{y_1}{x_1}$

Equation of line through $(0,0)$ and perpendicular to

$xx_1 + yy_1 = r^2$ is

$y - 0 = \frac{y_1}{x_1}(x - 0)$

$yx_1 = y_1x$

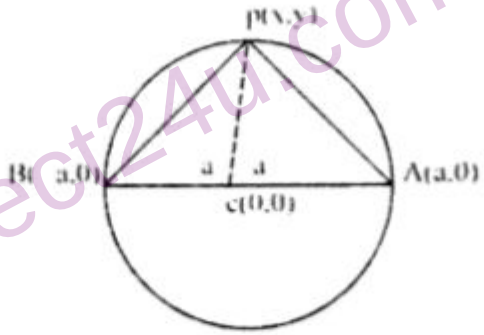
$yx_1 - y_1x = 0$

Which passes through point of tangency (x_1, y_1). hence the result

Q3 Prove that the midpoint of the hypotenuse of a right-angled triangle is the circum centre of the triangle.

Solution:

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Equation of the circle

$x^2 + y^2 = a^2$.....(1)

Slope of $AP = m_1 = \frac{y-0}{x-a} = \frac{y}{x-a}$

Slope of $BP = m_2 = \frac{y-0}{x+a} = \frac{y}{x+a}$

AP perpendicular to BP

Given $m_1 - m_2 = -1 \Rightarrow \left(\frac{y}{x-a}\right)\left(\frac{y}{x+a}\right) = -1$

$\frac{y^2}{x^2-a^2} = -1 \Rightarrow y^2 = -x^2 + a^2 \Rightarrow x^2 + y^2 = a^2$.....(2)

$|CA| = |CB| = |CP|$ (to prove)

If $|CA| = |CP|$

Then $a = \sqrt{(x-0)^2 + (y-0)^2} \Rightarrow x^2 + y^2 = a^2$.....(3)

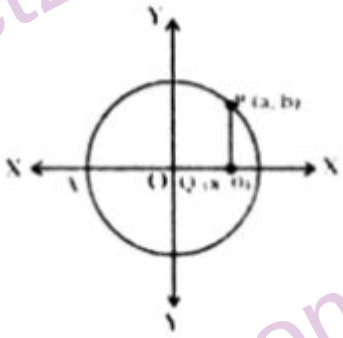
If $|CB| = |CP|$

Then $-a = \sqrt{(x-0)^2 + (y-0)^2} \Rightarrow x^2 + y^2 = a^2$.....(4)

Equations (1),(2),(3) and (4) are same which proves what was required.

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Q4 Prove that the perpendicular dropped from a point of a circle on a diameter is a mean proportional between the segments into which its divides the diameters.



Solution:

Let the circle be $x^2 + y^2 = r^2$.....(1)

And AB be a diameter of the circle such that if coincides with x axis.

Let $P(a, b)$ be a point on the circle and PQ must be the perpendicular dropped from P on AB.

Equation of PQ is $x = a$

Coordinates of the foot Q of the perpendicular are $(a, 0)$

$$|PQ| = |b| \Rightarrow |PQ|^2 = |b|^2$$

Since $P(a, b)$ lies on (1) we have

$$a^2 + b^2 = r^2 \quad \text{or} \quad b^2 = r^2 - a^2$$

Now $A = (-r, 0),\ B = (r, 0)$

Therefore $|AQ| = r + a$ and $|BQ| = r - a$

Now $|AQ|.|BQ| = r^2 - a^2$

$= b^2$ from (3)

$= |PR|^2$ from (2) Hence the result

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