

Exercise 4.2

- The two points P and O' are given in xy-coordinates system. Find the xy-coordinate of P referred to the translated axes O'X and O'Y.

i. $P(3,2); O'(1,3)$

Solution

Here $h = 1, k = 3$

Coordinates of P referred to the translated

$$x = x - h = 3 - 1 = 2$$

$$y = y - k = 2 - 3 = -1$$

i.e (2,-1)

ii. $P(-2,6); O'(-3,2)$

Solution

Here $(x, y) = (-2, 6), (h, k) = (-3, 2)$

Coordinates of referred to XY- coordinate system are

$$X = x - h = -2 - (-3) = -2 + 3 = 1$$

$$Y = y - k = 6 - 2 = 4$$

i.e (1,4)

Solution

$$\text{Here } (x, y) = (-6, -8)$$

$$(h, k) = (-4, -6)$$

Coordinates of referred to XY- coordinate system ...

$$X = x - h = -6 - (-4) = -6 + 4 = -2$$

$$Y = y - k = -8 - (-6) = -8 + 6 = -2$$

$$\Rightarrow (-2, -2)$$

iv. $P\left(\frac{3}{2}, \frac{5}{2}\right); O'\left(\frac{-1}{2}, \frac{7}{2}\right)$

$$\text{Here } (x, y) = \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$(h, k) = O'\left(\frac{-1}{2}, \frac{7}{2}\right)$$

Co-ordinates of P referred to XY- Plane are

$$\begin{aligned} X = x - h &= \left(\frac{3}{2}\right) - \left(\frac{-1}{2}\right) \\ &= \frac{3}{2} + \frac{1}{2} = 2 \end{aligned}$$

$$Y = y - k = \frac{5}{2} - \frac{7}{2} = \frac{5-7}{2} = \frac{-2}{2} = -1$$

$$P(x, y) = P(2, -1)$$

Q2. The xy- coordinate axes are translated through the point O' whose co-ordinates of P are given in the xy co-ordinate system. Find the co-ordinate of P in xy- co -ordinate

i. $P(8, 10), O(3, 4)$

Solution

$$(h, k) = (3, 4)$$

$$X = x + h = 8 + 3$$

$$Y = y + k = 10 + 4$$

$$\Rightarrow P(x, y) = P(11, 4)$$

ii. **P(-5, -3), O(3, 4)**

Solution

$$\text{Here } (x, y) = (-5, -3)$$

$$(h, k) = (-2, -6)$$

$$\Rightarrow x = x + h = -5 + (-2) \\ = -7$$

$$y = y + k = -3 + (-6) \\ = -3 - 6 = -9$$

Thus P(-7, -9)

iii. **P($\frac{-3}{4}, \frac{-7}{6}$), O'($\frac{1}{4}, \frac{-1}{6}$)**

Solution

$$\text{Here } (x, y) = \left(\frac{-3}{4}, \frac{-7}{6}\right)$$

$$x = x + k = \frac{-3}{4} + \frac{1}{4} = \frac{-2}{4} = \frac{1}{2}$$

Type equation here.

$$y = y + h = \frac{-7}{6} + \left(\frac{-1}{6}\right) \\ = \frac{-8}{6} = \frac{-4}{3}$$

$$\text{Thus } P(x, y) = \left(\frac{1}{2}, \frac{-4}{3}\right)$$

Solution

$$\text{Here } (x, y) = (4, -3)$$

$$(h, k) = (-2, 3)$$

$$x = x + h = 4 + (-2) = 4 - 2 = 2$$

$$y = y + k = -3 + 3 = 0$$

$$\text{Thus } P(x, y) = P(2, 0)$$

Q3. The xy-co-ordinates axes are rotated about the origin through the indicated angel. The new axes are OX and OY. Find XY-co-ordinates of the point P with the given XY coordinates

i. **$P(5, 3), \theta = 45^\circ$**

Solution

Let (x, y) be the co-ordinates of the point P referred to xy axes

Here **$\theta = 45^\circ$**

$$\begin{aligned} X \cos \theta + y \sin \theta &= 5 \cos 45^\circ + 3 \sin 45^\circ \\ &= \frac{5}{\sqrt{2}} + \frac{3}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \frac{2 \times 2}{\sqrt{2}} = 4\sqrt{2} \end{aligned}$$

$$\begin{aligned} Y &= -x \sin \theta + y \cos \theta \\ &= -5 \sin 45^\circ + 3 \cos 45^\circ \\ &= \frac{-5}{\sqrt{2}} + \frac{3}{\sqrt{2}} \\ &= \frac{-2}{\sqrt{2}} = \sqrt{-2} - \sqrt{2} \end{aligned}$$

i.e. $(x, y) = (4\sqrt{2}, -\sqrt{2})$

ii. **$P(3, -7), \theta = 45^\circ$**

Solution

Here $\theta = 30^\circ$

$$\begin{aligned} \therefore X &= x \cos \theta + y \sin \theta \\ &= 3 \cos 30^\circ + (-7) \sin 30^\circ \\ &= 3 \left(\frac{\sqrt{3}}{2} \right) - 7 \left(\frac{1}{2} \right) = \frac{3\sqrt{3}-7}{2} \\ Y &= -x \sin \theta + y \cos \theta \\ &= -3 \sin 30^\circ + (-7) \cos 30^\circ \\ &= -3 \left(\frac{1}{2} \right) - 7 \left(\frac{\sqrt{3}}{2} \right) = \frac{-7-3\sqrt{3}}{2} \\ \Rightarrow (x, y) &= \left(\frac{3\sqrt{3}-7}{2}, \frac{-7-3\sqrt{3}}{2} \right) \end{aligned}$$

ii. $p(11, -15), \theta = 60^\circ$

Solution

Let (x, y) be the co-ordinates of the point P referred to xy axes

Here $\theta = 60^\circ$

$$\begin{aligned} X &= x \cos \theta + y \sin \theta \\ &= 11 \cos 60^\circ + (-15) \sin 60^\circ \\ &= 11 \left(\frac{1}{2} \right) - 15 \left(\frac{\sqrt{3}}{2} \right) = \frac{11-15\sqrt{3}}{2} \\ Y &= -x \sin \theta + y \cos \theta \\ &= -11 \sin 60^\circ + (-15) \cos 60^\circ \\ &= -11 \left(\frac{\sqrt{3}}{2} \right) - 15 \left(\frac{1}{2} \right) = \frac{-11\sqrt{3}-15}{2} \\ \Rightarrow (x, y) &= \left(\frac{11-15\sqrt{3}}{2}, \frac{-11\sqrt{3}-15}{2} \right) \end{aligned}$$

iv. $PC(15, 10) : \theta = \arctan \left[\frac{1}{2} \right]$

Solution

Let (x, y) be the co-ordinates of the point P referred to xy axes

$$X = 15, y = 10 \text{ and } \tan \theta = \frac{1}{3}$$

$$\sin \theta = \frac{1}{\sqrt{10}}, \cos \theta = \frac{3}{\sqrt{10}}$$

$$\begin{aligned} X &= x \cos \theta + y \sin \theta = 15\left(\frac{3}{\sqrt{10}}\right) + 10\left(\frac{1}{\sqrt{10}}\right) \\ &= \frac{\sqrt{45}}{\sqrt{10}} + \frac{10}{\sqrt{10}} = \frac{55}{\sqrt{10}} \end{aligned}$$

$$P\left(\frac{55}{\sqrt{10}}, \frac{15}{\sqrt{10}}\right)$$

Q4. The xy-co-ordinates axes are rotated about the origin through the indicated angel. The new axes are OX and OY. Find XY-co-ordinates of the point P with the given XY coordinates

ii. $P(-5, 3), \theta = 30^\circ$

Solution

$$\sin \theta = \frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$X = x \cos \theta + y \sin \theta$$

$$\Rightarrow -3 = x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right)$$

$$\Rightarrow y = -10 \quad \text{_____ (1)}$$

and $Y = -x \sin \theta + y \cos \theta$

$$y = -x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow -x + \sqrt{3} y = 6 \quad \text{_____ (2)}$$

$$(1) \Rightarrow y = -10 - \sqrt{3} x \text{ in } \text{_____ (2)}$$

$$-4x = 6 - 10 + \sqrt{3}$$

$$-2x = 3 + 5\sqrt{3}$$

$$x = \frac{-3-5\sqrt{3}}{2} \text{ put in (1)}$$

$$\sqrt{3}\left(\frac{3+5\sqrt{3}}{2}\right) + y = -10$$

$$\frac{-3\sqrt{3}+15}{2} + y = -10$$

$$y = -10 - \frac{-3\sqrt{3}+15}{2} = \frac{-20+3\sqrt{3}+15}{2}$$

$$= -10 + \frac{3\sqrt{3}-5}{2}$$

$$\Rightarrow P\left(\frac{-3-5\sqrt{3}}{2}, \frac{3\sqrt{3}-5}{2}\right)$$

ii $P(-7\sqrt{2}, 5\sqrt{2}), \theta = 45^\circ$

Solution

Let $P(x, y)$ be the coordinates of P referred to xy -co-ordinates system.

$$\theta = 45^\circ \text{ given}$$

$$\Rightarrow \sin \theta = \frac{1}{\sqrt{2}}$$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

$$X = x \cos \theta + y \sin \theta$$

$$\Rightarrow -7\sqrt{2} = x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow x + y = -14 \quad (1)$$

and $Y = -x \sin \theta + y \cos \theta$

$$5\sqrt{2} = -x\left(\frac{1}{\sqrt{2}}\right) + y\left(\frac{1}{\sqrt{2}}\right)$$

From (1) $y = -14 - x$ in put in (2)

$$-x - 14 - x = 10$$

$$-2x - 14 = 10$$

$$-2x = 24$$

$$\Rightarrow x = -12 \quad \text{put in (1)}$$

$$-12 + y = -14$$

$$y = -14 + 12 = -2$$

$$\Rightarrow P(-12, -2)$$

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