

Solved Exercise 2.8

Q1. Apply the Maclaurin series expansion to prove that

$$\text{i. } \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \dots \dots \dots$$

Solution

$$\text{Let } f(x) = \ln(1+x)$$

$$f'(x) = \frac{1}{1+x}$$

$$f''(x) = -\frac{1}{(1+x)^2}$$

$$f'''(x) = \frac{(-1)(-2)}{(1+x)^3} = \frac{2}{(1+x)^3}$$

$$f^{(4)}(x) = \frac{-6}{(1+x)^4}$$

put $x = 0$ we have

$$f(0) = 1 \times (1+0) = \ln(1) = 0$$

$$f'(0) = \frac{1}{1+0} = 1$$

$$f''(0) = \frac{1}{(1+0)^2} = -1$$

$$f'''(0) = \frac{1}{(1+0)^3} = 2$$

$$f^{(4)}(0) = \frac{(-1)(-2)(-3)}{(1+0)^4} = -6$$

$$f^{(5)}(0) = \frac{(-1)(-2)(-3)}{(1+0)^4} = -6$$

By Maclaurin Expansion we have

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \dots \dots \dots$$

$$\ln(1+x) = 0 + \frac{1}{1!}x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3 + \frac{-6}{4!}x^4 + \dots \dots \dots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad \text{Ans}$$

$$\text{ii. } \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

Solution

$$\text{Let } f(x) = \cos x$$

$$\text{put } x = 0$$

$$f(x) = \cos x$$

$$f(0) = \cos 0 = 1$$

$$f'(x) = -\sin x$$

$$f'(0) = -\sin 0 = 0$$

$$f''(x) = -\cos x$$

$$f''(0) = -\cos 0 = -1$$

$$f'''(x) = \sin x$$

$$f'''(0) = \sin 0 = 0$$

$$f^{(4)}(x) = \cos x$$

$$f^{(4)}(0) = \cos 0 = 1$$

$$f^{(5)}(x) = -\sin x$$

$$f^{(5)}(0) = -\sin 0 = 0$$

Using Maclaurin Expansion we have

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \dots$$

$$\cos x = 1 + \frac{0}{1!}x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4 + \frac{0}{5!}x^5 + \dots$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad \text{Ans}$$

$$\text{iii. Prove that } \sqrt{1-x} = 1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16} + \dots$$

Solution

$$f(x) = (1+x)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2\sqrt{1+x}}$$

$$f''(x) = \frac{1}{2} \left(-\frac{1}{2} \right) (1+x)^{-\frac{3}{2}} = \frac{-1}{4(1+x)^{\frac{3}{2}}}$$

$$f'''(x) = \frac{1}{2} \left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right) (1+x)^{-\frac{5}{2}} = \frac{3}{8(1+x)^{\frac{5}{2}}}$$

$$f^{iv}(x) = \frac{1}{2} \left(-\frac{1}{2} \right) \left(-\frac{3}{2} \right) \left(-\frac{5}{2} \right) (1+x)^{-\frac{7}{2}} = \frac{-15}{8(1+x)^{\frac{7}{2}}}$$

Putting $x = 0$ we have

$$f(0) = (1+0)^{\frac{1}{2}} = 1$$

$$f'(0) = \frac{1}{2\sqrt{1+0}} = \frac{1}{2}$$

$$f''(0) = \frac{-1}{4(1+0)^{\frac{3}{2}}} = -\frac{1}{4}$$

$$f'''(0) = \frac{3}{8(1+0)^{\frac{5}{2}}} = \frac{3}{8}$$

$$f^{iv}(0) = \frac{-15}{16(1+0)^{\frac{7}{2}}} = -\frac{15}{16}$$

Using Maclaurin expansion, we have

$$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \dots$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \frac{-1}{(4)(2!)}x^2 + \frac{3}{(8)3!}x^3 - \frac{15}{(16)(4!)}x^4 + \dots$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{1}{16}x^3 - \frac{5}{128}x^4 + \dots \quad \text{Ans}$$

$$\text{iv. } e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Solution

$$\text{Let } f(x) = e^x$$

$$f(0) = e^0 = 1$$

$$f'(0) = e^x$$

$$f'(0) = e^0 = 1$$

$$f'(0) = e^x$$

$$f'(0) = e^0 = 1$$

$$f''(0) = e^x$$

$$f''(0) = e^0 = 1$$

By Maclaurin expansion we have

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \quad \text{Ans}$$

$$v.e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{x^3}{3!} + \dots$$

Solution

$$f(x) = e^{2x} \quad f(0) = e^0 = 1$$

$$f'(x) = 2e^{2x} \quad f'(0) = 2e^0 = 2$$

$$f''(x) = 2^2 e^{2x} \quad f''(0) = 4e^0 = 4$$

$$f'''(x) = 2^3 e^{2x} \quad f'''(0) = 8e^0 = 8$$

$$f^{iv}(x) = 2^4 e^{2x} \quad f^{iv}(0) = 16e^0 = 16$$

By Maclaurins expansion we have

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{iv}(0)}{4!} + \dots$$

$$e^{2x} = 1 + 2x + \frac{4}{2!}x^2 + \frac{8}{3!}x^3 + \frac{16}{4!}x^4 + \dots \quad \text{Ans}$$

Q2. Show that is

$$\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots \text{and Evaluate } \cos 61^\circ.$$

Solution

$$\text{Let } f(x+h) = f(x) + \frac{f'(x)}{1!}h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \dots$$

$$f(x) = \cos x$$

$$f'(x) = -\sin x$$

$$f''(x) = -\cos x$$

$$f'''(x) = \sin x$$

Using values, we have

$$\Rightarrow \cos(x+h) = \cos x - (\sin x)h - \frac{\cos x}{2!}h^2 + \frac{\sin x}{3!}h^3 + \dots B$$

$$= \cos x - h \sin x - \frac{h^2}{2!}\cos x + \dots$$

$$\text{Putting } x+h=61^\circ + 1^\circ = \frac{\pi}{3} + \frac{\pi}{180^\circ}. \text{ So that } x = 60^\circ = \frac{\pi}{3}$$

$$\cos 61^\circ = \cos \frac{\pi}{3} - \left(\sin \frac{\pi}{3}\right)\left(\frac{\pi}{180}\right) - \frac{\cos \frac{\pi}{3}}{2!}\left(\frac{\pi}{180}\right)^2 + \frac{\sin \frac{\pi}{3}}{3!}\left(\frac{\pi}{180}\right)^3$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2}\left(\frac{\pi}{180}\right) - \frac{1}{2!}\left(\frac{\pi}{180}\right)^2 + \frac{\sqrt{3}}{3!}\left(\frac{\pi}{180}\right)^3 +$$

$$\cos 61^\circ = \frac{1}{2} - \frac{\sqrt{3}}{2}(0.017485) - \frac{1}{2!}(0.017485)^2 +$$

$$= .5 - (.866)(.017485) - \frac{1}{4}(.0003045)$$

$$= .5 - .015116 - .000076$$

$$= .4848 \quad \text{Ans}$$

Q3. Show that $2^{x+h} = 2^x\{1 + \ln 2\}h + (\ln 2)^2 + (\ln 2)^3 h^3 + \dots\}$

Solution

$$\text{So. Let } (x+h) = 2^{x+h}$$

$$\Rightarrow f(x) = 2^x$$

Diff. w.r. t x we have

$$f'''(x) = 2^x \ln 2$$

$$f''(x) = 2^x \ln 2^2$$

$$f'''(x) = 2^x \ln 2^3$$

$$f^{iv}(x) = 2^x \ln 2^4$$

By using Taylors series, we have

$$f(x+h) = f(x) + \frac{f'(x)}{1!}h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \frac{f^{iv}(x)}{4!}h^4 + \dots$$

$$2^{x+h} = 2^x + (2^x \ln 2)h + \frac{2^x (\ln 2)^2}{2!}h^2 + \frac{2^x (\ln 2)^3}{3!}h^3 + \dots$$

$$2^{x+h} = 2^x \left[1 + (\ln 2)h + \frac{(\ln 2)^2}{2!}h^2 + \frac{(\ln 2)^3}{3!}h^3 + \dots \right] \quad \text{Ans}$$

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