

## Solved Exercise 2.7

**Q1. Find  $y^2$  if**

**i.  $y = 2x^5 - 3x^4 + 4x^3 + x - 2$**

**Solution**

Diff w.r.t  $x$

$$\frac{dy}{dx} = 10x^4 - 12x^3 + 12x^2 + 1 - 0$$

$$y_1 = 10x^4 - 12x^3 + 12x^2 + 1$$

Diff. again w.r.t  $x$

$$y_1 = 40x^3 - 36x^2 + 24x \quad \text{Ans}$$

**ii.  $y = (2x + 5)^{3/2}$**

**Solution**

$$y = (2x + 5)^{3/2}$$

Diff. w.r.t

$$y_1 = \frac{3}{2}(2x + 5) \frac{d}{dx}(2x + 5)$$

$$= \frac{3}{2}(2x + 5)^{1/2}(2)$$

$$y_1 = 3(2x + 5)^{1/2} \dots \dots \dots (1)$$

Diff. again w.r.t  $x$

$$y_2 = 3 \frac{1}{2}(2x + 5)^{1/2-1} \frac{d}{dx}(2x + 5)$$

$$= \frac{3}{2}(2x + 5)^{1/2} \dots\dots\dots(2)$$

$$y_2 = \frac{3}{(\sqrt{2x+5})} \quad \text{Ans}$$

iii.  $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

**Solution**

$$\frac{dy}{dx} = \frac{d}{dx} \left( x^{\frac{1}{2}} + x^{\frac{1}{2}} \right)$$

$$y_1 = \frac{1}{2}x^{\frac{1}{2}-1} - \frac{1}{2}x^{-\frac{1}{2}-1}$$

$$= \frac{1}{2}x^{\frac{1}{2}-1} - \frac{1}{2}x^{-\frac{3}{2}}$$

$$y_1 = \frac{1}{2} \left( x^{\frac{1}{2}-1} - \frac{1}{2}x^{-\frac{3}{2}} \right)$$

Diff. again w.r.t x

$$y_2 = \frac{1}{2} \left( -\frac{1}{2}x^{\frac{1}{2}-2} - \left( -\frac{3}{2}x^{-\frac{3}{2}-1} \right) \right)$$

$$= \frac{1}{2} \left( -\frac{1}{2}x^{\frac{3}{2}} + \frac{3}{2}x^{\frac{3}{2}} \right)$$

$$= -\frac{1}{4x^{\frac{3}{2}}} + \frac{3}{4x^{\frac{3}{2}}}$$

$$= \frac{-x+3}{4x^{\frac{3}{2}}} \quad \text{Ans}$$

**Q2. Find  $y_2$  if  $(1)y = x^2e^{-x}$**

**Solution**

$$y = x^2e^{-x}$$

Diff. w.r.t x

$$y = e^{-x}(2x) + (x^2)(-e^{-x})$$

$$= e^{-x}(2x - x^2)$$

Diff. w.r.t x

$$y_2 = e^{-x}(2 - 2x) + (2x - x^2)(-e^{-x})$$

$$= e^{-x}(2 - 2x - 2x + x^2)$$

$$= e^{-x}(x^2 - 4x + 2) \quad \text{Ans}$$

ii.  $y = \ln\left(\frac{2x+3}{3x+2}\right)$

**Solution**

$$y = \ln\left(\frac{2x+3}{3x+2}\right)$$

$$y = \ln(2x+3) - \ln(3x+2)$$

Diff. w.r.t x

$$y = \frac{1}{2x+3} \cdot 2 - \frac{1}{3x+2}$$

$$= \frac{2}{2x+3} - \frac{3}{3x+2}$$

Diff. again w.r.t x

$$y_2 = 2 \left[ \frac{d}{dx}(2x+3)^{-1} \right] - 3 \left[ \frac{d}{dx}(3x+2)^{-1} \right]$$

$$= 2(-1)(2x+3)^{-1-1} \times 2 - 3((-1)(3x+2)^{-2}(3))$$

$$y_2 = \frac{-4}{(2x+3)^2} + \frac{9}{(3x+2)^2}$$

$$y_2 = -\frac{[4(3x+2)^2 - 9(2x+3)^2]}{(2x+3)^2(3x+2)^2}$$

$$y_2 = \frac{[4(9x^2+12x+4)-9(4x^2+12x+9)]}{(2x+3)^2(3x+2)^2}$$

$$= \frac{[36^2 + 48x + 16 - 36^2 - 108x - 81]}{(2x+3)^2(3x+2)^2}$$

$$y_2 = \frac{60x+65}{(2x+3)^2(3x+2)^2} \quad \text{Ans}$$

**Q3. Find  $y_2$  if**

i.  $x^2 + y^2 = a^2$

**Solution**

Diff. both sides w.r.t x

$$2x + 2y \frac{dy}{dx} = 0$$

$$x + y \frac{dy}{dx} = 0$$

$$y \frac{dy}{dx} = -x$$

$$y \frac{dy}{dx} = -x$$

$$y \frac{dy}{dx} = -\frac{x}{y}$$

$$\Rightarrow \frac{dy}{dx} y_1 = -\frac{x}{y} \dots\dots\dots(1)$$

Now diff.(1) w.r.t x

$$y_2 = \frac{y(-1) - (-x) \frac{dy}{dx}}{(y)^2}$$

$$= \frac{-y + x \frac{dy}{dx}}{y^2}$$

$$= \frac{-y + x \left(-\frac{x}{y}\right)}{y^2}$$

$$= \frac{-y^2 - x^2}{y^2}$$

$$y_2 = \frac{(x^2+y^2)}{y^2} = \frac{a^2}{y^2} \quad \text{Ans}$$

ii.  $x^3 - y^3 = a^3$

**Solution**

Diff. both sides of above equation w.r.t x

$$3x^2 - 3y^2 \frac{dy}{dx} = 0$$

$$3y^2 \frac{dy}{dx} = 3x^2$$

$$y^2 \frac{dy}{dx} = x^2$$

$$\frac{dy}{dx} = \frac{x^2}{y^2}$$

$$\Rightarrow y_1 \frac{dy}{dx} = \frac{x^2}{y^2} \dots\dots\dots (1)$$

Now diff.(1) w.r.t x

$$y_2 = \frac{2xy^2 - (2y \frac{dy}{dx})x^2}{y^4}$$

$$= \frac{2xy^2 - x^2(2y)(\frac{x^2}{y^2})}{y^4} \text{ (using 1)}$$

$$= \frac{2xy^2 - \frac{2x^4}{y}}{y^4}$$

$$= \frac{2xy^3 - 2x^4}{y^5}$$

$$= \frac{2x(y^3 - x^3)}{y^5}$$

$$= \frac{2x(x^3 - y^3)}{y^5}$$

$$= \frac{2a^3x}{y^5} \quad \text{Ans}$$

iii.  $x = a \cos \phi, y = a \sin \phi$

**Solution**

$$x = a \cos \phi, y = a \sin \phi$$

$$\frac{dx}{d\phi} = -a \sin \phi, \frac{dy}{d\phi} = a \cos \phi$$

$$\frac{dy}{dx} = \frac{dy}{d\phi} \times \frac{d\phi}{dx}$$

$$= a \cos \phi \times \frac{1}{-a \sin \phi}$$

$$y_1 = \frac{dy}{dx} = -\cot \phi$$

Diff. again

$$y_2 = -(-\operatorname{cosec}^2 \phi) \frac{d\phi}{dx}$$

$$= \operatorname{cosec}^2 \phi \times \left( -\frac{1}{a \sin \phi} \right)$$

$$= -\frac{1}{a \sin^3 \phi} \quad \text{Ans}$$

iv.  $x = at^2, y = bt^4$

**Solution**

$$x = at^2, \frac{dy}{dt} = 4bt^3$$

$$\frac{dx}{dt} = 2at$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= 4bt^3 \times \frac{1}{2at}$$

$$y_1 = \frac{dy}{dx} = \frac{2bt^2}{a}$$

Diff. again

$$y_2 = \frac{2b}{a} \left( 2t \cdot \frac{dt}{dx} \right)$$

$$= \frac{2b}{a} \left( 2t \cdot \frac{1}{2at} \right)$$

$$= \frac{2b}{a^2} \quad \text{Ans}$$

$$v \cdot x^2 + y^2 + 2gx + 2fy + c = 0$$

**Solution**

Diff. w.r.t x

$$2x + 2y \frac{dy}{dx} + 2g + 2f \frac{dy}{dx} = 0$$

$$2(y + f) \frac{dy}{dx} = -2(x + g)$$

$$(y + f)y_1 = -(x + g)$$

$$y_1 = -\frac{(x+g)}{y+f}$$

Diff. again

$$\frac{d}{dx} y_1 = \frac{d}{dx} \left( \frac{x+g}{y+f} \right)$$

$$y_2 = \frac{(y+f)(1) - (x+g) \frac{dy}{dx}}{(y+f)^2} (y + f)$$

$$y_2 = \frac{(y+f) - (x+g) \left( -\frac{x+g}{y+f} \right)}{(y+f)^2}$$

$$y_2 = \frac{(y+f)^2 + (x+g)^2}{(y+f)^2}$$

$$\begin{aligned}
 &= \left[ \frac{y^2 + f^2 + 2fy + x^2 + g^2 + 2xg}{(y+f)^3} \right] \\
 &= \frac{x^2 + y^2 + 2gx + 2fy + c + f^2 + g^2}{(y+f)^2} \\
 &= \frac{c - f^2 - g^2}{(y+f)^3} \quad \text{Ans}
 \end{aligned}$$

**Q4. Find  $y^4$  if**

**i.  $y = \sin 3x$**

**Solution**

$$\begin{aligned}
 y_1 &= \cos 3x \cdot 3 = 3 \cos 3x \\
 y_2 &= 2(-\sin 3x) \cdot 3 = -9 \sin 3x \\
 y_3 &= -27 \cos 3x \\
 y_4 &= -27(-\sin 3x) \cdot 3 \\
 &= 81 \sin 3x \quad \text{Ans}
 \end{aligned}$$

**ii.  $y = \cos^3 x$**

**Solution**

As  $\cos^3 x = 3 \cos x - 4 \cos^3 x$

$\Rightarrow y = 3 \cos x - 4 \cos^3 x$

$y_1 = -3 \sin x + 12 \cos^2 x \times \sin x$

$= -3 \sin x + 3 \sin^2 x \cdot \cos x$

$y_2 = -3 \cos x + 9 \sin^2 x \cdot \cos x$

$$= -3\cos x + 9(1 - \cos^2 x)\cos^4$$

$$= 6\cos x - 9\cos^3 x$$

$$y_3 = -6\sin x \cdot 27\cos^2 x(-\sin x)$$

$$= -6\sin + 27(1 - \sin^2 x)\sin x$$

$$= 21\sin - 27\sin x$$

$$y_4 = 21\cos x - 81\sin^2 x \cos x$$

$$= 21\cos x - 81(1 - \cos^2 x)\cos^4$$

$$= 21\cos x - 81\cos x + 81\cos^3 x$$

$$= -60\cos x + 81\cos^3 x \quad \text{Ans}$$

iii.  $y = \ln(x^2 - 9)$

**Solution**

$$= \ln[(x + 3)(x - 3)]$$

$$= \ln(x + 3) + \ln(x - 3)$$

$$\frac{dy}{dx} = \frac{d}{dx}[\ln(x + 3)] + \frac{d}{dx}[\ln(x - 3)]$$

$$= \frac{1}{x+3}(1 + 0) + \frac{1}{x-3}(1 - 0)$$

$$= (x + 3)^{-1}(x - 3)^{-1}$$

$$y_2 = \frac{d^2y}{dx^2} = \frac{d}{dx}[(x + 3)^{-1}] + \frac{d}{dx}[(x - 3)^{-1}]$$

$$= -(x + 3)^{-2}(1 + 0) - (x - 3)^{-2}(1 - 0)$$

$$= -(x + 3)^{-2} - (x - 3)^{-2}$$

$$y_3 = \frac{d^3y}{dx^3} = \frac{d}{dx}[-(x + 3)^{-2}] - \frac{d}{dx}[(x - 3)^{-2}]$$

$$= 2(x + 3)^{-2}(1 + 0) + 2(x - 3)^{-3}(1 - 0)$$

$$= 2(x+3)^{-3} + 2(x-3)^{-3}$$

$$y_4 = \frac{d^4 y}{dx^4} = \frac{d}{dx} [2(x+3)^{-3}] + \frac{d}{dx} [2(x-3)^{-3}]$$

$$= -6(x+3)^{-4}(1+0) - 6(x-3)^{-4}(1-0)$$

$$= \frac{-6}{(x+3)^4} - \frac{6}{(x-3)^4}$$

$$= -6 \left[ \frac{1}{(x+3)^4} + \frac{1}{(x-3)^4} \right] \quad \text{Ans}$$

**Q5.** if  $x = \sin \theta$ ,  $y = \sin m\theta$  show that  $(1-x^2)y_2 - xy_1 + m^2y = 0$

**Solution**

$$x = \sin \theta \Rightarrow \sin^{-1} x = \theta$$

$$y = \sin m\theta \Rightarrow \sin (m \sin^{-1} x)$$

Diff. both sides w.r.t  $x$

$$\frac{dy}{dx} = \frac{d}{dx} [\sin (m \sin^{-1} x)]$$

$$= \frac{m \cos(m \sin^{-1} x)}{\sqrt{1-x^2}}$$

$$(\sqrt{1-x^2})y_1 = m \cos(m \sin^{-1} x)$$

$$\Rightarrow (1-x^2)y_1^2 = m^2 \cos^2(m \sin^{-1} x)$$

$$(1-x^2)y_1^2 = m^2 [1 - \sin^2(m \sin^{-1} x)]$$

$$(1-x^2)y_1^2 = m^2 (1-y^2)$$

Diff. again w.r.t  $x$

$$(1-x^2)2y_1y_2 - (2x)(y_1^2) = m^2(-2yy_1)$$

$$(1-x^2)y_2 - xy_1 + m^2y = 0 \quad \text{Ans}$$

**Q6.** If  $y = e^x \sin x$  show that

$$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0$$

**Solution**

$$y = e^x \sin x \dots\dots\dots(1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^x \sin x)$$

$$\frac{dy}{dx} = e^x \sin x + e^x \cos x \dots\dots\dots(2)$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^x \sin x + e^x \cos x + e^x \cos x - e^x \sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2e^x \cos x$$

$$= 2 \left( \frac{dy}{dx} - e^x \sin x \right) \text{ using 2}$$

$$= 2 \frac{dy}{dx} - 2y \text{ using (1)}$$

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0 \quad \text{Ans}$$

**Q7.** If  $y = e^{ax} \sin bx$  show that  $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$

**Solution**

$$y = e^{ax} \sin bx \quad (1)$$

$$\frac{dy}{dx} = \sin bx \cdot e^{ax} \cdot a + e^{ax} \cos bx \times b$$

$$\frac{dy}{dx} = e^{ax}(a \sin bx + b \cos bx) \quad (2)$$

$$\frac{dy}{dx} = e^{ax}(a \sin bx + b \cos bx) \quad (2)$$

$$\frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} [e^{ax}(0 \sin bx + b \cos bx)]$$

$$\begin{aligned}\frac{d^2y}{dx^2} &= \left(\frac{d}{dx}e^{ax}\right)[a\sin bx + b\cos bx] \\ &= e^{ax} \frac{d}{dx}[(a\sin bx + b\cos bx)] \\ \frac{d^2y}{dx^2} &= ae^{ax}[a\sin bx + b\cos bx] \\ &= e^{ax}[ab\cos bx - b^2\sin bx] \\ &= e^{ax}[a^2\sin bx + ab\cos bx + ab\cos bx - b^2\sin bx] \\ &= e^{ax}[2ab\cos bx + 2a^2\sin bx - 2a^2\sin bx + a^2\sin bx - \\ &\quad b^2\sin bx] \\ &= 2ae^{ax}[a\sin bx + b\cos bx] - e^{ax}(a^2 + b^2)\sin bx\end{aligned}$$

$$\frac{d^2y}{dx^2} = 2a \frac{dy}{dx} - (a^2 + b^2)y$$

$$\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0 \quad \text{Ans}$$

**Q8. If  $y = (\cos^{-1}x)^2$  prove that  $(1 - x^2)y_2 - xy_1 - 2 = 0$**

**Solution**

$$y = (\cos^{-1}x)^2 \quad \dots \dots (1)$$

$$\frac{dy}{dx} = 2\cos^{-1}x \left(-\frac{1}{\sqrt{1-x^2}}\right)$$

$$-\sqrt{1-x^2} y_1 = 2\cos^{-1}x$$

Diff. again w.r.t  $x$

$$\frac{2x}{2\sqrt{1-x^2}} y_1 + \sqrt{1-x^2} y_2 = \frac{-2x}{\sqrt{1-x^2}}$$

$$\frac{-xy_1}{\sqrt{1-x^2}} + \sqrt{1-x^2} y_2 = \frac{2x}{\sqrt{1-x^2}}$$

$$\Rightarrow -xy + (1-x^2)y_2 = 2$$

$$\Rightarrow (1-x^2)y_2 - xy_1 - 2 = 0 \quad \text{Ans}$$

**Q9.** if  $y = a \cos(\ln x) + b \sin(\ln x)$

**prove that**  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$

**Solution**

$$y = a \cos(\ln x) + b \sin(\ln x) \dots\dots\dots(1)$$

$$\frac{dy}{dx} = -\frac{a \sin(\ln x)}{x} + \frac{b \cos(\ln x)}{x}$$

$$\Rightarrow x \frac{dy}{dx} = -a \sin(\ln x) + b \cos(\ln x)$$

Diff. again w.r.t we have

$$\frac{d}{dx} \left( x \frac{dy}{dx} \right) = \frac{d}{dx} (-a \sin(\ln x) + b \cos(\ln x))$$

$$\frac{dy}{dx} + x \frac{d^2 y}{dx^2} = -\frac{a \cos \ln x}{x} + \frac{-b \sin \ln x}{x}$$

$$\Rightarrow x \frac{dy}{dx} + x^2 \frac{d^2 y}{dx^2} = -[a \cos(\ln x) + b \sin(\ln x)]$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -y \quad \text{by (i)}$$

$$\Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0 \quad \text{Ans}$$

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