

## Solved Exercise 2.6

**Q1. Find  $f'(x)$  if**

**i.  $f(x) = e^{\sqrt{x}-1}$**

**Solution**

Diff. w.r.t  $x$

$$\begin{aligned} f'(x) &= e^{\sqrt{x}-1} \cdot \frac{d}{dx}(\sqrt{x}-1) \\ &= e^{\sqrt{x}-1} \cdot \frac{1}{2\sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} e^{\sqrt{x}-1} \end{aligned}$$

**ii.  $f(x) = x^3 e^{\frac{1}{x}}$**

**Solution**

$$\begin{aligned} \Rightarrow f'(x) &= e^{\frac{1}{x}}(3x^2) + x^3 e^{\frac{1}{x}} \left(-\frac{1}{x^2}\right) \\ &= 3x^2 e^{\frac{1}{x}} - x e^{\frac{1}{x}} \\ &= x e^{\frac{1}{x}}(3x - 1) \end{aligned}$$

**iii.  $f(x) = e^x(1 + \ln x)$**

**Solution**

$$\begin{aligned} \Rightarrow f'(x) &= \frac{d}{dx}[e^x(1 + \ln x)] \\ &= (1 + \ln x)e^x + e^x \frac{1}{x} \end{aligned}$$

$$= \frac{(x(1+\ln x)+1)e^x}{x}$$

iv.  $f(x) = \frac{e^x}{e^{-x}+1}$

**Solution**

$$\begin{aligned} f'(x) &= \frac{d}{dx} \left( \frac{e^x}{e^{-x}+1} \right) \\ &= \frac{(e^{-x}+1) \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(e^{-x}+1)}{(e^{-x}+1)^2} \\ &= \frac{(e^{-x}+1)(e^x - e^x(-e^{-x}+1))}{(e^{-x}+1)^2} \\ &= \frac{e^{-x}e^x + e^x + e^x e^{-x}}{(e^{-x}+1)^2} \\ &= \frac{1+e^x+1}{(e^{-x}+1)^2} = \frac{e^x+2}{(e^{-x}+1)^2} \end{aligned}$$

v.  $f(x) = \ln(e^x + e^{-x})$

**Solution**

$$\begin{aligned} f(x) &= \frac{1}{e^x + e^{-x}} [e^x + e^{-x} (1)] \\ &= \frac{e^x - e^{-x}}{e^x + e^{-x}} \\ &= \frac{e^x \frac{1}{e^x}}{e^x + \frac{1}{e^x}} \\ &= \frac{\frac{e^{2x}-1}{e^x}}{\frac{e^{2x}+1}{e^x}} \end{aligned}$$

$$f(x) = \frac{e^{2x}-1}{e^{2x}+1}$$

$$\text{vi. } f(x) = \frac{e^{ax} - e^{-ax}}{e^{ax} + e^{-ax}}$$

**Solution**

$$\begin{aligned} \Rightarrow f(x) &= \frac{(e^{ax} + e^{-ax})(e^{ax} \cdot a + e^{-ax} \cdot a) - (e^{ax} - e^{-ax})(e^{ax} \cdot a) - (e^{ax} \cdot a)}{(e^{ax} + e^{-ax})^2} \\ &= \frac{a[(e^{ax} + e^{-ax})(e^{ax} + e^{-ax}) - (e^{ax} - e^{-ax})(e^{ax} - e^{-ax})]}{(e^{ax} + e^{-ax})^2} \\ &= a \left[ \frac{(e^{2ax} + e^{-2ax} + 2) - (e^{2ax} + e^{-2ax} - 2)}{(e^{ax} + e^{-ax})^2} \right] \\ &= a \left[ \frac{4}{(e^{ax} + e^{-ax})^2} \right] \\ &= \frac{4}{(e^{ax} + e^{-ax})^2} \end{aligned}$$

$$\text{vi. } f(x) = \sqrt{\ln(e^{2x} + e^{-2x})} = [\ln(e^{2x} + e^{-2x})]^{1/2}$$

**Solution**

$$\begin{aligned} f(x) &= \frac{d}{dx} (\ln(e^{2x} + e^{-2x}))^{-1/2} \frac{d}{dx} [\ln(e^{2x} + e^{-2x})] \\ &= \frac{1}{2\sqrt{\ln(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} \cdot \frac{d}{dx} (e^{2x} + e^{-2x}) \\ &= \frac{1}{2\sqrt{\ln(e^{2x} + e^{-2x})}} \cdot \frac{1}{e^{2x} + e^{-2x}} (2e^{2x} - 2e^{-2x}) \\ &= \frac{2(e^{2x} - e^{-2x})}{2(e^{2x} + e^{-2x})\sqrt{\ln(e^{2x} + e^{-2x})}} \\ &= \frac{e^{2x} - e^{-2x}}{(e^{2x} + e^{-2x})\sqrt{\ln(e^{2x} + e^{-2x})}} \end{aligned}$$

$$\text{viii. } f(x) = \ln(\sqrt{e^{2x} + e^{-2x}})$$

**Solution**

$$\begin{aligned}\Rightarrow f(x) &= \frac{1}{\sqrt{e^{2x} + e^{-2x}}} \\ &= \frac{1}{\sqrt{e^{2x} + e^{-2x}}} \frac{d}{dx} \frac{1}{2\sqrt{e^{2x} + e^{-2x}}} (e^{2x} \cdot 2 - e^{-2x} \cdot 2) \\ &= \frac{1}{2(e^{2x} + e^{-2x})} \times 2e^{2x} - e^{-2x} \cdot 2 \\ &= \frac{2(e^{2x} - e^{-2x})}{2(e^{2x} + e^{-2x})} \\ &= \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} = \tanh 2x\end{aligned}$$

**Q2. Find  $\frac{dy}{dx}$  if**

i.  $y = x^2 \ln \sqrt{x}$

**Solution**

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} \frac{d}{dx} (x^2 \ln x) \\ &= \frac{1}{2} \left( x^2 \cdot \frac{d}{dx} + \ln x (2x) \right) \\ &= \frac{x}{2} (1 + 2 \ln x) \quad \text{Ans}\end{aligned}$$

ii.  $y = x \sqrt{\ln x}$

**Solution**

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= \frac{d}{dx} (x \sqrt{\ln x}) \\ &= x \frac{d}{dx} (\sqrt{\ln x}) + \sqrt{\ln x} \frac{d}{dx} (x) \\ &= x \frac{1}{2\sqrt{\ln x}} \cdot \frac{1}{x} + \sqrt{\ln x} (1)\end{aligned}$$

$$= \frac{1}{2\sqrt{\ln x}} + \sqrt{\ln x}$$

$$= \frac{1+2\ln x}{2\sqrt{\ln x}} \quad \text{Ans}$$

iii.  $y = \frac{x}{\ln x}$

**Solution**

$$\Rightarrow \frac{dy}{dx} = \frac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{(\ln x)^2}$$

$$= \frac{\ln x - 1}{(\ln x)^2} \quad \text{Ans}$$

iv.  $y = x^2 \cdot \ln \left( \frac{1}{x} \right)$

**Solution**

$$\Rightarrow \frac{dy}{dx} = \ln \frac{1}{x} \cdot 2x + x^2 \cdot \frac{1}{x} \left( -\frac{1}{x^2} \right)$$

$$= 2x \ln \left( \frac{1}{x} \right) - x$$

$$= x \left[ 2 \ln \left( \frac{1}{x} \right) - 1 \right] \quad \text{Ans}$$

v.  $y = \ln \sqrt{\frac{x^2-1}{x^2+1}}$

**Solution**

$$\Rightarrow y = \ln \left( \frac{x^2-1}{x^2+1} \right)^{1/2}$$

$$= \frac{1}{2} \ln \left( \frac{x^2-1}{x^2+1} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \left[ \frac{1}{\frac{x^2-1}{x^2+1}} \cdot \frac{d}{dx} \left( \frac{x^2-1}{x^2+1} \right) \right]$$

$$\begin{aligned}
 &= \frac{1}{2} \left[ \frac{x^2+1}{x^2-1} \left( \frac{2x(x^2+1) - (2x)(x^2-1)}{(x^2+1)^2} \right) \right] \\
 &= \frac{1}{2} \left[ \frac{x^2+1}{x^2-1} \frac{2x^3+2x-2x^3+2x}{(x^2+1)^2} \right] \\
 &= \frac{1}{2} \left[ \frac{4x}{(x^2-1)(x^2+1)} \right] \\
 &= \frac{2x}{x^4-1} \quad \text{Ans}
 \end{aligned}$$

vi.  $y = \ln(x + \sqrt{x^2 + 1})$

**Solution**

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{1}{x + \sqrt{x^2 + 1}} \left( 1 + \frac{1}{2\sqrt{x^2 + 1}}(2x) \right) \\
 &= \frac{1}{x + \sqrt{x^2 + 1}} \left( 1 + \frac{x}{\sqrt{x^2 + 1}} \right) \\
 &= \frac{1}{x + \sqrt{x^2 + 1}} \left[ \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}} \right] \\
 &= \frac{1}{\sqrt{x^2 + 1}} \quad \text{Ans}
 \end{aligned}$$

vii.  $y = \ln(9 - x^2)$

**Solution**

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{1}{9 - x^2} (-2x) \\
 &= \frac{-2x}{9 - x^2} \quad \text{Ans}
 \end{aligned}$$

viii.  $y = e^{-2x} \sin 2x$

**Solution**

$$\begin{aligned}
 \frac{dy}{dx} &= e^{-2x}(\cos 2x \cdot 2) + \sin 2x(e^{-2x} - 2) \\
 &= 2e^{-2x}(\cos 2x - \sin 2x) \quad \text{Ans}
 \end{aligned}$$

ix.  $y = e^{-x}(x^3 + 2x^2 + 1)$

**Solution**

$$\begin{aligned}\frac{dy}{dx} &= e^{-x} \frac{d}{dx}(x^3 + 2x^2 + 1) + (x^3 + 2x^2 + 1) \frac{d}{dx}e^{-x} \\ &= e^{-x}(3x^2 + 4x) + (x^3 + 2x^2 + 1)(-e^{-x}) \\ &= e^{-x}[3x^2 + 4x - x^3 - 2x^2 - 1] \\ &= e^{-x}(-x^3 + x^2 + 4x - 1) \\ &= e^{-x}(x^3 - x^2 - 4x + 1) \quad \text{Ans}\end{aligned}$$

x.  $y = xe^{\sin x}$

**Solution**

$$\begin{aligned}\frac{dy}{dx} &= x \frac{d}{dx}(e^{\sin x}) + e^{\sin x} \frac{d}{dx}(x) \\ &= xe^{\sin x} \cdot \cos x + e^{\sin x} - 1 \\ &= (x \cos x + 1)e^{\sin x} \quad \text{Ans}\end{aligned}$$

xi.  $y = 5e^{3x-4}$

**Solution**

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(5e^{3x-4}) \\ &= 5 \frac{d}{dx}(e^{3x-4}) \\ &= 5e^{3x-4} \frac{d}{dx}(3x - 4) \\ &= 5e^{3x-4}(3) \\ \frac{dy}{dx} &= 15e^{3x-4} \quad \text{Ans}\end{aligned}$$

**xii.  $y = (x - 1)^2$**

**Solution**

$$\ln y = \ln(x + 1)^2$$

$$\ln y = x \ln(x + 1)$$

Diff.w.r.t. x

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx}(x \cdot \ln(x + 1))$$

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{x+1} + \ln(x + 1) \cdot 1$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{x}{x+1} + \ln(x + 1) \cdot 1$$

$$\frac{dy}{dx} = y \left[ \frac{x}{x+1} + \ln(x + 1) \right]$$

As  $y = (x + 1)$

So,  $\frac{dy}{dx} = (x + 1) \left[ \left( \frac{x}{x+1} \right) + \ln(x + 1) \right]$  **Ans**

**xiii.  $y = (\ln x)^{\ln x}$**

**Solution**

Taking in both sides

$$\ln y = \ln(\ln x)^{\ln x}$$

$$\ln y = \ln x \cdot \ln(\ln x)$$

Diff.w.r.t. x

$$\frac{1}{y} \frac{dy}{dx} = \ln x \cdot \frac{1}{\ln x} + \frac{1}{x} + \frac{1}{x} (\ln(\ln x))$$

$$= \frac{1}{x} + \frac{\ln(\ln x)}{x}$$

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \frac{\ln(\ln x) + 1}{x} \\ \frac{dy}{dx} &= y \left[ \frac{\ln(\ln x) + 1}{x} \right] \\ &= (\ln x)^{\ln x} \left[ \frac{\ln(\ln x) + 1}{x} \right] \quad \text{Ans}\end{aligned}$$

xiv.  $y = \frac{(\sqrt{x^2-1})(x+1)}{(x^3+1)^{1/2}}$

**Solution**

$$\begin{aligned}y &= \frac{\sqrt{(x+1)}(\sqrt{x-1})(x+1)}{(x+1)^{3/2}(x^2-x+1)^{3/2}} \\ &= \frac{(x-1)^{1/2}(x+1)^{3/2}}{(x+1)^{3/2}(x^2-x+1)^{3/2}}\end{aligned}$$

$$y = \frac{(x-1)^{1/2}}{(x^2-x+1)^{3/2}} \dots\dots\dots (1)$$

Taking in both sides

$$\begin{aligned}\ln y &= \ln \left[ \frac{(x-1)^{1/2}}{(x^2-x+1)^{3/2}} \right] \\ &= \ln(x-1)^{1/2} - \ln(x^2-x+1)^{3/2} \\ &= \frac{1}{2} \ln(x-1) - \frac{3}{2} \ln(x^2-x+1)\end{aligned}$$

Diff w.r.t x

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= \frac{1}{2} - \frac{3}{2} \frac{(2x-1)}{x^2-x+1} \\ &= \frac{1}{2} \left[ \frac{1}{x-1} - \frac{3(2x-1)}{x^2-x+1} \right] \\ &= \frac{1}{2} \left[ \frac{-x^2-x+1-3(2x-1)(x-1)}{(x-1)(x^2-x+1)} \right]\end{aligned}$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{2} \left[ \frac{-x^2-x+1-3(2x^2-3x+1)}{(x-1)(x^2-x+1)} \right]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{y}{2} \left[ \frac{x^2 - x + 1 - 6x^2 - 9x - 3}{(x-1)(x^2 - x + 1)} \right]$$

From above

$$= \left( \frac{\sqrt{x-1} - 5x^2 + 8x - 2}{2(x^2 - x + 1)^{\frac{3}{2}}(x-1)(x^2 - x + 1)} \right)$$

$$= - \frac{+5x^2 - 8x + 2}{2(x^2 - x + 1)^{\frac{3}{2}}\sqrt{x-1}} \quad \text{Ans}$$

**Q3. Find  $\frac{dy}{dx}$  if**

**1.  $y = (\cos h 2x)$**

**Solution**

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} (\cos h 2x)$$

$$= \sin h 2x \cdot 2$$

$$= 2 \sin h 2x \quad \text{Ans}$$

**ii.  $y = \sin h 3x$**

**Solution**

$$\Rightarrow \frac{dy}{dx} = \cos h 3x \cdot 3$$

$$= 3 \cos h 3x \quad \text{Ans}$$

**iii.  $y = \tan h^{-1}(\sin x)$**

**Solution**

$$\Rightarrow \tan hy = \sin x$$

Diff w.r.t x we have

$$\frac{d}{dx}(\tan hy) = \frac{d}{dx} \sin x$$

$$\sec h^2 y \frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{\cos x}{\sec h^2 y}$$

$$= \frac{\cos x}{1 - \tan h^2 y} = \frac{\cos x}{1 - \sin^2 x}$$

$$= \frac{\cos x}{\cos^2 x} = \frac{1}{\cos x}$$

$$= \sec x \quad \text{Ans}$$

iv.  $y = \sin h^{-1}(x^3)$

**Solution**

$$\Rightarrow \sin hy = x^3 \dots\dots\dots(1)$$

$$\frac{d}{dx}(\sin hy) = \frac{d}{dx}(x^3)$$

$$\cos hy \frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = 3x^2 \times \frac{1}{\cos hy}$$

$$= \frac{3x^2}{\sqrt{1 + \sin h^2 y}} = \frac{3x^2}{\sqrt{1 + (x^3)^2}}$$

$$= \frac{3x^2}{\sqrt{1 + x^6}} \quad \text{Ans}$$

v.  $y = \ln(\tan hx)$

**Solution**

$$\frac{dy}{dx} = \frac{1}{\tan hx} \frac{d}{dx} (\tan hx)$$

$$= \frac{1}{\tan hx} \sec^2 hx$$

$$= \frac{\cos hx}{\sin hx} \frac{1}{\cos^2 hx}$$

$$= \frac{1}{\sin x \cdot \cos hx}$$

$$= \frac{2}{2 \sin x \cdot \cos hx}$$

$$= \frac{2}{\sinh 2x}$$

$$= 2 \operatorname{cosec} h 2x$$

**Ans**

vi.  $y = \sinh^{-1} \left( \frac{x}{2} \right)$

**Solution**

$$y = \sinh^{-1} \left( \frac{x}{2} \right)$$

$$\Rightarrow \sinh y = \frac{x}{2} \quad (1)$$

Diff w.r.t x both sides

$$\frac{d}{dx} (\sinh y) = \frac{d}{dx} \left( \frac{x}{2} \right)$$

$$\cosh y \frac{dy}{dx} = \frac{1}{2} \quad (2)$$

$$\Rightarrow = \sqrt{1 + \sinh^2 y} \frac{dy}{dx} = \frac{1}{2}$$

$$\Rightarrow = \sqrt{1 + \left( \frac{x}{2} \right)^2} \frac{dy}{dx} = \frac{1}{2}$$

$$= \sqrt{\frac{4+x^2}{4}} \quad \frac{dy}{dx} = \frac{1}{2}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\frac{4+x^2}{2}}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{4+x^2}} \quad \text{Ans}$$

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