

Solved Exercise 2.3

- Differential w.r.t x

Q1. $x^4 + 2x^3 + x^2$

Solution

Let $Y = x^4 + 2x^3 + x^2$

Diff w.r.t x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [x^4 + 2x^3 + x^2] \\ &= \frac{d}{dx} x^4 + 2 \frac{d}{dx} x^3 + \frac{d}{dx} x^2 \\ &= 4x^3 + 6x^2 + 2x \quad \text{Ans}\end{aligned}$$

Q2. $x^{-3} + 2x^{\frac{3}{2}} + 3$

Solution

Let $Y = x^{-3} + 2x^{\frac{3}{2}} + 3x^{-3} + 2x^{\frac{3}{2}} + 3$

Diff.w.r.t. x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [x^{-3} + 2x^{\frac{3}{2}} + 3] \\ &= \frac{d}{dx} x^{-3} + 2 \frac{d}{dx} x^{-3/2} + \frac{d}{dx} (3) \\ &= -3x^{-3-1} + 2 \left(\frac{-3}{2} \right) x^{-5/2} + 0 \\ &= -3x^{-4} - 3x^{\frac{5}{2}} \\ &= \frac{-3}{x^4} - \frac{3}{x^{1/2}} = -3 \left[\frac{1}{x^4} + \frac{1}{x^{5/2}} \right] \quad \text{Ans}\end{aligned}$$

Q3. $\frac{a+x}{a-x}$

Solution

Let $Y = \frac{a+x}{a-x}$

Diff Both Sides w.r.t x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{a+x}{a-x} \right) \\ &= \frac{(1)(a-x) - (1)(a+x)}{(a-x)^2} \\ &= \frac{a-x+a+x}{(a-x)^2} \\ &= \frac{2a}{(a-x)^2}\end{aligned}$$

Ans

Q4. $\frac{2x-3}{2x+1}$

Solution

Let $Y = \frac{2x-3}{2x+1}$

Diff.w.r.t. x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{2x-3}{2x+1} \right) \\ &= \frac{(2+1) \frac{d}{dx}(2x-3) - (2x-3) \frac{d}{dx}(2x+1)}{(2x+1)^2} \\ &= \frac{(2x+1)(2) - (2x-3)(2)}{(2x+1)^2} \\ &= \frac{4x+2-4x+6}{(2x+1)^2} \\ &= \frac{8}{(2x+1)^2}\end{aligned}$$

Ans

Q5. $(x-5)(3-x)$

Solution

$$\text{Let } Y = (x-5)(3-x)$$

Diff.w.r.t. x

$$\frac{dy}{dx} = (x-5) \frac{d}{dx}(3-x) + (3-x) \frac{d}{dx}(x-5)$$

$$= (x-5)(-1) + (3-x)(1)$$

$$= -x + 5 + 3 - x = -2x + 8 \quad \text{Ans}$$

Q6. $(\sqrt{x} - \frac{1}{\sqrt{x}})^2$

Solution

$$\text{Let } Y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2$$

$$Y = (\sqrt{x})^2 + \left(\frac{1}{\sqrt{x}}\right)^2 - 2\sqrt{x} \frac{1}{\sqrt{x}}$$

$$= x + \frac{1}{x} - 2$$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}\left(\frac{1}{x}\right) - \frac{d}{dx}(2)$$

$$= 1 - \frac{1}{x^2} - 0$$

$$= \frac{x^2-1}{x^2} \quad \text{Ans}$$

Q7. $\frac{(1+\sqrt{x})(x-x^{\frac{5}{2}})}{\sqrt{x}}$

Solution

$$\begin{aligned}\text{Let } y &= \frac{(a+\sqrt{x})x(1-\sqrt{x})}{\sqrt{x}} \\ &= \sqrt{x}(1+\sqrt{x})(1-\sqrt{x}) \\ &= \sqrt{x}(1-x) \\ &= (\sqrt{x}-x)^{\frac{5}{2}} = \sqrt{x} - x\sqrt{x} \\ &= x^{1/2} - x^{3/2}\end{aligned}$$

Diff. w.r.t x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} (x^{1/2} - x^{3/2}) \\ &= \frac{1}{2\sqrt{x}} - \frac{3}{2}x^{\frac{1}{2}} \\ &= \frac{1}{2} \left(\frac{1-3x}{\sqrt{x}} \right) \\ &= \frac{1-3x}{2\sqrt{x}}\end{aligned}$$

Q8. $\frac{(x^2+1)^2}{x^2-1}$

Solution

Let $y = \frac{(x^2-1)^2}{x^2-1}$

Diff. w.r.t x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{(x^2+1)^2}{x^2-1} \right) \\ &= \frac{(x^2-1) \frac{d}{dx} (x^2+1)^2 - (x^2+1)^2 \frac{d}{dx} (x^2-1)}{(x^2-1)^2} \\ &= \frac{(x^2-1)2(x^2+1)(2x) - (x^2+1)^2(2x)}{(x^2-1)^2}\end{aligned}$$

$$= \frac{4x(x^4-1)-2x(x^4+2x^2+1)}{(x^2-1)^2}$$

$$\frac{dy}{dx} = \frac{2x(x^2+1)(x^2+3)}{(x^2-1)^2}$$

Q9. $\frac{x^2+1}{x^2-3}$

Solution

Let $y = \frac{x^2+1}{x^2+3}$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{x^2+1}{x^2+3} \right)$$

$$\frac{dy}{dx} = \frac{2x(x^2-3)-(2x)(x^2-1)}{(x^2-3)^2}$$

$$= \frac{2x[x^2-3-x^2-1]}{(x^2-3)^2}$$

$$= \frac{2x(-4)}{(x^2-3)^2}$$

$$= \frac{-8x}{(x^2-3)^2}$$

Ans

Q10. $\frac{\sqrt{1+x}}{\sqrt{1-x}}$

Solution

Let $y = \frac{\sqrt{1+x}}{\sqrt{1-x}}$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\sqrt{1+x}}{\sqrt{1-x}} \right)$$

$$= \frac{(\sqrt{1-x}) \left[\frac{1}{2\sqrt{1+x}} - \sqrt{1+x} \frac{1}{2\sqrt{1-x}} \right]}{(1+x)}$$

$$= \frac{1}{1-x} \frac{(1-x) + (1+x)}{2\sqrt{(1+x)(1-x)}}$$

$$= \frac{(1-x)^{1/2} + (1+x)^{1/2}}{2(1+x)^{1/2} \cdot 2(1-x)^{1/2}}$$

$$= \frac{1-x+1+x}{2(1+x)^{1/2}(1-x)^{1/2}(1-x)}$$

$$= \frac{2}{2\sqrt{1+x}(1-x)^{1/2}}$$

$$= \frac{1}{\sqrt{1+x}(1-x)^{1/2}} \quad \text{Ans}$$

Q11. $\frac{2x-1}{\sqrt{x^2+1}}$

Solution

Let $Y = \frac{2x-1}{\sqrt{x^2+1}}$

Diff.w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{2x-1}{\sqrt{x^2+1}} \right)$$

$$\frac{dy}{dx} = \frac{2(\sqrt{x^2+1}) - (2x-1) \frac{2x}{2\sqrt{x^2+1}}}{(\sqrt{x^2+1})^2}$$

$$= \frac{2x^2+1 - \frac{(2x-1) \cdot x}{\sqrt{x^2+1}}}{(x^2+1)}$$

$$\frac{dy}{dx} = \frac{2(x^2+1) - x(2x-1)}{(\sqrt{x^2+1})(x^2+1)}$$

$$= \frac{2x^2+2-2x^2+x}{(x^2+1)^{1/2}(x^2+1)}$$

$$\frac{dy}{dx} = \frac{x+2}{(x^2+1)^{3/2}} \quad \text{Ans}$$

Q12. $\sqrt{\frac{a-x}{a+x}}$

Solution

Let $Y = \sqrt{\frac{a-x}{a+x}}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left[\frac{(a-x)^{1/2}}{(a+x)^{1/2}} \right] \\ &= \frac{(a+x)^{1/2} \frac{d}{dx}(a-x)^{1/2} - (a-x)^{1/2} \frac{d}{dx}(a+x)^{1/2}}{[(a+x)^{1/2}]^2} \\ &= \frac{(a+x)^{1/2} \cdot \frac{1}{2}(a-x)^{-1/2} \cdot (-1) - (a-x)^{1/2} \cdot \frac{1}{2}(a+x)^{-1/2} \cdot (1)}{a+x} \\ &= \frac{\frac{(a+x)^{1/2} (a-x)^{-1/2}}{2} - \frac{(a-x)^{1/2} (a+x)^{-1/2}}{2}}{a+x} \\ &= \frac{\frac{(a+x)^{1/2} (a-x)^{-1/2} (a-x)^{1/2} (a+x)^{1/2}}{2(a-x)^{1/2} 2(a+x)^{1/2}}}{a+x} \\ &= \frac{\frac{(a+x)^{1/2} (a+x)^{1/2} (a-x)^{1/2} (a-x)^{1/2}}{2(a-x)^{1/2} (a+x)^{1/2}}}{a+x} \\ &= \frac{(a+x) - (a-x)}{2(a-x)^{1/2} (a+x)^{1/2} (a+x)} \\ &= \frac{a+x-a+x}{2\sqrt{a-x}(a+x)^{3/2}} \\ &= \frac{x}{\sqrt{(a-x)(a+x)^{3/2}}}\end{aligned}$$

Q13. $\frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

Solution

Let $Y = \frac{\sqrt{x^2+1}}{\sqrt{x^2-1}}$

Diff. w.r.t x

$$\begin{aligned}\frac{dy}{dx} &= \frac{\left(\frac{1}{2}\right)(x^2+1)^{-1/2} 2x(\sqrt{x^2-1}) - \frac{1}{2}(x^2-1)^{-1/2} 2x\sqrt{x^2+1}}{(\sqrt{x^2-1})^2} \\ &= \frac{\frac{x(x^2-1)}{(\sqrt{x^2+1})(\sqrt{x^2-1})} - \frac{x(x^2+1)}{(\sqrt{x^2-1})(\sqrt{x^2+1})}}{x^2-1} \\ &= \frac{x^3-x^3-x-x}{(x^2-1)^{3/2}\sqrt{x^2-1}} \\ &= \frac{-2x}{(x^2-1)^{3/2}\sqrt{x^2-1}} \quad \text{Ans}\end{aligned}$$

Q14. $\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

Solution

Let $Y = \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$$\begin{aligned}Y &= \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} \times \frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}} \\ &= \frac{(\sqrt{1+x}-\sqrt{1-x})^2}{(\sqrt{1+x})^2 - (\sqrt{1-x})^2} \\ &= \frac{1+x+1-x-2(\sqrt{1+x})(\sqrt{1-x})}{1+x-1-x} \\ &= \frac{2-2\sqrt{1-x^2}}{2x} \\ &= \frac{1-\sqrt{1-x^2}}{x}\end{aligned}$$

Diff.w.r.t. x

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \left(\frac{1-\sqrt{1-x^2}}{x} \right) \\ &= \frac{\frac{1}{2}(1-x^2)^{-1/2}(-2x) - (1-\sqrt{1-x^2}) \times 1}{(x)^2}\end{aligned}$$

$$\begin{aligned}
 &= \frac{\frac{x^2}{\sqrt{1-x^2}} - (1 - \sqrt{1-x^2})}{(x)^2} \\
 &= \frac{\frac{x^2}{\sqrt{1-x^2}} - 1 + \sqrt{1-x^2}}{(x)^2} \\
 &= \frac{\frac{x^2 - \sqrt{1-x^2} + 1 - x^2}{x^2 \sqrt{1-x^2}}}{(x)^2} \\
 &= \frac{1 - \sqrt{1-x^2}}{x^2 \sqrt{1-x^2}}
 \end{aligned}$$

Ans

Q15. $\frac{x\sqrt{a+x}}{\sqrt{a-x}}$

Solution

Let $y = \frac{x\sqrt{a+x}}{\sqrt{a-x}}$

Diff. both sides w.r.t x

$$\begin{aligned}
 \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{x\sqrt{a+x}}{\sqrt{a-x}} \right) \\
 &= \frac{x\sqrt{a+x}}{\sqrt{a-x}} \frac{d}{dx} (x) + x \frac{d}{dx} \left(\frac{\sqrt{a+x}}{\sqrt{a-x}} \right) \\
 &= \frac{\sqrt{a+x}}{\sqrt{a-x}} \times 1 + x \left[\frac{a-x \cdot \frac{1}{2\sqrt{a+x}} - \frac{\sqrt{a+x} \cdot (-1)}{2\sqrt{a-x}}}{(\sqrt{a-x})^2} \right] \\
 &= \frac{\sqrt{a+x}}{\sqrt{a-x}} + x \left[\frac{(a-x)(a+x)}{2(\sqrt{a-x})(\sqrt{a+x})(a-x)} \right] \\
 &= \frac{\sqrt{a+x}}{\sqrt{a-x}} + x \left[\frac{2a}{2(\sqrt{a-x})(\sqrt{a+x})(a-x)} \right] \\
 &= \frac{\sqrt{a+x}}{\sqrt{a-x}} + \frac{ax}{2(\sqrt{a-x})(\sqrt{a+x})(a-x)} \\
 &= \frac{(a+x)(a-x) + xa}{(\sqrt{a+x})(a-x)^{\frac{3}{2}}}
 \end{aligned}$$

$$= \frac{a^2 - x^2 + ax}{(\sqrt{a+x})(a-x)^{\frac{3}{2}}}$$

Q16. If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, which shows that $2x \frac{dy}{dx} + y = 2\sqrt{x}$

Solution

$$y = \sqrt{x} - \frac{1}{\sqrt{x}}$$

Diff. w.r.t x

$$\frac{dy}{dx} = \frac{d}{dx} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)$$

$$= \frac{1}{2} x^{-\frac{1}{2}} - \frac{d}{dx} x^{-\frac{1}{2}}$$

$$= \frac{1}{2} x^{-\frac{1}{2}} - \left(-\frac{1}{2} x^{-\frac{3}{2}} \right)$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2} x^{\frac{3}{2}}$$

$$= \frac{1}{2} x^{\frac{1}{2}} + \frac{1}{2} x^{\frac{3}{2}}$$

$$2 \frac{dy}{dx} = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x} \cdot x}$$

$$2x \frac{dy}{dx} = \frac{x}{\sqrt{x}} + \frac{x}{\sqrt{x} \cdot x}$$

$$2x \frac{dy}{dx} = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$2x \frac{dy}{dx} + y = \sqrt{x} + \frac{1}{\sqrt{x}} + \sqrt{x} - \frac{1}{\sqrt{x}}$$

$$2x \frac{dy}{dx} = 2\sqrt{x} \quad \text{Hence proved}$$

Q17. If $y = x^4 + 2x^2 + 2$

Prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$

Solution

$$y = x^4 + 2x^2 + 2$$

$$y = x^4 + 2x^2 + 1 + 1$$

$$\Rightarrow y - 1 = x^4 + 2x^2 + 1$$

$$y-1=(x^2+1)^2$$

Taking square root Both sides

$$\sqrt{y-1} = x^2 + 1 \quad (1)$$

$$\text{Now } y = x^4 + 2x^2 + 2 \quad (2)$$

Diff. both sides w.r.t x $4x^3 + 4x^2 = \frac{dy}{dx}$

$$\frac{dy}{dx} = 4x(x^2 + 1) \quad (3)$$

From (1) and (3)

$$\frac{dy}{dx} = 4x\sqrt{y-1}$$

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