

Solved Exercise 2.2

Q1. Find from the first principles, the derivative, of the following expressions w.r.t their respective independent variables.

i. $(ax + b)^3$

Solution

Let $y = (ax + b)^3$

$$y + \delta y = (a(x + \delta x) + b)^3$$

$$\delta y = [a(x + \delta x) + b]^3 - (ax + b)^3$$

$$= ((ax + b) + a\delta x)^3 - (ax + b)^3$$

$$= (ax + b)^3 + 3(ax + b)^2(a\delta x) + 3(ax + b)(a^2\delta x^2) + a^3\delta x^3$$

$$= a\delta x[3(ax + b)^2 + 3a\delta x(ax + b) + a^2\delta x^2]$$

dividing both sides by δx and take limit as $\delta x \rightarrow 0$

$$\Rightarrow \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} = \lim_{\delta x \rightarrow 0} (ax + b)^3 \left[\frac{3a}{ax + b} + \frac{3a^2\delta x}{(ax + b)^2} + \dots \right]$$

$$= \lim_{\delta x \rightarrow 0} \frac{\delta y}{\delta x} a[3(ax + b)^2 + 3a\delta x(ax + b) + a^2\delta x^2]$$

$$\frac{dy}{dx} = 3(ax + b)^2 a$$

$$= 3a(ax + b)^2 \quad \text{Ans}$$

ii. $(2x + 3)^5$

Solution

Let $y = (2x + 3)^5$

$$y + \delta y = [2(x + \delta x) + 3]^5$$

$$\begin{aligned}\Rightarrow \delta y &= (2(x + \delta x) + 3)^5 - (2x + 3)^5 \\ &= (2x + 3 + 2\delta x)^5 - (2x + 3)^5 \\ \text{Using Binominal theorem} \\ &= (2x + 3)^5 + 5(2x + 3)(2\delta x)^4 - (2x + 3)^5 \\ &= 2\delta x[5(2x + 3)^4 + 10(2x + 3)^3(2\delta x) + 10(2x + 3)^2(2\delta x)^2 + 5(2x + 3)(2\delta x)^3 + (2\delta x)^4]\end{aligned}$$

Dividing both sides by δx and take limit as $\delta x \rightarrow 0$

$$\begin{aligned}\Rightarrow \delta x \rightarrow 0 \frac{\delta y}{\delta x} \\ &= \delta x \rightarrow 0[5(2x + 3)^4 + 10(2x + 3)^3(2\delta x) + 10(2x + 3)^2(2\delta x)^2 + 5(2x + 3)(2\delta x)^3 + (2\delta x)^4] \\ \Rightarrow \frac{dy}{dx} &= (2x + 3)^5 \left[\frac{10}{2x+3} \right] \\ &= 10(2x + 3)^4 \quad \text{Ans}\end{aligned}$$

iii. $(3t + 2)^{-2}$

Solution

Let $y = (3t + 2)^{-2}$

$$Y + \delta y = (3(t + \delta t) + 2)^{-2}$$

$$\delta y = (3(t + \delta t) + 2)^{-2} - (3t + 2)^{-2}$$

$$= (3t + 3\delta t + 2)^{-2} - (3t + 2)^{-2}$$

$$= -(3t + 2)^{-2} \left[\left\{ 1 + (-2) \left(\frac{\delta t}{3t+2} \right) + \frac{(-2)(-2-1)}{2!} \left(\frac{3\delta t}{3t+2} \right)^2 + \dots - 1 \right\} \right]$$

dividing both sides by δt and take limit as $\delta t \rightarrow 0$

$$\delta t \rightarrow 0 \frac{\delta y}{\delta t}$$

$$\begin{aligned}
 &= (3t+2)^{-2} \left[\left(\frac{(-2)(3)}{3t+2} \right) + \frac{(-2)(-2-1)}{2!} \left(\frac{(3)\delta t}{(3t+2)} \right) + \dots \right] \\
 &= (3t+2)^{-2} \left(\frac{-2(3)}{3t+2} \right) \\
 &= -2(3)(3t+2)^{-3} \\
 &= -6(3t+2)^{-3} \quad \text{Ans}
 \end{aligned}$$

iv. $(ax+b)^{-5}$

Solution

Let $y = (ax+b)^{-5}$

$$y + \delta y = [a(x + \delta x) + b]^{-5} - (ax+b)^{-5}$$

$$\Rightarrow \delta y = (a(x + \delta x) + b)^{-5} - (ax+b)^{-5}$$

$$= (ax+b+a\delta x)^{-5} - (ax+b)^{-5}$$

$$= (ax+b)^{-5} \left[1 + \frac{a\delta x}{ax+b} \right] - (ax+b)^{-5}$$

$$= (ax+b)^{-5} \left\{ \left(1 + \frac{a\delta x}{ax+b} \right)^{-5} - 1 \right\}$$

Using Binomial series

$$= (ax+b)^{-5} \left[1 + (-5) \left(\frac{a\delta x}{ax+b} \right) + \frac{(-5)(-5-1)a^2\delta x^2}{(ax+b)^2} \dots - 1 \right]$$

Taking limit $\delta x \rightarrow 0$

$$= -5(ax+b)^{-5} \frac{a}{ax+b}$$

$$= -5a(ax+b)^{-6}$$

$$= \frac{-5a}{(ax+b)^6} \quad \text{Ans}$$

v. $y = \frac{1}{(ax+b)^7}$

Solution

$$\text{let } y = (az - b)^{-7}$$

$$y + \delta y = (a(z + \delta z) - b)^{-7}$$

$$\Rightarrow \delta y = (az - b + a\delta z)^{-7} - (az - b)^{-7}$$

$$= (az - b)^{-7} \left(\frac{1 + a\delta z}{az - b} \right)^{-7} - (az - b)^{-7}$$

$$= (az - b)^{-7} \left[\left(1 + \frac{a\delta z}{az - b} \right)^{-7} - 1 \right]$$

Using Binomial series

$$= (az - b)^{-7} \left[1 + (-7) \left(\frac{a\delta z}{az - b} \right) + \frac{(-7)(-7-1)}{2!} \left(\frac{a\delta z}{az - b} \right)^2 \dots - 1 \right]$$

Dividing both sides by δz and take limit as $\delta z \rightarrow 0$

$$\lim_{\delta z \rightarrow 0} \frac{\delta y}{\delta z} = \lim_{\delta z \rightarrow 0} (az - b)^{-7} \left[-7 \left(\frac{a}{az - b} \right) + \frac{(-7)(-7-1)}{2!} \frac{a^2 \delta z}{(az - b)^2} + \dots \right]$$

$$\frac{dy}{dz} = \frac{-7a}{(az - b)} (az - b)^{-7}$$

$$= \frac{-7a}{(az - b)^8} \quad \text{Ans}$$

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com

Perfect24u.com