SOME IMPORTANT POINTS OF CHAPTER NO: 11

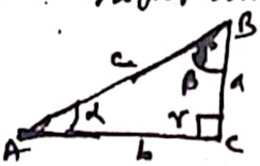
The following points will enable you to solve the problems of this chapter.

(i) A triangle has six elements, 3 sides and 3 angles.

(ii). Solving a triangle means, we will be given 3 elements (in which at least one must be a side) and we will find the remaining 3 elements.

(iii) Types of triangles

right triangle



Having one angle of 90°.

Use: the six trigonometric identities & pythagoras theorem to solve it.

$\sin\theta = \frac{P}{H} = \frac{a}{c}$ | $\csc\theta = \frac{H}{P} = \frac{c}{a}$

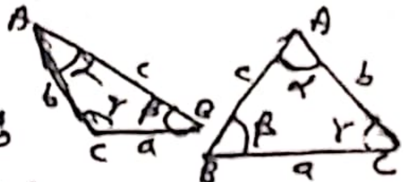
$\cos\theta = \frac{B}{H} = \frac{b}{c}$ | $\sec\theta = \frac{H}{B} = \frac{c}{b}$

$\tan\theta = \frac{P}{B} = \frac{a}{b}$ | $\cot\theta = \frac{B}{P} = \frac{b}{a}$

Trick to remember these Identities

Some people have curly brown hair through proper brushing

Oblique



triangle having no angle of 90°

To solve oblique triangle, Use

(I) The law of cosine (Cosine-Law)

(i) $a^2 = b^2 + c^2 - 2bc\cos\alpha$

(ii) $b^2 = a^2 + c^2 - 2ac\cos\beta$

(iii) $c^2 = a^2 + b^2 - 2ab\cos\gamma$

when two sides & included angle are known

From (i), (ii), (iii), we get

(iv) $\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$

(v) $\cos\beta = \frac{a^2 + c^2 - b^2}{2ac}$

(vi) $\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$

when all the three sides are given

(II) The Law of sines (Sine-Law)

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

{when {two angles & one side or two sides & an angle opposite to one of them are known}

(III) The Law of tangents

(i) $\frac{a+b}{a-b} = \frac{\tan\frac{1}{2}(\alpha+\beta)}{\tan\frac{1}{2}(\alpha-\beta)}$ (ii) $\frac{b+c}{b-c} = \frac{\tan\frac{1}{2}(\beta+\gamma)}{\tan\frac{1}{2}(\beta-\gamma)}$

(iii) $\frac{c+a}{c-a} = \frac{\tan\frac{1}{2}(\gamma+\alpha)}{\tan\frac{1}{2}(\gamma-\alpha)}$ {when two sides & included angle are known}
(وقتِ استعمال)

(IV) Half angle formulas

$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$, $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$

where $s = \frac{a+b+c}{2}$ {when all the three sides are known}

$\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{ac}}$, $\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

$\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $\tan\frac{\beta}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$, $\tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

(iv) Area of triangle

(i) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$ (Hero Formula or Heron Formula)

(ii) $\Delta = \frac{1}{2}ab\sin\gamma = \frac{1}{2}ac\sin\beta = \frac{1}{2}bc\sin\alpha$ {when two sides & Included angle are known}

(iii) $\Delta = \frac{1}{2}a^2\frac{\sin\beta\sin\gamma}{\sin\alpha} = \frac{1}{2}b^2\frac{\sin\alpha\sin\gamma}{\sin\beta} = \frac{1}{2}c^2\frac{\sin\alpha\sin\beta}{\sin\gamma}$ {when one side & two angles are known}

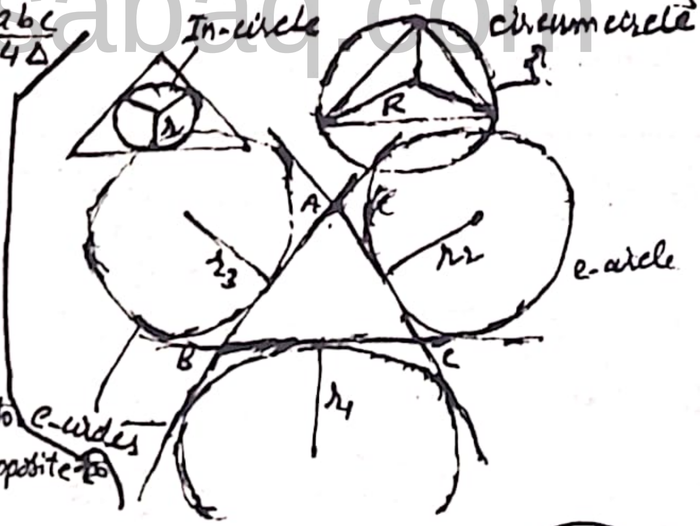
(V) circum-radius: $R = \frac{abc}{4\Delta}$

(VI) In-radius: $r = \frac{\Delta}{s}$

(VII) e-radii: $r_1 = \frac{\Delta}{s-a}$

$r_2 = \frac{\Delta}{s-b}$

$r_3 = \frac{\Delta}{s-c}$

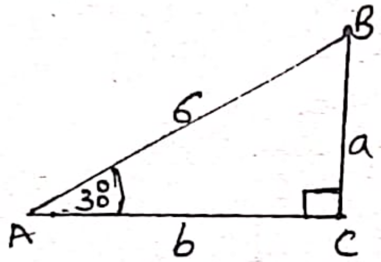


(viii) Smallest angle opposite to smallest side and Largest angle is opposite to Largest side.

EXERCISE 11.1

Q1: Solve the following right triangles.

(i)



Sol: Here $\alpha = 30^\circ, \gamma = 90^\circ, c = 6$

For β

$\beta = 180^\circ - (\alpha + \gamma) = 180^\circ - (30^\circ + 90^\circ)$

$= 180^\circ - 120^\circ = \boxed{60^\circ}$

For a

$\sin 30^\circ = \frac{a}{6}$

$\Rightarrow \frac{1}{2} = \frac{a}{6} \Rightarrow \boxed{a = 3}$

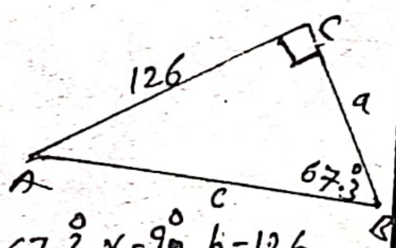
For b

$\cos 30^\circ = \frac{b}{6}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{b}{6} \Rightarrow b = \frac{6\sqrt{3}}{2} = \boxed{3\sqrt{3}}$

Hence $\boxed{a = 3,\ b = 3\sqrt{3},\ \beta = 60^\circ}$ Ans

(ii)



Sol: Here $\beta = 67.3^\circ, \gamma = 90^\circ, b = 126$

For α

$\alpha = 180^\circ - (\beta + \gamma)$

$= 180^\circ - (67.3^\circ + 90^\circ)$

$= 180^\circ - 67.3^\circ - 90^\circ$

$= 90^\circ - 67.3^\circ = \boxed{22.7^\circ}$

For a

$\tan 67.3^\circ = \frac{126}{a}$

$a = \frac{126}{\tan 67.3^\circ} = \boxed{52.7}$

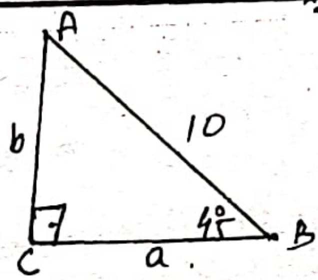
For c

$\sin 67.3^\circ = \frac{126}{c}$

$c = \frac{126}{\sin 67.3^\circ} = 136.6$

Hence $\boxed{a = 52.7,\ c = 136.6,\ \alpha = 22.7^\circ}$ Ans

(iii)



Sol: Here $\beta = 45^\circ, \gamma = 90^\circ, c = 10$

For a

$$\cos 45^\circ = \frac{a}{10}$$
$$\Rightarrow a = 10\cos 45^\circ$$
$$= 10\left(\frac{1}{\sqrt{2}}\right) = \boxed{5\sqrt{2}}$$

For b

$$\sin 45^\circ = \frac{b}{10}$$
$$\Rightarrow b = 10\sin 45^\circ$$
$$= 10\left(\frac{1}{\sqrt{2}}\right) = \boxed{5\sqrt{2}}$$

For α

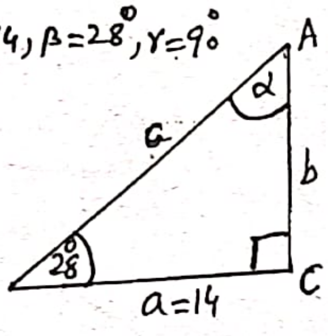
$$\alpha = 180^\circ - (\beta + \gamma)$$
$$= 180^\circ - (45^\circ + 90^\circ) = \boxed{45^\circ}$$

Hence $\boxed{\alpha = 45^\circ, a = 5\sqrt{2}, b = 5\sqrt{2}}$ Ans

Q.2: Solve the right triangles ABC in which $\gamma = 90^\circ$ and

(i) $a = 14, \beta = 28^\circ$

Sol: Here $a = 14, \beta = 28^\circ, \gamma = 90^\circ$



For b

$$\tan 28^\circ = \frac{b}{14}$$
$$\Rightarrow b = 14\tan 28^\circ$$
$$= \boxed{7.44}$$

For c

$$\cos 28^\circ = \frac{14}{c} \Rightarrow c = \frac{14}{\cos 28^\circ} = \boxed{15.86}$$

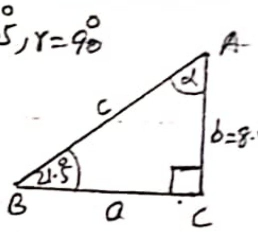
For α

$$\alpha = 180^\circ - (\beta + \gamma)$$
$$= 180^\circ - (28^\circ + 90^\circ)$$
$$= 90^\circ - 28^\circ = \boxed{62^\circ}$$

Hence $\boxed{\alpha = 62^\circ, b = 7.44, c = 15.86}$ Ans

(ii) $b = 8.9, \beta = 21.5^\circ$

Sol: Here $b = 8.9, \beta = 21.5^\circ, \gamma = 90^\circ$



For α

$$\alpha = 180^\circ - (\beta + \gamma)$$
$$= 180^\circ - (21.5^\circ + 90^\circ)$$
$$= 90^\circ - 21.5^\circ = \boxed{68.5^\circ}$$

For a

$$\tan 21.5^\circ = \frac{8.9}{a} \Rightarrow a = \frac{8.9}{\tan 21.5^\circ} = \boxed{22.59}$$

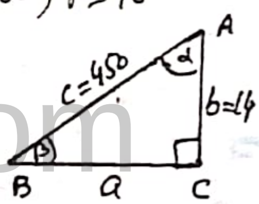
For c

$$\sin 21.5^\circ = \frac{8.9}{c} \Rightarrow c = \frac{8.9}{\sin 21.5^\circ} = \boxed{24.28}$$

Hence $\boxed{\alpha = 68.5^\circ, a = 22.59, c = 24.28}$ Ans

(iii) $b = 14, c = 450$

Sol: Here $b = 14, c = 450, \gamma = 90^\circ$



For a

$$a = \sqrt{c^2 - b^2}$$
$$= \sqrt{450^2 - 14^2}$$
$$= \boxed{449.78}$$

For α

$$\cos\alpha = \frac{14}{450} \Rightarrow \alpha = 88.22^\circ$$

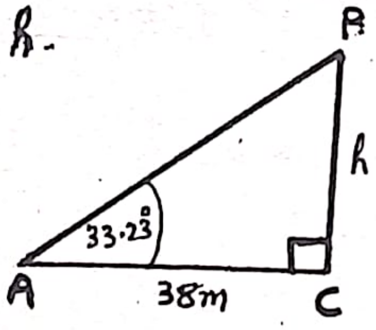
For β

$$\beta = 180^\circ - (\alpha + \gamma)$$
$$= 180^\circ - (88.22^\circ + 90^\circ)$$
$$= 90^\circ - 88.22 = 1.78^\circ$$

Hence $\boxed{\alpha = 88.22^\circ, \beta = 1.78^\circ, a = 449.78}$

Q3. The angle of elevation of the top of a post from a point on level ground 38m away is 33.23°. Find the height of the post.

Sol: Let the height of the post $= h$.



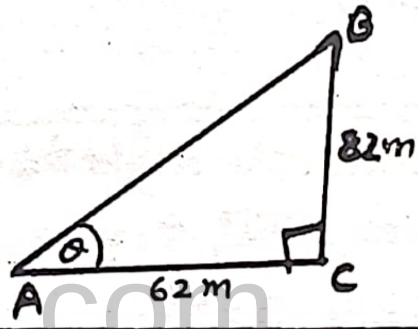
From rt $\triangle ABC$,

$$\tan(33.23^\circ) = \frac{h}{38}$$

$$\Rightarrow h = 38\tan 33.23^\circ = \boxed{24.89\,m}$$ Ans

Q.4: A masjid minar 82 meters high casts a shadow 62 meters long. Find the angle of elevation of the sun at that moment.

Sol: Let the angle of elevation $= \theta$.



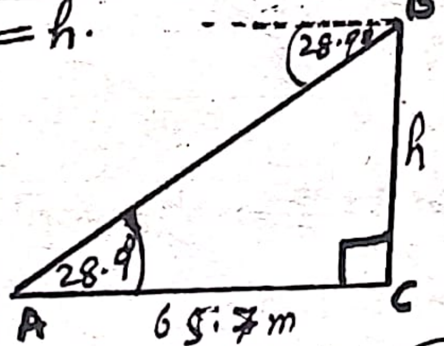
From rt $\triangle ABC$,

$$\tan\theta = \frac{82}{62}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{82}{62}\right) = \boxed{52.9^\circ}$$ Ans

Q.5: The angle of depression of a boat 65.7m from the base of a cliff is 28.9°. How high is the cliff?

Sol: Let height of the cliff $= h$.

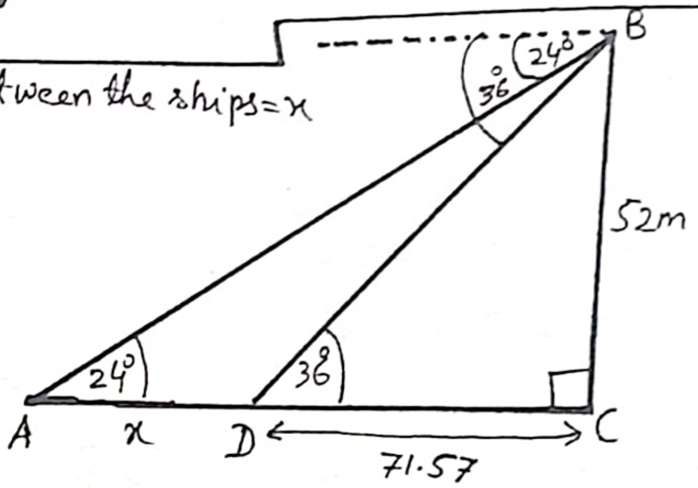


From rt $\triangle ABC$,

$$\tan 28.9^\circ = \frac{h}{65.7}$$

$$\Rightarrow h = 65.7\tan 28.9^\circ = \boxed{36.3\,m}$$ Ans

Q.6: From the top of a cliff 52m high the angles of depression of two ships due east of it are 36° and 24° respectively. Find the distance between the ships.



Sol: Let the distance between the ships $= x$

We first find DC.

From rt $\triangle$ BCD,

$$\tan 36^\circ = \frac{52}{DC}$$

$$\Rightarrow DC = \frac{52}{\tan 36^\circ} = 71.57$$

Now,

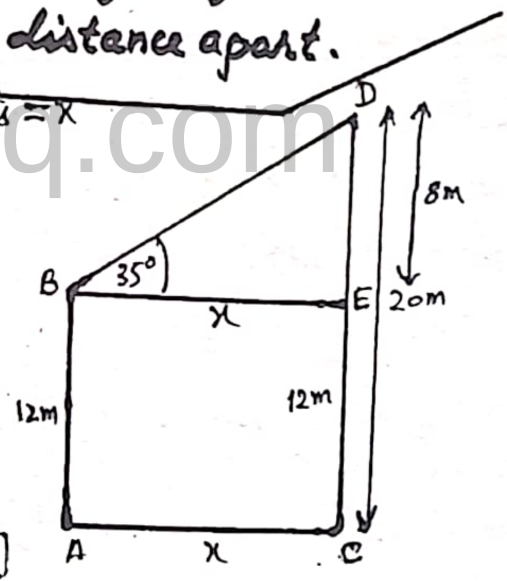
From rt $\triangle$ ABC,

$$\tan 24^\circ = \frac{52}{x+71.57} \Rightarrow x + 71.57 = \frac{52}{\tan 24^\circ} \Rightarrow x = \frac{52}{\tan 24^\circ} - 71.57 = \boxed{45.2}$$

Thus the required distance is $\boxed{45.2m}$ Ans

Q.7: Two mast are 20m and 12m high. If the line joining their tops makes an angle of 35° with the horizontal, find their distance apart.

Sol: Suppose distance b/w the two masts $= x$



From rt $\triangle$ BDE,

$$\tan 35^\circ = \frac{DE}{x} = \frac{DC - EC}{x}$$

$$= \frac{20-12}{x}$$

$$= \frac{8}{x}$$

$$\Rightarrow x = \frac{8}{\tan 35^\circ} = \boxed{11.43m}$$

Thus the required distance is $\boxed{11.43m}$

Q.8: The measure of the angle of elevation of a kite is 35°. The string of the kite is 340 meters long. If the sag in the string is 10 meters, find the height of the kite.

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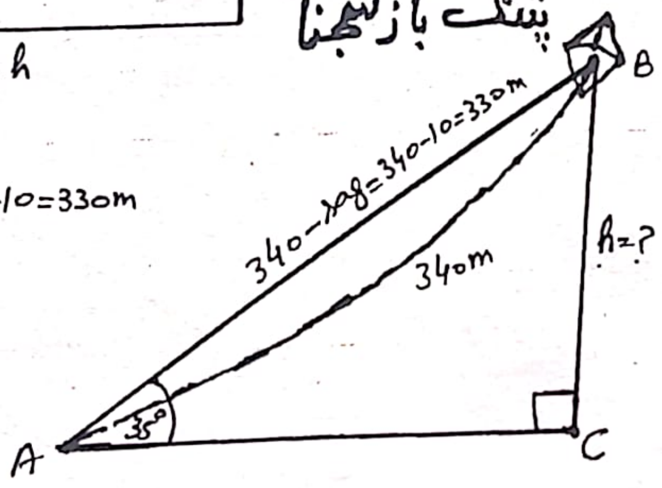


Sol: Let height of kite $= h$

$AB =$ net string

$= 340 - \text{sag} = 340 - 10 = 330\,m$

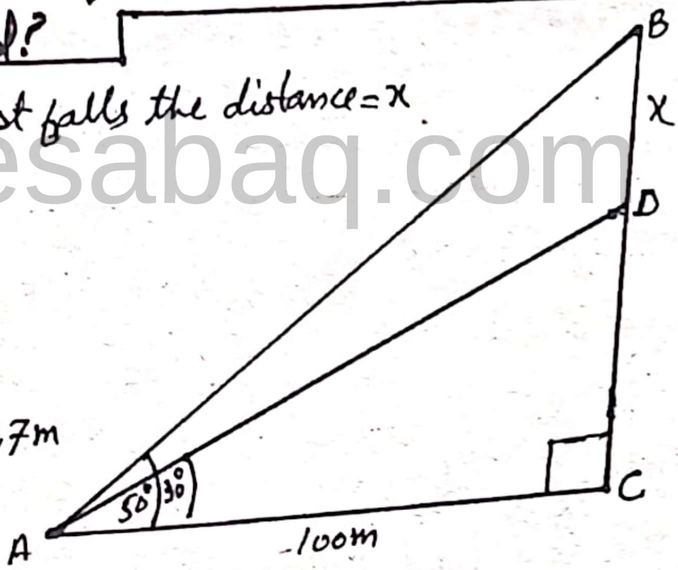
From rt ΔABC,

$\sin 35^\circ = \frac{h}{330}$

$\Rightarrow h = 330 \sin 35^\circ = \boxed{189.3\,m}$

Thus the kite $\boxed{189.3\,m}$ high.

Q.9: A parachutist is descending vertically. How far does the parachutist fall as the angle of elevation changes from 50° to 30° which is observed from a point 100 m away from the feet of the parachutist where he touches the ground?



Sol: Let the parachutist falls the distance $= x$

We first find DC.

From rt ΔACD,

$\tan 30^\circ = \frac{DC}{100}$

$\Rightarrow DC = 100 \tan 30^\circ = 100 \times \frac{1}{\sqrt{3}} = 57.7\,m$

Now, From rt ΔABC,

$\tan 50^\circ = \frac{x + 57.7}{100}$

$x + 57.7 = 100 \tan 50^\circ$

$\Rightarrow x = 100 \tan 50^\circ - 57.7 = \boxed{61.47}$

Thus the parachutist falls the distance $\boxed{61.47\,m}$ Ans

Q.10: An isosceles triangle has a vertical angle of 108° and a base 20 cm long. Calculate its altitude.

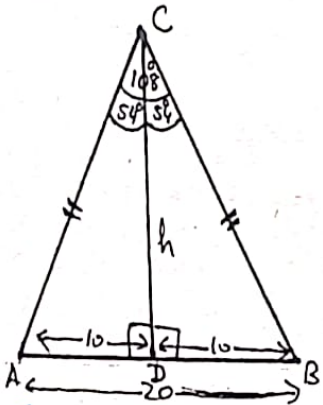
Sol: Let the altitude $= h$

From rt $\triangle ACD$,

$\tan 54^\circ = \frac{10}{h}$

$\Rightarrow h = \frac{10}{\tan 54^\circ} = \boxed{7.27}$

Thus altitude $= \boxed{7.27\,cm}$ Ans



EXERCISE 11.2

Q.1 Solve the triangles with dimensions.

(i) $a = 209, b = 120, c = 241$

Sol: $\alpha ?, \beta ?, \gamma ?$

Using Law of Cosines.

For α: $\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(120)^2 + (241)^2 - (209)^2}{2(120)(241)} = \frac{14400 + 58081 - 43681}{57840}$

$= \frac{28800}{57840} \Rightarrow \alpha = \cos^{-1}\left(\frac{28800}{57840}\right) = \boxed{60.14^\circ}$

For β $\cos\beta = \frac{a^2+c^2-b^2}{2ac} = \frac{(209)^2+(241)^2-(120)^2}{2(209)(241)} = \frac{43861+58081-14400}{100738} = \frac{87362}{100738}$

$\Rightarrow \beta = \cos^{-1}\left(\frac{87362}{100738}\right) = \boxed{29.86^\circ}$

For γ $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \gamma = 180^\circ-(\alpha+\beta) = 180^\circ-(60.14^\circ+29.86^\circ) = 180^\circ-90^\circ = \boxed{90^\circ}$

Hence $\boxed{\alpha = 60.14^\circ, \beta = 29.86^\circ, \gamma = 90^\circ}$ Ans

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(ii) $a=120, b=240, \gamma=32^\circ$

Sol: c? α? β?

Using cosine Law.

For c:

$c^2 = a^2+b^2-2ab\cos\gamma = (120)^2+(240)^2-2(120)(240)\cos 32^\circ$

$= 14400+57600-57600\cos 32^\circ = 72000-57600\cos 32^\circ$

$c = \sqrt{72000-57600\cos 32^\circ} = \boxed{152.2}$

For α: $\cos\alpha = \frac{b^2+c^2-a^2}{2bc} = \frac{(240)^2+(152.2)^2-(120)^2}{2(240)(152.2)}$

$\Rightarrow \alpha = \cos^{-1}\left[\frac{(240)^2+(152.2)^2-(120)^2}{2(240)(152.2)}\right] = \boxed{24.7^\circ}$

For β: $\gamma = 180^\circ-(\alpha+\gamma) = 180^\circ-(24.7^\circ+32^\circ) = \boxed{123.3^\circ}$

Thus $\boxed{c=152.2, \alpha=24.7^\circ, \gamma=123.3^\circ}$

(iii) $\alpha=100^\circ, c=345, \gamma=56.4^\circ$

Sol: β? a? b?

For β: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \beta = 180^\circ-(\alpha+\gamma)$

$= 180^\circ-(100^\circ+56.4^\circ) = 180^\circ-156.4^\circ = \boxed{23.6^\circ}$

Using Sine Law,

For a: $\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma} \Rightarrow a=\frac{c\sin\alpha}{\sin\gamma}=\frac{345\sin 100^\circ}{\sin 56.40}=\boxed{408}$

For b: $\frac{a}{\sin\alpha}=\frac{b}{\sin\beta} \Rightarrow b=\frac{a\sin\beta}{\sin\alpha}=\frac{408\sin 23.6^\circ}{\sin 100^\circ}=\boxed{166}$

Thus $\boxed{\beta=23.6^\circ,\ a=408,\ b=166}$ Ans

(iv) $a=24.5,\ c=43.8,\ \beta=112^\circ$

Sol: $b?\ \alpha?\ \gamma?$

Using Cosine Law.

For b: $b^2=a^2+c^2-2ac\cos\beta$

$\Rightarrow b=\sqrt{a^2+c^2-2ac\cos\beta}=\sqrt{(24.5)^2+(43.8)^2-2(24.5)(43.8)\cos 112^\circ}=\boxed{57.6}$

For α: $\cos\alpha=\frac{b^2+c^2-a^2}{2bc} \Rightarrow \alpha=\cos^{-1}\left[\frac{(57.6)^2+(43.8)^2-(24.5)^2}{2(57.6)(43.8)}\right]=23^\circ$

For γ: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \gamma=180^\circ-(\alpha+\beta)=180^\circ-(23^\circ+112^\circ)=180^\circ-135^\circ=\boxed{45^\circ}$

Thus $\boxed{b=57.6,\ \alpha=23^\circ,\ \gamma=45^\circ}$ Ans

(v) $b=1.6,\ c=3.2,\ \alpha=100^\circ 24'$

Sol: $a?\ \beta?\ \gamma?$

Using Cosine Law.

For a: $a^2=b^2+c^2-2bc\cos\alpha$

$\Rightarrow a=\sqrt{b^2+c^2-2bc\cos\alpha}=\sqrt{(1.6)^2+(3.2)^2-2(1.6)(3.2)\cos 100^\circ 24'}=\boxed{3.83}$ Ans

For β: $\cos\beta=\frac{a^2+c^2-b^2}{2ac} \Rightarrow \beta=\cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right)=\cos^{-1}\left[\frac{(3.83)^2+(3.2)^2-(1.6)^2}{2(3.83)(3.2)}\right]=24.3^\circ$

For γ: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \gamma=180^\circ-(\alpha+\beta)=180^\circ-(100^\circ24'+24.3^\circ)=55.3^\circ$

Hence $\boxed{\alpha=3.83, \beta=24.3^\circ, \gamma=55.3^\circ}$ Ans

(vi) $\beta=39^\circ30'$, $\gamma=34^\circ10'$, $a=240$

Sol: α? b? c?

For α: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \alpha=180^\circ-(\beta+\gamma)=180^\circ-(39^\circ30'+34^\circ10')=180^\circ-73^\circ40'$

$\Rightarrow \alpha=\boxed{106^\circ20'}$

For b: Using Sine Law:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta} \Rightarrow b=\frac{a\sin\beta}{\sin\alpha}=\frac{240\sin 39^\circ30'}{\sin 106^\circ20'}=\boxed{159}$$

For c:

$$\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma} \Rightarrow c=\frac{a\sin\gamma}{\sin\alpha}=\frac{240\sin 34^\circ10'}{\sin 106^\circ20'}=\boxed{140}$$

Thus $\boxed{\alpha=106^\circ20', b=159, c=140}$ Ans

(vii) $\alpha=35^\circ$, $\beta=70^\circ$, $c=115$

Sol: γ? a? b?

For γ: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \gamma=180^\circ-(\alpha+\beta)=180^\circ-(35^\circ+70^\circ)=180^\circ-105^\circ=\boxed{75^\circ}$

For a: Using Sine Law,

$$\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma} \Rightarrow a=\frac{c\sin\alpha}{\sin\gamma}=\frac{115\sin 35^\circ}{\sin 75^\circ}=\boxed{68}$$

For b:

$$\frac{b}{\sin\beta}=\frac{c}{\sin\gamma} \Rightarrow b=\frac{c\sin\beta}{\sin\gamma}=\frac{115\sin 70^\circ}{\sin 75^\circ}=\boxed{112}$$

Hence $\boxed{\gamma=75^\circ, a=68, b=112}$ Ans

(viii) $a = 37.5,\ b = 12.4,\ \beta = 72^\circ$

Sol: $c?\ \alpha?\ \gamma?$

For α: Using Sine Law:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \Rightarrow \alpha = \sin^{-1}\left(\frac{a\sin\beta}{b}\right) = \sin^{-1}\left(\frac{37.5\sin 72^\circ}{12.4}\right)$$

$$\Rightarrow \alpha = \sin^{-1}(2.88) \text{ is undefined. } (\because \text{Dom } \sin = [-1,1])$$

$\therefore$ solution is not possible. Ans

(ix) $b = 12.5,\ c = 23,\ \alpha = 38^\circ 20'$

Sol: $a?\ \beta?\ \gamma?$

For β & γ: $\because \alpha + \beta + \gamma = 180^\circ \Rightarrow \beta + \gamma = 180^\circ - \alpha = 180^\circ - 38^\circ 20' = 141^\circ 40'$

$$\Rightarrow \beta + \gamma = 141^\circ 40' \longrightarrow (1)$$

Using Tangent Law:

$$\frac{b+c}{b-c} = \frac{\tan\left(\frac{\beta+\gamma}{2}\right)}{\tan\left(\frac{\beta-\gamma}{2}\right)}$$

$$\Rightarrow \frac{12.5+23}{12.5-23} = \frac{\tan\left(\frac{141^\circ 40'}{2}\right)}{\tan\left(\frac{\beta-\gamma}{2}\right)} \Rightarrow \frac{35.5}{-10.5} = \frac{\tan(70^\circ 50')}{\tan\left(\frac{\beta-\gamma}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{\beta-\gamma}{2}\right) = \frac{-10.5\tan(70^\circ 50')}{35.5}$$

$$\Rightarrow \frac{\beta-\gamma}{2} = \tan^{-1}\left[\frac{-10.5\tan 70^\circ 50'}{35.5}\right]$$

$$\Rightarrow \frac{\beta-\gamma}{2} = -40^\circ 23' \Rightarrow \beta - \gamma = -80^\circ 46' \longrightarrow (2)$$

$(1)+(2) \Rightarrow$

$$\begin{aligned} \beta + \cancel{\gamma} &= 141^\circ 40' \\ \beta - \cancel{\gamma} &= -80^\circ 46' \\ \hline 2\beta &= 60^\circ 54' \end{aligned}$$

$$\Rightarrow \beta = \frac{60^\circ 54'}{2} = \boxed{30^\circ 27'}$$

$(1)-(2) \Rightarrow$

$$\begin{aligned} \cancel{\beta} + \gamma &= 141^\circ 40' \\ -\cancel{\beta} + \gamma &= +80^\circ 46' \\ \hline 2\gamma &= 222^\circ 26' \end{aligned}$$

$$\Rightarrow \gamma = \frac{222^\circ 26'}{2} = \boxed{111^\circ 13'}$$

For a: Using Sine Law:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow a = \frac{b\sin\alpha}{\sin\beta} = \frac{12.5\sin 38^\circ 20'}{\sin 30^\circ 27'} = \boxed{15.3}$$

Hence $\boxed{\beta = 30^\circ 27', \gamma = 111^\circ 13', a = 15.3}$ Ans

(x) $a = 168, c = 319, \beta = 110^\circ 22'$

Sol: $b? \alpha? \gamma?$

For α, γ: $\because \alpha + \beta + \gamma = 180^\circ \Rightarrow \gamma + \alpha = 180^\circ - \beta = 180^\circ - 110^\circ 22' = 69^\circ 38'$

$$\Rightarrow \gamma + \alpha = 69^\circ 38' \longrightarrow (1)$$

Using Tangent Law:

$$\frac{c+a}{c-a} = \frac{\tan\left(\frac{\gamma+\alpha}{2}\right)}{\tan\left(\frac{\gamma-\alpha}{2}\right)} \Rightarrow \frac{319+168}{319-168} = \frac{\tan\left(\frac{69^\circ 38'}{2}\right)}{\tan\left(\frac{\gamma-\alpha}{2}\right)}$$

$$\Rightarrow \frac{487}{151} = \frac{0.695}{\tan\left(\frac{\gamma-\alpha}{2}\right)} \Rightarrow 3.225 = \frac{0.695}{\tan\left(\frac{\gamma-\alpha}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{\gamma-\alpha}{2}\right) = \frac{0.695}{3.225} \Rightarrow \frac{\gamma-\alpha}{2} = \tan^{-1}(0.216)$$

$$\Rightarrow \gamma - \alpha = 2\tan^{-1}(0.216) \Rightarrow \gamma - \alpha = 2(12^\circ 11')$$

$$\Rightarrow \gamma - \alpha = 24^\circ 22' \longrightarrow (2)$$

(1)+(2) $\Rightarrow$

$$\begin{aligned} \gamma + \cancel{\alpha} &= 69^\circ 38' \\ \gamma - \cancel{\alpha} &= 24^\circ 22' \\ \hline 2\gamma &= 94^\circ \\ \Rightarrow \gamma &= \boxed{47^\circ} \end{aligned}$$

(1)−(2) $\Rightarrow$

$$\begin{aligned} \cancel{\gamma} + \alpha &= 69^\circ 38' \\ \pm\cancel{\gamma} \mp \alpha &= \pm 24^\circ 22' \\ \hline 2\alpha &= 45^\circ 16' \\ \Rightarrow \alpha &= \boxed{22^\circ 38'} \end{aligned}$$

For b:
Using Sine Law: $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

$$\Rightarrow b = \frac{c \sin\beta}{\sin\gamma} = \frac{319 \sin 110^\circ 22'}{\sin 47^\circ} = \boxed{409}$$

Hence $\boxed{b = 409,\ \gamma = 47^\circ,\ \alpha = 22^\circ 38'}$ Ans

Q.2 Find the angle of largest measure (Using half Cosine Law).

(i) $a = 74,\ b = 52$ and $c = 47$

Sol: The largest angle is opposite to the largest side.

Since $a = 74$ is the largest side, so the largest angle will be α.

For α: Using the half angle formula for cosine.

$$S = \frac{a+b+c}{2} = \frac{74+52+47}{2} = \frac{173}{2} = 86.5$$

Now $$\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} = \sqrt{\frac{86.5(86.5-74)}{(52)(47)}} = 0.665$$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.665) = 48^\circ 19'$$

$$\Rightarrow \boxed{\alpha = 96^\circ 38'}$$ Ans

(ii) $a = 7,\ b = 9$ and $c = 7$

Sol: The largest angle is opposite to the largest side.

Since $b = 9$ is the largest side, so the largest angle will be β.

For β: Using the half angle formula for cosine

$$S = \frac{a+b+c}{2} = \frac{7+9+7}{2} = \frac{23}{2} = 11.5$$

Now $$\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} = \sqrt{\frac{11.5(11.5-9)}{(7)(7)}} = 0.766$$

$$\cos\frac{\beta}{2} = 0.766 \Rightarrow \frac{\beta}{2} = \cos^{-1}(0.766) = 40^\circ 0' 14''$$
$$\Rightarrow \boxed{\beta = 80^\circ 0' 28''} \text{ Ans}$$

(iii) $a = 2.3$, $b = 1.5$ and $c = 2.7$

Sol: The largest angle is opposite to the largest side. Since $c = 2.7$ is the largest side, so the largest angle will be γ.

For γ: Using the half angle formula for cosine.

$$S = \frac{2.3+1.5+2.7}{2} = \frac{6.5}{2} = 3.25$$

Now $\cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}} \Rightarrow \cos\frac{\gamma}{2} = \sqrt{\frac{3.25(3.25-2.7)}{(2.3)(1.5)}} = 0.720$

$$\Rightarrow \frac{\gamma}{2} = \cos^{-1}(0.720) = 43^\circ 56' \Rightarrow \boxed{\gamma = 87^\circ 52'} \text{ Ans}$$

Q3: Solve the triangle for which length of these sides are given (Using half cosine law)

(i) $a = 9$, $b = 7$ and $c = 5$

Sol: α? β? γ?

Using the half angle formula for cosine

$$S = \frac{a+b+c}{2} = \frac{9+7+5}{2} = \frac{21}{2} = 10.5$$

For α: $\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} = \sqrt{\frac{10.5(10.5-9)}{(7)(5)}} = 0.671$

$$\frac{\alpha}{2} = \cos^{-1}(0.671) = 47.8^\circ \Rightarrow \boxed{\alpha = 95.6^\circ}$$

For β: $\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} = \sqrt{\frac{10.5(10.5-7)}{(9)(5)}} = 0.904$

$$\frac{\beta}{2} = \cos^{-1}(0.904) = 25.3^\circ \Rightarrow \boxed{\beta = 50.6^\circ}$$

For γ: $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}} = \sqrt{\frac{3.25(3.25-2.7)}{(2.3)(1.5)}} = 0.720$

$\frac{\gamma}{2} = \cos^{-1}(0.720) = 43^\circ 56' \Rightarrow \boxed{\gamma = 87^\circ 52'}$

Hence $\boxed{\alpha = 95.6^\circ, \beta = 50.6^\circ, \gamma = 87^\circ 52'}$ Ans

(ii) $a = 1.2$, $b = 9$, and $c = 10$

Sol: α? β? γ?

Using half angle formula for cosine

$S = \frac{a+b+c}{2} = \frac{1.2+9+10}{2} = \frac{20.2}{2} = 10.1$

For α: $\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} = \sqrt{\frac{10.1(10.1-1.2)}{(9)(10)}} = \sqrt{\frac{(10.1)(8.9)}{90}} = 0.99$

$\Rightarrow \cos\frac{\alpha}{2} = 0.99 \Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.99) = 2^\circ \Rightarrow \boxed{\alpha = 4^\circ}$

For β: $\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} = \sqrt{\frac{10.1(10.1-9)}{(1.2)(10)}} = \sqrt{\frac{10.1(1.1)}{12}} = 0.962$

$\frac{\beta}{2} = \cos^{-1}(0.962) \Rightarrow \beta = 2\cos^{-1}(0.962) = 31.6^\circ = \boxed{31^\circ 36'}$

For γ: $\because \alpha+\beta+\gamma = 180^\circ \Rightarrow \gamma = 180^\circ - (\alpha+\beta) = 180^\circ - (4^\circ + 31.6^\circ)$
$= 180^\circ - 35.6^\circ = 144.4^\circ = \boxed{144^\circ 24'}$

Hence $\boxed{\alpha = 4^\circ, \beta = 31^\circ 36', \gamma = 144^\circ 24'}$ Ans

(iii) $a = 6$, $b = 8$ and $c = 12$

Sol: α? β? γ?

Using half angle formula for cosine

$$S=\frac{a+b+c}{2}=\frac{6+8+12}{2}=\frac{26}{2}=13$$

For α: $\cos\frac{\alpha}{2}=\sqrt{\frac{S(S-a)}{bc}}=\sqrt{\frac{13(13-6)}{(8)(12)}}=\sqrt{\frac{(13)(7)}{96}}=\sqrt{\frac{91}{96}}=0.974$

$$\Rightarrow \frac{\alpha}{2}=\cos^{-1}(0.974)\Rightarrow \alpha=2\cos^{-1}(0.974)=26.2^\circ=\boxed{26^\circ12'}$$

For β: $\cos\frac{\beta}{2}=\sqrt{\frac{S(S-b)}{ac}}=\sqrt{\frac{13(13-8)}{(6)(12)}}=\sqrt{\frac{13(5)}{6(12)}}=\sqrt{\frac{65}{72}}=0.950$

$$\frac{\beta}{2}=\cos^{-1}(0.950)=18.2^\circ\Rightarrow \beta=36.4^\circ=\boxed{36^\circ24'}$$

For γ: $\because \alpha+\beta+\gamma=180^\circ\Rightarrow\gamma=180^\circ-(\alpha+\beta)=180^\circ-(26.2^\circ+36.4^\circ)=180^\circ-62.6^\circ$

$$\Rightarrow\boxed{\gamma=117.4^\circ \text{ or } 117^\circ24'}$$

Hence $\boxed{\alpha=26^\circ12',\ \beta=36^\circ24',\ \gamma=117^\circ24'}$ Ans

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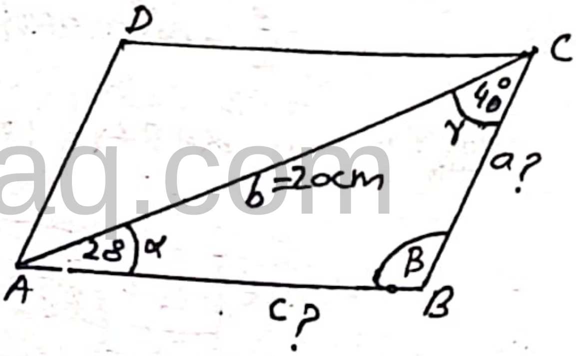
Q.4: One diagonal of a parallelogram is 20cm long and at one end forms angles 20° and 40° with the sides of the parallelogram. Find the length of the sides.

Sol: Let ABCD is a llogram.

$a?$ $c?$



From $\triangle ABC$,

$$20^\circ+\beta+40^\circ=180^\circ$$

$$\beta+60^\circ=180^\circ$$

$$\Rightarrow\beta=180^\circ-60^\circ=120^\circ$$

For a: using Sine-Law:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}\Rightarrow\frac{a}{\sin20^\circ}=\frac{20}{\sin120^\circ}\Rightarrow a=\frac{20\sin20^\circ}{\sin120^\circ}=\boxed{7.9}$$

For c: $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow \frac{20}{\sin 120^\circ} = \frac{c}{\sin 40^\circ} \Rightarrow c = \frac{20 \sin 40^\circ}{\sin 120^\circ} = \boxed{14.84}$

Hence the lengths of the sides are $\boxed{14.84\text{cm and } 7.9\text{cm}}$ Ans

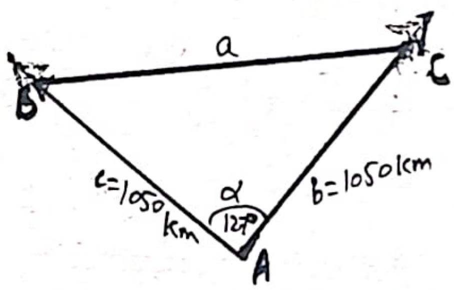
Q.5: Two planes start from Karachi International Airport at the same time and fly in directions that make an angle of 127° with each other. Their speeds are 525km/h. How far apart they are at the end of 2 hours of flying time.

Sol: a?

Here $V = 525\text{Km/h}$

$t = 2$ hours

$S = vt = 525 \times 2 = 1050\text{km}$



Using Cosine Law:

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

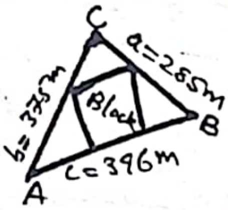
$a^2 = (1050)^2 + (1050)^2 - 2(1050)(1050)\cos 127^\circ = 3532410$

$\therefore a = \sqrt{3532410} = \boxed{1879\text{km}}$ Ans

Q.6: A city block is bounded by three streets. If the measure of the sides of the block are 285, 375 and 396 meters, find the measure of the angles of the streets making with each other.

Sol: Let $a = 285\text{ m}$, $b = 375\text{ m}$, $c = 396\text{ m}$

α, β, γ?



Using half angle formula

$$S = \frac{a+b+c}{2} = \frac{285+375+396}{2} = \frac{1056}{2} = 528$$

For α: $\cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} = \sqrt{\frac{528(528-285)}{(375)(396)}} = 0.929$

$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.929) = 21^\circ 43'$

$\Rightarrow \boxed{\alpha = 43^\circ 26'}$

For β: $\cos\frac{\beta}{2} = \sqrt{\frac{S(S-b)}{ac}} = \sqrt{\frac{528(528-375)}{285 \times 396}} = 0.846$

$\Rightarrow \frac{\beta}{2} = \cos^{-1}(0.846) = 32^\circ 13' \Rightarrow \boxed{\beta = 64^\circ 26'}$

For γ: $\because \alpha+\beta+\gamma = 180^\circ \Rightarrow \gamma = 180^\circ - (\alpha+\beta) = 180^\circ - (43^\circ 26' + 64^\circ 26')$

$= 180^\circ - 107^\circ 52' = \boxed{72^\circ 8'}$

Hence $\boxed{\alpha = 43^\circ 26',\ \beta = 64^\circ 26',\ \gamma = 72^\circ 8'}$ Ans

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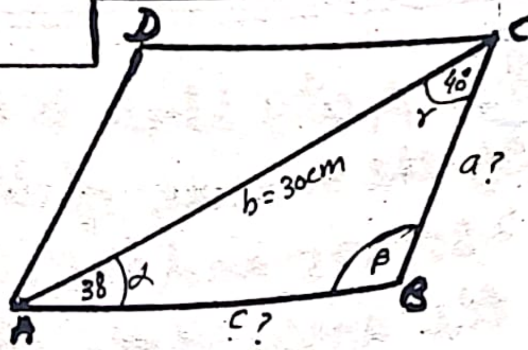
Q.7: The diagonal of a parallelogram meets the sides at angle of 30° and 40°. If the length of the diagonal is 30.0 cm, then find the perimeter of the parallelogram.



Sol: Let ABCD is a llogram.

From $\triangle ABC$,

$30^\circ + \beta + 40^\circ = 180^\circ$

$\beta + 70^\circ = 180^\circ \Rightarrow \beta = 110^\circ$

First we find a & c.

For a: Using Sine-Law:

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} \Rightarrow \frac{a}{\sin 30^\circ} = \frac{30}{\sin 110^\circ} \Rightarrow a = \frac{30\sin 30^\circ}{\sin 110^\circ} = 16$

For c: $\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow \frac{30}{\sin 118} = \frac{c}{\sin 40^\circ} \Rightarrow c = \frac{30 \sin 40^\circ}{\sin 110^\circ} = 20.5$

Perimeter of the parallelogram ABCD $= 2(a+b) = 2(16+20.5)$
$= 2(36.5) = \boxed{73\text{cm}}$ Ans

Q.8: Use the law of cosines to prove

(i) $1+\cos\alpha = \frac{(b+c+a)(b+c-a)}{2bc}$

Sol: By Law of cosines,

$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$

Adding 1 on both sides.

$1+\cos\alpha = 1+\frac{b^2+c^2-a^2}{2bc}$

$= \frac{2bc+b^2+c^2-a^2}{2bc}$

$= \frac{b^2+c^2+2bc-a^2}{2bc}$

$= \frac{(b+c)^2-a^2}{2bc}$

$= \frac{(b+c+a)(b+c-a)}{2bc}$ Proved

(ii) $1-\cos\alpha = \frac{(a-b+c)(a+b-c)}{2bc}$

Sol: By Law of cosines,

$\cos\alpha = \frac{b^2+c^2-a^2}{2bc}$

subtracting both sides from 1.

$1-\cos\alpha = 1-\frac{b^2+c^2-a^2}{2bc}$

$= \frac{2bc-b^2-c^2+a^2}{2bc}$

$= \frac{a^2-(b^2+c^2-2bc)}{2bc}$

$= \frac{a^2-(b-c)^2}{2bc}$

$= \frac{[a+(b-c)][a-(b-c)]}{2bc}$

$= \frac{(a+b-c)(a-b+c)}{2bc}$ Proved

EXERCISE 11.3

Full

Q.1 Find the area of the triangle ABC in each case.

(i) $a=15, b=80, \gamma=38^\circ$

Sol: $\Delta = \frac{1}{2}ab\sin\gamma$

$= \frac{1}{2}(15)(80)\sin 38^\circ$

$= \boxed{369.6 \text{ sq. units}}$ Ans

(ii) $a=14, c=12, \alpha=82^\circ$

Sol: $\Delta = \frac{1}{2}ac\sin\alpha$

$= \frac{1}{2}(14)(12)\sin 82^\circ$

$= \boxed{83.16 \text{ sq. units}}$ Ans

(iii) $a=30, \beta=50^\circ, \gamma=100^\circ$

Sol: We first find α.

$\because \alpha+\beta+\gamma = 180^\circ$

$\Rightarrow \alpha = 180^\circ-(\beta+\gamma)$

$= 180^\circ-(50^\circ+100^\circ)$

$= 180^\circ-150^\circ$

$= 30^\circ$

Now, $\Delta = \frac{1}{2}a^2\frac{\sin\beta\sin\gamma}{\sin\alpha}$

$= \frac{1}{2}(30)^2\frac{\sin 50^\circ\sin 100^\circ}{\sin 30^\circ}$

$= \boxed{679.06 \text{ sq. units}}$ Ans

(iv) $b=40, \alpha=50^\circ, \gamma=60^\circ$

Sol: We first find β.

$\because \alpha+\beta+\gamma = 180^\circ$

$\Rightarrow \beta = 180^\circ-(\alpha+\gamma)$

$= 180^\circ-(50^\circ+60^\circ)$

$= 180^\circ-110^\circ = 70^\circ$

Now, $\Delta = \frac{1}{2}b^2\frac{\sin\alpha\sin\gamma}{\sin\beta}$

$= \frac{1}{2}(40)^2\frac{\sin 50^\circ\sin 60^\circ}{\sin 70^\circ}$

$\Delta = \boxed{564.56 \text{ sq. units}}$ Ans

(v) $a=7.0, b=8.0, c=2.0$

Sol: $S = \frac{a+b+c}{2} = \frac{7+8+2}{2} = \frac{17}{2} = 8.5$

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{8.5(8.5-7)(8.5-8)(8.5-2)}$

$\Delta = \boxed{6.44 \text{ sq. units}}$ Ans

(vi) $a=11, b=9.0, c=8.0$

Sol: $S = \frac{a+b+c}{2} = \frac{11+9+8}{2} = \frac{28}{2} = 14$

$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{14(14-11)(14-9)(14-8)}$

$= \sqrt{14(3)(5)(6)} = \sqrt{1260} = \boxed{35.5 \text{ sq. units}}$ Ans

(vii) $b = 414,\ c = 485,\ \alpha = 49^\circ 47'$

Sol: $\Delta = \frac{1}{2} bc \sin\alpha$

$= \frac{1}{2}(14)(485) \sin 49^\circ 47'$

$= \boxed{76662 \text{ sq. units}}$ Ans

(viii) $a = 32,\ \beta = 47^\circ 24',\ \gamma = 70^\circ 16'$

Sol: We first find α.

$\because \alpha + \beta + \gamma = 180^\circ$

$\Rightarrow \alpha = 180^\circ - (\beta + \gamma)$

$= 180^\circ - (47^\circ 24' + 70^\circ 16')$

$= 180^\circ - 117^\circ 40'$

$= 62^\circ 20'$

Now, $\Delta = \frac{1}{2} a^2 \frac{\sin\beta \sin\gamma}{\sin\alpha}$

$= \frac{1}{2}(32)^2 \frac{\sin 47^\circ 24' \sin 70^\circ 16'}{\sin 62^\circ 20'}$

$= \boxed{400.5 \text{ sq. units}}$ Ans

(ix) $b = 47,\ \alpha = 60^\circ 25',\ \gamma = 41^\circ 35'$

Sol: We first find β.

$\because \alpha + \beta + \gamma = 180^\circ \Rightarrow \beta = 180^\circ - (\alpha + \gamma)$

$= 180^\circ - (68^\circ 25' + 41^\circ 35')$

$= 180^\circ - 102^\circ = 78^\circ$

Now, $\Delta = \frac{1}{2} b^2 \frac{\sin\alpha \sin\gamma}{\sin\beta}$

$= \frac{1}{2}(47)^2 \frac{\sin 68^\circ 25' \sin 41^\circ 35'}{\sin 78^\circ}$

$= \boxed{651.7 \text{ sq. units}}$ Ans

(x) $c = 57,\ \alpha = 23^\circ 24',\ \beta = 71^\circ 36'$

Sol: We first find γ.

$\because \alpha + \beta + \gamma = 180^\circ \Rightarrow \gamma = 180^\circ - (\alpha + \beta)$

$= 180^\circ - (23^\circ 24' + 71^\circ 36') = 180^\circ - 95^\circ$

$= 85^\circ$

$\Delta = \frac{1}{2} c^2 \frac{\sin\alpha \sin\beta}{\sin\gamma}$

$= \frac{1}{2}(57)^2 \frac{\sin 23^\circ 24' \sin 71^\circ 36'}{\sin 85^\circ}$

$= \boxed{614.5 \text{ sq. units}}$ Ans

(xi) $a = 925,\ c = 433,\ \beta = 42^\circ 17'$

Sol: $\Delta = \frac{1}{2} ac \sin\beta$

$= \frac{1}{2}(925)(433) \sin 42^\circ 17'$

$= \boxed{134736.6 \text{ sq. units}}$

Ans

(xii) $a = 92,\ b = 71,\ \gamma = 56^\circ 44'$

Sol: $\Delta = \frac{1}{2} ab \sin\gamma$

$= \frac{1}{2}(92)(71) \sin(56^\circ 44')$

$= \boxed{2730.7 \text{ sq. units}}$ Ans

Q2: The area of a triangle is 121.34. If $\alpha = 32^\circ 25'$, $\beta = 65^\circ 27'$ then find c and angle γ.

Sol: Given $\Delta = 121.34$, $\alpha = 32^\circ 25'$, $\beta = 65^\circ 27'$, $c?$ $\gamma?$

For γ: $\because \alpha+\beta+\gamma = 180^\circ \Rightarrow \gamma = 180^\circ - (\alpha+\beta) = 180^\circ - (32^\circ 25' + 65^\circ 27')$

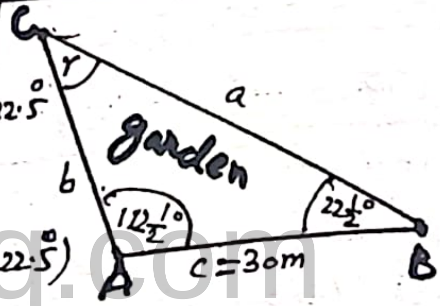
$= 180^\circ - 97^\circ 52' = \boxed{82^\circ 8'}$

For c: $\because \Delta = \frac{1}{2} c^2 \frac{\sin\alpha \sin\beta}{\sin\gamma}$

$\Rightarrow c = \sqrt{\frac{2\Delta \sin\gamma}{\sin\alpha \sin\beta}} = \sqrt{\frac{2(121.34)\sin 82^\circ 8'}{\sin 32^\circ 25' \sin 65^\circ 27'}} = \boxed{22.24}$

Hence $\boxed{c = 22.24, \ \gamma = 82^\circ 8'}$ Ans

Q.3: One side of a triangular garden is 30m. If its two corner angles are $22\frac{1}{2}^\circ$ and $112\frac{1}{2}^\circ$, find the cost of planting the grass at the rate of Rs. 5 per square meter.



Sol: Given $c = 30m$, $\alpha = 112\frac{1}{2}^\circ = 112.5^\circ$, $\beta = 22\frac{1}{2}^\circ = 22.5^\circ$

we first find γ.

$\because \alpha+\beta+\gamma = 180^\circ \Rightarrow \gamma = 180^\circ - (\alpha+\beta) = 180^\circ - (112.5^\circ + 22.5^\circ)$

$= 180^\circ - 135^\circ = 45^\circ$

Now, $\Delta = \frac{1}{2} c^2 \frac{\sin\alpha \sin\beta}{\sin\gamma} = \frac{1}{2}(30)^2 \frac{\sin 112.5^\circ \sin 22.5^\circ}{\sin 45^\circ} = 225\ m^2$

Rate of planting the grass $=$ Rs. $5/m^2$

Total cost $= 225 \times 5 = \boxed{\text{Rs. } 1125}$ Ans

Q.4: A new home owner has a triangular-shaped back yard. Two of three sides measure 53ft and 42ft and form an included angle of $135°$. To determine the amount of fertilizer and grass seed to be purchased, the owner has to know the area of the yard. Find the area of the yard to the nearest square foot.

Sol: Let $a = 53ft$, $b = 42ft$, $\gamma = 135°$

$$\Delta = \frac{1}{2}ab\sin\gamma = \frac{1}{2}(53)(42)\sin 135° = \boxed{787\,ft^2} \text{ Ans}$$

EXERCISE 11.4

Q.1: Compute the in-radius (r) and circum-radius (R) of the triangles whose sides are given;

(i) 3, 5, 6

Sol: Let $a=3, b=5, c=6$

$$S = \frac{a+b+c}{2} = \frac{3+5+6}{2} = \frac{14}{2} = 7$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$
$$= \sqrt{7(7-3)(7-5)(7-6)}$$
$$= \sqrt{7(4)(2)(1)} = \sqrt{56} = 7.48$$

(i) $r = \frac{\Delta}{S} = \frac{7.48}{7} = \boxed{1.1}$ Ans

(ii) $R = \frac{abc}{4\Delta} = \frac{(3)(5)(6)}{4(7.48)} = \boxed{3}$ Ans

(ii) 21, 20, 29

Sol: Let $a=21, b=20, c=29$

$$S = \frac{a+b+c}{2} = \frac{21+20+29}{2} = \frac{70}{2} = 35$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)}$$
$$= \sqrt{35(35-21)(35-20)(35-29)}$$
$$= \sqrt{35(14)(15)(6)} = \sqrt{44100} = 210$$

(i) $r = \frac{\Delta}{S} = \frac{210}{35} = \boxed{6}$ Ans

(ii) $R = \frac{abc}{4\Delta} = \frac{21(20)(29)}{4(210)} = \boxed{14.5}$ Ans

Q.2 Find the area of the inscribed circle of the triangle with measures of the sides 55m, 25m and 70m.

Sol: Let $a = 55m$, $b = 25m$, $c = 70m$.

We first find r.

$$S = \frac{a+b+c}{2} = \frac{55+25+70}{2} = \frac{150}{2} = 75$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{75(75-55)(75-25)(75-70)} = \sqrt{37500}$$
$$= 612.4$$

Now $r = \frac{\Delta}{S} = \frac{612.4}{75} = 8.16m$

Area of Incircle $= \pi r^2 = (3.14)(8.16)^2 = \boxed{209m^2}$ Ans

Q.3 The measures of the sides of a triangle are 20, 25, 30 decimeter. Find the radius of the described circles.

Sol: Let $a = 20dm$, $b = 25dm$, $c = 30dm$

$$S = \frac{a+b+c}{2} = \frac{20+25+30}{2} = \frac{75}{2} = 37.5$$

$$\Delta = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{37.5(37.5-20)(37.5-25)(37.5-30)}$$
$$= \sqrt{37.5(17.5)(12.5)(7.5)} = 248.04$$

(i) Opposite to larger side

$$r_3 = \frac{\Delta}{S-c}$$
$$= \frac{248.04}{37.5-30}$$
$$= \frac{248.04}{7.5} = \boxed{33.07dm}$$ Ans

(ii) Opposite to smaller side

$$r_1 = \frac{\Delta}{S-a}$$
$$= \frac{248.04}{37.5-20}$$
$$= \frac{248.04}{17.5} = \boxed{14.17dm}$$ Ans

Q.4: Show that (i) $\sqrt{r r_1 r_2 r_3} = \Delta$

Sol: $L.H.S = \sqrt{r r_1 r_2 r_3}$

$$= \sqrt{\frac{\Delta}{s} \cdot \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}} = \sqrt{\frac{\Delta^4}{s(s-a)(s-b)(s-c)}}$$

$$= \sqrt{\frac{\Delta^4}{\Delta^2}} = \sqrt{\Delta^2} = \Delta = R.H.S \text{ Proved.}$$

(ii) $abc(\sin\alpha + \sin\beta + \sin\gamma) = 4\Delta s$

Sol: $L.H.S = abc(\sin\alpha + \sin\beta + \sin\gamma)$

$$= abc\sin\alpha + abc\sin\beta + abc\sin\gamma$$

$$= a(bc\sin\alpha) + b(ac\sin\beta) + c(ab\sin\gamma)$$

$$= a(2\Delta) + b(2\Delta) + c(2\Delta)$$

$$= 2\Delta(a+b+c)$$

$$= 2\Delta(2s)$$

$$= 4\Delta s = R.H.S \text{ Proved}$$

(iii) $r_1 r_2 r_3 = r s^2$

Sol: $L.H.S = r_1 r_2 r_3$

$$= \frac{\Delta}{s-a} \cdot \frac{\Delta}{s-b} \cdot \frac{\Delta}{s-c}$$

$$= \frac{\Delta^3}{(s-a)(s-b)(s-c)} = \frac{\Delta^3 s}{s(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta^3 s}{\Delta^2} = \Delta s = \frac{\Delta}{s} s^2 = r s^2 = R.H.S$$

Proved.

Q.5: Prove that for any triangle ABC.

(i) $r_1+r_2+r_3-r=4R$

Sol: $L.H.S = r_1+r_2+r_3-r$

$$= \frac{\Delta}{s-a}+\frac{\Delta}{s-b}+\frac{\Delta}{s-c}-\frac{\Delta}{s}$$

$$=\Delta\left[\left(\frac{1}{s-a}+\frac{1}{s-b}\right)+\left(\frac{1}{s-c}-\frac{1}{s}\right)\right]$$

$$=\Delta\left[\frac{s-b+s-a}{(s-a)(s-b)}+\frac{s-(s-c)}{s(s-c)}\right]$$

$$=\Delta\left[\frac{2s-a-b}{(s-a)(s-b)}+\frac{\cancel{s}-\cancel{s}+c}{s(s-c)}\right]$$

$$=\Delta\left[\frac{\cancel{a}+\cancel{b}+c-\cancel{a}-\cancel{b}}{(s-a)(s-b)}+\frac{c}{s(s-c)}\right] \quad (\because 2s=a+b+c)$$

$$=c\Delta\left[\frac{1}{(s-a)(s-b)}+\frac{1}{s(s-c)}\right]$$

$$=c\Delta\left[\frac{s(s-c)+(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right]$$

$$=c\cancel{\Delta}\left[\frac{s^2-cs+s^2-bs-as+ab}{\Delta^{\cancel{2}}}\right]$$

$$=\frac{c}{\Delta}\left[2s^2-s(a+b+c)+ab\right]$$

$$=\frac{c}{\Delta}\left[2s^2-s(2s)+ab\right]$$

$$=\frac{c}{\Delta}\left[\cancel{2s^2}-\cancel{2s^2}+ab\right]$$

$$=\frac{abc}{\Delta}$$

$$=4\frac{abc}{4\Delta}=4R=R.H.S \text{ Proved}$$

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(ii) $r_1r_2+r_2r_3+r_3r_1=S^2$

Sol: $L.H.S = r_1r_2+r_2r_3+r_3r_1$

$$=\frac{\Delta}{S-a}\cdot\frac{\Delta}{S-b}+\frac{\Delta}{S-b}\cdot\frac{\Delta}{S-c}+\frac{\Delta}{S-c}\cdot\frac{\Delta}{S-a}$$

$$=\frac{\Delta^2}{(S-a)(S-b)}+\frac{\Delta^2}{(S-b)(S-c)}+\frac{\Delta^2}{(S-c)(S-a)}$$

$$=\Delta^2\left[\frac{1}{(S-a)(S-b)}+\frac{1}{(S-b)(S-c)}+\frac{1}{(S-c)(S-a)}\right]$$

$$=\Delta^2\left[\frac{S-c+S-a+S-b}{(S-a)(S-b)(S-c)}\right]$$

$$=\Delta^2\left[\frac{3S-(a+b+c)}{(S-a)(S-b)(S-c)}\right]=\Delta^2\left[\frac{3S-2S}{(S-a)(S-b)(S-c)}\right]$$

$$=\frac{\Delta^2 S}{(S-a)(S-b)(S-c)}=\frac{\Delta^2S^2}{S(S-a)(S-b)(S-c)}$$

$$=\frac{\Delta^2S^2}{\Delta^2}=S^2=R.H.S \text{ Proved.}$$

(iii) $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}=\frac{1}{r}$

Sol: $L.H.S=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$

$$=\frac{S-a}{\Delta}+\frac{S-b}{\Delta}+\frac{S-c}{\Delta}=\frac{S-a+S-b+S-c}{\Delta}$$

$$=\frac{3S-(a+b+c)}{\Delta}=\frac{3S-2S}{\Delta}=\frac{S}{\Delta}=\frac{1}{r}=R.H.S$$

Proved

Q.6: Show that

(i) $r_1 = s\tan\frac{\alpha}{2}$	$r_2 = s\tan\frac{\beta}{2}$	$r_3 = s\tan\frac{\gamma}{2}$
Sol: $R.H.S = s\tan\frac{\alpha}{2}$ $= s\left(\frac{r}{s-a}\right)$ $= \frac{rs}{s-a}$ $= \frac{\Delta}{s-a}$ $= r_1 = L.H.S$ Proved	**Sol:** $R.H.S = s\tan\frac{\beta}{2}$ $= s\left(\frac{r}{s-b}\right)$ $= \frac{rs}{s-b}$ $= \frac{\Delta}{s-b}$ $= r_2 = L.H.S$ Proved	**Sol:** $R.H.S = s\tan\frac{\gamma}{2}$ $= s\left(\frac{r}{s-c}\right)$ $= \frac{rs}{s-c}$ $= \frac{\Delta}{s-c}$ $= r_3 = L.H.S$ Proved

Q.7: The sides of a triangle are in the ratio 3:7:8. The radius of the inscribed circle is 2m. Find the sides of the triangle.

Sol: Let $a = 3x,\ b = 7x,\ c = 8x$

$$s = \frac{a+b+c}{2} = \frac{3x+7x+8x}{2} = \frac{18x}{2} = 9x$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{9x(9x-3x)(9x-7x)(9x-8x)}$$

$$= \sqrt{9x(6x)(2x)(x)} = \sqrt{9\times6\times2\,x^4} = 6\sqrt{3}\,x^2$$

$$\because r = 2 \Rightarrow \frac{\Delta}{s} = 2 \Rightarrow \Delta = 2s \Rightarrow 6\sqrt{3}\,x^2 = 2(9x)$$

$$\Rightarrow 6\sqrt{3}\,x^{\cancel{2}} = 18\cancel{x} \Rightarrow x = \sqrt{3}$$

$$\therefore a = 3x = 3\sqrt{3},\ b = 7x = 7\sqrt{3},\ c = 8x = 8\sqrt{3}$$

Hence the sides are $\boxed{3\sqrt{3},\ 7\sqrt{3},\ 8\sqrt{3}}$ Ans

REVIEW EXERCISE 11

Q.1: Choose the correct option.

(i) In right triangle ABC, find b if $a=2$, $c=5$, and $\gamma = 90°$.

(a) 7 (b) 3 (c) $\sqrt{21}$ (d) $\sqrt{29}$

Sol: $b = \sqrt{c^2 - a^2} = \sqrt{5^2 - 2^2} = \sqrt{25-4} = \sqrt{21}$

Thus the correct option is (c) Ans

(ii) An escalator in a department store makes an angle of 45° with the ground. How long is the escalator if it carries people a vertical distance of 24 feet?

(a) $12\sqrt{2}$ ft (b) $24\sqrt{2}$ ft (c) $8\sqrt{3}$ ft (d) 48 ft

Sol:

$\sin 45° = \frac{24}{c}$

$\frac{1}{\sqrt{2}} = \frac{24}{c}$

$\Rightarrow c = 24\sqrt{2}$

Thus the correct option is (b). Ans

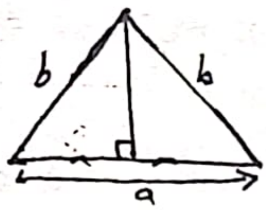
(iii) If in an isosceles triangle, 'a' is the length of the base and 'b' the length of one of the equal sides, then its area is

(a) $\frac{a}{4}\sqrt{4b^2-a^2}$ (b) $\frac{b}{4}\sqrt{4b^2-a^2}$ (c) $\frac{a+b}{4}\sqrt{a^2-b^2}$ (d) $\frac{a-b}{4}\sqrt{b^2-a^2}$

Sol: $s = \frac{a+b+b}{2} = \frac{a+2b}{2}$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{a+2b}{2}\left(\frac{a+2b}{2}-a\right)\left(\frac{a+2b}{2}-b\right)\left(\frac{a+2b}{2}-b\right)}$



$$= \sqrt{\frac{a+2b}{2}\left(\frac{a+2b-2a}{2}\right)\left(\frac{a+2b-2b}{2}\right)\left(\frac{a+2b-2b}{2}\right)}$$

$$= \sqrt{\frac{a+2b}{2}\left(\frac{2b-a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)} = \frac{a}{2}\sqrt{\frac{(2b)^2-a^2}{4}} = \frac{a}{4}\sqrt{4b^2-a^2}$$

Thus the correct option is (a). Ans

(iv) If Heron's formula is used to find the area of triangle ABC having $a=3$ meters, $b=5$ meters, and $c=6$ meters, which of the following shows the correct way to set up the formula?

(a) $\Delta = 7\sqrt{(10)(12)(13)}$ (b) $\Delta = \sqrt{(4)(2)(1)}$

(c) $\Delta = \sqrt{7(3)(5)(6)}$ (d) $\Delta = \sqrt{7(4)(2)(1)}$

Sol: $S = \frac{a+b+c}{2} = \frac{3+5+6}{2} = \frac{14}{2} = 7$

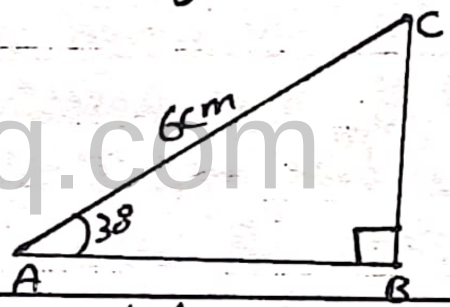
$$\Delta = \sqrt{S(S-a)(S-b)(S-c)} = \sqrt{7(7-3)(7-5)(7-6)} = \sqrt{7(4)(2)(1)}$$

Thus the correct option is (d). Ans

(v) In the adjoining figure, the length of $\overline{BC}$ is

(a) $2\sqrt{3}$ cm (b) $3\sqrt{3}$ cm

(c) $4\sqrt{3}$ cm (d) 3cm

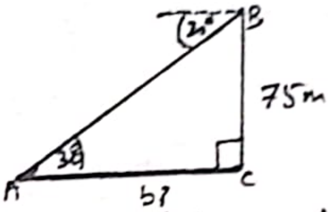


Sol: $\sin 30^\circ = \frac{BC}{6} \Rightarrow \frac{1}{2} = \frac{BC}{6} \Rightarrow BC = 3$

Thus the correct option is (d). Ans

(vi) If the angle of depression of an object from a 75m high tower is 30° then the distance of the object from the tower is

(a) $25\sqrt{3}$ m (b) $50\sqrt{3}$ m (c) $75\sqrt{3}$ m (d) 150 m



Sol: $\tan 30^\circ = \frac{75}{b} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{b}$

$\Rightarrow b = 75\sqrt{3}\,m$

Thus the correct option is (c). Ans

(vii) The point of concurrency of the right bisectors of the sides of a triangle is called

(a) In-centre (b) Orthocentre (c) Circumcentre (d) Centroid

Sol: The correct option is (c). Ans

(viii) With usual notations $r r_1 r_2 r_3 =$

(a) Δ (b) Δ^2 (c) $\frac{abc}{\Delta}$ (d) $\frac{\Delta}{abc}$

Sol: The correct option is (b). Ans Note (For Proof see Q.4(i) Exercise 11.4)

Q2: Solve the triangles.

(i) $a = 0.7,\ c = 0.8,\ \beta = 141^\circ 30'$

Sol: $b?\ \alpha?\ \gamma?$

Using Cosine Law:

For b: $b^2 = a^2 + c^2 - 2ac\cos\beta$

$\Rightarrow b = \sqrt{a^2 + c^2 - 2ac\cos\beta} = \sqrt{(0.7)^2 + (0.8)^2 - 2(0.7)(0.8)\cos 141^\circ 30'}$

$= \boxed{1.42}$

For α: $\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \alpha = \cos^{-1}\left[\frac{(1.42)^2 + (0.8)^2 - (0.7)^2}{2(1.42)(0.8)}\right] = \boxed{17^\circ 30'}$

For γ: $\because \alpha + \beta + \gamma = 180^\circ \Rightarrow \gamma = 180^\circ - (\alpha + \beta) = 180^\circ - (17^\circ 30' + 141^\circ 30')$

$= 180^\circ - 159^\circ = \boxed{21^\circ}$

Hence $\boxed{b = 1.42,\ \alpha = 17^\circ 30',\ \gamma = 21^\circ}$ Ans

(ii) $a = 34,\ b = 23,\ c = 58$

Sol: $\because a + b = 34 + 23 = 57 < 58$

$\therefore$ No triangle is possible. Ans

(iii) $a = 15.6,\ b = 18,\ \gamma = 35^\circ 10'$

Sol: $c?\ \alpha?\ \beta?$

For c: Using cosine Law:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$\Rightarrow c = \sqrt{a^2 + b^2 - 2ab\cos\gamma} = \sqrt{(15.6)^2 + (18)^2 - 2(15.6)(18)\cos 35^\circ 10'} = \boxed{10.4}$$

For α: $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc} \Rightarrow \alpha = \cos^{-1}\left[\dfrac{(18)^2 + (10.4)^2 - (15.6)^2}{2(18)(10.4)}\right] = \boxed{59^\circ 44'}$

For β: $\because \alpha + \beta + \gamma = 180^\circ \Rightarrow \beta = 180^\circ - (\alpha + \gamma) = 180^\circ - (59^\circ 44' + 35^\circ 10')$

$= 180^\circ - 94^\circ 54' = \boxed{85^\circ 6'}$

Hence $\boxed{c = 10.4,\ \alpha = 59^\circ 44',\ \beta = 85^\circ 6'}$ Ans

(iv) $a = 48,\ b = 32,\ \gamma = 57^\circ$

Sol: $c?\ \alpha?\ \beta?$

For α & β: $\because \alpha + \beta + \gamma = 180^\circ \Rightarrow \alpha + \beta = 180^\circ - \gamma = 180^\circ - 57^\circ = 123^\circ$

$\Rightarrow \alpha + \beta = 123^\circ \longrightarrow (1)$

Using Tangent Law:

$$\frac{a+b}{a-b} = \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} \Rightarrow \frac{48+32}{48-32} = \frac{\tan\left(\frac{123^\circ}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

$$\Rightarrow \frac{80}{16} = \frac{\tan 61.5^\circ}{\tan\left(\frac{\alpha-\beta}{2}\right)} \Rightarrow 5 = \frac{\tan 61.5^\circ}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

$$\Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right) = \frac{\tan 61.5^\circ}{5} = 0.368$$

$$\Rightarrow \alpha-\beta = 2\tan^{-1}(0.368)$$

$$\Rightarrow \alpha-\beta = 40.4^\circ \longrightarrow (2)$$

$(1)+(2) \Rightarrow$

$$\alpha+\cancel{\beta} = 123^\circ$$
$$\alpha-\cancel{\beta} = 40.4^\circ$$
$$2\alpha = 163.4^\circ$$
$$\alpha = \frac{163.4^\circ}{2} = 81.7^\circ$$

$(1)-(2) \Rightarrow$

$$\cancel{\alpha}+\beta = 123^\circ$$
$$\cancel{\alpha}-\beta = 40.4^\circ$$
$$- \quad + \quad -$$
$$2\beta = 82.6^\circ$$
$$\Rightarrow \beta = 41.3^\circ$$

For c: Using Sine Law:

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma} \Rightarrow \frac{48}{\sin 81.7^\circ} = \frac{c}{\sin 57^\circ}$$

$$\Rightarrow c = \frac{48\sin 57^\circ}{\sin 81.7^\circ} = 42.7$$

Hence $\boxed{\alpha = 81.7^\circ,\ \beta = 41.3^\circ,\ c = 42.7}$ Ans

(v) $b=35,\ c=37,\ \alpha = 23^\circ 25'$

Sol: $a?\ \beta?\ ,\gamma?$

For c: Using Cosine Law:

$$a^2 = b^2+c^2-2bc\cos\alpha$$

$$\Rightarrow a = \sqrt{b^2+c^2-2bc\cos\alpha} = \sqrt{(35)^2+(37)^2-2(35)(37)\cos 23^\circ 25'} = \boxed{14.74}$$

For β: $\cos\beta = \frac{a^2+c^2-b^2}{2ac} = \frac{(14.74)^2+(37)^2-(35)^2}{2(14.74)(37)} = 0.33 \Rightarrow \beta = \cos^{-1}(0.33) = 70^\circ 43'$

For γ: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \gamma=180^\circ-(\alpha+\beta)=180^\circ-(23^\circ25'+70^\circ43')$
$=180^\circ-94^\circ8'$

Hence $\boxed{c=14.74,\ \beta=70^\circ43',\ \gamma=85^\circ52'}$ Ans

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(vi) $a=58.4,\ \beta=37.2^\circ,\ \gamma=100^\circ$

Sol: $b?\ c?\ \alpha?$

For α: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \alpha=180^\circ-(\beta+\gamma)=180^\circ-(37.2^\circ+100^\circ)$
$=180^\circ-137.2^\circ=\boxed{42.8^\circ}$

For b: Using Sine Law:

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}\Rightarrow b=\frac{a\sin\beta}{\sin\alpha}=\frac{58.4\sin37.2^\circ}{\sin42.8^\circ}=\boxed{52}$$

For c: $$\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}\Rightarrow c=\frac{a\sin\gamma}{\sin\alpha}=\frac{58.4\sin100^\circ}{\sin42.8^\circ}=\boxed{84.7}$$

Hence $\boxed{\alpha=42.8^\circ,\ b=52,\ c=84.7}$ Ans

(vii) $c=13.6,\ \alpha=30^\circ24',\ \beta=72^\circ6'$

Sol: $\gamma?\ a?\ b?$

For γ: $\because \alpha+\beta+\gamma=180^\circ \Rightarrow \gamma=180^\circ-(\alpha+\beta)=180^\circ-(30^\circ24'+72^\circ6')$
$=180^\circ-102^\circ30'=\boxed{77^\circ30'}$

For a: $$\frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}\Rightarrow a=\frac{c\sin\alpha}{\sin\gamma}=\frac{13.6\sin30^\circ24'}{\sin77^\circ30'}=\boxed{7.1}$$

For b: $$\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}\Rightarrow b=\frac{c\sin\beta}{\sin\gamma}=\frac{13.6\sin72^\circ6'}{\sin77^\circ30'}=\boxed{13.3}$$

Hence $\boxed{\gamma=77^\circ30',\ a=7.1,\ b=13.3}$ Ans

Q.3: Find the measure of the smallest angle of the triangle whose sides have lengths

(i) 4.3, 5.1 and 6.3

Sol: Let $a = 4.3$, $b = 5.1$, $c = 6.3$

The smallest angle is opposite to the smallest side.

Since $a = 4.3$ is the smallest side, so the smallest angle will be α.

For α: Using the half angle formula for cosine.

$$S = \frac{a+b+c}{2} = \frac{4.3+5.1+6.3}{2} = \frac{15.7}{2} = 7.85$$

$$\text{Now } \cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} = \sqrt{\frac{7.85(7.85-4.3)}{(5.1)(6.3)}} = \sqrt{0.87} = 0.93$$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.93) = 21.52^\circ$$

$$\Rightarrow \boxed{\alpha = 43.04^\circ} \text{ Ans}$$

(ii) 3, 4.2 and 3.8

Sol: Let $a = 3$, $b = 4.2$, $c = 3.8$

The smallest angle is opposite to the smallest side.

Since $a = 3$ is the smallest side, so the smallest angle will be α.

For α: Using the half angle formula for cosine.

$$S = \frac{a+b+c}{2} = \frac{3+4.2+3.8}{2} = \frac{11}{2} = 5.5$$

$$\text{Now, } \cos\frac{\alpha}{2} = \sqrt{\frac{S(S-a)}{bc}} = \sqrt{\frac{5.5(5.5-3)}{(4.2)(3.8)}} = 0.93$$

$$\Rightarrow \frac{\alpha}{2} = \cos^{-1}(0.93) = 21.57$$

$$\Rightarrow \boxed{\alpha = 43.14} \text{ Ans}$$

Q.4: Find the measure of the largest angle of the triangle whose sides have lengths

(i) 2.9, 3.3 and 4.1

Sol: Let $a = 2.9$, $b = 3.3$, $c = 4.1$

The largest angle is opposite to the largest side.

Since $c = 4.1$ is the largest side, so the largest angle will be γ.

For γ: Using the half angle formula for cosine.

$$S = \frac{a+b+c}{2} = \frac{2.9+3.3+4.1}{2} = \frac{10.3}{2} = 5.15$$

$$\text{Now, } \cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}} = \sqrt{\frac{5.15(5.15-4.1)}{(2.9)(3.3)}} = 0.75$$

$$\frac{\gamma}{2} = \cos^{-1}(0.75) = 41.4^\circ$$

$$\Rightarrow \boxed{\gamma = 82.8^\circ} \text{ Ans}$$

(ii) 6.0, 8 and 9.4

Sol: Let $a = 6.0$, $b = 8$, $c = 9.4$

The largest angle is opposite to the largest side.

Since $c = 9.4$ is the largest side, so the larest angle will be γ.

For γ: Using the half angle formula for cosine.

$$S = \frac{a+b+c}{2} = \frac{6.0+8+9.4}{2} = \frac{23.4}{2} = 11.7$$

$$\text{Now, } \cos\frac{\gamma}{2} = \sqrt{\frac{S(S-c)}{ab}} = \sqrt{\frac{11.7(11.7-9.4)}{(6)(8)}} = 0.75$$

$$\frac{\gamma}{2} = \cos^{-1}(0.75) = 41.52^\circ$$

$$\boxed{\gamma = 83.04} \text{ Ans}$$

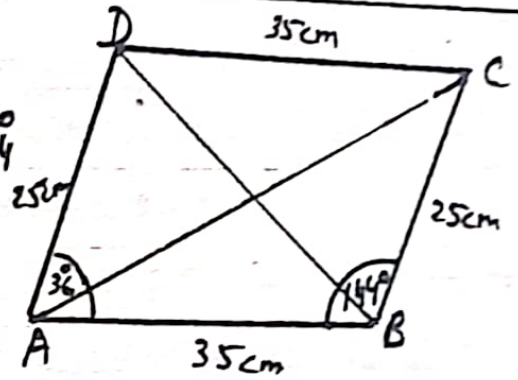
Q.5: The sides of a parallelogram are 25cm and 35cm and one of its angles is 36°. Find the lengths of its diagonals.

Sol: Let ABCD be a llogram.

Let $AB = 35cm$, $BC = 25cm$

Let $A = 36^\circ$. Then $B = 180^\circ - 36^\circ = 144^\circ$

We need to find AC and BD.



For AC: From $\triangle ABC$,

using Cosine Law:

$$(AC)^2 = (AB)^2 + (BC)^2 - 2(AB)(BC)\cos B$$

$$\Rightarrow AC = \sqrt{(AB)^2 + (BC)^2 - 2(AB)(BC)\cos B}$$

$$= \sqrt{(35)^2 + (25)^2 - 2(35)(25)\cos 144^\circ} = \boxed{57.15}$$

For BD: From $\triangle ABD$,

using Cosine Law:

$$BD^2 = (AB)^2 + (AD)^2 - 2(AB)(AD)\cos A$$

$$\Rightarrow BD = \sqrt{(AB)^2 + (AD)^2 - 2(AB)(AD)\cos A}$$

$$= \sqrt{(35)^2 + (25)^2 - 2(35)(25)\cos 36^\circ} = \boxed{20.84}$$

Hence the diagonals are $\boxed{57.15cm \text{ and } 20.84cm}$ Ans

Q.6. A man is flying a kite. He has let out 50m of string, and he notices that the string makes an angle of 60° with the ground. How high is the kite?

Sol: Let $h = PK$.

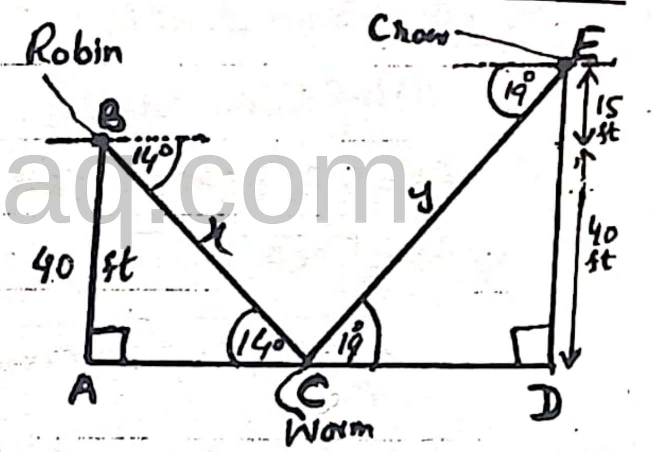
From rt ΔSPK,

$\sin 60^\circ = \frac{h}{50} \Rightarrow h = 50\sin 60^\circ = 50\left(\frac{\sqrt{3}}{2}\right) = \boxed{25\sqrt{3}}$

Hence the kite is $25\sqrt{3}$ m high.

Q.7: A robin on a branch 40ft up in a tree spots a worm at an angle of depression of 14°. From a branch 15ft above the robin, a crow spots the same worm at an angle of depression of 19°. How far is each bird from the worm.

Sol:



Let x = Distance b/w robin & worm

y = Distance b/w crow & worm

For x: From rt ΔABC,

$\sin 14^\circ = \frac{40}{x}$

$\Rightarrow x = \frac{40}{\sin 14^\circ} = \boxed{165.3\,ft}$ Ans

For y: From rt ΔCDE,

$\sin 19^\circ = \frac{55}{y} \Rightarrow y = \frac{55}{\sin 19^\circ} = \boxed{168.93\,ft}$ Ans

Q.8: The angle of elevation of a building is 48° from A and 61° from B. If AB is 20m, find the height of the building.

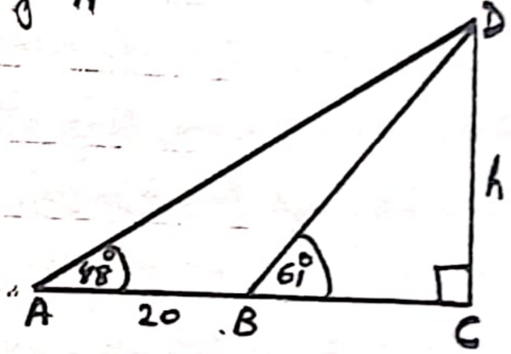
Sol: Let height of the building $= h$

We first BC.

From rt $\triangle$ BCD,

$\tan 61^\circ = \frac{h}{BC}$

$\Rightarrow BC = \frac{h}{\tan 61^\circ} \longrightarrow (1)$



Now From rt $\triangle$ ACD,

$\tan 48^\circ = \frac{h}{20+BC} \Rightarrow h = (20+BC)\tan 48^\circ$

$\Rightarrow h = \left(20 + \frac{h}{\tan 61^\circ}\right)\tan 48^\circ$ (using (1))

$\Rightarrow h = 20\tan 48^\circ + h\frac{\tan 48^\circ}{\tan 61^\circ}$

$\Rightarrow h - h\frac{\tan 48^\circ}{\tan 61^\circ} = 20\tan 48^\circ$

$\Rightarrow h\left(1 - \frac{\tan 48^\circ}{\tan 61^\circ}\right) = 20\tan 48^\circ$

$\Rightarrow h\left(\frac{\tan 61^\circ - \tan 48^\circ}{\tan 61^\circ}\right) = 20\tan 48^\circ$

$\Rightarrow h = \frac{20\tan 48^\circ \tan 61^\circ}{\tan 61^\circ - \tan 48^\circ} = \boxed{57.9}$

Hence the height of the building is $\boxed{57.9m}$ Ans