



COMPREHENSIVE NOTES

Physics

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Including

- Theory
- Solution of Examples, Assignments, Quiz
- Solution of Exercise
- Solution of difficult MCQ's
- ETEA Test MCQ's
- Dimension of Physical Quantities
- Collection of Formulas
- Board Papers



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CHAPTER

01

MEASUREMENTS

Q1: What is physics? Discuss the importance of physics. (OR) Define physics. Explain the scope and importance of physics in today's technological world.

Ans. (a) Physics:

"The branch of science which deals with the properties of matter, energy and their mutual relationship is known as physics."

(b) Scope and Importance of Physics in today's Technological World:

Physics has played an important role in our daily life. For example;

- 1. The electronic media like T.V, radio, loud speaker, cell phone, photo phone, fax machine etc are the important sources of communication which works on the principles of physics.*
- 2. Information technology is based on the principle of physics which is the scientific method of storing, protecting, processing, transmitting, receiving and retrieving information.*
- 3. The whole world has become a global village due to use of computer networks which works on the principle of physics.*
- 4. Atomic bomb is the invention of physics which can be used for the production of huge amount of energy. It can be also used for defence purposes.*
- 5. X-rays, γ -rays, laser rays etc are the inventions of physics which can be used in industry, health and agriculture department for various purposes.*
- 6. Cathode Ray Oscilloscope [C.R.O] is the invention of physics which can be used as graph plotting device to see the wave form of a repetitive electronic signal. The C.R.O can be also used in T.V circuit, in computers system and in A.C circuits or D.C circuits.*
- 7. All means of transport, like aeroplane, buses, motor cycles, railways etc work on the principles of physics.*

8. Electricity is the invention of physics, without which we cannot imagine any machine in working conditions.
9. Rockets and satellites are the invention of physics with the help of which we can get information about the upper atmosphere.
10. The simple machine like pulley, wedge, lever, screw-jack etc are the invention of physics which can be used for various purposes in daily life.
11. Physics explains the natural phenomena in terms of fundamental principles and basic laws.
12. Electron microscope is the invention of physics which is used by various departments for various purposes.
13. Telescope is the invention of physics which is used to see distant objects clearly.
14. Microscope is the invention of physics which can be used to study micro organisms clearly and easily.
15. Physics employs the essential steps of scientific method i.e. observation, recording, analysis, prediction and verification.

Q2: Define measurement, unit and magnitude?

Ans: (a) Measurement:

The comparison of something with some standard is known as measurement.

(b) Unit:

The standard with which things are compared, is known as unit.

(c) Magnitude:

A number with proper unit is known as magnitude. For example "15kg", here 15 represent proper number and "kg" represent proper unit.

Q3: (a) Define physical quantities. (b) Explain base physical quantities and derived physical quantities.

Ans: (a) Physical Quantities:

All the measurable quantities are known as physical quantities. For example, speed, mass, force, velocity, momentum, temperature etc.

(b) (i) Base Physical Quantities:

These are those physical quantities in terms of which other physical

quantities can be defined. The base quantities cannot be defined in terms of other physical quantities.

Example of Base Physical Quantities:

There are seven base quantities, which are given below:

- Mass
- Length
- Time
- Current
- Temperature
- Amount of substance and
- Luminous intensity

(ii) Derived Physical Quantities:

Those physical quantities which can be derived from base quantities, are known as derived physical quantities.

Examples of Derived Physical Quantities:

Force, work, power, energy, momentum, torque, speed, acceleration etc are the examples of derived physical quantities.

Q4: Define system of units. Give the name of various systems of units.

Ans: A complete set of units for all physical quantities is known as system of units.

Explanation:

There are several system of units. For example,

- Foot - pound - second system [FPS]
- Meter - kilogram - second system [MKS]
- Centimeter - gram - second system [CGS]

But the most popular system is the system international [SI].

System International [SI]:

In 1960, an international committee agreed on a set of definitions and standards to describe the physical quantities. The system that was established is called the system international [SI].

The system international is used throughout the world. Its units are convenient for daily life. The system international [SI] is build-up from three kinds of units i.e.

- Base units

- Derived units
- Supplementary units

Q5: Define (a) base units (b) derived units (c) supplementary units.

Ans: (a) Base Units:

The units of base quantities are known as base units. The base quantities and their units are given in the following table:

S.No	Base Quantities	Symbol	Unit of Base Quantity	Symbol
1	Length	l	Meter	m
2	Mass	M	Kilogram	Kg
3	Time	t	Second	S
4	Electric current	I	Ampere	A
5	Temperature	T	Kelvin	K
6	Intensity of light	L	Candela	Cd
7	Amount of substance	n	Mole	mol

(b) Derived Units:

The units of derived quantities are known as derived units. The derived units can be derived from base units and supplementary units. Some of the derived quantities and their units are given in the following table:

Physical Quantities	Unit	Symbol	Terms of base units
Force	Newton	N	$Kg \cdot m \cdot sec^{-2}$
Work	joule	J	$N \cdot m = Kg \cdot m^2 / sec^2$
Power	Watt	W	$J/sec = Kg \cdot m^2 / sec^3$
Pressure	Pascal	Pa	$N/m^2 = Kg/m \cdot sec^2$
Electric charge	Coulomb	C	$Amp \cdot Sec = A \cdot S$

(c) Supplementary Units:

The general conference on weights and measures has not yet classified certain units of the SI under either base units or derived units. Such SI units are called supplementary units. This class contains only two units of purely geometrical quantities, which are plane angle and the solid angle.

The supplementary unit of plane angle is radian while that of the

solid angle is steradian.

1. Radian: (ETEA 2005, BISE Pesh 2019)

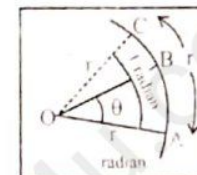
It is the unit of plane angle. One radian represents a plane angle subtended at the centre of a circle by an arc of length equal to radius of the circle.

$$1 \text{ revolution} = 2\pi \text{ radian} = 360^\circ$$

$$\Rightarrow 2\pi \text{ radian} = 360^\circ$$

$$\Rightarrow 1 \text{ radian} = \frac{360^\circ}{2\pi}$$

$$\Rightarrow 1 \text{ radian} = 57.3^\circ$$



2. Steradian: (ETEA 2011)

Steradian is the unit for a solid angle in a three dimensional space. Steradian is defined as "the solid angle subtended at the centre of a sphere by an area of its surface equal to the square of the radius of that sphere."

The surface area of a closed sphere of radius "r" is $4\pi r^2$. So according to the definition of steradian, the solid angle subtended by a closed sphere at its centre is $4\pi r^2$ divided by r^2 , which is " 4π " i.e.

$$1 \text{ steradian} = \frac{4\pi r^2}{r^2}$$

$$\Rightarrow 1 \text{ steradian} = 4\pi$$

In fact any closed surface subtends a solid angle of " 4π " at any of its interior point.



Physical Quantity	Unit	Symbol
1. Plane angle	radian	rad
2. Solid angle	Steradian	Sr

Q6: What is scientific notation? Explain. (ETEA 2015)

ETEA 2015

Ans: Writing numbers in power of ten or standard form $N = M \times 10^n$ is called scientific notation.

Explanation:

With the help of scientific notation, we can express very large number or small number very easily. A large or small number "N" can be expressed in terms of a number "M" and a power of "10" i.e.

$$N = M \times 10^n \quad (1)$$

In equation (1), "M" represents a number whose first digit is non-zero digit. "n" represents the power of 10, which may be positive or negative.

For example, 150,000,000,000m can be expressed in scientific notation as under,

$$150,000,000,000\text{m} = 1.5 \times 10^{11}\text{m}$$

Similarly, 0.000,000,000,000,0016m can be expressed in scientific notation as under,

$$0.000,000,000,000,0016\text{m} = 1.6 \times 10^{-15}\text{m}$$

Q7: What do you mean by "prefixes of the power of ten"? Explain.

ETEA 2013, 2014

Ans. These are certain specific powers of ten which have been given special names and symbols and are frequently used in measurement.

Explanation:

Sometimes, we have to deal with quantities that are very large or very small as compared to the base units. Under such conditions, we use higher or lower unit of the base unit.

For convenience, we express a large or small numbers by certain prefixes, such as centi, milli, micro, kilo etc to the number. They are given special names and symbols and certain power of 10 to multiply a quantity.

Prefix	Equivalent to	Symbol	Prefix	Equivalent to	Symbol
Atto	10^{-18}	a	kilo	10^3	K
Femto	10^{-15}	f	mega	10^6	M
Pico	10^{-12}	p	giga	10^9	G
Nano	10^{-9}	n	tera	10^{12}	T
Micro	10^{-6}	μ	peta	10^{15}	P
Milli	10^{-3}	m	exa	10^{18}	E
Centi	10^{-2}	c			
Deci	10^{-1}	d			
Deca	10^1	da			

Q8: Discuss the conventions for using SI units.

Ans. The various conventions for using SI units are given below:

1. Full name of the unit does not begin with a capital letter even if named after a scientist. For example "Newton" represent the name of the scientist while "newton" represent the name of a unit used for force.
2. Symbol of unit name after scientist has initial capital letter. For example, N for newton, Pa for pascal, Hz for hertz etc.
3. The prefix [μ, m, p etc] is written before an adjacent to the unit. For example, 10μm, 15kwatt, 100pc, 10mm etc.
4. The combination of units is written each with one space apart. For example, Torque = 55Nm
5. Double, triple etc prefixes are not allowed. For example, 5μF may be written as, 1pF.
6. When a multiple of base unit is raised to a power. The power applies to the whole multiple and not just the base unit. For example,
 $1\text{Km}^2 = 1(\text{Km})^2 = 1[10^3\text{m}]^2 = 1 \times 10^6\text{m}^2$
7. Before substituting values in the formula to calculate the final result, make sure that all the included physical quantities are expressed in their SI units. For example, if a car is covering a distance of 2km in 2 minutes. then its speed can be calculated in the following way:

$$\text{Distance travelled} = S = 2\text{km} = 2 \times 10^3\text{m}$$

$$\text{Time taken} = t = 2\text{min} = 120\text{sec}$$

[m, sec represents SI units]

$$\text{Speed of car } v = ?$$

We know that,

$$S = vt \Rightarrow v = \frac{S}{t}$$

$$\Rightarrow v = \frac{2 \times 10^3}{120} \Rightarrow v = \frac{2000}{120} = 16.66\text{m/sec}$$

$$\Rightarrow v = 16.66\text{m/sec} \text{ [m/sec is also SI unit of speed]}$$

Q9: Discuss the conversion of units in detail.

Ans. We can convert the unit of one system into the unit of another system of the same physical quantity. For this purpose we should know

about relations between various units, some of which are given below:

10mm = 1cm	1mile = 1.609km
100cm = 1m	1m = 3.281 ft
1000m = 1km	1 pound force = 4.4N
1 inch = 2.54cm	1gm = 10^{-3} kg
12 inch = 1 foot	1 milli sec = 10^{-3} sec
3 foot = 1 yard	1 min = 60 sec
1760 yards = 1 mile	1 hr = 3600 sec
	1N = 0.22481b
24 hr = 86400 sec	1 micro sec = 10^{-6} sec
60 min = 1 hr	1 day = 24 hr
1 year = 3.5×10^7 sec	1 rad = 57.30°
	1o = 0.01745 rad

Example: Convert 2 minutes into seconds:

Answer: We know that,

$$1 \text{ min} = 60 \text{ sec then}$$

$$2 \text{ min} = ?$$

Now by cross multiplication, we get,

$$\frac{(2 \times 60)}{1} = 120 \text{ sec}$$

$$\Rightarrow 2 \text{ min} = 120 \text{ sec}$$

Do you know:

There are some non-SI related units for distances extensively used around the world. Some of these units are discussed below:

1. Light Year: A light-year (ly), is a non-SI unit of length. It is defined as the distance that light travels in a vacuum in one year, which is 9.4607×10^{15} m. For example spiral galaxies like our own Milky Way measure approximately 2×10^5 light-years in diameter. Our nearest-neighbor galaxy is the great spiral galaxy Andromeda, which has been determined to be approximately 2.5 million light years away.

2. Angstrom: The angstrom or angstrom (Å) is also a non-SI and internationally recognized unit of length equal to 1×10^{-10} meters (m) or 0.1 nanometer (nm). For example, a helium atom has a size of about 1 Angstrom, while its nucleus is only 1 femtometer (10^{-15} meters) in diameter.

3. Micron: A micron (μ) is an obsolete name of a micrometer, which is a decimal fraction of the meter equal to 1×10^{-6} meters. For examples red blood cells (RBCs) are approximately 10 microns in diameter. Human hair is between 10 and 100 microns in diameter.

Example: Convert 5 inch into centimeter.

Answer: We know that,

$$1 \text{ inch} = 2.54 \text{ cm then}$$

$$5 \text{ inch} = ?$$

Now by cross multiplication, we get,

$$\frac{(5 \times 2.54)}{1} = 12.72 \text{ cm}$$

$$\Rightarrow 5 \text{ inch} = 12.70 \text{ cm}$$

Q10: Define error. Discuss the various types of error?

BISE DI Khan 2019

Ans: (a) Error:

The difference between measured value and actual value is known as error.

Types of Errors:

There are three types of errors which are given below:

1. Personal Error:

The error due to carelessness or improper knowledge about an instrument or incorrect reading of a scale by an experimenter is known as personal error.

Elimination:

The personal error can be removed to some extent if the experimenter take care during the measurement of any quantity and have proper knowledge about the measuring instrument.

2. Systematic Error:

The error due to faulty apparatus and poor calibration of the instrument is known as systematic error. The systematic error also arises from zero error between scales as in vernier calipers, screw gauge and spherometer.

Least Count Error:

The smallest value that can be measured by the measuring instrument is called its least count. All the readings or measured values are good only up to this value. The least count error is the error associated with the resolution of the instrument. For example, a vernier calipers has the least count of 0.01cm; a spherometer may have a least count of 0.001cm. Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors. We can reduce the least count error by using instruments with higher resolution, improving experimental techniques etc.

Elimination:

The systematic error can be removed by comparing the faulty instrument with some accurate and standard instrument. The difference of the two can be added or subtracted to the measurement.

If the error is positive, then it will be subtracted from faulty instrument reading for correctness. If the error is negative, then it will be added to the faulty instrument reading for correctness.

3. Random Error (or) Accidental Error (or) Statistical Error:

The error due to repeated measurement of same quantity under same condition that results in different values of the same quantity is known as random error. (OR)

The error which occurs due to accidental changes in experimental conditions is known as random error.

The random error can arise due to the change in temperature, voltage, humidity or other unknown causes.

Elimination:

The random error can be removed by maintaining strict controlled conditions in the laboratory. Such error can be also reduced by taking several readings and then take their average value.

Q11: What is uncertainty in measurement? Write down its types.

Ans: Uncertainty:

Definition: Uncertainty is the error due to natural imperfection of a person, limitation of apparatus and abrupt changes in surrounding. (OR)

The range of possible values within which the true value of the measurement lies is called uncertainty in measurement.

Explanation:

Uncertainty estimates how small or large is the error in a measurement. In general we have,

$$\text{Measurement} = \text{best estimate} + \text{uncertainty}$$

Example: Let the length of a solid rod is $(10 \pm 0.1)\text{cm}$. In general form we can write this measurement as,

$$\text{Measurement of length of solid rod} = L \pm \Delta L$$

Here $L = 10\text{cm}$ represent the best estimated value of length and ΔL

$= 0.1\text{cm}$ represents the uncertainty in the measurement. The result $(10 \pm 0.1)\text{cm}$ shows that the actual value of length of the rod lie between 9.9cm and 10.1cm .

Types of Uncertainty:

There are three types of uncertainty which are given below:

1. Absolute Uncertainty:

It represents the maximum possible uncertainty in any measurement. It is approximately equal to the least count of the instrument. It is represented by the symbol " Δ " followed by the symbol of quantity. For example let the length of a rod $= (L \pm \Delta L)\text{cm} = (10 \pm 0.1)\text{cm}$

Here $\Delta L = 0.1\text{cm}$ represented the absolute uncertainty in a measurement. As $\Delta L = 0.1\text{cm}$ represent the magnitude of error which is also called precision. Thus the precision represent the absolute uncertainty in a measurement.

2. Relative or Fractional Uncertainty:

The absolute uncertainty divided by the actual value is called relative or fractional uncertainty. i.e.

$$\text{Relative uncertainty} = \frac{\text{absolute uncertainty}}{\text{actual value}}$$

For example in the measurement $(10 \pm 0.1)\text{cm}$, the relative uncertainty can be determined as,

$$\text{Relative uncertainty} = \frac{0.1\text{cm}}{10\text{cm}}$$

$$\Rightarrow \text{Relative uncertainty} = 0.01$$

Relative uncertainty has no unit because it is the ratio between two quantities having same units. The relative uncertainty gives us the accuracy of the measurement. i.e. smaller the relative uncertainty, greater will be the accuracy of the measurement and vice versa.

3. Percentage Uncertainty:

The relative uncertainty multiplied by 100 gives us the percentage uncertainty. i.e.

$$\text{Percentage uncertainty} = \text{relative uncertainty} \times 100$$

$$\text{OR Percentage uncertainty} = \frac{\text{absolute uncertainty}}{\text{actual value}} \times 100$$

Q12. Differentiate between precision and accuracy in the measurement.

Ans. Precision and accuracy tells us about the quality of a measurement. Accuracy reflects how much a measured value is closer to the actual value. Precision reflects how the measured values of different experimenters are close to each other, even if they are far from the actual value.

1. Precision:

Precision refers to how the measurements of different persons about the same thing are close to each other. Precision does not need any standard value of a quantity for comparison.

Explanation:

Case-1: If we made the measurements with same apparatus, then we take those results to be more precise which are close to each other.

Example: Let the diameter of sphere measured by four persons with the help of screw gauge are 2.12cm, 2.13cm, 2.14cm and 2.19cm.

Now out of these measurements, 2.12cm, 2.13cm and 2.14cm are more precise because these three results are close to each other.

Case-2: If we made the measurement with different apparatus, then the measurement of that apparatus will be more precise whose least count is smaller.

Example: Let we measure the length of solid cylinder with three different instruments. Let the results are 3.2cm, 3.25cm and 3.253cm. Now out of these readings, "3.253 cm" will be more precise reading because it has smallest least count as compare to the other two results.

2. Accuracy:

Accuracy refers to how a measured value is closer to the actual value. That is the measurement will be more accurate if the measured value is closer to the actual value and vice versa. Accuracy needs a standard value of a quantity for comparison.

Explanation:

Accuracy is inversely proportional to the relative error. i.e. greater will be the accuracy of a measurement if the relative error is smaller and vice versa.

The accuracy also depends upon the number of significant digits given in a measurement i.e. greater the number of significant digits, greater will be the accuracy and vice versa.

Example 1:

Case-1: Let the actual value of atmospheric pressure be 75.8cm of mercury. Suppose we determine the atmospheric pressure by using Boyle's law and get 74.5cm of mercury. Now in such a case the absolute error is 1.3cm of mercury. Now the relative error is given by,

$$\text{Relative error} = \frac{\text{absolute error}}{\text{actual value}} = \frac{1.3}{75.8}$$

$$\Rightarrow \text{Relative error} = 0.017$$

Case-2: Let the actual value of specific heat of copper is 0.39 J/gm·C° and experimental value is 0.38 J/gm·C°. Now the absolute error in this case is 0.01 J/gm·C°.

Now the relative error is given by,

$$\text{Relative error} = \frac{\text{absolute error}}{\text{actual value}}$$

$$\Rightarrow \text{Relative error} = \frac{0.01}{0.39} = 0.026$$

$$\Rightarrow \text{Relative error} = 0.026$$

Conclusion: In case 1, the absolute error is greater, while in case 2, the absolute error is less. But the relative error in case 1 is smaller than case 2. So in case 1, we get more accurate result than case 2. So it is the relative error with the help of which we can judge the accuracy of the result.

Example 2:

Let the length of a rod measured in different experiments are 5.2cm, 5.25cm, 5.253cm respectively. Now out of these readings the reading 5.253cm is more accurate because this reading contain more significant figures.

Q13: Explain "rounding off numbers"?

Ans: The quantities in a measurement are always rounded off to certain level of accuracy. The following rules must be observed while rounding off a number.

1. If the dropping digit is less than 5, then the last retained digit is kept as it is. For example, 863.73 is rounded to 863.7.
2. If the dropping digit is greater than 5, then the last retained digit is increased by 1. For example, 9.66 is rounded to 9.7.
3. If the dropping digit is 5, and the last retained digit is odd, then it will be increased by 1. For example, 76.35 is rounded to 76.4.
4. If the dropping digit is "5" and the last retained digit is even, then it will remain unaffected. For example, 66.45 is rounded to 66.4.

Q14: What do you mean by significant figures? Give the rules for determining significant figures.

ETEA 2006-2007, 2008, 2009, 2012, 2014

Ans: Significant Figures [اعداد کے معنی فہم ہوتے]:

The combination of accurately known digits and the first doubtful digit during a measurement are known as significant figures.

Explanation:

Suppose we want to measure the length of a rod with the help of meter rod. Let the length of the rod measured by three persons are 10.73cm, 10.72cm and 10.74cm respectively.

Now in this case, we agree with 10.7, so these digits are known as accurately known digits. But there is doubt about the fourth digit. We are in suspense that which one among 3, 2, 4 is accurate digit. Such digit is known as doubtful digit.

Thus, in above example, there are four significant figures, out of which three are accurately known and fourth one is doubtful.

Rules for determining significant figures:

The following rules should be kept in mind while determining significant figures in a given data.

1. All non-zero digits from 1 to 9 are significant. For example, in 36.473m, there are five significant figures.
2. The zeros between two non-zero digits are significant. For ex-

ample in $y = 4.03\text{m}$ there are three significant figures.

3. The zeros to the right of a decimal symbol and to the left of non-zero digits are not significant. For example, in 0.000084m , there are two significant figures, i.e. 8 and 4.
4. All zeros to the right of a decimal that appears after non-zero digit are significant. For example, in $x = 3.5000$, there are five significant figures.
5. In case of scientific notation, the figures before the power of ten are significant. For example, in $m = 8.02 \times 10^5\text{kg}$, there are three significant figures i.e. 8, 0 and 2.

6. Addition and Subtraction:

In case of addition and subtraction, the result obtained is retained to the measurement which is least precise in the data. For example, let $x_1 = 3.5\text{cm}$, $x_2 = 8.735\text{cm}$ and $x_3 = 9.73\text{cm}$ represents the lengths of three objects respectively. Let we calculate the total length. For this purpose, we will add x_1 , x_2 and x_3 , so we have,

$$x = x_1 + x_2 + x_3$$

$$\Rightarrow x = 3.5\text{cm} + 8.735\text{cm} + 9.73\text{cm}$$

$$\Rightarrow x = 21.965\text{cm}$$

Now in given data, we see that 3.5cm is least precise, so we round off the final result [i.e. $x = 21.965\text{cm}$] to two significant figures as,

$$x = 21.965\text{cm} = 21.96\text{cm} = 22\text{cm}$$

$$\Rightarrow x = 22\text{cm}$$

7. Multiplication and Division:

In case of multiplication and division, the final result is retained to the measurement which contains the least number of significant figures. For example, let $x_1 = 3.6\text{cm}$ and $x_2 = 3.856\text{cm}$ represents the lengths of two objects respectively. Let we multiply x_1 with x_2 to get the final result, then we have,

$$x = x_1 \times x_2$$

$$\Rightarrow x = 3.6\text{cm} \times 3.856\text{cm} = 13.8816\text{cm}^2$$

$$\Rightarrow x = 13.8816\text{cm}^2$$

Now in given data, we see that 3.6cm contains the least number of significant figures, i.e. it contains only two significant figures, so the

final result [i.e. $x = 13.8816\text{cm}^2$] is also rounded off to two significant figures as,

$$x = 13.8816\text{cm}^2 = 13.882\text{cm}^2 = 13.88\text{cm}^2$$

$$\Rightarrow x = 13.9\text{cm}^2 = 14\text{cm}^2$$

$$\Rightarrow x = 14\text{cm}^2$$

8. Rounding off Non-Significant Figures:

In case of rounding off non-significant figures, the following rules should be followed.

- 1) If the dropping digit is less than 5, then it will be just ignored. For example, 863.73 is rounded to 863.7.
- 2) If the dropping digit is greater than 5 then the last retained digit is increased by 1. For example, 8.76 is rounded to 8.8.
- 3) If the dropping digit is 5 and the last retained digit is even, then it will remain unaffected. For example, 56.45 is rounded to 56.4.
- 4) If the dropping digit is 5 and the last retained digit is odd, then it will be increased by 1. For example, 46.35 is rounded to 46.4.

Q15: Discuss the assessment of uncertainty in the final result?

Ans. We can calculate an error or uncertainty in the final result by the following method:

1. Sum (OR) Difference Rule:

Suppose x_1 and x_2 are two quantities and we want to add them or subtract them to get the final result x .

$$\text{Let } x_1 = 4.5 \pm 0.1\text{cm}$$

$$\text{And } x_2 = 14.6 \pm 0.1\text{cm}$$

Let we add x_1 and x_2 to get the final result x , then we have,

$$x = x_1 + x_2$$

$$\Rightarrow x = [14.6 \pm 0.1] + [4.5 \pm 0.1]$$

$$\Rightarrow x = [(14.6 + 4.5)] \pm (0.1 + 0.1)\text{cm}$$

$$\Rightarrow x = (19.1 \pm 0.2)\text{cm}$$

2. Product and Quotient Rule:

In this case, the quantities are multiplied (or) divided by each other, while the percentage uncertainty in the final result is equal to the sum of the separate percentage uncertainties. For example, we

want to calculate the resistance of a conductor, then we have,

$$V = IR \Rightarrow R = \frac{V}{I} \quad (1)$$

$$\text{Let } V = (5.2 \pm 0.1)\text{V}$$

$$\text{And } I = (0.841 \pm 0.05)\text{A}$$

Now the percentage uncertainty for "V" is given by,

$$V = \left(\frac{0.1}{5.2}\right) \times 100 = 1.9\% = 2\%$$

While the percentage uncertainty for "I" is given by,

$$I = \left(\frac{0.05}{0.841}\right) \times 100 = 5.9\% = 6\%$$

Now the total uncertainty in "R" = 2% + 6%

$$\Rightarrow \text{Total uncertainty in "R"} = 8\%$$

Now equation (1)

$$\Rightarrow R = \left(\frac{5.2}{0.841}\right) \pm 8\% = 6.2 \pm 8\% \Rightarrow R = (6.2 \pm 8\%) \Omega \quad (2)$$

In terms of fractional uncertainty, equation (2) can be written as,

$$R = \left[6.2 \pm \left(6.2 \times \frac{8}{100}\right)\right] \Omega = (6.2 \pm 0.5) \Omega$$

$$\Rightarrow R = (6.2 \pm 0.5) \Omega \quad (3)$$

3. Power of a Quantity: (ETEA 2012)

In case of power, the individual percentage uncertainty is multiplied with the power in the final result. For example, let we want to calculate the volume of a sphere by using the following formula, i.e.

$$V = \frac{4}{3} \pi r^3 \quad (1)$$

$$\text{Let } r = (2.25 \pm 0.01)\text{cm}$$

Now the percentage uncertainty in "r" is given by,

$$\left(\frac{0.01}{2.25}\right) \times 100 = 0.4\%$$

Now total uncertainty in "r" of power "3" is given by,

$$0.4\% \times 3 = 1.2\%$$

Now equation (1)

$$\Rightarrow V = \frac{4}{3} \times 3.14 \times (2.25)^3 \pm 1.2\%$$

$$\Rightarrow V = (47.689 \pm 1.2\%) \text{cm}^3 \quad (2)$$

In terms of fractional uncertainty, equation (2) becomes,

$$V = \left(47.689 \pm 47.689 \times \frac{1.2}{100} \right) \text{cm}^3$$

$$\Rightarrow V = (47.689 \pm 0.57) \text{cm}^3$$

$$\Rightarrow V = (47.69 \pm 0.6) \text{cm}^3$$

$$\Rightarrow V = (47.7 \pm 0.6) \text{cm}^3 \quad (3)$$

4. Uncertainty in Average Value:

In case of average, the following steps should be followed:

1. First we find the average of individual values.
2. Then we find the deviation of each individual value from average value i.e. we subtract each individual value from mean value by ignoring negative sign.
3. Then we find the average of deviated values. This average gives us the uncertainty. For example, let $x_1 = 1.5 \text{cm}$, $x_2 = 1.51 \text{cm}$ and $x_3 = 1.52 \text{cm}$

(i) Now the average of these individual values is given by,

$$x = \frac{x_1 + x_2 + x_3}{3} = \frac{1.50 + 1.51 + 1.52}{3}$$

$$\Rightarrow x = 1.51 \text{cm}$$

(ii) Now deviation of $x_1 = \Delta x_1 = x - x_1$

$$\Rightarrow \Delta x_1 = 1.51 - 1.50 \Rightarrow \Delta x_1 = 0.01 \text{cm}$$

Now deviation of $x_2 = \Delta x_2 = x - x_2$

$$\Rightarrow \Delta x_2 = 1.51 - 1.51 = 0 \Rightarrow \Delta x_2 = 0 \text{cm}$$

Now deviation of $x_3 = \Delta x_3 = x - x_3$

$$\Rightarrow \Delta x_3 = 1.51 - 1.52 = -0.01 \text{cm} = 0.01 \text{cm}$$

$$\Rightarrow \Delta x_3 = 0.01 \text{cm}$$

(iii) Now the average of deviated values is given by,

$$x = \frac{\Delta x_1 + \Delta x_2 + \Delta x_3}{3}$$

$$\Rightarrow \Delta x = \frac{0.01 + 0 + 0.01}{3} = 0.0067 \text{cm} = 0.007 \text{cm} \Rightarrow \Delta x = 0.007 \text{cm}$$

Now the final result is given by,

$$\text{Final result} = x + \Delta x = (1.51 \pm 0.007) \text{cm}$$

$$\Rightarrow \text{Final result} = (1.51 \pm 0.007) \text{cm}$$

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Uncertainty in Timing Experiment:

Let "T" be the time period of a simple pendulum (or) any other vibrating body, then "T" can be calculated as,

$$T = \left[\frac{\text{total time of vibrations}}{\text{total no. of vibrations}} \pm \text{Fractional uncertainty} \right] \quad (1)$$

Let a simple pendulum completes 30 vibrations in 54.6 sec. Let the least count of the stopwatch is 0.1 sec. Now here we have,

$$\text{Total time of vibration} = 54.6 \text{sec}$$

$$\text{Total number of vibrations} = 30$$

$$\text{Fractional uncertainty} = \frac{0.1}{30} = 0.003 \text{sec}$$

Now equation (1)

$$T = \left(\frac{54.6}{30} \pm 0.003 \right) \text{sec}$$

$$\Rightarrow T = (1.82 \pm 0.003) \text{sec}$$

Q16: (a) What is meant by dimensions of physical quantities? (b) What are the applications and (c) the limitations of dimensional analysis?

Ans. (a) Dimension of Physical Quantity:

The powers [positive or negative] to which the fundamental units of first system must be raised to give the unit of a physical quantity [derived unit] are called dimensions of that quantity.

Explanation:

The dimension represents the nature of a physical quantity. It tells us that how a quantity is related to the base units. It is represented by capital letters enclosed in a square brackets. For example;

1. Dimension of mass, length, time are respectively written as, [M], [L], [T].

2. The dimension of area = $A = \text{m}^2$

$$\Rightarrow A = (\text{length})^2 = \text{length} \times \text{length}$$

$$\Rightarrow \text{Area} = L \times L = [L^2]$$

So the dimension of area = $[L^2]$

3. Dimension of speed = $\frac{\text{meter}}{\text{sec}}$

$$\Rightarrow [V] = \frac{\text{dimension of length}}{\text{dimension of time}} = \frac{[L]}{[T]}$$

$$\Rightarrow [V] = [LT^{-1}]$$

$$4. \text{ Dimension of acceleration} = \frac{\text{meter}}{\text{sec}^2}$$

$$\Rightarrow [a] = \frac{\text{dimension of length}}{[\text{dimension of time}]^2} = \frac{[L]}{[T^2]} \Rightarrow [a] = [LT^{-2}]$$

b. Uses of Dimensions (OR) Advantages of Dimensions (OR)

Importance of Dimensions (OR) Applications of Dimensions:

1. The dimensions are used for checking the correctness of an equation or formula.
2. The dimensions are used for deriving certain formula for a physical quantity.
3. The dimension are used to check the homogeneity of a physical equation.

c. Limitations of Dimensional Analysis:

1. Using this method, we cannot find the value of dimensionless constant.
2. We cannot apply this method to equations involving exponential and trigonometric function.
3. It tells us only that whether a physical relation is dimensionally correct or not. It cannot tell whether the relation is absolutely correct or not. For example, $S = vt + \frac{1}{3}at^2$ is dimensionally correct but the absolute correct relation is, $S = vt + \frac{1}{2}at^2$.

Fundamental Dimensions	
Quantity	Symbol
Length	[L]
Mass	[M]
Time, duration	[T]
Electric current	[I]
Thermodynamic temperature	[Θ]
Amount of substance	[N]
Luminous intensity	[J]

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Some Important Derived Dimensions

Quantity	Dimension	Quantity	Dimension
Velocity	[LT ⁻¹]	Area	[L ²]
Acceleration	[LT ⁻²]	Volume	[L ³]
Force	[MLT ⁻²]	Pressure	[ML ⁻¹ T ⁻²]
Work or energy	[ML ² T ⁻²]	Period	[T]
Power	[ML ² T ⁻³]	Frequency	[T ⁻¹]

Q17: Explain the following terms with examples:

- a. Dimensional variables
- b. Dimensional constants
- c. Dimensionless variables
- d. Dimensionless constants

Ans. (a) Dimensional Variables:

Those physical quantities which have dimensions but variable magnitude are known as dimensionally variable quantities. For example, momentum, force, acceleration, torque, energy, speed etc.

(b) Dimensional Constants:

The physical quantities which have dimensions and having constant magnitude are known as dimensional constant. For example, planks constant, gravitational constant, speed of light in vacuum, ideal gas constant "R" etc.

(c) Dimensionless Variables:

Those physical quantities which have no dimensions and having variable magnitude are known as dimensionless quantities. For example, coefficient of friction, linear magnification, strain, solid angle, plane angle etc.

(2) Dimensionless Constants:

Those quantities which have no dimensions and are constants are known as dimensionless constants. For example, pure numbers i.e. 1, 2, 3, and $\pi = \frac{22}{7}$ etc.

Solution of Examples & Assignments
EXAMPLE 1.1:

Two connected gears are rotating. The smaller gear has a radius of 0.4m and the larger gear's radius is 0.7m. What is the angle through which the larger gear has rotated when the smaller gear has made one complete rotation?

Solution:

Given data: Larger gear's radius = " r_L " = 0.7m

Smaller gear's radius " r_s " 0.4m

Required: Angle of rotation for larger gear = " θ_L " = ?

Calculation: The smaller gear complete one revolution, so we have,

$$\theta_s = 2\pi \text{ radian}$$

The length of the arc traveled is given by,

$$S = r\theta \Rightarrow S = r_s\theta_s \quad (1)$$

Putting the values in equation (1), we get,

$$S = 0.4\text{m} \times 2\pi \Rightarrow S = 0.8\pi\text{m}$$

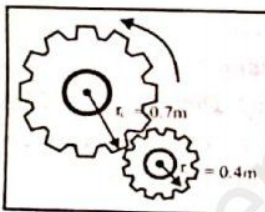
Now for larger gear, we have,

$$S = r_L\theta_L \Rightarrow \theta_L = \frac{S}{r_L} \quad (2)$$

Putting the values of " S " and " r_L " in equation (2), we get,

$$\theta_L = \frac{0.8\pi\text{m}}{0.7\text{m}} \Rightarrow \theta_L = 1.143\pi \text{ radian}$$

$$\Rightarrow \theta_L = 1.143 \times 3.14 \text{ radian} \Rightarrow \theta_L = 3.6 \text{ radian}$$

**EXTENSION EXERCISE**

What is the angle in degrees through which the larger gear has rotated?

Solution:

Given data: $\theta_L = 3.6$ radian

Required: θ_L in degree = ?

Calculation: We know that

$$2\pi \text{ radian} = 360^\circ$$

$$3.6 \text{ radian} = ?$$

By cross multiplication, we get

$$\frac{3.6 \text{ rad} \times 360^\circ}{2\pi \text{ rad}} = \frac{3.6 \times 360^\circ}{2 \times 3.14} = \frac{1296^\circ}{6.28}$$

$$\Rightarrow \theta_L = 206.4^\circ$$

ASSIGNMENT 1.1:

A pulley of radius 0.9m is used to lift a bucket from the well. If it took 3.6 rotations for the pulley to take water out of the well, how deep is water in the well? (Answer = 20m)

Solution:

Given data: Radius of pulley = $r = 0.9\text{m}$

Number of rotation = $n = 3.6$

Required: Depth of water = $S = ?$

Calculation: As we get water from the well after 3.6 rotation [revolution] so the length of arc traveled will give in the depth of water. So we have,

$$S = r\theta \quad (1)$$

We know that,

1 rotation [revolution] = 2π rad, then 3.6 ro-

tation = $3.6 \times 2\pi$ radian

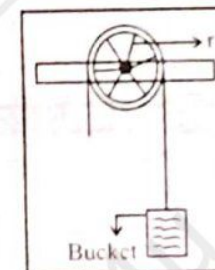
$$\Rightarrow \theta = 3.6 \times 2\pi \text{ rad}$$

Now putting the values of " r " and " θ " in equation (1), we get,

$$S = 0.9\text{m} \times 3.6 \times 2\pi$$

$$\Rightarrow S = (0.9 \times 3.6 \times 2 \times 3.14)\text{m} = 20.3\text{m} \text{ or } S = 20\text{m}$$

$$\text{Depth of water} = S = 20\text{m}$$

**EXAMPLE 1.2:**

If $d_1 \pm \Delta d_1 = (101.41 \pm 0.05)\text{mm}$ represents the internal diameter of the metal pipe and $d_2 \pm \Delta d_2 = (102.79 \pm 0.05)\text{mm}$ represents the external diameter of the metal pipe, then find the thickness of the metal part of the pipe including uncertainty in it.

Solution:**Given data:**

Internal diameter = $d_1 \pm \Delta d_1 = (101.41 \pm 0.05)\text{mm}$

External diameter = $d_2 \pm \Delta d_2 = (102.79 \pm 0.05)\text{mm}$

Required: Thickness of pipe = " t " = ?

Calculation: We have, $d_1 = 101.41\text{mm}$

Uncertainty in internal diameter $= \Delta d_1 = 0.05\text{mm}$

$$d_2 = 102.79\text{mm}$$

Uncertainty in external diameter $= \Delta d_2 = 0.05\text{mm}$

Now net diameter is given by,

$$d = (d_2 - d_1) \pm (\Delta d_1 + \Delta d_2) \quad (1)$$

Putting the values in equation (1), we get

$$d = (102.79 - 101.41)\text{mm} \pm (0.05 + 0.05)\text{mm}$$

$$\Rightarrow d = 1.38\text{mm} \pm 0.1\text{mm}$$

Now the thickness of the pipe is given by,

$$t = \frac{d}{2} = \frac{1.38\text{mm} \pm 0.1\text{mm}}{2}$$

$$\Rightarrow t = \frac{1.38\text{mm}}{2} \pm \frac{0.1\text{mm}}{2}$$

$$\Rightarrow t = 0.69\text{mm} \pm 0.05\text{mm}$$

ASSIGNMENT 1.2:

A physicist calculated the wall width of half brick thickness (the brick is laid in a flat position, lengthwise called stretcher position), as $13.6\text{cm} \pm 0.1\text{cm}$. And one brick thickness (the brick is placed in flat position, lengthwise orthogonal to wall, called header position), as $23.6\text{cm} \pm 0.1\text{cm}$. Calculate the difference in width of walls with uncertainty in it?

Solution:

Given data: Width of 1st brick $= w_1 = 13.6\text{cm} \pm 0.1\text{cm}$

Width of 2nd brick $= w_2 = 23.6\text{cm} \pm 0.1\text{cm}$

Required: Difference in width $= w = ?$

Calculation: We know that

$$w = w_2 - w_1 = (23.6\text{cm} - 13.6\text{cm}) \pm (0.1\text{cm} + 0.1\text{cm})$$

$$\Rightarrow w = 10\text{cm} \pm 0.2\text{cm}$$

EXAMPLE 1.3:

The length and width of a rectangular room are measured to be $l = (4.050 \pm 0.005\text{m})$ and $(w \pm \Delta w) = (2.995 \pm 0.005\text{m})$. Calculate the area $A = (A \pm \Delta A)$ of the room and its uncertainty.

Solution:

Given data: Length $= L = (4.050 \pm 0.005)\text{m}$

$$\text{Width} = w = (2.995 \pm 0.005)\text{m}$$

Required: Area $= "A" = (A \pm \Delta A) = ?$

Calculation: Area of room is given by,

$$\text{Area of room} = A \pm \Delta A \quad (1)$$

First we calculate the percentage uncertainty in length and width.

$$\text{For length, we have } \Delta L = \frac{0.005}{4.050} \times 100\% \Rightarrow \Delta L = 0.12\%$$

$$\text{For length, we have } \Delta w = \frac{0.005}{2.995} \times 100\% \Rightarrow \Delta w = 0.17\%$$

Now uncertainty in area $= \Delta A = \Delta L + \Delta w = 0.12\% + 0.17\%$

$$\Rightarrow \Delta A = 0.29\%$$

Now putting $A = L \times w$ and $\Delta A = 0.29\%$ in equation (1), we get,

$$\text{Area of room} = L \times w \pm 0.29\%$$

$$\Rightarrow \text{Area of room} = (4.050 \times 2.995)\text{m}^2 \pm 0.29\%$$

$$\Rightarrow \text{Area of room} = 12.13\text{m}^2 \pm 0.29\%$$

In terms of fractional uncertainty, the answer can be written as,

$$\text{Area of room} = 12.13\text{m}^2 \pm \left(\frac{0.29}{100} \times 12.13\text{m}^2 \right)$$

$$\Rightarrow \text{Area of room} = 12.13\text{m}^2 \pm 0.035\text{m}^2$$

ASSIGNMENT 1.3:

The voltage " $V(V \pm \Delta V)$ " is measured as $7.3\text{V} \pm 0.1\text{V}$ and current " $I(I \pm \Delta I)$ " is measured as $2.73\text{A} \pm 0.5\text{A}$. Calculate the resistance " R "

by using Ohm's law as $R = \frac{V}{I}$.

Solution:

Given data: Voltage $= V = 7.3\text{V} \pm 0.1\text{V}$

Current $= I = 2.73\text{A} \pm 0.5\text{A}$

Required: Resistance $= R \pm \Delta R = ?$

$$\text{We know that, } V = IR \Rightarrow R = \frac{V}{I}$$

$$\text{So resistance} = R \pm \Delta R = \frac{V}{I} \pm \Delta R$$

$$\Rightarrow \text{Resistance} = \frac{V}{I} \pm \Delta R \quad (1)$$

First we calculate the percentage uncertainty in voltage and current.

$$\Delta V = \frac{0.1}{7.3} \times 100\% \Rightarrow \Delta V = 1.37\%$$

$$\text{And } \Delta I = \frac{0.05}{2.73} \times 100\% \Rightarrow \Delta I = 1.83\%$$

Now uncertainty in resistance is given by,

$$\Delta R = \Delta V + \Delta I = 1.37\% + 1.83\%$$

$$\Rightarrow \Delta R = 3.2\%$$

Now putting the values in equation (1), we get,

$$\text{Resistance} = \frac{7.3}{2.73} \pm 3.2\% = 2.7 \text{ ohm} \pm 3.2\%$$

$$\Rightarrow \text{Resistance} = 2.7 \text{ ohm} \pm 3.2\%$$

In terms of fractional uncertainty, the answer can be written as,

$$\text{Resistance} = 2.7 \text{ ohm} \pm \frac{3.2}{100} \times 2.7 \text{ ohm}$$

$$\Rightarrow \text{Resistance} = 2.7 \text{ ohm} \pm 0.086 \text{ ohm}$$

$$\text{OR } \text{Resistance} = 2.7 \text{ ohm} \pm 0.09 \text{ ohm}$$

EXAMPLE 1.4:

A ball drops from rest from an unknown height "h". The time "t" it takes for the ball to hit the ground is measured to be $(t \pm \Delta t)$ = $1.3 \text{ s} \pm 0.2 \text{ s}$. The height is related to this time by the equation $h = \frac{1}{2}gt^2$ (where $g = 9.81 \text{ m/s}^2$). Assume that the value for "g" carries no uncertainty and calculate the height "h" including its uncertainty.

Solution: Given data: Time $t(t \pm \Delta t) = 1.3 \text{ s} \pm 0.2 \text{ s}$

Acceleration due to gravity "g" = 9.81 m/s^2

Required: Height "h" = $(h \pm \Delta h)$ = ?

Calculation: Height = $h \pm \Delta h$ _____ (1)

$$\text{Here } h = \frac{1}{2}gt^2 \text{ _____ (2)}$$

First we calculate the percentage uncertainty in time, i.e.

$$\Delta t = \frac{0.2}{1.3} \times 100\% \Rightarrow \Delta t = 15.4\%$$

As the power of "t" is "2", so according to rules, we multiply the value of " Δt " by "2", which gives the total uncertainty in height, so

$$\Delta h = 15.4\% \times 2 \Rightarrow \Delta h = 30.8\%$$

Now putting the actual values in equation (2), we get,

$$h = \frac{1}{2} \times 9.81 \times (1.3)^2 = 8.3 \text{ m}$$

Putting the values of "h" and " Δh " in equation (1), we get,

$$\text{Height} = 8.3 \pm 30.8\%$$

In terms of fractional uncertainty, the answer can be written as,

$$\text{Height} = 8.3 \text{ m} \pm \left(\frac{30.8}{100} \times 8.3 \text{ m} \right)$$

$$\Rightarrow \text{Height} = 8.3 \text{ m} \pm 2.6 \text{ m}$$

ASSIGNMENT 1.4:

The radius of sphere "r" is measured with vernier calipers as $(r \pm \Delta r) = 2.25 \text{ cm} \pm 0.01 \text{ cm}$. Calculate the volume of sphere.

Solution:

Given data: Radius = $r \pm \Delta r = 2.25 \text{ cm} \pm 0.01 \text{ cm}$

Required: Volume of sphere = $V \pm \Delta V$ = ?

Calculation: Volume of sphere = $V \pm \Delta V$ _____ (1)

The volume of sphere is given by,

$$V = \frac{4}{3}\pi r^3 \text{ _____ (2)}$$

First we calculate the percentage uncertainty in radius, for which we

$$\text{have, } \Delta r = \frac{0.01}{2.25} \times 100\% \Rightarrow \Delta r = 0.44\%$$

As in equation (2), the power of "r" is "3", so according to rules, we multiply the value of " Δr " by "3" which gives the total uncertainty in volume. So

$$\Delta V = 3 \times 0.44\% \Rightarrow \Delta V = 1.32\%$$

Now putting the actual values of "r" and " π " in equation (2), we get,

$$V = \frac{4}{3} \times 3.14 \times (2.25)^3 \text{ cm}^3 = 47.7 \text{ cm}^3$$

Putting the values of "V" and " ΔV " in equation (1), we get,

$$\text{Volume of sphere} = 47.7 \text{ cm}^3 \pm 1.32\%$$

Now in terms of fractional uncertainty, the answer can be written as,

$$\text{Volume of sphere} = 47.7 \text{ cm}^3 \pm \left(\frac{1.32}{100} \times 47.7 \text{ cm}^3 \right)$$

$$\Rightarrow \text{Volume of sphere} = 47.7 \text{ cm}^3 \pm 0.63 \text{ cm}^3$$

EXAMPLE 1-5:

Calculate the answers to the appropriate number of significant figures: (a) $0.35 - 0.1$

(b) $32.567 + 135.0 + 1.4567$

(c) $420.03 + 299.270 + 99.068$ (d) 14×8

(e) $(2400) (3.45) (16.21)$ (f) $\frac{32.09 + 1.2 - 17.035}{19.8}$

Solution:

(a) $0.35 - 0.1$

$\Rightarrow 0.35 - 0.1 = 0.25$

Now in given data, we see that "0.1" consist of one significant figures, which is the least significant figure, so we will round off the final result to one significant figures. So we get,

$0.35 - 0.1 = 0.25 = 0.2$

$\Rightarrow 0.35 - 0.1 = 0.2$

(b) $32.567 + 135.0 + 1.4567$

$\Rightarrow 32.567 + 135.0 + 1.4567 = 169.0237$

Now see that in the given data, 135.0 represents three significant figures, which is the least significant figures, so we will round off the answer to three significant figures. So we get,

$32.567 + 135.0 + 1.4567 = 169.0$

(c) $420.03 + 299.270 + 99.068$

$\Rightarrow 420.03 + 299.270 + 99.068 = 818.368$

Now we see that in the given data, we see that, 420.03 consists of five significant figures which represents the least number of significant figure. So we will round off our answer to five significant figures. So we get,

$420.03 + 299.270 + 99.068 = 818.37$

(d) 14×8

$\Rightarrow 14 \times 8 = 112$

Now we in the given data, "8" represents the least number of significant figure i.e. "8" represents one significant figure. So we round off our answer to one significant figure. So we get,

$14 \times 8 = 100$

(e) $(2400) (3.45) (16.21)$

$\Rightarrow 2400 \times 3.45 \times 16.21 = 134218.8$

Now in the given data, 2400 contain two significant digits which represent the least significant figures, so the answer will be round off to two significant digits. So we get,

$2400 \times 3.45 \times 16.21 = 130000$

(f) $\frac{30.09 + 1.2 - 17.035}{19.8} = \frac{16.255}{19.8} = 0.820959596$

$\Rightarrow \frac{16.255}{19.8} = 0.820959596$

Now in above division 16.255 contain five significant figures, while 19.8 contain three significant figures, so we will round off our answer to three significant figures. So we get,

$\frac{16.255}{19.8} = 0.821$

ASSIGNMENT 1-5:

Calculate the answers to the appropriate number of significant figures: (a) $0.31 + 0.1$

(b) $658.0 + 23.5478 + 1345.29$

(c) 8×7 (d) $0.9935 \times 10.48 \times 13.4$

(e) $5.5 / 1.1$ (f) $\frac{73.2 + 18.72 \times 6.1}{3.4}$

Solution:

(a) $0.31 + 0.1$

$\Rightarrow 0.31 + 0.1 = 0.41$

We see that in the given data we see that, "0.1" consist of one significant figures which represent the least number of significant figures, so we round off our answer to one significant figure, so we get,

$0.31 + 0.1 = 0.4$

(b) $658 + 23.5478 + 1345.29$

$\Rightarrow 658 + 23.5478 + 1345.29 = 2026.8378$

Now in the given data we see that, 658.0 consist of three significant figures which represent the least number of significant figures, so we round off our answer to three significant figures. So we get,

$658 + 23.5478 + 1345.29 = 2030$

(c) 8×7

$\Rightarrow 8 \times 7 = 56$

Now we see that the given data contain one significant figure, so we round off our answer to one significant figure. So we get,

$$8 \times 7 = 60$$

$$(d) 0.9935 \times 10.48 \times 13.4$$

$$\Rightarrow 0.9935 \times 10.48 \times 13.4 = 139.519192$$

Now in given data we see that, 13.4 consist of three significant figures which represents the least number of significant figures, so we round off our answer to three significant figures. So we get,

$$0.9935 \times 10.48 \times 13.4 = 139$$

$$(e) \frac{5.5}{1.1} = \frac{5.5}{1.1} = 5$$

$$(f) \frac{73.2 + 18.72 \times 6.1}{3.4}$$

$$\Rightarrow \frac{73.2 + (18.72 \times 6.1)}{3.4} = \frac{73.2 + 114.192}{3.4} = \frac{187.392}{3.4}$$

$$\Rightarrow \frac{187.392}{3.4} = 55.11529412$$

Now in $\left(\frac{187.392}{3.4}\right)$, "3.4" represent the least number of significant figures, so we round off our answer to two significant figures. So we get,

$$\frac{187.392}{3.4} = 55$$

EXAMPLE 1.6:

Using the dimension analysis to find the correct relation:

$$(a) F = \frac{mv^2}{r^2} \quad \text{or} \quad (b) F = \frac{mv^2}{r}$$

Solution: Given data: Dimensions of force " F " = $[M^1 L^1 T^2]$

Dimensions of mass " m " = $[M^1]$

Dimensions of velocity " v " = $[L^1 T^{-1}]$

Dimensions of radius " r " = $[L^1]$

Required: Correct relation for force " F " = ?

Calculation:

$$(a) F = \frac{mv^2}{r^2}$$

Taking L.H.S, we get,

$$F = [M^1 L^1 T^{-2}] \quad (1)$$

Now taking R.H.S, we get,

$$\frac{mv^2}{r^2} = \frac{[M^1][L^1 T^{-1}]^2}{[L^1]^2} = \frac{[M^1 L^2 T^{-2}]}{[L^2]}$$

$$\Rightarrow \frac{mv^2}{r^2} = [M^1 L^2 T^{-2}]$$

$$\Rightarrow \frac{mv^2}{r^2} = [M^1 L^0 T^{-2}] \quad (2)$$

Comparing equation (1) and equation (2), we get,

$$\text{Dimension of force} \neq \text{Dimension of } \frac{mv^2}{r^2}$$

Thus the relation " $F = \frac{mv^2}{r^2}$ " is not correct dimensionally.

Part (b) $F = \frac{mv^2}{r}$ Taking L.H.S, we get,

$$F = [M^1 L^1 T^{-2}] \quad (3)$$

Now taking R.H.S, we get,

$$\frac{mv^2}{r} = \frac{[M^1][L^1 T^{-1}]^2}{[L^1]} = \frac{[M^1][L^2 T^{-2}]}{[L^1]}$$

$$\Rightarrow \frac{mv^2}{r} = [M^1 L^{2-1} T^{-2}] = [M^1 L^1 T^{-2}]$$

$$\Rightarrow \frac{mv^2}{r} = [M^1 L^1 T^{-2}] \quad (4)$$

Comparing equation (3) and equation (4), we get,

$$\text{Dimension of force} = \text{Dimension of } \frac{mv^2}{r}$$

Thus the relation " $F = \frac{mv^2}{r}$ " is dimensionally correct.

EXAMPLE 1.7:

Show that the equations (a) $v_f = v_i + at$ (b) $S = v_i t + \frac{1}{2}at^2$ are dimensionally correct.

Solution: (a) $v_f = v_i + at$ (1)

Taking L.H.S, we get,

$$v_f = \frac{L^1}{T^1} = [L^1 T^{-1}] \Rightarrow v_i = [L^1 T^{-1}] \quad (2)$$

Now taking R.H.S, we get,

$$v_i + at = \left[\frac{L'}{T'} + \frac{L'}{T'^2} \cdot T' \right] = \frac{L'}{T'} + \frac{L'}{T'} \\ \Rightarrow v_i + at = \left[\frac{L'T^{-1}}{T'} + \frac{L'T^{-2}}{T'} \cdot T' \right] = \left[\frac{L'}{T'} + \frac{L'}{T'} \right] \Rightarrow v_i + at = 2 \left[\frac{L'T^{-1}}{T'} \right] \quad (3)$$

Comparing equation (2) and equation (3), we get,

$$\text{Dimension of "v_i" = Dimension of (v_i + at)}$$

Thus the equation, " $v_i = v_i + at$ " is dimensionally correct.

Part (b) $S = v_i t + \frac{1}{2} at^2$ (4)

Taking L.H.S, we get,

$$S = \text{Length} = [L'] \Rightarrow S = [L'] \quad (5)$$

Now taking R.H.S, we get,

$$v_i t + \frac{1}{2} at^2 = \left[\frac{L'}{T'} \cdot T' + \frac{1}{2} \frac{L'}{T'^2} \cdot T'^2 \right] = \left[L' + \frac{L'}{2} \right] = \left[\frac{2L' + L'}{2} \right] = \left[\frac{3L'}{2} \right] = \frac{3}{2} [L'] \\ \Rightarrow v_i t + \frac{1}{2} at^2 = \frac{3}{2} [L'] \quad (6)$$

Comparing equation (5) and (6), we get,

$$\text{Dimension of "S" = Dimension of (v_i t + \frac{1}{2} at^2)}$$

Thus the equation, " $S = v_i t + \frac{1}{2} at^2$ " is dimensionally correct.

EXAMPLE 1.7:

When a solid sphere moves through a liquid, the liquid opposes the motion with a force F . The magnitude of F depends on the coefficient of viscosity η (having dimensions $\eta = [M'L^{-1}T^{-1}]$) of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Given Data:

Dimensions of coefficient of viscosity " η " $[M'L^{-1}T^{-1}]$

Dimensions of radius " r " $[L']$

Dimensions of velocity " v " $[L'T^{-1}]$

Dimensions of force " F " $[MLT^{-2}]$

Required:

Possible formula for drag force " F " = ?

Solution:

Let the drag force depends upon coefficient of viscosity " η ", radius " r " and velocity " v " by the following equation:

$$F_D \propto \eta^a r^b v^c \text{ or } F_D = k \eta^a r^b v^c \quad (1)$$

Where k is constant of proportionality. Now putting appropriate dimensions

$$[M'L'T^{-2}] = [M'L^{-1}T^{-1}]^a [L']^b [L'T^{-1}]^c = [M^a \cdot L^{-a} \cdot T^{-a} \cdot L^b \cdot L^c \cdot T^{-c}]$$

$$[M]^a [L]^b [T]^{-2} = [M]^a [L]^{b+c} [T]^{-a-c}$$

Comparing the powers of similar physical quantities on both sides, we get,

For M	For T	For L
$[M]^a = [M]^1$ $a = 1$	$[T]^{-2} = [T]^{-a-c}$ $-a - c = -2$ As $a = 1$, so $-1 - c = -2$ $-c = -2 + 1$ $-c = -1$ $c = 1$	$[L]^b = [L]^{b+c}$ $-a + b + c = 1$ As $a = 1$ and $c = 1$, so $-1 + b + 1 = 1$ $b = 1$

Putting the values of a, b, c in equation (1), we get,

$$F_D = k \eta^1 r^1 v^1 \quad (2)$$

Here $k = 6\pi$, so equation (2) becomes

$$F_D = 6\pi \eta r v \quad (3)$$

Equation (3) represents the formula for drag force.

ASSIGNMENT 1.7:

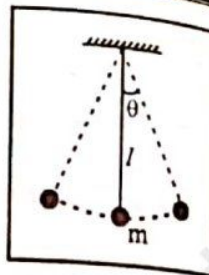
Find an expression for the time period " T " of a simple pendulum. The time period " T " may depend upon (i) mass " m " of the bob of the pendulum, (ii) length " l " of pendulum, (iii) acceleration due to gravity " g " at the place where the pendulum is suspended.

Solution:

Consider a simple pendulum as shown in the figure. Let " T " be the time period of simple pendulum which possibly depends upon the following factors:

- " T " is directly proportional to the length " l " of the pendulum i.e.

- $T \propto l^a$ (1) where $a = \text{constant}$
2. " T " is directly proportional to acceleration due to gravity " g " i.e.
 $T \propto g^b$ (2) [where $b = \text{constant}$]
3. " T " is directly proportional to the mass " m " of the bob, i.e.
 $T \propto m^c$ (3) [where $c = \text{constant}$]



Combining relations (1), (2) and (3), we get,

$$T \propto l^a g^b m^c$$

$$\Rightarrow T = k l^a g^b m^c \text{ (4) [where } k = \text{constant}]$$

Now put the dimensions of l , g and m in equation (4), we get,

$$[T] = k [L]^a \left[\frac{L}{T^2} \right]^b [M]^c$$

$$\Rightarrow [M^0 L^0 T^1] = k [L]^a [L]^b [T^{-2b}] [M]^c$$

$$\Rightarrow [M^0 L^0 T^1] = k [L]^{a+b} [T]^{-2b} [M]^c \text{ (5)}$$

Comparing the powers of M , L , T on both sides of equation (5), we get,

$$\text{For } M, c = 0 \text{ (6)}$$

$$\text{For } L, a + b = 0 \text{ (7)}$$

$$\text{For } T, -2b = 1 \Rightarrow b = -\frac{1}{2} \text{ (8)}$$

Putting equation (8) in equation (7), we get,

$$a - \frac{1}{2} = 0 \Rightarrow a = \frac{1}{2} \text{ (9)}$$

Putting the values of a , b , c in equation (4), we get,

$$T = k l^{\frac{1}{2}} g^{-\frac{1}{2}} m^0 \quad [m^0 = 1]$$

$$\Rightarrow T = k \left(\frac{l}{g} \right)^{\frac{1}{2}} \cdot 1$$

$$\Rightarrow T = k \sqrt{\frac{l}{g}} \text{ (10)}$$

In equation (10) " k " is constant and its experimental value is equal to " 2π " i.e. $k = 2\pi$ so equation (10) $\Rightarrow$

$$T = 2\pi \sqrt{\frac{l}{g}} \text{ (11)}$$

Exercise

Choose the best possible answer:

1. What is the radian measure between the arm of watch at 5:00 pm?
 a. 1 radian b. 2 radian ✓ c. 3 radian d. 4 radian

Sol: At 5:00 pm, the minute arm will be at "12" and hour arm will beat "5" as shown in the figure. There is a gap of 5 hour and each hour space is 30° . Thus there will be an angle of 150° between the two arms of the watch.

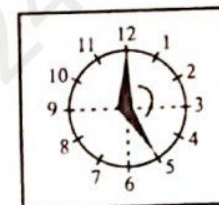
$$\text{Now } 180^\circ = \pi \text{ radian}$$

$$\text{Then } 150^\circ = \frac{\pi \text{ radian} \times 150}{180}$$

$$\Rightarrow 150^\circ = \frac{3.14 \times 150}{180} \text{ radian}$$

$$\Rightarrow 150^\circ = 2.6 \text{ radian}$$

$$\Rightarrow 150^\circ = 3 \text{ radian}$$



$$2. 1^\circ = \dots\dots\dots$$

$$\checkmark \text{ a. } 0.01745 \text{ radian}$$

$$\text{b. } 1 \text{ radian}$$

$$\text{c. } 3.14 \text{ radian}$$

$$\text{d. } 2\pi \text{ radian}$$

$$\text{Sol: } 360^\circ = 2\pi \text{ radian}$$

$$1^\circ = ?$$

$$\Rightarrow 1^\circ = \frac{2\pi \text{ radian} \times 1^\circ}{360^\circ} \Rightarrow 1^\circ = 0.01745 \text{ radian}$$

3. The metric prefix for 0.000001 is:

$$\text{a. hector}$$

$$\checkmark \text{ b. micro}$$

$$\text{c. deka}$$

$$\text{d. nano}$$

4. Which of the following is the correct way of writing units?

$$\text{a. } 71 \text{ Newton}$$

$$\text{b. } 12 \text{ m}\mu\text{s}$$

$$\text{c. } 8 \text{ kg}$$

$$\checkmark \text{ d. } 43 \text{ kgm}^{-3}$$

5. A student measures a distance several times. The readings lie between 49.8 cm and 50.2 cm. This measurement is best recorded as:

$$\text{a. } 49.8 \pm 0.2 \text{ cm}$$

$$\text{b. } 49.8 \pm 0.4 \text{ cm}$$

$$\checkmark \text{ c. } 50.0 \pm 0.2 \text{ cm}$$

$$\text{d. } 50.0 \pm 0.4 \text{ cm}$$

6. The percent uncertainty in the measurement of $3.763 \pm 0.25\text{m}$ is:
 a. 4% ✓ b. 6.6% c. 25% d. 376%

Sol: $3.76 \pm 0.25\text{m}$

$$\text{Percentage uncertainty} = \frac{0.25}{3.76} \times 100\% = 6.6\%$$

7. The temperature of two bodies measured by a thermometer are $t_1 = 20^\circ\text{C} \pm 0.5^\circ\text{C}$ and $t_2 = 50^\circ\text{C} \pm 0.5^\circ\text{C}$. The temperature difference and the error therein is:

- a. $30^\circ\text{C} \pm 0.0^\circ\text{C}$ b. $30^\circ\text{C} \pm 0.5^\circ\text{C}$
 ✓ c. $30^\circ\text{C} \pm 1^\circ\text{C}$ d. $30^\circ\text{C} \pm 1.5^\circ\text{C}$

Sol: $t_2 - t_1 = 50^\circ\text{C} - 20^\circ\text{C} \pm (0.5 \pm 0.5)^\circ\text{C} = 30^\circ\text{C} \pm 1^\circ\text{C}$

8. $(5.0\text{m} \pm 4.0\%) \times (3.0\text{s} \pm 3.3\%) =$

- a. $15.0\text{ms} \pm 13.2\%$ ✓ b. $15.0\text{ms} \pm 7.3\%$
 c. $15.0\text{ms} \pm 0.7\%$ d. $15.0\text{ms} \pm 15.3\%$

Sol: $(5.0\text{m} \pm 4.0\%) \times (3.0\text{s} \pm 3.3\%)$

$$= 5 \times 3\text{ms} \pm (4\% + 3.3\%) = 15\text{ms} \pm 7.3\%$$

9. $(2.0\text{m} \pm 2.0\%)^3 =$

- a. $8.0\text{m}^3 \pm 1.0\%$ b. $8.0\text{m}^3 \pm 2.0\%$
 c. $8.0\text{m}^3 \pm 5.0\%$ ✓ d. $8.0\text{m}^3 \pm 6.0\%$

Sol: $(2.0 \pm 2.0\%)^3 = (2.0\text{m})^3 \pm (3 \times 2\%) = 8.0\text{m}^3 \pm 6.0\%$

10. The number of significant figures in measurement of 0.00708600 cm are:

- a. 3 b. 4 ✓ c. 6 d. 9

11. How many significant figures does $1.362 + 25.2$ have?

- a. 2 ✓ b. 3 c. 5 d. 8

12. Compute the result to correct number of significant digits $1.513\text{m} + 27.3\text{m} =$

- a. 29m ✓ b. 28.8m c. 28.81m d. 28.813m

Sol: $1.513\text{m} + 27.3\text{m} = 28.813\text{m}$

In given data " 27.3m " represents the least number of significant figure, so we round off the answer to three significant figures, so $28.813\text{m} = 28.8\text{m}$

13. If 7.635 and 4.81 are two significant numbers. Their multipli-

cation in significant digits is:

- a. 36.72435 b. 36.724 c. 36.72 ✓ d. 36.7

Sol: $7.635 \times 4.81 = 36.72435$

Here " 4.81 " is the least number of significant figures, so we round off the answer to three significant figures, so $36.72435 = 36.7$.

14. The precision of the measurement $385,000\text{km}$ is:

- a. 10km b. 100km ✓ c. 1000km d. 1000000km

15. $[M^0 L^0 T^0]$ are dimension of:

- a. strain b. refractive index
 c. magnification ✓ d. all of these

16. The dimensions of torque are:

- a. $[MLT]$ b. $[M^2 L^2 T]$
 ✓ c. $[ML^2 T^2]$ d. $[ML^2 T^2]$

Conceptual Questions

Give short response to the following questions:

Q1: For an answer to be complete, the units need to be specified. Why?

Ans. For an answer to be complete, the units must be specified.

Reason and Explanation:

Physics deals with physical quantities. Physical quantities are those quantities which can be measured. For measurement we need proper number and proper unit. Without proper number and proper unit we cannot measure any physical quantity.

Example: Let suppose we want to buy 5kg sugar from a shopkeeper. If we tell to a shopkeeper to give us 5kg sugar he would easily understand to measure 5kg sugar for us. This is the main reason that we discuss an appropriate unit "kilogram".

Instead of it if we ignore the unit "kilogram", and tell the shopkeeper to give us 5 sugar then in this case the shopkeeper would not be able to understand our point. He must tell to us about the unit. He would tell us that in which unit you need the sugar i.e. in kilogram, milligram or gram.

Conclusion: From the above discussion it is clear that for complete explanation of any physical quantity, we need proper unit.

Q2: What are the advantages of using international system of units (SI)?

Ans: Advantages of using international system of unit (SI):

1. The international system of units are used throughout the world.
2. It has only one unit for each quantity. This means that it is never necessary to convert from one unit to another within the system. For example, the one and only SI unit of length is meter "m". Numerical prefixes may be attached, but they do not form a separate unit.
3. SI units are coherently derived as the simple algebraic quotients or product of a few independent base units, using the same equation as the quantity being measure. There is no numerical definitions for students to memorize. For example, power can be defined as "rate of doing work". Its units is "watt" and is equal to "joule per second".
4. SI uses decimals exclusively, eliminating clumsy fractions and mixed numbers.
5. The prefixes used in SI are short, convenient and unambiguous and can be pronounce easily. For example, kilo stands for 1000, mega stands for 1000000 etc.
6. SI unit are limited in number and one can learn and memorize them easily in short time.
7. Using SI units, we can express physical quantities in simple way. For example, 500 watts (500w) is much simpler than non-SI expressions of power such as 1700 British thermal units per hour (1700 Btu/hr).

Q3: (a) How many radians account for circumference of a circle?

(b) How many steradians account for circumference of a sphere?

Ans: Part (a) There are 2π radians account for circumference of a circle.

Explanation:

Consider a particle of mass "m" is moving along a circular path of radius

"r" as shown in the figure. When the particle moves from "A" to "B", it make angle " θ " with the centre which is known as angular displacement. While the curved segment between "A" and "B" represent the arc "S". The relation between "S", " θ " and "r" is given by,

$$s = r\theta \Rightarrow \theta = \frac{S}{r} \quad (1)$$

Now when the particle complete one rotation, then we put "S" equal circumference of the circle i.e.

$$S = 2\pi r \quad (2)$$

Putting equation (2) in equation (1), we get,

$$\theta = \left(\frac{2\pi r}{r} \right) \text{ radian}$$

$$\Rightarrow \theta = 2\pi \text{ radian}$$

Thus there are 2π radians account for circumference of a circle.

Part (b) There are 4π steradians account for surface area of a sphere.

Explanation:

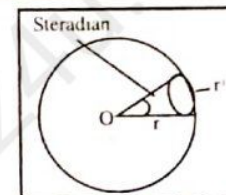
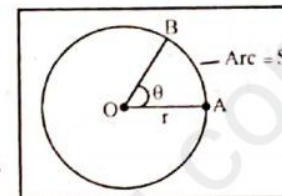
Steradian is the unit of solid angle in a three dimensional space. It is defined as "the solid angle subtended at the centre of a sphere by an area of its surface equal to the square of the radius of that sphere." We can obtain a solid angle by dividing the area of the sphere with square of its radius i.e.

$$1 \text{ steradian} = \frac{\text{Area of sphere}}{(\text{radius})^2}$$

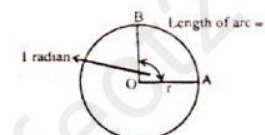
$$\Rightarrow 1 \text{ steradian} = \frac{4\pi r^2}{r^2}$$

$$\Rightarrow 1 \text{ steradian} = 4\pi$$

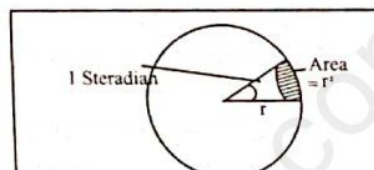
Conclusion: Any closed surface subtends a solid angle of " 4π " at any of its interior point.



For Your Information:



A radian "cuts out" a length of a circle's circumference equal to the radius.



A steradian "cut out" an area of a sphere equal to $(\text{radius})^2$.

Q4: What is least count error? How can least count error be reduced?

Ans: (a) Least Count and Least Count Error:

The smallest value that can be measured by the measuring instrument is called its least count. While the least count error is the error associated with the resolution of the instrument.

(b) How to reduce Least Count Error?

1. We can reduce least count error by using instrument of higher precision. The least count of an instrument is inversely proportional to the precision of the instrument i.e. smaller the least count of the instrument, more precise will be the measurement and vice versa.
2. We should improve our experimental techniques for reducing the error.
3. We should take several readings and then calculate their mean value. The mean value reduces the error and is nearest to the actual value.

Q5: Why including more digits in answers, does not make it more accurate?

Ans: Including more digits in an answer makes the answer to be more precise. But it cannot make the answer more accurate. It is because the accuracy is independent of the precision. The answer will be more accurate if the measured value is nearest to the actual value and vice versa.

Example: Let Anmol and Nayab want to measure the length of a rod, whose actual length is 15.5cm. Let the measured value of Nayab is 15.4cm and the measured value of Anmol is 15.26cm.

Case -1: The difference between actual value and measured value of Nayab is given by, difference = $15.5\text{cm} - 15.4\text{cm} = 0.1\text{cm}$

Case -2: The difference between actual and measured value of Anmol is given by,

$$\text{Difference} = 15.5\text{cm} - 15.26\text{cm} = 0.24\text{cm}$$

Conclusion: The answer of Nayab consists of less digits and we get very less difference between measured value and actual value. So we

get more accurate result. While the answer of Anmol consists of more digits and we get comparatively large difference between measured value and actual value. So the answer of Anmol is less accurate as compared to the result of Nayab. Thus including more digits in answer does not make it more accurate.

Q6: What determines the precision of a measurement?

Ans: The precision of a measurement can be determined by the instrument used for that measurement.

Reason and Explanation:

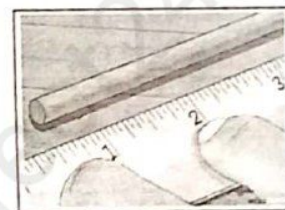
The precision of a measurement depends upon the least count of the measuring instrument. That is the measurement will be more precise if the least count of the instrument is smaller and vice versa.

Example: Let Abdul Basit want to measure the length of a table with the help of two different types of meter rods. That is one meter rod having division in millimeter, and the other having division in centimeters.

Now the least count of 1st meter rod will be 1mm while that of 2nd meter rod will be 1cm. Let the length of the table measured with two meter rods are 170.4cm and 170cm respectively.

Now out of these readings 170.4cm is more precise because this reading was taken with meter rod having minimum least count i.e. 1mm.

Conclusion: From the above discussion it is clear that the precision of measurement depends upon the least count of the measuring instrument.



Q7: If two quantities have different dimensions, is it possible to multiply and / or divide? Can we add and / or subtract them?

Ans: We can multiply or divide two quantities having different dimensions. But we cannot add or subtract two quantities having different dimensions.

Example: Let "A" and "B" represent two quantities having different dimensions.

$$\text{Let } A = [M^1 L^2 T^{-1}] \text{ (1) and } B = [L^1] \text{ (2)}$$

Case -1: Multiplication of "A" and "B":

Multiplying equation (1) and equation (2), we get,

$$A \cdot B = [M^1 L^2 T^{-2}] [L^1] \Rightarrow A \cdot B = [M^1 L^3 T^{-2}]$$

Case -2: Division of "A" and "B":

Dividing equation (1) by equation (2), we get,

$$\frac{A}{B} = \frac{[M^1 L^2 T^{-2}]}{[L^1]} = [M^1 L^{2-1} T^{-2}]$$

$$\Rightarrow \frac{A}{B} = [M^1 L^1 T^{-2}]$$

Case -3: Addition of "A" and "B":

As we see that quantity "A" and quantity "B" have different dimension. So we cannot add "A" with "B" or "B" with "A".

Case -4: Subtraction of "A" and "B":

Since "A" and "B", both having different dimensions, so we cannot subtract "A" from "B" or "B" from "A".

Q8: The human pulse and the swing of a pendulum are possible time units. Why are they not often used?

Ans: We cannot use the human pulse and the swing of pendulum as standard units of time. The reasons are given below:

(a) Reasons for Human Pulse:

1. Sudden changes may occur in human pulse rate, due to variation in heart beats.
2. The shortness of breath effect human pulse rate.
3. Chest pain can effect human pulse rate.
4. The presence of cholesterol can effect the pulse rate of human being.
5. The pulse rate of running person, stationary person are different.
6. The pulse rate in sitting position and supine position is different.
7. The pulse rate also changes due to increase in ages, because the blood pressure may change during different stages of life.

Conclusion: Due to above reasons, we cannot use the human pulse as standard unit of time.

(b) Reasons for Swing of a Pendulum:

1. The time period of a simple pendulum is given by,

$$T = 2\pi \sqrt{\frac{l}{g}} \quad (1)$$

Equation (1) shows that the value of time period will be effected due to change in value of "g". As the value of "g" is different at different places [i.e. "g" varies with altitude and depth], so time period will be effected.

2. The time period also depends upon the length of pendulum. If the wire is metallic, then expansion or contraction may occurs in size of wire and bob due to heating effect or cooling effect. As a result length of pendulum will be affected which will affect the time period of pendulum.
3. Air resistance affects the swinging of pendulum. So time period will be also affected.

Conclusion: Due to above reason, we cannot use swing of pendulum as standard unit of time.

Q9: If an equation is dimensionally correct, is that equation a right equation?

Ans: If an equation is dimensionally correct, then the equation may or may not be a right equation.

Reasons and Explanation:

Dimensions are used to determine the nature of physical quantities mathematically. According to the principle of homogeneity, the dimension of both sides of an equation is said to be dimensionally correct but it is not necessary that it is also correct physically.

Examples:

$$1. \text{ Let } S = vt \quad (1)$$

This equation is true both physical and dimensionally.

$$2. \text{ Let } S = vt + \frac{1}{2} \quad (2)$$

Equation (2) is correct dimensionally but not correct physically.

3. 1 monkey = 2 monkey is dimensionally correct but not correct physically.

Conclusion: From the above discussion it is clear that, if an equation is dimensionally correct, then the equation may or may not be correct physically. Moreover if an equation is not correct dimensionally, then it will be also not correct physically.

Numerical Problems

Q1: A circular pizza into 3 equal parts, one piece of pizza is taken out. Estimate the degree measure of the single piece of pizza and convert the measure into radians. What is the radian measure of the angle of the remaining part of pizza?

Ans. Consider a circular pizza as shown in the figure (a). Let we divide the pizza into three parts at point "A", "B" and "C" as shown in figure (a). Under such condition each arc [i.e. arc AB, arc BC, arc CA] will make $\frac{360^\circ}{3} = 120^\circ$ with centre.

Required: (a) Degree measure of single piece = $\theta = ?$

(b) Degree measure of remaining two pieces = $\theta' = ?$

Calculation:

Part (a): A single part of pizza is shown in figure (b). For this part we have,

$$\theta = 120^\circ$$

Conversion into radian:

We know that,

$$360^\circ = 2\pi \text{ radian}$$

Then $120^\circ = ?$

$$\Rightarrow 120^\circ = \frac{120^\circ \times 2\pi \text{ radian}}{360^\circ} = 2.09 \text{ radian}$$

$$\Rightarrow 120^\circ = 2.09 \text{ radian}$$

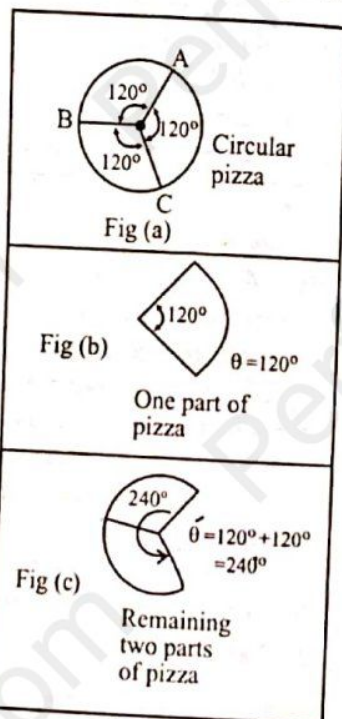
Part (b): The remaining two parts of pizza are shown in figure (c). For this we have,

$$\theta' = 240^\circ$$

Conversion into radian:

We know that,

$$360^\circ = 2\pi \text{ radian}$$



Then $240^\circ = ?$

$$\Rightarrow 240^\circ = \frac{240^\circ \times 2\pi \text{ radian}}{360^\circ} = 4.19 \text{ rad}$$

$$\Rightarrow 240^\circ = 4.19 \text{ radian} \text{ or } 240^\circ = 4.2 \text{ rad}$$

Q2: The length of a pendulum is $(1.5 \pm 0.01)\text{m}$ and the acceleration due to gravity is taken into account as $(9.8 \pm 0.1)\text{ms}^{-2}$. Calculate the time period of the pendulum with uncertainty in it.

Solution:

Given data: Length of pendulum = $\ell = (1.5 \pm 0.01)\text{m}$

$$g = (9.8 \pm 0.1)\text{m/sec}^2$$

Required: Time period with uncertainty = $T \pm \Delta T = ?$

Calculation: We know that,

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T \pm \Delta T = 2\pi\sqrt{\frac{\ell}{g}} \pm \Delta T \quad \text{--- (1)}$$

First we find the percentage uncertainty in length " ℓ " and " g ".

For " ℓ ", we have,

$$\Delta\ell = \frac{0.01}{1.5} \times 100\% \Rightarrow \Delta\ell = 0.67\%$$

For " g ", we have,

$$\Delta g = \frac{0.1}{9.8} \times 100\% \Rightarrow \Delta g = 1.02\%$$

As the power of $\sqrt{\frac{\ell}{g}} = \left[\frac{\ell}{g}\right]^{\frac{1}{2}}$ is $\frac{1}{2}$, so we multiply the total

uncertainty in time by $\frac{1}{2}$ so we get,

$$\Delta T = \frac{1}{2}(\Delta\ell + \Delta g) = \frac{1}{2}(0.67\% + 1.02\%)$$

$$\Rightarrow \Delta T = \frac{1}{2}(1.69\%) = 0.8\% \Rightarrow \Delta T = 0.8\%$$

$$\text{Now } T = 2\pi\sqrt{\frac{\ell}{g}} = 2 \times 3.14 \times \sqrt{\frac{1.5}{9.8}} = 6.28\sqrt{0.15} = 6.28 \times 0.4$$

$$\Rightarrow T = 2.512 \text{ sec}$$

$$\Rightarrow T = 2.5 \text{ sec}$$

Now time period with uncertainty = $T \pm \Delta T$

$$\Rightarrow T \pm \Delta T = 2.5 \text{ sec} \pm 0.8\%$$

Q3: Determine the area of a rectangular sheet with length $(l \pm \Delta l) = (1.50 \text{ m} \pm 0.02 \text{ m})$ and width $(w \pm \Delta w) = (0.20 \pm 0.01 \text{ m})$. Calculate the area $(A \pm \Delta A)$?

Solution:

Given data: Length of rectangular sheet

$$= l \pm \Delta l = (1.50 \pm 0.02) \text{ m}$$

Width of rectangular sheet = $w \pm \Delta w = (0.20 \pm 0.01) \text{ m}$

Required: Area of sheet with uncertainty = $A \pm \Delta A = ?$

Calculation: First we calculate the percentage uncertainty in length and width. For length, we have,

$$\Delta l = \frac{0.02}{1.50} \times 100\% \Rightarrow \Delta l = 1.3\%$$

For width, we have,

$$\Delta w = \frac{0.01}{0.20} \times 100\% \Rightarrow \Delta w = 5\%$$

Now total uncertainty in area is given by,

$$\Delta A = (\Delta l + \Delta w) = (1.3\% + 5\%)$$

$$\Rightarrow \Delta A = 6.3\%$$

Now area

$$= A = L \times w = (1.50 \times 0.20) \text{ m}^2 \Rightarrow A = 0.3 \text{ m}^2$$

$$\text{Now } A \pm \Delta A = 0.3 \text{ m}^2 \pm 6.3\%$$

In terms of fractional uncertainty, the answer can be written as,

$$A \pm \Delta A = 0.3 \text{ m}^2 \pm \left(\frac{6.3}{100} \times 0.3 \text{ m}^2 \right)$$

$$\Rightarrow A \pm \Delta A = 0.3 \text{ m}^2 \pm 0.02 \text{ m}^2$$

Q4: Calculate the answer upto appropriate numbers of significant digits: (a) $246.24 + 238.278 + 98.3$

$$(b) (1.4 \times 2.639) + 117.25$$

$$(c) (2.66 \times 10^4) - (1.03 \times 10^3)$$

$$(d) (112 \times 0.456) / (3.2 \times 120)$$

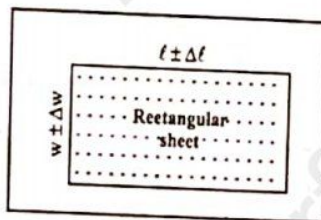
$$(e) 168.99 \times 9$$

$$(f) 1023 + 8.5489$$

Solution: (a) $246.24 + 238.278 + 98.3$

$$\Rightarrow 246.24 + 238.278 + 98.3 = 582.818 = 582.8$$

Now in given data we see that, 98.3 consists of three significant



figures which represent the least number of significant figures, so we round off the answer to three significant figures. So we get,

$$246.24 + 238.27 + 98.3 = 583$$

$$(b) (1.4 \times 2.639) + 117.25$$

$$\Rightarrow (1.4 \times 2.639) + 117.25 = 3.6946 + 117.25 = 120.9446 = 120.9$$

Now in given data we see that, 1.4 consist of two significant figures which represent the least number of significant figures, so we round off the answer to two significant figures. So we get,

$$(1.4 \times 2.639) + 117.25 = 120$$

$$(c) (2.66 \times 10^4) - (1.03 \times 10^3)$$

$$\Rightarrow (2.66 \times 10^4) - (1.03 \times 10^3) = 26600 - 1030 = 25570$$

Now in figure data the least number of significant figures are three, so we round off our answer to three significant figures. So we get,

$$(2.66 \times 10^4) - (1.03 \times 10^3) = 25600 = 2.56 \times 10^4$$

$$(d) \frac{(112 \times 0.456)}{(3.2 \times 120)} \Rightarrow \frac{112 \times 0.456}{3.2 \times 120} = \frac{51.072}{384}$$

$$\Rightarrow \frac{51.072}{384} = 0.133 = 0.13$$

$$(e) 168.99 \times 9$$

$$\Rightarrow 168.99 \times 9 = 1520.91$$

Now in given data we see that, 9 consist of one significant figures which represent the least number of significant figures, so we round off our answer to one significant figure. So we get,

$$168.99 \times 9 = 2000$$

$$(f) 1023 + 8.5489 \Rightarrow 1023 + 8.5489 = 1031.5489$$

Now in given data we see that, 1023 consists of four significant figures which represent the least number of significant figures, so we round off our answer to four significant figures, so we get,

$$1023 + 8.5489 = 1032$$

Q5: Calculate the answer up to appropriate numbers of significant digits:

(a) The ratio of mass of proton " m_p " to the mass of electron " m_e "

$$\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.1096 \times 10^{-31} \text{ kg}}$$

(b) The ratio of charge on electron " q_e " to mass of electron " m_e "

$$\frac{q_e}{m_e} = \frac{1.6 \times 10^{-19} \text{ C}}{9.1096 \times 10^{-31} \text{ kg}}$$

Solution:

Part (a): $\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27} \text{ kg}}{9.1096 \times 10^{-31} \text{ kg}}$

$$\Rightarrow \frac{m_p}{m_e} = 0.18332 \times 10^4 \Rightarrow \frac{m_p}{m_e} = 1.8332 \times 10^3$$

Now in given data, "1.67" represents the least number of significant figures [i.e. three significant figures], so the final result is rounded off to three significant figures. So we get,

$$\frac{m_p}{m_e} = 1.8332 \times 10^3 = 1.83 \times 10^3$$

$$\Rightarrow \frac{m_p}{m_e} = 1.83 \times 10^3$$

Part (b): $\frac{q_e}{m_e} = \frac{1.6 \times 10^{-19} \text{ C}}{9.1096 \times 10^{-31} \text{ kg}}$

$$\Rightarrow \frac{q_e}{m_e} = 0.1756388864 \times 10^{12} \text{ C} \cdot \text{kg}^{-1}$$

$$\Rightarrow \frac{q_e}{m_e} = 1.756388864 \times 10^{11} \text{ C} \cdot \text{kg}^{-1}$$

Now in given data, "1.6" represents the least number of significant figures, so we rounded off the answer up to two significant figures. So we get,

$$\frac{q_e}{m_e} = 1.8 \times 10^{11} \text{ C} \cdot \text{kg}^{-1}$$

Q6: Find the dimensions of:

(a) Planck's constant "h" from formula ($E = hf$) where "E" is the energy and "f" is frequency.

(b) Gravitational constant "G" from the formula ($F = G \frac{m_1 m_2}{r^2}$) where "F" is force, "m₁" and "m₂" are masses of objects and "r" is the distance between centers of objects.

BISE Malakand 2019, BISE Kohat 2019, BISE Peshawar 2019

Solution:

Part (a): We have, $E = hf$

$$\Rightarrow h = \frac{E}{f} \quad (1)$$

Now we find the dimensions of "E" and "f", let $E = mgh$

$$\Rightarrow E = M \left[\frac{L}{T^2} \right] [L] \Rightarrow E = [M^1 L^2 T^{-2}] \quad (2)$$

We also know that,

$$f = \frac{1}{T} \Rightarrow f = [T^{-1}] \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$\Rightarrow h = \frac{[M^1 L^2 T^{-2}]}{[T^{-1}]} \Rightarrow h = [M^1 L^2 T^{-2+1}]$$

$$\Rightarrow h = [M^1 L^2 T^{-1}]$$

Part (b): $F = \frac{G m_1 m_2}{r^2}$

$$\Rightarrow G = \frac{F r^2}{m_1 m_2} \quad (1)$$

$$\text{As } F = ma = M \cdot \frac{L}{T^2} = [M^1 L^1 T^{-2}]$$

$$\Rightarrow F = [M^1 L^1 T^{-2}] \quad (2)$$

$$\Rightarrow r^2 = [L]^2 = [L^2] \Rightarrow r^2 = [L^2] \quad (3)$$

$$m_1 m_2 = [M^1 \cdot M^1] = [M^2]$$

$$\Rightarrow m_1 m_2 = [M^2] \quad (4)$$

Putting equation (2), equation (3) and equation (4) in equation (1), we get,

$$G = \frac{[M^1 L^1 T^{-2}] [L^2]}{[M^2]} = [M^{1-2} L^{1+2} T^{-2}]$$

$$\Rightarrow G = [M^{-1} L^3 T^{-2}]$$

Q7: Show that: (a) $KE = \frac{1}{2} mv^2$ and (b) $PE_p = mgh$ are dimensionally correct.

BISE Peshawar 2017

Solution: Part (a) $K \cdot E = \frac{1}{2} mv^2 \quad (1)$

Taking L.H.S, we get,

$$K \cdot E = \text{workdone}$$

$$\Rightarrow K \cdot E = F \cdot d = \text{mad}$$

$$\Rightarrow K \cdot E = \text{mad} \quad (2)$$

Here $m = [M]$, $a = \frac{L}{T^2} = [L^1 T^{-2}]$ and $d = [L]$

So equation (2) becomes

$$K \cdot E = [M] [L^1 T^{-2}] [L] = [M^1 L^1 T^{-2}]$$

$$\Rightarrow K \cdot E = [M^1 L^2 T^{-2}] \quad (3)$$

Now taking R.H.S, we have,

$$\frac{1}{2} mv^2 = \frac{1}{2} \left[M \cdot \left(\frac{L}{T} \right)^2 \right] = \frac{1}{2} \left[M^1 \frac{L^2}{T^2} \right]$$

$$\Rightarrow \frac{1}{2} mv^2 = \frac{1}{2} [M^1 L^2 T^{-2}] \quad (4)$$

Comparing equation (3) and equation (4), we get,

$$\text{Dimension of } K \cdot E = \text{Dimension of } \frac{1}{2} mv^2$$

Thus the expression $(K \cdot E = \frac{1}{2} mv^2)$ is dimensionally correct.

Part (b): $P \cdot E = mgh$ (1)

Taking L.H.S, we have,

$$P \cdot E = \text{workdone} = Fd = mad$$

$$\Rightarrow P \cdot E = mad$$

$$\Rightarrow P \cdot E = [M] \cdot \frac{L}{T^2} \cdot L = [M^1 L^2 T^{-2}]$$

$$\Rightarrow P \cdot E = [M^1 L^2 T^{-2}] \quad (2)$$

Now taking R.H.S, we have,

$$mgh = [M] \cdot \frac{L}{T^2} \cdot L \Rightarrow mgh = [M^1 L^1 T^{-2}]$$

$$\Rightarrow mgh = [M^1 L^2 T^{-2}] \quad (3)$$

Comparing equation (2) and equation (3), we get,

$$\text{Dimension of } P \cdot E = \text{Dimension of } mgh$$

Thus the expression $P \cdot E = mgh$ is dimensionally correct.

VECTORS AND EQUILIBRIUM

Q1: Define (a) physical quantities, (b) magnitude?

Ans: (a) Physical Quantities:

Those quantities which can be measured are known as physical quantities. For example, length, mass, temperature, speed, velocity, acceleration, energy, torque etc.

(b) Magnitude:

A number with proper unit is known as magnitude. For example 100kg, 27°C, 50N, 1000m etc. Here 100, 27, 50, 1000 represent number which kg, °C, N, m represent the unit.

Q2: Define and explain scalar and vector quantities. Give examples in each case?

Ans: (a) Scalar Quantities:

Definition: Those physical quantities which are completely specified by their magnitude only are known as scalar quantities.

Explanation:

For the complete explanation of scalar quantities, the unit and number both are necessary. If we ignore any one of them, then our explanation will be incomplete. The scalars can be added, subtracted, multiplied and divided by ordinary mathematical method.

Examples: Speed, mass, energy, work, power, distance, temperature, electric flux etc are the examples of scalar quantities.

(b) Vector Quantities:

Definition: Those physical quantities which are completely specified by their magnitude and proper direction are known as vector quantities.

Explanation:

For the complete explanation of vector quantities, the number, unit and proper direction all are necessary. If we ignore any one of them,

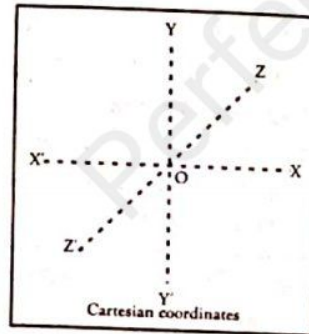
then our explanation will be incomplete. The vectors can be added, subtracted, divided and multiplied by graphical method or geometrical method.

Examples: Momentum, acceleration, torque, displacement, velocity, weight, force etc are the examples of vector quantities.

Q3: (a) What is Cartesian coordinate system? (b) Write the steps to represent a vector in Cartesian coordinate system.

Ans: (a) Cartesian Coordinate System:

The Cartesian coordinate system is also called number plane. It consists of lines xox' , yoy' , zoz' which are mutually perpendicular to each other. The xox' [x-axis] and yoy' [y-axis] lines are usually drawn in the plane of the page. While the line zoz' [z-axis] comes directly out of the page at the origin "o" as shown in the figure. All the three axis [i.e. x-axis, y-axis, z-axis] are making an angle of 90° with each other.



(b) Steps to represent a Vector in Cartesian Coordinate System:

The following steps are taken for representing a vector in Cartesian coordinate system:

Step-1: Draw a Cartesian coordinate system.

Step-2: Select a suitable scale in case of large values.

Step-3: Draw a line according to the scale selected.

Step-4: Put an arrow at the end of the line, which represent the required vector.

Q4: Discuss the representation of a vector in Cartesian coordinate system?

Ans: There are two methods for representing a vector in Cartesian coordinate system. These methods are given below:

- Representing a vector in plane
- Representing a vector in space

1. Representing a Vector in Plane:

To represent a vector in plane, we draw horizontal and vertical lines [i.e. xox' and yoy'] perpendicular to each other. Then we select a suitable scale. We draw the representative line according to the given scale. We put arrow at the end of the line which represent the required vector.

Example: A force of 50N is acting on xy-plane by making an angle of 30° with x-axis. Draw the representative vector of the given force.

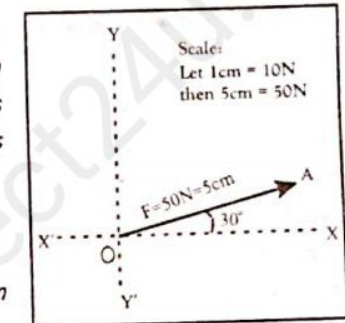
Solution:

Step-1: First we draw the Cartesian coordinate system. i.e. we draw x-axis and y-axis perpendicular to each other as shown in the figure.

Step-2: Now we select a suitable scale. Let $10N = 1cm$ then $50N = 5cm$

Step-3: Now we draw a line $OA = 5cm$ at an angle of 30° with x-axis.

Step-4: Now we put an arrow at the end of the line OA . Thus $\vec{OA} = 5cm$ [at 30° with x-axis] represents the force of 50N acting in xy-plane.

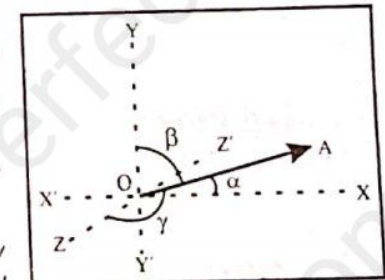


2. Representing of a Vector in Space:

To represent a vector in space, we draw x-axis, y-axis and z-axis [i.e. xox' , yoy' and zoz'] which are mutually perpendicular to each other.

Now we draw a vector $\vec{OA}$ in space making an angle " α " with x-axis, " β " with y-axis and " γ " with z-axis respectively as shown in the figure.

Note: The line zoz' cannot be drawn actually on the paper. It is directed out of the paper at the origin "o". All these axis i.e. x-axis, y-axis and z-axis are mutually perpendicular to each other.



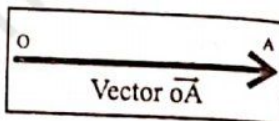
✍️ اے ایم اے کے دل اللہ کی یاد سے سکون پاتے ہیں۔ خبردار ہو کہ دلوں کا چین اللہ کے ذکر ہی میں ہے۔ ✍️

Q5: Discuss how a vector is represented?

Ans. A vector is usually represented by the following two methods:

1. Graphical Representation:

Graphically, a vector is represented by a bold straight line having an arrow head at its one end. The arrow indicates the direction of the given vector. For example, in the given figure, the line " $\vec{OA}$ " represents a vector.



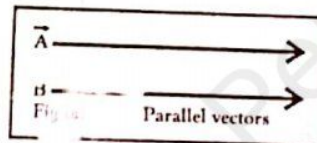
2. Symbolic Representation:

Symbolically, a vector is represented by any English alphabet having an arrow head upon it. For example, $\vec{A}, \vec{a}, \vec{B}, \vec{b}, \vec{S}, \vec{T}$ etc represents the vector symbolically.

Q6: Define the following: (i) Parallel vectors, (ii) Anti-parallel vectors, (iii) Equal vectors, (iv) Negative of a vector, (v) Null vector (or) Zero vector?

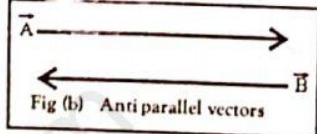
Ans. (i) Parallel Vectors:

Vectors are said to be parallel to each other if they are acting in the same direction, as shown in the figure (a). The angle between parallel vectors is 0° .



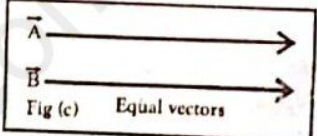
(ii) Anti-Parallel Vectors:

Vectors are said to be anti-parallel to each other if they are acting in opposite direction, as shown in the figure (b). The angle between anti-parallel vectors is 180° .



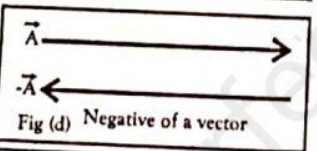
(iii) Equal Vectors:

Two vectors are said to be equal, if they have the same magnitude and same direction, as shown in the figure (c).



(iv) Negative of a Vectors:

Negative of a vector is that vector whose magnitude is the same to that of the given vector but opposite in direction, as shown in the figure (d).



(v) Null Vector or Zero Vectors:

A vector whose magnitude is zero is known as null vector or zero vector. It is arbitrary in direction.

Q7: Define and explain unit vector?

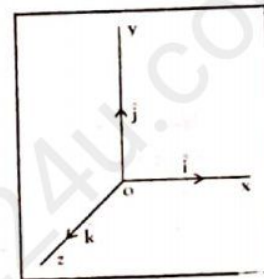
ETEA 2009-2015

Ans. Definition: A vector whose magnitude is one is known as unit vector.

Explanation:

A unit vector is represented by an English alphabet having a cap upon it. For example, $\hat{a}, \hat{b}, \hat{c}$ etc represents unit vectors. A unit vector can be obtained by dividing a vector with its magnitude i.e.

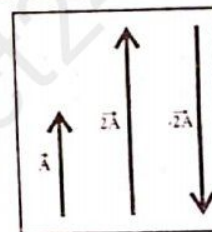
$$\hat{a} = \frac{\vec{a}}{a}$$



The unit vector is used for the specification of directions. In Cartesian coordinates, we take unit vector " $\hat{i}$ " along x-axis, " $\hat{j}$ " along y-axis and " $\hat{k}$ " along z-axis.

Q8: Discuss the multiplication of a vector by a number?

Ans. We can multiply a vector by a number. If the number is positive, then the direction of given vector and product vector will remain the same. If the number is negative, then the given vector and product vector will be acting in opposite directions to each other, as shown in the figure.



Generally, if a vector " $\vec{A}$ " is multiplied by a number " n ", then its magnitude will be equal to " nA ".

Q9: Discuss the addition of vectors by graphical method (or) geometrical method?

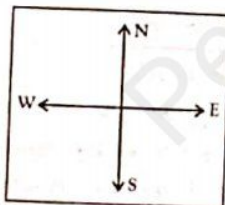
Ans. We can add two or more than two vectors by geometrical method or graphical method to get their resultant vector. We use "head-to-tail rule" for addition of vectors by graphical method or geometrical method.

Steps to add two Vectors by Head-to-Tail Rule:

The following steps must be followed to add vectors by head-to-tail rule:

1. Select a suitable scale for the representation of given vectors.
2. Draw the 1st vector according to the selected scale in the given direction.
3. Draw the 2nd vector according to the selected scale in the specified direction such that its tail touches the head of the first vector.
4. Now draw the 3rd vector according to the selected scale in the specified direction such that its tail touches the head of the second vector.
5. Continue the above process for given number of vectors.
6. For getting the resultant vector, combine the tail of the first vector to the head of the last vector. Represent the resultant vector by the symbol " $\vec{R}$ ".
7. Measure the length of " $\vec{R}$ " which will give its magnitude.
8. To determine the direction of " $\vec{R}$ ", measure the angle of " $\vec{R}$ " with respect to positive x-direction in anticlockwise sense.

Example: Consider two vectors $\vec{A}$ and $\vec{B}$ such that $\vec{A} = 25\text{km}$ and directed towards East. $\vec{B} = 20\text{km}$ and directed towards North. Draw their resultant vector and find its magnitude and direction?



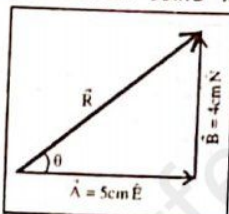
Solution: (i) First we select a suitable scale
Let $5\text{km} = 1\text{cm}$ then $25\text{km} = 5\text{cm}$ and $20\text{km} = 4\text{cm}$

(ii) Now we specify the directions as shown in the figure.

(iii) Now we draw vector " $\vec{A}$ " according to the selected scale in the specified direction.

(iv) Now we draw vector " $\vec{B}$ " according to the selected scale in the specified direction such that its tail touches the head of $\vec{A}$

(v) Now we combine the tail of $\vec{A}$ with the



head of $\vec{B}$ which gives us the resultant vector " $\vec{R}$ ".

(vi) Now measure the length of " $\vec{R}$ " with the help of meter rod which is about 6.4cm . This represents the magnitude of vector " $\vec{R}$ ", which is in fact 32km according to the selected scale.

(vii) Now measure the angle made by vector " $\vec{R}$ " with x-axis or East direction with the help of protractor which is about 38.6° . This gives the direction of resultant vector $\vec{R}$.

Q10: Discuss the addition of more than two vectors.

Ans. We can add any number of vectors by "head-to-tail rule" method.

Example: Consider four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ in xy-plane as shown in the figure:

To add these vectors, we take the following steps:

Step-1: We draw x-axis and y-axis perpendicular to each other.

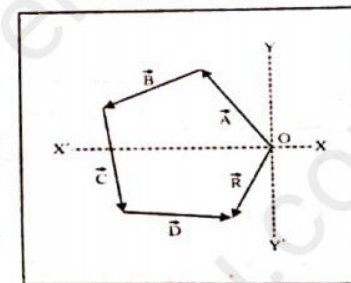
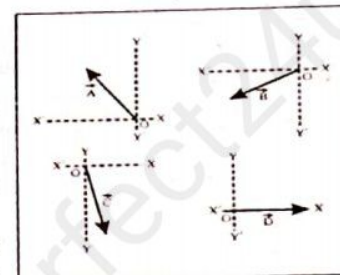
Step-2: We select a suitable scale.

Step-3: We combine the tail of " $\vec{B}$ " with the head of " $\vec{A}$ " according to the specified direction.

Step-4: We combine the tail of " $\vec{C}$ " with the head of " $\vec{B}$ " according to the specified direction.

Step-5: We combine the tail of " $\vec{D}$ " with the head of " $\vec{C}$ " according to the specified direction.

Step-6: Now for finding the resultant, we combine the tail of 1st vector " $\vec{A}$ " with the head of last vector " $\vec{D}$ " as shown in the figure. Thus vector " $\vec{R}$ " represents the resultant vector due to combination of four vectors.



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

Using this method we can add any number of vectors.

Q11: Discuss the commutative property of addition of vectors.

(OR) Show that, $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Ans: Consider two vectors $\vec{A}$ and $\vec{B}$ lying in xy-plane in such a way

that both represents the adjacent sides of a parallelogram as shown in the figure (a).

Let we add vector $\vec{B}$ to vector $\vec{A}$ by head-to-tail rule and get their resultant vector $\vec{R}$ as shown in figure (b).

So we have,

$$\vec{R} = \vec{A} + \vec{B} \quad (1)$$

Now if we add vector $\vec{A}$ to vector $\vec{B}$ we get the same resultant vector $\vec{R}$ as shown in figure (c). So we have,

$$\vec{R} = \vec{B} + \vec{A} \quad (2)$$

$(\vec{A} + \vec{B})$ and $(\vec{B} + \vec{A})$ construct a parallelogram as shown in figure (c).

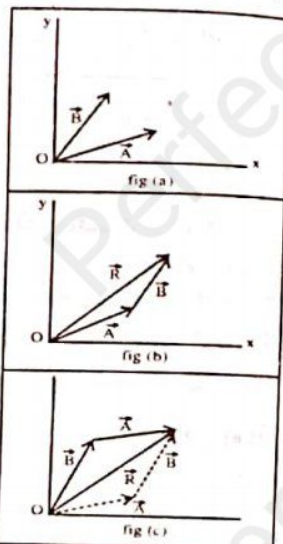
Now comparing equation (1) and equation (2), we get,

$$\vec{A} + \vec{B} = \vec{B} + \vec{A} \quad (3)$$

Equation (3) shows that vectors obey the commutative property of addition.

Q12: Discuss the subtraction of vectors?

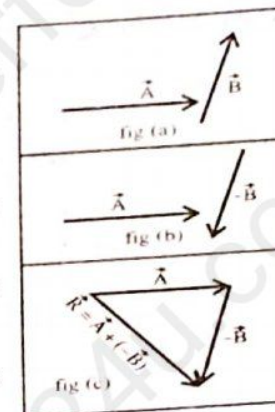
Ans: There is no direct method for determining the result of subtraction of two vectors. Consider two vectors $\vec{A}$ and $\vec{B}$ as shown in figure (a).



Now if we want to subtract vector $\vec{B}$ from vector $\vec{A}$ then first we find the negative of vector $\vec{B}$ as shown in figure (b).

Now to subtract $\vec{B}$ from $\vec{A}$ we add the negative of $\vec{B}$ to $\vec{A}$ by head-to-tail rule as shown in figure (c). For getting their resultant, we combine the tail of $\vec{A}$ to the head of $(-\vec{B})$.

Thus $\vec{R} = \vec{A} + (-\vec{B})$ represents the resultant due to subtraction of two vectors.



Q13: Define and explain position vector?

ETEA 2007

Ans: Position Vector:

Definition:

The vector with the help of which we can locate the position of a point with respect to some fixed point is known as position vector.

Explanation:

In the given figure, $\vec{OP} = \vec{r}$ represents the position vector. In two dimensions, the position vector can be expressed as,

$$\vec{r} = x\hat{i} + y\hat{j}$$

In three dimensions, it can be expressed as,

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

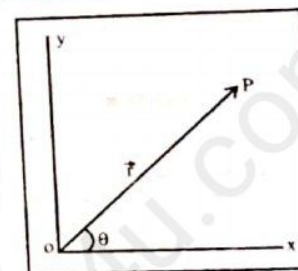
Q14: Define and explain the resolution of a vector?
(ETEA 2005, 2006)

Ans: Definition: The splitting of a vector into its components is known as resolution of a vector.

Explanation:

Consider a vector $\vec{F}$ lying in xy-plane by making an angle " θ " with x-axis, as shown in the figure:

In order to resolve this vector into its com-



ponents, we draw a perpendicular "AB" from the head of vector $\vec{F}$ on x-axis.

Thus $\vec{OB} = \vec{F}_x$ represents the x-component or horizontal component of vector $\vec{F}$ while $\vec{BA} = \vec{F}_y$ represents the y-component or vertical component of vector $\vec{F}$. Now in $\triangle AOB$, we have,

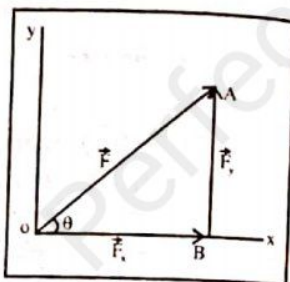
$$\cos \theta = \frac{OB}{OA} \Rightarrow \cos \theta = \frac{F_x}{F}$$

$$\Rightarrow F_x = F \cos \theta \quad (1)$$

Similarly, in $\triangle AOB$, we have,

$$\sin \theta = \frac{AB}{OA} \Rightarrow \sin \theta = \frac{F_y}{F}$$

$$\Rightarrow F_y = F \sin \theta \quad (2)$$



Equation (1) represents the magnitude of horizontal component of vector $\vec{F}$. While equation (2) represents the magnitude of vertical component of vector $\vec{F}$.

Magnitude of Vector $\vec{F}$:

Applying pathagorem theorem to $\triangle AOB$, we get,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$\Rightarrow (OA)^2 = (OB)^2 + (BA)^2$$

$$\Rightarrow F^2 = F_x^2 + F_y^2$$

$$\Rightarrow F = \sqrt{F_x^2 + F_y^2} \quad (3)$$

Equation (3) represents the magnitude of vector $\vec{F}$.

Direction of Vector $\vec{F}$: (ETEA 2005)

We know that,

$$\tan \theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\Rightarrow \tan \theta = \frac{AB}{OB}$$

$$\Rightarrow \tan \theta = \frac{F_y}{F_x} \Rightarrow \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) \quad (4)$$

In three dimension the magnitude of $\vec{F}$ is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

(ETEA 2010)

Equation (4) represents the direction of vector $\vec{F}$.

Q15: Discuss the addition of vectors by rectangular components method. (OR) Discuss the addition of vectors by analytical method?

(BISE Mardan 2019, BISE Kohat 2019, BISE DI Khan 2019)

Ans. **Definition:** The rectangular components of a vector are those components which are making an angle of 90° with each other.

Explanation:

The analytical method gives us more accurate result as compare to geometrical method. It is because, the analytical method is more mathematical method than geometrical method. The result obtained by geometrical method [head-to-tail rule method] is always approximately correct, because drawing sketches can never be perfect.

Here we will consider two vectors and will find an expression for their addition. Then we will generalize the expression for addition of "n" number of vectors.

Now let us consider two vectors $\vec{A}$ and $\vec{B}$ making an angle ' θ_1 ' and ' θ_2 ' with x-axis respectively, as shown in the figure.

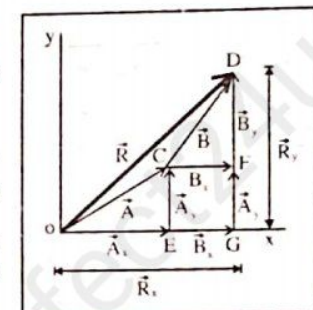
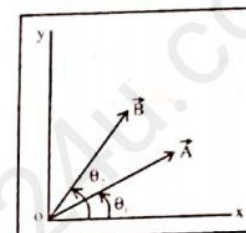
Now we add $\vec{B}$ to $\vec{A}$ by geometrical method. For this purpose, we combine the tail of $\vec{B}$ with the head of $\vec{A}$ as shown in the figure. Now to get their resultant, we combine the tail of $\vec{A}$ with the head of $\vec{B}$

as shown in the figure. Thus $\vec{OD} = \vec{R}$ represents the resultant of $\vec{A}$ and $\vec{B}$ so,

$$\vec{R} = \vec{A} + \vec{B} \quad (1)$$

Now for finding the rectangular components, we resolve $\vec{A}$ and $\vec{B}$ into their horizontal and vertical components. For this purpose we draw perpendiculars from "C" and "D" on x-axis as shown in the figure. We also draw a perpendicular from "C" on "DG" which cuts "DG" at point "F" as shown in the figure.

Now we see that, $\vec{OE} = \vec{A}_x$ and $\vec{CE} = \vec{A}_y$ represent the horizontal and



vertical components of $\vec{A}$ respectively. While $\vec{CE} = \vec{B}_x$ and $\vec{FD} = \vec{B}_y$, represents the horizontal and vertical components of $\vec{B}$ respectively.

Now from the figure, we have, $\left[\vec{CF} = \vec{EG} = \vec{B}_x \text{ and } \vec{CE} = \vec{FG} = \vec{B}_y \right]$

$$\vec{R}_x = \vec{A}_x + \vec{B}_x$$

$$\Rightarrow R_x \hat{i} = A_x \hat{i} + B_x \hat{i} \Rightarrow \boxed{R_x = A_x + B_x} \quad (2)$$

Equation (2) represents the magnitude of x-component of $\vec{R}$.

Similarly, $\vec{R}_y = \vec{A}_y + \vec{B}_y$

$$\Rightarrow R_y \hat{j} = A_y \hat{j} + B_y \hat{j} \Rightarrow \boxed{R_y = A_y + B_y} \quad (3)$$

Equation (3) represents the magnitude of y-component of $\vec{R}$.

Now for any number of co-planar vectors ($\vec{A}, \vec{B}, \vec{C}, \vec{D}, \dots$) equation (2) and equation (3) can be written as,

$$\boxed{R_x = A_x + B_x + C_x + D_x + \dots} \quad (4)$$

Similarly,

$$\boxed{R_y = A_y + B_y + C_y + D_y + \dots} \quad (5)$$

Magnitude of $\vec{R}$:

The magnitude of $\vec{R}$ is given by,

$$R = \sqrt{R_x^2 + R_y^2} \quad (6)$$

Putting equation (4) and equation (5) in equation (6), we get,

$$\boxed{R = \sqrt{(A_x + B_x + C_x + D_x + \dots)^2 + (A_y + B_y + C_y + D_y + \dots)^2}} \quad (7)$$

Equation (7) represents the magnitude of $\vec{R}$

Direction of $\vec{R}$:

We know that,

$$\tan \theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1} \left(\frac{R_y}{R_x} \right) \quad (8)$$

Putting equations (4), (5) in equation (8), we get,

$$\boxed{\theta = \tan^{-1} \left(\frac{A_y + B_y + C_y + D_y + \dots}{A_x + B_x + C_x + D_x + \dots} \right)} \quad (9)$$

Equation (9) represents the direction of vector $\vec{R}$

Steps for Vector Addition by Rectangular Components Method:

1. Draw the individual vectors being summed and the axis being used.
2. Find the x-components [horizontal component] and y-components [vertical component] of all the given vectors.
3. Add the x-components of all the given vectors to get the x-component " R_x " of the resultant vector $\vec{R}$.
4. Add the y-components of all the given vectors to get the y-component " R_y " of the resultant vector $\vec{R}$.
5. Find the magnitude of the resultant vector $\vec{R}$ i.e.
 $R = \sqrt{R_x^2 + R_y^2}$
6. Find the direction of the resultant vector $\vec{R}$ i.e.
 $\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$

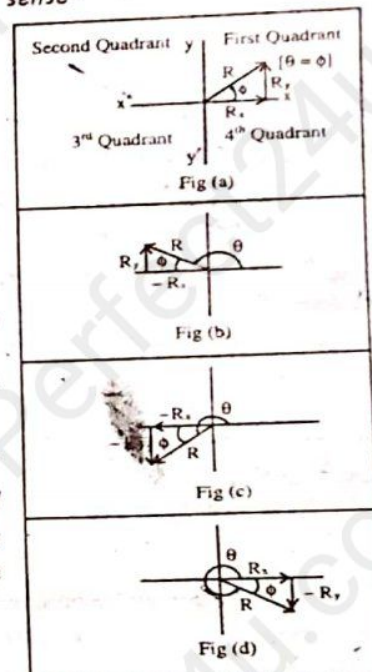
Where θ represents the angle which is made by the resultant vector with positive x-axis in anti-clockwise sense.

Procedure for determining " θ ":

Let " ϕ " be the angle made by the vector $\vec{R}$ in any quadrant and θ be the angle made by the vector $\vec{R}$ with positive x-axis in anti-clockwise sense. We can determine the value of θ by using the following procedure:

1. Find the value of $\tan^{-1} \left(\frac{R_y}{R_x} \right)$ by using calculator and call this angle as " ϕ ".
2. If " R_x " and " R_y " both are positive, then the resultant " $\vec{R}$ " will lie in 1st quadrant as shown in the figure (a). In such a case, we have

$$\boxed{\theta = \phi}$$



3. If " R_x " is negative and " R_y " is positive, then " $\vec{R}$ " will lie in 2nd quadrant, as shown in figure (b). In such a case, θ can be calculated as,

$$\theta = 180^\circ - \phi$$

4. If " R_x " and " R_y " both are negative, then " $\vec{R}$ " will lie in 3rd quadrant, as shown in figure (c). In such a case, θ can be calculated as,

$$\theta = 180^\circ + \phi$$

5. If " R_x " is positive and " R_y " is negative, then " $\vec{R}$ " will lie in the 4th quadrant, as shown in figure (d). In such a case, θ can be calculated as,

$$\theta = 360^\circ - \phi$$

Q16: Define and explain scalar product (or) dot product?

(ETE 2007-2017, BISE DI Khan 2019)

Ans. Scalar Product:

Definition: If we multiply two vectors and we get a scalar quantity, then such product of two vectors is known as scalar product or dot product.

Explanation:

The scalar product of two vectors is equal to the product of the magnitude of the two vectors and the cosine of the angle between them.

Let $\vec{A}$ and $\vec{B}$ represents the two vectors, then we have,

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad (1)$$

Similarly,

$$\vec{B} \cdot \vec{A} = BA \cos \theta \quad [AB = BA]$$

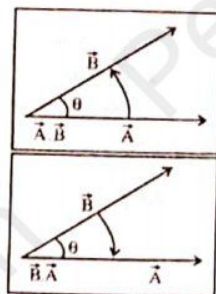
$$\Rightarrow \vec{B} \cdot \vec{A} = AB \cos \theta \quad (2)$$

Comparing equation (1) and equation (2), we get,

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad (3)$$

Equation (3) shows that the scalar product (or) dot product obeys commutative law. Now in case of unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ we have,

$$\hat{i} \cdot \hat{i} = 1 \cos 0 \quad [\text{Magnitude of unit vector} = 1]$$



$$\Rightarrow \hat{i} \cdot \hat{i} = 1 \times 1 \times \cos 0^\circ$$

$$\Rightarrow \hat{i} \cdot \hat{i} = 1 \times 1 \times 1 \quad [\cos 0^\circ = 1]$$

$$\Rightarrow \hat{i} \cdot \hat{i} = 1$$

Similarly,

$$\hat{j} \cdot \hat{j} = 1 \quad \text{and} \quad \hat{k} \cdot \hat{k} = 1$$

$$\text{Now } \hat{i} \cdot \hat{j} = ij \cos \theta$$

$$\Rightarrow \hat{i} \cdot \hat{j} = 1 \times 1 \times \cos 90^\circ \quad [\cos 90^\circ = 0]$$

$$\Rightarrow \hat{i} \cdot \hat{j} = 1 \times 1 \times 0 = 0$$

$$\Rightarrow \hat{i} \cdot \hat{j} = 0$$

Similarly,

$$\hat{j} \cdot \hat{i} = 0, \hat{j} \cdot \hat{k} = 0, \hat{k} \cdot \hat{j} = 0, \hat{k} \cdot \hat{i} = 0, \hat{i} \cdot \hat{k} = 0$$

Example: If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ then find $\vec{A} \cdot \vec{B} = ?$

Solution:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}, \quad \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\text{Now } \vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\Rightarrow \vec{A} \cdot \vec{B} = [A_x B_x (\hat{i} \cdot \hat{i}) + A_x B_y (\hat{i} \cdot \hat{j}) + A_x B_z (\hat{i} \cdot \hat{k}) + A_y B_x (\hat{j} \cdot \hat{i}) + A_y B_y (\hat{j} \cdot \hat{j}) + A_y B_z (\hat{j} \cdot \hat{k}) + A_z B_x (\hat{k} \cdot \hat{i}) + A_z B_y (\hat{k} \cdot \hat{j}) + A_z B_z (\hat{k} \cdot \hat{k})]$$

$$\Rightarrow \vec{A} \cdot \vec{B} = [A_x B_x \times 1 + A_x B_y \times 0 + A_x B_z \times 0 + A_y B_x \times 0 + A_y B_y \times 1 + A_y B_z \times 0 + A_z B_x \times 0 + A_z B_y \times 0 + A_z B_z \times 1]$$

$$\Rightarrow \vec{A} \cdot \vec{B} = A_x B_x + 0 + 0 + 0 + A_y B_y + 0 + 0 + 0 + A_z B_z$$

$$\Rightarrow \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \quad (4)$$

The R.H.S of equation (4) represents a scalar quantity, so such product of two vector $\vec{A}$ and $\vec{B}$ is known as scalar product (or) dot product.

Characteristics of Scalar Product:

1. The scalar product obeys the commutative law i.e.

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

2. The scalar product obeys the condition of perpendicularity i.e.

$$\vec{A} \cdot \vec{B} = AB \cos \theta = AB \cos 90^\circ = AB \times 0 = 0 \Rightarrow \vec{A} \cdot \vec{B} = 0$$

3. The scalar product obeys the condition of parallelism i.e.

$$\vec{A} \cdot \vec{B} = AB \cos 0^\circ = AB \times 1 \Rightarrow \vec{A} \cdot \vec{B} = AB$$

4. The scalar product obeys the condition of anti parallelism i.e.

$$\vec{A} \cdot \vec{B} = AB \cos 180^\circ = AB \cos(-1) \Rightarrow \vec{A} \cdot \vec{B} = -AB$$

5. If $\vec{A} \cdot \vec{B} = 0$ and $\theta \neq 90^\circ$, then either $A = 0$ or $B = 0$.

6. If $\vec{A} \cdot \vec{B} = 0$ and $\theta \neq 90^\circ$, also $A \neq 0$ then $\vec{B}$ must be a null vector i.e. $B = 0$.

7. The scalar product obeys distributive law i.e.

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

Examples of Scalar Product:

1. Workdone = $w = \vec{F} \cdot \vec{d} \Rightarrow$

$$w = \vec{F} \cdot \vec{d}$$

2. Electric flux = $\phi_e = \vec{E} \cdot \vec{A} \Rightarrow$

$$\phi_e = \vec{E} \cdot \vec{A}$$

3. Power = $P = \vec{F} \cdot \vec{v} \Rightarrow$

$$P = \vec{F} \cdot \vec{v}$$

4. Magnetic flux = $\phi_m = \vec{B} \cdot \vec{A} \Rightarrow$

$$\phi_m = \vec{B} \cdot \vec{A}$$

Q17: Define and explain vector product (or) cross product?

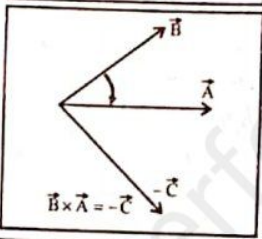
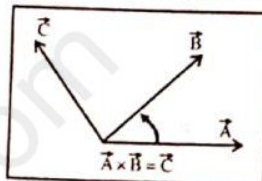
ETEA 2006-2008, BISE Malakand 2019, BISE Peshawar 2019

Ans. Vector Product:

Definition: If we multiply two vectors and we get a vector quantity, then such product of two vectors is known as vector product (or) cross product.

Explanation: The vector product of two vectors is equal to the product of the magnitude of the two vectors and the sine of the angle between them. Let $\vec{A}$ and $\vec{B}$ represents the two vectors, then $(\vec{A} \times \vec{B})$ gives us another vector $\vec{C}$ which is perpendicular to both $\vec{A}$ and $\vec{B}$ i.e.

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} \quad (1)$$



$$(OR) \vec{A} \times \vec{B} = \vec{C} \quad (2)$$

$$\text{But } \vec{B} \times \vec{A} = -\vec{C} \quad (3)$$

Comparing equation (2) and equation (3), we get,

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A} \quad (4)$$

Equation (4) shows that the vector product do not obeys the commutative law. Now in case of unit vectors $\hat{i}, \hat{j}, \hat{k}$ we have,

$$\hat{i} \times \hat{i} = \hat{i} \sin 0^\circ \hat{n}$$

$$\Rightarrow \hat{i} \times \hat{i} = \hat{i} \sin 0^\circ \hat{n}$$

$$= 1 \times 1 \times 0 \times \hat{n}$$

$$\Rightarrow \hat{i} \times \hat{i} = 0$$

Similarly,

$$\hat{j} \times \hat{j} = 0, \hat{k} \times \hat{k} = 0$$

Now $\hat{i} \times \hat{j} = \hat{i} \hat{j} \sin 90^\circ \hat{k}$

$$= \hat{i} \hat{j} \sin 90^\circ (\hat{k}) = 1 \times 1 \times 1 (\hat{k})$$

$$\Rightarrow \hat{i} \times \hat{j} = \hat{k}$$

Similarly,

$$\hat{j} \times \hat{i} = -\hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{j} = -\hat{i}, \hat{k} \times \hat{i} = \hat{j}, \hat{i} \times \hat{k} = -\hat{j}$$

Example: If $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ and $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$ then find $(\vec{A} \times \vec{B})$?

Solution: $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}, \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

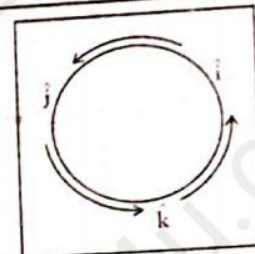
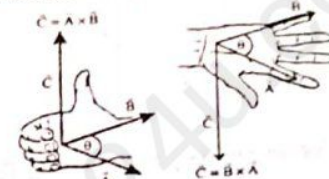
$$\text{Now } \vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$\Rightarrow \vec{A} \times \vec{B} = \begin{bmatrix} A_x B_y (\hat{i} \times \hat{j}) + A_x B_z (\hat{i} \times \hat{k}) + A_y B_x (\hat{j} \times \hat{i}) + A_y B_z (\hat{j} \times \hat{k}) + A_z B_x (\hat{k} \times \hat{i}) + A_z B_y (\hat{k} \times \hat{j}) + A_x B_z (\hat{k} \times \hat{k}) \end{bmatrix}$$

$$\Rightarrow \vec{A} \times \vec{B} = \begin{bmatrix} A_x B_y \hat{k} + A_x B_z (-\hat{j}) + A_y B_x (-\hat{k}) + A_y B_z \hat{k} + A_z B_x \hat{j} + A_z B_y (-\hat{i}) + A_x B_z \times 0 \end{bmatrix}$$

For your information:

You can determine the direction of vector " $\vec{C}$ " by Right Hand Rule as shown in the figure.



$$\Rightarrow \vec{A} \times \vec{B} = \begin{bmatrix} 0 + A_x B_y \hat{k} - A_x B_z \hat{j} - A_y B_x \hat{k} + 0 + A_y B_z \hat{j} + \\ A_z B_x \hat{j} - A_z B_y \hat{i} + 0 \end{bmatrix}$$

$$\Rightarrow \vec{A} \times \vec{B} = A_x B_y \hat{k} - A_x B_z \hat{j} - A_y B_x \hat{k} + A_y B_z \hat{j} + A_z B_x \hat{j} - A_z B_y \hat{i}$$

$$\Rightarrow \vec{A} \times \vec{B} = A_y B_z \hat{j} - A_z B_y \hat{i} + A_z B_x \hat{j} - A_x B_z \hat{j} + A_x B_y \hat{k} - A_y B_x \hat{k}$$

$$\Rightarrow \vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \quad (5)$$

The R.H.S of equation (5) represents a vector quantity. So such product of the two vectors is known as vector product (or) cross product.

Characteristics of Vector Product:

1. The vector product do not obeys the commutative law i.e.
 $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$
2. The vector product obeys the condition of parallelism i.e.
 $\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = AB \sin 0^\circ \hat{n} \quad [\sin 0^\circ = 0]$
 $\Rightarrow \vec{A} \times \vec{B} = 0$
3. If $\vec{A}$ and $\vec{B}$ are anti-parallel, then we put
 $\theta = 180^\circ$, so we get,
 $\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = AB \sin 180^\circ \hat{n} \quad [\sin 180^\circ = 0]$
 $\Rightarrow \vec{A} \times \vec{B} = 0$
4. If $\vec{A} \times \vec{B} = 0$ and $A \neq 0, \theta \neq 0^\circ$ or $\theta \neq 180^\circ$, then $\vec{B}$ must be a null vector.
5. The vector product obeys the distributive law i.e.
 $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$

Examples of Vector Product:

1. Angular momentum $= \vec{L} = \vec{r} \times \vec{p} \Rightarrow \vec{L} = \vec{r} \times \vec{p}$
2. Linear velocity in terms of angular velocity i.e. $\vec{v} = \vec{\omega} \times \vec{r}$
3. Torque $= \vec{r} \times \vec{F} \Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$
4. Magnetic force $= q(\vec{v} \times \vec{B}) \Rightarrow \vec{F}_m = q(\vec{v} \times \vec{B})$

Physical Significance of Vector Product:

1. Using vector product, we can determine the area of a parallelogram.

2. Using vector product, we can determine the torque or moment of force i.e. $\vec{\tau} = \vec{r} \times \vec{F}$.
3. Using vector product, we can determine the angular momentum of a body during angular motion i.e. $\vec{L} = \vec{r} \times \vec{p}$.

Q18: Define and explain torque (or) moment of force.

Ans: Torque (or) Moment of Force:

Definition: The turning effect produced in a body about a fixed point due to an applied force is known as torque (or) moment of force.

Explanation:

The torque is an agent which produces the angular acceleration in a body. It depends upon the moment arm $\vec{r}$ and applied force $\vec{F}$.

The moment arm represents the perpendicular distance between the line of action of force and the point of rotation. Mathematically, we have,

Torque = Moment arm $\times$ effective force

$$\Rightarrow \vec{\tau} = \vec{r} \times \vec{F}$$

$$\Rightarrow \tau = rF \sin \theta \quad (1)$$

Equation (1) represents the vector form of torque. The magnitude of torque is given by,

$$\tau = rF \sin \theta \quad (2)$$

Equation (2) shows that for given values of "r" and "F", the torque also depends upon " θ ".

(i) If $\theta = 0^\circ$ i.e. $\vec{r}$ and $\vec{F}$ are parallel to each other, then,

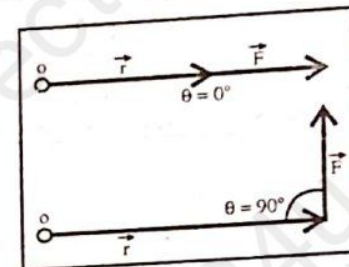
$$\tau = rF \sin 0^\circ$$

$$\Rightarrow \tau = rF \times 0 \quad [\sin 0^\circ = 0]$$

$$\Rightarrow \tau = 0 \quad (3)$$

Equation (3) shows that, there will be no torque, if $\vec{r}$ and $\vec{F}$ are parallel to each other.

(ii) If $\theta = 90^\circ$ i.e. $\vec{r}$ and $\vec{F}$ are perpendicular to each other, then



Equation (2)

$$\Rightarrow \tau = rF \sin 90^\circ \quad [\sin 90^\circ = 1]$$

$$\Rightarrow \tau = rF \times 1$$

$$\Rightarrow \tau_{\max} = rF \quad (4)$$

Equation (4) shows that torque will be maximum if $\vec{r}$ and $\vec{F}$ are perpendicular to each other. The torque is a vector quantity. Its unit is $\text{N}\cdot\text{m}$. Clockwise torque is taken as negative while anti-clockwise torque is taken as positive.

Dimension of Torque: (ETEA 2006-2008)As torque $= \tau = rF$

$$\Rightarrow \tau = rma \quad [F = ma]$$

$$\Rightarrow \tau = m \times \text{kg} = \text{m} / \text{sec}^2$$

$$\Rightarrow \tau = \text{length} \times \text{mass} \times \text{length}/(\text{time})^2$$

$$\Rightarrow \tau = [L^2 MT^{-2}]$$

Torque is perpendicular to the plane of $\vec{r}$ and $\vec{F}$.
(ETEA 2009)

Q19: Discuss the application of torque?

Ans.

1. When we apply equal and opposite forces on the steering wheel of a vehicle with both hands, torque is produced and steering wheel can be rotated easily.
2. When we apply forces on the paddles of a cycle by means of our feet, then these forces act as couple forces. As a result, the wheels of cycle begin to rotate and cycle moves in the forward direction.
3. Similarly in an electrical instrument such as galvanometer, ammeter and voltmeter torque is exerted on the rectangular coil by means of electric current flowing through it. Under the action of this torque the coil rotates and comes to rest where the torque is balanced by opposing torque. In this way needle move on the calibrated scale.
4. Applying torque we can open a tightly closed bottle having threaded closures on the neck of the bottle.
5. A cross bar wrench is used to change a tyre of a car. A certain force is being exerted on each handle of the wrench in opposite direction. These forces constitute a couple due to which the

net torque is equal to the product of any one force and the couple arm.

$$\tau = FL \quad (1)$$

Where "L" is called a couple arm.

6. Similarly applying torque on a pulley by means of a rope we can take water from a well in daily life.

Q20: Define equilibrium. Explain with examples?ETEA 2012Ans. Equilibrium:

Definition: The state of a body, under the action of several forces acting together and there is no change in the translational motion as well as its rotational motion is known as equilibrium. (OR)
A body is said to be in state of equilibrium, if it is at rest or moving with uniform velocity.

Explanation:

There are two types of equilibrium which are given below:

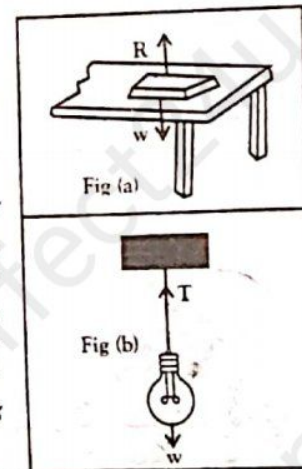
1. Static Equilibrium:

When a body is at rest then its acceleration will be zero and it will be in state of static equilibrium.

Examples of Static Equilibrium:

A book lying on the table is the example of static equilibrium. In such a case, the weight of the book "w" is acting in downwards direction while the normal-reaction "R" of the table on the book is acting in upwards direction, as shown in figure (a). Both forces balance each other and as a result, the book remains in state of static equilibrium.

A bulb hanging from a rigid support by means of an inextensible thread is another example of static equilibrium. In this case, the weight of the bulb "w" is acting in downwards direction while the tension [reaction] "T" in the string [thread] is acting in upwards direction. Both forces balance each other and as a result the bulb remains in state of static equilibrium.



2. Dynamic Equilibrium:

When a body is moving with uniform velocity, then the body will have no acceleration and the body will be in state of dynamic equilibrium.

Types of Dynamic Equilibrium:

1. Dynamic Translational Equilibrium: When a body is moving uniform linear velocity then the body is said to be in state of dynamic translational equilibrium.

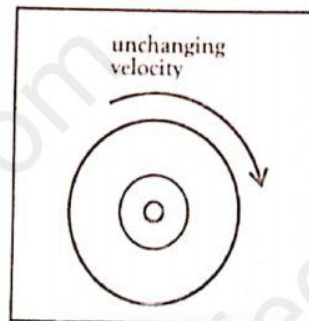
Example: A paratrooper falling down with constant velocity is the example of translational dynamic equilibrium.

When a paratrooper jumps from an aeroplane in air, its weight " w " is acting in downwards direction while the reaction " R " of air is acting in upwards direction. Both forces balance each other and as a result the paratrooper moves downwards with uniform velocity. Such velocity is known as terminal velocity of paratrooper. In this way, the paratrooper remains in state of translational dynamic equilibrium.



2. Dynamic Rotational Equilibrium: When a body is moving with uniform angular velocity, then the body is said to be in state of dynamic rotational equilibrium.

Examples: A compact disk [CD] rotating in CD-player with constant angular velocity is the example of dynamic rotational equilibrium. A rotating wheel and grinding rotating about its axis with constant angular speed are also the examples of dynamic rotational equilibrium.



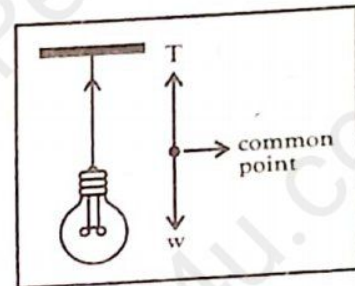
Q21: What do you mean by concurrent forces? Explain with examples.

Ans. Concurrent Forces:

Definition: Two or more than two forces are said to be concurrent forces, if they are acting upon a body and their lines of action pass through a common point.

Explanation:

Consider a bulb which is suspended by means of a thread from a rigid support as shown in the figure. In this case, the weight " w " of the bulb is acting in downwards direction while the tension [reaction] " T " in the thread is acting in upwards direction. The line of action of both forces passes through the same point or common point as shown in the figure. Both forces balance each other and as a result, the bulb remains in state of equilibrium. Such forces are known as concurrent forces.



Q22: Explain couple forces?

ETEA 2006-2008, 2012

Ans. Couple Forces:

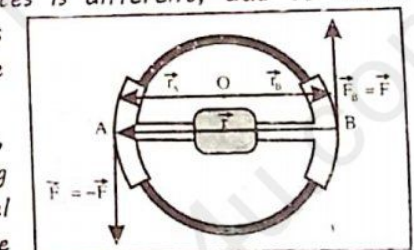
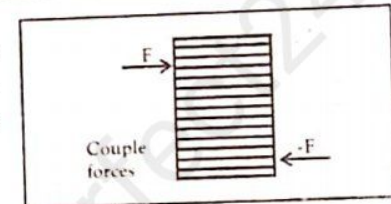
Definition: Two forces having same magnitude, acting in opposite direction having different line of action are called couple force.

Explanation:

Consider two forces " F " and " $-F$ " of equal magnitude acting on a body in opposite direction as shown in the figure.

The line of action of both these forces is different, due to which the body begins to rotate and thus produce torque. Such forces are known as couple forces.

Similarly, when we are driving a car, with both hands on the steering wheel, we push one side of the wheel upward with one hand and push the



other side downward with the other hand, as shown in the figure. Both forces are equal in magnitude but opposite in direction. So as a result, the steering wheel begins to rotate due to couple forces. The couple forces produce angular motion in the body and results in torque. Such forces are always non-collinear and non-concurrent.

Q23: Discuss the conditions of equilibrium?

(BISE Peshawar 2017)

Ans. There are two conditions of equilibrium which are given below:

1. First Condition of Equilibrium:

According to the first condition of equilibrium, a body is said to be in equilibrium, if the vector sum of all external forces acting on the body is zero i.e. they cancel the effect of each other i.e.

$$\sum \vec{F} = \vec{0}$$

In components form, we have,

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = \vec{0}$$

$$\Rightarrow \sum \vec{F}_x = \vec{0}$$

Similarly,

$$\vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y} + \dots + \vec{F}_{ny} = \vec{0}$$

$$\Rightarrow \sum \vec{F}_y = \vec{0}$$

A book lying on the table and a paratrooper moving downwards with uniform velocity satisfies the first condition of equilibrium.

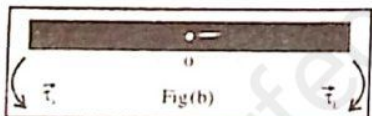
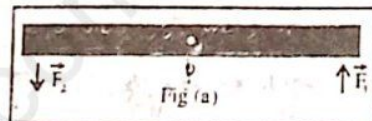
2. Second Condition of Equilibrium:

According to the second condition of equilibrium, a body is said to be in equilibrium if the vector sum of all the torques acting on a body is equal to zero i.e. $\sum \vec{\tau} = \vec{0}$

Explanation:

There are certain situations, in which an extended body will not be in equilibrium even when the first condition of equilibrium is satisfied.

Example: In the figure (a) two equal forces are acting on a meter rod in opposite direction. As the



lines of action of these forces are not the same, so the meter rod do not remain in state of equilibrium and begins to rotate in anti-clockwise direction. So the first condition of equilibrium is not sufficient in such cases. Under such situations we should arrange the forces on the body in such a way, that they causes clockwise torque and anti-clockwise torque acting on the body simultaneously and cancel the effect of each other, as shown in figure (b). Thus we have,

Sum of clockwise torque = Sum of anti-clockwise torque

$$\Rightarrow \sum \vec{\tau}_{\text{clock}} = \sum \vec{\tau}_{\text{anticlock}} \quad (1)$$

In general form, equation (1) can be written as,

$$\sum \vec{\tau} = \vec{0} \quad (2)$$

NOTE: When the first condition is satisfied, there will be no linear acceleration and the body will be in state of translational equilibrium.

When the second condition of equilibrium is satisfied, there will be no rotational acceleration and the body will be in state of rotational equilibrium.

Thus we can say that, a body will be in complete equilibrium, if both conditions are satisfied. Thus in general, a body is said to be in state of equilibrium if it has no acceleration.

Q24: Explain centre of gravity and centre of mass.

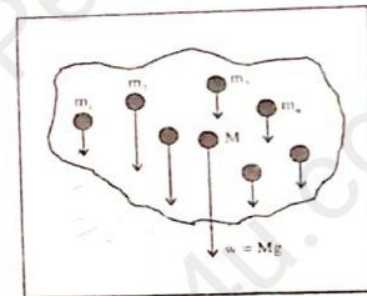
ETEA 2017

Ans. **Definition:** The centre of gravity is that point in a rigid body, where total weight of the body act, while the point at which the external force can produce translatory motion in the body is known as centre of mass. Both these points exists in the same region of the rigid body.

Explanation:

Consider a rigid body having total weight "w" and total mass "M" as shown in the figure.

Let the body is divided into large number of small elements each having mass $m_1, m_2, m_3, \dots, m_n$ respectively.



Now due to the force of gravity, each element will experience gravitational force, i.e. $w_1, w_2, w_3, \dots, w_n$ respectively.

Now the total resultant force experienced by this body will be equal to the sum of forces experienced by the individual elements i.e.

$$\text{Total weight} = w = w_1 + w_2 + w_3 + \dots + w_n$$

$$\Rightarrow w = m_1g + m_2g + m_3g + \dots + m_ng$$

$$\Rightarrow w = [m_1 + m_2 + m_3 + \dots + m_n]g$$

$$\Rightarrow w = [\text{total mass}]g \quad [\text{total mass} = M]$$

$$\Rightarrow w = Mg \quad (1)$$

In equation (1), "w" represents the resultant weight which acts at a single point. Such point is known as centre of gravity.

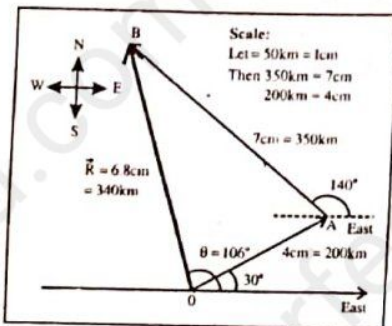
Solution of Examples & Assignments

EXAMPLE 2.1:

A ship leaves port and travels 200km at 30° North of East. Then it changes its direction and travels 350km in a direction 140° North of East to reach destination. Calculate straight line distance covered by ship.

Solution:

1. First we specify the direction.
2. Now we select a suitable scale. Let $50\text{km} = 1\text{cm}$ then $200\text{km} = 4\text{cm}$, $350\text{km} = 7\text{cm}$
3. Now we draw a vector which is 4cm long and making an angle of 30° with east. Now we draw another vector which is 7cm long and making an angle of 140° with east as shown in the figure.
4. Now for finding the resultant vector, we combine the tail of first vector $\vec{OA}$ with the head of last vector $\vec{AB}$. Thus $\vec{OB} = \vec{R}$ represent the straight line distance covered by the



ship.

5. We measure the length of $\vec{OB}$ with the help of meter rod which is about 6.8cm [340km].
6. We measure the angle made by vector $\vec{OB}$ with east with the help of protector which is 106° with east.

Ans. (a) $\text{Straight line distance} = 6.8\text{cm} = 340\text{km}$

(b) $\text{Angle of straight line with east} = 106^\circ$

ASSIGNMENT 2.1:

An airplane is moving at 120m/s at an angle of 10° with x-axis, through a 30m/s cross wind, blowing at angle of 260° with x-axis. Determine the resultant velocity of the airplane.

Solution:

Given Data:

Let $\vec{v}_1 = 120\text{m/sec}$ making an angle of 10° with x-axis and $\vec{v}_2 = 30\text{m/sec}$ making an angle of 260° with x-axis.

Required: Resultant velocity of airplane = $\vec{v} = ?$

Calculation: The resultant velocity is given by,

$$v = \sqrt{v_x^2 + v_y^2} \quad (1)$$

First we resolve $\vec{v}_1$ and $\vec{v}_2$ into its horizontal and vertical components as shown in figure (b). Now we have,

$$v_{1x} = v_1 \cos \theta_1 = 120 \times \cos 10^\circ \text{ m/sec}$$

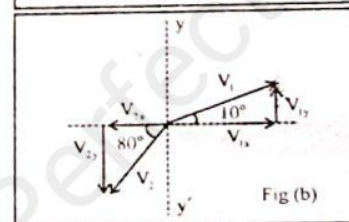
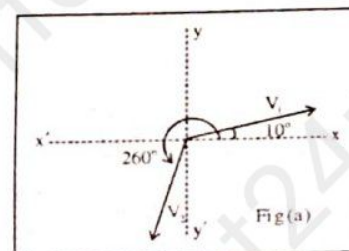
$$\Rightarrow v_{1x} = 120 \times 0.985 \text{ m/sec}$$

$$\Rightarrow v_{1x} = 118.2 \text{ m/sec}$$

$$\text{And } v_{1y} = v_1 \sin \theta_1 = 120 \times \sin 10^\circ = 120 \times 0.174 \text{ m/sec}$$

$$\Rightarrow v_{1y} = 20.88 \text{ m/sec}$$

Similarly,



$$v_{2x} = v_2 \cos \theta_2 = 30 \times \cos 80^\circ$$

$$= 30 \times 0.174 \text{ m/sec}$$

$$\Rightarrow v_{2x} = 5.22 \text{ m/sec}$$

$$\text{And } v_{2y} = v_2 \sin \theta_2 = 30 \times \sin 80^\circ$$

$$= 30 \times 0.985 \text{ m/sec}$$

$$\Rightarrow v_{2y} = 29.55 \text{ m/sec}$$

Now the x-component of resultant velocity is given by,

$$v_x = v_{1x} + (-v_{2x}) = v_{1x} - v_{2x} = (118.2 - 5.22) \text{ m/sec}$$

$$\Rightarrow v_x = 112.98 \text{ m/sec}$$

Now the y-component of resultant velocity is given by,

$$v_y = v_{1y} + (-v_{2y}) = v_{1y} - v_{2y} = (20.88 - 29.55) \text{ m/sec}$$

$$\Rightarrow v_y = -8.67 \text{ m/sec}$$

Putting the values of " v_x " and " v_y " in equation (1)

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(112.98)^2 + (-8.67)^2}$$

$$\Rightarrow v = \sqrt{12764.5 + 75.2} = \sqrt{12839.7}$$

$$\Rightarrow v = 113.3 \text{ m/sec}$$

The direction of " v " is given by,

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{-8.67}{112.98} \right)$$

$$\Rightarrow \theta = \tan^{-1}(0.077) \Rightarrow \theta = 4^\circ$$

Now the actual angle of " v " with positive x-axis is given by,

$$\theta' = 360^\circ - 4^\circ \Rightarrow \theta' = 356^\circ$$

EXAMPLE 2.2:

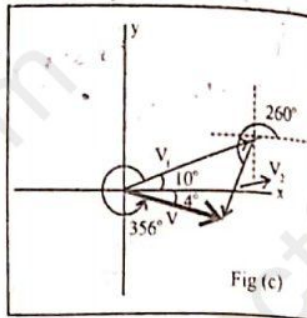
Two forces $\vec{F}_1 = 15\text{N}$ making an angle $\theta_1 = 70^\circ$ with positive x-axis and force $\vec{F}_2 = 25\text{N}$ making an angle $\theta_2 = 220^\circ$ with positive x-axis, act at a point, calculate the resultant force $\vec{F}_R$.

Solution:

Given Data:

Force $F_1 = 15\text{N}$, angle $\theta_1 = 70^\circ$

Force $F_2 = 25\text{N}$, angle $\theta_2 = 220^\circ$



6

Required: Resultant force $\vec{F}_R = ?$

Calculation: The magnitude of the resultant force is given by,

$$F = \sqrt{F_x^2 + F_y^2} \quad (1)$$

First we resolve " $\vec{F}_1$ " and " $\vec{F}_2$ " into its components. For " $\vec{F}_1$ ", we have,

$$F_{1x} = F_1 \cos \theta_1 = 15 \times \cos 70^\circ = 15 \times 0.342\text{N}$$

$$\Rightarrow F_{1x} = 5.13\text{N}$$

$$\text{And } F_{1y} = F_1 \sin \theta_1 = 15 \times \sin 70^\circ = 15 \times 0.939\text{N}$$

$$\Rightarrow F_{1y} = 14.09\text{N}$$

For $\vec{F}_2$ we have,

$$F_{2x} = -F_2 \cos \theta_2 = -25 \times \cos 40^\circ = -25 \times 0.766\text{N}$$

$$\Rightarrow F_{2x} = -19.15$$

$$\Rightarrow F_{2x} = -19.15\text{N} \text{ [negative sign shows that}$$

" $\vec{F}_2$ " is acting along x' -axis]

$$\text{And } F_{2y} = -F_2 \sin \theta_2 = -25 \times \sin 40^\circ$$

$$= -25 \times 0.643\text{N}$$

$$\Rightarrow F_{2y} = -16.07\text{N}$$

Now the x-component of the resultant is given by,

$$F_x = F_{1x} + F_{2x} = 5.13\text{N} + (-19.15\text{N}) = 5.13\text{N} - 19.15\text{N}$$

$$\Rightarrow F_x = -14.02\text{N}$$

And the y-component of the resultant is given by,

$$F_y = F_{1y} + F_{2y} = 14.09\text{N} + (-16.07\text{N}) = 14.09\text{N} - 16.07\text{N}$$

$$\Rightarrow F_y = -1.98\text{N}$$

Putting the values of " F_x " and " F_y " in equation (1) we get,

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(-14.02\text{N})^2 + (-1.98\text{N})^2}$$

$$\Rightarrow F_R = \sqrt{196.6 + 3.9} \text{N} = \sqrt{200.5\text{N}}$$

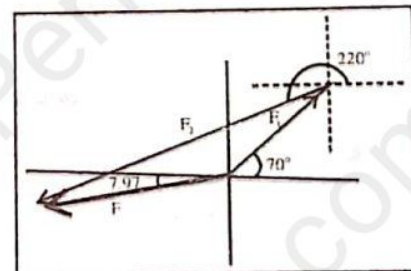
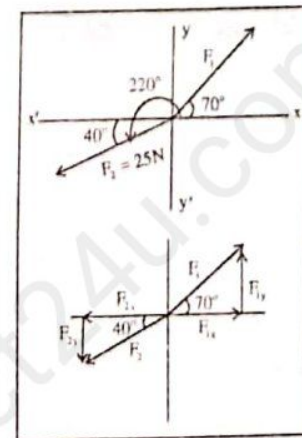
$$\Rightarrow F_R = 14.16\text{N}$$

The direction of " $\vec{F}_R$ " is given by,

$$\theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) = \tan^{-1} \left(\frac{-1.98}{-14.02} \right) = \tan^{-1}(0.14)$$

$$\Rightarrow \theta = 7.97^\circ$$

As " F_x " and " F_y " both are negative so the resultant force lies in



third quadrant. So the actual angle of "F" with positive x-axis is given by,

$$\theta' = 180^\circ + \theta = 180^\circ + 7.97^\circ \Rightarrow \theta = 187.97^\circ$$

ASSIGNMENT 2.2:

A force $F_1 = 20\text{N}$ making an angle $\theta_1 = 30^\circ$ with positive x-axis and force $F_2 = 30\text{N}$ making an angle $\theta_2 = 150^\circ$ with positive x-axis, acts at a point, calculate the resultant force?

Solution:

Given Data:

$F_1 = 20\text{N}$ at angle of $\theta_1 = 30^\circ$ with positive x-axis

$F_2 = 30\text{N}$ at angle of $\theta_2 = 150^\circ$ with positive x-axis

Required: Resultant force = $F = ?$

Calculation: The magnitude of resultant force is given by,

$$F = \sqrt{F_x^2 + F_y^2} \quad (1)$$

First we resolve $\vec{F}_1$ and $\vec{F}_2$ into its components, as shown in figure (b). For " F_1 ", we have,

$$F_{1x} = F_1 \cos \theta_1 = 20 \times \cos 30^\circ = 20 \times 0.866\text{N}$$

$$\Rightarrow F_{1x} = 17.32\text{N}$$

And $F_{1y} = F_1 \sin \theta_1 = 20 \times \sin 30^\circ = 20 \times 0.5\text{N}$

$$\Rightarrow F_{1y} = 10\text{N}$$

For F_2 , we have,

$$F_{2x} = -F_2 \cos \theta_2 = -30 \times \cos 30^\circ = -30 \times 0.866\text{N}$$

$$\Rightarrow F_{2x} = -25.98\text{N}$$

And $F_{2y} = F_2 \sin \theta_2 = 30 \times \sin 30^\circ = 30 \times 0.5\text{N}$

$$\Rightarrow F_{2y} = 15\text{N}$$

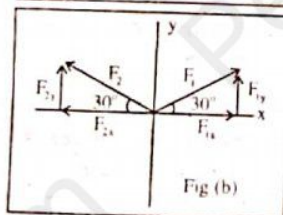
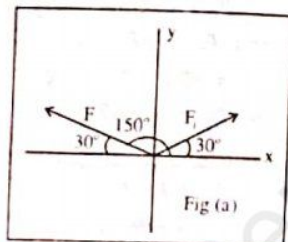
Now the horizontal component [x-component] of the resultant "F" is given by,

$$F_x = F_{1x} + F_{2x} = [17.32 + (-25.98)]\text{N} = (17.32 - 25.98)\text{N}$$

$$\Rightarrow F_x = -8.66\text{N}$$

Now the y-component [vertical component] of the resultant force "F" is given by,

$$F_y = F_{1y} + F_{2y} = 10\text{N} + 15\text{N} \Rightarrow F_y = 25\text{N}$$



Putting the values of " F_x " and " F_y " in equation (1), we get,

$$F = \sqrt{(-8.66\text{N})^2 + (25)^2} = \sqrt{74.9956 + 625}\text{N}$$

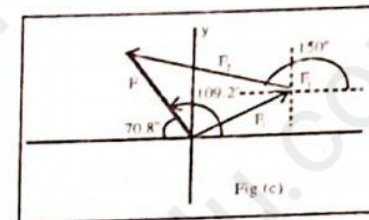
$$\Rightarrow F = \sqrt{699.9956}\text{N} \Rightarrow F = 26.46\text{N} \text{ or } F = 26.5\text{N}$$

Direction of $\vec{F}$:

$$\text{We have, } \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{25}{-8.66} \right) = \tan^{-1} (2.886)$$

$$\Rightarrow \theta = 70.8^\circ$$



As x-component of $\vec{F}$ is negative, y-component is positive, so resultant force $\vec{F}$ will be acting in second quadrant by making an angle of 70.8° with negative x-axis, as shown in the figure (c). So the actual angle made by $\vec{F}$ with positive x-axis is given by,

$$\theta' = 180^\circ - 70.8^\circ = 109.2^\circ \Rightarrow \theta' = 109.2^\circ$$

EXAMPLE 2.3:

Find the angle between two forces of equal magnitude such that the magnitude of their resultant is also equal to either of them.

Solution:

Let $\vec{F}_1$ and $\vec{F}_2$ represents the two vectors such that $\vec{F}_1$ is acting along x-axis while $\vec{F}_2$ is making an angle θ with positive x-axis, as shown in the figure. [θ also represents the angle between the two vectors]. First we resolve $\vec{F}_1$ and $\vec{F}_2$ into their horizontal and vertical components. As $\vec{F}_1$ is acting along x-axis, so it possesses horizontal component and no vertical component. So we have,

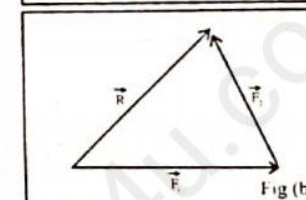
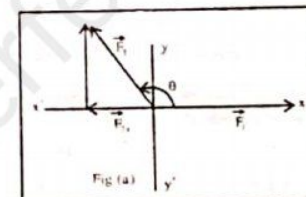
$$F_{1x} = F_1 \cos \theta = F_1 \cos 0^\circ = F_1 \times 1 = F_1$$

$$\Rightarrow F_{1x} = F_1$$

While $F_{1y} = 0\text{N}$

Now, $F_{2x} = F_2 \cos \theta$, $F_{2y} = F_2 \sin \theta$

Now the horizontal component of resultant is given by,



$$R_x = F_1 + F_2$$

$$\Rightarrow R_x = (F_1 + F_2 \cos \theta) \quad (1)$$

While the vertical component of the resultant is given by,

$$R_y = F_1 + F_2 \sin \theta = 0 + F_2 \sin \theta$$

$$\Rightarrow R_y = F_2 \sin \theta \quad (2)$$

Now the magnitude of $\vec{R}$ is given by,

$$R = \sqrt{R_x^2 + R_y^2} \quad (3)$$

Putting equations (1), (2) in equation (3), we get,

$$R = \sqrt{(F_1 + F_2 \cos \theta)^2 + (F_2 \sin \theta)^2}$$

$$\Rightarrow R = \sqrt{F_1^2 + F_2^2 \cos^2 \theta + 2F_1 F_2 \cos \theta + F_2^2 \sin^2 \theta} \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow R = \sqrt{F_1^2 + F_2^2 (\cos^2 \theta + \sin^2 \theta) + 2F_1 F_2 \cos \theta}$$

$$\Rightarrow R = \sqrt{(F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta)} \quad (4)$$

As in equation, we have given that, $R = F_1 = F_2$

So we generally put $R = F_1 = F_2 = F$

So equation (4) becomes,

$$\Rightarrow F = \sqrt{F^2 + F^2 + 2F^2 \cos \theta}$$

$$\Rightarrow F = F\sqrt{1 + 1 + 2 \cos \theta} = F\sqrt{2 + 2 \cos \theta}$$

$$\Rightarrow F = F\sqrt{2 + 2 \cos \theta}$$

$$\Rightarrow 1 = \sqrt{(2 + 2 \cos \theta)} \quad (5)$$

Squaring both sides of equation (5), we get,

$$1 = 2 + 2 \cos \theta$$

$$\Rightarrow 2 \cos \theta = -1 \Rightarrow \cos \theta = \frac{-1}{2} = -0.5 \quad [\cos(-x) = \cos x]$$

$$\Rightarrow \cos \theta = -0.5 \Rightarrow \theta = \cos^{-1}(0.5) \Rightarrow \theta = 120^\circ$$

Thus the angle between the two forces is equal to 120° .

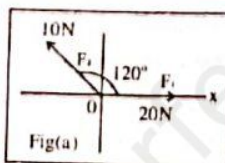
ASSIGNMENT 2-3:

Two forces of 20N and 10N are making an angle of 120° with each other. Find a single pull that would (a) replace the given forces system (b) balance the given forces system.

Solution:

Let $\vec{F}_1 = 20\text{N}$ acting along x-axis, $\vec{F}_2 = 10\text{N}$ making an angle of 120° with positive x-axis

(a) (i) Magnitude of resultant of $\vec{F}_1$ and



$$\vec{F}_2 = R = ?$$

(ii) Direction of $\vec{R} = \theta = ?$

(b) Balance force of $\vec{R} = \vec{F} = ?$

(a) First we resolve $\vec{F}_1$ and $\vec{F}_2$ into its horizontal and vertical components. As $\vec{F}_1$ is lying only along x-axis, so it has only horizontal component and its vertical component will be equal to zero. So we have,

$$F_{1x} = F_1 \cos \theta = 20 \times \cos 0^\circ$$

$$\Rightarrow F_{1x} = 20 \times 1\text{N}$$

$$\Rightarrow F_{1x} = 20\text{N and } F_{1y} = 0\text{N}$$

Now we resolve $\vec{F}_2$ into its horizontal and vertical components, as shown in figure (b). So we have,

$$F_{2x} = F_2 \cos \theta = -10 \times \cos 60^\circ$$

$$\Rightarrow F_{2x} = -10 \times 0.5 = -5\text{N}$$

$$\Rightarrow F_{2x} = -5\text{N}$$

$$\text{While, } F_{2y} = F_2 \sin \theta = 10 \times \sin 60^\circ$$

$$\Rightarrow F_{2y} = 10 \times 0.866\text{N}$$

$$\Rightarrow F_{2y} = 8.66\text{N}$$

Now the x-component of the resultant is given by,

$$R_x = F_{1x} + F_{2x}$$

$$\Rightarrow R_x = 20 + (-5) = 20 - 5 = 15\text{N}$$

$$\Rightarrow R_x = 15\text{N}$$

Now y-component of $\vec{R}$ [resultant] is given by,

$$R_y = F_{1y} + F_{2y}$$

$$\Rightarrow R_y = 0\text{N} + 8.66\text{N}$$

$$\Rightarrow R_y = 8.66\text{N}$$

(i) Now the magnitude of the resultant is given by,

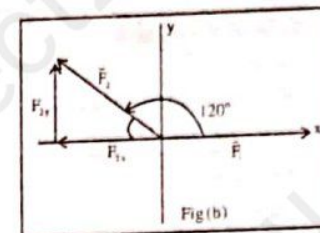
$$R = \sqrt{R_x^2 + R_y^2}$$

$$\Rightarrow R = \sqrt{15^2 + (8.66)^2} = \sqrt{225 + 74.9956}$$

$$\Rightarrow R = \sqrt{299.9956}$$

$$\Rightarrow R = 17.32\text{N}$$

(ii) The direction of $\vec{R}$ is given by,



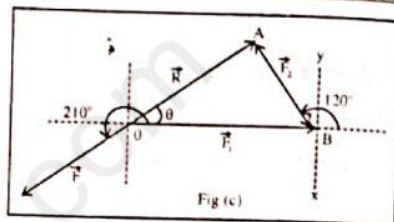
F_{2x} is in second quadrant so it is taken as negative.

$$\theta = \tan^{-1} \left(\frac{R_y}{R_x} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{8.66}{15} \right) = \tan^{-1}(0.577)$$

$$\Rightarrow \theta = 29.98^\circ = 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$



(b) Let $\vec{F}$ be the force which balance the resultant $\vec{R}$ [i.e. resultant of given forces] then $\vec{F}$ must have the same magnitude as $\vec{R}$ but opposite to $\vec{R}$ as shown in the figure. So $F = 17.32\text{N}$ and making an angle of 210° with positive x -axis, as shown in the figure (c). $\vec{R}$ and $\vec{F}$ are opposite, so the angle between them is 180° . As $\vec{R}$ make 30° with x -axis, so the angle made by $\vec{F}$ with x -axis $= 180^\circ + 30^\circ = 210^\circ$.

EXAMPLE 2.4:

Vector 'A' having magnitude 3.2 makes 50° with x -axis and vector 'B' with magnitude 5.1 makes 110° with x -axis. What is the magnitude of their dot and cross products?

Solution:Given Data:Vector $|\vec{A}| = 3.2$, angle $\theta_1 = 50^\circ$ with x -axisVector $|\vec{B}| = 5.1$, angle $\theta_2 = 110^\circ$ with x -axisRequired: (a) $|\vec{A} \cdot \vec{B}| = ?$ (b) $|\vec{A} \times \vec{B}| = ?$ Calculation: (a) Let ' θ ' be angle between two vectors, then we have,

$$\theta = \theta_2 - \theta_1 = 110^\circ - 50^\circ = 60^\circ$$

We know that the magnitude of dot product is given by,

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta \quad (1)$$

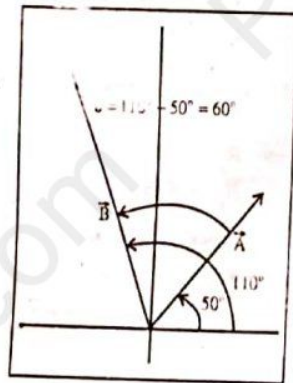
Putting the values in equation (1), we get,

$$|\vec{A} \cdot \vec{B}| = (3.2)(5.1)(\cos 60^\circ)$$

$$\Rightarrow |\vec{A} \cdot \vec{B}| = 3.2 \times 5.1 \times 0.5 = 8.16 = 8.2$$

$$\Rightarrow |\vec{A} \cdot \vec{B}| = 8.2$$

(b) The magnitude of cross-product of two vectors is given by,



$$|\vec{A} \times \vec{B}| = AB \sin \theta \quad (2)$$

Putting the values in equation (2), we get,

$$|\vec{A} \times \vec{B}| = 3.2 \times 5.1 \times \sin 60^\circ = 3.2 \times 5.1 \times 0.866 \Rightarrow |\vec{A} \times \vec{B}| = 14.7$$

ASSIGNMENT 2.4:Show that $|\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2 B^2$ Proof:The magnitude of dot product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta \quad (1)$$

Squaring both sides of equation (1), we get,

$$|\vec{A} \cdot \vec{B}|^2 = A^2 B^2 \cos^2 \theta \quad (2)$$

The magnitude of cross-product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$$|\vec{A} \times \vec{B}| = AB \sin \theta \quad (3)$$

Squaring both sides of equation (3), we get,

$$|\vec{A} \times \vec{B}|^2 = A^2 B^2 \sin^2 \theta \quad (4)$$

Adding equation (2) and equation (4), we get,

$$|\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2 B^2 \cos^2 \theta + A^2 B^2 \sin^2 \theta$$

$$\Rightarrow |\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2 B^2 [\cos^2 \theta + \sin^2 \theta] \quad \boxed{\cos^2 \theta + \sin^2 \theta = 1}$$

$$\Rightarrow |\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2 B^2 \times 1$$

$$\Rightarrow |\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2 B^2 \times 1$$

$$\Rightarrow |\vec{A} \cdot \vec{B}|^2 + |\vec{A} \times \vec{B}|^2 = A^2 B^2$$

EXAMPLE 2.5:

Hina weighing 450N is standing at the edge of the uniform diving board 5m in length. Weight of the board is 150N, and is bolted down at the left end, while being supported 1.50m away by a fulcrum, as in figure 2.35. Find the forces and that the bolt and the fulcrum, exert on the board.

Solution:Given Data:Weight of Hina $= F_H = 450\text{N}$ Weight of board $= F_C = 150\text{N}$

Center of gravity of board
= $l_c = 2.5m$

Distance of fulcrum = $l_f = 1.5m$

Length of board

$$L = l_u = 5m$$

Required: (a)

Force exerted by fulcrum = $F_f = ?$

(b) Force exerted by bolt = $F_b = ?$

Calculation: For the axis of rotation at point A, let the torque produced by support fulcrum is " τ_f " the torque produced by weight of board is " τ_c " and the torque produced by weight of girl Hina is " τ_u ".

By second condition of equilibrium,

$$\Sigma \tau = 0$$

$$\Rightarrow \tau_f - \tau_c - \tau_u = 0$$

$$\Rightarrow (F_f)(l_f) - (F_c)(l_c) - (F_u)(l_u) = 0$$

$$\Rightarrow (F_f)(l_f) = (F_c)(l_c) + (F_u)(l_u) \Rightarrow F_f = \frac{(F_c)(l_c) + (F_u)(l_u)}{(l_f)} \quad (1)$$

Putting the values in equation (1), we get,

$$F_f = \frac{(150N)(2.5m) + (450N)(5m)}{1.5m}$$

$$\Rightarrow F_f = \frac{375Nm + 2250Nm}{1.5m} \Rightarrow F_f = \frac{2625Nm}{1.5m} \quad \boxed{F_f = 1750N}$$

(b) Now the force due to bolt " F_b " can be easily find out by solving first condition of equilibrium along y-axis. Such that

$$\Sigma F_y = 0 \Rightarrow -F_b + F_f + (-F_c) + (-F_u) = 0$$

$$\Rightarrow -F_b + F_f - F_c - F_u = 0$$

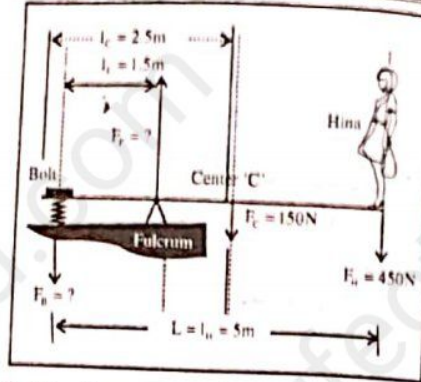
$$\Rightarrow F_b = F_f - F_c - F_u \quad (2)$$

Putting the values in equation (3), we get,

$$F_b = 1750N - 150N - 450N \Rightarrow \boxed{F_b = 1150N}$$

ASSIGNMENT 2-6:

A uniform plank of 200N and length 6m is supported horizontally by a rope as shown in figure (a). If the breaking tension of the rope is 400N, how far can a boy of weight 400N walk from "A" to "B" i.e. towards the support?



Solution: Given Data:

Weight of beam = $w_1 = 200N$

Weight of man = $w_2 = 400N$

Tension of the rope = $T = 400N$

Length of beam = $r = 6m$

Required: Distance of boy from "A" = $r_2 = ?$

Calculation: Now for balance condition, let the weight of the beam is acting at point "C" and weight of the man is acting at point "D" as shown in figure (b). The torque due to " w_1 " is given by,

$$\tau_1 = r_1 \times w_1 = 3 \times 200 \Rightarrow \tau_1 = 600N \cdot m$$

The torque due to " w_2 " is given by,

$$\tau_2 = r_2 \times w_2 = r_2 \times 400 \Rightarrow \tau_2 = 400r_2N$$

The torque due to tension " T " is given by,

$$\tau_3 = r \times T = 6 \times 400 \Rightarrow \tau_3 = 2400N \cdot m$$

Here we see that " w_1 " and " w_2 " produces torque towards left, so total torque towards left is given by,

$$\tau_L = \tau_1 + \tau_2 = 600N \cdot m + 400 \times r_2N$$

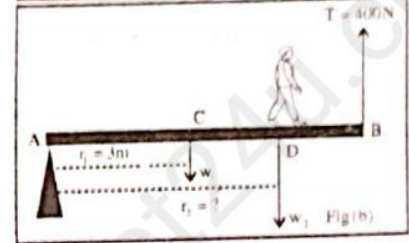
While " τ_3 " represents the torque towards right, so

$$\tau_R = \tau_3 = 2400N \cdot m$$

Now for balance conditions, we have, $\tau_L = \tau_R$

$$\Rightarrow 600N \cdot m + 400 \times r_2N = 2400N \cdot m \Rightarrow 400 \times r_2N = 2400N \cdot m - 600N \cdot m$$

$$\Rightarrow 400 \times r_2N = 1800N \cdot m \Rightarrow r_2 = \left(\frac{1800}{400} \right) m = 4.5m \Rightarrow \boxed{r_2 = 4.5m}$$



❖ قرآن مجید میں ارشاد ہے "والدین، رشتہ داروں، مسکینوں اور مسافروں سے نیک سلوک کرو۔"

❖ قرآن میں ارشاد ہے "بے شرمی کی باتیں نہ کہی ہوں یا بھیجی ان کے پاس تک نہ پہنچو۔"

❖ قرآن میں ارشاد ہے "اللہ ہی تمہارا مالک ہے اور وہ سب سے اچھا و دانا ہے۔"

Exercise

Choose the best possible answer:

1. Two vectors lie with their tails at the same point. When the angle between them is increased by 20° their scalar product has the same magnitude but changes from positive to negative. The original angle between them was:

a. 0° b. 60° c. 70° ☒ d. 80°

Sol: We know that, $\vec{A} \cdot \vec{B} = AB \cos \theta$ _____ (1)

Putting $\theta = 80^\circ$ we get,

$$\vec{A} \cdot \vec{B} = AB \cos 80^\circ = 0.174AB$$

$$\Rightarrow \vec{A} \cdot \vec{B} = 0.174AB \text{ _____ (2)}$$

Now put $\theta = 80^\circ + 20^\circ = 100^\circ$ in equation (1),

$$\vec{A} \cdot \vec{B} = AB \cos 100^\circ AB (-0.174)$$

$$\Rightarrow \vec{A} \cdot \vec{B} = -0.174AB \text{ _____ (3)}$$

Equation (2) shows that the dot product for 80° and 100° has the same magnitude but changes from positive to negative, so the original angle is 80° .

2. The minimum number of vectors of unequal magnitude required to produce a zero resultant is:

a. 2 ☒ b. 3 c. 4 d. 5

3. If the resultant of two vectors, each of magnitude "A" is also a magnitude of "A". The angle between the two vectors will be:

a. 30° b. 45° c. 60° ☒ d. 120°

Sol: The resultant of two force " F_1 " and " F_2 " is given by,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos \theta} \text{ _____ (1)}$$

According to given condition, we have,

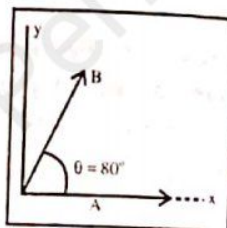
$$R = F_1 + F_2 = F, \text{ so equation (1) becomes}$$

$$F = \sqrt{F^2 + F^2 + 2F^2 \cos \theta} = F\sqrt{(1+1+2\cos \theta)}$$

$$\Rightarrow 1 = \sqrt{2+2\cos \theta} \Rightarrow 1^2 = (\sqrt{2+2\cos \theta})^2$$

$$\Rightarrow 1 = 2 + 2\cos \theta \Rightarrow 2\cos \theta = -1 \Rightarrow \cos \theta = -\frac{1}{2}$$

$$\Rightarrow \theta = \cos^{-1}\left(-\frac{1}{2}\right) \Rightarrow \boxed{\theta = 120^\circ}$$



4. The magnitude of vector $\vec{A} = 2\hat{i} + \hat{j} + 2\hat{k}$ is:

a. 9 b. 5 ☒ c. 3 d. 1

$$\text{Sol: } \vec{A} = 2\hat{i} + \hat{j} + 2\hat{k} \Rightarrow |\vec{A}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{9} = 3$$

5. When $F_x = 3N$ and $F = 5N$ then $F_y =$

a. 6N ☒ b. 4N c. 2N d. 0N

Sol: $F_x = 3N, F = 5N$, we know that

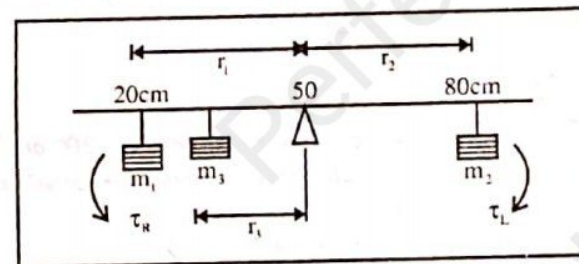
$$F^2 = F_x^2 + F_y^2 \Rightarrow F_y^2 = F^2 - F_x^2 = (5N)^2 - (3N)^2 = 25N^2 - 9N^2$$

$$\Rightarrow F_y^2 = 16N^2 \Rightarrow \boxed{F_y = 4N}$$

6. A meter stick is supported by a knife-edge at the 50cm mark. Arif hangs masses of 0.40 and 0.60kg from the 20cm and 80cm marks, respectively. Where should Arif hang a third mass of 0.30kg to keep the stick balanced?

☒ a. 20cm b. 70cm c. 30cm d. 25cm

Sol: $m_1 = 0.40\text{kg}, m_2 = 0.60\text{kg}, m_3 = 0.3\text{kg}$



$$r_1 = 30\text{cm}, r_2 = 30\text{cm}, r_3 = ?$$

For balance conditions, we have, $\tau_R = \tau_L$

$$\Rightarrow (w_1 \times r_1) + (w_3 r_3) = w_2 r_2$$

$$\Rightarrow (m_1 g r_1) + (m_3 g r_3) = m_2 g r_2$$

$$\Rightarrow m_1 r_1 + m_3 r_3 = m_2 r_2$$

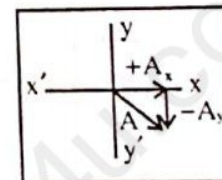
$$\Rightarrow r_3 = \frac{m_2 r_2 - m_1 r_1}{m_3} = \left(\frac{0.6 \times 30 - 0.4 \times 30}{0.3} \right) \text{cm} = \left(\frac{18 - 12}{0.3} \right) \text{cm}$$

$$= \left(\frac{6}{0.3} \right) \text{cm} \Rightarrow \boxed{r_3 = 20\text{cm}}$$

7. If $A_x = 1.5\text{cm}, A_y = -1.0\text{cm}$, into which quadrant do the vector "A" point?

a. I b. II c. III ☒ d. IV

Sol: $A_x = 1.5\text{cm}, A_y = -1.0\text{cm}$



$$8. \vec{A} \cdot (\vec{A} \times \vec{B}) = ?$$

$$\checkmark a. 0 \quad b. 1 \quad c. AB \quad d. A^2B$$

$$\text{Sol: We know that } (\vec{A} \times \vec{B}) = \vec{C} \quad (1)$$

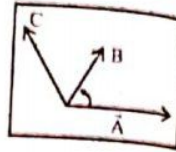
$$\text{Now } \vec{A} \cdot (\vec{A} \times \vec{B}) = \vec{A} \cdot \vec{C}$$

$$\Rightarrow \vec{A} \cdot (\vec{A} \times \vec{B}) = AC \cos \theta \quad (2)$$

As vector $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$ so put $\theta = 90^\circ$ in equation (1), we get,

$$\vec{A} \cdot (\vec{A} \times \vec{B}) = AC \times 0 = 0$$

$$\Rightarrow \vec{A} \cdot (\vec{A} \times \vec{B}) = 0$$



9. Two forces of magnitude 20N and 50N act simultaneously on a body. Which one of the following forces cannot be a resultant of the two forces?

$$\checkmark a. 20N \quad b. 30N \quad c. 40N \quad d. 70N$$

$$\text{Sol: } F_1 = 20N, F_2 = 50N$$

$$\text{We have, } R_{\min} \leq R \leq R_{\max}$$

$$\Rightarrow (F_2 - F_1) \leq R \leq (F_2 + F_1)$$

$$\Rightarrow (50 - 20) \leq R \leq 50 + 20$$

$$\Rightarrow 30 \leq R \leq 70$$

This shows that the range of resultant will be 30 or 70 or between them. So 20N does not come among the possible values of resultant.

10. If the dot product of two non-zero vectors "A" and "B" is zero then the magnitude of their cross product is.....

$$a. 0 \quad b. 1 \quad \checkmark c. AB \quad d. -AB$$

$$\text{Sol: } \vec{A} \cdot \vec{B} = AB \cos \theta \quad (1)$$

For $\vec{A} \cdot \vec{B} = 0$, put $\theta = 90^\circ$, so equation (1) becomes

$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ = AB \times 0 \Rightarrow \vec{A} \cdot \vec{B} = 0 \quad (1)$$

$$\text{Now } \vec{A} \times \vec{B} = AB \sin \theta = AB \sin 90^\circ = AB \times 1 = AB$$

11. The sum of magnitude of two forces is 16N. If the resultant force is 8N and its direction is perpendicular to minimum force then the force are:

$$\checkmark a. 6N \text{ and } 10N \quad b. 8N \text{ and } 8N \\ c. 4N \text{ and } 12N \quad d. 2N \text{ and } 14N$$

$$\text{Sol: We have given that, resultant } = R = 8N \text{ and } F_1 + F_2 = 16 \Rightarrow F_2 = (16 - F_1) \quad (1)$$

According to equation, we have,

$$R^2 = F_2^2 - F_1^2 \Rightarrow F_2 = \sqrt{R^2 + F_1^2} \quad (2)$$

Comparing equation (1), equation (2), we get,

$$(16 - F_1) = \sqrt{R^2 + F_1^2} \Rightarrow (16 - F_1)^2 = R^2 + F_1^2 = 8^2 + F_1^2$$

$$\Rightarrow 256 + F_1^2 - 32F_1 = 64 + F_1^2 \Rightarrow 256 - 64 - 32F_1 = 0$$

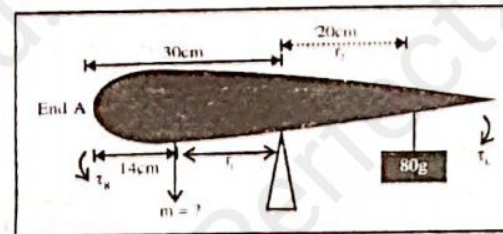
$$\Rightarrow 192 - 32F_1 = 0 \Rightarrow -32F_1 = -192 \Rightarrow F_1 = 6N$$

Putting $F_1 = 6N$ in equation (1), we get,

$$F_2 = (16 - 6)N \Rightarrow F_2 = 10N$$

Thus the two forces are (6N, 10N).

12. Find the mass of the uneven rod shown in the figure. If its center of gravity is 14cm from end A is.....



$$\checkmark a. 100g \quad b. 150g \quad c. 80g \quad d. 5g$$

$$\text{Sol: Mass of rod} = m_1 = ?, r_1 = 30cm - 14cm = 16cm$$

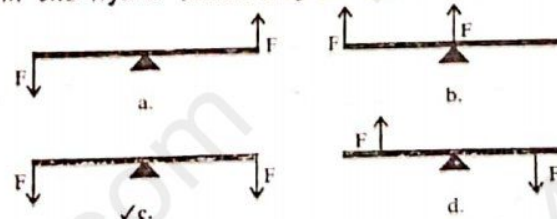
$$\text{Mass of object} = m_2 = 80gm, r_2 = 20cm$$

For balance condition, we have, $\tau_{\text{right}} = \tau_{\text{left}}$

$$\Rightarrow w_1 \times r_1 = w_2 \times r_2 \Rightarrow m_1 g r_1 = m_2 g r_2 \Rightarrow m_1 r_1 = m_2 r_2$$

$$\Rightarrow m_1 = \frac{m_2 r_2}{r_1} = \left(\frac{80 \times 20}{16} \right) gm = 100gm \Rightarrow m_1 = 100gm$$

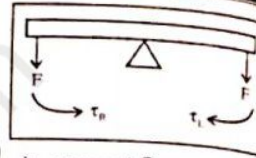
13. The following diagrams show a uniform rod with its midpoint on the pivot. Two equal forces "F" are applied on the rod, as shown in the figure. Which diagram shows the rod in equilibrium?



Sol: The torque produce towards left balance

the torque produce towards right i.e.

$$\tau_R = \tau_L \text{ or } \Sigma \tau = 0$$



14. For which angle the equation $|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}|$ is correct?

- a. 30° ✓ b. 45° c. 60° d. 90°

Sol: $|\vec{A} \cdot \vec{B}| = AB \cos \theta = AB \cos 45^\circ = AB(0.707) = 0.707AB$

$$\Rightarrow |\vec{A} \cdot \vec{B}| = 0.707AB \quad (1)$$

Now $|\vec{A} \times \vec{B}| = AB \sin \theta = AB \sin 45^\circ = AB(0.707) = 0.707AB$

$$\Rightarrow |\vec{A} \times \vec{B}| = 0.707AB \quad (2)$$

Comparing equation (1), equation (2), we get,

$$|\vec{A} \cdot \vec{B}| = |\vec{A} \times \vec{B}| \text{ for } \theta = 45^\circ$$

15. What is the net torque on wheel radius 2m as shown:

- a. 10N anti-clockwise
b. 10Nm anti-clockwise
✓ c. 10Nm clockwise
d. 5Nm clockwise

Sol: Clockwise torque

$$= \tau_c = r \times F_1 = 2 \times 10 = 20\text{N}\cdot\text{m}$$

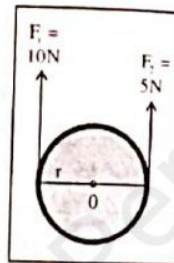
Anti-clockwise torque

$$= \tau_{acw} = r \times F_2 = 2 \times 5 = 10\text{N}\cdot\text{m}$$

Net torque

$$= \tau_c - \tau_{acw} = 20\text{N}\cdot\text{m} - 10\text{N}\cdot\text{m} = 10\text{N}\cdot\text{m clockwise}$$

$$\Rightarrow \tau_{net} = 10\text{N}\cdot\text{m clockwise}$$



❖ قرآن مجید میں ارشاد ہے "اچھی بات کہنا اور درگزر کرنا ایسی خیرات سے بہتر ہے جس کے بعد آزاد پہنچایا جائے۔"

❖ نبی کریم ﷺ کا ارشاد ہے "بڑی مجلس سے احتراز کرو۔"

❖ نبی کریم ﷺ کا ارشاد ہے "انسان کی کجھادری یہ ہے کہ وہ کفایت شعار ہو۔"

❖ نبی کریم ﷺ کا ارشاد ہے "مکر، دھوکہ اور خیانت کرنے والا دوزخ میں جائے گا۔"

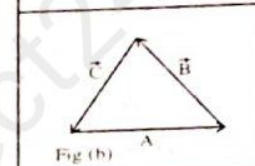
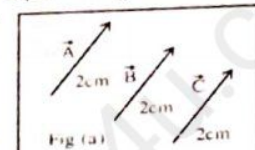
Conceptual Questions

Q1: Is it possible to add three vectors of equal magnitude but in different directions to get a null vector? Illustrate with a diagram.

Ans: Yes, it is possible to add three vectors of equal magnitude but different in directions to get a null vector or zero vector.

Example and Explanation:

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ having same magnitude as shown in the figure (a). Let we add these vectors by head to tail rule. For this purpose we combine the head of $\vec{A}$ with the tail of $\vec{B}$. Then we combine the head of $\vec{B}$ with the tail of $\vec{C}$ as shown in the figure (b). Now for resultant, we will combine the tail of first vector [i.e. $\vec{A}$] to the head of last vector [i.e. $\vec{C}$]. But we see that the tail of $\vec{A}$ already touches the head of $\vec{C}$. We can put only a dot at their point of contact. So the resultant vector in this case will be a null vector or zero vector. Thus we have $\vec{R} = \vec{A} + \vec{B} + \vec{C} = 0$



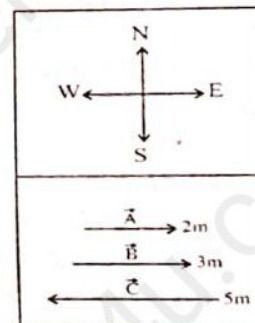
Conclusion: Thus it is possible to add three vectors of equal magnitude but different in directions to get a null vector.

Q2: The magnitudes of three vectors are 2m, 3m and 5m respectively. The directions are at your disposal. Can these vectors be added to yield zero? Illustrate with a diagram.

Ans: Yes, it is possible to add these vectors to yield zero or null vector.

Reason and Explanation:

Consider three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ having magnitude 2m, 3m and 5m respectively. Let $\vec{A}$ and $\vec{B}$ are directed towards East and $\vec{C}$ is directed towards West as shown in the figure. Now by adding $\vec{A}$ and $\vec{B}$ according to head to tail rules, we get,



$$\vec{A} + \vec{B} = 5\text{m (East)}$$

Now adding vector $\vec{C}$ with $\vec{A} + \vec{B}$ in opposite direction, i.e. towards West, we get $\vec{A} + \vec{B} + (-\vec{C}) = 0$

Thus we get null vector or zero vector because the tail of $(\vec{A} + \vec{B})$ and head of $\vec{C}$ just touches each other. For resultant we can put only a dot which will give zero magnitude. Mathematically we have,

$$\begin{aligned}\vec{R} &= \vec{A} + \vec{B} + (-\vec{C}) \\ \Rightarrow \vec{R} &= 2\hat{i}\text{m} + 3\hat{i}\text{m} + (-5\hat{i}\text{m}) \\ \Rightarrow \vec{R} &= 5\hat{i}\text{m} - 5\hat{i}\text{m} \\ \Rightarrow \vec{R} &= 0\end{aligned}$$

Which is thus proved.

Q3: What units are associated with the unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$?

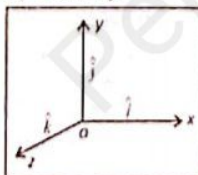
Ans: The unit vectors $\hat{i}, \hat{j}, \hat{k}$ are unitless because they are used for the specification of directions only.

Reason and Explanation:

A unit vector is that vector whose magnitude is "1". The unit vectors along x, y and z-axis are represented by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

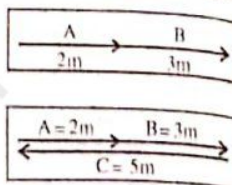
The magnitude of $\hat{i}$, $\hat{j}$ and $\hat{k}$ is given by,

$$\begin{aligned}|\hat{i}| &= 1 \quad (1) \\ |\hat{j}| &= 1 \quad (2) \\ |\hat{k}| &= 1 \quad (3)\end{aligned}$$



Thus no unit is associated with $\hat{i}, \hat{j}$ and $\hat{k}$. The unit vectors specify the direction of the given vector only. It does not change the magnitude or dimensions of the vectors.

Example: Let there is a force of $20\text{N}\hat{i}$ i.e. $F = 20\hat{i}\text{N}$. Here the unit "N" is not the unit of " $\hat{i}$ ". The unit "N" goes with magnitude and not with direction. Here " $\hat{i}$ " tells us that the vector, $F = 20\text{N}$ is acting along x-axis. Similarly, $F = 30\text{N}\hat{j}$ tells us that force of 30N is acting along y-axis. Similarly, $F = 30\text{N}\hat{k}$ tells us that force of 30N is acting along z-axis. Thus the unit vectors $\hat{i}, \hat{j}, \hat{k}$ are used for the specification of direction only.

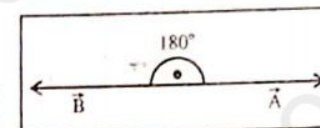


Q4: Can a scalar product of two vectors be negative? Provide a proof and give an example.

Ans: Yes a scalar product of two vectors may be negative if they are antiparallel.

Proof and Explanation:

Consider two vectors $\vec{A}$ and $\vec{B}$ making an angle of 180° with each other, as shown in the figure.



The scalar product of these two vectors is given by,

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad (1)$$

Putting $\theta = 180^\circ$ in equation (1), we get,

$$\vec{A} \cdot \vec{B} = AB \cos 180^\circ = AB(-1) = -AB$$

$$\Rightarrow \vec{A} \cdot \vec{B} = -AB \quad (2)$$

Which is the required proof. Equation (2) shows that the scalar product of two vectors will be negative if they are antiparallel i.e. the angle between them is 180° .

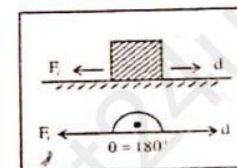
Example: Workdone against force of friction is given by,

$$W = \vec{F}_f \cdot \vec{d}$$

$$\Rightarrow W = F_f d \cos \theta$$

$$\Rightarrow W = F_f d \cos 180^\circ = F_f d(-1)$$

$$\Rightarrow W = -F_f d \quad (3)$$



Equation (3) shows that the scalar product of two vectors i.e. force of friction and displacement is negative. Thus the workdone against the force of friction is the example of negative scalar product.

Q5: $\vec{A}$ and $\vec{B}$ are two non-zero vectors. (a) How can their scalar product be zero? And (b) How can their vector product be zero?

Ans: The scalar product of two vectors $\vec{A}$ and $\vec{B}$ will be equal to zero if they are perpendicular to each other. While their vector product will be equal to zero if they are parallel or anti-parallel to each other.

Reason and Explanation:

(a) The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta \quad (1)$$

As we have given that $\vec{A}$ and $\vec{B}$ are non-zero vectors, so for $|\vec{A} \cdot \vec{B}| = 0$, the other possibility is that we must put $\theta = 90^\circ$ i.e. when the two vectors are perpendicular to each other. So equation (1) becomes,

$$|\vec{A} \cdot \vec{B}| = AB \cos 90^\circ = AB \times 0 \Rightarrow |\vec{A} \cdot \vec{B}| = 0$$

(b) The vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$$|\vec{A} \times \vec{B}| = AB \sin \theta \quad (2)$$

As we have given that $\vec{A}$ and $\vec{B}$ are non-zero vectors, so for getting $|\vec{A} \times \vec{B}| = 0$ the value of θ must be "0" i.e. the two vectors must be parallel to each other. Put $\theta = 0^\circ$ in equation (2), we get,

$$|\vec{A} \times \vec{B}| = AB \sin 0^\circ = AB \times 0 \Rightarrow |\vec{A} \times \vec{B}| = 0$$

Also if the two vectors are anti-parallel to each other, then we put $\theta = 180^\circ$, so equation (2) becomes

$$|\vec{A} \times \vec{B}| = AB \sin 180^\circ = AB \times 0 = 0$$

Q6: Suppose you are given a known non-zero vector A . The scalar product of A with an unknown vector B is zero. Likewise, the vector product of A with B is zero. What can you conclude about B ?

Ans: In such a case vector B must be a null vector i.e. $|\vec{B}| = 0$

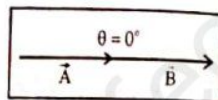
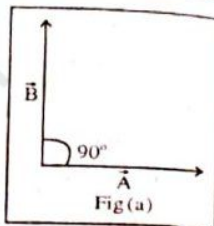
Reason and Explanation:

Case-1: We know that the scalar product of two vectors $\vec{A}$ and $\vec{B}$ is given by,

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta \quad (1)$$

Equation (1) shows that the scalar product will be equal to zero, if $\vec{A}$ or $\vec{B}$ is a null vector or if the two vectors are perpendicular to each other. As we have given that $\vec{A}$ is non-zero vector, so if $|\vec{A} \cdot \vec{B}| = 0$, then $\vec{B}$ will be a null vector or both vectors will be perpendicular to each other [$\cos 90^\circ = 0$].

Case-2: We know that the vector product of two vectors $\vec{A}$ and $\vec{B}$ is given by,



$$|\vec{A} \times \vec{B}| = AB \sin \theta \quad (2)$$

According to equation (2), if $|\vec{A} \times \vec{B}| = 0$, then $\vec{A}$ will be a null vector, or $\vec{B}$ will be a null vector or the two vectors will be parallel to each other. But we have given that $\vec{A}$ is non-zero vector. So if $|\vec{A} \times \vec{B}| = 0$, then $\vec{B}$ will be a null vector or both vectors will be parallel to each other.

Conclusion about Case-1 and Case-2:

In case (1) there is condition of perpendicularity and case (2) there is condition of parallelism. These two conditions are contradictory to each other. That is we cannot imagine the two vectors to be perpendicular and parallel at the same time. So we ruled out the condition of perpendicularity and parallelism. So under such conditions, there is only one possibility that is vector $\vec{B}$ must be a null vector or zero vector.

Q7: Why a particle experiencing only one force cannot be in equilibrium?

Ans: A particle experiencing only one force cannot be in state of equilibrium, because it does not satisfy the condition of equilibrium.

Reason and Explanation:

According to Newton's second law, we have,

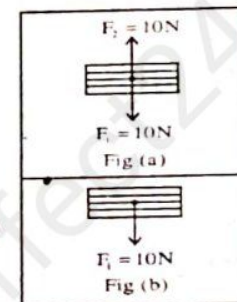
$$F = ma \quad (1)$$

According to first condition of equilibrium, a body will be in state of equilibrium if the algebraic sum of forces acting on the body is equal to zero i.e.

$$\Sigma F = 0 \quad (2)$$

Equation (2) will be satisfied, if forces of equal magnitude are acting at the same line in opposite direction. Now in case of single force, equation (2) will not be satisfied. According to equation (1), such single force will produce certain acceleration in the body and as a result the body will not remain in state of equilibrium.

Example: In case figure (a) two equal and opposite forces are acting along the same line on a uniform body, so the body remains in



state of equilibrium.

In case of figure (b), only a single force is acting on body. There is no equal and opposite force which balance the effect of " F ". So the body will move with certain acceleration and thus will not remain in state of equilibrium.

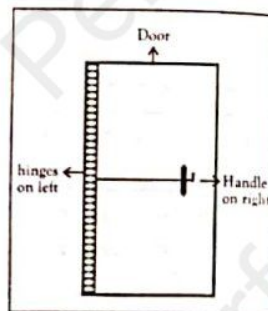
Q8: To open a door that has the handle on the right and the hinges on the left a torque must be applied. (a) Is the torque clockwise or counterclockwise when viewed from above? (b) Does your answer depend on whether the door opens toward or away from you?

Ans: (a) The direction of the torque depends on whether the door opens outside or inside, when viewed from above. For example, if the door opens from inside of the room then anticlockwise torque will be produced.

(b) Yes the direction of torque depends on whether the door open towards or away from us.

Reason and Explanation:

Consider a door having handle on the right and hinges on the left as shown in the figure.



Case-1: When we pull the door towards us:

If we apply the force $\vec{F}$ on the door towards ourselves for opening, then according to figure, clockwise torque will be produced.

Case-2: When we push the door away from us:

If we apply the force $\vec{F}$ on the door away from ourselves for opening, then according to figure, counterclockwise torque will be produced.

Q9: Explain the warning, never use a large wrench to tighten a small bolt?

Ans: We should never use a large wrench to tighten a small bolt. It is because the bolt will be damaged due to large torque.

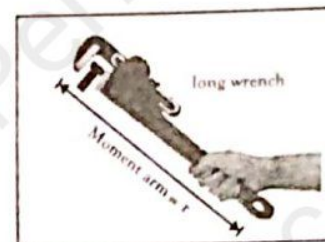
Reasons and Explanation:

We know that the magnitude of torque depends upon the magnitude of applied force, magnitude of moment arm and value of θ

between them i.e.

$$\tau = rF \sin \theta \quad (1)$$

Equation (1) shows that, for given values of " F " and θ the torque depends upon moment arm ' r ' i.e. greater the moment arm, greater will be the torque and vice versa.



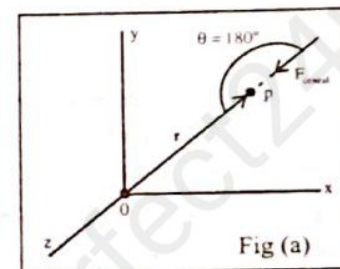
Now if we tight a small bolt with large wrench, then greater torque will act on the bolt due large moment arm. The bolt may be damaged due to excessive torque. So we should avoid the use of large wrench to tighten a small bolt. To keep the bolt in safe condition, we should use a wrench of proper size.

Q10: A central force is one that is always directed towards the same point. Can a central force give rise to a torque about that point?

Ans: A central force cannot give rise to a torque because it is anti-parallel or parallel to the moment arm at that point.

Reasons and Explanation:

Consider a central force " F_{central} " is acting on a body at point " P " at a distance " r " from the origin " O " as shown in the figure.



Case-1:

We know that the torque acting on a body is given by,

$$\tau = rF \sin \theta$$

$$\Rightarrow \tau = rF_{\text{central}} \sin \theta \quad (1)$$

Now in the given figure, we see that the angle between " F_{central} " and moment arm " r " is 180° , so equation (1) becomes

$$\tau = rF_{\text{central}} \times \sin 180^\circ$$

$$\Rightarrow \tau = rF_{\text{central}} \times 0 = 0 \quad [\sin 180^\circ = 0]$$

$$\Rightarrow \tau = 0 \quad (2)$$

Equation (2) shows that the central force produce no torque.

Case-2:

When " F_{central} " and " r " are parallel, then we put $\theta = 0^\circ$ so, equation

(1) becomes,

$$\tau = rF \sin 0^\circ = rF \times 0 = 0$$

$$\Rightarrow \tau = 0$$

Answers 2: The central force produce no torque because the line of action of force passes through the pivot point.

Comprehensive Questions

Give extended response to the following questions:

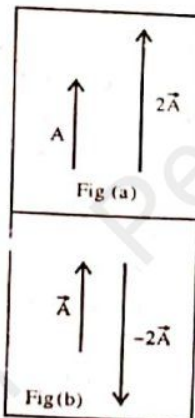
Q1: How are vectors added and subtracted geometrically?

Ans: See answer of questions no 7 and question no 9 in the text.

Q2: If a vector is multiplied by a positive scalar, how is the result related to the original vector? What if the scalar is zero? Negative?

Ans: We can multiply a vector by any number.

Case-1: If the number is positive, then the product vector and original vector will be acting along the same direction. For example in figure (a), the original vector $\vec{A}$ is multiplied with positive number "2". As a number we get a product vector which is twice than original vector and both acting along the same direction.



Case-2: If we multiply a given vector with zero, then we will get a null vector or zero vector.

Case-3: If we multiply a given vector with negative number, then the original vector and product vector will be acting in opposite direction. In figure (b) an original vector $\vec{A}$ is multiplied with "-2". As a result we get a product vector which is twice than original vector but opposite in direction.

Q3: What are rectangular components of a vector? How rectangular components are used to represent a vector?

Ans: See answer of question no 11 in the text.

Q4: Explain addition of vectors by rectangular components.

Ans: See answer of questions no 12 in the text.

Q5: Define the dot product [scalar product] of two vectors. What geometric interpretation does the dot product have? Give examples.

Ans: See answer of question no 13 in the text.

Q6: Define the cross product [vector product] of two vectors. What geometric interpretation does the cross product have? Give examples.

Ans: See answer of question no 14 in the text.

Q7: Define torque. Show that torque is the vector product of force and position vector.

Ans: See answer of question no 15 in the text.

Q8: What is mechanical equilibrium? Explain different types of equilibrium.

Ans: See answer of question no 16 in the text.

Q9: What type of equilibrium is guaranteed by each condition of equilibrium?

Ans: Static and dynamic equilibrium is guaranteed by each condition of equilibrium. For detail see "types of equilibrium" in equations no 16 in the text.

❖ ارشاد نبوی ﷺ ہے "عورت کی عزت شریف الطبع ہی کرتے ہیں اور اس کی اہانت کہنے لوگوں کے سوا کوئی نہیں کرتا۔"

❖ ارشاد نبوی ﷺ ہے "عورتوں میں سب سے اچھی عورت وہ ہے جسے اس کا شوہر دیکھے تو خوش ہو جائے۔"

❖ ارشاد نبوی ﷺ ہے "اللہ کو ماننے کے بعد بہترین داتا کی انسانوں سے محبت کرنا ہے۔"

❖ ارشاد نبوی ﷺ ہے "دولت مندر پر حسد نہ کرو دولت کی لذت فانی و عارضی ہیں۔"

❖ ارشاد نبوی ﷺ ہے "اللہ کے نزدیک بہترین دوست وہ ہے جو اپنے دوست کا خیر خواہ ہو۔"

Numerical Problems

Q1: A person throws a ball straight up with a speed of 12m/s . If the bus is moving at 25m/s , what is the velocity of the ball to an observer on ground?

Solution:

Given Data:

Velocity of ball
 $= V_{\text{ball}} = 12\text{m/sec}$

Velocity of bus
 $= V_{\text{bus}} = 25\text{m/sec}$

Required: Velocity of ball to an observer on ground
 $= V_{\text{relative}} = ?$

Calculation: Let the ball is thrown straight North and the bus is moving East. Here we add the velocities of ball and bus by head to tail rule as shown in the figure.

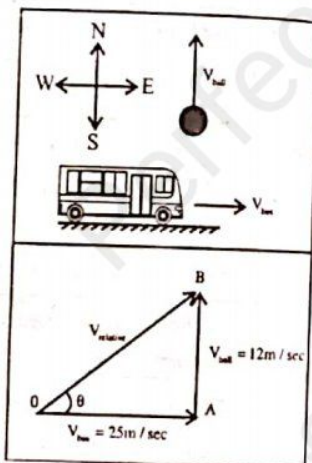
Now for finding the resultant velocity, we combine the tail of first vector $\vec{OA}$ with the head of last vector $\vec{AB}$. Thus $\vec{OB}$ gives us the resultant velocity or relative vector of the ball to an observer on ground. Now

$$\begin{aligned}(OB)^2 &= (OA)^2 + (AB)^2 \\ \Rightarrow v_{\text{rel}}^2 &= v_{\text{bus}}^2 + v_{\text{ball}}^2 = (25\text{m/sec})^2 + (12\text{m/sec})^2 \\ \Rightarrow v_{\text{rel}}^2 &= (625\text{m}^2/\text{sec}^2) + (144\text{m}^2/\text{sec}^2) \\ \Rightarrow v_{\text{rel}}^2 &= 769\text{m}^2/\text{sec}^2 \Rightarrow v_{\text{rel}} = \sqrt{769\text{m}^2/\text{sec}^2} \\ \Rightarrow v_{\text{rel}} &= 27.7\text{m/sec} \\ \text{OR } v_{\text{rel}} &= 28\text{m/sec}\end{aligned}$$

Direction of V_{rel} :

$$\begin{aligned}\text{As } \theta &= \tan^{-1} \left(\frac{V_{\text{ball}}}{V_{\text{bus}}} \right) \Rightarrow \theta = \tan^{-1} \left(\frac{12}{25} \right) = 25.6^\circ \\ \Rightarrow \theta &= 25.6^\circ \text{ or } \theta = 26^\circ\end{aligned}$$

Thus the relative velocity of ball will make an angle of 25.6° with



Perpendicular = V_{ball}
 Base = V_{bus}

horizontal [i.e. with direction of bus].

Q2: A football leaves the foot of a punter at an angle of 54° positive x-direction at a speed of 21m/s . Determine the horizontal and vertical components of the velocity.

Solution:

Given Data: Velocity of football

$= V = 21\text{m/sec}$

Angle of football with x-axis = $\theta = 54^\circ$

Required:

(a) Horizontal component of velocity = $V_x = ?$

(b) Vertical component of velocity = $V_y = ?$

Calculation: (a) We know that,

$$V_x = V \cos \theta \quad (1)$$

Putting the values in equation (1), we get;

$$V_x = [21 \times \cos 54^\circ] \text{m/sec} = [21 \times 0.588] \text{m/sec}$$

$$\Rightarrow V_x = 12.4\text{m/sec} \text{ or } V_x = 12\text{m/sec}$$

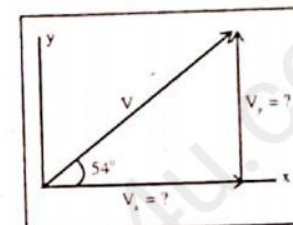
(b) We also know that,

$$V_y = V \sin \theta \quad (2)$$

Putting the values in equation (2), we get,

$$V_y = (21 \times \sin 54^\circ) \text{m/sec} = (21 \times 0.809) \text{m/sec}$$

$$V_y = 16.99\text{m/sec} \text{ or } V_y = 17\text{m/sec}$$



Q3: A 1.84kg school bag hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50^\circ$. Find the tension, T in the clothesline.

Solution:

Given Data:

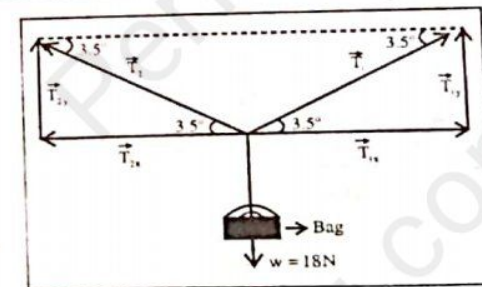
Mass of bag = $m = 1.84\text{kg}$

Weight of bag
 $= w = mg = 1.84 \times 9.8\text{N} = 18\text{N}$

Angle = $\theta = 3.50^\circ$

Required: Tension in the clothesline

Calculation: Let $T_1 = T_2 = T$ be the tension in each part of the cloth line. Here we resolve the tension in each part into their horizontal



and vertical components, as shown in the figure. As the system is in equilibrium state, so we apply the first condition of equilibrium i.e.

$$\Sigma F_x = 0 \Rightarrow T_{1x} + (-T_{2x}) = 0 \Rightarrow T_{1x} = T_{2x} \quad (1)$$

$$\text{And } \Sigma F_y = 0 \Rightarrow \Sigma T_y = 0 \Rightarrow T_{1y} + T_{2y} + (-w) = 0$$

$$\Rightarrow T_{1y} + T_{2y} - w = 0 \Rightarrow T_1 \sin \theta_1 + T_2 \sin \theta_2 = w$$

$$\Rightarrow T \sin \theta + T \sin \theta = w \Rightarrow 2T \sin \theta = w \quad \begin{cases} \theta_1 = \theta_2 = \theta \\ T_1 = T_2 = T \end{cases}$$

$$\Rightarrow T = \frac{w}{2 \sin \theta} = \left(\frac{18}{2 \times \sin 3.5^\circ} \right) N$$

$$\Rightarrow T = \left(\frac{18}{2 \times 0.061} \right) N = \left(\frac{18}{0.122} \right) N = 147.5 N$$

$$\Rightarrow \boxed{T = 147.5 N} \text{ or } \boxed{T = 148 N}$$

Q4: Find the magnitude and direction of vector represented by the following pair of components: (a) $A_x = -2.3 \text{ cm}$, $A_y = +4.1 \text{ cm}$ (b) $A_x = +3.9 \text{ m}$, $A_y = -1.8 \text{ m}$.

Solution:

Given Data:

$$(a) A_x = -2.3 \text{ cm}, A_y = +4.1 \text{ cm}$$

$$(b) A_x = +3.9 \text{ m}, A_y = -1.8 \text{ m}$$

Required: Magnitude of vector in each case = $A = ?$

Direction of $\vec{A}$ in each case = $\theta' = ?$

Calculation: (a) We know that the magnitude of a vector is given by,

$$A = \sqrt{A_x^2 + A_y^2} \quad (1)$$

Putting $A_x = -2.3 \text{ cm}$ and $A_y = +4.1 \text{ cm}$ in equation (1), we get,

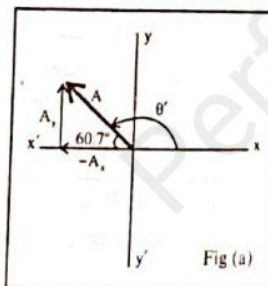
$$A = \sqrt{(-2.3 \text{ cm})^2 + (4.1 \text{ cm})^2} = \sqrt{5.29 \text{ cm}^2 + 16.81 \text{ cm}^2}$$

$$\Rightarrow A = \sqrt{(5.29 + 16.81) \text{ cm}^2} = \sqrt{22.1 \text{ cm}^2} = 4.7 \text{ cm}$$

$$\Rightarrow \boxed{A = 4.7 \text{ cm}}$$

We also know that,

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right) = \tan^{-1} \left(\frac{4.1}{-2.3} \right) = \tan^{-1} (1.78) \Rightarrow \boxed{\theta = 60.7^\circ}$$



As the x-component of $\vec{A}$ is negative and y-component is positive, so vector $\vec{A}$ lies in second quadrant as shown in figure (a). Thus the actual angle made by the vector $\vec{A}$ with positive x-axis is given by,

$$\theta' = 180^\circ - \theta \Rightarrow \theta' = 180^\circ - 60.7^\circ \Rightarrow \boxed{\theta' = 119.3^\circ}$$

Part (b): Now putting $A_x = +3.9 \text{ m}$ and $A_y = -1.8 \text{ m}$ in equation (1),

we get,

$$A = \sqrt{(3.9 \text{ m})^2 + (-1.8 \text{ m})^2} = \sqrt{15.21 \text{ m}^2 + 3.24 \text{ m}^2}$$

$$\Rightarrow A = \sqrt{18.45 \text{ m}^2} \Rightarrow \boxed{A = 4.3 \text{ m}}$$

Now the direction of vector $\vec{A}$ is given by,

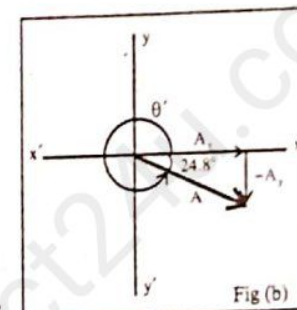
$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{-1.8}{3.9} \right) \Rightarrow \boxed{\theta = 24.8^\circ}$$

As the x-component of $\vec{A}$ is positive and y-component is negative, so the vector $\vec{A}$ lies in fourth quadrant as shown in figure (b). Thus the actual angle made by the vector $\vec{A}$ with positive x-axis is given by,

$$\theta' = 360^\circ - \theta \Rightarrow \theta' = 360^\circ - 24.8^\circ$$

$$\Rightarrow \boxed{\theta' = 335.2^\circ}$$



Q5: Vector " $\vec{F}$ " having magnitude 5.5 N makes 10° with x-axis and vector " $\vec{r}$ " with magnitude 4.3 m makes 80° with x-axis. What is the magnitude of their dot and cross products?

Solution:

Given Data: Magnitude of vector $\vec{F} = F = 5.5 \text{ N}$, $\theta_1 = 10^\circ$

Magnitude of vector $\vec{r} = r = 4.3 \text{ m}$, $\theta_2 = 80^\circ$

Required:

$$(a) |\vec{F} \cdot \vec{r}| = ?$$

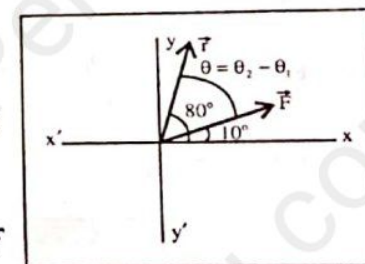
$$(b) |\vec{F} \times \vec{r}| = ?$$

Calculation: The angle between $\vec{F}$ and $\vec{r}$ is given by,

$$\theta = \theta_2 - \theta_1 = 80^\circ - 10^\circ \Rightarrow \theta = 70^\circ$$

(a) The magnitude of dot product of $\vec{F}$ and $\vec{r}$ is given by,

$$|\vec{F} \cdot \vec{r}| = Fr \cos \theta \quad (1)$$



Putting the values in equation (1), we get,

$$|\vec{F} \cdot \vec{r}| = 5.5\text{N} \times 4.3\text{m} \times \cos 70^\circ$$

$$= 5.5\text{N} \times 4.3\text{m} \times 0.342$$

$$\Rightarrow |\vec{F} \cdot \vec{r}| = 8.09\text{N}\cdot\text{m} \text{ or } |\vec{F} \cdot \vec{r}| = 8.1\text{N}\cdot\text{m}$$

(b) The magnitude of cross product of $\vec{F}$ and $\vec{r}$ is given by,

$$|\vec{F} \times \vec{r}| = Fr \sin \theta \quad (2)$$

Putting the values in equation (2), we get,

$$|\vec{F} \times \vec{r}| = 5.5\text{N} \times 4.3\text{m} \times \sin 70^\circ = 5.5\text{N} \times 4.3\text{m} \times 0.939$$

$$\Rightarrow |\vec{F} \times \vec{r}| = 22.2\text{N}\cdot\text{m}$$

Q6: The magnitude of dot and cross product of two vectors is $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.

Solution:

Given Data:

$$|\vec{A} \cdot \vec{B}| = 6\sqrt{3}, |\vec{A} \times \vec{B}| = 6$$

Required:

Angle between two vectors = $\theta = ?$

Calculation: We know that

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta \quad (1)$$

$$\text{And } |\vec{A} \times \vec{B}| = AB \sin \theta \quad (2)$$

Dividing equation (2) by equation (1), we get,

$$\frac{|\vec{A} \times \vec{B}|}{|\vec{A} \cdot \vec{B}|} = \frac{AB \sin \theta}{AB \cos \theta}$$

$$\Rightarrow \frac{|\vec{A} \times \vec{B}|}{|\vec{A} \cdot \vec{B}|} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\Rightarrow \tan \theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A} \cdot \vec{B}|} = \frac{6}{6\sqrt{3}}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

$$\Rightarrow \theta = 30^\circ$$

Q7: A uniform rod 1m long with weight 6N can be supported in a horizontal position on a sharp edge with weights of 10N and 5N suspended from its ends. What is the position of point of balance?

Solution:

Given Data:

Length of rod = $r = 1\text{m}$

Weight of rod = $w = 6\text{N}$

Weight suspended from one end of rod = $w_1 = 10\text{N}$

Weight suspended from other end of rod = $w_2 = 5\text{N}$

Let the midpoint of the rod represents the pivot point, then

$$r_1 = \left(\frac{1}{2} \right) \text{m} \Rightarrow r_1 = 0.5\text{m} \text{ and } r_2 = \left(\frac{1}{2} \right) \text{m} \Rightarrow r_2 = 0.5\text{m}$$

Let the center of gravity is situated at a distance " r_3 " from pivot point.

Required: Position of point of balance = $r_3 = ?$

Calculation: For balance condition, we have, $\tau_{\text{right}} = \tau_{\text{left}}$

$$\Rightarrow \tau_R = \tau_L \Rightarrow w_1 r_1 = w r_3 + w_2 r_2$$

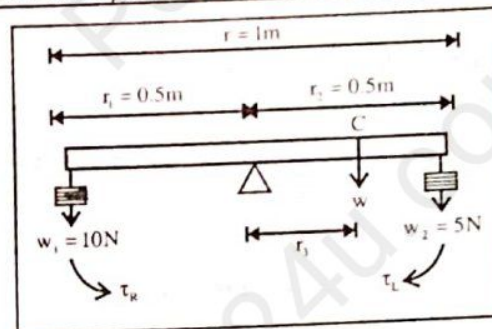
$$\Rightarrow w r_3 = w_1 r_1 - w_2 r_2 \Rightarrow r_3 = \frac{w_1 r_1 - w_2 r_2}{w} \quad (1)$$

Putting the values in equation (1), we get,

$$r_3 = \left(\frac{10 \times 0.5 - 5 \times 0.5}{6} \right) \text{m} \Rightarrow r_3 = \left(\frac{5 - 2.5}{6} \right) \text{m}$$

$$\Rightarrow r_3 = \left(\frac{2.5}{6} \right) \text{m} = 0.416\text{m}$$

$$\Rightarrow \text{Position of point of balance} = r_3 = 0.42\text{m}$$



Q8: A 4.0m long uniform ladder with weight of 120N leans against a wall making 70° above a cement floor as shown in figure. Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

Solution:

Given Data:

Length of ladder = $AB = L = 4\text{m}$

 Weight of ladder = $w = 120\text{N}$

Centre of gravity from ground

$$= AG = \frac{L}{2}$$

Required: (a) Force of reaction on the ladder due to wall = $F_{\text{wall}} = ?$

(b) Force of reaction on the ladder due to ground = $F = ?$

Calculation: (a) According to first condition of equilibrium, we have,

$$\Sigma F_x = 0 \Rightarrow F_x + (-F_{\text{wall}}) = 0$$

$$\Rightarrow F_x - F_{\text{wall}} = 0 \Rightarrow F_x = F_{\text{wall}} \quad (1)$$

$$\& \Sigma F_y = 0 \Rightarrow F_y + (-w) = 0 \Rightarrow F_y - w = 0$$

$$\Rightarrow F_y - 120\text{N} = 0 \Rightarrow F_y = 120\text{N} \quad (2)$$

Now applying second condition of equilibrium, we get,

Sum of anticlockwise moments = Sum of clockwise moments

$$\Rightarrow F_{\text{wall}} \times F_{\text{wall}} = AD \times w$$

$$\Rightarrow F_{\text{wall}} = \frac{AD \times w}{AE} \quad (3)$$

Now from the figure, we have, $AD = AG \cos \theta$

$$\Rightarrow AD = \frac{L}{2} \cos 70^\circ = \frac{4}{2} \cos 70^\circ = 2 \cos 70^\circ = 2 \times 0.342 = 0.684$$

$$\Rightarrow AD = 0.684\text{m}$$

$$\text{And } AE = CB = AB \sin \theta = L \sin 70^\circ = 4 \times 0.939\text{m}$$

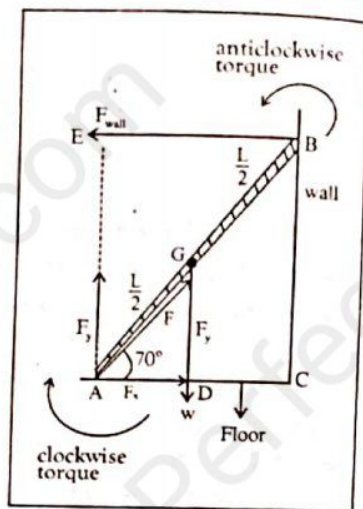
$$\Rightarrow AE = 3.756\text{m}$$

Putting the values of "AD", "AE" and "w" in equation (3), we get,

$$F_{\text{wall}} = \left(\frac{0.684 \times 120}{3.756} \right) \text{N} \Rightarrow F_{\text{wall}} = 21.8\text{N}$$

Putting the value of " F_{wall} " in equation (1), we get,

$$F_x = 21.8\text{N} \quad (4)$$



In $\triangle AGD$,

$$\cos \theta = \frac{AD}{AG}$$

$$\Rightarrow AD = AG \cos \theta$$

$$\frac{AG}{L} = \frac{1}{2}$$

In $\triangle ABC$,

$$\sin \theta = \frac{BC}{AB}$$

$$\Rightarrow BC = AB \sin \theta$$

$$\Rightarrow BC = AE = AB \sin \theta$$

$$(b) \text{ Now } F = \sqrt{F_x^2 + F_y^2} \quad (5)$$

Putting equation (2) and equation (4) in equation (5), we get,

$$F = \sqrt{(21.8)^2 + (120)^2} \text{N}$$

$$\Rightarrow F = \sqrt{475.24 + 14400} \text{N} = \sqrt{14875.24} \text{N}$$

$$\Rightarrow F = 121.96\text{N} \text{ or } F = 122\text{N}$$

$AB = \text{length of Ladder} = L$

Q9: The 450kg uniform 1-beam supports the load of 220kg as shown. Determine the reactions at the supports.

Solution:

Given Data:

Mass of beam = $M = 450\text{kg}$

Weight of beam

$$= w = mg = 450 \times 9.8\text{N}$$

$$\Rightarrow w = 4410\text{N}$$

Mass of load = $m = 220\text{kg}$

Weight of load

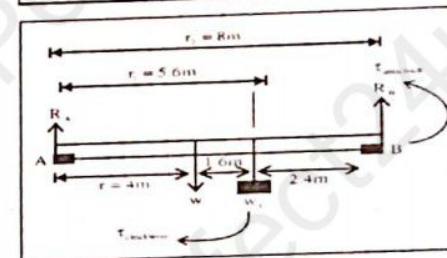
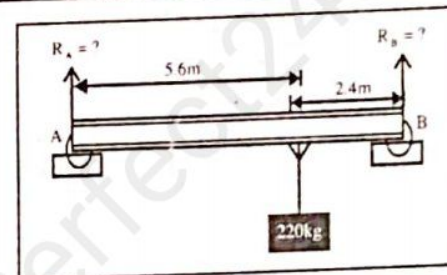
$$= w_1 = mg = 220 \times 9.8\text{N}$$

$$\Rightarrow w_1 = 2156\text{N}$$

Moment arm of " w " = $r = 4\text{m}$

Moment arm of " w_1 " = $r_1 = 5.6\text{m}$

Moment arm of " R_B " = $r_2 = 8\text{m}$



Required:

(a) Reaction at "B" = $R_B = ?$

(b) Reaction of "A" = $R_A = ?$

Calculation:

(a) Applying second condition of equilibrium, we get,

$$\tau_{\text{anticlockwise}} = \tau_{\text{clockwise}}$$

$$\Rightarrow R_B \times r_2 = (w \times r) + (w_1 \times r_1)$$

$$\Rightarrow R_g = \frac{(wr) + (w_1 r_1)}{r_2} \quad (1)$$

Putting the values in equation (1), we get,

$$R_g = \left(\frac{(4410 \times 4) + (2156 \times 5.6)}{8} \right) N = \left(\frac{17640 + 12073.6}{8} \right) N$$

$$\Rightarrow R_g = \left(\frac{29713.6}{8} \right) N \Rightarrow R_g = 3714.2 N \text{ or } R_g = 3714 N$$

Now applying first condition of equilibrium, we get,

$$\Sigma F = 0 \Rightarrow R_g + R_g + (-w) + (-w_1) = 0$$

$$\Rightarrow R_g + R_g - w - w_1 = 0 \Rightarrow R_g = w + w_1 - R_g$$

$$\Rightarrow R_g = (4410 + 2156 - 3714) N = (6566 - 3714) N$$

$$\Rightarrow R_g = 2852 N$$

❖ ارشاد نبوی ﷺ ہے "جو شخص عیب جوئی کرتا ہے اور لوگوں پر آوازیں کستا ہے اس کیلئے بڑی تباہی ہے۔"

❖ ارشاد نبوی ﷺ ہے "وہ شخص کامل مومن نہیں ہو سکتا جو خود تو سیر ہو کر کھائے لیکن اس کا مہیا بھوکا ہے۔"

❖ ارشاد نبوی ﷺ ہے "جو شخص جھوٹی قسم کھائے اپنا ٹھکانہ جہنم میں بنائے۔"

❖ ارشاد نبوی ﷺ ہے "تم میں سے اچھے لوگ وہی ہیں جو اپنی بیویوں کے ساتھ اچھا سلوک کرتے ہیں۔"

❖ ارشاد نبوی ﷺ ہے "دولت خرچ کرتے رہو روکو مت، ورنہ ہو سکتا ہے کہ اللہ بھی دولت کے دروازے تم پر

بند کر دے۔"

❖ ارشاد نبوی ﷺ ہے "دو شخصوں پر رہ کر پناہ پانے، ایک وہ دولت مند جو اللہ کے راستے میں مال خرچ کرتا ہے،

دوسرے وہ عالم جو علم کے ذریعے فیصلے کرتا ہے۔"

❖ ارشاد نبوی ﷺ ہے "اپنے گھر کی دیوار اتنی بلند نہ کرو کہ پردہ کی ہوا تک جائے۔"

CHAPTER

03

MOTION & FORCE

Q1: Define rest and motion?

Ans. (i) Rest:

A body is said to be at rest if it do not change its position with respect to the surrounding.

(ii) Motion:

A body is said to be in motion if it changes its position with respect to the surrounding.

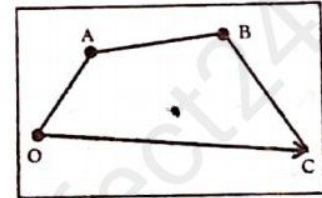
Point to Ponder:

When sitting on a chair, your speed is zero relative to earth. But you are moving at 30km/sec relative to the sun, because the earth is revolving around sun continuously.

Q2: Define distance and displacement?

Ans. (i) Distance:

The length of actual path traversed by a body in motion is known as distance. The actual path may be straight or curved. The distance is a scalar quantity. Its SI unit is meter.



(ii) Displacement:

The shortest distance between two points is known as displacement.

Explanation:

In given figure, a body reaches from point "O" to "C" through "A" and "B". Thus the path "OABC" represents the actual path traversed by the body during its motion. So it is called distance. While $\vec{OC}$ represents the shortest distance between the initial point "O" and final point "C". Thus $\vec{OC}$ is known as displacement. It is a vector quantity. Its SI unit is meter.

Q3: Define the following terms: position, speed, uniform speed, variable speed, average speed, instantaneous speed.

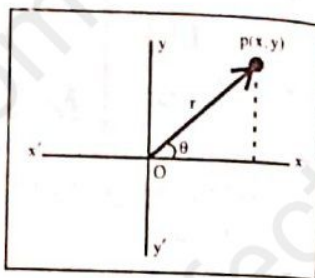
Ans. 1. Position:

The location of an object relative to some reference point [origin] is

known as position of that object.

Explanation:

In the given figure, an object is placed at point "p". If we join the point with origin "o", then $op = r$ represents the distance of object from the origin "o". From this line, we can also find the angle made by point "p" with x-axis or y-axis. The position of point "p" is represented by $P(x, y)$, where "x" represents the perpendicular distance of point "p" from y-axis and "y" represents the perpendicular distance of point "p" from x-axis respectively. In vector terminology, $\vec{OP} = \vec{r}$ is known as position vector.



2. Speed:

The distance covered by a moving body in one second is known as speed. i.e.

$$\text{Speed} = \text{Distance covered} / \text{time}$$

$$\Rightarrow v = S / t$$

The speed is a scalar quantity. Its S.I unit is "m/sec".

3. Uniform Speed [Constant Speed]:

A body is said to be moving with uniform speed, if it covers equal distance in equal intervals of time.

4. Variable Speed:

A body is said to be moving with variable speed if it covers unequal distance in equal intervals of time.

5. Average Speed:

The average speed of a moving body can be obtained by dividing total distance covered by total time taken i.e.

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$\Rightarrow \langle v \rangle = S / T$$

6. Instantaneous Speed:

The speed of a body at any particular instant of time is known as instantaneous speed. For such speed, we take time interval " Δt " to be very small such that " Δt " approaches to zero i.e. $\lim_{\Delta t \rightarrow 0}$

Mathematically, we have,

$$v_{\text{ins}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta S}{\Delta t}$$

Q4: Define the following terms: velocity, uniform velocity, variable velocity, average velocity, instantaneous velocity.

Ans. 1. Velocity:

The distance covered by a body in one second along a particular direction is known as velocity. (OR)

The rate of change of displacement is known as velocity.

Mathematically, we have,

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\Rightarrow \vec{v} = \frac{\vec{S}}{t}$$

Velocity is a vector quantity. Its standard unit is "m/sec".

2. Uniform Velocity:

A body is said to be moving with uniform velocity, if it covers equal displacement in equal intervals of time.

3. Variable Velocity:

A body is said to be moving with variable velocity, if it covers unequal displacement in equal intervals of time.

4. Average Velocity:

The average velocity can be obtained by dividing total displacement by total time taken i.e.

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

$$\Rightarrow \langle \vec{v} \rangle = \frac{\vec{S}}{t}$$

5. Instantaneous Velocity:

The velocity of a body at any particular instant of time is known as instantaneous velocity. i.e.

$$\vec{v}_{\text{ins}} = \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta \vec{S}}{\Delta t} \right)$$

Q5: Define the following terms: acceleration, uniform acceleration, variable acceleration, radial acceleration, linear acceleration, positive acceleration, negative acceleration, instantaneous acceleration?

Ans: 1. Acceleration:

The rate of change of velocity is known as acceleration. Mathematically, we have,

$$\text{Acceleration} = \frac{\text{velocity}}{\text{time}}$$

$$\Rightarrow \vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

It is a vector quantity. Its S.I unit is m/sec^2 .

2. Uniform Acceleration:

A body is said to be moving with uniform acceleration if equal change occurs in velocity in equal intervals of time.

3. Variable Acceleration:

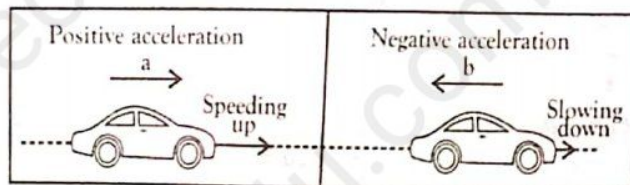
A body is said to be moving with variable acceleration if unequal change occurs in velocity in equal intervals of time.

4. Radial Acceleration:

The acceleration which is produced due to change in direction of velocity is known as radial acceleration.

5. Linear Acceleration:

The acceleration which is produced due to change in magnitude of velocity is known as linear acceleration.



6. Positive Acceleration:

The acceleration is said to be positive, if the magnitude of velocity increases with respect to time.

For Your Information:
Dimensions of Velocity:

We know that,

$$v = \frac{\text{displacement}}{\text{time}} = \text{m/sec}$$

$$= \frac{\text{length}}{\text{time}} \quad v = [L T^{-1}]$$

Dimensions of Acceleration:

We know that,

$$a = \frac{\text{velocity}}{\text{time}} = \text{m/sec} \cdot \text{sec} = \text{m/sec}^2$$

$$\Rightarrow a = \frac{\text{length}}{\text{time}^2} \Rightarrow a = [L T^{-2}]$$

7. Negative Acceleration:

The acceleration will be negative, if the magnitude of velocity decreases with respect to time.

8. Instantaneous Acceleration:

The acceleration of a body at any particular instant of time is known as instantaneous acceleration. i.e.

$$\vec{a}_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}$$

Q6: Discuss the displacement - time graph.

Ans: The displacement - time graph gives us a certain slope or gradient from which we can calculate the velocity of a body.

For example, figure (a) shows the displacement - time graph of a moving car. From the slope of the graph, the velocity of the car can be calculated in the following manner i.e.

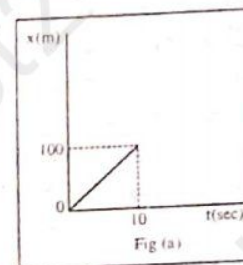
Velocity = Slope of the graph

$$\Rightarrow \text{Velocity} = \frac{\Delta y}{\Delta x} \quad (1) \quad \left[\text{Slope} = \frac{\Delta y}{\Delta x} \right]$$

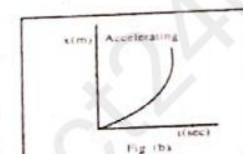
From figure, $\Delta y = 100\text{m}$, $\Delta x = 10\text{sec}$, so eq. (1)

$$\Rightarrow \text{Velocity} = \frac{100}{10} \text{ m/sec}$$

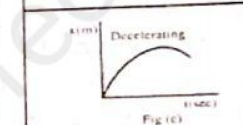
$$\Rightarrow v = 10 \text{ m/sec}$$



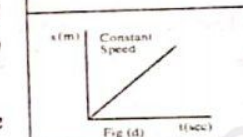
1. If the slope of the displacement - time graph increases, it means that the object is speed up or accelerating, as shown in figure (b)



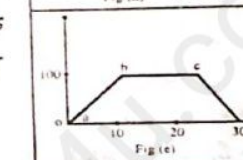
2. If the slope of displacement - time graph decreases, it means that the object is slowing down (or) decelerating as shown in figure (c).



3. If the slope of the displacement - time graph remains constant, then it means that the body is moving with constant velocity as shown in figure (d).



In figure (e),

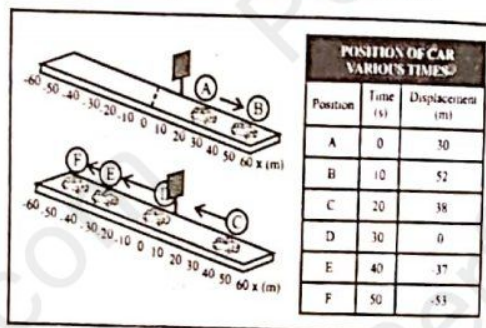


- (a) The line 'ab' shows that the object is moving with constant velocity.
- (b) The line 'bc' shows that there is no change in displacement of the body, so the body is at rest.
- (c) The line 'cd' shows that the body begins reverse its journey with constant velocity.

Q7: Discuss how can you calculate the magnitude of velocity from displacement - time graph?

Ans: We can calculate the magnitude of velocity of a moving body from displacement - time graph. For this purpose we need the magnitude of displacement covered by the body and time interval taken by the body in which the concern displacement is covered.

Example: Consider a car which is moving backward forth along a straight path as shown in the figure. Here we measure the displacement of the car after every 10sec. The magnitude of displacement covered after every 10sec are given in the table.



Selection of Scale for Representing the V_{avg} vs of Time and Displacement:

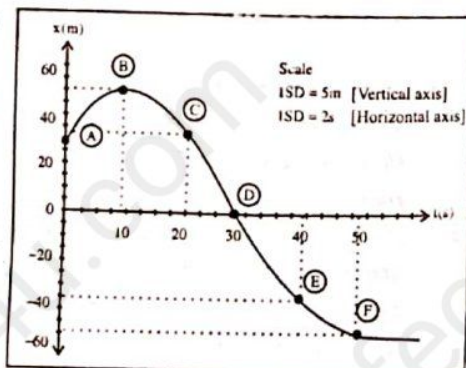
We draw horizontal and vertical axis. We take the displacement "x" on vertical axis and time taken on horizontal axis. Now we select a suitable scale.

Let one small division on vertical axis = 5m

And one small division on horizontal axis = 2sec

Now we put dots at point A,

B, C, D, E and "F" for the given values of displacement and time.



Now we joint the dots by free hand curve and get a graph as shown in the figure.

Calculation of Velocity between point "A" and point "B":

The average velocity between point "A" and "B" is given by,

$$v_{AB} = \frac{x_B - x_A}{t_B - t_A} \quad (1)$$

Putting the values in equation (1), we get,

$$v_{AB} = \left[\frac{52 - 30}{10 - 0} \right] \text{ m / sec}$$

$$\Rightarrow v_{AB} = \left(\frac{22}{10} \right) \text{ m / sec} \Rightarrow v_{AB} = 2.2 \text{ m / sec}$$

Calculation of Velocity between point "B" and point "C":

The average velocity between "B" and "C" is given by,

$$v_{BC} = \frac{x_C - x_B}{t_C - t_B} \quad (2)$$

Putting the values in equation (2), we get,

$$v_{BC} = \left(\frac{38 - 52}{20 - 10} \right) \text{ m / sec}$$

$$\Rightarrow v_{BC} = \left(\frac{-14}{10} \right) \text{ m / sec} \Rightarrow v_{BC} = -1.4 \text{ m / sec}$$

In this case the car moves backward from "B" to "C".

Calculation of Velocity between point "C" and point "D":

The average velocity between "C" and "D" is given by,

$$v_{CD} = \frac{x_D - x_C}{t_D - t_C} \quad (3)$$

Putting the values in equation (3), we get,

$$v_{CD} = \left(\frac{0 - 38}{30 - 20} \right) \text{ m / sec}$$

$$v_{CD} = \left(\frac{-38}{10} \right) \text{ m / sec}$$

$$\Rightarrow v_{CD} = -3.8 \text{ m / sec}$$

Calculation of Velocity between point "D" and point "E":

The average velocity between point "D" and "E" is given by,

$$\Rightarrow v_{DE} = \frac{x_E - x_D}{t_E - t_D} \quad (4)$$

Putting the values in equation (4), we get,

$$v_{DE} = \left(\frac{-37 - 0}{40 - 30} \right) \text{ m / sec}$$

$$v_{DE} = \frac{-37}{10} \text{ m / sec} \Rightarrow v_{DE} = -3.7 \text{ m / sec}$$

Calculation of Velocity between point "E" and point "F":

The average velocity between point "E" and "F" is given by,

$$v_{EF} = \frac{x_F - x_E}{t_F - t_E} \quad (5)$$

Putting the values in equation (5), we get,

$$v_{EF} = \left(\frac{-53 - (-37)}{50 - 40} \right) \text{ m / sec}$$

$$v_{EF} = \left(\frac{-53 + 37}{10} \right) \text{ m / sec} = \left(\frac{-16}{10} \right) \text{ m / sec}$$

$$\Rightarrow v_{EF} = -1.6 \text{ m / sec}$$

Q8: Discuss the velocity - time graph?

Ans. With the help of velocity - time graph, we can discuss the acceleration of a body and can calculate the distance travelled by a body.

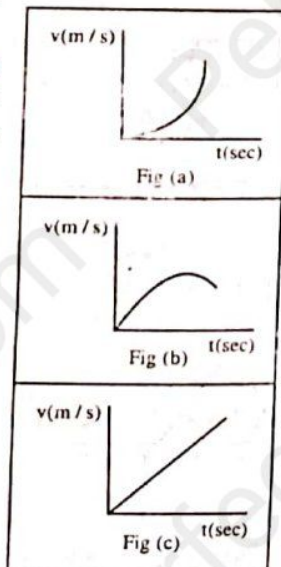
1. The slope or gradient of velocity - time graph is called acceleration.
2. The area under velocity - time graph is called distance travelled.

a. If the slope of velocity - time graph increases, then it shows increasing acceleration, as shown in figure (a).

b. If the slope of velocity - time graph decreases, then it shows decreasing acceleration, as shown in figure (b).

If the slope of velocity - time graph remains straight, then it shows constant or uniform acceleration as shown in figure (c).

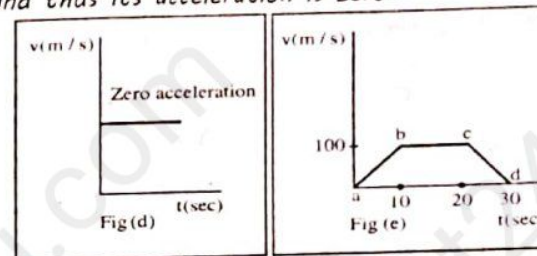
d. If the body is moving with constant velocity, then the acceleration of the body will be zero. In



such a case we get a straight line parallel to time - axis, as shown in figure (d).

In figure (e), the line "ab" shows that the body is moving with constant acceleration.

3. The line "bc" shows that the body is moving with constant velocity and thus its acceleration is zero.

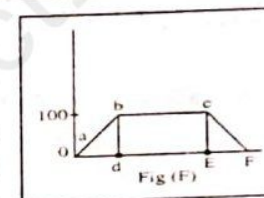


4. The line "cd" shows that the acceleration of the body decreases uniformly.

Calculation of Distance travelled:

The area under velocity - time graph gives us the distance travelled. The distance covered by a body according to figure "F" can be calculated as,

$$\text{Distance covered} = \text{Area of } \triangle abd + \text{Area of rectangle } bcde + \text{Area of } \triangle cfe$$



Q9: Discuss how you can calculate the magnitude of acceleration from velocity - time graph?

Ans. We can calculate the magnitude of acceleration of a moving body from velocity - time graph.

Example: Consider a graph between given values of velocity and time interval as shown in the figure.

Calculation of Acceleration between points "A" and "B":

The acceleration of the body between points "A" and "B" is given by,

$$a = \frac{v_B - v_A}{t_B - t_A} = \left(\frac{20 - 0}{10 - 0} \right) \text{ m / sec}^2$$

$$\Rightarrow a_{AB} = 2 \text{ m / sec}^2$$

Calculation of Acceleration between points "B" and "C":

The acceleration of the body between points "B" and "C" is given by,

$$a_{BC} = \left(\frac{v_C - v_B}{t_C - t_B} \right) = \left(\frac{20 - 20}{20 - 10} \right) m / sec = 0 \Rightarrow a = 0$$

This shows that the body moves with uniform velocity from point "B" to point "C".

Calculation of Acceleration between points "C" & "D":

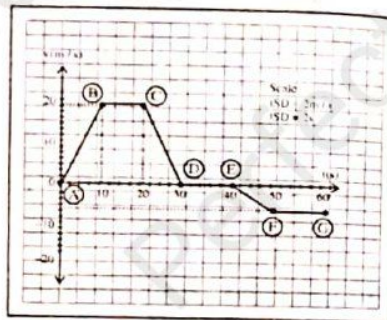
The acceleration of the body between points "C" and "D" is given by,

$$a_{CD} = \left(\frac{v_D - v_C}{t_D - t_C} \right) = \left(\frac{0 - 20}{30 - 20} \right) m / sec$$

$$a_{CD} = \left(\frac{-20}{10} \right) m / sec = -2m / sec^2$$

$$a_{CD} = -2m / sec^2$$

The negative sign shows that the velocity of the body decreases while moving from "C" to "D".



Calculation of Acceleration between points "D" and "E":

The acceleration of the body between points "D" and "E" is given by,

$$a_{DE} = \frac{v_E - v_D}{t_E - t_D} \Rightarrow a_{DE} = \left(\frac{0 - 0}{40 - 30} \right) m / sec^2$$

$$\Rightarrow a_{DE} = 0$$

As v_D, v_E are zero, so the body is at rest in this region.

Calculation of Acceleration between points "E" and "F":

The acceleration between points "E" and "F" is given by,

$$a_{EF} = \frac{v_F - v_E}{t_F - t_E} \Rightarrow a_{EF} = \left(\frac{-6 - 0}{50 - 40} \right) m / sec^2$$

$$\Rightarrow a_{EF} = \frac{-6}{10} m / sec^2 \Rightarrow a_{EF} = -0.6m / sec^2$$

Calculation of Acceleration between points "F" and "G":

The acceleration between points "F" and "G" is given by,

$$a_{FG} = \frac{v_G - v_F}{t_G - t_F} = \left(\frac{-6 - (-6)}{60 - 50} \right) m / sec^2 = \left(\frac{-6 + 6}{10} \right) m / sec^2$$

$$\Rightarrow a_{FG} = 0$$

This shows that the body is moving constant velocity from "F" to "G".

﴿حضرت ابو بکر صدیقؓ کا قول ہے "پرہیز کرو، پرہیز کرنا نفع دیتا ہے، عمل کرو، عمل قبول کیا جاتا ہے۔﴾

Q10: Derive first equation of motion (OR) Show that $(v_f = v_i + at)$

Ans. Consider a body of mass "m" is moving with initial velocity " v_i ". Let " v_f " be the final velocity of the body after time "t" then the acceleration of the body is given by,

Acceleration = rate of change of velocity

$$\Rightarrow a = \frac{(v_f - v_i)}{t} \Rightarrow at = v_f - v_i$$

$$\Rightarrow at + v_i + v_i \Rightarrow v_f = at + v_i$$

$$\Rightarrow v_f = v_i + at \quad (1)$$

Equation (1) represents the first equation of motion.

Q11: Derive second equation of motion (OR) Show that,

$$(S = v_i t + \frac{1}{2} at^2)$$

Ans. Consider a body of mass "m" is moving with initial velocity " v_i ". Let the body covers distance "S" in time interval "t". Now the distance covered by the body in terms of average velocity " $\langle v \rangle$ " is given by,

$$S = \langle v \rangle t \quad (1)$$

The average velocity is given by,

$$\langle v \rangle = \frac{(v_f + v_i)}{2} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$S = \frac{(v_f + v_i)t}{2} \quad (3)$$

Now from first equation of motion, we have,

$$v_f = v_i + at \quad (4)$$

Putting equation (4) in equation (3), we get,

$$S = \frac{(v_i + at + v_i)t}{2}$$

$$\Rightarrow S = \frac{(2v_i + at)t}{2} = \frac{2v_i t}{2} + \frac{1}{2} at^2$$

$$\Rightarrow S = v_i t + \frac{1}{2} at^2 \quad (5)$$

Equation (5) represents the second equation of motion.

Q12: Derive third equation of motion (OR) Show that,

$$(2aS = v_f^2 - v_i^2)$$

Ans. Consider a body of mass "m" is moving with initial velocity

" v_i ". Let the body covers a distance " S " in time interval " t " and let " v_f " be its final velocity. Now the distance covered by the body in terms of average velocity ' $\langle v \rangle$ ' is given by,

$$S = \langle v \rangle t \quad (1)$$

The average velocity is given by,

$$\langle v \rangle = \frac{(v_f + v_i)}{2} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$S = \frac{(v_f + v_i)t}{2} \quad (3)$$

Now from first equation of motion, we have,

$$v_f = v_i + at$$

$$\Rightarrow at = v_f - v_i \Rightarrow t = \frac{(v_f - v_i)}{a} \quad (4)$$

Putting equation (4) in equation (3), we get, $[(a + b)(a - b) = a^2 - b^2]$

$$S = \frac{(v_f + v_i)}{2} \cdot \frac{(v_f - v_i)}{a}$$

$$\Rightarrow 2aS = (v_f + v_i)(v_f - v_i)$$

$$\Rightarrow \boxed{2aS = v_f^2 - v_i^2} \quad (5)$$

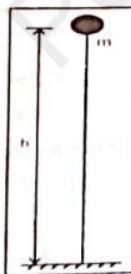
Equation (5) represents the third equation of motion.

Q13: Define free fall motion. Also define the acceleration due to gravity?

Ans. Definition: If we drop an object from certain height " h " and ignore the air resistance, then such motion of the body is known as free fall motion.

Acceleration due to Gravity:

The acceleration of free fall body is known as acceleration due to gravity. It is represented by " g ". Its value varies with altitude and depth. On earth surface or near earth surface, its value is 9.8 m/sec^2 .



Q14: State and explain the Newton's laws of motion with examples?

ETEA 2011

Ans. 1. Newton's 1st Law of Motion:

Statement: This law states that, "A body at rest will remain at rest

$\vec{a} = 0$ means, the body will be either at rest or will be moving with uniform velocity.

and a body in motion will continue its motion with uniform velocity unless it is acted upon by the some unbalanced external force." i.e. If $\vec{F} = 0$, then $\vec{a} = 0$

Explanation:

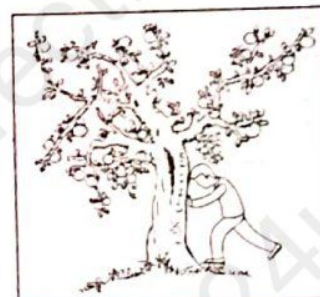
The 1st law of motion is also known as law of inertia. The inertia is the property of a body which opposes any change in its state of motion or rest.

Actually, when some force is applied on a body for changing its state of rest or uniform motion, then the body will offer resistance against this change.

In other words, we can say that all the bodies try to maintain its state of rest or continue its uniform motion due to inertia. That is why, the Newton's 1st law is also known as law of inertia.

Examples:

1. When we give a jerk to the branch of a tree, then the fruit falls down. It is because, when we give a jerk, the branch of the tree comes into motion suddenly. While the fruits are at rest and want to remain at rest due to inertia. So in this attempt, the fruit falls down.



2. When a bus starts motion suddenly, the passengers experience a push in the backward direction. It is because, when the bus starts motion suddenly, the lower parts of the passenger in contact with the seats also come into motion. While the upper parts of the passengers are at rest and according to inertia, they try to remain at rest. In this attempt, they fall in the backwards direction.



Similarly, when a moving bus stops suddenly, the passenger experience a push in forward direction. It is due to inertia. When bus stops suddenly, the lower parts of passengers are in contact with seats also comes to rest suddenly. While the upper

parts are in motion and according to inertia try to continue their motion. So we the passenger experience a push in forward direction.

3. When a cyclist applies brakes suddenly, he experiences a push in the forward direction. It is because, when he apply the brakes suddenly, his lower parts in contact with cycle also comes to rest suddenly. While the upper parts are in state of motion and try to maintain its state of motion due to inertia. In this attempt, he experience a push in the forward direction.



2. Newton's 2nd Law of Motion: (ETEA 2010)

Statement: This law states that, "when a force is applied on a body, then acceleration is produced in the body in the direction of applied force. The magnitude of this acceleration is directly proportional to the applied force and inversely proportional to the mass of the body."

Explanation:

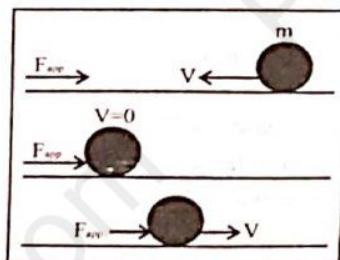
The 1st law of motion gives us the qualitative measurement of force, while the 2nd law of motion gives us the quantitative measurement of force.

Now if the direction of motion of the body and direction of force are the same, then its velocity and acceleration increases.

If the direction of motion of the body and direction of force are opposite in direction, then the velocity and acceleration of the body will decrease gradually and finally becomes zero and the body will come to rest. Then due to applied force, the body will start motion in the direction of force i.e. the body will start motion opposite to its initial direction.

Mathematical form of Newton's 2nd Law:

Let "F" represents the magnitude of applied force on the body, "a" represents the magnitude of acceleration produced in the body



and "m" represents the mass of the body, then we have,

$$a \propto F \quad (1)$$

$$\text{And } a \propto \frac{1}{m} \quad (2)$$

Combining relations (1), (2), we get,

$$a \propto \frac{F}{m}$$

$$\Rightarrow a = \text{const.} \cdot \frac{F}{m} \quad (3)$$

Put constant = 1 in equation (3), we get,

$$a = 1 \times \frac{F}{m}$$

$$\Rightarrow a = \frac{F}{m} \Rightarrow F = ma \quad (4)$$

In vector form, equation (4) can be written as,

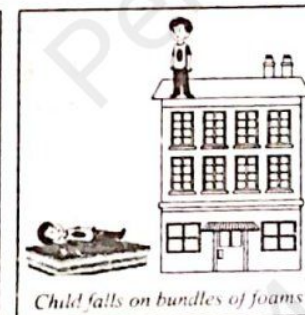
$$\vec{F} = m\vec{a} \quad (5)$$

Equation (5) represents the mathematical form of Newton's 2nd law of motion.

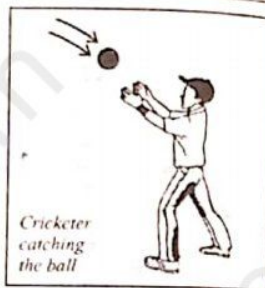
Examples of Newton's 2nd Law:

1. When a body falls on a solid floor from the top of a building, the body experiences or feels great pain. It is because, when the body falls on the floor, its acceleration becomes zero suddenly and the body faces huge force for a very short period of time. Thus the body feel great pain.

On the other hand, if the body falls upon the bundles of foam or cotton, then the body feel less pain or no pain. It is because, when the body falls on the foams, its acceleration decreases gradually and the body does not face large force for a short period of time. Thus the body feels no pain or less pain.



2. When a cricketer catches a ball, he moves his hands in the backward direction. It is because, in doing so, the acceleration of the ball does not become zero suddenly and the cricketer will not face huge force for a short period of time. Thus the cricketer will be able to catch the ball easily and will feel no pain or less pain.



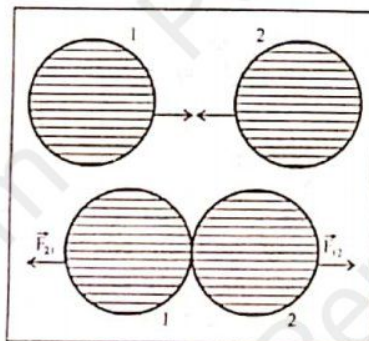
3. Newton's 3rd Law of Motion:

Statement: This law states that, "for every action, there is always a reaction. These forces are equal in magnitude but opposite in direction."

Explanation:

This law tells us that nobody can experience a force from its surrounding, until this body exerts any force on its surrounding. It means that forces are exerted in pairs.

A single isolated force is impossible. In other words, we can say that, if a body exerts some force on another body, then the 2nd body will also exert an equal and opposite force on the 1st body. The force which is exerted on 2nd body due to 1st body is known as action. While the force on 1st body due to 2nd body is known as reaction.



Mathematical Form:

Consider two bodies "1" and "2" moving towards each other as shown in the figure. Let the two bodies collide with each other. Now if $\vec{F}_{12}$ represents the force exerted by body "1" on body "2" and $\vec{F}_{21}$ be the force exerted by body "2" on body "1", then according to Newton's 3rd law, we have,

$$\vec{F}_{12} = -\vec{F}_{21} \quad (1)$$

Equation (1) represents the mathematical form of Newton's 3rd law of motion. The negative sign shows that both forces are opposite in

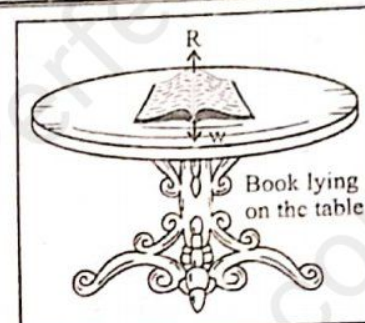
directions.

Examples of Newton's 3rd Law:

1. When a book is lying on the table, then its weight "w" is acting on the table in downwards direction, which is action. While the table exerts an equal and opposite force "R" on the book in upwards direction which is reaction. Thus the book remains in equilibrium condition.
2. When we walk on the ground, we push the ground in backward direction, which is action. While in reaction, the ground pushes us in the forward direction.
3. When a bullet is fired from a gun, it moves in forward direction due to action of gases which are produced due to explosion of chemicals. While in reaction, the bullet pushes the gun in backwards direction.
4. When a football is kicked, the foot exerts the force F_{AB} on the football which is action while in reaction the football exerts an equal and opposite force F_{BA} on the foot as shown in figure. i.e.

$$F_{AB} = -F_{BA}$$

The force of A on B is equal in magnitude and opposite in direction of the force of B on A.



Q15: Define force. Define its SI unit. Also write the dimension of force.

BISE Mardan 2019

Ans. Force: "A force is an agent which stops a moving body (or) try to stop a moving body (or) which moves a stationary body (or) try to move a stationary body." (OR)

"The product of mass and acceleration is known as force." i.e.

$$F = ma \quad (1)$$

Force is a vector quantity.

Unit of Force:

From equation (1), we have

$$F = ma = \text{kg} \cdot \text{m} / \text{sec}^2$$

$$\Rightarrow F = \text{kg} \cdot \text{m} / \text{sec}^2 = \text{Newton}$$

So the unit of force is "Newton", which is defined as, "force is said to be 1 Newton, if it moves a body of mass 1kg with acceleration of 1m/sec^2 ."

Dimension of Force:

$$\text{As } F = \text{kg} \cdot \text{m} / \text{sec}^2$$

$$\Rightarrow F = \frac{\text{mass} \cdot \text{length}}{\text{time}^2}$$

$$\Rightarrow F = [MLT^{-2}]$$

Q16: Define linear momentum. Express Newton's 2nd law in terms of momentum?

Ans. Linear Momentum:

"The quantity of motion in a body is known as momentum." (OR)

"The product of mass and velocity is known as linear momentum."

Explanation:

The momentum depends upon the mass and velocity of the body.

Mathematically, we have,

$$P = mv \quad (1)$$

In vector form, equation (1) can be written as,

$$\vec{P} = m\vec{v} \quad (2)$$

The momentum is a vector quantity.

Unit of Linear Momentum:

$$\text{As } P = mv = \text{kg} \cdot \text{m} / \text{sec}$$

$$\Rightarrow P = \text{kg} \cdot \text{m} / \text{sec}$$

For Your Information:

Newtonian mechanics are limited to situations where speeds are less than about 1% of the speed of light - that is, less than 3,000km/s. Most things we encounter in daily life move much slower than this speed, therefore we can safely apply Newton's Laws. However they were refined further at the beginning of the 20th century when Einstein developed his theories of relativity. His theories of relativity extended to concept of Newtonian mechanics to be applied to all objects, even objects travelling close to the speed of light.

Dimensions of Linear Momentum:

$$\text{As } P = \text{kg} \cdot \text{m} / \text{sec}$$

$$\Rightarrow P = \frac{\text{mass} \cdot \text{length}}{\text{time}}$$

$$\Rightarrow P = [MLT^{-1}]$$

Newton's 2nd Law in terms of Momentum:

From Newton's 2nd law, we have,

$$\vec{F} = m\vec{a} \quad (1)$$

We also know that,

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$\vec{F} = m \frac{\Delta \vec{v}}{\Delta t}$$

$$\Rightarrow \vec{F} = \frac{\Delta(m\vec{v})}{\Delta t} \quad (3)$$

We know that,

$$\vec{P} = m\vec{v} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$\vec{F} = \frac{\Delta \vec{P}}{\Delta t} \quad (5)$$

Equation (5) represents Newton's 2nd law in terms of momentum. Equation (5) shows that "force is equal to the rate of change of momentum."

Q17: (a) Define impulse. Explain with example. (b) Show that impulse is equal to change in momentum. i.e. show that, $J = \Delta P$.

BISE Kohat 2019

Ans. (a) Impulse:

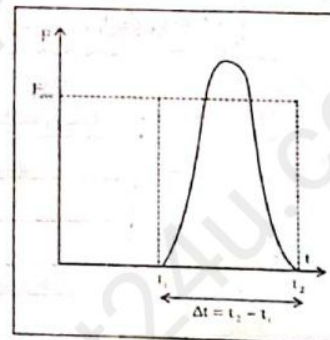
"The product of force and duration of time in which the force acts is known as impulse." Mathematically, we have,

$$\text{Impulse} = \text{Force} \times \text{Duration of time}$$

$$\Rightarrow J = F \times \Delta t \quad (1)$$

In case of variation of force, we take its average value. So equation (1) can be written as,

$$J = F_{\text{ave}} \times \Delta t \quad (2)$$



N·sec is the unit of linear momentum.

$$\text{Hints: } \frac{\Delta P}{\Delta t} = F \Rightarrow \Delta P = F \cdot \Delta t = \text{N} \cdot \text{sec}$$

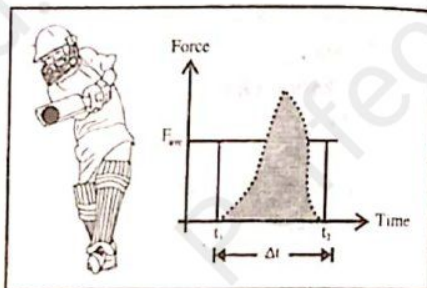
(ETEA 2015)

Graph of Impulse:

The variation in force during time interval ($t_2 - t_1$) is shown in the figure. If we take average value of force, then the area under the " $F_{ave} - t$ " curve gives us rectangular area, as shown by the dotted lines in the given figure.

Example of Impulse:

Consider Jannat Gul is hitting a ball with his bat as shown in the figure. In this case, Jannat Gul applies large force on the ball for a short time " Δt ". That is the ball remain in contact with the bat for short time " Δt ". The graph between force and time of contact shows that, large impulse produces a large response i.e. if the impulse is large, then the ball will depart from the bat with greater velocity. However a massive ball will depart with less velocity as compare to lighter ball for same impulse.

**(b) To show that $J = \Delta P$:**

From Newton's 2nd law of motion, we have,

$$F = \frac{\Delta P}{\Delta t}$$

$$\Rightarrow F \times \Delta t = \Delta P \quad (3)$$

We also know that,

$$J = F \times \Delta t \quad (4)$$

Putting equation (3) in equation (4), we get,

$$J = \Delta P \quad (5)$$

Equation (5) shows that, impulse is equal to change in momentum.

Q18: (a) What is isolated system? (b) State and explain the law of conservation of linear momentum with an example.

Ans: (a) Isolated System:

"A system which is not affected by external forces is known as an isolated system."

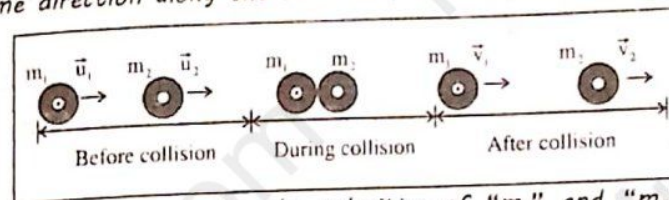
(b) Law of Conservation of Linear Momentum:

Statement: This law states that, "in the absence of external

forces, the total linear momentum of the system remains constant."

Example:

Consider two bodies of masses " m_1 " and " m_2 " which are moving in the same direction along the same line, as shown in the figure.



Let $\vec{u}_1$ and $\vec{u}_2$ represents the velocities of " m_1 " and " m_2 " before collision respectively.

Now let F_{12} be the force exerted by " m_2 " on " m_1 ", then due to this force, the momentum of " m_1 " will change and is given by,

$$\vec{F}_{12} = \left[\frac{m_1 \vec{v}_1 - m_1 \vec{u}_1}{\Delta t} \right] \quad (1)$$

Similarly,

$$\vec{F}_{21} = \left[\frac{m_2 \vec{v}_2 - m_2 \vec{u}_2}{\Delta t} \right] \quad (2)$$

Now according to Newton's 3rd law, we have,

$$\vec{F}_{12} = -\vec{F}_{21} \quad (3)$$

Putting equations (1), (2) in equation (3), we get,

$$\left[\frac{m_1 \vec{v}_1 - m_1 \vec{u}_1}{\Delta t} \right] = - \left[\frac{m_2 \vec{v}_2 - m_2 \vec{u}_2}{\Delta t} \right]$$

$$\Rightarrow m_1 \vec{v}_1 - m_1 \vec{u}_1 = -m_2 \vec{v}_2 + m_2 \vec{u}_2$$

$$\Rightarrow m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{u}_1 + m_2 \vec{u}_2$$

$$\Rightarrow m_1 \vec{u}_1 + m_2 \vec{u}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2 \quad (4)$$

Equation (4) shows that, total momentum before collision is equal to total momentum after the collision, which satisfies the law of conservation linear momentum.

Q19: (a) Define collision. (b) Discuss the types of collision. Give examples in each case.

BISE DI Khan 2019

Ans: (a) Collision:

When two or more than two bodies come so closer to each other

Do You Know!

An isolated system is a collection of particles that can interact with each other but whose interactions with the environment outside the collection have a negligible effect on their motions.

that there exists some type of interaction between them in the presence or absence of external forces, then such type of interaction is known as collision.

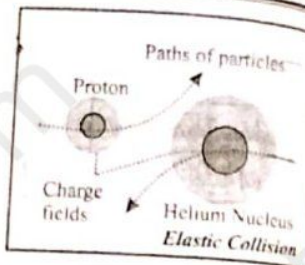
(b) Types of Collision:

There are two types of collision, which are given below:

(i) **Elastic Collision:** The collision in which the momentum and kinetic energy both are conserved is known as elastic collision.

Examples:

- Collision of steel balls,
- Collision of atoms,
- Collision of molecules



(ii) **Inelastic Collision:** The collision in which the momentum is conserved but K.E is not conserved, is known as inelastic collision. In such type of collision, some of the K.E is converted into internal energy, sound energy and the work needed to permanently deform the objects involved, such as cars in a car crash.



Examples:

- Collision between two vehicles,
- Collision between two mud balls

Q20: (a) What is meant by perfectly elastic head-on collision? (b) Discuss the elastic collision in one dimension and find the mathematical equations for calculating the final velocities of colliding objects in one dimension.

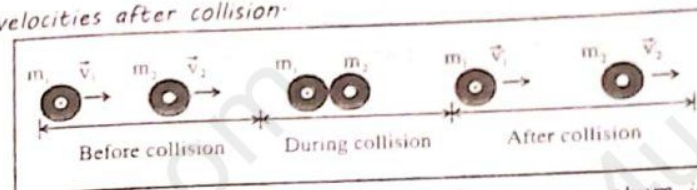
(ETE 2011, BISE Malakand 2019, BISE Peshawar 2019)

Ans. (a) Perfect Elastic Head-on-Collision:

The collision in which the motion of the objects occurs along a straight line is known as perfect elastic head-on-collision.

(b) Determination of Mathematical Equations for Calculating the Final Velocities of Colliding Objects in one Dimension:

Consider two bodies of masses " m_1 " and " m_2 " moving with initial velocities v_1 and v_2 respectively, as shown in the figure. Let these bodies suffer elastic collision and let v_1' and v_2' represent their velocities after collision.



Now we apply the law of conservation of linear momentum i.e.

Total momentum before collision = Total momentum after collision

$$\Rightarrow m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\Rightarrow m_1 v_1 - m_1 v_1' = m_2 v_2' - m_2 v_2$$

$$\Rightarrow (v_1 - v_1') m_1 = (v_2' - v_2) m_2 \quad (1)$$

Now we apply the law of conservation of energy i.e.

Total K.E before collision = Total K.E after collision

$$\Rightarrow \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$\Rightarrow m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$\Rightarrow m_1 v_1^2 - m_1 v_1'^2 = m_2 v_2'^2 - m_2 v_2^2$$

$$\Rightarrow m_1 (v_1^2 - v_1'^2) = m_2 (v_2'^2 - v_2^2)$$

$$\Rightarrow (v_1 + v_1')(v_1 - v_1') m_1 = (v_2' + v_2)(v_2' - v_2) m_2 \quad (2) \quad [a^2 - b^2 = (a+b)(a-b)]$$

Dividing equation (2) by equation (1), we get,

$$\frac{(v_1 + v_1')(v_1 - v_1') m_1}{(v_1 - v_1') m_1} = \frac{(v_2' + v_2)(v_2' - v_2) m_2}{(v_2' - v_2) m_2}$$

$$\Rightarrow (v_1 + v_1') = (v_2' + v_2) \quad (3)$$

$$\Rightarrow v_1 - v_2 = v_2' - v_1' \Rightarrow -(v_2 - v_1) = (v_2' - v_1')$$

$$\Rightarrow (v_2 - v_1) = -(v_2' - v_1') \quad (4)$$

In equation (4), we have,

$v_2 - v_1$ = relative velocity of colliding objects before collision = v_{rel}

$$\Rightarrow v_2 - v_1 = v_{rel} \quad (5)$$

While $v_2' - v_1'$ = relative velocity of colliding objects after collision = v_{rel}'

$$\Rightarrow v_2' - v_1' = v_{rel}' \quad (6)$$

Putting equations (5), (6) in equation (4), we get,

$$v_{rel} = -v'_{rel} \quad (7)$$

Equation (7) shows that, the relative velocity of approach of two colliding objects is equal to the relative velocity of recession. Now from equation (3), we get,

$$v_1 + v'_1 = v'_2 + v_2$$

$$\Rightarrow v'_2 = (v_1 + v'_1 + v_2) \quad (8)$$

Putting equation (8) in equation (1), we get,

$$(v_1 - v'_1)m_1 = (v_1 + v'_1 + v_2 - v_2)m_2 = (v_1 + v'_1 - 2v_2)m_2$$

$$\Rightarrow m_1v_1 = m_1v'_1 = m_2v_1 + m_2v'_1 - 2m_2v_2$$

$$\Rightarrow -m_1v'_1 - m_2v'_1 = m_2v_1 - m_1v_1 - 2m_2v_2$$

$$\Rightarrow -[m_1v'_1 + m_2v'_1] = m_2v_1 - m_1v_1 - 2m_2v_2$$

$$\Rightarrow m_1v'_1 + m_2v'_1 = m_1v_1 - m_2v_1 + 2m_2v_2$$

$$\Rightarrow (m_1 + m_2)v' = (m_1 - m_2)v_1 + 2m_2v_2$$

$$\Rightarrow v'_1 = \left[\frac{m_1 - m_2}{m_1 + m_2} \right] v_1 + \left[\frac{2m_2}{m_1 + m_2} \right] v_2 \quad (9)$$

Equation (9) represents the velocity of " m_1 " after the collision.

Now from equation (3), we have,

$$v'_1 = (v'_2 + v_2 - v_1) \quad (10)$$

Putting equation (10) in equation (1), we get,

$$(v_1 - v'_2 - v_2 + v_1)m_1 = (v'_2 - v_2)m_2$$

$$\Rightarrow (2v_1 - v'_2 - v_2)m_1 = m_2v'_2 - m_2v_2$$

$$\Rightarrow 2m_1v_1 - m_1v'_2 - m_1v_2 = m_2v'_2 - m_2v_2$$

$$\Rightarrow -m_1v'_2 - m_2v'_2 = -m_2v_2 + m_1v_2 - 2m_1v_1$$

$$\Rightarrow -[m_1 + m_2]v'_2 = -m_2v_2 + m_1v_2 - 2m_1v_1$$

$$\Rightarrow (m_1 + m_2)v'_2 = m_2v_2 - m_1v_2 + 2m_1v_1$$

$$\Rightarrow v'_2 = \left[\frac{m_2 - m_1}{m_1 + m_2} \right] v_2 + \left[\frac{2m_1}{m_1 + m_2} \right] v_1 \quad (11)$$

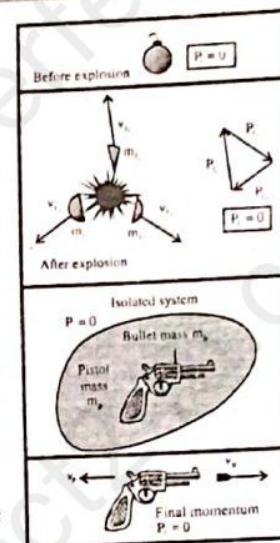
Equation (11) represents the velocity of " m_2 " after the collision.

Q21: Discuss the momentum changes produced by explosive forces.

ETEA 2007

Ans. There are many cases where momentum changes are produced by explosive forces within an isolated system. It can be explain with the help of following examples:

Example 1: When a shell explodes in mid-air, its fragments fly apart in different directions. The various forces which occur in the explosion have zero resultant, because they cancel the effect of each other. So the total momentum of the fragments is equal to the initial momentum of the shell. Thus the law of conservation of momentum is satisfied in this case.



Example 2: The firing of a gun is another example of explosive forces and conservation of momentum. In the gun fire, the gun powder explodes due to internal forces. Before firing, the gun and the bullet are at rest. So the initial momentum of the system is zero. When the gun is fired, the bullet moves with a forward momentum and the gun recoils backward. In this way, the total momentum is conserved.

Q22: (a) What do you mean by motion in two dimensions? (b) Define projectile. (c) Derive equation of motion for projectile.

Ans. (a) Motion in two Dimension:

The motion in which a body moves along two axis at the same time is known as motion in two dimension.

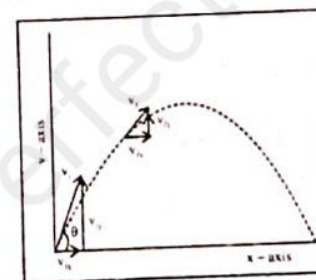
(b) Projectile:

A body moving in two dimensions making an angle ' θ ' with horizontal and having no mechanism of propulsion in it, is known as projectile.

The motion of projectile is known as projectile motion. The angle made by projectile with horizontal is known as angle of projection.

Equation of Motion for Projectile:

Consider a projectile is projected with initial velocity " v_i " making an angle ' θ ' with horizontal, as shown in the figure. Here we consider only gravitational force. Also in projectile motion, the x-component



of velocity remains constant. So we put $a_x = 0$ and $a_y = -g$.

Now the equation of motion for projectile along x-axis [horizontal direction] is given by,

$$v_x = v_{ix} + a_x t \Rightarrow v_{ix} = v_{ix} + a_x t \quad [a_x = 0]$$

$$\Rightarrow v_{ix} = v_{ix} + 0$$

$$\Rightarrow \boxed{v_{ix} = v_{ix}} \quad (1)$$

$$\text{And } S = v_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x = v_{ix} t + \frac{1}{2} a_x t^2 = v_{ix} t + 0$$

$$\Rightarrow \boxed{x = v_{ix} t} \quad (2)$$

Now the equation of motion for projectile along y-axis [vertical direction] is given by,

$$v_y = v_{iy} + a_y t \Rightarrow \boxed{v_{iy} = v_{iy} - gt} \quad (3)$$

$$\text{And } S = v_y t + \frac{1}{2} a_y t^2 \Rightarrow \boxed{y = v_{iy} t - \frac{1}{2} gt^2} \quad (4)$$

Equations (1), (2), (3), (4) represents the equation of motion for projectile.

Q23: Find a relation for magnitude of velocity of projectile and direction of projectile?

Ans: (a) Magnitude of Velocity of Projectile:

The final velocity of projectile is given by,

$$\vec{v}_r = \vec{v}_{ix} + \vec{v}_{iy}$$

$$\Rightarrow \vec{v}_r = (v_{ix} \hat{i} + v_{iy} \hat{j}) \quad (1)$$

From equations of projectile motion, we have,

$$v_{ix} = v_{ix} \quad (2)$$

$$\text{And } v_{iy} = v_{iy} - gt \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$\vec{v}_r = v_{ix} \hat{i} + (v_{iy} - gt) \hat{j} \quad (4)$$

Now in magnitude form, equation (4) can be written as,

$$\Rightarrow v_r = \sqrt{v_{ix}^2 + (v_{iy} - gt)^2} \quad [(a-b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow v_r = \sqrt{v_{ix}^2 + v_{iy}^2 + g^2 t^2 - 2v_{iy} gt} \quad [v_{iy} = v_i \sin \theta, v_{ix} = v_i \cos \theta]$$

$$\Rightarrow v_r = \sqrt{v_i^2 \cos^2 \theta + v_i^2 \sin^2 \theta + g^2 t^2 - 2(v_i \sin \theta) gt}$$

$$\Rightarrow v_r = \sqrt{v_i^2 [\cos^2 \theta + \sin^2 \theta] + g^2 t^2 - 2v_i gt \sin \theta}$$

$$\Rightarrow v_r = \sqrt{v_i^2 + g^2 t^2 - 2v_i gt \sin \theta} \quad [\cos^2 \theta + \sin^2 \theta = 1]$$

$$\Rightarrow \boxed{v_r = \sqrt{v_i^2 + g^2 t^2 - 2v_i gt \sin \theta}} \quad (5)$$

Equation (5) represents the magnitude of velocity of projectile.

(b) Direction of Projectile:

We know that,

$$\tan \theta = \frac{v_{ry}}{v_{rx}}$$

$$[v_{ry} = v_{iy} - gt, v_{rx} = v_{ix}]$$

$$\Rightarrow \tan \theta = \frac{(v_{iy} - gt)}{v_{ix}}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{v_{iy} - gt}{v_{ix}} \right) \quad (6)$$

Equation (6) represents the direction of projectile.

Q24: Derive an equation for the position vector of projectile.

Ans: In two dimensions, the position vector is given by,

$$\vec{r} = x \hat{i} + y \hat{j} \quad (1)$$

Now from equations of projectile, we have,

$$x = v_{ix} t \quad (2)$$

$$\text{And } y = (v_{iy} t - \frac{1}{2} gt^2) \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$\vec{r} = v_{ix} t \hat{i} + \left(v_{iy} t - \frac{1}{2} gt^2 \right) \hat{j} = v_{ix} t \hat{i} + v_{iy} t \hat{j} - \frac{1}{2} gt^2 \hat{j}$$

$$\Rightarrow \vec{r} = (v_{ix} \hat{i} + v_{iy} \hat{j}) t - \frac{1}{2} (g \hat{j}) t^2 \quad [g \hat{j} = \vec{g}]$$

$$[v_{ix} \hat{i} + v_{iy} \hat{j} = \vec{v}_i]$$

$$\Rightarrow \boxed{\vec{r} = \vec{v}_i t + \frac{1}{2} (-\vec{g}) t^2} \quad (4)$$

Equation (4) represents the equation for the position vector of projectile.

Q25: Define maximum height of projectile. Derive a mathematical expression for maximum height of projectile.

ETEA 2006-2008, BISE DI Khan 2019

Ans: Definition: The maximum vertical distance of projectile from horizontal plane is known as maximum height.

Mathematical Expression:

Consider a projectile is thrown in xy -plane by making an angle ' θ ' with horizontal [x -axis], as shown in the figure. Now we know that,

$$2aS = v_f^2 - v_i^2 \quad (1)$$

Here we have,

$$a = -g, v_f = v_{fy} = 0$$

$$S = H, v = v_{iy} = v_i \sin \theta$$

So equation (1) becomes,

$$-2gH = 0^2 - v_i^2 \sin^2 \theta$$

$$\Rightarrow -2gH = -v_i^2 \sin^2 \theta$$

$$\Rightarrow H = \frac{v_i^2 \sin^2 \theta}{2g} \quad (2)$$

Equation (2) represents the maximum height of projectile.

Q26: Define total time of flight. Derive a mathematical expression for it.

Ans. Definition: The time taken by projectile from point of projection to the striking point is known as total time of flight.

Mathematical Expression:

From equation of projectile, we have,

$$y = v_{iy}t - \frac{1}{2}gt^2 \quad (1)$$

Here we put, $y = 0, v_{iy} = v_i \sin \theta, t = T$

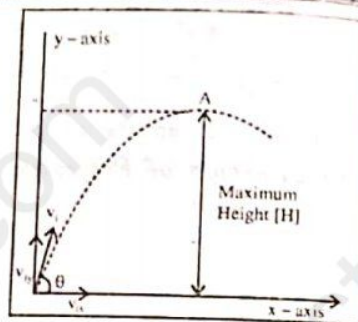
So equation (1) becomes

$$0 = v_i \sin \theta T - \frac{1}{2}gT^2$$

$$\Rightarrow \frac{1}{2}gT^2 = v_i \sin \theta T \Rightarrow gT = 2v_i \sin \theta$$

$$\Rightarrow T = \frac{2v_i \sin \theta}{g} \quad (2)$$

Equation (2) represents the mathematical expression for time of flight of projectile.



As height is parallel to y -axis, so we consider vertical components of velocity.

At highest point, $v_{fy} = 0$

Q27: Define the time of summit. Show that the time of summit is equal to the half of total time of flight. (OR) Show that, $t = T/2$.

Ans. Definition: The time taken by the projectile from point of projection to its maximum height is known as time of summit.

Mathematical Expression:

From equation of projectile, we have,

$$v_{fy} = v_{iy} - gt \quad (1)$$

At highest point, $v_{fy} = 0$, also $v_{iy} = v_i \sin \theta$

So equation (1) becomes,

$$0 = v_i \sin \theta - gt \Rightarrow -gt = -v_i \sin \theta \Rightarrow gt = v_i \sin \theta$$

$$\Rightarrow t = \frac{v_i \sin \theta}{g} \quad (2)$$

Equation (2) represents the time of summit.

To Show that, $t = T/2$:

The total time of flight is given by,

$$T = \frac{2v_i \sin \theta}{g} \Rightarrow T = 2 \left(\frac{v_i \sin \theta}{g} \right) \quad (3)$$

Putting equation (2) in equation (3), we get,

$$T = 2t$$

$$\Rightarrow t = \frac{T}{2} \quad (4)$$

Equation (4) shows that the time of summit is equal to half of total time of flight.

Q28: Define horizontal range. Find an expression for horizontal range of projectile.

ETEA 2009, BISE Malakand 2019, BISE Peshawar 2019

Ans. Definition: The horizontal distance between point of projection and striking point is known as horizontal range.

Mathematical Expression:

From equation of projectile, we have,

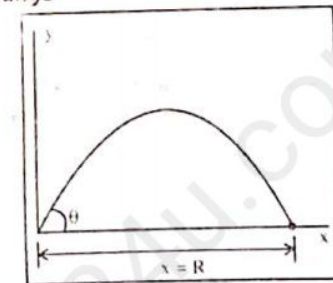
$$x = v_{ix}t \quad (1)$$

Here we put,

$$x = R, v_{ix} = v_i \cos \theta, t = T$$

So equation (1) becomes;

$$R = v_i \cos \theta T \quad (2)$$



We know that the total time of flight is given by,

$$T = \frac{2v \sin \theta}{g} \quad (3)$$

Putting equation (3) in equation (2), we get,

$$R = \frac{v \cos \theta \times 2v \sin \theta}{g}$$

$$\Rightarrow R = \frac{v^2}{g} [2 \sin \theta \cos \theta] \quad (4)$$

We know that,

$$2 \sin \theta \cos \theta = \sin 2\theta \quad (5)$$

Putting equation (5) in equation (4), we get,

$$R = \frac{v^2 \sin 2\theta}{g} \quad (6)$$

Equation (6) represents the horizontal range of projectile.

Maximum Horizontal Range:

The range will be maximum, if $\sin 2\theta = 1$ and it is possible only, when $2\theta = 90^\circ$

$$\Rightarrow \theta = \frac{90^\circ}{2} \Rightarrow \theta = 45^\circ \quad [\sin 90^\circ = 1]$$

Thus the range will be maximum, if $\theta = 45^\circ$

So equation (6) becomes

$$R_{\max} = \frac{v^2 \sin 2 \times 45^\circ}{g} \Rightarrow R_{\max} = \frac{v^2 \sin 90^\circ}{g} = \frac{v^2 \times 1}{g}$$

$$\Rightarrow R_{\max} = \frac{v^2}{g} \quad (7)$$

Equation (7) represents the maximum horizontal range.

Q29: Show that for the same initial speed and same range of projectile, there are two mutually complementary angles of projection.

ETEA 2007

Ans: We know that the range of projectile is given by,

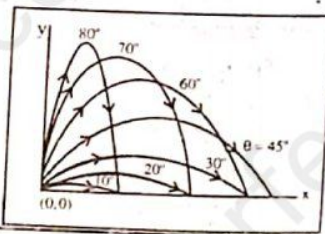
$$R = \frac{v^2 \sin 2\theta}{g} \quad (1)$$

Now we can write that,

$$\sin 2\theta = \sin(180^\circ - 2\theta)$$

$$\Rightarrow \sin 2\theta = \sin 2(90^\circ - \theta) \quad (2)$$

Equation (2) shows that, for same range, there will be two values of ' θ ' for example,



- (a) If $\theta = 30^\circ$, then the other possible value of ' θ ' for same projection will be, $90^\circ - 30^\circ = 60^\circ$
- (b) If $\theta = 10^\circ$, then the other possible value of ' θ ' for same projection will be, $90^\circ - 10^\circ = 80^\circ$
- (c) If $\theta = 20^\circ$, then the other possible value of ' θ ' for same projection will be, $90^\circ - 20^\circ = 70^\circ$

Thus for same range, the projectile possesses two values of ' θ ' which are mutually complementary.



Point to Ponder:

How rockets accelerate in space?

As there is no air in space to push against such that as a reaction rocket is pushed forward. The answer lies in conservation of momentum principle. The rocket ejects gases from its tail at a high velocity, as a result rocket's mass decreases. Thus giving acceleration to the rocket called thrust. Any space vehicle is maneuvered in empty space by firing its rockets in the direction opposite to that in which it needs to accelerate.

Solution of Examples & Assignments

EXAMPLE 3.1:

The velocity time graph shows the motion of bicyclist in a straight line. (a) From the slope of the graph calculate the acceleration of the bicyclist between segment A and B, B and C, C and D and D and E. (b) Calculate the average acceleration of the bicyclist. Also (c) Plot the acceleration time graph for this motion.

Solution:

- (a) (i) The acceleration between points "A" and "B" is given by,

$$a_{AB} = \frac{v_B - v_A}{t_B - t_A} = \left(\frac{10 - 0}{4 - 0} \right) \text{ m / sec}^2$$

$$\Rightarrow a_{AB} = 2.5 \text{ m / sec}^2$$

(ii) The acceleration between points "B" and "C" is given by,

$$a_{BC} = \frac{v_C - v_B}{t_C - t_B} = \frac{(15 - 10)}{(8 - 4)} \text{ m / sec}^2$$

$$\Rightarrow a_{BC} = 1.5 \text{ m / sec}^2$$

(iii) The acceleration between point "C" and "D" is given by,

$$a_{CD} = \frac{v_D - v_C}{t_D - t_C} = \frac{(0 - 15)}{(12 - 8)} \text{ m / sec}^2$$

$$\Rightarrow a_{CD} = -3.75 \text{ m / sec}^2$$

(iv) The acceleration between points "D" and "E" is given by,

$$a_{DE} = \frac{v_E - v_D}{t_E - t_D} = \frac{(-5 - 0)}{(16 - 12)} \text{ m / sec}^2$$

$$\Rightarrow a_{DE} = -1.25 \text{ m / sec}^2$$

(b) The average acceleration of the bicyclist for whole journey is equal to the acceleration between initial point "A" and final point "E" i.e.

$$a_{AE} = \frac{v_E - v_A}{t_E - t_A} = \frac{(-5 - 0)}{(16 - 0)} \text{ m / sec}^2$$

$$= \frac{-5}{16} \text{ m / sec}^2$$

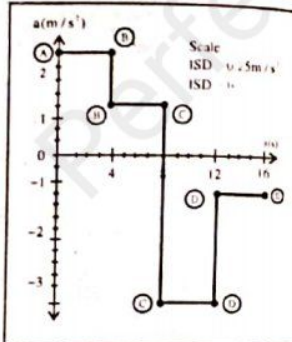
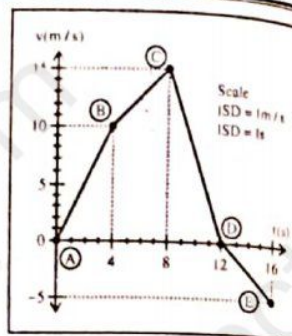
$$\Rightarrow a_{AE} = -0.3125 \text{ m / sec}^2$$

(c) First we select a suitable scale.

Let 1 small division on vertical axis = 0.25 m / sec^2

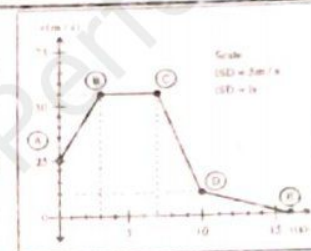
And 1 small division on horizontal axis = 1 sec

Now we put dots for $a = 2.5 \text{ m / sec}^2$, 1.25 m / sec^2 , -3.75 m / sec^2 , -1.25 m / sec^2 , $-0.3125 \text{ m / sec}^2$ and $t = 0 \text{ sec}, 4 \text{ sec}, 8 \text{ sec}, 12 \text{ sec}, 16 \text{ sec}$ respectively as shown in the figure. Finally we join the point and get a graph as shown in the figure.



ASSIGNMENT 3.1

The velocity time graph shows the motion of a car in a straight line. By reading the scale carefully, calculate (a) the acceleration of the car between segment A and B, B and C, C and D and D and E, from the slope of the graph. Also (b) calculate the car's average acceleration for the complete journey.



Solution:

(a) (i) Acceleration of the car between points "A" and "B" is given by,

$$a_{AB} = \frac{v_B - v_A}{t_B - t_A} = \frac{(55 - 25)}{(3 - 0)} \text{ m / sec}^2$$

$$\Rightarrow a_{AB} = 10 \text{ m / sec}^2$$

(ii) The acceleration of the car between points "B" and "C" is given by,

$$a_{BC} = \frac{v_C - v_B}{t_C - t_B} = \frac{(55 - 55)}{(7 - 3)} \text{ m / sec}^2$$

$$\Rightarrow a_{BC} = 0$$

(iii) The acceleration of the car between points "C" and "D" is given by,

$$a_{CD} = \frac{v_D - v_C}{t_D - t_C} = \frac{(10 - 55)}{(10 - 7)} \text{ m / sec}^2$$

$$\Rightarrow a_{CD} = -15 \text{ m / sec}^2$$

(iv) The acceleration of the car between points "D" and "E" is given by,

$$a_{DE} = \frac{v_E - v_D}{t_E - t_D} = \frac{(0 - 10)}{(15 - 10)} \text{ m / sec}^2 = -2 \text{ m / sec}^2$$

$$\Rightarrow a_{DE} = -2 \text{ m / sec}^2$$

(b) The acceleration of the car between points "A" and "E" gives us the average acceleration for the whole journey. So we have,

$$a_{AE} = \frac{v_E - v_A}{t_E - t_A} = \frac{(0 - 25)}{(15 - 0)} \text{ m / sec}^2$$

$$\Rightarrow a_{AE} = -1.666 \text{ m / sec}^2 \text{ or } a_{AE} = -1.67 \text{ m / sec}^2$$

EXAMPLE 3.2

A (joint fighter-17) JF Thunder 17 aircraft takes off at 70.0 m/s (252 km/h). After accelerating uniformly at 3.90 m/s^2 from rest

that lasts 6.5s during the initial phase or takeoff. The afterburner engines are then turned up to full power for an acceleration of 7.1m/s^2 . Calculate the length of runway needed and the total time of takeoff.

Solution:

Given Data:

Acceleration " a_1 " for phase (1) = 3.9m/s^2

Acceleration " a_2 " for phase (2) = 7.1m/s^2

Time " t_1 " for phase (1) = $t_1 = 6.5\text{sec}$

Final velocity " v_f " for phase (2) = 70.0m/s

Required:

(a) Length of runway " s " = ?

(b) Time of takeoff " t " = ?

Calculation: For finding the length of runway, we will calculate the distance covered during 1st phase of takeoff and then during 2nd phase of takeoff.

(a) For 1st phase of takeoff, we have,

$$S_1 = v_i t_1 + \frac{1}{2} a_1 t_1^2$$

$$\Rightarrow S_1 = \left[0 \times 6.5 + \frac{1}{2} \times 3.9 \times (6.5)^2 \right] \text{m} \Rightarrow S_1 = 82.4\text{m}$$

The final velocity at 1st phase is given by,

$$v_f = v_i + a_1 t_1 = [0 + 3.9 \times 6.5] \text{m/s} \Rightarrow v_f = 25.35\text{m/s}$$

Now for finding the distance covered during 2nd phase of takeoff is given by,

$$2a_2 S_2 = v_f^2 - v_i^2 \quad (1)$$

Here the final velocity of 1st phase represents the initial velocity of 2nd phase, so we have,

$$v_i = v_f = 25.35\text{m/s}$$

So equation (1) becomes,

$$2 \times 7.1 \times S_2 = (70)^2 - (25.35)^2$$

$$\Rightarrow 14.2 S_2 = 4900 - 642.6 = 4257.4$$

$$\Rightarrow S_2 = \left(\frac{4257.4}{14.2} \right) \text{m} \Rightarrow S_2 = 299.8\text{m}$$

Now the length of runway will be equal to the distance covered during 1st phase and 2nd phase. So we have,

$$S = S_1 + S_2$$

$$\Rightarrow S = (82.4 + 299.8)\text{m} \Rightarrow S = 382.2\text{m}$$

(b) Time for 1st phase = $t_1 = 6.5\text{sec}$

Now for finding the total time of takeoff, we have,

$$t = t_1 + t_2 \quad (2)$$

First we calculate the time for 2nd phase, for which we have,

$$v_f = v_i + a_2 t_2 \Rightarrow 70 = 25.35 + 7.1 \times t_2 \Rightarrow 7.1 t_2 = 70 - 25.35$$

$$\Rightarrow t_2 = \left(\frac{70 - 25.35}{7.1} \right) \text{sec} \Rightarrow t_2 = 6.3\text{sec}$$

Putting the values in equation (2), we get,

$$t = 6.5\text{sec} + 6.3\text{sec} \Rightarrow t = 12.8\text{sec}$$

ASSIGNMENT 3.2:

A proton moving with a speed of $1.0 \times 10^7\text{ms}^{-1}$ passes through a 0.020cm thick sheet of paper and emerges with a speed of $2.0 \times 10^6\text{ms}^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Solution: Initial speed of proton = $v_i = 1.0 \times 10^7\text{m/s}$

Final speed of proton = $v_f = 2.0 \times 10^6\text{m/s}$

Distance covered by proton = $S = 0.020\text{cm} = 0.0002\text{m}$

Required: (a) Acceleration = $a = ?$

(b) Time taken = $t = ?$

Calculation: We know that,

$$2aS = v_f^2 - v_i^2$$

$$\Rightarrow a = \frac{v_f^2 - v_i^2}{2S} \quad (1)$$

Putting the values in equation (1), we get,

$$a = \left[\frac{(2 \times 10^6)^2 - (1 \times 10^7)^2}{2 \times 0.0002} \right] \text{m/s}^2 = \frac{(4 \times 10^{12} - 1 \times 10^{14})}{4 \times 10^{-4}} \text{m/s}^2$$

$$\Rightarrow a = \frac{10^{12} [4 - 1 \times 10^2]}{4 \times 10^{-4}} \text{m/s}^2 = \frac{10^{16}}{4} [4 - 100] \text{m/s}^2$$

$$\Rightarrow a = \frac{-10^6 \times 96}{4} \text{m/s}^2 = -24 \times 10^{16} \text{m/s}^2$$

$$\Rightarrow a = -24 \times 10^{16} \text{m/s}^2 \text{ or } a = -2.4 \times 10^{17} \text{m/s}^2$$

(b) We know that,

$$v_f = v_i + at$$

$$\Rightarrow t = \frac{v_f - v_i}{a} = \left(\frac{2 \times 10^6 - 1 \times 10^7}{-2.4 \times 10^{17}} \right) \text{sec}$$

$$\Rightarrow t = \frac{10^6(2 - 10)}{-2.4 \times 10^{17}} \text{sec} = \frac{10^6 \times (-8)}{-2.4 \times 10^{17}} \text{sec}$$

$$\Rightarrow t = 3.33 \times 10^{-11} \text{sec}$$

EXAMPLE 3.3:

Hassan and Umar are standing face to face on ice wearing ice skates. If Hassan apply a force of 10N [E] on Umar [assume no other opposing force exists], what are their respective accelerations? If mass of Umar is 80kg and Hassan is 50kg.

Solution:

Given Data:

Hassan's mass = $m_H = 50\text{kg}$

Umar's mass = $m_U = 80\text{kg}$

Force = $F = 10\text{N [E]}$

Required:

(a) Hassan's acceleration = $a_H = ?$

(b) Umar's acceleration = $a_U = ?$

Calculation: (a) We know that,

$$F = ma \quad (1)$$

As Hassan apply a force of 10N [East] on Umar, so due to this force Umar will be accelerated towards East, so equation (1)

$$F = m_U a_U \Rightarrow a_U = \frac{F}{m_U} \quad (2)$$

Putting the values in equation (2), we get,

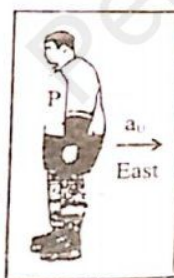
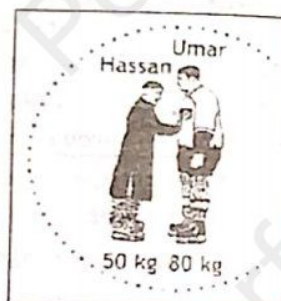
$$a_U = \left(\frac{10}{80} \right) \text{m / sec}^2$$

$$\Rightarrow a_U = 0.125 \text{m / sec}^2 \text{ towards East}$$

✓ (b) Now according to Newton's 3rd law, Umar will exert the same force on Hassan in opposite direction, i.e. towards West, so we put

$$F = m_H a_H \Rightarrow a_H = \frac{F}{m_H} \quad (3)$$

Putting the values in equation (3), we get,



$$a_H = \left(\frac{-10}{50} \right) \text{m / sec}^2$$

$$\Rightarrow a_H = -0.2 \text{m / sec}^2 \text{ towards West}$$

ASSIGNMENT 3.3:

Suppose that the mass of the spacecraft " m_s " is 11000kg and that the mass of the astronaut " m_a " is 92kg. In addition, assume that the astronaut pushes with a force $F = +36\text{N}$ (along x-axis) on the spacecraft. Find the acceleration of the spacecraft and the astronaut.

Solution:

Mass of spacecraft = $m_s = 11000\text{kg}$

Mass of astronaut = $m_a = 92\text{kg}$

Force exerted by astronaut on spacecraft = $F_a = 36\text{N}$

Force exerted by spacecraft on astronaut = $F_s = -36\text{N}$

Required: (a) Acceleration of spacecraft = $a_s = ?$

(b) Acceleration of astronaut = $a_a = ?$

Calculation: (a) Let " F_a " represents

the force exerted by the astronaut on spacecraft and " F_s " represents the force exerted by the spacecraft on astronaut. According to Newton's 3rd

law, these forces must be equal and opposite in direction i.e.

$$F_a = -F_s$$

Now according to Newton's 2nd law, we have, $F = ma$

$$\Rightarrow F_a = m_s a_s \Rightarrow a_s = \frac{F_a}{m_s} \quad (1)$$

Putting the values in equation (1), we get,

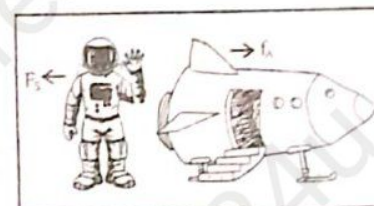
$$a_s = \left(\frac{36}{11000} \right) \text{m / sec}^2 \Rightarrow a_s = 0.0033 \text{m / sec}^2$$

Similarly, $F_s = m_a a_a$

$$\Rightarrow a_a = \frac{F_s}{m_a} \quad (2)$$

Putting the values in equation (2), we get,

$$a_a = \left(\frac{-36}{92} \right) \text{m / sec}^2 \Rightarrow a_a = -0.39 \text{m / sec}^2$$



EXAMPLE 3.4:

A cricket ball of mass 0.163kg has an initial velocity of -36m/s as it approaches a bat. The batsman hits the ball hard and the ball moves away from the bat with velocity of $+47\text{m/s}$. (a) Determine the impulse applied to the ball by the bat. (b) Assuming that the time of contact is 1.6ms , find the average force exerted on the ball by the bat.

Solution:**Given Data:**Mass of ball = $m = 0.163\text{kg}$ Initial velocity of ball = $v_i = -36.2\text{m/s}$ Final velocity = $v_f = +47.0\text{m/s}$ Time of contact = $\Delta t = 1.6\text{ms} = 1.6 \times 10^{-3}\text{s}$ **Required:**(a) Impulse applied = $J = ?$ (b) Average force exerted = $F_{\text{ave}} = ?$ **Calculation:** (a) We know that,

Impulse = change in momentum

$$\Rightarrow J = \Delta P = mv_f - mv_i \Rightarrow J = mv_f - mv_i \quad (1)$$

Putting the values in equation (1), we get,

$$J = [0.163 \times 47 - 0.163 \times (-36.2)] \text{N}\cdot\text{sec}$$

$$= 7.661 + 5.9006 \Rightarrow J = 13.6\text{N}\cdot\text{sec}$$

(b) We also know that, $J = F_{\text{ave}} \times \Delta t$

$$\Rightarrow F_{\text{ave}} = \frac{J}{\Delta t} = \left(\frac{13.6}{1.6 \times 10^{-3}} \right) \text{N} = 8.5 \times 10^3 \text{N} \Rightarrow F_{\text{ave}} = 8500\text{N}$$

**ASSIGNMENT 3.4:**

A girl of mass 48.0kg is rescued from a building fire by leaping into a firefighters' net. The window from which she leapt was 12.0m above the net. She lands in the net so that she is brought to a complete stop in 0.45s . During this interval (a) What is her change in momentum? (b) What is the impulse on the net due to the girl? (c) What is the average force on the net due to the girl?

Solution:Mass of girl = $m = 48\text{kg}$ Initial velocity of girl = $v_i = 0$ Height of net = $h = 12\text{m}$ Time = $\Delta t = 0.45\text{sec}$ **Required:** (1) Change in momentum = $\Delta P = ?$ (2) Impulse on the net due to girl = $J = ?$ (3) Average force on the net due to girl = $F_{\text{ave}} = ?$ **Part (1):** The change in momentum is given by,

$$\Delta P = P_f - P_i \Rightarrow \Delta P = mv_f - mv_i \quad (1)$$

Case (1): First we calculate the final velocity of the girl when she touches the net. We have,

$$2as = v_f^2 - v_i^2 \Rightarrow 2gh = v_f^2 - v_i^2$$

$$\Rightarrow v_f^2 = 2gh + v_i^2 \Rightarrow v_f^2 = 2 \times 9.8 \times 12 + 0^2 = 235.2 + 0$$

$$\Rightarrow v_f^2 = 235.2 \text{m}^2/\text{sec}^2 \Rightarrow v_f = \sqrt{235.2 \text{m}^2/\text{sec}^2} \Rightarrow v_f = 15.33 \text{m/sec}$$

Case (2): Now when the girl touches the net, then she takes 0.45sec in coming to rest, so for 2nd case, the velocity $v_f = 15.33\text{m/sec}$ becomes initial velocity and the final velocity of the girl after coming into rest becomes zero. So we have,

$$v_i = 15.33 \text{m/sec}, v_f = 0$$

Putting the values in equation (1), we get,

$$\Delta P = [48 \times 0 - 48 \times 15.33] \text{kg}\cdot\text{m/sec} \Rightarrow \Delta P = -735.84 \text{kg}\cdot\text{m/sec}$$

The negative sign shows that the momentum is up.

Part (2): We know that, impulse = change in momentum

$$\Rightarrow J = \Delta P = 735.84 \text{kg}\cdot\text{m/sec}$$

$$\Rightarrow J = 735.84 \text{kg}\cdot\text{m/sec} \text{ (down)}$$

Part (3): We know that,

$$J = F_{\text{ave}} \cdot \Delta t \Rightarrow F_{\text{ave}} = 1635.2\text{N} = 1.635 \times 10^3 \text{N}$$

$$\Rightarrow F_{\text{ave}} = 1.6 \times 10^3 \text{N}$$

EXAMPLE 3.5:

In a nuclear reactor a neutron of mass $1u$ ($1u = 1.66 \times 10^{-27}\text{kg}$) moving with a velocity of $2,000\text{km/s}$ to the right and a heavy water molecule mass $20.0u$ moving with a velocity of 0.40km/s to the left collide head-on. What are the velocities of the neutron and water molecule after the collision?

Solution:**Given Data:**Mass of neutron = $m_1 = 1u = 1.66 \times 10^{-27} \text{ kg}$

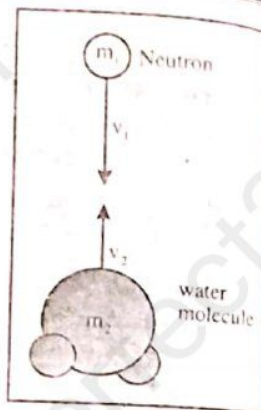
Mass of water molecule

 $= m_2 = 20u = 20 \times 1.66 \times 10^{-27} \text{ kg} = 33.2 \times 10^{-27} \text{ kg}$

Velocity of neutron before collision

 $= v_1 = 2000 \text{ km/s} = 2000 \times 10^3 \text{ m/sec}$ $= 2 \times 10^6 \text{ m/sec}$

Velocity of water molecule before collision

 $= v_2 = 0.40 \text{ km/s} = 0.40 \times 10^3 \text{ m/sec}$ **Required:**(a) Velocity of neutron after collision = $v_1' = ?$ (b) Velocity of water molecule after collision = $v_2' = ?$ **Calculation:** (a) We know that,

$$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \frac{2m_2 v_2}{(m_1 + m_2)} \quad (1)$$

Putting the values in equation (1), we get,

$$v_1' = \left[\frac{1.66 \times 10^{-27} - 33.2 \times 10^{-27}}{1.66 \times 10^{-27} + 33.2 \times 10^{-27}} \right] 2 \times 10^6 + \frac{2 \times 33.2 \times 10^{-27} \times 0.40 \times 10^3}{(1.66 \times 10^{-27} + 33.2 \times 10^{-27})} \text{ m/sec}$$

$$\Rightarrow v_1' = \left[\frac{(1.66 - 33.2) \times 10^{-27} \times 2 \times 10^6}{(1.66 + 33.2) \times 10^{-27}} + \frac{2 \times 33.2 \times 10^{-27} \times 0.4 \times 10^3}{(1.66 + 33.2) \times 10^{-27}} \right] \text{ m/sec}$$

$$\Rightarrow v_1' = \frac{-63.08 \times 10^6}{34.86} + \frac{26.56 \times 10^3}{34.86}$$

$$\Rightarrow v_1' = [-1.8 \times 10^6 + 0.76 \times 10^3] \text{ m/sec}$$

$$\Rightarrow v_1' = [-1.8 \times 10^6 + 0.00076 \times 10^6] \text{ m/sec} = 10^6 [-1.8 + 0.00076] \text{ m/sec}$$

$$\Rightarrow v_1' = -1.80076 \times 10^6 \text{ m/sec} = -1800.76 \times 10^3 \text{ m/sec}$$

$$\Rightarrow v_1' = -1800.76 \text{ km/sec}$$

 $10^3 = \text{kilo}$

(b) We also know that,

$$v_2' = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2 + \frac{2m_1 v_1}{(m_1 + m_2)} \quad (2)$$

Putting the values in equation (2), we get,

$$\Rightarrow v_2' = \left[\frac{33.2 \times 10^{-27} - 1.66 \times 10^{-27}}{1.66 \times 10^{-27} + 33.2 \times 10^{-27}} \right] 0.40 \times 10^3 + \left[\frac{2 \times 1.66 \times 10^{-27} \times 2 \times 10^6}{(1.66 \times 10^{-27} + 33.2 \times 10^{-27})} \right]$$

$$\Rightarrow v_2' = \left[\frac{31.54 \times 10^{-27} \times 0.40 \times 10^3}{34.86 \times 10^{-27}} + \left(\frac{6.64 \times 10^{-27} \times 10^6}{34.86 \times 10^{-27}} \right) \right] \text{ m/sec}$$

$$\Rightarrow v_2' = \left(\frac{12.616 \times 10^3}{34.86} + 0.190476 \times 10^6 \right) \text{ m/sec}$$

$$\Rightarrow v_2' = (0.36190476 \times 10^3 + 0.190476 \times 10^6) \text{ m/sec}$$

$$\Rightarrow v_2' = (0.00036190476 + 0.190476 \times 10^6) \text{ m/sec}$$

$$\Rightarrow v_2' = (0.00036190476 + 0.190476) \times 10^6 \text{ m/sec}$$

$$\Rightarrow v_2' = 0.19083 \times 10^6 \text{ m/sec} = 190.83 \times 10^3 \text{ m/sec} = 190.83 \text{ km/sec}$$

$$\Rightarrow v_2' = 190.83 \text{ km/sec}$$

ASSIGNMENT 3.5:

On a highway a car of mass 1500kg is stopped at traffic signal. A pickup of mass 2000kg comes up from behind and hits the stopped car. Assuming the collision is elastic, the pickup stops with collision and push the car ahead onto the highway at 10.0m/s. How fast was the pickup going just before the collision?

Solution: Mass of pickup $= m_1 = 2000 \text{ kg}$ Mass of car = $m_2 = 1500 \text{ kg}$ Initial velocity of car = $v_2 = 0$ Final velocity of car = $v_2' = 10 \text{ m/sec}$ Final velocity of pickup = $v_1' = 0$ **Required:** Initial velocity of pickup = $v_1 = ?$ **Calculation:**

As collision is elastic, so we have,

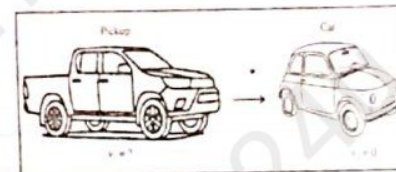
Total momentum before collision = total momentum after collision

$$\Rightarrow m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\Rightarrow (2000 \times v_1) + (1500 \times 0) = (2000 \times 0) + (1500 \times 10)$$

$$\Rightarrow 2000 v_1 + 0 = 0 + 15000$$

$$\Rightarrow v_1 = \left(\frac{15000}{2000} \right) \text{ m/sec} \Rightarrow v_1 = 7.5 \text{ m/sec}$$

**EXAMPLE 3.6:**

A cricket ball is hit and moves initially at an angle of 35° above the horizontal ground with a velocity of 25.0m/s. (a) How high will the ball go? (b) How long will the ball be in the air? (c)

What will be the range for this projectile?

Solution: Given Data:

Angle of projection = $\theta = 35^\circ$

Initial velocity = $v_i = 25.0 \text{ m/sec}$

Acceleration due to gravity = $g = 9.8 \text{ m/s}^2$

Required:

(a) Maximum height = $H = ?$

(b) Time of flight = $T = ?$

(c) Horizontal range = $R = ?$

Calculation: (a) We know that, maximum height for projectile is given by,

$$H = \frac{v_i^2 \sin^2 \theta}{2g} \quad (1)$$

Putting the values in equation (1), we get,

$$H = \left[\frac{25^2 \times [\sin(35^\circ)]^2}{2 \times 9.8} \right] \text{ m} = \left[\frac{625 \times (0.574)^2}{19.6} \right] \text{ m}$$

$$\Rightarrow H = 10.5 \text{ m}$$

(b) The time of flight is given by,

$$T = \frac{2v_i \sin \theta}{g} \quad (2)$$

Putting the values in equation (2), we get,

$$T = \left[\frac{2 \times 25 \times \sin(35^\circ)}{9.8} \right] \text{ sec} = \frac{28.679}{9.8} \text{ sec}$$

$$\Rightarrow T = 2.93 \text{ sec}$$

(c) The horizontal range is given by,

$$R = \frac{v_i^2 \sin 2\theta}{g} \quad (3)$$

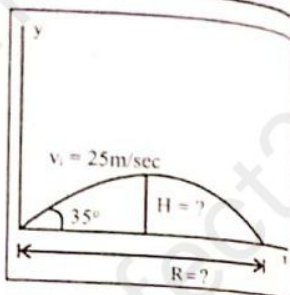
Putting the values in equation (3), we get,

$$R = \frac{25^2 \times \sin(2 \times 35^\circ)}{9.8} = \frac{625 \times 0.94}{9.8} = 59.9 \text{ m}$$

$$\Rightarrow R = 59.9 \text{ m}$$

ASSIGNMENT 3.6:

At Arbab Niaz Cricket Stadium Peshawar a batsman hits the shot at initial velocity of 28 m/s . If the boundary is 72 m from the batsman, will the ball cross the boundary for a six? If the angle with the hori-



zontal is (a) 30° (b) 45° and (c) 70° . (Ignore air resistance)

Solution:

Initial velocity of ball = $v_i = 28 \text{ m/sec}$

Required: (a) Horizontal range = $R = ?$ for $\theta = 30^\circ$

(b) Horizontal range = $R = ?$ for $\theta = 45^\circ$

(c) Horizontal range = $R = ?$ for $\theta = 70^\circ$

Calculation: (a) We know that,

$$R = \frac{v_i^2 \sin 2\theta}{g} \quad (1)$$

Putting $v_i = 28 \text{ m/sec}$, $\theta = 30^\circ$ in equation (1), we get,

$$R = \frac{28^2 \times \sin(2 \times 30^\circ)}{9.8} \text{ m} = \frac{784 \times 0.866}{9.8} \text{ m} = 69.28 \text{ m}$$

$$\Rightarrow R = 69.28 \text{ m}$$

As the range is less than the length of boundary $[72 \text{ m}]$, so for $\theta = 30^\circ$, the ball will not cross the boundary.

(b) For $\theta = 45^\circ$ equation (1) becomes

$$R = \frac{28^2 \times \sin(2 \times 45^\circ)}{9.8} = \left(\frac{784 \times 1}{9.8} \right) \text{ m}$$

$$\Rightarrow R = 80 \text{ m}$$

In this case the range is greater than the length of the boundary $[72 \text{ m}]$, so the ball will cross the boundary.

(c) Put $\theta = 70^\circ$ in equation (1),

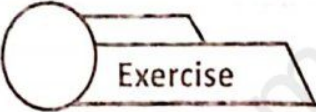
$$R = \frac{28^2 \times \sin(2 \times 70^\circ)}{9.8} = \left(\frac{784 \times 0.643}{9.8} \right) \text{ m}$$

$$\Rightarrow R = 51.44 \text{ m}$$

The range is less than the length of boundary, so in this case, the ball will not cross the boundary.

✪ حضرت ابو بکر صدیق کا قول ہے "مصیبت میں صبر کرنا سخت ہے مگر صبر کے ثواب کو ضائع نہ ہونے دینا سخت تر ہے۔"

✪ حضرت ابو بکر صدیق کا قول ہے "علم کی قوت جب حد سے بڑھ جائے تو مکاری اور سیار دانی پیدا کرتی ہے اور جب ناقص ہو تو حماقت اور ابلہی پیدا کرتی ہے۔"



Exercise

Choose the best possible answer:

1. A ball is thrown vertically upwards at 19.6 m/s. For its complete trip (up and back down to the starting position), its average speed is:

a. 19.6 m/s ✓ b. 9.8 m/s c. 6.5 m/s d. 4.9 m/s

Sol: $v_i = 19.6 \text{ m/sec}$

At highest point, $v_f = 0$, $g = -9.8 \text{ m/sec}^2$

First we calculate the distance covered during upward motion, we have

$$2gS_1 = v_i^2 - v_f^2 \Rightarrow -2 \times 9.8 \times S_1 = 0^2 - 19.6^2$$

$$\Rightarrow -19.6 \times S_1 = -384.16 \Rightarrow S_1 = \frac{384.16}{19.6} \text{ m} = 19.6 \text{ m}$$

$$\Rightarrow S_1 = 19.6 \text{ m}$$

During downward motion, same distance will be covered, so

$$S_2 = 19.6 \text{ m}$$

Now we calculate the time for upward motion, so we have,

$$v_f = v_i + gt_1 \Rightarrow 0 = 19.6 - 9.8t_1$$

$$\Rightarrow -9.8t_1 = -19.6 \Rightarrow t_1 = \frac{19.6}{9.8} \text{ sec} \Rightarrow t_1 = 2 \text{ sec}$$

Similarly for downward motion, the ball will take 2 sec i.e. $t_2 = 2 \text{ sec}$

$$\text{Now average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{S_1 + S_2}{t_1 + t_2}$$

$$\Rightarrow \langle v \rangle = \frac{(19.6 + 19.6)}{2 + 2} \text{ m/sec} \Rightarrow \langle v \rangle = 9.8 \text{ m/sec}$$

2. If you throw a ball downward, then its acceleration immediately after leaving your hand, assuming no air resistance, is:

✓ a. 9.8 m/s² b. more than 9.8 m/s²
c. less than 9.8 m/s² d. speed of throw is required for answer

3. The time rate of change of momentum gives:

✓ a. Force b. Impulse
c. Acceleration d. Power

4. The area between the velocity-time graph and the time axis is numerically equal to:

a. Velocity ✓ b. Displacement
c. Acceleration d. Time

5. If the slope of velocity-time graph gradually decreases, then the body is said to be moving with:

a. Positive acceleration ✓ b. Negative acceleration
c. Uniform velocity d. Zero acceleration

6. If the velocity of an object changes from 3 m/s to 4 m/s during 0.5 sec the acceleration of the object is:

a. 1.5 m/s² ✓ b. 2.0 m/s²
c. 3.5 m/s² d. 6.0 m/s²

Sol: $v_i = 3 \text{ m/sec}$, $v_f = 4 \text{ m/sec}$, $t = 0.5 \text{ sec}$

$$\text{As } a = \frac{v_f - v_i}{t} = \frac{4 - 3}{0.5} = \frac{1}{0.5} = 2 \text{ m/sec}^2$$

$$\Rightarrow a = 2 \text{ m/sec}^2$$

7. A 7.0 kg bowling ball experiences a net force of 5.0 N. What will be its acceleration?

a. 35 m/s² b. 7.0 m/s²
c. 5.0 m/s² ✓ d. 0.71 m/s²

Sol: $m = 7 \text{ kg}$, $F = 5 \text{ N}$, $a = \frac{F}{m} = \frac{5}{7} \Rightarrow a = 0.71 \text{ m/sec}^2$

8. SI unit of impulse is:

a. kg ms⁻² ✓ b. Ns c. Ns⁻¹ d. Nm

Sol: $J = F \cdot \Delta t = \text{N} \cdot \text{sec} = \text{Ns}$

9. A ball with original momentum +4.0 kg·m/s hits a wall and bounces straight back without losing any kinetic energy. The change in momentum of the ball is:

a. +4Ns b. -4Ns c. +8Ns² ✓ d. -8Ns

Sol: Momentum before collision, $P_i = 4 \text{ kg} \cdot \text{m/sec}$

Momentum after collision, $P_f = -4 \text{ kg} \cdot \text{m/sec}$

$$\Delta P = P_f - P_i = -4 \text{ kg} \cdot \text{m/sec} - 4 \text{ kg} \cdot \text{m/sec} = -8 \text{ kg} \cdot \text{m/sec} = -8 \text{ N} \cdot \text{s}$$

10. A body is traveling with a constant acceleration of 10 m/s². If S_1 is the distance traveled in 1st second and S_2 is the distance traveled in 2nd second, which of the following shows a correct relation between S_1 and S_2 ?

a. $S_1 = S_2$ b. $S_1 = 3S_2$ ✓ c. $S_2 = 3S_1$ d. $2S_2 = 3S_1$

Sol: In 1st second, we have,

$v_1 = 0, t_1 = 1, a = 10 \text{ m/sec}^2$, so we have,

$S_1 = v_1 t + \frac{1}{2} a t^2 = 0 + \frac{1}{2} \times 10 \times 1^2 \Rightarrow S_1 = 5 \text{ m}$

In 2nd second, we have,

$v_1 = 10 \text{ m/sec}, t_2 = 1, a = 10 \text{ m/sec}^2$, so we get

$S_2 = v_1 t_2 + \frac{1}{2} a t_2^2 = 10 \times 1 + \frac{1}{2} \times 10 \times 1^2 = 10 + 5 = 15 \text{ m}$

$\Rightarrow S_2 = 15 \text{ m}$

Now $\frac{S_2}{S_1} = \frac{15 \text{ m}}{5 \text{ m}} \Rightarrow \frac{S_2}{S_1} = 3 \Rightarrow S_2 = 3S_1$

At the end of 1st second, we have
 $v_r = v_1 + at$
 $= 0 + 10 \times 1$
 $\Rightarrow v_r = 10 \text{ m/sec}$
 At the beginning of 2nd second,
 v_r acts as initial velocity, so we put
 $v_1 = v_r = 10 \text{ m/sec}$

11. During projectile motion, the horizontal component of velocity:
 a. Changes with time b. Becomes zero
 ✓ c. Remains constant d. Increases with time

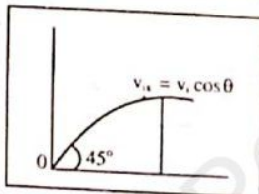
12. A projectile is thrown horizontally from a 490m high cliff with a velocity of 100 ms^{-1} . The time taken by projectile to reach the ground is:

- a. 2.5s b. 5.0s
 c. 7.5s ✓ d. 10s

Sol: $v_{iy} = 100 \text{ m/sec}, g = 9.8 \text{ m/sec}^2, v_{fy} = 0$

We have, $v_{fy} = v_{iy} - gt \Rightarrow 0 = 100 - 9.8t$

$\Rightarrow -9.8t = -100 \Rightarrow t = \frac{100}{9.8} \text{ sec} \Rightarrow t = 10 \text{ sec}$



13. A projectile is launched at 45° to the horizontal with an initial kinetic energy E . Assuming air resistance to be negligible what will be the kinetic energy of the projectile when it reaches its highest point?

- ✓ a. $0.50E$ b. $0.71E$ c. $0.70E$ d. E

Sol: At point of projection,

$K.E = E = \frac{1}{2} m v_i^2$

$\Rightarrow E = \frac{1}{2} m v_i^2$ (1)

At highest point the vertical component of velocity become zero, i.e. $v_{iy} = 0$ while $v_{ix} = v_{ix} = v_i \cos \theta$

So $K.E$ at highest point $= K.E_x = \frac{1}{2} m v_{ix}^2 = \frac{1}{2} m v_i^2 \cos^2 \theta$

$\Rightarrow K.E_x = \left(\frac{1}{2} m v_i^2 \right) \cos^2 \theta \Rightarrow K.E_x = E \cdot \cos^2 \theta$

$\Rightarrow K.E_x = E \times [\cos 45^\circ]^2 = E \times [0.707]^2 = E \times (0.5)$

$\Rightarrow K.E_x = 0.5E$

14. To improve the jumping record the long jumper should jump at an angle of:

- a. 30° ✓ b. 45° c. 60° d. 90°

Sol: As $R = \frac{v_i^2 \sin 2\theta}{g}$ (1)

Equation (1) shows that range of jump will be maximum, if $\sin 2\theta$ gives us maximum value i.e. "1", and "1" value of $\sin 2\theta$ is possible if we put $\theta = 45^\circ$, then

$R_{\max} = \frac{v_i^2 \sin(2 \times 45^\circ)}{g} = \frac{v_i^2 \sin 90^\circ}{g} = \frac{v_i^2 \times 1}{g}$

$\Rightarrow R_{\max} = \frac{v_i^2}{g}$

Thus for improving the range of jump, the angle of projection should be 45° .

15. Range of a projectile on a horizontal plane is same for the following pair of angles:

- a. 15° and 18° ✓ b. 43° and 47°
 c. 30° and 80° d. 52° and 62°

Sol: As $R = \frac{v_i^2}{g} \sin 2(90^\circ - \theta)$ (1)

In equation (1), $90^\circ - \theta$ gives us the two possible angles for same range. Now if $\theta = 43^\circ$, then $90^\circ - 43^\circ = 47^\circ$

So for $(43^\circ, 47^\circ)$ the range will be the same.

✽ حضرت عمر فاروقؓ کا قول ہے "کم گوئی میں حکمت، کم خوری میں صحت اور لوگوں سے کم ملنے جلنے سے عافیت ہے۔"

✽ حضرت عمر فاروقؓ کا قول ہے "جو پیچھے ہٹا وہ پھر آگے نہیں بڑھے گا۔"

✽ حضرت عمر فاروقؓ کا قول ہے "لا لُح سے زیادہ (ذہنی طور پر) عقل کو برباد کرنے والی کوئی چیز نہیں، شراب بھی نہیں۔"

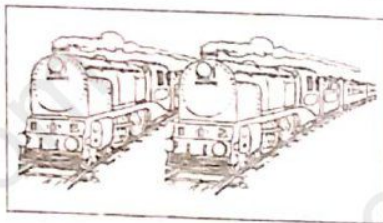
Conceptual Questions

Q1: If you are riding on a train that speeds past another train moving in the same direction on an adjacent track, it appears that the other train is moving backward, why?

Ans: When we are riding on a train that speeds past another train moving in the same direction on an adjacent track, then the other train appears to us to be moving in backward direction. It is because rest and motion are relative terms.

Reasons and Explanation:

When we are sitting in a train which is moving with uniform velocity, then it appears to us that we are at rest because we are not moving relative to the passenger sitting inside the same train. But when our train speeds past another train moving on the adjacent track, it will appear to us that the second train is moving backwards relative to us. Seeing the other train go past your window from front to rear makes it look like the train is going backwards.



Example: Let " $\vec{V}_1$ " is the velocity of 1st train where observer is sitting and " $\vec{V}_2$ " is the velocity of the other train such that " $V_1 > V_2$ ", then the relative velocity between the two trains is given by,

$$\vec{V}_{rel} = \vec{V}_1 - \vec{V}_2 \quad (1)$$

Due to this relative velocity, the 2nd train appears to be moving backward observed by an observer sitting in the 1st train. There is no absolute rest or motion and everybody is in motion with respect to other bodies.

Q2: Can the velocity of a body reverse the direction when the acceleration is constant? If you think so, give an example.

Ans: Yes, the velocity of a body reverse its direction when the acceleration is constant.

Reason and Explanation:

Consider a ball of mass " m " is thrown upward with certain velocity " v_i " as shown in the figure. During the motion the body possesses acceleration due to gravity i.e. $g = 9.8 \text{ m/sec}^2$. The ball reaches a certain height and comes to rest momentarily and then moves downward with same acceleration due to gravity i.e. $g = 9.8 \text{ m/sec}^2$.

In this way, the velocity of the ball reverses its direction while its acceleration due to gravity " g " remains constant.



Q3: When you stand still on the ground, how large a force does the ground exert on you? Why does not this force make you rise up into the air?

Ans: (a) When we stand on the ground, the ground exerts a force on our body which is equal to our weight.

b) This force does not make us rise up into the air, because no net force is acting on us and 1st condition of equilibrium is satisfied.

Reasons and Explanation:

When we are standing still on the ground, two types of forces exist which are given below:

1. Our weight " w " acting in downward direction.
2. The normal force " F_N " exerted by the ground on us vertically in upward direction. Both these forces are equal in magnitude but opposite in direction. So we have,

$$F_N = -w$$

$$\Rightarrow F_N + w = 0$$

$$\Rightarrow \sum F = 0 \quad (1)$$

Equation (1) satisfies the 1st condition of equilibrium. This equation shows that there is no net force acting on us and hence we will not be able to rise up in the air.



Q4: A man standing on the top of a tower throws a ball vertically up with certain velocity. He also throws another ball vertically downward with the same speed. Neglecting air resistance, which ball will hit the ground with higher speed?

Ans: In this case both balls will hit the ground with same speed.

Reason and Explanation:

Consider a man standing on the top of a tower as shown in the figure. Let the man throws ball "A" upward and ball "B" downwards with same initial speed " v_i ".

Consider Ball A:

For upward motion of the ball we have,

$$v_f = 0, S = h_1, a = -g$$

so we have,

$$2aS = v_f^2 - v_i^2 \Rightarrow -2gh_1 = 0^2 - v_i^2$$

$$\Rightarrow v_i^2 = 2gh_1 \quad (1)$$

Now when the ball "A" reaches to highest point, it comes to rest momentarily and then starts motion downwards. For downward motion, we have,

$$v_f = 0, S = h_1 + h, a = g, \text{ so we get,}$$

$$2aS = v_f^2 - v_i^2 \Rightarrow 2g(h_1 + h) = v_f^2 - 0^2$$

$$\Rightarrow v_f^2 = 2gh_1 + 2gh \quad (2)$$

Putting equation (1) in equation (2), we get,

$$v_f^2 = v_i^2 + 2gh \Rightarrow (v_f)_{\text{ball A}} = \sqrt{v_i^2 + 2gh} \quad (3)$$

Consider Ball B:

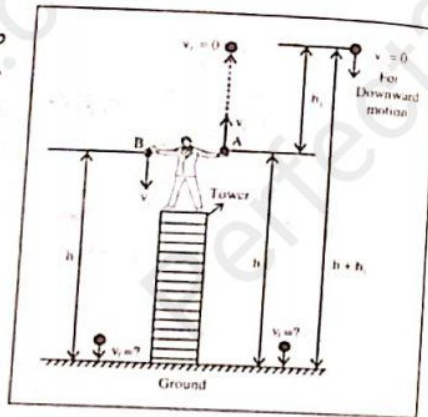
For downward motion of ball "B" thrown with initial velocity " v_i ", we have,

$$a = g, S = h, \text{ so we get,}$$

$$2aS = v_f^2 - v_i^2 \Rightarrow 2gh = v_f^2 - v_i^2$$

$$\Rightarrow v_f^2 = v_i^2 + 2gh \Rightarrow (v_f)_{\text{ball B}} = \sqrt{v_i^2 + 2gh} \quad (4)$$

Comparing equation (3) and equation (4), we see that both balls will hit the ground with same speed.



Q5: The cricket coach explains that the follow-through with the shot will make the ball travel a greater distance. Explain the reasoning in terms of the impulse-momentum theorem.

Ans: The cricket coach explains that the follow-through with the shot will make the ball travel a greater distance.

Reason and Explanation:

We know that the momentum in terms of impulse is given by,

$$J = \Delta P$$

$$\Rightarrow J = m\Delta v \quad (1)$$

According to 2nd equation of motion, the distance covered by a body is given by,

$$S = v_i t + \frac{1}{2} at^2 \quad (2)$$

Put $t = \Delta t$ and $a = \frac{\Delta v}{\Delta t}$ in equation (2), we get,

$$S = v_i \Delta t + \frac{1}{2} \frac{\Delta v}{\Delta t} \cdot (\Delta t)^2$$

$$\Rightarrow S = v_i \Delta t + \frac{1}{2} \Delta v \cdot \Delta t \quad (3)$$

Equation (1) shows that greater the impulse larger will be the increase in velocity and vice versa.

Equation (3) shows that larger the increase or change in velocity " Δv " greater will be distance travel by the ball and vice versa.

Conclusion:

The above discussion shows that for large impulse there will be greater change in velocity, and as a result the ball will travel greater distance. Thus according to impulse-momentum theorem the statement of the coach is true.

Q6: When you release an inflated but untied balloon, why does it fly across the room?

Ans: When we release an inflated but untied balloon, it fly across the room. The law of conservation of momentum is involved in this case.



Reason & Explanation:

The pressure of air inside a balloon is greater than the outside pressure. So when we release an inflated but untied balloon at rest, the gas inside the balloon rushes out the open end of the balloon. The escaping gas and the balloon form a closed system to conserve momentum. The escaping air gain momentum " P_1 " in the direction of the escaping air. Now to conserve momentum, the balloon with remaining air acquire momentum " P_2 " in opposite direction of the escaping air. Thus the balloon is pushed forward i.e. across the room to conserve momentum. i.e.

$$P_1 = -P_2 \quad \text{or} \quad P_1 + P_2 = 0$$

Thus when we release an inflated but untied balloon, it fly across the room to conserve momentum.

Q7: Modern cars are not rigid but are designed to have "crumple zones" that collapse upon impact. What is the advantage of this new design?

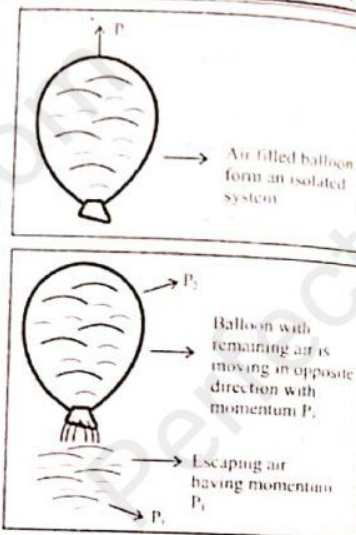
Ans: Modern cars are not rigid but are designed to have "crumple zones" that collapse upon impact. It reduces the chances of serious injuries and keep most part of the car in safe condition.

Reason and Explanation:

We know that, impulse is given by,

$$J = F_{ave} \cdot \Delta t \Rightarrow F_{ave} = \frac{J}{\Delta t} \quad (1)$$

In modern cars, crumple zone or crush zone or crash zones are designed which reduce the chances of great losses. The crumple zones increases the contact time during the collision between two vehicles. When time of contact ' Δt ' increases then according to equation (1), the vehicles and passenger inside it faces less force. The crumple zones located in front of the car absorb energy from the impact of



a head-on collision.

In this way the crumple zone slow down the crash. It reduces the difference between speed of the car occupants and the car. The momentum and velocity of the vehicles decreases gradually. In this way very less part of the colliding vehicles is damaged. Also the driver and passenger remain safe from serious injuries.

Q8: Why we can hit a long sixer in a cricket match rather than if we toss a ball for ourselves?

Ans: We can hit a long sixer in a cricket match rather than if we toss a ball for ourselves.

Reason and Explanation:

We know that the horizontal range of an object is given by,

$$R = \frac{v_i^2 \sin 2\theta}{g} \quad (1)$$

Equation (1) shows that the horizontal range will be maximum if the initial velocity of the body is maximum and also the value of $\sin 2\theta$ is maximum. It is possible when we keep $\theta = 45^\circ$, so we get $\sin 2\theta = \sin(2 \times 45^\circ) = \sin 90^\circ = 1$ which is the maximum value of $\sin \theta$. So equation (1) becomes,

$$R_{max} = \frac{v_i^2 \sin 90^\circ}{g} = \frac{v_i^2 \times 1}{g}$$

$$\Rightarrow R_{max} = \frac{v_i^2}{g} \quad (2)$$

Thus if we hit a ball with a bat with greater momentum or greater velocity by making an angle of 45° with ground, then under such condition range of ball will be maximum and we will be able to hit a long sixer in cricket match rather than if we toss a ball for ourselves.

Q9: An aeroplane while travelling horizontally, dropped a bomb when it was exactly above the target, the bomb missed the target. Explain.

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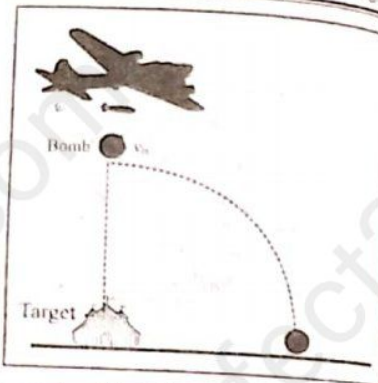
Ans: An aeroplane while travelling horizontally, dropped a bomb when



it was exactly above the target, the bomb missed the target due to horizontal component of its velocity.

Reason and Explanation:

The bomb in the aeroplane has exactly the same horizontal speed as that of aeroplane. When the aeroplane drops the bomb, the bomb will continue its horizontal motion with velocity " v_x " due to inertia. Thus the bomb will follow projectile motion and will fall away from the target. Thus the bomber misses the target due to horizontal component of its velocity.



Q10: Calculate the angle of projection for which K.E at summit [highest point] of its trajectory is equal to one-fourth of its K.E at point of projection?

Solution:

Angle of projection = $\theta = ?$

According to the given conditions, we have,

$$(K.E)_{\text{summit}} = \frac{1}{4} (K.E)_{\text{projection}}$$

$$\Rightarrow \frac{1}{2} m v_x^2 = \frac{1}{4} \times \frac{1}{2} m v_i^2$$

$$\Rightarrow v_x^2 = \frac{1}{4} v_i^2 \quad (1)$$

At summit, we have $v_y = 0$ and $v_x = v_{ix}$, so equation (1) becomes

$$v_{ix}^2 = \frac{1}{4} v_i^2$$

$$\Rightarrow v_x^2 = \frac{1}{4} v_i^2 \quad (2)$$

As $v_{ix} = v_i \cos \theta$, so equation (2) becomes

$$v_i^2 \cos^2 \theta = \frac{1}{4} v_i^2$$

$$\Rightarrow \cos^2 \theta = \frac{1}{4} \Rightarrow \cos \theta = \frac{1}{2} = 0.5$$

$$\Rightarrow \theta = \cos^{-1}(0.5) \Rightarrow \theta = 60^\circ$$

Thus the angle of projection for which K.E at summit of its trajectory is equal to one-fourth of its K.E at point of projection is 60° .

At point of projection we take initial velocity " v_i " and at summit, we take final velocity " v_x ".

At summit the vertical component becomes zero i.e. $v_{iy} = 0$ so we take horizontal component of " v_i " i.e. we put $v_i = v_{ix}$.

Equation of projectile i.e.
 $v_{ix} = v_{ix}$

Q11: For any specific velocity of projection, the range of a projectile cannot exceed from a value equal to four times of the corresponding height. Discuss.

Solution:

We know that the maximum height of projectile is given by,

$$H = \frac{v_i^2 \sin^2 \theta}{2g} \quad (1)$$

We also know that the horizontal range of projectile is given by,

$$R = \frac{v_i^2 \sin 2\theta}{g} \quad (2)$$

$$\Rightarrow R = \frac{v_i^2 2 \sin \theta \cos \theta}{g} \quad (3)$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

Dividing and multiplying the R.H.S of equation (3) by $\frac{\sin \theta}{2}$, so we get,

$$R = \frac{v_i^2 2 \sin \theta \cos \theta}{g} \times \frac{(\sin \theta) / 2}{(\sin \theta) / 2}$$

$$\Rightarrow R = \frac{4 \cos \theta}{\sin \theta} \left[\frac{v_i^2 \sin^2 \theta}{2g} \right] \quad (4)$$

Putting equation (1) in equation (4), we get,

$$R = \frac{4 \cos \theta}{\sin \theta} \times H \Rightarrow R = 4 \cos \theta \times H \Rightarrow R = 4 \times \frac{1}{\tan \theta} \times H$$

$$\Rightarrow R \tan \theta = 4H \quad (5)$$

Now we know that the range of projectile is maximum, if $\theta = 45^\circ$,

so equation (5) becomes,

$$R_{\max} \times \tan 45^\circ = 4H \quad (6)$$

As $\tan 45^\circ = 1$, so equation (1) becomes

$$\Rightarrow R_{\max} \times 1 = 4H$$

$$\Rightarrow R_{\max} = 4H \quad (7)$$

Equation (6) shows that for any specific velocity of projection, the range of a projectile cannot exceed from a value equal to four times of the corresponding height.

Q12: What is the angle for which the maximum height reached and corresponding range are equal?

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Solution:

According to given conditions, we have,

Maximum height = Horizontal range

$$\Rightarrow H = R \quad (1)$$

We know that

$$H = \frac{v^2 \sin^2 \theta}{2g} \quad (2)$$

$$\& R = \frac{v^2 \sin 2\theta}{g} \quad (3)$$

Putting equation (2), (3) in equation (1), we get,

$$\Rightarrow \frac{v^2 \sin^2 \theta}{2g} = \frac{v^2 \sin 2\theta}{g} \Rightarrow \frac{\sin^2 \theta}{2} = \sin 2\theta$$

$$\Rightarrow \frac{\sin^2 \theta}{2} = 2 \sin \theta \cos \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\Rightarrow \sin \theta = 4 \cos \theta \Rightarrow \frac{\sin \theta}{\cos \theta} = 4$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

$$\Rightarrow \tan \theta = 4$$

$$\Rightarrow \theta = \tan^{-1}(4) = 75.96^\circ = 76^\circ \Rightarrow \theta = 76^\circ$$

Thus the angle of projection for which the maximum height reached and corresponding range are equal is 76° .

⑤ حضرت عمر فاروقؓ کا قول ہے "ایمان کے بعد بڑی نعمت عورت ہے۔"

⑥ حضرت عمر فاروقؓ کا قول ہے "قبل اس کے کہ بزرگ بنو علم حاصل کرو۔"

⑦ حضرت عمر فاروقؓ کا قول ہے "تین چیزیں محبت بڑھانے کا ذریعہ ہیں: (۱) سلام کرنا، (۲) دوسروں کے لیے

مجلس میں جگہ خالی کرنا، (۳) مخاطب کو بہترین نام سے پکارنا۔"

⑧ حضرت عمر فاروقؓ کا قول ہے "جو آدمی اپنے آپ کو عالم کہے وہ جاہل ہے اور جو اپنے آپ کو جنتی بتائے وہ جہنمی ہے۔"

Comprehensive Questions

Give extended response to the following questions:

Q1: Explain displacement - time graph and velocity - time graph. In each type give brief details along with appropriate diagram for illustration.

Ans: See answer of questions no 6, 8 in the text.

Q2: Apply Newton's laws to explain the motion of objects in a variety of context.

Ans: See answer of questions no 14 in the text.

Q3: What is linear momentum? Derive and state Newton's 2nd law in terms of linear momentum.

Ans: See answer of question no 16 in the text.

Q4: State and explain law of conservation of linear momentum for an isolated system of bodies.

Ans: See answer of questions no 18 in the text.

Q5: Define elastic and inelastic collisions. Give examples in each case. Derive mechanical equations for calculating the final velocities of the elastically colliding bodies in one dimension.

Ans: See answer of question no 19 and 20 in the text.

Q6: What is projectile motion? Give examples. Find out the expression of instantaneous velocity for a projectile.

Ans: See answer of question no 22 and 23 in the text.

Q7: What is maximum height and time of flight for projectile? Derive mathematical equations for maximum height attained and time of flight.

Ans: See answer of question no 25, 26 in the text.

Q8: What is range of a projectile? State in which condition the range will be maximum if speed of projectile is kept constant in a uniform gravitational field. Also show that there are two projection angles for the same range.

Ans: See answer of question no 28 and 29 in the text.

Numerical Problems

Q1: An object is falling freely under gravity. How much distance will it travel in 2nd and 3rd second of its journey?

Solution:

Given Data:

Initial velocity of the object = $v_i = 0$

Acceleration due to gravity
= $g = 9.8 \text{ m/sec}^2$

Required:

(a) Distance travelled in 2nd sec = $S = ?$

(b) Distance travelled in 3rd sec = $S' = ?$

Calculation: (a) We know that,

First we calculate the final distance at the end of 1st second, so we have, $v_f = v_i + gt = [0 + 9.8 \times 1] \text{ m/sec} \Rightarrow v_f = 9.8 \text{ m/sec}$

This velocity " v_f " acts as initial velocity at the starting of 2nd second. So we have,

$$v_i = 9.8 \text{ m/sec}$$

Now the distance travelled in 2nd second is given by,

$$S = v_i t + \frac{1}{2} g t^2 = \left[9.8 \times 1 + \frac{1}{2} \times 9.8 \times (1)^2 \right] \text{ m} = 14.7 \text{ m}$$

$$\Rightarrow S = 15 \text{ m}$$

(b) First we calculate the velocity as the 3rd of 2nd second, for which we have,

$$v_i = 9.8 \text{ m/sec}, g = 9.8 \text{ m/sec}^2 \text{ and } t = 1 \text{ sec}$$

So we have,

$$v_f = v_i + gt = (9.8 + 9.8 \times 1) \text{ m/sec} \Rightarrow v_f = 19.6 \text{ m/sec}$$

This velocity $v_f = 19.6 \text{ m/sec}$ acts as initial velocity at the starting of 3rd second, so we have,

$$v_i = 19.6 \text{ m/sec}$$

Now the distance travel in 3rd second is given by,

$$S' = v_i t + \frac{1}{2} g t^2 = \left[19.6 \times 1 + \frac{1}{2} \times 9.8 \times (1)^2 \right] \text{ m}$$

$$\Rightarrow S' = (19.6 + 4.9) = 24.5 \text{ m} \Rightarrow S = 25 \text{ m} \text{ approximately}$$



Q2: A helicopter is ascending vertically at a speed of 19.6 m/sec . When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

Solution:

Given Data:

Initial velocity of helicopter and stone
= $v_i = 19.6 \text{ m/sec}$

Acceleration due to gravity
= $g = -9.8 \text{ m/sec}^2$

vertical distance covered by helicopter and stone = $h = 156.8 \text{ m}$

Required:

Time taken by the stone to reach the ground = $T = ?$

Calculation:

As the stone inside helicopter is moving upward with 19.6 m/sec , so when it is released at point "A", it first moves upwards toward point "B" due to inertia and then moves down from point "B" towards ground. So first we calculate the distance travelled by stone from point "A" towards point "B", we have,

$$2aS = v_f^2 - v_i^2 \quad (1)$$

Put $a = -g$, $S = h$, $v_f = 0$, $v_i = 19.6$ in equation (1), we get

$$-2 \times 9.8 h = 0^2 - (19.6)^2$$

$$-19.6 h = -19.6 \times 19.6$$

$$\Rightarrow h = 19.6 \text{ m}$$

Now we calculate the time taken by stone from "A" towards "B", for which we have,

$$v_f = v_i + at \quad (2)$$

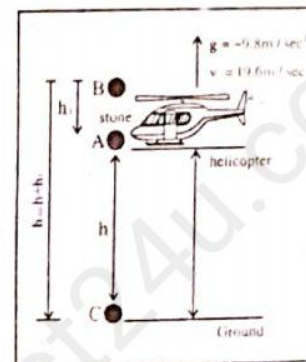
Put $v_f = 0$, $v_i = 19.6 \text{ m/sec}$, $a = -g = -9.8 \text{ m/sec}^2$ in equation (2), we get,

$$0 = 19.6 - 9.8 \times t \Rightarrow -9.8t = -19.6$$

$$\Rightarrow t = \frac{19.6}{9.8} = 2 \text{ sec}$$

$$\Rightarrow t_1 = 2 \text{ sec}$$

Now the total vertical distance between point "B" and ground



At highest point "B" the stone comes to rest, so we put $v_f = 0$.

(point c) is given by,

$$H = h_1 + h = (156.8 + 19.6)m$$

$$\Rightarrow H = 176.4m$$

Now we calculate the time taken by the stone from point "B" to ground, for which we have,

$$S = vt + \frac{1}{2}at^2 \quad (3)$$

Here $S = H$, $v = 0$, $a = g$

So equation (3) becomes

$$\Rightarrow H = 0 \times t + \frac{1}{2} \times 9.8 t^2$$

$$\Rightarrow H = 0 + 4.9 t^2$$

$$\Rightarrow 4.9 t^2 = H$$

$$\Rightarrow t^2 = \frac{H}{4.9} = \frac{176.4}{4.9} \text{ sec}^2 \Rightarrow t^2 = 36 \text{ sec}^2$$

$$\Rightarrow t = \sqrt{36 \text{ sec}^2} = 6 \text{ sec} \Rightarrow t_2 = 6 \text{ sec}$$

Now the total time taken by the stone in air (that is from point "A" to "B" and then from "B" to ground point "C") is given by,

$$T = t_1 + t_2 = 2 \text{ sec} + 6 \text{ sec} = 8 \text{ sec}$$

$$\Rightarrow T = 8 \text{ sec}$$

At highest point "B" when the stone moves back towards ground then its initial velocity will be equal to 0. $v = 0$

Q3: A car moving at 20.0 m/s (72.0 km/h) crashes into a tree. Find the magnitude of the average force acting on a passenger of mass 70 kg in each of the following cases. (a) The passenger is not wearing a seat belt. He is brought to rest by a collision with the windshield and dashboard that lasts 2.0 ms . (b) The car is equipped with a passenger-side air bag. The force due to the air bag acts for 45 ms , bringing the passenger to rest.

Solution:

Given Data: Initial speed of car $= v_i = 20 \text{ m/sec}$

Final velocity $= v_f = 0$

Mass of passenger $= m = 70 \text{ kg}$

Required: (a) Average force on passenger for

$$\Delta t_1 = 2 \text{ ms} = 2 \times 10^{-3} \text{ sec} = F_{ave} = ?$$

(b) Average force on passenger for

$$\Delta t_2 = 45 \text{ ms} = 45 \times 10^{-3} \text{ sec} = F_{ave} = ?$$

Calculation: (a) We know that, $J = F_{ave} \cdot \Delta t$

$$\Rightarrow F_{ave} = \frac{J}{\Delta t} \quad (1)$$

$$\text{Also } J = \Delta P = mv_f - mv_i = (70 \times 0 - 70 \times 20) \text{ N}\cdot\text{s}$$

$$\Rightarrow J = -1400 \text{ N}\cdot\text{s}$$

Now putting $J = -1400 \text{ N}\cdot\text{s}$ and $\Delta t = \Delta t_1 = 2 \times 10^{-3} \text{ sec}$ in equation (1), we get,

$$F_{ave} = \left(\frac{-1400}{2 \times 10^{-3}} \right) \text{ N} = -700 \times 10^3 \text{ N}$$

$$\Rightarrow F_{ave} = -7.00 \times 10^5 \text{ N}$$

Now the tree will act equal and opposite force on passenger, so we have

$$F'_{ave} = 7.00 \times 10^5 \text{ N}$$

(b) Putting $J = -1400 \text{ N}\cdot\text{s}$ and $\Delta t = \Delta t_2 = 45 \times 10^{-3} \text{ sec}$ in equation (1), we get,

$$F_{ave} = \left(\frac{-1400}{45 \times 10^{-3}} \right) \text{ N} = -31.1 \times 10^3 \text{ N}$$

$$\Rightarrow F_{ave} = -3.11 \times 10^4 \text{ N}$$

Now the tree will exert equal and opposite force on passenger, so we have

$$F'_{ave} = 3.11 \times 10^4 \text{ N}$$

Q4: A 0.4 kg ball travelling with the speed of 15 ms^{-1} strikes a rigid wall and rebounds elastically. If the ball is in contact with the wall for 0.045 s , what is (a) the momentum imparted to the wall and (b) the average force exerted on the wall?

Solution:

Given Data:

Mass of ball $= m = 0.4 \text{ kg}$

Initial speed of ball $= v_i = -15 \text{ m/sec}$

Final speed of ball $= v_f = 15 \text{ m/sec}$

Time interval $= \Delta t = 0.045 \text{ sec}$

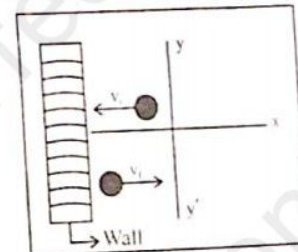
Required:

(a) Momentum imparted to the wall $= \Delta P = ?$

(b) Average force exerted on the wall $= F_{ave} = ?$

Calculation: (a) We know that, $\Delta P = mv_f - mv_i$

$$\Rightarrow \Delta P = [0.4 \times 15 - 0.4 \times (-15)] \text{ kg}\cdot\text{m/sec}$$



$$\Rightarrow \Delta P = (6 + 6) \text{ kg} \cdot \text{m} / \text{sec} \Rightarrow \Delta P = 12 \text{ kg} \cdot \text{m} / \text{sec}$$

(b) We know that,

$$F_{\text{ave}} = \frac{\Delta P}{\Delta t}$$

$$\Rightarrow F_{\text{ave}} = \left(\frac{12}{0.045} \right) \text{ N} \Rightarrow F_{\text{ave}} = 266.66 \text{ N} \text{ or } F_{\text{ave}} = 266.7 \text{ N}$$

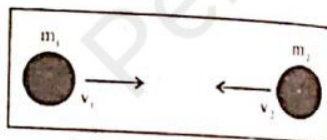
Q5: One ball of mass 0.600 kg travelling 9.00 m/s to the right collides head on elastically with a second ball of mass 0.300 kg travelling 8.00 m/s to the left. After the collision, what are their velocities after collision?

Solution: Given Data: Mass of 1st ball = $m_1 = 0.600 \text{ kg}$

Speed of 1st ball = $v_1 = 9.00 \text{ m} / \text{sec}$

Mass of 2nd ball = $m_2 = 0.300 \text{ kg}$

Speed of 2nd ball = $v_2 = -8 \text{ m} / \text{sec}$



The negative sign is used because both bodies move in opposite direction.

Required:

(a) Velocity of 1st ball after collision = $v'_1 = ?$

(b) Velocity of 2nd ball after collision = $v'_2 = ?$

Calculation: (a) We know that,

$$v'_1 = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_1 + \frac{2m_2 v_2}{m_1 + m_2} \quad (1)$$

$$\text{Similarly, } v'_2 = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_2 + \frac{2m_1 v_1}{m_1 + m_2} \quad (2)$$

Putting the values in equation (1), we get,

$$v'_1 = \left[\frac{0.600 - 0.300}{0.600 + 0.300} \right] 9.00 + \left[\frac{2 \times 0.300 \times (-8)}{0.600 + 0.300} \right] \text{ m} / \text{sec}$$

$$\Rightarrow v'_1 = \left[\frac{0.3 \times 9}{0.9} \right] - \left[\frac{4.8}{0.9} \right] \text{ m} / \text{sec} = (3 - 5.33) \text{ m} / \text{sec}$$

$$\Rightarrow v'_1 = -2.33 \text{ m} / \text{sec}$$

(b) Putting the values in equation (2), we get,

$$v'_2 = \left[\frac{0.300 - 0.600}{0.600 + 0.300} \right] (-8) + \left[\frac{2 \times 0.600 \times 9}{0.600 + 0.300} \right] \text{ m} / \text{sec}$$

$$\Rightarrow v'_2 = \left[\frac{-0.3 \times (-8)}{0.9} \right] + \left[\frac{10.8}{0.9} \right] \text{ m} / \text{sec} = (2.666 + 12) \text{ m} / \text{sec}$$

$$= (2.67 + 12) \text{ m} / \text{sec} = 14.67 \text{ m} / \text{sec}$$

$$\Rightarrow v'_2 = 14.67 \text{ m} / \text{sec}$$

Q6: In a wedding a bullet is fired in air at a speed of 500 m/s making an angle of 60° with horizontal from an AK 47 rifle. (a) How high will the bullet rise? (b) What time would it take to reach ground? (c) How far would it go? (Ignore air resistance)

Solution:

Given Data:

Initial speed of bullet in air = $v_i = 500 \text{ m} / \text{sec}$

Angle of projection = $\theta = 60^\circ$

Required:

(a) Maximum height = $H = ?$

(b) Time of flight = $T = ?$

(c) Horizontal range = $R = ?$

Calculation: (a) We know that, the maximum height is given by,

$$H = \frac{v_i^2 \sin^2 \theta}{2g} \quad (1)$$

Putting the values in equation (1), we get,

$$H = \left[\frac{(500)^2 \times (\sin 60^\circ)^2}{2 \times 9.8} \right] \text{ m} = \left(\frac{250000 \times 0.75}{19.6} \right) \text{ m}$$

$$\Rightarrow H = 9566.3 \text{ m}$$

(b) We know that, the time of flight is given by,

$$T = \frac{2v_i \sin \theta}{g} \quad (2)$$

Putting the values in equation (2), we get,

$$T = \left(\frac{2 \times 500 \times \sin 60^\circ}{9.8} \right) \text{ sec} = \left(\frac{1000 \times 0.866}{9.8} \right) \text{ sec}$$

$$\Rightarrow T = \left(\frac{866}{9.8} \right) \text{ sec} = 88.36 \text{ sec} = 88.4 \text{ sec} \Rightarrow T = 88.4 \text{ sec}$$

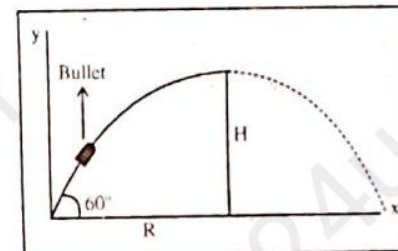
(c) We know that, the horizontal range is given by,

$$R = \frac{v_i^2 \sin 2\theta}{g} \quad (3)$$

Putting the values in equation (3), we get,

$$R = \frac{(500)^2 \times \sin(2 \times 60^\circ)}{9.8} = \left[\frac{250000 \times \sin(120^\circ)}{9.8} \right] \text{ m}$$

$$\Rightarrow R = \left[\frac{250000 \times 0.866}{9.8} \right] \text{ m} = 22091.8 \text{ m} \Rightarrow R = 22091.8 \text{ m}$$



Q7: The catapult hurls a stone of mass 32.0 kg with a velocity of 50.0 m/s at a 30.0° angle of elevation. (a) What is the maximum height reached by the stone? (b) What is its range? (c) How long has the stone been in the air when it returns to its original height?

Solution:

Given Data: Mass of stone = $m = 32 \text{ kg}$

Initial velocity of stone = $v = 50 \text{ m/sec}$

Angle of elevation = $\theta = 30^\circ$

Required:

(a) Maximum height = $H = ?$ (b) Horizontal range = $R = ?$

(c) Time of flight = $T = ?$

Calculation: (a) We know that maximum height is given by,

$$H = \frac{v^2 \sin^2 \theta}{2g} \quad (1)$$

Putting the values in equation (1), we get,

$$H = \left[\frac{(50)^2 (\sin 30^\circ)^2}{2 \times 9.8} \right] m = \left[\frac{2500 \times (0.5)^2}{19.6} \right] m$$

$$\Rightarrow H = \left[\frac{2500 \times 0.25}{19.6} \right] m = 31.88 \text{ m} \Rightarrow \boxed{H = 31.88 \text{ m}}$$

(b) We know that horizontal range is given by,

$$R = \frac{v^2 \sin 2\theta}{g} \quad (2)$$

Putting the values in equation (2), we get,

$$R = \frac{50^2 \times \sin(2 \times 30^\circ)}{9.8} = \left[\frac{2500 \times \sin 60^\circ}{9.8} \right] m$$

$$\Rightarrow R = \left[\frac{2500 \times 0.866}{9.8} \right] m = 220.9 \text{ m} \Rightarrow \boxed{R = 220.9 \text{ m}}$$

(c) We know that time of flight is given by,

$$T = \frac{2v \sin \theta}{g} \quad (3)$$

Putting the values in equation (3), we get,

$$T = \frac{2 \times 50 \times \sin(30^\circ)}{9.8} = \left(\frac{100 \times 0.5}{9.8} \right) \text{ sec}$$

$$\Rightarrow \boxed{T = 5.10 \text{ sec}}$$

Q1: Define and explain work.

ETEA 2010

Ans. Definition:

The scalar product of force and displacement is known as work. (OR)

A body is said to do work when it apply certain force on another body and moves it through some displacement.

Explanation:

Consider a force " F " is acting on a body of mass " m " and moves it from point " A " to point " B " through a displacement " d " as shown in the figure. Now the workdone on the body is given by,

Work = Force $\cdot$ Displacement

$$\Rightarrow w = F \cdot d$$

$$\Rightarrow \boxed{w = Fd \cos \theta}$$

(1)

Equation (1) shows that the workdone depends upon the magnitude of force and displacement. i.e. greater the magnitude of force and displacement, greater will be the amount of work and vice versa.

Equation (1) also shows that for given values of " F " and " d ", the work also depends upon value of " θ ". For example,

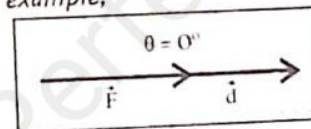
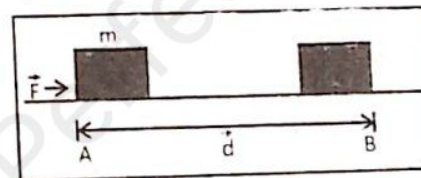
Case-1: If $\theta = 0^\circ$, i.e. the force and displacement are acting along the same direction along the same line, then equation (1) becomes,

$$w = Fd \cos \theta = Fd \cos 0^\circ \quad [\cos 0^\circ = 1]$$

$$\Rightarrow \boxed{w = Fd} \quad (2)$$

Equation (2) shows that the workdone will be positive and maximum if $\theta = 0^\circ$.

Example of Positive Work: Lifting a suitcase through a height " h " by applying force " F " is an example of positive work.



Case-2: If $\theta = 90^\circ$, i.e. the force and displacement are perpendicular to each other, then equation (1) becomes,

$$W = Fd \cos 90^\circ$$

$$\Rightarrow W = Fd \times 0 \quad [\cos 90^\circ = 0]$$

$$\Rightarrow \boxed{W = 0} \quad (3)$$

Equation (3) shows that no work will be done if the force and displacement are perpendicular to each other.

Case-3: If $\theta = 180^\circ$, i.e. the force and displacement are acting in opposite direction, then equation (1) becomes,

$$W = Fd \cos 180^\circ \quad [\cos 180^\circ = -1]$$

$$\Rightarrow W = Fd \times (-1)$$

$$\Rightarrow \boxed{W = -Fd} \quad (4)$$

Equation (4) shows that workdone will be taken as negative, if the force and displacement are opposite to each other.

Example of Negative Work: Workdone against the force of friction is the example of negative work.

Unit of Work:

$$\text{As } w = Fd \Rightarrow w = \text{N} \cdot \text{m} = \text{joule}$$

So the unit of work is joule which is defined as, "workdone is said to be 1 joule if one Newton force is acting on a body and displaces it through one meter."

Dimensions of Work:

$$\text{As } W = FS \Rightarrow W = \text{ma} \cdot S = \text{kg} \cdot \text{m} / \text{sec}^2 \cdot \text{m}$$

$$\Rightarrow W = \frac{\text{Mass} \cdot \text{length} \cdot \text{length}}{\text{time}^2}$$

$$\Rightarrow W = [ML^2T^{-2}]$$

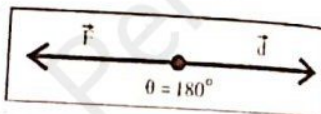
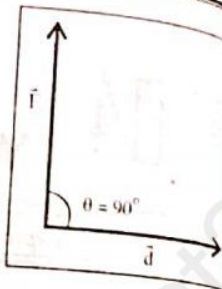
Work is a scalar quantity.

Q2: (a) What is conservative field? Give examples. **(b)** Define closed path (or) loop.

ETEA 2007, BISE Mardan 2019

Ans: (a) Conservative Field:

A field is said to be conservative field if the workdone in it along a closed path is equal to zero. (OR)



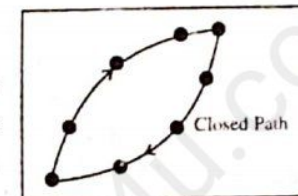
A field in which workdone is independent of the path followed is known as conservative field.

Examples:

1. Gravitational field
2. Electrostatic field
3. Magnetic field

(b) Closed Path (or) Loop:

A closed path (or) loop is that path in which a body reaches to its starting point again after passing through several points, as shown in the figure.



Q3: Discuss the workdone by a variable force?

Ans: Consider a body of mass "m" is allowed to move from point "A" to "B" under the action of variable force.

Here we divide the length between point "A" and point "B" into large number of parts i.e.

$$\Delta d_1, \Delta d_2, \Delta d_3, \dots, \Delta d_n$$

respectively.

Now the workdone through Δd_1 is given by,

$$\Delta w_1 = F_1 \cdot \Delta d_1 = F_1 \Delta d_1 \cos \theta_1$$

$$\Rightarrow \Delta w_1 = (F_1 \cos \theta_1) \Delta d_1$$

Similarly,

$$\Delta w_2 = (F_2 \cos \theta_2) \Delta d_2$$

$$\Delta w_3 = (F_3 \cos \theta_3) \Delta d_3$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\& \Delta w_n = (F_n \cos \theta_n) \Delta d_n$$

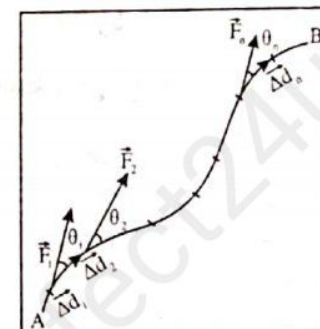
Now the total workdone between "A" and "B" is given by,

$$w_T = \Delta w_1 + \Delta w_2 + \Delta w_3 + \dots + \Delta w_n$$

$$\Rightarrow \Delta_T = (F_1 \cos \theta_1) \Delta d_1 + (F_2 \cos \theta_2) \Delta d_2 + \dots + (F_n \cos \theta_n) \Delta d_n$$

$$\Rightarrow \boxed{w_T = \sum_{i=1}^{n} (F_i \cos \theta_i) \Delta d_i} \quad (1)$$

Equation (1) represents the workdone by a variable force.



Q4: Discuss the workdone in gravitational field. (OR) Show that workdone along a closed path in gravitational field is equal to zero?

Ans. Consider a body of mass "m" placed at point "A" as shown in the figure.

Let the body is allowed to move from point "A" towards point "C" in gravitational field, as shown in the figure. Now the workdone on the body from "A" to "C" is given by,

$$w_{A \rightarrow C} = \vec{w} \cdot \vec{d} = wd \cos \theta \quad (1)$$

Now from the figure, we have,

$$\cos \theta = \frac{d_1}{d} \Rightarrow d \cos \theta = d_1 \quad (2)$$

Putting equation (2) in equation (1), we get,

$$w_{A \rightarrow C} = wd_1 \quad (3)$$

Now let the body is allowed to move from "C" towards "B", then we have,

$$w_{C \rightarrow B} = \vec{w} \cdot \vec{d}_2 = wd_2 \cos \theta = wd_2 \cos 90^\circ$$

$$\Rightarrow w_{C \rightarrow B} = 0 \quad (4) \quad [\cos 90^\circ = 0]$$

Now let the body is allowed to move from "B" towards "A", then we have,

$$w_{B \rightarrow A} = \vec{w} \cdot \vec{d}_1 = wd_1 \cos \theta = wd_1 \cos 180^\circ$$

$$\Rightarrow w_{B \rightarrow A} = -wd_1 \quad (5) \quad [\cos 180^\circ = -1]$$

Now the total workdone is given by,

$$w_T = w_{A \rightarrow C} + w_{C \rightarrow B} + w_{B \rightarrow A} \quad (6)$$

Putting equations (3), (4), (5) in equation (6), we get,

$$w_T = wd_1 + 0 - wd_1 = 0$$

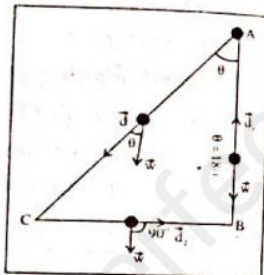
$$\Rightarrow w_T = 0 \quad (7)$$

Equation (7) shows that workdone on a body along a closed path in gravitational field is equal to zero.

Q5: Show that workdone in a gravitational field is independent of the path followed?

Ans. Consider a body of mass "m" placed at point "A" as shown in the figure.

Let the body is allowed to move from "A" towards "B" in gravitational field, then the workdone on the body is given by,



$$w_{A \rightarrow B} = \vec{w} \cdot \vec{d}_1 = wd_1 \cos 0^\circ \quad [F = \vec{w}]$$

$$\Rightarrow w_{A \rightarrow B} = wd_1 \cos 0^\circ = wd_1 \quad [\cos 0^\circ = 1]$$

$$\Rightarrow w_{A \rightarrow B} = wd_1 \quad (1)$$

Now the workdone in moving the body from "B" towards "C" is given by,

$$w_{B \rightarrow C} = \vec{w} \cdot \vec{d}_2 = wd_2 \cos 90^\circ = wd_2 \cos 90^\circ$$

$$\Rightarrow w_{B \rightarrow C} = 0 \quad (2)$$

Now the total workdone from "A" to "C" through "B" is given by,

$$(w)_{A \rightarrow B \rightarrow C} = w_{A \rightarrow B} + w_{B \rightarrow C} = wd_1 + 0$$

$$\Rightarrow w_{A \rightarrow B \rightarrow C} = wd_1 \quad (3)$$

Now again we consider the body at point "A" and allowed it to move from "A" to "C" directly, then the workdone on the body in gravitational field is given by,

$$w_{A \rightarrow C} = \vec{w} \cdot \vec{d} = wd \cos \theta$$

$$\Rightarrow w_{A \rightarrow C} = wd \cos \theta \quad (4)$$

Now from the figure, we have,

$$\cos \theta = \frac{d_1}{d} \Rightarrow d \cos \theta = d_1 \quad (5)$$

Putting equation (5) in equation (4),

we get,

$$w_{A \rightarrow C} = wd_1 \quad (6)$$

Comparing equation (3) and equation (6), we get,

$$w_{A \rightarrow B \rightarrow C} = w_{A \rightarrow C} \quad (7)$$

Equation (7) shows that workdone on a body in gravitational field is independent of the path followed.

Q6: (a) Define power and instantaneous power.

(b) Show that, $P = \vec{F} \cdot \vec{V}$

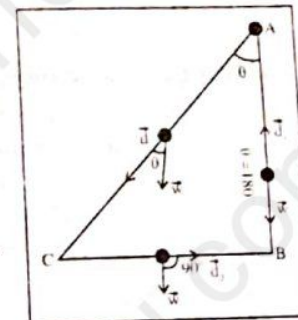
(c) Write the units and dimensions of power.

ETEA 2006-2008, 2011

Ans. (a) Power:

The rate of doing work is known as power. It is represented by "P" and is given by,

$$\text{Power} = \frac{\text{work}}{\text{time}}$$



For Your Information:

The frictional force is a non-conservative force, because if an object is moved over a rough surface between two points along different paths, the workdone against the frictional force certainly depends on the path followed.

$$\Rightarrow P = \frac{W}{t} \quad (1)$$

Instantaneous Power:

The rate of doing work in the limit when " Δt " approaches to zero is known as instantaneous power i.e.

$$P_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} \quad (2)$$

(b) To show that, $P = \vec{F} \cdot \vec{V}$:

We know that,

$$P = \frac{W}{t} \quad (1)$$

We also know that,

$$W = \vec{F} \cdot \vec{d} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$P = \vec{F} \cdot \left(\frac{\vec{d}}{t} \right) \quad (3)$$

We know that,

$$\frac{\vec{d}}{t} = \text{rate of change of displacement} = \text{velocity}$$

$$\Rightarrow \frac{\vec{d}}{t} = \vec{V} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$P = \vec{F} \cdot \vec{V}$$

Which is the required proof.

(c) Unit of Power:

$$\text{As } P = \frac{W}{t} = \frac{J}{\text{sec}} = \text{watt}$$

So the S.I unit of power is watt, which is defined as, "power is said to be one watt, if 1 joule work is done in 1 second."

Other Units of Power:

Kilo watt, mega watt, horse power etc are the other units of power.

$$1 \text{ kilo watt} = 1000 \text{ watt} = 10^3 \text{ watt}$$

$$1 \text{ mega watt} = 10^6 \text{ watt}$$

$$1 \text{ horse power (h.p)} = 746 \text{ watt}$$

For Your Information:

In British engineering system, the unit of power is called horse power (h.p) and numerically, 1hp = 746 watts.

Dimensions of Power:

$$\text{As } P = \frac{W}{t} = \frac{F \cdot d}{t}$$

$$\Rightarrow P = m a \cdot \frac{d}{t} = \text{kg} \cdot \text{m/sec}^2 \cdot \text{m/sec}$$

$$\Rightarrow \text{kg} \cdot \text{m}^2/\text{sec}^3 = \frac{\text{Mass} \cdot \text{length}^2}{\text{time}^3}$$

$$\Rightarrow P = [ML^2T^{-3}]$$

Q7: Define energy. Write the units of energy?

Ans. Energy:

The capacity of a body to do work is known as energy.

(OR)

Energy means power to work.

(OR)

Energy is an agent which causes some change in the state of a system.

Units of Energy:

Joule, kilo joule, mega joule, kilo watt-hour etc are the units of energy.

$$1 \text{ kilo joule} = 10^3 \text{ joule}$$

$$1 \text{ mega joule} = 10^6 \text{ joule}$$

$$1 \text{ kilo watt-hour}$$

$$= (1000 \times 3600) \text{ watt} \cdot \text{sec}$$

$$\Rightarrow 1 \text{ kilo watt-hour} = 3.6 \times 10^6 \text{ joule}$$

$$\Rightarrow 1 \text{ kilo watt-hour} = 3.6 \text{ mega joule}$$

$$\therefore [\text{watt} \cdot \text{sec} = \text{joule}]$$

For Your Information:

The food we eat in one day has about the same energy as 0.33 liters of petrol. Experiments shows that the average power of a man walking upstairs at an ordinary speed is only about 0.33kW.

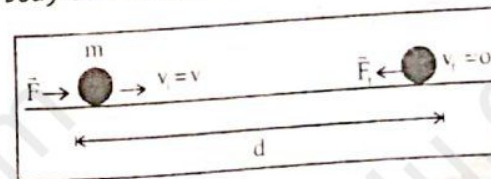
Q8: Define kinetic energy. Show that $K.E = \frac{1}{2}mv^2$?

Ans. Kinetic Energy (K.E):

Definition: The energy in the body due to its motion is known as K.E.

Explanation:

Consider a body of mass "m" moving on the ground with initial velocity " v_i " as shown in the figure.



The motion of the body is opposing by the force of friction " F_f ".

due to which the velocity of the body decreases gradually and finally comes to rest. So the final velocity of the body becomes zero. Let "d" be the distance covered by the body before coming to rest. Now we know that,

$$2aS = v_f^2 - v_i^2 \quad (1)$$

Here $a = \frac{-F}{m}$, $S = d$, $v_f = 0$, $v_i = v$

So equation (1) becomes,

$$-2 \times \frac{F}{m} \times d = 0^2 - v^2$$

$$\Rightarrow \frac{-2Fd}{m} = -v^2 \Rightarrow 2Fd = mv^2$$

$$\Rightarrow Fd = \frac{1}{2}mv^2 \quad (2)$$

Here $Fd = \text{workdone}$ so equation (2) becomes,

$$\text{Workdone} = \frac{1}{2}mv^2 \quad (3)$$

Due to this workdone, the moving body comes to rest, so this workdone must be equal to the K.E of the body. So equation (3) becomes,

$$K.E = \frac{1}{2}mv^2 \quad (4)$$

Equation (4) represents the K.E of a moving body. K.E is a scalar quantity.

Dimension of Energy:

$$\text{As } K.E = \frac{1}{2}mv^2 = \text{kg} \cdot (\text{m/sec})^2$$

$$\Rightarrow K.E = \frac{\text{Mass} \times \text{Length}^2}{\text{Time}^2} \Rightarrow K.E = [ML^2 T^{-2}]$$

حضرت عثمان غنی کا قول ہے "تجب ہے اس پر جو موت کو حق جانتا ہے اور پھر بھی ہنستا ہے۔"

$$F = ma$$

$$\Rightarrow a = F / m$$

$$\Rightarrow a = -F / m$$

The negative sign shows that velocity of the body decreases.

$$* Fd = \text{work} = K.E$$

Q9: Define potential energy. Show that P.E = mgh?

Ans. Definition:

The energy of a body due to its position is known as potential energy.

Explanation:

Consider a body of mass "m" at point "A" and a height "h" from earth surface as shown in the figure.

Now if the body is allowed to move from point "A" towards point "B", then the force of gravity on the body is equal to the weight of the body. Now the workdone on the body due to force of gravity is given by,

$$\text{Workdone} = Fd = wd$$

$$\Rightarrow \text{Workdone} = mgh \quad (1)$$

Here the workdone is equal to the P.E of the body at point "A" at height "h" with respect to point "B". So equation (1)

$$P.E = mgh \quad (2)$$

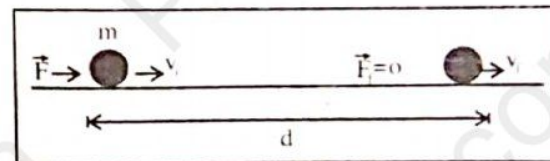
Equation (2) represents the P.E of the body at point "A". It is a scalar quantity and depends upon the height of the body from earth surface.

Q10: Discuss the work-energy principle. (OR) Show that, workdone = change in K.E of a body.

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Ans. Consider a body of mass "m" placed on the ground as shown in the figure.

Now if some force "F" is applied on the body, then the body will start motion with initial velocity "v".



in the direction of applied force. If we neglect the force of friction, then the body will continue its motion. Now we know that,

$$2aS = v_f^2 - v_i^2 \quad (1)$$

Here $S = d$, $a = \frac{F}{m}$



Here $d = h$
 $F = w = mg$

So equation (1) becomes,

$$\frac{2Fd}{m} = v_f^2 - v_i^2 \Rightarrow 2Fd = mv_f^2 - mv_i^2$$

$$\Rightarrow Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad [Fd = \text{work}]$$

$$\Rightarrow \text{workdone} = (K \cdot E)_f - (K \cdot E)_i \quad (2)$$

Equation (2) represents the work-energy principle. Equation (2) shows that workdone is equal to the change in K.E of a body.

Q11: What is a machine? Discuss the input, output and efficiency of a mechanical machine?

Ans. (a) Machine:

A machine is a thing or device with the help of which we can do more work than we can do by hands. (OR)

A machine is a device for multiplying forces or simply changing the direction of forces.

(b) Input of Mechanical Machines:

The amount of workdone on a machine by a given effort [force] is known as input of a machine. i.e.

$$\text{Input} = \text{effort} \times \text{effort arm}$$

$$\Rightarrow \text{Input} = p \times d$$

(c) Output of Mechanical Machines:

The amount of workdone by a machine on the load [weight] is known as output of the machine. i.e.

$$\text{Output} = \text{load} \times \text{load arm}$$

$$\Rightarrow \text{Output} = w \times D$$

(d) Efficiency:

The ratio of output and input of a machine is known as efficiency. i.e.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

$$\Rightarrow \eta = \frac{w \times D}{p \times d} \quad (1)$$

In terms of percentage, equation (1) can be written as,

$$\eta = \left(\frac{w \times D}{p \times d} \right) \times 100\% \quad (2)$$

Equation (1), (2) represents the efficiency of a machine. It has no unit because it is a ratio between two similar quantities.

Q12: What is an ideal machine? Discuss.

Ans. An ideal machine is a hypothetical machine whose output is equal to its input. For an ideal machine, we have

$$\text{Output} = \text{Input}$$

The efficiency of an ideal machine is 100% because there is no loss of energy in an ideal machine due to friction or any other means that can waste useful energy.

Q13: Discuss the efficiency of (a) transformer, (b) pulley system.

Ans. (a) Efficiency of a Transformer:

A transformer is a device which is used for the cheaper transmission of electrical energy from one place to another. (OR)

A transformer is a device which is used for increasing or decreasing the alternating voltage. For an ideal transformer, we have

$$\text{Power input} = \text{Power output}$$

Thus the efficiency of an ideal transformer is 100%. But practically, the efficiency of a transformer cannot be 100% due to the following reasons:

1. Some amount of energy is lost due to resistance of the primary and secondary coils of the transformer. The energy losses can be minimized by using conducting wires of large cross sectional area or by using high purity copper wires.
2. Some eddy current losses in the core. We can reduce such losses by laminating the core.
3. The energy required to magnetized the core is not entirely recovered during demagnetization, because some energy is lost as heat in the core.

(b) Efficiency of Pulley:

A pulley is a simple machine with the help of which we can lift loads easily. The efficiency of a pulley system cannot be 100% because of the forces of friction present in the pulley system.

For example, if we do 100J of work, then about 60J of energy is lost as thermal energy due to friction and the remaining 40J appears as work output. So in this case, the efficiency of the pulley system will be 40%.

Mathematically we have,

$$\text{Efficiency} = \frac{\text{useful energy output}}{\text{total energy input}}$$

The efficiency of some practical devices are shown in the table given below:

Practical devices	Efficiency
Petrol heat engine	(25-30)%
Diesel engine	(34-40)%
Steam locomotive	(35-40)%
Incandescent lamp	5%
Fluorescent lamp	20%
Steam turbine	(34-46)%
Air craft gas turbine	36%
Nuclear power plant	(30-35)%
Fossil fuel power plant	(34-40)%
Electric generator	(70-98)%
Electric motor	(50-92)%
Storage reactor	(38-40)%
Dry cell battery	90%
Electric heater	100%
Hair drier	100%
Battery	90%
Home coal furnace	55%
Steam turbine	45%
Steam locomotive	10%

For Your Information:

The pyramids in Egypt are thought to have been built by slaves hauling loads to height by inclined planes. It was the first use of inclined plane in construction happened around 2600BC. A pyramid at Egypt, believed to have been built by hauling stones up inclined planes.



For Your Information:

Efficiency of some electrical equipments:

LED light bulbs have been introduced to replace ordinary light bulbs, as they are much more efficient. Let's take a look at a standard 50watt. The energy consumption to use a light bulb like this would cost about 1278 Rs over the course of 1 year. An LED, running over the course of 1 year would cost only 260 Rs to operate. Recently developed, AC/DC fans can operate on less energy while producing a high air flow. In fact, they can cut down your power consumption by up to 65% and can operate on solar panel. AC/DC fans are designed to run on 12V and consume around 26-35W. Ordinary fan consume 75 watt while AC/DC fans consume about 35 to 40 watt so AC/DC fans are more efficient.



Q14: What is absolute potential energy? Derive an expression for absolute potential energy.

Ans: Absolute Potential Energy:

Definition: The absolute potential energy of a body on the surface of earth is equal to the workdone in lifting the body from earth surface to infinity.

Explanation:

In order to find an expression for this work, first we find the workdone from point "1" to "n". Then according to the given condition, we will consider point "1" at earth surface and point "n" at infinity. For this purpose, we divide the distance between point "1" and "n" into large number of parts each of length " Δr " as shown in the figure.

Let $r_1, r_2, r_3, \dots, r_n$ be the distance between centre of earth and points 1, 2, 3, ..., n respectively. Now consider the body of mass "m" at point "1" as shown in the figure. If we move the body from point "1" to point "2", then the work done in such a case is given by,

$$w_{1 \rightarrow 2} = \Delta F \cdot r = \frac{GM_e m}{r^2} \cdot \Delta r$$

$$\Rightarrow w_{1 \rightarrow 2} = GM_e m \frac{\Delta r}{r^2} \quad (1)$$

$$\text{Here, } r = \frac{r_1 + r_2}{2}$$

$$\Rightarrow r^2 = \frac{1}{4} (r_1 + r_2)^2$$

$$\Rightarrow r^2 = \frac{1}{4} [r_1^2 + r_2^2 + 2r_1 r_2] \quad (2)$$

From figure, we have,

$$r_2 = r_1 + \Delta r \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\Rightarrow r^2 = \frac{1}{4} [r_1^2 + (r_1 + \Delta r)^2 + 2r_1(r_1 + \Delta r)]$$

$$\Rightarrow r^2 = \frac{1}{4} [r_1^2 + r_1^2 + \Delta r^2 + 2r_1 \Delta r + 2r_1^2 + 2r_1 \Delta r]$$

$$\Rightarrow r^2 = \frac{1}{4} [4r_1^2 + 4r_1 \Delta r + \Delta r^2]$$

$$\Rightarrow r^2 = \frac{1}{4} [4r_1^2 + 4r_1 \Delta r]$$

$$\Rightarrow r^2 = \frac{1}{4} [4r_1^2 + 4r_1(r_2 - r_1)]$$

$$\Rightarrow r^2 = \frac{1}{4} [4r_1^2 + 4r_1 r_2 - 4r_1^2]$$

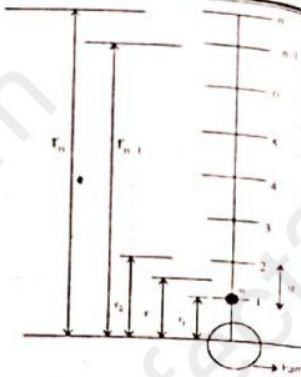
$$\Rightarrow r^2 = \frac{1}{4} [4r_1 r_2] = r_1 r_2$$

$$\Rightarrow r^2 = r_1 r_2 \quad (4)$$

Putting equation (4) in equation (1), we get

$$w_{1 \rightarrow 2} = GM_e m \frac{\Delta r}{r_1 r_2} = GM_e m \left(\frac{r_2 - r_1}{r_1 r_2} \right) \quad \left[\begin{array}{l} r_2 = r_1 + \Delta r \\ \Rightarrow \Delta r = r_2 - r_1 \end{array} \right]$$

Here Δr is very small, so we neglect its high powers i.e. Δr^2 etc.



$$\Rightarrow w_{1 \rightarrow 2} = GM_e m \left[\frac{1}{r_1} - \frac{1}{r_2} \right] \quad (5)$$

$$\text{Similarly, } w_{2 \rightarrow 3} = GM_e m \left[\frac{1}{r_2} - \frac{1}{r_3} \right] \quad (6)$$

$$w_{3 \rightarrow 4} = GM_e m \left[\frac{1}{r_3} - \frac{1}{r_4} \right] \quad (7)$$

$$w_{4 \rightarrow 5} = GM_e m \left[\frac{1}{r_4} - \frac{1}{r_5} \right] \quad (8)$$

$$w_{5 \rightarrow 6} = GM_e m \left[\frac{1}{r_5} - \frac{1}{r_6} \right] \quad (9)$$

$$\& w_{(n-1) \rightarrow n} = GM_e m \left[\frac{1}{r_{n-1}} - \frac{1}{r_n} \right] \quad (n)$$

Now the total workdone from point "1" to "n" is given by,

$$w_{1 \rightarrow n} = w_{1 \rightarrow 2} + w_{2 \rightarrow 3} + w_{3 \rightarrow 4} + w_{4 \rightarrow 5} + w_{5 \rightarrow 6} + \dots + w_{(n-1) \rightarrow n}$$

$$\Rightarrow w_{1 \rightarrow n} = GM_e m \left[\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_3} - \frac{1}{r_4} + \frac{1}{r_4} - \frac{1}{r_5} + \frac{1}{r_5} - \frac{1}{r_6} + \dots + \frac{1}{r_{n-1}} - \frac{1}{r_n} \right]$$

$$\Rightarrow w_{1 \rightarrow n} = GM_e m \left[\frac{1}{r_1} - \frac{1}{r_n} \right] \quad (A)$$

Now we apply the conditions for absolute P.E i.e. we consider the point "1" to be at earth's surface and point "n" at infinity. So equation (A) becomes,

$$w_{e \rightarrow \infty} = GM_e m \left[\frac{1}{R_e} - \frac{1}{\infty} \right] \quad \left[\begin{array}{l} n = \infty \\ r_n = \infty \end{array} \right]$$

$$\Rightarrow w_{e \rightarrow \infty} = GM_e m \left[\frac{1}{R_e} - 0 \right] = \frac{GM_e m}{R_e} \quad \left[\frac{1}{\infty} = 0 \right]$$

$$\Rightarrow w_{e \rightarrow \infty} = \frac{-GM_e m}{R_e} \quad (B)$$

Equation (B) represents the workdone against the earth's gravitational field in displacing a body from the earth's surface to a point at infinity. The negative sign is used which shows that work is done

against gravity.

Q15: Define and explain escape velocity.

ETEA 2011, BISE Mardan 2019, BISE Abbottabad 2019

Ans: Definition:

When a body is thrown vertically upwards and it goes out of earth's gravitational field, then the velocity of the body is known as escape velocity.

Explanation:

We know that the workdone in lifting a body from earth's surface to infinity is given by,

$$W_{\infty} = \frac{GM_e m}{R_e} \quad (1)$$

Here the workdone " W_{∞} " must be equal to the K.E of the body.

So equation (1) becomes,

$$K.E = \frac{GM_e m}{R_e} \quad \left[\text{Work} = K.E = \frac{1}{2} mv^2 \right]$$

$$\Rightarrow \frac{1}{2} mv^2 = \frac{GM_e m}{R_e}$$

$$\Rightarrow \frac{v^2}{2} = \frac{GM_e}{R_e} \Rightarrow v^2 = \frac{2GM_e}{R_e} \quad (2)$$

Dividing and multiplying the R.H.S of equation (2) by " R_e ", we get,

$$\Rightarrow v^2 = 2R_e \left(\frac{GM_e}{R_e^2} \right) \quad \left[\frac{GM_e}{R_e^2} = g \right]$$

$$\Rightarrow v^2 = 2R_e g$$

$$\Rightarrow v_{es} = \sqrt{2gR_e} \quad (3)$$

Equation (3) represents the relation for escape velocity. Now we know that,

$$g = 9.8 \text{ m/sec}^2, R_e = 6.4 \times 10^6 \text{ m}$$

So equation (3) becomes,

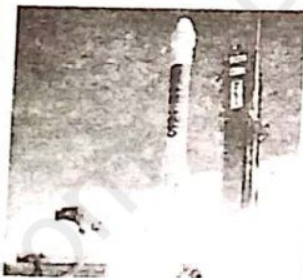
$$v_{es} = \left[\sqrt{2 \times 9.8 \times 6.4 \times 10^6} \right] \text{ m/sec}$$

$$\Rightarrow v_{es} = 11.2 \times 10^3 \text{ m/sec} \quad [10^3 = \text{kilo}]$$

$$(OR) v_{es} = 11.2 \text{ km/sec}$$

NOTE: The above value of escape velocity is for earth's gravitational field. For

For Your Information:



A delta II rocket blasting off. A rocket going into space needs large amount of energy to achieve escape velocity in order to make it off earth and get into space.

any other planet the value of escape velocity will be different and can be computed on the value of mass and radius of the planet or on the value of " g " in the field of that planet and radius of that planet.

Q16: State and explain the law of conservation of energy.

Ans: Statement:

This law states that energy cannot be created or destroyed. It may be changed from one form to another, but its total value remains constant.

Explanation: If we allow a body to fall freely under gravity and we ignore the friction of the air, then during downwards motion, the loss in P.E will be equal to the gain in K.E. i.e.

$$\text{loss in P.E} = \text{gain in K.E} \quad (1)$$

Now according to the law of conservation of energy, equation (1) can be also written as,

$$\text{total energy} = P.E + K.E = \text{constant} \quad (2)$$

According to Einstein mass-energy equation, we have,

$$E = mc^2 \quad (3)$$

Equation (3) shows that mass and energy are interconvertible. That is due to decrease in energy, there will be increase in mass. While due to decrease in mass, there will be increase in energy. Thus we can say that conservation of mass and energy occurs together rather than separately.

Q17: Discuss the inter-conversion of potential energy and kinetic energy.

Ans: Consider a body of mass " m " and weight " w " is placed at point "A", as shown in the figure.

(a) Consider Point "A":

At point "A", the P.E of the body is given by,

$$(P.E)_A = mgh \quad (1)$$

The K.E of the body at point "A" is given by,

$$(K.E)_A = 0 \quad (2)$$

Now the total energy at "A" is given by,

$$(E_T)_A = (K \cdot E)_A + (P \cdot E)_A = 0 + mgh$$

$$\Rightarrow (E_T)_A = mgh \quad (3)$$

(b) Consider Point "B":

Let the body is allowed to fall freely from "A" and reaches another point "B" by covering a distance "x" as shown in the figure. Now at point "B", the P·E is given by,

$$(P \cdot E)_B = mg(h - x) \quad (4)$$

Now the workdone on the body from "A" to "B" is given by,

$$\text{Workdone} = Fd \quad \left[\begin{array}{l} F = w = mg \\ d = x \end{array} \right]$$

$$\Rightarrow \text{Workdone} = mgx$$

$$\Rightarrow \text{Workdone} = mgx \quad (5)$$

This workdone must be equal to the K·E of the body at point "B", so equation (5) becomes,

$$(K \cdot E)_B = mgx \quad (6)$$

Now the total energy at "B" is given by,

$$(E_T)_B = (K \cdot E)_B + (P \cdot E)_B$$

$$\Rightarrow (E_T)_B = mgx + mg(h - x)$$

$$\Rightarrow (E_T)_B = \cancel{mgx} + mgh - \cancel{mgx}$$

$$\Rightarrow (E_T)_B = mgh \quad (6)$$

(c) Consider Point "C":

When the body reaches at point "C", then the P·E of the body at "C" is given by,

$$(P \cdot E)_C = mgh \Rightarrow (P \cdot E)_C = mg \times 0 \quad [\text{At "C", } h = 0]$$

$$\Rightarrow (P \cdot E)_C = 0 \quad (7)$$

Now the workdone on the body in reaching point "C" is given by,

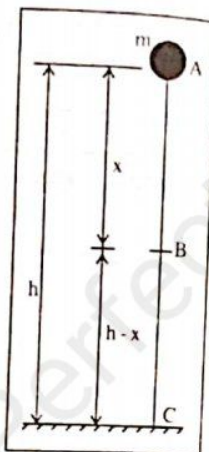
$$\text{Workdone} = fd = mgh \quad \left[\begin{array}{l} F = w = mg \\ d = h \end{array} \right]$$

$$\Rightarrow \text{Workdone} = mgh \quad (8)$$

This work must be equal to the K·E of the body, so equation (8) becomes,

$$(K \cdot E)_C = mgh \quad (9)$$

Now the total energy at "C" is given by,



$$(E_T)_C = (K \cdot E)_C + (P \cdot E)_C$$

$$\Rightarrow (E_T)_C = mgh + 0$$

$$\Rightarrow (E_T)_C = mgh \quad (10)$$

Conclusion:

Comparing equations (3), (6), (10), we get,

$$(E_T)_A = (E_T)_B = (E_T)_C$$

This shows that energy may be changed from one form to another but its total value remains the same. Thus the law of conservation of energy is satisfied here.

NOTE: When there is no force of friction, then we have,

$$\text{Loss in P·E} = \text{Gain in K·E} \quad (A)$$

But when there is force of friction, then same part of P·E is used in workdone against force of friction. So equation (A) can be written as,

$$\text{Loss in P·E} = \text{Gain in K·E} + F_f h \quad (B)$$

Equation (B) is valid for downwards motion of a body. For upwards motion, equation (B) can be written as,

$$\text{Loss in K·E} = \text{Gain in P·E} + F_f h \quad (C)$$

Workdone against the force of friction = $F_f h$

Q18: (a) What is energy? Discuss its importance. (b) Discuss the non-renewable and renewable sources of energy.

Ans: (a) Energy means power to work. (OR)

The ability of a body to do work is known as energy.

Importance of Energy:

As we know that energy represents the ability of a body to do work i.e. work is possible at the expenditure of energy. The energy is used to operate industry, automobiles, stoves, washing machines, fans, air conditioners, computing system, radio sets, T·V sets etc in daily life. Human beings use the various forms of energy for their needs and desires. Presently, about 6×10^{13} kilo watt-hour energy is being used in the world annually.

In short, energy plays a tremendous role in our daily life and without energy, we cannot imagine any machine in working conditions.

(b) (i) Non-Renewable Sources of Energy:

Those sources of energy which cannot be renewed are known as non-renewable sources of energy. About 85% of the energy is used in the world from non-renewable sources. Some of the important non-renewable sources of energy are given below:

(i) **Coal:** Coal is the most abundant fossil fuel in the world with an estimated reserve of one trillion metric tons. The coal is formed slowly over million of years from the buried remains of ancient swamp plants. The coal results from differences in the pressure and temperature that prevailed during formation. Currently, the world is consuming coal at the rate of about 5 billion metric tons per year. The coal is mainly used for power generation, heating and cooking purposes. If the consumption continues at the same rate, the current reserves will last for more than 200 years. The burning of coal results in significant atmospheric pollution.

(ii) **Oil:** Liquid petroleum or crude oil is a fossil fuel that is refined into many different energy products. These products include gasoline, diesel fuel, jet fuel, heating oil etc. The oil is formed in rocks such as shale, which is rich in organic materials. The oil is mainly used as a source of energy. It is also used as a chemical in various industries.

(iii) **Natural Gas:** The natural gas is a mixture of gases, the most common of which is methane. It is supplied to industry and houses through pipe lines system. The natural gas can be also supplied in cylinder. The cylinders are used in those areas, where pipe line system is not available. For this purpose, the natural gas is made into liquefied petroleum gas [LPG]. LPG is shipped in special pressurized tanks as a fuel source.

The use of natural gas is growing rapidly. It is used in industries for various purposes. It is also used for cooking, heating and powering vehicles. It is also used in a process for making ammonia fertilizer. The current estimate of natural gas reserves is about 100 million metric tons. At current usage levels, this supply will last an estimated 100 years.

In Pakistan oil is being extracted from areas situated in Pothohar

Punjab and Sindh. The natural gas [sui gas] is being supplied for household and industrial use from huge reservoirs of gas located at sui in Baluchistan. Some smaller deposits have been found in Sindh, Punjab, Khyber Pakhtunkhwa and other parts of the country.

(ii) Renewable Energy Sources:

Those sources of energy which can be renewed are known as renewable sources of energy. Some of the renewable energy sources are given below:

(i) **Energy from Biomass:** The biomass is a matter which is also called as garbage. It includes dead trees, yard clippings, left-over crops, wood chips, bark and sawdust from lumber mills.

The trucks bring the waste from factories and from farms to a biomass power plant. This is then fed into a furnace where it is burned. The heat is used to boil water in the boiler and the energy in the steam is used to turn turbines and generators.

In another process, the garbage is decomposed to produce methane gas. The animals like cattles, crows, kitchens etc. produce manure. When manure decomposes, it also gives off methane gas. This gas can be burned right at the farm to make energy to run the farm.

(ii) **Electrical Energy:** Electrical energy is the scientific form of electricity. It refers to the flow of power or the flow of charges along a conductor to create energy. These other forms of energy are known as the primary sources of energy and can be used from coal, nuclear energy, natural gas, or oil.

Hydro-electric plants are installed at big dams. The big dams produce electric power in the range of thousands of mega watt (MW). Presently Terbela Dam produces thousands of mega watt power of electricity. Mangla Dam on the other hand produces more than 900MW. Besides these, Warsak Dam, Malakand Dam also produces huge amount electricity.

(iii) **Energy from Waves:** The oceans are vast and full of energy produced by the sun and wind. This makes the wave energy as a potential power platform. The waves having large amplitude possess large amount of energy. The wave energy is in fact a stored and concentrated form of solar energy.

(iv) **Geothermal Energy:** The sun rays beating down on the land surface is the major source of geothermal energy. The geothermal energy is obtained by tapping the energy of the hot, molten areas inside the earth crust when water is pumped down to the hot rocks, it becomes as steam. This steam is returned and is used to drive turbines. This steam is also used for direct heating.

(v) **Nuclear Energy:** The nuclear energy is obtained from nuclear reactions. Nuclear fusion and nuclear fission are nuclear reactions in which tremendous amount of energy is produced. Sun and other stars are the main sources of nuclear fusion in which huge amount of energy is released. The nuclear fission reaction occurs when a particle [projectile] is allowed to fall on a nuclear material like uranium. When the particle is absorbed by uranium, a compound nucleus is formed. This compound nucleus is very much unstable and disintegrates to give various outgoing product with the release of tremendous amount of energy known as nuclear energy. This energy can be obtained in a controlled manner with the help of moderators in a nuclear reactor.

(vi) **Tidal Energy:** The tidal energy is produced in tidal power stations due to rise and fall of water in narrow tunnels. The tidal energy is used for power generation.

(vii) **Solar Energy:** Sun is one of the major sources of solar energy. The solar energy is concentrated and converges with the help of mirrors, lenses, solar cells etc and are used for heating, cooking and production of electricity.

(viii) **Wind Energy:** The non uniform distribution of heat due to solar energy in different regions causes the movement of hot and cold air over the earth's surface, winds blows from area of high pressure to areas of low pressure. At sea, the winds are even stronger than on the land surface. An equivalent of 100 billion watts per year of power in the shape of wind energy is available on the earth. In the windy regions, wind mills are installed to produce mechanical energy. This mechanical energy may be used in tube wells or flour mills. In Pakistan places like Chitral, Gilgit, coastal areas and high mountain valleys are suitable for the production of wind power.

سب سے بڑا جابلو وہ ہے جو گناہ سے باخبر ہوئے ہوئے بھی گناہ کرتا ہے۔

Solution of Examples & Assignments

EXAMPLE 4-1:

Calculate the power required of a 1400kg car under the following circumstances: (a) the car climbs a 10° hill (a fairly steep hill) at a steady speed 80km/hr, (b) the car accelerates along a level road from 90 kmh⁻¹ to 110 kmh⁻¹ in 6.0s. Assume the average retarding force on the car is $F_r = 700\text{N}$ throughout.

Solution:

Given data:

$$\text{Speed on hill} = v = 80\text{km/hr} = \left(\frac{20 \times 1000}{3600}\right) \text{m/sec} = 22\text{m/sec}$$

$$\text{Mass} = m = 1400\text{kg}, \text{ angle of climb} = 10^\circ, \text{ Initial speed} = v_i = 90\text{kmh}^{-1}$$

$$\Rightarrow v_i = \frac{90 \times 1000}{3600} \text{m/sec} = 25\text{m/sec}$$

$$\text{Final velocity} = v_f = 110\text{km/hr}$$

$$= \frac{110 \times 1000}{3600} \text{m/sec} = 30.5\text{m/sec}$$

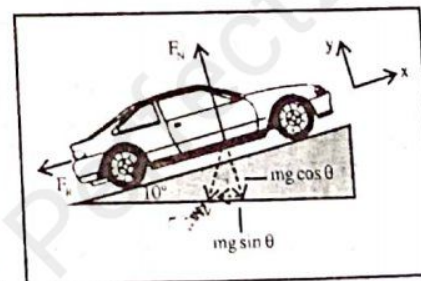
$$\text{Time taken} = t = 6\text{sec}, \text{ retarding force} = F_r = 700\text{N}$$

Required: Power $P = ?$

Calculation:

(a) When the car climb the hill, the normal force " F_N " on the car is acting in upward direction. But the weight " w " of car is not exactly acting in opposite direction to normal for " F_N ". So we resolve the weight " w " into its horizontal and vertical components. The component " $mg \cos \theta$ " balance the normal force " F_N " while the component " $mg \sin \theta$ " is acting in opposite direction of the car motion. Thus the applied force on the car must be equal to the sum of retarding force " F_r " and " $mg \sin \theta$ ". i.e.

$$F_{app} = F_r + mg \sin \theta = 700 + (1400 \times 9.8 \times \sin 10^\circ)$$

$$\Rightarrow F_{app} = (700 + 2382.5)\text{N} \Rightarrow F_{app} = 3082.5\text{N}$$


$$\text{Now } P = F \cdot v = (3082.5 \times 22) \text{ watt} \Rightarrow P = 67815.0 \text{ watt}$$

$$\Rightarrow P = 67.8 \times 10^3 \text{ watt} = 68 \times 10^3 \text{ watt} \Rightarrow \boxed{P = 68 \text{ kWatt}} \quad [10^3 = \text{kilo}]$$

Power in Horsepower:

$$P = 68 \times 10^3 \text{ watt} \quad [\text{We have } 746 \text{ watt} = 1 \text{ h.p.}]$$

$$\Rightarrow P = \frac{68 \times 10^3}{746} = \frac{68000}{746} \text{ h.p.} \Rightarrow \boxed{P = 91 \text{ h.p.}}$$

(b) The acceleration of car on level road is given by,

$$a_s = \frac{(v_f - v_i)}{t} = \left[\frac{30.5 - 25}{6} \right] \text{ m/sec}^2 \Rightarrow a_s = 0.92 \text{ m/sec}^2$$

Now the force required for producing the horizontal acceleration "a_s" is given by, $F_s = ma_s = [1400 \times 0.92] \text{ N} = 1288 \text{ N}$

Now the total force exerted by the car must be equal to the retarding force plus the force for producing acceleration "a_s", so we get,

$$F = F_s + F_r = [1288 + 700] \text{ N}$$

$$\Rightarrow \boxed{F = 1988 \text{ N}}$$

Now power in this case is given by,

$$P = F \cdot v_s = [1988 \times 30.5] \text{ watt}$$

$$\Rightarrow P = 60634 \text{ watt} = 60.634 \times 10^3 \text{ watt} = 60.6 \times 10^3 \text{ watt}$$

$$\Rightarrow P = 61 \times 10^3 \text{ watt} = 61 \text{ kWatt} \Rightarrow \boxed{P = 61 \text{ kWatt}}$$

$$(\text{OR}) P = \left[\frac{61 \times 10^3}{746} \right] \text{ h.p.} \Rightarrow P = 81.8 \text{ h.p.} \Rightarrow \boxed{P = 82 \text{ h.p.}}$$

EXAMPLE 4.2:

Find the work required to lift a mass of 5 tonnes to a height of 30m. If this is done in 2 minutes, what power is being used?

Solution:

Given data:

$$\text{Mass} = m = 5 \text{ ton} = 5 \times 1000 \text{ kg}$$

$$[1 \text{ ton} = 1000 \text{ kg}]$$

$$\Rightarrow \text{Mass} = m = 5000 \text{ kg, height} = h = 30 \text{ m}$$

$$\text{Time} = t = 2 \text{ min} = 2 \times 60 \text{ s} = 120 \text{ s}$$

Required: (a) Work = w = ?, (b) Power = P = ?

Calculation: We know that,

$$\text{Work} = F \cdot d \quad (1)$$

$$\Rightarrow \text{Work} = w \cdot h = mgh$$

So equation (1) becomes

$$[\text{Put } F = w = mg, d = h]$$

$$\text{Work} = mgh \quad (2)$$

Putting the values in equation (2), we get,

$$\text{Work} = [5000 \times 9.81 \times 30] \text{ J} = [1471500] \text{ J}$$

$$\Rightarrow \boxed{\text{Work} = 1471500 \text{ J}}$$

(b) We know that,

$$P = \frac{w}{t} \quad (1)$$

Putting the values in equation (3), we get,

$$P = \left(\frac{1471500}{120} \right) \text{ watt} = 12262.5 \text{ watt}$$

$$\text{Ans. } \boxed{P = 12262.5 \text{ watt}}$$

ASSIGNMENT 4.1:

What is the power of an airplane of mass 3000kg if when on the runway it is capable of reaching a speed of 80ms⁻¹ from rest in 4.0s? (1.2MW)

Solution:

Given data:

$$\text{Mass of airplane } m = 3000 \text{ kg}$$

$$\text{Speed of airplane} = v = 80 \text{ m/sec, time taken} = t = 4 \text{ sec}$$

Required: Power of airplane = P = ?

Calculation: We know that,

$$P = F \cdot v = w \cdot v$$

$$[F = w = mg]$$

$$\Rightarrow P = mgv \quad (1)$$

Putting the values in equation (1), we get,

$$P = [3000 \times 9.8 \times 80] \text{ watt} = 2352000.0 \text{ watt}$$

$$\Rightarrow P = 2.35 \times 10^6 \text{ watt} = 2.35 \text{ mega watt} \quad [10^6 = \text{Mega}]$$

$$\text{Ans. } \boxed{P = 2.35 \text{ Mega watt}}$$

EXAMPLE 4.3:

A machine needed 1000J of energy to raise a 10kg block at a distance of 6.0m. What is the machine efficiency?

Solution:

Given data:

$$\text{Input work} = 1000 \text{ J, mass} = m = 10 \text{ kg, distance} = d = 6.0 \text{ m}$$

Required: Machine efficiency = η = ?

Calculation: We know that the efficiency of a machine is given by,

$$\eta = \frac{\text{output work}}{\text{input work}} = \frac{F \cdot d}{\text{input work}} = \frac{mgh}{\text{input work}} \quad \left[\begin{array}{l} F = w = mg \\ d = h \end{array} \right]$$

$$\Rightarrow \eta = \frac{mgh}{\text{input work}} \quad (1)$$

Putting the values in equation (1), we get,

$$\eta = \left(\frac{10 \times 9.8 \times 6}{1000} \right) \Rightarrow \boxed{\eta = 0.588} \quad (2)$$

In terms of percentage equation (2) can be written as,

$$\Rightarrow \eta = (0.588 \times 100)\%$$

$$\Rightarrow \eta = 58.8\% = 59\% \Rightarrow \boxed{\eta = 59\%}$$

EXAMPLE 4.4:

Block and tackle system of pulleys is used to raise a load of 500N through a height of 20m. The workdone against friction is 2000J. Calculate the (a) workdone by the effort (b) the efficiency of the system.

Solution:

Given data:

Load = $w = 500\text{N}$, height = $h = 20\text{m}$, work against friction = 2000J

Required: (a) Workdone by effort = ? (b) efficiency = η = ?

Calculation: (a) Workdone by effort = input work = workdone in raising load + workdone against friction

$$\Rightarrow \text{Input work} = [F \cdot h + 2000] \text{ J} \quad [F = w = 500\text{N}]$$

$$\Rightarrow \text{Input work} = (500 \times 20 + 2000) \text{ J}$$

$$\Rightarrow \text{Input work} = 12000 \text{ J}$$

$$\Rightarrow \boxed{\text{Input work} = 12000 \text{ J}}$$

(b) The efficiency of pulley system is given by,

$$\eta = \frac{\text{output work}}{\text{input work}} = \frac{F \cdot d}{\text{input work}} = \frac{w \cdot h}{\text{input work}}$$

$$\Rightarrow \eta = \left[\frac{500 \times 20}{12000} \right] = 0.83 \Rightarrow \boxed{\eta = 0.83} \quad (1)$$

In terms of percentage, equation (1) can be written as,

$$\eta = 0.83 \times 100\% \Rightarrow \boxed{\eta = 83\%}$$

EXAMPLE 4.5:

The moon's radius is $1.6 \times 10^6 \text{ m}$ and the acceleration due to gravity, $g = 1.6 \text{ ms}^{-2}$ on its surface. Find out the escape velocity from moon's surface.

Solution:

Given data:

Radius of moon = $R_m = 1.6 \times 10^6 \text{ m}$

Acceleration due to gravity = $g_m = 1.6 \text{ ms}^{-2}$

Required: Escape velocity = v_{esc} = ?

Calculation: We know that,

$$v_{esc} = \sqrt{2gR_s} \quad (1)$$

For moon, equation (1) can be written as,

$$v_{esc} = \sqrt{2g_m R_m} \quad (2)$$

Putting the values in equation (2), we get,

$$v_{esc} = \left[\sqrt{2 \times 1.6 \times 1.6 \times 10^6} \right] \text{ m/sec} = \left[\sqrt{5.12 \times 10^6} \right] \text{ m/sec}$$

$$\Rightarrow v_{esc} = 2.26 \times 10^3 \text{ m/sec} = 2.3 \times 10^3 \text{ m/sec}$$

$$\text{Ans: } \boxed{v_{esc} = 2.3 \times 10^3 \text{ m/sec}}$$

EXAMPLE 4.6:

Compare the escape speed of a rocket launched from the moon with earth. The mass of the moon is $7.35 \times 10^{22} \text{ kg}$ and the radius is $1.74 \times 10^6 \text{ m}$.

Solution:

Given data: Mass of moon = $M_m = 7.35 \times 10^{22} \text{ kg}$

Radius of moon = $R_m = 1.74 \times 10^6 \text{ m}$

Required: Speed = v = ?

Calculation: We know that,

$$v_{escape} = \sqrt{\frac{2GM_s}{R_s}} \quad (1)$$

For moon, equation (1) can be written as,

$$v_{es} = \sqrt{\frac{2GM_m}{R_m}} \quad (2)$$

Putting the values in equation (2), we get,

$$v_{es} = \left[\sqrt{\frac{2 \times 6.673 \times 10^{-11} \times 7.35 \times 10^{22}}{1.74 \times 10^6}} \right] \text{ m/sec}$$

$$\Rightarrow v_{es} = \left[\sqrt{\frac{98.0931 \times 10^{-11+22-6}}{1.74}} \right] \text{ m/sec}$$

$$= \left[\sqrt{56.375 \times 10^5} \right] \text{ m/sec}$$

$$\Rightarrow v_{es} = \left[\sqrt{563.75 \times 10^4} \right] \text{ m/sec}$$

$$v_{es} = 23.7 \times 10^2 \text{ m/sec} = 2370.0 \text{ m/sec}$$

$$\text{Ans. } v_{es} = 2370.0 \text{ m/sec}$$

ASSIGNMENT 4.2:

How fast would be the moon need to travel in order to escape the gravitational pull of earth, if earth has a mass of $5.98 \times 10^{24} \text{ kg}$ and the distance from earth to the moon is $3.84 \times 10^8 \text{ m}$? (1441 ms^{-1})

Solution: Given data: Mass of earth = $M_e = 5.98 \times 10^{24} \text{ kg}$

Distance between earth and moon = $r = 3.84 \times 10^8 \text{ m}$

Required: Escape velocity of moon = $v_{es} = ?$

Calculation: We know that,

$$v_{es} = \sqrt{\frac{2GM_e}{r}} \quad (1)$$

Putting the values in equation (1), we get,

$$v_{es} = \left[\sqrt{\frac{2 \times 6.673 \times 10^{-11} \times 5.98 \times 10^{24}}{3.84 \times 10^8}} \right]$$

$$\Rightarrow v_{es} = \left[\sqrt{\frac{79.80908 \times 10^{-11+24-8}}{3.84}} \right] \text{ m/sec} = \left[\sqrt{20.78 \times 10^5} \right] \text{ m/sec}$$

$$\Rightarrow v_{es} = \left[\sqrt{207.8 \times 10^4} \right] \text{ m/sec} = 14.41 \times 10^2 \text{ m/sec}$$

$$\text{Ans. } v_{es} = 1441.0 \text{ m/sec}$$

EXAMPLE 4.7:

A ball of mass 100 g is thrown vertically upward at a speed of 25 ms^{-1} . If no energy is lost, determine the height it would reach. If the ball only rises to 25 m , calculate the workdone against air resistance. Also calculate the force of friction?

Solution: Given data:

(a) Mass = $m = 100 \text{ g} = 0.1 \text{ kg}$

Speed of ball = $v = 25 \text{ m/sec}$

Required: (a) Height = $h = ?$

(b) If height = $h = 25 \text{ m}$

Then workdone against air resistance = $w = ?$

(c) Force of friction = $F_f = ?$

Calculation: (a) If no energy is lost, then

Loss in K.E = Gain in P.E

$$\Rightarrow \frac{1}{2} mv^2 = mgh \Rightarrow \frac{v^2}{2} = gh$$

$$\Rightarrow h = \frac{v^2}{2g} = \frac{25^2}{2 \times 9.8} = 31.9 \text{ m} \Rightarrow \boxed{h = 31.9 \text{ m}}$$

(b) In presence of air resistance, we have,

Loss in K.E = Gain in P.E + Workdone

$\Rightarrow$ Workdone = Loss in K.E - Gain in P.E

$$\Rightarrow \text{Workdone} = \frac{1}{2} mv^2 - mgh$$

$$\Rightarrow w = \left(\frac{1}{2} \times 0.1 \times 25^2 \right) - (0.1 \times 9.8 \times 25)$$

$$\Rightarrow w = (31.25 - 24.5) = 6.75 \text{ J} \Rightarrow \boxed{w = 6.75 \text{ J}}$$

(c) Now we know that,

$$\text{Work} = F_f \cdot h \Rightarrow F_f = \frac{\text{work}}{h}$$

$$\Rightarrow F_f = \left(\frac{6.75}{25} \right) \text{ N} \Rightarrow \boxed{F_f = 0.27 \text{ N}}$$

ASSIGNMENT 4.3:

Consider a person on a sled sliding down a 100 m long hill on a 30° incline. The mass is 20 kg , and the person has a velocity of 2 ms^{-1} down the hill when they're at the top. (a) How fast is the person travelling at the bottom of the hill? (b) If, the velocity at

the bottom of the hill is 10ms^{-1} , because of friction. How much work is done by friction?

Solution: Given data:

Length of hill = $L = 100\text{m}$

Angle of inclination = $\theta = 30^\circ$

Mass of person = 20kg

Velocity at the top of hill

$$= v_i = 2\text{m/sec}^{-1}$$

Required:

(a) Velocity at the bottom = $v_f = ?$

(b) Workdone against friction

$$= \text{Work} = ? \text{ [if } v_f = 10\text{m}\cdot\text{s}^{-1}]$$

Calculation: (a) In $\triangle ABC$, we have,

$$\sin \theta = \frac{h}{L} \Rightarrow h = L \sin \theta = 100 \times \sin 30^\circ = 100 \times 0.5$$

$$\Rightarrow h = 50\text{m}$$

Now by law of conservation of energy, we have,

Total energy at bottom = total energy at top

$$\Rightarrow (K\cdot E)_{\text{bottom}} = (P\cdot E)_{\text{top}} + (K\cdot E)_{\text{top}}$$

$$\Rightarrow \frac{1}{2}mv_b^2 = mgh + \frac{1}{2}mv_i^2 \Rightarrow \frac{1}{2}v_b^2 = gh + \frac{1}{2}v_i^2$$

$$\Rightarrow v_b^2 = 2gh + v_i^2 = [2 \times 9.8 \times 50 + 2^2] \text{m}^2\cdot\text{s}^{-2} = (980 + 4)\text{m}^2\cdot\text{s}^{-2}$$

$$\Rightarrow v_b^2 = 984\text{m}^2\cdot\text{s}^{-2} \Rightarrow v_b = \sqrt{984\text{m}^2\cdot\text{s}^{-2}} = 31.3\text{m}\cdot\text{s}^{-1}$$

$$\Rightarrow \boxed{v_b = 31.3\text{m}\cdot\text{s}^{-1}}$$

Part (b): In case of force of friction, we have,

Total energy at top = $K\cdot E$ at bottom + work against friction

$$\Rightarrow (P\cdot E)_{\text{top}} + (K\cdot E)_{\text{top}} = (K\cdot E)_{\text{bottom}} + \text{work}$$

$$\Rightarrow \text{Work} = (P\cdot E)_{\text{top}} + (K\cdot E)_{\text{top}} - (K\cdot E)_{\text{bottom}}$$

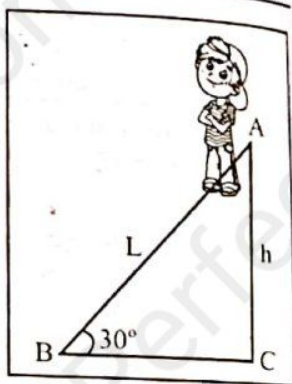
$$\Rightarrow \text{Work} = mgh + \frac{1}{2}mv_i^2 - \frac{1}{2}mv_b^2$$

$$= 20 \times 9.8 \times 50 + \frac{1}{2} \times 20 \times 2^2 - \frac{1}{2} \times 20 \times 10^2$$

$$\Rightarrow \text{Work} = (9800 + 40 - 1000) \text{J} = (9840 - 1000) \text{J} = 8840\text{J}$$

$$\Rightarrow \text{Work} = 8840\text{J}$$

$$\Rightarrow \boxed{\text{Work} = -8840\text{J}} \text{ [Workdone against friction is taken as negative]}$$



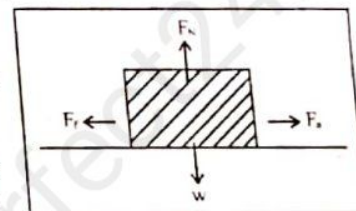
Exercise

Choose the best possible answer:

1. You push a heavy crate down a ramp at a constant velocity. Only four forces act on the crate. Which force does the greatest magnitude of work on the crate?

- a. The force of friction b. The force of gravity
c. The normal force d. The force you pushing

Sol: The normal force " F_N " does not work as it is perpendicular to direction of motion of crate. The applied force and a component of gravity are responsible for the motion of the crate. The force of friction opposes the motion of the crate. Under balanced condition, the force of friction is equal to the sum of applied force and component of gravity. So the frictional force is greater than any other individual force.



2. The force constant of a wire is k and that of the another wire is $3k$ when both the wires are stretched through same distance, if workdone are w_1 and w_2 , then:

- a. $w_2 = w_1$ b. $w_2 = 9w_1$
c. $w_1 = 3w_2$ d. $w_2 = 3w_1$

Sol: Workdone in stretching the string of constant " K " is through displacement " x " is given by,

$$w_1 = Kx^2 \quad (1)$$

While the workdone in stretching the string of constant " $3K$ " through the same displacement " x " is given by,

$$w_2 = (3K)x^2 \quad (2)$$

Dividing equation (2) by equation (1), we get,

$$\frac{w_2}{w_1} = \frac{(3K)x^2}{Kx^2} \Rightarrow \frac{w_2}{w_1} = 3 \Rightarrow \boxed{w_2 = 3w_1}$$

3. Escape velocity on the surface of the earth is 11.2kms^{-1} . If the mass of the earth increases to twice its value and the radius

of the earth becomes half the escape velocity is:

- a. 5.6kms⁻¹ b. 11.2kms⁻¹
c. 22.4kms⁻¹ ✓ d. 33.6kms⁻¹

Sol: $v_{es} = 11.2 \text{ km/sec}$ _____ (1)

We also know that,

$$v_{es} = \sqrt{\frac{2GM_e}{R_e}} \text{ _____ (2)}$$

When $M_e = 2M_e$ and $R_e = \frac{R_e}{2}$

The equation (2) becomes,

$$v'_{es} = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{2G \times 2M_e}{\left(\frac{R_e}{2}\right)}} = \sqrt{4 \left(\frac{2GM_e}{R_e}\right)}$$

$$\Rightarrow v'_{es} = 2 \sqrt{\frac{2GM_e}{R_e}} \text{ _____ (3)}$$

Putting equation (2) in equation (3), we get,

$$v'_{es} = 2v_{es} \text{ _____ (4)}$$

Putting equation (1) in equation (4), we get,

$$v'_{es} = (2 \times 11.2) \text{ km/sec} \Rightarrow v'_{es} = 22.4 \text{ km/sec}$$

4. An example of non-conservative force is:

- a. Electric force b. Gravitational force
c. Frictional force ✓ d. Magnetic force

5. When the speed of your car is doubled, by what factor does its kinetic energy increase?

- a. $\sqrt{2}$ b. 2 c. 4 ✓ d. 8

Sol: We know that, $K.E = \frac{1}{2}mv^2$ _____ (1)

When speed is double, then equation (1) become,

$$K.E' = \frac{1}{2}m(2v)^2 = \frac{1}{2}m(4v^2)$$

$$\Rightarrow K.E' = 4 \left(\frac{1}{2}mv^2 \right) \text{ _____ (2)}$$

Putting equation (1) in equation (2), we get,

$$K.E' = 4K.E$$

6. One horse power is given by:

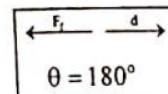
- a. 746 watts ✓ b. 746J

d. 746 foot pounds

7. Work is said to be negative when $\vec{F}$ and $\vec{d}$ are:
a. Parallel b. Anti parallel ✓
c. Perpendicular d. at 45°

$$\text{Sol: } W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

$$\Rightarrow W = Fd \cos 180^\circ = Fd(-1)$$



NOTE: Work done against force of friction is the example of negative work.

$$\Rightarrow W = -Fd$$

So work is said to be negative if " $\vec{F}$ " and " $\vec{d}$ " are anti parallel.

8. Two bodies of masses m_1 and m_2 have equal momentum their kinetic energies E_1 and E_2 are in the ratio:

- a. $\sqrt{m_1} : \sqrt{m_2}$ b. $m_1 : m_2$
c. $m_2 : m_1$ ✓ d. $\sqrt{m_1^2} : \sqrt{m_2^2}$

Sol: Momentum of 1st body of mass " m_1 " is given by,

$$p_1 = \sqrt{2m_1E_1} \text{ _____ (1)}$$

$$\text{Similarly, } p_2 = \sqrt{2m_2E_2} \text{ _____ (2)}$$

$$\text{As } p_1 = p_2 \Rightarrow \sqrt{2m_1E_1} = \sqrt{2m_2E_2}$$

$$\Rightarrow \cancel{2}m_1E_1 = \cancel{2}m_2E_2 \Rightarrow \frac{E_1}{E_2} = \frac{m_2}{m_1}$$

$$\Rightarrow E_1 : E_2 = m_2 : m_1$$

9. The atmosphere is held to the earth by:

- a. Winds b. Gravity ✓
c. Clouds d. The rotation of earth

10. If momentum is increased by 20% then K.E increases by:

- a. 44% ✓ b. 55% c. 66% d. 77%

Sol: The initial K.E is given by,

$$K.E = \frac{p^2}{2m} \text{ _____ (1)}$$

When the initial momentum is increased by 20% then change in momentum is given by,

$$\Delta p = 20\%p, \Rightarrow \Delta p = \frac{20}{100}p, \Rightarrow \Delta p = 0.2p,$$

Now final momentum is given by,

$$p_f = p + \Delta p = p + 0.2p, \Rightarrow p_f = 1.2p$$

Now the final K.E is given by,

$$K \cdot E_i = \frac{p_i^2}{2m} = \frac{(1.2p_i)^2}{2m} = \frac{1.44p_i^2}{2m}$$

Now percentage change in $K \cdot E$ is given by,

$$\% \text{ increase in } K \cdot E = \left[\frac{K \cdot E_f - K \cdot E_i}{K \cdot E_i} \right] \times 100\%$$

$$\Rightarrow \% \text{ increase in } K \cdot E = \left[\frac{1.44 \frac{p_i^2}{2m} - \frac{p_i^2}{2m}}{\frac{p_i^2}{2m}} \right] \times 100\%$$

$$\Rightarrow \% \text{ increase in } K \cdot E = \frac{p_i^2}{2m} \left[\frac{1.44 - 1}{\frac{p_i^2}{2m}} \right] \times 100\%$$

$$\Rightarrow \% \text{ increase in } K \cdot E = (0.44 \times 100)\% = 44\%$$

$$\Rightarrow \% \text{ increase in } K \cdot E = 44\%$$

11. If the $K \cdot E$ of a body becomes four times of the initial value, then new momentum will:

- a. become twice its initial value ✓
- b. become three times, its initial value
- c. become four times, its initial value
- d. remains constant

Sol: We know that,

$$p_i = \sqrt{2mK \cdot E_i} \quad (1)$$

When $K \cdot E$ becomes four times, equation (1) become,

$$p_2 = \sqrt{2m(4K \cdot E_i)} \Rightarrow p_2 = 2\sqrt{2mK \cdot E_i} \quad (2)$$

Putting equation (1) in equation (2), we get,

$$p_2 = 2p_i$$

12. Two bodies with kinetic energies in the ratio of 4:1 are moving with equal linear momentum. The ratio of their masses is:

- a. 1:2
- b. 1:1
- c. 4:1
- d. 1:4 ✓

Sol: According to the question, we have,

$$K \cdot E_1 : K \cdot E_2 = 4 : 1 \Rightarrow \frac{K \cdot E_1}{K \cdot E_2} = \frac{4}{1} \quad (1)$$

$$\text{Now } K \cdot E_1 = \frac{p_1^2}{2m_1} \text{ and } K \cdot E_2 = \frac{p_2^2}{2m_2}$$

So equation (1) becomes,

$$\frac{\frac{p_1^2}{2m_1}}{\frac{p_2^2}{2m_2}} = \frac{4}{1} \Rightarrow \frac{p_1^2}{2m_1} \times \frac{2m_2}{p_2^2} = \frac{4}{1}$$

$$\Rightarrow \frac{p_1^2}{p_2^2} = \frac{4m_1}{m_2} \Rightarrow \frac{p_1}{p_2} = \sqrt{4 \left(\frac{m_1}{m_2} \right)} \Rightarrow \frac{p_1}{p_2} = 2 \sqrt{\frac{m_1}{m_2}}$$

As we have given that, $p_1 = p_2 = p$ so we get,

$$\frac{p}{p} = 2 \sqrt{\frac{m_1}{m_2}} \Rightarrow 2 \sqrt{\frac{m_1}{m_2}} = 1$$

$$\Rightarrow \sqrt{\frac{m_1}{m_2}} = \frac{1}{2} \Rightarrow \frac{m_1}{m_2} = \frac{1}{4} \Rightarrow m_1 : m_2 = 1 : 4$$

13. A body of mass 5kg is moving with a momentum of 10kgms⁻¹. A force of 0.2N acts on it in the direction of motion of the body for 10s. The increase in its kinetic energy is:

- a. 2.8J
- b. 3.2J
- c. 3.8J
- d. 4.4J ✓

Sol: The change in $K \cdot E$ is given by,

$$\Delta K \cdot E = K \cdot E_f - K \cdot E_i \quad (1)$$

$$\Rightarrow \Delta K \cdot E = \frac{p_f^2}{2m} - \frac{p_i^2}{2m} = \frac{1}{2m} [p_f^2 - p_i^2] \quad (2)$$

We have given that,

$$m = 5\text{kg}, p_i = 10\text{kgm/sec}, F = 0.2\text{N}, \Delta t = 10\text{sec}$$

$$\text{Now } F = \frac{\Delta p}{\Delta t} \Rightarrow \Delta p = F \cdot \Delta t \Rightarrow p_f - p_i = F \cdot \Delta t$$

$$\Rightarrow p_f = F \cdot \Delta t + p_i = (0.2 \times 10 + 10)\text{kg} \cdot \text{m/sec}$$

$$\Rightarrow p_f = 12\text{kgm/sec}$$

Putting the values in equation (2), we get,

$$\Delta K \cdot E = \frac{1}{2 \times 5} [12^2 - 10^2] \text{J} = \frac{1}{10} (144 - 100)$$

$$\Rightarrow \Delta K \cdot E = \frac{44}{10} \Rightarrow \Delta K \cdot E = 4.4\text{J}$$

14. If force and displacement of particle in the direction of force are doubled. Work would be:

- a. Double
- b. 4 times ✓
- c. Half
- d. 1/4 times

$$\text{Sol: Work} = F \cdot d \Rightarrow W = F \cdot d \quad (1)$$

Now when force and displacement in the direction of force are doubled, then

$$F' = 2F \text{ and } d' = 2d$$

For " F " and " d ", equation (1) becomes

$$w' = F'd' \quad (1)$$

Putting the values in equation (2), we get,

$$w' = 2F \cdot 2d = 4(F \cdot d) \Rightarrow w' = 4w$$

Conceptual Questions

Give short answers to the following questions:

Q1: A bucket is taken to bottom of a well, does the bucket possess any potential energy?

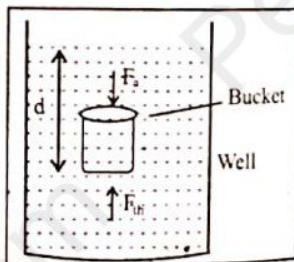
BISE Kohat 2019

Ans: Yes, when a bucket is taken to the bottom of a well, it possesses potential energy.

Reasons and Explanation:

When a bucket is taken to the bottom of a well, it is opposed by the upthrust of the water. Thus work must be done on the bucket against the upthrust of water. This workdone is stored in the form of potential energy in the bucket.

In the given figure, a bucket is moved through a depth " d " below the surface of water in a well under the action of applied force " F_a ". The bucket is opposed by the upthrust of water which is acting in upward direction. Thus work must be done with the help of applied force " F_a "



against the upthrust of water. This work is stored in the form of potential energy in the bucket. Mathematically, we have,
Workdone = $F_a \cdot d = F_a d \cos \theta$

$$\Rightarrow \text{Workdone} = F_a d \cos 0^\circ = F_a d \times 1$$

$$\Rightarrow \text{Workdone} = F_a d \quad (1)$$

[" F_a " & " d " are parallel so $\theta = 0^\circ$]

Here workdone = P.E, $F_a = w = mg$, $d = h$

So equation (1) becomes,

$$P.E = mgh \text{ or } P.E = -mgh \quad (2)$$

The negative sign is used because the bucket is situated below the reference point.

Note: If we consider the reference point at the bottom, then $h=0$ and thus the bucket will possess no P.E in such a case.

Q2: When an arrow is shot from a bow, it has K.E. From where does it get the K.E?

Ans: When an arrow is shot from a bow, it moves with certain K.E which the arrow gains due to the loss of P.E stored in the bow.

Reasons & Explanation:

When we want to shoot an arrow from a bow, we pull the string of the bow towards us. Thus we have to do work against the restoring force of the string. Mathematically, we have,

$$\text{Work} = F_{\text{avg}} \cdot x \Rightarrow \text{work} = \left(\frac{F_i + F_f}{2} \right) \cdot x \quad [F_i = 0, F_f = F]$$

$$\Rightarrow \text{Work} = \left(\frac{0 + F}{2} \right) \cdot x = \frac{Fx}{2} \quad (1)$$

According to Hooke's law, we have,

$$F = kx \quad (2)$$

Putting equation (2) in equation (1),

we get,

$$\text{Work} = \frac{kx \cdot x}{2} = \frac{1}{2} kx^2 \Rightarrow \text{Work} = \frac{1}{2} kx^2 \quad (3)$$

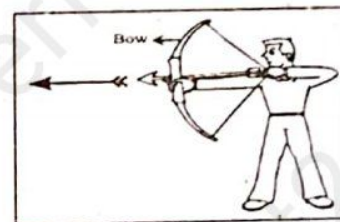
This workdone on the string is stored in the form of P.E in the string. So equation (3) becomes,

$$P.E = \frac{1}{2} kx^2 \quad (4)$$

Now when the string is released, it loses its P.E which appears in the form of K.E in the arrow. So equation (4) becomes,

$$K.E = P.E = \frac{1}{2} kx^2$$

With the help of this K.E, the arrow moves in the forward direction. Thus the arrow gains K.E from the P.E stored in the bow.



Q3: Does a hydrogen filled balloon possess P.E? Explain.

BISE Bannu 2017

Ans: Yes, a hydrogen filled balloon possesses P.E due to upthrust of the air.

Reasons and Explanation:

Hydrogen is the lightest gas. When it is filled in a balloon and the balloon is released in air, the balloon moves upwards. It is because the hydrogen filled balloon is lighter in weight and the upthrust of the air is greater than its weight. Thus the upthrust of air does work on the balloon which appears in the form of P.E in the balloon. Mathematically, if " F_{th} " represent the upthrust of air and " w " represents the weight of the balloon, then the net force on the balloon is given by,

$$F_{net} = F_{th} - w$$

The hydrogen filled balloon moves upwards under the action of this net force. This net force does work against gravity and air resistance which is stored in the balloon in the form of P.E. Thus we can say that a hydrogen filled balloon possesses P.E.

Q4: Is K.E. a vector quantity? Explain.

Ans: No, K.E. is not a vector quantity. It is a scalar quantity.

Reasons and Explanation:

K.E. is a scalar quantity, because it depends upon the mass and magnitude of velocity of the body. It does not depend upon the direction of velocity of the body. It can be stored in a body by doing workdone in moving a stationary body or by stopping a moving body. Mathematically, we have,

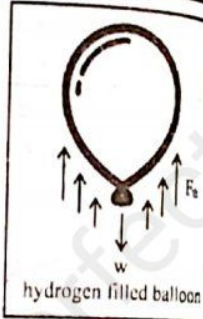
$$K.E = \frac{1}{2}mv^2 \quad (1)$$

In equation (1),

$$v^2 = \vec{v} \cdot \vec{v} = v v \cos \theta = v^2 \cos 0^\circ$$

$$\Rightarrow \vec{v} \cdot \vec{v} = v^2 \times 1 \Rightarrow \boxed{\vec{v} \cdot \vec{v} = v^2}$$

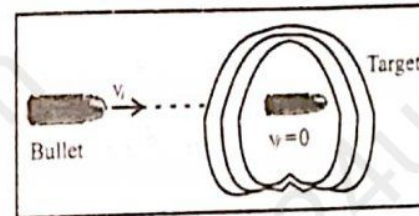
Now according to the rules of product of two vectors, the quantity which exists due to dot product or scalar product of two vectors



must be a scalar quantity. Thus the term " m " and " v^2 " in equation (1) both are scalar quantities. In other words, we can say that K.E. is a scalar quantity not a vector quantity.

Q5: What happens to K.E. of a bullet when it penetrates into a target?

Ans: When a bullet penetrates into a target, it loses its K.E. Some part of the K.E. is utilized to increase the internal energy of the target material. Some part of K.E. is converted into sound energy while most of the K.E. is used to do work against the resistance of the target.



Reasons and Explanation:

Let a bullet of mass " m " and having velocity " v_i " is allowed to hit a target as shown in the figure. When the bullet hits the target, it does work against the resistance of the target. As a result its K.E. decreases and finally comes to rest after covering some distance inside the target. So the final velocity will become zero. So $v_f = 0$. Now according to work-energy principle, we have,

Work = Change in K.E. of bullet

$$\Rightarrow \text{Work} = K.E_f - K.E_i$$

$$\Rightarrow \text{Work} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad (1)$$

Put $v_f = 0$ in equation (1), we get,

$$\text{Work} = 0 - \frac{1}{2}mv_i^2 \Rightarrow \boxed{\text{Work} = -\frac{1}{2}mv_i^2} \quad (2)$$

In equation (2) the negative sign shows that the K.E. of the bullet is utilized by doing workdone against the force of friction of the target.

Q6: Does the tension in the string of a swinging pendulum do any work? Explain.

Ans: No, the tension in the string of a swinging pendulum does not do any work.

Reasons and Explanation: When a pendulum is swinging, the ten-

sion in the string is acting in upwards direction i.e. towards its point of suspension. There is no displacement of the bob in the direction of the string. The displacement of the bob exists due to applied force not due to tension in the string.

Mathematically, we have,

$$\text{Work} = \vec{T} \cdot \vec{x}$$

$$\Rightarrow \text{Work} = T x \cos \theta \quad (1)$$

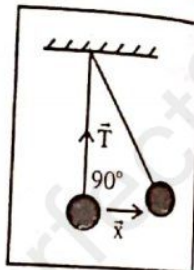
Now in the given figure, we see that the displacement of the bob is perpendicular to the tension in the string, so put $\theta = 90^\circ$ in equation (1), we get,

$$\text{Work} = T x \cos 90^\circ$$

$$\cos 90^\circ = 0$$

$$\Rightarrow \text{Work } T x = \cos \theta \Rightarrow \boxed{\text{Work} = 0} \quad (2)$$

Equation (2) shows that the tension in the string does not work on the bob during the swinging of the pendulum.



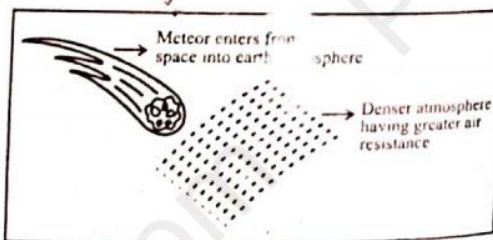
Q7: Meteor when enters into earth's atmosphere it burns. What happens to its energy and momentum?

Ans: When a meteor enters into earth's atmosphere, it burns and its energy and momentum is transferred to the surrounding particles in atmosphere.

Reasons and Explanation:

When a meteor enters into earth atmosphere, it is burns. It because when the meteor is travelling in space, it posses high energy and momentum. When this high energy meteor enters into the denser atmosphere of the earth, it faces huge air resistance. Due to huge frictional forces of the air and other particles the meteor catches fire and burns. The energy of meteor appears in the form of heat energy, light energy and sound energy. The energy and momentum of the meteor is transferred to the surrounding particles.

As a result the K.E and momentum of the surrounding particles



present in atmosphere increases.

Q8: What type of energy is stored in the spring of a watch?

Ans: Elastic P.E is stored in the spring of a watch.

Reasons and Explanation:

A watch consists of an elastic spring inside it. When we are winding a watch, we do work against the restoring force of the elastic spring. Let we apply certain force and compress the spring through a displacement "x" in the direction of applied force. Now the workdone in such a case is given by,

$$\text{Work} = F_{\text{ave}} \cdot x$$

$$\Rightarrow \text{Work} = \left(\frac{F_i + F_f}{2} \right) x \quad (1)$$

In compressing the spring, we increase the applied force from ON to certain value F. So we put $F_i = 0$ and $F_f = F$ in equation (1), so we get,

$$\text{Work} = \left(\frac{0 + F}{2} \right) x = \frac{Fx}{2} \Rightarrow \text{Work} = \frac{Fx}{2} \quad (1)$$

According to Hook's law,

$$F = K x \quad (2)$$

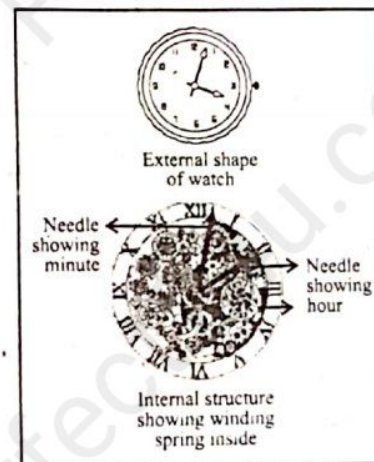
Putting equation (2) in equation (1), we get,

$$\text{Work} = \frac{K x \cdot x}{2} \Rightarrow \text{Work} = \frac{1}{2} K x^2 \quad (3)$$

This workdone on compressing the spring of a watch is stored in the spring in the form of elastic potential energy. So equation (1) can be written as,

$$P.E_{\text{elastic}} = \frac{1}{2} K x^2$$

NOTE: This elastic P.E is then converted into K.E which keeps the needle of the watch in moving condition on the dial.



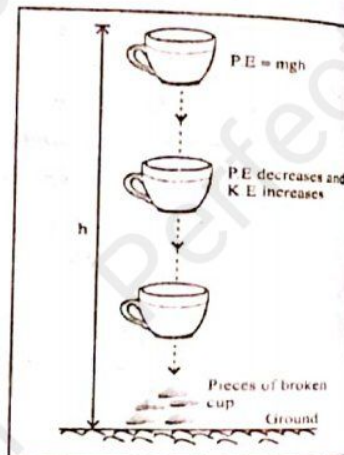
Q9: A man drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

BISE Peshawar 2017

Ans: Let a cup is held at a height "h" above the earth surface. At height "h" the cup posses P.E, which is given by,
 $P.E = mgh$ (1)

Now when the cup is released downwards its P.E decreases and K.E increases. When the cup reaches the earth surface, its P.E is totally converted into K.E. As a result, the cup posses high K.E and momentum just above the earth surface.

Now according to Newton's 3rd law of motion, when the cup strikes the ground, it exerts force of action on the ground. The ground exerts force of reaction on the cup. As a result of this reaction of earth, the cup breaks into pieces. Its energy is converted into heat energy, sound energy. Some energy is imparted into the pieces of the cup while some energy is transferred to the earth on striking point.



Q10: A man rowing boat upstream is at rest with respect to the shore. Is he doing work?

Ans: No work is done by the man while rowing a boat upstream when he is at rest with respect to the shore.

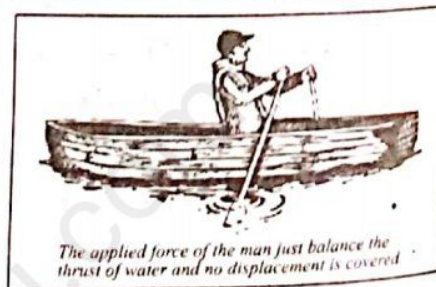
Reasons and Explanation:

We know that the workdone is equal to the scalar product of force and displacement i.e.

$$Work = \vec{F} \cdot \vec{d}$$

$$\Rightarrow Work = Fd \cos \theta \quad (1)$$

Equation (1) shows that the workdone depends upon the magnitude of force, displacement and value of " θ " between force and dis-



placement.

Now when a man rowing boat upstream is at rest with respect to the shore, then no displacement is covered in such a case. The applied force of the man on the boat just balance the thrust of water.

Now put $d = 0$ in equation (1), we get,

$$Work = F \times 0 \times \cos \theta = 0$$

$$\Rightarrow Work = 0 \quad (2)$$

Equation (2) shows that no work is done by the man keeping the boat at rest with respect to the shore.

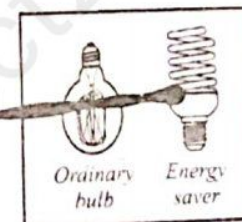
Q11: Why energy savers are used instead of normal bulb?

BISE Abbottabad 2018

Ans: Now a days the energy savers are used more frequently instead of ordinary normal bulb. It is because of the following reasons:

Reasons and Explanation:

1. The resistance of ordinary bulb is very high than energy saver bulb. Due to high resistance, the ordinary bulb can convert about 2% of electrical energy into light energy. The remaining 98% electrical energy is converted into heat energy. While the energy saver bulbs are less resistive and can convert 80% of electrical energy into light energy.
2. Due to low resistance, the power loss in case of energy saver bulb is very low as compare to ordinary bulb.
3. The energy saver bulb saves energy due to its high efficiency as compare to ordinary bulb.
4. The life time of energy saver bulb is 10 time longer than ordinary bulb. Thus energy saver bulbs are more economical than ordinary bulb.
5. The energy saver bulb produces very less amount of heat due to low resistance. So these bulbs are fit for standard fitting in homes, offices and industry.
6. The use of energy saver bulbs can reduce the global warming, because the production of heat due to power dissipation in energy saver bulb is very low as compare to ordinary bulb.



Comprehensive Questions

Give extended response to the following questions:

Q1: Define work and show that it is the dot product of force and displacement. At what conditions workdone will be maximum or minimum?

Ans: See answer of questions no 1 in the text.

Q2: Define power and show that power is the dot product of force and velocity. What are the different units of power used in our daily life?

Ans: See answer of questions no 6 in the text.

Q3: Explain the work energy principle in the cases of the change in K.E or body.

Ans: See answer of question no 14 in the text.

Q4: Prove that absolute P.E = $\frac{GmM}{R}$.

Ans: See answer of questions no 14 in the text.

Q5: Calculate the values of the escape velocity of a body and show that it equal 11.2 kms^{-1} .

Ans: See answer of question no 15 in the text.

Q6: Describe briefly various non-conventional sources of energy.

Ans: See answer of question no 18 in the text.

- عظمت صرف ایک فیصد ودیعت کی جاتی ہے اور ۹۹ فیصد محنت اور ریاضت سے ملتی ہے۔
- چٹائی کی مشعل سے فائدہ اٹھاؤ، یہ مت دیکھو کہ مشعل بردار کون ہے؟

Numerical Problems

Q1: A 70kg man runs up a long flight of stairs in 4sec. The vertical height of the stair is 4.5m. Calculate his power.

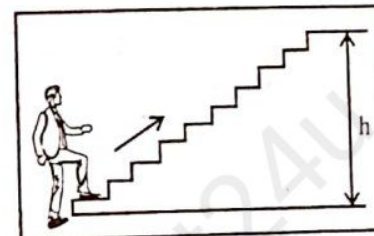
Solution:

Given Data:

Mass of the man
= $m = 70\text{kg}$

Time interval = $t = 4\text{sec}$,

Height of stairs
= $h = 4.5\text{m}$



Required: Power = $P = ?$

Calculate: We know that the power is given by,

$$P = \frac{\text{work}}{\text{time}} \quad (1)$$

Here workdone = $P \cdot E = mgh$, so equation (1)

$$P = \frac{P \cdot E}{t} = \frac{mgh}{t} \quad (2)$$

Putting the values in equation (2), we get,

$$P = \left(\frac{70 \times 9.8 \times 4.5}{4} \right)$$

$$\Rightarrow P = 70 \times 9.8 \times \frac{4.5}{4}$$

$$\Rightarrow P = 771.75 \text{ watt}$$

$$\Rightarrow P = 7.7175 \times 10^2 \text{ watt}$$

Ans: $P = 7.7 \times 10^2 \text{ watt}$

Q2: A body of mass 2kg is dropped from a rest position 5m above the ground. What is its velocity at height of 3m above the ground?

Solution:

Given Data:

Mass of body = $m = 2\text{kg}$

Total height = $h = 5\text{m}$

Required: Velocity at height of 3m = $v = ?$

Calculation: Let the body reaches from "A" to "B". If we ignore the air resistance, then at point "B" we have,

Loss in P.E = Gain in K.E

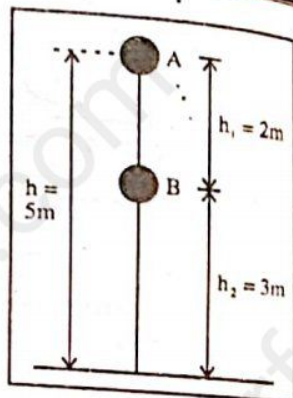
$$\Rightarrow mgh_1 = \frac{1}{2}mv^2 \Rightarrow gh_1 = \frac{v^2}{2}$$

$$\Rightarrow v^2 = 2gh_1 \Rightarrow v = \sqrt{2gh_1} \quad (1)$$

Putting the values in equation (1), we get,

$$\Rightarrow v = \sqrt{2 \times 9.8 \times 2} \text{ m/sec}$$

$$\Rightarrow v = 6.26 \text{ m/sec} \Rightarrow \boxed{v = 6.3 \text{ m/sec}}$$



Q3: A man pulls a trolley through a distance of 50m by applying a force of 100N which makes an angle of 30° with horizontal. Calculate the workdone by the man.

Solution: **Given Data:**

Distance travelled

$$= d = 50\text{m}$$

Force applied

$$= F = 100\text{N}, \theta = 30^\circ$$

Required:

Workdone = $w = ?$

Calculate: We know that,

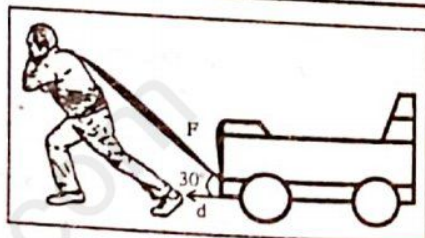
$$w = Fd \cos \theta \quad (1)$$

Putting the values in equation (1), we get,

$$w = 100 \times 50 \times \cos 30^\circ$$

$$\Rightarrow 5000 \times 0.866 \text{ J} = 4330 \text{ J}$$

$$\text{Ans. } \boxed{w = 4330 \text{ J}}$$



Q4: The roller-coaster car starts its journey from a vertical height of 40m on the first hill and reaches a vertical height of only 25m on the second hill, where it slows to a momentary stop. It travelled a total distance of 400m. Determine the thermal energy produced and estimate the average friction force on the car whose mass is 1000kg. (147000J, 368N)

Solution:

Given Data:

Height of 1st hill

$$= h_1 = 40\text{m}$$

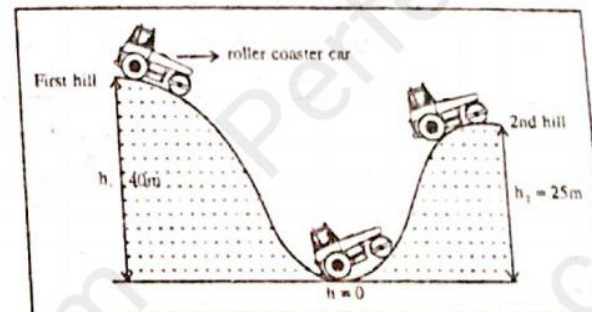
Height of 2nd hill

$$= h_2 = 25\text{m}$$

Mass of car

$$= m = 1000\text{kg}$$

Distance travelled = $d = 400\text{m}$



Required:

(a) Thermal energy = $E_{\text{thermal}} = ?$

(b) Average force of friction = $F_f = ?$

Calculation: (a) The potential energy of car on top of 1st hill is given by,

$$P.E_1 = mgh_1 = (1000 \times 9.8 \times 40) \text{ J}$$

$$\Rightarrow P.E_1 = 392000 \text{ J}$$

The P.E at the top of 2nd hill is given by,

$$P.E_2 = mgh_2 = (1000 \times 9.8 \times 25) \text{ J}$$

$$= P.E_2 = 245000 \text{ J}$$

Now when the roller coaster car moves from 1st hill to 2nd hill then inter-conversion of K.E and P.E takes place. So we have,

$$\Delta K.E = \Delta P.E \Rightarrow \Delta K.E = P.E_1 - P.E_2 = (392000 - 245000) \text{ J}$$

$$\Rightarrow \Delta K.E = 147000 \text{ J}$$

Now the thermal energy produced is equal to the change in mechanical energy $[\Delta K.E]$ so we have,

$$E_{\text{thermal}} = \Delta K.E = 147000 \text{ J} \Rightarrow \boxed{E_{\text{thermal}} = 147000 \text{ J}}$$

(b) According to work-energy theorem,

$$\text{Work} = \Delta K.E \Rightarrow F_f d = 147000 \text{ J}$$

$$\Rightarrow F_f = \left(\frac{147000}{d} \right) \text{ N} = \left(\frac{147000}{400} \right) \text{ N}$$

$$\Rightarrow \boxed{F_f = 367.5 \text{ N}} \text{ or } \boxed{F_f = 368 \text{ N}}$$

Q5: A man whose mass is 70kg walks up to the 3rd floor of a building which is 12m above the ground in 20sec. Find his power in watts and horse power.

Solution:

Given Data: Mass of man = $m = 70\text{kg}$

Height of building = $h = 12\text{m}$

Time taken = $t = 20\text{sec}$

Required:

(a) Power in watt = ?

(b) Power in horse power = ?

Calculation: (a) We know that,

$$P = \frac{\text{work}}{\text{time}} = \frac{W}{t}$$

$$\Rightarrow P = \frac{P \cdot E}{t} \Rightarrow P = \frac{mgh}{t} \quad (1)$$

$$[W = P \cdot E = mgh]$$

Putting the values in equation (1), we get,

$$P = \frac{(70 \times 9.8 \times 12)}{20} = 411.6 \text{ watt}$$

$$\text{Ans. } P = 411.6 \text{ watt}$$

$$[1 \text{ h.p} = 746 \text{ watts}]$$

(b) As $P = 411.6 \text{ watt}$

$$\Rightarrow P = \left(\frac{411.6}{746} \right) \text{ h.p} = 0.55 \text{ h.p}$$

$$\text{Ans. } P = 0.55 \text{ h.p}$$

Q6: To what height can a 400 watt engine lift a 100kg mass in 3sec?

Solution:

Given Data:

Power of engine = $P = 400\text{watt}$

Mass of load = $m = 100\text{kg}$

Time taken = $t = 3\text{sec}$

Required: Height of load = $h = ?$

Calculation: We know that the workdone by the engine in lifting the body to height "h" is equal to its P.E at that height. So we have,

$$\text{Work} = P \cdot E = mgh$$

$$[P \cdot E = mgh]$$

$$\Rightarrow \text{Work} = mgh \quad (1)$$

We also know that,

$$\text{Power} = \frac{\text{work}}{\text{time}} \Rightarrow \text{Work} = \text{power} \times \text{time}$$

$$\Rightarrow W = P \times t \quad (2)$$

Comparing equation (1) and equation (2), we get,

$$mgh = P \times t \Rightarrow h = \frac{P \times t}{mg} \quad (3)$$

Putting the values in equation (3), we get,

$$h = \left(\frac{400 \times 3}{100 \times 9.8} \right) \text{m} \Rightarrow h = 1.22\text{m} \Rightarrow \boxed{\text{Height} = h = 1.22\text{m}}$$

Q7: A ball of mass 100gm is thrown vertically upwards at a speed of 25m/sec. If no energy is lost, determine the height it would reach. If the ball only rises to 25m, calculate the workdone against air resistance. Also calculate the force of friction.

Solution:

Given Data:

Mass of ball = $m = 100\text{gm} = 0.1\text{kg}$

Speed of ball = $v = 25\text{m/sec}$

Required:

(a) Height to which the ball would reach = $h = ?$

(b) If height = $h' = 25\text{m}$, then

(i) Workdone = ? (ii) Force of friction = $F_f = ?$

Calculation: (a) If we ignore the air resistance, then,

Loss in K.E = Gain in P.E

$$\Rightarrow \frac{1}{2}mv^2 = mgh \Rightarrow \frac{v^2}{2} = gh \Rightarrow h = \frac{v^2}{2g} \quad (1)$$

Putting the values in equation (1), we get,

$$h = \frac{25^2}{2 \times 9.8} \Rightarrow \boxed{h = 31.9\text{m}}$$

(b) (i) $h' = 25\text{m}$, in case of air resistance, we have,

Loss in K.E = gain in P.E + workdone against air resistance

$$\Rightarrow \text{Workdone} = \text{loss in K.E} - \text{gain in P.E}$$

$$\Rightarrow w = \frac{1}{2}mv^2 - mgh'$$

$$\Rightarrow w = \frac{1}{2} \times 0.1 \times 25^2 - 0.1 \times 9.8 \times 25 = (31.25 - 24.5) \text{ J}$$

$$\Rightarrow w = 6.7 \text{ J}$$

(ii) We know that,

$$\text{Work} = Fd \Rightarrow w = F_r h'$$

$$[d = h']$$

$$\Rightarrow F_r = \frac{w}{h'} = \frac{6.7}{25} \Rightarrow F_r = 0.26 \text{ N} \Rightarrow \boxed{F_r = 0.3 \text{ N}}$$

Q8: An object of mass 1000g falls from a height of 30m on the sand below. If it penetrates 4cm into the sand, what opposing force is exerted on it by the sand? Neglect air friction. ($F = 7350 \text{ N}$)

Solution:

Given Data: Mass of object

$$= m = 1000 \text{ gm} = 1 \text{ kg}$$

Height of ball from sand surface = $h = 3 \text{ m}$

Distance covered by ball inside sand

$$= d = 4 \text{ cm} = 0.04 \text{ m}$$

Required: Force exerted by sand on the ball
= $F = ?$

Calculation: Let the object fall down from point "A" and reaches the point "B" on sand surface. Now neglecting air friction, we have,

Loss in P.E = Gain in K.E

$$\Rightarrow mgh = K.E$$

$$\Rightarrow K.E = mgh \quad (1)$$

Putting the values in equation (1), we get,

$$K.E = [1 \times 9.8 \times 30] \text{ J} \Rightarrow \boxed{K.E = 294 \text{ J}}$$

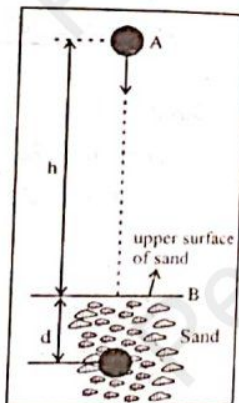
When the object comes to rest, the workdone on the object must be equal to its K.E. So we have,

$$\text{Work} = K.E \Rightarrow \text{Work} = 294 \text{ J}$$

$$[\text{Work} = Fd]$$

$$\Rightarrow F \cdot d = 294 \text{ J} \Rightarrow F = \left(\frac{294}{d} \right) \text{ N}$$

$$\Rightarrow F = \left(\frac{294}{0.04} \right) \text{ N} = 7350 \text{ N} \Rightarrow \boxed{\text{Force} = F = 7350 \text{ N}}$$



Q9: A body of mass "m" drops from bridge into water of the river. The bridge is 10m high from the water surface. (a) Find the speed of the body 5m above the water surface. (b) Find the speed of the body before it strikes the water.

Solution:

Given Data:

Height of bridge = $h = 10 \text{ m}$, $v_i = 0$

Required:

(a) $h_2 = 5 \text{ m}$, $v_f = v = ?$

(b) $h_2 = 0$, $v_f = v = ?$

Calculation:

(a) According to the law of conservation of mechanical energy, we have,

Loss in P.E = Gain in K.E

$$\Rightarrow \Delta P.E = \Delta K.E$$

$$\Rightarrow mgh_1 - mgh_2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\Rightarrow (gh_1 - gh_2)m = \frac{m}{2}[v_f^2 - v_i^2]$$

$$\Rightarrow 2gh_1 - 2gh_2 = v_f^2 - v_i^2$$

$$\Rightarrow v_f^2 = 2gh_1 - 2gh_2 + v_i^2 = (2 \times 9.8 \times 10) - (2 \times 9.8 \times 5) + (0)$$

$$\Rightarrow v_f^2 = 196 - 98 = 98 \text{ m}^2 / \text{sec}^2$$

$$\Rightarrow v_f = \sqrt{98 \text{ m}^2 / \text{sec}^2} = 9.89 \text{ m / sec}$$

$$\Rightarrow \boxed{v_f = 9.9 \text{ m / sec}}$$

(b) When the body reaches just the water surface, then we have,

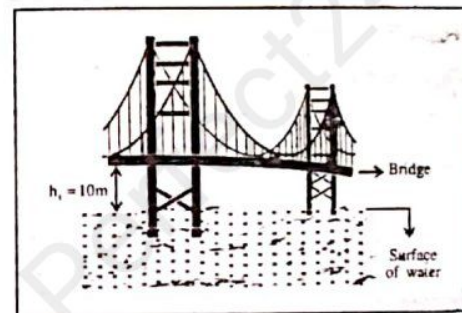
$$h_1 = 10 \text{ m}, h_2 = 0$$

Again we have,

Loss in P.E = Gain in K.E

$$\Rightarrow mgh_1 - mgh_2 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

When the body hits the surface of water, its height becomes zero i.e. $h_2 = 0$



$$\Rightarrow mgh_i - mg \times 0 = \frac{1}{2}mv_f^2 - \frac{1}{2}m \times 0^2$$

$$\Rightarrow mgh_i = \frac{1}{2}mv_f^2$$

$$\Rightarrow gh_i = \frac{v_f^2}{2} \Rightarrow v_f^2 = 2gh_i$$

$$\Rightarrow v_f = \sqrt{2gh_i} = \sqrt{2 \times 9.8 \times 10}$$

$$\Rightarrow v_f = \sqrt{196} \text{ m/sec} \Rightarrow \boxed{v_f = 14 \text{ m/sec}}$$

Q10: The engine of a JF-thunder fighter develops a thrust of 3000N. What horse power does it at a velocity of 600m/sec?

Solution:

Given Data:

Thrust of engine = $F = 3000\text{N}$

Velocity = $V = 600\text{m/sec}$

Required:

Power in horse power = $P = ?$

Calculate:

We know that,

$$P = \vec{F} \cdot \vec{v} \Rightarrow P = Fv \cos \theta$$

$$\Rightarrow P = 3000 \times 600 \times \cos 0^\circ$$

$$\Rightarrow P = 1800000 \times 1 = 1800000 \text{ watt}$$

$$\Rightarrow P = 1800000 \text{ watt}$$

$$\Rightarrow P = \left[\frac{1800000}{746} \right] \text{ h.p}$$

Power in horsepower:

As $P = 1800000 \text{ watt}$

$$\Rightarrow P = \left(\frac{1800000}{746} \right) \text{ h.p}$$

$$\Rightarrow \boxed{P = 2413 \text{ h.p}}$$

$$\boxed{\theta = 0^\circ}$$

$$\boxed{\cos 0^\circ = 1}$$

$$\boxed{1 \text{ h.p} = 746 \text{ watt}}$$

Q11: The mass of the moon is $1/80$ of the mass of the earth and corresponding radius is $1/4$ of the earth. Calculate the escape velocity on the surface of moon. (2.5 kms^{-1})

Solution:

Given Data:

Mass of moon = $\frac{1}{80}$ times the mass of earth

$$\Rightarrow M_m = \frac{1}{80} M_e$$

Radius of moon = $R_m = \frac{1}{4}$ times the radius of earth

$$\Rightarrow R_m = \frac{1}{4} R_e$$

Values of $G = 6.673 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$

Required:

Escape velocity = $v_{\infty} = ?$

Calculation:

We know that,

$$v_{\infty} = \sqrt{\frac{2GM_e}{R_e}} \quad (1)$$

For moon, equation (1) can be written as,

$$v_{\infty} = \sqrt{\frac{2GM_m}{R_m}} \quad (2)$$

Putting the values of M_m , R_m in equation (2), we get,

$$v_{\infty} = \sqrt{\frac{2G \times \frac{1}{80} M_e}{\frac{1}{4} R_e}} = \sqrt{\frac{2 \times 4 GM_e}{80 R_e}}$$

$$\boxed{R_e = 6.4 \times 10^6 \text{ m}}$$

$$\boxed{M_e = 6 \times 10^{24} \text{ kg}}$$

$$\Rightarrow v_{\infty} = \sqrt{\frac{GM_e}{10 R_e}} \quad (3)$$

Putting the values of " R_e ", " G " and " M_e " in equation (3), we get,

$$v_{..} = \left[\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{10 \times 6.4 \times 10^6} \right] \text{ m/sec}$$

$$\Rightarrow v_{..} = \left[\frac{0.63 \times 10^{-11+24-6}}{10 \times 6.4 \times 10^6} \right] \text{ m/sec} = \sqrt{0.63 \times 10^7}$$

$$\Rightarrow v_{..} = \left[\sqrt{6.3 \times 10^6} \right] \text{ m/sec} = 2.5 \times 10^3 \text{ m/sec}$$

$$\Rightarrow v_{..} = 2.5 \times 10^3 \text{ m/sec}$$

$$(OR) \quad v_{..} = 2.5 \text{ km/sec}$$

$$10^3 = \text{kilo}$$

+++

- جس نے طلب علم میں وفات پائی وہ شہید مراد۔ (رسول اللہ ﷺ)
- خاموشی عالم کے لیے زیور اور جاہل کے لیے پردہ ہے۔ (حضرت عین)
- تم بڑوں کی عزت کرو، چھوٹے تمہاری عزت کریں گے۔ (حضرت عین)
- جس نے انسان کا حق نہیں پہچانا اس نے اللہ تعالیٰ کا حق نہیں پہچانا۔ (حضرت عین)
- جاہلوں کی بات پر حمل کرنا عقل کو زکوۃ دیتا ہے۔ (حضرت علی)
- دنیا کی سب سے بڑی خیانت قوم کے ساتھ غداری ہے۔ (حضرت علی)
- پرہیزگاری سے بڑھ کر کوئی عزت نہیں۔ (حضرت علی)

CHAPTER

05

ROTATIONAL & CIRCULAR MOTION

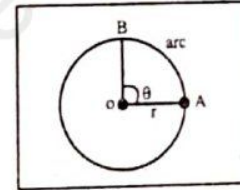
Q1: Define rotational motion, angular displacement, angular velocity and angular acceleration?

Ans: (i) Rotational Motion:

The motion of a body about its axis of rotation is known as rotational motion or angular motion.

(ii) Angular Displacement:

The angle subtended by an arc at the centre during angular motion is known as angular displacement. In given figure, " θ " represents the angular displacement, such that, $\theta = 1$ radian. The units of angular displacement are degree, revolution and radian [rad].



$$1 \text{ revolution} = 360^\circ$$

$$1 \text{ revolution} = 2\pi \text{ radian}$$

$$360^\circ = 2\pi \text{ radian} \quad [1 \text{ rad} = 57.3^\circ]$$

NOTE: If the length of an arc is equal to the radius of the circle, then the angle subtended at the centre of the circle is equal to 1 radian.

(iii) Angular Velocity:

The rate of change of angular displacement is known as angular velocity. It is represented by " $\vec{\omega}$ " and is given by,

$$\vec{\omega} = \frac{\Delta\theta}{\Delta t}$$

The units of angular velocity are rad/sec, rev/sec and degree/sec.

Average Angular Velocity:

The average angular velocity can be obtained by dividing total angular displacement by total time taken. i.e.

$$\langle \vec{\omega} \rangle = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Velocity:

The angular velocity at any particular instant of time is known as instantaneous angular velocity. (OR)

The angular velocity for which time intervals approaches to zero is known as instantaneous angular velocity. i.e.

$$\omega_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t}$$

Dimension of angular velocity:

$$\text{As } \omega = \frac{\Delta \theta}{\Delta t} = \frac{\text{rad}}{\text{sec}} = \frac{\text{rad}}{\text{time}} = [T^{-1}]$$

(iv) Angular Acceleration:

The rate of change of angular velocity is known as angular acceleration. It is represented by " α " and is given by,

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

The units of angular acceleration are, rad/sec^2 , rev/sec^2 , degree/sec^2 .

Average Angular Acceleration:

The average angular acceleration can be obtained by dividing total change in angular velocity by total time taken. i.e.

$$\langle \alpha \rangle = \frac{\Delta \omega}{\Delta t}$$

Instantaneous Angular Acceleration:

The angular acceleration of a body at any particular instant of time is known as instantaneous angular acceleration. i.e.

$$\alpha_{ins} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \omega}{\Delta t}$$

Dimension of angular acceleration: (ETEA 2007)

$$\text{As } \alpha = \frac{\Delta \omega}{\Delta t} = \frac{\text{rad}}{\text{sec}^2} = [T^{-2}]$$

Q2: Discuss the relation between linear displacement and angular displacement. (OR) Show that, $S = r\theta$

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Ans. Consider a body of mass "m" is moving along a circular path of radius "r" as shown in the figure.

Let "AB" and "AC" represents the two arc making an angle $\angle AOB$ and $\angle AOC$ with the centre respectively. Now we have,

$$\angle AOB \propto \text{Arc AB} \Rightarrow \angle AOB = \text{constant arc AB} \quad (1)$$

$$\text{Similarly, } \angle AOC = \text{constant arc AC} \quad (2)$$

Dividing equation (1) by equation (2), we get

$$\frac{\angle AOB}{\angle AOC} = \frac{\text{Arc AB}}{\text{Arc AC}} \quad (3)$$

$$\text{Here } \angle AOB = \theta$$

$$\angle AOC = 1 \text{ radian}$$

$$\text{Arc AB} = S$$

$$\text{Arc AC} = r$$

So equation (3) becomes,

$$\frac{\theta}{1} = \frac{S}{r} \Rightarrow S = r\theta \quad (4)$$

Equation (4) represents the relation between linear displacement and angular displacement.

Q3: Discuss the relation between linear velocity and angular velocity. (OR) Show that $\vec{v} = \vec{\omega} \times \vec{r}$

Ans. We know that,

$$S = r\theta \quad (1)$$

We also know that,

$$S = vt \quad (2) \text{ Comparing equations (1), (2), we get,}$$

$$vt = r\theta$$

$$\Rightarrow v = r \frac{\theta}{t} \quad (3)$$

$$\text{As } \frac{\theta}{t} = \text{angular velocity} = \omega$$

So equation (3) becomes,

$$v = r\omega \quad (4)$$

In vector form equation (4) can be written as,

$$\vec{v} = \vec{\omega} \times \vec{r} \quad (5)$$

Equations (4), (5) represent the relation between linear velocity and angular velocity.

Q4: Discuss the relation between linear acceleration and angular acceleration. (OR) Show that, $\vec{a} = \vec{\alpha} \times \vec{r}$

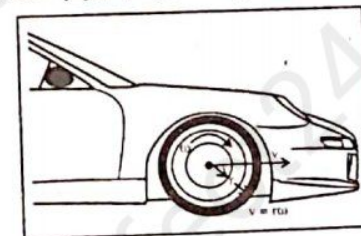
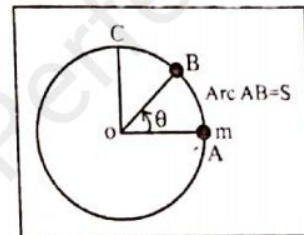
Ans. By definition, we have,

$$\text{Linear acceleration} = \frac{(\vec{v}_f - \vec{v}_i)}{t}$$

$$\Rightarrow \vec{a} = \frac{(\vec{v}_f - \vec{v}_i)}{t} \quad (1)$$

Here we have,

$$[\vec{v} = \vec{\omega} \times \vec{r}]$$



$$\vec{v}_r = \vec{\omega}_r \times \vec{r} \text{ and } \vec{v}_t = \vec{\omega}_t \times \vec{r}$$

So equation (1) becomes,

$$\vec{a} = \frac{(\vec{\omega}_r - \vec{\omega}_t)}{t} \times \vec{r} \quad (2)$$

We know that,

$$\frac{(\vec{\omega}_r - \vec{\omega}_t)}{t} = \text{rate of change of angular acceleration}$$

$$\Rightarrow \frac{(\vec{\omega}_r - \vec{\omega}_t)}{t} = \vec{\alpha} \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\vec{a} = \vec{\alpha} \times \vec{r} \quad (4)$$

Equation (4) represents the relation between linear acceleration and angular acceleration.

Q5: Discuss the linear kinematic equation in rotational equation form.

LINEAR MOTION	ROTATORY MOTION
1. $S = vt$ $\longrightarrow$	$\theta = \omega t$
2. $v_f = v_i + at$ $\longrightarrow$	$\omega_f = \omega_i + \alpha t$
3. $\langle v \rangle = \frac{(v_f + v_i)}{2}$ $\longrightarrow$	$\langle \omega \rangle = \frac{(\omega_f + \omega_i)}{2}$
4. $2aS = v_f^2 - v_i^2$ $\longrightarrow$	$2\alpha\theta = \omega_f^2 - \omega_i^2$
5. $S = v_i t + \frac{1}{2}at^2$ $\longrightarrow$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

Q6: (a) Define circular motion. (b) Define centripetal force and centripetal acceleration. Derive mathematical relations for centripetal force and centripetal acceleration.

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Ans. (a) Circular Motion: The motion of a body along a circular path is known as circular motion.

(b) Centripetal Force and Centripetal Acceleration: (ETEA 2006-8)

Centripetal Force: The force which compels the body to move in circle is known as centripetal force. It is also known as centre seeking force.

Centripetal Acceleration: The acceleration of the body during circular motion is known as centripetal acceleration. It is always directed towards the centre of the circle.

Mathematical Form:

Consider a body of mass "m" moving along a circular path of radius "r" as shown in the figure.

Let " $\vec{v}_1$ " and " $\vec{v}_2$ " represents the velocity of the body at points "A" and "B" respectively. Here changes occur in velocity due to change in its direction, which is given by,

$$\Delta \vec{v} = (\vec{v}_2 - \vec{v}_1) \quad (1)$$

We also know that,

$$S = r\theta \quad (2)$$

Here we put,

$$S = \Delta v, r = v, \theta = \Delta\theta$$

So equation (2) becomes,

$$\Delta v = v\Delta\theta \quad (3)$$

Dividing both sides of equation (3) by " Δt " and taking the limit i.e. $\lim_{\Delta t \rightarrow 0}$ we get,

$$\lim_{\Delta t \rightarrow 0} \left(\frac{\Delta v}{\Delta t} \right) = (v) \lim_{\Delta t \rightarrow 0} \left(\frac{\Delta\theta}{\Delta t} \right)$$

$$\Rightarrow \boxed{a = v\omega} \quad (4)$$

We also know that, $v = r\omega$ so equation (4)

$$a = r\omega \cdot \omega \Rightarrow \boxed{a = r\omega^2} \quad (5)$$

As $v = r\omega \Rightarrow \omega = \frac{v}{r}$, so equation (4) becomes,

$$a = v \cdot \frac{v}{r} \Rightarrow \boxed{a = \frac{v^2}{r}} \quad (6)$$

Equations (4), (5), (6) represent the magnitude of centripetal acceleration. In vector form, equations (4), (5), (6) can be written as,

$$\boxed{\vec{a} = -v\omega \vec{r}} \quad (7)$$

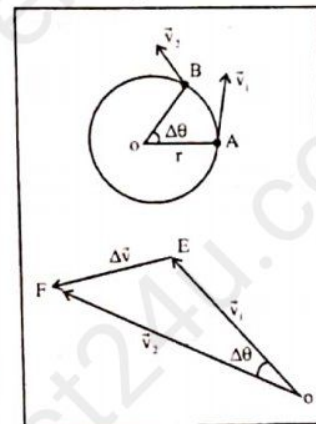
$$\boxed{\vec{a} = -r\omega^2 \vec{r}} \quad (8)$$

$$\& \boxed{\vec{a} = -\frac{v^2}{r} \vec{r}} \quad (9)$$

The negative sign shows that the acceleration is directed towards the centre of the circle.

Expressions for Centripetal Force: (ETEA 2007)

We know that, $F = ma$ $\quad (10)$



Putting equations (4), (5), (6) in equation (10) one by one, we get,

$$F = mv\omega \quad (11)$$

$$F = mr\omega^2 \quad (12)$$

$$\& F = \frac{mv^2}{r} \quad (13)$$

Equations (11), (12), (13) represent the magnitude of centripetal force. In vector form, equations (11), (12), (13) can be written as,

$$\vec{F} = -v\omega \hat{r} \quad (14)$$

$$\vec{F} = -mr\omega^2 \hat{r} \quad (15)$$

$$\& \vec{F} = -\frac{mv^2}{r} \hat{r} \quad (16)$$

Q7: Define and explain centrifugal force or reaction force.

Ans: Centrifugal Force (or) Reaction Force:

Definition: The force which is equal in magnitude but opposite in direction to the centripetal force is known as centrifugal force.

Explanation:

Consider a piece of stone is tied to a string as shown in the figure.

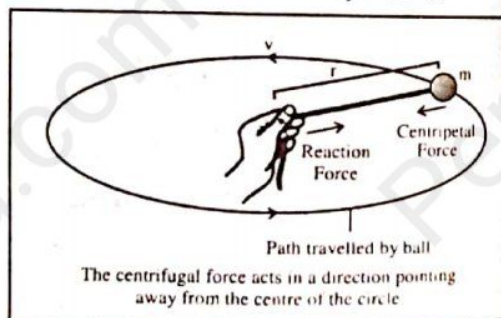
Now if we whirl the stone, then we provide centripetal force to the stone. During this process, our hands experience a push in the outwards direction. Such outwards push or force is known as centrifugal force. Its magnitude is equal to that of centripetal force but opposite in direction. Mathematically, we have,

$$F_{\text{centrifugal}} = \frac{mv^2}{r}$$

Q8: Explain the "bending of road" or curved road?

Ans: Consider a car is moving along a circular path [banked curve] of radius "r" as shown in the figure.

During the turn, the following forces are acting on the car.



1. The weight of the car "w" is acting downwards.

2. The normal force " F_N " is acting at angle " θ " with normal in upwards direction.

3. The centripetal force " F_c " is acting towards the centre.

Now if we resolve " F_N " into its components i.e. " $F_N \cos \theta$ " and " $F_N \sin \theta$ ", then we see that " $F_N \cos \theta$ "

balance the weight "w" of the car. While the component " $F_N \sin \theta$ " is acting towards the centre. So " $F_N \sin \theta$ " acts as centripetal force. So we have,

$$F_c = F_N \sin \theta$$

$$\Rightarrow \frac{mv^2}{r} = F_N \sin \theta \quad (1)$$

$$\& w = F_N \cos \theta$$

$$\Rightarrow mg = F_N \cos \theta \quad (2)$$

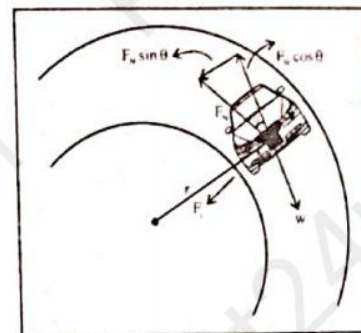
Now dividing equation (1) by equation (2), we get,

$$\frac{mv^2}{rmg} = \frac{F_N \sin \theta}{F_N \cos \theta} = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow \frac{v^2}{rg} = \tan \theta \Rightarrow v^2 = rg \tan \theta$$

$$\Rightarrow v = \sqrt{rg \tan \theta} \quad (3)$$

Equation (3) shows that for given values of r, g and θ we can determine the speed of the car during the turn.



$$F_c = mv^2 / r \quad w = mg$$

$$\frac{\sin \theta}{\cos \theta} = \tan \theta$$

Q9: What is moment of inertia? Discuss the importance of moment of inertia.

ETEA 2015

Ans: Moment of Inertia:

The reluctance of a rotating body to change its angular velocity is known as moment of inertia. (OR)

The property of a rotating body which maintains its state of rest or uniform rotational motion is called moment of inertia.

Significance of Moment of Inertia:

We know that the mass of a body plays an important role in linear motion. In the same way, the moment of inertia plays an important role in rotatory motion. The moment of inertia is the reluctance

tance of a rotating body to change its angular velocity. It opposes the angular acceleration of a rotating body. It depends upon the distribution of mass from the axis of rotation. The moment of inertia is represented by "I" and is given by,

$$I = mr^2 \quad (1)$$

In equation (1), "m" represents the mass of the rotating body and "r" represents the distance of the body from the axis of rotation.

Q10: Discuss the relation between torque and moment of inertia of a rotating body. (OR) Show that $\tau = I\alpha$

Ans. Consider a body of mass "m" attached to the end of a light rod, as shown in the figure.

Now if some force is applied on the body, then the body begins to rotate about the origin "O". In this way, torque is produced which is given by,

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\Rightarrow \tau = rF \sin \theta \Rightarrow \tau = rF \sin \theta \quad (1)$$

Now if the force " $\vec{F}$ " is perpendicular to " $\vec{r}$ ", then we put $\theta = 90^\circ$, so equation (1)

$$\Rightarrow \tau = rF \sin 90^\circ \quad [\sin 90^\circ = 1]$$

$$\Rightarrow \tau = rF \quad (2)$$

We also know that,

$$F = ma \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\tau = rma \quad (4)$$

We also know that,

$$a = r\alpha \quad (5)$$

Putting equation (5) in equation (4), we get,

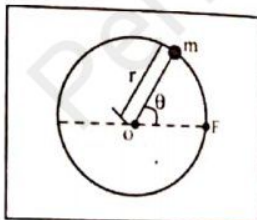
$$\tau = rm \times r \alpha$$

$$\Rightarrow \tau = mr^2 \alpha$$

$$[mr^2 = \text{moment of inertia} = I]$$

$$\Rightarrow \tau = I\alpha \quad (6)$$

Equation (6) represents the relation between torque and moment of inertia during rotatory motion. Equation (6) shows that the product of moment of inertia and angular acceleration is equal to torque.



Torque produces angular acceleration in a body. (ETEA 2010-11)

Torque and Moment of Inertia of a Non-Uniform Rigid Body:

In case of a rigid body, we suppose the whole body to be consists of large number of small parts having masses $m_1, m_2, m_3, m_4, \dots$, m_n at distance $r_1, r_2, r_3, \dots, r_n$ from the axis of rotation respectively. Now the torque acting on " m_1 " is given by,

$$\tau_1 = m_1 r_1^2 \alpha$$

Similarly,

$$\tau_2 = m_2 r_2^2 \alpha$$

$$\tau_3 = m_3 r_3^2 \alpha$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

Now the total torque is given by,

$$\tau_t = \tau_1 + \tau_2 + \tau_3 + \dots + \tau_n$$

$$\Rightarrow \tau_t = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + \dots + m_n r_n^2 \alpha$$

$$\Rightarrow \tau_t = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2) \alpha$$

$$\Rightarrow \tau_t = \left(\sum_{i=1}^n m_i r_i^2 \right) \alpha \quad (A)$$

In equation (A), we have,

$$\sum_{i=1}^n m_i r_i^2 = \text{total inertia} = I$$

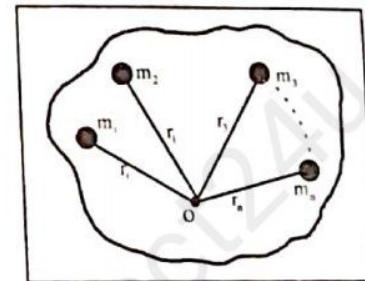
So equation (A) becomes,

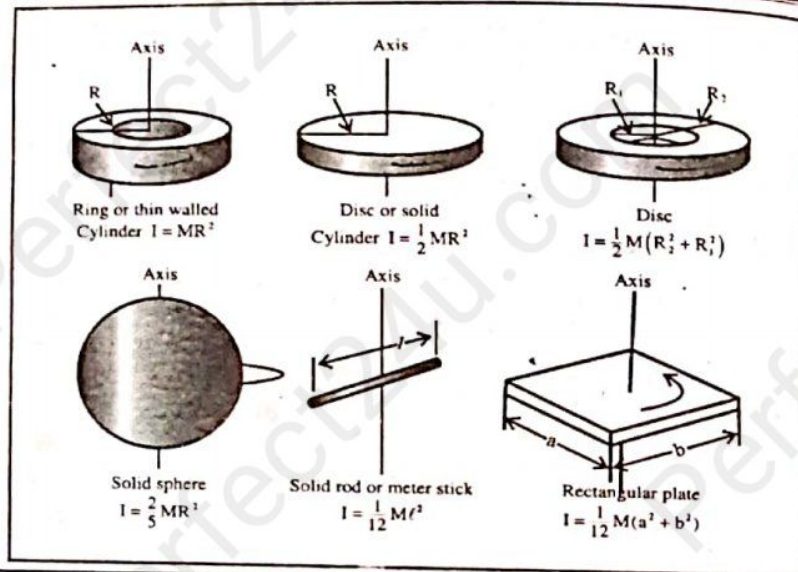
$$\tau_t = I\alpha \quad (B)$$

Equation (B) represents the torque and moment of inertia of a rigid body.

NOTE:

The mathematical expression of moment of inertia for bodies of various geometrical shapes are given below:



**ACTIVITY:**

Find the torque acting on bike wheel of the given figure.

Ans: Let "F" represents the applied force on the wheel which rotates with angular acceleration " α " as shown in the figure. Then the torque produced is given by,

$$\tau = rF \quad (1)$$

$$\text{Also } F = ma \quad (2)$$

Putting equation (2) in equation (1), we get,

$$\tau = rma \quad (3)$$

$$\text{Also } a = r\alpha \quad (4)$$

Putting equation (4) in equation (3), we get

$$\tau = rm \cdot r\alpha = r^2 m \alpha \quad (5)$$

In equation (5), $mr^2 = \text{moment of inertia} = I$

So equation (5) becomes,

$$\tau = I\alpha \quad (6)$$

Equation (6) represents the magnitude of torque acting on the bike wheel of radius "r" and having angular acceleration " α ".



Q11: Define and explain angular momentum.

BISE DI Khan 2019

Ans: **Definition:**

The momentum of a body due to rotatory motion is known as angular momentum. (OR)

The cross product of moment arm and linear momentum is known as angular momentum.

Explanation:

The angular momentum is represented by " $\vec{L}$ " and is given by,

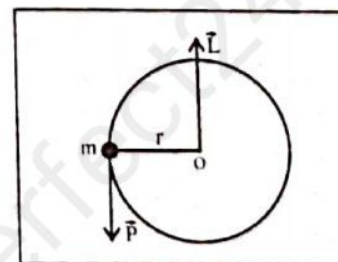
$$\vec{L} = \vec{r} \times \vec{p} \quad (1)$$

$$\Rightarrow \vec{L} = r p \sin \theta \hat{n}$$

$$\Rightarrow L = r p \sin \theta \quad (2)$$

Equation (2) represents the magnitude of angular momentum. Equation (2) shows that for given values of r, p, the angular momentum depends upon " θ ".

The angular momentum is a vector quantity. Its unit is $\text{kg} \cdot \text{m}^2 / \text{sec}$. It is always perpendicular to the plane determined by " $\vec{r}$ " and " $\vec{p}$ ".



Dimension of Angular Momentum:

$$\text{As } L = \text{kg} \cdot \text{m}^2 / \text{sec}$$

$$\Rightarrow L = \frac{\text{mass} \cdot \text{length}^2}{\text{time}}$$

$$\Rightarrow L = [ML^2T^{-1}]$$

Dimension of angular momentum and Plank's constant are same. (ETEA 2009, 10)

$$\text{Hints: } L = \frac{nh}{2\pi} \Rightarrow h = \frac{2\pi L}{n} = \frac{2\pi}{n} \cdot \frac{\text{kg} \cdot \text{m}^2}{\text{sec}} \Rightarrow h = [ML^2T^{-1}]$$

Q12: Discuss the relation between angular momentum and moment of inertia. (OR) Show that, $L = I\omega$

Ans: Consider a particle of mass "m" having linear momentum " $\vec{p}$ " moving along a circular path of radius "r" as shown in the figure.

Now the angular momentum " $\vec{L}$ " of the particle is given by,

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Rightarrow \vec{L} = r p \sin \theta \hat{n} \Rightarrow L = r p \sin \theta \quad (1)$$

For maximum angular momentum, we put $\theta = 90^\circ$ so equation (1) becomes,

$$L = r p \sin 90^\circ$$

$$[\sin 90^\circ = 1]$$

$$\Rightarrow L = rp \quad (2)$$

We know that,

$$p = mv \quad (3)$$

Putting equation (3) in equation (2), we get,

$$L = rmv \quad (4)$$

We also know that,

$$v = r\omega \quad (5)$$

Putting equation (5) in equation (4), we get,

$$L = rm \times r\omega$$

$$\Rightarrow L = mr^2\omega \quad [mr^2 = \text{moment of inertia} = I]$$

$$\Rightarrow \boxed{L = I\omega} \quad (5)$$

Equation (6) represents the relation between angular momentum and moment of inertia.

Angular Momentum of a Rotating Rigid Body:

In case of rigid body, we suppose the body to be consist of large number of parts having masses $m_1, m_2, m_3, m_4, \dots, m_n$ at distance $r_1, r_2, r_3, \dots, r_n$ from axis of rotation "o" respectively. Now the angular momentum of " m_1 " is given by,

$$L_1 = m_1 r_1^2 \omega$$

Similarly,

$$L_2 = m_2 r_2^2 \omega$$

$$L_3 = m_3 r_3^2 \omega$$

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$$L_n = m_n r_n^2 \omega$$

Now the total angular momentum of the rigid body is given by,

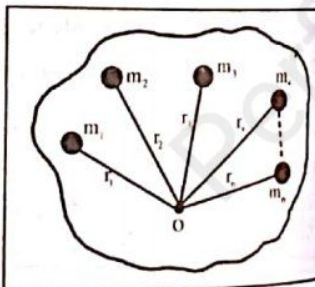
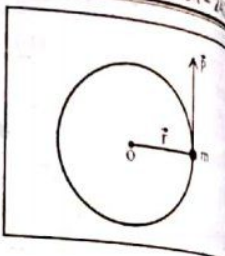
$$L_e = L_1 + L_2 + L_3 + \dots + L_n$$

$$\Rightarrow L_e = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega + \dots + m_n r_n^2 \omega$$

$$\Rightarrow L_e = (m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2) \omega$$

$$\Rightarrow L_e = \left[\sum_{i=1}^n m_i r_i^2 \right] \omega$$

$$\Rightarrow \boxed{L_e = I\omega} \quad (A) \quad \left[\sum_{i=1}^n m_i r_i^2 = \text{total moment of inertia} = I \right]$$



Equation (A) represents the angular momentum of a rotating rigid body.

Q13: Discuss the relation between torque and angular momentum.

(OR) Show that, $\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$

BISE Malakand 2019, DI Khan 2019, BISE Peshawar 2019

Ans. We know that the angular momentum is given by,

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow \vec{L} = rp \sin \theta \hat{n} \Rightarrow L = rp \sin \theta$$

$$\Rightarrow L = rp \sin 90^\circ = rp \times 1 \quad [\text{put } \theta = 90^\circ]$$

$$\Rightarrow L = rp \Rightarrow \Delta L = r \Delta p \quad (1)$$

Dividing both sides of equation (1) by " Δt ", we get,

$$\frac{\Delta L}{\Delta t} = r \left(\frac{\Delta p}{\Delta t} \right) \quad (2)$$

According to Newton's 2nd law, we get,

$$\frac{\Delta p}{\Delta t} = F \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\frac{\Delta L}{\Delta t} = rF \quad (4)$$

As $rF = \text{torque}$, so equation (4) becomes

$$\frac{\Delta L}{\Delta t} = \tau \Rightarrow \tau = \frac{\Delta L}{\Delta t} \Rightarrow \boxed{\vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}} \quad (5)$$

Equation (5) represents the relation between torque and angular momentum. Equation (5) shows that the torque is equal to the rate of change of angular momentum.

Q14: What do you mean by conservation of a physical quantity?

Ans. The conservation of a physical quantity means that a physical quantity remains conserved during any change of an isolated system. For example, mass, charge, energy, momentum etc are the physical quantities which remain constant or conserved during any change taking place in an isolated system.

Q15: State and explain the law of conservation of angular momentum with examples.

ETEA 2005

Ans. **Statement:**

This law states that, the angular momentum of a system remains constant in the absence of external torque.

Explanation: We know that,

$$\dot{L} = \frac{\Delta L}{\Delta t} \quad (1)$$

Now in the absence of external torque, we have,

$$\dot{L} = 0 \quad (2)$$

Putting equation (2) in equation (1), we get,

$$0 = \frac{\Delta L}{\Delta t} \Rightarrow \frac{\Delta L}{\Delta t} = 0 \Rightarrow \Delta L = 0 \quad (3)$$

Equation (3) shows that change in angular momentum is equal to zero. In other words, we can say that, there is no change in angular momentum i.e. the angular momentum remains constant. So equation (2) can be written as,

$$L = \text{constant} \quad (4)$$

Equation (4) represents the mathematical form of law of conservation of angular momentum.

Examples: The examples of law of conservation of angular momentum are:

1. Motion of planets around the sun.
2. Motion of moon and artificial satellites around the earth.
3. Orbital motion of electrons around the nucleus.
4. Motion of divers, ice skaters, dancers etc.

Q16: Discuss the K.E of a rotating rigid body.

BISE Bannu 2017

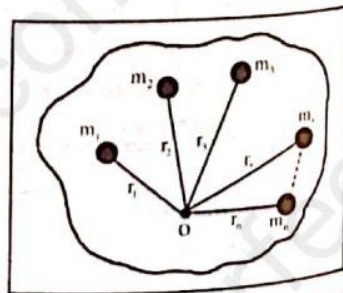
Ans: Consider a rigid body which consists of large number of parts having masses $m_1, m_2, m_3, \dots, m_n$ at distance $r_1, r_2, r_3, \dots, r_n$ from fixed origin "O" as shown in the figure. Now the K.E of " m_1 " is given by,

$$(K.E)_1 = \frac{1}{2} m_1 v_1^2 \quad [v = r \omega]$$

$$\Rightarrow (K.E)_1 = \frac{1}{2} m_1 r_1^2 \omega^2$$

$$\text{Similarly, } (K.E)_2 = \frac{1}{2} m_2 r_2^2 \omega^2$$

$$(K.E)_3 = \frac{1}{2} m_3 r_3^2 \omega^2$$



$$(K.E)_n = \frac{1}{2} m_n r_n^2 \omega^2$$

$$(K.E)_n = \frac{1}{2} m_n r_n^2 \omega^2$$

Now the total K.E of the rotating rigid body is given by,

$$(K.E)_t = (K.E)_1 + (K.E)_2 + \dots + (K.E)_n$$

$$\Rightarrow (K.E)_t = (K.E)_1 + (K.E)_2 + \dots + \frac{1}{2} m_n r_n^2 \omega^2$$

$$\Rightarrow (K.E)_t = \frac{1}{2} \omega^2 [m_1 r_1^2 + m_2 r_2^2 + \dots + m_n r_n^2]$$

$$\Rightarrow (K.E)_t = \frac{1}{2} \omega^2 \left[\sum_{i=1}^n m_i r_i^2 \right] \quad \left[\sum_{i=1}^n m_i r_i^2 = \text{total moment of inertia} = I \right]$$

$$\Rightarrow (K.E)_t = \frac{1}{2} \omega^2 I$$

$$\Rightarrow (K.E)_t = \frac{1}{2} I \omega^2 \quad (1)$$

Equation (1) represents the K.E of a rotating rigid body.

Q17: Discuss the K.E of a rolling body.

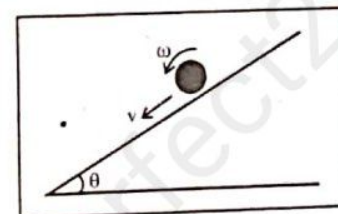
Ans: When an object rolls down on an inclined plane, it possesses linear motion as well as rotational motion. Thus the body possesses transla-

tional K.E due to linear velocity " $\vec{v}$ " and rotational K.E due to rotational velocity " $\vec{\omega}$ ".

Thus the total K.E of the rolling body is given by,

$$(K.E)_{\text{total}} = (K.E)_{\text{linear}} + (K.E)_{\text{rotational}}$$

$$\Rightarrow (K.E)_t = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \quad (1)$$



Equation (1) represents the K.E of a rolling body.

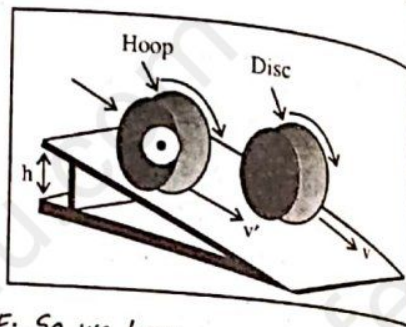
Q18: A disc and a hoop of the same radius starts moving down from the top of an inclined plane at the same time. Which will be moving faster on reaching the bottom of the inclined?

BISE Mardan 2019

Ans: A piece of hollow sphere is known as hoop or thin ring. While a

piece of solid cylinder is known as disc.

Consider a hoop and disc of the same radius start moving down from the top of an inclined plane at the same time as shown in the figure. At the top, the hoop and disc possess P.E. At the bottom of the inclined plane, their P.E is totally converted into translational K.E and rotational K.E. So we have,



$$P.E = (K.E)_{trans} + (K.E)_{rot}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{v^2}{2r^2}I \quad (1) \quad \left[v = r\omega, \omega = \frac{v}{r} \right]$$

For Disc: For disc, the moment of inertia is given by,

$$I = \frac{1}{2}mr^2 \quad (2)$$

Putting equation (2) in equation (1), we get,

$$mgh = \frac{1}{2}mv^2 + \frac{v^2}{2r^2} \times \frac{1}{2}mr^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{2mv^2 + mv^2}{4}$$

$$\Rightarrow mgh = \frac{3mv^2}{4} \Rightarrow gh = \frac{3v^2}{4}$$

$$\Rightarrow v^2 = \frac{4gh}{3} \Rightarrow v = \sqrt{\frac{4}{3}gh}$$

$$\Rightarrow v = \sqrt{1.33gh} \quad (3)$$

For Hoop:

For hoop, the moment of inertia is given by,

$$I = mr^2 \quad (4)$$

Putting equation (4) in equation (1), we get,

$$mgh = \frac{1}{2}mv^2 + \frac{v^2}{2r^2} \times mr^2$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{2mv^2}{2} = mv^2$$

$$\Rightarrow mgh = mv^2 \Rightarrow v^2 = gh$$

$$\Rightarrow v = \sqrt{gh} \quad (5)$$

Comparing equation (3), (5), we get,
 $v > v'$ i.e. $(v)_{disc} > (v)_{hoop}$

Thus the disc will reach the bottom of inclined plane faster than hoop.

Q19: (a) What is an artificial satellite? (b) Discuss the critical velocity of satellite. (c) Discuss the orbital velocity of satellite.

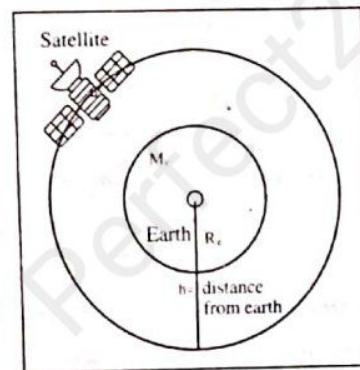
ETEA 2006, 2009, 2010, 2013, 2015, BISE Peshawar 2015, BISE Mardan 2017

Ans. (a) Artificial Satellite:

Artificial satellites are spacecraft that stay in orbit around the earth.

Applications of Artificial Satellites:

1. With the help of artificial satellites, we can take the pictures of the earth.
2. Artificial satellites relay communications around the earth.
3. Some satellites are used for navigation. The signals they send out are used by the crews of ships and aircraft to keep on course as they travel around the earth.
4. The global positioning system [GPS] is a system of 24 satellites which can tell a person his or her position on the earth very accurately.
5. Some satellites are used to investigate space.



(b) Critical Velocity of Satellite:

The minimum velocity required to put a satellite into the orbit is known as critical velocity of satellites.

When a satellite is moving in a circle of radius "R" from centre of earth of mass "M_e", it has centripetal acceleration, which is given by,

$$a_c = \frac{v^2}{R}$$

$$[a_c = g]$$

$$\Rightarrow g = \frac{v^2}{R}$$

$$\Rightarrow v^2 = gR \Rightarrow v = \sqrt{gR} \quad (1)$$

Put $g = 9.8 \text{ m/sec}^2$, $R = 6.4 \times 10^6 \text{ m}$ [radius of earth] in equation (1), we get,

$$v = \sqrt{9.8 \times 6.4 \times 10^6} = \sqrt{62.72 \times 10^3} \text{ m/sec}$$

$$\Rightarrow v = 7.9 \times 10^3 \text{ m/sec}$$

$$\Rightarrow v = 7.9 \text{ km/sec} \quad [10^3 = \text{kilo}]$$

This is the minimum velocity required to put a satellite into the orbit and is known as critical velocity.

(c) Orbital Velocity of Satellite:

A satellite of mass "m" moving around the earth in a circular orbit due to gravitational force between earth and satellite. So this force acts as centripetal force. So we have,

$$F_c = F_g$$

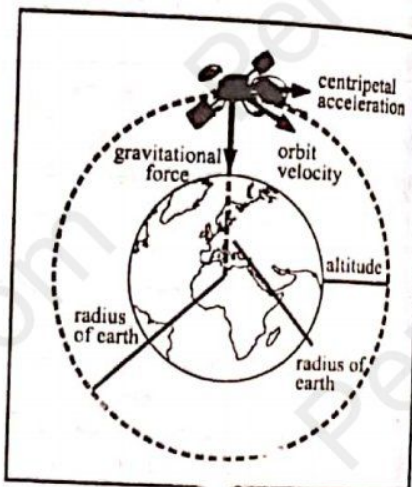
$$\Rightarrow \frac{mv^2}{r} = \frac{GM_e m}{r^2} \Rightarrow v^2 = \frac{GM_e}{r}$$

$$\Rightarrow v = \sqrt{\frac{GM_e}{r}} \quad (2)$$

In equation (2) $\sqrt{GM_e}$ = constant, so equation (2) becomes,

$$\Rightarrow v \propto \frac{1}{\sqrt{r}} \quad (3)$$

Equation (2) shows that the velocity of satellite is inversely proportional to the distance of satellite from the planet. i.e. a satellite in orbit will move faster when it is closer to the planet and slower when it is farther away. When a satellite falls from high altitude to lower altitude, it gains speed, and when it rises from low altitude to higher altitude, it loses speed.



Q20: Write a note on geostationary satellite.

Ans: It is a satellite which moves around the earth about its own axis. Its time period is 24 hours and appears stationary as seen from the earth. These satellites are very useful for world wide communication, weather observations and military purposes.

Now we know that the orbital speed of satellite is given by,

$$v = \sqrt{\frac{GM_e}{r}} \quad (1)$$

We also know that,

$$S = vt \Rightarrow v = \frac{S}{t}$$

$$\Rightarrow v = \frac{2\pi r}{T} \quad (2)$$

Comparing equations (1), (2), we get,

$$\frac{2\pi r}{T} = \sqrt{\frac{GM_e}{r}}$$

Squaring both sides, we get,

$$\Rightarrow \left(\frac{2\pi r}{T}\right)^2 = \left(\sqrt{\frac{GM_e}{r}}\right)^2$$

$$\Rightarrow \frac{4\pi^2 r^2}{T^2} = \frac{GM_e}{r} \Rightarrow r^3 = \frac{GM_e T^2}{4\pi^2} \quad (3)$$

Time Period of Satellite:

From equation (3), we have,

$$T^2 = \frac{4\pi^2 r^3}{GM_e} = \frac{4\pi^2 r}{\left(\frac{GM_e}{r^2}\right)} = \frac{4\pi^2 r}{g} \quad \left[\frac{GM_e}{r^2} = g\right]$$

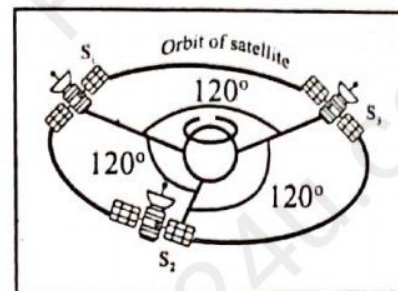
$$\Rightarrow T^2 = 4\pi^2 \frac{r}{g} \Rightarrow T = 2\pi \sqrt{\frac{r}{g}} \quad (4)$$

Equation (4) represents the time period of satellite.

Frequency of Satellite:

$$\text{As } f = \frac{1}{T} \quad f = \frac{1}{2\pi \sqrt{\frac{r}{g}}} \quad (5)$$

Equation (5) represents the frequency of satellite.



$$\left[\begin{array}{l} S = \text{circumference} = 2\pi r \\ t = \text{time period} = T \end{array} \right]$$

Radius of Satellite:

From equation (3), we have,

$$r^3 = \frac{GM_e T^2}{4\pi^2} \Rightarrow r = \left[\frac{GM_e T^2}{4\pi^2} \right]^{1/3} \quad (3)$$

Put $G = 6.673 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$, $M_e = 6 \times 10^{24} \text{ kg}$

$T = 365 \text{ days} = 365 \times 24 \text{ hr} = 365 \times 24 \times 3600 \text{ sec}$

$\pi = 3.14$ in equation (3), we get,

$$r = \left[\frac{6.673 \times 10^{-11} \times 6 \times 24 \times (365 \times 24 \times 3600)^2}{4 \times (3.14)^2} \right]^{1/3}$$

$$\Rightarrow r = 4.23 \times 10^7 \text{ m} \quad [10^3 = \text{kilo}]$$

$$\Rightarrow r = 4.23 \times 10^4 \times 10^3 \text{ m} = 4.23 \times 10^4 \text{ km}$$

$$\Rightarrow r = 4.23 \times 10^4 \text{ km} \quad (4)$$

Equation (4) represents the orbital radius for geostationary satellite which is measured from the centre of the earth. At this height, a satellite will always stay over a particular point on the surface of earth. This height above the equator comes to be 36000 km. The whole surface of the earth can be covered by using three geostationary satellites as shown in the figure. Each satellite covers a longitude of 120° .

Q21: (a) What do you mean by real weight and apparent weight?

(b) Discuss the weight of a body in a lift or elevator?

Ans: (a) Real and Apparent Weight:

The earth's gravitational force on the body represents a real weight. While the force needed to support the body against gravity, i.e. to prevent the body from falling represent its apparent weight.

(b) Weight of a Body in a Lift or Elevator:

Let Anmol weight a fish with a spring scale attached to the ceiling of lift as shown in the figure.

Case-1: When the lift is at rest: When the lift is at rest, then the weight of fish is acting in downwards direction while the tension in the string is acting in upward direction. Both the forces balance each other and the net force on the fish is zero. So we have,

$$T = w \Rightarrow T = mg \quad (1)$$

Case-2: When the lift is moving upward: When the lift is moving upward, then the net force on the fish is given by,

$$F_{\text{net}} = T - w$$

$$\Rightarrow ma = T - mg \Rightarrow T = ma + mg \quad (2)$$

Equation (2) shows that when the lift is moving upward, then the spring balance measures the weight which is greater than actual weight "W". This increase in weight of the fish is called apparent weight of the fish.

Case-3: When the lift is moving downwards: When the lift is moving downwards, then the net force on the fish is given by,

$$F_{\text{net}} = w - T$$

$$\Rightarrow ma = mg - T \Rightarrow -T = ma - mg$$

$$\Rightarrow T = mg - ma \quad (3)$$

Equation (3) shows that when the lift is moving downwards, the spring balance measures the weight which is less than actual weight "w" of the fish. This decrease in weight of the fish represents the apparent weight of the fish.

Case-4: When the lift is falling freely: When the lift is falling freely, then we put $a = g$, so equation (3) becomes,

$$T = mg - mg \Rightarrow T = 0 \quad (4)$$

Equation (4) shows that the spring balance shows zero reading of weight when the lift falls freely. Thus the apparent weight of the fish becomes zero when the lift is falling freely under gravity.



Fig. a: When the elevator is at rest, the spring scale reads the true weight of the fish.



Fig. b: When the elevator accelerates upward, the spring scale reads a value greater than the weight of the fish.

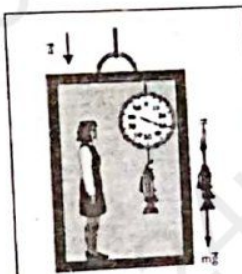


Fig. c: When the elevator accelerates downward, the spring scale reads a value less than the weight of the fish.

Q22: Discuss the weightlessness in a satellite.

Ans: Consider a satellite of mass "m" moving around the earth in a circular orbit of radius "r" as shown in the figure.

Here the weight of satellite is responsible for providing centripetal force. So we have,

$$w = F_c \Rightarrow mg = \frac{mv^2}{r}$$

$$\Rightarrow g = \frac{v^2}{r} \quad (1)$$

Now consider a body of mass "m" suspended by a spring from the ceiling of satellite. Here two forces are acting on the body. One is the force of gravity and the other is the normal reaction or tension in the spring, which acts in upwards direction. Thus the net force on the body is given by,

$$F_{net} = w - T$$

$$\Rightarrow F_{net} = mg - T \quad (2)$$

This force provides the necessary centripetal force to keep the body in circular orbit. So equation (2) becomes,

$$\frac{mv^2}{r} = mg - T \quad (3)$$

Putting equation (1) in equation (3), we get,

$$\frac{mv^2}{r} = \frac{mv^2}{r} - T \Rightarrow -T = 0$$

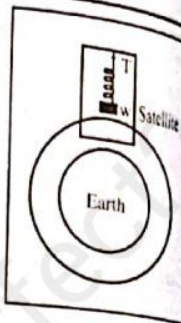
$$\Rightarrow T = 0 \quad (4)$$

Equation (4) shows that the tension or normal reaction on the body is zero, which shows that the body inside the satellite is weightless. Thus the body inside the satellite will appear in state of weightlessness.

Q23: Write a note on artificial gravity.

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Ans: As we know that the body inside the satellite is weightless, so the body faces certain problems while working inside the satellite. To overcome these problems, an artificial gravity is produced inside the satellite. For this purpose, some spin motion is given to the



satellite due to which the body inside the satellite experiences some centrifugal force. Due to centrifugal force, the body experiences a push away from the axis of rotation and thus finally comes in contact with the walls of satellite. In this way, an artificial acceleration is produced which is equal to the centrifugal acceleration and is given by,

$$a = \frac{v^2}{r}$$

$$\Rightarrow g = \frac{(r\omega^2)}{r}$$

$$\Rightarrow g = r\omega^2 \quad (1)$$

We know that,

$$f = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi f \quad (2)$$

Putting equation (2) in equation (1), we get,

$$g = r \times (2\pi f)^2$$

$$\Rightarrow g = 4\pi^2 r f^2 \Rightarrow f^2 = \frac{1}{4\pi^2} \frac{g}{r}$$

$$\Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{r}} \quad (3)$$

Equation (3) represents the frequency of satellite with which we get gravity equal to earth gravity and as a result the astronaut will feel no difficulty in performing their duties.

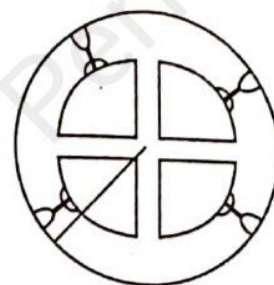
Q24: Discuss the various types of artificial satellite.

Ans: Types of Satellites:

There are several types of satellite which are given below:

1. Navigation Satellites:

The GPS (global positioning system) is made up of 24 satellites that orbit at an altitude of 20,000km above the surface of the earth. The difference in time for signals received from four satellites is used to calculate the exact location of a GPS receiver on earth.



The spaceship rotates with angular velocity " ω " and frequency $f = \frac{1}{2\pi} \sqrt{\frac{g}{r}}$ giving us the artificial gravity like earth.

For producing gravity equal to earth gravity, we put $a = g$

$$\text{As } \omega = \frac{\theta}{t}$$

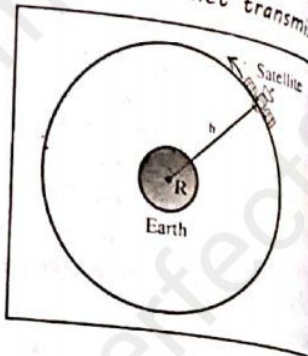
$$\Rightarrow \omega = \frac{2\pi}{T}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

$$\Rightarrow f = \frac{1}{T} = \frac{\omega}{2\pi}$$

2. Communication Satellites:

These satellites are used for television, phone or internet transmissions, for example, the Optus D1 satellite is in a geostationary orbit above the equator and has a coverage footprint to provide signals to all of Australia and New Zealand.



Communications satellites allow radio, television and telephone transmissions to be sent live anywhere in the world. Before satellites, transmissions were difficult or impossible at long distances. The signals, which travel in straight lines, could not bend around the round earth to reach a destination far away. Because satellites are in orbit, the signals can be sent instantaneously into space and then redirected to another satellite or directly to their destination.

Communications satellites are often in geostationary orbit. At the high orbital altitude of 35,800 kilometers, a geostationary satellite orbits the earth in the same amount of time it takes the earth to revolve once. From earth, therefore, the satellite appears to be stationary, always above the same area of the earth. The area to which it can transmit is called a satellite's footprint.

3. Weather Satellites:

These satellites are used to image clouds and measure temperature and rainfall. Both geostationary and low earth orbits are used depending on the type of weather satellite. Weather satellites are used to help with more accurate weather forecasting.

- مشکلات میں گھبرا کر نہیں چاہئے کیونکہ ستارے ہمیشہ اندھیرے میں ہی چمکتے ہیں۔
- اصل طاقت برداشت کرنے میں ہے انتقام لینے میں نہیں۔

Solution of Examples & Assignments

EXAMPLE 5.1:

An electric motor turns at 400rpm. What is the angular velocity in rad/sec? What is the angular displacement after 4s?

Solution:

Given data:

Angular velocity = $\omega = 400 \text{ rev / min}$, time = $t = 4 \text{ sec}$

Required:

(a) Angular velocity in rad/sec = $\omega = ?$

(b) Angular displacement = $\theta = ?$

Calculation:

(a) Angular velocity = $\omega = 400 \text{ rev / min}$

$$\Rightarrow \omega = 400 \times \frac{2\pi}{60} \text{ rad/sec}$$

$$1 \text{ revolution} = 2\pi \text{ radian}$$

$$\Rightarrow \omega = 41.9 \text{ rad / sec}$$

(b) We know that, $\theta = \omega t$ (2)

Putting the values in equation (2), we get,

$$\theta = (41.9 \times 4) \text{ rad} \Rightarrow \theta = 167.6 \text{ rad}$$

ASSIGNMENT 5.1:

A rotating pulley completes 12rev in 4s. Determine the average angular velocity in rev/s, rpm, and in rad/s? (3.00 rev/s, 28.6 rpm, and 18.8 rad/s)

Solution:

Given data:

Total angular displacement = $\theta = 12 \text{ rev}$

Total time = $t = 4 \text{ sec}$

Required:

(a) Average angular velocity in rev/sec = ?

(b) Average angular velocity in rev/min = ?

(c) Average angular velocity in rad/sec = ?

Calculation:

(a) We know that average angular velocity is given by,

$$60 \text{ sec} = 1 \text{ min}$$

$$1 \text{ sec} = \frac{1}{60} \text{ min}$$

$$1 \text{ rev} = 2\pi \text{ rad}$$

$$\omega_{\text{avg}} = \frac{\text{total angular displacement}}{\text{total time taken}}$$

$$\Rightarrow \omega_{\text{avg}} = \frac{12 \text{ rev}}{4 \text{ sec}} \Rightarrow \omega_{\text{avg}} = 3 \text{ rev / sec}$$

$$(b) \omega_{\text{avg}} = 3 \text{ rev / sec} = \frac{3 \text{ rev}}{\left(\frac{1}{60}\right) \text{ min}} = 3 \times 60 \text{ rev / min}$$

$$\Rightarrow \omega_{\text{avg}} = 180 \text{ rev / min}$$

$$(c) \omega_{\text{avg}} = 180 \text{ rev / min} = 180 \times \frac{2\pi}{60} \text{ rad / sec}$$

$$\Rightarrow \omega_{\text{avg}} = 18.8 \text{ rad / sec}$$

EXAMPLE 5.2:

In a carnival ride, the passenger travel in a circle of radius 5.0m, making one complete cycle in 4.0s. What is its acceleration?

Solution:

Given data:

Radius of circle = $r = 5.0\text{m}$, time = $t = 4.0\text{s}$

Required:

Acceleration = $a = ?$

Calculation:

We know that,

$$a_c = \frac{v^2}{r} \quad (1)$$

We also know that,

$$v = \frac{s}{t} \quad (2)$$

Putting equation (2) in equation (1), we get

$$a_c = \frac{s^2}{t^2 \cdot r}$$

$$\Rightarrow a_c = \frac{(2\pi r)^2}{t^2 \cdot r} = \frac{4\pi^2 r^2}{t^2 \cdot r}$$

$$\Rightarrow a_c = \frac{4\pi^2 r}{t^2} \quad (3)$$

Putting the values in equation (3), we get,

$$a_c = \frac{4 \times (3.14)^2 \times 5}{4^2} = 12.3 \text{ rad/sec}^2 \Rightarrow a_c = 12.3 \text{ rad / sec}^2$$

For complex cycle,
 $S = 2\pi r$

ASSIGNMENT 5.2:

An airplane dives along a curved path of radius "R" and velocity "v". The centripetal acceleration is 10ms^{-2} . If both the velocity and the radius are doubled, what will be the new acceleration? (20ms^{-2})

Solution:

Given data:

Centripetal acceleration = $a_c = 10\text{m / sec}^2$

Radius of curved path = R, Velocity of airplane = v

If velocity and radius are doubled, then, $R' = 2R$ and $v' = 2v$

Required:

New centripetal acceleration = $a'_c = ?$

Calculation:

We know that,

$$a_c = \frac{v^2}{R} \quad (1)$$

For new acceleration equation (1) becomes,

$$a'_c = \frac{v'^2}{R'} \quad (2)$$

Putting the values of "v" and "R" in equation (1), we get,

$$a_c = \frac{(2v)^2}{2R} = \frac{4v^2}{2R} = 2 \left(\frac{v^2}{R} \right)$$

$$\Rightarrow a'_c = 2 \left(\frac{v^2}{R} \right) \quad (3)$$

From equation (1),

$$\frac{v^2}{R} = a_c = 10 \text{ m/sec}^2 \Rightarrow \frac{v^2}{R} = 10 \text{ m/sec}^2 \quad (4)$$

Putting equation (4) in equation (3), we get,

$$a'_c = 2 \times 10 \text{ m/sec}^2 \Rightarrow a'_c = 20 \text{ m / sec}^2$$

$$\Rightarrow a'_c = 20 \text{ m / sec}^2$$

EXAMPLE 5.3:

The curved roadway is designed in such a way that a car will not have to rely on friction to round the curve even when the road is covered with ice. Suppose the designated speed for the road is to be 12m/s and the radius of the curve is 36.0m . At what angle

should the curve be banked?

Solution:

Given data:

Radius of circle = $r = 36.0\text{m}$, speed = $v = 12\text{m/s}$

Required: Angle of bank = $\theta = ?$

Calculation: We know that, $v = \sqrt{rg \tan \theta}$

$$\Rightarrow v^2 = rg \tan \theta \Rightarrow \tan \theta = \frac{v^2}{rg}$$

$$\Rightarrow \theta = \tan^{-1} \left(\frac{v^2}{rg} \right) = \tan^{-1} \left[\frac{12^2}{36 \times 9.8} \right] = \tan^{-1} \left[\frac{144}{352.8} \right]$$

$$\Rightarrow \theta = \tan^{-1}(0.408) = 22.1^\circ = 22^\circ \Rightarrow \boxed{\theta = 22^\circ}$$

ASSIGNMENT 5.3:

At what speed (in km/h) is a bank angle of 45° required for an aeroplane to turn on a radius of 60m? (87.34km/h)

Solution:

Given data:

Bank angle = $\theta = 45^\circ$, radius of turn = $r = 60\text{m}$

Required:

Speed of aeroplane = $v = ?$

Calculation:

We know that,

$$v = \sqrt{rg \tan \theta} \quad (1)$$

Putting the values in equation (1), we get,

$$v = \sqrt{60 \times 9.8 \times \tan(45^\circ)} = \sqrt{588 \times 1}$$

$$= 24.3 \text{ m/sec}$$

$$\Rightarrow \boxed{v = 24.3 \text{ m/sec}}$$

$$\text{OR } v = 24.3 \times \frac{3600}{1000} \text{ km/hr} = 87.3 \text{ km/hr}$$

$$\Rightarrow \boxed{\text{Speed} = v = 87.3 \text{ km/hr}}$$

EXAMPLE 5.4:

A 2kg mass swings in a circle of radius 50cm at the end of a light rod. What resultant torque is required to give an angular acceleration of 2.5 rad/s^2 .

Solution:

Given data: Radius = $r = 50\text{cm} \Rightarrow r = 0.5\text{m}$,
Mass = $m = 2\text{kg}$

Angular acceleration = $\alpha = 2.5 \text{ rad/sec}^2$

Required: Torque = $\tau = ?$

Calculation:

We know that, $\tau = I\alpha$ _____ (1)

Also we know that, moment of inertia of a circular hoop is given by,

$$I = mr^2 \quad (2)$$

Putting equation (2) in equation (1), we get,

$$\tau = mr^2 \alpha \quad (3)$$

Putting the values in equation (3), we get,

$$\tau = [2 \times (0.5)^2 \times 2.5] \text{ N}\cdot\text{m} = 1.25 \text{ N}\cdot\text{m}$$

$$\Rightarrow \boxed{\tau = 1.25 \text{ N}\cdot\text{m}}$$

ASSIGNMENT 5.5:

A cord is wrapped around the rim of a cylinder that has a mass of 10kg and a radius of 30cm. If the rope is pulled with a force of 60N, what is the angular acceleration of the cylinder? (40 rad/s^2)

Solution:

Given data: Mass = $m = 10\text{kg}$, Radius = $30\text{cm} = 0.3\text{m}$

Force = $F = 60\text{N}$

Required:

Angular acceleration = $\alpha = ?$

Calculation:

For cylinder, we have,

$$I = \frac{1}{2} mr^2 \quad (1)$$

We know that, $\tau = I\alpha$ _____ (2)

Also $\tau = rF$ _____ (3)

Comparing equation (1), (2), (3), we get,

$$I \propto rF \Rightarrow \alpha = \frac{rF}{I} \quad (4)$$

Putting equation (1) in equation (4), we get,

$$\alpha = \frac{rF}{\frac{1}{2} mr^2} = \frac{2rF}{mr^2} = \frac{2F}{mr}$$

$$\Rightarrow \alpha = \frac{2F}{mr} \quad (5)$$

Putting the values in equation (5), we get,

$$\alpha = \left[\frac{2 \times 60}{10 \times 0.3} \right] \text{rad/sec}^2 = 40 \text{rad/sec}^2 \Rightarrow \boxed{\alpha = 40 \text{rad/sec}^2}$$

ASSIGNMENT 5.6:

A belt is wrapped around the edge of a pulley that is 40cm in diameter. The pulley rotates with a constant angular acceleration of 3.50 rad/s^2 . At $t = 0$, the rotational speed is 2 rad/s . What is the angular displacement and angular velocity of the pulley 2s later? ($\theta = 238 \text{ rad}$, 9.00 rad/s)

Solution: Given data:

Diameter of pulley = $d = 40 \text{ cm} = 0.4 \text{ m}$

Angular acceleration = $\alpha = 3.5 \text{ rad/sec}^2$

Initial speed = $\omega_i = 2 \text{ rad/sec}$, time = $t = 2 \text{ sec}$

Required:

(a) Final angular velocity = $\omega_f = ?$

(b) Angular displacement = $\theta = ?$

Calculation:

(a) We know that, $\omega_f = \omega_i + \alpha t$ (1)

Putting the values in equation (1), we get,

$$\omega_f = (2 + 3.5 \times 2) \text{ rad/sec} = 9 \text{ rad/sec}$$

$$\Rightarrow \boxed{\omega_f = 9 \text{ rad/sec}}$$

(b) We also know that,

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2 \quad (2)$$

Putting the values in equation (2), we get,

$$\theta = \left[2 \times 2 + \frac{1}{2} \times 3.5 \times 2^2 \right] \text{ rad} = (4 + 7) \text{ rad}$$

$$\Rightarrow \boxed{\theta = 11 \text{ rad}}$$

EXAMPLE 5.5:

Find earth angular momentum using earth-sun distance and mass of earth. Earth-sun distance $149.6 \times 10^9 \text{ m}$, mass of the earth $5.9742 \times 10^{24} \text{ kg}$

Solution:

Given data: Mean distance from earth to sun = $r = 149.6 \times 10^9 \text{ m}$

Mass of the earth
= $M_e = 5.9742 \times 10^{24} \text{ kg}$

Required:

Angular momentum = $L = ?$

Calculation:

We know that,

$$L = mvr \quad (1)$$

We also know that, $S = vt \Rightarrow v = \frac{S}{t}$

$$\Rightarrow v = \frac{2\pi r}{T} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$L = m \times \frac{2\pi r}{T} \times r$$

$$\Rightarrow L = \frac{2\pi mr^2}{T} \quad (3)$$

Putting the values in equation (3), we get,

$$L = \left[\frac{2 \times 3.14 \times 5.9742 \times 10^{24} \times (149.6 \times 10^9)^2}{31536000} \right] \text{ kgm}^2 / \text{sec}$$

$$\Rightarrow L = \left[\frac{839658.3058 \times 10^{24} \times 10^{18}}{31536000} \right] = 0.02663 \times 10^{42} \text{ kgm}^2 / \text{sec}$$

$$\Rightarrow \boxed{L = 2.663 \times 10^{40} \text{ kg} \cdot \text{m}^2 / \text{sec}}$$

ASSIGNMENT 5.7:

A DVD disc has a radius of 0.0600 m , and a mass of 0.0200 kg . The moment of inertia of a solid disc is $I = (1/2) MR^2$, where M is the mass of the disc, and R is the radius. When a DVD in a certain machine starts playing, it has an angular velocity of 160.0 rads^{-1} . What is the angular momentum of this disc? ($0.00576 \text{ kg} \cdot \text{m}^2/\text{s}^{-1}$)

Solution:

Given data:

Radius = $r = 0.0600$, Mass = $m = 0.0200 \text{ kg}$

Angular velocity = $\omega = 160 \text{ rad/sec}$

Required:

Angular momentum of disc = $L = ?$

Earth complete one rotation around the sun in 365 days, so
Time period = $T = 365 \text{ days}$
 $\Rightarrow T = 365 \times 24 \times 3600 \text{ sec}$
 $\Rightarrow T = 31536000 \text{ sec}$
And distance covered in one rotation
= $S = \text{circumference}$
 $\Rightarrow S = 2\pi r$

Calculation:

We know that, $L = I\omega$ _____ (1)

For disc, $I = \frac{1}{2}mr^2$ _____ (2)

Putting equation (2) in equation (1), we get,

$$L = \frac{1}{2}mr^2 \times \omega$$
 _____ (3)

Putting the values in equation (3), we get,

$$L = \left[\frac{1}{2} \times 0.0200 \times (0.0600)^2 \times 160 \right] \text{kg} \cdot \text{m}^2 / \text{sec}$$

$$\Rightarrow L = 5.76 \times 10^{-3} \text{kg} \cdot \text{m}^2 / \text{sec}^2$$

$$\Rightarrow L = 0.005763 \text{kg} \cdot \text{m}^2 / \text{sec}$$

EXAMPLE 5.6:

A body of moment of inertia $0.80 \text{kg} \cdot \text{m}^2$ about a fixed axis, rotates with constant angular velocity of 100rad/s^{-1} . Calculate (i) its angular momentum = $\bar{L}$ (ii) Torque to sustain this position = $\bar{\tau}$

Solution:Given data:

Moment of inertia = $I = 0.80 \text{kg} \cdot \text{m}^2$

Angular velocity = $\omega = 100 \text{rad/s}^{-1}$

Required:

(i) Angular momentum = $L = ?$, (ii) Torque = $\tau = ?$

Calculation:

We know that, $L = I\omega$ _____ (1)

Putting the values in equation (1), we get,

$$L = [0.80 \times 100] \text{kg} \cdot \text{m}^2 / \text{sec}^2 \Rightarrow L = 80 \text{kg} \cdot \text{m}^2 / \text{sec}$$

(ii) Also $\tau = I\alpha$ _____ (2)

As the body is rotating with constant angular velocity so angular acceleration will be zero. So put $\alpha = 0$ in equation (2), we get,

$$\tau = [0.80 \times 0] \text{N} \cdot \text{m} \Rightarrow \tau = 0$$

EXAMPLE 5.7:

A child of mass 25kg stands at the edge of a rotating platform of mass 150kg and radius 4.0m . The platform with the child on it rotates with an angular speed of 6.2rad/s . The child jumps off in a radial direction. What happens to the angular speed of the plat-

form? Treat the platform as a uniform disk.

Solution:Given data:

Mass of child = $m_1 = 25 \text{kg}$, mass of platform = $m_2 = 150 \text{kg}$,
Radius = $r = 4.0 \text{m}$, initial angular speed = $\omega_i = 6.2 \text{rad/s}^{-1}$

Required:

Final angular speed = $\omega_f = ?$

Calculation:

By law of conservation of angular momentum, we have,

Total initial angular momentum = Total final angular momentum

$$\Rightarrow L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f \Rightarrow \omega_f = \frac{I_i \omega_i}{I_f}$$
 _____ (1)

$$L = I\omega$$

Here $I_i = (I_{\text{platform}} + I_{\text{child}})$

$$\Rightarrow I_i = \frac{1}{2}m_2 r^2 + m_1 r^2$$

$$\Rightarrow I_i = \left(\frac{1}{2} \times 150 \times 4^2 \right) + (25 \times 4^2)$$

$$\Rightarrow I_i = (1200 + 400) \text{kg} \cdot \text{m}^2$$

$$\Rightarrow I_i = 1600 \text{kg} \cdot \text{m}^2$$

$$\text{While } I_f = \frac{1}{2}m_2 r^2 = \frac{1}{2} \times 150 \times 4^2 = 1200 \text{kg} \cdot \text{m}^2$$

Putting the values of I_f , I_i and ω_i in equation (1), we get,

$$\omega_f = \left(\frac{1600 \times 6.2}{1200} \right) \text{rad/sec} = 8.27 \text{rad/sec}$$

$$\Rightarrow \omega_f = 8.27 \text{rad/sec}$$

EXAMPLE 5.8:

A 70kg man is standing on a scale in an elevator which is accelerating, as it heads for the top floor of a building at 4m/s^2 . What apparent weight will show on the scale?

Solution:Given data:

Mass of man = $m = 70 \text{kg}$, Acceleration = $a = 4 \text{m/s}^2$

Required: Apparent weight = $T = ?$

Calculation:

For platform

$$I_{\text{platform}} = \frac{1}{2}m r^2$$

For child

$$I_{\text{child}} = m r^2$$

For upward motion, we have,

$$T = mg + ma \quad (1)$$

Putting the values in equation (1), we get,

$$T = (90 \times 9.8 + 70 \times 4) \text{ N} = 686 \text{ N} + 280 \text{ N}$$

$$\Rightarrow T = 966 \text{ N}$$

$$\Rightarrow \text{Apparent weight on the scale} = 966 \text{ N}$$

ASSIGNMENT 5.8:

A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2 , what will be the reading of the spring balance? (24N)
[Take $g = 10 \text{ m/sec}^2$]

Solution:

Given data: Weight of bag = $w = 49 \text{ N}$,

Acceleration = $a = 5 \text{ m/sec}^2$

Required:

Apparent weight of bag = $T = ?$

Calculation:

First we calculate the mass of the bag, at stationary condition, we have,

$$w = 49 \text{ N} \Rightarrow mg = 49 \text{ N}$$

$$\Rightarrow m = \frac{49}{10} \text{ kg} \Rightarrow m = 4.9 \text{ kg}$$

Now when the lift is moving downwards, then the apparent weight is given by,

$$T = mg - ma = (4.9 \times 10 - 4.9 \times 5) \text{ N}$$

$$\Rightarrow T = 49 - 24.5 \Rightarrow T = 24.5 \text{ N}$$

ASSIGNMENT 5.9:

In an elevator that has a constant acceleration upward, Abdur Rahman is standing on a scale. The scale reads 960N. When Abdur Rahman picks up a 20kg box, the scale reads 1200N. The acceleration of the elevator stays the same. (a) Find the acceleration of the elevator (b) Find Abdur-Rahman's weight. (784N, 2.2 m/s^2)

Solution:

Given data:

Let mass of Abdur-Rahman = $m \text{ kg}$

Normal force before picking box = $F_N = 960 \text{ N}$

Normal force after picking box = $F'_N = 1200 \text{ N}$

Required:

(a) Mass of Abdur-Rahman = $m = ?$

(b) Weight of Abdur-Rahman = $w = ?$

(c) Acceleration = $a = ?$

Calculation:

(a) For upward motion, we have,

$$F_N = mg + ma$$

$$\Rightarrow 960 = mg + ma \quad (1)$$

When Abdur-Rahman picks up 20 kg box, then we have,

$$F'_N = (m + 20)g + (m + 20)a \quad (2)$$

From equation (1), we have,

$$960 = mg + ma \Rightarrow ma = 960 - mg$$

$$\Rightarrow a = \frac{960}{m} - g \quad (3)$$

Putting equation (3) in equation (2), we get,

$$F'_N = (m + 20)g + (m + 20)\left(\frac{960}{m} - g\right)$$

$$\Rightarrow 1200 = \cancel{mg} + 20g + m \times \frac{960}{m} - \cancel{mg} + \frac{20 \times 960}{m} - 20g$$

$$\Rightarrow 1200 = 960 + \frac{19200}{m} \Rightarrow \frac{19200}{m} = 1200 - 960 = 240$$

$$\Rightarrow \frac{19200}{m} = 240 \Rightarrow m = \frac{19200}{240} \text{ kg} \Rightarrow m = 80 \text{ kg}$$

$$\Rightarrow \text{Mass of Abdur - Rahman} = m = 80 \text{ kg}$$

$$(b) \text{ Weight of Abdur - Rahman} = w = mg = 80 \times 9.8 \text{ N} = 784 \text{ N}$$

(c) For upward motion, we have,

$$F_N = mg + ma \Rightarrow a = \frac{F_N - mg}{m}$$

$$= \left(\frac{960 - 80 \times 9.8}{80} \right) \text{ m/sec}^2$$

$$\Rightarrow a = \left(\frac{960 - 784}{80} \right) \text{ m/sec}^2 \Rightarrow a = 2.2 \text{ m/sec}^2$$

EXAMPLE 5.9:

Determine the orbital speed for the international space station (ISS). If its orbit $4.0 \times 10^2 \text{ km}$ above the earth surface.

Solution:

Given data:

Height of space station from earth surface

$$= h = 4 \times 10^2 \text{ km} = 4 \times 10^2 \times 10^3 \text{ m} = 4 \times 10^5 \text{ m}$$

$$\text{Radius of earth} = R = 6.36 \times 10^6 \text{ m}$$

Total distance of space station from centre of earth is given by,

$$R = R_e + h = (6.36 \times 10^6 + 0.4 \times 10^6) \text{ m}$$

$$\Rightarrow R = 10^6 (6.36 + 0.4) \text{ m} = 6.76 \times 10^6 \text{ m}$$

Value of gravitational constant

$$= G = 6.673 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$$

$$\text{Mass of earth} = M_e = 6 \times 10^{24} \text{ kg}$$

Required:

$$\text{Orbital speed} = v = ?$$

Calculation:

The orbital speed is given by,

$$v = \sqrt{\frac{GM_e}{R}} \quad (1)$$

Putting the values in equation (1), we get,

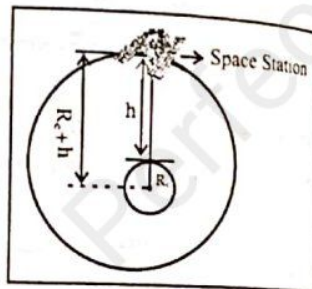
$$v = \left[\sqrt{\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{6.76 \times 10^6}} \right] \text{ m/sec}$$

$$\Rightarrow v = \left[\sqrt{5.9 \times 10^{-11+24-6}} \right] \text{ m/sec} = \left[\sqrt{5.9 \times 10^7} \right] \text{ m/sec}$$

$$\Rightarrow v = \left[\sqrt{59.0 \times 10^6} \right] \text{ m/sec} = 7.67 \times 10^3 \text{ m/sec}$$

$$\Rightarrow v = 7.67 \times 10^3 \text{ m/sec}$$

$$\Rightarrow \text{Orbital speed} = v = 7.67 \times 10^3 \text{ m/sec}$$

**EXAMPLE 5.10:**

What should be the orbital speed to launch a satellite in a circular orbit 900 km above the surface of the earth?

Solution:

Given data:

$$\text{Height above the surface of earth is } = h = 900 \text{ km}$$

$$= 900 \times 10^3 \text{ m} = 9 \times 10^2 \times 10^3 \text{ m} = 9 \times 10^5 \text{ m}$$

$$\text{Radius of earth} = R_e = 64 \times 10^5 \text{ m}$$

$$\text{So radius of orbit will be } r_o = R_e + h$$

$$= (64 \times 10^5 + 9 \times 10^5) \text{ m}$$

$$= (64 + 9) \times 10^5 \text{ m} = 73 \times 10^5 \text{ m}$$

$$\text{Here } G = 6.673 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$$

$$\text{And mass of earth} = M_e = 6 \times 10^{24} \text{ kg}$$

$$\text{Required: Orbital speed} = v_o = ?$$

Calculation:

We know that,

$$v_o = \sqrt{\frac{GM_e}{r_o}} \quad (1)$$

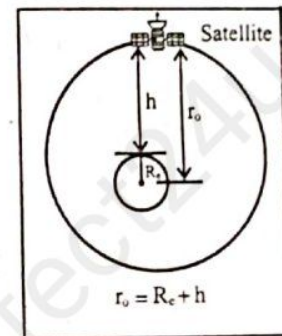
Putting the values in equation (1), we get,

$$v_o = \sqrt{\frac{6.673 \times 10^{-11} \times 6 \times 10^{24}}{73 \times 10^5}}$$

$$\Rightarrow v_o = \left[\sqrt{0.548 \times 10^5} \right] \text{ m/sec}$$

$$\Rightarrow v_o = 0.740 \times 10^4 \text{ m/sec}$$

$$\Rightarrow v_o = 7.40 \times 10^3 \text{ m/sec}$$



✳ حضرت علیؑ کا قول ہے: "خندہ روئی سے پیش آنا سب سے پہلی نیکی ہے۔"

✳ حضرت علیؑ کا قول ہے: "کارخانہ قدرت میں فکر کرنا بھی عبادت ہے۔"

✳ حضرت علیؑ کا قول ہے: "بے موقع حیا بھی باعثِ محرومی ہے، قابلِ محبت بہت کم لوگ ہیں۔"

Exercise

Choose the best possible answer:

1. The angular speed in radians / hours for daily rotation of our earth is:

a. 2π b. 4π c. $\frac{\pi}{6}$ ☒ d. $\frac{\pi}{12}$

Sol: As $T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T}$ (1)

Put $T = 24$ hour in equation (1), we get,

$$\omega = \frac{2\pi}{24} \text{ rad/hour} \Rightarrow \omega = \frac{\pi}{12} \text{ rad/hr}$$

2. Linear acceleration $a = r\alpha$ when " θ " is:

a. 0° b. 180° c. 360° ☒ d. 90°

3. What is moment of inertia of a sphere?

a. MR^2 b. $\frac{1}{2}MR^2$ ☒ c. $\frac{2}{5}MR^2$ d. $\frac{1}{2}MR^2$

4. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of particle. The motion of the particle takes place in a horizontal plane. It follows:

a. Linear momentum is constant b. Velocity is constant
☒ c. It moves in a circular path d. None of these

5. A body moving in a circular path with constant speed has:

a. Constant acceleration b. Constant retardation
☒ c. Variable acceleration

d. Variable speed and constant velocity

Sol: The body is moving with constant speed but its direction is changes continuously due to which the body posses variable acceleration.

6. Astronauts appear weightless in space because:

a. There is no gravity in space
 b. There is no floor pushing upwards

☒ c. Satellite is freely falling

d. There is no air in space

7. Which one is constant for a satellite in orbit?

a. Velocity

☒ c. Angular momentum

b. $K \cdot E$

d. Potential energy

8. If the earth suddenly stops rotating the value of " g " at equator would:

a. Decrease

b. Remain unchanged

☒ c. Increase

d. Become zero

Sol: If the earth stops rotating then the value of " g " at equator will increase because when the earth rotates the gravity at every point remains uniform. And when it stops rotating, the gravity increases because the pressure is exerted on only one point at the equator.

9. If solid sphere and solid cylinder of same mass and density rotate about their own axis, the moment of inertia will be greater for:

a. Solid sphere

☒ b. Solid cylinder

c. The one that has the largest mass arrives first

d. The one that has the largest radius arrives first

Sol: For sphere, $I_{\text{sphere}} = \frac{2}{5}MR^2$ (1)

For cylinder, $I_{\text{cylinder}} = \frac{1}{2}MR^2$ (2)

Dividing equation (2) by equation (1)

$$\frac{I_{\text{cylinder}}}{I_{\text{sphere}}} = \frac{\frac{1}{2}MR^2}{\frac{2}{5}MR^2} = \frac{1 \cdot 2}{2 \cdot 5} = \frac{1}{2} \times \frac{5}{2} = \frac{5}{4}$$

$$\Rightarrow \frac{I_{\text{cylinder}}}{I_{\text{sphere}}} = \frac{5}{4} \Rightarrow I_{\text{cylinder}} = \left(\frac{5}{4}\right) I_{\text{sphere}} \text{ (3)}$$

As $\frac{5}{4} > 1$, so equation (3) $\Rightarrow I_{\text{cylinder}} > I_{\text{sphere}}$

So option (b) is correct.

10. The gravitational force exerted on an astronaut on earth's surface is 650N down. When she is in the international space station, is the gravitational force on her:

a. Larger

b. Exactly the same

☒ c. Smaller

d. Nearly but not exactly zero

e. Exactly zero

Sol: Gravitational force is inversely proportion to the square of the distance between two bodies. The distance of internal space station is situated very far from earth, so distance is greater and force will be smaller.

11. A solid cylinder of mass "M" and radius "R" rolls down an incline without slipping. Its moment of inertia about an axis through its center of mass is $MR^2/2$. At any instant while in motion, its rotational kinetic energy about its center of mass is what fraction of its total kinetic energy?

- (a) $\frac{1}{2}$ b. $\frac{1}{4}$ ✓ c. $\frac{1}{3}$ d. $\frac{2}{5}$ e. None of these

Sol: According to the question, the fraction of rotational K.E and total K.E is given by,

$$\frac{(K.E)_r}{(K.E)_{total}} = \frac{K.E_r}{K.E_r + K.E_{trans}}$$

$$K.E_{total} = K.E_{rot} + K.E_{trans}$$

$$\Rightarrow \frac{(K.E)_r}{(K.E)_{total}} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} m v^2} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + \frac{1}{2} M R^2 \omega^2} \quad [v = R\omega]$$

$$\Rightarrow \frac{(K.E)_r}{(K.E)_{total}} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 + I \omega^2}$$

For cylinder
 $I = \frac{1}{2} MR^2$

$$\Rightarrow \frac{(K.E)_r}{(K.E)_{total}} = \frac{\frac{1}{2} I \omega^2}{\frac{1}{2} I \omega^2 [1 + 2]} = \frac{1}{1 + 2} = \frac{1}{3}$$

So option (c) is correct.

- ① حضرت علی کا قول ہے: "مگر نعمتِ حصولِ کما ہٹ ہے اور ناشکری حصولِ زحمت کا باعث ہے۔"
- ② حضرت علی کا قول ہے: "ادب بہترین کمالات اور خیرات افضل ترین عبادات سے ہے۔"
- ③ حضرت علی کا قول ہے: "موت ایک بے خبر ساتھی ہے۔"

Conceptual Questions

Give short answers to the following questions:

Q1: Why is the fly wheel of an engine made heavy in the rim?

Ans: The fly wheel of an engine is made heavy in the rim for increasing the moment of inertia of fly wheel.

Reasons and Explanation:

When a fly wheel is put into motion, torque is acting on it which is given by,

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} \quad (1)$$

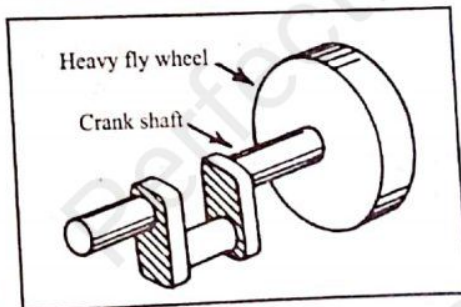
We also know that the moment of inertia is given by,

$$I = mr^2 \quad (2)$$

When the engine is started, the crank gets energy from the engine in the form of jerks. The crank is connected to the heavy flywheel. Now to control the jerky motion, the flywheel is needed to rotate with steady angular acceleration " α ".

Now according to equation (1), for getting smaller value of angular acceleration, we need greater moment of inertia. Equation (2) shows that keeping radius "r" constant, we need greater mass for getting more moment of inertia.

Thus for getting more moment of inertia the flywheel is made heavy or massive in the rim. Such heavy flywheel possesses large moment of inertia which avoids the jerky motion of the crank. In this way the engine remains in working conditions and remains safe from serious damages.



Q2: Why is a rifle barrel "rifled"?

Ans: The barrel of a rifle is rifled to keep the bullet along the re-



quired direction and to hit the target accurately.

Reasons and Explanation:

Rifling is the process in which helical grooves are made inside the barrel of a rifle. When the rifle is fired, the helical grooves provide spin to the bullet around its long axis. The spin motion improves the projectile motion of the bullet. It also improves the aerodynamic stability and accuracy of the bullet. The bullet fired from rifled rifle posses rotational K.E as well as translation K.E. Thus the total energy of the bullet is given by,

$$(K.E)_{total} = (K.E)_{trans} + (K.E)_{rot}$$

$$\Rightarrow (K.E)_{total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Now due to rotational and translational K.E, the bullet posses both spin angular momentum as well as linear momentum. Both momentums are parallel to each other and as result the bullet gain large momentum. As a result the bullet travel fast and can hit the target with accuracy. Such bullet imparts greater momentum to the target.

Q3: Is it possible for a person to distinguish between a raw egg and a hard boiled one by spinning each on a table? Explain.

Ans: Yes, it is possible for a person to distinguish between a raw egg [uncooked] and a hard boil one by spinning each on a table.

Reasons and Explanation:

We know that the angular momentum of a spinning body is given by,

$$L = I\omega \Rightarrow \omega = \frac{L}{I} \quad (1)$$

Equation (1) shows the angular speed of spinning object is inversely proportional to the moment of inertia of the spinning object i.e. greater the moment of inertia of the object, smaller will be its angular velocity and vice versa.

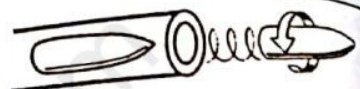


Fig a: Rifled bore fires spinning projectile on a stable course

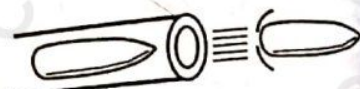


Fig b: Smooth bore fires tumbling projectile on unstable course.

Now we can distinguish between a raw egg and boiled hard egg on the basis of their moment of inertia and angular velocities.

Case1: Spinning of raw egg [uncooked egg]:

When a raw egg is allowed to spin, the liquid molecules inside the shell moves away from the axis of rotation towards the inner surface of the shell as shown in figure (a). As a result the mass distribution is disturbed and the egg becomes more massive near the shell as compare to the central region. We also know that the moment of inertia increases with increase in mass. So the moment of inertia of raw egg increases due to non-uniform distribution of the mass of liquid inside the shell. Now according to equation (1), the angular velocity of the raw egg will decreases due to greater moment of inertia. Thus the raw egg will rotate slowly and will comes to rest sooner.

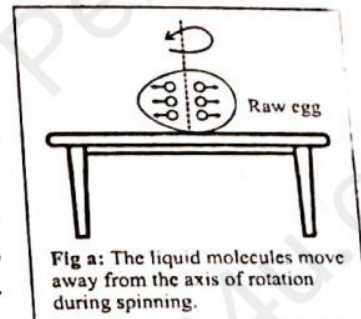


Fig a: The liquid molecules move away from the axis of rotation during spinning.

Case2: Spinning of hard boil egg:

The liquid inside the shell appears in solid state after boiling the egg. Now when the egg is allowed to spin, the solid molecule does not move towards the shell. As a result mass distribution remains the same. Due to same mass distribution, the moment of inertia of the boiled egg remains low as compare to raw egg.

Now according to equation (1), the angular speed of the boiled egg will be greater due to its low moment of inertia. i.e. the boiled egg will rotate with greater angular speed and will comes to rest later as compare to raw egg.

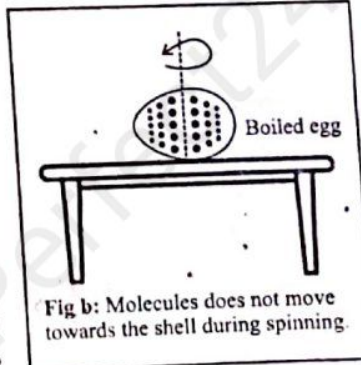


Fig b: Molecules does not move towards the shell during spinning.

Conclusion:

From the above discussion, it is clear that boiled egg will posses more angular velocity and will comes to rest later as compare to

raw egg. Thus we can distinguish between a raw egg and hard boil egg due to difference in moment of inertia and angular speed of the two eggs.

Q4: Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?

Ans: The acceleration of a body moving uniformly in a circle is always directed towards the centre of the circle.

Reasons and Explanation:

When a body of mass "m" is moving along a circle uniformly, then the magnitude of the velocity of the body (i.e. speed) remains constant while the direction of its velocity changes at each point.

Now when the magnitude of velocity remains constant, then there will be no tangential acceleration in the motion of the body. Only radial acceleration is produced due to change in the direction of velocity.

The change in the direction of velocity is due to centripetal force. So the radial acceleration is produced due to action of centripetal force. Thus the radial acceleration is also known as centripetal acceleration. As the centripetal force is directed towards the centre of the circular path, so the centripetal acceleration is also directed towards the centre of the circle.

Mathematical Form:

We know that,

$$F_{\text{centripetal}} = -m \frac{v^2}{r} \quad (1)$$

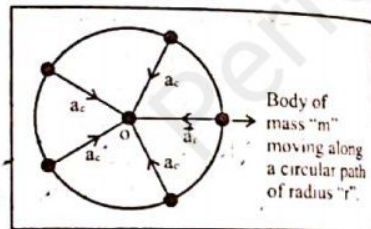
We also know that,

$$F_{\text{centripetal}} = ma_c \quad (2)$$

Comparing equation (1) and equation (2), we get,

$$ma_c = -m \frac{v^2}{r} \Rightarrow a_c = -\frac{v^2}{r} \quad (3)$$

The negative sign in equation (3) shows that the centripetal acceleration "a_c" is directed towards the centre of the circular path.



Q5: A ball is just supported by a string without breaking. If it is set swinging, it breaks. Why?

Ans: A swinging ball can break the string due to the influence of centrifugal force.

Reasons and Explanation:

When a ball is just supported by a string without breaking, then under such condition, the weight "w" of the ball is acting in downward direction while the tension in the string "T" is acting in upward direction. So we have,

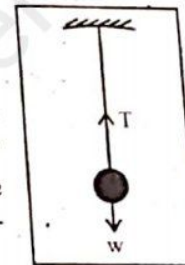
$$T = w \quad (1)$$

Equation (1) shows that the tension in the string remains within its elastic limit and hence remains un-breaking.

Now when the string is set swinging, then centripetal and centrifugal forces exist. The tension in the string acts as centripetal force. The centrifugal force is acting in opposite direction of centripetal force. So centrifugal force is added to the weight. So equation (1) becomes,

$$T = w + \text{centrifugal force} \quad (2)$$

Equation (2) shows that the tension in the swinging string increases. Such increases in tension cross the elastic limit of the string which causes the string to break.

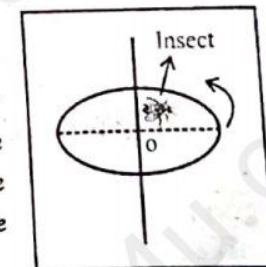


Q6: An insect is sitting close to the axis of a wheel. If the friction between the insect and the wheel is very small, describe the motion of the insect when the wheel starts rotating.

Ans: When the wheel starts rotation, the insect will fly away from the axis of rotation due to lack of necessary centripetal force.

Reasons and Explanation:

Let the insect is sitting on the wheel near the axis of rotation. Now if the wheel starts rotation, and there is sufficient force of friction between surface of wheel and insect, then the force of friction in this case will act as centripetal force and as a result the insect will remain stick to the surface of the wheel during rotation.



But here we are given that the force of friction between the insect and wheel is very very small. So when the wheel begins to rotate, the smaller force of friction will be unable to provide necessary centripetal force to the insect. As a result the insect will not remain stick to the surface of the wheel and will move away from the axis of rotation of the wheel due to centrifugal force. The insect will follow a spiral motion and will fall off the rotating wheel tangentially after a short period of time.

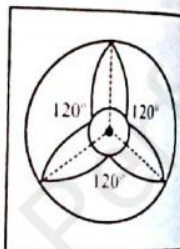


Q7: Explain how many minimum number of geo-stationary satellites are required for global coverage of T-V transmission?

Ans: At least three geo-stationary satellites are required for the global coverage of T-V transmission.

Reasons and Explanation:

We know that the geo-stationary satellites move around the earth from west to east in the equatorial plane at the same rate as that of the rotation of the earth about its own axis. A single geo-stationary satellite covers a longitude of 120° [$360^\circ/3=120^\circ$] so at least three geo-stationary satellites will be required to the whole earth longitude. These satellites are placed at three proper position in such a way that they can transmit T-V signals from any part of the world. These satellite have made the whole world as global village.



Q8: Explain the significance of moment of inertia in rotatory motion?

Ans: The moment of inertia play the same role in rotary motion as the mass perform in the translatory motion.

Reasons and Explanation:

The Newton, 2nd Law for translatory motion is given by,

$$F = ma \quad (1)$$

The Newton's 2nd law for rotatory motion is given by,

$$\tau = I\alpha \quad (2)$$

Using equation (1), we can explain various phenomenon related to translator motion. In equation (1), 'm' represents the mass which is also known as "measure of inertia". Equation (1) shows that massive objects posses more reluctance and greater force will be required to change the state of rest or motion of a that object. Equation (1) also shows that acceleration produced in a body is inverse proportional to its mass i.e. heavier body can be accelerated by applying larger force and lighter body can be accelerated by applying smaller force. Equation (1) can be used to explain various phenomenon like momentum, energy, weight, and many other interaction b/w various objects.

In the same way, in equation (2) 'I' represents the moment of inertia of rotating objects. The moment of inertia of various rotating objects depends upon their mass, geometrical shape and distance from the axis of rotation. Heavier objects posses greater moment of inertia and vice versa.

The moment of inertia plays an important role in daily life. Their applications are use in vehicles for comfortable journey. We can express the torque, angular velocity, angular acceleration, angular momentum, rotational K.E of various objects in terms of moment of inertia. These parameters are used to explain various phenomenon in case of satellite and planetary motion.

Q9: Why does the coasting rotating system slow down as water drops into the beaker?

Ans: The coasting rotating system becomes slowed down when water is added to the beaker. It is because the moment of inertia of the rotating system increases which opposes the rotation of system.

Reasons and Explanation:

We know that the angular momentum of a system having moment of inertia 'I' rotating with certain angular velocity ' ω ' is given by,

$$L = I\omega \Rightarrow \omega = \frac{L}{I} \quad (1)$$

We also know that the moment of inertia of a rotating system depends upon its mass. i.e. greater the mass, greater will be the its

moment of inertia and vice versa.

Now when water is added into the beaker, the mass of rotating system increases. Due to increase in mass of the rotating system, its moment of inertia " I " increases.

Now according to equation (1), when moment of inertia increases, then the angular velocity " ω " of the system decreases. Thus by adding water into the beaker, the angular velocity of coasting rotating system decreases and becomes slowed down.

Q10: A body will be weightless when the elevator falls down just like a free falling body. Explain.

Ans. A body will be weightless when the elevator falls down just like a free falling body.

Reasons and Explanation:

Let a fish is hanged from the ceiling of a lift with the help of spring balance. Let a man is standing on the floor of lift and observe the weight of the fish with the help of the spring balances.

We know that when the lift is moving downwards with certain acceleration " a ", then the tension in spring balance is given by,

$$T = mg - ma \quad (1)$$

In equation (1), the tension in the spring represents the apparent weight of the fish. Equation (1) shows that the apparent weight of the fish decreases when the lift is falling down with certain acceleration " a ".

Now when the elevator is falling freely, then we put $a = g$ so equation (1) becomes,

$$T = mg - mg \Rightarrow T = 0 \quad (2)$$

Equation (2) shows that the spring balance will read zero tension if the lift is falling freely. In other words we can say that the fish becomes weightless under the influence of free fall motion. Thus the body will be weightless when the elevator (lift) falls down just like



Fig. 6: When the elevator accelerates downward, the spring scale reads the a value less than the weight of the fish

a free falling body.

Q11: When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel, why? Explain.

Ans. When a tractor moves with uniform velocity, its heavier wheel rotates slowly than its lighter wheel.

Reason and Explanation:

First Answer: Let " I_1 ", " ω_1 " represents the moment of inertia and angular velocity of lighter wheel and " I_2 ", " ω_2 " represents the moment of inertia and angular velocity of heavier wheel respectively.

Now the angular momentum of lighter wheel is given by,

$$L_1 = I_1 \omega_1 \quad (1)$$

Similarly, the angular momentum of heavier wheel is given by,

$$L_2 = I_2 \omega_2 \quad (2)$$

Now by law of conservation of momentum we have,

$$L_1 = L_2 \Rightarrow I_1 \omega_1 = I_2 \omega_2$$

$$\Rightarrow \omega_1 = \left(\frac{I_2}{I_1} \right) \omega_2 \quad (3)$$

As the heavier wheel has greater mass than lighter wheel, so its moment of inertia " I_2 " will be also greater than the moment of inertia " I_1 " of the lighter wheel. So we

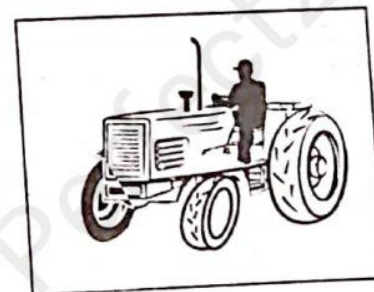
$$\text{have, } \frac{I_2}{I_1} > 1$$

So equation (3) becomes, $\omega_1 > \omega_2$ or $\omega_2 < \omega_1$

This shows that due to greater mass and greater moment of inertia, the heavier wheel will rotate slowly as compare to the lighter wheel.

Second Answer: Let " r " represents the radius of the lighter wheel and " R " represents the radius of heavier wheel. Now the distance covered by the lighter wheel during one revolution is equal to the circumference, so we have,

$$S_1 = 2\pi r \quad (1)$$



Similarly, distance covered by the heavier wheel during one rotation is given by,

$$S_2 = 2\pi R \quad (2)$$

Dividing equation (1) by equation (2), we get,

$$\frac{S_1}{S_2} = \frac{2\pi r}{2\pi R} \Rightarrow \frac{S_1}{S_2} = \frac{r}{R}$$

$$\Rightarrow S_1 = \left(\frac{r}{R}\right) S_2 \quad (3)$$

As $R > r$, so $\frac{r}{R} < 1$, so equation (3) shows that $S_1 < S_2$

This shows that the distance covered by lighter wheel during one rotation is less than the distance covered by heavier wheel during one rotation. Now for covering the same distance in same time the lighter wheel needs to make more rotation than the heavier wheel. i.e. the lighter wheel will rotate more quickly as compare to heavier wheel. In other words we can say that, when a tractor moves with a uniform velocity, its heavier wheel rotates slowly than its lighter wheel.

Comprehensive Questions

Give extended response to the following questions:

Q1: What are centripetal acceleration and centripetal force? Derive their equations.

Ans: See answer of question no 6 part-b in text.

Q2: Show that angular momentum in magnitude is given by, $|\vec{L}| = |\vec{r} \times \vec{p}| = mvr = mr^2\omega$

Ans: The cross product of moment arm " $\vec{r}$ " and linear momentum " $\vec{p}$ " is known as angular momentum. i.e.

$$\vec{L} = \vec{r} \times \vec{p} \quad (1)$$

In magnitude form, equation (1) can be written as,

$$|\vec{L}| = |\vec{r} \times \vec{p}| \quad (2)$$

As $\vec{p} = m\vec{v}$, so equation (2) becomes,

$$|\vec{L}| = |\vec{r} \times m\vec{v}| \Rightarrow |\vec{L}| = mvr \quad (3)$$

As $v = r\omega$, so equation (3) becomes,

$$\begin{aligned} |\vec{L}| &= m(r\omega)r = mr^2\omega \\ \Rightarrow |\vec{L}| &= mr^2\omega \quad (4) \end{aligned}$$

Equations (2), (3), (4) represent the magnitude form of angular momentum.

Q3: Show that role playing by mass in linear motion is playing by moment of inertia in rotatory motion.

Ans: See answer of short question no. 8.

Q4: What do you mean by "INTELSAT". At what frequencies it operates? For how many T.V stations this system is used?

Ans: "INTELSAT" means international telecommunication satellite organization. It is the largest satellite system which is managed by 126 countries. It operates at microwaves of frequencies 4, 6, 11 and 14 GHz and has a capacity of 30000 two way telephone circuits plus three T.V channels.

There are 200 earth stations which transmit the signals to satellites and receive signals via satellites from other countries. We can pick up the signals from the satellite using a dish antenna on our houses.

Q6: Show that in angular form, centripetal acceleration is $\vec{a}_c = -\omega^2 \vec{r}$

Proof: We know that,

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad (1)$$

We also know that, $v = r\omega$ (2)

Putting equation (2) in equation (1), we get,

$$\vec{a}_c = -\frac{r^2\omega^2}{r} \hat{r} = -r\omega^2 \hat{r}$$

$$\Rightarrow \vec{a}_c = -\omega^2(r\hat{r}) \quad (3)$$

The unit vector " $\hat{r}$ " can be expressed as,

$$\hat{r} = \frac{\vec{r}}{r} \Rightarrow \vec{r} = r\hat{r} \Rightarrow r\hat{r} = \vec{r} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$\vec{a}_c = -\omega^2 \vec{r}$$

Which is the required proof.

Q7: Show that the centripetal force is also shown by,

$$\vec{F}_c = -\frac{mv^2}{r} \hat{r} = -\frac{mv^2}{r^2} \vec{r} = -m\omega^2 \vec{r}$$

Ans. We know that,

$$\vec{a}_c = -\frac{v^2}{r} \hat{r} \quad (1)$$

We also know that,

$$\vec{F}_c = m\vec{a}_c \quad (2)$$

Putting equation (1) in equation (2), we get,

$$\vec{F}_c = -m \frac{v^2}{r} \hat{r} \quad (3)$$

We know that,

$$\hat{r} = \frac{\vec{r}}{r} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$\vec{F}_c = -m \frac{v^2}{r^2} \vec{r} \quad (5)$$

We also know that, $\omega = \frac{v}{r}$ (6)

Putting equation (6) in equation (5), we get,

$$\vec{F}_c = -\frac{mr^2\omega^2}{r^2} \vec{r} \Rightarrow \vec{F}_c = -m\omega^2 \vec{r} \quad (7)$$

Comparing equations (3), (5), (7), we get,

$$\vec{F}_c = -\frac{mv^2}{r} \hat{r} = -\frac{mv^2}{r^2} \vec{r} = -m\omega^2 \vec{r}$$

Which is the required proof.

Q8: Show that a satellite near the earth will have greater velocity.

Ans. Consider a satellite of mass "m" is moving around the earth in a circular orbit of radius "r" as shown in the figure. The gravitational force between earth and satellite is given by,

$$F = \frac{GM_e m}{r^2} \quad (1)$$

Due to this gravitational force, the satellite performs circular motion around the earth, so this force acts as centripetal force. So we have,

$$F = \frac{mv^2}{r} \quad (2)$$

Comparing equations (1), (2), we get,

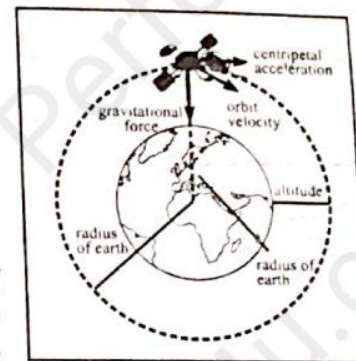
$$\frac{mv^2}{r} = \frac{GM_e m}{r^2}$$

$$\Rightarrow v^2 = \frac{GM_e}{r} \Rightarrow v = \sqrt{\frac{GM_e}{r}}$$

$$\Rightarrow v = \sqrt{\frac{\text{constant}}{r}} \quad [GM_e = \text{constt}]$$

$$\Rightarrow v \propto \frac{1}{\sqrt{r}} \quad (3)$$

Relation (3) shows that the orbital speed of satellite is inversely proportional to the distance "r" of satellite from the centre of earth. That is, the higher the satellite, the slower will be its orbital velocity and will take longer time to complete one revolution around the earth. In other words, we can say that a satellite near the earth will have greater velocity.



Q9: What do you mean by weight of a body? Using examples to distinguish between real weight and the apparent weight of a body.

Ans. **Weight:** The gravitational force exerted on the body due to earth or other planet is known as weight of that body. Weight is a vector quantity. Its SI unit is Newton.

Dimensions of Weight: As weight = Newton = kg·m/sec²

$$\Rightarrow \text{Weight} = \frac{\text{mass} \times \text{length}}{\text{time}^2} \Rightarrow \text{Weight} = [MLT^{-2}]$$

Real Weight and Apparent Weight:

The weight of an object due to the gravitational pull of the earth is called real weight. While the weight measured by a spring balance is called apparent weight. The weight of a person, ball, stone etc due to earth's gravity represents their real weight. While the weight of a person in a pond of water, and in moving elevator represents their apparent weights.

Q10: Explain how gravity is provided to the occupants of the space ship?

Ans. See answer of question 23 in the text.

Q11: Give three examples to illustrate the phenomenon of conservation of angular momentum.

Ans. See answer of question 15 in the text.

Q12: Explain why mud guards are used on the wheels of cycles, motor cars and other driving vehicles.

Ans: The mud guards are used on the wheels of driving vehicles to prevent the falling of mud on the surrounding parts of vehicles near the wheels.

Explanation:

When the tyre rotates, each point on the tyre, including mud clinging to the tyre has a tangential velocity " v " as well as angular velocity " ω ". There is adhesive force [لصق قوت] between the tyre and mud. When the tyre rotates, a centripetal force acts on the mud which is equal to the adhesive force between tyre and mud. When the angular speed of the tyre increases, the centripetal force on the mud also increases. When the centripetal force becomes greater than adhesive force, the mud leaves the tyre and fly off tangentially along a straight line due to centrifugal force which is simply the reaction of the centripetal force. The mud strikes the mud guard and thus the other parts of the vehicle is prevented from falling of mud on them.



Numerical Problems

Q1: If the plate microwave oven has a radius of 0.15m and rotates at 6.0 rev/min, calculate the total distance travelled by the fly during a 2.0m cooking period. (11m)

Solution:

Radius = $r = 0.15\text{m}$

Angular velocity = $\omega = 6\text{ rev/min} = 6 \times \frac{2\pi}{60} \text{ rad/sec} = 0.628\text{ rad/sec}$

Time period = $T = 2\text{ min} = 2 \times 60\text{ sec} = 120\text{ sec}$

Required:

Distance travelled = $S = ?$

Calculation:

We know that, $S = r\theta$ (1)

We also know that, $\theta = \omega T$ (2)

Putting equation (2) in equation (1), we get,
 $S = r\omega T$ (3)

Putting the values in equation (3), we get,
 $S = [0.15 \times 0.628 \times 120]\text{m} \Rightarrow S = 11.3\text{m}$

OR $S = 11\text{m}$ approximately

Distance travelled by fly = $S = 11\text{m}$

Q2: A circular drum of radius 40cm is initially rotating at 400 revolution/min. It is brought to stop after making 50 revolutions. What is the angular acceleration and the stopping time? (2.79 rad/s^2 , 15.0s)

Solution: Radius of drum = $r = 40\text{cm} = 0.4\text{m}$

Initial angular velocity

$$\omega_i = 400\text{ rev/min} = \frac{400 \times 2\pi}{60} \text{ rad/sec}$$

$$\Rightarrow \omega_i = 41.866 \text{ rad/sec}$$

Angular displacement = $\theta = 50\text{ revolution}$
 $= 50 \times 2\pi \text{ rad}$

$$\Rightarrow \theta = 314\text{ rad, final angular velocity} = \omega_f = 0$$

Required:

(a) Angular acceleration = $\alpha = ?$

(b) Stopping time = $t = ?$

Calculation:

(a) We know that,

$$2\alpha\theta = \omega_f^2 - \omega_i^2 \quad (1)$$

Putting the values in equation (1), we get,

$$2 \times \alpha \times 314 = 0^2 - (41.866)^2 \Rightarrow 628\alpha = -1752.76$$

$$\Rightarrow \alpha = \left[\frac{-1752.76}{628} \right] \text{ rad/sec}^2 = -2.79 \text{ rad/sec}^2 \quad \alpha = -2.79 \text{ rad/sec}^2$$

(b) We know that,

$$\omega_f = \omega_i + \alpha t \quad (2)$$

Putting the values in equation (2), we get,

$$0 = 41.866 - 2.79t$$

$$\Rightarrow -2.79t = -41.866$$

$$\Rightarrow t = \left(\frac{41.866}{2.79} \right) \text{ sec} \Rightarrow t = 15.0 \text{ sec}$$

Drum comes to rest,
so put $\omega_f = 0$

$$2as = v_f^2 - v_i^2$$

$$2\alpha\theta = \omega_f^2 - \omega_i^2$$

The negative sign shows
that the drum becomes
slowed down.

$$\text{Stopping time} = 15.0 \text{ sec}$$

Q3: A string 1m long is used to whirl a 100g stone in a horizontal circle at a speed of 2ms^{-1} . Find the tension in the string. ($T=0.4\text{N}$)

Solution:

Length of string = $l = r = 1\text{m}$

Mass of stone = $m = 100\text{gm} = 0.1\text{kg}$

Speed of stone = $v = 2\text{m/sec}$

Required:

Tension in the string = $T = ?$

Calculation:

Here the tension in the string must be equal to the centripetal force i.e.

$$T = F_c \Rightarrow T = \frac{mv^2}{r} \quad (1)$$

Putting the values in equation (1), we get.

$$T = \left(\frac{0.1 \times 2^2}{1} \right) \text{N} = 0.4\text{N} \Rightarrow T = 0.4\text{N}$$

$$\text{Tension in the string} = T = 0.4\text{N}$$

Q4: The moon revolves around the earth in almost a circle of radius 382400 km in 27.3 days. What is the centripetal acceleration?

Solution:

Given Data:

Radius of circle = $r = 382400\text{km} \Rightarrow r = 382400 \times 10^3\text{m}$

Time period = $T = 27.3 \text{ day} = 27.3 \times 24 \times 3600 = 2358720\text{sec}$

Required: Centripetal acceleration $a_c = ?$

Calculation: We know that, $a_c = \frac{v^2}{r} = \frac{r^2 \omega^2}{r}$

$$\Rightarrow a_c = r \omega^2 \Rightarrow a_c = r \left(\frac{2\pi}{T} \right)^2 = \frac{4\pi^2 r}{T^2}$$

$$\Rightarrow a_c = \frac{4\pi^2 r}{T^2} \quad (1)$$

Putting the values in equation (1), we get,

$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ \Rightarrow \omega &= \frac{2\pi}{T} \\ v &= r\omega \end{aligned}$$

$$\begin{aligned} a_c &= \left[\frac{4 \times 3.14^2 \times 382400 \times 10^3}{(2358720)^2} \right] \text{m/sec}^2 \\ &= 0.00271 \text{m/sec}^2 \\ \Rightarrow a_c &= 0.00271 \text{m/sec}^2 \end{aligned}$$

Q5: A modern FI car can accelerate from 0 to 62 mile/h (100km/h) in 2.50s. What is the angular acceleration of its 170mm radius wheels? (65.17 rad/s^2)

Solution:

Given Data:

Initial velocity = $v_i = 0\text{m/sec}$

Final velocity = $v_f = 100\text{km/h} = 100 \times \frac{10^3\text{m}}{3600\text{sec}}$

$\Rightarrow v_f = 27.77\text{m/sec} = 27.8\text{m/sec}$

Time taken = $t = 2.5\text{sec}$

Radius of wheel = $r = 170\text{mm} = 170 \times 10^{-3}\text{m} = 0.170\text{m}$

Required:

Angular acceleration = $\alpha = ?$

Calculation:

We know that,

$$\omega_f = \omega_i + \alpha t \Rightarrow \alpha t = \omega_f - \omega_i \quad (1)$$

$$\text{Also } v = r\omega \Rightarrow v_f = r\omega_f \Rightarrow \omega_f = \frac{v_f}{r} \quad (2)$$

$$\text{Similarly, } \omega_i = \frac{v_i}{r} \quad (3)$$

Putting equation (2) and equation (3) in equation (1), we get,

$$\alpha t = \frac{v_f}{r} - \frac{v_i}{r} = \frac{1}{r} (v_f - v_i)$$

$$\Rightarrow \alpha = \frac{1}{rt} (v_f - v_i) \quad (4)$$

Putting the values in equation (4), we get,

$$\alpha = \frac{1}{(0.170 \times 2.5)} [27.8 - 0] = \left(\frac{27.8}{0.425} \right) \text{rad/sec}^2 \Rightarrow \alpha = 65.4 \text{rad/sec}^2$$

Q6: An electric motor is running at 1800rev/min. It comes to rest in 20sec. If the angular acceleration is uniform, find the number of revolutions it made before stopping?

Solution: Initial angular speed = $\omega_i = 1800 \text{ rev/min}$

$$\Rightarrow \omega_i = \frac{1800}{60} \text{ rev/sec} = 30 \text{ rev/sec}$$

$$\text{Final angular speed} = \omega_f = 0, \text{ time taken} = t = 20 \text{ sec}$$

Required:

$$\text{Number of revolutions} = \theta = ?$$

Calculation:

We know that,

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \theta = \omega_i t + \frac{1}{2} (\omega_f - \omega_i) t$$

$$\Rightarrow \theta = \omega_i t + \frac{1}{2} (\omega_f - \omega_i) t \quad (1)$$

Putting the values in equation (1), we get,

$$\theta = 30 \times 20 + \frac{1}{2} (0 - 30) 20$$

$$\Rightarrow \theta = 600 - \frac{600}{2} = 600 - 300 = 300 \text{ rev}$$

$$\boxed{\text{Number of revolution} = \theta = 300 \text{ rev}}$$

Q7: What is the moment of inertia of a 100kg sphere whose radius is 50cm?

Solution:

$$\text{Mass of sphere} = M = 100 \text{ kg}$$

$$\text{Radius of sphere} = R = 50 \text{ cm} = 0.5 \text{ m}$$

Required:

$$\text{Moment of inertia} = I = ?$$

Calculation:

We know that the moment of inertia of a solid sphere is given by,

$$I = \frac{2}{5} MR^2 \quad (1)$$

Putting the values in equation (1), we get,

$$\Rightarrow I = \left[\frac{2}{5} \times 100 \times (0.5)^2 \right] \text{ kg} \cdot \text{m}^2 \Rightarrow \boxed{I = 10 \text{ kg} \cdot \text{m}^2}$$

$$\Rightarrow \boxed{I = 10 \text{ kg} \cdot \text{m}^2}$$

$$S = v_i t + \frac{1}{2} \alpha t^2$$

$$\theta = \omega_i t + \frac{1}{2} \alpha t^2$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

Q8: A rope is wrapped several times around a cylinder of radius 0.2m and mass 30kg. What is the angular acceleration of the cylinder if the tension in the rope is 40N and it turns without friction? (13.3 rad/s²).

Solution: Radius of cylinder = $r = 0.2 \text{ m}$, mass of cylinder = $m = 30 \text{ kg}$
Tension in the rope = $T = F = 40 \text{ N}$

Required:

$$\text{Angular acceleration} = \alpha = ?$$

Calculation:

We know that,

$$\tau = I \alpha \quad (1)$$

We also know that,

$$\tau = r F \quad (2)$$

Comparing equation (1) and equation (2), we get,

$$I \alpha = r F \Rightarrow \alpha = \frac{r F}{I} \quad (3)$$

The moment of inertia of cylinder is given by,

$$I = \frac{1}{2} m r^2 \quad (4)$$

Putting equation (4) in equation (3), we get,

$$\alpha = \frac{r F}{\left(\frac{1}{2} m r^2 \right)} = \frac{2 r F}{m r^2} = \frac{2 F}{m r} \Rightarrow \alpha = \frac{2 F}{m r} \quad (5)$$

Putting the values in equation (5), we get,

$$\alpha = \left(\frac{2 \times 40}{30 \times 0.2} \right) \text{ rad/sec}^2 \Rightarrow \boxed{\alpha = 13.3 \text{ rad/sec}^2}$$

Q9: What is the kinetic energy of a 5.0kg solid ball whose diameter is 15cm if it rolls across a level surface with a speed of 2ms⁻¹? (14J)

Solution: Given Data:

$$\text{Mass of solid ball} = M = 5 \text{ kg, diameter of ball}$$

$$= D = 15 \text{ cm} = 0.15 \text{ m}$$

$$\text{Radius of solid ball}$$

$$= R = \frac{D}{2} = \frac{0.15}{2} = 0.075 \text{ m}$$

$$\text{Speed of ball} = v = 2 \text{ m/sec}$$

Required:

K.E of rolling ball = K.E = ?

Calculation:

We know that rolling body posses both linear K.E and rotational K.E, so its total K.E is given by,

$$(K.E)_{\text{total}} = (K.E)_{\text{linear}} + (K.E)_{\text{rot}}$$

$$\Rightarrow (K.E)_{\text{total}} = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2 \quad (1)$$

For solid sphere or ball, we have,

$$I = \frac{2}{5} MR^2 \quad (2)$$

$$\text{Also } v = R\omega \Rightarrow \omega = \frac{v}{R} \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

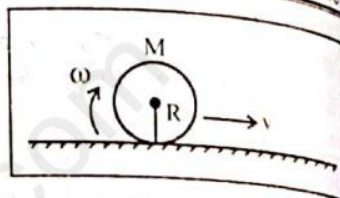
$$K.E = \frac{1}{2} Mv^2 + \frac{1}{2} \times \left(\frac{2}{5} MR^2 \right) \times \frac{v^2}{R^2}$$

$$\Rightarrow K.E = \frac{1}{2} Mv^2 + \frac{1}{5} Mv^2 \quad (4)$$

Putting the values in equation (4), we get,

$$\Rightarrow K.E = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{5} \times 5 \times 2^2$$

$$\Rightarrow K.E = (10 + 4)J \Rightarrow \boxed{K.E = 14 \text{ joules}}$$



Q10: A cylinder of 50cm diameter at the top of an incline 29.4cm high and 10m long is released and rolls down the incline. Find its linear and angular speed at the bottom. Neglect friction.

Solution: Given Data:

Diameter of cylinder

$$= D = 50\text{cm} = 0.5\text{m}$$

Radius of solid cylinder

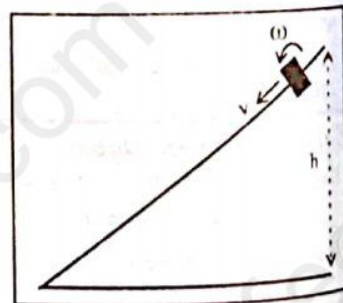
$$= R = \frac{D}{2} = \frac{0.5}{2} = 0.25\text{m}$$

Height = h = 29.4cm = 0.294m

Required:

(a) Linear speed = v = ?

(b) Angular speed = ω = ?



Calculation:

(a) As the cylinder is rolling down from top to bottom, so according to the law of conservation of energy, we have,

Loss in P.E = Gain in K.E

$$\Rightarrow mgh = (K.E)_{\text{linear}} + (K.E)_{\text{rot}}$$

$$\Rightarrow mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \quad (1)$$

or solid cylinder, we have,

$$I = \frac{1}{2} mR^2 \quad (2)$$

$$\text{So we know that, } v = R\omega \Rightarrow \omega = \frac{v}{R} \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} \times \frac{1}{2} mR^2 \times \frac{v^2}{R^2}$$

$$\Rightarrow gh = \frac{v^2}{2} + \frac{v^2}{4} \Rightarrow gh = \frac{3v^2}{4} \Rightarrow v^2 = \frac{4}{3} gh$$

$$\Rightarrow v = \sqrt{\frac{4}{3} gh} \quad (4)$$

Putting the values in equation (4), we get,

$$v = \sqrt{\frac{4 \times 9.8 \times 0.294}{3}} \text{ m/sec} = 1.96 \text{ m/sec}$$

$$\Rightarrow \boxed{v = 1.96 \text{ m/sec}}$$

$$\text{b) As } v = R\omega \Rightarrow \omega = \frac{v}{R} = \frac{1.96}{0.25} \Rightarrow \boxed{\omega = 7.84 \text{ rad/sec}}$$

Q11: A disc without slipping rolls down a hill of vertical height of 1000cm. If the disc starts from rest at the top of the hill, what is its magnitude of velocity at the bottom?

Solution:

Given Data:

Height of hill = h = 1000cm

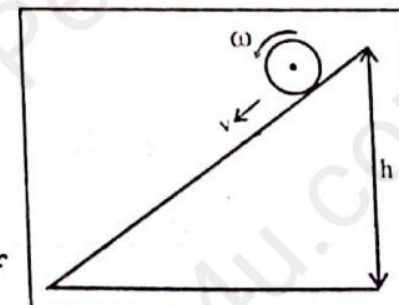
$$\Rightarrow h = 10\text{m}$$

Required:

Speed at the bottom = v = ?

According to the law of conservation of energy, we have,

Loss in P.E = Gain in



$$(K \cdot E)_{\text{linear}} + (K \cdot E)_{\text{rot}}$$

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \quad (1)$$

$$\text{For disc, we have, } I = \frac{1}{2}mr^2 \quad (2)$$

$$\text{Also, } v = r\omega \Rightarrow \omega = \frac{v}{r} \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{1}{2}mr^2 \times \frac{v^2}{r^2}$$

$$\Rightarrow gh = \frac{v^2}{2} + \frac{v^2}{4} = \frac{3v^2}{4} \Rightarrow gh = \frac{3v^2}{4} \Rightarrow v^2 = \frac{4gh}{3}$$

$$\Rightarrow v = \sqrt{\frac{4}{3}gh} = \sqrt{\frac{4 \times 9.8 \times 10}{3}}$$

$$\Rightarrow v = 11.4 \text{ m/sec}$$

Q12: A motor car is travelling at a speed of 30m/sec. If its wheel has a diameter of 1.5m, find its angular speed in rad/sec and in rev/sec.

Solution:

Speed of the car = $v = 30 \text{ m/sec}$, diameter of wheel = $d = 1.5 \text{ m}$

Radius of wheel = $r = \frac{d}{2} = \frac{1.5}{2} = 0.75 \text{ m}$

Required:

(a) Angular speed in rad/sec = $\omega = ?$

(b) Angular speed in rev/sec = $\omega = ?$

Calculation:

(a) We know that, $v = r\omega$

$$\Rightarrow \omega = \frac{v}{r} = \frac{30}{0.75} \Rightarrow \omega = 40 \text{ rad/sec}$$

(b) $\omega = 40 \text{ rad/sec}$

$$1 \text{ rev} = 2\pi \text{ rad} \Rightarrow 1 \text{ rad} = \frac{1}{2\pi} \text{ rev}$$

$$\Rightarrow \omega = 40 \times \frac{1}{2\pi} \text{ rev/sec}$$

$$\Rightarrow \omega = 6.369 \text{ rev/sec} \Rightarrow \omega = 6.37 \text{ rev/sec}$$

• اچھی صورت کے مقابلے میں اچھی سیرت کا رتبہ بلند ہوتا ہے۔
• جو کسی کا بڑا نہیں چاہتے، ان کے ساتھ کوئی بڑا نہیں کر سکتا، یہ میرے رتب کا وعدہ ہے۔

Q1: Define the following terms: (a) Fluid dynamics, hydrostatics, (c) Fluid friction, (d) Ideal fluid

Ans: (a) Fluid:

A substance which can flow is known as fluid. For example, oil, milk, water, gas etc are fluids.

(b) Hydrostatics and Fluid Dynamics:

The study of fluid at rest is called hydrostatic. While the study of properties and effect of moving fluids is known as fluid dynamics. In fluid dynamics, 'hydrodynamics' deals with the study of liquids in motion, while 'aerodynamics' deals with the study of gases in motion.

(c) Fluid Friction:

When a body moves through a fluid, then an opposing force acts on the body. This opposing force is known as fluid friction or dragging force. This force depends upon the:

- Size of the body
- Geometric shape of the body
- Direction of velocity of the body and
- Viscosity of the fluid

(d) Ideal Fluid: (ETEA 2016)

An incompressible, non-viscous fluid having constant density at any pressure, is called an ideal fluid.

Q2: What is a fluid? Discuss briefly, the viscosity of a fluid and explain how the flow of a fluid is characterized by viscosity.

Ans: Fluid:

A substance which flows is known as fluid. For example, oil, water, gas etc.

Viscosity of a Fluid:

The resistance offered by different layers of a fluid to its flow is known as viscosity. When fluids are in motion, the viscosity is a key characteristic of fluids, which determines the rate of flow of a fluid.

Coefficient of Viscosity:

The numerical value of resistance to the flow of fluids between its different layers is called coefficient of viscosity. Greater the coefficient of viscosity of a fluid, the fluid is said to be more viscous and will offer greater dragging force. The coefficient of viscosity is represented by the symbol " η ". Its numerical values for different fluids are given in the table given below:

Fluid Name	Viscosity (Pa·S)	Fluid Name	Viscosity (Pa·S)
Air	1.8×10^{-5}	Acetone	2.9×10^{-4}
Methanol	5.1×10^{-4}	Benzene	1.00×10^{-3}
Water	8.91×10^{-4}	Ethanol	1.00×10^{-3}
Blood	1.6×10^{-3}	Honey	1.42

Effect of Temperature on Viscosity:

The viscosity of a fluid depends upon the temperature of fluid. In case of liquids, the viscosity is produced due to intermolecular forces of attraction. If we increase the temperature, the intermolecular forces decrease and hence viscosity decreases.

In case of gases, the viscosity depends upon the velocity of molecules. Greater the velocity of molecules of gases, smaller will be the viscosity and vice versa. If we increase the temperature of the gas, the velocity of molecules of the gas increases and hence viscosity decreases.

Units of Viscosity:

The SI unit of viscosity is Pascal second (Pa·S), Dyne/cm². Poise is another unit of viscosity.

$$1 \text{ Pa} \cdot \text{S} = 1 \text{ poise}, 1 \text{ centipoise} = 10^{-3} \text{ Pa} \cdot \text{S}$$

Q3: (a) State and explain stokes law. (b) Derive a mathematical equation for acceleration and terminal velocity by using stokes law.

Ans: (a) Stokes Law:

Statement:

This law states that the dragging force " F_D " acting on a spherical body inside a fluid is directly proportional to the radius of spherical body, velocity of spherical body and viscosity of the fluid.

Explanation:

Stoke did his research and study on spherical shaped objects of uniform density. Let a spherical body of radius " r " is allowed to move slowly with velocity " v " inside a fluid of viscosity " η " as shown in the figure.

Now according to stokes law, we have,

$$F_D \propto r \quad (1)$$

$$F_D \propto v \quad (2)$$

$$\text{And } F_D \propto \eta \quad (3)$$

Combining relations (1), (2), (3),

we get,

$$F_D \propto \eta r v \Rightarrow F_D = \text{constant} \cdot \eta r v$$

$$\Rightarrow F_D = 6\pi\eta r v \quad (4) \quad [6\pi = \text{constant}]$$

Equation (4) represents the dragging force acting on spherical body inside a fluid.

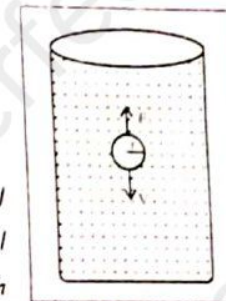
(b) Calculation of Acceleration of Spherical Droplet:
(ETEA 2009)

When a spherical body [droplet] is moving through a fluid, then its weight " w " is acting downwards while the dragging " F_D " is acting upwards. So the net force on the body is given by,

$$F_{\text{net}} = w - F_D \Rightarrow ma = mg - 6\pi\eta r v$$

$$\Rightarrow a = \frac{mg - 6\pi\eta r v}{m}$$

$$\Rightarrow a = g - \frac{6\pi\eta r v}{m} \quad (5)$$



Dimension of viscosity:

$$F_D = 6\pi\eta r v \quad (\text{ETEA 2011})$$

$$\Rightarrow \eta = \frac{F_D}{6\pi r v} = \frac{\text{kg} \cdot \frac{\text{m}}{\text{sec}^2}}{\text{m} \cdot \frac{\text{m}}{\text{sec}}}$$

$$\Rightarrow \eta = \text{kg} \cdot \frac{\text{m}}{\text{sec}^2} = \frac{\text{sec}}{\text{m}^2}$$

$$\Rightarrow \eta = \text{kg/sec} \cdot \text{m}$$

$$\Rightarrow \eta = [MT^{-1}L^{-1}]$$



$$\begin{aligned} F_{\text{net}} &= ma \\ w &= mg \\ F_D &= 6\pi\eta r v \end{aligned}$$

Equation (5) represents the acceleration of the droplet moving through the fluid.

Calculation of Terminal Velocity:

The maximum constant velocity of a body, when forward directed forces are balanced by backward directed forces is known as terminal velocity of that body.

When a droplet is moving downward its velocity increases until a stage comes, when the weight of the droplet and dragging force balance each other. At this stage, there is no further increase in velocity and the body moves down with constant velocity. Such velocity is known as terminal velocity. At this stage, we put $a=0$ and $v=v_t$ so equation (5) becomes,

$$0 = g - \frac{6\pi\eta r v_t}{m}$$

$$\Rightarrow \frac{-6\pi\eta r v_t}{m} = -g \Rightarrow 6\pi\eta r v_t = mg$$

$$\Rightarrow v_t = \frac{mg}{6\pi\eta r} \quad \text{--- (6)}$$

Equation (6) represents the terminal velocity in terms of mass of the droplet.

Terminal Velocity in terms of Density:

We know that,

Mass = density $\times$ volume

$$\Rightarrow m = \delta \times \frac{4}{3}\pi r^3 \quad \text{--- (7)}$$

Putting equation (7) in equation (6), we get,

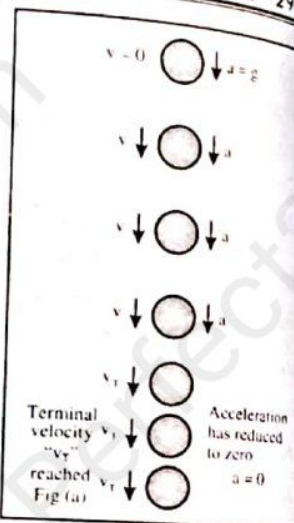
$$v_t = \frac{4\pi\delta r^3 g}{18\pi\eta r}$$

$$\Rightarrow v_t = \frac{2\delta g r^2}{9\eta} \quad \text{--- (8)}$$

Equation (8) represents the terminal velocity of the droplet in terms of density.

Graphical Representation of Terminal Velocity:

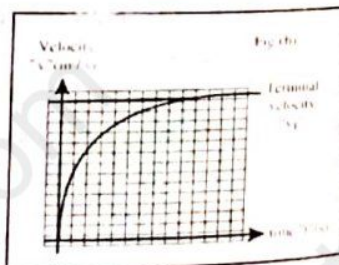
The graphical representation of terminal velocity is shown in the figure (b). The graph shows that the velocity increases with the passage of time and finally becomes constant.



For spherical body,

$$\text{volume} = \frac{4}{3}\pi r^3$$

Rain drops falling from 5kg reach the ground with terminal velocity.
ETEA 2010



Object	Mass (kg)	Cross-sectional Area (m^2)	Terminal speed (m/s)
Sky diver	70	0.70	54
Base ball (radius 3.7cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1cm)	0.046	1.4×10^{-3}	44
Hail stone (radius 0.5cm)	4.8×10^{-4}	7.9×10^{-5}	14
Rain drop (radius 0.2cm)	3.4×10^{-5}	1.3×10^{-5}	09

Q4: Discuss the terminal velocity of a paratroopers?

ETEA 2009

Ans. Terminal Velocity of a Paratrooper:

The constant velocity gained by a paratrooper falling under the action of gravity at the stage when drag force on his body become equal to his weight is known as terminal velocity of the paratrooper. It is denoted by v_t . Paratrooper gains terminal velocity twice.

Explanation:

At $t=0$ when a paratrooper jumps from an aeroplane, at the beginning the air drag force acting on his body is less than his weight and the net force is given by,

$$F_n = w - F_d \quad \text{--- (1)}$$

$$\Rightarrow ma = w - F_d$$

$$a = \frac{w - F_d}{m} \quad \text{--- (2)}$$

With this acceleration the paratrooper moves down which is shown in the $v-t$ graph.

1st Terminal Velocity:

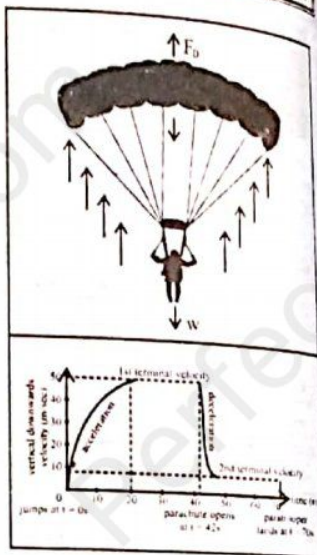
But as the paratrooper moves down, his velocity increases due to the above acceleration. Since drag force is directly proportional to



velocity, so drag force also increases. A stage is reached when drag force becomes equal to the weight of paratrooper. At this stage the paratrooper gains maximum constant velocity which is called 1st terminal velocity. In the graph the paratrooper gains first terminal velocity of 50m/s after 20 seconds.

2nd Terminal Velocity:

When a paratrooper opens his parachute, it dramatically increases the area of intercept and the air resistance force becomes much greater than the weight of paratrooper. This causes a rapid deceleration in the 1st terminal velocity of the paratrooper as shown in the graph. But as the speed of the paratrooper decreases, the air drag force also decreases and once again a stage is reached when drag force becomes equal to the weight of paratrooper. This time the paratrooper gains 2nd terminal velocity which is slow enough and he lands safely on the ground. In graph the parachute is opened after 40sec and large drag force decelerates his motion. Within 50sec or so the paratrooper gains 2nd terminal velocity which is quite low and nearly 8m/sec.

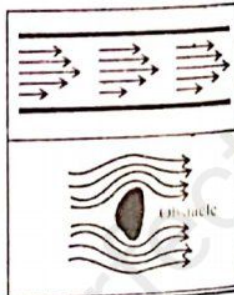


Q5: Explain the difference between stream line flow and turbulent flow of fluids.

Ans: 1. Stream Line Flow (or) Laminar Flow:

The flow of a fluid in which layers of fluid slide smoothly past each other is known as stream line flow or laminar flow.

The pattern of laminar flow is shown in the figure. Such flow occurs in steady state. If an obstacle is placed in the path of the fluid, then the fluid does not cross the obstacle and its adjacent layer slide smoothly past each other.



2. Turbulent Flow:

The flow of a fluid in which there is a greater disorder and the layer of fluid does not slide smoothly past each other is known as turbulent flow.

The pattern of turbulent flow is shown in the figure. If an obstacle is placed in front of the fluid, then the adjacent layers of the fluid do not slide smoothly past each other. The turbulent flow does not occur in steady state and continuous changes occur in the pattern of the flow.



Q6: State the equation of continuity. Show that how it is based on law of conservation of mass? Give two examples from daily life which are related with the principle of equation of continuity.

ETEA 2015, 2016, BISE Mardan 2019, BISE Bannu 2017

Ans: Statement: It states that for an incompressible fluid in steady flow, the net rate of flow of mass inward across any closed surface is equal to the net rate of flow of mass outward.

Explanation:

Consider a pipe having non-uniform area of cross-section " A_1 " and " A_2 ", as shown in the figure.

Let " V_1 " and " V_2 " be the speed of a steady fluid at " A_1 " and " A_2 " respectively. Now the distance covered by the fluid across " A_1 " during time interval " Δt " is given by,

$$\Delta x_1 = V_1 \Delta t \quad (1)$$

Now the volume of the fluid through the cross-sectional area " A_1 " is given by,

$$\Delta V_1 = A_1 \cdot \Delta x_1 \quad (2)$$

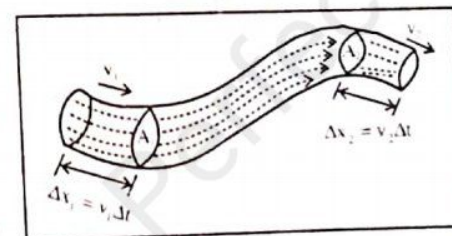
Putting equation (1) in equation (2), we get,

$$\Delta V_1 = A_1 \cdot V_1 \Delta t \quad (3)$$

Similarly, the volume of the fluid across " A_2 " during time interval " Δt " is given by,

$$\Delta V_2 = A_2 \cdot V_2 \Delta t \quad (4)$$

Now the mass of the fluid across " A_1 " is given by,



$$\Delta m_1 = \delta A_1 V_1 \Delta t \quad (5)$$

Putting equation (3) in equation (5), we get,

$$\Delta m_1 = \delta A_1 V_1 \Delta t \quad (6)$$

Similarly, the mass of the fluid across " A_2 " is given by,

$$\Delta m_2 = \delta A_2 V_2 \Delta t \quad (7)$$

Now in case of steady flow, the total mass of the fluid in the pipe remains constant. So we have,

$$\Delta m_1 = \Delta m_2 \Rightarrow \delta A_1 V_1 \Delta t = \delta A_2 V_2 \Delta t$$

$$\Rightarrow A_1 V_1 = A_2 V_2 \quad (8)$$

Equation (8) represents the equation of continuity. Equation (8) shows that the volume flow rate across any cross-section of flow tube remains constant. The equation of continuity is the mathematical expression of the principle of conservation of mass. This principle states that for steady state flow, the mass flow into the volume must be equal to the mass flow rate out.

Examples related to equation of continuity:

- Flowing of fluid in pipes
- Flowing of gases in pipes and tubes
- Flowing of blood in aorta and capillaries

Q7: Show that the speed of fluid through any pipe is inversely proportional to the cross-sectional area of the pipe.

BISE Malakand 2019, BISE Peshawar 2019

Ans: From equation of continuity, we have,

$$A_1 V_1 = A_2 V_2 \Rightarrow \frac{V_1}{V_2} = \frac{A_2}{A_1} \quad (1)$$

In general form, equation (1) can be written as,

$$V \propto \frac{1}{A} \quad (2)$$

Relation (2) shows that the speed of fluid through any pipe is inversely proportional to the cross-sectional area of the pipe.

The deeper part of a river has larger cross-sectional area as compare to shallow part of the river. So according to relation (2), the deep water flows slowly in comparison to shallow water. However, the volume flow rates in both cases are the same.

Q8: Show that the volume flow rate (or) time rate of flow of volume is constant.

ETEA 2015

Ans: According to the equation of continuity, we have,

$$A_1 V_1 = A_2 V_2 = \text{constant} \quad (1)$$

Generally equation (1) can be written as,

$$\Rightarrow AV = \text{constant} \quad (2)$$

In equation (2), we have,

V = velocity = rate of change of displacement

$$\Rightarrow V = \frac{\Delta x}{\Delta t} \quad (3)$$

Putting equation (3) in equation (2), we get,

$$A \cdot \frac{\Delta x}{\Delta t} = \text{constant} \Rightarrow \frac{\Delta}{\Delta t} (A \cdot x) = \text{constant} \quad (4)$$

In equation (4), $A \cdot x$ = volume = V , so equation (4) becomes,

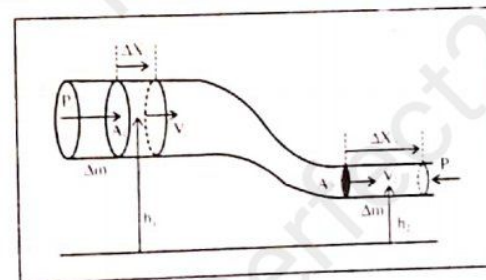
$$\frac{\Delta V}{\Delta t} = \text{constant} \quad (5)$$

Equation (5) shows that the volume flow rate [time rate of flow of volume] is constant. In other words we can say that, the volume of an incompressible fluid passing through any point in unit time through a pipe of non-uniform cross-section is constant in the steady flow.

Q9: What is Bernoulli's equation? Show that how it is based on the law of conservation of energy?

ETEA 2010, BISE Mardan 2019, BIE DI Khan 2019

Ans: The Bernoulli's equation was derived by a Swiss physicist Daniel Bernoulli in 1738. This equation represents the relationship between pressure, speed and elevation of an ideal fluid.



Explanation:

Consider an incompressible, non-viscous fluid flowing in a pipe in steady state manner. Let " v_1 " and " v_2 " be the speed of fluid at cross-sectional area " A_1 " and " A_2 " respectively. The workdone on the fluid at " A_1 " and " A_2 " are respectively given as,

$$\Delta w_1 = F_1 \cdot \Delta x_1 \quad (1)$$

$$\& \Delta w_2 = F_2 \cdot \Delta x_2 \quad (2)$$

Now the net workdone on the fluid is given by,

$$\Delta w = \Delta w_1 - \Delta w_2 \quad (3)$$

Putting equations (1), (2) in equation (3), we get,

$$\Delta w = F_1 \Delta x_1 - F_2 \Delta x_2 \quad (4)$$

Now we know that,

$$P = \frac{F}{A} \Rightarrow P_1 = \frac{F_1}{A_1} \Rightarrow F_1 = P_1 A_1 \quad (5)$$

$$\text{Similarly, } F_2 = P_2 A_2 \quad (6)$$

Putting equations (5), (6) in equation (4), we get,

$$\Delta w = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2 \quad (7)$$

Here, $A_1 \Delta x_1 = \text{volume} = V_1$

Similarly, $A_2 \Delta x_2 = V_2$

So equation (7) becomes,

$$\Delta w = P_1 V_1 - P_2 V_2 \quad (8)$$

For incompressible fluid, we have,

$$V_1 = V_2 = V$$

So equation (8) becomes,

$$\Delta w = P_1 V - P_2 V$$

$$\Rightarrow \Delta w = (P_1 - P_2) V \quad (9)$$

We know that, mass = density $\times$ volume

$$\Rightarrow m = \delta \times V \Rightarrow V = \frac{m}{\delta}$$

So equation (9) becomes,

$$\Delta w = (P_1 - P_2) \frac{m}{\delta} \quad (10)$$

Now according to work-energy theorem, we have,

$$\Delta w = \Delta K \cdot E + \Delta P \cdot E \quad (11)$$

$$\text{Here } \Delta K \cdot E = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \quad (12)$$

$$\& \Delta P \cdot E = m g h_2 - m g h_1 \quad (13)$$

Putting equations (12), (13) in equation (11), we get,

$$\Delta w = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + m g h_2 - m g h_1 \quad (14)$$

Comparing equations (10), (14), we get,

$$(P_1 - P_2) \frac{m}{\delta} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + m g h_2 - m g h_1$$

$$\Rightarrow P_1 - P_2 = \delta \left[\frac{v_2^2}{2} - \frac{v_1^2}{2} + g h_2 - g h_1 \right]$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2} \delta v_2^2 - \frac{1}{2} \delta v_1^2 + \delta g h_2 - \delta g h_1$$

$$\Rightarrow P_1 + \frac{1}{2} \delta v_1^2 + \delta g h_1 = P_2 + \frac{1}{2} \delta v_2^2 + \delta g h_2 \quad (15)$$

Unit of pressure:

$$\text{Hint: } P = \frac{F}{A} = \frac{m a}{A}$$

$$\Rightarrow P = \frac{\text{kg} \cdot \text{m}}{\text{m}^2}$$

$$\Rightarrow P = \frac{\text{m}^2 \cdot \text{sec}^{-2}}{\text{m}^2} = \text{kg} \cdot \text{m}^{-1} \cdot \text{sec}^{-2}$$

ETEA 2013

Nayab Physics Notes - 1st Year

In general form, equation (15) can be written as,

$$P + \frac{1}{2} \delta v^2 + \delta g h = \text{constant} \quad (16)$$

Equations (15), (16) represent the Bernoulli's equation. As the Bernoulli's equation is derived by using work-energy theorem, so it is not a free standing law of physics. It is a consequence of the law of conservation of energy, applied to an ideal fluid.

In other words, we can say that the Bernoulli's equation is based on the law of conservation of energy.

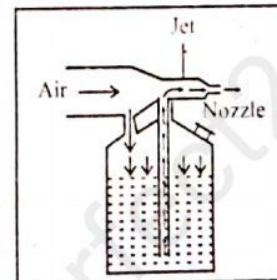
Importance of Bernoulli's Equation:

The Bernoulli's equation plays an important role in analyzing plumbing system, hydroelectric generations and flight of aeroplanes. The "jet and nozzle" works on the principle of Bernoulli's equation. Using Bernoulli's equation, we can determine the speed of flow of a fluid through a pipe. The device which is used for the measurement of blood pressure is based on Bernoulli's equation.

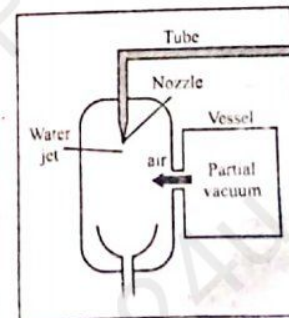
Q10: Discuss the applications of Bernoulli's equation.

Ans-1: Jets and Nozzles (or) Atomizers:

The Bernoulli's equation plays an important role in our daily life. We can apply its principle to many devices like jets and nozzles. For this purpose, the air from the pipe is evacuated due to which the pressure in the pipe decreases. As a result, the velocity of the fluid increases.



The velocity of the fluid can be further increased by minimizing the area of cross-section of the exit opening of the device. Using the above phenomenon, we can draw water, oil etc from a container.



2. Filter Pumps:

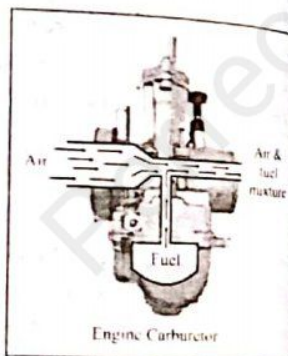
Pumps are used to transfer liquids from low-pressure zones to high pressure zones. "A filter pump is a device used to produce partial vacuum in vessel attached to it".

A filter pump consists of a tube with jet

attached to it, in which water flows from the tube toward the jet as shown in figure. When water reaches the jet section its speed increases, as a result the pressure drops near it. This drop in pressure allows air to flow in from the side tube to which the vessel is connected, thus air and water are forced together at the bottom of the filter pump. In this way a partial vacuum is created in the vessel attached to it.

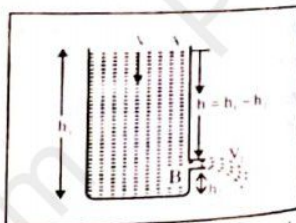
3. Engine Carburetor:

The principle of engine carburetor is the same as that of filter pump. The carburetor of an engine uses the Bernoulli's theorem to feed a suitable amount of mixture of air and fuel to the cylinder. The engine draws air through a horizontal pipe to the cylinder. This pipe is connected to the fuel tank by a side tube. When the air through the pipe is flowing with high speed, the pressure of air inside the pipe decreases. As a result, the fuel moves to the pipe and a proper mixture of the fuel vapours and air is formed which is then sprayed to the engine.



4. Torricelli's Theorem [Speed of Efflux]: ETEA 2011, 2015, BISE Malakand 2019, BISE Peshawar 2019

Statement: This theorem states that the velocity of efflux from an opening at a depth "h" below the top surface of the fluid is equal to the vertical velocity of a freely falling body from height "h".



Explanation:

Torricelli's theorem is a special case of Bernoulli's equation with the help of which we can determine the velocity of a fluid coming out through narrow opening of a container.

Consider a fluid inside a container as shown in the figure. Let there is a narrow opening at point "B" through which the fluid flows out of the container and let "h" be the height of the fluid above point "B". Now we apply Bernoulli's equation to point "A" and "B" of the given figure.

At Point A: We have, $P_1 =$ atmospheric pressure $= P$

Height $= h_1, v_1 = 0$

At Point B: We have,

$P_2 = P$, height $= h_2$

$v_2 =$ velocity of efflux of the fluid $= v$

Now the Bernoulli's equation is given by,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$\Rightarrow P + 0 + \rho gh_1 = P + \frac{1}{2}\rho v^2 + \rho gh_2$$

$$\Rightarrow \frac{1}{2}\rho v^2 = \rho gh_1 - \rho gh_2 \Rightarrow \frac{1}{2}v^2 = g(h_1 - h_2)$$

$$\Rightarrow \frac{v^2}{2} = g(h_1 - h_2) \quad \boxed{h_1 - h_2 = h}$$

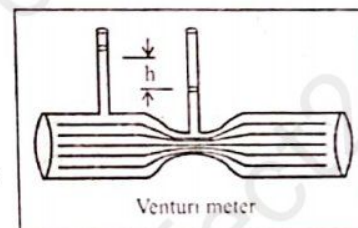
$$\Rightarrow v^2 = 2gh \Rightarrow v = \sqrt{2gh} \quad \text{--- (1)}$$

Equation (1) represents the speed of efflux of a fluid. This expression was first obtained by Torricelli and is known as Torricelli's theorem.

The speed of efflux is the same as the speed of a ball that falls freely through a height "h".

5. Flow Meter (or) Venturi Meter:

The venturi meter is a device which works on the principle of Bernoulli's equation and with the help of which we can determine the speed of flow of a fluid through a pipe.



Explanation:

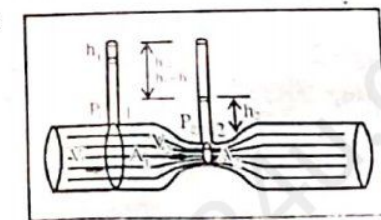
The venturi meter consists of a horizontal tube having a narrow part in the middle, which is known as throat. Now from Bernoulli's equation, we have,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \quad \text{--- (1)}$$

Here both ends are the same height from ground level, so we put

$h_1 = h_2 = h$, so equation (1) becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh$$



We put $v_1 = 0$, because the upper surface of fluid moves downwards very slowly.

$$\Rightarrow P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\Rightarrow v_2^2 - v_1^2 = \frac{2}{\rho} (P_1 - P_2) \quad (2)$$

From equation of continuity, we have,

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1 v_1}{A_2} \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\frac{A_1^2 v_1^2}{A_2^2} - v_1^2 = \frac{2}{\rho} (P_1 - P_2) \Rightarrow v_1^2 \left(\frac{A_1^2}{A_2^2} - 1 \right) = \frac{2}{\rho} (P_1 - P_2)$$

$$\Rightarrow \frac{v_1^2 (A_1^2 - A_2^2)}{A_2^2} = \frac{2}{\rho} (P_1 - P_2) \Rightarrow v_1^2 = \frac{2 A_2^2 (P_1 - P_2)}{\rho (A_1^2 - A_2^2)}$$

$$\Rightarrow v_1 = A_2 \sqrt{\frac{2 (P_1 - P_2)}{\rho (A_1^2 - A_2^2)}} \quad (4)$$

Equation (4) represents the speed of fluid in terms of pressure difference.

We also know that,

$$P_1 - P_2 = \rho g h \quad (5)$$

Putting equation (5) in equation (4), we get,

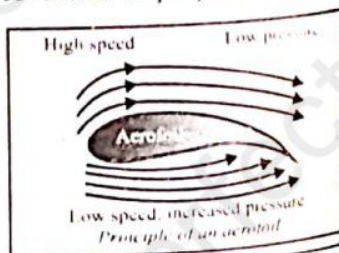
$$v_1 = A_2 \sqrt{\frac{2 \rho g h}{\rho (A_1^2 - A_2^2)}} = A_2 \sqrt{\frac{2 g h}{(A_1^2 - A_2^2)}}$$

$$\Rightarrow v_1 = A_2 \sqrt{\frac{2 g h}{(A_1^2 - A_2^2)}} \quad (6)$$

Equation (6) represents the speed of fluid in terms of difference of liquid level in the two tubes.

6. Aerofoil [Lift on an Aeroplane Wing]:

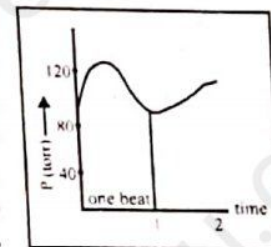
It is a device which is designed in such a way that the relative motion between it and a fluid produce a force which is perpendicular to the flow. One of the examples of aerofoil is the aeroplane wing which is designed in such a way that fluid above it flows faster than the fluid flowing below it. As a result, the pressure above the wing becomes minimum and below the wing becomes maximum.



In this way, an upwards force exists which is perpendicular to the flow of the fluid. This provides "lift" for the aeroplane.

7. Blood Flow:

Blood is an incompressible fluid having a density nearly equal to that of water. The red blood cells increase the viscosity of blood from 3 to 5 times than that of water. Blood vessels are not rigid but they stretch like the rubber hose. Under normal circumstances the volume of blood is sufficient to keep the vessels in flattened all the times. This means that there is tension in the walls of the blood vessels. As a result the pressure of blood inside the vessel is greater than the external atmospheric pressure.



Variation of Blood Pressure with Heart Beats:

The variation in blood pressure as the heart beats is shown in the figure. The pressure varies from high [systolic pressure] of 120 torr to a low pressure [diastolic pressure] of about 75 - 80 torr between beats in a normal healthy person. The number tends to increase with age due to the decrease in flexibility of vessel walls.

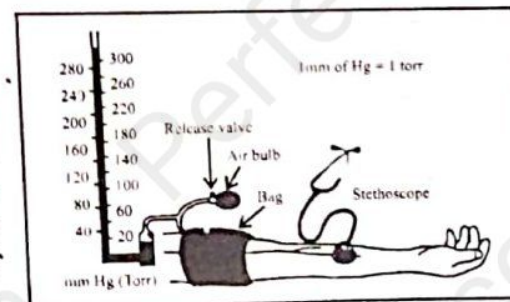
Unit of Blood Pressure:

Torr or millimeter [mm] of Hg [mercury] are the units of blood pressure due to their extensive use in medical equipments. Although the unit of pressure in SI system is Pascal or N/m^2 .

$$1 \text{ torr} = 133.3 \text{ N/m}^2$$

Blood Flow Meter:

The human blood flows in flexible blood vessels. The blood possesses certain pressure due to heart beats. The range of this pressure lies between 75 mm of Hg to 120 mm of Hg. Its high value is known as



systolic pressure and its low value is known as diastolic pressure. For measuring the blood pressure, an inflatable bag is wound around the arm of a person. The external pressure is increased by inflating

the bag, due to which the arm squeezes and the blood is allowed to compress inside. When the external pressure becomes larger than systolic pressure, then the vessels collapse and the flow of blood cuts off at this stage.

Now the external pressure is allowed to decrease gradually by opening the release valve. When the external pressure and systolic pressure becomes equal, then at this point the first surges of the blood flow through the narrow stricture with very high speed. This flow occurs in turbulent manner. In this way, the systolic pressure of the person is recorded. Now when the external pressure decreases further, then a stage comes when the external pressure becomes equal to the diastolic pressure. At this stage, the vessels become fully relaxed and the flow of blood occurs in steady state, i.e. in laminar form. The gurgle in the stethoscope disappears. In this way, the diastolic pressure is recorded.

Solution of Examples & Assignments

EXAMPLE 6.1:

The radius of small fog droplet in air is found to be $5.1 \times 10^{-6} \text{ m}$. The coefficient of viscosity of air is $1.9 \times 10^{-5} \text{ kg/m} \cdot \text{sec}$. Find out the settling speed of the droplet in air.

Solution:

Given data: Radius of droplet = $r = 5.1 \times 10^{-6} \text{ m}$

Coefficient of viscosity = $\eta = 1.9 \times 10^{-5} \text{ kg/m} \cdot \text{s}$

Density of water = $\delta = 10^3 \text{ kg/m}^3$

Required:

Settling speed of the droplet = $v_r = ?$

Calculation:

We know that, $v_r = \frac{2\delta g r^2}{9\eta}$ (1)

Putting the values in equation (1), we get,

$$\Rightarrow v_r = \frac{2 \times 10^3 \times 9.8 \times (5.1 \times 10^{-6})^2}{9 \times 1.9 \times 10^{-5}} \text{ m/sec}$$

$$\Rightarrow v_r = 29.8 \times 10^{-4} \text{ m/sec} \Rightarrow v_r = 2.98 \times 10^{-3} \text{ m/sec}$$

ASSIGNMENT 6.1:

A certain globular protein particle has a density of 1246 kg/m^3 . It falls through water (having coefficient of viscosity $8.91 \times 10^{-4} \text{ Pa} \cdot \text{s}$) with a terminal speed of $8.33 \times 10^{-6} \text{ m/s}$. Find the radius of the particle.

Solution:

Given data: Density of protein particle = $\delta = 1246 \text{ kg/m}^3$

Coefficient of viscosity of water = $\eta = 8.91 \times 10^{-4} \text{ Pa} \cdot \text{s}$

Terminal speed of protein particle = $v_r = 8.33 \times 10^{-6} \text{ m/sec}$

Required:

Radius of the particle = $r = ?$

Calculation:

We know that, $v_r = \frac{2\delta g r^2}{9\eta} \Rightarrow r^2 = \frac{9\eta v_r}{2\delta g}$ (1)

Putting the values in equation (1), we get,

$$r^2 = \frac{9 \times 8.91 \times 10^{-4} \times 8.33 \times 10^{-6}}{2 \times 1246 \times 9.8} = \frac{667.9 \times 10^{-10}}{24421.6} \text{ m}^2$$

$$\Rightarrow r^2 = 0.0273 \times 10^{-10} \text{ m}^2 \Rightarrow r = \sqrt{0.0273 \times 10^{-10} \text{ m}^2}$$

$$\Rightarrow r = 0.165 \times 10^{-5} \text{ m} \Rightarrow r = 1.65 \times 10^{-6} \text{ m}$$

EXAMPLE 6.2:

A garden hose of inner radius 1.25cm carries water at 2.60m/s. The nozzle at the end has radius 0.30cm. How fast does the water emerge out through the nozzle?

Solution:

Given data:

Radius of garden hose = " r_1 " = 1.25cm = 0.0125m

Radius of the nozzle = " r_2 " = 0.30cm = 0.0030m

Speed through garden hose

Required:

Speed out of nozzle = " v_2 " = ?

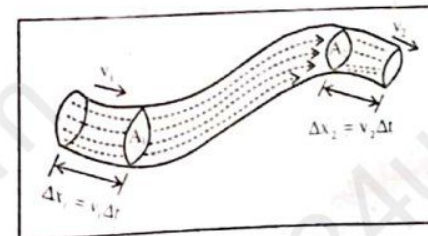
Calculation:

The equation of continuity is,

$$A_1 v_1 = A_2 v_2 \text{ (1)}$$

We know that area = πr^2

$$\Rightarrow A_1 = \pi r_1^2 \text{ and } A_2 = \pi r_2^2$$



So equation (1) becomes,

$$\pi r_1^2 v_1 = \pi r_2^2 v_2 \Rightarrow r_1^2 v_1 = r_2^2 v_2$$

$$\Rightarrow v_2 = \frac{r_1^2 v_1}{r_2^2} \quad (2)$$

Putting values in equation (2), we get,

$$v_2 = \frac{(0.0125\text{m})^2 \times 2.60\text{ms}^{-1}}{(0.0030\text{m})^2}$$

$$\text{Hence } v_2 = 45.14\text{ms}^{-1}$$

The speed of the water from the nozzle is 45.14m/s.

ASSIGNMENT 6-2:

The heart pumps blood into the aorta, which has an inner radius of 1.0cm. The aorta feeds 32 major arteries (each have an inner radius of 0.21cm). If blood in the aorta travels at a speed of 25cm/s, at approximately what average speed does it travel in the arteries? Assume that blood can be treated as an ideal fluid?

Solution:

Given data:

$$\text{Radius of aorta} = r_1 = 1\text{cm} = 0.01\text{m}$$

$$\text{Speed of blood in aorta} = v_1 = 25\text{cm/sec} = 0.25\text{m/sec}$$

$$\text{Radius of artery} = r_2 = 0.21\text{cm} = 0.0021\text{m}$$

$$\text{Area of cross-section of aorta} = A_1 = \pi r_1^2$$

$$\text{Area of cross-section of 32 arteries} = A_2 = 32\pi r_2^2$$

Required:

$$\text{Speed of blood in artery} = v_2 = ?$$

Calculation:

From equation of continuity, we have,

$$A_1 v_1 = A_2 v_2 \Rightarrow \pi r_1^2 v_1 = 32\pi r_2^2 v_2$$

$$\Rightarrow r_1^2 v_1 = 32 r_2^2 v_2$$

$$\Rightarrow v_2 = \frac{r_1^2 v_1}{32 r_2^2} \quad (1) \quad \boxed{\text{Area} = \pi r^2}$$

Putting the values in equation (1), we get,

$$v_2 = \left[\frac{(0.01)^2 \times 0.25}{32(0.0021)^2} \right] \text{m/sec} = 0.18\text{m/sec}$$

$$\Rightarrow v_2 = 0.18\text{m/sec}$$

EXAMPLE 6-3:

Water is flowing smoothly through a pipe. At one point the pressure is 33.2 kPa and the speed of water is 2m/s. While at another point 2.3m higher the pressure is 3.7 kPa, at what speed is the water flowing through this point?

Solution:

Given data:

$$\text{Pressure} = "P_1" = 33.2\text{kPa} = 33.2 \times 10^3 \text{Nm}^{-2}$$

$$\text{Pressure} = "P_2" = 3.7\text{kPa} = 3.7 \times 10^3 \text{Nm}^{-2}$$

$$\text{Speed of water} = "v_1" = 2\text{ms}^{-1}$$

$$\text{Height} = "h_1" = 0\text{m}, \text{Height} = "h_2" = 2.3\text{m}$$

$$\text{Density of water} = "\rho" = 1000\text{kgm}^{-3}$$

Required:

$$\text{Speed of water} = "v_2" = ?$$

Calculation:

We know that the Bernoulli's equation is given by,

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

$$\Rightarrow \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 - P_2 - \rho gh_2$$

$$\Rightarrow v_2 = \sqrt{\frac{2}{\rho} [P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 - P_2 - \rho gh_2]} \quad (1)$$

Putting the values in equation (1), we get,

$$v_2 = \sqrt{\frac{2}{1000} [(33.2 \times 10^3) + (\frac{1}{2} \times 1000 \times 2^2) + (1000 \times 9.8 \times 0) - (3.7 \times 10^3) - (1000 \times 9.8 \times 2.3)]}$$

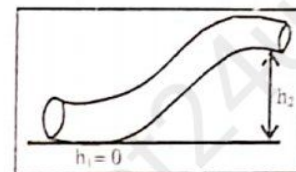
$$\Rightarrow v_2 = 2 \times 10^3 [(33.2 \times 10^3) + (2 \times 10^3) + 0 - (3.7 \times 10^3) - (22.5 \times 10^3)]$$

$$\Rightarrow v_2 = 2 \times 10^3 \times 10^3 [33.2 + 2 - 3.7 - 22.5] = 2[10.3]$$

$$= 18.8\text{m}^2/\text{sec}^2$$

$$\Rightarrow v_2 = 18.8\text{m}^2/\text{sec}^2 \Rightarrow \sqrt{v_2^2} = \sqrt{18.8\text{m}^2/\text{sec}^2}$$

$$\Rightarrow v = 4.33\text{m/sec} \Rightarrow v_2 = 4\text{m/sec} \text{ Approx.}$$



ASSIGNMENT 6-3:

Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3ms⁻¹, while at another point 3m higher, the speed is 4ms⁻¹. At lower point the pressure is 80 kPa. Find the pressure at the upper point.

Solution:

Given data:

Speed of water at one point of pipe = $v_1 = 3 \text{ m/sec}$

Speed of water at other point of pipe = $v_2 = 4 \text{ m/sec}$

Initial height = $h_1 = 0$,

Height at other end = $h_2 = 3 \text{ m}$

Pressure at lower point

$$= P_1 = 80 \text{ kPa} = 80 \times 10^3 \text{ Pa} = 80 \times 10^3 \text{ N/m}^2$$

Density of water = $\delta = 1000 \text{ kg/m}^3$

Required:

Pressure at upper point = $P_2 = ?$

Calculation:

We know that Bernoulli's equation is given by,

$$P_1 + \frac{1}{2} \delta v_1^2 + \delta g h_1 = P_2 + \frac{1}{2} \delta v_2^2 + \delta g h_2$$

$$\Rightarrow P_2 = P_1 + \frac{1}{2} \delta v_1^2 + \delta g h_1 - \frac{1}{2} \delta v_2^2 - \delta g h_2 \quad (1)$$

Putting the values in equation (1), we get,

$$P_2 = (80 \times 10^3) + \left(\frac{1}{2} \times 1000 \times 3^2 \right) + (1000 \times 9.8 \times 0) - \left(\frac{1}{2} \times 1000 \times 4^2 \right) - (1000 \times 9.8 \times 3)$$

$$\Rightarrow P_2 = \left[(80 \times 10^3) + \left(\frac{9}{2} \times 10^3 \right) + 0 - (8 \times 10^3) - (29.4 \times 10^3) \right] \text{ N/m}^2$$

$$\Rightarrow P_2 = 10^3 [80 + 4.5 - 8 - 29.4] \text{ N/m}^2 = 10^3 [47.1] \text{ N/m}^2 \quad [10^3 = \text{kilo}]$$

$$\Rightarrow P_2 = 47.1 \times 10^3 \text{ N/m}^2 \quad \text{or} \quad P_2 = 47.1 \text{ KPa}$$

EXAMPLE 6.4:

A cylindrical water storage tank has a horizontal spigot near the bottom, at a depth of 1.2m beneath the water surface. (a) When the spigot opened, how fast does the water come out? (b) If the radius of spigot is $6.0 \times 10^{-3} \text{ m}$, what will be the volume flow rate?

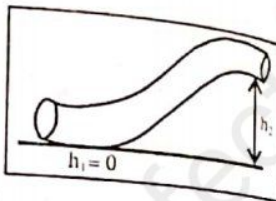
Solution:

Given data:

Height of water in tank = $h = 1.2 \text{ m}$

Radius of spigot = $r = 6.0 \times 10^{-3} \text{ m}$

Acceleration due to gravity = $g = 9.8 \text{ ms}^{-2}$



Required: (i) Speed of water = $v = ?$

(ii) Volume flow rate = $\frac{\Delta V}{\Delta t} = ?$

Calculation:

(i) According to Torricelli's theorem, we have,

$$v = \sqrt{2gh} \quad (1)$$

Putting the values in equation (1), we get,

$$v = \sqrt{2 \times 9.8 \times 1.2} = \sqrt{23.52} = 4.85 \text{ m/s}$$

$$\Rightarrow v = 4.85 \text{ m/sec}$$

(ii) We know that,

$$\text{Volume flow rate} = \frac{\Delta V}{\Delta t} = Av$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \pi r^2 v \quad (2) \quad [A = \pi r^2]$$

Putting the values in equation (2), we get,

$$\frac{\Delta V}{\Delta t} = [3.14 \times (6 \times 10^{-3})^2 \times 4.85] \text{ m}^3 / \text{sec} = (3.14 \times 36 \times 10^{-6} \times 4.85) \text{ m}^3 / \text{sec}$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = 548.2 \times 10^{-6} \text{ m}^3 / \text{sec} = 5.48 \times 10^{-4} \text{ m}^3 / \text{sec}$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = 5.48 \times 10^{-4} \text{ m}^3 / \text{sec}$$

EXTENSION EXERCISE:

If the opening from spigot points upward, how high does the resulting fountain go?

Sol: Speed of efflux = $v = 4.85 \text{ m/sec}$

Height of resulting fountain = $h = ?$

We know that,

$$v = \sqrt{2gh} \Rightarrow v^2 = 2gh$$

$$\Rightarrow h = \frac{v^2}{2g} \Rightarrow h = \frac{(4.85)^2}{2 \times 9.8} = \frac{23.5}{19.6} \Rightarrow h = 1.2 \text{ m}$$

ASSIGNMENT 6.4:

A tank full of water has a (small) hole near its bottom at a depth of 2.0m from the top surface, which is open to air. What is the speed of the stream of water emerging from the hole?

Solution:

Given data:

Depth of hole = $h = 2 \text{ m}$

Required:Speed of efflux of water = $v = ?$ **Calculation:** According to Torricelli's theorem, we have,

$$V = \sqrt{2gh} \quad (1)$$

Putting the values in equation (1), we get,

$$V = \sqrt{2 \times 9.8 \times 2} \text{ m/sec} = (\sqrt{39.2}) \text{ m/sec}$$

$$\Rightarrow V = 6.3 \text{ m/sec}$$

EXAMPLE 6-5:

A venturi meter measures the flow speed in the pipe. Suppose water flows through the pipe with cross-sectional area of 0.001257 m^2 , if the throat has an area of 0.000314 m^2 , and the fluid heights in the vertical tubes are $h_1 = 1.20 \text{ m}$ and $h_2 = 0.80 \text{ m}$. (a) Find the gauge pressure P_1 and P_2 . (b) Find the flow speed in the pipe.

Solution:**Given data:**Fluid height = $h_1 = 1.2 \text{ m}$, Fluid height = $h_2 = 0.80 \text{ m}$ Cross-sectional area = $A_1 = 0.001257 \text{ m}^2$,Cross-sectional area = $A_2 = 0.000314 \text{ m}^2$,Density = $\rho = 1000 \text{ kg m}^{-3}$ **Required:**(a) Gauge pressures = P_1 and $P_2 = ?$ (b) Flow speed = $v = ?$ **Calculation:** (a) We know that, $P = \rho gh$

$$\Rightarrow P_1 = \rho gh_1 \quad (1)$$

Putting the values in equation (1), we get,

$$P_1 = (1000 \times 9.8 \times 1.2) \text{ Pa} = 11760 \text{ Pa}$$

$$\Rightarrow P_1 = 11.76 \times 10^3 \text{ Pa} = 11.76 \text{ kPa} \Rightarrow P_1 = 11.76 \text{ kPa}$$

Similarly, $P_2 = \rho gh_2 \quad (2)$

Putting the values in equation (2), we get,

$$P_2 = (1000 \times 9.8 \times 0.80) \text{ Pa} = 7840 \text{ Pa} = 7.84 \times 10^3 \text{ Pa}$$

$$\Rightarrow P_2 = 7.84 \text{ kPa}$$

(b) For venturi meter relation, $P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$

$$10^3 = \text{kilo}$$

$$V_1 = A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}} \quad (3)$$

Putting the values in equation (3), we get,

$$V_1 = 0.314 \times 10^{-3} \sqrt{\frac{2 \times 9.8 \times 0.4}{(1.257 \times 10^{-3})^2 - (0.314 \times 10^{-3})^2}}$$

$$\Rightarrow V_1 = \left[0.314 \times 10^{-3} \sqrt{\frac{7.84}{(1.6 \times 10^{-6} - 0.1 \times 10^{-6})}} \right] \text{ m/sec}$$

$$\Rightarrow V_1 = \left[0.314 \times 10^{-3} \sqrt{\frac{7.84}{(1.6 - 0.1) \times 10^{-6}}} \right] \text{ m/sec}$$

$$\Rightarrow V_1 = \left[\frac{0.314 \times 10^{-3}}{10^{-3}} \sqrt{\frac{7.84}{1.5}} \right] \text{ m/sec}$$

$$\Rightarrow V_1 = \left[0.314 \sqrt{5.23} \right] \text{ m/sec} = (0.314 \times 2.3) \text{ m/sec}$$

$$\Rightarrow V_1 = 0.722 \text{ m/sec}$$

$$h = h_1 - h_2$$

$$h = 1.2 \text{ m} - 0.8 \text{ m}$$

$$\Rightarrow h = 0.4 \text{ m}$$

ASSIGNMENT 6-5:

A venturi meter is measuring the flow of water in a pipe having cross-sectional area of 0.0038 m^2 , a throat will cross-sectional area of 0.00031 m^2 is connected to it. If the pressure difference is measured to be 2.4 kPa , what is the speed of the water in the pipe?

Solution:**Given data:**Cross-sectional area of pipe = $A_1 = 0.0038 \text{ m}^2$ Cross-sectional area of throat = $A_2 = 0.00031 \text{ m}^2$ Pressure difference = $P_1 - P_2 = 2.4 \text{ kPa} = 2.4 \times 10^3 \text{ Pa}$ Density of water = $\rho = 1000 \text{ kg/m}^3 = 10^3 \text{ kg/m}^3$ **Required:**Speed of water in the pipe = $v = ?$ **Calculation:**

We know that speed of water in a pipe in terms of pressure difference is given by,

$$V_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

$$\Rightarrow V_1 = 0.00031 \sqrt{\frac{2 \times 2.4 \times 10^3}{10^3 [(0.0038)^2 - (0.00031)^2]}}$$

$$\Rightarrow V_1 = 0.00031 \sqrt{\frac{4.8}{(1.4 \times 10^{-5} - 9.61 \times 10^{-6})}}$$

$$= 0.00031 \sqrt{\frac{4.8}{(1.4 \times 10^{-5} - 0.00961 \times 10^{-5})}}$$

$$\Rightarrow V_1 = 0.00031 \sqrt{\frac{4.8}{10^{-5}(1.4 - 0.00961)}} = 0.00031 \sqrt{\frac{4.8 \times 10^{-5}}{1.39}}$$

$$\Rightarrow V_1 = 0.00031 \sqrt{3.45 \times 10^5} = 0.00031 \sqrt{34.5 \times 10^4}$$

$$\Rightarrow V_1 = 0.00031 \times 10^2 \sqrt{34.5} = 0.031 \times 5.9 \text{ m/sec}$$

$$\Rightarrow V_1 = 0.18 \text{ m/sec}$$

EXAMPLE 6-6:

What is the aerofoil's lift (in Newtons) on a wing of area 88m^2 if the air passes at speed over its top surface at 280m/s and bottom surface at 150m/s ?

Solution:

Given data: Surface area = $A = 88\text{m}^2$,

Speed at top of wing = $v_2 = 280\text{m/s}$

Speed at bottom of wing = $v_1 = 150\text{m/s}$

Density of the air = $\delta = 1.28\text{kg/m}^3$

Required: Force = $F = ?$

Calculation: We know that, Pressure = $\frac{\text{Force}}{\text{Area}} \Rightarrow P_1 - P_2 = \frac{F}{A}$

$$\Rightarrow F = A(P_1 - P_2) \quad (1)$$

From venturimeter, we have,

$$P_1 - P_2 = \frac{1}{2} \delta (v_2^2 - v_1^2) \quad (2)$$

Putting equation (2) in equation (1), we get,

$$F = A \times \frac{1}{2} \delta (v_2^2 - v_1^2) \quad (3)$$

Putting the values in equation (3), we get,

$$F = 88 \times \frac{1}{2} \times 1.28 [(280)^2 - (150)^2] \text{ N}$$

$$\Rightarrow F = 56.32 [78400 - 22500] \text{ N} = 56.32 (55900) \text{ N}$$

$$\Rightarrow F = 56.32 \times 55900 \text{ N} = 3148288 \text{ N}$$

$$\Rightarrow F = 3.148288 \times 10^6 \text{ N} = 3.2 \times 10^6 \text{ N} \Rightarrow \boxed{F = 3.2 \times 10^6 \text{ N}}$$

ASSIGNMENT 6-6:

An airplane wing is designed so that the speed of the air across the top of the wing is 240m/s when the speed of the air below the wing is 215m/s . The density of the air is 1.28kg/m^3 . What is the lifting force on a wing of area 24.0m^2 ?

Solution: Given data: Speed below the wing = $v_1 = 215\text{m/sec}$

Speed of air above the wing = $v_2 = 240\text{m/sec}$

Density of air = $\delta = 1.28\text{kg/m}^3$, Area of wing = 24m^2

Required: Lift force on wing = $F = ?$

Calculation:

We know that,

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} \Rightarrow P_1 - P_2 = \frac{F}{A}$$

$$\Rightarrow F = A(P_1 - P_2) \quad (1)$$

From venturimeter, we have,

$$P_1 - P_2 = \frac{1}{2} \delta (v_2^2 - v_1^2) \quad (2)$$

Putting equation (2) in equation (1), we get,

$$F = A \times \frac{1}{2} \delta (v_2^2 - v_1^2) \quad (3)$$

Putting the values in equation (3), we get,

$$F = 24 \times \frac{1.28}{2} [(240)^2 - (215)^2] \text{ N}$$

$$\Rightarrow F = 15.36 [57600 - 46225] = 15.36 \times 11375 \text{ N}$$

$$\Rightarrow F = 174720.0 \text{ N} = 1.75 \times 10^5 \text{ N} \Rightarrow \boxed{F = 1.75 \times 10^5 \text{ N}}$$

Exercise

Choose the best possible answer:

1. For substances that do not flow easily [like honey] have the..... value for the coefficient of viscosity.
 a. Low ✓ b. High c. Zero d. Negative
2. A unit for viscosity, the centipoises, is equal to which of the following?

✓ a. 10^{-3}Ns/m^2 b. 10^{-2}Ns/m^2
 c. 10^2Ns/m^2 d. 10^3Ns/m^2

Sol: We know that,

$$1 \text{ centipoise} = 10^{-3} \text{ Pascal} \cdot \text{sec} \quad \boxed{1 \text{ Pascal} = 1 \text{N/m}^2}$$

$$\Rightarrow 1 \text{ centipoise} = 10^{-3} (\text{N/m}^2) \cdot \text{sec}$$

$$\Rightarrow \boxed{1 \text{ centipoise} = 10^{-3} \text{N} \cdot \text{sec/m}^2}$$

So option (a) is correct.

3. Stokes law is applicable only if a body is moving through a liquid with slow speed and has...shape.

a. A cubical ✓ b. A spherical
 c. A rough d. Any

Sol: According to Stokes law,

$$F_{\text{dragging}} = 6\pi\eta r v \quad (1)$$

In equation (1), "r" represents the radius of spherical body, so option (b) is correct.

4. The net force that acts on a 10-N falling object, when it counters 4N of air resistance is:..

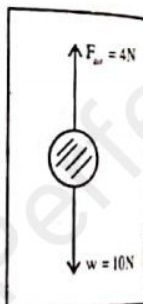
a. 0N b. 4N ✓ c. 6N d. 10N

Sol: In given figure, the weight "w" of the object is acting downward while force of friction " F_{air} " is acting upwards, so net force is given by,

$$F_{\text{net}} = w - F_{\text{air}} = 10\text{N} - 4\text{N} = 6\text{N}$$

So equation (c) is correct.

5. A skydiver jumps from a high-flying helicopter. Before reaching terminal velocity, her acceleration:



- a. Increase ✓ b. Decrease
 c. Remain the same d. Is zero

Sol: We know that terminal velocity of an object is given by,

$$v_t = \frac{mg}{6\pi\eta r} \quad (1)$$

Equation (1) shows that for terminal velocity, we take value of "g" equal $+9.8 \text{m/sec}^2$.

6. At terminal velocity the acceleration of a falling object is:

✓ a. 0ms^{-2} b. 1ms^{-2}
 c. -9.8ms^{-2} d. $+9.8 \text{ms}^{-2}$

7. According to equation of continuity $Av = \text{constant}$. This constant is equal to:

a. Volume of fluid b. Mass of fluid
 c. Density of fluid ✓ d. Volume flow rate

Sol: According to equation of continuity, we have,

$$\frac{\Delta V}{\Delta t} = Av = \text{constant} \quad (1)$$

In equation (1) $\frac{\Delta V}{\Delta t}$ represents the volume flow rate of a fluid.

So option (d) is correct.

8. At the constriction in the cross-section for ideal flow, from equation of continuity it follows that, the speed of fluid is:

✓ a. Greater b. Less c. Same d. Zero

Sol: According to equation of continuity, we have,

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1 v_1}{A_2} \quad (1)$$

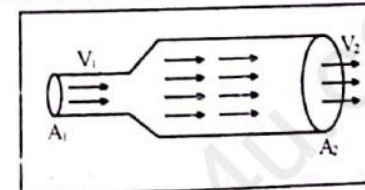
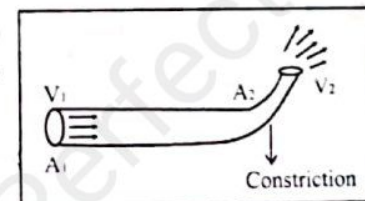
Due to constriction, the area of cross-section " A_2 " of the pipe decreases. Now according to equation (1) when " A_2 " decreases,

the speed " v_2 " at that end increases. So option (a) is correct.

9. As water in a level pipe passes from a narrow cross-section of pipe to a wider cross-section, the pressure against the wall:

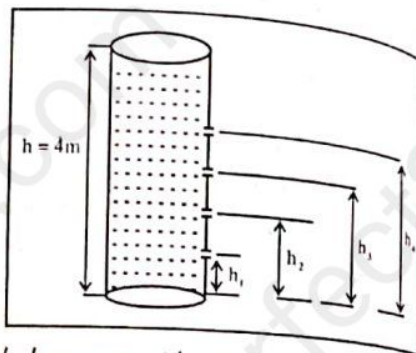
✓ a. Increases b. Decrease
 c. Remains the same d. Is zero

Sol: We know that, $A_1 v_1 = A_2 v_2$



$$\Rightarrow v_2 = \frac{A_1 v_1}{A_2}$$

Equation (1) shows that greater the area of cross-section, smaller will be speed of the fluid and vice versa. As in the given figure, " A_2 " > " A_1 ", so speed of fluid in the wider region will reduce. Now according to Bernoulli's



principle, when speed of fluid decreases, the pressure of fluid in that region increases. So option (a) is correct.

10. A 4m high tank filled with water is drilled with four identical small holes at 1m, 1.5m, 2m and 2.5m from the bottom of tank, the speed of efflux will be greatest from the hole at:
 ✓ a. 1m b. 1.5m c. 2m d. 2.5m

Sol: Height of tank = $h = 4\text{m}$

Let $h_1 = 1\text{m}$ from bottom, $h_2 = 1.5\text{m}$ from bottom, $h_3 = 2\text{m}$ from bottom, $h_4 = 2.5\text{m}$ from bottom

Now speed at 1st hole is given by,

$$v = \sqrt{2gh_{\text{net}}} \quad (1)$$

$$\Rightarrow v_1 = \sqrt{2g(h-h_1)} = \sqrt{2 \times 9.8 \times (4-1)} \text{ m/sec}$$

$$\Rightarrow v_1 = 7.7 \text{ m/sec}$$

Similarly, for 2nd hole, $v_2 = \sqrt{2g(h-h_2)}$

$$\Rightarrow v_2 = \sqrt{2 \times 9.8 \times (4-1.5)} \text{ m/sec} \Rightarrow v_2 = 7 \text{ m/sec}$$

Similarly, for 3rd hole, $v_3 = \sqrt{2g(h-h_3)}$

$$= \sqrt{2 \times 9.8 \times (4-2)} \text{ m/sec}$$

$$\Rightarrow v_3 = 6.3 \text{ m/sec}$$

Now for 4th hole, $v_4 = \sqrt{2g(h-h_4)} = \sqrt{2 \times 9.8 \times (4-2.5)}$

$$\Rightarrow v_4 = 5.4 \text{ m/sec}$$

As $v_1 > v_2 > v_3 > v_4$

So the speed at height of 1m from bottom of the tank will be greater. So option (a) is correct.

11. Venturimeter is a device used to measure the:

- a. Mass of fluid
 ✓ c. Speed of fluid

12. A certain pipe has a cross-sectional area of 0.0001m^2 in which water is flowing at 10m/s . The volume flow rate is:

- a. $0.00001\text{m}^3/\text{s}$
 ✓ b. $0.001\text{m}^3/\text{s}$
 c. $1\text{m}^3/\text{s}$
 d. $10.0001\text{m}^3/\text{s}$

Sol: Area = $A = 0.0001\text{m}^2$, speed = $v = 10\text{m/sec}$

Volume flow rate is given by,

$$\frac{\Delta V}{\Delta t} = AV \Rightarrow \frac{\Delta V}{\Delta t} = 0.0001 \times 10$$

$\Rightarrow \frac{\Delta V}{\Delta t} = 0.001\text{m}^3/\text{sec}$, so option (b) is correct.

13. At sufficiently high speeds the flow of viscous fluid becomes:

- a. Unexpected
 ✓ d. Turbulent
 b. Stream line
 c. Non-viscous

14. The water in the tank is 10m above the leak point. The speed with which the water emerges from the leak is:

- a. 10m/s
 ✓ b. 14m/s
 c. 194m/s
 d. 0.1m/s

Sol: $h = 10\text{m}$, speed = $v = ?$

$$\text{As } v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 10} = 14\text{m/sec} \Rightarrow v = 14\text{m/sec}$$

So option (b) is correct

15. When the radius of the artery is reduced, the blood pressure:

- a. Increased
 ✓ b. Decreased
 c. Remain the same
 d. Is zero

Sol: From equation of continuity, we have,

$$A_1 v_1 = A_2 v_2$$

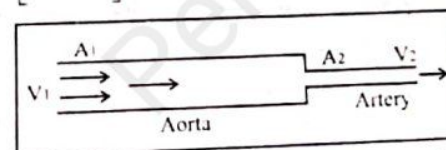
$$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 \Rightarrow r_1^2 v_1 = r_2^2 v_2$$

$$[A = \pi r^2]$$

$$\Rightarrow v_2 = \frac{r_1^2 v_1}{r_2^2} \quad (1)$$

Equation (1) shows that when radius " r_2 " of artery is reduced,

velocity of blood in artery will increase. Now according to the Bernoulli's principle, when velocity of fluid increases, the pressure of fluid decreases. Thus the pressure of blood in artery will decrease if its radius is reduced, so option (b) is correct.



Conceptual Questions

Give short answers to the following questions:

Q1: From the top of a tall building, you drop two table-tennis balls, one filled with air and the other with water. Which ball reaches terminal velocity first and why?

Ans: When we drop two table-tennis balls from the top of a tall building such that one is filled with air and the other with water, then the ball filled with air will reach to its terminal velocity first.

Reasons and Explanation:

Consider two balls such that one is filled with air and the other is filled with water as shown in the figure. Now we know that the dragging force is given by,

$$F_d = 6\pi\eta r v \quad (1)$$

Now when velocity becomes terminal velocity, then we put $v = v_t$ and $F_d = w$

So equation (1) becomes,

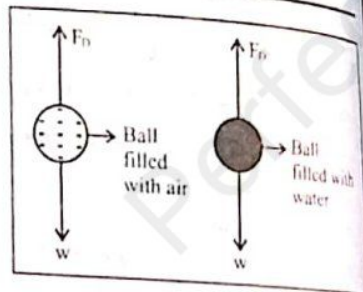
$$w = 6\pi\eta r v_t \Rightarrow 6\pi\eta r v_t = w$$

$$\Rightarrow v_t = \frac{w}{6\pi\eta r} \quad (2)$$

Equation (2) shows that the terminal velocity of the body is directly proportional to the weight of the object. i.e. greater the weight of the body, greater will be its velocity for attaining terminal velocity and vice versa. As the ball filled with water is more heavier than ball filled with air. So water filled ball will take more time for attaining its terminal velocity while the air filled ball will take less time for attaining its terminal velocity. Thus the ball filled with air will reach to its terminal velocity first.

Q2: Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

Ans: A squirrel jumps from a tree branch to the ground and runs



away undamaged while a human could break a bone in such a fall.

Reasons and Explanation:

We know that the terminal velocity is given by,

$$v_t = \frac{mg}{6\pi\eta r} \quad (1)$$

Equation (1) shows that the terminal velocity of a falling body depends upon its mass. i.e. greater the mass, high will be its terminal velocity and vice versa.

Now the mass of a squirrel is less than the mass of a man. Also when it jumps from the tree, it falls down with extended legs and tail. Due to less mass and extended legs its terminal velocity remains low. As a result, the squirrel gains its terminal velocity before reaching the ground. Thus it lands on ground in safe and sound conditions.

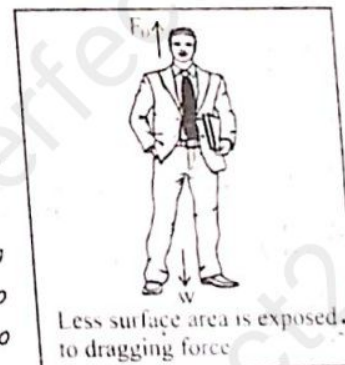
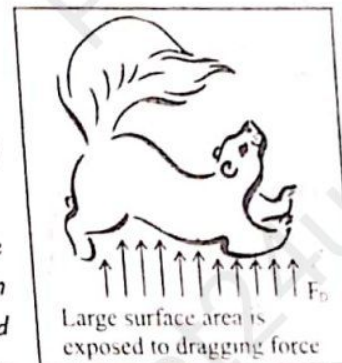
On the other hand, the mass of the man is greater than the mass of a squirrel. So its terminal velocity remains high due to its heavier mass. So when the man jumps from the tree he cannot gain its terminal velocity before reaching the ground. As a result, he feels great pain while landing on ground due to accelerated motion.

Q3: How does the terminal speed of a parachutist before opening a parachute compare to the terminal speed afterward? Why is there a difference?

Ans: The terminal velocity of a parachutist before opening the parachute is different from its terminal velocity after opening the parachute.

Reasons and Explanation:

The dragging force depends upon the exposed surface area of the fall-



ing object. i.e. greater the exposed surface area of the body, greater will be the dragging force and low will be its terminal velocity and vice versa.

Case-1: Now when the parachutist is falling down without opening parachute, the exposed surface of parachutist to air resistance is minimum. So the parachutist experience less dragging force than its weight. Thus it posses high terminal velocity as shown in figure (a).

Case-2: When the parachutist is falling down with open parachute, large surface area is exposed to air resistance. As a result the paratrooper experience large dragging force and hence posses low terminal velocity as shown in figure (b). Thus there is a difference in terminal velocities of paratrooper before opening the parachute and after opening the parachute due to different values of dragging force.

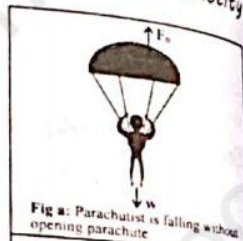


Fig a: Parachutist is falling without opening parachute



Fig b: Parachutist is falling with open parachute

Q4: You can squirt water over a greater distance by placing your thumb over the end of a garden hose, than by leaving it completely uncovered. Explain how this works?

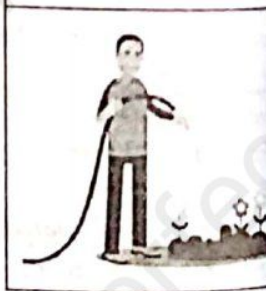
Ans: Water squirt with greater distance by placing a thumb on the end of garden hose than by leaving it completely uncovered.

Reasons and Explanation:

According to the equation of continuity, we have,

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1 v_1}{A_2} \quad (1)$$

Equation (1) shows that the speed of fluid emerging out from an opening is inversely proportional to the area of cross-section of that opening. i.e. smaller the area of cross-section of the opening, greater will be the speed of outgoing fluid and vice versa.



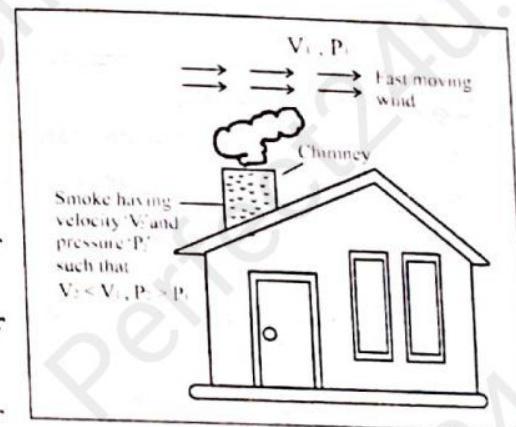
Now when we place our thumb over the end of a garden hose, its cross-sectional area decreases. As a result the speed of outgoing water increases. In this way, the water can be squirt over a greater distance.

Q5: Why does smoke rise faster in a chimney on a windy day?

Ans: The smoke rises faster in a chimney on a windy day due to difference in pressure inside the chimney and outside the chimney.

Reasons and Explanation:

Let " v_1 " represents the velocity of wind outside the chimney and " v_2 " represents the velocity of smoke inside the chimney. Let " P_1 " represents the pressure of fast moving wind and " P_2 " represents the pressure of smoke inside the chimney.



Now when wind passes over

the chimney with high speed " v_1 ", then according to Bernoulli's theorem the pressure " P_1 " in that region remains minimum. While the speed of " v_2 " of the smoke inside the chimney is low, so the pressure " P_2 " inside is high as compared to the pressure outside the chimney. i.e.

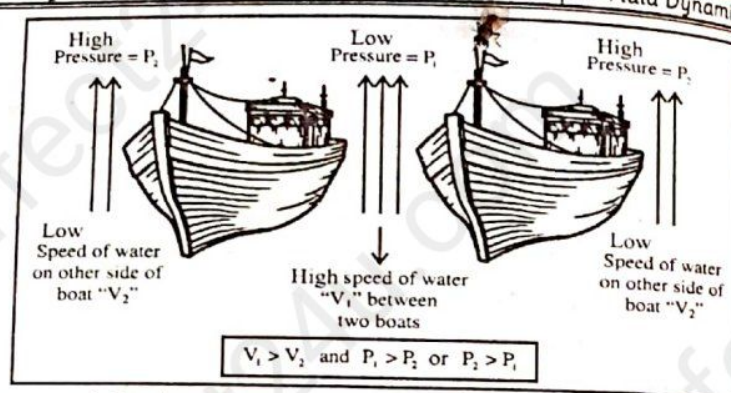
When ' $v_1 > v_2$ ' and ' $P_2 > P_1$ '

Due to the pressure difference, the smoke rises in chimney faster on windy day, i.e. when wind are moving fast over the chimney.

Q6: Two boats moving in parallel paths closed to one another risk colliding. Why?

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Ans: Two boats moving in parallel path closed to one another risk colliding. It is due to pressure difference between the two boats and outside the two boats.

**Reasons and Explanation:**

When two boats are moving parallel in water, the water between the two boats is moving with high speed " v_1 ". While the water present on the other side of both boats are moving with low speed " v_2 ". Due to difference in velocities, a pressure difference exists between the boats and on other sides of the boats. The pressure " P_1 " between boats is minimum and pressure " P_2 " on the outer sides of the boats is maximum. So both the boats are pushed towards low pressure region i.e. towards each other. In this way risk of collision between the two boats increases due to pressure difference. According to Bernoulli's principle, this difference in pressure is given by,

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

Q7: A cricket ball moves past an observer from left to right, spinning counter clockwise. In which direction will the ball tend to deflect?

Ans. When a cricket ball moves past an observer from left to right, spinning counter clockwise, then the ball will move away from the observer, due to pressure difference above and below the ball.

Reasons and Explanation:

Let a cricket ball is moving with velocity " v " from left to right and spinning counter clockwise as shown in the figure.

Now it is clear from the figure, that the air above the ball will pass with the greater speed " v_2 ". It is because the spin motion of the ball and motion of air are parallel. While the air below the ball will pass with low speed " v_1 ". It is because the spin motion of the ball and motion of air are opposite in direction. In this way, pressure difference exists on the top of ball and below the ball such that $v_2 > v_1$ and $P_2 < P_1$ or $P_1 > P_2$.

Now according to Bernoulli's theorem, the difference in pressure is given by,

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\Rightarrow \Delta P = \frac{1}{2} \rho (v_2^2 - v_1^2) \Rightarrow \frac{F}{A} = \frac{1}{2} \rho (v_2^2 - v_1^2) \quad \left[\Delta P = \frac{F}{A} \right]$$

$$\Rightarrow F = \frac{1}{2} \rho A (v_2^2 - v_1^2) \quad \text{————— (1)}$$

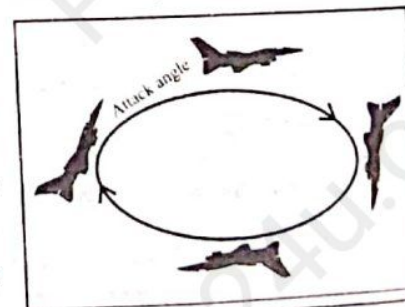
Equation (1) represents the force which gives an extra curvature to the cricket ball and as a result the ball will be deflected from high pressure side towards low pressure side. Thus the ball will move away from the observer.

Q8: If aerofoil lift the aeroplane in upright position, how do the pilots make the aeroplane fly upside down?

Ans. The pilots make the aeroplane fly upside down by increasing the angle of attack by pointing their nose up or down.

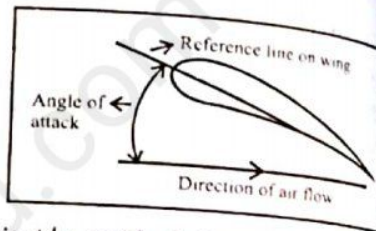
Reason and Explanation:

To fly upside down, we need a wing which is designed in such a way that it can still provide lift even when inverted. On a conventional aircraft, the aerofoil is curved on the upper side and flat on the underside. In such a case, the air over the top flows more quickly as compared to the air below the wing. The



difference in pressure gives the wing lift upwards.

The wings of aerobatic planes are curved on both upper and lower sides. With this symmetric design, the plane can fly either normally or inverted. The pilot can flip from one to the other by altering the angle of attack. The angle of attack is the angle between the flowing air and a reference line on the wing. The angle of attack can increase the lift as well as the drag and thus provide more thrust with the aircraft engine and thus keep the aircraft in safe condition.

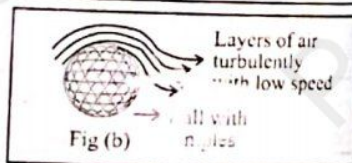
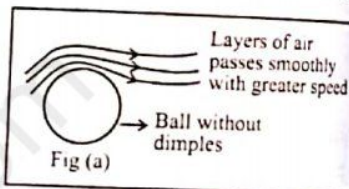


Q9: Why do the golf balls have dimples?

Ans: Golf balls have dimples, so that can move up in the air very high and can travel larger distance.

Reasons and Explanation:

Consider two balls as shown in the figure. Figure (a) represents a ball without dimples while figure (b) represents a ball with dimples. In case of ball without dimples, the air passes smoothly with greater speed. While in case of ball with dimples, the air bends more around the backside of ball turbulently due to dimples. In this way, a thin layer of air clings to the surface of the ball, which reduces the drag force. Due to reduction of dragging force, the golf ball cannot be slowed down easily. As a result, the golf ball moves up in the air easily and travel larger distance as compare to a ball without dimples.



Q10: How by using wind deflectors on the top truck cabs reduce fuel consumption?

Ans: By using wind deflectors on the top of truck cabs reduce drag

ging force and the consumption of fuel.

Reasons and Explanation:

In figure (a) a truck is moving on highway without having wind deflectors. The air cannot pass smoothly over the truck and as result the truck faces larger air resistance. Thus the engine of the truck needs to do more work against air resistance and hence consume more fuel.

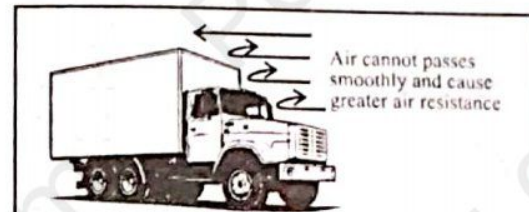


Fig a: Truck without wind deflector on top of their cab



Fig b: Truck with wind deflector on top of their cab

In figure (b) a truck is moving on highway having wind deflectors on the top of their cabs. The deflectors enable the air to pass over the truck smoothly. As a result, the truck faces less forces of friction of air. So the engine do less work against the low air resistance and thus consume less amount of fuel. Thus by using wind deflectors on the top of truck cabs reduce the consumption of fuel.

- ⊗ حضرت علی کا قول ہے "احسان کی خوبی اس کے نہ جتنا کہ پر منحصر ہے۔"
- ⊗ حضرت علی کا قول ہے "جو شخص کسی کے احسان کا شکر گزار نہیں ہے، وہ آئندہ ضرور اس سے محروم ہو جاتا ہے۔"
- ⊗ حضرت علی کا قول ہے "جب کسی احسان کا بدلہ ادا کرنے سے تیرے ہاتھ قاصر ہوں تو زبان سے اس کا شکر یہ ضرور ادا کرو۔"
- ⊗ حضرت علی کا قول ہے "معافی نہایت اچھا احسان ہے اور احسان انسان کو غلام بنالیتا ہے۔"

Comprehensive Questions

Give extended response to the following questions:

Q1: What is viscous drag? State and explain Stokes law.

Ans: See answer of question no 3 in the text.

Q2: What is terminal velocity? Derive mathematical relation for terminal velocity by using Stokes law.

Ans: See answer of question no 3 in the text.

Q3: Derive mathematically the equation of continuity, and relate it to the time rate of volume flow. How equation of continuity is based on conservation of mass?

Ans: See answer of question no 6 in the text.

Q4: Derive mathematical expression for the Bernoulli's equation. How Bernoulli's equation is based on conservation of energy?

Ans: See answer of question no 9 in the text.

Q5: Using Bernoulli's equation, what is the speed of efflux from a leak at the bottom of large storage tank?

Ans: See Torricelli's theorem in the text.

Q6: By Bernoulli's equation, how we can determine the speed of the fluid in a pipe?

Ans: See Venturi meter in the text.

Q7: What is aero-foil? Explain aero-foil lift on the wing of an aeroplane.

Ans: See article aerofoil in the text.

Q8: Use Bernoulli's equation to explain the working of engine carburetor and perfume bottle spray.

Ans: See article engine carburetor and jet and nozzle in the text.

حضرت علی رضی اللہ عنہ کا فرمان ہے کہ مہر ایک ایسی سواری ہے جو اپنے سوار کو گرنے نہیں دیتی تاکہ اس کے قدموں میں نہ کسی کی نظروں سے ...

Numerical Problems

Q1: Eight equal drops of oil are falling through air with steady velocity of 0.1 ms^{-1} . The drops recombine to form a single drop, what should be the new terminal velocity?

Solution:

Terminal velocity of single oil drop $= v_t = 0.1 \text{ m/sec}$

Let "m" represents the mass of each individual oil drop and "M" represents the mass of single drop after the combination of eight drops. So we have,

$$M = 8m \quad (1)$$

Similarly, if "V" represents the volume of each individual drop, then the volume of the single drop after the combination of eight drops is given by,

$$V' = 8V$$

$$\Rightarrow \frac{4}{3}\pi R^3 = 8 \left[\frac{4}{3}\pi r^3 \right]$$

$$\Rightarrow R^3 = 8r^3 \Rightarrow R = (2r)^3$$

$$\Rightarrow R = 2r \quad (2)$$

Now the terminal velocity of big drop of radius "R" is given by,

$$V_t' = \frac{Mg}{6\pi\eta R} \quad (3)$$

Putting equation (1) and equation (2) in equation (3), we get,

$$V_t' = \frac{8m \times g}{6\pi\eta \times 2r} = \frac{8}{2} \left[\frac{mg}{6\pi\eta r} \right] \quad (4)$$

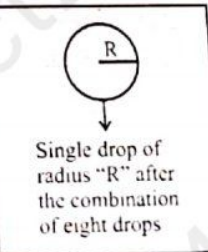
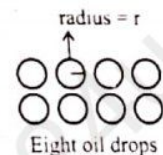
In equation (4), $\frac{mg}{6\pi\eta r}$ = terminal velocity of individual drop $= v_t$

So equation (4) becomes,

$$V_t' = \frac{8}{2}(v_t) \Rightarrow V_t' = 4v_t \quad (5)$$

Putting the value of " v_t " in equation (5), we get,

$$V_t' = (4 \times 0.1) \text{ m/sec} \Rightarrow V_t' = 0.4 \text{ m/sec}$$



Q2: Water travels through a 9.6 cm diameter fire hose with a speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose diameter is 2.5 cm. (a) What is the speed of the water coming out of the nozzle? (b) What diameter nozzle is required to give water speed of 21 m/s?

Solution:

Given Data:

Diameter of fire hose = $d_1 = 9.6 \text{ cm} = 0.096 \text{ m}$

Speed of water at one end = $v_1 = 1.3 \text{ m/sec}$

Diameter of fire hose at other end = $d_2 = 2.5 \text{ cm} = 0.025 \text{ m}$

Required:

(a) Speed of water at other end = $v_2 = ?$

(b) Diameter of nozzle [other end] for speed of 21 m/sec = $d_2 = ?$

Calculation:

(a) From equation of continuity, we have,

$$A_1 v_1 = A_2 v_2 \Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 \quad [A = \pi r^2]$$

$$\Rightarrow r_1^2 v_1 = r_2^2 v_2 \Rightarrow v_2 = \frac{r_1^2 v_1}{r_2^2} \quad (1)$$

Here $r_1 = \frac{d_1}{2}$ and $r_2 = \frac{d_2}{2}$ so equation (1) becomes,

$$v_2 = \frac{\left(\frac{d_1}{2}\right)^2 v_1}{\left(\frac{d_2}{2}\right)^2} = \frac{d_1^2 v_1}{d_2^2} \Rightarrow v_2 = \frac{d_1^2 v_1}{d_2^2} \quad (2)$$

Putting the values in equation (2), we get,

$$v_2 = \frac{(0.096)^2 \times 1.3}{(0.025)^2} \text{ m/sec} = 19.2 \text{ m/sec}$$

$$\Rightarrow v_2 = 19.2 \text{ m/sec}$$

(b) For speed $v_2 = 21 \text{ m/sec}$, the diameter of nozzle " d_2 " is given by,

$$d_2^2 = \frac{d_1^2 v_1}{v_2} \quad (3)$$

Putting the values in equation (3), we get,

$$d_2^2 = \frac{(0.096)^2 \times 1.3}{21} \Rightarrow d_2^2 = 5.7 \times 10^{-4}$$

$$\Rightarrow \sqrt{d_2^2} = \sqrt{5.7 \times 10^{-4}}$$

$$\Rightarrow d_2 = 2.4 \times 10^{-2} \text{ m} \text{ or } d_2 = 0.024 \text{ m} \text{ or } d_2 = 2.4 \text{ cm}$$

$$100 \text{ cm} = 1 \text{ m}$$

Q3: A fish tank has dimensions 0.30m wide by 1.0m long by 0.60m high. If the filter should process all the water in the tank once every 3 hour, what should the flow speed be in the 3.0cm diameter input tube for the filter?

Solution: Given Data:

Width of tank = $w = 0.30 \text{ m}$, Length of tank = $L = 1 \text{ m}$

Height of tank = $h = 0.6 \text{ m}$, Volume of tank

$$\Delta V = L \times w \times h = (1 \text{ m} \times 0.30 \text{ m} \times 0.6 \text{ m})$$

$$\Rightarrow \Delta V = 0.18 \text{ m}^3$$

$$\text{Time} = 3 \text{ hr} = 3 \times 3600 = 10800 \text{ sec}$$

$$\Rightarrow \Delta t = 10800 \text{ sec}$$

$$\text{Diameter} = d = 3.0 \text{ cm} = 0.03 \text{ m}$$

Required:

Flow speed = $v = ?$

Calculation:

We know that volume flow rate is given by,

$$\frac{\Delta V}{\Delta t} = Av$$

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \frac{\pi d^2}{4} v \Rightarrow v = \frac{4}{\pi d^2} \left(\frac{\Delta V}{\Delta t}\right) \quad (1)$$

Putting the values in equation (1), we get,

$$100 \text{ cm} = 1 \text{ m}$$

$$\Rightarrow v = \frac{4}{3.14 \times (0.03)^2} \left[\frac{0.18}{10800}\right] \text{ m/sec} = 0.02359 \text{ m/sec}$$

$$\Rightarrow v = 0.024 \text{ m/sec} \text{ or } v = 2.4 \text{ cm/sec}$$

Q4: A venturi meter is measuring the flow of water; it has a main diameter of 3.5cm tapering down to a throat diameter of 1.0cm. If the pressure difference is measured to be 18mm Hg, what is the speed of the water entering the venturi throat?

Solution: Given Data:

Diameter of one end = $d_1 = 3.5 \text{ cm} = 0.035 \text{ m}$

Diameter of other end = $d_2 = 1.0 \text{ cm} = 0.01 \text{ m}$

Pressure difference = $P_1 - P_2 = 18 \text{ mm Hg}$

$$\Rightarrow P_1 - P_2 = (133 \times 18) \text{ Pascal} = 2394 \text{ Pascal}$$

$$1 \text{ mm-Hg} = 133 \text{ Pascal}$$

$$\text{Area of one end} = A_1 = \pi r_1^2 = \pi \frac{d_1^2}{4}$$

$$\Rightarrow A_1 = \left(\frac{3.14 \times (0.035)^2}{4} \right) \text{m}^2 = 9.62 \times 10^{-4} \text{m}^2$$

$$A = \pi r^2, r = \frac{d}{2}$$

$$\text{Area of other side} = A_2 = \frac{\pi d_2^2}{4} = \frac{3.14 \times (0.01)^2}{4}$$

$$\Rightarrow A_2 = 7.85 \times 10^{-5} \text{m}^2$$

$$\text{Density of water} = \delta = 1000 \text{kg/m}^3$$

Required:

$$\text{Velocity of water} = v_1 = ?$$

Calculation:

We know that the speed of water by venturi meter in terms of pressure is given by,

$$v_1 = A_2 \sqrt{\frac{2(P_1 - P_2)}{\delta(A_1^2 - A_2^2)}} \quad (1)$$

Putting the values in equation (1), we get,

$$v_1 = 7.85 \times 10^{-5} \sqrt{\frac{2(2394)}{1000[(9.62 \times 10^{-4})^2 - (7.85 \times 10^{-5})^2]}}$$

$$\Rightarrow v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4788}{1000[92.6 \times 10^{-8} - 61.6 \times 10^{-10}]}}$$

$$\Rightarrow v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4.8}{10^{-8}(92.6 - 61.6 \times 10^{-2})}}$$

$$\Rightarrow v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4.8 \times 10^8}{(92.6 - 0.616)}}$$

$$\Rightarrow v_1 = 7.85 \times 10^{-5} \sqrt{\frac{4.8 \times 10^8}{91.98}} \Rightarrow v_1 = 7.85 \times 10^{-5} \times 10^4 \sqrt{\frac{4.8}{92}}$$

$$\Rightarrow v_1 = 7.85 \times 10^{-1} \sqrt{0.05} \Rightarrow v_1 = 7.85 \times 10^{-1} \times 0.23 = \frac{7.85 \times 0.23}{10}$$

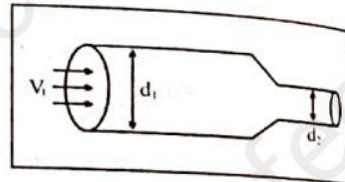
$$\Rightarrow v_1 = 0.18 \text{m/sec}$$

Q5: A small circular hole 6.00mm in diameter is cut in the side of a large water tank, 14.0m below the water level in the tank. The top of the tank is open to the air. Find (a) the speed of efflux of the water and (b) the volume discharged per second.

Solution:

Given Data:

$$\text{Diameter of hole} = d = 6 \text{mm} = 6 \times 10^{-3} \text{m}$$



$$\text{Height of tank} = h = 14 \text{m}$$

Required:

$$(a) \text{ Speed of efflux of water} = v = ?$$

$$(b) \text{ Volume discharge per second} = \frac{\Delta V}{\Delta t} = ?$$

$$10^{-3} = \text{milli}$$

Calculation:

(a) We know that,

$$v = \sqrt{2gh} \quad (1) \quad [\text{Torricelli's theorem}]$$

Putting the values in equation (1), we get,

$$v = (\sqrt{2 \times 9.8 \times 14}) \text{m/sec} \Rightarrow v = 16.6 \text{m/sec}$$

(b) We know that

$$\text{Volume flow rate} = \frac{\Delta V}{\Delta t} = Av$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \pi r^2 v \Rightarrow \frac{\Delta V}{\Delta t} = \pi \left(\frac{d}{2} \right)^2 v$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = \frac{\pi d^2 v}{4} \quad (1)$$

Putting the values in equation (1), we get,

$$\frac{\Delta V}{\Delta t} = \frac{3.14 \times (6 \times 10^{-3})^2 \times 16.6}{4}$$

$$\Rightarrow \frac{\Delta V}{\Delta t} = 469.12 \times 10^{-6} \text{m}^3 / \text{sec}$$

$$\text{or } \frac{\Delta V}{\Delta t} = 469.12 \times 10^{-6} \times 10^6 \text{cm}^3 / \text{sec}$$

$$\text{or } \frac{\Delta V}{\Delta t} = 0.469 \text{cm}^3 / \text{sec}$$

$$\begin{aligned} 1 \text{m} &= 100 \text{cm} \\ 1 \text{m}^3 &= (100 \text{cm})^3 \\ \Rightarrow 1 \text{m}^3 &= 10^6 \text{cm}^3 \end{aligned}$$

Q6: What is the aerofoil lift (in newtons) due to Bernoulli's principle on a paper plane of wing area 0.01m² if the air passes over the top and bottom surfaces at speeds of 9m/s and 7m/s respectively? (Take the density of air as 1.28kg/m³)

Solution:

$$\text{Given Data: Area of wing} = 0.01 \text{m}^2 \Rightarrow A = 0.01 \text{m}^2$$

$$\text{Speed at top of wing} = v_2 = 9 \text{m/sec}$$

$$\text{Speed below the wing} = v_1 = 7 \text{m/sec}$$

$$\text{Density of air} = \delta = 1.28 \text{kg/m}^3$$

Required: Aerofoil lift = $F = ?$

Calculation: We know that,

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$\Rightarrow P_1 - P_2 = \frac{F}{A} \Rightarrow F = A(P_1 - P_2) \quad (1)$$

From venturi meter, we have,

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) \quad (2)$$

Putting equation (2) in equation (1), we get,

$$F = A \times \frac{1}{2} \rho (v_2^2 - v_1^2) \quad (3)$$

Putting the values in equation (3), we get,

$$F = 0.01 \times \frac{1}{2} \times 1.28 (9^2 - 7^2) = 6.4 \times 10^{-3} (81 - 49)$$

$$\Rightarrow F = 6.4 \times 10^{-3} \times 32 \text{ N} = 204.8 \times 10^{-3}$$

$$\Rightarrow \boxed{F = 204.8 \times 10^{-3} \text{ N}} \text{ or } F = 0.2048 \text{ N} \text{ or } \boxed{F = 0.2 \text{ N}}$$

Q7: During a windstorm, a 25m/s wind blows across the flat roof of a small home. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is 1.28 kg/m³)

Solution:

Given Data: Speed of air = $v_2 = 25 \text{ m/sec}$

Density of air = $\rho = 1.28 \text{ kg/m}^3$

As the doors and windows of the house are closed, so we take the speed of inside air to be zero i.e. put $v_1 = 0$

Required: Pressure difference = $P_1 - P_2 = ?$

Calculation: We know that,

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) \quad (1)$$

Putting the value in equation (1), we get,

$$P_1 - P_2 = \left[\frac{1}{2} \times 1.28 (25^2 - 0^2) \right] \text{ N/m}^2$$

$$\Rightarrow P_1 - P_2 = 0.64 \times (625 - 0) \text{ N/m}^2$$

$$\Rightarrow \boxed{P_1 - P_2 = 400 \text{ N/m}^2} \text{ or } \boxed{P_1 - P_2 = 400 \text{ Pa}}$$

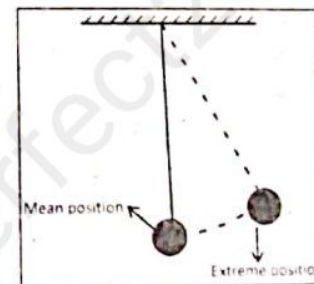
Q1: Define and explain oscillatory motion. Give examples of oscillatory motion.

Ans: Oscillatory Motion:

Definition: The back and forth motion of a body over the same path about its mean position is known as oscillatory motion.

Explanation:

The oscillatory [vibratory] motion can be initiated when a body is displaced from its mean position due to external applied force. When the external force is removed, the body moves back towards its mean position. On the way its velocity increases and becomes maximum at mean position. As a result the body does not stop at mean position and moves towards the other side of mean position. On the way its velocity decreases and finally comes to rest at extreme position. The body then moves back towards mean position due to restoring force. In this way, the body performs oscillatory motion about mean position.



Examples of Oscillatory Motion:

- Motion of a body attached to an elastic spring.
- Motion of wires of a sitar.
- Motion of simple pendulum.

Q2: Define the following terms: Periodic motion, vibratory motion, vibration, time period, frequency, displacement, amplitude and angular frequency

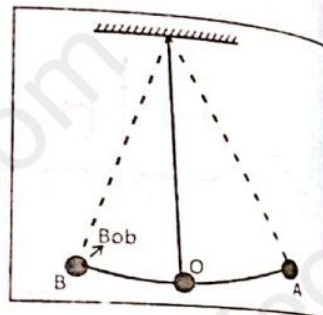
Ans-1. Periodic Motion:

Motion, which is repeated in equal intervals of time is known as

periodic motion.

2. Vibratory Motion:

The to and fro [i.e. back and forth] motion of a body about the mean position is known as vibratory motion. In the given figure the motion of the bob from point "A" towards point "B" through mean position "O" and then from "B" towards "A" through "O" represents vibratory motion.



3. Vibration:

The complete round trip of a body about mean position is known as vibration. In the given figure, if the body moves from "A" towards "B" and then again reaches "A" from "B", then under such conditions, the body [i.e. bob] completes one vibration.

4. Time Period:

The time required by a body to complete one vibration is known as time period. It is represented by "T". It is usually measured in seconds.

5. Frequency:

The number of vibrations in one second is known as frequency. (OR)

The reciprocal of time period is known as frequency. i.e. $f = \frac{1}{T}$

The S.I unit of frequency is Hertz or cycles per second.

$$1 \text{ Hertz} = 1 \text{ cycle/sec} \Rightarrow 1 \text{ Hz} = 1 \text{ c.p.s}$$

6. Angular Frequency:

The number of revolution per second of a body is known as angular frequency. If " ω " represents the angular velocity, then the angular frequency is given by, $f = \frac{\omega}{2\pi}$

7. Displacement:

The distance of a vibrating body from its mean position at any instant is known as displacement. It is represented by "x".

8. Amplitude:

The maximum displacement of a vibrating body from mean position is known as amplitude. It is represented by " x_0 ".

Q3: (a) Define simple harmonic motion [S.H.M].

(b) Show that motion of a mass attached to an elastic spring executes S.H.M.

(OR) Show that mass-spring system execute S.H.M?

BISE Abbottabad 2019

Ans: (a) Simple Harmonic Motion [S.H.M]: ETEA 2010, 2013

Definition: The type of motion in which the acceleration of the body is always directly proportional to the displacement of the body from mean position and is always directed towards the mean position is known as simple harmonic motion [S.H.M]. i.e.

$$a \propto (-x)$$

The negative sign shows that the acceleration is always directed towards the mean position.

(b) Motion of a mass attached to an Elastic Spring:

Consider a body of mass "m" is attached to an elastic spring lying on a smooth frictionless surface as shown in the figure. Point "O" represents the mean position or equilibrium position of the mass.

Now if we displace the body towards right and bring it to point "A" and then released, the body moves back towards mean position

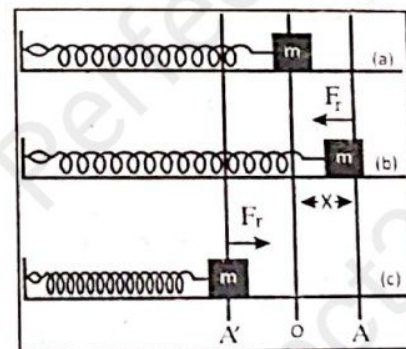
"O" due to restoring force " F_r ". In this way, the body begins to vibrate about its mean position "O".

Similarly, if we displace the body towards left and bring it to point "A'" and then released, the body moves back towards mean position

"O" due to restoring force " F_r ". In this way, the body begins to vibrate about its mean position "O". Now the restoring force which acts on the mass-spring system is given by,

$$F_{\text{restoring}} = kx \quad (1)$$

According to Newton's 2nd law, the applied force is given by,



"k" is called spring constant. Its value depends upon the physical shape and structure of the material.

$$F = ma \quad (2)$$

Here the applied force and restoring force are opposite in direction so we have,

$$F_{\text{applied}} = -F_{\text{restoring}} \quad (3)$$

Putting equation (1) and (2) in equation (3), we get,

$$ma = -kx$$

$$\Rightarrow a = -\left(\frac{k}{m}\right)x \quad (4)$$

As $\left(\frac{k}{m}\right) = \text{constant}$, so equation (4) becomes

$$a \propto (-x) \quad (5)$$

Relation (5) represents the mathematical form of S.H.M. Thus the motion of a body attached to an elastic spring executes simple harmonic motion [S.H.M].

Time Period of Mass-Spring System:

ETEA 2012

We know that,

$$S = vt \Rightarrow t = \frac{S}{v} \quad (6)$$

Here we put, $S = \theta = 2\pi$, $v = \omega$ & $t = T$

So equation (6) becomes,

$$T = \frac{2\pi}{\omega} \quad (7)$$

Put $\omega = \sqrt{\frac{k}{m}}$ in equation (7), we get,

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (8)$$

Equation (8) represents the time period of mass-spring system.

Conclusion about Time Period: Equation (8) shows that the time period depends upon the mass of the body attached to the spring: i.e. greater the mass, greater will be the time period and vice versa.

Frequency of Mass-Spring System:

We know that,

$$f = \frac{1}{T} \quad (9)$$

Putting equation (8) in equation (9), we get,

For complete round trip
 $\theta = 1 \text{ revolution} = 2\pi \text{ radian}$

$$a = -\left(\frac{k}{m}\right)x \quad (a)$$

$$\text{Also } a = -(\omega^2)x \quad (b)$$

Comparing equations (a), (b), we get

$$\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$$

F = kx
 ETEA 2013

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (10)$$

Equation (1) represents the frequency of mass-spring system.

Conclusion about Frequency: Equation (10) shows that frequency depends upon mass of the body inversely: i.e. greater then mass of the body, smaller will be the frequency and vice versa.

Q4: Discuss the relation between S.H.M and circular motion.

(OR) Show that if a particle perform circular motion then its projection executes S.H.M.

(OR) Show that motion of projection of a particle is simple harmonic motion.

ETEA 2009, BISE Peshawar 2017

Ans. Consider a body of mass "m" moving along a circular path of radius "r" as shown in the figure.

Now when the body is at point "A", its projection will be also at point "A". When the body moves from "A" towards "B", its projection moves from "A" to "O". Similarly, when the body moves from "B" towards "C" its projection moves from "O" towards "C".

Now when the body moves from "C" towards "A" through "D", its projection moves from "C" towards "A" through "O". In this way, the particle perform circular motion and its projection executes S.H.M along the diameter of the circle.

Now in ΔOPQ of figure (a), we have,

$$\cos \theta = \frac{x}{r}$$

$$\Rightarrow x = r \cos \theta \quad (1)$$

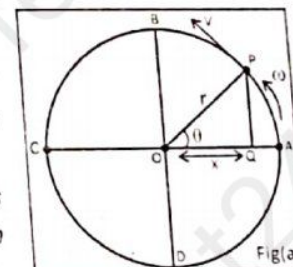
We know that, $\theta = \omega t$ (2)

Putting equation (2) in equation (1), we get,

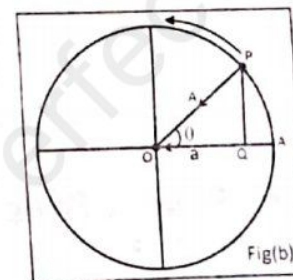
$$\Rightarrow x = r \cos(\omega t) \quad (3)$$

Equation (3) represents the displacement of projection from "O".

Now in ΔOPQ of figure (b), we have,



Fig(a)



Fig(b)

$$\cos \theta = \frac{a}{A} \Rightarrow a = A \cos \theta \quad (4)$$

a = acceleration
of projection
 A = centripetal
acceleration

Putting equation (2) in equation (4), we get,

$$a = A \cos(\omega t) \quad (5)$$

In figure (b), "A" represents the centripetal acceleration, so we have,

$$A = \frac{v^2}{r} \quad (6)$$

centripetal acceleration
 $= a_c = \frac{v^2}{r}$

Putting equation (6) in equation (5), we get,

$$a = \frac{v^2}{r} \cos(\omega t) \quad (7)$$

We also know that, $v = r\omega$ (8)

Putting equation (8) in equation (7), we get,

$$a = \frac{r^2 \omega^2}{r} \cos(\omega t) \Rightarrow a = r\omega^2 \cos(\omega t)$$

$$\Rightarrow a = \omega^2 (r \cos \omega t) \quad (9)$$

Putting equation (3) in equation (9), we get,

$$a = \omega^2 x$$

$$(OR) \boxed{a = -\omega^2 x} \quad (10)$$

Equation (10) represents the acceleration of projection of the body at any instant. In equation (10), $\omega^2 = \text{constant}$

So equation (3) $\Rightarrow a = \text{constant} (-x)$

$$\Rightarrow \boxed{a \propto (-x)} \quad (11)$$

Relation (11) represents the mathematical form of S.H.M, so the motion of projection of a body is simple harmonic motion.

Time Period of Projection:

We know that, $S = vt$

$$\Rightarrow t = \frac{S}{v} \quad (12)$$

Here we put, $S = \theta = 2\pi \Rightarrow S = 2\pi$ (13)

Putting equation (13) in equation (12), we get,

$$t = \frac{2\pi}{v} \quad (14)$$

In case of circular motion
we put $S = \theta$
For one trip we have
 $\theta = 1 \text{ revolution} = 2\pi$
radian

Also we replace "v" by " ω " and "t" by "T", so equation (14) becomes,

$$T = \frac{2\pi}{\omega} \quad (15)$$

Equation (15) represents the time period of projection.

Conclusion about Time Period:

- Equation (15) shows that the time period depends upon the angular speed of the body. i.e. greater the angular speed, smaller will be the time period and vice versa.
- Equation (15) shows that the time period does not depend upon mass of the body.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

Also $V_y = r\omega$

$$V = r\omega$$

Put $r = x_0$

$$V = x_0 \omega$$

ETEA 2009

Frequency of Projection: ETEA 2015

$$\text{As } f = \frac{1}{T} \quad (16)$$

Putting equation (15) in equation (16), we get,

$$f = \frac{\omega}{2\pi} \quad (17)$$

Equation (17) represents the frequency of projection in S.H.M.

Conclusion about Frequency: Equation (17) shows that:

- The frequency depends upon the angular velocity of the body i.e. greater the angular speed, greater will be the frequency and vice versa.
- The frequency does not depend upon the mass of the body.

Q5: Find an expression for the velocity of projection of a body in S.H.M. (OR) Show that, $v = \omega \sqrt{r^2 - x^2}$

Ans: If we resolve the velocity " V_0 " of the body into horizontal and vertical components, then we have,

$$V_{0x} = V_0 \cos \theta \quad (1)$$

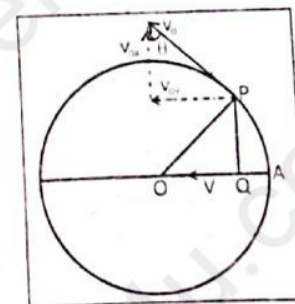
$$\& \ V_{0y} = V_0 \sin \theta \quad (2)$$

Now we see that the component " V_{0y} " is parallel to the velocity " V " of projection, so we have,

$$V = V_{0y} \quad (3)$$

Putting equation (2) in equation (3), we get,

$$V = V_0 \sin \theta \quad (4)$$



We know that,

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \quad (5)$$

$$\text{And } V_o = r\omega \quad (6)$$

Putting equation (5) and (6) in equation (4), we get,

$$V = r\omega \sqrt{1 - \cos^2 \theta} \quad (7)$$

As we know that the displacement of projection from mean position "O" at any instant is given by,

$$x = r \cos \theta \Rightarrow \cos \theta = \frac{x}{r} \quad (8)$$

Putting equation (8) in equation (7), we get,

$$V = r\omega \sqrt{1 - \frac{x^2}{r^2}}$$

$$\Rightarrow V = \frac{r\omega \sqrt{r^2 - x^2}}{r}$$

$$\Rightarrow V = \omega \sqrt{r^2 - x^2} \quad (9)$$

Equation (9) represents the velocity of projection of the body in S.H.M.

Q6: What is simple pendulum? Show that motion of simple pendulum is simple harmonic motion [S.H.M].

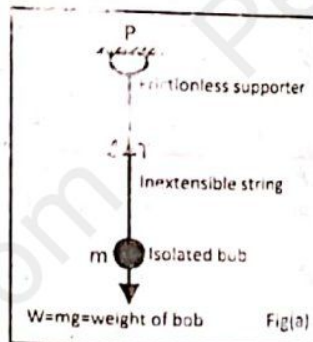
Ans. Simple Pendulum:

Definition: A simple pendulum consists of a small heavy mass "m" [called bob] suspended by a light string of length "l" fixed with its upper end, as shown in the figure (a).

Explanation:

Consider a simple pendulum as shown in figure (b). At point "A" the weight of the pendulum is acting downwards while the tension in the string is acting upwards. Both forces balance each other and thus the simple pendulum remains in state of equilibrium.

Now if we displace the bob from its mean position "A" towards point "B" through a displacement "x" due to external applied force



$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

$$\Rightarrow \sin \theta = \sqrt{1 - \cos^2 \theta}$$

Also $V_o = r\omega$

"F_{applied}" then at point "B" its weight "w" is resolved into two components i.e. "wcosθ" and "wsinθ".

The component "wcosθ" balance the tension "T" in the string, while the component "wsinθ" is directed towards the mean position and thus it acts as restoring force. So we have,

$$T = w \cos \theta \quad (1)$$

$$\& F_{\text{restoring}} = w \sin \theta \quad (2)$$

Now according to Newton's 2nd law, the applied force is given by,

$$F_{\text{applied}} = ma \quad (3)$$

Here "F_{restoring}" and "F_{applied}" are opposite in direction, so we have,

$$F_{\text{applied}} = -F_{\text{restoring}} \quad (4)$$

Putting equation (2) and equation (3) in equation (4), we get,

$$ma = -w \sin \theta \quad (5)$$

$$\text{We know that, } w = mg \quad (6)$$

Putting equation (6) in equation (5), we get,

$$ma = -mg \sin \theta \Rightarrow a = -g \sin \theta \quad (7)$$

Now for correct time period, we keep the amplitude to be very small. For smaller amplitude, we keep "θ" to be very small i.e. $\theta \ll 1$

Under such condition, we put $\sin \theta \approx \theta$, so equation (7) becomes,

$$a = -g\theta \quad (8)$$

Now according to the figure, the displacement "x" of the body is proportional to "θ", so we have,

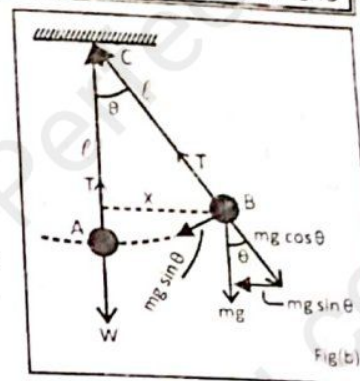
$$x \propto \theta \Rightarrow x = \text{constant } \theta \Rightarrow x = l\theta \quad (9)$$

In equation (9), "l" is constant which represents the length of the string. Now from equation (9), we have,

$$\theta = \frac{x}{l} \quad (10)$$

Putting equation (10) in equation (8), we get,

$$a = -g \cdot \frac{x}{l} \Rightarrow a = \frac{g}{l} (-x) \quad (11)$$



In equation (11), $\left(\frac{g}{l}\right) = \text{constant}$, so equation (11) becomes,

$$a = \text{constant} (-x) \Rightarrow a \propto (-x) \quad (12)$$

Equation (12) represents the mathematical form of simple harmonic motion, so the motion of simple pendulum is simple harmonic. The negative sign shows that the bob is always directed towards the mean position.

Time Period of Simple Pendulum: (ETEA 2007, 9, 10, 11)

We know that,

$$T = \frac{2\pi}{\omega} \quad (a)$$

In case of circular motion, we have,

$$a = -\omega^2 x \quad (b)$$

In case of simple pendulum, we have,

$$a = -\left(\frac{g}{l}\right) x \quad (c)$$

Comparing equations (b), (c), we get,

$$\omega^2 = \frac{g}{l}$$

$$\Rightarrow \omega = \sqrt{\frac{g}{l}} \quad (d)$$

Putting equations (d) in equation (a), we get,

$$T = 2\pi \sqrt{\frac{l}{g}} \quad (e)$$

Equation (e) represents the time period of simple pendulum.

Conclusion about Time Period:

- Equation (e) shows that the time period of simple pendulum depends upon the length of pendulum i.e. longer the pendulum, greater will be the time period and vice versa.
- The time period of the pendulum does not depend on the mass of the bob.

Frequency of Simple Pendulum:

$$\text{As } f = \frac{1}{T} \quad (f)$$

Putting equation (e) in equation (f), we get,

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \quad (g)$$

Points to be Noted:

For a body executing S.H.M the following conditions must be satisfied:

- The restoring force must be act on the system.
- The system should have inertia.
- S.H.M is performed along a straight line.
- The system should be frictionless.
- The Hook's law should be satisfied in case of mass-spring system.

Equation (g) represents the frequency of simple pendulum.

Conclusion about Frequency:

- Equation (g) shows that longer the pendulum, smaller will be the frequency and vice versa.
- The frequency of the pendulum does not depend on the mass of the bob.

Q7: (a) Derive equations for K.E and P.E of a body of mass "m" executing S.H.M. (b) Show that energy is conserved in case of S.H.M.

BISE Kohat 2019

Ans: (a) Kinetic Energy:

We know that,

$$K.E = \frac{1}{2} mv^2 \quad (1)$$

In case of circular motion, we know that,

$$v = \omega \sqrt{r^2 - x^2} \quad (2)$$

In case of mass-spring system, we put

$$r = x_0 \quad (3)$$

$$\text{And } \omega = \sqrt{\frac{k}{m}} \quad (4)$$

Putting equation (3) and equation (4) in equation (2), we get,

$$v = \sqrt{\frac{k}{m}} \sqrt{x_0^2 - x^2} \quad (5)$$

Putting equation (5) in equation (1), we get,

$$K.E = \frac{1}{2} m \left(\sqrt{\frac{k}{m}} \sqrt{x_0^2 - x^2} \right)^2$$

$$\Rightarrow K.E = \frac{1}{2} m \times \frac{k}{m} (x_0^2 - x^2) = \frac{1}{2} (kx_0^2 - kx^2)$$

$$\Rightarrow K.E = \frac{1}{2} k (x_0^2 - x^2) \quad (6)$$

Equation (6) represents the K.E of the body executing S.H.M.

Maximum Kinetic Energy:

The K.E will be maximum if we displace the body through maximum displacement " x_0 ". So under such conditions we consider " x " and ignore " x ". So we put $x = 0$ in equation (6), we get,

$$(K.E)_{\text{max}} = \frac{1}{2} k (x_0^2 - 0^2)$$

$$\Rightarrow (K.E)_{\max} = \frac{1}{2} k x_0^2 \quad (7)$$

Equation (7) represents the maximum K.E of the body executing S.H.M.

(b) Potential Energy: (ETEA 2009)

Consider a body of mass "m" attached to an elastic spring, as shown in the figure. Initially when the body is at mean position "O", then the initial force acting on mass spring system will be zero. i.e.

$$F_{\text{initial}} = 0 \quad (1)$$

Now if we increase the applied force and finally displace the body towards point "A" through a displacement "x", then the final force acting on mass-spring system is given by,

$$F_{\text{final}} = kx \quad (2)$$

Now the average force on mass-spring system is given by,

$$F_{\text{ave}} = \left(\frac{F_{\text{final}} + F_{\text{initial}}}{2} \right) \quad (3)$$

Putting equation (1) and equation (2) in equation (3), we get,

$$F_{\text{ave}} = \left(\frac{kx + 0}{2} \right) \Rightarrow F_{\text{ave}} = \frac{1}{2} kx \quad (4)$$

Now the workdone in compressing the spring through a displacement "x" is given by,

$$\text{Work} = F_{\text{ave}} \cdot x \quad (5)$$

Putting equation (4) in equation (5), we get,

$$\Rightarrow \text{Work} = \frac{1}{2} kx \cdot x \Rightarrow \text{Work} = \frac{1}{2} kx^2 \quad (6)$$

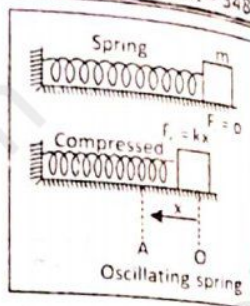
This work appears in the form of P.E in the spring, so equation (6) becomes,

$$P.E = \frac{1}{2} kx^2 \quad (7)$$

Equation (7) represents the P.E of the body attached to an elastic spring at any instant.

Maximum Potential Energy: (ETEA 2009)

The P.E will be maximum, if the spring is compressed through maximum displacement " x_0 ", under such condition, we replace "x" by " x_0 ", so equation (7) becomes,



$$(P.E)_{\max} = \frac{1}{2} kx_0^2 \quad (8)$$

Equation (8) represents the maximum P.E at extreme position " x_0 ".

Conservation of Energy in S.H.M: (ETEA 2015, 2016)

The law of conservation of energy is satisfied in simple harmonic motion i.e. K.E may be converted into P.E and P.E may be converted into K.E but its total value remains the same.

Now the total energy in simple harmonic motion is given by,

$$E_t = K.E + P.E \quad (1)$$

We know that,

$$K.E = \frac{1}{2} k(x_0^2 - x^2) \quad (2)$$

$$\& P.E = \frac{1}{2} kx^2 \quad (3)$$

Putting equations (2), (3), in equation (1), we get,

$$E_t = \frac{1}{2} k(x_0^2 - x^2) + \frac{1}{2} kx^2$$

$$\Rightarrow E_t = \frac{1}{2} kx_0^2 - \frac{1}{2} kx^2 + \frac{1}{2} kx^2$$

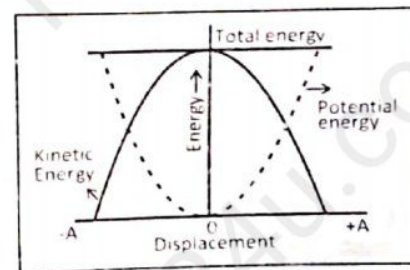
$$\Rightarrow E_t = \frac{1}{2} kx_0^2 \quad (4)$$

Equation (4) represents the total energy of vibrating body in S.H.M. This energy is totally present in the form of K.E at mean position $\left[K.E_{\max} = \frac{1}{2} kx_0^2 \right]$ and in the form of P.E at extreme position

$\left[P.E_{\max} = \frac{1}{2} kx_0^2 \right]$. Thus energy is conserved in simple harmonic motion.

Graphical Representation of Law of Conservation of Energy:

The graph between K.E and P.E is shown in the figure. The graph shows that when the body moves from mean position "O" toward extreme position "+A" or "-A", the P.E increases and K.E decreases. Similarly when the body moves



from extreme position "+A" or "-A" towards mean position "O", the P.E decreases and K.E increases.

Conclusion: The above discussion shows that during the motion of mass-spring system, the K.E is converted into P.E and P.E is converted into K.E, but the total energy remains constant. So the mass-spring system obeys the law of conservation of energy.

Q8: Differentiate between free and forced oscillations.

Ans: 1. Free Oscillation:

Definition: A body is said to be executing free oscillation if it oscillates with its natural frequency without the interference of an external force.

Explanation:

Free vibrations are oscillations where the total energy stays the same over time. This means that the amplitude of the vibration stays the same. For example, a simple pendulum vibrates freely with its natural frequency in the absence of external forces. This is a theoretical idea because in real systems the energy is dissipated to the surroundings over time and the amplitude decays away to zero. This dissipation of energy is called damping.

2. Forced Oscillation:

Definition: If a freely oscillating system is subjected to an external force, then forced oscillation will take place.

Explanation:

Forced oscillation takes place under the action of external applied force. For example, when the bob of vibrating pendulum is pushed repeatedly, then forced oscillations are produced. The vibrations of a factory floor due to running of heavy machinery, the loud music produced by sounding wooden boards of strings instruments are the examples of forced oscillations.

Q9: What is resonance? Give some applications of resonance in daily life.

Ans: Resonance:

Definition: The phenomenon in which large amplitude of vibration is

produced due to small periodic driving force is known as resonance.

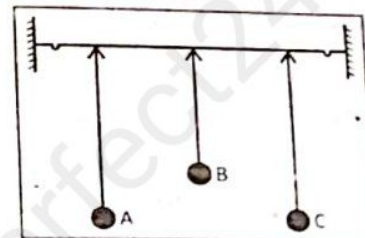
Explanation:

It has been found that when the frequency of the external driving force becomes equal to the natural frequency of oscillating body, then the body oscillates with larger amplitude. Such phenomenon is known as resonance.

The resonance also occurs when the applied force has a frequency which are integral multiple of the natural frequency of the body i.e. $2f, 3f, 4f, 5f, \dots, nf$.

Experimental Demonstration of Resonance:

Consider three pendulums A, B, C which are suspended from a stretched rubber cord, as shown in the figure.



Now if the length of pendulum "A" and "B" are the same, then by disturbing pendulum "A" slightly, the pendulum "B" also starts vibration due to the phenomenon of resonance.

Similarly, if the length of pendulum "A" and pendulum "C" are the same, then by disturbing "A" slightly, the pendulum "C" also begins to vibrate due to the phenomenon of resonance.

Applications of Resonance:

1. Radio and Resonance: Tuning a radio is the best example of electrical resonance. When we turn the knob of a radio to tune a station, we are changing the natural frequency of the electric circuit of the receiver to make it equal to the transmission frequency of the radio station. When the two frequencies match each other, resonance will occur. Under such conditions, the absorption of energy becomes maximum and as a result we hear the program of concert station clearly.



2. Magnetic Resonance Image [M.R.I]: Magnetic resonance scan-

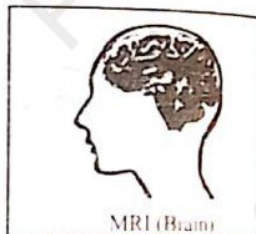
ning has greatly improved medical diagnoses. With the help of M.R.I machine, we can get photographic film of brain, backbone and other parts of the body. For this purpose, radio waves of high frequency are used to oscillate the nuclei of atoms. When resonance occurs, the molecules absorb the energy. The pattern of absorbed energy can be used to produce a computer enhanced photograph.

3. Cooking of Food: (BISE Mardan 2019)

The microwave oven is an important application of resonance. In a microwave oven, typical microwaves of wavelength 12cm and frequency of 2450 MHz is used. Such frequency of microwaves is similar to the natural frequency of vibration of water molecules and fat molecules in the food i.e.

$$\text{Frequency of microwaves} = \text{Frequency of water molecules \& fat molecules}$$

Now when the food containing water is placed in the oven, the water molecules and fat molecules in the food resonates under the action of periodic force of microwaves. As a result, the molecules absorb energy from the microwaves due to resonance. The energy of microwaves heats up the food items. In this way, the food items are heated or cooked in the microwaves oven due to resonance.



MRI (Brain)



Microwave oven

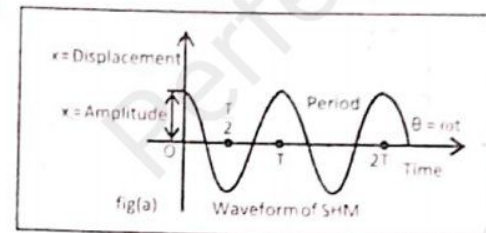
4. The amplitude of a swing can be increased due to the phenomenon of resonance by applying small periodic force.
5. We can use resonance to find natural frequency of different bodies.
6. The resonance phenomenon can be used in vehicles for comfortable journey.
7. With the help of resonance apparatus, we can find speed of sound.

Q10: Explain the waveform of S.H.M.

Ans. In case of circular motion, we know that the displacement of projection of a body from mean position at any instant is given by.

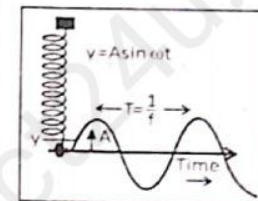
$$x = x_0 \cos(\omega t) \quad (1)$$

The waveform of equation (1) is shown in figure (a). If we determine the value of "x" at different instant and plot it against time, we get a curve as shown in figure (a). This graph gives the displacement as a function of time and is known as waveform of S.H.M.



Experiment for getting the Wave form of S.H.M:

Consider a body of mass "m" is attached to an elastic spring as shown in the figure. A sheet of paper is placed behind the mass in such a way that it can move horizontally with constant speed. The mass is provided with a pen which can touch the paper slightly.



When the spring is compressed through a displacement "y" and then released, the mass execute S.H.M having amplitude "A" as shown in the figure. During oscillation, the pen would mark the position of the mass "m" at each instant on the paper. In this way, the pen will trace out the waveform of S.H.M on the paper, as shown in the figure.

Q11: What is phase? Explain.

ETEA 2011

Ans. Phase:

Phase is the quantity which shows the state of motion of an oscillator. (OR)

The angle $\theta = \omega t$ which satisfies the displacement "x" as well as the direction of motion of a point executing S.H.M is called phase.

Explanation:

We know that the equation for instantaneous displacement of an oscillating body executing S.H.M is given by,

$$x = x_0 \cos \theta \quad (1)$$

In equation (1), "x" represents the instantaneous displacement, "x₀" represents the amplitude, "θ" represents the phase angle. Generally, we put,

$$\theta = \omega t + \phi \quad (2)$$

In equation (2), " ω " represents the angular velocity, " ϕ " represents an angle which gives us information about the initial phase of an oscillator. The quantity " ϕ " represents the phase difference between the states of motion of two oscillators. Now putting equation (2) in equation (1), we get,

$$x = x_0 \cos(\omega t + \phi) \quad (3)$$

Case-1: If we put $\phi = 0$, then equation (3) becomes,

$$x = x_0 \cos \omega t \quad (4)$$

Now for different values of " t " we get a graph between " x " and " t " as shown in figure (a).

Case-2: If $\phi = 90^\circ$, then equation (3) can be written as,

$$x = x_0 \cos(\omega t + 90^\circ) \quad (5)$$

Now for different values of " t ", we get a graph between " x " and " t " as shown in figure (b).

Now comparing figure (a), (b), we see that the curve in figure (a) leads the curve in figure (b) by 90° [i.e. $\pi/2$].

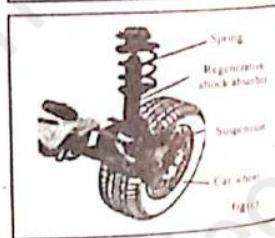
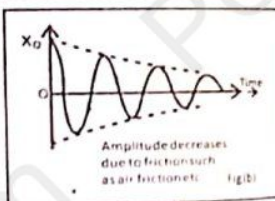
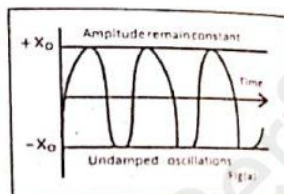
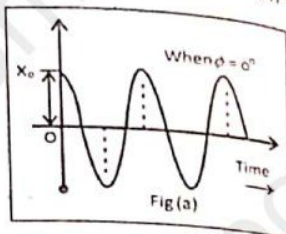
Case-3: If $\phi = 180^\circ$, then equation (3) becomes,

$$x = x_0 \cos(\omega t + 180^\circ) \quad (6)$$

Now for different values of " t ", we get a graph between " x " and " t " as shown in figure (c).

Now comparing the figures (a), (c), we see that the curve in figure (a) and curve in figure (c) are out of phase by 180° [i.e. π]. In other words, we can say that the phase difference between the two oscillations is 180° .

Thus the angle $\theta = \omega t$ which specifies the displacement " x " as well as the direction of motion of the point executing S.H.M is called phase.



Q12: Explain what is meant by damped oscillations?

BISE Abbottabad 2019

Ans. Definition: The oscillation in which the amplitude decreases steadily with time is known as damped oscillations. (OR)

A process in which energy is dissipated from the oscillating system is known as damped oscillation.

Explanation:

In case of mass-spring system, motion of a body along a circular path, motion of simple pendulum, we say that the total mechanical energy of the system remains constant. This is an ideal situation for which we ignore the frictional forces such as air friction.

In real situation, some part of mechanical energy of the oscillating system is used against the frictional forces. As a result, the amplitude of oscillation decreases steadily with time. Such oscillation is known as damped oscillations.

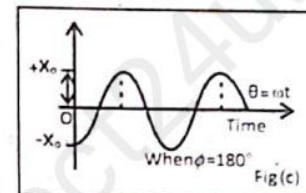
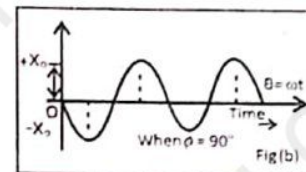
The undamped oscillation is shown in figure (a). The damped oscillation is shown in figure (b).

Applications of Damped Oscillation:

1. **Shock Absorber of Car:** The shock absorber of car is the application of damped oscillations which provides a damping force to prevent excessive oscillations as shown in figure (c). If the shock absorbers are worn badly, then the car becomes too bouncy and thus becomes uncomfortable.

2. **Shock Absorber System of Human Body:** Another application of damped oscillation is the shock absorber system of human body. Skier's body moves over the bumpy snow smoothly while his / her thighs and calves act like a damping spring. That is why skiing instructors always saying "bend zee knees".

3. The racing cars and hydrofoils can be fitted with active suspensions. These involve computer operated hydraulic systems in which bumps or waves are sensed and the suspension system is adjusted accordingly.



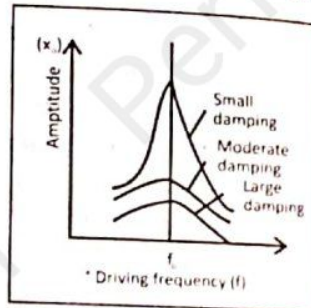
4. The computers are used to control the angle of the waves through the water to achieve remarkably stable movement of the hover craft.

Q13: Discuss the sharpness of resonance.

Ans: At resonance, the amplitude of vibrations becomes very large when damping is small. Thus the damping prevents the amplitude from becoming excessively large. The amplitude decreases rapidly at a frequency which is slightly different from the resonant frequency. The sharpness of the resonance curve of a resonating system depends on the frictional loss of energy. It can be explain with the help of following experiment:

Experiment:

Consider two bobs of simple pendulum that one is a pith - ball having lighter mass and the other is a lead bob having heavier mass. Let both are attached to a light, incompressible string to make two simple pendulums. These two pendulums are set into vibrations by a third pendulum of same length, attached to the same rod. It is observed that the amplitude of the lead bob is much greater than that of the pith - ball. It is because the damping effect for the lead bob due to air resistance is much smaller than that for the pith - ball. Thus, the sharpness of resonance curve of a resonating system depends on the frictional loss of energy.



④ زیادہ ہٹنے سے دل مر رہا ہو جاتا ہے۔ (حضور ﷺ)

④ انسان کا سب سے بڑا کارنامہ یہ ہے کہ وہ اپنے دل اور زبان کو قابو میں رکھے۔ (حضرت امام غزالی)

④ جو شخص زمین کا سفر کرتا ہے اس کے پاؤں میں ابلے پڑتے ہیں۔ جو آسمان کا سفر کرتا ہے اس کے دل میں ابلے پڑتے ہیں۔ (حضرت ابو الحسن خرقانی)

④ غنہ ہے جس کا دل اللہ کے سوا ہر چیز سے خالی ہو۔ (حضرت داتا گنج بخش)

Solution of Examples & Assignments

EXAMPLE 7.1:

A mass of 0.5kg is suspended from a spring. The spring is stretched by 0.098m. Calculate the spring constant of the mass when it is given a small displacement.

Solution:

Given data: Mass of body = $m = 0.5\text{kg}$

Force acting on body = $F = w = mg = 0.5 \times 9.8 = 4.9\text{N}$

Extension in the spring = $x = 0.098\text{m}$

Required: Spring constant = $k = ?$

Calculation:

We know that, $F = kx \Rightarrow k = \frac{F}{x}$ (1)

Putting the values in equation (1), we get,

$$k = \frac{4.9}{0.098} = 50\text{N/m} \Rightarrow \text{Spring constant} = k = 50\text{N/m}$$

ASSIGNMENT 7.1:

A 2.0kg block hung from a vertical spring causes it to stretch by 20.0cm. If the 2.0kg block is replaced by a 0.50kg mass, and the spring is stretched and released, what are the frequency and period of the oscillations?

Solution:

Given data:

Mass of block = $m_1 = 2.0\text{kg}$

Displacement of block = $x = 20.0\text{cm} = 0.20\text{m}$

$$\text{Spring constant} = k = \frac{F}{x} = \frac{w}{x} = \frac{m_1 g}{x} = \frac{2 \times 9.8}{0.20} \text{N/m} = 98\text{N/m}$$

$$F = w = mg$$

Mass of another block = $m_2 = 0.50\text{kg}$

Required:

1. Time period of oscillation = $T = ?$
2. Frequency of oscillation = $f = ?$

Calculation:

1. We know that,

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m_2}{k}} \Rightarrow T = 2\pi\sqrt{\frac{m_2}{k}} \quad (1)$$

Putting the values in equation (1), we get,

$$T = \left[2 \times 3.14 \times \sqrt{\frac{0.50}{98}} \right] \text{sec} = 0.0448 \text{sec} = 0.045 \text{sec}$$

$$\Rightarrow T = 0.045 \text{ sec}$$

2. We know that,

$$f = \frac{1}{T} \quad (2)$$

Putting the values of "T" in equation (2), we get,

$$f = \left(\frac{1}{0.45} \right) \text{Hz} = 2.2 \text{Hz} \Rightarrow f = 2.2 \text{Hz}$$

$$1. \text{ Time period of oscillation} = T = 0.45 \text{ sec}$$

$$2. \text{ Frequency of oscillation} = f = 2.2 \text{ Hz}$$

EXAMPLE 7.2:

A spider swings in the breeze from a silk thread with a period of 0.6s. How long is the spider's strand of silk?

Solution: Given data:

Time period of spider = $T = 0.60 \text{ sec}$

Acceleration due to gravity = $g = 9.8 \text{ m/sec}^2$

Required: Length of thread = $\ell = ?$

Calculation:

$$\text{We know that, } T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T^2 = 4\pi^2 \frac{\ell}{g} \Rightarrow \ell = \frac{gT^2}{4\pi^2} \quad (1)$$

Putting the values in equation (1), we get,

$$\ell = \left[\frac{9.8 \times (0.60)^2}{4 \times (3.14)^2} \right] \text{m} \Rightarrow \ell = 0.089293 \text{m}$$

$$\text{Length of thread} = \ell = 0.089293 \text{m}$$

ASSIGNMENT 7.2:

A pendulum extending from the ceiling almost touches the floor and that its period is 12s. How tall is the tower? ($\ell = 36 \text{m}$)

Solution:

Given data:

Time period of pendulum = $T = 12 \text{ sec}$

Acceleration due to gravity = $g = 9.8 \text{ m/sec}^2$

Required:

Length of tower = $\ell = ?$

Calculation:

$$\text{We know that, } T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow T^2 = 4\pi^2 \frac{\ell}{g} \Rightarrow \ell = \frac{gT^2}{4\pi^2} \quad (1)$$

Putting the values in equation (1), we get,

$$\ell = \left[\frac{9.8 \times (12)^2}{4 \times (3.14)^2} \right] \text{m} = 35.8 \text{ m} = 36 \text{m} \Rightarrow \ell = 36 \text{m}$$

As the pendulum almost touches the floor, so the length of pendulum will give us the length of tower.

$$\text{Length of tower} = \ell = 36 \text{m}$$

EXAMPLE 7.3:

A mass at the end of a spring vibrates up and down with a frequency of 0.600Hz and an amplitude of 5cm. What is its displacement 2.56 sec after it reaches a maximum?

Solution:

Given data:

Frequency = $f = 0.600 \text{ Hz}$,

Amplitude = $x_0 = 5 \text{ cm} = 0.05 \text{ m}$

Time interval = $t = 2.56 \text{ sec}$

Required:

Displacement of mass = $x = ?$

Calculation:

$$\text{We know that, } x = x_0 \cos(2\pi f t) \quad (1)$$

Putting the values in equation (1), we get,

$$\begin{aligned} x &= x_0 \cos(\omega t) \\ x &= x_0 \cos(2\pi f t) \\ f &= \frac{\omega}{2\pi} \\ \Rightarrow \omega &= 2\pi f \end{aligned}$$

$$x = [0.05 \times \cos(2 \times 3.14 \times 0.600 \times 2.56)] \text{ m}$$

$$= 0.05 \times 0.9858615948 \text{ m}$$

$$\Rightarrow x = 0.04929307974 \text{ m} = 0.0493 \text{ m approximately}$$

$$\Rightarrow x = 0.0493 \text{ m}$$

$$\Rightarrow x = 4.93 \text{ cm}$$

$$\text{Displacement of mass} = x = 4.93 \text{ cm}$$

ASSIGNMENT 7-3:

Determine the period and frequency of a car whose mass is 1400 kg and whose shock absorbers have a spring constant of $6.5 \times 10^4 \text{ N/m}$ after hitting a bump. Assume the shock absorbers are poor, so the car really oscillates up and down.

Solution:Given data:

Mass of car = $m = 1400 \text{ kg}$, Spring constant = $6.5 \times 10^4 \text{ N/m}$

Required:

1. Time period = $T = ?$ 2. Frequency = $f = ?$

Calculation:

1. We know that,

$$T = 2\pi \sqrt{\frac{m}{k}} \quad (1)$$

Putting the values in equation (1), we get,

$$T = 2 \times 3.14 \times \sqrt{\frac{1400}{6.5 \times 10^4}} = \left[2 \times 3.14 \times \sqrt{\frac{1400}{65000}} \right] \text{ sec}$$

$$\Rightarrow T = (6.28 \times 0.147) \text{ sec} = 0.9216520284 \text{ sec}$$

$$\Rightarrow T = 0.921652 \text{ sec}$$

2. We know that,

$$f = \frac{1}{T} \quad (2)$$

Putting the values of "T" in equation (2), we get,

$$f = \left(\frac{1}{0.921652} \right) \text{ Hz} = 1.085 \text{ Hz} \Rightarrow f = 1.085 \text{ Hz}$$

اللہ تعالیٰ کا ذکر ہر قسم کے ذکر کو ختم کر دیتا ہے۔

ASSIGNMENT 7-4:

An object vibrates with an amplitude of 6 cm and a frequency of 0.490 Hz. Starting from maximum displacement in the positive direction, when will be the first time that its displacement is 2 cm?

Solution:Given data:

Amplitude = $x_0 = 6 \text{ cm} = 0.06 \text{ m}$,

Frequency = $f = 0.490 \text{ Hz}$

Displacement = $x = 2 \text{ cm} = 0.02 \text{ m}$

Required:

Time interval = $t = ?$

Calculation:

We know that, $x = x_0 \cos(\omega t)$

$$\Rightarrow x = x_0 \cos(2\pi f t) \Rightarrow \cos(2\pi f t) = \frac{x}{x_0} \quad [\omega = 2\pi f]$$

$$\Rightarrow 2\pi f t = \cos^{-1} \left(\frac{x}{x_0} \right) \Rightarrow 2 \times 180 \times 0.490 \times t = \cos^{-1} \left(\frac{0.02}{0.06} \right)$$

$$\Rightarrow 176.4 t = 70.5 \Rightarrow t = \left(\frac{70.5}{176.4} \right) \text{ sec}$$

$$\Rightarrow t = 0.399 \text{ sec} = 0.400 \text{ sec}$$

$$\Rightarrow t = 0.400 \text{ sec}$$

$$\pi \text{ radian} = 180^\circ$$

- ⊕ علم آسمانی بھی بند ہی ہے ہوا اللہ کے پاس انصاف ضرور ہوتا ہے۔
- ⊕ بے شک زندگی تھکا دینے والی چیز ہے اور جب تم تھک جاؤ تو اپنے رب کا ذکر کیا کرو کیونکہ سکون تو صرف اللہ کے ذکر میں ہے۔
- ⊕ قابلیت اتنی بڑھاؤ کہ تمہیں ہر آنے کے لیے کوششیں نہیں سارا شیں کرنی پڑیں۔
- ⊕ خوش رہنا چاہیے ہو تو دوسروں کو خوش کرو۔ (حضرت غوث اعظم)
- ⊕ بعض احمق اس غم میں گھٹتے ہیں کہ اللہ تعالیٰ نے فلاں کو دولت کیوں دی۔ (امام غزالی)
- ⊕ تم خوشی چاہتے ہو اور اسے نہیں پاتے یہ جنت میں ہو گی۔ (حضرت ابو تراب بخاری)

Solution of Quiz

QUIZ 7-1:

A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is measured (1) If the elevator moves with constant velocity, does the period (a) increase, (b) decrease, or (c) remain the same. (2) If the elevator accelerates upward, does the period (a) increase, (b) decrease, or (c) remain the same?

Ans: Part-1: We know that,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

When the elevator is moving with constant velocity, its acceleration "a" is zero. So there will be no change in value of "g". So according to equation (1), when "g" remain the same, time period will remain the same. Thus in part (1) the option (c) is correct i.e. "T" remain the same.

Part-2: When the elevator is moving upward with acceleration "a" then value of acceleration due to gravity becomes "g + a". So equation (1) can be written as,

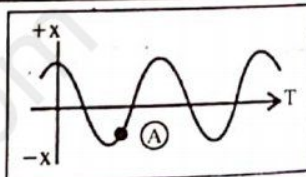
$$T = 2\pi\sqrt{\frac{l}{(g+a)}} \quad (2)$$

Equation (2) shows that "g + a" increases, so time period will decrease. Thus option (b) is correct i.e. "T" decreases.

QUIZ 7-2:

Consider a graphical representation of simple harmonic motion as described mathematically in equation 7-21 (ii).

When the particle is at point A on the graph, what can you say about its position and velocity? (a) The position and velocity are both positive. (b) The position and velocity are both negative. (c) The position is positive, and the velocity is zero. (d) The position is negative, and the velocity is zero.



ro. (e) The position is positive, and the velocity is negative. (f) The position is negative, and the velocity is positive.

Ans: 1. Conclusion about Position of Point "A":

It is clear from the figure that point "A" is in negative x-direction. Therefore, the position of point "A" is negative.

Conclusion about Velocity of Point "A":

Now the velocity of the particle is equal to the slope of position time graph at any point. So we have,

$$v = \frac{\Delta x}{\Delta t} \quad (1)$$

In equation (1), " Δx " represents the change in position and " Δt " represents the time interval. Now at point "A", the slope of the graph is positive. Therefore, the velocity at point "A" will be also positive. Thus option (f) is correct i.e. the position is negative and velocity is positive.

Exercise

Choose the best possible answer:

- Tuning of a radio set is an example of:

a. Mechanical resonance	b. Musical resonance
✓ c. Electrical resonance	d. Free vibrations
- The heating and cooking of food evenly by microwave oven is an example of:

a. S.H.M.	✓ b. Resonance
c. Damped oscillation	d. Free oscillation
- The time period of the same pendulum at Karachi and Murree are related as:

a. $T_k = T_m$	b. $T_k > T_m$
✓ c. $T_k < T_m$	d. $2T_k = 3T_m$

Sol: The time period of a simple pendulum is given by,

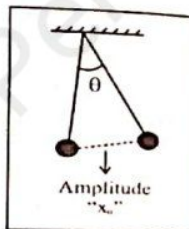
$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

Equation (1) shows that greater the value of "g", smaller will be the time period and vice versa. Now we know that value of "g" at

Karachi will be greater than the value of "g" at Murree. It is because the value of "g" decreases with altitude. Now according to equation (1), if "g" is greater than "T" will be smaller. Thus value of time period pendulum at Karachi will be smaller due to greater value of "g". While value of time period of pendulum at Murree will be greater due to smaller value of "g". Thus

$$T_k < T_m \text{ or } T_m > T_k$$

4. In an isolated system the total energy of vibrating mass and spring is:
 a. Variable b. Low c. High ✓ d. Constant
5. While deriving the equation of time period for simple pendulum which quantity should be kept small:
 a. Length of simple pendulum
 ✓ b. Amplitude
 c. Mass of simple pendulum
 d. Gravitational acceleration g



Sol: When we are deriving equation for time period of simple pendulum, we put $\sin \theta \approx \theta$ i.e. we keep "θ" very very small. Now for keep "θ" to be small, we should keep the amplitude to be small as clear from the given figure.

6. If the period of oscillation of mass (M) suspended from a spring is 2s, then the period of mass 4M will be:
 a. 1s b. 2s c. 3s ✓ d. 4s

Sol: We know that

$$T = 2\pi\sqrt{\frac{M}{k}} \quad (1)$$

For mass "M" the time period is 2sec, so we have,

$$T = 2\text{sec}$$

Now for mass "4M", equation (1) can be written as,

$$T' = 2\pi\sqrt{\frac{4M}{k}} = 2\pi\sqrt{4}\sqrt{\frac{M}{k}} = \sqrt{4} \cdot 2\pi\sqrt{\frac{M}{k}}$$

$$\Rightarrow T' = 2 \left(2\pi\sqrt{\frac{M}{k}} \right) \quad (2)$$

Putting equation (1) in equation (2), we get,

$$T' = 2T \quad (3)$$

Putting $T = 2\text{sec}$ in equation (3), we get,

$$T' = 2 \times 2\text{sec} \Rightarrow T' = 4\text{sec}$$

7. The time period of a simple pendulum is 2 seconds. If its length is increased by 4 times, then its period becomes:
 a. 16s b. 12s c. 8s ✓ d. 4s

Sol: We know that,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

For length 'l', we have given that, $T = 2\text{sec}$

Now length becomes four times, then we put

$l' = 4l$, so equation (1) becomes

$$T' = 2\pi\sqrt{\frac{l'}{g}} \Rightarrow T' = 2\pi\sqrt{\frac{4l}{g}} = 2 \left(2\pi\sqrt{\frac{l}{g}} \right) \quad (2)$$

Putting equation (1) in equation (2), we get,

$$\Rightarrow T' = 2T \quad (3)$$

Putting $T = 2\text{sec}$ in equation (3), we get,

$$T' = 2 \times 2\text{sec} \Rightarrow T' = 4\text{sec}$$

8. To make the frequency double of a spring oscillation, we have to:

- ✓ a. Reduce the mass to one fourth
 b. Quadruple the mass
 c. Double the mass d. Half the mass

Sol: We know that, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (1)$

Now when frequency is doubled, then we have,

$$f' = 2f \quad (2)$$

For "f", we represent the mass by "m", so we get,

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{m'}} \quad (3)$$

Putting equation (1) and equation (3) in equation (2), we get,

$$\frac{1}{2\pi}\sqrt{\frac{k}{m'}} = 2 \times \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{1}{m'}} = 2 \times \sqrt{\frac{1}{m}} \quad (4)$$

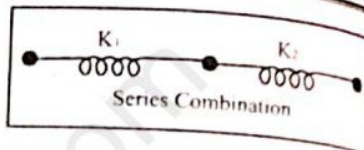
Squaring both sides, we get, $\frac{1}{m'} = \frac{4}{m} \Rightarrow m' = \frac{m}{4}$

9. The restoring force of S.H.M is maximum when particle:

- ✓ a. Displacement is maximum b. Half way between them
 c. Crossing mean position d. At rest

10. Two springs of spring constants k_1 and k_2 are joined in series. The effective spring constant of the combination is given by,

- a. $(k_1 + k_2) / 2$ b. $k_1 + k_2$
 ✓ c. $k_1 k_2 / (k_1 + k_2)$ d. $\sqrt{k_1 k_2}$



Sol: When two springs are connected in series then the effective spring constant 'K' is given by,

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} \Rightarrow \frac{1}{K} = \frac{K_2 + K_1}{K_1 K_2} \Rightarrow K = \frac{K_1 K_2}{K_1 + K_2}$$

Short Questions

Q1: Give two applications in which resonance plays an important role.

Ans: 1. Microwave Oven: BISE Mardan 2017

The microwave oven is an important application of resonance. In microwave oven, typical microwave with a wavelength 12cm frequency of 2450 MHz is produced. Such frequency of microwave is similar to the natural frequency of vibrations of water molecules and fat molecules in the food i.e.



Microwave oven

Frequency of microwaves = Frequency of water molecules & fat molecules

Now when food is placed in the oven, and the set is ON then the water molecules and fat molecules in the food resonates under the action of periodic force of microwaves. As a result, these molecules absorb energy of microwaves due to resonance. The energy of microwaves heats up the food item. In this way, the food items are heated or cooked in the microwave oven due to resonance.

2. Radio Receiver:

Tuning of radio is another important application of resonance. When we turn the knob of a radio to tune a station, we are changing the frequency of the electrical circuit of the receiver, to make it equal

to the transmission frequency of radio station. When the two frequencies match each other then resonance occur. Under such conditions maximum absorption of energy takes place and thus we hear the concern station very clearly due to resonance.



Q2: What happens to the time period of a simple pendulum if its length is doubled?

Ans: The time period of a simple pendulum will increase by a factor $\sqrt{2}$ -times if its length is doubled.

Reason and Explanation:

We know that the time period of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

Now when the length is doubled, then we have,

$$l' = 2l \quad (2)$$

Now for " l' ", equation (1) can be written as,

$$T' = 2\pi\sqrt{\frac{l'}{g}} \quad (3)$$

Putting equation (2) in equation (3), we get,

$$T' = 2\pi\sqrt{\frac{2l}{g}} \Rightarrow T' = \sqrt{2} \left(2\pi\sqrt{\frac{l}{g}} \right) \quad (4)$$

Putting equation (1) in equation (4), we get,

$$T' = \sqrt{2}T \quad (5)$$

Equation (5) shows that the time of simple pendulum will increase by a factor $\sqrt{2}$ - times if its length is doubled.

Q3: What will be the frequency of a simple pendulum if its length is 1m?

Ans: The frequency of a simple pendulum will be 0.5 Hz if its length is 1m.

Reason and Explanation: We know that the frequency of simple pendulum is given by,

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \quad (1)$$

Now we put, $l = 1\text{m}$, $g = 9.8\text{m/sec}^2$ in equation (1), we get,

$$f = \frac{1}{2 \times 3.14} \sqrt{\frac{9.8}{1}} \Rightarrow f = \frac{3.13}{6.28} \Rightarrow f = 0.498\text{Hz}$$

$$\Rightarrow \boxed{f = 0.5\text{Hz}}$$

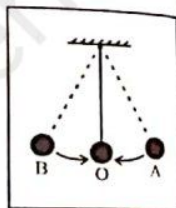
Conclusion: The frequency of a simple pendulum will be 0.5 Hz if its length is 1m.

Q4: Give one practical example each of free and forced oscillation.

Ans. Free Oscillation:

When a system oscillates with its own natural frequency and no net force acts on it, then such oscillation of the system is known as free oscillation.

Example 1: When a simple pendulum is displaced from its mean position and then left free, it executes free oscillations, in the absence of external or resistive forces. For the oscillation, the time period of simple pendulum is given by,



$$T_0 = 2\pi\sqrt{\frac{l}{g}}$$

While the frequency of simple pendulum is given by,

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

Example 2: Oscillation of a swing in the absence of external force is another example of free oscillation.

Forced Oscillation:

When a system oscillates under the influence of a net external force, then such oscillation is known as forced oscillation.

Example 1: If the mass of a vibrating pendulum is struck again and again then such oscillations of the pendulum is called forced oscillations.

Example 2: The vibrations of a factory floor caused by the running of heavy machinery are another example of forced oscillations.

Example 3: Another example is a child's swing that is pushed on each downswing.

Q5: How can you compare the masses of two bodies by observing their frequencies of oscillation when supported by a spring?

Ans. Consider a mass-spring system as shown in figure (a). The frequency of mass-spring system is given by,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{--- (1)}$$

Now consider two bodies of mass " m_1 " and " m_2 " attached to an elastic spring as shown in figure (b) and figure (c).

Case-1: For " m_1 " equation (1) can be written as,

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}} \quad \text{--- (2)}$$

Case-2: For " m_2 " equation (1) can be written as,

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m_2}} \quad \text{--- (3)}$$

Now dividing equation (2) by equation (3), we get,

$$\frac{f_1}{f_2} = \frac{\frac{1}{2\pi}\sqrt{\frac{k}{m_1}}}{\frac{1}{2\pi}\sqrt{\frac{k}{m_2}}} = \frac{\sqrt{\frac{k}{m_1}}}{\sqrt{\frac{k}{m_2}}} = \sqrt{\frac{m_2}{m_1}}$$

$$\Rightarrow \frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}} \Rightarrow \frac{f_1^2}{f_2^2} = \frac{m_2}{m_1} \Rightarrow \frac{m_1}{m_2} = \frac{f_2^2}{f_1^2} \quad \text{--- (4)}$$

Squaring both sides of equation (4), we get,

$$\frac{f_1^2}{f_2^2} = \frac{m_2}{m_1} \Rightarrow \frac{m_2}{m_1} = \frac{f_1^2}{f_2^2} \Rightarrow \boxed{\frac{m_1}{m_2} = \frac{f_2^2}{f_1^2}} \quad \text{--- (5)}$$

Using equation (3), we can compare the masses of two bodies by observing their frequencies of oscillation when supported by a spring.

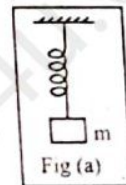


Fig (a)

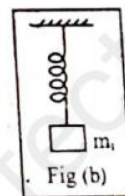


Fig (b)

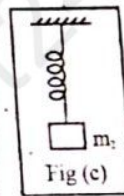


Fig (c)

Q6: A wire hangs from the top of a dark high tower, so that the top of the tower is not visible. How you would be able to determine the height of that tower?

Ans. We can determine the height of the tower by using the relation of simple pendulum. For this purpose we connect a bob to the lower end of the wire so that the arrangement becomes as a simple pendulum.

Reason and Explanation:

Consider a wire of length "l" is hanging from top of the tower as shown in the figure (a). Now for finding the height of the tower, we attach a bob to the lower end of the wire so that the arrangement behaves like a simple pendulum, as shown in figure (b).

The length of this pendulum will give us the length or height of the tower. Now the time period of the simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

Squaring both sides of equation (1), we get,

$$T^2 = 4\pi^2 \frac{l}{g} \Rightarrow l = \frac{gT^2}{4\pi^2} \quad (2)$$

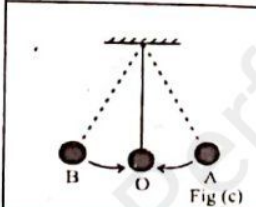
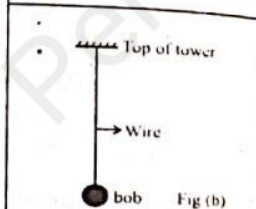
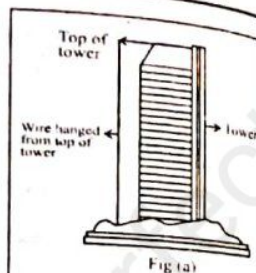
Now make the pendulum to vibrate about mean position "O" as shown in figure (c). Note the time for one vibration with the help of stopwatch. This will give us the time period "T" of the pendulum. Now by putting the values of "g", "π" and "T" in equation (2), we can calculate the length of the pendulum. Such length of the pendulum will represent the height of the tower.

Q7: Why in S.H.M the acceleration is zero and the velocity is greatest?

Ans. In S.H.M when velocity is greatest, the acceleration is zero.

Reason and Explanation:

The velocity of a simple harmonic oscillator is given by,



$$V = \omega\sqrt{x_0^2 - x^2} \quad (1)$$

While the acceleration is given by,

$$a = -\omega^2 x \quad (2)$$

Now when the body is displaced to extreme position "x₀" and then released, the body executes S.H.M and its velocity becomes maximum at its mean position. For maximum displacement, we consider "x₀" and ignore "x" i.e. we put x = 0, so equation (1) becomes,

$$V_{max} = \omega\sqrt{x_0^2 - 0^2} = \omega\sqrt{x_0^2} \Rightarrow V_{max} = \omega x_0 \quad (3)$$

Now put x = 0 in equation (2), we get,

$$a = -\omega^2 x = 0 \Rightarrow a = 0 \quad (4)$$

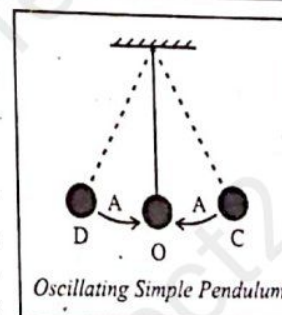
Equation (3) and equation (4) shows that in S.H.M, the acceleration is zero when the velocity is greatest.

Q8: What is the total distance covered by a simple harmonic oscillator in a time equal to its period? The amplitude of oscillation is A?

Ans. The total distance travelled by a simple harmonic oscillator in a time equal to its time period is equal to "4A", where "A" is its amplitude.

Explanation:

In this question, the total distance in a time equal to period means the distance covered during one complete vibration. Consider the oscillation of simple pendulum as shown in the figure.



When the pendulum moves from mean position "O" towards extreme position "C" and then moves back from "C" towards "O", it completes half vibration. During this half vibration, the pendulum covers a distance "A+A=2A". Similarly, when the pendulum moves from "O" towards "D" and then from "D" towards "O", it completes half vibration. During this half vibration, the pendulum covers a distance "A+A=2A". So during one vibration, the time taken by the oscillator is equal to its time period and the total distance covered during one vibration is equal to "2A+2A=4A".

Q9: What happens to the frequency of a simple pendulum as its oscillations die down from large amplitude to small one?

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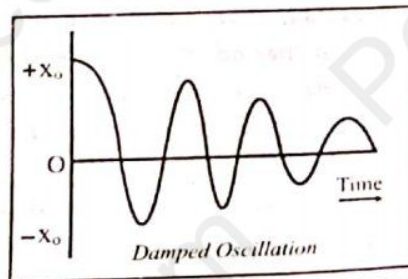
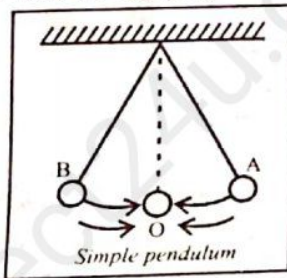
Ans. When the amplitude of a simple pendulum dies down from large amplitude to small, it behaves as a damped oscillator. Damping not only reduces the amplitude of the oscillator but also reduces the frequency of the oscillation while increasing the period of oscillation.

Explanation:

If we set a simple pendulum in a laboratory and put it into oscillations. Its amplitude decreases with time and finally comes to rest due to air friction. Since, the amplitude of the pendulum decreases as a result the energy of the oscillating system decreases due to work against the air resistive force. This energy is dissipated in the particles of the surrounding medium. Damping not only has effect of reducing the amplitude but also the effect of reducing the frequency or increasing the time period of oscillation. We know that,

$$f = \frac{1}{T} \quad (1)$$

Equation (1) shows that frequency decreases when time period increases due to damping.



Q10: A singer holding a note of right frequency, can shatter a glass? Explain.

Ans. A singer holding a note of right frequency can shatter a glass.

Reason and Explanation:

It is possible for a singer to shatter a glass, because every solid body has certain natural frequency of vibration depending upon its elasticity, mass, shape and size. A singer sings a note of particular

frequency ' f_s ' and when this frequency of the note becomes equal to the natural frequency ' f_o ' of the air in glass i.e.

$$f_s = f_o \quad (1)$$

So resonance takes place. Therefore, the air in glass will also begin to vibrate with large amplitude due to resonance. This large amplitude vibration may shatter the glass.

Comprehensive Questions

Q1: Show that motion of a mass attached with a spring executes S.H.M.

Ans. See answer of question no 3 in the text.

Q2: Prove that the projection of a body motion in a circle describes S.H.M.

Ans. See answer of question no 4 in the text.

Q3: Show that energy is conserved in case of S.H.M.

Ans. See answer of question no 7 in the text.

Q4: Differentiate free and forced oscillations.

Ans. See answer of question no 8 in the text.

Q5: What is resonance give three of its applications in our daily life?

Ans. See answer of question no 9 in the text.

Q6: Derive equations for kinetic and potential energy of a body of mass " m " executing S.H.M.

Ans. See answer of question no 7 in the text.

Q7: Explain what is mean by damped oscillations?

Ans. See answer of question no 12 in the text.

فرمانِ مصطفیٰ ﷺ: مسلمان کا صدقہ عمر میں زیادتی کا سبب ہے اور بڑی موت کو دفع کرتا ہے۔

کسی شخص کا آپ پر اعتماد کرنا آپ کے لیے باعثِ فخر ہے اور آپ پر جب کوئی اعتماد کرے تو کوشش کریں کہ اس کا مان نہ لیں۔

Numerical Problems

Q1: A force of 0.4N is required to displace a body attached to a spring through 0.1m from its mean position. Calculate the spring constant of the spring.

Solution:

Given data:

Force = $F = 0.4\text{N}$, Displacement = $x = 0.1\text{m}$

Required:

Spring constant = $k = ?$

Calculation:

We know that,

$$F = kx \Rightarrow k = \frac{F}{x} \quad (1)$$

Putting the values in equation (1), we get,

$$k = \frac{0.4}{0.1} \text{N/m} = 4\text{N/m} \Rightarrow \text{Spring constant} = 4\text{N/m}$$

Q2: A pendulum clock keeps perfect time at a location where the acceleration due to gravity is exactly 9.8ms^{-2} . When the clock is moved to a higher altitude, it loses 80.0s per day. Find the value of ' g ' at this new location. (9.78m/s^2)

Solution: **Given data:** Acceleration due to gravit $g = 9.8\text{m/sec}^2$

Loss in time per day = 80.0sec,

$$\text{Loss in time per second} = \frac{80}{86400}$$

$$1\text{day} = 86400\text{sec}$$

Loss in time per second = 0.000926sec

The loss in time shows that swinging of clock becomes slower. Thus if the time period " T " at the location where $g = 9.8\text{m/sec}^2$ is 1sec, then the time period " T " at new altitude will be equal to $(1+0.000926)\text{sec}$ i.e. $T' = 1.000926\text{sec}$

Required:

Acceleration due to gravity at new altitude = $g' = ?$

Calculation:

We know that, the time period of pendulum is given by,

$$T = 2\pi\sqrt{\frac{l}{g}} \quad (1)$$

For new time period " T' ", equation (1) can be written as,

$$T' = 2\pi\sqrt{\frac{l}{g'}} \quad (2)$$

Dividing equation (1) by equation (2), we get,

$$\frac{T}{T'} = \frac{2\pi\sqrt{\frac{l}{g}}}{2\pi\sqrt{\frac{l}{g'}}} = \frac{\sqrt{\frac{l}{g}}}{\sqrt{\frac{l}{g'}}} \Rightarrow \frac{T}{T'} = \sqrt{\frac{l}{g}} \div \sqrt{\frac{l}{g'}} = \sqrt{\frac{l}{g}} \times \sqrt{\frac{g'}{l}} = \sqrt{\frac{g'}{g}}$$

$$\Rightarrow \frac{T}{T'} = \sqrt{\frac{g'}{g}} \quad (3)$$

Squaring both sides of equation (3), we get,

$$\frac{T^2}{T'^2} = \frac{g'}{g} \Rightarrow g' = \frac{gT^2}{T'^2} \quad (4)$$

Putting the values of " g ", " T " and " T' " in equation (4), we get,

$$g' = \frac{9.8 \times 1^2}{(1.000926)^2} \Rightarrow g = 9.78\text{m/sec}^2 \quad g' = 9.78\text{m/sec}^2$$

Q3: Calculate the length of a second pendulum having time period 2sec at a place where $g = 9.8\text{m/sec}^2$.

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Solution: **Given data:**

Time period = $T = 2\text{sec}$, Acceleration due to gravity = $g = 9.8\text{m/sec}^2$

Required:

Length of second pendulum = $l = ?$

Calculation:

$$\text{We know that, } T = 2\pi\sqrt{\frac{l}{g}}$$

Squaring both sides, we get,

$$\Rightarrow T^2 = 4\pi^2 \left(\frac{l}{g} \right) \Rightarrow l = \frac{gT^2}{4\pi^2} \quad (1)$$

$$\Rightarrow l = \frac{9.8 \times (2)^2}{4 \times (3.14)^2} \Rightarrow l = 0.992\text{m}$$

$$\Rightarrow l = 0.992\text{m}$$

Q4: A body of mass "m" suspended from a spring with force constant "k" vibrates with " f_1 ". When its length is cut into half and the same body is suspended from one of the halves, the frequency is " f_2 ". Find out f_1/f_2 .

Solution:

Given data:

Mass attached = m, spring constant = k

Frequency of whole spring = f_1 , frequency of half spring = f_2

Displacement for whole spring = x_1

Displacement for half spring = $x_2 = \frac{x_1}{2}$

Required:

Relation of $\frac{f_1}{f_2} = ?$

Calculation:

For mass-spring system, we have,

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (1)$$

We know that, $F = kx$

$$\Rightarrow w = kx \Rightarrow mg = kx$$

$$\Rightarrow k = \frac{mg}{x} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$f = \frac{1}{2\pi} \sqrt{\frac{mg}{mx}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{g}{x}} \quad (3)$$

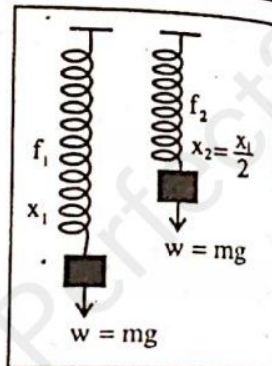
Now for 1st spring, equation (3) can be written as,

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{g}{x_1}} \quad (4)$$

While for 2nd spring, equation (3) can be written as,

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{g}{x_2}}$$

$$\Rightarrow f_2 = \frac{1}{2\pi} \sqrt{\frac{2g}{x_1}} \Rightarrow f_2 = \sqrt{2} \left(\frac{1}{2\pi} \sqrt{\frac{g}{x_1}} \right) \quad (5)$$



F = weight of suspended mass = $w = mg$

Putting equation (4) in equation (5), we get,

$$f_2 = \sqrt{2} f_1 \Rightarrow f_1 = \frac{1}{\sqrt{2}} f_2 \Rightarrow \frac{f_1}{f_2} = \frac{1}{\sqrt{2}} = 0.707$$

$$\Rightarrow \frac{f_1}{f_2} = 0.707$$

Q5: A mass at the end of spring describes S.H.M with $T = 0.40 \text{ sec}$. Find out acceleration when the displacement is 0.04 m .

Solution:

Given data:

Time period = $T = 0.40 \text{ sec}$, Displacement = $x = 0.04 \text{ m}$

Required:

Acceleration = $a = ?$

Calculation:

We know that, $a = -\omega^2 x$ (1)

$$\text{Also } T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{T} \quad (2)$$

Putting equation (2) in equation (1), we get,

$$a = -\left(\frac{2\pi}{T}\right)^2 x \Rightarrow a = -\frac{4\pi^2 x}{T^2} \quad (3)$$

Putting the values in equation (3), we get,

$$a = \frac{-4 \times 3.14^2 \times 0.04}{(0.40)^2} = -9.86 \text{ m/sec}^2$$

$$\text{Acceleration} = a = -9.86 \text{ m/sec}^2$$

Q6: A block weighing 4 kg extends a spring by 0.16 m from its unstretched position. The block is removed and a 0.50 kg body is hung from same spring. If the spring is now stretched and released, what is its period of vibrations?

Solution:

Given data:

(i) Mass of block = $m_1 = 4 \text{ kg}$, Displacement = $x = 0.16 \text{ m}$

(ii) Mass of body = $m_2 = 0.50 \text{ kg}$

Required: Time period = $T = ?$

Calculation:

First we calculate the spring constant, we know that

$$F = kx \Rightarrow w = kx$$

$$F = w = mg$$

$$\Rightarrow m_1 g = kx \Rightarrow k = \frac{m_1 g}{x} \quad (1)$$

Putting the values in equation (1), we get,

$$k = \frac{4 \times 9.8}{0.16} \Rightarrow k = 245 \text{ N/m}$$

Now we know that,

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m_2}{k}} \Rightarrow T = 2\pi \sqrt{\frac{m_2}{k}} \quad (2)$$

Putting the values in equation (2), we get,

$$T = 2 \times 3.14 \sqrt{\frac{0.50}{245}} = 0.28 \text{ sec} \quad \boxed{T = 0.28 \text{ sec}}$$

Q7: What should be the length of simple pendulum whose time period is one second? What is its frequency?

Solution:

Given data:

Time period = $T = 1 \text{ sec}$

Required:

(a) Length of pendulum = $l = ?$

(b) Frequency of simple pendulum = $f = ?$

Calculation:

(a) We know that,

$$T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2 \frac{l}{g} \Rightarrow l = \frac{gT^2}{4\pi^2} \quad (1)$$

Putting the values in equation (1), we get,

$$l = \frac{9.8 \times (1)^2}{4 \times (3.14)^2} = 0.25 \text{ m} \Rightarrow l = 0.25 \text{ m}$$

(b) We also know that,

$$f = \frac{1}{T} \quad (2)$$

Putting the values in equation (2), we get,

$$f = \frac{1}{1} \text{ cycles/sec} \Rightarrow f = 1 \text{ c.p.s}$$

$$\boxed{l = 0.25 \text{ m}} \quad \text{or} \quad \boxed{f = 1 \text{ c.p.s}}$$

Q8: A spring whose spring constant is 80N/m vertically supports a mass of 1kg is the rest position. Find the distance by which the mass must be pulled down, so that on being released, it may pass the mean position with velocity of one meter per second?

Solution:

Given data: Spring constant = $k = 80 \text{ N/m}$

Mass = $m = 1 \text{ kg}$, Maximum velocity = $V_{\max} = 1 \text{ m/sec}$

Required:

Amplitude = $x_0 = ?$

Calculation:

We know that, $V_{\max} = \omega x_0$

$$\Rightarrow V_{\max} = \sqrt{\frac{k}{m}} x_0 \quad (1)$$

Putting the values in equation (1), we get,

$$1 = \sqrt{\frac{80}{1}} x_0 \Rightarrow 1 = 8.944 x_0$$

$$\Rightarrow x_0 = \frac{1}{8.944} \Rightarrow \boxed{x_0 = 0.11 \text{ m}}$$

$$\Rightarrow \boxed{x_0 = 0.11 \text{ m}}$$

$$\begin{aligned} a &= \omega^2 x \\ a &= \frac{k}{m} x \\ \Rightarrow \omega^2 &= \frac{k}{m} \\ \omega &= \sqrt{\frac{k}{m}} \end{aligned}$$

Q9: A 8000gm body vibrates S.H.M with amplitude 0.30m. The restoring force is 60N and the displacement is 0.30m. Find out, (i) time period (ii) acceleration (iii) velocity (iv) K.E (v) P.E when the displacement is 12cm.

Solution:

Given data:

Mass of body = $m = 8000 \text{ gm} = 8.0 \text{ kg}$

Amplitude = $x_0 = 0.3 \text{ m}$, Restoring force = $F = 60 \text{ N}$

Displacement = $x = 12 \text{ cm} = 0.12 \text{ m}$

Required:

- (i) Time period = $T = ?$
 (ii) Acceleration = $a = ?$
 (iii) Velocity = $v = ?$
 (iv) K.E = ? (v) P.E = ?

Calculation:

(i) As $T = 2\pi\sqrt{\frac{m}{k}}$

$\Rightarrow T = 2\pi\sqrt{\frac{mx_0}{F}} \quad \text{--- (1)}$

Putting the values in equation (1), we get,

$T = 2 \times 3.14 \sqrt{\frac{3.0 \times 0.3}{60}} \Rightarrow T = 1.3 \text{ sec}$

(ii) As $a = \omega^2 x \Rightarrow a = \frac{k}{m} \cdot x \quad \text{--- (2)}$

Putting the values in equation (2), we get,

$a = \frac{200}{2} \times 0.12 \Rightarrow a = 3 \text{ m/sec}^2$

(iii) As $v = \omega \sqrt{x_0^2 - x^2} \Rightarrow v = \sqrt{\frac{k}{m}} \sqrt{x_0^2 - x^2} \quad \text{--- (3)}$

Putting the values in equation (3), we get,

$v = \sqrt{\frac{200}{8}} \times \sqrt{0.3^2 - 0.12^2}$

$\Rightarrow v = (5 \times 0.274) \text{ m/sec} \Rightarrow v = 1.4 \text{ m/sec}$

(iv) We know that,

$K.E = \frac{1}{2} k(x_0^2 - x^2) \quad \text{--- (4)}$

Putting the values in equation (4), we get,

$K.E = \frac{1}{2} \times 200(0.3^2 - 0.12^2) = 100(0.0756) \text{ joules}$

$\Rightarrow K.E = 7.6 \text{ J}$

(v) $P.E = \frac{1}{2} kx^2 \quad \text{--- (5)}$

Putting the values in equation (5), we get,

$P.E = \frac{1}{2} \times 200 \times (0.12)^2 \Rightarrow P.E = 1.44 \text{ J}$

$F = kx_0$

$\Rightarrow k = \frac{F}{x_0}$

Also $\omega = \sqrt{\frac{k}{m}}$

$\omega^2 = \frac{k}{m}$

$F = 60 \text{ N}$

$x_0 = 0.3$

$k = \frac{F}{x_0}$

$k = \frac{60}{0.3} = 200 \frac{\text{N}}{\text{m}}$

Q10: Find the amplitude, frequency and time period of an object oscillating at the end of a spring, if the equation for its position at any instant "t" is given by, $x = 0.25 \cos(\frac{\pi}{8})t$. Find the displacement of the object after 2.0 seconds.

Solution:Given data:

The given equation is,

$x = 0.25 \cos(\frac{\pi}{8})t \quad \text{--- (1)}$

Required:

- (i) Amplitude = $x_0 = ?$
 (ii) Frequency = $f = ?$
 (iii) Time period = $T = ?$
 (iv) Displacement after 2sec = ?

Calculation:

The instantaneous displacement in S.H.M is given by,

$x = x_0 \cos \omega t \quad \text{--- (2)}$

Comparing equation (1), (2), we get,

$x_0 = 0.25 \text{ m}, \omega = \frac{\pi}{8}$

(i) Amplitude = $x_0 = 0.25 \text{ m}$

(ii) We know that,

$f = \frac{\omega}{2\pi} \quad \text{--- (3)}$

Putting the values in equation (3), we get,

$f = \frac{\pi}{8 \times 2\pi} = \left(\frac{1}{16}\right) \text{ Hz}$

$\Rightarrow f = \frac{1}{16} \text{ Hz}$

(iii) We know that,

$T = \frac{1}{f} \quad \text{--- (4)}$

Putting the values in equation (4), we get,

$$T = \frac{1}{(1/16)} = 16 \text{ sec}$$

$$\Rightarrow T = 16 \text{ sec}$$

(iv) Put $t = 2 \text{ sec}$ in equation (1), we get,

$$x = 0.25 \times \cos\left(\frac{\pi \times 2}{8}\right) = 0.25 \times \cos\left(\frac{\pi}{4}\right)$$

$$\Rightarrow x = 0.25 \times \cos\left(\frac{180^\circ}{4}\right) = 0.25 \times \cos(45^\circ)$$

$$\pi \text{ rad} = 180^\circ$$

$$\Rightarrow x = 0.25 \times 0.707 \text{ m}$$

$$\Rightarrow x = 0.17 \text{ m}$$

+++

④ ہم کسی شخص کی طاقت سے زیادہ اس پر بوجھ نہیں ڈالتے، اگر تم اللہ پر ایمان رکھتے ہو تو شرط فرماؤ کہ یہ ہے کہ اس پر بھروسہ رکھو۔ (قرآن خداوندی)

⑤ رسول اللہ ﷺ نے فرمایا: "دنیا سے بے رشتی دل اور روح کو حقیقی راحت دیتی ہے اور دنیا کی رغبت و محبت زندگی میں رنج و الم کو بڑھاتی ہے۔"

⑥ رسول اللہ ﷺ نے فرمایا: "جب تم تین آدمی ساتھ ہو تو تم میں سے دو آدمی تیسرے کو اکیلا چھوڑ کر کان میں بات اور سرگوشی نہ کریں، کیونکہ یہ اسے رنج و غم میں مبتلا کر دے گا۔" (سنن ابوداؤد / اسلم حج)

⑦ رسول اللہ ﷺ نے فرمایا: "یاد رکھو! اللہ تعالیٰ کی مدد و نصرت میرے ساتھ ہی ہے اور ہر تکلیف و مصیبت کے بعد کشادگی ہے اور بلاشبہ ہر تنگی و مشکل کے بعد آسانی ہے۔" (مسند احمد: ۲۸۰۳، عن حضرت ابن عباس)

⑧ ایمان و نصف ہیں، نصف میر اور نصف لکڑی (ارشاد نبوی ﷺ)

⑨ ایمان و نصف ہے جسے لوگ اپنے مال و جان کو محفوظ سمجھیں۔ (ارشاد نبوی ﷺ)

⑩ میر ایمان سے اسلام ہے جسے سر جسم سے۔ (ارشاد نبوی ﷺ)

⑪ کجا بولنا ایمان کی پہچان کی علامت ہے۔ (حکیم بزرگمہر)

Q1: (a) What is a wave? (b) Define and explain wave motion.

Ans: (a) Wave:

Definition: Wave is a disturbance of some kinds that propagates through a medium and carries energy.

(b) Wave Motion:

Definition: The mechanism by which energy is transferred from one point to another point is known as wave motion.

Explanation:

When we throw a stone on the still surface of water in a pond, a disturbance is produced in water. In another words energy is transferred to the particles of water due to which they begins to oscillate about their mean position. The oscillating molecules of water transfer the energy to their neighboring molecules. This process continues and finally the energy is transferred to the edge of the pond. This motion of the disturbance is due to the transfer of energy from particle to particle is known as wave motion.

The waves transport energy from one place to another but the particles of the medium does not change their position permanently. They perform only vibratory motion about their mean position.

Q2: What is a periodic wave? Explain with examples.

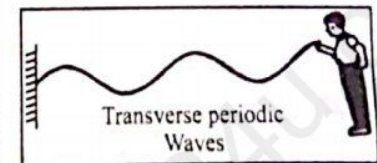
Ans: Periodic Waves:

Definition: If a medium is disturbed periodically [i.e. disturbed after equal intervals of time], then the waves with same frequency and amplitude are produced, which are known as periodic waves.

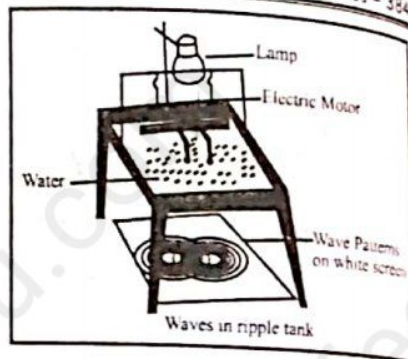
Examples of Periodic Waves:

1. Waves produced in a rope:

1. Consider an ordinary rope whose one end is fastened to a fixed



support, as shown in the figure. If we hold the other end of the rope in hand and give a sudden up and down motion periodically, then a train of transverse periodic waves are produced in the string, as shown in the figure. The waves travel towards the fixed end, while particles of the rope vibrate up and down about their mean position.



2. The circular periodic waves can be produced in a ripple tank when spherical dippers just touching the water surface. This is done by means of a mechanical arrangement driven by a small electric motor. The rate of dipping the rods is controlled by changing the speed of motor. The pattern of waves obtained at any instant of time is shown in the figure.

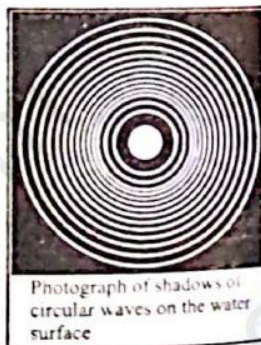
Q3: What are progressive waves? Explain.

Ans: Progressive Waves (or) Travelling Waves:

Definition: Those waves which transmit energy from one place to another place are known as progressive waves or travelling waves.

Explanation:

When we drop a stone on the still surface of water in a pond, a disturbance is produced in water. Due to this disturbance, the particles of water are set into motion. These particles transfer their energy to the neighboring particles. This process continues and finally, the energy reaches to the edge of the pond of water. Such waves which are produced in water and carry energy from one place to another are called transverse progressive waves.



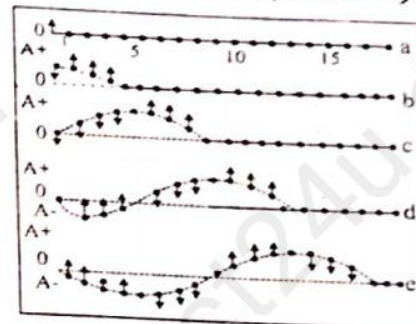
Photograph of shadows of circular waves on the water surface

Q4: Discuss the propagation of waves in detail.

Ans: Propagation of waves:

The propagation of mechanical waves occurs due to the interaction of the particles of the medium with each other. The particle by particle wave propagation can occur only in an elastic medium.

Now consider a mechanical mode of vibration of a stretched long elastic rod which consists of equally spaced particles such as 1, 2, 3, 4, 5, 6, along its length as shown in figure (a)



Now if the particle "1" is given an upward motion at right angle to the cord, the particle "2" will follow it due to elasticity of the cord. The particle "2" will lag behind the particle "1" due to its inertia.

Now when the particle "2" rises, the particle "3" will follow it but will lag behind the particle "2" due to its inertia. Similarly the particle "4" will follow particle "3" and so on.

When the particle "1" reaches to its highest position "+A", the particle "5" just begins to move up while at that instant particle "1" starts downwards motion. The particle "2" will continue its upward motion due to its inertia until it reaches to its highest position "+A".

When the particle "1" reaches its mean position "O" it does not stop there and moves downwards from mean position "O" as shown in figure (c).

When particle "1" reaches to position "-A", then it returns towards its mean position "O". When it reaches "O", one vibration is completed and as a result the other particles become disturbed as shown in figure (d). The particle "1" is now ready for second vibration. In this way, the waves propagate towards right and the particles of the medium [cord] oscillate only about their mean position.

Q5: Discuss the types of progressive mechanical waves. (OR)
Describe transverse and longitudinal waves with examples and clearly differentiate between them.

ETEA 2006, 2009, BISE Abbottabad 2019

Ans: 1. Transverse Waves:

Definition: Those waves in which the particle of the medium vibrates along a line perpendicular to the direction of propagation of the waves, are known as transverse waves.

Explanation:

Whenever a disturbance is propagating through a medium, then the particles of the medium are disturbed and start vibration. If the vibration of the particle is perpendicular to the direction of the propagation of the wave then it is called transverse wave pulse. The particle of medium does not change their position permanently and performs only vibratory motion about their mean position. In transverse waves, the particles vibrate with the period and frequency of the source.

The transverse waves consist of crests and troughs which are produced one after the other in a certain order. The distance between two consecutive crests or two consecutive troughs is known as wavelength. It is represented by " λ ", while the distance between a crest and nearest trough is equal to the half of the wavelength i.e. " $\frac{\lambda}{2}$ ".

The number of waves passing through a certain point is known as frequency of the waves. It is represented by " f ".

The velocity, frequency and wavelength of the waves are related to each other by the following equation i.e.

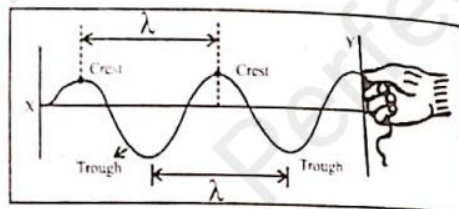
$$v = f\lambda$$

The transverse waves can propagate through a medium whose elasticity is larger. So these waves need a material medium for their propagation.

Examples of Transverse Waves:

The examples of transverse waves are given below:

- Water waves



- Light waves
- Radio waves
- Micro waves
- Waves produced in a stretched string

2. Longitudinal Waves [Compressional Waves]:

Those waves in which the particles of the medium vibrate about their mean position along the direction of propagation of the waves is known as longitudinal waves or compressional waves.

Explanation:

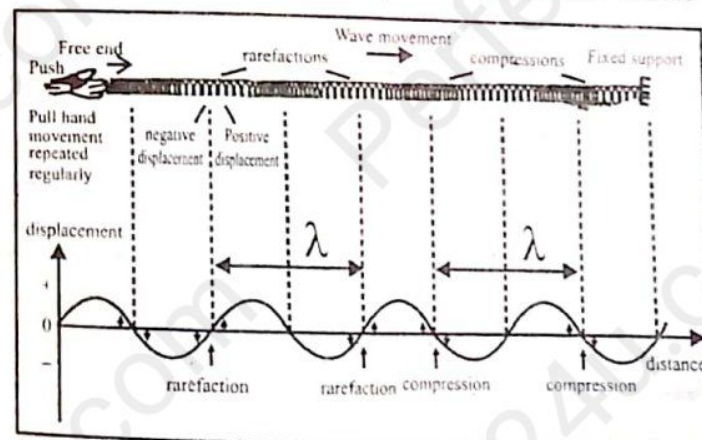
Consider a slinky spring placed on a smooth table top as shown in the figure. Let one end of the spring is free and the other end is connected to a fixed support, as shown in the figure. When we push the free end of the spring towards right, then the loops near the end are compressed towards right due to the applied force. The compressed loops then exert a force on the loops to the right side of them. In this way, a compressional pulse travels towards right along the spring.

Now when the spring is pulled suddenly in backward direction, [i.e. towards left], the loops near the end are rarefacted due to the applied force. The rarefacted loops then exert a force on the loops of the right of them. In this way, a rarefaction travels along the spring.

Now if the spring is pushed forward and backward at a constant rate, a series of longitudinal waves is setup. As the wave moves

through the slinky spring, compression and rarefaction can be seen along the spring.

The longitudinal waves consist of compression and rarefactions which are pro-



duced one after the other in a certain order. The distance between two consecutive compressions or two consecutive rarefactions is known as wavelength " λ ". The wavelength " λ ", frequency " f ", and velocity " v " of the waves are related to each other by the following equation i.e.

$$v = f\lambda$$

Examples of Longitudinal Waves:

- Sound waves
- Compressional waves in a spring
- Railway engine connected to boggies with buffer springs in between them.
- Shock waves [seismic waves] produced during an earthquake.

Q6: Define the following terms: Wave speed, frequency of waves, time period of waves, wavelength, amplitude of wave, intensity of wave, phase.

Ans. 1. Wave Speed: (ETEA 2006)

The distance travelled by a wave per unit time is known as speed of a wave. The speed of a wave depends upon the type of wave as well as the properties of the medium.

For example, the speed of transverse wave pulse in a stretched string depend upon the mass of string " M ", length of string " L " and tension in the string " T " and is given by,

$$v = \sqrt{\frac{TL}{M}} \quad (1)$$

If " m " represents mass per unit length, i.e.

$$m = \frac{M}{L} \Rightarrow M = mL \quad (2)$$

Putting equation (2) in equation (1), we get,

$$v = \sqrt{\frac{TL}{mL}} \Rightarrow v = \sqrt{\frac{T}{m}} \quad (3)$$

These equations show that the speed of transverse waves in well stretched and thin string is greater as compared to a loosely and thick one. While in case of longitudinal waves, the speed of wave depends upon the modulus of elasticity " E " and density " δ " of the medium, and is given by,

$$v = \sqrt{\frac{E}{\delta}} \quad (4)$$

As the gases are more compressible, so the elastic modulus " E " is smaller than solids. That is why, the longitudinal waves travel faster in solids as compared to gases. The unit of velocity is m/sec.

2. Frequency of Waves:

The number of waves passing through a certain point in unit time is known as frequency of the wave. When a wave progresses, each particle of the medium oscillates periodically with the time period and frequency of the source. Thus the frequency " f " of the wave is equal to the frequency of the source.

$$\text{Frequency} = \frac{\text{number of waves passing through a point}}{\text{time taken by waves}}$$

The unit of frequency is Hz or cycle per sec (c.p.s).

3. Time Period of Waves:

The time during which a wave passes through a certain point is called the time period of the wave. When a wave progresses, each particle of the medium oscillates periodically with the time period and frequency of the source. Thus the period " T " of the wave is equal to the time period of the source. The time period is equal to the reciprocal of frequency i.e. $T = \frac{1}{f}$

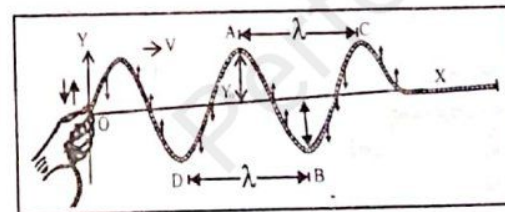
The unit of time period is sec.

4. Wavelength:

The distance between the two successive particles which are exactly in the same state of vibration is known as wavelength. It is represented by " λ ". Its

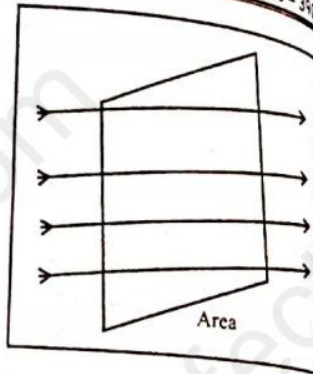
unit is meter. The relation between velocity, frequency and wavelength of a wave is given by,

$$v = f\lambda$$



5. Amplitude of Wave:

The maximum displacement covered by a vibrating particle from its mean position on either side is known as amplitude of the wave. In above figure, "y₀" represents the amplitude.



6. Intensity of Wave:

The amount of energy transmitted per second per unit area placed perpendicular to the direction of propagation of waves is called the intensity of the waves. It is represented by "I". Its unit is "J/sec.m²".

7. Phase:

The state of motion in a wave is known as phase.

Q7: Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace?

BISE Malakand 2019, BISE Mardan 2019, BISE Peshawar 2019

Ans. Speed of Sound:

The distance covered by sound wave per unit time is called speed of sound wave.

Newton's Formula for Speed of Sound:

The sound waves are longitudinal waves [mechanical waves] which need a material medium for their propagation. The speed of sound waves in a medium depends upon the density "δ" of the medium and elasticity "E" of the medium i.e.

$$v = \sqrt{\frac{E}{\delta}} \quad (1)$$

According to Newton, the temperature of air does not change when sound waves pass through it. Newton told that heat which is produced in the compression region leaks to the surrounding, while the decrease in the temperature which occurs in the rarefaction region is compensated by the flow of heat from the surrounding.

In this way, the temperature of the medium [air] remains constant. So under such conditions, the elasticity "E" of air may be equal to the pressure of the air i.e. $E = P$

Thus equation (1) becomes,

$$v = \sqrt{\frac{P}{\delta}} \quad (2)$$

To Show that, $E = P$:

Consider a layer of air whose volume is "V" and pressure is "P". Let compression is produced in this layer due to which its volume decreases by "v" and pressure increases by "p", so the final volume becomes "V-v" and the final pressure becomes "P+p".

Now as the temperature was assumed to remain constant, so the process is isothermal for which we apply Boyle's law i.e., we have,

$$PV = (P + p)(V - v)$$

$$\Rightarrow PV = PV - Pv + pV - pv$$

$$\Rightarrow -Pv + pV - pv = 0 \Rightarrow -Pv + pV - 0 = 0$$

$$\Rightarrow -Pv = -pV$$

$$\Rightarrow P = \frac{pV}{v} = \frac{P}{(v/V)}$$

$$\Rightarrow P = \frac{P}{(v/V)} = \frac{\text{stress}}{\text{volumetric strain}} = \text{Elasticity} = E$$

$$\Rightarrow P = E$$

Which is the required proof.

Now to calculate the speed of sound in air, we put,

$$P = \delta'gh \quad (3)$$

In equation (3) "δ'" represents the density of standard fluid mercury and "h" represents height of mercury at atmospheric pressure.

Putting equation (3) in equation (2), we get,

$$v = \sqrt{\frac{\delta'gh}{\delta}} \quad (4)$$

Put $\delta' = 13.6 \text{ gm/cm}^3$, $g = 980 \text{ cm/sec}^2$, $h = 76 \text{ cm of Hg}$,

$\delta = \text{density of air} = 0.001293 \text{ gm/cm}^3$ in equation (4), we get,

$$v = \sqrt{\frac{13.6 \times 980 \times 76}{0.001293}} \Rightarrow v = 28000 \text{ cm/sec [approximately]}$$

$$\Rightarrow v = 280 \text{ m/sec}$$

But the above value is about 16% less than the experimental result which is about 332 m/sec. So we cannot ignore it as an experimental error. This error was then removed by Laplace and is known as Laplace correction.

Laplace Correction:

According to Laplace, Newton was wrong in assuming that the temperature of air does not change when sound waves pass through it. He told that sound waves consist of compression and rarefactions which are produced in air so rapidly, that the temperature of air does not remain constant i.e. the rise in temperature during compression cannot balance the fall in temperature during rarefaction. So Laplace assumes an adiabatic process instead of an isothermal process, for which he put, $E = \gamma P$.

Where " γ " is constant and its value depends upon the nature of the gas. For air, $\gamma = 1.42$.

To show that, $E = \gamma P$:

Consider a layer of air having pressure " P " and volume " V ". Let compression be produced in it, such that the pressure increases by a factor " p " and volume decreases by a factor " v ". Now we have,

$$PV' = (P + p)(V - v)$$

$$\Rightarrow PV' = (P + p)V \left(1 - \frac{v}{V}\right)$$

$$\Rightarrow P = (P + p) \left[1 + \left(-\frac{v}{V}\right)\right] \quad \text{--- (5)}$$

Expanding the second term of R.H.S of equation (5), binomially, we get,

$$\left[1 + \left(-\frac{v}{V}\right)\right] = 1 + \gamma \left(-\frac{v}{V}\right) + \text{neglected terms}$$

$$\Rightarrow \left[1 + \left(-\frac{v}{V}\right)\right] = 1 - \frac{\gamma v}{V} \quad \text{--- (6)}$$

Putting equation (6) in equation (5), we get,

$$P = (P + p) \left(1 - \frac{\gamma v}{V}\right)$$

$$\Rightarrow P = P - \frac{\gamma P v}{V} + p - \frac{\gamma p v}{V}$$

$$\Rightarrow -\frac{\gamma P v}{V} + p - \frac{\gamma p v}{V} = 0$$

$$\Rightarrow -\frac{\gamma P v}{V} = -p$$

The term $\frac{\gamma p v}{V}$ is very very small, so we neglect it.

Binomial Formula:

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots$$

As $\frac{v}{V} \ll 1$ so we neglect its high power terms

$$\Rightarrow \gamma P = \frac{P}{\left(\frac{V}{V}\right)} = \frac{\text{stress}}{\text{volumetric strain}} = \text{Elasticity} = E$$

$$\Rightarrow \gamma P = E \Rightarrow E = \gamma P \quad \text{--- (7)}$$

Putting equation (7) in equation (1), we get,

$$V = \sqrt{\frac{\gamma P}{\delta}}$$

$$P = \delta'gh$$

$$\Rightarrow V = \sqrt{\gamma \times \frac{\delta'gh}{\delta}} \Rightarrow V = \sqrt{\frac{1.42 \times 13.6 \times 980 \times 76}{0.001293}}$$

$$\Rightarrow V = 33353 \text{ cm/sec}$$

$$\Rightarrow V = 333.53 \text{ m/sec} \Rightarrow V = 333 \text{ m/sec}$$

This value agrees with experimental value which is 332 m/sec.

Q8: Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.

BISE Abbottabad 2019, BISE DI Khan 2019, BISE Mardan 2017

Ans: The speed of sound in a gas depends upon various factors which are given below:

1. Effect of Density:

We know that,

$$V = \sqrt{\frac{\gamma P}{\delta}} \quad \text{--- (1)}$$

Put $\gamma P = \text{constant}$ in equation (1), we get,

$$V \propto \sqrt{\frac{1}{\delta}} \quad \text{--- (2)}$$

Relation (2) shows that at constant temperature and pressure, the speed of sound in gases is inversely proportional to the square root of their density i.e. smaller the density of a gas, greater will be the speed of sound through it. As hydrogen is less denser than oxygen, so sound waves travel faster in hydrogen as compared to oxygen.

2. Effect of Moisture:

The speed of sound increases with the presence of moisture in the air. It is because the moisture reduces the resultant density of air. When the density decreases, the speed of sound will increase. Thus the velocity of sound in damp air is greater than its value in dry air.

3. Effect of Pressure: (ETEA 2007, 2011)

It has been found that the speed of air does not depend upon the

pressure of the air i.e. $V = \text{constant}$

Proof: We know that,

$$V = \sqrt{\frac{\gamma P}{\delta}} \quad (1)$$

We also know that,

$$\delta = \frac{m}{V} \quad (2)$$

Also $PV = nRT$ [Put $n = 1$]

$$\Rightarrow PV = RT \Rightarrow V = \frac{RT}{P} \quad (3)$$

Putting equation (3) in equation (2), we get,

$$\delta = \frac{mP}{RT} \quad (4)$$

Putting equation (4) in equation (1), we get,

$$V = \sqrt{\frac{\gamma P \frac{RT}{mP}}{\frac{mP}{RT}}} = \sqrt{\frac{\gamma RT}{m}} \quad (5)$$

Putting $\frac{\gamma PT}{m} = \text{constant}$, so equation (5) becomes,

$$V = \text{constant} \quad (6)$$

Equation (6) shows that the speed of sound is independent of the pressure of the air.

4. Effect of Temperature: (BISE Mardan 2019)

The speed of sound increases with increase in temperature. It has been found that the speed of sound in gas increases about 0.6m/sec for each 1°C rise in temperature.

We know that,

$$V = \sqrt{\frac{\gamma RT}{m}} \quad (1)$$

Put $\frac{\gamma R}{m} = \text{constant}$, so equation (1) becomes,

$$V = \text{constt.} \cdot \sqrt{T} \Rightarrow V \propto \sqrt{T} \quad (2)$$

Relation (2) shows that the speed of sound in a gas is directly proportional to the square root of the absolute temperature of the gas i.e. when temperature of a gas increases, the speed of sound also increases and vice versa.

5. Effect of Wind:

If " V " is the velocity of sound and " V_w " is the velocity of wind, then if the wind is moving in the direction of sound waves, then the relative velocity becomes $(V + V_w)$. So relative speed of sound in such a case increases, and if the wind is moving in opposite direction of the sound waves, then the relative speed becomes $(V - V_w)$. So the relative speed of sound in such a case decreases.

Q9: How the speed of sound in the air varies with temperature and hence show that for each one degree centigrade rise in temperature the speed of sound increases by 0.61m/sec?

Ans: The speed of sound in air depends upon temperature of the air. If the temperature of air increases, then the velocity of speed of sound wave also increases. It has been found that for each one degree centigrade rise in temperature, the speed of sound increases by 0.61m/sec.

Proof:

Let " V_0 " represents the speed of sound at temperature " T_0 " and " V " represents the speed of sound at temperature " T ". Then we have,

$$\frac{V}{V_0} = \sqrt{\frac{T}{T_0}} \quad (1)$$

We have, $T_0 = (0^\circ\text{C} + 273) = 273\text{K}$ and $T = (t^\circ\text{C} + 273)\text{K}$

So equation (1) becomes,

$$\frac{V}{V_0} \sqrt{\frac{(t^\circ\text{C} + 273)\text{K}}{273\text{K}}} = \sqrt{\frac{t^\circ\text{C} + 273}{273}}$$

$$\Rightarrow \frac{V}{V_0} = \sqrt{\frac{t^\circ\text{C}}{273} + 1} = \left(1 + \frac{t^\circ\text{C}}{273}\right)^{1/2}$$

$$\Rightarrow \frac{V}{V_0} = \left(1 + \frac{t^\circ\text{C}}{273}\right)^{1/2} \quad (2) \quad \left[\text{Binomial formula: } (1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \dots \right]$$

Expanding the R.H.S of equation (2) binomially and neglecting the higher power terms, we get,

$$\left(1 + \frac{t^\circ\text{C}}{273}\right)^{1/2} = 1 + \frac{1}{2} \left(\frac{t^\circ\text{C}}{273}\right) + \dots$$

$$\Rightarrow \left(1 + \frac{t^\circ\text{C}}{273}\right)^{1/2} = 1 + \frac{t^\circ\text{C}}{546}$$

So equation (2) becomes,

$$\frac{V}{V_0} = \left(1 + \frac{t^\circ\text{C}}{546}\right) \Rightarrow V = V_0 + \frac{V_0 t^\circ\text{C}}{546} \quad (3)$$

At 0°C , we have, $V_0 = 332\text{m/sec}$

Put $V_0 = 332\text{m/sec}$ in 2nd term of R.H.S of equation (3), we get,

$$V = V_0 + \frac{332t^\circ\text{C}}{546} \Rightarrow V = V_0 + 0.61t^\circ\text{C} \quad (4)$$

Equation (4) shows that the speed of sound increases by 0.61m/sec for each one degree centigrade rise in temperature.

Q10: State and explain the principle of superposition of waves.

Ans. Statement:

It states that whenever two waves are travelling in the same region, the total displacement at any point is equal to the vector sum of their individual displacements at that point.

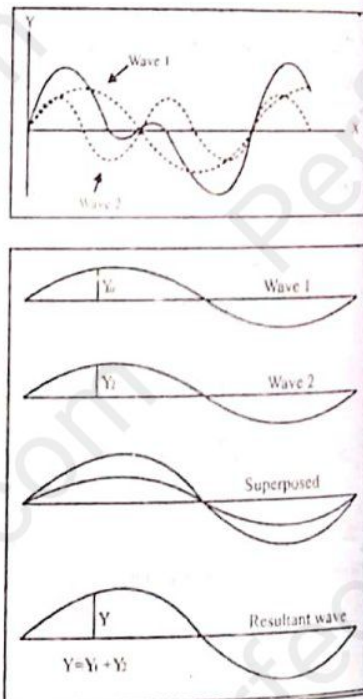
Explanation:

When two waves travelling in the same direction are superposed, the shape of the string at any instant can be determined by applying the principle of superposition.

In the given figure, the displacement " y_1 " arising from the wave "1" are given by the dotted curve. While the displacement " y_2 " arising from the wave "2" are given by the broken curve. The resultant displacements are given by the solid curve which is obtained by adding the displacement " y_1 " and " y_2 " for every point along the string i.e.

$$y = y_1 + y_2$$

The resultant wave will travel along the string with same speed as that



of component waves. The principle holds for mechanical wave and all electromagnetic waves.

The phenomena of interference, diffraction, beats and stationary waves are the consequences of superposition of waves.

1. If the two waves possess same frequency and same phase, then phenomenon of interference will take place.
2. If the two waves possess slightly different frequencies and are travelling in same direction, when phenomenon of beats will take place.
3. If the two waves of same frequency are travelling in opposite directions, then standing or stationary waves will be produced.

Q11: (a) What is meant by the interference of waves?

(b) Discuss the types of interference.

(c) Discuss the conditions for interference.

Ans. (a) Interference of Waves:

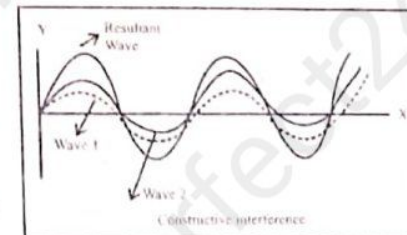
Definition: The superposition of two waves coming from two coherent sources at the same time through the same medium having the same frequency and amplitude is known as interference of waves.

(b) Types of Interference:

There are two types of interference, which are given below:

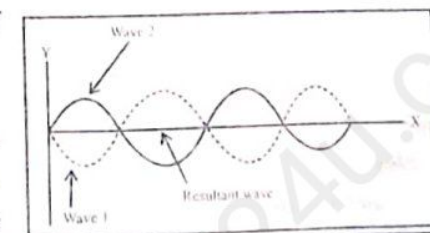
1. Constructive Interference:

When two waves arrive at a certain point in the same phase i.e. when the two crests and two troughs falls upon each other, they reinforce each other. As a result, the intensity of waves increases and a resulting wave with large amplitude is produced. In the given figure, wave 1 and wave 2 interfere each other constructively due to which a resultant wave of large amplitude is produced.



2. Destructive Interference:

When two waves arrive at a certain point in opposite phase i.e. when the crest of one wave falls



upon the trough of other wave, then they are said to be interfere destructively. In such a case, they cancel the effect of each other and the resultant intensity becomes minimum or zero. In given figure, wave 1 and wave 2 interfere destructively and as a result we get a resultant wave with zero amplitude.

(c) Conditions for Interference:

For interference, the following conditions must be satisfied.

1. The two waves must be phase coherent.
2. They must arrive at the same place at the same time.
3. The two waves must be travelling in the same direction.
4. The principle of linear superposition must be satisfied.
5. The constructive interference takes place only, if the path difference is zero i.e. an integral multiple of " λ ". i.e.
Path difference = $m\lambda$.

(OR) Path difference = $0\lambda, 2\lambda, 3\lambda, \dots, m\lambda$.

[Where $m = 0, 1, 2, 3, 4, \dots$]

6. The destructive interference takes place only, if the path difference between the two waves is an odd multiple of half of the wavelength i.e.

$$\text{Path difference} = m \frac{\lambda}{2}$$

(OR) $p.d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \dots, \left(m + \frac{1}{2}\right)\lambda$.

[Where $m = 0, 1, 2, 3, 4, \dots$]

Q12: Explain the interference of sound waves. Also discuss its experimental verification.

Ans: The effect produced by the superposition of sound waves from two coherent sources, passing through the same region is known as interference of sound waves.

If the phase of the two waves is the same, i.e. when two compressions and two rarefactions falls upon each other, they reinforce each other and constructive interference will take place. The intensity and amplitude of the resultant wave increases and we hear a loud sound.

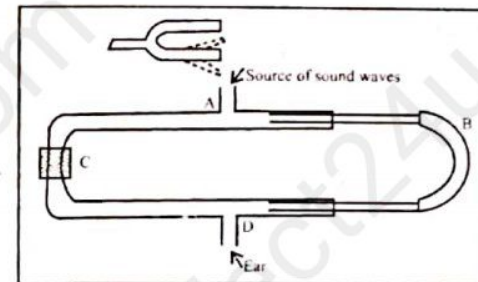
If the phase of the two sound waves is not the same, then the compression of one wave falls upon the rarefaction of the other

wave, then they cancel the effect of each other due to which the amplitude of the resultant wave becomes zero and thus we hear no sound. Such type of interference is known as destructive interference.

Experimental Verification of Interference of Sound Waves:

The interference of sound waves can be verified experimentally by using a U-shaped tube apparatus as shown in the figure.

The sound waves are produced with the help of tuning fork at "A". These waves reach the ear which is kept near at point "D" by travelling along the path "ACD" and "ABD", as shown in the figure.



Now if the lengths of the two paths are the same, then the waves reaching at "D" along the two paths will interfere constructively and a loud sound will be heard.

Now if the U-shaped tube is moved towards right such that the length of path "ABD" increases gradually, then a stage comes when the ear will hear no sound because at this stage destructive interference takes place at point "D". This happens when the length of the path "ABD" is greater than the length of the path "ACD" by " $\frac{\lambda}{2}$ ".

Now if the pinch cork at "C" is closed, then the ear will hear the sound which is due to the waves coming to the point "D" along the path "ABD" only.

Q13: What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources?

Ans. Beats: The periodic variation in the loudness of sound between maximum and minimum due to superposition of two waves of slightly different frequencies is known as beats. (OR)
The rise and fall in the intensity of sound waves per second is called beats.

Explanation:

One maximum and one minimum form one beat. The number of beats produced in a unit time is equal to the difference between the frequencies of the two sources.

For beats to be heard clearly, the difference between the frequencies should be small. The difference between beats and interference is that, in case of interference two tuning forks of equal frequencies are used, while in case of beats phenomenon, two tuning forks of slightly different frequencies are used.

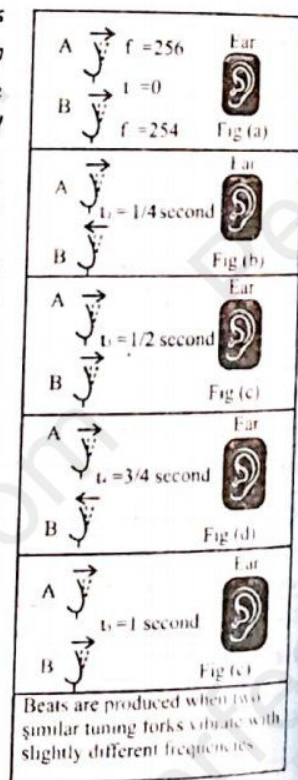
Experimental Demonstration of Beats:

Consider two tuning forks "A" and "B" having frequencies of 256Hz and 254Hz respectively. Both the tuning forks are placed at the same distance from ear, as shown in the figure. Let the two tuning forks are sounded together by striking them on a rubber pad.

Case-1: At $t = 0$, both the tuning forks send compression towards right as shown in figure (a). These compressions reinforce each other and as a result, a loud sound will be heard.

Case-2: At $t = \frac{1}{4}$ sec, the tuning fork "A" will send compression while the tuning fork "B" will send rarefaction, because at $t = \frac{1}{4}$ sec, tuning fork "A" will complete 64 vibrations while the tuning fork "B" complete 63.5 vibrations. The compression and rarefaction cancel the effect of each other and thus no sound is heard. Case number 2 is shown in figure (b).

Case-3: At $t = \frac{1}{2}$ sec, the tuning fork "A" complete 128 vibrations and thus it will send compression towards right. While the tuning fork "B" complete 127 vibrations. So it will also send compression towards



right. Both the compressions reinforce each other and as a result a loud sound is produced. Case number 3 is shown in figure (c).

Case-4: At $t = \frac{3}{4}$ sec, the fork "A" complete 192 vibrations and thus it will send compression towards right. While the tuning fork "B" complete 190.5 vibrations and thus it will send rarefaction towards right. The compression and rarefaction cancel the effect of each other and thus no sound will be heard. Case number 4 is shown in figure (d).

Case-5: At $t = 1$ sec, the tuning fork "A" will complete 256 vibrations and thus it will send compression towards right. While the tuning fork "B" will complete 254 vibrations and thus it will also send compression towards right. Both the compressions reinforced each other and as a result a loud sound is produced. Case number 5 is shown in figure (e).

Beats Frequency:

The difference between the frequencies of the two waves is called beat frequency. It is represented by "N" and is given by,

$$N = f_1 - f_2 \quad (1)$$

Now if "T" is the time interval between two successive loud sounds, then the number of oscillation of the first wave will be equal to " $f_1 T$ " and that of the second wave will be equal to " $f_2 T$ ". But the first wave should have made one oscillation more than the second one. So we have,

$$f_1 T - f_2 T = 1$$

$$\Rightarrow (f_1 - f_2)T = 1 \Rightarrow f_1 - f_2 = \frac{1}{T} \quad (2)$$

Comparing equations (1), (2), we get,

$$N = f_1 - f_2 = \frac{1}{T}$$

Uses of Beats:

1. We can use beats to tune a string instruments such as piano or violin by beating a note [sound] against a note of known frequency. The string can then be adjusted to the desired frequency by tightening or loosening it until no beat are heard.
2. Beats phenomenon is used for finding the unknown frequency.

3. Beats are used to produce variety in music.

Q14: Discuss the reflection of waves and phase changes that occurs during reflection.

Ans: Reflection of Waves:

Definition: The bouncing back of waves from the boundary of a certain medium is known as reflection of waves.

(a) Reflection of Mechanical Waves:

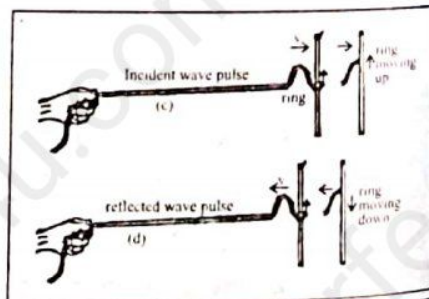
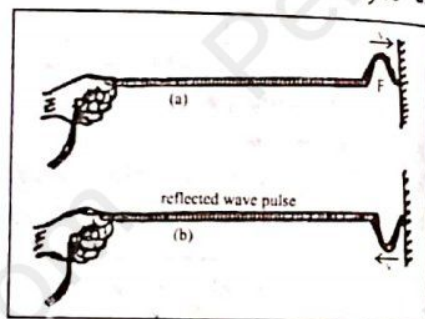
Consider a string whose one end is free and the other end is connected to a fixed support [wall] as shown in the figure.

If we hold the other end of the string in hand and move it up and down, then a crest [an upward pulse] is sent towards right as shown in the figure.

When the crest strike the fixed support [i.e. wall → denser medium], some part of its energy is absorbed and the rest is reflected. As the wall does not move up with the crest in same way as it pulls the string upwards, so the wall exert downward pull on the string. As a result the crest is reflected in the form of trough as shown in figure (b). In this way, a phase change of 180° or π radian occurs which is equal to half of the wavelength i.e. " $\frac{\lambda}{2}$ " between the incident crest and reflected trough.

On the other hand, if the end of the string is connected to a ring [rare medium], then when the crest strike the ring moves up and down and reflect the crest as crest, as shown in the figure. Thus there is no phase change in this case.

From the above discussion it is clear that when a transverse wave on a string is reflected from a denser medium, there is



a phase change of 180° or π radian. While when a transverse wave on a string is reflected from the boundary of a rare medium, it suffers no phase change. The same principle is applicable for longitudinal waves.

(b) Reflection of Electromagnetic Waves:

The phase change also occurs when electromagnetic waves such as light waves are reflected from the boundary of a denser medium. Various techniques have been developed for locating the position of objects by reflecting the waves of known speed from them as radar waves.

Q15: Discuss the reflection of sound waves.

Ans: Reflection of Sound Waves:

Definition: The bouncing back of sound waves from certain surface is known as reflection of sound waves.

Explanation:

The sound waves can be reflected from large flat and hard surfaces, such as wall, cliffs, clouds, ground etc. Like other waves, the sound waves also obey the laws of reflection.

Applications of Reflection of Sound Waves:

1. The reflection of sound waves can be used to determine the depth of the sea. For this purpose a sound pulse having frequency greater than 20KHz is sent out under water from a ship. When the sound waves reach the bottom of the sea, they reflect and moves back towards the receiver fitted on the ship. When the receiver detects the sound, the time interval " T " is noted. Now we take the half of " T " [i.e. $t = \frac{T}{2}$] which gives us the time taken by the sound waves in reaching the bottom of the sea. Now we use the following formula for determining the depth of the sea. i.e. $S = vt$. The velocity of sound in sea water is 1450m/sec.
2. The reflection of sound gives us information about an obstacle in the path of various insects and birds. When an insect travels in a dark night, it emits ultrasonic waves whose frequency is greater than 20KHz. These waves travel faster ahead the in-

sect and strike any obstacle in its way, they reflect and travel back along the same path. The insect detects the reflected sound and thus changes their direction. In this way, the insect continues their journey in safe and sound conditions.

3. The reflection of sound is used in defence and detection departments for various purposes.
4. The reflection of sound waves gives us information about echo, reverberation etc.

Q16: Write a note on, (a) Echo, (b) Reverberation

Ans. (a) Echo:

The reflection of an original sound from a certain object is received at 0.1 sec later than the direct sound is known as an echo.

Explanation:

Echo is a factor due to which we cannot hear the sound clearly. This phenomenon occurs in those halls, which have large size and smooth walls.

To avoid or minimize echo, the dimension and size of the hall should not be too large. Also the distance between source of sound and the wall of hall should be less than 17m. It is because, if the distance is greater than 17m, then the time intervals will be more than 0.1 seconds and under such conditions, our ear will hear two separate sounds. To avoid such two separate sounds, the time intervals should be less than 0.1 seconds.

The speed of sound in air is mostly taken as 340m/s, so the effective distance for echo is
$$is = \frac{340 \times 0.1}{2} = 17m$$

(b) Reverberation:

It is that effect due to which we cannot distinguish between the original sound and reflecting sound and thus causes a general confusion of the sound impression on the air.

Q17: Write a detail note on stationary waves or standing waves.

ETEA 2006, 7, 11, 13, 14, 16, BISE Peshawar 2017

Ans. Standing Waves [Stationary Waves]:

When two waves of same frequency, period and amplitude are travelling along the same line in opposite direction and superimpose each other, then standing or stationary waves are produced.

Explanation:

Consider an elastic cord whose one end is fixed and the other is held

in hand as shown in the figure.

Now if we wiggle the cord up and down then waves are set up in the cord. These waves travel towards the fixed end of the cord, suffer reflections and travel back towards the hand. The incoming waves and reflected waves overlap each other and thus form a system of standing waves.

There are certain points on stationary waves at which the amplitude of vibration is zero and tension is maximum. Such points are known as nodes. In the given figure, N_1, N_2, N_3 etc represent the nodes. While the points where the amplitude of vibration is maximum and the tension is minimum, are known as antinodes. In the given figure, A_1, A_2 etc represent the antinodes.

The node and antinodes are produced in such a manner that there is one node between two antinodes and there is one antinode between two nodes.

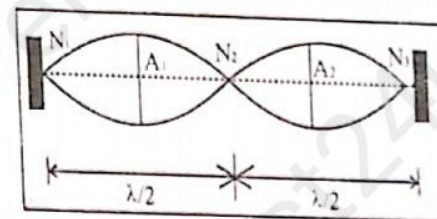
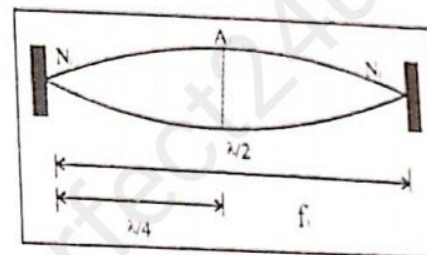
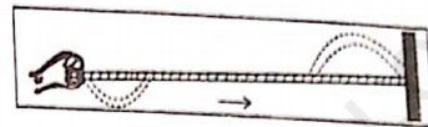
The distance between two consecutive nodes or two consecutive antinodes is equal to half of the wavelength i.e. $\frac{\lambda}{2}$. While the distance between a node and its neighboring antinode is equal to one fourth of the wavelength i.e. $\frac{\lambda}{4}$. In standing waves, the energy is not transferred from one particle to another.

When the cord vibrates in one loop, then the frequency is called 1st harmonic. It is represented by " f_1 ".

In case of two loops, the frequency is called second harmonic. It is represented by " f_2 " and is given by,

$$f_2 = 2f_1$$

Similarly, $f_3 = 3f_1, f_4 = 4f_1, f_5 = 5f_1, f_6 = 6f_1, \dots, f_n = nf_1$



Thus we can say that the frequencies with which the string can vibrate are quantized. That is the string can vibrate with discrete set of frequencies i.e. $f_1, 2f_1, 3f_1, 4f_1, \dots, nf_1$.

Q18: Discuss the formation of transverse stationary waves in a stretched string. (OR) What do you mean by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decrease.

Ans. Stationary Waves:

When two waves of same frequency, period, amplitude and velocity are travelling along the same line in opposite direction and overlap each other, then standing or stationary waves are produced.

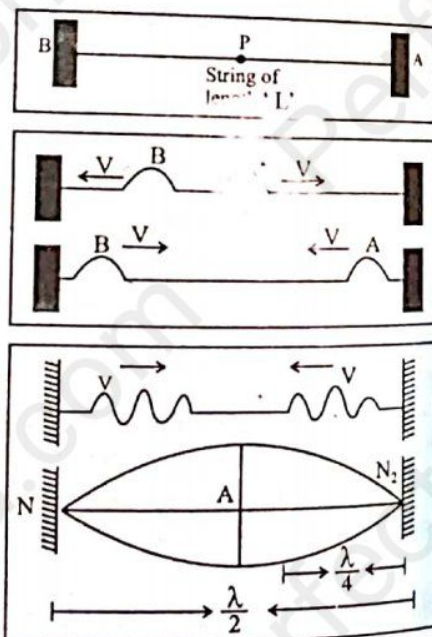
Transverse Stationary Waves in a Stretched String:

A standing wave obtained due to the superposition of transverse waves is known as transverse stationary wave.

Now consider a string of length " L " stretched between two supports " A " and " B " as shown in the figure. This string can be made to vibrate in any number of loops.

Case-1: Vibration of String in one Loop:

In this case, the string is made to vibrate in one loop. For this purpose, the string is plucked at its mid point " P " due to which two transverse waves originates from point " P ". One of these waves move towards the right end and the other move towards the left end. When these waves reach the fixed ends, they are reflected back. They meet at the middle where they superpose each other and as a result a stationary wave is setup. The whole string will vibrate in one loop, with two nodes and one antinode, as shown in the figure.



Now in case of one loop, the frequency is called fundamental frequency or first harmonic. It is represented by " f_1 " and is given by,

$$f_1 = \frac{V}{\lambda_1} \quad (1)$$

Now for one loop, we have,

$$L = \frac{\lambda_1}{2} = \frac{\lambda_1}{2}$$

$$\Rightarrow \lambda_1 = 2L \quad (2)$$

Putting equation (2) in equation (1), we get,

$$f_1 = \frac{V}{2L} \quad (3)$$

Equation (3) represents the lowest frequency which is called fundamental frequency or first harmonic.

Now we know that the velocity of wave in a stretched string in terms of tension " T " and mass per unit length " m " is given by,

$$V = \sqrt{\frac{T}{m}} \quad (4)$$

Putting equation (4) in equation (3), we get,

$$f_1 = \frac{1}{2L} \sqrt{\frac{T}{m}} \quad (5)$$

Equation (5) represents the fundamental frequency or first harmonic with which the string vibrates in one loop.

Case-2: Vibration of String in two Loops:

In this case, the same string of length " L " is made to vibrate in two loops. For this purpose, the string is plucked from one quarter of its length and as a result the string vibrates with frequency " f_2 " in two loops as shown in the figure. Here we have,

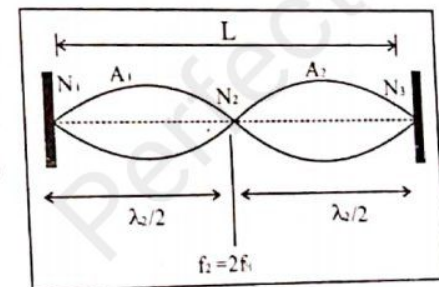
$$f_2 = \frac{V}{\lambda_2} \quad (6)$$

In case of two loops, we see that,

$$L = \frac{\lambda_2}{2} + \frac{\lambda_2}{2} = \frac{2\lambda_2}{2}$$

$$\Rightarrow \lambda_2 = \frac{2L}{2} \quad (7)$$

As $V = f \lambda$
 $\Rightarrow f = \frac{V}{\lambda}$
 $\Rightarrow f = \frac{V}{\lambda_1}$



Putting equation (7) in equation (6), we get,

$$f_2 = \frac{v}{\left(\frac{2L}{2}\right)} = 2 \frac{v}{2L}$$

$$\Rightarrow f_2 = 2 \left(\frac{v}{2L}\right) \text{ ————— (8)}$$

Putting equation (3) in equation (8), we get,

$$f_2 = 2f_1 \text{ ————— (9)}$$

In equation (9) " f_2 " is called second harmonic.

Case-3: Vibration of String in three Loops:

In this case the same string is made to vibrate in three loops. For this purpose, the string is plucked at one sixth of its length and as a result the string vibrates with frequency " f_3 " in three loops, as shown in the figure.

Here we have,

$$f_3 = \frac{v}{\lambda_3} \text{ ————— (10)}$$

In case of three loops, we have,

$$L = \frac{\lambda_3}{2} + \frac{\lambda_3}{2} + \frac{\lambda_3}{2} \Rightarrow L = \frac{3\lambda_3}{2}$$

$$\Rightarrow \lambda_3 = \frac{2L}{3} \text{ ————— (11)}$$

Putting equation (11) in equation (10), we get,

$$f_3 = \frac{v}{\left(\frac{2L}{3}\right)} = 3 \frac{v}{2L} \Rightarrow f_3 = 3 \left(\frac{v}{2L}\right) \text{ ————— (12)}$$

Putting equation (3) in equation (12), we get,

$$f_3 = 3f_1 \text{ ————— (13)}$$

Similarly, in case of 4 loops, we have,

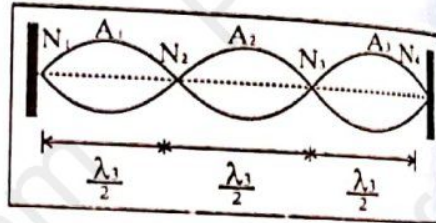
$$f_4 = 4f_1 \text{ ————— (14)}$$

⋮
⋮
⋮

And in case of N-number of loops, we have,

$$f_n = nf_1$$

Conclusion: From above discussion it is clear that the frequencies with which the string can vibrate are quantized. That is the string



can vibrate with discrete set of frequencies i.e.

$$1f_1, 2f_1, 3f_1, 4f_1, 5f_1, 6f_1, \dots, nf_1$$

Now comparing equations (2), (7), (11) etc, we see that $\lambda_3 < \lambda_2 < \lambda_1$.

This shows that the wavelength decreases as the number of loops increases. Now comparing equations (9), (13), (14) etc we see that $f_4 > f_3 > f_2 > f_1$.

This shows that, the frequency of vibrating string increases as the number of loop increases.

Q19: Explain the following: (a) Fundamental frequency, (b) Overtones, (c) Quantization of frequencies

Ans: (a) Fundamental Frequency or First Harmonic:

We know that a string fixed firmly at its two ends resonates to only certain very special frequencies i.e. $f_1, f_2, \dots, f_n$. The lowest characteristics frequency of vibration f_1 is called the fundamental frequency or first harmonic.

(b) Overtones or Harmonics:

The other possible modes of vibration whose frequencies are all integral multiple of a lowest frequency are called overtones or harmonics.

When a stretched string is excited by a small periodic force having a frequency equal to any of the quantized frequencies of the string, the phenomena of resonance will take place and stationary wave will be setup on the string.

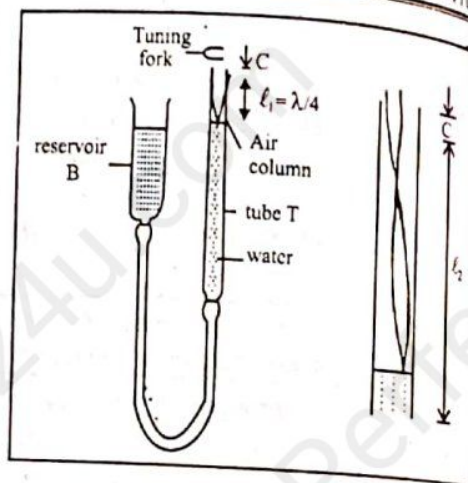
(c) Quantization of Frequencies:

It is observed that stationary waves on the string can be setup only with a discrete set of frequencies $f_1, f_2, f_3, \dots$. This shows that the resonant frequencies of the string are quantized, meaning that they are separated by frequency gaps. In other words quantum jumps in frequency exist between the resonance frequencies. This phenomenon is known as the quantization of frequency.

Q20: Discuss how stationary waves are produced in the air column?

Ans: The stationary wave can be also setup in the air column. In the given arrangement, a glass tube filled with water is connected to a reservoir.

Now if we bring a vibrating tuning fork over the open end of the glass tube and at the same time we slowly lower the water surface in the tube with the help of reservoir. During this process, a stage comes when at a certain height of water level, the air column in the tube will resonate loudly due to sound sent into it by the tuning fork.



The resonance occurs when the frequency "f" of the periodic force due to tuning fork becomes equal to the fundamental frequency " f_0 " of the air column. The resonance can occur several times by changing the height of water in the tube.

In above process, the waves produced in the air column are reflected from the surface of water. The reflected waves and incoming waves superpose each other and as a result standing or stationary waves are produced. The antinode is formed at open end of the tube while the node is formed at the other end.

Note: Using the resonance tube apparatus, we can also calculate the speed of sound through air column. For this purpose we consider the distance between node and antinode which is equal to " $\frac{\lambda}{4}$ ". This distance represents the length "l" of air column in the tube. So we have,

$$l = \frac{\lambda}{4} \Rightarrow \lambda = 4l \quad (1)$$

We also know that, $v = f\lambda$ (2)

Putting equation (1) in equation (2), we get,

$$v = 4fl \quad (3)$$

In equation (3) f = frequency of air column = frequency of tuning fork and l = length of air column, which can be noted from graduated glass tube.

Putting the values of "f" and "l" in equation (3), we can calculate the speed of sound in air.

Q21: (i) What are organ pipes? Discuss the formation of stationary waves in an organ pipe. (ii) Also show that an open organ pipe is richer in harmonics than a closed organ pipe.

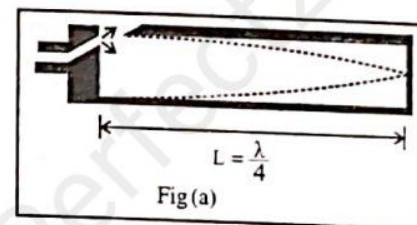
Ans: (i) Organ Pipes:

An organ pipe is the simplest example of an instrument which produces sound by means of vibrating air column. The organ pipes are of two types i.e.

- > Closed organ pipe
- > Open organ pipe

(a) Closed Organ Pipes:

A closed organ pipe is that which is closed at one end and open at the other end. When a jet of air is sent through the open end inside the pipe, then the columns of air



are set into vibrations. These waves are reflected at the closed end and forms a system of standing waves. There is an antinode at open end and node at the closed end.

Case-1: In first case, a half loop is formed as shown in figure (a). We know that the distance between a node and its neighboring antinode is equal to " $\frac{\lambda}{4}$ " so we have,

$$L = \frac{\lambda}{4} \Rightarrow \lambda = 4L$$

$$\lambda_1 = 4L \quad (1)$$

Now the frequency in this case is given by,

$$f_1 = \frac{v}{\lambda_1} \quad (2)$$

Putting equation (1) in equation (2), we get,

$$f_1 = \frac{v}{4L} \quad (3)$$

Equation (3) represents the fundamental frequency or first harmonic.

Case-2: In second case one complete loop and one half loop is formed, as shown in figure (b). In this case, we have,

$$L = \frac{\lambda}{2} + \frac{\lambda}{4} \Rightarrow L = \frac{2\lambda + \lambda}{4}$$

$$\Rightarrow L = \frac{3\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{3} \Rightarrow \lambda_2 = \frac{4L}{3} \quad (4)$$

Now the frequency in this case is given by,

$$f_2 = \frac{V}{\lambda_2} \quad (5)$$

Putting equation (4) in equation (5), we get,

$$f_2 = \frac{V}{\left(\frac{4L}{3}\right)} = 3 \frac{V}{4L} \Rightarrow f_2 = 3 \left(\frac{V}{4L}\right) \quad (6)$$

Putting equation (3) in equation (3), we get,

$$f_2 = 3f_1 \quad (7)$$

Equation (7) represents the second harmonic or first overtone.

Case-3: In third case, two complete loops and one half loop is formed, as shown in figure (c). In this case, we have,

$$L = \frac{\lambda + 2\lambda + 2\lambda}{4}$$

$$\Rightarrow L = \frac{\lambda + 2\lambda + 2\lambda}{4} \Rightarrow L = \frac{5\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{5}$$

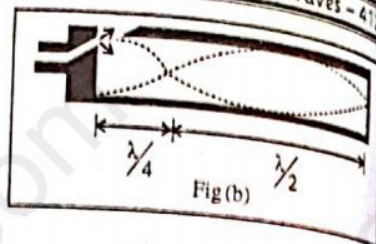
$$\Rightarrow \lambda_3 = \frac{4L}{5} \quad (8)$$

Now the frequency in this case is given by,

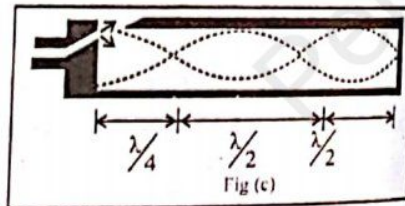
$$f_3 = \frac{V}{\lambda_3} \quad (9)$$

Putting equation (8) in equation (9), we get,

$$f_3 = \frac{V}{\left(\frac{4L}{5}\right)} = 5 \frac{V}{4L} \Rightarrow f_3 = 5 \left(\frac{V}{4L}\right) \quad (10)$$



For 2nd case, we put $\lambda = \lambda_2$



For 3rd case, we put $\lambda = \lambda_3$

Putting equation (3) in equation (10), we get,

$$f_3 = 5f_1 \quad (11)$$

Similarly, $f_4 = 4f_1 \quad (12)$

$$\text{And } f_n = (2n - 1)f_1 \quad (A)$$

As $f_1 = \frac{V}{4L}$ so equation (A) can be also written as,

$$f_n = (2n - 1) \frac{V}{4L} \quad (B)$$

Thus the possible frequencies are,

$$\frac{V}{4L}, 3 \frac{V}{4L}, 5 \frac{V}{4L}, \dots, (2n - 1) \frac{V}{4L}$$

(b) Open Organ Pipes:
(ETE 2009, 2013)

It is that pipe which is open at both ends. In such a case, antinodes will be formed at both ends.

Case-1: In this case, the air column

in the pipe oscillates for stationary wave and form certain loop with antinodes at both ends, as shown in figure (d). In this case, we have,

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = 2 \frac{\lambda}{4}$$

$$\Rightarrow L = \frac{\lambda}{2} \Rightarrow \lambda_1 = 2L \quad (1)$$

The frequency in this case is given by,

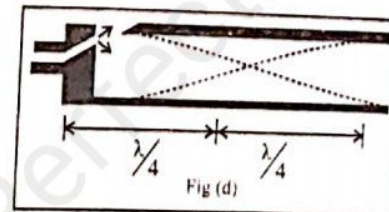
$$f_1 = \frac{V}{\lambda_1} \quad (2)$$

Putting equation (1) in equation (2), we get,

$$f_1 = \frac{V}{2L} \quad (3)$$

Equation (3) represents the first harmonic or fundamental frequency.

Case-2: In this case, the air column in the pipe oscillate for stationary waves and form certain loops, as shown in figure (e). In this case, we have,



$$L = \frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{4}$$

$$\Rightarrow L = \frac{(\lambda + 2\lambda + \lambda)}{4} = \frac{4\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{4}$$

$$\Rightarrow \lambda_2 = \frac{4L}{4} \quad (4)$$

Now the frequency in this case is given by,

$$f_2 = \frac{V}{\lambda_2} \quad (5)$$

Putting equation (4) in equation (5), we get,

$$f_2 = \frac{V}{\left(\frac{4L}{4}\right)} = 4 \left(\frac{V}{4L}\right) = 2 \left(\frac{V}{2L}\right)$$

$$\Rightarrow f_2 = 2 \left(\frac{V}{2L}\right) \quad (6)$$

Putting equation (3) in equation (6), we get,

$$f_2 = 2f_1 \quad (7)$$

Equation (7) represents the second harmonic or first overtone.

Case-3: In this case, the air column in the pipe oscillate for stationary waves and form certain loops as shown in figure (f). In this case, we have,

$$L = \frac{\lambda}{4} + \frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{4}$$

$$\Rightarrow L = \frac{(\lambda + 2\lambda + 2\lambda + \lambda)}{4} = \frac{6\lambda}{4} = \frac{3\lambda}{2} \Rightarrow L = \frac{3\lambda}{2}$$

$$\Rightarrow \lambda = \frac{2L}{3} \Rightarrow \lambda_3 = \frac{2L}{3} \quad (8)$$

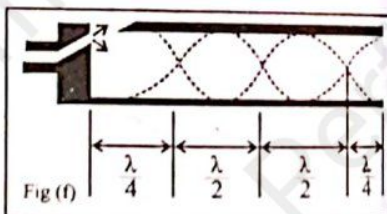
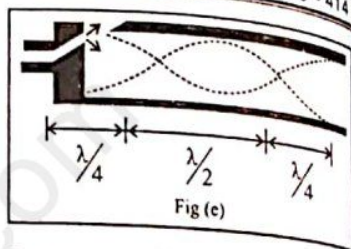
Now the frequency in this case is given by,

$$f_3 = \frac{V}{\lambda_3} \quad (9)$$

Putting equation (8) in equation (9), we get,

$$f_3 = \frac{V}{\left(\frac{2L}{3}\right)} = 3 \left(\frac{V}{2L}\right) \Rightarrow f_3 = 3 \left(\frac{V}{2L}\right) \quad (10)$$

Putting equation (3) in equation (10), we get,



$$f_3 = 3f_1 \quad (11)$$

$$\text{Similarly, } f_4 = 4f_1 \quad (12)$$

$$\text{And } f_n = nf_1 \quad (A)$$

Thus the possible frequencies are,

$$\frac{V}{2L}, 2 \frac{V}{2L}, 3 \frac{V}{2L}, 4 \frac{V}{2L}, \dots, n \left(\frac{V}{2L}\right)$$

NOTE:

The set of frequencies in case of closed organ pipe is given by,

$$f_2 = 3f_1, f_3 = 5f_1, f_4 = 7f_1, \dots, f_n = (2n-1)f_1$$

While the set of frequencies in case of open pipe is given by,

$$f_2 = 2f_1, f_3 = 3f_1, f_4 = 4f_1, f_5 = 5f_1, \dots, f_n = nf_1$$

Comparing the above sets of frequencies, we see that an open organ pipe is richer in harmonics than a closed organ pipe.

Q22: What is Doppler's effect? Derive expressions for the frequencies heard: (a) When the sounding source approaches a stationary listener? (b) When listener move towards a stationary sounding source? (c) When the source and listener are moving towards each other?

ETEA 2010, BISE DI Khan 2019

Ans. (a) Doppler's Effect:

Definition: The apparent change in pitch of sound due to the relative motion between the source and listener [observer] is known as Doppler effect.

Explanation:

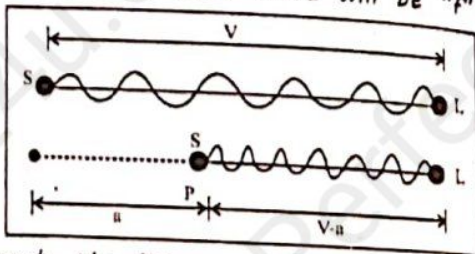
Consider a listener standing on a railway platform. An engine sounding whistle is approaching the listener. The listener will find that the pitch of the note of the whistle appears to increase as the engine approaches the listener. The pitch of sound will be maximum when the engine is just crossing the listener. This pitch will decrease as the engine moves away from the listener.

Let "f" be the actual frequency of the whistle and "f'" be the apparent frequency of the whistle appears to the listener. The value of

"f'" will be different from "f" due to the relative motion between source and the listener. Let "v" be the velocity of sound in air.

Case 1: When the source is moving towards a stationary listener:
Consider a listener "L" situated at a distance "V" from the source "S" as shown in the figure.

Let the sound waves starting from "S" reach "L" in exactly one second. Now when the source is at rest, then there will be "f" waves in distance "v". Now the wavelength of sound is given by,



$$\lambda = \frac{\text{total distance}}{\text{total no. of waves}}$$

$$\Rightarrow \lambda = \frac{V}{f} \quad (1)$$

Let the source is moving towards the listener with velocity "a" such that it reaches the point "P" in one second. Now the same number of waves "f" will be compressed in a distance "V-a". Now the wavelength "λ'" will be,

$$\lambda' = \frac{\text{distance}}{\text{no. of waves}} \Rightarrow \lambda' = \frac{(V-a)}{f} \quad (2)$$

Now the apparent frequency heard by the listener is given by,

$$f' = \frac{V}{\lambda'} \quad (3)$$

Putting equation (1) in equation (3), we get,

$$f' = \left(\frac{V}{V-a} \right) f \quad (4)$$

$$\text{Now as } (V-a) < V \Rightarrow \left(\frac{V}{V-a} \right) > 1$$

$$\text{So equation (4)} \Rightarrow f' > f$$

Thus the apparent frequency increases when the source of sound is moving towards the listener and as a result the pitch of sound increases. Similarly, when the source of sound is moving away from the listener, then equation (4) can be written as,

$$f' = \left(\frac{V}{V+a} \right) f \quad (5)$$

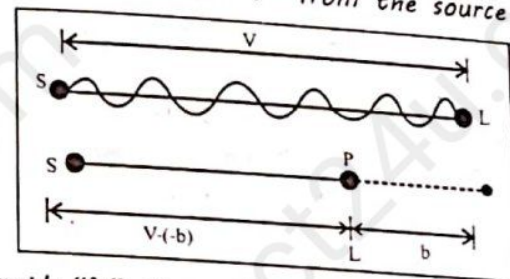
$$\text{As } (V+a) > V \Rightarrow \left(\frac{V}{V+a} \right) < 1, \text{ so equation (5)} \Rightarrow f' < f$$

Thus the apparent frequency decreases when the source of sound is moving away from the listener and as a result the pitch of sound decreases.

Case 2: When listener is moving towards stationary source:

Consider a listener "L" situated at a distance "V" from the source of sound "S" as shown in the figure.

Let the sound waves starting from "S" and reaches "L" in exactly one second. When the listener is at rest, then there will be "f" waves in distance "V". Now the wavelength "λ" of sound waves is given by,



$$\lambda = \frac{\text{total distance}}{\text{total no. of waves}}$$

$$\Rightarrow \lambda = \frac{V}{f} \quad (1)$$

Now let the listener is moving with velocity "b" towards the source and reaches the point "P", as shown in the figure. Now the relative velocity of sound waves with respect to the listener is given by,

$$V - (-b) = V + b$$

Now the apparent frequency in this case is given by,

$$f' = \frac{(V+b)}{\lambda} \quad (2)$$

Putting equation (1) in equation (2), we get,

$$f' = \left(\frac{V+b}{V} \right) f \quad (3)$$

$$\text{As } (V+b) > V \Rightarrow \left(\frac{V+b}{V} \right) > 1$$

$$\text{So equation (3)} \Rightarrow f' > f$$

Thus the apparent frequency increases when the listener is moving towards a stationary source of sound, and thus the pitch increases. Similarly, when the listener is moving away from the source, then equation (3) can be written as,

$$f' = \left(\frac{V - b}{V} \right) f \quad \text{--- (4)}$$

As $\left(\frac{V - b}{V} \right) < 1$, so equation (4) $\Rightarrow f' < f$

Thus the apparent frequency decreases when the listener is moving away from the stationary source and thus the pitch of sound decreases.

Case 3: When source and listener both are moving towards each other:

Let the source and listener both are moving towards each other with velocities 'a' and 'b' respectively. The apparent frequency in such a case is given by,

$$f' = \frac{v'}{\lambda'} \quad \text{--- (1)}$$

Here 'v' represents the speed of sound relative to the listener and is given by,

$$v' = v + b \quad \text{--- (2)}$$

While ' λ' ' represents the apparent wavelength and is given by,

$$\lambda' = \frac{v - a}{f} \quad \text{--- (3)}$$

Putting equation (2) and equation (3) in equation (1), we get,

$$f' = \frac{v + b}{\left(\frac{v - a}{f} \right)} = \left(\frac{v + b}{v - a} \right) f \Rightarrow f' = \left(\frac{V + b}{V - a} \right) f \quad \text{--- (4)}$$

In equation (4), $\left(\frac{V + b}{V - a} \right) > 1$, so equation (4) $\Rightarrow f' > f$

This shows that the apparent frequency increases when the source and listener both are moving towards each other.

Similarly, when the source and listener both are moving away from each other, then equation (4) can be written as,

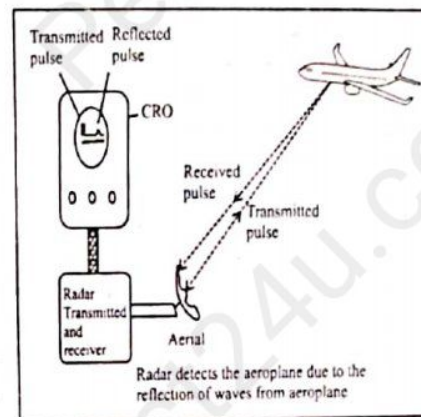
$$f' = \left(\frac{v - b}{v + a} \right) f \quad \text{--- (5)}$$

Here $\left(\frac{V - b}{V + a} \right) < 1$, so equation (5) $\Rightarrow f' < f$

This shows that the apparent frequency decreases when the source and listener both are moving away from each other.

Applications of Doppler Effect:

1. The Doppler Effect is not limited to only sound waves. It is also applicable to light waves. The frequency of light being received from certain stars which are moving towards earth or moving away from the earth is found to be slightly different than the frequency of the same light emitted by a source on the earth.



2. The reflection of radar waves from an aeroplane is another application of Doppler Effect.

The frequency of reflected waves is decreased, if the aeroplane is moving away from the source. The frequency of reflected waves is increased if the plane is moving towards the source. From this frequency shift ' Δf ', the speed and direction of the aeroplane can be determined.

3. When sound waves are reflected from a moving submarine, their frequency is changed. By this change in frequency, we can calculate the speed and direction of the submarine.

4. The Doppler shift in the frequency of radio waves [which they transmit] can be used to calculate velocities of earth satellites.

Q23: What are ultrasonic waves? Discuss the uses of ultrasonic waves.

Ans. Ultrasonic Waves:

Definition: Those sound whose frequencies are higher than 20 kHz are called ultrasonic waves.

Explanation:

A normal human ear can hear a sound whose frequency lies between 20 Hz and 20 kHz. If the frequency of sound is below 20 Hz or above 20 kHz, then human ear cannot hear it, because such sounds cannot vibrate the membrane of our ear.

The sounds whose frequencies below 20 Hz are known as infrasonic waves while the sound whose frequencies are above 20 kHz are

called ultrasonic waves.

The term ultrasonic means above or beyond sound. The ultrasonic waves can be produced by an object whose frequency of vibration is higher than 20 kHz. An ultrasonic wave is a pressure wave which has an extremely short wavelength because of its high frequency. In fact an ultrasonic wave of frequency 25 billion Hz has a wavelength of 10^{-8} m, which is smaller than the wavelength of visible light (10^{-6} m) and is comparable to the wavelength of x-rays (10^{-10} m).

Uses of Ultrasonic Waves:

As the ultrasonic waves possess shorter wavelength, higher frequency and energy, so these waves are widely used in various departments for various purposes. For example:

1. The ultrasonic waves have a great importance in bloodless surgery and diagnostic work in medicine. In diagnostic work an ultrasonic signal is transmitted through a patient. By the analysis of reflected or refracted signals, the cysts and tumor can be located in the body. It has also a great importance in surgical departments.
2. The ultrasonic waves are used in industry for the testing of materials. For this purpose a beam of ultrasonic waves of high frequency is allowed to penetrate through the material. The reflection of these waves from certain spots explores the physical defects in that material.
3. The ultrasonic waves are used for cleaning purposes. These waves can remove the traces of foreign matter sticking to the metal.
4. The ultrasonic waves of high intensity can be used to destroy bacteria and other microorganisms present in liquids and air.
5. The ultrasonic waves play an important role in neurosurgery. The ultrasonic wave of high intensity can be focused in the required region without disturbing the surrounding tissues. For this purpose, special type of ultrasonic equipments are used for the treatment of arthritis, muscular rheumatism and sciatic diseases.
6. The ultrasonic waves are used in the process of cavitations, which helps in degassing. When the ultrasonic waves are focused on a small space in liquids, very high intensity can be produced. The liquid is rapidly volatilized and a large number of bubbles are formed. This process is called cavitations. The collapse of these bubbles produces destructive forces near the surface in a liquid.

7. The ultrasonic waves can be used to determine the quantity of water in a container.
8. The ultrasonic waves can be used to determine the depth of the sea.
9. The ultrasonic waves are used to check an unborn baby in the womb [بطن] of mother. For this purpose, an ultrasonic transmitter is moved over the body of the mother. The ultrasonic waves which are reflected from different layers inside the body are picked up by a detector. The signals are then focused on the screen and thus the doctor can see its final image. From the final image, the doctor can get information about the nature, size, age, movement etc of the unborn baby in the womb of the mother.

Q24: Discuss the following: (i) Generation of ultrasonic waves.

(ii) Detection of ultrasonic waves.

Ans: (a) Generation of Ultrasonic Waves:

Ultrasonic waves can be generated by any object which is capable of oscillation at a frequency higher than 20 kHz.

Method a:

In most of applications, ultrasonic waves are generated by applying an electric current (electric field) to a specialized kind of crystal known as piezoelectric crystal. This crystal converts electrical energy into mechanical energy, as a result the crystal is set to vibrate at high frequency, generating ultrasonic waves.

Method b:

In another technique a magnetic field is applied to a special crystal which causes it to oscillate at a frequency higher than 20 kHz, emitting ultrasonic waves.

(b) Detection of Ultrasonic Waves:

There are so many methods by which ultrasonic waves can be detected. Some methods are given below:

(i) Piezoelectric Detection Method: As ultrasonic waves consist of compressions and rarefaction so when they are allowed to fall on a quartz crystal, a certain potential difference is produced across the crystal faces. This difference is amplified by an amplifier and the ultrasonic waves are detected.

(ii) Kundt's Tube Method: Kundt's tube is a long glass tube supported horizontally with an air column in it. The lycopodium powder

is sprinkled in the tube. When ultrasonic waves are allowed to pass through this Kundt's tube, the lycopodium powder in the tube collects at the nodes and blown off at the antinodes. This method is used when the wavelength is not very short.

Solution of Examples & Assignments

EXAMPLE 8.1:

A wave generator produces 500 pulses in 10sec. Find the period and frequency of the pulses it produces.

Solution:

Given data:

Number of pulses = $n = 500$, Time taken = $t = 10\text{sec}$

Required: (i) Frequency of pulse = $f = ?$

(ii) Time period of pulse = $T = ?$

Calculation:

(i) We know that,

$$f = \frac{\text{No. of pulses}}{\text{time}} = \frac{500}{10} \Rightarrow f = 50 \text{ pulse / sec}$$

$$(ii) \text{ As } T = \frac{1}{f} = \frac{1}{50} \text{ sec} \Rightarrow T = \frac{1}{50} \text{ sec}$$

ASSIGNMENT 8.1:

Compute the speed of sound in sea water. If its density is 1025kg/m^3 and elastic modulus is $2.1 \times 10^9 \text{N/m}^2$

Solution:

Given data:

Density of sea water = $\delta = 1025 \text{kg / m}^3$

Elastic modulus = $E = 2.1 \times 10^9 \text{N/m}^2$

Required:

Speed of sound = $V = ?$

Calculation:

We know that,

$$V = \sqrt{\frac{E}{\delta}} \quad (1)$$

Putting the values in equation (1), we get,

$$V = \sqrt{\frac{2.1 \times 10^9}{1025}} = 0.1430 \times 10^4 \text{ m/sec}$$

$$\Rightarrow V = 1430 \text{ m / sec}$$

EXAMPLE 8.2:

In a ripple tank 40 waves passes through a certain point in one second. If the wavelength of the waves is 5cm, then find the speed of the waves?

Solution:

Given data:

Number of waves per second = $f = 40 \text{ wave/sec}$

Wavelength = $\lambda = 5 \text{cm} = 0.05 \text{m}$

Required: Speed of the mass = $V = ?$

Calculation:

We know that, $V = f \lambda$ _____ (1)

Putting the values in equation (1), we get,

$$V = (40 \times 0.05) \text{ m/sec} \Rightarrow V = 2 \text{ m/sec} \Rightarrow V = 2 \text{ m / sec}$$

ASSIGNMENT 8.2:

How many beats per second are heard when two tuning forks of 256 Hz and 259 Hz are sounded together?

Solution: Given data:

Frequency of 1st tuning fork = $f_1 = 259 \text{Hz}$

Frequency of 2nd tuning fork = $f_2 = 256 \text{Hz}$

Required:

Beat per sec = $N = ?$

Calculation:

We know that, $N = f_1 - f_2$ _____ (1)

Putting the values in equation (1), we get,

$$N = 259 - 256 \Rightarrow N = 3 \text{ beats per sec}$$

EXAMPLE 8.3:

Find the speed of sound in a steel railway track, if the density of steel is 7800kg/m^3 and elastic modulus is $2 \times 10^{11} \text{N/m}^2$.

Solution: Given data:

Density of steel = $\delta = 7800 \text{kg / m}^3$,

$$\text{Elastic modulus} = E = 2 \times 10^{11} \text{ N/m}^2$$

Required:

$$\text{Speed of sound} = V = ?$$

Calculation:

$$\text{We know that, } V = \sqrt{\frac{E}{\rho}} \quad (1)$$

Putting the values in equation (1), we get,

$$V = \sqrt{\frac{2 \times 10^{11}}{7800}} = 0.0506 \times 10^5 \text{ m/sec} \Rightarrow V = 5.06 \times 10^3 \text{ m/sec}$$

ASSIGNMENT 8.3:

A 40g string 2m in length vibrates in three loops. The tension in the string is 270N. What is the wavelength and frequency? (1.33m, 87.1Hz)

Solution:

Given data:

$$\text{Length of string} = L = 2\text{m, Mass} = M = 40\text{g} = 0.04\text{kg}$$

$$\text{Mass per unit length} = m = \frac{M}{L} = \frac{0.04}{2} = 0.02\text{kg/m}$$

$$\text{Number of loops} = n = 3, \text{ Tension} = T = 270\text{N}$$

Required:

$$(i) \text{ Wavelength} = \lambda_3 = ?$$

$$(ii) \text{ Frequency} = f_3 = ?$$

Calculation:

(i) We know that,

$$\lambda_n = \frac{2L}{n} \Rightarrow \lambda_3 = \frac{2 \times 2}{3} = \frac{4}{3} \text{ m} \Rightarrow \lambda_3 = 1.33\text{m}$$

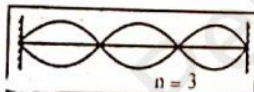
(ii) We know that, $v = f\lambda$

$$\Rightarrow v = f_3 \lambda_3 \Rightarrow f_3 = \frac{v}{\lambda_3} \quad (1)$$

$$\text{Also } v = \sqrt{\frac{T}{m}} = \sqrt{\frac{270}{0.02}} \text{ m/sec} = (\sqrt{13500}) \text{ m/sec} = 116.2 \text{ m/sec} = 116 \text{ m/sec}$$

Putting the values of "v" and " λ_3 " in equation (1), we get,

$$f_3 = \left(\frac{116}{1.33} \right) \text{ Hz} = 87.2 \text{ Hz} \Rightarrow f_3 = 87.2 \text{ Hz}$$



EXAMPLE 8.4:

Find the speed of sound in a neon gas at 0°C . γ for monatomic gas = 1.66. [Take $R = 8314 \text{ J/K}\cdot\text{mol}\cdot\text{K}$, molar mass of neon gas = $M = 20.18 \text{ kg/K}\cdot\text{mol}$]

Solution:

Given Data:

$$\gamma = 1.66, T = 0^\circ\text{C} = 0 + 273 = 273\text{K}$$

$$R = 8314 \text{ J/K}\cdot\text{mol}\cdot\text{K}, M = 20.18 \text{ kg/K}\cdot\text{mol}$$

Required:

$$\text{Speed of sound} = V = ?$$

Calculation:

$$\text{We know that, } V = \sqrt{\frac{\gamma RT}{M}} \Rightarrow V = \sqrt{\frac{1.66 \times 8314 \times 273}{20.18}}$$

$$\Rightarrow V = 432 \text{ m/sec}$$

ASSIGNMENT 8.4:

What length of closed pipe will produce a fundamental frequency of 256Hz at 20°C ?

Solution: Given data:

$$\text{Fundamental frequency} = f_1 = 256\text{Hz}$$

$$\text{Speed of sound at } 20^\circ\text{C} = 343 \text{ m/sec}$$

Required:

$$\text{Length of closed pipe} = L = ?$$

Calculation:

We know that for closed pipe,

$$f_1 = \frac{v}{4L} \Rightarrow L = \frac{v}{4f_1} \Rightarrow L = \frac{343}{4 \times 256} = 0.3349\text{m}$$

$$\Rightarrow L = 0.3349\text{m} = 0.3349 \times 100\text{cm} = 33.49\text{cm} \Rightarrow L = 33.5\text{cm}$$

EXAMPLE 8.5:

Two pianos sound the same note. If the vibration from one is 221.60Hz and that of the other is 221.40 Hz. What is the beat frequency between the two tones?

Solution: Given Data:

$$\text{Frequency of 1st piano} = f_1 = 221.60\text{Hz}$$

$$\text{Frequency of 2nd piano} = f_2 = 221.40\text{Hz}$$

Required: Beat frequency = $N = ?$

Calculation:

We know that, $N = f_1 - f_2$ (1)

Putting the values in equation (1), we get,

$$N = (221.60 - 221.40)\text{Hz} = 0.20\text{Hz}$$

$$\text{Beat frequency} = N = 0.20\text{Hz}$$

ASSIGNMENT 8-5:

- (a) What frequency is received by a woman watching an oncoming ambulance moving at 110km/h and emitting a steady 800Hz sound from its siren? The speed of sound on this day is 345m/s. (b) What frequency does she receive after the ambulance has passed? (a) 878Hz, (b) 735Hz

Solution: Given data:

Speed of sound = $v = 345\text{m/sec}$, Frequency of sound = $f = 800\text{Hz}$
Speed of source

$$= a = 110\text{km/hr} = \frac{110 \times 1000}{3600}\text{m/sec} = 30.5\text{m/sec}$$

Required:

- (a) Apparent frequency when source is approaching = $f' = ?$
(b) Apparent frequency when source is receding = $f'' = ?$

Calculation:

- (a) We know that, when source is approaching the listener, then

$$f' = \left(\frac{v}{v - a} \right) f = \left(\frac{345}{345 - 30.5} \right) 800 = 878\text{Hz} \Rightarrow f' = 878\text{Hz}$$

- (b) When source is moving away, then we have,

$$f'' = \left(\frac{v}{v + a} \right) f = \left(\frac{345}{345 + 30.5} \right) 800 = 735\text{Hz} \Rightarrow f'' = 735\text{Hz}$$

EXAMPLE 8-6:

The speed of a wave on a particular string is 24m/sec. If the string is 6m long, to what driving frequency will it resonate?

Solution: Given Data: Length of string = $L = 6\text{m}$,

Speed of the wave = $V = 24\text{m/sec}$

Calculation:

$$\text{We know that, } \lambda_n = \frac{2L}{n} \quad (1)$$

Put $n = 1$, so equation (1) becomes,
 $\lambda_1 = \frac{2 \times 6}{1} = 12\text{m} \Rightarrow \lambda_1 = 12\text{m}$

Put $n = 2$, in equation (1), we get,
 $\lambda_2 = \frac{2 \times 6}{2} \Rightarrow \lambda_2 = 6\text{m}$

Put $n = 3$ in equation (1), we get,
 $\lambda_3 = \frac{2 \times 6}{3} \Rightarrow \lambda_3 = 4\text{m}$

Now the possible frequencies are given below:

$$f_1 = \frac{v}{\lambda_1} = \frac{24}{12} \Rightarrow f_1 = 2\text{Hz}$$

$$f_2 = \frac{v}{\lambda_2} = \frac{24}{6} \Rightarrow f_2 = 4\text{Hz}$$

$$\& f_3 = \frac{v}{\lambda_3} = \frac{24}{4} \Rightarrow f_3 = 6\text{Hz}$$

EXAMPLE 8-7:

A string 4m long has a mass of 3gm. One end of the string is fasted to a stop and the other end hangs over a pulley with a 20kg mass attached. What is the speed of a transverse wave in this string?

Solution:

Given Data: Mass of pulley = $m = 20\text{kg}$,

Length of string = $L = 4\text{m}$

Mass of string = $M = 3\text{gm} = 0.003\text{kg}$

$$\text{Mass per unit length} = m' = \frac{M}{L} = \frac{0.003}{4} = 7.5 \times 10^{-4}\text{kg/m}$$

Tension in the string = $T = w = mg$

$$\Rightarrow T = 20 \times 9.8 = 19.6\text{N}$$

Required:

Speed of transverse waves = $V = ?$

Calculation:

$$\text{We know that, } V = \sqrt{\frac{T}{m'}} \Rightarrow V = \sqrt{\frac{19.6}{7.5 \times 10^{-4}}}$$

$$\Rightarrow V = 1.60 \times 10^2\text{m/sec}$$

$$\Rightarrow V = 160\text{m/sec}$$

EXAMPLE 8-8:

The lowest resonance frequency for a guitar string of length 0.75m is 400Hz. Calculate the speed of a transverse wave on the string.

Solution: Given Data:

String's length = $L = 0.75\text{m}$, Lowest frequency = $f_1 = 400\text{Hz}$

Required: Speed of transverse waves = $V = ?$

Calculation: We know that, $\lambda_n = \frac{2L}{n} \Rightarrow \lambda_1 = \frac{2L}{1}$

$$\Rightarrow \lambda_1 = 2 \times 0.75 \Rightarrow \lambda_1 = 1.50\text{m}$$

$$\text{Now } V = f\lambda \Rightarrow V = f_1\lambda_1 = 400 \times 1.5 \Rightarrow \boxed{V = 600\text{m/sec}}$$

EXAMPLE 8-9:

A pipe open at one end and closed at the other end, is 82cm long. What are the three lowest frequencies to which it will resonate? Take the speed of sound as 340m/sec.

Solution: Given Data:

Length of pipe = $L = 82\text{cm} = 0.82\text{m}$

Speed of sound = $V = 340\text{m/sec}$

Required:

Three lowest possible frequencies = ? i.e.

(i) $f_1 = ?$ (ii) $f_2 = ?$ (iii) $f_3 = ?$

Calculation:

(i) In case of closed pipe, we have,

$$f_1 = \frac{V}{4L} = \frac{340}{(4 \times 0.82)} \Rightarrow \boxed{f_1 = 104\text{Hz}}$$

$$\text{Now (ii) } f_2 = 3f_1 = 3 \times 104 \Rightarrow \boxed{f_2 = 312\text{Hz}}$$

$$\text{(iii) } f_3 = 5f_1 = 5 \times 104 \Rightarrow \boxed{f_3 = 520\text{Hz}}$$

EXAMPLE 8-10:

A car is moving at 20m/sec along a straight road with its frequency 500Hz horn sounding. You are standing at the road side. What receding from you at 20m/sec? Take the speed of sound as

340m/sec

Solution:

Given Data:

$$f = 500\text{Hz}, V = 340\text{m/sec}$$

Speed of sound source = $a = 20\text{m/sec}$

Required:

(i) Apparent frequency when car approaches = $f' = ?$

(ii) Apparent frequency when car receding = $f'' = ?$

Calculation:

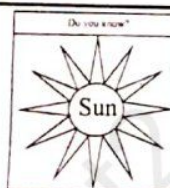
(i) We know that,

$$f' = \left(\frac{V}{V - a} \right) f = \left(\frac{340}{340 - 20} \right) \times 500 \Rightarrow \boxed{f' = 531\text{Hz}}$$

$$\text{(ii) And } f'' = \left(\frac{V}{V + a} \right) f = \left(\frac{340}{340 + 20} \right) \times 500$$

$$\Rightarrow \boxed{f'' = 472\text{Hz}}$$

Explosions, occurs on the surface of the sun due to fission and fusion reactions but we can't hear, why?

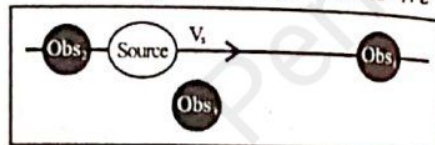


Ans: There is about 150 million kilometer distance between sun and earth. Sound waves are longitudinal waves which needs a material medium for their propagation. But the space between sun and the earth is mostly a vacuum through which sound waves cannot pass. There is not sufficient matter to transfer the sound of explosion on sun surface. Thus we cannot hear the sound of explosion occurs on sun's surface.

Quiz-1: Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.
Ans: If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

Quiz-2: Three stationary observers observe the Doppler shift from a source moving at a constant velocity. The observers are stationed as shown below. Which observer will observe the highest frequency? Which observer will observe the lowest frequency? What can be said about the frequency observed by observer 3?

Ans. (1) Observer (1) will observe the highest frequency
 (2) Observer (2) will observe the lowest frequency
 (3) Observer (3) will hear a higher frequency than the source frequency, but lower than the frequency observed by observer (1), as the source approaches and a lower frequency than the source frequency, but higher than the frequency observed by observer (1), as the source moves away from observer (3).



Exercise

Choose the best possible answer:

1. When a wave goes from one medium to another medium, which one of the following characteristics of the wave remains constant?

- a. Velocity ✓ b. Frequency
 c. Wavelength d. Phase

2. When a transverse wave is reflected from the boundary of a denser to a rarer medium, it undergoes a phase change of:

- a. 0 b. $\frac{\pi}{2}$ ✓ c. π d. 2π

3. If the tension in the string is doubled and its mass per unit length is reduced to half. Then the speed of transverse wave on it is:

- ✓ a. Doubled b. Halved
 c. Constant d. One fourth

Sol: We know that,

$$v = \sqrt{\frac{T}{m}} \quad (1)$$

Now if tension becomes double and mass becomes half, then we have,

$$T' = 2T \text{ and } m' = \frac{m}{2}$$

For " T " and " m ", equation (1) can be written as,

$$v' = \sqrt{\frac{T'}{m'}} \quad (2)$$

Putting the values of " T " and " m " in equation (2), we get,

$$v' = \sqrt{\frac{2T}{\frac{m}{2}}} = \sqrt{\frac{4T}{m}} = \sqrt{\frac{T}{m}} \Rightarrow v' = 2\sqrt{\frac{T}{m}} \quad (3)$$

Putting equation (1) in equation (3), we get,

$$v' = 2v$$

Thus the speed becomes double, so option (a) is correct.

4. Which one of the following properties is not exhibited by the longitudinal waves?

- a. Reflection b. Interference
c. Diffraction ✓ d. Polarization
5. A sounding source and a listener are both at rest relative to each other. If wind blows from the listener towards the source, then which one of the following of sound will change?
a. Frequency ✓ b. Speed
c. Phase d. Wavelength
6. Which one of the following factors has no effect on the speed of sound in a gas?
a. Humidity ✓ b. Pressure
c. Temperature d. Density
7. There is no net transfer of energy by particles of medium in:
a. Longitudinal wave b. Transverse wave
c. Progressive wave ✓ d. Stationary wave
8. Which one of the following could be the frequency of ultraviolet radiation?
a. $1.0 \times 10^6 \text{ Hz}$ b. $1.0 \times 10^9 \text{ Hz}$
c. $1.0 \times 10^{12} \text{ Hz}$ ✓ d. $1.0 \times 10^{15} \text{ Hz}$
9. When a stationary wave is formed then its frequency is:
✓ a. Same as that of the individual waves
b. Twice that of the individual waves
c. Half that of the individual waves
d. $\sqrt{2}$ that of the individual waves
10. The fundamental frequency of a closed organ pipe is f . If both the ends are opened then its fundamental frequency will be:
a. f b. $0.5f$ ✓ c. $2f$ d. $4f$

Sol: Case-1: When pipe is closed then

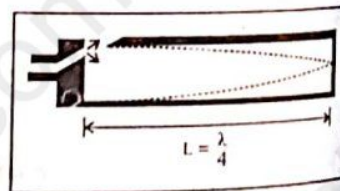
$$L = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4L$$

$$\text{Now } f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \Rightarrow f_1 = \frac{v}{4L} \text{ ————— (1)}$$

Case-2: When pipe is open, then

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{2\lambda}{4}$$

$$\Rightarrow \lambda = \frac{4L}{2}$$



$$\text{Now } f_1 = \frac{v}{\lambda_1} = \frac{v}{(4L/2)}$$

$$\Rightarrow f_1 = 2 \left(\frac{v}{4L} \right) \text{ ————— (2)}$$

Putting equation (1) in equation (2),

we get,

$$f_1 = 2f_1$$

11. If the amplitude of a wave is doubled, then its intensity is:

- a. Doubled b. Halved
✓ c. Quadrupled d. One fourth

Sol: Intensity is proportional to the square of amplitude. So when amplitude becomes double, then intensity will become four times i.e. quadrupled.

12. A sound source is moving towards stationary listener with $1/10^{\text{th}}$ of the speed of sound. The ratio of apparent to real frequency is:

a. $\frac{11}{10}$ b. $\left[\frac{11}{10} \right]^2$ c. $\left[\frac{9}{10} \right]^2$ ✓ d. $\frac{10}{9}$

Sol: Speed of sound = v , speed of source = $a = \frac{v}{10}$

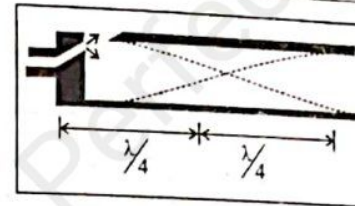
When source is moving towards listener, then we have,

$$f' = \frac{v}{(v - a)} f \text{ ————— (1)}$$

Putting the values in equation (1), we get,

$$f' = \frac{v}{\left(v - \frac{v}{10} \right)} f \Rightarrow \frac{f'}{f} = \frac{v}{\left(\frac{10v - v}{10} \right)} = \frac{10v}{9v}$$

$$\Rightarrow \frac{f'}{f} = \frac{10v}{9v} \Rightarrow \frac{f'}{f} = \frac{10}{9} \text{ so option (d) is correct.}$$



Short Questions

Q1: What is the difference between progressive and stationary waves?

Ans: The difference between progressive and stationary waves are given below:

Progressive Waves	Stationary Waves
1. A disturbance of some kind produced due to a source as a result of which energy is transmitted from one place to another place is called progressive waves.	1. When two waves having the same amplitude and frequency with proper phase relationship travelling with the same speed in opposite direction through a medium are superposed and a wave obtained, is known as stationary or standing wave.
2. Progressive waves propagate in the medium and transmit energy from one point to another.	2. Stationary waves do not propagate, so they do not transmit energy from one place to another place. Stationary wave can occur only in a system which has boundaries such as string or spring stretched between two clamps.
3. Progressive waves are not confined in bounded system but travel unrestricted.	3. Stationary waves of discreet set of frequencies can be produced in any medium when its particles are disturbed.
4. In progressive waves we have no nodes and anti-nodes.	4. In stationary waves we have nodes and anti-nodes.
5. The frequency and wavelength of a progressive is equal to the frequency and wavelength of the source producing the wave.	5. The frequency and wavelength of a stationary wave are equal to that of either of the superposing wave.

Q2: Clearly explain the difference between longitudinal and transverse waves.

Ans: The difference between transverse and longitudinal waves is given below:

Transverse Waves	Longitudinal Waves
1. In transverse waves the particles of the medium execute SHM perpendicular to the direction of the propagation of the waves.	1. In longitudinal waves the particles of the medium execute SHM along the direction of the propagation of the waves.
2. Transverse waves consist of crests and troughs.	2. Longitudinal waves consist of compressions and rarefactions.
3. The distance between two consecutive crests or troughs is called wavelength ' λ '.	3. The distance between two consecutive compressions or rarefactions is called wavelength ' λ '.
4. Transverse waves may be mechanical or electromagnetic in nature.	4. Longitudinal waves are only mechanical waves.
5. Water waves, waves along a string, light waves, radio waves and T.V waves etc are the example of transverse waves.	5. Sound waves, waves along a spring or slinky etc are the examples of longitudinal waves.
6. Transverse waves can be polarized.	6. Longitudinal waves cannot be polarized.
7. Transverse wave equation is given by, $x = x_0 \sin \omega t$ _____ (1)	7. The longitudinal wave equation is given by, $x = x_0 \cos \omega t$ _____ (1)

Q3: How are beats useful in tuning a musical instrument?

Ans: Beats are useful in tuning a musical instrument.

Reason and Explanation:

Beats represents the periodic variation in the loudness of sound which can be heard when two notes of slightly different frequencies are played simultaneously. When two sound sources of slightly dif-

ferent frequencies are sounded at the same time, a single note is heard which rises and falls in intensity. As a result, a loud and fast sound is heard periodically. The phenomenon of beats plays an important role in tuning of musical instruments.

Tuning is the process of adjusting the pitch of one or more tones from musical instruments until they form a desired arrangement. Tuning may be carried out by sounding two instruments and adjusting one of them to match the frequency of the other. Thus when we are tuning, we are trying to match the frequency of one note to another.

Example: Consider a tuning fork and a guitar as shown in the figure. Let " f_A " represents the frequency of tuning fork and " f_B " represents the frequency of guitar. Let the two instruments are sounded simultaneously. In the beginning when " f_A " and " f_B " are slightly different, then beats occurs and we have,

$$N = f_A - f_B \quad (1)$$

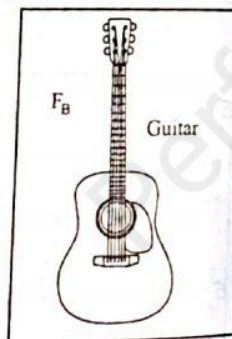
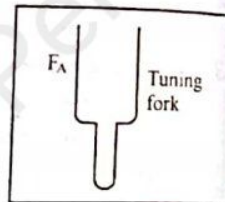
Now for tuning the guitar, we should match the frequency of guitar to the frequency of tuning fork. For this purpose, the tension in the string of guitar is adjusted until the beats disappear. Under such condition, we have,

$$f_A = f_B \text{ and } N = 0$$

Thus the frequencies of both instruments match each other and the guitar is tuned.

Q4: Two wave pulses travelling in opposite direction completely cancel each other as they pass. What happens to the energy possessed by the waves?

Ans: The energy remains conserved when two wave pulses travelling in opposite direction.



Reason and Explanation:

Consider two waves pulses "A" and "B" travelling in opposite direction, as shown in figure (1). Let the two pulses overlap in figure (2). The two pulses cancel the effect of each other and the net displacement becomes zero at the point of superposition (overlapping). Thus the total energy becomes zero at the point of superposition. Now when the two pulses cross each other, they travel in their own direction with same shape and speed as shown in figure (3). Each pulse maintain its initial energy and thus the law of conservation of energy is satisfied in the above process.

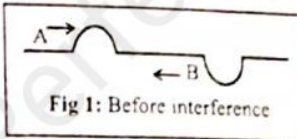


Fig 1: Before interference

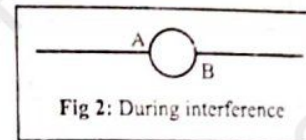


Fig 2: During interference

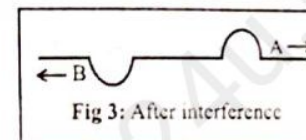


Fig 3: After interference

Q5: What are the conditions of constructive and destructive interference?

Ans: Conditions for Constructive and Destructive Interference:

For constructive and destructive interference, the following conditions must be satisfied.

7. The two waves must be phase coherent.
8. They must arrive at the same place at the same time.
9. The two waves must be travelling in the same direction.
10. The principle of linear superposition must be satisfied.

Extra Special Conditions for Constructive Interference:

They constructive interference takes place only, if the path difference is zero i.e. an integral multiple of " λ ". i.e.

$$\text{Path difference} = m\lambda$$

$$(\text{OR}) \text{ Path difference} = 0, \lambda, 2\lambda, 3\lambda, \dots, m\lambda$$

[Where $m = 0, 1, 2, 3, 4, \dots$]

Extra Special Conditions for Destructive Interference:

The destructive interference takes place only, if the path difference between the two waves is an odd multiple of half of the wavelength i.e.

$$\text{Path difference} = m \frac{\lambda}{2}$$

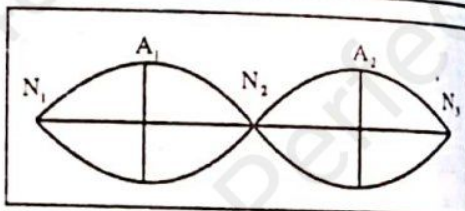
$$(OR) p.d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}, \dots, \left(m + \frac{1}{2}\right)\lambda$$

[Where $m = 0, 1, 2, 3, 4, \dots$]

Q6: How might one can locate the position of nodes and antinodes in a vibrating string?

Ans No-1: We can locate the position of nodes in a vibrating string easily. It is because, at nodes point, the tension is maximum and the amplitude of vibration is zero.

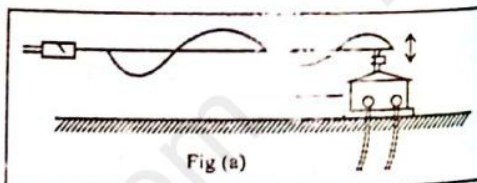
The nodes are at rest and the string appears to us in the form of loops with nodes which are located at the ends of each loop as shown in the figure.



Similarly, we can locate the position of antinodes in a vibrating string easily. It is because, at antinodes point, the tension is minimum and amplitude of vibration is maximum. The string appears to us in the form of loops. At midpoint of each loop, the amplitude is maximum and thus the antinodes are located at the midpoint of each loop above or below the mean position, as shown by "A₁" and "A₂" in the given figure.

Ans No-2: We can locate the position of nodes and antinodes in a vibrating string with the help of Melde's experiment. In this experiment,

stationary waves are produced in the string by means of electric vibrator as shown in figure (a).



Location of Nodes: The points of the string which do not vibrate represent the nodes. The displacement of the particles is zero at the nodes and can be located easily. The nodes are denoted by "N".

Location of Antinodes: The points of the string at which the amplitude of vibration is maximum represents the antinodes. The dis-

placement of the particles is maximum at antinodes and can be located easily. The antinodes are denoted by "A".

Q7: Is it possible for an object which is vibrating transversely to produce sound wave?

Ans: Yes, it is possible for an object which is vibrating transversely to produce sound wave.

Reason and Explanation:

All the stringed and percussion instruments, when played, produce transverse waves. And the energy from these vibrating instruments travel through air in the form of sound waves. The human ear can detect sound whose frequency range is from 20Hz to 20kHz. So if the frequency range of the transverse waves of the musical instrument is in between 20Hz and 20kHz, we can hear the sound of that instrument which were vibrated transversely.

Q8: Why does a sound wave travel faster in solid than in gases?

Ans: Sound waves travel faster in solids than in gases, because for solids the ratio of elastic modulus to density is greater than for gases.

Reason and Explanation:

We know that the speed of sound waves in a medium of elasticity "E" and density "δ" is given by,

$$V = \sqrt{\frac{E}{\delta}} \quad (1)$$

In case of solids, "E" is taken as its Young's modulus while in case of gases, "E" is taken as its bulk modulus.

It is true that density "δ" of solids is greater than that of a gas, but at the same time, the Young modulus of a solid or elasticity of solids is far greater than the bulk modulus of a gas or elasticity of gases. So the term "E/δ" becomes greater for solids than in gases i.e. sound will travel faster in solids than gases.

Q9: Why does the speed of a sound wave in a gas changes with temperature?

Ans: The speed of sound in a gas changes with temperature.

Reason and Explanation:

We know that the speed of sound in a gas is given by,

$$v = \sqrt{\frac{E}{\delta}} \quad (1)$$

Equation (1) shows that the speed of sound is inversely proportional to the square root of the density i.e.

$$v \propto \frac{1}{\sqrt{\delta}} \quad (2)$$

Now we know that when temperature increases, the volume of the gas increases and its density decreases. So according to relation (2), the speed of sound will increase. And when the temperature of gas decreases, its volume decreases and density increases. So according to relation (2), the speed of sound decreases, that is why the speed of a sound wave in a gas change with temperature.

Ans-2: We know that the speed of sound in the gas at temperature "T" is given by,

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (3)$$

In equation (1) $\frac{\gamma R}{M} = \text{constant}$, so equation (1) becomes,

$$v = \sqrt{\text{constant} \times T}$$

$$\Rightarrow v = \text{constant} \times \sqrt{T} \Rightarrow v \propto \sqrt{T} \quad (2)$$

Relation (2) shows that the speed of sound is directly proportional to the square root of absolute temperature of the gas. i.e. when temperature increases, speed of sound also increases. It is because at higher temperature, the molecules of the gas have greater mobility to transport energy from particle to particle.

Relation (2) shows that the speed of sound increases by a factor 0.6m/sec for each one degree rise in temperature. Thus the speed of sound wave in a gas changes with temperature.

Q10: Is it possible for two astronauts to talk directly to one another even if they remove their helmets?

Ans: It is not possible for two astronauts to talk directly to one another even if they remove their helmets.

Reason and Explanation:

It is not possible for two astronauts to talk directly to one another when they remove their helmets in space, because ordinary sound waves are longitudinal mechanical waves. They always require medium for their propagation. Since in space there is no atmospheric gas, so it is not possible for astronaut to talk directly to one another, even if they could remove their helmets in the space. They can use electronic communication system for contact with each other.



Q11: Estimate the frequencies at which a test tube 15cm long resonates when you blow across its lips.

Ans: Length of tube = $L = 15\text{cm} = 0.15\text{m}$

Speed of sound in air = $v = 332\text{m/sec}$

Here the test tube behave like a closed pipe for which the fundamental frequency is given by,

$$f_1 = \frac{v}{4L} \quad (1)$$

Putting the values in equation (1), we get,

$$f_1 = \left(\frac{332}{4 \times 0.15} \right) \text{Hz} \Rightarrow f_1 = 553\text{Hz}$$

The second harmonic is given by,

$$f_2 = 3f_1 = 3 \times 553\text{Hz} \Rightarrow f_2 = 1659\text{Hz}$$

$$\text{Similarly, } f_3 = 5f_1 = 5 \times 553\text{Hz} \Rightarrow f_3 = 2765\text{Hz}$$

$$\& f_n = (2n - 1)f_1 = (2n - 1)553\text{Hz}$$

رسول اللہ ﷺ نے ارشاد فرمایا: جس کی زندگی کا آخری لمحہ لا الہ الا اللہ ہو گا وہ جنت میں ضرور جائے گا۔

حضرت علیؓ کا فرمان ہے: تمہارا ایک رتب ہے پھر بھی تم اسے یاد نہیں کرتے لیکن اس کے کتنے بندے ہیں پھر بھی تم کو نہیں بھولتے۔

oscillation of the particles is maximum is known as anti-node. It is represented by $A_1, A_2, A_3, \dots$, as shown in the figure.

Q4: What do you mean by stationary waves? Show that as the string vibrates in more and more loops, its frequency increases and wavelength decreases.

Ans: See answer of question no 18 in the text.

Q5: Explain Newton's formula for the speed of sound. Show that how it was corrected by a French scientist Laplace.

Ans: See answer of question no 7 in the text.

Q6: Explain the speed of sound in a gas and give all the factors which affect the speed of sound in the air.

Ans: See answer of question no 8 in the text.

Q7: How the speeds of sound in the air varies with temperature and hence show that for each one degree centigrade rise in temperature the speed of sound increases by 0.61 ms^{-1} ?

Ans: See answer of question no 9 in the text.

Q8: What are beats? Explain how they are produced and show that the number of beats per second is equal to the difference in frequencies of the two sources.

Ans: See answer of question no 13 in the text.

Q9: What is Doppler's Effect? Derive expression for the frequencies heard. (a) When the sounding source approaches a stationary listener. (b) When listener moves towards a stationary sounding source.

Ans: See answer of question no 21 in the text.

Q10: What are organ pipes? Show that an open organ pipe is richer in harmonics than a closed organ pipe.

Ans: See answer of question no 20 in the text.

Q11: Explain the vibrations in a closed organ pipe and show that the frequency of third harmonic is $5v/4L$.

Ans: See answer of question no 20 part (a) in the text.

Numerical Problems

Q1: What are the wavelengths of a television station which transmits vision on 500 MHz and sound on 505 MHz respectively? Take speed of electromagnetic wave as $3 \times 10^8 \text{ ms}^{-1}$. (60.0 cm, 59.4 cm)

Solution:

Given data:

$$\text{Frequency} = f_1 = 500 \text{ MHz} = 500 \times 10^6 \text{ Hz}$$

$$\text{Frequency} = f_2 = 505 \text{ MHz} = 505 \times 10^6 \text{ Hz}$$

$$\text{Speed of electromagnetic waves} = c = 3 \times 10^8 \text{ m/sec}$$

$$\text{Mega} = 10^6$$

Required:

$$(a) \text{ Wavelength} = \lambda_1 = ? \quad (b) \text{ Wavelength} = \lambda_2 = ?$$

Calculation:

(a) We know that,

$$v = f\lambda \Rightarrow c = f\lambda \Rightarrow \lambda = \frac{c}{f} \Rightarrow \lambda_1 = \frac{c}{f_1} \quad (1)$$

$$1 \text{ m} = 100 \text{ cm}$$

Putting the values in equation (1), we get,

$$\lambda_1 = \frac{3 \times 10^8}{500 \times 10^6} \Rightarrow \lambda_1 = 0.006 \times 10^2 \text{ m} = 0.6 \text{ m} = 60 \text{ cm}$$

$$\Rightarrow \lambda_1 = 60 \text{ cm}$$

$$(b) \text{ Now } \lambda_2 = \frac{c}{f_2} \quad (2)$$

Putting the values in equation (2), we get,

$$\lambda_2 = \frac{3 \times 10^8}{505 \times 10^6} \Rightarrow \lambda_2 = 0.00594 \times 10^2 \text{ m} = 0.594 \text{ m} = 59.4 \text{ cm}$$

$$\Rightarrow \lambda_2 = 59.4 \text{ cm}$$

$$\lambda_1 = 60 \text{ cm} \quad \text{and} \quad \lambda_2 = 59.4 \text{ cm}$$

Q2: A person on the sea shore observes that 48 waves reach the shore in one minute. If the wavelength of the waves is 10 meter, then find the velocity of the waves. (8 ms^{-1})

Solution: Given data:

$$\text{Number of waves} = 48, \text{ Time} = t = 1 \text{ min} = 60 \text{ sec}$$

$$\text{Frequency of waves} = \frac{\text{no. of waves}}{\text{time}}$$

$$\Rightarrow f = \frac{48}{60} = 0.8 \text{ Hz} \Rightarrow f = 0.8 \text{ Hz}$$

Wavelength of the waves = $\lambda = 10 \text{ m}$

Required:

Velocity of waves = $v = ?$

Calculation:

We know that, $v = f\lambda$ _____ (1)

Putting the values in equation (1), we get,

$$v = 0.8 \times 10 \text{ m/sec} = 8 \text{ m/sec} \Rightarrow \boxed{v = 8 \text{ m/sec}}$$

Q3: In a ripple tank 500 waves pass through a certain point in 10 seconds, if the speed of the wave is 3.5 ms^{-1} , then find the wavelength of the waves. (7cm)

Solution:

Given data:

Number of waves = 500, Time = $t = 10 \text{ sec}$

$$\text{Frequency of waves} = f = \frac{\text{no. of waves}}{\text{time}} \Rightarrow f = \frac{500}{10} = 50 \text{ Hz}$$

Speed of waves = $v = 3.5 \text{ m/sec}$

Required: Wavelength of the waves = $\lambda = ?$

Calculation:

$$\text{We know that, } v = f\lambda \Rightarrow \lambda = \frac{v}{f} \text{ _____ (1)}$$

$$\boxed{1 \text{ m} = 100 \text{ cm}}$$

Putting the values in equation (1), we get,

$$\lambda = \frac{3.5}{50} \Rightarrow \lambda = 0.07 \text{ m} \Rightarrow \boxed{\lambda = 7 \text{ cm}}$$

$$\boxed{\text{Wavelength} = \lambda = 7 \text{ cm}}$$

Q4: A string of a guitar is 1.3m long vibrates with 4 nodes, 2 of them at the two ends. Find the wavelength and speed of the wave in the string if it vibrates at 500Hz.

Solution:

Given data:

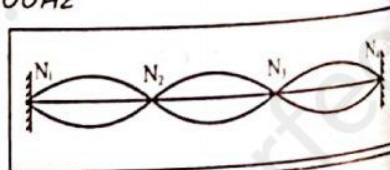
In case of 4 nodes, the string will vibrate in three loops, i.e. $n = 3$

Length of string = $L = 1.3 \text{ m}$, $f_3 = 500 \text{ Hz}$

Required:

(i) Wavelength = $\lambda_3 = ?$

(ii) Speed of the wave = $v = ?$



Calculation:

(i) We know that,

$$\lambda_n = \frac{2L}{n} \Rightarrow \lambda_3 = \frac{2L}{3} \text{ _____ (1)}$$

Put $n = 3$, for 3 loops

Putting the values in equation (1), we get,

$$\lambda_3 = \frac{2 \times 1.3}{3} = 0.866 \text{ m} \Rightarrow \lambda_3 = 0.866 \text{ m}$$

(ii) As $v = f\lambda \Rightarrow v = f_3\lambda_3$ _____ (2)

Putting the values in equation (2), we get,

$$v = (500 \times 0.866) \text{ m/sec} \Rightarrow v = 433 \text{ m/sec}$$

$$\boxed{\lambda_3 = 0.866 \text{ m}} \quad \boxed{v = 443 \text{ m/sec}}$$

Q5: A tension of 400N causes a 300g wire of length 1.6m to vibrate with a frequency of 40Hz. What is the wavelength of the transverse waves? (1.15m)

Solution:

Given data:

Tension in wire = $T = 400 \text{ N}$,

Mass of wire = $m = 300 \text{ gm} = 0.3 \text{ kg}$

Length of wire = 1.6 m , Frequency of vibration = $f = 40 \text{ Hz}$

Required:

Wavelength of the waves = $\lambda = ?$

Calculation:

$$\text{We know that, } v = f\lambda \Rightarrow \lambda = \frac{v}{f} \text{ _____ (1)}$$

We also know that speed of transverse waves in a string is given by,

$$v = \sqrt{\frac{T}{m}} \text{ _____ (2)}$$

Putting the values in equation (2), we get,

$$v = \sqrt{\frac{400 \times 1.6}{0.3}} = \sqrt{\frac{640}{0.3}} = 46 \text{ m/sec} \Rightarrow v = 46 \text{ m/sec}$$

Now putting the values of " f " and " v " in equation (1), we get,

$$\lambda = \left(\frac{46}{40}\right) \text{ m} = 1.15 \text{ m} \Rightarrow \lambda = 1.15 \text{ m}$$

$$\Rightarrow \boxed{\text{Wavelength} = \lambda = 1.15 \text{ m}}$$

Q6: Compare the theoretical speeds of sound in hydrogen ($M_H = 2.0 \text{ g/mol}$, $\gamma_H = 1.4$) with helium ($M_{He} = 4.0 \text{ g/mol}$, $\gamma_{He} = 1.66$ and $R = 8334 \text{ J/k}^\circ\text{mol}$) at 0°C .
($v_{He} = 0.77 v_H$)

Solution:

Given data:

Mass of hydrogen $M_H = 2.0 \text{ g/mol}$

Value of " γ " for hydrogen = $\gamma_H = 1.4$

Mass of helium = $M_{He} = 4.0 \text{ g/mol}$

Value of " γ " for helium = $\gamma_{He} = 1.66$

Gas constant = $R = 8334 \text{ J/k}^\circ\text{mol}$

Temperature = $0^\circ\text{C} = 0 + 273 = 273 \text{ K}$

Required: Comparison of speed of "H" with "He" = ?

Calculation:

We know that the speed of sound is given by,

$$v = \sqrt{\frac{\gamma RT}{M}} \quad (1)$$

For hydrogen, equation (1) becomes,

$$v_H = \sqrt{\frac{\gamma_H RT}{M_H}} \quad (2)$$

For helium, equation (1) becomes,

$$v_{He} = \sqrt{\frac{\gamma_{He} RT}{M_{He}}} \quad (3)$$

Dividing equation (3) by equation (2), we get,

$$\frac{v_{He}}{v_H} = \sqrt{\frac{\gamma_{He} RT}{M_{He}}} \times \sqrt{\frac{M_H}{\gamma_H RT}} \Rightarrow \frac{v_{He}}{v_H} = \sqrt{\frac{M_H \gamma_{He}}{M_{He} \gamma_H}} \quad (4)$$

Putting the values in equation (4), we get,

$$\frac{v_{He}}{v_H} = \sqrt{\frac{2 \times 1.66}{4 \times 1.4}} = \sqrt{\frac{3.32}{5.6}} = 0.77$$

$$\Rightarrow \frac{v_{He}}{v_H} = 0.77 \Rightarrow v_{He} = 0.77 v_H \Rightarrow v_{He} = 0.77 v_H$$

Q7: The speed of sound in air at 0°C is 332 ms^{-1} . What will be the speed of sound at 22°C ? (345.2 ms^{-1})

Solution:

Given data:

Initial temperature = $T_0 = 0^\circ\text{C} = 273 \text{ K}$

Speed of sound at $0^\circ\text{C} = v_0 = 332 \text{ m/sec}$

Final temperature = $T = 22^\circ\text{C} = 22 + 273 = 295 \text{ K}$

Required: Speed of sound at $22^\circ\text{C} = v = ?$

Calculation: We know that,

$$\frac{v}{v_0} = \sqrt{\frac{T}{T_0}} \Rightarrow v = \sqrt{\frac{T}{T_0}} \times v_0 \quad (1)$$

Putting the values in equation (1), we get,

$$v = \sqrt{\frac{295}{273}} \times 332 \Rightarrow v = 345.2 \text{ m/sec}$$

$$\text{Velocity} = v = 345.2 \text{ m/sec}$$

Q8: Two tuning forks "P" and "Q" give 4 beats per second. On loading "Q" lightly with wax, we get 3 beats per second. What is the frequency of "Q" before and after loading if the frequency of "P" is 512 Hz . (516 Hz , 515 Hz)

Solution:

Given data: Number of beats before loading = $n = 4$

Number of beats after loading = $n' = 3$

Frequency of tuning fork "P" = $f_p = 512 \text{ Hz}$

Required:

(i) Frequency of tuning fork "Q" before loading = $f_q = ?$

(ii) Frequency of tuning fork "Q" after loading = $f'_q = ?$

Calculation:

(i) We know that, $n = f_2 - f_1$

$$\Rightarrow n = f_q - f_p$$

$$\text{Put } n = 4, f_p = 512 \text{ Hz}$$

$$\Rightarrow 4 = f_q - 512 \Rightarrow f_q = 4 + 512$$

$$\Rightarrow f_q = 516 \text{ Hz}$$

(ii) After loading, we put $n' = 3, f_p = 512 \text{ Hz}$

So we get, $n' = f'_q - f_p$

$$\Rightarrow 3 = f'_q - 512 \Rightarrow f'_q = 3 + 512 \Rightarrow f'_q = 515 \text{ Hz}$$

Q9: On a sunny day, the speed of sound in the air is 340 ms^{-1} , tuning forks A and B are sounded simultaneously. The wavelength of the sounds emitted are 1.5 m and 1.68 m respectively. How many beats will produce per second? (24 beats approx)

Solution: **Given data:**

Speed of sound = $v = 340 \text{ m/sec}$

Wavelength of tuning fork "A" = $\lambda_A = 1.5\text{m}$

Wavelength of tuning fork "B" = $\lambda_B = 1.68\text{m}$

Required: Number of beats = $n = ?$

Calculation: We know that, $n = f_A - f_B$

$$\Rightarrow n = \frac{v}{\lambda_A} - \frac{v}{\lambda_B} = \left[\frac{1}{\lambda_A} - \frac{1}{\lambda_B} \right] \Rightarrow n = 340 \left[\frac{1}{1.5} - \frac{1}{1.68} \right]$$

$$\Rightarrow n = 340 [0.666 - 0.595] = 340(0.071)$$

Ans. $n = 24$ beat approximately

$$v = f\lambda, f = \frac{v}{\lambda}$$

Q10: A sound source vibrates at 200Hz and is receding from a stationary observer at 18ms^{-1} . If the speed of sound is 331ms^{-1} then what frequency does the observer hear? (189.68Hz)

Solution:

Given data: Frequency of sound source = $f = 200\text{Hz}$

Velocity of sound = $v = 331\text{m/sec}$,

Velocity of source = $a = 18\text{m/sec}$

Required: Apparent frequency = $f' = ?$

Calculation: When sound source is moving away from the stationary observer [listener], then we have,

$$f' = \left(\frac{v}{v + a} \right) f \quad (1)$$

Putting the values in equation (1), we get,

$$f' = \frac{331 \times 200}{(331 + 18)} \Rightarrow f' = 189.68\text{Hz}$$

Ans. $f' = 189.68\text{Hz}$

Q11: Suppose a train that has a 150Hz horn is moving at 35.0m/s in still air on a day when the speed of sound is 340m/s . (a) What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes? (b) What frequency is observed by the train's engineer traveling on the train? (167Hz, 136Hz)

Solution:

Given data: Frequency of horn = $f = 150\text{Hz}$,

Velocity of train = $a = 35\text{m/sec}$

Velocity of sound = $v = 340\text{m/sec}$

Required:

(a) Apparent frequency of horn, when

(i) Train approaches the person = $f' = ?$

(ii) Train moving away from person = $f'' = ?$

(b) Frequency observed by engineer traveling on the train = ?

Calculation:

(a) (i) When train approaches the person, then the apparent frequency is given by,

$$f' = \left(\frac{v}{v - a} \right) f \quad (1)$$

Putting the values in equation (1), we get,

$$f' = \left(\frac{340}{340 - 35} \right) \times 150\text{Hz} = \left(\frac{340 \times 150}{305} \right) \text{Hz} \Rightarrow f' = 167\text{Hz}$$

(ii) When train is moving away from person, then equation (1) can be written as,

$$f'' = \left(\frac{v}{v + a} \right) f \quad (2)$$

Putting the values in equation (2), we get,

$$f'' = \left(\frac{340}{340 + 35} \right) 150\text{Hz} = \left(\frac{340 \times 150}{375} \right) \Rightarrow f'' = 136\text{Hz}$$

(b) As engineer is present on the train, so both are the part of the same system. So there will be no apparent change for engineer in frequency of horn.

Q12: The first overtone of an open organ pipe has the same frequency as the first overtone of a closed pipe 3.6m in length. What is the length of the open organ pipe? (7.3m)

Solution:

Let the frequency of 1st overtone of an open organ pipe = f_1 and let the frequency of 1st overtone of the closed pipe = f_1'

Length of open organ pipe = $L = ?$

Length of closed organ pipe = $L' = 3.6\text{m}$

By given condition,

$$f_1 = f_1' \Rightarrow \frac{v}{\lambda_1} = \frac{v}{\lambda_1'} \Rightarrow \frac{v}{2L} = \frac{v}{4L'} \Rightarrow \frac{1}{2L} = \frac{1}{4L'}$$

$$\Rightarrow 2L = 4L' \Rightarrow L = 2L' = 2 \times 3.6 \Rightarrow L = 7.2\text{m}$$

For open pipe,

$$\lambda_1 = 2L$$

For closed pipe,

$$\lambda_1' = 4L'$$

Q13: What length of open pipe will produce a frequency of 1200Hz as its first overtone on a day when the speed of sound is 340ms⁻¹? (28.3cm)

Solution: Given data: Speed of sound = $v = 340\text{m/sec}$

First overtone = $f_2 = 1200\text{Hz}$

Required:

Length of open pipe = L ?

Calculation:

In case of 1st overtone, we have,

$$f_2 = 2 \left(\frac{v}{2L} \right) \Rightarrow f_2 = \frac{v}{L} \Rightarrow L = \frac{v}{f_2} \quad (1)$$

Putting the values in equation (1), we get,

$$L = \left(\frac{340}{1200} \right) \text{m} = 0.283\text{m}$$

$$\Rightarrow L = (0.283 \times 100)\text{cm}$$

$$100\text{cm} = 1\text{m}$$

$$\Rightarrow L = 28.3\text{cm}$$

Ans: Length of open pipe = $L = 28.3\text{cm}$

- ⑤ رسول اللہ ﷺ نے ارشاد فرمایا: اللہ تعالیٰ نے تمہاری شہادتیں میں رکھی۔
- ⑤ رسول اللہ ﷺ نے ارشاد فرمایا: جس شخص کے جسم سے روح اس حال میں نکلی کہ وہ تین چیزوں سے بڑی تھا تو وہ جنت میں داخل ہوگا: تکبر، قرض اور خیانت۔
- ⑤ ہم تکلیف کے جس مقام پر بھی ہوں، ہم سے آگے ضرور کچھ لوگ ہوتے ہیں۔ انہیں دیکھ کر ہمیں شکر کی لالچھی تمام لگتی چاہیے تاکہ راستہ سہولت سے کٹ سکے۔
- ⑤ اگر تم اس وقت مسکرا سکتے ہو جب تم پوری طرح نوٹ چکے ہو تو یقین چاہو کہ دنیا میں تمہیں کبھی کوئی توفیق نہیں سکتا۔
- ⑤ خوبصورتی تو فتنہ اور تبدیل ہو جاتی ہے لیکن سیرت نہ تو فتنہ ہوتی ہے نہ تبدیل۔
- ⑤ مشکوں سے لونا نہیں بلکہ سنہبل کر گزرتا سیکھو نہ آپ کی شخصیت تہا ہو جائے گی۔
- ⑤ حضرت عثمان غنی کا قول ہے "محبوبین کی کمائیں بھی دل کی طرح نرم ہو جاتی ہیں۔ ان کے روکنے کھڑے ہو جاتے ہیں۔ ان کے دل اور جلد نرم ہو جاتے ہیں اور یاد خدا سے ان کو راحت ہوتی ہے۔"

Q1: What is meant by the dual nature of light? Discuss the history about the nature of light in detail.

Ans: Dual Nature of Light:

Light is a form of energy with the help of which we can see objects in our surrounding. The light shows dual nature i.e. in some cases, it behaves like a wave and in some cases it behaves like a particle. In case of Compton effect, photoelectric and pair production phenomenon's, the light behaves like a particle. While in case of interference, polarization, reflection, diffraction, the light behaves like a wave. Thus the light shows dual nature.

History about the Nature of Light:

As the light shows dual nature, so in the beginning it was a great problem for the scientists to explain its real nature. The scientists gave their ideas about light in the form of following theories:

1. Newton's Corpuscular Theory of Light:

The corpuscular theory of light was put forward by Newton. It is also known as particle theory of light. According to Newton, light travels from one place to another in the form of extremely small particles known as corpuscles. On the basis of this theory, Newton was able to explain the phenomenon of reflection and refraction of light.

2. Wave Theory of Light:

This theory was put forward by Huygen. According to this theory, light travels from one place to another in the form of waves. This theory was able to explain reflection and refraction. However, this theory was failed to explain the phenomenon of polarization, diffraction and interference of light.

3. Maxwell's Theory of Light:

This theory was put forward by Maxwell in 1873. According to

Maxwell, light is electromagnetic in nature. It consists of both electric vector " $\vec{E}$ " and magnetic vector " $\vec{H}$ " and both are oscillating perpendicularly to the direction of propagation of the waves. The speed of light is $3 \times 10^8 \text{ m/sec}$. This theory tells us that light waves needs no material medium for their propagation.

Beside it, "Thomas Young" performed an experiment on the basis of which he explained the diffraction of light.

In 1907, "Plank" introduced the "Quantum theory" of light. According to this theory, light travels from one place to another in the form of pockets or bundles of energy which are known as Quanta or photons. The energy of photon is equal to " hf ". The rest mass of photon is zero.

The Quantum theory was able to explain the phenomenon of photoelectric effect, Compton effect and pair production.

Q2: (a) What is wave front? What is the difference between spherical and plane wave fronts? (b) State and explain Huygens principle.

Ans: (a) Wavefront:

The locus of points in a medium with same phase of vibration is known as wave front.

Explanation:

When we throw a piece of stone into a pond of water, then circular waves are produced. Now the crest produced here is a wave front, because all the points on the crest will have same phase of vibrations.

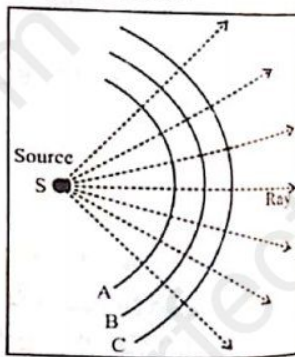
Types of Wavefront:

There are two types of wave fronts which are given below:

(i) Spherical Wave Front:

A wave front which have concentric sphere with centre at its source is known as spherical wave front, as shown in the figure.

In case of spherical wave front, the energy of the wave is transmitted equally in all directions as the wave propagates. The direction in which the energy travel is known as the ray.



(ii) Plane Wavefront:

At very large distance, some portion of spherical wavefront appears as plane. Such straight portion of the wavefront is known as plane wavefront.

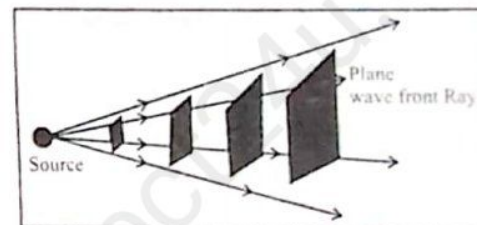
The plane surfaces are parallel to each other. The rays are perpendicular to the plane surfaces.

(b) Huygen's Principle:

According to this principles, "all points on a spherical wavefront acts as point sources for the production of secondary spherical wave which are known as wavelets."

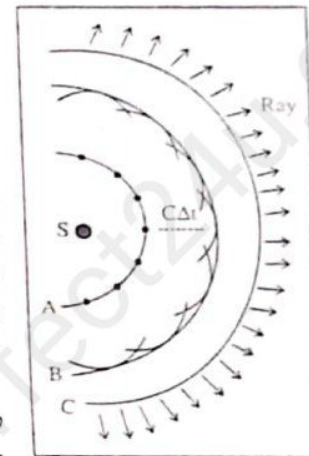
Explanation:

Consider a source "S" which produces a wavefront "A" as



shown in the figure. Let us take several points in the form of dots on the wavefront "A", as shown in the figure. These dots act as secondary sources and each produces spherical wave, as shown in the figure. Thus using these points as secondary sources, we can draw spherical circles each having radius " $C\Delta t$ ", where "C" is the velocity of light and " Δt " represents the time interval during which a wave propagates from one wavefront to another.

Thus the wavefront "A" give rises to another wavefront "B" as shown in the figure. Similarly, the wavefront "B" will give rise to another wavefront "C" and so on. This phenomena also occurs in case of plane wavefront. Such principle is known as Huygen's principle.



Q3: What is meant by coherent and incoherent sources of light?
Explain how coherent sources can be produced?

Ans: (i) Coherent Sources:

Two sources of light are said to be coherent, if there is a constant

phase relationship between the waves produced by them.

(ii) Incoherent Sources:

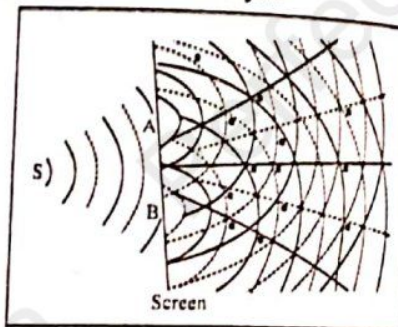
Two sources of light are said to be incoherent sources, if there is no constant phase relationship between the waves produced by them.

Production of Coherent Sources:

When light from a single source "S" is allowed to fall on a screen having two narrow openings, then the light which emerges out through the narrow openings becomes coherent. Thus the openings or slits of the screen behaves as coherent sources of light.

Examples:

1. The two vibrating spheres that produce the water waves are coherent sources because the spheres vibrate up and down together.
2. The two slits in Young's double slit experiment are coherent sources because the light from these sources originates from the same primary source.



Q4: (a) What is interference of light? (b) Discuss the types of interference. (c) Discuss the conditions for interference.

Ans: (a) Interference of Light: (ETEA 2012-2013)

The superposition of two coherent light waves travelling along the same medium is known as interference of light.

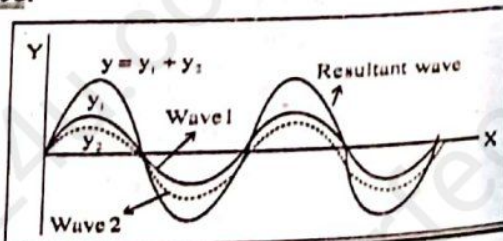
(b) Types of Interference of Light:

There are two types of interference which are given below:

(i) Constructive Interference:

Waves are said to be interfere constructively when they reinforce the effect of each other.

Due to constructive interference, the resultant in-

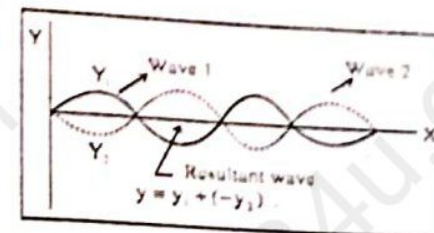


tensity of light increases and as a result bright bands [fringes] can be seen on the screen.

For constructive interference, the crest of one wave should coincide the trough of the other wave.

(ii) Destructive Interference:

Waves are said to be interfere destructively, if they cancel the effect of each other. Due to destructive interference, the intensity of light decreases and as a result dark bands [fringes] are produced on the screen. In case of destructive interference, the amplitude of the resultant wave will be less than either of the individual waves. For destructive interference, the crest of one wave coincide the trough of the other wave.



(c) Conditions for Interference: (ETEA 2006-2008)

1. The waves having same wavelength and period should be emitted continuously from the two sources.
2. The distance between the sources should be minimum.
3. The amplitude of the two waves should be the same.
4. Both light sources should be narrow.
5. For constructive interference, the path difference between the two waves is an integral multiple of the wavelength i.e.

$$P.d = m\lambda \quad (1)$$

Where, $m = 0, 1, 2, 3, \dots$

And $\lambda =$ wavelength of light coming from the sources.

6. For destructive interference, the path difference between the two waves is an odd integral multiple of half of wavelength i.e.

$$P.d = \left(m + \frac{1}{2}\right)\lambda \quad (2)$$

Where $m = 0, 1, 2, 3, \dots$

And $\lambda =$ wavelength of light

Do you know!

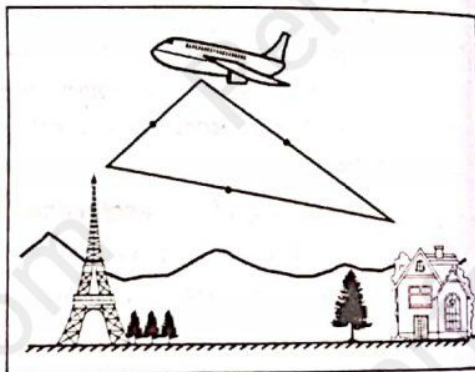
Different colors can be seen on the soap bubble whenever the light of the sun is falling on its surface. It is because the colors seen in a soap bubble arise from interference of light reflecting from the front and back surfaces of the thin soap film.



For your information:

Distortion of the picture on a television receiver:

This occurs when an aircraft passes low overhead. The waves traveling directly from the transmitting antenna to the receiving aerial, interferes with the waves reflected from the aircraft. Usually the reflected wave is much weaker and so the interference is never completely destructive as a result the distortion of the picture on television screen takes place.



Q5: Describe the experimental arrangement for the production of interference fringes by Young's double slits method, and get an expression for the fringes space.

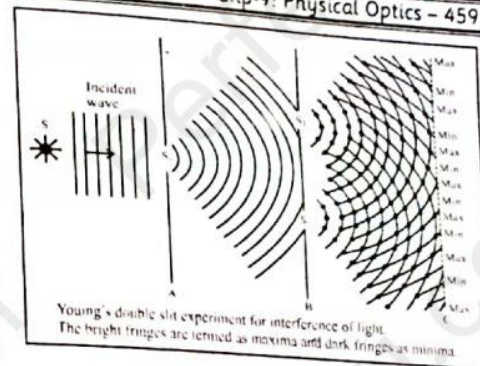
ETEA 2009, 2016, BISE Mardan 2019

Ans. The Young's double slits experiment gives the experimental verification of wave theory of light. This experiment was performed by Thomas Young in 1801 for studying the interference phenomenon of light.

Experimental Arrangement:

The experimental arrangement consists of light source "S", two slits "S₁" and "S₂" and a screen as shown in the figure.

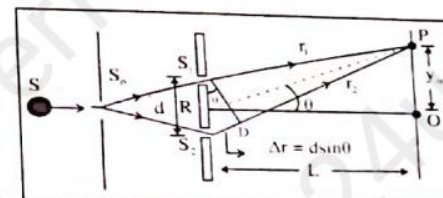
A point "P" on the screen is situated at a distance "r₁" and at a distance "r₂" from slit "S₂" respectively. While the separation between the two slits is represented by "d". There is another narrow slit "S₀" placed closer to the source of monochromatic light, as shown in the figure.



Now in the given arrangement, a beam of monochromatic light is allowed to pass through the narrow slit "S₀" and then through slits "S₁" and "S₂", as shown in the figure. The waves pass through "S₁" and "S₂" produce interference at point "P" on the screen.

Path Difference: (ETEA 2013, 2016)

From the figure, we see that, $r_2 > r_1$. The difference between them is known as the path difference. To find the path difference, we draw a perpendicular from "S₁" on "r₂", as shown in the figure. Now in "ΔS₁DS₂", we have,



$$\sin \theta = \frac{\Delta r}{d} \Rightarrow \Delta r = d \sin \theta \quad (1)$$

Equation (1) represents the path difference. Now there will be constructive interference, if

$$\Delta r = d \sin \theta = m \lambda \quad (2)$$

Where, $m = 0, \pm 1, \pm 2, \pm 3, \dots$

And λ = wavelength

Now there will be destructive interference if

$$\Delta r = d \sin \theta = \left(m + \frac{1}{2}\right) \lambda \quad (3)$$

Position of Bright Fringes [Band] and Dark Fringes on the Screen:
In order to find out the position of bright band [fringes] or dark fringes on the screen we use the triangle "ΔROP", in which,

$$\tan(i) = \frac{y_m}{L} \Rightarrow y_m = L \tan(i) \quad (4)$$

Now if $L \gg d$ and point "P" is very close to point "O", we put $\tan(i) = \sin(i)$ so equation (4) becomes,

$$y_m = L \sin(i) \quad (5)$$

Now for constructive interference, we have,

$$d \sin(i) = m\lambda$$

$$\Rightarrow \sin(i) = \frac{m\lambda}{d} \quad (6)$$

Putting equation (6) in equation (5), we get,

$$y_m = L \frac{m\lambda}{d} \Rightarrow y_m = \frac{L}{d} m\lambda \quad (7)$$

Equation (7) represents the position of m th bright fringe on the screen. Now for destructive interference, we have,

$$d \sin(i) = \left(m + \frac{1}{2}\right)\lambda \Rightarrow \sin(i) = \frac{1}{d} \left(m + \frac{1}{2}\right)\lambda \quad (8)$$

Putting equation (8) in equation (5), we get,

$$y_m = \frac{L}{d} \left(m + \frac{1}{2}\right)\lambda \quad (9)$$

Equation (9) represents the position of m th dark fringe on the screen.

Fringe Width: (ETEA 2014)

The distance between two consecutive bright fringes is known as fringe width or fringe spacing.

Now for finding the fringe spacing for dark fringe, we subtract 1st order dark fringe from 2nd order dark fringe i.e.

Fringe spacing = $y_2 - y_1$

$$\Rightarrow [\text{Fringe spacing}]_{\text{Dark}} = \frac{5L\lambda}{2d} - \frac{3L\lambda}{2d}$$

$$\Rightarrow (\text{Fringe spacing})_{\text{Dark}} = \frac{L\lambda}{d} \quad (10)$$

Similarly for bright fringes, we have,

(Fringe spacing)_{Bright} = $y_2 - y_1$

$$\Rightarrow (\text{Fringe spacing})_{\text{Bright}} = \frac{2L\lambda}{d} - \frac{L\lambda}{d}$$

$$\Rightarrow (\text{Fringe spacing})_{\text{Bright}} = \frac{L\lambda}{d} \quad (11)$$

$$y_m = \frac{L}{d} \left(m + \frac{1}{2}\right)\lambda$$

For 1st order we put $m = 1$

$$\text{So } y_1 = \frac{3L\lambda}{2d}$$

For $m = 2$

$$y_2 = \frac{5L\lambda}{2d}$$

Uses of Young's Double Slits Experiments:

1. It gives us the experimental verification of wave theory of light.
2. It can be used to study the phenomenon of interference.
3. It is used for the determination of fringe width.
4. It can be used for finding the wavelength of light.

$$y_m = \frac{L}{d} m\lambda$$

For $m = 1$

$$y_1 = \frac{L}{d} \lambda$$

For $m = 2$

$$y_2 = \frac{2L}{d} \lambda$$

Quiz:

Young's double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distant car?

Ans. No, two independent light sources do not have coherent phase.

Q6: Explain the interference effect produced by thin film.

Ans. Thin Film:

A layer of extremely small thickness of certain transparent medium is known as thin film.

Examples of Thin Film:

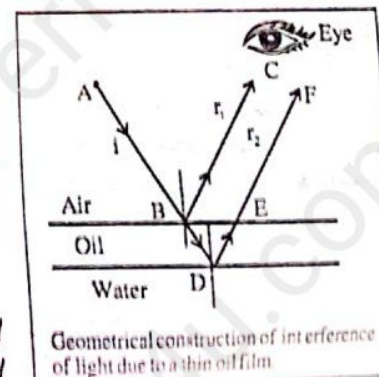
Surface of soap bubbles, a thin layer of oil on the surface of water, cracks in glass plate etc are the examples of thin film in which phenomenon of interference takes place.

Explanation:

Consider a thin film of a refracting medium, as shown in the figure.

Let a beam "AB" of monochromatic light is allowed to fall on the upper surface of the film. At point "B" the incident ray is divided into two parts. One part is reflected in the form of beam "BC", while the other part is refracted in the form of beam "BD" as shown in the figure.

Now at "D" the ray "BD" is divided into two parts. One part is refracted



and the other part is reflected in the form of beam "DE". At "E" the beam is refracted in the form of ray "EF" as shown in the figure. As the beam "BC" and "EF" are the parts of the same primary beam "AB", so both beams are phase coherent. As the film is thin, so the separation between the beam "BC" and "EF" will be very small. Thus both beams superpose each other and as a result, interference takes place which can be observed with eye, as shown in the figure.

Now if the two beams reinforce each other, then constructive interference will take place and the film will appear as bright.

And if the two beams cancel each other, then destructive interference will take place and the film will appear as dark.

Film of Irregular Thickness:

If we allowed a white light to fall on a film of irregular thickness at all possible angles, then the interference can be observed due to each spectral colour separately. It is quite possible that at a certain place on the film, its thickness and the angle of incidence of light are such that the condition of destructive interference of one colour is being satisfied. Hence that portion of the film will exhibit the remaining constituent colours of white light.

Q7: Describe the construction and working of Michelson's interferometer.

ETEA 2006, 2010

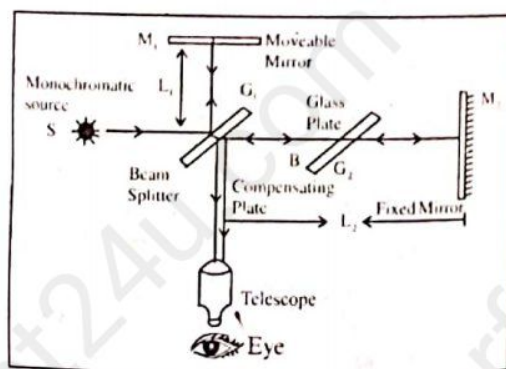
Ans. Michelson's interferometer is an optical instrument which is used to study the interference of light waves and find its wavelength.

Principle:

It splits a light beam into two parts and then recombines them to form an interference pattern after they have travelled over different paths.

Construction:

It consists of the following parts:



1. Source of monochromatic light.
2. Two plane mirrors M_1 and M_2 .
3. Half silvered glass plate " G_1 ".
4. Glass plate [compensator plate] " G_2 ".

Working:

In the given arrangement, a beam of monochromatic light originates from the source "S" as shown in the figure. This beam of light is then allowed to fall on a glass plate " G_1 " which divides this ray into two parts. One of these rays is directed towards the mirror " M_1 " and the other is directed towards the mirror " M_2 ". These rays are reflected from the mirrors and travel back towards the glass plate " G_1 ". Both rays transmit through " G_1 " and enter the eye through telescope. Both waves superpose each other and thus the phenomenon of interference takes place.

Now if the paths " L_1 " and " L_2 " are the same, then we will observe constructive interference. As a result bright fringes will be produced which can be seen with the help of telescope, as shown in the figure. We can change the length of path " L_1 " by moving the mirror " M_1 " and as a result, we will observe both constructive and destructive interference.

Alternate Bright and Dark Fringes:

If the mirror " M_1 " is moved through a distance " $\frac{\lambda}{4}$ " backward, then the path difference between the two beams will be equal to " $\frac{\lambda}{2}$ " and destructive interference will take place and thus we will observe dark fringe.

If the mirror " M_1 " is further moved through a distance " $\frac{\lambda}{4}$ " then the path difference will become $\left[\frac{\lambda}{2} + \frac{\lambda}{2} = \lambda\right]$ " λ " and as a result constructive interference will take place. So we will observe bright fringe.

In this way, if we move the mirror " M_1 " slowly through distance " $\frac{\lambda}{4}$ " each time, then bright and dark fringes will appear alternately.

Determination of Wavelength:

To determine the wavelength " λ " of the given light, let " m " be the total number of observed fringes. Now if the mirror " M_1 " is moved backward through distance " $\frac{\lambda}{4}$ " each time, then the total distance " p " is given by,

$$p = m \left(\frac{\lambda}{4} + \frac{\lambda}{4} \right) \Rightarrow p = \frac{m\lambda}{2} \Rightarrow \lambda = \frac{2p}{m} \quad (1)$$

Using equation (1), we can calculate the wavelength of given light for given values of " p " and " m ".

Uses of Michelson Interferometer:

1. It is used to observe the interference of light.
2. It is used to calculate the wavelength of light.

QB: Define and explain the diffraction of light.

ETEA 2005, 2013

Ans. Diffraction of Light:

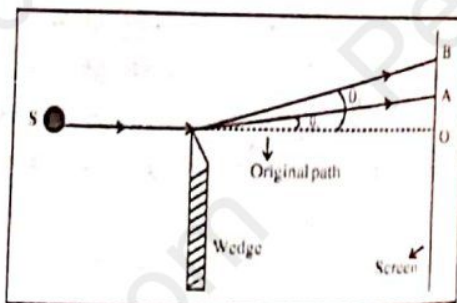
The spreading of light waves round the edges of a narrow opening is known as diffraction. (OR)

The spreading of light into the region behind an obstacle is known as diffraction. (OR)

The deviation of light from its original path is known as diffraction.

Explanation:

Consider a wedge placed in the path of a beam of light coming from its source " S " as shown in the figure. When the beam of light strikes the wedge, it deviates from its path through an angle θ_1, θ_2 etc as shown in the figure. As a result, the original position " O " on the screen will appear dark and we will observe the diffraction fringes at points " A " and " B " above the central point " O ". Such deviation of light from its original path is known as diffraction of light.



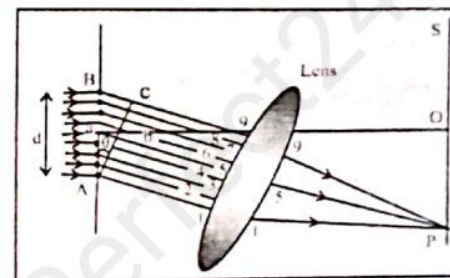
جب اللہ تعالیٰ کا حکم ہو جاتا ہے تو سب کچھ بن جاتے ہیں اور مانگنے کا طریقہ بھی آجاتا ہے۔

Q9: Discuss Fraunhofer diffraction at a single slit. (OR) Discuss the diffraction due to a narrow slit.

BISE Mardan 2019

Ans. Consider a beam of monochromatic light of wavelength " λ " falls on a slit AB having width " d " as shown in the figure. In order to observe the diffraction pattern, a screen " S " is placed parallel to the slit " AB "; the rays [wavelets] are focused on the screen with the help of a convex lens. A small portion of the incident wavefront passes through the narrow slit. Each point of this section of the wavefront sends out secondary wavelets to the screen.

The wavelets interfere to produce the diffraction pattern on the screen. Now let the waves " 1 " and " 5 " are in phase. When these waves reach the wavefront " AC ", wave " 5 " would have a path difference " ab ".



If $ab = \frac{\lambda}{2}$, then the two rays will reach the point " p " on the screen and destructive interference will take place. Similarly, each pair 2 and 6, 3 and 7, 4 and 8, differ in path by $\frac{\lambda}{2}$ and hence they will also interfere destructively.

Now to find the value of path difference " ab ", we use the ΔabA , in which we have,

$$\sin \theta = \frac{ab}{\left(\frac{AB}{2}\right)} \quad \left[\sin \theta = \frac{\text{perp}}{\text{hyp}} \right]$$

$$\Rightarrow ab = \frac{AB}{2} \sin \theta \Rightarrow ab = \frac{d}{2} \sin \theta \quad (1) \quad [AB = d]$$

For destructive interference, we have,

$$ab = \frac{\lambda}{2} \quad \text{So equation (1)}$$

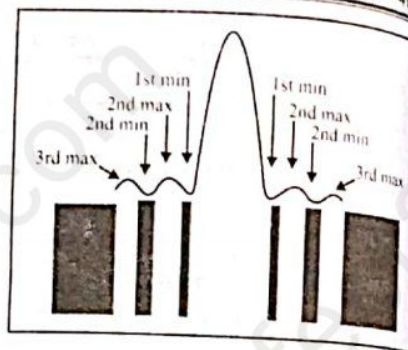
$$\Rightarrow \frac{\lambda}{2} = \frac{d}{2} \sin \theta \Rightarrow \boxed{d \sin \theta = \lambda} \quad (2)$$

Equation (2) represents the first order minimum [dark]. In general, equation (2) can be written as,

$$d \sin \theta = m \lambda \quad (3)$$

Where $m = 1, 2, 3, 4, \dots$

The region between any two consecutive minima will be maxima [bright]. Thus a series of bright and dark [maxima and minima] regions are observed on the screen. The first bright region is situated at the centre of the pattern, as shown in the figure.



Q10: Discuss the difference between interference and diffraction.

INTERFERENCE	DIFFRACTION
1. Interference is the superposition of two waves.	1. Diffraction is the bending of waves around an obstacle.
2. Two separate wave fronts originating from two coherent sources produce interference.	2. Secondary wavelets originating from different parts of the same wave front cause diffraction.
3. The region of minimum intensity is perfectly dark in interference.	3. In diffraction they are not perfectly dark.
4. The width of the fringes is equal in interference.	4. In diffraction the width of the fringes is never equal.
5. The intensity of all positions of maxima are of the same intensity in interference.	5. In diffraction, only zeroth order maxima has maximum intensity while all other maxima are of lower intensity than the zeroth order maxima.

Q11: (a) What is diffraction grating? (b) What is grating element? (c) How can the wavelength of a beam of light be measured with diffraction grating?

BISE Kohat 2017, 2019, BISE DI Khan 2019, BISE Bannu 2017

Ans. (a) Diffraction Grating:

A glass plate having large number of close parallel equidistant slits mechanically ruled on it is known as diffraction grating. (OR)

A diffraction grating is a device that can separate a beam of light into its constituent colours or wavelengths.

(b) Grating Element:

The separation between two slits is known as grating element. It is represented by " d ". Its value can be obtained by dividing the unit length of grating by the total number of lines ruled on it. i.e.

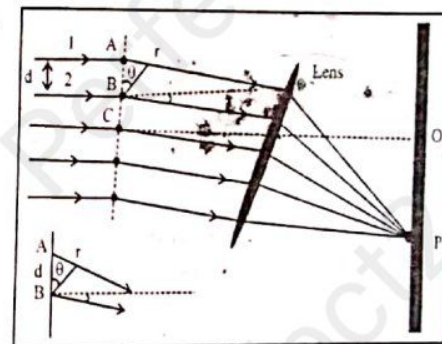
$$d = \frac{\text{unit length of grating}}{\text{total number of lines ruled on it}} \Rightarrow d = \frac{l}{N}$$

(c) Grating Equation for Measuring Wavelength of Light:

Consider a grating plate which consists of a number of parallel slits, as shown in the figure. Let parallel rays of light are allowed to pass through this diffraction grating, as shown in the figure.

The diffraction grating diffracts the rays towards a convex lens which converges these rays to a single point "O" on the screen as shown in the figure.

Now consider rays "1" and "2", we see that ray "1" covers longer distance than ray "2". So for finding their path difference, we draw a perpendicular "Br" from ray "2" on ray "1". Thus "Ar" represents path difference between the two waves.



Now in " $\triangle ABr$ ", we have,

$$\sin \theta = \frac{Ar}{AB} \Rightarrow Ar = AB \sin \theta \quad (1)$$

In equation (1), Ar = path difference, $AB = d$

So equation (1) becomes,

$$\text{Path difference} = d \sin \theta \quad (2)$$

For constructive interference, we have,

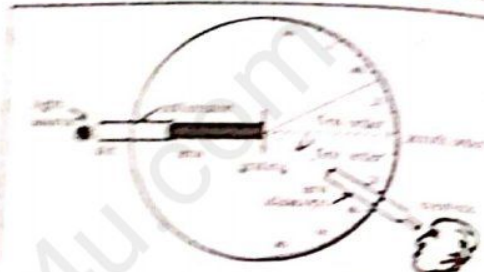
$$\text{Path difference} = m \lambda \quad (3)$$

Comparing equation (2), (3), we get,

$$d \sin \theta = m \lambda \quad (4)$$

Where $m = 0, 1, 2, 3, 4, \dots$

Equation (4) can be used to calculate the wave length of light. For this purpose, grating spectrometer is used as shown in the figure. Here monochromatic light falls on grating normally through the collimator. The light deviates from original path through an angle " θ " and its image can be viewed with the help of telescope. The value of " θ " can be measured with the help of telescope on both sides of central maxima for the same order " m ". For one side we take " θ_1 " and for other side we take " θ_2 ". We take their average to get " θ ". Then using equation (4), wavelength " λ " of the light can be calculated.



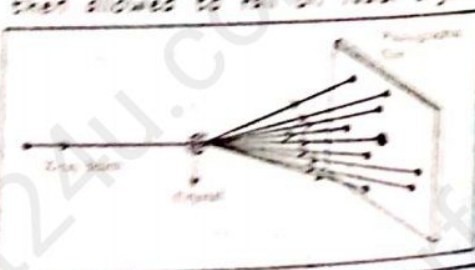
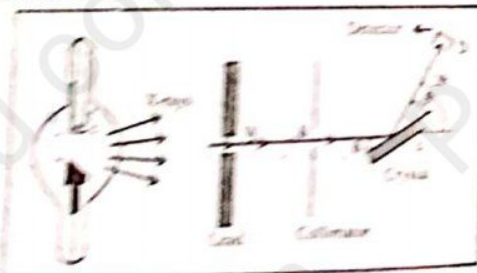
Q2: (a) Explain the diffraction of x-rays by crystal. (b) Derive an expression for Bragg's law to find the wavelength of light used.

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Ans: (a) Diffraction of X-rays by Crystal

In case of crystal, the layers of atoms are situated very closely to each other. The separation between these layers is less than 10Å . Thus the diffraction of crystals is possible only if we use wave of shorter wavelength like x-rays.

In the given arrangement, a beam of x-ray is obtained from x-ray source. The beam of x-rays is then allowed to fall on NaCl crystal. After passing through the narrow opening of lead and on narrow slit "S" as shown in the figure. When a beam strikes the crystal, x-rays are transmitted and hit the detector "D" as



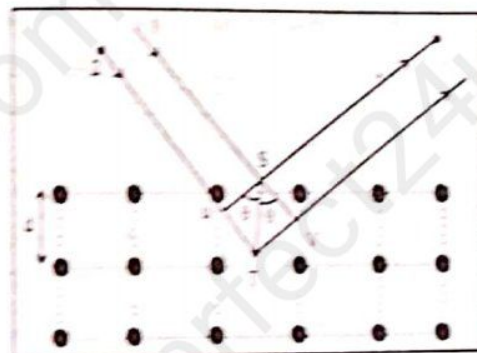
shown in the figure.

The main photograph obtained consists of the central spot surrounded by many other spots arranged in proper manner. These spots are known as lattice spots.

The arrangement of lattice spots for different crystal is different depending on their structure. The above experiment shows that x-rays are electromagnetic waves and the atoms are arranged in three dimensional lattice.

(b) Bragg's Law

Consider a rock salt's crystal as shown in the figure. The arrangements of atoms of the crystal are shown by dots and let " d " be the distance of separation between the two atoms.



Now consider two rays "1" and "2" which are allowed to fall on the crystal. The ray "1" is reflected at atom "5" and ray "2" is reflected at atom "4", as shown in the figure.

We see that, ray "2" covers large distance than ray "1". So there is a path difference between them. To find this path difference, we draw two perpendiculars "Su" and "Sv", as shown in the figure.

Now the total path difference is given by,
Path difference = $uT + TV$ (1)

Now in " ΔSuT ", we have,

$$\sin \theta = \frac{uT}{ST} \Rightarrow uT = ST \sin \theta$$

$$\Rightarrow uT = d \sin \theta \quad (2)$$

[$ST = d$]

Similarly, in " ΔSTV ", we have,

$$TV = d \sin \theta \quad (3)$$

Putting equations (2), (3), in equation (1), we get,

$$p.d = d \sin \theta + d \sin \theta$$

$$\Rightarrow p.d = 2d \sin \theta \quad (4)$$

Now for constructive interference, we have,

A polarized wave is transverse wave.
BTEA 2013

$$p \cdot d = m\lambda \quad (5)$$

Comparing equations (4), (5), we get,

$$2d \sin \theta = m\lambda \quad (6)$$

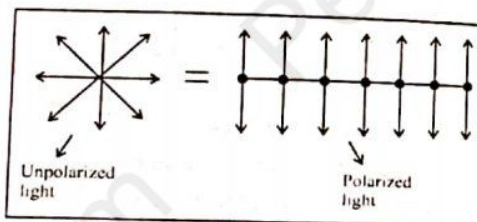
Equation (6) represents the Bragg's equation or Bragg's law. For given values of m , d , θ , we can calculate the wavelength " λ " of the given light. Similarly, for the given values of λ , θ , m we can calculate the interplanar spacing " d " between the smaller parallel planes of a crystal.

Q13: (a) What is polarization of light? (b) Define plane polarized light. (c) Explain that how this phenomenon decide that light waves are transverse in nature?

ETEA 2005, 2010, 2011, 2012, 2013

Ans: (a) Polarization of Light:

The process in which an ordinary light is allowed to pass through calcite crystal, then the light which is emergent out is known as polarized light and the process is called polarization of light.



(b) Plane Polarized Light:

The process of forcing or confining of light waves to vibrate only in one plane is known as plane polarization and the light is called plane polarized light.

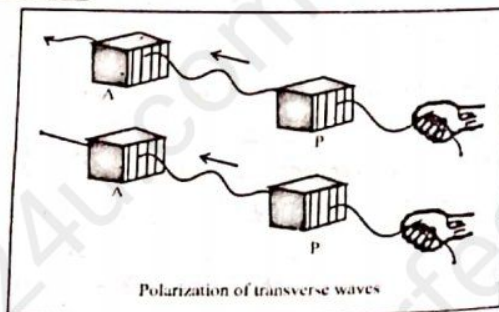
In polarization process, the electric and magnetic vibrations of light waves are restricted to a single plane of vibration.

(c) Polarization is the property exhibited by transverse waves only. It does not occur for longitudinal waves such as sound waves.

Experiment with Transverse Waves:

To study that only transverse wave exhibits polarization property, we consider a transverse wave on a string which passes through a slot "p" in a wooden board, as shown in the figure.

When the slot is parallel to



the direction of the transverse vibrations, the wave passes through the slot un-disturbed. But if we held another slot at right angle to the first one, then no wave will pass through the 2nd one. If the string is replaced by a spring and longitudinal wave is sent into it, then the wave will pass through the slots regardless of the orientation of the slots. This shows that longitudinal waves do not show polarization property.

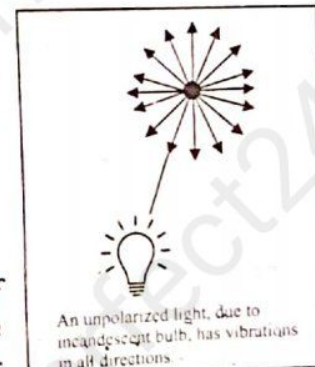
Conclusion about the Nature of Light:

From the above discussion, it is clear that polarization phenomenon is applicable to transverse waves only. It is not applicable to longitudinal waves. Now as we can convert ordinary light into polarized light through the process of polarization and polarization is applicable to transverse waves only, so the phenomenon of polarization clearly and successfully decides that light waves are transverse in nature.

Q14: (a) What is meant by unpolarized light and polarized light? (b) Discuss the various process through we can obtain polarized light.

Ans: (a) Unpolarized Light:

The light which vibrates in all directions is known as unpolarized light. The ordinary sun light, light emitted from ordinary incandescent bulb etc are the examples of unpolarized light.

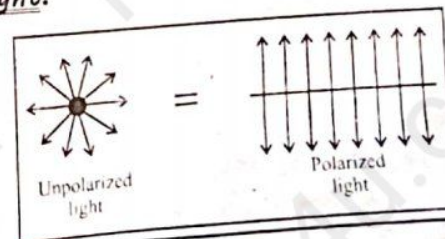


Polarized Light:

If the electric and magnetic vibrations of light waves are restricted to a single plane of vibrations, then such light is known as polarized light.

(b) Production of Polarized Light:

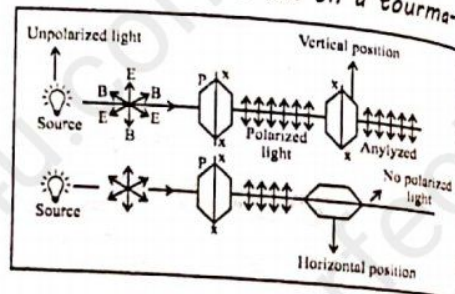
We can obtained plane polarized light from un-polarized light by removing all the waves from the beam except those having vibrations along one particular direction. This can be achieved



by various process given below:

(i) Polarization by Selective Absorption:

Consider a beam of un-polarized light is allowed to fall on a tourmaline crystal as shown in the figure.



The intermolecular structure of the crystal is such that it allows only those electric and magnetic vibrations which are parallel to its crystallographic axis and absorb all the remaining vibrations. Thus the light passed through this crystal has all the fields of vibrations in one plane and is known as the plane polarized light.

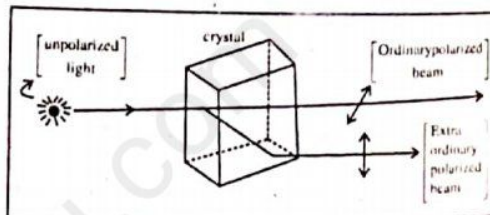
Now when another tourmaline crystal is placed in the path of this polarized light parallel to the 1st tourmaline crystal, then the polarized light will completely transmit through it.

Now if the 2nd crystal is rotated gradually, then the intensity of transmitted light decreases and when it becomes perpendicular to the 1st crystal, then no light will transmit through it, as shown in the figure.

If we rotate the 2nd crystal further, the transmitted light will be appear again. Thus the 2nd crystal is known as analyzer. In this way, we can obtain a beam of polarized light.

(ii) Polarization by Refraction:

The polarization by refraction can be obtained by using calcite crystal. In the given arrangement, un-polarized beam of light is allowed to fall on calcite crystal. This beam after

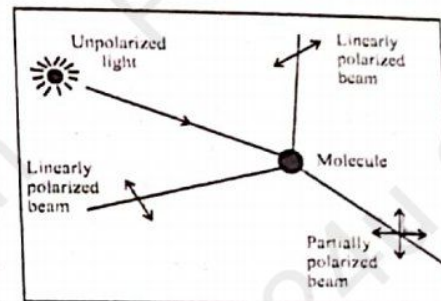


falling on the crystal is divided into two parts. One part is known as ordinary ray and the other part is known as extraordinary ray, as shown in the figure. When these two parts of a single beam were

observed by analyzer, they were found to be polarized beam. Such phenomenon is known as polarization by double refraction.

(iii) Polarization by Scattering:

When an un-polarized light is allowed to fall on a certain tiny particle or molecule, then this beam is absorbed by the molecule. This molecule emits the absorbed beam in the form of linearly polarized beam and partially polarized beam, as shown in the figure. Such phenomenon is known as polarization by scattering.

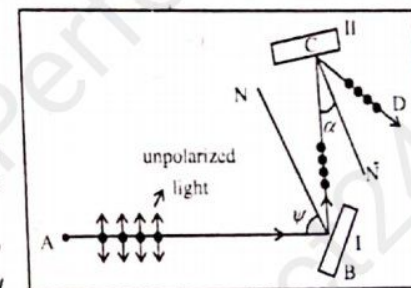


(iv) Polarization by Reflection:

Consider two glass plate "I" and "II" of the same dimension which are placed in such a way that their normals "N", "N'" are parallel to each other as shown in the figure.

Let un-polarized beam "AB" is allowed to fall on the 1st glass plate as shown in the figure.

Now the reflected beam "BC" from 1st glass plate and reflected beam "CD" from 2nd glass plate becomes polarized. Such phenomenon is known as polarization by reflection.



Q15: Describe Brewster's law of polarization.

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Ans. The Brewster's law gives us the relation between refractive index of two medium and tangent of angle of incidence. i.e.

$$\frac{n_2}{n_1} = \tan i_p$$

Proof:

Let an un-polarized beam of light is allowed to fall on the surface of medium (1) as shown in the figure. At point of incidence, one

part of incident beam is reflected 2nd is found to be polarized totally. The other part is refracted and is found to be partially polarized. Let " i_p " represent the angle of incidence or polarization angle and " r " represents the angle of refraction, then from figure, we have,

$$i_p + 90^\circ + r = 180^\circ$$

$$\Rightarrow r = 90^\circ - i_p \quad (1)$$

Now from Snell's law, we have,

$$n_1 \sin i_p = n_2 \sin r \quad (2)$$

Putting equation (1) equation (2), we get,

$$n_1 \sin i_p = n_2 \sin [90^\circ - i_p]$$

$$\Rightarrow n_1 \sin i_p = n_2 [\sin 90^\circ \cos i_p + \sin i_p \cos 90^\circ]$$

$$\Rightarrow n_1 \sin i_p = n_2 [1 \times \cos i_p + \sin i_p \times 0] = n_2 \cos i_p$$

$$\Rightarrow n_1 \sin i_p = n_2 \cos i_p \Rightarrow \frac{n_2}{n_1} = \frac{\sin i_p}{\cos i_p}$$

$$\Rightarrow \frac{n_2}{n_1} = \tan i_p \Rightarrow \boxed{\frac{n_2}{n_1} = \tan i_p} \quad (3)$$

Equation (3) represents the Brewster's law. This equation shows that for given values of n_1 , n_2 , we can calculate the angle of incidence or polarization angle " i_p ".

Do you know!

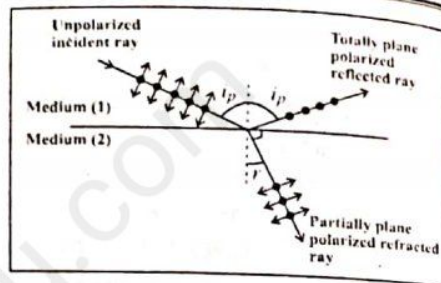
When light is reflecting from the glass then it is partially polarized. It is because some part of it refracts and some part reflects.

Do you know!

It is very effective to use a polarized sun glasses than using ordinary sun glasses. It is because the polarized sun glasses protect our eyes from the harmful glare of the sun light.

Q16: Discuss the application of polarized light.

Ans. Some important applications of polarized light in our daily life are given below:



Formula:

$$\begin{aligned} &[\sin(\theta_1 + \theta_2)] \\ &= \sin \theta_1 \cos \theta_2 + \sin \theta_2 \cos \theta_1 \end{aligned}$$

1. Reducing Glare:

Glare caused by light reflected from a smooth surface such as roads and lakes can be reduced by using polarizing materials as the reflected light is partially or completely polarized.

In case of photography, the Polaroid discs are placed in front of the camera lens which enabling us to see the details which would be otherwise hidden by glare.

2. Optical Activity:

When a beam of light is made to pass through certain crystal like quartz or certain liquids like sugar solution, then the directions of vibration of the transmitted polarized light is found to be rotated. This phenomenon is known as optical activity. The angle of rotation for a solution depends upon its concentration. The concentration of the solution is measured by using a device known as polarimeter. For example, the polarimeter is used in sugar mills to measure the sugar concentration in the solution which is obtained from sugar cane.

3. Curtianless Window:

Two polarizing sheets are fixed in a window, one inside and the other outside. The outside is kept fixed and the inner one is rotated in such a way to adjust the amount of light to be admitted.

4. Control of Head Light Glare:

The head light glare in night driving can be controlled by using polarizing viewer.

5. Stress Analysis:

When a glass or plastic material is subjected to certain stress, then strain is produced. If the strain is viewed in the presence of white light, we see Polaroid's colour fringes around the strain. This effect is known as photo elasticity and is used to analyze stresses in plastic model of various structures.

6. As the polarized light is restricted to one plane only, so such light is dangerous for human eyes. To overcome this problem, we should use sunglasses which are made of polarized sheet. The light after passing through this sheet becomes partially polarized and thus our eye sight will remain unaffected.

Solution of Examples & Assignments

EXAMPLE 9-1:

Yellow light from a sodium vapour lamp of wavelength $5893 \times 10^{-10} \text{ m}$ is directed upon two narrow slits 0.10 cm apart of the Young's experiment. Find the position of the first bright and dark fringes on a screen 100 cm away?

Solution:

Given data: Wavelength of light $= \lambda = 5893 \times 10^{-10} \text{ m}$

Separation of slits $= d = 0.10 \text{ cm}, 0.001 \text{ m}$,

Distance of slits from screen $= L = 100 \text{ cm} = 1 \text{ m}$

Order of fringe $= m = 1$

Required:

(i) Position of 1st bright fringe $= y_{\text{Bright}} = ?$

(ii) Position of 1st dark fringe $= y_{\text{Dark}} = ?$

Calculation:

(i) For bright fringe, we have,

$$y_s = \frac{L\lambda}{d} \quad (1)$$

Putting the values in equation (1), we get,

$$y_s = \frac{1 \times 5893 \times 10^{-10}}{0.001}$$

$$\Rightarrow y_s = 5893000 \times 10^{-10} \text{ m} \Rightarrow y_s = 5.893 \times 10^{-4} \text{ m}$$

(ii) For dark fringe, we have,

$$y_D = \frac{L\lambda}{2d} \quad (2)$$

Putting the values in equation (2), we get,

$$y_D = \frac{1 \times 5893 \times 10^{-10}}{(2 \times 0.001)} \Rightarrow y_D = 2946500 \times 10^{-10} \text{ m}$$

$$\Rightarrow y_D = 2.9465 \times 10^{-4} \text{ m} \Rightarrow y_D = 2.95 \times 10^{-4} \text{ m}$$

$$\text{Ans. } y_s = 5.893 \times 10^{-4} \text{ m} \text{ and } y_D = 2.9465 \times 10^{-4} \text{ m}$$

ASSIGNMENT 9-1:

The 3rd bright fringe in a double slit experiment makes a 2.4° angle with respect to the central line. The wavelength of the monochromatic light used is 480 nm . Find the distance "d" between the two sources. (0.0344 mm)

Solution:

Given data: For 3rd bright fringe, we put $m = 3$

$$\text{Angle } = \theta = 2.4^\circ$$

$$\text{Wavelength } = \lambda = 480 \text{ nm} = 480 \times 10^{-9} \text{ m}$$

$$\text{Nano} = 10^{-9}$$

Required:

Distance between two sources $= d = ?$

Calculation: We know that for bright fringe, we have,

$$d \sin \theta = m \lambda \Rightarrow d = \frac{m \lambda}{\sin \theta} \quad (1)$$

Putting the values in equation (1), we get,

$$d = \left(\frac{3 \times 480 \times 10^{-9}}{\sin(2.4^\circ)} \right) \text{ m} = \left(\frac{1440 \times 10^{-9}}{0.042} \right) \text{ m}$$

$$\Rightarrow d = 34285.7 \times 10^{-9} \text{ m} = 0.0342857 \times 10^{-3} \text{ m}$$

$$\Rightarrow d = 0.0343 \text{ mm}$$

$$10^{-3} = \text{milli}$$

EXAMPLE 9-2:

Light of wavelength 546 nm produces Young's interference pattern. The 2nd order dark fringe is along the direction that makes an angle of 18 min , related to the direction to the central maximum. What is the distance between the slits?

Solution:

Given data: Wavelength $= \lambda = 546 \text{ nm} = 546 \times 10^{-9} \text{ m}$

Order of dark fringe $= m = 2$

$$\text{Angle } = \theta = 18 \text{ min} = \frac{18}{60} = 0.30^\circ$$

$$1^\circ = 60 \text{ minutes}$$

Required: Distance between the slits $= d = ?$

Calculation:

For mth dark fringe, we have,

$$d \sin \theta = \left(m + \frac{1}{2} \right) \lambda \quad (1)$$

Putting the values in equation (1), we get,

$$\Rightarrow d \times \sin(0.30^\circ) = \left(2 + \frac{1}{2} \right) \times 546 \times 10^{-9} = \frac{5}{2} \times 546 \times 10^{-9}$$

$$\Rightarrow d \times 5.236 \times 10^{-3} = 1365 \times 10^{-9}$$

$$\Rightarrow d = \frac{1365 \times 10^{-9}}{5.236 \times 10^{-3}} = 260.7 \times 10^{-6} \text{ m}$$

$$\Rightarrow d = 260.7 \mu \text{m}$$

$$10^{-6} = \text{micro} = \mu$$

ASSIGNMENT 9-2:

A beam of x-rays of wavelength 0.3nm is incident on a crystal, and gives a first-order maximum when the glancing angle is 9.0° . Find the atomic spacing.

Solution:

Given data:

Wavelength of x-rays $= \lambda = 0.3\text{nm} = 0.3 \times 10^{-9}\text{m}$

Order of image $= m = 1$, Angle $= \theta = 9^\circ$

Required:

Atomic spacing $= d = ?$

Calculation: We know that, $2d\sin\theta = m\lambda$

$$\Rightarrow d = \frac{m\lambda}{2\sin\theta} \quad (1)$$

Putting the values in equation (1), we get,

$$d = \frac{1 \times 0.3 \times 10^{-9}}{2 \times \sin(9^\circ)} = \frac{0.3 \times 10^{-9}}{2 \times 0.16} \Rightarrow d = 0.9375 \times 10^{-9}\text{m} \text{ OR}$$

$$d = 0.9375\text{nm}$$

EXAMPLE 9-3:

A red laser light of wavelength 630nm is used in a Michelson interferometer. While keeping the mirror M_1 fixed, mirror M_2 is moved. The fringes are found to move past a fixed cross-hair in the viewer. Find the distance the mirror M_2 is moved for a single fringe to move past the reference line.

Solution:

Given data: Wavelength of laser light $= \lambda = 630\text{nm} = 630 \times 10^{-9}\text{m}$

Order of fringe $= m = 1$

Required:

Distance travelled by mirror " M_2 " $= p = ?$

Calculation: We know that,

$$p = \frac{m\lambda}{2} \quad (1)$$

$$10^{-9} = \text{nano}$$

Putting the values in equation (1), we get,

$$p = \left(\frac{1 \times 630 \times 10^{-9}}{2} \right) m = 315 \times 10^{-9}\text{m} \Rightarrow p = 315\text{nm}$$

EXAMPLE 9-4:

The deviation of the second order diffracted image formed by an optical grating having 5000 lines per centimeter is 32° . Calculate the wavelength of light used.

Solution:

Given data: Number of lines per centimeter $= 5000$ lines

Angle $= \theta = 32^\circ$

Order of image $= m = 2$

Grating element $= d = \frac{1\text{cm}}{5000} = 0.00020\text{cm}$

Required:

Wavelength $= \lambda = ?$

Calculation: We know that, $d\sin\theta = m\lambda$

$$\Rightarrow \lambda = \frac{d\sin\theta}{m} \quad (1)$$

Putting the values in equation (1), we get,

$$\lambda = \frac{0.00020 \times \sin(32^\circ)}{2} = 5.3 \times 10^{-5}\text{cm}$$

$$\text{Ans. Wavelength} = \lambda = 5.3 \times 10^{-5}\text{cm}$$

EXAMPLE 9-5:

X-rays of wavelength 3nm are incident on a crystal for which the lattice spacing is 5nm . Calculate the angle at which the first Bragg diffraction is observed.

Solution:

Given data: Wavelength $= \lambda = 3\text{nm} = 3 \times 10^{-9}\text{m}$

Lattice spacing $= d = 5\text{nm} = 5 \times 10^{-9}\text{m}$, order of the image $= m = 1$

Required: Angle $= \theta = ?$

Calculation: As $2d\sin\theta = m\lambda$

$$\Rightarrow \sin\theta = \frac{m\lambda}{2d} \quad (1)$$

Putting the values in equation (1), we get,

$$\sin\theta = \frac{1 \times 3 \times 10^{-9}\text{m}}{2 \times 5 \times 10^{-9}\text{m}} \Rightarrow \sin\theta = 0.3$$

$$\Rightarrow \theta = \sin^{-1}(0.3) = 17^\circ \Rightarrow \theta = 17^\circ$$

$$\text{Ans. } \theta = 17^\circ$$

Exercise

Choose the best possible answer:

1. The principle of Young's double slits experiment is based on the division of:

(a) Amplitude
(b) Frequency

(c) Velocity
✓ (d) Wavelength
2. Which one of the following properties proves the transverse wave nature of light?

(a) Interference
(b) Refraction

✓ (c) Polarization
(d) Diffraction
3. Coloured fringes observed in soap bubbles are the examples of:

(a) Diffraction
✓ (b) Interference

(c) Polarization
(d) Reflection
4. During a sunny day we see the objects in a class room even when all the electric lights are off, due to:

(a) Reflection of light
(b) Refraction of light

✓ (c) Diffraction of light
(d) Interference of light
5. The principle of Michelson interferometer is based on the division of:

(a) Wavefront
✓ (b) Amplitude

(c) Frequency
(d) Speed of light
6. In the Young's double slit experiment the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is:

(a) Halved
(b) Unchanged

(c) Doubled
✓ (d) Quadrupled

Sol: We know that the fringes space is given by,

$$\text{Fringe space} = \frac{L\lambda}{d} \quad (1)$$

Now when slit separation is halved, then

$$d' = \frac{d}{2} \quad (2)$$

And when distance of slits from screen becomes double, then

$$L' = 2L \quad (3)$$

Now for d' , L' equation (1) becomes

$$\text{New fringe space} = \frac{L'\lambda}{d'} \quad (4)$$

Putting equation (2), equation (3) in equation (4), we get,

$$\text{New fringe space} = \frac{2L \times \lambda}{\frac{d}{2}} = \frac{4L\lambda}{d}$$

$$\Rightarrow \text{New fringe space} = 4 \left(\frac{L\lambda}{d} \right) \quad (5)$$

Putting equation (1) in equation (5), we get,

$$\text{New fringe space} = 4 (\text{initial fringe space})$$

Thus the new fringe space increases quadruple i.e. four times

7. Signal from a remote control to the device operated by it travels with the speed of:

(a) Sound
✓ (b) Light

(c) Ultrasonic
(d) Supersonics

8. Light of wavelength " λ " is incident normally on a diffraction grating for which the slit spacing is equal to " 3λ ". What is the sine of the angle between the second order maximum and the normal?

(a) $\frac{1}{6}$
(b) $\frac{1}{3}$
✓ (c) $\frac{2}{3}$
(d) 1

Sol: For diffracting grating, we have,

$$d \sin \theta = m \lambda \quad (1)$$

When $d = 3\lambda$ and $m = 3$, equation (1) becomes

$$3\lambda \sin \theta = 3\lambda \Rightarrow 3 \sin \theta = 3$$

$$\Rightarrow \sin \theta = \frac{3}{3} \quad \text{so option (c) is correct.}$$

9. Which of the following gives three regions of the electromagnetic spectrum in order of increasing wavelength, visible radiation?

(a) Gamma rays, microwaves, visible radiation

(b) Radio waves, ultraviolet, x-rays

✓ (c) Ultraviolet, infrared microwaves

(d) Visible radiation, gamma rays, radio waves

10. Two monochromatic radiations X and Y are incident normally on a diffraction grating. The second order intensity maximum

for X coincides with the third order intensity maximum for Y.
What is the ratio $\frac{\text{wavelength of X}}{\text{wavelength of Y}}$?

- (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ ✓ (c) $\frac{3}{2}$ (d) $\frac{2}{1}$

Sol: We know that,

$$d \sin \theta = m \lambda \quad (1)$$

For x-radiation, we have, $m = 2$ and $\lambda = \lambda_x$, so equation (1) becomes,

$$d \sin \theta = m \lambda_x \quad (2)$$

For y-radiation, we have, $m = 3$ and $\lambda = \lambda_y$, so equation (1) becomes,

$$d \sin \theta = 3 \lambda_y \quad (3)$$

Dividing equation (2) by equation (3), we get,

$$\frac{d \sin \theta}{d \sin \theta} = \frac{2 \lambda_x}{3 \lambda_y} \Rightarrow \frac{2 \lambda_x}{3 \lambda_y} = 1 \Rightarrow \boxed{\frac{\lambda_x}{\lambda_y} = \frac{3}{2}}$$

So option (c) is correct.

11. The tip of a needle does not give a sharp image, it is due to:

- (a) Polarization (b) Interference
✓ (c) Diffraction (d) Refraction

Short Questions

Write short answers of the following questions:

Q1: A soap bubble looks black when it bursts, why?

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Ans: When a soap bubble bursts, it looks black due to destructive interference.

Explanation:

Here the soap bubble behaves like a thin film. Let " λ " represents the wavelength of light, " n " represents the refractive index of water, " t " represents the thickness of film and " m " represents the order of fringe, then we have,

$$2nt = m \lambda \quad (1)$$

Now when bubble bursts, its thickness become so smaller such that we put $t = 0$, so equation (1) becomes,

$$2n \times 0 = m \lambda \Rightarrow m \lambda = 0 \Rightarrow \boxed{m = 0} \quad (2)$$

Equation (2) shows that we get the first order dark fringe due to destructive interference. Thus when a soap bubble burst, the thickness of the film becomes zero. Under such conditions radiations of all wavelength disappears, and the soap bubble appears black when it bursts due to destructive interference.

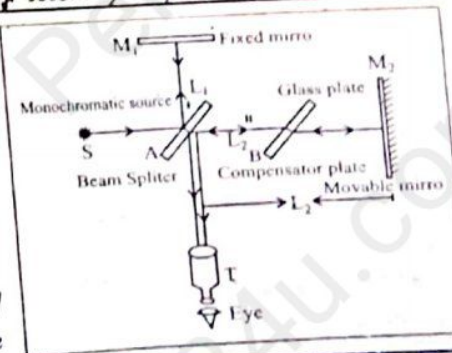
Q2: What is the difference between interference and diffraction?

Ans:

INTERFERENCE	DIFFRACTION
1. Interference is the superposition of two waves.	1. Diffraction is the bending of waves around an obstacle.
2. Two separate wave fronts originating from two coherent sources produce interference.	2. Secondary wavelets originating from different parts of the same wave front cause diffraction.
3. The region of minimum intensity is perfectly dark in interference.	3. In diffraction they are not perfectly dark.
4. The width of the fringes is equal in interference.	4. In diffraction the width of the fringes is never equal.
5. The intensity of all positions of maxima are of the same intensity in interference.	5. In diffraction, only zeroth order maxima has maximum intensity while all other maxima are of lower intensity than the zeroth order maxima.

Q3: In a Michelson interferometer a second glass plate is also used, why?

Ans: The principle of Michelson's interferometer is based on the division of amplitude, usually by partial reflection and transmission. Therefore, a second glass plate B, equal in thickness to plate A is placed in the path of the horizontal ray 2. So that to introduce



the same retardation in ray 2 as is introduced in ray 1 by its two passages through plate A. With this arrangement each ray will then pass through the same thickness of glass which equalizes the path length of the two rays. Therefore, the second glass plate is used as an equalizer of path length of the two light rays.

Q4: How you can explain Brewster's law of polarization?

The Brewster's law gives us the relation between refractive index of two medium and tangent of angle of incidence. i.e.

$$\frac{n_2}{n_1} = \tan i_p$$

Proof:

Let an un-polarized beam of light is allowed to fall on the surface of medium (1) as shown in the figure. At point of incidence, one part of incident beam is reflected 2nd is found to be polarized totally. The other part is refracted and is found to be partially polarized. Let " i_p " represent the angle of incidence or polarization angle and " r " represents the angle of refraction, then from figure, we have,

$$i_p + 90^\circ + r = 180^\circ \Rightarrow r = 90^\circ - i_p \quad (1)$$

Now from Snell's law, we have,

$$n_1 \sin i_p = n_2 \sin r \quad (2)$$

Putting equation (1) equation (2), we get,

$$n_1 \sin i_p = n_2 \sin [90^\circ - i_p]$$

$$\Rightarrow n_1 \sin i_p = n_2$$

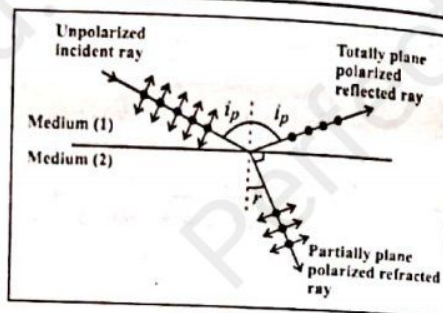
$$[\sin 90^\circ \cos i_p + \sin i_p \cos 90^\circ]$$

$$\Rightarrow n_1 \sin i_p = n_2 [1 \times \cos i_p + \sin i_p \times 0]$$

$$\Rightarrow n_1 \sin i_p = n_2 [\cos i_p + 0] = n_2 \cos i_p$$

$$\Rightarrow n_1 \sin i_p = n_2 \cos i_p \Rightarrow \frac{n_2}{n_1} = \frac{\sin i_p}{\cos i_p}$$

$$\Rightarrow n_1 \sin i_p = n_2 \cos i_p \Rightarrow \frac{n_2}{n_1} = \frac{\sin i_p}{\cos i_p}$$



Formula:

$$[\sin(\theta_1 + \theta_2)] \\ = \sin \theta_1 \cos \theta_2 + \sin \theta_2 \cos \theta_1$$

$$\Rightarrow \frac{n_2}{n_1} = \tan i_p \Rightarrow \frac{n_2}{n_1} = \tan i_p \quad (3)$$

Equation (3) represents the Brewster's law. This equation shows that for given values of n_1 , n_2 , we can calculate the angle of incidence or polarization angle " i_p ".

Q5: What is meant by the path difference with reference to the interference of two wave motion?

Ans: The difference between the lengths of the path covered by two waves having the same initial and terminal points is called as path difference.

Explanation:

When two waves pass through a medium at a point, then the resultant displacement at that point is the vector sum of the displacements due to two component waves. This modification in displacement as a result of superposition of two waves is called interference. For constructive interference, the phase difference is:

$$\text{Phase difference} = 0 = 0, 2\pi, 4\pi, 6\pi, \dots$$

Which correspond respectively to the path difference of:

$$\text{Path difference} = d = 0, \lambda, 2\lambda, 3\lambda, \dots = m\lambda$$

Where $m = 0, 1, 2, 3, \dots$

For destructive interference, the phase difference is:

$$\text{Phase difference} = 0 = 3\pi, 5\pi, \dots$$

Which correspond respectively to the path difference of:

$$\text{Path difference} = d = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots = \left(m + \frac{1}{2}\right)\lambda$$

Where $m = 0, 1, 2, 3, \dots$

Q6: Why it is not possible to see the interference where the light beams from the head lamps of a car overlap?

Ans: It is not possible to see the interference pattern due to light coming from the head lamps of a car.

Explanation:

In above case, we cannot see the interference pattern due to the following reasons:

1. The light beams coming from the head lamps of the car are

not phase coherent. The crest of wave of one head lamp does not match the crest of wave of the 2nd head lamp. So we cannot observe the interference.

2. We know that the fringe space is given by,

$$y = \frac{\lambda L}{d} \quad (1)$$

Here the separation "d" between the two head lamps is very large as compare to the wavelength of light emitted. So due to larger value of "d" the fringe space becomes too smaller. Under such condition the interference maxima and minima would be too closely spaced to be observable. Thus it is not possible to see the interference due to light coming from the head lamps of a car.

Q7: A telephone pole casts a clear shadow in the light from a distant head lamp of a car, but no such effect is noticed for the sound from the car horn. Why?

Ans: A telephone pole casts a clear shadow in the light from a distance head lamp of a car but no such effect is noticed for the sound from the car horn. This is because of the diffraction of waves.

Explanation:

The diffraction of waves depends upon their wavelength. Sound and light both travel as waves. The properties of these waves differ quite considerably. Sound waves travel a million times slower than light waves and have wavelengths ranging from 1cm to 10m, and will easily diffract round corners. Light waves have much smaller wavelengths, and only diffract through very small holes. Since diffraction depends upon the wavelength of waves and the wavelength of light waves is very much shorter than those of sound waves, so light waves cannot diffract around the telephone pole, hence casts clear shadow, while sound waves diffract about the pole, therefore no such effect is observed.

Q8: Why it is not possible to obtain the diffraction of x-rays by Young's double slits experiment?

BISE Kihat, 2017, 2019

Ans: It is not possible to attain the diffraction of x-rays by young's double slits experiment.

Reason and Explanation:

The diffraction of radiation depends upon the wavelength of radiation and the diameter of the slit. The diffraction will occur if the diameter of the slit is either equal to or less than the wavelength of light used.

We know that the wavelength of light is of the order of 10^{-6} m while the wavelength of x-rays is of the order of 10^{-10} m. But in case of Young's double slits experiment, we use slit whose diameter is of the order of 10^{-6} m. As the wavelength of light is comparable to the diameter of the slit, so light can be used to produce diffraction in Young's double slit experiment. We cannot use x-rays in Young double slit experiment, because the diameter of the slit is not comparable to wavelength of x-rays.

The diffraction of x-rays can occur in case of crystals where the space between atoms is of the order of 10^{-10} m. For example the space between atoms of NaCl crystal is about 10^{-10} m. So the x-rays can be diffracted by NaCl crystal.

Q9: Can we apply Huygen's principle to radar waves?

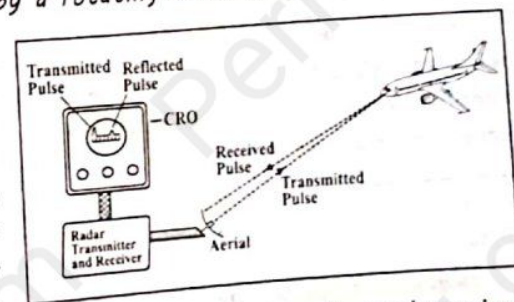
Ans: Yes, we can apply the Huygen's principle to radar waves.

Reason and Explanation:

Each point on the wavefront of radar wave behaves as a source for the secondary spherical wavelet. In this way the wavefront propagates from one region to another in space.

The radar waves are microwaves. Its wavelength is 3cm. These waves are emitted and picked up by a rotating aerial of radar after reflection from an aircraft.

The received signals are displayed on a CRO with a calibrated time base. On the CRO if the amplitude of the reflected signal received increases, it means that the aircraft or missile is coming. But if the amplitude of the reflected signal received on CRO decreases, it shows that the object is moving away.



we can apply Huygen's principle to radar waves.

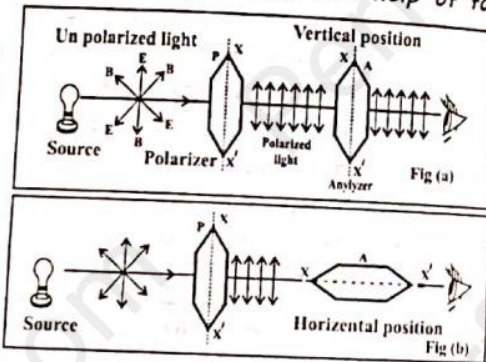
Q10: How would you justify that light waves are transverse in nature?

Ans: The transverse nature of light can be verified by the phenomenon of polarization. The theory of electricity and magnetism tells us that light is transverse in nature. The light consists of electric and magnetic field which are oscillating at right angle to the direction of propagation of light waves.

Experimental Verification:

We can justify the transverse nature of light with the help of following experiment:

In the given arrangement, a beam of un-polarized light is allowed to fall on a tourmaline crystal which acts as polarizer i.e. it converts the un-polarized light to polarized lighter. This polarized light transmitted through the 2nd



tourmaline crystal which acts as analyzer i.e. it is used for the confirmation of polarized light, as shown in the figure (a). Now if the analyzer is rotated gradually, then the amount of transmitted light decreases. When the analyzer becomes perpendicular to the 1st crystal [Polaroid], then no light will be transmitted through it as shown in the figure (b).

If we rotate the analyzer further, the transmitted light will appear again. In this way, we can obtain a beam of polarized light. As polarization occurs in case of transverse waves, so it shows that light waves are transverse in nature.

① حضرت علیؓ کا فرمان ہے: رشتے خون کے نہیں احساس کے ہوتے ہیں اگر احساس ہو تو چٹنی بھی اپنے ہو جاتے ہیں، اگر احساس نہ ہو تو اپنے بھی انجینی ہو جاتے ہیں...

② حضرت علیؓ نے فرمایا: اللہ تعالیٰ کے نزدیک کسی انسان کی عزت جتنی زیادہ ہوتی جاتی ہے اس کا استخوان بھی اس قدر سخت ہوتا چلا جاتا ہے...

③ کفر کے بعد سب سے بڑا گناہ دل دکھانا ہے چاہے وہ دل مؤمن کا ہو یا کافر کا...

Comprehensive Questions

Q1: What is meant by the dual nature of light? Discuss the history about the nature of light in detail.

Ans: See answer of question number 1 in text.

Q2: Explain the diffraction of x-rays by crystal and derive an expression for Bragg's law to find the wavelength of light used?

Ans: See answer of question number 12 in text.

Q3: Describe the experimental arrangement for the production of interference fringes by Young's double slits method, and get an expression for the fringes space.

Ans: See answer of question number 5 in text.

Q4: State and explain Huygen's principle. What is the difference between spherical and plane wavefronts?

Ans: See answer of question number 2 in text.

Q5: Explain the interference effect produced by thin film.

Ans: See answer of question number 6 in text.

Q6: What is the principle of interference of light? Discuss the necessary condition for interference of light.

Ans: See answer of question number 4 in text.

Q7: What is diffraction grating? How can the wavelength of a beam of light be measured with it?

Ans: See answer of question number 11 in text.

Q8: Describe the construction and working of Michelson's interferometer. How one can determine the wavelength of light used by this instrument?

Ans: See answer of question number 7 in text.

Q9: What is meant by plane polarized light? How does this phenomenon decide that light waves are transverse in nature?

Ans: See answer of question number 13 in text.

Numerical Problems

Q1: In a Young double slit experiment the separation of the slits is 1mm and red light of wavelength 620nm is falling on it. Determine the distance between the central bright band and the fifth bright fringe on the screen which is 3m away from the slit. (9.3mm)

Solution:

Given data: Separation between slits

$$= d = 1\text{mm} = 1 \times 10^{-3}\text{m}$$

$$\text{Wavelength} = \lambda = 620\text{nm} = 620 \times 10^{-9}\text{m}$$

$$\text{Distance of screen from slit} = L = 3\text{m}$$

$$\begin{aligned} 10^{-3} &= \text{milli} \\ 10^{-9} &= \text{nano} \end{aligned}$$

Required:

Distance between central bright band and fifth bright fringe = $\Delta y = y_5 - y_0 = ?$

Calculation: For m th bright fringe, we have,

$$y_m = \frac{L}{d} m \lambda \quad (1)$$

Now for central bright fringe, we put $m = 0$ in equation (1), so we get,

$$y_0 = 0$$

For 5th bright fringe, we put $m = 5$ in equation (1), so we get,

$$y_5 = \frac{3}{10^{-3}} \times 5 \times 620 \times 10^{-9}$$

$$\Rightarrow y_5 = 9300 \times 10^{-6}\text{m} \Rightarrow y_5 = 9.3 \times 10^{-3}\text{m}$$

$$\text{Now } \Delta y = (y_5 - y_0) \quad (2)$$

Putting the values in equation (2), we get,

$$\Delta y = (9.3 \times 10^{-3} - 0)$$

$$\Rightarrow \Delta y = 9.3 \times 10^{-3}\text{m}$$

$$\text{Ans. } \Delta y = 9.3\text{mm}$$

$$10^{-3} = \text{milli}$$

Q2: Two parallel slits are illuminated by light of two wavelengths, one of which is $5.8 \times 10^{-7}\text{m}$. On the screen the fourth dark line of the known wavelength coincides with the fifth bright line of the light of unknown wavelength. Find the unknown wavelength. ($5.2 \times 10^{-7}\text{m}$)

Solution: **Given data:** Wavelength = $\lambda = 5.8 \times 10^{-7}\text{m}$

Required:

Unknown wavelength = $\lambda' = ?$

Calculation: Let " y_m " represents the position of fourth dark fringe and " y'_m " represents the position of 5th bright fringe, then according to the given conditions, we have,

$$y_m = y'_m$$

$$\Rightarrow \frac{L}{d} \left(m + \frac{1}{2} \right) \lambda = \frac{L}{d} m' \lambda'$$

$$\Rightarrow \left(m + \frac{1}{2} \right) \lambda = m' \lambda'$$

$$\Rightarrow \left(3 + \frac{1}{2} \right) \lambda = 5 \lambda'$$

$$\Rightarrow \frac{7\lambda}{2} = 5\lambda' \Rightarrow \lambda' = \frac{7\lambda}{10}$$

$$\Rightarrow \lambda' = \frac{7 \times 5.8 \times 10^{-7}}{10} = 4.06 \times 10^{-7}\text{m}$$

$$\Rightarrow \lambda' = 4.06 \times 10^{-7}\text{m}$$

- For 4th dark fringe, we put $m = 3$
- For 5th bright fringe, we put $m = 5$

Q3: When the movable mirror of a Michelson interferometer is moved 0.1mm. How many dark fringes pass through the reference point, if light of wavelength 580nm is used? (345 fringes)

Solution:

Given data:

$$\text{Total distance} = p = 0.1\text{mm} = 0.1 \times 10^{-3}\text{m} = 1 \times 10^{-4}\text{m}$$

$$\text{Wavelength} = \lambda = 580\text{nm} = 580 \times 10^{-9}\text{m}$$

$$\begin{aligned} \text{milli} &= 10^{-3} \\ \text{nano} &= 10^{-9} \end{aligned}$$

Required:

Number of dark fringes = $m = ?$

Calculation: In case of Michelson interferometer, we have,

$$\lambda = \frac{2p}{m} \Rightarrow m = \frac{2p}{\lambda} = \frac{1 \times 10^{-4}}{580 \times 10^{-9}} \Rightarrow m = 0.003448 \times 10^5$$

$$\Rightarrow m = 0.00345 \times 10^5 = 345 \text{ fringes}$$

Q4: A soap film has a refractive index of 1.40. How thick must the film be, if it appears black, when mercury light of wavelength 546.1nm falls on it normally? ($1.95 \times 10^{-7}\text{m}$)

Solution:

Given data: Refractive index = $n = 1.40$

Wavelength of light = $\lambda = 546.1 \text{ nm}$

$$\Rightarrow \lambda = 546.1 \times 10^{-9} \text{ m}, m = 1$$

Required:

Thickness of film = $t = ?$

Calculation: We know that,

$$2nt = m\lambda \Rightarrow t = \frac{m\lambda}{2n} \Rightarrow t = \frac{1 \times 546.1 \times 10^{-9}}{2 \times 1.40}$$

$$\Rightarrow t = 1.95 \times 10^{-7} \text{ m}$$

$$\text{Ans: } t = 1.95 \times 10^{-7} \text{ m}$$

Q5: A diffraction grating has 5000 lines per centimeter. At what angle does the second order spectrum of the sodium yellow light of wavelength 589 nm occur?

Solution:

Given data: $N = 5000 \text{ lines/cm} = 500000 \text{ lines/m}$

$$\Rightarrow N = 5 \times 10^5 \text{ lines/m}$$

$$\Rightarrow d = \frac{1}{N} = \frac{1}{5 \times 10^5} = 0.2 \times 10^{-5} \text{ m}$$

For 2nd order, we put $m = 2$

Wavelength = $589 \text{ nm} = 589 \times 10^{-9} \text{ m}$

Required: Angle = $\theta = ?$

Calculation: We know that, $d \sin \theta = m\lambda$

$$\Rightarrow \sin \theta = \frac{m\lambda}{d} = \left(\frac{2 \times 589 \times 10^{-9}}{0.2 \times 10^{-5}} \right)$$

$$\Rightarrow \sin \theta = \frac{2 \times 589 \times 10^{-9 \times 5}}{0.2} \Rightarrow \sin \theta = 5890 \times 10^{-4} = 0.5890$$

$$\Rightarrow \theta = \sin^{-1}(0.5890) \Rightarrow \theta = 36^\circ$$

$$\text{Ans: } \theta = 36^\circ$$

Q6: Light is incident normally on a grating which has 250 lines/mm. Find the wavelength of spectral line for which the deviation in second order is 12° . ($\lambda = 4158 \times 10^{-10} \text{ m}$)

Solution:

Given data: $N = 250 \text{ lines/mm} = 250000 \text{ lines/m}$

$$\Rightarrow N = 2.5 \times 10^5 \text{ lines/m}$$

For 2nd order, $m = 2$, $\theta = 12^\circ$

Required: Wavelength = $\lambda = ?$

Calculation: We know that, $d \sin \theta = m\lambda$

$$\left[d = \frac{1}{N} \right]$$

$$\Rightarrow \frac{1}{N} \sin \theta = m\lambda \Rightarrow \sin \theta = N \cdot m\lambda$$

$$\Rightarrow \sin \theta = 2.5 \times 10^5 \times 2 \times \lambda$$

$$\Rightarrow \sin 12^\circ = 5 \times 10^5 \lambda \Rightarrow 0.20791169 = 5 \times 10^5 \lambda$$

$$\Rightarrow \lambda = \frac{0.20791169}{5 \times 10^5} = 0.041580 \times 10^{-5} \text{ m}$$

$$\text{Ans: } \lambda = 4158.0 \times 10^{-10} \text{ m}$$

Q7: In a certain x-rays diffraction experiment the first order image is observed at an angle of 5° for a crystal plane spacing of $2.8 \times 10^{-10} \text{ m}$. What is the wavelength of x-rays used? ($\lambda = 0.49 \times 10^{-10} \text{ m}$)

Solution:

Given data: $m = 1$ [First order], $\theta = 5^\circ$, $d = 2.8 \times 10^{-10} \text{ m}$

Required: Wavelength = $\lambda = ?$

Calculation: According to Bragg's law, we have,

$$2d \sin \theta = m\lambda \Rightarrow \lambda = \frac{2d \sin \theta}{m} = \frac{2 \times 2.8 \times 10^{-10} \times \sin 5^\circ}{1}$$

$$\Rightarrow \lambda = 0.4880 \times 10^{-10} \text{ m} \Rightarrow \lambda = 0.49 \times 10^{-10} \text{ m}$$

Q8: An x-ray beam of wavelength $0.48 \times 10^{-10} \text{ m}$ is used to get Bragg reflection from a crystal at an angle of 20° for the first order maximum. What are the possible layer plane spacing which give rise to this maximum? ($0.70 \times 10^{-10} \text{ m}$)

Solution:

Given data: Wavelength = $\lambda = 0.48 \times 10^{-10} \text{ m}$

$\theta = 20^\circ$, $m = 1$ [First order]

Required: Layer plane spacing = $d = ?$

Calculation: According to Bragg's law, we have,

$$2d \sin \theta = m\lambda \Rightarrow d = \frac{m\lambda}{2 \sin \theta} = \frac{1 \times 0.48 \times 10^{-10}}{2 \times \sin 20^\circ}$$

$$\Rightarrow d = \frac{0.48 \times 10^{-10}}{2 \times 0.342020143}$$

$$\Rightarrow d = \frac{0.35 \times 10^{-9}}{0.68 - 0.6286} = 0.70 \times 10^{-9} \text{ m}$$

Ans: $d = 0.70 \times 10^{-9} \text{ m}$

Q9: The spacing of one set of crystal planes in NaCl (table salt) is $d = 0.282 \text{ nm}$. A monochromatic beam of x-rays produces a Bragg maximum when its glancing angle with these planes is $\theta = 7^\circ$. Assume that this is a first order maximum ($n = 1$), find the wavelength of the x-rays. (0.069 nm)

Solution:

Given data:

Plane spacing = $d = 0.282 \text{ nm} = 0.282 \times 10^{-9} \text{ m}$,

Angle = $\theta = 7^\circ$, order = $m = 1$

Required:

Wavelength = $\lambda = ?$

Calculation: According to Bragg's law, we have,

$$2d \sin \theta = m\lambda \Rightarrow \lambda = \frac{2d \sin \theta}{m} \quad (1)$$

Putting the values in equation (1), we get,

$$\lambda = \frac{2 \times 0.282 \times 10^{-9} \times \sin(7^\circ)}{1} = 2 \times 0.282 \times 10^{-9} \times 0.122 \text{ m}$$

$$\Rightarrow \lambda = 0.069 \times 10^{-9} \text{ m}$$

Ans: $\lambda = 0.069 \text{ nm}$

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حضرت علیؑ قول ہے "دنیا اور آخرت کی مثال دو بیجوں کی طرح ہے، جب ایک کو راضی کرو تو دوسری ناخوش ہو جاتی ہے۔"

انہی سے کلام کیے بغیر دعا پر بھروسہ کرنا حماقت ہے اور اپنی محنت پر بھروسہ کر کے کاماتے کرنے کرنا تکبر ہے۔۔۔

انسان دلی محنت کا اصلی ذریعہ رہنا چاہیے نہیں بلکہ حیا اور پردہ ہے۔۔۔

ہاں کہہ دیتا ہے جو اللہ تعالیٰ پر یقین نہیں رکھتا اور محروم ہو جاتا ہے جو اللہ تعالیٰ کی نعمتوں کا شکر ادا نہیں کرتا۔۔۔

نیک اس کے نصیب میں آئی جس نے اپنی خواہشات کو چھوڑا اور محروم ہی ہے جس نے دنیا کے مقابلے میں آخرت سے مت

مولانا (ابن الجوزی)

مصیبت میں آرام کی تلاش مصیبت کو ترقی دیتی ہے۔ (امام جعفر صادقؑ)

Q1: Define and explain thermal equilibrium.

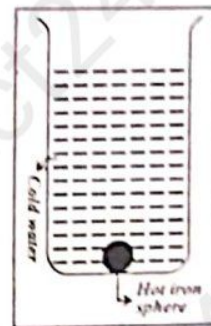
Ans: Thermal Equilibrium:

Bodies are said to be in thermal equilibrium if they are at the same temperature and there is no net exchange of heat between them.

Explanation:

Consider a solid hot sphere is placed in cold water as shown in the figure. In the beginning, there is a temperature difference between water and solid sphere, i.e. the solid sphere is at high temperature and cold water is at low temperature.

Due to the temperature difference, the hot sphere loses heat and the cold water gains the heat. As a result, the temperature of sphere decreases and that of the water increases gradually with the passage of time. This process goes on till the water and sphere attain the same temperature. At this stage, the net flow of heat stops and they are said to be in thermal equilibrium.



Point to Ponder:

Two bodies, one hot and the other cold, are kept in vacuum. What will happen to the temperature of the hot body after some time?

Ans: Hot bodies emit thermal radiations which are electromagnetic in nature. Electromagnetic waves don't require material medium to pass through. Heat energy is transmitted even through vacuum. We get heat energy from sun in this way. Hence the temperature of hot body will fall until it comes in thermal equilibrium with the cold body.

Q2: Define and explain the following terms used in thermodynamics:

(a) Work (b) Heat (c) Internal energy

Ans: (a) Work:

The energy which is added to the system in such a way, that a dif

ference of temperature is not directly involved, is known as work.

Explanation:

Besides supplying heat to a body, there is another very general method of raising the temperature of a body. The method is to perform work on the body, for example:

1. When a tyre is pumping up with the help of a heat pump, the pump becomes heated.
2. When we churn curd in a vessel, the contents becomes heated.
3. When we hit a nail with a hammer into wood or wall, the nail becomes heated.
4. When we rub our hands together, our hands become warm.

From the above examples, it is clear that work can be exchanged between an object and its surroundings by the overall displacement of the object without requiring the difference of temperature. The work is not a state variable. It depends on the process performed on the system.

Quick Quiz:

The temperature of a normal healthy person is 37°C . What will be the temperature of a dead person?

Ans. After the heart stops beating, the body immediately starts turning cold. This phase is known as algor mortis, or the death chill. Each hour, the body temperature falls about 1.5 degrees Fahrenheit (0.83 degrees Celsius) until it reaches room temperature. At the same time, without circulation to keep it moving through the body, blood starts to pool and settle.

(b) Heat:

The energy which flows from hot body to cold body due to temperature difference between them is called heat.

Explanation:

Heat represents the total kinetic energy of molecules of a body. It gives us the sensation of warmth. Heat always flows from hot body to cold body. The flow of heat from cold body to hot body is possible only with expenditure of energy [like refrigerator].

Let us take a piece of ice in a vessel and keep it at room temperature. After some time, we see that the ice begins to melt and is

converted into water.

The temperature of room and temperature of water becomes the same and in this way, a thermal equilibrium, is established between the system and its surroundings. In above case, the ice [cold body] absorbs heat from its surroundings [hot body]. In other words, we can say that heat is a form of energy which flows from one body to another body due to temperature difference between them. Heat is not a state variable.

Units of Heat:

The units of heat are, joule, calorie, British thermal unit [B.T.U]

$$1 \text{ B.T.U} = 1254.87 \text{ joules}$$

$$1 \text{ B.T.U} = 251.996 \text{ calories}$$

$$1 \text{ calorie} = 4.186 \text{ joules}$$

$$1 \text{ joule} = 0.238 \text{ calories}$$

(c) Internal Energy: (ETEA 2013)

The sum of kinetic and potential energies associated with the random motion of the atoms of the substance is known as internal energy of that substance.

Explanation:

We know that matter consists of molecules which are in state of constant motion. Due to their motion, each molecule possesses certain kinetic energy and potential energy. The kinetic energy may be in the form of translational, rotational or vibrational kinetic energy. When we heat a substance, the motion and energy of its molecules increase. Thus in this case, the heat energy is converted into internal energy of the substance.

Similarly, when work is performed on a substance in such a way that the random motion of the molecules of the substance increases, then the work is converted into the internal energy of the substance.

The internal energy resides in the system and is represented by "u". It is a state variable and possesses a unique value in a given state of the system.

Brain Teaser:

Does the temperature of a child increase with the growth of the

child?

Ans: When a child is born its temperature is low and it is kept in a warmer for about an hour. After sometime its temperature becomes normal and remains 98.6° Fahrenheit for rest of its life.

Can you tell?

Bore with a small drill into a hard board. The drill becomes hot to touch. Why? What happens if hard board is replaced by a soft board?

Ans: The drill becomes hot to touch while drilling because some part of work done against force of friction is converted into heat. If a hard board is replaced by a soft board then less resistance is offered during drilling. Hence drill will be less hot as compared to previous case.

Quiz:

Why a spark is produced when two stones collide against each other?

Ans: When two stones hit against each other with very high speed, the total energy of the impact gets concentrated at a small spot at the point of contact of the hit. In many such occasions, the energy is sufficient to chip off a tiny and hot mass of the stone carrying enormous amount of energy, thus moving with very high velocity. This motion in air further heats up the spark due to friction with the atmosphere. This heated particle chemically reacts with the oxygen of the atmosphere and glows as a bright spark.

Brain Teaser:

Is the temperature of a normal healthy old man is less than that of a normal healthy young man?

Ans: Humans are warm blooded species and their body temperature remains constant throughout their lives. There is no difference in the temperature of a normal healthy old man than that of a normal healthy young man.

A glass full of water contains ice cubes floating in it. What will happen to the water level when ice melts?

Ans: The water level does not change. When the ice is floating,

[According to Archimedes] it displaces water equal to the weight of ice. For example,

Let say 1m³ ice is floating

Displaced water weight = weight of ice = density of ice $\times$ volume $\times g$
 $= 900 \text{ kg/m}^3 \times 1 \text{ m}^3 \times 10 \text{ m/s}^2 = 9000 \text{ N}$

Displaced water volume = $\frac{m}{\rho} = \frac{mg}{\rho g} = \frac{9000 \text{ N}}{(1000 \times 10)}$

(Raised water level) = 0.9 cubic metre

When the ice melts, it melts equal to its weight [i.e. mass], not to its volume [ice shrinks as it changes its state to water].

Weight of newly formed water = weight of ice = 900 kg

So new water level raise = $\frac{m}{\rho} = \frac{900}{1000} = 0.9 \text{ cubic metre}$

Which is same so no change in water level.

Q3: Explain equivalence of heat and work. (OR) What is mechanical equivalent of heat?

Ans: Mechanical Equivalent of Heat:

The ratio of the workdone in joules to the heat produced in calories is known as mechanical equivalent of heat. (OR)

The amount of mechanical energy or work required to produce a unit quantity of heat is known as mechanical equivalent of heat.

Explanation:

According to the mechanical theory of heat, the heat is a form of energy which is associated with molecular motion of the substance.

This theory tells us that when two bodies are rubbed together, then mechanical energy is converted into heat energy. The relation between work and heat is given by,

$$W \propto H \Rightarrow W = \text{constant} \times H$$

$$\Rightarrow W = JH \quad (1)$$

In equation (1), "J" is constant and is known as mechanical equivalent of heat. When 4.186 joules of work is done, then one calorie of heat is produced. i.e.

$$J = 4.186 \text{ joules / calories}$$

Note: The nature of heat can be also explained with the help of caloric theory of heat. According to this theory, heat is a form of

fluid which is weightless, invisible and self repellent i.e. heat can flow from one substance to another. Such fluid is called calorie. When calorie enters a substance its temperature rises. When calorie is removed from a body, its temperature falls down. The calorie is produced during combustion or when matter is broken down into pieces.

Quick Quiz:

What is the significant of Joule's experiment for determining the value of J?

Ans. It gives us the relation between the amount of work done and heat energy thus produced.

How can you measure the temperature of the mosquito?

Ans. We can measure the temperature of an insect with an infrared detector, perhaps a pyro-electric sensor - depends on how hot is the environment around the insect.

Q4: Define the following terms: (a) Thermodynamics, (b) System, (c) Surrounding, (d) Boundary, (e) Closed system, (f) Open system, (g) Isolated system

Ans. (a) **Thermodynamics:** The branch of physics in which we study the relationship between heat and other forms of energy is known as thermodynamics.

(b) System:

The quantity of matter or region of space whose behavior is being studied is called system. (OR)

Anything which is under scientific observation is known as a system. For example, a gas enclosed in a cylinder represents a system.

(c) Surroundings:

Anything outside the system is known as surrounding.

(d) Boundary:

The region which separates a system from its surrounding, is known as boundary of the system.

(e) Closed System:

The system in which there is no transfer of mass across its boundary is known as closed system.

In closed system, the transfer of heat energy can take place from the system to the surrounding and vice versa. For example, hot water in a closed vessel.

(f) Open System:

The system in which there is a transfer of mass across its boundary is known as open system.

In open system, the transfer of mass takes place from the system to the surrounding and vice versa. Plants and animals are the examples of open system because they exchange materials [food, oxygen, waste products] with the surroundings.

(g) Isolated System: (ETEA 2007)

The system in which there is no transfer of mass and heat energy across its boundary is known as isolated system.

In an isolated system, both mass and energy cannot enter or leave the system across its boundary. Tea contained in a well insulating thermo flask is an example of an isolated system.

For your information:

An empty polythene bag burns or melts on a flame of a stove. However we can make a few cups of tea by placing water filled polythene bag on a flame of a stove.

Point to Ponder:

An ink dot on a white porcelain dish appears dark. When the dish is raised to a very large temperature the dot appears brighter than the surroundings. Why?

Ans. When dish is raised to a very high temperature it emits all the possible wavelengths because of which the ink dot starts to appear brighter than its surrounding.

Point to Ponder:

Why do heels crack in winter? What effect does the application of lubricants have on the heels?

Ans. Since we don't drink a lot of water during winter seasons so the arteries contract to save water loss. When the feet become too dry, heel fissures can develop quite easily. The heel pad wants to expand outward, but the skin is not pliable enough to expand with it, so it cracks.

Q5: Define thermodynamic state and thermodynamic state variables.

Ans: The particular condition when a system has specified values of pressure "P", volume "V" and temperature "T" etc is known as thermodynamic state of the system.

The variables or functions which determine the physical state of the system are called state variables or state functions of the system. If the system has the same composition everywhere [i.e. the system is homogenous], then pressure, volume and temperature are sufficient to determine the state of the system. The mathematical relation between the variables is given by,

$$PV = nRT \quad (1)$$

Equation (1) is known as equation of state of a system. In equation (1) "n" represent the number of moles.

Equation (1) is applicable to an ideal gas only. It is not applicable to solids and liquids. In addition to pressure, volume, temperature, the other state variables are internal energy "u", entropy "S", free energy function "F", Gibb's function "G", enthalpy of the system "H".

Q6: State and explain first law of thermodynamics.

ETE 2006, 2012, BISE Mardan 2019

Ans. Statement:

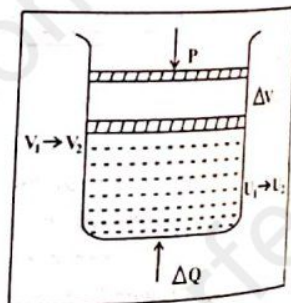
This law states that when heat energy is converted into mechanical energy or when mechanical or some other form of energy is converted into heat energy, then its total value remains the same.

Thus the first law of thermodynamics satisfies the law of conservation of energy.

Explanation:

Consider an ideal gas filled in a cylinder having a movable piston as shown in the figure. Now if we supply some amount of heat " ΔQ " to the cylinder, then this heat is absorbed by the system to increase its internal energy as well as to do work against the piston. Mathematically, we have,

$$\Delta Q = \Delta u + \Delta w \quad (1)$$



Equation (1) represents the mathematical form of first law of thermodynamics.

Quick Quiz:

Two blocks of ice when pressed together, combine to form a single piece. Explain how this happens?

Ans: When two blocks of ice are compressed, the ice at the point of contact melts but when compressing force is removed, the melted water again freezes and a single block of ice is formed.

Point to Ponder:

The earth is receiving heat from the sun continuously. In this case, the earth will be as hot as a furnace in no time. As such thing does not happen, what compensatory measures does earth have to maintain the temperature?

Ans: Several things regulate the temperature but the two biggest factors are:

- The oceans and
- Atmosphere

Water is able to absorb and hold a lot of energy [heat from the sun] and give it off slowly, thus keeping the earth warm at night and moving warm water to cold regions, like the poles. The atmosphere regulates it by allowing heat to radiate out into space, countered by green-house gases that keep it in. Thus earth maintains a relatively constant temperature by being in radiative balance between the incoming short wave solar radiation and the outgoing long wave infrared radiation. Each of these, short wave and long wave, are modulated by clouds, the atmosphere and the earth's surface such that a balance is achieved.

Q7: Discuss the applications of first law of thermodynamics. (OR)
In the light of first law of thermodynamics, describe the following processes (a) Isobaric process (b) Isochoric process (c) Isothermal process (d) Adiabatic process

BISE Mardan 2019

Ans. (a) Isobaric Process:

Definition: The process in which the pressure is kept constant is known as isobaric process.

Explanation:

Consider an ideal gas filled in a cylinder having movable piston as shown in the figure. Let amount of heat " ΔQ " is supplied to the system, then the gas absorbs this heat and use it to increase its internal energy and to do work against the piston. Now from first law of thermodynamics, we have,

$$\Delta Q = \Delta u + \Delta w \quad (1)$$

Here we have, $\Delta w = P\Delta V$

So equation (1) becomes,

$$\Delta Q = \Delta u + P\Delta V \quad (2)$$

Equation (2) shows that the amount of heat supplied to the system is used to increase the internal energy of the system as well as to do work against the piston.

(b) Isochoric Process:

Definition: The process in which the volume of the system remains constant is known as isochoric process.

Explanation:

Consider an ideal gas filled in a cylinder having movable piston, as shown in the figure. Let amount of heat " ΔQ " is supplied to the system. As the volume remains constant so no work will be done on the piston, i.e. $\Delta w = 0$.

Now from first law of thermodynamics, we have,

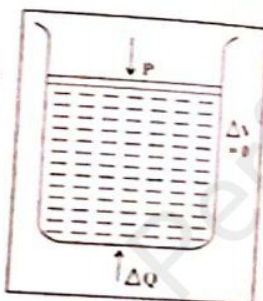
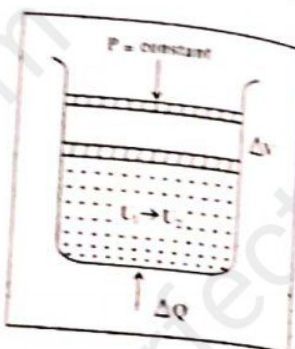
$$\Delta Q = \Delta u + \Delta w \Rightarrow \Delta Q = \Delta u + 0$$

$$\Rightarrow \Delta Q = \Delta u \quad (3)$$

Equation (3) shows that the amount of heat supplied to the system under isochoric process is totally used to increase the internal energy of the system.

(c) Isothermal Process: (ETEA 2015)

Definition: The process in which the internal energy of the system remains constant is known as isothermal process.



Explanation:

Consider an ideal gas filled in a cylinder having movable piston, as shown in the figure. Let amount of heat " ΔQ " is supplied to the system. As the internal energy in this process is kept constant, so we have, $\Delta u = 0$. Now according to the first law of thermodynamics, we have,

$$\Delta Q = \Delta u + \Delta w \Rightarrow \Delta Q = 0 + \Delta w$$

$$\Rightarrow \Delta Q = \Delta w \quad (4)$$

Equation (4) shows that the amount of heat supplied to the system under isothermal process is totally used to do work against the piston.

(d) Adiabatic Process: (ETEA 2007, 2011, 2015)

Definition: The process in which heat is neither given to the system nor removed out of the system is known as adiabatic process.

Explanation:

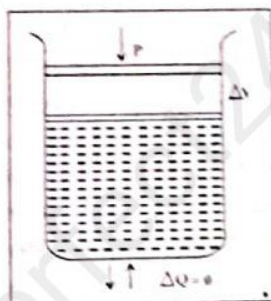
Consider an ideal gas filled in a cylinder having movable piston as shown in the figure. If no heat is supplied to the system, then we have, $\Delta Q = 0$.

Now by first law of thermodynamics, we have,

$$\Delta Q = \Delta u + \Delta w \Rightarrow 0 = \Delta u + \Delta w$$

$$\Rightarrow \Delta w = -\Delta u \quad (5)$$

Equation (5) shows that under adiabatic expansion process, the work is done at the cost of internal energy.



Can you tell?

The rate of formation of ice on ponds decreases gradually as more and more ice is formed, why?

Ans. The ice formed on the surface of water floats on water surface as density of ice is less than water and therefore it serves as shield from the cold climate to the water below it. This results in decreased rate of formation of ice.

Q8: Define and explain the following terms:

- Specific heat of a substance,
- Molar specific heat of a gas.

Ans: (a) Specific Heat of a Substance:

Definition: The amount of heat required to raise the temperature of 1kg of a substance through one degree Kelvin (1K) is known as specific heat of a substance.

Explanation:

Let "m" represents the mass of a substance, " ΔQ " represents the amount of the heat supplied to the substance and " ΔT " represents the rise in temperature, then we have,

$$\Delta Q \propto m \quad (1)$$

$$\& \Delta Q \propto \Delta T \quad (2)$$

Combining relations (1), (2), we get,

$$\Delta Q \propto m\Delta T \Rightarrow \Delta Q = \text{constant} \cdot m\Delta T$$

$$\Rightarrow \Delta Q = Cm\Delta T \quad (3)$$

In equation (3), "C" is constant and is known as specific heat of a substance. From equation (3), we have,

$$C = \frac{\Delta Q}{m\Delta T} = \frac{\Delta Q}{1\text{kg} \cdot 1\text{K}}$$

So the specific heat "C" of a substance can be defined as the amount of heat required to raise the temperature of 1kg of a substance through 1K is called specific heat of a substance. The unit of specific heat of a substance is $\text{J/kg} \cdot \text{K}$.

(b) Molar Specific Heat of a Gas:

Definition: The amount of heat required to raise the temperature of 1 mole of a gas through 1K is known as molar specific heat of a gas.

Explanation:

Let "n" represents the number of moles of a gas, " ΔQ " represents the amount of heat supplied to the gas and " ΔT " represents the rise in temperature then we have,

$$\Delta Q \propto n \quad (1)$$

$$\& \Delta Q \propto \Delta T \quad (2)$$

Combining relations (1), (2), we get,

The amount of heat required to raise the temperature of a body through 1K is called heat capacity. (ETEA 2010)

$$\Delta Q \propto n\Delta T \Rightarrow \Delta Q = C_n n\Delta T \quad (3)$$

In equation (3) " C_n " is constant and is known as molar specific heat of a gas. From equation (3), we have,

$$C_n = \frac{\Delta Q}{n\Delta T} = \frac{\Delta Q}{1\text{mol} \cdot \text{K}}$$

So the molar specific heat of a gas is defined as the amount of heat required to raise the temperature of 1 mole of a gas through 1K, is called molar specific heat of a gas.

Q9: Define and explain the following terms: (a) Molar specific heat at constant volume, (b) Molar specific heat at constant pressure.

Ans: (a) Molar Specific Heat at Constant Volume:

Definition: The amount of heat required to raise the temperature of 1 mole of a gas through 1K by keeping the volume constant is called molar specific heat at constant volume.

Explanation:

Let " ΔQ_v " represents the amount of heat supplied to "n" moles of a gas enclosed in a cylinder at constant volume. Now if " ΔT " represents the rise in temperature of the system, then

$$\Delta Q_v \propto n \quad (1)$$

$$\& \Delta Q_v \propto \Delta T \quad (2)$$

Combining relations (1), (2), we get,

$$\Delta Q_v \propto n\Delta T$$

$$\Rightarrow \Delta Q_v = \text{constant} \cdot n\Delta T$$

$$\Rightarrow \Delta Q_v = C_v n\Delta T \quad (3)$$

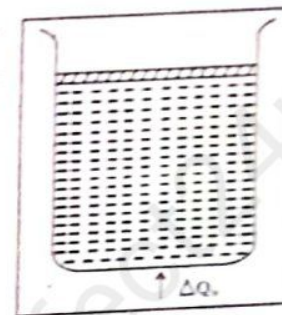
In equation (3) " C_v " is constant and is known as molar specific heat at constant volume. Its unit is " $\text{J/mole} \cdot \text{K}$ ".

(b) Molar Specific Heat at Constant Pressure:

The amount of heat required to raise the temperature of 1 mole of a gas through 1K at constant pressure is known as molar specific heat at constant pressure.

Explanation:

Let " ΔQ_p " represents the amount of heat supplied to "n" moles of



a gas by keeping the pressure constant, and let " ΔT " represents the rise in temperature, then we have,

$$\Delta Q_p = n \Delta T \quad (1)$$

$$\& \Delta Q_v = n \Delta T \quad (2)$$

Combining relations (1), (2), we get,

$$\Delta Q_p = n \Delta T \Rightarrow \Delta Q_p = \text{constant} \times n \Delta T$$

$$\Rightarrow \Delta Q_p = C_p n \Delta T \quad (3)$$

Equation (3) represents amount of heat " ΔQ_p " supplied to the system at constant pressure. In equation (3) " C_p " is constant and is known as molar specific heat at constant pressure. Its units is " $\text{J/mole}\cdot\text{K}$ ".

Q10: Show that, for one mole of an ideal gas $C_p - C_v = R$.

BISE Malakand 2019, BISE DI Khan 2019, BISE Pesh 2019, BISE Bannu 2017

Proof:

We can supply heat to a gas enclosed in a cylinder at constant pressure as well as at constant volume. Now consider " n " moles of an ideal gas enclosed in a cylinder as shown in the figure. Now at constant volume, we have,

$$\Delta Q_v = n C_v \Delta T \quad (1)$$

Now according to first law of thermodynamics, we have,

$$\Delta Q = \Delta u + \Delta w \quad (2)$$

At constant volume, we have,

$$\Delta w = 0, \text{ so equation (2) becomes,}$$

$$\Delta Q_v = \Delta u \quad (3)$$

Putting equation (3) in equation (1), we get,

$$\Delta u = n C_v \Delta T \quad (4)$$

Now consider the same system at constant pressure, for which we have,

$$\Delta Q_p = n C_p \Delta T \quad (5)$$

Now according to first law of thermodynamics, we have,

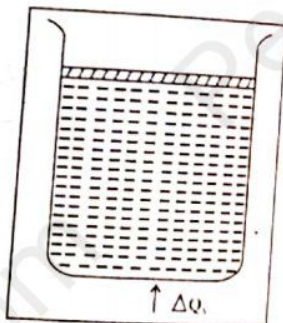
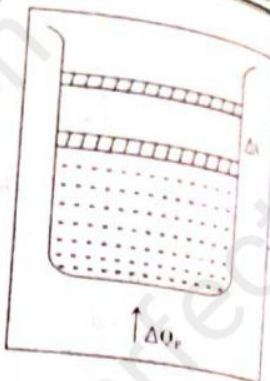
$$\Delta Q_p = \Delta u + \Delta w \quad (6)$$

Putting equation (4), (5) in equation (6), we get,

$$n C_p \Delta T = n C_v \Delta T + \Delta w$$

$$\Rightarrow n C_p \Delta T = n C_v \Delta T + P \Delta V \quad (7) \quad [\Delta w = P \Delta V]$$

We also know that,



$$PV = nRT$$

$$\Rightarrow P \Delta V = n R \Delta T \quad (8)$$

Putting equation (8) in equation (7),

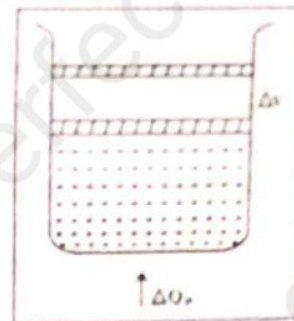
$$\text{We get,}$$

$$n C_p \Delta T = n C_v \Delta T + n R \Delta T$$

$$\Rightarrow \Delta T [C_p] = \Delta T [C_v + R]$$

$$\Rightarrow C_p = C_v + R \quad (9)$$

$$\Rightarrow C_p - C_v = R \quad (10)$$



Note: Equation (9) $\Rightarrow C_p > C_v$, it is due to the fact, when a gas is heated at constant volume, then heat supplied is totally used to increase the internal energy of the system. While at constant pressure, the same amount of heat is used to increase the internal energy of the system as well as to do work against the piston. Now for getting the same rise in internal energy in both cases, we must supply more heat at constant pressure than at constant volume. That is why, $C_p > C_v$.

Point to Ponder:

A football is inflated in a warm room. It is used out of door on a cold day. What happens to the ball? Why?

Ans: Since the temperature outside the room is cold therefore when football is taken outdoor, the gas inside it will contract and the football will be less inflated.

Q11: Explain with examples reversible and irreversible processes.

BISE Bannu 2017

Ans: (a) Reversible Process:

Definition: A process is said to be reversible if it can be retraced exactly in reverse order without producing any change in the surroundings.

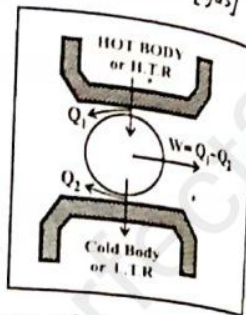
Explanation:

In the reverse process, the working substance [system or gas] passes through the same stages as in the direct process but thermal and mechanical effects at each state are exactly reversed. It means that during a reversible process, if heat is absorbed in the direct process, it will be given out in the reverse process.

Similarly, if work is done by the working substance in the direct process, an equal amount of work will be done on the working substance in the reverse process. Thus the working substance [gas] is restored to its original condition.

Examples of Reversible Process:

1. The liquification and evaporation of a substance performed slowly are reversible process.
2. A gas enclosed in a cylinder having frictionless piston is another example of reversible process. If the gas is compressed gradually by increasing pressure on the piston, then on decreasing the pressure, the gas can expand and can regain its original condition.
3. The gradual compression and extension of a spring with elastic limit is reversible process.



(b) Irreversible Process:

Definition: A process which cannot be retraced in the backward [opposite] direction by reversing the controlling factors is known as irreversible process.

Explanation:

Most of the processes in nature are irreversible due to dissipation of energy through friction, conduction, convection and radiation phenomenon.

Examples of Irreversible Process:

1. An explosion of bomb and other explosive materials is irreversible process.
2. Production of heat in electric circuits is an irreversible process.
3. Workdone against the force of friction is an irreversible process.
4. Rusting of iron is an irreversible process.

Q12: What is meant by a heat engine? What is its main purpose? Describe an expression for its efficiency.

ETEA 2016, BISE Abbottabad 2019

Ans. Heat Engine:

It is a device which converts heat energy into mechanical work.

Construction:

It consists of hot body [or source of heat, or high temperature reservoir H.T.R], cold body [or sink or low temperature reservoir

L.T.R] and engine as shown in the figure.

Principle of Working:

The principle of heat engine is that an amount of heat " Q_1 " is taken from hot body. Some part of " Q_1 " is converted into mechanical work " W " and some part " Q_2 " is rejected to cold body.

Working of Heat Engine:

The heat engine consists of certain gas. This engine undergoes various processes and then come back to its original state. These processes form one complete cycle. After one cycle, the internal energy of the gas regains its original value.

Efficiency of Heat Engine:

The ratio between output and input is known as the efficiency of heat engine i.e.

$$\eta = \frac{\text{output}}{\text{input}} \quad (1)$$

During the whole process, amount of heat " Q_1 " is taken from hot body which is taken as input i.e.

$$\text{Input} = Q_1 \quad (2)$$

As " Q_2 " is the amount of heat rejected to cold body, so the energy converted into mechanical work will be equal to " $Q_1 - Q_2$ ". This mechanical work is taken as output i.e.

$$\text{Output} = Q_1 - Q_2 \quad (3)$$

Putting equations (2), (3) in equation (1), we get,

$$\eta = \frac{(Q_1 - Q_2)}{Q_1} = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$$

$$\Rightarrow \eta = 1 - \frac{Q_2}{Q_1} \quad (4)$$

In terms of temperature, equation (4) can be written as,

$$\eta = 1 - \frac{T_2}{T_1} \quad (5)$$

Equations (4), (5) represents the efficiencies of heat engine which is always less than 100%.

Brain Teaser:

What is the function of spark plug in a petrol engine?

Ans. Spark plugs are used in the piston chambers of petrol engines for ignition of air-petrol mixture. When the cylinder draws fuel in and compresses it, the spark plug creates a spark [much like the ones created on a gas stove] which ignites the petrol and pushes the piston back up. The piston is connected to the crank shaft which, through the flywheel and the propeller shaft makes your wheels turn. The spark plugs don't actually help your car start up. Instead, there is a separate motor [the starter motor surprisingly] which is electric and helps turn the main engine over so that the pistons will start moving in the first place.

There is no spark plug in a diesel engine, then how does fuel burn in it?

Ans. When compressed beyond a certain point, an air-fuel mixture becomes so unstable [due to heat and closeness of the molecules] that it ignites without spark. The energy produced from the compression and the subsequent collisions of atoms takes the place of the energy of the spark. To withstand this extra compression the diesel engine is made of stronger construction and thus is much heavier.

Q13: State the 2nd law of thermodynamics in its alternative forms. Discuss the assertions of the first and second laws about heat and work energies.

ETEA 2011, 2016

Ans. The second law of thermodynamics is a generalization of the law of conservation of energy. It tells us that heat and mechanical work are mutually interconvertible.

The second law of thermodynamics tells us that how heat energy can be converted into useful work. This law can be stated in a number of forms, two of which are given below:

1. Lord Kelvin Statement:

It states that, it is impossible to construct a heat engine, operating continuously in a cycle, which takes heat from hot body at higher temperature and performs an equivalent amount of work without rejecting any heat to a cold body at low temperature.

Explanation:

According to Kelvin statement, a heat source at high temperature and a heat sink [cold body] at low temperature are essentials for the conversion of heat into mechanical work. Heat cannot be completely converted into useful work during a complete cycle. A heat engine absorbs an amount of heat from hot body and converts some parts of it into useful work [mechanical work] and rejects the remaining heat to cold body.

Therefore, the work performed by the heat engine will be less than the heat absorbed. Thus the efficiency of heat engine will be always less than 100%. The Kelvin statement gives us the principle of heat engine.

2. Rudolf Clausius Statement:

It states that, it is impossible to make the flow of heat from a cold body to a hot body without the expenditure of energy.

Explanation:

From our daily life experience we know that heat flows from hot body to a cold body when the two are in thermal contact. But heat can be made to flow from cold body to a hot body by an expenditure of energy. For example, in case of refrigerator, heat is given out from the cool interior part to the warmer surroundings. So the Clausius statement gives us the principle of refrigerator.

Assertions of the First and Second Laws of Thermodynamics about Heat and Work Energies:

The first law of thermodynamics tells us of the equivalence of heat and work. The first law asserts that for a cyclic process the heat energy flowing into the system is equal to the workdone by the system i.e.

$$Q = W$$

The second law is concerned with the conversion of heat into work. The second law asserts that the work performed will be less than the heat absorbed i.e.

$$Q > W \text{ or } W < Q$$

Q14: What were the basic questions that led Carnot to invent Carnot engine?

Ans. A heat engine is a closed system that exchanges only heat and

work with its surrounding and that operates in cycles. All cyclic heat engines derive heat from hot body and convert some of it into mechanical work and reject the remaining heat to the cold body. Due to dissipative effects, the efficiency of all heat engines is reduced sufficiently, and thus the efficiency of a heat engine is always less than 100%.

But here, some questions arise about the efficiency of the engine i.e.

1. What should be the maximum possible efficiency of an engine working between two heat reservoirs at different temperatures?
2. Can we improve the efficiency of heat engine?

These were the basic questions that led Carnot to invent Carnot engine. Carnot was a French military engineer. In 1824, he considered a hypothetical, idealized heat engine which was free from all sorts of heat losses and friction and is known as "Carnot heat engine". The efficiency of an actual heat engine is always less than that of Carnot heat engine. He told that the efficiency of an engine can be improved by increasing the temperature of the hot reservoir and by reducing the frictional effects.

Q15: Discuss the construction, working and efficiency of Carnot heat engine in detail.

ETEA 2009, 2015, BISE Peshawar 2017-18m BISE Mardan 2017

Ans: The Carnot heat engine is a hypothetical, idealized heat engine which is free from all sorts of heat losses and friction and possesses maximum efficiency than all other heat engines.

Construction:

The Carnot engine consists of the following parts:

1. High Temperature Reservoir [H.T.R]:

This part was used for supplying heat energy to operate the engine for mechanical work. The heat absorbed from H.T.R is taken as input.

2. Low Temperature Reservoir [L.T.R]:

This part was used to accept the amount of heat rejected by the engine during its operation.

3. A Block of Insulator:

This part was used to prevent the flow of heat into the system or out of the system during adiabatic process.

4. Cylinder:

It was used for maintaining gas inside it.

Work:

The working of Carnot engine consists of four processes which are performed one after the other in a certain order. These four processes form one cycle which is known as Carnot cycle. These processes are given below:

a. Isothermal Expansion:

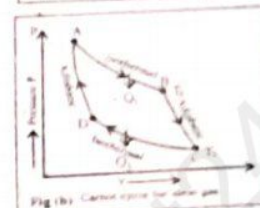
In this process, the gas is allowed to expand isothermally by placing the base of the cylinder on "H.T.R", as shown in the figure (a). The isothermal expansion is shown by the curve "AB" in the given figure.



b. Adiabatic Expansion:

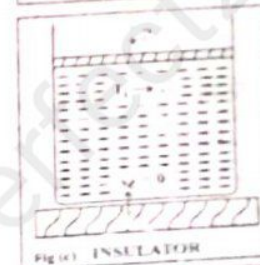
In this process, the gas is allowed to expand adiabatically by placing the base of the cylinder on an insulator, as shown in the figure (c). As no heat enters the system, so work is done at the cost of internal energy and thus the temperature of the system falls from " T_1 " to " T_2 ".

The adiabatic expansion is represented by the curve "BC" in the given figure.



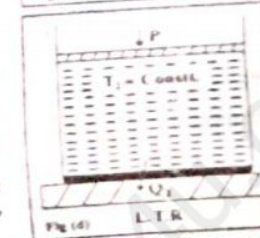
c. Isothermal Compression:

In this process, the gas is allowed to compress isothermally by placing the base of the cylinder on L.T.R as shown in the figure (d). In this case the amount of heat " Q_2 " is rejected to cold body for keeping the temperature " T_2 " constant. The isothermal compression is represented by the curve "CD" in the given figure.



d. Adiabatic Compression:

In this case, the gas is allowed to compress adiabatically by placing the base of the cylinder on an insulator, as shown in the figure (e). As no heat leaves the system, so the temperature of the system rises from " T_2 ".



to " T_1 ".

The adiabatic compression is shown by the curve "DA" in the given figure. In this way, one cycle is complete which is known as Carnot cycle.

Efficiency of Carnot Engine:

During Carnot cycle, the amount of heat " Q_1 " supplied to the system is taken as input. While the amount of heat " Q_2 " is rejected to cold body. Thus " $Q_1 - Q_2$ " will be the amount of heat used for mechanical work. So " $Q_1 - Q_2$ " is taken as output. Now the efficiency of Carnot engine is given by,

$$\eta = \frac{\text{output}}{\text{input}} \Rightarrow \eta = \frac{(Q_1 - Q_2)}{Q_1} = \frac{Q_1}{Q_1} - \frac{Q_2}{Q_1}$$

$$\Rightarrow \boxed{\eta = 1 - \frac{Q_2}{Q_1}} \quad (1)$$

In terms of temperature, the efficiency can be written as,

$$\boxed{\eta = 1 - \frac{T_2}{T_1}} \quad (2)$$

Equations (1), (2) represents the efficiencies of Carnot engine. These equations show that the efficiency of the engine is never 100% or more than 100%. It may be nearer to 100%, if the temperature of hot body [H.T.R] is taken as larger and the frictional effects are reduced as much as possible.

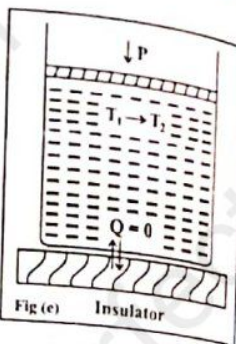
Interesting Information:

The first successful petrol engine was invented by Nikolaus Otto in the year 1876 and the Diesel engine was invented by Rudolph Diesel in 1892.

Q16: State and explain Carnot theorem.

Ans. Carnot Theorem:

Statement: This theorem states that, no real heat engine operating between two heat reservoirs can be more efficient than a Carnot engine, operating between the same two reservoirs.



Explanation:

We know that the efficiency of Carnot engine is given by,

$$\eta = 1 - \frac{T_2}{T_1} \quad (1)$$

Equation (1) shows that the efficiency of Carnot heat engine depends upon the temperature difference between the two reservoirs. The efficiency can be improved by raising the temperature of H.T.R [hot body] or by decreasing the temperature of L.T.R [cold body]. If we put $T_2 = 0K$, then equation (1) becomes,

$$\eta = 1 - \frac{0}{T_1} = 1 - 0 \Rightarrow \eta = 1 = 100\%$$

i.e. if the temperature of cold body is 0K, then the efficiency of Carnot engine will be 100%. But practically, it is not possible, because we cannot achieve the temperature of absolute zero i.e. 0K. From this, Carnot deduced that no engine could have 100% efficiency. He showed that the efficiency of the best heat engine that could be made was a function of the two temperatures at which heat was supplied and rejected in the cycle and was independent of the nature of the working substance. This statement is called Carnot theorem.

The real heat engine cannot attain Carnot efficiency due to presence of friction and heat losses by conduction and radiation.

Q17: What do you mean by a refrigerator? How does it function? Derive an expression for the coefficient of performance of a refrigerator.

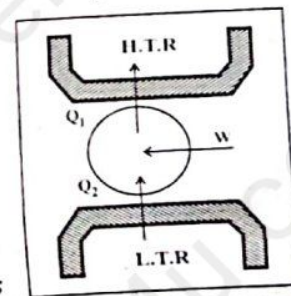
ETEA 2019

Ans. Refrigerator:

The device in which the working substance performs cycle in a direction opposite to that of a heat engine is known as refrigerator.

Explanation:

The refrigerator can keep a body cool or it can maintain a body temperature below that of the surroundings. When a refrigerator is used for cooling purposes in summer, then it is called refrigerator. When a refrigerator is



used for heating purposes in winter, then it is called a heat pump.

Principle of Refrigerator:

The principle of refrigerator is that, a working substance [called refrigerant] is taken through a cycle in such a way that some amount of heat " Q_2 " is removed from a low temperature source at temperature T_2 with expenditure of work energy " w ". This work is performed by the compressor of the refrigerator on the refrigerant. If " Q_1 " is the amount of heat rejected to the atmosphere [high temperature source] by the radiator of refrigerator, then we have,

$$Q_1 = w + Q_2 \quad (1)$$

Coefficient of Performance of Refrigerator:

The ratio of the amount of heat removed from cold body to the work required to do so is known as coefficient of performance of refrigerator.

(a) Now in case of cooling, the coefficient of performance is given by,

$$E_{\text{cooling}} = \frac{Q_2}{w} \quad (2)$$

From equation (1), we have,

$$Q_1 = w + Q_2 \Rightarrow w = Q_1 - Q_2 \quad (3)$$

Putting equation (3) in equation (2), we get,

$$E_{\text{cooling}} = \frac{Q_2}{Q_1 - Q_2} \quad (4)$$

In terms of temperature, equation (4) can be written as,

$$E_{\text{cooling}} = \frac{T_2}{(T_1 - T_2)} \quad (5)$$

(b) In case of heating, the coefficient of performance is given by,

$$E_{\text{heating}} = \frac{Q_1}{w} \quad (6)$$

$$\Rightarrow E_{\text{heating}} = \frac{Q_1}{(Q_2 - Q_1)} \quad (7) \quad [w = Q_2 - Q_1]$$

In terms of temperature, equation (7) can be written as,

$$E_{\text{heating}} = \frac{T_1}{(T_2 - T_1)} \quad (8)$$

No cyclic device has ever been built that will extract heat " Q_2 " from a cold reservoir and rejects it entirely to a hotter reservoir

without the expenditure of work. This statement is referred to as the Clausius statement of the second law of thermodynamics.

Point to Ponder:

Why is the freezer in the upper part of the refrigerator?

Ans. The freezer is in the upper part of the refrigerator due to the fact that most units have the compressor at the bottom. The compressor generates heat and would interfere more with a freezer than it does with the frig section.

Why do we not keep bananas in the refrigerator?

Ans. The cold temperature of your fridge encourages an enzyme polyphenyl oxidase, naturally found in the banana to polymerize phenols in the banana skin into polyphenols. Polyphenols are similar to melanin, the pigment responsible for the blackish colour in our skin. This is what blackens the skin of bananas.

For your Information:

Despite the color, the cold temperature will keep your banana firmer than a banana that was left at room temperature for the same amount of time. The enzymes that break the starch into sugar [which makes the banana soft and ripe] work better at room temperature. If you want to make other fruit ripen quickly, stick the unripe fruit in a bag with a blackened fridge banana. The ethylene being given off by the damaged banana skin will facilitate the quick ripening of other fruits.

Why does the refrigerator switch itself OFF intermittently with some noise?

Ans. That is actually the noise of a thermocouple called Relay. It is set in the refrigerator which switches OFF and ON it automatically producing some noise. Relay switches ON and OFF for temperature regulation.

Q18: Explain the concept of entropy. How is the second law of thermodynamics expressed in terms of entropy?

Ans. Entropy:

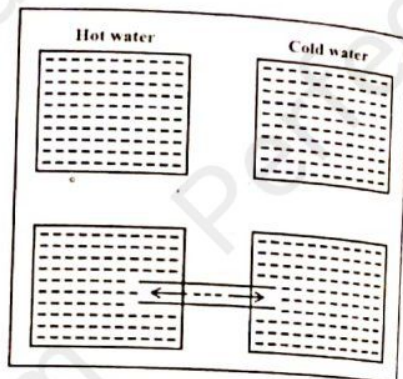
The unavailability of energy for its conversion into useful work is known as entropy. (OR)

The measurement of the disorder of the system is known as entropy.

Example-1:

Consider two tanks of water such that one containing cold water and the other containing hot water, as shown in the figure. Thus there is an ordered arrangement.

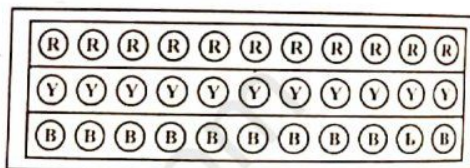
Now if we connect both tanks by means of a pipe, then the water molecules of both tanks will mix up and thus transfer of energy will



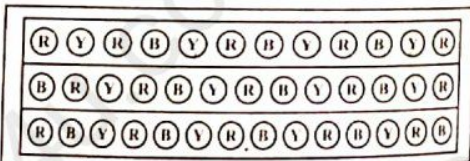
take place. Finally, a stage comes when both the tanks will acquire the same temperature. In this way, thermal equilibrium exists between the two tanks and under such conditions there will be no transformation of heat energy and no useful work will be done. This unavailability of energy for its conversion into useful work is known as entropy.

Example-2:

Consider a box which contains red, blue and yellow colour balls as shown in the figure.



In the beginning, there is an ordered arrangement. i.e. all red balls are present in first row, all yellow balls are



present in second row while all blue balls are present in third row.

Now if we shrink the box, then all the balls will be mixed up and there will be no chance for each ball to occupy their original position. Such disorder of the balls is known as entropy.

Second Law of Thermodynamics in terms of Entropy:

It states that, the entropy of the universe either remains constant or increases. The temperature of universe increases gradually with the passage of time. Finally a stage will come when the temperature of the universe will become the same. Thus thermal equilibrium will exist and no energy will be available for its conversion into useful work. Every living things will face certain death which is known as heat death of universe.

Now if " ΔS " represents the change in entropy of the system, when an amount of heat " ΔQ " is supplied to the system at temperature " T ", then

$$\Delta S = \pm \frac{\Delta Q}{T} \quad \text{--- (1)}$$

Equation (1) represents the mathematical form of second law of thermodynamics in terms of entropy.

The change in entropy is taken as positive when heat is added to the system and is taken as negative when heat is removed from the system.

Unit of Entropy: (ETEA 2015)

$$\text{As } \Delta S = \frac{\Delta Q}{T} = \frac{J}{K}$$

Solution of Examples & Assignments

EXAMPLE 10-1:

A heat engine operates with 65.0 Kcal of heat supplied and exhausts 40.0 Kcal of heat. How much work did the engine do?

Solution: Given data: Input heat = $Q_1 = 65 \text{ Kcal}$

Heat rejected = $Q_2 = 40 \text{ Kcal}$

Mechanical equivalent of heat = $J = 4184 \text{ J}$

Required: Workdone by engine = $w = ?$

Calculation: The relation between " Q_1 ", " Q_2 " " J " and " w " is given by,
 $w = J(Q_1 - Q_2)$ (1)

Putting the values in equation (1), we get,

$$w = 4184(65 - 40) \text{ J} = (4184 \times 25) \text{ J}$$

$$\Rightarrow w = 104600.0 \text{ J} = 104.6 \times 10^3 \text{ J}$$

$$\Rightarrow w = 105 \times 10^3 \text{ J} = 105 \text{ KJ}$$

$$[10^3 = \text{kilo}]$$

$$\text{Ans. } w = 105 \text{ KJ}$$

ASSIGNMENT 10-1:

An ideal gas absorbs $5.00 \times 10^3 \text{ J}$ of energy while doing $2.00 \times 10^3 \text{ J}$ of work on the environment during a constant pressure process. (a) Compute the change in the internal energy of the gas. (b) If the internal energy now drops by $4.50 \times 10^3 \text{ J}$ and $7.50 \times 10^3 \text{ J}$ is expelled from the system, find the change in volume, assuming a constant pressure process at $1.01 \times 10^5 \text{ Pa}$. ($3.00 \times 10^5 \text{ J}$, $-2.97 \times 10^{-2} \text{ m}^3$)

Solution:

Given data: Part-a: Heat supplied = $\Delta Q = 5 \times 10^3 \text{ J}$
 Workdone = $\Delta w = 2 \times 10^3 \text{ J}$

Required:

(i) Change in internal energy = $\Delta u = ?$

Part-b: Change in internal energy = $\Delta u = 4.5 \times 10^3 \text{ J}$

Heat given out from the system = $\Delta Q = 7.5 \times 10^3 \text{ J}$

Constant pressure = $P = 1.01 \times 10^5 \text{ Pa}$

Required: (ii) Change in volume = $\Delta V = ?$

Calculation: Part-a: We know that,

$$\Delta Q = \Delta u + \Delta w \Rightarrow \Delta u = \Delta Q - \Delta w \quad (1)$$

Putting the values in equation (1), we get,

$$\Delta u = (5 \times 10^3 - 2 \times 10^3) \text{ J} = (5 - 2) \times 10^3 \text{ J}$$

$$\Rightarrow \Delta u = 3 \times 10^3 \text{ J}$$

Part-b: For constant pressure process [isobaric process] we have, $[\Delta w = P\Delta V]$

$$\Delta Q = \Delta u + \Delta w \Rightarrow \Delta Q = \Delta u + P\Delta V$$

$$\Rightarrow P\Delta V = \Delta Q - \Delta u \Rightarrow \Delta V = \frac{\Delta Q - \Delta u}{P} \quad (2)$$

Putting the values in equation (2), we get,

$$\Delta V = \left[\frac{7.5 \times 10^3 - 4.5 \times 10^3}{1.01 \times 10^5} \right] \text{ m}^3$$

$$\Rightarrow \Delta V = \frac{(7.5 - 4.5)}{1.01 \times 10^5} \times 10^3 \text{ m}^3 = \frac{3}{1.01} \times 10^3 \times 10^{-5} \text{ m}^3$$

$$\Rightarrow \Delta V = 2.97 \times 10^{-2} \text{ m}^3$$

As heat is given out from the system, so the volume decreases. Thus we take " ΔV " as negative, so

$$\Delta V = -2.97 \times 10^{-2} \text{ m}^3$$

EXAMPLE 10-2:

In a certain process, 400 J of heat energy is supplied to a system and at the same time 150 J of work is done by the system. What is the increase in internal energy of the system?

Solution:

Given data: Heat supplied = $\Delta Q = 400 \text{ J}$

Workdone = $\Delta w = 150 \text{ J}$

Required: Increase in internal energy = $\Delta u = ?$

Calculation: We know that,

$$\Delta Q = \Delta u + \Delta w \Rightarrow \Delta u = \Delta Q - \Delta w \quad (1)$$

Putting the values in equation (1), we get,

$$\Delta u = 400 \text{ J} - 150 \text{ J} = 250 \text{ J}$$

$$\text{Ans. } \Delta u = 250 \text{ J}$$

ASSIGNMENT 10-2:

During one cycle, an engine extracts $2.00 \times 10^3 \text{ J}$ of energy from a hot reservoir and transfers $1.50 \times 10^3 \text{ J}$ to a cold reservoir. (a) Find the

thermal efficiency of the engine. (b) How much work does this engine do in one cycle? (c) What average power does the engine generate if it goes through four cycles in 2.50s? (0.250, or 25.0%, $5.00 \times 10^2 \text{ J}$, $8.00 \times 10^2 \text{ W}$)

Solution:

Given data: Input heat = $Q_1 = 2 \times 10^3 \text{ J}$

Heat rejected = $Q_2 = 1.5 \times 10^3 \text{ J}$, time = $t = 2.5 \text{ sec}$

Required: (a) Efficiency of engine = $\eta = ?$

(b) Workdone in one cycle = ?

(c) Power generated in four cycles = ?

Calculation: (a) We know that,

$$\eta = 1 - \frac{Q_2}{Q_1} \quad (1)$$

Putting the values in equation (1), we get,

$$\eta = 1 - \frac{1.5 \times 10^3}{2 \times 10^3} = 1 - \frac{1.5 \times 10^3}{2 \times 10^3} = 1 - \frac{1.5}{2} = 1 - 0.75 \Rightarrow \boxed{\eta = 0.25}$$

$$(OR) \eta = (0.25 \times 100)\% \Rightarrow \boxed{\eta = 25\%}$$

(b) We know that,

$$\eta = \frac{\text{output}}{\text{input}} \Rightarrow \text{output} = \eta \times \text{input} \quad (2)$$

Here output of the engine represents the workdone in one cycle, so equation (2) becomes,

$$\text{Workdone in one cycle} = \eta \times \text{input} \quad (3)$$

Putting the values in equation (3), we get,

$$\text{Workdone in one cycle} = 0.25 \times 2 \times 10^3 \text{ J}$$

$$\Rightarrow \text{Workdone in one cycle} = 0.5 \times 10^3 \text{ J}$$

$$\Rightarrow \boxed{\text{Workdone in one cycle} = 5.0 \times 10^2 \text{ J}}$$

(c) We know that,

$$\text{Power} = \frac{\text{workdone in four cycles}}{\text{time}} \quad (4)$$

We are asked to calculate the power generated in four cycles, so workdone in four cycles given by,

$$\text{Workdone in four cycles} = 4 \times \text{workdone in one cycle}$$

$$\Rightarrow \text{Workdone in four cycles} = 4 \times 5 \times 10^2 \text{ J} = 20 \times 10^2 \text{ J}$$

Putting these values in equation (4), we get,

$$P = \left(\frac{20 \times 10^2}{2.50} \right) \text{ watt} = 8 \times 10^2 \text{ watt}$$

$$\text{Ans. } \boxed{P = 8 \times 10^2 \text{ watt}}$$

EXAMPLE 10.3:

What is the change in internal energy of 200gm of nitrogen as it is heated from 10°C to 30°C at constant volume? For nitrogen gas, $C_v = 20.815 \text{ J/mol}\cdot\text{K}$.

Solution:

Given data: Mass of nitrogen = $m = 200 \text{ gm}$

Molecular mass of nitrogen = $M = 28 \text{ gm}$

Number of mole of nitrogen = $n = \frac{\text{given mass}}{\text{molecular mass}}$

$$\Rightarrow n = \frac{200}{28} = 7.143 \text{ moles}$$

Initial temperature of N_2 gas = $T_1 = 10^\circ\text{C}$

Final temperature of N_2 gas = $T_2 = 30^\circ\text{C}$

Change in temperature = $\Delta T = T_2 - T_1 = 30 - 10 = 20^\circ\text{C}$

$$\Rightarrow \Delta T = 20 \text{ K}$$

Specific heat of nitrogen = $C_v = 20.815 \text{ J/mol}\cdot\text{K}$

Required: Change in internal energy of N_2 gas = $\Delta u = ?$

Calculation: At constant volume, we have,

$$\Delta Q_v = \Delta u$$

$$\Rightarrow nC_v\Delta T = \Delta u \Rightarrow \Delta u = nC_v\Delta T \quad (1)$$

Putting the values in equation (1), we get,

$$\Delta u = 7.143 \times 20.815 \times 20 \text{ J} \Rightarrow \boxed{\Delta u = 2973.6 \text{ J}}$$

$$\text{Ans. } \boxed{\Delta u = 2973.6 \text{ J}}$$

ASSIGNMENT 10.3:

(a) Find the change in entropy of $3.00 \times 10^2 \text{ g}$ of lead when it melts at 327°C . Lead has a latent heat of fusion of $2.45 \times 10^4 \text{ J/kg}$. (b) Suppose the same amount of energy is used to melt part of a piece of silver, which is already at its melting point of 961°C . Find the change in the entropy of the silver. (12.3 J/K , 5.96 J/K)

Solution: **Given data:**

(a) Mass of lead = $m = 3 \times 10^2 \text{ gm} = 3 \times 10^2 \times 10^{-3} \text{ kg}$

$$\Rightarrow m = 3.0 \times 10^{-1} \text{ kg} = 0.3 \text{ kg}$$

Melting temperature of lead = $T_c = 327^\circ\text{C}$

$$= (327 + 273) \text{ K} = 600 \text{ K}$$

Heat of fusion of lead = $H_f = 2.45 \times 10^4 \text{ J / kg}$

Required: (a) Change in entropy of lead = $\Delta S_L = ?$

(b) Melting temperature of silver = $T_s = 961^\circ\text{C}$

$$= (961 + 273) \text{ K} = 1234 \text{ K}$$

Required: Change in entropy of silver = $\Delta S_s = ?$

Mass of silver = $m = 3 \times 10^{-2} \text{ gm} = 0.3 \text{ kg}$

Heat of fusion of silver = $H_f = 2.45 \times 10^4 \text{ J / kg}$

Calculation: (a) We know that,

$$\Delta S = \frac{\Delta Q}{T} \quad (1)$$

Also $\Delta Q = mH_f$, so equation (1) becomes,

$$\Delta S_L = \frac{mH_f}{T_L} \quad (2)$$

Putting the values in equation (2), we get,

$$\Delta S_L = \frac{0.3 \times 2.45 \times 10^4}{600} \Rightarrow \Delta S_L = 1.23 \times 10^{-3} \times 10^4 \text{ J / K}$$

$$\Rightarrow \Delta S_L = 1.23 \times 10^1 \text{ J / K} \Rightarrow \boxed{\Delta S_L = 12.3 \text{ J / K}}$$

(b) For silver, equation (2) becomes,

$$\Delta S_s = \frac{mH_f}{T_s} \quad (3)$$

Putting the values in equation (3), we get,

$$\Delta S_s = \frac{0.3 \times 2.45 \times 10^4}{1234} \Rightarrow \Delta S_s = (5.96 \times 10^{-4} \times 10^4) \text{ J / K}$$

$$\Rightarrow \boxed{\Delta S_s = 5.96 \text{ J / K}}$$

EXAMPLE 10.4:

A reversible engine works between two temperatures whose difference is 100°C . If it absorbs 746 J of heat from the source and rejects 546 J to sink, calculate the temperature of the source and the sink.

Solution: Given data:

Change in temperature = $\Delta T = 100^\circ\text{C} = 100 \text{ K}$

Heat absorbed = $Q_1 = 746 \text{ J}$, heat rejected = $Q_2 = 546 \text{ J}$

Required: Temperature of source = $T_1 = ?$

Temperature of sink = $T_2 = ?$

Calculation: We know that,

$$\eta = 1 - \frac{T_2}{T_1} \quad (1)$$

We also know that,

$$\eta = 1 - \frac{Q_2}{Q_1} \quad (2)$$

Comparing equations (1), (2), we get,

$$1 - \frac{T_2}{T_1} = 1 - \frac{Q_2}{Q_1} \quad [T_1 - T_2 = \Delta T]$$

$$\Rightarrow \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow \frac{\Delta T}{T_1} = \frac{Q_1 - Q_2}{Q_1} \Rightarrow \frac{T_1}{\Delta T} = \frac{Q_1}{Q_1 - Q_2}$$

$$\Rightarrow T_1 = \frac{Q_1}{(Q_1 - Q_2)} \times \Delta T = \frac{746 \times 100}{(746 - 546)}$$

$$\Rightarrow T_1 = \left(\frac{74600}{200} \right) \text{ K} \Rightarrow T_1 = 373 \text{ K}$$

$$\Rightarrow T_1 = (373 - 273)^\circ\text{C} = 100^\circ\text{C} \Rightarrow \boxed{T_1 = 100^\circ\text{C}}$$

As $T_1 - T_2 = \Delta T$

$$\Rightarrow T_2 = T_1 - \Delta T = 100 - 100 \Rightarrow \boxed{T_2 = 0^\circ\text{C}}$$

EXAMPLE 10.5:

A Carnot heat engine has a maximum efficiency of 55% . It takes certain amount of heat from a source, converts a part of it into work and rejects the remaining heat towards the heat sink. If the temperature of the heat source is 600 K then find the temperature of the heat sink.

Solution: Given data: Temperature of source = $T_1 = 600 \text{ K}$, $\eta = 55\% = 0.55$

Required: Temperature of sink = $T_2 = ?$

Calculation: We know that,

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow 0.55 = 1 - \frac{T_2}{600} \Rightarrow \frac{-T_2}{600} = 0.55 - 1 = -0.45$$

$$\Rightarrow \frac{-T_2}{600} = -0.45 \Rightarrow T_2 = 0.45 \times 600$$

$$\text{Ans. } \boxed{T_2 = 270 \text{ K}}$$

فرمان مصطفیٰ ﷺ: "جو کسی مسلمان کو زیادتی تکلیف سے رہائی دے تو اللہ تعالیٰ اس سے قیامت کے دن کی معیبت ڈور فرمائے گا۔" (ترمذی)

EXAMPLE 10-6:

A refrigerator has a coefficient of performance 8. If the temperature in the freezer is -23°C . What is the temperature at which it rejects heat?

Solution:

Given data: Coefficient of performance = $E = 8$,
 $T_2 = -23^{\circ}\text{C} = (-23 + 273)\text{K} = 250\text{K}$

Required: Temperature of heat source = $T_1 = ?$

Calculation: We know that,

$$E = \frac{T_2}{(T_1 - T_2)} \Rightarrow E = \frac{250}{(T_1 - 250)}$$

$$\Rightarrow T_1 - 250 = \frac{250}{E}$$

$$\Rightarrow T_1 - 250 = \frac{250}{8} = 31.25 \Rightarrow T_1 = 31.25 + 250$$

$$\Rightarrow T_1 = 281.25\text{K}$$

$$\Rightarrow T_1 = (281.25 - 273)^{\circ}\text{C} \Rightarrow T_1 = 8.25^{\circ}\text{C}$$

EXAMPLE 10-7:

What is the change in entropy of 30gm of water at 0°C as it is changed into ice at 0°C ?

Solution:

Given data: Mass of water

$$= m = 30\text{gm} = 0.03\text{kg}$$

Latent heat of fusion of ice

$$= H_f = 336000\text{J/kg}$$

Heat removed from water

$$= \Delta Q = mH_f$$

$$\Rightarrow \Delta Q = 0.03 \times 336000\text{J} = 10080\text{J}$$

Constant temperature at fusion point = $T = 0^{\circ}\text{C}$

or $T = 273\text{K}$

Required: Change in entropy = $\Delta S = ?$

Calculation: We know that,

$$\Delta S = -\frac{\Delta Q}{T} \Rightarrow \Delta S = -\frac{10080}{273}$$

$$\text{Ans. } \Delta S = -36.92\text{J/K}$$

When heat is removed from the system, then the change in entropy is taken as negative.

Exercise

Choose the best possible answer:

- In assume we can change the equilibrium state of a system via two different processes. Assume that the initial and the final state are the same. Which of the quantities $\Delta U, \Delta Q, \Delta W$, and ΔT must be the same for the two processes?
 a. Only ΔQ and ΔW ✓ (b) Only ΔU and ΔT
 c. Only ΔQ and ΔT (d) Only ΔU and ΔW
- In any process the maximum amount of mechanical energy that can be converted to heat:
 ✓ (a) Depends upon the amount of friction
 (b) Depends upon the intake and exhaust temperature
 (c) Depends upon whether kinetic or potential energy is involved
 (d) Is 100%
- In an isothermal change, internal energy:
 (a) Decreases (b) Increases
 (c) Becomes zero ✓ (d) Remains constant
- A thermos bottle containing hot coffee is vigorously shaken. Consider coffee as the system, then its temperature:
 ✓ (a) Increases (b) Decreases below than 0°C
 (c) Remains the same (d) Decreases
- Maximum work can be obtained in the process called:
 (a) Cyclic ✓ (b) Isothermal
 (c) Adiabatic (d) Isochoric
- A heat engine takes in 800J of heat at 1000K and exhausts 600J of heat at 400K. What is the actual efficiency of this engine?
 ✓ (a) 25% (b) 40%
 (c) 50% (d) 75%
- If the temperature of the heat source is increased, the efficiency of a Carnot's engine:
 ✓ (a) Increases (b) Decreases
 (c) Remains same (d) First increases then becomes constant

8. Triple point of water is:
 (a) 273.16°F (b) 372.16K
 (c) 273.16°F ✓ (d) 273.16K
9. A real gas can be approximated to an ideal gas at:
 ✓ (a) Low density (b) High pressure
 (c) High density (d) Low temperature
10. If the volume of the gas is to be increased by 4 times, then:
 (a) Temperature and pressure must be double
 ✓ (b) At constant P the temperature must be increased by four times
 (c) At constant T the pressure must be increased by four times
 (d) It cannot be increased
11. In which of the systems listed below is the entropy decreasing?
 ✓ (a) A gas is cooled (b) A plate is shattered
 (c) An egg is scrambled
 (d) A drop of dye diffuses in a cup of water
12. If the temperature of source and sink of a Carnot engine having efficiency " η " are each decreased by 100K , then the efficiency " η "
 (a) Remain constant (b) become 1
 ✓ (c) Increases (d) Decreases

حاصل اور لا حاصل کے دائروں میں سب گھوم رہے ہیں۔ حاصل کی تندرستی اور لا حاصل کی حریت قسم ہی نہیں ہوتی...

Short Questions

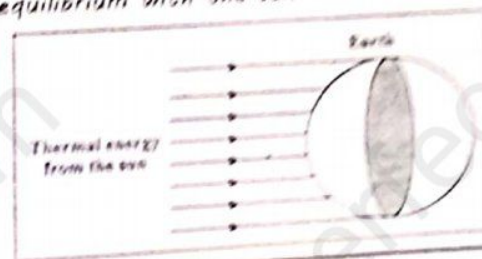
Write short answers of the following questions:

Q1: Why is the earth not in thermal equilibrium with the sun?

Ans: The earth is not in thermal equilibrium with the sun.

Reason and Explanation:

- During rotation of the earth, half of its surface is exposed to sun's radiations as shown in the figure. Thus sun's radiation does not fall on all parts of the earth at the same time.
- The temperature of the sun is about 6000K , while the average temperature of the earth is about 300K due to various factors. The process of sea breeze and land breeze also affect the temperature of earth.
- Due to fusion reactions, the temperature of sun is very high i.e. in the range of 6000K . As a result, the radiations coming from the sun possess mostly shorter wavelength. These radiations can pass freely through the earth's atmosphere without being absorbed. Thus the temperature of earth remains low as compared to the temperature of the sun. Therefore, the earth is not in thermal equilibrium with the sun.



Q2: When a block with a hole in it is heated, why does not the material around the hole expand into the hole and make it small?

Ans: When a block with a hole in it is heated, the material around the hole does not expand with the hole.

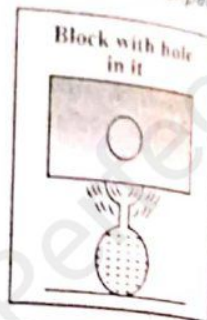
Explanation:

We know that most of the material expands on heating and contracts on cooling. In case of solid materials they oscillate about

their equilibrium position [mean position].

Now when we supply heat to a solid block, the K.E of its molecules increases. Due to increase in K.E, the amplitude of vibration of the molecules increases. Due to increase in amplitude of vibration of the molecules, the block expands and its volume increases. Such expansion is known as volumetric expansion. The molecules situated around the hole are pulled outward due to which the diameter of the hole increases.

A useful analogy to thermal expansion is that of photographic enlargement. Every linear dimension of the object undergoes the same fractional change. Thus increase in temperature will expand the hole and will not reduce it.



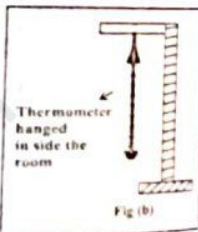
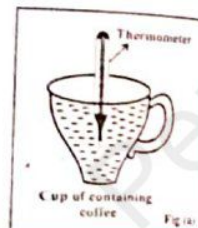
Q3: A thermometer is placed in direct sun light. Will it read the temperature of the air, or of the sun, or of something else?

Ans: When a thermometer is placed in direct sun light, it will read the temperature of the surrounding air because the thermometer is in contact with the air.

Explanation:

When we want to measure the temperature of an object, we place the mercury bulb of a thermometer in that object. For example,

1. When we want to measure the temperature of coffee, we place the mercury bulb of thermometer inside a cup containing coffee as shown in the figure (a).
2. When we want to measure the temperature of the room, we suspend the thermometer inside the room as shown in the figure (b).
3. When we want to measure the temperature of a patient, the doctor placed the thermometer below the tongue inside the mouth of the patient as shown in the figure (c).



Conclusion:

The above discussion shows that when a thermometer is placed in direct sun light, it will read the temperature of the surrounding air. It is because the thermometer is in direct contact with surrounding air and not with sun.

Q4: The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen, why?

Ans: The pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen.

Reason and Explanation:

According to Graham's law of diffusion, the rate of diffusion of a certain gas is inversely proportional to its molecular mass and density, i.e. smaller the molecular mass and density, greater will be the rate of diffusion and vice versa. As the hydrogen is lighter than oxygen i.e. its molecular mass and density is lower than that of oxygen, so the pressure in a gas cylinder containing hydrogen will leak more quickly than if it is containing oxygen.

Mathematical Proof:

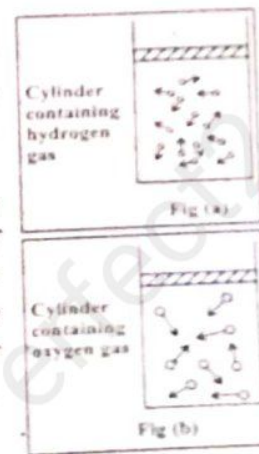
Consider two cylinders such that one containing hydrogen gas and the other containing oxygen gas as shown in the figure (a) and figure (b) respectively.

Let " V_H " represents the velocity of hydrogen molecule and " V_O " represents the velocity of oxygen molecule inside the cylinder. Now according to ideal gas equation, the velocity of a gas molecule is inversely proportional its molar mass, so we have,

$$V \propto \frac{1}{\sqrt{M}} \Rightarrow V = \text{constant} \frac{1}{\sqrt{M}} \quad (1)$$

For hydrogen molecule, equation (1) can be written as, $V_H = \text{constant} \frac{1}{\sqrt{M_H}}$ (2)

And for oxygen molecule, equation (1) can be written as,



$$V_o = \text{constant} \sqrt{\frac{T}{m_o}} \quad (3)$$

Dividing equation (2) by equation (3), we get,

$$\frac{V_u}{V_o} = \frac{\text{constant} \sqrt{\frac{T}{m_u}}}{\text{constant} \sqrt{\frac{T}{m_o}}} \Rightarrow \frac{V_u}{V_o} = \sqrt{\frac{m_o}{m_u}} \quad (4)$$

As molar mass of oxygen = 32gm and molar mass of hydrogen = 2gm
so equation (4) becomes,

$$\frac{V_u}{V_o} = \sqrt{\frac{32gm}{2gm}} = \sqrt{16} = 4$$

$$\Rightarrow \frac{V_u}{V_o} = 4 \Rightarrow V_u = 4V_o \quad (5)$$

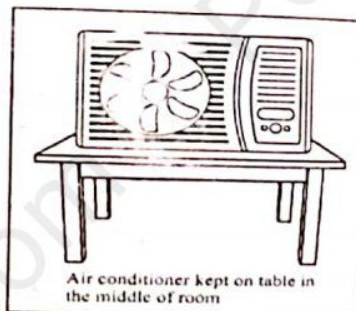
Equation (5) shows that velocity of hydrogen molecule is four times greater than velocity of oxygen molecule. Thus hydrogen molecules will leak more quickly than oxygen molecules.

Q5: What happens to the temperature of a room in which an air conditioner is left running on a table in the middle of the room?

Ans: When an air conditioner is left running on a table in the middle of the room, it will increase the temperature of the room slightly rather than cooling.

Explanation:

Consider an air conditioner is kept on a table in the middle of the room as shown in the figure. When the air conditioner is turned ON, it absorbs heat from the room from one side and expels the same amount of heat into the room through coil of the condenser at the same time. Thus the same amount of heat which is absorbed from the room is transferred to the room again. During this process, some heat is produced due to workdone by the compressor. This heat is also given out from compressor into the same room. As a result, the temperature of the room increases slightly.

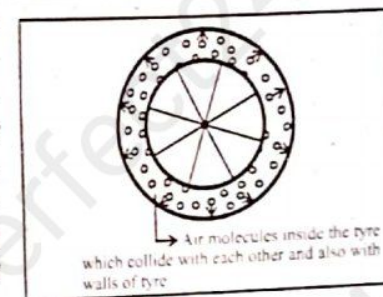


Q6: Why does the pressure of the air in automobile tyre increase if the automobile is driven for a while?

Ans: The pressure of the air in an automobile tyre increases if the automobile is driven for a while.

Reasons:

When an automobile is driven on a road for a while a force of friction exists between road and tyre. To overcome this force of friction, work is done. This workdone appears as heat which increases the temperature of the tyre. Due to increase in temperature of the tyre, the temperature of air molecules inside the tyre also increases. As a result, the kinetic energy of molecules increase due to which the velocity of molecules increases. They collide with each other and also with the walls of the tyre and exert more pressure. Thus the pressure of the air in an automobile tyre increases if the automobile is driven for a while.



Q7: On removing the valve, the air escaping from a cycle tube cools, why?

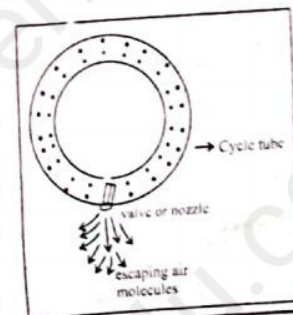
BISE Kohat 2017, 2019

Ans: On removing the valve, the air escaping from a cycle tube becomes cool.

Reasons:

When we open the valve of a cycle tube, the inside air escapes from the tube. It is an adiabatic process, because here work is done by the system and no heat enters the tube [system] from outside. So we have, $\Delta Q = 0$

Now according to first law of thermody-



namics, we have,

$$\Delta Q = \Delta u + \Delta w \quad (1)$$

Putting $\Delta Q = 0$ in equation (1), we get,

$$\Delta u = \Delta w = 0 \Rightarrow \Delta w = -\Delta u \quad (2)$$

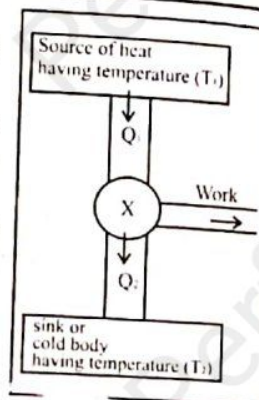
Equation (2) shows that work is done at the expenditure of internal energy. We know that the internal energy is a function of temperature. So when internal energy decreases the temperature also decreases. Thus the temperature of air decreases and becomes cool after escaping from a cycle tube.

Q8: Write the limitations of first law of thermodynamics.

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Ans.

1. It does not indicate the direction of heat transfer.
2. It does not tell us anything about the conditions under which heat can be converted into work.
3. It gives no information about frictional effects.
4. It does not tell us that why the whole of heat energy cannot be converted into mechanical work continuously.
5. It gives no information about the principle of heat engine and refrigerator.
6. It gives no informations about thermal equilibrium and entropy of the system.



Q9: Is it possible, according to the second law of thermodynamics, to construct a heat engine that is free from thermal pollution?

Ans: According to the second law of the thermodynamics, it is not possible to construct a heat engine that is free from thermal pollution.

Reasons:

According to second law of thermodynamics, it is not possible to construct a heat engine which would convert whole heat " Q_1 " taken from the source into mechanical work. It means that during operation, some heat " Q_2 " is rejected to the sink i.e. surrounding. Thus when heat en-

gine operates, it converts some part of the input heat " Q_1 " into useful work and reject the remaining heat to the surrounding. As a result, the temperature of the surrounding increases. This effect is known as thermal pollution. Thus the heat engine causes thermal pollution. In other words we can say that, it is impossible to construct a heat engine which is free from thermal pollution.

Q10: Can specific heat of a gas be zero or infinity? Can specific heat be negative?

Ans: Yes, the specific of a gas may be zero or infinity or negative under certain conditions.

Explanation:

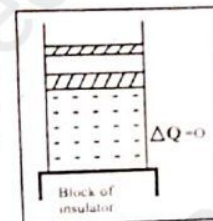
We know that, $\Delta Q = mC\Delta T$

$$\Rightarrow C = \frac{\Delta Q}{m\Delta T} \quad (1)$$

CASE-1: Zero Specific Heat:

Let a gas enclosed in a cylinder is allowed to expand adiabatically. Under such conditions we place the base of the cylinder on block of insulator. So no heat is supplied to the system. So we put $\Delta Q = 0$ in equation (1), we get,

$$C = \frac{0}{m\Delta T} \Rightarrow C = 0$$

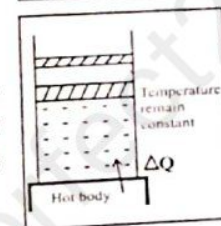


CASE-2: Infinite Specific Heat:

Let the gas is allowed to expand isothermally i.e. the fall in temperature due to expansion is balanced by rise in temperature due to heat supplied from hot body. So we have, $\Delta T = 0$

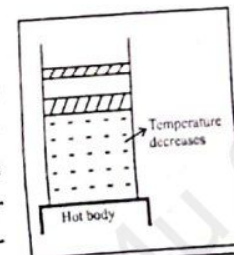
Thus equation (1) becomes,

$$C = \frac{\Delta Q}{m \times 0} = \frac{\Delta Q}{0} = \infty \Rightarrow C = \infty$$



CASE-3: Negative Specific Heat:

Let the gas is allowed to expand isobarically such that the fall in temperature due to expansion is greater than the rise in temperature due to heat supplied from hot body. Under such conditions we take " ΔT " negative, so equ-



ation (1) becomes,

$$C = \frac{\Delta Q}{m(-\Delta T)} = \frac{-\Delta Q}{m\Delta T}$$

$$\Rightarrow C = -\frac{\Delta Q}{m\Delta T} \quad (2)$$

Equation (2) shows that specific heat is negative.

Q11: An inventor claims to have developed a heat engine, working between 27°C and 227°C having an efficiency of 45%. Is the claim valid? Why?

Ans: In order to check the claim of the inventor, we calculate the efficiency of his engine in the light of given data:

Given Data:

Temperature of hot body = $T_1 = 227^\circ\text{C}$

$$\Rightarrow T_1 = (227 + 273)\text{K} = 500\text{K}$$

Temperature of cold body = $T_2 = 27^\circ\text{C}$

$$\Rightarrow T_2 = (27 + 273)\text{K} \Rightarrow T_2 = 300\text{K}$$

We know that the efficiency of heat engine is given by,

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \eta = 1 - \frac{300}{500}$$

$$\Rightarrow \eta = \frac{(500 - 300)}{500} = \frac{200}{500}$$

$$\Rightarrow \eta = 0.4 \Rightarrow \eta = 0.4 \times 100\% = 40\% \Rightarrow \eta = 40\%$$

Thus the maximum possible efficiency is 40% which is less than the claimed efficiency i.e. 45%. Thus the claim of the inventor is not valid.

Q12: Entropy has often called "time arrow". Explain.

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Ans: We often called the entropy as a time arrow.

Reason and Explanation:

When a certain process is performed, time must be consumed i.e. we say that how much time will be taken during the completion of a certain process. Similarly we explain the law of increase of entropy by giving the reference of time.

The law of increase of entropy states that all natural processes undergo certain changes. They proceed towards a state of greater entropy as well as towards greater disorder. This law of increase of entropy tells us that the entropy of all natural processes increases with time. In other words, we can say that the increase in entropy describes the direction of time. Thus we call the entropy as a time arrow.

Comprehensive Questions

Q1: Explain briefly, the following terms used in thermodynamics: System, Surroundings, Boundary and State variables

Ans: See answer of question number 4 in text.

Q2: Distinguish among the three forms of energy: work, heat and internal energy

Ans: See answer of question number 2 in text.

Q3: State and explain the first law of thermodynamics.

Ans: See answer of question number 6 in text.

Q4: In the light of the first law of thermodynamics describe the processes: (a) Isothermal process (b) Isobaric process

Ans: See answer of question number 7 in text.

Q5: Define the molar heat capacities C_p and C_v for a gas, show that for a mole of an ideal gas, $C_p - C_v = R$

Ans: See answers of questions number 9, 10 in text.

Q6: Explain with examples reversible and irreversible processes.

Ans: See answer of question number 11 in text.

Q7: What is meant by a heat engine? What is its main purpose? How is its efficiency defined?

Ans: See answer of question number 12 in text.

Q8: State the second law of thermodynamics in its alternative forms. Discuss the assertions of the first and second laws about heat and work energies.

Ans: See answer of question number 13 in text.

Q9: What were the basic questions that led Carnot to invent Carnot engine?

Ans: See answer of question number 14 in text.

Q10: What is meant by Carnot cycle and by Carnot engine?

Ans: See answer of question number 15 in text.

Q11: State Carnot theorem about the characteristics of a Carnot engine.

Ans: See answer of question number 16 in text.

Q12: What do you mean by a refrigerator? How does it function? Derive an expression for the coefficient of performance of a refrigerator.

Ans: See answer of question number 17 in text.

Q13: Explain the concept of entropy. Mention its major properties. How is the second law of thermodynamics expressed in terms of entropy?

Ans: See answer of question number 18 in text.

Numerical Problems

Q1: Water at 20°C falls from a height of 854 meters. If the whole energy is used in increasing the temperature, find out the final temperature. Specific heat of water is 4200 J K⁻¹ kg⁻¹. (22°C)

Solution:

Given data: Specific heat of water = $C = 4200 \text{ J/kg} \cdot \text{K}$

Initial temperature of water = $T_1 = 20^\circ\text{C} = (20 + 273) \text{K}$

$\Rightarrow T_1 = 293 \text{K}$

Height of water = $h = 854 \text{m}$

Acceleration due to gravity = $g = 9.8 \text{m/sec}^2$

Required:

Final temperature of water = $T_2 = ?$

Calculation: When the water falls from height "h", then according to the conservation of energy, we have,

Loss in P.E = Gain in heat energy

$$\Rightarrow mgh = \Delta Q$$

$$\Rightarrow mgh = mC\Delta T$$

$$\Rightarrow gh = C(T_2 - T_1)$$

$$\Rightarrow T_2 - T_1 = \frac{gh}{C} \Rightarrow T_2 = \frac{gh}{C} + T_1 \quad (1)$$

Putting values in equation (1), we get,

$$T_2 = \left(\frac{9.8 \times 854}{4200} + 293 \right) \text{K} = (1.99 + 293) \text{K}$$

$$\Rightarrow T_2 = 294.99 \text{K} \Rightarrow T_2 = 295 \text{K}$$

$$\Rightarrow T_2 = (295 - 273) \text{K} \Rightarrow T_2 = 22^\circ\text{C}$$

Ans: $T_2 = 22^\circ\text{C}$

Q2: 25200J of heat is supplied to the system while the system does 6000J of work. Calculate the change in internal energy of the system. (19200J)

Solution:

Given data: Heat supplied to the system = $\Delta Q = 25200 \text{J}$

Workdone by the system = $\Delta w = 6000 \text{J}$

Required:

Change in internal energy of system = $\Delta u = ?$

Calculation: According to the first law of thermodynamics, we have,

$$\Delta Q = \Delta u + \Delta w \Rightarrow \Delta u = \Delta Q - \Delta w \quad (1)$$

Putting the values in equation (1), we get,

$$\Rightarrow \Delta u = (25200 - 6000) \text{J} \Rightarrow \Delta u = 19200 \text{J}$$

Ans: $\Delta u = 19200 \text{J}$

Q3: A sample of ideal gas is uniformly heated at constant pressure. if the amount of 180J of heat is supplied to the gas. Calculate the change in internal energy of the gas and workdone by the gas. Take $\gamma = 1.41$ (27.66J, 52.34J)

Solution:

Given data:

Heat supplied at constant pressure = $\Delta Q_p = 180 \text{J}$, $\gamma = 1.41$

Required:

(i) Change in internal energy = $\Delta u = ?$

(ii) Workdone by the gas = $\Delta w = ?$

Calculation:

(i) From the first law of thermodynamics, we have,

$$\Delta Q = \Delta u + \Delta w \Rightarrow \Delta Q_p = \Delta u + \Delta w$$

$$\Rightarrow \Delta u = \Delta Q_p - \Delta w$$

$$\Rightarrow \Delta u = \Delta Q_p - P\Delta V$$

$$\Rightarrow \Delta u = \Delta Q_p - nR\Delta T$$

$$\Rightarrow \Delta u = \Delta Q_p - nR \cdot \frac{\Delta Q_p}{nC_p}$$

$$\Rightarrow \Delta u = \Delta Q_p \left[1 - \frac{R}{C_p} \right]$$

$$\Rightarrow \Delta u = \Delta Q_p \left[1 - \frac{(C_p - C_v)}{C_p} \right]$$

$$\begin{aligned} \Delta w &= P\Delta V \\ PV &= nRT \\ \Rightarrow P\Delta V &= nR\Delta T \\ \Delta Q_p &= nC_p\Delta T \\ \Rightarrow \Delta T &= \frac{\Delta Q_p}{nC_p} \\ C_p - C_v &= R \\ \frac{C_p}{C_v} &= \gamma \\ \Rightarrow \frac{C_p}{C_v} &= \frac{1}{\gamma} \\ \Delta T &= \frac{\Delta Q_p}{nC_p} \end{aligned}$$

$$\Rightarrow \Delta u = \Delta Q_p \left[1 - 1 + \frac{C_v}{C_p} \right]$$

$$\Rightarrow \Delta u = \Delta Q_p \left[\frac{C_v}{C_p} \right]$$

$$\Rightarrow \Delta u = \frac{\Delta Q_p}{\gamma} \quad (1)$$

Putting the values in equation (1), we get,

$$\Delta u = \frac{180}{1.41} \Rightarrow \Delta u = 127.66 \text{ J}$$

$$(b) \text{ As } \Delta w = P \Delta V \Rightarrow \Delta w = n R \Delta T$$

$$\Rightarrow \Delta w = n R \times \frac{\Delta Q_p}{n C_p} \Rightarrow \Delta w = \frac{R \Delta Q_p}{C_p}$$

$$\Rightarrow \Delta w = \frac{(C_p - C_v)}{C_p} \Delta Q_p$$

$$\Rightarrow \Delta w = \left(1 - \frac{C_v}{C_p} \right) \Delta Q_p$$

$$\Rightarrow \Delta w = \left(1 - \frac{1}{\gamma} \right) \Delta Q_p \quad (2)$$

Putting the values in equation (2), we get,

$$\Delta w = \left(1 - \frac{1}{1.41} \right) \times 180 \Rightarrow \Delta w = (1 - 0.709219858) \times 180$$

$$\Rightarrow \Delta w = (0.290780141 \times 180) \text{ J} \Rightarrow \Delta w = 52.34 \text{ J}$$

Q4: Find the efficiency of a Carnot's heat engine working between steam and ice points.

Solution:

Given data: At steam point, $T_1 = 100^\circ\text{C}$

$$\Rightarrow T_1 = (100 + 273) \text{ K} = 373 \text{ K}$$

At ice point, $T_2 = 0^\circ\text{C} = 0 + 273 = 273 \text{ K}$

Required: Efficiency of Carnot's heat engine = $\eta = ?$

Calculation: We know that,

$$\eta = 1 - \frac{T_2}{T_1} \quad (1)$$

Putting the values in equation (1), we get,

$$\Rightarrow \eta = 1 - \frac{273}{373} = \frac{373 - 273}{373} = \frac{100}{373} \Rightarrow \eta = 0.268$$

$$\Rightarrow \eta = 0.268 \times 100\% \Rightarrow \eta = 26.8\%$$

$$\text{Ans: } \eta = 26.8\%$$

Q5: A Carnot heat engine absorbs 2000J of heat from the source of heat engine at 227°C and rejects 1200J of heat during each cycle to sink. Calculate efficiency of engine temperature of sink and amount of work done during each cycle. (40%, 27°C , 800J)

Solution: Given data: Heat absorbed = $Q_1 = 2000 \text{ J}$

Heat rejected = $Q_2 = 1200 \text{ J}$

Temperature of source = $T_1 = 227^\circ\text{C}$

$$\Rightarrow T_1 = (227 + 273) \text{ K} = 500 \text{ K}$$

Required: (i) Efficiency of engine = $\eta = ?$

(ii) Temperature of sink = $T_2 = ?$

(iii) Mechanical work = $w = ?$

Calculation: (i) We know that,

$$\eta = 1 - \frac{Q_2}{Q_1} \quad (1)$$

Putting the values in equation (1), we get,

$$\Rightarrow \eta = 1 - \frac{1200}{2000} = \frac{2000 - 1200}{2000} \Rightarrow \eta = \frac{800}{2000}$$

$$\Rightarrow \eta = 0.4 \Rightarrow \eta = 0.4 \times 100\% \Rightarrow \eta = 40\%$$

$$(ii) \text{ As } \eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \eta - 1$$

$$\Rightarrow \frac{T_2}{T_1} = 1 - \eta \Rightarrow T_2 = (1 - \eta) \times T_1 \quad (2)$$

Putting the values in equation (2), we get,

$$\Rightarrow T_2 = (1 - 0.4) \times 500 \text{ K} \Rightarrow T_2 = 300 \text{ K}$$

$$\Rightarrow T_2 = (300 - 273)^\circ\text{C} \Rightarrow T_2 = 27^\circ\text{C}$$

$$\text{As } w = Q_1 - Q_2 \quad (3)$$

Putting the values in equation (3), we get,

$$\Rightarrow w = (2000 - 1200) \text{ J} \Rightarrow w = 800 \text{ J}$$

Q6: What is the least amount of work that must be performed to freeze one gram of water at 0°C by means of a refrigerator? Take the temperature of the surrounding as 37°C . How much heat is passed on to the surrounding during this process? (45.54J, 381.54J)

Solution:

Given data: Mass of water = $m = 1\text{gm} \Rightarrow m = 1 \times 10^{-3}\text{kg}$

Temperature of L.T.R (cold body) = $T_2 = 0^\circ\text{C} = 273\text{K}$

$$\Rightarrow T_2 = 273\text{K}$$

Temperature of H.T.R (hot bod) = $T_1 = 37^\circ\text{C}$

$$\Rightarrow T_1 = 37 + 273 = 310\text{K} \Rightarrow T_1 = 310\text{K}$$

Required:

(i) Amount of work = $w = ?$

(ii) Heat transferred to surroundings = $Q_1 = ?$

Calculation: (i) For refrigerator, we have,

$$E = \frac{Q_2}{w} \quad (1)$$

$$\text{Also } E = \frac{T_2}{T_1 - T_2} \quad (2)$$

Comparing equations (1), (2), we get,

$$\frac{Q_2}{w} = \frac{T_2}{T_1 - T_2} \Rightarrow \frac{w}{Q_2} = \frac{T_1 - T_2}{T_2}$$

$$\Rightarrow w = \left(\frac{T_1 - T_2}{T_2} \right) Q_2 \quad (3)$$

Here $Q_2 = mL_f$, so equation (3) becomes,

$$w = \left(\frac{T_1 - T_2}{T_2} \right) \times mL_f \quad (4)$$

Putting the values in equation (4), we get,

$$\Rightarrow w = \left[\left(\frac{310 - 273}{273} \right) \times 10^{-3} \times 336000 \right] \text{J}$$

$$\Rightarrow w = \left[\frac{37 \times 10^{-3} \times 336 \times 10^3}{273} \right] \text{J} \Rightarrow w = 45.54\text{J}$$

(ii) In case of refrigerator, we have,

$$Q_1 = Q_2 + w \Rightarrow Q_1 = mL_f + w$$

$$\Rightarrow Q_1 = 10^{-3} \times 336 \times 10^3 + 45.54$$

$$\Rightarrow Q_1 = 336 + 45.54 \Rightarrow Q_1 = 381.54\text{J}$$

Q7: Calculate the change in entropy when 10kg of water is heated from 90°C to 100°C . Specific heat of water is $4180\text{J/mol}\cdot\text{K}$.

Solution:

Given data:

$$\text{Mass of water} = m = 10\text{kg}$$

L_f = Latent heat of fusion

For water,
 $L_f = 336000\text{J/kg}$

Initial temperature $T_1 = 90^\circ\text{C}$

$$\Rightarrow T_1 = 90 + 273 = 363\text{K}$$

Final temperature = $T_2 = 100^\circ\text{C} \Rightarrow T_2 = 100 + 273 = 373\text{K}$

Specific heat of water = $C = 4180\text{J/mol}\cdot\text{K}$

Required:

Change in entropy = $\Delta S = ?$

Calculation:

We know that,

$$\Delta S = \frac{\Delta Q}{T} \quad (1)$$

Here $\Delta Q = mC\Delta T \Rightarrow \Delta Q = mC(T_2 - T_1)$

$$\Rightarrow \Delta Q = 10 \times 4180(373 - 363)$$

$$\Rightarrow \Delta Q = 41800 \times 10 = 418000\text{J}$$

$$\Rightarrow \Delta Q = 418000\text{J}$$

Here " T " represents the average temperature, so we put,

$$T = \frac{(T_1 + T_2)}{2} = \frac{(363 + 373)}{2} \Rightarrow T = 368\text{K}$$

Putting the values of " ΔQ " and " T " in equation (1), we get,

$$\Delta S = \left[\frac{418000}{368} \right] \text{J/K} \Rightarrow \Delta S = 1135.8\text{J/K}$$

Q8: A system absorbs 1176J of heat and at the same time does 352.8J of external work. Find the change in internal energy of the system. Find the change in internal energy of the system when it absorbs 1050J of heat while 84J of work is done? What will be the change in internal energy of the gas if 210J of heat is removed at constant volume? (1260J, 1092J, -210J)

Solution:

Given data & Requirement:

(a) Heat supplied = $\Delta Q = 1176\text{J}$

$$\text{Work} = \Delta w = 352.8\text{J}$$

Change in internal energy = $\Delta u = ?$

Given data & Requirement:

(b) Heat supplied = $\Delta Q = 1050\text{J}$

$$\text{Work} = \Delta w = 84\text{J}, \text{ Change in internal energy} = \Delta u = ?$$

Given data & Requirement:

 (c) Heat removed = $\Delta Q = -210\text{J}$

 At constant volume, $\Delta w = 0$,

 Change in internal energy = $\Delta u = ?$

 Calculation: (a) $\Delta Q = 1176\text{J}$, $\Delta w = 352.8\text{J}$

 We know that, $\Delta Q = \Delta u + \Delta w$

$$\Rightarrow \Delta u = \Delta Q - \Delta w = 1176 - 352.8 \Rightarrow \Delta u = 823.2\text{J}$$

 (b) $\Delta Q = 1050\text{J}$, $\Delta w = 84\text{J}$

 Again we have, $\Delta Q = \Delta u + \Delta w$

$$\Rightarrow \Delta u = \Delta Q - \Delta w \Rightarrow \Delta u = (1050 - 84)\text{J} \Rightarrow \Delta u = 966\text{J}$$

 (c) $\Delta Q = -210\text{J}$, $\Delta w = 0\text{J}$, $\Delta u = ?$

 Again we have, $\Delta Q = \Delta u + \Delta w$

$$\Rightarrow \Delta u = \Delta Q - \Delta w \Rightarrow \Delta u = -210 - 0 \Rightarrow \Delta u = -210\text{J}$$

Q9: An ideal gas at 20.0°C and a pressure of $1.50 \times 10^5\text{Pa}$ is in a container having a volume of 1.00L . (a) Determine the number of moles of gas in the container. (b) The gas pushes against a piston, expanding to twice its original volume, while the pressure falls to atmospheric pressure. Find the final temperature. ($6.16 \times 10^{-2}\text{mol}$, 395K)

Solution:
Given data: Temperature = 20°C ,

$$\text{Pressure} = 1.5 \times 10^5 \text{ Pascal} = 1.5 \times 10^5 \text{ N/m}^2$$

$$\text{Volume} = V = 1.00\text{L} = 0.001\text{m}^3, R = 8.36\text{J/mol}\cdot\text{K}$$

Required:

 (a) Number of moles = $n = ?$

$$1\text{Pascal} = 1\text{N} / \text{m}^2$$

(b) Final volume = twice the initial volume

$$\Rightarrow V' = 2V = 2 \times 0.001\text{m}^3$$

$$\text{Pressure} = P' = 1\text{atm} = 1.01 \times 10^5 \text{ Pascal} = 1.01 \times 10^5 \text{ N} / \text{m}^2$$

 Final temperature = $T' = ?$
Calculation:

(a) We know that,

$$PV = nRT \Rightarrow n = \frac{PV}{RT} \quad (1)$$

Putting the values in equation (1), we get,

$$n = \left(\frac{1.5 \times 10^5 \times 0.001}{8.36 \times 20} \right) \text{ moles} = 8.97 \times 10^{-6} \times 10^5 \text{ moles}$$

$$\Rightarrow n = 8.97 \times 10^{-1} \text{ mol}$$

 (b) Again $PV' = nRT'$

$$\Rightarrow T' = \frac{PV'}{nR} \quad (2)$$

Putting the values in equation (2), we get,

$$T' = \frac{1.01 \times 10^5 \times 2 \times 0.001}{8.97 \times 10^{-1} \times 8.36} = 2.69 \times 10^5 \times 10^{-4}$$

$$\Rightarrow T' = 2.69 \times 10^1 = 26.9^\circ\text{C} = 26.9 + 273 = 27^\circ\text{C}$$

$$= 27 + 273 = 300\text{K} \Rightarrow T' = 300\text{K}$$

Q10: A block of ice at 273K is put in thermal contact with a container of steam at 373K , converting 25.0g of ice to water at 273K while condensing some of the steam to water at 373K . (a) Find the change in entropy of the ice. (b) Find the change in entropy of the steam. (c) Find the change in entropy of the universe. (30.5J/K , -22.3J/K , $+8.2\text{J/K}$)

Solution:
Given data: Temperature of ice $T_{ice} = 273\text{K}$

 Temperature of steam = $T_{steam} = 373\text{K}$

 Mass of ice = $m_{ice} = 25\text{gm} = 0.025\text{kg}$

 Latent heat of fusion of ice = $H_f = 3.33 \times 10^5\text{J}$

First we calculate the thermal energy gained by ice for converting into water, which is given by,

$$\Delta Q_{ice} = mH_f = 0.025 \times 3.33 \times 10^5\text{J} = 8.33 \times 10^3\text{J}$$

Required: (a) Change in entropy of ice = $\Delta S_{ice} = ?$

 (b) Change in entropy of steam = $\Delta S_{steam} = ?$

 (c) Change in entropy of universe = $\Delta S_{universe} = ?$
Calculation:

$$(a) \text{ We know that, } \Delta S = \frac{\Delta Q}{T} \quad (1)$$

For ice, equation (1) becomes,

$$\Delta S_{ice} = \frac{\Delta Q_{ice}}{T_{ice}} \quad (2)$$

Putting the values in equation (2), we get,

$$\Delta S_{ice} = \frac{8.33 \times 10^3}{273} = 0.0305 \times 10^3 \text{ J / K} = 30.5 \text{ J / K} \Rightarrow \boxed{\Delta S_{ice} = 30.5 \text{ J / K}}$$

(b) The change in entropy of steam is given by,

$$\Delta S_{steam} = \frac{\Delta Q_{steam}}{T_{steam}} \quad (3)$$

Here the thermal energy gain by ice is equal to the thermal energy lost by steam, so we have,

$$\Delta S_{steam} = -\Delta Q_{ice} \Rightarrow \Delta Q_{steam} = -8.33 \times 10^3 \text{ J}$$

So equation (3) becomes,

$$\Delta S_{steam} = \frac{-8.33 \times 10^3}{373} \text{ J / K} = -0.0223 \times 10^3 \text{ J / K}$$

$$\Rightarrow \boxed{\Delta S_{steam} = -22.3 \text{ J / K}}$$

(c) The change in entropy of universe is equal to the sum of change in entropy of ice and change in entropy of steam i.e.

$$\Delta S_{universe} = \Delta S_{ice} + \Delta S_{steam} \quad (4)$$

Putting the values in equation (4), we get,

$$\Delta S_{universe} = 30.5 \text{ J / K} + (-22.3 \text{ J / K})$$

$$\Rightarrow \Delta S_{universe} = 30.5 \text{ J / K} - 22.3 \text{ J / K} = +8.2 \text{ J / K}$$

$$\Rightarrow \boxed{\Delta S_{universe} = +8.2 \text{ J / K}}$$



① تم بہت کام کسی جماعت یا مولوی کا کام نہیں بلکہ یہ حضرت محمد خاتم النبیین ﷺ سے وفاداری نبھانے کا کام ہے۔

② کسی کی پہچان علم سے نہیں ہوتی بلکہ ادب سے ہوتی ہے کیونکہ علم تو ابلیس کے پاس بھی تھا لیکن وہ ادب سے محروم تھا۔

③ علم اس وقت تک حاصل نہیں ہوتا جب تک کوئی عطا کرنے والا نہ ہو کیونکہ اصل علم اللہ والے کی نگاہ سے ملتا ہے کتاب سے نہیں۔

④ ہم تب نہیں سیکھتے جب چیزیں آسان ہوتی ہیں، ہم تب سیکھتے ہیں جب ہم مشکلات کا سامنا کرتے ہیں۔

⑤ وہ شخص کامیاب نہیں ہو پاتا جس میں ناکافی کاغذات کامیابی کی چاہت سے زیادہ ہو۔

⑥ علم ایک پاداش ہے جسے دل و دماغ کی سر زمین میں لگانے سے عقل کے پھل ملتے ہیں۔

⑦ لکھائیں وہ تو تمہیں ہیں، ایک گوار اور دوسرا قلم۔