

The following points will enable you to solve the problems of this chapter.

(iv) Factorial Notation

$$n! = n(n-1)(n-2) \dots 3 \cdot 2 \cdot 1 \quad | \quad 0! = 1, 1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120 \text{ and so on}$$

(ii) How to decide whether a question is of permutation or of combination? Ask yourself a question.

Is the order important?

If the answer is YES $\implies$ it is permutation

If the answer is No $\Rightarrow$ it is combination.

For example, Q: How many teams of playing eleven can be made out of 14 players?

Is the order important? Answer will be No. Because by the shuffling of names the team remains the same.

Q2: How many batting orders are possible of a cricket team players?

Is the order important? The answer must be YES. Because you would not like to do opening from a baller.

$$(ii) \quad nPr = \frac{n!}{(n-r)!}$$

(iv) ${}^nC_r = \frac{n!}{r!(n-r)!}$. nC_r can also be written as $\binom{n}{r}$

Permutation with repetition;

$$\frac{n!}{p! q! r!}$$

$$\alpha^{(n)}(b, g, h)$$

where one thing occurs p times

(vi) If you are taking r things at a time from n things then the following are the ways.

I (i) Order is important (ii) repetition is not allowed. <u>Permutation:</u> ${}^n P_r = \frac{n!}{(n-r)!}$	II (i) order is important (ii) repetition is allowed <u>Permutation:</u> $\frac{n!}{r! (n-r)!}$
III (i) Order is not important (ii) repetition is not allowed <u>Combination</u> ${}^n C_r = \frac{n!}{r! (n-r)!}$	IV (i) you are taking all the n things (ii) repetition is allowed n^n

(v) ${}^n P_n = n!$; ${}^n P_0 = 1$; ${}^n C_n = 1$; ${}^n C_0 = 1$, ${}^n P_1 = n$

(vi) ${}^n C_r = {}^n C_{n-r}$

(vii) Probability: $P(A) = \frac{n(A)}{n(S)}$ where S is sample space

Conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

(ix) $0 \leq P(A) \leq 1$; $P(\emptyset) = 0$, $P(S) = 1$

(x) (i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) $P(A \cup B) = P(A) + P(B)$ (for mutually exclusive case)

(xi) $P(A') = 1 - P(A)$: $P(A')$ is also denoted as $P(\bar{A})$.

(xii) For independent events. $P(A \cap B) = P(A) P(B)$

EXERCISE 6.1

Q.1: Evaluate the following.

(i) $\frac{10!}{3!3!4!}$

Sol: $\frac{10!}{3!3!4!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 4!}$
 $= 4200$ Ans

(ii) $\frac{3!+4!}{5!-4!}$

Sol: $\frac{3!+4!}{5!-4!} = \frac{6+24}{120-24} = \frac{30}{96} = \frac{5}{16}$ Ans

(iii) $\frac{(n-1)!}{(n+1)!}$

Sol: $\frac{(n-1)!}{(n+1)!} = \frac{(n-1)!}{(n+1)n(n-1)!} = \frac{1}{(n+1)n}$ Ans

(iv) $\frac{10!}{(5!)^2}$

Sol: $\frac{10!}{(5!)^2} = \frac{10!}{5!5!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5 \times 4 \times 3 \times 2 \times 1 \times 5!}$
 $= 252$ Ans

Q.2: Write the following in terms of factorials.

(i) $19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14$

Sol: $19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14$
 $= \frac{19!}{13!}$ Ans

(ii) $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12$

Sol: $2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12$
 $= 1 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 2 \cdot 4 \cdot 2 \cdot 5 \cdot 2 \cdot 6 \cdot 2$
 $= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$
 $= 2^6 \cdot 6!$ Ans

(iii) $n(n^2-1)$

Sol: $n(n^2-1) = n(n+1)(n-1)$
 $= (n+1)n(n-1)$
 $= \frac{(n+1)n(n-1)(n-2)!}{(n-2)!}$
 $= \frac{(n+1)!}{(n-2)!}$ Ans

(iv) $\frac{n(n+1)(n+2)}{3}$

Sol: $\frac{n(n+1)(n+2)}{3} = \frac{(n+2)(n+1)n}{3}$
 $= \frac{(n+2)(n+1)n(n-1)!}{3(n-1)!}$
 $= \frac{(n+2)!}{3(n-1)!}$ Ans

Q.3 Prove the following.

$$(i) \frac{1}{6!} + \frac{2}{7!} + \frac{3}{8!} = \frac{75}{8!}$$

Sol:

$$\begin{aligned} \text{L.H.S} &= \frac{1}{6!} + \frac{2}{7!} + \frac{3}{8!} \\ &= \frac{8 \times 7 + 8 \times 2 + 3}{8!} \\ &= \frac{56 + 16 + 3}{8!} \\ &= \frac{75}{8!} = \text{R.H.S. Proved} \end{aligned}$$

$$(ii) \frac{(n+5)!}{(n+3)!} = n^2 + 9n + 20$$

Sol:

$$\begin{aligned} \text{L.H.S} &= \frac{(n+5)!}{(n+3)!} \\ &= \frac{(n+5)(n+4)(n+3)!}{(n+3)!} \\ &= (n+5)(n+4) \\ &= n^2 + 4n + 5n + 20 \\ &= n^2 + 9n + 20 \\ &= \text{R.H.S. Proved} \end{aligned}$$

Q.4: Find the value of n , when

$$(i) \frac{n(n!)}{(n-5)!} = \frac{12(n!)}{(n-4)!}$$

$$\text{Sol: } \frac{n(n!)}{(n-5)!} = \frac{12(n!)}{(n-4)!}$$

$$n(n-4)! = 12(n-5)!$$

$$n(n-4)(n-5)! = 12(n-5)!$$

$$n(n-4) = 12$$

$$n^2 - 4n - 12 = 0$$

$$n^2 - 6n + 2n - 12 = 0$$

$$n(n-6) + 2(n-6) = 0$$

$$(n-6)(n+2) = 0$$

$$\Rightarrow \begin{array}{l|l} n-6=0 & n+2=0 \\ \Rightarrow n=6 & n=-2 \\ & \text{(Not possible)} \end{array}$$

$$\therefore \boxed{n=6} \text{ Ans}$$

$$(ii) \frac{n!}{(n-4)!} : \frac{(n-1)!}{(n-4)!} = 9:1$$

$$\text{Sol: } \frac{n!}{(n-4)!} : \frac{(n-1)!}{(n-4)!} = 9:1$$

$$\frac{n!}{(n-4)!} = \frac{9}{1}$$

$$\frac{n!}{(n-1)!} = \frac{9}{1}$$

$$\frac{n(n-1)!}{(n-1)!} = 9$$

$$\Rightarrow \boxed{n=9} \text{ Ans}$$

Q.5: Show that

$$(i) \frac{(2n)!}{n!} = 2^n (1 \cdot 3 \cdot 5 \dots (2n-1))$$

$$\begin{aligned} \text{Sol: } (2n)! &= 1 \cdot 2 \cdot 3 \dots (2n-1)(2n) \\ &= [2 \cdot 4 \cdot 6 \dots 2n] [1 \cdot 3 \cdot 5 \dots (2n-1)] \\ &= [(1 \cdot 2) \cdot (2 \cdot 2) \cdot (2 \cdot 3) \dots (2 \cdot n)] [1 \cdot 3 \cdot 5 \dots (2n-1)] \\ &= 2^n [1 \cdot 2 \cdot 3 \dots n] [1 \cdot 3 \cdot 5 \dots (2n-1)] \\ &= 2^n n! [1 \cdot 3 \cdot 5 \dots (2n-1)] \\ \frac{(2n)!}{n!} &= 2^n (1 \cdot 3 \cdot 5 \dots (2n-1)) \quad \text{Proved.} \end{aligned}$$

$$(ii) \frac{(2n+1)!}{n!} = 2^n [1 \cdot 3 \cdot 5 \dots (2n-1)(2n+1)]$$

$$\begin{aligned} \text{Sol: L.H.S} &= \frac{(2n+1)!}{n!} \\ &= \frac{(2n+1)(2n)!}{n!} \\ &= (2n+1) \frac{(2n)!}{n!} \\ &= (2n+1) 2^n (1 \cdot 3 \cdot 5 \dots (2n-1)) \\ &= 2^n [1 \cdot 3 \cdot 5 \dots (2n-1)(2n+1)] \\ &= R.H.S \quad \text{Proved.} \end{aligned}$$

EXERCISE 6.2

Q.1:

Evaluate

Sol: (i) ${}^6P_5 = \frac{6!}{(6-5)!}$

$= \frac{6!}{1!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = \boxed{720} \text{ Ans}$

(ii) ${}^{20}P_2 = \frac{20!}{(20-2)!}$

$= \frac{20!}{18!} = \frac{20 \times 19 \times 18!}{18!} = 20 \times 19 = \boxed{380} \text{ Ans}$

(iii) ${}^{16}P_3 = \frac{16!}{(16-3)!}$

$= \frac{16!}{13!} = \frac{16 \times 15 \times 14 \times 13!}{13!} = 16 \times 15 \times 14 = \boxed{3360} \text{ Ans}$

Q.2:

Solve for n.

(i) ${}^nP_5 = 56({}^nP_3)$

Sol: ${}^nP_5 = 56({}^nP_3)$

$\frac{n!}{(n-5)!} = 56 \times \frac{n!}{(n-3)!}$

$(n-3)! = 56 \times (n-5)!$

$(n-3)(n-4)(n-5)! = 56 \times (n-5)!$

$(n-3)(n-4) = 56$

$n^2 - 7n + 12 - 56 = 0$

$n^2 - 7n - 44 = 0$

$n^2 - 11n + 4n - 44 = 0$

$n(n-11) + 4(n-11) = 0$

$(n-11)(n+4) = 0$

$\Rightarrow \begin{array}{l|l} n-11=0 & n+4=0 \\ n=11 & n=-4 \text{ (not possible)} \end{array}$

$\therefore \boxed{n=11} \text{ Ans}$

(ii) ${}^nP_5 = 9({}^{n-1}P_4)$

Sol: ${}^nP_5 = 9({}^{n-1}P_4)$

$\frac{n!}{(n-5)!} = 9 \frac{(n-1)!}{(n-1-4)!}$

$\frac{n!}{(n-5)!} = 9 \frac{(n-1)!}{(n-5)!}$

$n! = 9(n-1)!$

$n(n-1)! = 9(n-1)!$

$\Rightarrow \boxed{n=9} \text{ Ans}$

(iii) ${}^n P_2 = 600$

Sol: ${}^n P_2 = 600$

$$\frac{(n^2)!}{(n^2-2)!} = 600$$

$$\frac{n^2(n^2-1)(n^2-2)!}{(n^2-2)!} = 600$$

$$n^4 - n^2 - 600 = 0$$

$$n^4 - 25n^2 + 24n^2 - 600 = 0$$

$$n^2(n^2 - 25) + 24(n^2 - 25) = 0$$

$$(n^2 - 25)(n^2 + 24) = 0$$

$$\begin{array}{l|l} n^2 - 25 = 0 & n^2 + 24 = 0 \\ n^2 = 25 & n^2 = -24 \\ n = \pm 5 & \text{(Not possible)} \\ n = -5 & \\ \text{(Not possible)} & \end{array}$$

$\therefore \boxed{n = 5}$ Ans

Q.3: Prove the following by fundamental principle of counting.

(i) ${}^n P_n = n({}^{n-1} P_{n-1})$

Sol: ${}^n P_n = n({}^{n-1} P_{n-1})$

$$\begin{aligned} \text{R.H.S} &= n({}^{n-1} P_{n-1}) \\ &= n \times \frac{(n-1)!}{[(n-1)-(n-1)]!} \\ &= n \times \frac{(n-1)!}{(n-1-n+1)!} \\ &= \frac{n(n-1)!}{(n-n)!} \\ &= \frac{n!}{(n-n)!} \\ &= {}^n P_n = \text{L.H.S} \quad \text{Proved} \end{aligned}$$

(ii) ${}^n P_n = {}^{n-1} P_{n-1} + n({}^{n-1} P_{n-1})$

$$\begin{aligned} \text{Sol: R.H.S} &= {}^{n-1} P_{n-1} + n({}^{n-1} P_{n-1}) \\ &= \frac{(n-1)!}{[(n-1)-(n-1)]!} + n \times \frac{(n-1)!}{[(n-1)-(n-1)]!} \\ &= \frac{(n-1)!}{(n-1-n+1)!} + \frac{n(n-1)!}{(n-1-n+1)!} \\ &= \frac{(n-1)!}{(n-n)!} + \frac{n(n-1)!}{(n-n)(n-n)!} \\ &= \frac{(n-1)!}{(n-n-1)!} \left[1 + \frac{n}{n-n} \right] \\ &= \frac{(n-1)!}{(n-n-1)!} \left[\frac{n-n+n}{n-n} \right] \\ &= \frac{n(n-1)!}{(n-n)(n-n-1)!} \\ &= \frac{n!}{(n-n)!} = {}^n P_n = \text{L.H.S} \quad \text{Proved} \end{aligned}$$

Q.4: In how many ways can a police department arrange eight suspects in a line up?

Sol: Total number of suspects = 8

Required number of arrangements = ${}^8P_8 = 8!$

= $\boxed{40320}$ Ans

Q.5: In how many ways can letters of the word 'Fasting' be arranged?

Sol: Total number of letters = 7

Total number of ways = ${}^7P_7 = 7! = \boxed{5040}$ Ans

Q.6: How many 4 digit numbers can be formed with the digits 2, 4, 5, 7, 9. (Repetitions not being allowed). How many of these are even?

Sol: 1st part

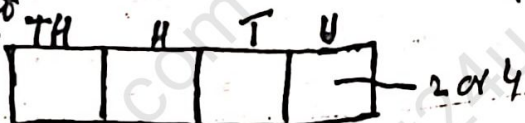
2, 4, 5, 7, 9

Total number of digits = 5

Total number of 4-digit numbers (without repetition)

= ${}^5P_4 = \frac{5!}{(5-4)!} = \frac{5!}{1!} = 5! = \boxed{120}$ Ans

2nd part



If we fix 2 or 4 on the right most position, then the rest of these positions can be filled by any of the remaining 4 digits.

$\therefore$ Total number of even numbers = ${}^4P_3 \times {}^2P_1$
 = $\frac{4!}{1!} \times \frac{2!}{1!} = 24 \times 2 = \boxed{48}$ Ans

Q.7: How many three digit numbers can be formed from the digits 1, 2, 3, 4 and 5 if repetitions (i) are allowed (ii) are not allowed.

Sol: Total number of digits = 5

(i) When repetitions are allowed.

$$\text{The no. of 3-digit numbers} = 5^3 = 5 \times 5 \times 5 = \boxed{125} \quad \text{Ans}$$

(ii) When repetitions are not allowed.

$$\text{The no. of 3-digit numbers} = {}^5P_3 = \frac{5!}{(5-3)!} = \frac{5 \times 4 \times 3 \times 2!}{2!} = \boxed{60} \quad \text{Ans}$$

Q.8: How many different arrangements can be formed of the word "Equation" if all the vowels are to be kept together?

Sol: There are 8 letters in the word "Equation", in which 5 i.e. e, u, a, i, o are vowels.

If we consider the 5 vowels as one letter then,

the total number of arrangements = ${}^4P_4 \times {}^5P_5$

$$= 4! \times 5! = 24 \times 120 = \boxed{2880} \quad \text{Ans}$$

Q.9: How many signals can be given by six flags of different colors when any number of them are used at a time.

Sol: Total no. of flags = 6

$$\begin{aligned} \text{Total no. of signals} &= {}^6P_1 + {}^6P_2 + {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6 \\ &= 6 + 30 + 120 + 360 + 720 + 720 = \boxed{1956} \text{ Ans} \end{aligned}$$

Q.10: In how many ways can five students be seated in a row of eight seats if a certain two students

- (i) insist on sitting next to each other?
- (ii) refuse to sit next to each other?

Sol: Total no. of seats = $n = 8$
 No. of students = $r = 5$
 Total number of ways = ${}^n P_r = {}^8 P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!} = \boxed{6720}$

(i) If a certain two students insist on sitting next to each other.

then the no. of ways in this case = ${}^8 P_4 \times 2!$

$$= \frac{8!}{(8-4)!} \times 2 = \frac{8!}{4!} \times 2 = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} \times 2 = \boxed{3360} \text{ Ans}$$

(ii) If a certain two students refuse to sit next to each other.

$$\text{Then the no. of ways in this case} = 6720 - 3360 = \boxed{3360} \text{ Ans}$$

Q.11 How many numbers each lying between 10 and 1000 can be formed with digits 2, 3, 4, 0, 8, 9 using only once.

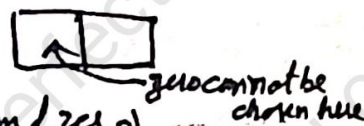
Sol: Total no. of digits = 6

We need the no. of 2-digit and 3-digit numbers, greater than 10 and less than 1000 in which no digit is repeated.

For the 2-digit numbers:

First digit could be chosen in 5 ways (excluding 0)

2nd digit could be chosen in 5 ways (remaining 4 digits and zero)



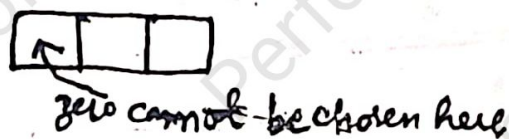
Thus the required 2-digit numbers have $= 5 \times 5 = \boxed{25}$ ways.

For the 3-digit numbers:

For 1st digit, we have 5 choices

For 2nd digit, we have 5 =

For 3rd digit, we have 4 =



Thus we have $5 \times 5 \times 4 = 100$ 3-digit numbers (meeting the criteria)

Total number of the required numbers $= 25 + 100 = \boxed{125}$ Ans

Q.12. How many different words can be formed from the letters of the following words if the letters are taken all at a time.

(i) Bookworm

Sol: The total no. of letters $= n = 8$

In which O occurs 3 times and the rest of the letters only once.

$$\therefore \text{total no. of words formed} = \frac{8!}{3!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!} = \boxed{6720} \text{ Ans}$$

(ii) Bookkeeper

Sol: Total no. of letters = $n=10$.

In which O occurs 2 times, K occurs 2 times, E occurs 3 times.

$$\therefore \text{total no. of words formed} = \binom{10}{2,2,3} = \frac{10!}{2!2!3!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{2 \times 2 \times 3!} = \boxed{151200} \text{ Ans}$$

(iii) Abbottabad

Sol: Total no. of letters = $n=10$

In which A occurs 3 times, B occurs 3 times, T occurs 2 times

$$\therefore \text{total no. of words formed} = \binom{10}{3,3,2} = \frac{10!}{3!3!2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 3! \times 2!} = \boxed{50400} \text{ Ans}$$

(iv) Letter

Sol: Total no. of letters = $n=6$

In which E occurs 2 times, T occurs 2 times

$$\therefore \text{total no. of words formed} = \binom{6}{2,2} = \frac{6!}{2!2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2 \times 2!} = \boxed{180} \text{ Ans}$$

Q.13: Find the number of permutations of the word 'EXCELLENCE'.

Sol: Total no. of letters in 'EXCELLENCE' = 10

In which E occurs 4 times, C occurs 2 times, L occurs 2 times

$$\begin{aligned} \text{Number of permutations} &= \binom{10}{4,2,2} = \frac{10!}{4!2!2!} \\ &= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 2 \times 2} = \boxed{37800} \text{ Ans} \end{aligned}$$

How many of these permutations (i) begin with E

Sol: Fix E at the extreme left position and arrange the rest.
Now, no. of letters = 9. In which E occurs 3 times, L occurs twice and C occurs twice.

$$\begin{aligned}\text{Number of permutations} &= \frac{9!}{3!2!2!} \\ &= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3}{3! \times 2 \times 2} = \boxed{15,120} \text{ Ans}\end{aligned}$$

(ii) begin with E and end with C.

Sol: Fix E at the extreme left position and C at the extreme right position, and arrange the rest.

Now, no. of letters = 8. In which E occurs 3 times, L occurs twice.

$$\begin{aligned}\text{Number of permutations} &= \frac{8!}{3!2!} \\ &= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3}{3! \times 2} = \boxed{3,360} \text{ Ans}\end{aligned}$$

(iii) begin with E and end with E.

Sol: Fix one E at the extreme left and another at extreme right positions.

Now, no. of letters = 8.

In which E occurs twice, L occurs twice and C occurs twice.

$$\begin{aligned}\text{Number of permutations} &= \frac{8!}{2!2!2!} \\ &= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}{2! \times 2 \times 2} = \boxed{5,040} \text{ Ans}\end{aligned}$$

(iv) do not begin with E.

Sol: Number of permutations that do not begin with E
 $=$ Total number of permutations - Number of permutations that begin with E
 $= 37800 - 15120 = \boxed{22680}$ Ans

(v) contain 2 Ls together.

Sol: Consider 2 Ls as one letter. Then

No. of letters $= 9$

In which E occurs 4 times, C occurs 2 times

Number of permutations $= \frac{9!}{4!2!}$

$\frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1} = \boxed{7560}$ Ans

(vi) do not contain 2 Ls together.

Sol: Number of permutations that do not contain 2 Ls together
 $=$ Total number of permutations - Number of permutations that contain 2 Ls together
 $= 37800 - 7560 = \boxed{30240}$ Ans

Q. 14: If five distinct keys are placed on a key ring, how many different orders are possible?

Sol: Total number of keys $= n = 5$
 Total possible different orders $= \frac{(n-1)!}{2} = \frac{(5-1)!}{2} = \frac{4!}{2} = \frac{24}{2} = \boxed{12}$ Ans

Q.15: In how many ways can 7 people be arranged at a round table so that 2 particular persons always sit together?

Sol: Consider 2 particular persons as one person.

Then total number of persons becomes $= n = 6$

Total number of arrangements $= (n-1)! = (6-1)! = 5! = \boxed{120}$ Ans

EXERCISE 6.3

Q.1. Solve the following for n .

(i) ${}^nC_2 = 36$

Sol: ${}^nC_2 = 36$

$$\Rightarrow \frac{n!}{2!(n-2)!} = 36$$

$$\Rightarrow \frac{n(n-1)(n-2)!}{2 \times (n-2)!} = 36$$

$$\Rightarrow n(n-1) = 72$$

$$\Rightarrow n^2 - n - 72 = 0$$

$$\Rightarrow n^2 - 9n + 8n - 72 = 0$$

$$\Rightarrow n(n-9) + 8(n-9) = 0$$

$$\Rightarrow (n-9)(n+8) = 0$$

$$n-9=0 \quad | \quad n+8=0$$

$$n=9 \quad | \quad n=-8$$

(Not possible)

$$\therefore \boxed{n=9} \text{ Ans}$$

(ii) ${}^{n+1}C_4 = 6 \cdot {}^{n-1}C_2$

Sol: ${}^{n+1}C_4 = 6 \cdot {}^{n-1}C_2$

$$\frac{(n+1)!}{4!(n+1-4)!} = 6 \times \frac{(n-1)!}{2!(n-1-2)!}$$

$$\Rightarrow \frac{(n+1)n(n-1)!}{4 \times 3 \times 2 \times 1 \times (n-3)!} = 6 \frac{(n-1)!}{2(n-3)!}$$

$$\Rightarrow (n+1)n = 72$$

$$\Rightarrow n^2 + n - 72 = 0$$

$$\Rightarrow n^2 + 9n - 8n - 72 = 0$$

$$\Rightarrow n(n+9) - 8(n+9) = 0$$

$$\Rightarrow (n+9)(n-8) = 0$$

$$n+9=0 \quad | \quad n-8=0$$

$$\Rightarrow n=-9 \quad | \quad n=8$$

(Not possible)

$$\therefore \boxed{n=8}$$

$$(iii) {}^nC_2 = 30 \cdot {}^nC_3$$

$$\text{Sol: } {}^nC_2 = 30 \cdot {}^nC_3$$

$$\frac{n!}{2!(n-2)!} = 30 \cdot \frac{n!}{3!(n-3)!}$$

$$\frac{n^2(n-1)(n-2)!}{2!(n-2)!} = 30 \cdot \frac{n(n-1)(n-2)(n-3)!}{3 \cdot 2!(n-3)!}$$

$$n^2(n-1) = 10n(n-1)(n-2)$$

$$n^2(n-1)(n+1) = 10n(n-1)(n-2)$$

$$n(n+1) = 10(n-2)$$

$$n^2 + n - 10n + 20 = 0$$

$$n^2 - 9n + 20 = 0$$

$$n^2 - 5n - 4n + 20 = 0$$

$$n(n-5) - 4(n-5) = 0$$

$$(n-4)(n-5) = 0$$

$$\begin{array}{l|l} n-4=0 & n-5=0 \\ n=4 & n=5 \end{array}$$

$$\text{Hence } \boxed{n=4, 5} \text{ Ans}$$

$$\text{Q.2 Find } n \text{ and } r \text{ if } {}^nP_r = 840 \rightarrow (1)$$

$$\text{and } {}^nC_r = 35 \rightarrow (2)$$

$$\text{Sol: } \because {}^nP_r = r! {}^nC_r$$

$$\Rightarrow 840 = r!(35)$$

$$\Rightarrow r! = \frac{840}{35} = 24 = 4!$$

$$\Rightarrow r = 4!$$

$$\therefore \boxed{r=4}$$

So (1) becomes,

$${}_nP_4 = 840$$

$$\Rightarrow \frac{n!}{(n-4)!} = 840$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 840$$

$$\Rightarrow n(n-1)(n-2)(n-3) = 7 \cdot 6 \cdot 5 \cdot 4$$

$$\Rightarrow \boxed{n=7}$$

$$\text{Hence } \boxed{r=4, n=7} \text{ Ans}$$

$$\text{Q.3 Find } n \text{ when } {}^{2n}C_3 : {}^nC_2 = 36:3$$

$$\text{Sol: } {}^{2n}C_3 : {}^nC_2 = 36:3$$

$$\frac{{}^{2n}C_3}{{}^nC_2} = \frac{36}{3}$$

$$\frac{\frac{(2n)!}{(2n-3)!3!}}{\frac{n!}{(n-2)!2!}} = 12$$

$$\frac{(2n)!}{(2n-3)!3!} \times \frac{(n-2)!2!}{n!} = 12$$

$$\frac{2n(2n-1)(2n-2)(2n-3)! \times (n-2)!2!}{(2n-3)!3! \cdot n(n-1)(n-2)!} = 12$$

$$4(2n-1) = 36$$

$$\Rightarrow 2n-1 = 9$$

$$\Rightarrow 2n = 10 \Rightarrow \boxed{n=5} \text{ Ans}$$

Q.4: Prove that (i) ${}^{n-1}C_r + {}^{n-1}C_{r-1} = {}^nC_r$

Sol: L.H.S = ${}^{n-1}C_r + {}^{n-1}C_{r-1}$

$$= \frac{(n-1)!}{r![(n-1)-r]!} + \frac{(n-1)!}{(r-1)![(n-1)-(r-1)]!}$$

$$= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!}$$

$$= \frac{(n-1)!}{r(r-1)!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(r-1)!(n-r-1)!} \left[\frac{1}{r} + \frac{1}{n-r} \right]$$

$$= \frac{(n-1)!}{(r-1)!(n-r-1)!} \left[\frac{n-r+r}{r(n-r)} \right] = \frac{n(n-1)!}{r!(n-r)!(n-r-1)!}$$

$$= \frac{n!}{r!(n-r)!} = {}^nC_r = \text{R.H.S Proved}$$

(ii) $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$

Sol: L.H.S = $r \cdot {}^nC_r$

$$= r \cdot \frac{n!}{r!(n-r)!}$$

$$= r \cdot \frac{n!}{r(r-1)!(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!} \rightarrow (1)$$

R.H.S = $n \cdot {}^{n-1}C_{r-1}$

$$= n \cdot \frac{(n-1)!}{(r-1)![(n-1)-(r-1)]!}$$

$$= n \cdot \frac{(n-1)!}{(r-1)!(n-r)!}$$

$$= \frac{n(n-1)!}{(r-1)!(n-r)!}$$

$$= \frac{n!}{(r-1)!(n-r)!} \rightarrow (2)$$

From (1) and (2), L.H.S = R.H.S Proved

Q.5: How many (i) straight lines (ii) triangles are determined by 12 points, no three of which lie on the same straight line.

Sol: (i) Two points are needed for a straight line.

$$\therefore \text{Number of straight lines} = {}^{12}C_2 = \frac{12!}{2!10!} = \frac{12 \times 11 \times 10!}{2 \times 10!} = \boxed{66} \text{ Ans}$$

(ii) Three non-collinear points are needed for a triangle.

$$\therefore \text{Number of triangles} = {}^{12}C_3 = \frac{12!}{3!9!} = \frac{12 \times 11 \times 10 \times 9!}{3 \times 2 \times 1 \times 9!} = \boxed{220} \text{ Ans}$$

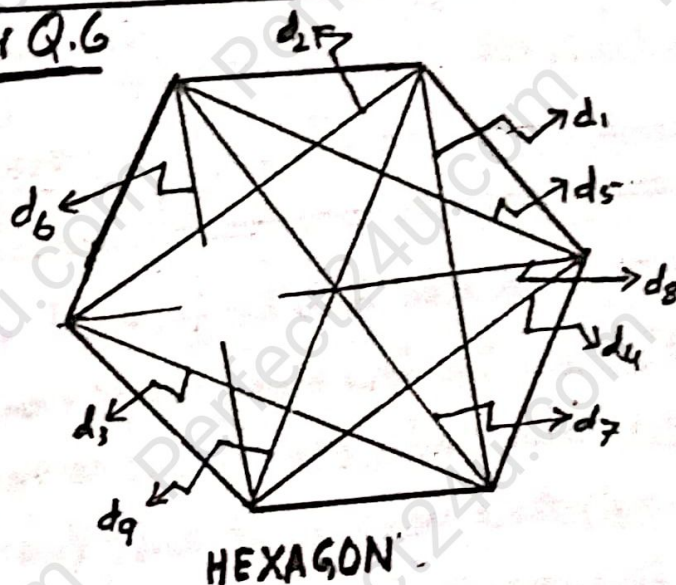
Q.6: Find the total number of diagonals of a hexagon.

Sol: Number of diagonals of a hexagon = ${}^6C_2 - 6$

$$= \frac{6!}{2!4!} - 6 = \frac{6 \times 5 \times 4!}{2 \times 4!} - 6 = 15 - 6 = \boxed{9} \text{ Ans}$$

For Q.6

The general formula for an n -sided polygon is ${}^nC_2 - n$



The 9 diagonals are shown.

Q.7: Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

Sol: For a handshake two people are needed.

$\therefore$ total number of handshakes = ${}^{20}C_2$

$$= \frac{20!}{2!18!}$$

$$= \frac{20 \times 19 \times 18!}{2 \times 18!} = \boxed{190} \text{ Ans}$$



Q.8: A student is to answer 7 out of 10 questions in an examination. How many choices has he, if he must answer the first 3 questions?

Sol: Total no. of questions = 10

Compulsory questions = first 3

Remaining questions = $10 - 3 = 7$

The student has to select 4 questions out of 7

Compulsory	{	Q1. ---
		Q2. ---
		Q3. ---
		Q4. ---
		Q5. ---
4 Questions From These 7 Questions	{	Q6. ---
		Q7. --- Paper
		Q8. ---
		Q9. ---
		Q10. ---

$$\text{Number of choices} = {}^7C_4 = \frac{7!}{4!3!} = \frac{7 \times 6 \times 5 \times 4!}{4! \times 3 \times 2 \times 1} = \boxed{35} \text{ ways}$$

Q.9: An 8-person committee is to be formed from a group of 6 women and 7 men. In how many ways can the committee be chosen if (i) the committee must contain four men and four women? (ii) there must be at least two women? (iii) there must be at least two women (iv) there must be more women than men?

Sol: Total no. of women = 6
Total no. of men = 7

(i) The committee must contain four men and four women
 $= {}^7C_4 {}^6C_4 = 35 \cdot 15 = \boxed{525}$ Ans

(ii) There must be at least two women

$$= {}^7C_2 {}^6C_6 + {}^7C_3 {}^6C_5 + {}^7C_4 {}^6C_4 + {}^7C_5 {}^6C_3 + {}^7C_6 {}^6C_2 + {}^7C_7 {}^6C_1$$

$$= 21 \cdot 1 + 35 \cdot 6 + 35 \cdot 15 + 21 \cdot 20 + 7 \cdot 15 + 1 \cdot 6$$

$$= 21 + 210 + 525 + 420 + 105 + 6 = \boxed{1,287}$$

(iii) There must be at least two women ———— Ans

$$= {}^6C_2 {}^7C_6 + {}^6C_3 {}^7C_5 + {}^6C_4 {}^7C_4 + {}^6C_5 {}^7C_3 + {}^6C_6 {}^7C_2$$

$$= 15 \cdot 7 + 20 \cdot 21 + 15 \cdot 35 + 6 \cdot 35 + 1 \cdot 21$$

$$= 105 + 420 + 525 + 210 + 21 = \boxed{1,281}$$
 Ans

(iv) There must be more women than men

$$= {}^6C_5 {}^7C_3 + {}^6C_6 {}^7C_2 = 6 \cdot 35 + 1 \cdot 21 = 210 + 21 = \boxed{231}$$
 Ans

EXERCISE 6.4

Q.1: Let $S = \{1, 2, 3, 4, 5, 6\}$ be the sample space of rolling a die. What is the probability of

(i) Rolling a 5?

Sol: $S = \{1, 2, 3, 4, 5, 6\}$

Let $A = \text{Rolling a } 5 = \{5\}$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{6} \quad \underline{\text{Ans}}$$

(ii) Rolling a number less than one?

Sol: Let $B = \text{Rolling a number less than one} = \{\}$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{0}{6} = \frac{0}{6} \quad \underline{\text{Ans}}$$

(iii) Rolling a number greater than 0?

Sol: Let $C = \text{Rolling a number greater than } 0 = S$

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{6}{6} = 1 \quad \underline{\text{Ans}}$$

(iv) Rolling a multiple of 3

Sol: Let $D = \text{Rolling a multiple of } 3 = \{3, 6\}$

$$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{2}{6} = \frac{1}{3} \quad \underline{\text{Ans}}$$

(v) Rolling a number greater than or equal to 4?

Sol: Let $E = \text{Rolling a number greater than or equal to } 4 = \{4, 5, 6\}$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2} \quad \underline{\text{Ans}}$$

Q.2: A bag contains 4 white, 5 red and 6 green balls. 3 balls are drawn at random. What is the probability that (i) All are green (ii) All are white.

Sol:

No. of white balls = 4

No. of red balls = 5

No. of green balls = 6

Total no. of balls in the bag = 15

$$\begin{aligned}
 \text{(i)} \quad P(\text{All are green}) &= \frac{{}^6C_3 {}^4C_0 {}^5C_0}{{}^{15}C_3} = \frac{{}^6C_3 \cdot 1 \cdot 1}{{}^{15}C_3} = \frac{{}^6C_3}{{}^{15}C_3} \\
 &= \frac{6!}{3!3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = \frac{6 \times 5 \times 4}{3 \times 3} = \frac{20}{3} = \frac{4}{91} \text{ Ans}
 \end{aligned}$$

$$\text{(ii)} \quad P(\text{All are white}) = \frac{{}^4C_3 {}^5C_0 {}^6C_0}{{}^{15}C_3} = \frac{{}^4C_3 \cdot 1 \cdot 1}{{}^{15}C_3} = \frac{{}^4C_3}{{}^{15}C_3} = \frac{4}{455} \text{ Ans}$$

Q.3: A True or false test contains eight questions. If a student guesses the answer for each question, find the probability.

Sol: There are 8 questions and a question can be either true or false.

$$\therefore n(S) = 2^8 = 256$$

(i) 8 answers are correct.

Let A denotes the event that 8 answers are correct.

$$\therefore n(A) = {}^8C_8 = 1$$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{1}{256} \text{ Ans}$$

(i) 7 answers are correct and 1 is an incorrect.

Sol: Let B denotes the event that 7 answers are correct and 1 is an incorrect $\Rightarrow n(B) = {}^8C_7 = 8 \therefore P(B) = \frac{n(B)}{n(S)} = \frac{8}{256} = \boxed{\frac{1}{32}}$ Ans

(ii) 6 answers are correct and 2 are incorrect.

Sol: Let C denotes the event that 6 answers are correct and 2 are incorrect $\Rightarrow n(C) = {}^8C_6 = \frac{8!}{6!2!} = \frac{8 \times 7 \times 6!}{6! \times 2} = 28$
 $\therefore P(C) = \frac{n(C)}{n(S)} = \frac{28}{256} = \boxed{\frac{7}{64}}$ Ans

(iii) At least 6 answers are correct.

Sol: Let D denotes the event that at least 6 answers are correct.

$$\Rightarrow n(D) = {}^8C_6 + {}^8C_7 + {}^8C_8 = 28 + 8 + 1 = 37$$

$$\therefore P(D) = \frac{n(D)}{n(S)} = \boxed{\frac{37}{256}}$$
 Ans

Q4: Three unbiased coins are tossed. What is the probability of obtaining (i) All heads (ii) two heads (iii) one head (iv) at least one head (v) at least two heads (vi) All tails

Sol: There are three coins and a toss can either be head or tail.

$$\therefore n(S) = 2^3 = 8, \text{ where } S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$$(i) \text{ Let } A = \{\text{All heads}\} = \{HHH\} \Rightarrow n(A) = 1 \therefore P(A) = \frac{n(A)}{n(S)} = \boxed{\frac{1}{8}}$$
 Ans

$$(ii) \text{ Let } B = \{\text{two heads}\} = \{HHT, HTH, THH\} \Rightarrow n(B) = 3$$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \boxed{\frac{3}{8}}$$
 Ans

$$(iii) \text{ Let } C = \{\text{one head}\} = \{HTT, THT, TTH\} \Rightarrow n(C) = 3$$

$$\therefore P(C) = \left[\frac{3}{8} \right] \underline{\text{Ans}}$$

(iv) Let $D = \{\text{at least one head}\} = \{HHH, HHT, HTH, THH, HTT, THT, TTH\}$
 $\Rightarrow n(D) = 7 \therefore P(D) = \frac{n(D)}{n(S)} = \left[\frac{7}{8} \right] \underline{\text{Ans}}$

(v) Let $E = \{\text{at least two heads}\} = \{HHH, HHT, HTH, THH\}$
 $\Rightarrow n(E) = 4 \therefore P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \left[\frac{1}{2} \right] \underline{\text{Ans}}$

(vi) Let $F = \{\text{all tails}\} = \{TTT\} \Rightarrow n(F) = 1 \therefore P(F) = \frac{n(F)}{n(S)} = \left[\frac{1}{8} \right] \underline{\text{Ans}}$

Q.5: A committee of 5 ^{persons} is to be selected at random from 6 men and 4 women. Find the probability that the committee will consist of

(i) 3 men and 2 women (ii) 2 men and 3 women.

Sol: No. of men = 6

No. of women = 4

Total persons = 10

(i) $P(3 \text{ men } 2 \text{ women}) = \frac{{}^6C_3 {}^4C_2}{{}^{10}C_5} = \frac{20 \times 6}{252} = \frac{120}{252} = \left[\frac{10}{21} \right] \underline{\text{Ans}}$

(ii) $P(2 \text{ men and } 3 \text{ women}) = \frac{{}^6C_2 {}^4C_3}{{}^{10}C_5} = \frac{15 \times 4}{252} = \left[\frac{5}{21} \right] \underline{\text{Ans}}$

Q.6: If one card is drawn at random from a well shuffled pack of 52 cards. Then find the probability of each of the following.

(i) Drawing an ace card

Sol: $P(\text{Drawing an ace card}) = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \boxed{\frac{1}{13}}$ Ans

(ii) Drawing either spade or hearts

Sol: $P(\text{Drawing either spade or hearts}) = \frac{{}^{26}C_1}{{}^{52}C_1} = \frac{26}{52} = \boxed{\frac{1}{2}}$ Ans

(iii) Drawing a diamond card

Sol: $P(\text{Drawing a diamond card}) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \boxed{\frac{1}{4}}$ Ans

(iv) Drawing a face card

Sol: $P(\text{Drawing a face card}) = \frac{{}^{12}C_1}{{}^{52}C_1} = \frac{12}{52} = \boxed{\frac{3}{13}}$ Ans

(v) Not drawing an ace of hearts.

Sol: Method I

$P(\text{Not drawing an ace of hearts}) = \frac{{}^{51}C_1}{{}^{52}C_1} = \boxed{\frac{51}{52}}$ Ans

Method II

$P(\text{Drawing an ace of hearts}) = \frac{1}{52}$

$P(\text{Not drawing an ace of hearts}) = 1 - \frac{1}{52} = \frac{52-1}{52} = \boxed{\frac{51}{52}}$ Ans

Q.7: Two dice are thrown simultaneously. Find the probability of getting:

The sample space is

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\} \Rightarrow n(S) = 36$$

(i) doublet of even numbers

Sol: Let $A = \{\text{doublet of even numbers}\} = \{(2,2), (4,4), (6,6)\} \Rightarrow n(A) = 3$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \boxed{\frac{1}{12}} \text{ Ans}$$

(ii) a sum less than 6

Sol: Let $B = \{\text{a sum less than 6}\}$

$$= \{(1,1), (2,1), (1,2), (3,1), (2,2), (1,3), (4,1), (3,2), (2,3), (1,4)\}$$

$$\Rightarrow n(B) = 10. \therefore P(B) = \frac{n(B)}{n(S)} = \frac{10}{36} = \boxed{\frac{5}{18}} \text{ Ans}$$

(iii) a sum more than 7

Sol: Let $C = \{\text{a sum more than 7}\}$

$$= \{(6,2), (5,3), (4,4), (3,5), (2,6), (6,3), (5,4), (4,5), (3,6), (6,4), (4,6), (6,5), (5,6), (6,6)\}$$

$$\Rightarrow n(C) = 15, \therefore P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \boxed{\frac{5}{12}} \text{ Ans}$$

(iv) a sum greater than 10

Sol: Let $D = \{\text{a sum greater than 10}\}$
 $= \{(6,5), (5,6), (6,6)\} \Rightarrow n(D) = 3 \quad \therefore P(D) = \frac{n(D)}{n(S)} = \frac{3}{36} = \boxed{\frac{1}{12}} \text{ Ans}$

(v) a sum at least 10

Sol: Let $E = \{\text{a sum at least 10}\}$
 $= \{(6,4), (5,5), (4,6), (6,5), (5,6), (6,6)\}$
 $\Rightarrow n(E) = 6 \quad \therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \boxed{\frac{1}{6}} \text{ Ans}$

(vi) six as the product

Sol: Let $F = \{\text{six as the product}\}$
 $= \{(3,2), (2,3), (6,1), (1,6)\} \Rightarrow n(F) = 4 \quad \therefore P(F) = \frac{n(F)}{n(S)} = \frac{4}{36} = \boxed{\frac{1}{9}} \text{ Ans}$

(vii) an even number as the sum

Sol: Let $G = \{\text{an even number as the sum}\}$
 $= \{(1,1), (2,1), (2,2), (1,3), (5,1), (4,2), (3,3), (2,4), (1,5),$
 $(6,2), (5,3), (4,4), (3,5), (2,6), (6,4), (5,5), (4,6), (6,6)\}$
 $\Rightarrow n(G) = 18 \quad \therefore P(G) = \frac{n(G)}{n(S)} = \frac{18}{36} = \boxed{\frac{1}{2}} \text{ Ans}$

(viii) an odd number as the sum

Sol: Let $H = \{\text{an odd number as the sum}\}$
 $= \{(2,1), (1,2), (4,1), (3,2), (2,3), (1,4), (6,1), (5,2), (4,3), (3,4), (2,5), (1,6),$
 $(6,3), (5,4), (4,5), (3,6), (6,5), (5,6)\} \Rightarrow n(H) = 18$
 $\therefore P(H) = \frac{n(H)}{n(S)} = \frac{18}{36} = \boxed{\frac{1}{2}} \text{ Ans}$

Method II $P(\text{an odd number as the sum}) = 1 - P(\text{an even number as the sum})$
 $= 1 - \frac{1}{2} = \boxed{\frac{1}{2}} \text{ Ans}$

(ix) a multiple of 3 as the sum

Sol: Let $I = \{\text{a multiple of 3 as the sum}\}$

$$= \{(2,1), (1,2), (5,1), (4,2), (3,3), (2,4), (1,5), (6,1), (5,4), (4,5), (3,6), (6,6)\} \Rightarrow n(I) = 12.$$

$$\therefore P(I) = \frac{n(I)}{n(S)} = \frac{12}{36} = \boxed{\frac{1}{3}} \text{ Ans}$$

(x) sum as a prime number

Sol: Let $J = \{\text{sum as a prime number}\}$

$$= \{(1,1), (2,1), (1,2), (4,1), (3,2), (2,3), (1,4), (6,1), (5,2), (4,3), (3,4), (2,5), (1,6), (6,5), (5,6)\} \Rightarrow n(J) = 15$$

$$\therefore P(J) = \frac{n(J)}{n(S)} = \frac{15}{36} = \boxed{\frac{5}{12}} \text{ Ans}$$

EXERCISE 6.5

Q.1: Suppose events A and B are such that $P(A) = \frac{2}{5}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$. Find $P(A \cap B)$.

Sol: $\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{1}{2} = \frac{2}{5} + \frac{2}{5} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{2}{5} + \frac{2}{5} - \frac{1}{2} = \frac{4+4-5}{10} = \boxed{\frac{3}{10}} \text{ Ans}$$

Q.2: If A and B are 2 events in a sample space S

such that $P(A) = \frac{1}{2}$, $P(\bar{B}) = \frac{5}{8}$, $P(A \cup B) = \frac{3}{4}$.

(i) $P(A \cap B)$

Sol: $\because P(B) = 1 - P(\bar{B}) = 1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$

Now $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\frac{3}{4} = \frac{1}{2} + \frac{3}{8} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{1}{2} + \frac{3}{8} - \frac{3}{4}$$

$$= \frac{4+3-6}{8} = \boxed{\frac{1}{8}} \text{ Ans}$$

(ii) $P(\bar{A} \cap \bar{B})$

Sol: $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$ (By using De-Morgan Law)

$$= 1 - P(A \cup B)$$

$$= 1 - \frac{3}{4} = \frac{4-3}{4} = \boxed{\frac{1}{4}} \text{ Ans}$$

✓ Q.3. Given $P(A) = 0.5$ and $P(A \cup B) = 0.6$, find $P(B)$ if A and B are mutually exclusive.

Sol: $\because$ A and B are mutually exclusive $\Rightarrow P(A \cap B) = 0$

$$\therefore P(A \cup B) = P(A) + P(B)$$

$$0.6 = 0.5 + P(B) \Rightarrow P(B) = 0.6 - 0.5 = \boxed{0.1} \text{ Ans}$$

Q.4. A bag contains 30 tickets numbered from 1 to 30. One ticket is selected at random. Find the probability that its number is either odd or the square of an integer.

Sol: Here $S = \{1, 2, 3, \dots, 30\} \Rightarrow n(S) = 30$

Method I: Let A denotes the event that the number is odd

$$\therefore A = \{1, 3, 5, \dots, 29\} \Rightarrow n(A) = 15 \therefore P(A) = \frac{n(A)}{n(S)} = \frac{15}{30} = \frac{1}{2}$$

Let B denotes the event that the number is the square of an integer $\therefore B = \{1, 4, 9, 16, 25\} \Rightarrow n(B) = 5 \therefore P(B) = \frac{n(B)}{n(S)} = \frac{5}{30} = \frac{1}{6}$

$$A \cap B = \{1, 9, 25\} \Rightarrow n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{30} = \frac{1}{10}$$

$P(\text{The number is odd or square of an integer})$
 $= P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{6} - \frac{1}{10} = \frac{15+5-3}{30} = \boxed{\frac{17}{30}} \text{ Ans}$$

Method II:

Let E denotes the event that the number is odd or square of an integer $\Rightarrow E = \{1, 3, 5, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 4, 16\}$

$$\Rightarrow n(E) = 17 \therefore P(E) = \frac{n(E)}{n(S)} = \boxed{\frac{17}{30}} \text{ Ans}$$

Q.5: A student finds that the probability of passing an algebra test is $\frac{8}{9}$. What is the probability of failing the test?

Sol: Let A denotes the event of passing the test $\Rightarrow P(A) = \frac{8}{9}$

$$P(\text{of failing the test}) = P(A') = 1 - P(A) = 1 - \frac{8}{9} = \boxed{\frac{1}{9}} \text{ Ans}$$

Q.6: In the two dice experiment, given that the first die shows 4, what is the probability that second die shows a number greater than 4?

Sol: The sample space is

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\} \Rightarrow n(S) = 6^2 = 36$$

Method I Let $A = \{ \text{1st die shows 4} \}$

$$= \{ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \}$$

$$\Rightarrow n(A) = 6 \quad \therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Let $B = \{ \text{the second die shows a number greater than 4} \}$

$$= \{ (1,5), (1,6), (2,5), (2,6), (3,5), (3,6), (4,5), (4,6), (5,5), (5,6), (6,6) \}$$

$$A \cap B = \{ (4,5), (4,6) \} \Rightarrow n(A \cap B) = 2 \quad \therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

Now $P(B|A) = \frac{P(A \cap B)}{P(A)}$ (By definition of conditional probability)

$$= \frac{\frac{1}{18}}{\frac{1}{6}} = \frac{1}{18} \times \frac{6}{1} = \frac{1}{3} \text{ Ans}$$

Method II Taking the sample space $S = \{ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \}$

$\therefore n(S) = 6$. Let $E = \{ \text{the second die shows a number greater than 4} \}$

$$\Rightarrow E = \{ (4,5), (4,6) \} \Rightarrow n(E) = 2$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3} \text{ Ans}$$

Q.7: One card is drawn from a pack of 52 cards, what is the probability that the card drawn is neither red nor king.

Sol: Let R = the card drawn is red.

K = the card drawn is king.

$$\begin{aligned} P(R \cup K) &= P(R) + P(K) - P(R \cap K) \\ &= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{26+4-2}{52} = \boxed{\frac{28}{52}} \end{aligned}$$

$$P(\bar{R} \cap \bar{K}) = P(\bar{R \cup K}) = 1 - P(R \cup K)$$

$$= 1 - \frac{28}{52} = \frac{52-28}{52} = \frac{24}{52} = \boxed{\frac{6}{13}} \text{ Ans}$$

Q.8 If a pair of dice is thrown, find the probability that the sum of digits is neither 7 nor 11.

Sol: The sample space is

$$S = \left\{ \begin{array}{cccccc} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{array} \right\} \Rightarrow n(S) = 6 \times 6 = 36$$

Let A = the sum is 7 $\Rightarrow A = \{(6,1), (5,2), (4,3), (3,4), (2,5), (1,6)\}$

$$\therefore n(A) = 6 \therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36}$$

Let $B = \text{The sum is 11} \Rightarrow B = \{(6,5), (5,6)\}$

$$\therefore n(B) = 2 \quad \therefore P(B) = \frac{n(B)}{n(S)} = \boxed{\frac{2}{36}}$$

$$P(A \cup B) = P(A) + P(B) \quad (\because A \text{ and } B \text{ are mutually exclusive events})$$

$$= \frac{6}{36} + \frac{2}{36} = \frac{8}{36}$$

$$\text{Now } P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - \frac{8}{36} = \frac{36-8}{36} = \frac{28}{36} = \boxed{\frac{7}{9}} \quad \underline{\text{Ans}}$$

Q.9: Ajmal and Bushra appear in an interview for 2 vacancies. The probability of their selection being $\frac{1}{7}$ and $\frac{1}{5}$ respectively. Find the probability that

Let $A = \text{Ajmal be selected}$

$B = \text{Bushra be selected}$

Given $P(A) = \frac{1}{7}$ and $P(B) = \frac{1}{5}$

(i) both will be selected.

$$\text{Sol: } P(\text{both will be selected}) = P(A \cap B)$$

$$= P(A)P(B) \quad [\because A \text{ and } B \text{ are independent events}]$$

$$= \frac{1}{7} \times \frac{1}{5} = \boxed{\frac{1}{35}} \quad \text{Ans}$$

(ii) only one is selected.

$$\text{Sol: } P(\text{only one is selected}) = P[(A \cap \bar{B}) \cup (\bar{A} \cap B)]$$

$$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$$

$$\begin{aligned}
 &= P(A)[1-P(B)] + [1-P(A)] \cdot P(B) \\
 &= \frac{1}{7} \cdot \left(1 - \frac{1}{5}\right) + \left(1 - \frac{1}{7}\right) \cdot \frac{1}{5} \\
 &= \frac{1}{7} \cdot \frac{4}{5} + \frac{6}{7} \cdot \frac{1}{5} = \frac{4}{35} + \frac{6}{35} = \frac{10}{35} = \boxed{\frac{2}{7}} \quad \underline{\text{Ans}}
 \end{aligned}$$

(iii) none will be selected.

$$\begin{aligned}
 \underline{\text{Sol:}} \quad P(\text{none will be selected}) &= P(\bar{A} \cap \bar{B}) \\
 &= P(\bar{A}) \cdot P(\bar{B}) \\
 &= [1-P(A)][1-P(B)] \\
 &= \left(1 - \frac{1}{7}\right) \left(1 - \frac{1}{5}\right) \\
 &= \frac{6}{7} \times \frac{4}{5} = \boxed{\frac{24}{35}} \quad \underline{\text{Ans}}
 \end{aligned}$$

(iv) at least one of them will be selected.

$$\begin{aligned}
 \underline{\text{Sol:}} \quad P(\text{at least one of them will be selected}) &= 1 - P(\text{none of them will be selected}) \\
 &= 1 - \frac{24}{35} = \boxed{\frac{11}{35}} \quad \underline{\text{Ans}}
 \end{aligned}$$

Q.10: A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are defective. If a person takes out 2 at random what is the probability that either both are apples or both are good.

Sol: No. of Apples = 20

No. of Oranges = 10

Total apples and oranges = 30

No. of defective apples = 5 $\Rightarrow$ No. of good apples = $20 - 5 = 15$

No. of defective oranges = 3

Total defective apples and oranges = 8

Total good apples and oranges = $30 - 8 = 22$

Now let A = both are apples

G = both are good

$P(\text{both are apples or both are good}) = P(A \cup G)$

$$= P(A) + P(G) - P(A \cap G)$$

$$= \frac{{}^{20}C_2}{{}^{30}C_2} + \frac{{}^{22}C_2}{{}^{30}C_2} - \frac{{}^{15}C_2}{{}^{30}C_2}$$

$$= \frac{190}{435} + \frac{231}{435} - \frac{105}{435}$$

$$= \frac{190 + 231 - 105}{435} = \frac{316}{435} \text{ Ans}$$

زہ رحمان چہ غشے اولم بیائے اولم - اوچہ تے نہ اولم چا بلارے چہ ما اولمے ہا ہا ہا

REVIEW EXERCISE 6

Q.1: Choose the correct option.

(i) In how many ways can we name the vertices of a pentagon using any five of the letters O, P, Q, R, S, T, U in any order.

(a) 2520

(b) 9040

(c) 5140

(d) 4880

Sol: ${}^7P_5 = \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 2520$. Thus the correct option is (a). Ans

(ii) How many two-digit odd numbers can be formed from the digits $\{1, 2, 3, 4, 5, 6, 7\}$ if repeated digits are allowed.

(a) 14

(b) 42

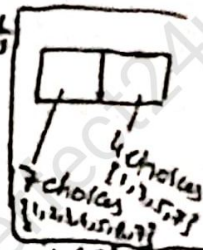
(c) 28

(d) 21

Sol: For odd numbers unit place can be filled by 4 digits

$\{1, 3, 5, 7\}$. Since repetitions are allowed so Ten's place can be

filled by any of the seven given numbers. So the total



numbers formed are $7 \times 4 = 28$. Thus the correct option is (c) Ans

(iii) How many six-digit numbers can be formed from the digits $\{2, 3, 4, 6, 7, 8\}$ without repetition if the digits 3 and 7 must be together?

(a) 120

(b) 180

(c) 144

(d) 96

Sol: Consider 3 and 7 as one digit then total no. of digits becomes

5, so the six-digit numbers formed are $= {}^5P_5 = 5! = 120$. Thus the correct option

is (a). Ans

(iv) Evaluate $\frac{(n+2)!(n-2)!}{(n+1)!(n-1)!}$

 (a) $(n-3)$

 (b) $(n-1)$

 (c) $\frac{(n+1)}{(n+2)}$

 (d) $\frac{(n+2)}{(n-1)}$

Sol: $\frac{(n+2)!(n-2)!}{(n+1)!(n-1)!} = \frac{(n+2)(n+1)!(n-2)!}{(n+1)!(n-1)(n-2)!} = \frac{n+2}{n-1}$. Thus the correct option is (d). Ans

(v) In how many different ways can 5 couples be seated around a circular table if the couples must not be repeated.

(a) 768

(b) 724

(c) 844

(d) 696

Sol: $(5-1)! 2! 2! 2! 2! 2! = 4! 2! 2! 2! 2! = 24 \times 2 \times 2 \times 2 \times 2 = 768$

Thus the correct option is (a). Ans

(vi) A committee of 4 people will be selected from 8 girls

and 12 boys in a class. How many different selections are possible if at least one boy must be selected?

- (a) 2865 (b) 3755 (c) 4225 (d) 4775

Sol: Total possible selections = ${}^{12}C_1 {}^8C_3 + {}^{12}C_2 {}^8C_2 + {}^{12}C_3 {}^8C_1 + {}^{12}C_4 {}^8C_0$
 $= 12 \times 56 + 66 \times 28 + 220 \times 8 + 495 \times 1$
 $= 672 + 1848 + 1760 + 495 = 4775$

Thus the correct option is (d). Ans

(vii) The number of all possible matrices of order 3×3 with each entry 0 and 1 is:

- (a) 18 (b) 27 (c) 512 (d) 81

Sol: $2^9 = 512$ matrices. Thus the correct option is (c) Ans

(viii) How many diagonals can be drawn in a plane figure of 8 sides?

- (a) 21 (b) 20 (c) 35 (d) 81

Sol: No. of diagonals = ${}^8C_2 - 8 = \frac{8 \times 7}{2} - 8 = 28 - 8 = 20$

Thus the correct option is (b). Ans

(ix) If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A|B)$ is

- (a) 0 (b) $\frac{1}{2}$ (c) not defined (d) 1

Sol: $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0} = \text{Not defined.}$

Thus the correct option is (c). Ans

(X) If A and B are events such that $P(A|B) = P(B|A)$ then

(a) $A \subset B$ but $A \neq B$ (b) $A = B$ (c) $A \cap B = \emptyset$ (d) $P(A) = P(B)$

Sol: $P(A|B) = P(B|A)$

$$\Rightarrow \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A) = P(B). \text{ Thus the correct option is (d).}$$

Ans

Q.21: (i) If ${}^{2n}C_n = {}^{2n}C_{n+2}$; find n . (ii) If ${}^{18}C_n = {}^{18}C_{n+2}$; find n .

Sol: ${}^{2n}C_n = {}^{2n}C_{n+2}$

$$\therefore n \neq n+2$$

$$\therefore n + n + 2 = 2n$$

$$\Rightarrow 2n + 2 = 2n$$

$$\Rightarrow 2n = 2n - 2$$

$$\Rightarrow \boxed{n = n-1} \text{ Ans}$$

Sol: Here $2n = 18 \Rightarrow n = 9$

$$\therefore n = n-1 = 9-1 = 8 \text{ (by part (i))}$$

Now ${}^{18}C_8 = {}^{18}C_{10} = \frac{18!}{8!10!}$
 $= \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{8!} = \frac{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$
 $= \boxed{165} \text{ Ans}$

Q.3: ${}^{56}P_{n+6} : {}^{54}P_{n+3} = 30800:1$, find n .

Sol: ${}^{56}P_{n+6} : {}^{54}P_{n+3} = 30800:1$

$$\frac{56!}{(56-n-6)!} : \frac{54!}{(54-n-3)!} = 30800:1$$

$$\frac{56!}{(50-n)!} : \frac{54!}{(51-n)!} = 30800:1 \Rightarrow \frac{56 \cdot 55 \cdot 54!}{(50-n)!} = \frac{54!}{(51-n)(50-n)!} = 30800:1$$

$$\Rightarrow \frac{3080}{1} : \frac{1}{51-n} = 30800:1 \Rightarrow \frac{3080}{\frac{1}{51-n}} = \frac{30800}{1} \Rightarrow 3080(51-n) = 30800$$

$$\Rightarrow 51-n = 10 \Rightarrow n = 51-10 \therefore \boxed{n = 41} \text{ Ans}$$

Q.4: In how many distinct ways can $x^4 y^3 z^5$ be expressed without exponents?

Sol: $x^4 y^3 z^5 = x x x x y y y z z z z z$

Total no. of letters = $n = 13$

In which x is repeated 4 times, y is repeated 3 times and z is repeated 5 times.

$$\begin{aligned} \text{Total no. of ways without exponents} &= \frac{13!}{4! 3! 5!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{24 \times 6 \times 5!} \\ &= \boxed{27720} \text{ Ans} \end{aligned}$$

Q.5: In how many different ways can be six children seated at a round table if a certain two children (i) refuse to sit next to each other? (ii) insist on sitting next to each other?

Sol: No. of children = $n = 6$

Total no. of ways = $(n-1)! = (6-1)! = 5! = 120$

(ii) consider the two children as one child who insist on sitting next to each other.

Then the no. of ways = $(5-1)! \times 2! = 4! \times 2! = 24 \times 2 = \boxed{48}$

(i) If the two children refuse to sit next to each other
 $= 120 - 48 = \boxed{72}$

Q.6 Six people including Faisal and Saima are to be seated around a circular table. Find the probability that Faisal and Saima are seated next to each other.

Sol: Six people can be seated around a circular table $= (6-1)! = 5! = 120$ ways $\Rightarrow n(S) = 120$

Faisal and Saima are seated next to each other $= (5-1)! \times 2! = 4! \times 2 = 24 \times 2 = 48$ ways

Let $A =$ Faisal and Saima are seated next to each other $\Rightarrow n(A) = 48$

$P(\text{Faisal and Saima are seated next to each other})$

$$= P(A) = \frac{n(A)}{n(S)} = \frac{48}{120} = \frac{4}{10} = \boxed{\frac{2}{5}} \text{ Ans}$$

Maths Wonders:

$$\begin{aligned} 12,345,679 \times 9 &= 111,111,111 \\ 12,345,679 \times 18 &= 222,222,222 \\ 12,345,679 \times 27 &= 333,333,333 \\ 12,345,679 \times 36 &= 444,444,444 \\ 12,345,679 \times 45 &= 555,555,555 \\ 12,345,679 \times 54 &= 666,666,666 \\ 12,345,679 \times 63 &= 777,777,777 \\ 12,345,679 \times 72 &= 888,888,888 \\ 12,345,679 \times 81 &= 999,999,999 \end{aligned}$$

Q.7: If $P(A) = 0.8$, $P(B) = 0.5$, $P(B|A) = 0.4$, find
 (i) $P(A \cap B)$ (ii) $P(A|B)$ (iii) $P(A \cup B)$

Sol: (i) $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$$\Rightarrow 0.4 = \frac{P(A \cap B)}{0.8} \Rightarrow P(A \cap B) = 0.4 \times 0.8 = \boxed{0.32} \text{ Ans}$$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$
 $= \frac{0.32}{0.5} = \boxed{0.64} \text{ Ans}$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
 $= 0.8 + 0.5 - 0.32 = 1.3 - 0.32 = \boxed{0.98} \text{ Ans}$

Q.8: How many 6-digit telephone numbers can be constructed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 if each number starts with 35 and no digit appears more than once.

Sol: Total no. of digits = $n = 10$



If we fix 3 and 5 at two positions at the extreme left. Then the remaining 4 places can be filled with the remaining 8 digits in ${}^8P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$ ways

Hence required number of telephone numbers are $\boxed{1680} \text{ Ans}$

Q.9: How many numbers greater than a million can be

formed with digits 2, 3, 0, 3, 4, 2, 3?

Sol: Total number of digits = $n = 7$

A number greater than a million has at least 7 digits, and thus all the 7 given digits are to be used.

Now we have 7-digits, in which 2 is repeated twice and 3 is repeated 3 times.

$$\text{Total number formed} = \frac{7!}{2!3!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{2 \times 3!} = 420$$

But numbers beginning with zero are no more 7-digit numbers. Such numbers are = $\frac{6!}{2!3!} = \frac{6 \times 5 \times 4 \times 3!}{2 \times 3!} = 60$

$$\text{Hence the required numbers formed} = 420 - 60 = \boxed{360} \text{ Ans}$$

Q.10 A party of n men is to be seated round a circular table. Find the probability that two particular men sit together.

Sol: n people can be seated around a circular table

$$= (n-1)! \text{ ways} \Rightarrow n(S) = (n-1)!$$

Two Particular men sit together = $(n-2)! \times 2!$ ways

Let A = Two particular men sit together

$$\Rightarrow n(A) = (n-2)! \times 2!$$

$$P(\text{Two particular men sit together}) = P(A) = \frac{n(A)}{n(S)} = \frac{(n-2)! \times 2!}{(n-1)!} \\ = \frac{(n-2)! \times 2}{(n-1)(n-2)!} = \boxed{\frac{2}{n-1}} \text{ Ans}$$

Q.11: Given the following spinners, determine the probability

$P(\text{orange})$

$P(\text{Red or Green})$

$P(\text{Not Red})$

$P(\text{Pink})$

Sol:

$$\boxed{\frac{1}{4}}$$

Ans

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

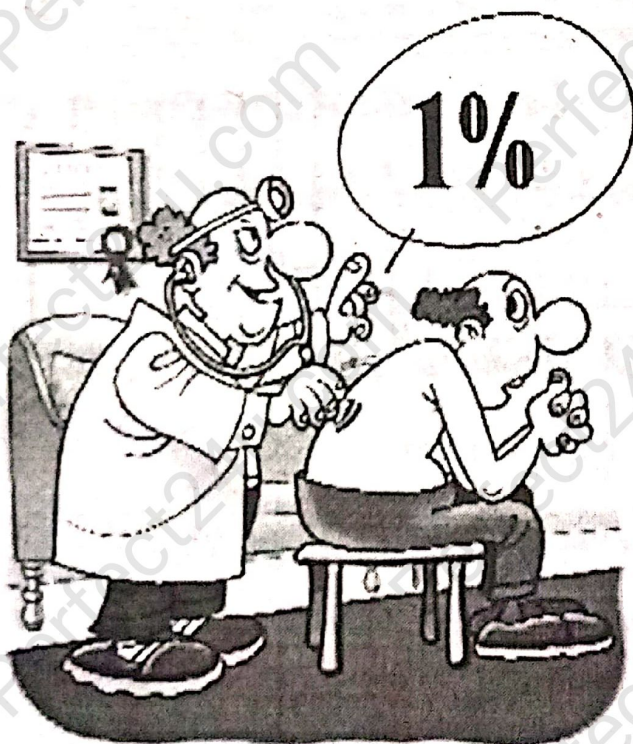
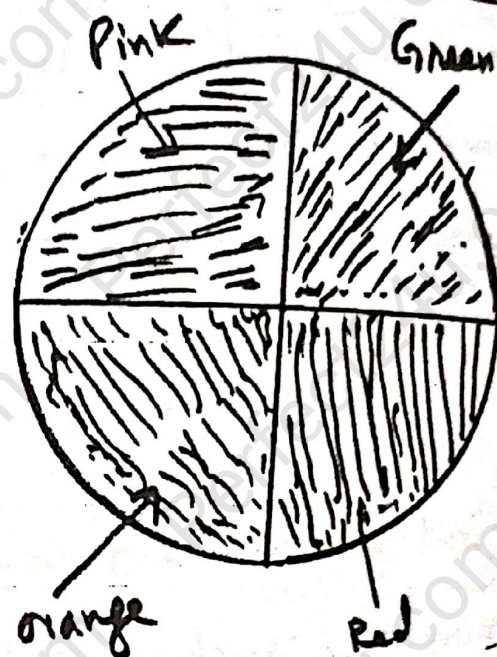
Ans

$$1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$

Ans

$$\boxed{\frac{1}{4}}$$

Ans



A patient is talking to his doctor about a necessary operation.

----- 'I'm worried about this operation, Doctor. They say it's 99 per cent risky.'

----- 'That's true, but you needn't worry.'

----- 'Why?'

----- 'Because you are the hundredth patient. The other 99 have already died!'