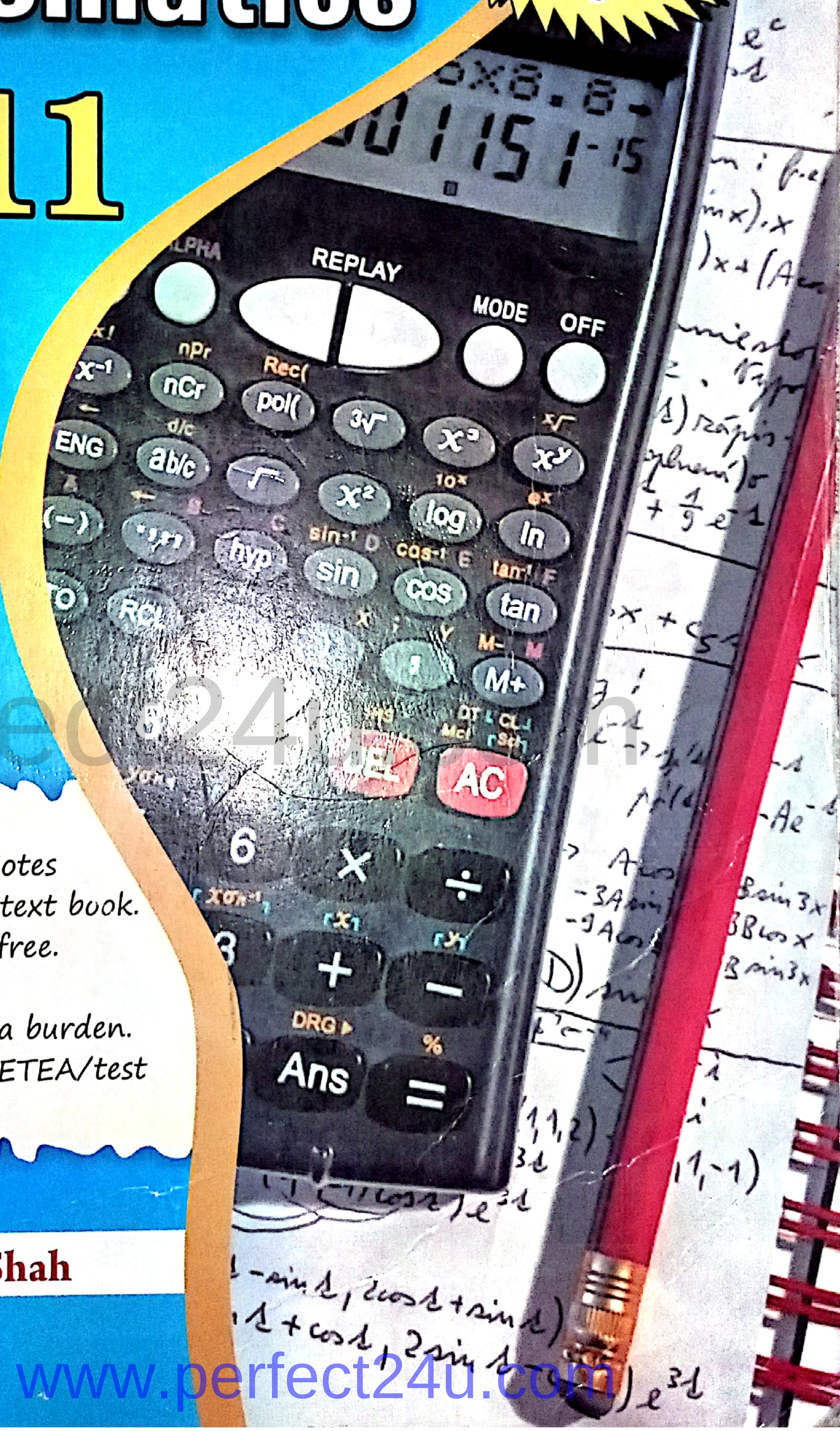
ISLAMIA NOTES

NEW COURSE 2020-21 & Onwards

Mathematics

11



Salient Features

- Hand written class Notes by the author of the text book.
- Accurate and error free.
- Simple and easy.
- To the point, no extra burden.
- Suitable for exam & ETEA/test preparation.

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SOME IMPORTANT POINTS OF CHAPTER NO:1

The following points will enable you to solve problems of chapter No1.

(i) If Z is a complex Number then

$$Z = x + iy, \; x, y \in R$$

(ii) If $x=0$ then Z is called pure imaginary

(iii) If $y=0$ then Z is called real.

(iv) $Re(Z) = x$, $Im(Z) = y$

Real part of Z   Imaginary part of Z

(v) $a+bi = c+di \Leftrightarrow a=c, b=d$ (Equating real & imaginary parts)

(vi) $i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, ...

(vii) Conjugate of a complex Number: $Z = x+iy \Rightarrow \bar{Z} = x - iy$

(Just change the sign of imaginary parts)

(viii) Modulus/absolute of a complex number: $Z = x+iy \Rightarrow |Z| = \sqrt{x^2+y^2}$

(ix) $Z = x+iy$ can be written as $Z = (x,y)$

(x) Set of complex numbers:

$$C = \{(x,y) \mid x, y \in R\} \text{ or } \{x+iy \mid x, y \in R\}$$

(xi) $(a+b)^2 = a^2 + 2ab + b^2$

(xii) $(a-b)^2 = a^2 - 2ab + b^2$

(xiii) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(xiv) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

(xv) $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

(xvi) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

(xvii) Quadratic formula, $az^2 + bz + c = 0, a \neq 0$ (Quadratic Equation)

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

EXERCISE 1.1

Q.1: Simplify the following.

(i) $i^9 + i^{19}$

Sol: $i^9 + i^{19} = i^8 \cdot i + i^{18} \cdot i = (i^2)^4 \cdot i + (i^2)^9 \cdot i = (-1)^4 \cdot i + (-1)^9 \cdot i = i - i = \boxed{0}$ Ans

(ii) $(-i)^{23}$

Sol: $(-i)^{23} = -i^{23} = -i^{22} \cdot i = -(i^2)^{11} i = -(-1)^{11} = \boxed{i}$ Ans

(iii) $(-1)^{\frac{-23}{2}}$

Sol: $(-1)^{\frac{-23}{2}} = \left[(-1)^{\frac{1}{2}}\right]^{-23} = (\sqrt{-1})^{-23} = i^{-23} = \frac{1}{i^{23}} = \frac{1}{i^{22} \cdot i}$

$= \frac{1}{(i^2)^{11} \cdot i} = \frac{1}{(-1)^{11} \cdot i} = \frac{1}{-i} = \frac{1 \cdot i}{-i \cdot i} = \frac{i}{-i^2}$

$= \frac{i}{-(-1)} = \frac{i}{1} = \boxed{i}$ Ans

(iv) $(-1)^{\frac{15}{2}}$

Sol: $(-1)^{\frac{15}{2}} = \left[(-1)^{\frac{1}{2}}\right]^{15} = (\sqrt{-1})^{15} = i^{15} = i^{14} \cdot i = (i^2)^7 \cdot i = (-1)^7 \cdot i = \boxed{-i}$ Ans

Full

Q.2: Prove that $i^{107} + i^{112} + i^{122} + i^{153} = 0$

Sol: $i^{107} + i^{112} + i^{122} + i^{153}$

$= i^{106} \cdot i + i^{112} + i^{122} + i^{152} \cdot i$

$= (i^2)^{53} \cdot i + (i^2)^{56} + (i^2)^{61} + (i^2)^{76} \cdot i$

$= (-1)^{53} \cdot i + (-1)^{56} + (-1)^{61} + (-1)^{76} \cdot i$

$= -i + 1 - 1 + i = 0$ Proved.

Q.3 Add the following complex numbers.

(i) $3(1+2i), -2(1-3i)$

Sol: $3(1+2i)+[-2(1-3i)] = 3(1+2i)-2(1-3i)$

$= 3+6i-2+6i = \boxed{1+12i}$ Ans

(ii) $\frac{1}{2}-\frac{2}{3}i, \frac{1}{4}-\frac{1}{3}i$

Sol: $\frac{1}{2}-\frac{2}{3}i+\frac{1}{4}-\frac{1}{3}i = \frac{1}{2}+\frac{1}{4}-\left(\frac{2}{3}+\frac{1}{3}\right)i = \frac{2+1}{4}-\frac{2+1}{3}i = \boxed{\frac{3}{4}-i}$ Ans

(iii) $(\sqrt{2},1), (1,\sqrt{2})$

Sol: $(\sqrt{2},1)+(1,\sqrt{2}) = \boxed{(\sqrt{2}+1, 1+\sqrt{2})}$ Ans

Q.4 Subtract the second complex number from first.

(i) $(a,0), (2,-b)$

Sol: $(a,0)-(2,-b) = (a-2, 0-(-b)) = \boxed{(a-2, b)}$ Ans

(ii) $(-3,\frac{1}{2}), (3,\frac{1}{2})$

Sol: $(-3,\frac{1}{2})-(3,\frac{1}{2}) = (-3-3, \frac{1}{2}-\frac{1}{2}) = \boxed{(-6,0)}$ Ans

(iii) $3\sqrt{3}-5\sqrt{7}i, \sqrt{3}+2\sqrt{7}i$

Sol: $3\sqrt{3}-5\sqrt{7}i-(\sqrt{3}+2\sqrt{7}i)$

$= 3\sqrt{3}-5\sqrt{7}-\sqrt{3}-2\sqrt{7}i = \boxed{2\sqrt{3}-7\sqrt{7}i}$ Ans

Q.5 Multiply the following complex numbers.

(i) $8i+11, -7+5i$

Sol: $(8i+11)(-7+5i) = -56i+40i^2-77+55i = -56i+40(-1)-77+55i$

$= -56i-40-77+55i = \boxed{-117+i}$ Ans

(ii) $3i, 2(1-i)$

Sol. $3i \times 2(1-i) = 6i(1-i) = 6i - 6i^2 = 6i - 6(-1)$
$= 6i + 6 = \boxed{6+6i}$ Ans

(ii) $\sqrt{2}+\sqrt{3}i,\ 2\sqrt{2}-\sqrt{3}i$

Sol: $(\sqrt{2}+\sqrt{3}i)(2\sqrt{2}-\sqrt{3}i) = 4 - \sqrt{6}i + 2\sqrt{6}i - 3i^2$
$= 4 + \sqrt{6}i - 3(-1)$
$= 4 + \sqrt{6}i + 3 = \boxed{7+\sqrt{6}i}$ Ans

Q.6: Perform the indicated division and write the answer in the form $a+bi$.

(i) $\dfrac{4+i}{3+5i}$

Sol: $\dfrac{4+i}{3+5i} = \dfrac{4+i}{3+5i} \times \dfrac{3-5i}{3-5i} = \dfrac{12-20i+3i-5i^2}{(3)^2-5^2i^2}$

$= \dfrac{12-17i-5(-1)}{9-25(-1)} = \dfrac{12-17i+5}{9+25} = \dfrac{17-17i}{34}$

$= \dfrac{17(1-i)}{34} = \dfrac{1-i}{2} = \boxed{\dfrac{1}{2}-\dfrac{1}{2}i}$ Ans

(ii) $\dfrac{1}{-8+i}$

Sol: $\dfrac{1}{-8+i} = \dfrac{1}{-8+i} \times \dfrac{-8-i}{-8-i} = \dfrac{-8-i}{(-8)^2-i^2} = \dfrac{-8-i}{64-(-1)}$

$= \dfrac{-8-i}{64+1} = \dfrac{-8-i}{65} = \boxed{\dfrac{-8}{65}-\dfrac{1}{65}i}$ Ans

(iii) $\dfrac{1}{7-3i}$

Sol: $\dfrac{1}{7-3i} = \dfrac{1}{7-3i} \times \dfrac{7+3i}{7+3i} = \dfrac{7+3i}{7^2-3^2i^2} = \dfrac{7+3i}{49-9(-1)} = \dfrac{7+3i}{49+9}$

$= \dfrac{7+3i}{58} = \boxed{\dfrac{7}{58}+\dfrac{3}{58}i}$ Ans

(iv) $\frac{6+i}{i}$

Sol:- $\frac{6+i}{i} = \frac{6+i}{i} \times \frac{i}{i} = \frac{6i+i^2}{i^2} = \frac{6i-1}{-1} = -6i+1 = 1-6i$ Ans

Full

Q.7 If $z_1 = 1+2i$ and $z_2 = 2+3i$, evaluate

(i) $|z_1+z_2|$

Sol: $z_1+z_2 = 1+2i+2+3i$

$= 3+5i$

$|z_1+z_2| = \sqrt{3^2+5^2} = \sqrt{9+25} = \boxed{\sqrt{34}}$ Ans

(ii) $|z_1 z_2|$

Sol:- $z_1 z_2 = (1+2i)(2+3i)$

$= 2+3i+4i+6i^2$

$= 2+7i+6(-1)$

$= 2+7i-6$

$= -4+7i$

$|z_1 z_2| = \sqrt{(-4)^2+7^2} = \sqrt{16+49} = \boxed{\sqrt{65}}$ Ans

(iii) $\left|\frac{z_1}{z_2}\right|$

Sol:- $\frac{z_1}{z_2} = \frac{1+2i}{2+3i}$

$= \frac{1+2i}{2+3i} \times \frac{2-3i}{2-3i} = \frac{2-3i+4i-6i^2}{(2)^2-(3^2i^2)}$

$= \frac{2+i-6(-1)}{4-9(-1)} = \frac{2+i+6}{4+9} = \frac{8+i}{13} = \frac{8}{13}+\frac{1}{13}i$

$$\left|\frac{z_1}{z_2}\right| = \sqrt{\left(\frac{8}{13}\right)^2+\left(\frac{1}{13}\right)^2} = \sqrt{\frac{64}{169}+\frac{1}{169}} = \sqrt{\frac{64+1}{169}} = \frac{1}{13}\sqrt{65} \text{ Ans}$$

Q.8: Express the following in the standard form a+ib.

(i) $\frac{1-2i}{2+i}+\frac{4-i}{3+2i}$

Sol:-

$$\frac{1-2i}{2+i}+\frac{4-i}{3+2i} = \frac{(1-2i)(3+2i)+(4-i)(2+i)}{(2+i)(3+2i)}$$

$$= \frac{3+2i-6i-4i^2+8+4i-2i-i^2}{6+4i+3i+2i^2}$$

$$= \frac{3+2i-6i-4(-1)+8+4i-2i-(-1)}{6+4i+3i+2(-1)}$$

$$= \frac{3+2i-6i+4+8+4i-2i+1}{6+4i+3i-2}$$

$$= \frac{16-2i}{4+7i}$$

$$= \frac{16-2i}{4+7i}\times\frac{4-7i}{4-7i}$$

$$= \frac{64-112i-8i+14i^2}{4^2-7^2i^2}$$

$$= \frac{64-112i-8i+14(-1)}{16-49(-1)}$$

$$= \frac{64-112i-8i-14}{16+49}$$

$$= \frac{50-120i}{65}$$

$$= \frac{5(10-24i)}{65} = \frac{10-24i}{13} = \boxed{\frac{10}{13} - \frac{24}{13}i} \text{ Ans}$$

(ii) $\frac{2+\sqrt{-9}}{-5-\sqrt{-16}}$

Sol:- $\frac{2+\sqrt{-9}}{-5-\sqrt{-16}} = \frac{2+3\sqrt{-1}}{-5-4\sqrt{-1}} = \frac{2+3i}{-5-4i}$

$$= \frac{2+3i}{-5-4i} \times \frac{-5+4i}{-5+4i}$$

$$= \frac{-10+8i-15i+12i^2}{(-5)^2-4^2i^2}$$

$$= \frac{-10-7i+12(-1)}{25-16(-1)}$$

$$= \frac{-22-7i}{25+16} = \frac{-22-7i}{41} = \boxed{\frac{-22}{41} - \frac{7}{41}i} \text{ Ans}$$

(iii) $\frac{(1+i)^2}{4+3i}$

Sol:- $\frac{(1+i)^2}{4+3i} = \frac{1+2i+i^2}{4+3i} = \frac{1+2i-1}{4+3i} = \frac{2i}{4+3i}$

$$= \frac{2i}{4+3i} \times \frac{4-3i}{4-3i} = \frac{8i-6i^2}{4^2-3^2i^2} = \frac{8i-6(-1)}{16-9(-1)}$$

$$= \frac{8i+6}{16+9} = \frac{6+8i}{25} = \boxed{\frac{6}{25} + \frac{8}{25}i} \text{ Ans}$$

Q.9: Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$.

Sol:- Let $z = \frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{6+9i-4i-6i^2}{2-i+4i-2i^2}$

$$= \frac{6+5i-6(-1)}{2+3i-2(-1)} = \frac{6+5i+6}{2+3i+2}$$

$$= \frac{12+5i}{4+3i}$$

$$= \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$$

$$= \frac{48-36i+20i-15i^2}{(4)^2-3^2i^2} = \frac{48-16i-15(-1)}{16-9(-1)}$$

$$= \frac{48-16i+15}{16+9} = \frac{63-16i}{25}$$

$$\therefore z = \frac{63}{25} - \frac{16}{25}i$$

$$\bar{z} = \boxed{\frac{63}{25} + \frac{16}{25}i} \quad \text{Ans}$$

Q.10: Evaluate $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

Sol:- $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

$$=\left[i^{18}+\frac{1}{i^{25}}\right]^3=\left[i^{18}+\frac{1}{i^{24}i}\right]^3=\left[(i^2)^9+\frac{1}{(i^2)^{12}\cdot i}\right]^3$$

$$=\left[(-1)^9+\frac{1}{(-1)^{12}\cdot i}\right]^3=\left[-1+\frac{1}{i}\right]^3=\left[-1+\frac{1}{i}\times\frac{i}{i}\right]^3$$

$$=\left[-1+\frac{i}{i^2}\right]^3=\left[-1+\frac{i}{-1}\right]^3=\left[-1-i\right]^3$$

$$=(-1)^3-3(-1)^2i+3(-1)i^2-i^3$$

$$=-1-3i-3(-1)-i^2i \quad (\because i^2=-1)$$

$$=-1-3i+3-(-1)\cdot i$$

$$=-1-3i+3+i$$

$$=\boxed{2-2i} \text{ Ans}$$

Q.11: Let $z_1=2-i$, $z_2=-2+i$, find

(i) $Re\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

Sol:- $z_1 z_2=(2-i)(-2+i)=-4+2i+2i-i^2=-4+4i-(-1)$

$$=-4+4i+1=-3+4i$$

$$\therefore \frac{z_1 z_2}{\bar{z}_1}=\frac{-3+4i}{2+i}$$

$$=\frac{-3+4i}{2+i}\times\frac{2-i}{2-i}$$

$$=\frac{-6+3i+8i-4i^2}{2^2-i^2}$$

$$=\frac{-6+11i-4(-1)}{4-(-1)} \quad (\because i^2=-1)$$

$$=\frac{-6+11i+4}{4+1}=\frac{-2+11i}{5}$$

$$\frac{z_1 z_2}{\bar{z}_1} = -\frac{2}{5} + \frac{11}{5}i$$

$$\therefore \quad Re\left(\frac{z_1 z_2}{\bar{z}_1}\right) = \boxed{-\frac{2}{5}} \text{ Ans}$$

(ii) $Im\left(\frac{1}{z_1\bar{z}_1}\right)$

Sol:- $z_1\bar{z}_1 = (2-i)(2+i)$

$$= 2^2 - i^2$$
$$= 4-(-1)$$
$$= 4+1$$
$$= 5$$
$$z_1\bar{z}_1 = 5$$
$$\frac{1}{z_1\bar{z}_1} = \frac{1}{5}$$
$$\frac{1}{z_1\bar{z}_1} = \frac{1}{5} + 0i$$
$$Im\left(\frac{1}{z_1\bar{z}_1}\right) = \boxed{0} \text{ Ans}$$

EXERCISE 1.2

Full

Q.1: If $z_1 = 2+i$ and $z_2 = 1-i$, then verify commutative property w.r.t addition and multiplication.

Sol:- $z_1 + z_2 = 2+i+1-i = 3 \longrightarrow$ (i)

$z_2 + z_1 = 1-i+2+i = 3 \longrightarrow$ (ii)

From (i) and (ii), $z_1 + z_2 = z_2 + z_1$ verified

Commutative property w.r.t multiplication

$z_1z_2 = (2+i)(1-i) = 2-2i+i-i^2 = 2-i-(-1)$

$= 2-i+1 = 3-i \longrightarrow (iii)$

$z_2z_1 = (1-i)(2+i) = 2+i-2i-i^2 = 2-i-(-1)$

$= 2-i+1 = 3-i \longrightarrow (iv)$

From (iii) and (iv), we have

$z_1z_2 = z_2z_1$ ∴ verified

Full Q.2 $z_1 = -1+i$, $z_2 = 3-2i$ and $z_3 = 2+3i$, verify associative property w.r.t addition and multiplication.

Sol:- Associative Property w.r.t addition

$z_1+(z_2+z_3) = -1+i+(3-2i+2+3i)$

$= -1+i+5+i$

$= 4+2i \longrightarrow (i)$

$(z_1+z_2)+z_3 = (-1+i+3-2i)+2+3i$

$= 2-i+2+3i$

$= 4+2i \longrightarrow (ii)$

From (i) and (ii), we have

$z_1+(z_2+z_3) = (z_1+z_2)+z_3$ verified

Associative Property w.r.t multiplication

$z_1(z_2z_3) = (-1+i)[(3-2i)(2+3i)]$

$= (-1+i)[6+9i-4i-6i^2]$

$= (-1+i)[6+5i-6(-1)]$ $(\because i^2=-1)$

$= (-1+i)[6+5i+6]$

$= (-1+i)(12+5i)$
$= -12-5i+12i+5i^2$
$= -12+7i+5(-1) \quad (\because i^2=-1)$
$= -12+7i-5$
$= -17+7i \longrightarrow$ (iii)

$(z_1z_2)z_3 = [(-1+i)(3-2i)](2+3i)$
$= (-3+2i+3i-2i^2)(2+3i)$
$= (-3+5i-2(-1))(2+3i) \quad (\because i^2=-1)$
$= (-3+5i+2)(2+3i)$
$= (-1+5i)(2+3i)$
$= -2-3i+10i+15i^2$
$= -2+7i+15(-1)$
$= -2+7i-15 \quad (\because i^2=-1)$
$= -17+7i \longrightarrow$ (iv)

From (iii) and (iv), we have.
$z_1(z_2z_3) = (z_1z_2)z_3$ Verified

Full **Q.3** $z_1 = \sqrt{3}+\sqrt{2}i$, $z_2 = \sqrt{2}-\sqrt{3}i$ and $z_3 = 2-2i$; verify distributive property of multiplication over addition.

Sol:- Distributive Property of multiplication over addition
$z_1(z_2+z_3) = (\sqrt{3}+\sqrt{2}i)[\sqrt{2}-\sqrt{3}i+2-2i]$
$= (\sqrt{3}+\sqrt{2}i)[2+\sqrt{2}-(2+\sqrt{3})i]$

$= \sqrt{3}(2+\sqrt{2}) - \sqrt{3}(2+\sqrt{3})i + \sqrt{2}(2+\sqrt{2})i - \sqrt{2}(2+\sqrt{3})i^2$

$= \sqrt{3}(2+\sqrt{2}) - \sqrt{3}(2+\sqrt{3})i + \sqrt{2}(2+\sqrt{2})i + \sqrt{2}(2+\sqrt{3})$

$= 2\sqrt{3} + \sqrt{6} - 2\sqrt{3}i - 3i + 2\sqrt{2}i + 2i + 2\sqrt{2} + \sqrt{6}$

$= 2\sqrt{3} + 2\sqrt{2} + 2\sqrt{6} - 2\sqrt{3}i + 2\sqrt{2}i - i$

$= 2\sqrt{3} + 2\sqrt{2} + 2\sqrt{6} - (2\sqrt{3} - 2\sqrt{2} + 1)i \longrightarrow (i)$

$z_1z_2 + z_1z_3 = (\sqrt{3}+\sqrt{2}i)(\sqrt{2}-\sqrt{3}i) + (\sqrt{3}+\sqrt{2}i)(2-2i)$

$= \sqrt{6} - 3i + 2i - \sqrt{6}i^2 + 2\sqrt{3} - 2\sqrt{3}i + 2\sqrt{2}i - 2\sqrt{2}i^2$

$= \sqrt{6} - i + \sqrt{6} + 2\sqrt{3} - 2\sqrt{3}i + 2\sqrt{2}i + 2\sqrt{2}$

$= 2\sqrt{3} + 2\sqrt{2} + 2\sqrt{6} - (2\sqrt{3} - 2\sqrt{2} + 1)i \longrightarrow (ii)$

From (i) and (ii), we have

$z_1(z_2+z_3) = z_1z_2 + z_1z_3$ verified.

Full **Q.4** Find the additive and multiplicative inverses of the following complex numbers.

(i) $5+2i$

Sol:- Let $z = 5+2i$

Additive inverse

$-z = -(5+2i) = \boxed{-5-2i}$ Ans

Multiplicative Inverse

$z^{-1} = \dfrac{1}{z} = \dfrac{1}{5+2i} = \dfrac{1}{5+2i} \times \dfrac{5-2i}{5-2i} = \dfrac{5-2i}{5^2-2^2i^2} = \dfrac{5-2i}{25-4(-1)}$

$= \dfrac{5-2i}{25+4} = \dfrac{5-2i}{29} = \boxed{\dfrac{5}{29} - \dfrac{2}{29}i}$ Ans

(ii) $(7,-9)$

Sol:- Let $z=(7,-9)$

Additive Inverse

$-z=-(7,-9)=(-7,9)$ Ans

Multiplicative Inverse

$z=(7,-9)=7-9i$

$$z^{-1}=\frac{1}{z}=\frac{1}{7-9i}=\frac{1}{7-9i}\times\frac{7+9i}{7+9i}=\frac{7+9i}{7^2-9^2i^2}=\frac{7+9i}{49-81(-1)}$$

$$=\frac{7+9i}{49+81}=\frac{7+9i}{130}=\frac{7}{130}+\frac{9}{130}i=\boxed{\left(\frac{7}{130},\frac{9}{130}\right)}\text{ Ans}$$

Full Q.5 (i) Let $z_1=2+4i$ and $z_2=1-3i$. Verify that $\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$

Sol:- $z_1+z_2=2+4i+1-3i$

$=3+i$

$\Rightarrow \overline{z_1+z_2}=3-i \longrightarrow (i)$

$\overline{z_1}+\overline{z_2}=2-4i+1+3i=3-i \longrightarrow (ii)$

From (i) & (ii)

$\overline{z_1+z_2}=\overline{z_1}+\overline{z_2}$

(ii) Let $z_1=2+3i$ and $z_2=2-3i$. Verify that $\overline{z_1z_2}=\overline{z_1}\,\overline{z_2}$

Sol:- $z_1z_2=(2+3i)(2-3i)=2^2-3^2i^2=4-9(-1)$

$=4+9=13$

$\overline{z_1z_2}=13 \longrightarrow (iii)$

$$\bar{z}_1\bar{z}_2=(2-3i)(2+3i)=2^2-3^2i^2=4-9(-1)$$
$$=4+9=13 \longrightarrow (iv)$$

From (iii) and (iv), we have

$$\overline{z_1z_2}=\bar{z}_1\bar{z}_2 \quad \text{verified.}$$

(iii) If $z_1=-a-3bi$, $z_2=2a-3bi$, then verify that
$$\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\bar{z}_1}{\bar{z}_2}$$

Sol:-
$$\frac{z_1}{z_2}=\frac{-a-3bi}{2a-3bi}$$
$$=\frac{-a-3bi}{2a-3bi}\times\frac{2a+3bi}{2a+3bi}=\frac{-2a^2-3abi-6abi-9b^2i^2}{(2a)^2-3^2b^2i^2}$$
$$=\frac{-2a^2-9abi-9b^2(-1)}{4a^2-9b^2(-1)}=\frac{-2a^2-9abi+9b^2}{4a^2+9b^2}$$
$$=\frac{-2a^2+9b^2}{4a^2+9b^2}-\frac{9abi}{4a^2+9b^2}$$
$$\overline{\left(\frac{z_1}{z_2}\right)}=\frac{-2a^2+9b^2}{4a^2+9b^2}+\frac{9abi}{4a^2+9b^2} \longrightarrow (i)$$

$$\frac{\bar{z}_1}{\bar{z}_2}=\frac{-a+3bi}{2a+3bi}$$
$$=\frac{-a+3bi}{2a+3bi}\times\frac{2a-3bi}{2a-3bi}=\frac{-2a^2+3abi+6abi-9b^2i^2}{(2a)^2-3^2b^2i^2}$$

$$= \frac{-2a^2+9abi-9b^2(-1)}{4a^2-9b^2(-1)} = \frac{-2a^2+9abi+9b^2}{4a^2+9b^2}$$

$$= \frac{-2a^2+9b^2}{4a^2+9b^2} + \frac{9abi}{4a^2+9b^2} \longrightarrow (iv)$$

From (iii) and (iv), we have

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}} \quad \text{verified}$$

feel **Q.6:** Show that for all complex numbers z_1 and z_2

(i) $|z_1z_2| = |z_1||z_2|$

Sol:- Let $z_1 = x_1+iy_1$, $z_2 = x_2+iy_2$ be any two complex numbers.

$$z_1z_2 = (x_1+iy_1)(x_2+iy_2)$$

$$= x_1x_2 + ix_1y_2 + ix_2y_1 + i^2y_1y_2$$

$$= x_1x_2 + i(x_1y_2+x_2y_1) - y_1y_2 \qquad (\because i^2=-1)$$

$$= x_1x_2 - y_1y_2 + i(x_1y_2+x_2y_1)$$

$$|z_1z_2| = \sqrt{(x_1x_2-y_1y_2)^2 + (x_1y_2+x_2y_1)^2}$$

$$= \sqrt{x_1^2x_2^2+y_1^2y_2^2-\cancel{2x_1x_2y_1y_2}+x_1^2y_2^2+x_2^2y_1^2+\cancel{2x_1x_2y_1y_2}}$$

$$= \sqrt{x_1^2x_2^2+y_1^2y_2^2+x_1^2y_2^2+x_2^2y_1^2} \longrightarrow (i)$$

$$|z_1||z_2| = \sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}$$

$$= \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$$

$$= \sqrt{x_1^2x_2^2+y_1^2y_2^2+x_1^2y_2^2+x_2^2y_1^2} \longrightarrow (ii)$$

From (i) and (ii), we have

$$|z_1 z_2| = |z_1||z_2| \quad \text{Proved}$$

(ii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, where $z_2 \neq 0$.

Sol:-

$$\frac{z_1}{z_2} = \frac{x_1+iy_1}{x_2+iy_2}$$

$$= \frac{x_1+iy_1}{x_2+iy_2} \times \frac{x_2-iy_2}{x_2-iy_2} = \frac{x_1x_2 - ix_1y_2 + ix_2y_1 - i^2y_1y_2}{x_2^2 - i^2y_2^2}$$

$$= \frac{x_1x_2 + i(x_2y_1 - x_1y_2) - (-1)y_1y_2}{x_2^2 - (-1)y_2^2}$$

$$= \frac{x_1x_2 + i(x_2y_1 - x_1y_2) + y_1y_2}{x_2^2 + y_2^2}$$

$$= \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + i\,\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}$$

$$\left|\frac{z_1}{z_2}\right| = \sqrt{\left(\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}\right)^2 + \left(\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}\right)^2}$$

$$= \sqrt{\frac{(x_1x_2 + y_1y_2)^2 + (x_2y_1 - x_1y_2)^2}{(x_2^2 + y_2^2)^2}}$$

$$= \sqrt{\frac{x_1^2x_2^2 + y_1^2y_2^2 + \cancel{2x_1x_2y_1y_2} + x_2^2y_1^2 + x_1^2y_2^2 - \cancel{2x_1x_2y_1y_2}}{(x_2^2 + y_2^2)^2}}$$

$$= \sqrt{\frac{x_1^2(x_2^2+y_2^2)+y_1^2(x_2^2+y_2^2)}{(x_2^2+y_2^2)^2}}$$

$$= \sqrt{\frac{(x_1^2+y_1^2)(x_2^2+y_2^2)}{(x_2^2+y_2^2)^2}}$$

$$= \sqrt{\frac{x_1^2+y_1^2}{x_2^2+y_2^2}} \longrightarrow (i)$$

$$\frac{|z_1|}{|z_2|} = \frac{\sqrt{x_1^2+y_1^2}}{\sqrt{x_2^2+y_2^2}} = \sqrt{\frac{x_1^2+y_1^2}{x_2^2+y_2^2}} \longrightarrow (ii)$$

From (i) and (ii), we have

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$ Proved

Q.7 Separate into real and imaginary parts.

(i) $\frac{2+3i}{5-2i}$

Sol: $\frac{2+3i}{5-2i} = \frac{2+3i}{5-2i} \times \frac{5+2i}{5+2i}$

$$= \frac{10+4i+15i+6i^2}{5^2-2^2i^2} = \frac{10+19i+6(-1)}{25-4(-1)}$$

$$= \frac{10+19i-6}{25+4} = \frac{4+19i}{29} = \boxed{\frac{4}{29}+\frac{19}{29}i} \text{ Ans}$$

(ii) $\frac{(1+2i)^2}{1-3i}$

Sol:- $\frac{(1+2i)^2}{1-3i} = \frac{1+2(2i)+4i^2}{1-3i} = \frac{1+4i-4}{1-3i}$

$= \frac{-3+4i}{1-3i} = \frac{-3+4i}{1-3i} \times \frac{1+3i}{1+3i}$

$= \frac{-3-9i+4i+12i^2}{1^2-3^2i^2}$

$= \frac{-3-5i+12(-1)}{1-9(-1)} = \frac{-3-5i-12}{1+9}$

$= \frac{-15-5i}{10} = \frac{-15}{10} - \frac{5}{10}i$

$= \boxed{-\frac{3}{2}-\frac{1}{2}i}$ Ans

(iii) $\frac{1-i}{(1+i)^2}$

Sol:- $\frac{1-i}{(1+i)^2} = \frac{1-i}{1+2i+i^2} = \frac{1-i}{1+2i-1} = \frac{1-i}{2i}$

$= \frac{1-i}{2i} \times \frac{i}{i} = \frac{i-i^2}{2i^2} = \frac{i-(-1)}{2(-1)}$

$= \frac{i+1}{-2} = \boxed{-\frac{1}{2}-\frac{1}{2}i}$ Ans

(iv) $(2a-bi)^{-2}$

Sol:- $(2a-bi)^{-2} = \frac{1}{(2a-bi)^2}$

$= \frac{1}{(2a)^2+(bi)^2-2(2a)(bi)}$

$$= \frac{1}{4a^2+b^2i^2-4abi} = \frac{1}{4a^2-b^2-4abi}$$

$$= \frac{1}{4a^2-b^2-4abi} \times \frac{4a^2-b^2+4abi}{4a^2-b^2+4abi} = \frac{4a^2-b^2+4abi}{(4a^2-b^2)^2-4^2a^2b^2i^2}$$

$$= \frac{4a^2-b^2+4abi}{16a^4+b^4-8a^2b^2+16a^2b^2} \quad (\because i^2=-1)$$

$$= \frac{4a^2-b^2+4abi}{16a^4+b^4+8a^2b^2} = \frac{4a^2-b^2+4abi}{(4a^2+b^2)^2}$$

$$= \frac{4a^2-b^2}{(4a^2+b^2)^2} + \frac{4ab}{(4a^2+b^2)^2}i$$

(v) $\left(\frac{3+4i}{4-3i}\right)^{-2}$

Sol:- $\left(\frac{3+4i}{4-3i}\right)^{-2} = \left(\frac{4-3i}{3+4i}\right)^2$

$$= \left(\frac{4-3i}{3+4i} \times \frac{3-4i}{3-4i}\right)^2 = \left(\frac{12-16i-9i+12i^2}{3^2-4^2i^2}\right)^2$$

$$= \left(\frac{12-25i+12(-1)}{9-16(-1)}\right)^2 = \left(\frac{12-25i-12}{9+16}\right)^2$$

$$= \left(\frac{-25i}{25}\right)^2 = (-i)^2 = i^2 = -1$$

(vi) $\left(\frac{4-5i}{2+3i}\right)^2$

Sol:- $\left(\frac{4-5i}{2+3i}\right)^2 = \left(\frac{4-5i}{2+3i} \times \frac{2-3i}{2-3i}\right)^2 = \left(\frac{8-12i-10i+15i^2}{2^2-3^2i^2}\right)^2$

$$= \left(\frac{8-22i+15(-1)}{4-9(-1)}\right)^2 = \left(\frac{8-22i-15}{4+9}\right)^2 = \left(\frac{-7-22i}{13}\right)^2$$

$$= \left(-\frac{7+22i}{13}\right)^2 = \frac{(7+22i)^2}{169} = \frac{49+484i^2+308i}{169}$$

$$= \frac{49-484+308i}{169} = \frac{-435+308i}{169}$$

$$= \boxed{-\frac{435}{169} + \frac{308}{169}i} \quad \underline{Ans}$$

Q.8: Show that

(i) $z+\bar{z} = 2Re(z)$

Sol:- Let $z = x+iy$

$\Rightarrow \bar{z} = x-iy$

$z+\bar{z} = x+\cancel{iy}+x-\cancel{iy} = 2x = 2Re(z)$

i.e $z+\bar{z} = 2Re(z)$ Proved

(ii) $z-\bar{z} = 2i\,Im(z)$

Sol:- $z-\bar{z} = x+iy-(x-iy) = x+iy-x+iy = 2iy = 2i\,Im(z)$

i.e $z-\bar{z} = 2i\,Im(z)$ Proved

(iii) $z\bar{z} = [Re(z)]^2+[Im(z)]^2$

Sol:- $z\bar{z} = (x+iy)(x-iy)$

$= x^2-i^2y^2 = x^2+y^2 = [Re(z)]^2+[Im(z)]^2$

i.e $z\bar{z} = [Re(z)]^2+[Im(z)]^2$ Proved

(iv) $z = \bar{z} \Rightarrow z$ is real

Sol:- $z = \bar{z} \Rightarrow x+iy = x-iy$

$\Rightarrow iy = -iy \Rightarrow iy+iy = 0 \Rightarrow 2iy = 0 \Rightarrow y = 0$

$\Rightarrow Im(z)=0$

i.e $z=\bar{z} \Rightarrow Im(z)=0$ Proved

(v) $\bar{z}=-z$ if and only if z is pure imaginary.

Sol:- $\bar{z}=-z \Leftrightarrow x-iy=-(x+iy)$

$\Leftrightarrow x-iy=-x-iy$

$\Leftrightarrow x=-x$

$\Leftrightarrow x+x=0$

$\Leftrightarrow 2x=0$

$\Leftrightarrow x=0$

$\Leftrightarrow z$ is pure imaginary. Proved

i.e $\bar{z}=-z \Leftrightarrow z$ is pure imaginary

Q.9: If $z=3+2i$ then verify that

(i) $-|z| \le Re(z) \le |z|$.

Sol:- $z=3+2i$

$\Rightarrow |z|=\sqrt{3^2+2^2}=\sqrt{9+4}=\sqrt{13}=3.6$

$Re(z)=3$

$\because -\sqrt{13} \le 3 \le \sqrt{13} \Rightarrow -|z| \le Re(z) \le |z|$ verified

(ii) $-|z| \le Im(z) \le |z|$

Sol:- $Im(z)=2$

$\because -\sqrt{13} \le 2 \le \sqrt{13}$

$\Rightarrow -|z| \le Im(z) \le |z|$

verified

EXERCISE 1.3

Q.1 Solve the simultaneous linear equations with complex coefficients.

(i) $z-4w=3i \longrightarrow (1)$

$2z+3w=11-5i \longrightarrow (2)$

Sol:- $(1)\times 2 - (2)\times 1 \Rightarrow$

$$\cancel{2z}-8w=6i$$
$$\cancel{\pm 2z}\pm 3w=\pm 11\mp 5i$$
$$-11w=-11+11i$$
$$\boxed{w=1-i} \quad (\div \text{ by } -11)$$

$(1)\Rightarrow$ $z-4(1-i)=3i$

$$z-4+4i=3i$$
$$z=4-4i+3i$$
$$\boxed{z=4-i}$$

Thus : $\boxed{z=4-i,\ w=1-i}$ Ans.

(ii) $z+w=3i \longrightarrow (1)$

$2z+3w=2 \longrightarrow (2)$

Sol:- $(1)\times 2 - (2)\times 1 \Rightarrow$

$$\cancel{2z}+2w=6i$$
$$\cancel{\pm 2z}\pm 3w=\pm 2$$
$$-w=6i-2$$
$$\Rightarrow \boxed{w=2-6i}$$

$(1)\Rightarrow z+2-6i=3i \Rightarrow z=-2+6i+3i \Rightarrow \boxed{z=-2+9i}$

Thus $\boxed{z=-2+9i,\ w=2-6i}$ Ans

(iii) $3z + (2+i)w = 11 - i \longrightarrow (1)$

$(2-i)z - w = -1 + i \longrightarrow (2)$

Sol:- ① × 1 + ② × (2+i) ⇒

$$3z + \cancel{(2+i)w} = 11 - i$$
$$(2+i)(2-i)z - \cancel{(2+i)w} = (2+i)(-1+i)$$

$$3z + (2+i)(2-i)z = 11 - i + (2+i)(-1+i)$$

$$3z + (4 - i^2)z = 11 - i - 2 + 2i - i + i^2$$

$$3z + (4 - (-1))z = 11 - i - 2 + 2i - i - 1$$

$$3z + 5z = 8$$

$$8z = 8 \Rightarrow \boxed{z = 1}$$

① ⇒ $3(1) + (2+i)w = 11 - i$

$$3 + (2+i)w = 11 - i$$

$$(2+i)w = 8 - i$$

$$w = \frac{8-i}{2+i}$$

$$w = \frac{8-i}{2+i} \times \frac{2-i}{2-i} = \frac{16 - 8i - 2i + i^2}{2^2 - i^2}$$

$$= \frac{16 - 10i - 1}{4 - (-1)} = \frac{15 - 10i}{5} = 3 - 2i$$

$$\boxed{w = 3 - 2i}$$

Thus $\boxed{z = 1,\ w = 3 - 2i}$

Q.2: Factorize the polynomials $P(z)$ into linear factors.

(i) $P(z) = z^2 + 6z + 20$

Sol:- $P(z) = z^3+6z+20$

$\because P(-2) = (-2)^3+6(-2)+20 = -8-12+20 = 0$

$\Rightarrow z=-2$ is a root.

For other roots, by Synthetic Division

$$\begin{array}{c|cccc} -2 & 1 & 0 & 6 & 20 \\ & 0 & -2 & 4 & -20 \\ \hline & 1 & -2 & 10 & 0 \end{array}$$

$\therefore P(z) = (z+2)(z^2-2z+10)$

$= (z+2)\left[z^2-2z+1+9\right]$

$= (z+2)\left[(z-1)^2+3^2\right]$

$= (z+2)\left[(z-1)^2-(-3^2)\right]$

$= (z+2)\left[(z-1)^2-(3i)^2\right]$

$= (z+2)\left[(z-1+3i)(z-1-3i)\right]$

$P(-2) = \boxed{(z+2)(z-1+3i)(z-1-3i)}$ Ans

(ii) $P(z) = 3z^2+7$

Sol:- $P(z) = 3z^2+7 = 3z^2-(-7) = (\sqrt{3}z)^2-(\sqrt{7}i)^2$

$= (\sqrt{3}z+\sqrt{7}i)(\sqrt{3}z-\sqrt{7}i)$ Ans

(iii) $P(z) = z^2+4$

Sol:- $P(z) = z^2+4 = z^2-(-4) = z^2-(4i^2) = z^2-(2i)^2$

$= \boxed{(z+2i)(z-2i)}$ Ans

(iv) $P(z)=z^3-2z^2+z-2$

Sol:- $P(z)=z^3-2z^2+z-2$

$=z^2(z-2)+1(z-2)$

$=(z-2)(z^2+1)$

$=(z-2)[z^2-(-1)]$

$=(z-2)[z^2-i^2]$

$=\boxed{(z-2)(z+i)(z-i)}$ Ans

Q.3: Show that each $z_1=-1+i$ and $z_2=-1-i$ satisfies the equation $z^2+2z+2=0$

Sol:- $z^2+2z+2=0$

For $z_1=-1+i$

$(-1+i)^2+2(-1+i)+2=0$

$(-1)^2+i^2+2(-1)(i)-2+2i+2=0$

$1-1-2i-2+2i+2=0$

$0=0$ (true)

$\therefore z_1=-1+i$ satisfies the given equation.

For $z_2=-1-i$

$(-1-i)^2+2(-1+i)+2=0$

$(-1)^2+i^2-2(-1)(-i)-2+2i+2=0$

$1-1-2i-2+2i+2=0$

$0=0$ (true)

$\therefore z_2=-1-i$ satisfies the given equation.

Q.4: Determine whether $1+2i$ is a solution of $z^2-2z+5=0$.

Sol:- $z^2-2z+5=0$

Put $z=1+2i$

$(1+2i)^2-2(1+2i)+5=0$

$1^2+(2i)^2+2(1)(2i)-2-4i+5=0$

$1-4+4i-2-4i+5=0$

$0 = 0$ (true)

$\Rightarrow 1+2i$ is a solution of the given equation. Ans

Q.5: Find all solutions to the following equations.

(i) $z^2+z+3=0$

Sol: $z^2+z+3=0$

Here $a=1, b=1, c=3$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2-4(1)(3)}}{2 \cdot 1}$$

$$= \frac{-1 \pm \sqrt{1-12}}{2}$$

$$= \frac{-1 \pm \sqrt{-11}}{2}$$

$$= \frac{-1 \pm i\sqrt{11}}{2}$$

$$= \boxed{\frac{-1}{2} \pm \frac{1}{2} i\sqrt{11}}$$ Ans

(ii) $z^2-1=iz$

Sol:- $z^2-iz-1=0$

Here $a=1, b=-i, c=-1$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-i) \pm \sqrt{(-i)^2-4(1)(-1)}}{2 \cdot 1}$$

$$= \frac{i \pm \sqrt{-1+4}}{2}$$

$$= \frac{i \pm \sqrt{3}}{2}$$

$$= \pm\frac{\sqrt{3}}{2} + \frac{i}{2}$$

$$= \boxed{\frac{\sqrt{3}}{2} + \frac{1}{2}i, -\frac{\sqrt{3}}{2} + \frac{1}{2}i}$$ Ans

(iii) $z^2-2z+i=0$

Sol: $z^2-2z+i=0$

Here $a=1, b=-2, c=i$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(i)}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{4-4i}}{2}$$

$$= \frac{2 \pm 2\sqrt{1-i}}{2}$$

$$= \boxed{1 \pm \sqrt{1-i}}$$ Ans

(iv) $z^2+4=0$

Sol:- $z^2+4=0$

$\Rightarrow z^2 = -4$

$\Rightarrow z = \pm\sqrt{-4}$

$= \boxed{\pm 2i}$ Ans

Q.6: Find the solutions to the following equations.

(i) $z^4+z^2+1=0$

Sol:- $z^4+z^2+1=0$

$z^4+1+z^2=0$

$z^4+1+2z^2-2z^2+z^2=0$

$(z^2+1)^2-z^2=0$

$(z^2+1+z)(z^2+1-z)=0$

$(z^2+z+1)(z^2-z+1)=0$

$z^2+z+1=0$

Here $a=1, b=1, c=1$

$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2\cdot1}$

$=\frac{-1\pm\sqrt{-3}}{2}$

$=-\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$

$z^2-z+1=0$

Here $a=1, b=-1, c=1$

$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2\cdot1}$

$=\frac{1\pm\sqrt{-3}}{2}$

$=\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$

$\therefore \boxed{z=-\frac{1}{2}\pm\frac{i\sqrt{3}}{2}, \frac{1}{2}\pm\frac{i\sqrt{3}}{2}}$ Ans

(ii) $z^3=-8$

Sol:- $z^3=-8 \Rightarrow z^3+8=0$

$z^3+2^3=0 \Rightarrow (z+2)(z^2-2z+4)=0$

$z+2=0$

$z=-2$

$z^2-2z+4=0$

$z=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2\cdot1}$

$=\frac{2\pm\sqrt{-12}}{2}$

$=\frac{2\pm2\sqrt{-3}}{2}$

$=1\pm i\sqrt{3}$

$\boxed{z=-2, 1\pm i\sqrt{3}}$ Ans

(iii) $(z-1)^3=-1$

Sol:- $(z-1)^3=-1$

$(z-1)^3+1=0$

$(z-1)^3+1^3=0$

$(z-1+1)[(z-1)^2-(z-1)+1]=0$

$z(z^2-2z+1-z+1+1)=0$

$z(z^2-3z+3)=0$

$z=0$

$z^2-3z+3=0$

$z=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(3)}}{2\cdot1}$

$=\frac{3\pm\sqrt{9-12}}{2}$

$=\frac{3\pm\sqrt{-3}}{2}=\frac{3}{2}\pm\frac{i\sqrt{3}}{2}$

$\therefore \boxed{z=0, \frac{3}{2}\pm\frac{i\sqrt{3}}{2}}$ Ans

(iv) $z^3=1$

Sol:- $z^3-1=0$

$(z-1)(z^2+z+1)=0$

$z-1=0$

$z=1$

$z^2+z+1=0$

$z=\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2}$

$=\frac{-1\pm\sqrt{-3}}{2}$

$=\frac{-1\pm i\sqrt{3}}{2}$

$=-\frac{1}{2}\pm\frac{i\sqrt{3}}{2}$

$\therefore \boxed{z=1, -\frac{1}{2}\pm\frac{i\sqrt{3}}{2}}$ Ans

REVIEW EXERCISE 1

Q.1: Choose the correct option.

(i) $\left(\frac{2i}{1+i}\right)^2$

(a) i (b) $2i$ (c) $1-i$ (d) $1-2i$

Sol:- $\left(\frac{2i}{1+i}\right)^2 = \left(\frac{2i}{1+i}\times\frac{1-i}{1-i}\right)^2 = \left(\frac{2i-2i^2}{1-i^2}\right)^2 = \left(\frac{2i+2}{1+1}\right)^2 = (1+i)^2$

$= 1+2i+i^2 = 1+2i-1 = 2i$

$\therefore$ the correct option is (b) Ans

(ii) Divide $\frac{5+2i}{4-3i}$

(a) $-\frac{7}{25}+\frac{26}{25}i$ (b) $\frac{5}{4}-\frac{2}{3}i$ (c) $\frac{14}{25}+\frac{23}{25}i$ (d) $\frac{26}{7}+\frac{23}{7}i$

Sol:- $\frac{5+2i}{4-3i} = \frac{5+2i}{4-3i}\times\frac{4+3i}{4+3i} = \frac{20+15i+8i+6i^2}{4^2-3^2i^2} = \frac{20+23i-6}{16+9}$

$= \frac{14+23i}{25} = \frac{14}{25}+\frac{23}{25}i$

$\therefore$ The correct option is (c) Ans

(iii) $i^{57}+\frac{1}{i^{25}}$ when simplified has the value

(a) 0 (b) $2i$ (c) $-2i$ (d) 2

Sol:- $i^{57}+\frac{1}{i^{25}} = i^{56}\cdot i+\frac{1}{i^{24}i} = (i^2)^{28}\cdot i+\frac{1}{(i^2)^{12}i} = (-1)^{28}\cdot i+\frac{1}{(-1)^{12}i}$

$= i+\frac{1}{i} = i+\frac{i}{i^2} = i-i = 0$

$\therefore$ the correct option is (a) Ans

(iv) $1+i^2+i^4+i^6+\cdots+i^{2n}$ is

(a) Positive (b) negative (c) 0 (d) cannot be determined

Sol:- For $n=1$, we have $1+i^2 = 1-1 = 0$

For $n=2$, we have $1+i^2+i^4 = 1-1+1 = 1$

so the answer depends on n.

$\therefore$ the correct option is **(d)**. Ans

(v) If $z = x+iy$ and $\left|\frac{z-5i}{z+5i}\right| = 1$ then z lies on

(a) X-axis (b) Y-axis (c) line $y=5$ (d) None of these

Sol:- $\left|\frac{z-5i}{z+5i}\right| = 1 \Rightarrow \frac{|z-5i|}{|z+5i|} = 1 \Rightarrow |z-5i| = |z+5i|$

$\Rightarrow |x+iy-5i| = |x+iy+5i| \Rightarrow |x+(y-5)i| = |x+(y+5)i|$

$\Rightarrow \sqrt{x^2+(y-5)^2} = \sqrt{x^2+(y+5)^2} \Rightarrow x^2+(y-5)^2 = x^2+(y+5)^2$

$\Rightarrow (y-5)^2 = (y+5)^2 \Rightarrow y=0$

$\therefore$ the correct option is **(c)**. Ans

(vi) The multiplicative inverse of $z = 3-2i$, is

(a) $\frac{1}{3}(3+2i)$ (b) $\frac{1}{13}(3+2i)$ (c) $\frac{1}{13}(3-2i)$ (d) $\frac{1}{4}(3-2i)$

Sol:- $z = 3-2i$

$z^{-1} = \frac{1}{z} = \frac{1}{3-2i} = \frac{1}{3-2i} \times \frac{3+2i}{3+2i} = \frac{3+2i}{9+4} = \frac{1}{13}(3+2i)$

$\therefore$ the correct option is **(b)**. Ans

(vii) If $(x+iy)(2-3i) = 4+i$, then

(a) $x = -14/13,\ y = 5/13$ (b) $x = 5/13,\ y = 14/13$

(c) $x = 14/13,\ y = 5/13$ (d) $x = 5/13,\ y = -14/13$

Sol: $(x+iy)(2-3i) = 4+i$

$x+iy = \frac{4+i}{2-3i}$

$= \frac{4+i}{2-3i} \times \frac{2+3i}{2+3i} = \frac{8+12i+2i-3}{4+9} = \frac{5}{13} + \frac{14}{13}i$

$\Rightarrow x = 5/13,\ y = 14/13$. So the correct option is **(b)**. Ans

Q.2: Show that $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0 \;\forall n \in N$.

Sol:- $i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n(1 + i + i^2 + i^3)$

$= i^n(1 + i - 1 - i) = i^n(0) = 0$ Proved

Q.3: Express the following complex numbers in the form $x+iy$.

(i) $(1+3i)+(5+7i)$

Sol:- $(1+3i)+(5+7i) = 1+5+(3+7)i = \boxed{6+10i}$ Ans

(ii) $(1+3i)-(5+7i)$

Sol:- $(1+3i)-(5+7i) = 1+3i-5-7i = \boxed{-4-4i}$ Ans

(iii) $(1+3i)(5+7i)$

Sol:- $(1+3i)(5+7i) = 5+7i+15i+21i^2 = 5+22i-21$

$= \boxed{-16+22i}$ Ans

(iv) $\dfrac{1+3i}{5+7i}$

Sol:- $\dfrac{1+3i}{5+7i} = \dfrac{1+3i}{5+7i} \times \dfrac{5-7i}{5-7i} = \dfrac{5-7i+15i-21i^2}{5^2-7^2i^2}$

$= \dfrac{5+8i-21(-1)}{25-49(-1)} = \dfrac{5+8i+21}{25+49} = \dfrac{26+8i}{74}$

$= \dfrac{2(13+4i)}{74} = \dfrac{13+4i}{37} = \boxed{\dfrac{13}{37}+\dfrac{4}{37}i}$ Ans

Q.4: If $z_1 = 2-i$, $z_2 = 1+i$, find $\left|\dfrac{z_1+z_2+1}{z_1-z_2+1}\right|$

Sol:- $\left|\dfrac{z_1+z_2+1}{z_1-z_2+1}\right| = \left|\dfrac{2-i+1+i+1}{2-i-(1+i)+1}\right| = \left|\dfrac{4}{2-2i}\right| = \left|\dfrac{2}{1-i}\right| = \left|\dfrac{2}{1-i} \times \dfrac{1+i}{1+i}\right|$

$= \left|\dfrac{2(1+i)}{1-i^2}\right| = \left|\dfrac{2(1+i)}{1+1}\right| = |1+i| = \sqrt{1^2+1^2} = \boxed{\sqrt{2}}$ Ans

Q.5: Find the modulus of $\dfrac{1+i}{1-i} - \dfrac{1-i}{1+i}$

Sol:- $\frac{1+i}{1-i}-\frac{1-i}{1+i}=\frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}=\frac{1+i^2+2i-(1+i^2-2i)}{1-i^2}=\frac{1-1+2i-(1-1-2i)}{1-(-1)}=\frac{2i+2i}{2}$

$=\frac{4i}{2}=2i \Rightarrow \frac{1+i}{1-i}-\frac{1-i}{1+i}=2i$

$\therefore \left|\frac{1+i}{1-i}-\frac{1-i}{1+i}\right|=\sqrt{0^2+2^2}=\sqrt{4}=\boxed{2}$ Ans

Q.6: Find the conjugate of $\frac{1}{3+4i}$

Sol:- Let $z=\frac{1}{3+4i}$

$=\frac{1}{3+4i}\times\frac{3-4i}{3-4i}=\frac{3-4i}{9+16}=\frac{3-4i}{25}=\frac{3}{25}-\frac{4}{25}i$

$\therefore \bar{z}=\boxed{\frac{3}{25}+\frac{4}{25}i}$ Ans

Q.7: Find the multiplicative inverse of $z=\frac{3i+2}{3-2i}$

Sol:- $z=\frac{3i+2}{3-2i}$

$z^{-1}=\frac{1}{z}=\frac{3-2i}{3i+2}=\frac{3-2i}{2+3i}=\frac{3-2i}{2+3i}\times\frac{2-3i}{2-3i}=\frac{6-9i-4i-6}{4+9}$

$=\frac{-13i}{13}=\boxed{-i}$ Ans

Q.8: Solve the quadratic equation $z+\frac{2}{z}=2$

Sol:- $z+\frac{2}{z}=2$

$\times$ by z

$z^2+2=2z$

$\Rightarrow z^2-2z+2=0$

$z=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2\cdot 1}$

$=\frac{2\pm\sqrt{4-8}}{2}=\frac{2\pm\sqrt{-4}}{2}=\frac{2\pm 2i}{2}$

$=\frac{2(1\pm i)}{2}$

$=\boxed{1\pm i}$ Ans