

CHAPTER 12

Angle on a Segment of a circle

Theorem: 12.1

The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

Given:

A circle with centre O, AB is a minor arc whose central angle is $\angle AOB$. P is any point on the major arc ADB. $\angle APB$ is the angle subtended by the arc at P.

To Prove:

$$m\angle AOB = 2m\angle APB$$

Figure:

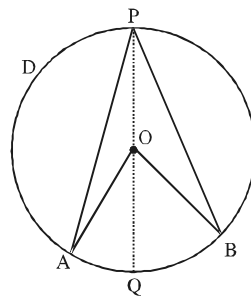


Fig. 1

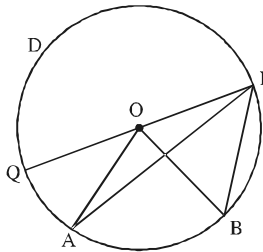


Fig. 2

We will prove the above theorem for the following cases according to the position of the point P on the major arc.

Case 1: When P is an arbitrary point of the major arc (General case) Figure 1.

Case 2:

When PQ is an extremity of minor arc AB properly as shown in the figure 2.

Construction:

Draw $\overline{PO}$ to meet the circumference at Q (for both cases).

Proof:

Statements	Reasons
Case1	
In the $\square OAP$	
$m\overline{OA} = m\overline{OP}$	Same circle having same radii.
$\therefore \triangle OAP$ is an isosceles triangle.	Definition of an isosceles triangle
$m\angle OAP = m\angle OPA$	$\therefore$ Triangles having two sides equal, the angle opposite to them are also equal

Corollary:

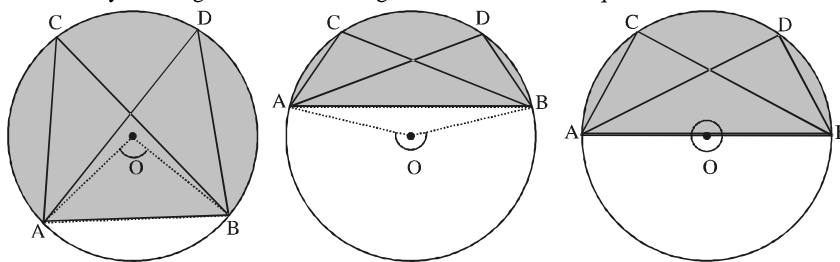
The angle which an arc of a circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference.

Theorem: 12.2

Any two angles in the same segment of a circle are equal

Figure:

Any two angles in the same segment of a circle are equal.



Given:

A circle with centre O. $\angle ACB$ and $\angle ADB$ are any two angles in the same segment ACDB (Shaded) of the circle.

Statements	Reasons
Also $\angle AOQ = m\angle OAP$	The measure of exterior angle of a triangle is equal to the sum of measure of opposite interior angles.
$\therefore m\angle AOQ = 2m\angle OPA$ (1)	$\therefore m\angle OPA = m\angle OPA$ (Proved above)
Similarly $m\angle BOQ = m\angle OPB$ (ii) $\therefore m\angle AOQ + m\angle BOQ = 2m\angle OPA + 2m\angle OPB$	Adding equation (1) and (ii)
OR $m\angle AOB = 2[m\angle OPA + m\angle OPB]$	Postulates of addition of angles
OR $m\angle AOB = 2m\angle APB$	

Case2:

All the statement and reason are same up to obtained the equation (1) from (ii) we get

$$m\angle BOQ - m\angle AOQ =$$

$$2m\angle OPB - 2m\angle OPA$$

$$m\angle BOQ - m\angle AOQ =$$

$$2[m\angle OPB - m\angle OPA]$$

$$\therefore m\angle AOB = 2m\angle APB$$

$$\therefore 2m\angle ACB = 2m\angle ADB$$

$$\therefore m\angle ACB = m\angle ADB$$

Postulate of subtraction of angle

The R.H.S of (i) and (ii) are equal to the same quantity

Halves of equal quantities are again equal

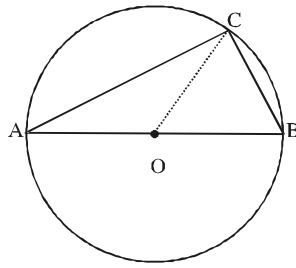
Theorem: 12.3.i

The angle in a semi circle is a right angle.

Given:

A circle with centre O, $\overline{AB}$ is a diameter of the circle and $\angle ACB$ is any angle in the semi-circle.

Figure:



To Prove:

$$m\angle ACB = m\angle ADB$$

Construction:

Join O to A and B respectively.

Proof:

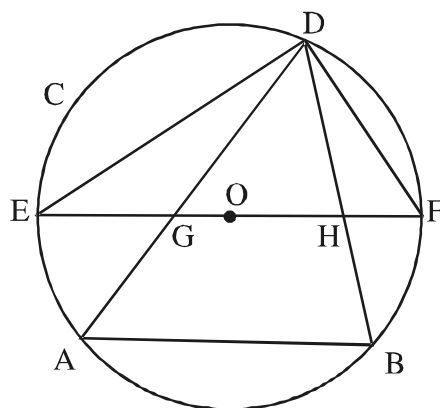
Statements	Reasons
In all the three figure we have $m\angle AOB = 2m\angle ADB$ (ii) (i)	The angle which an Arc of a circle subtends at the centre is twice that which it subtend at any point on the remaining part of the circumference.
Similarly $m\angle AOB = 2m\angle ACB$ (ii)	The angle which an Arc of the circle subtends at the centre is twice that which it subtends at any point on the remaining part of the circumference
OR $m\angle ACB + m\angle ACB = 180^\circ$ $\Rightarrow 2m\angle ACB = 180^\circ$	Adding the two equal numbers

$\therefore \angle ACB = \frac{180^\circ}{2}$ $\therefore \angle ACB = 90^\circ$ OR $\angle ACB$ is a right angle	Dividing "2" to both side of the equation
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Theorem: 12.3 ii

"The angle in a segment greater than a semi circle is less than a right angle."

Figure:



To Prove: $\angle ACB$ is a right angle OR $m\angle ACB \cong 90^\circ$

Conclusion:

Draw $\overline{OC}$, join O and C

Proof:

Statements	Reasons
In $\triangle OAC$	Same circle have same radii
$\overline{OA} = \overline{OC}$	
$\therefore \triangle OAC$ is an isosceles triangle	Isosceles triangle:.
and	If two sides of a triangle are

$$m\angle OAC = m\angle OCA$$

Similarly in $\triangle OCB$

$$m\angle OCB = m\angle OBC$$

$$\therefore m\angle OAC = m\angle OCB$$

$$\therefore m\angle OAC + m\angle OBC =$$

$$m\angle OCA + m\angle OCB$$

$$m\angle OCA + m\angle OCB = m\angle ACB$$

$$\text{But } m\angle AOB = 2m\angle ADB \text{ (ii)}$$

equal the angle which are opposite to them are also equal.

Circle having same radii

Adding (i) and (ii)

$$m\angle OCA + m\angle OCB = m\angle ACB$$

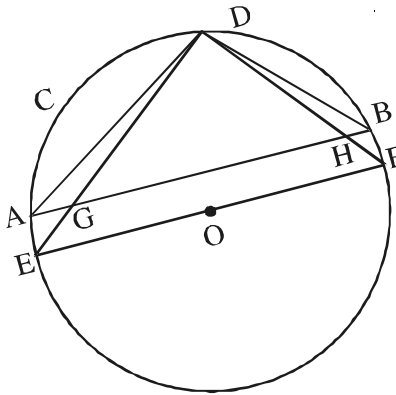
$\therefore$ The sum of three angles is equal to 180°

$$\therefore m\angle OAC + m\angle OBC = m\angle ACB$$

Theorem: 12.3.iii

The angle in a segment less than a semi circle is greater than a right angle

Figure:



Given:

A circle with centre O, $\overline{AB}$ is a chord and $\overline{EF}$ is the diameter parallel to $\overline{AB}$ such that $\overline{AB}$ lies above the diameter

$\overline{EF}$. The segment ACBD is less than the semi circular region.

$\angle ADB$ is any angle in the segment.

To Prove:

$\angle ABD$ is greater than a right angle i.e. $m\angle ADB > 90^\circ$,

Construction:

Join point D to E and F $\angle EDF$ is in the semi circle.

Proof:

Statements	Reasons
$m\angle EDF = 90^\circ$	Angle is always right angle inside the semi circle

Given:

A circle with centre O, $\overline{AB}$ is a chord and $\overline{EF}$ is the diameter parallel to $\overline{AB}$ such that $\overline{AB}$ lies below the diameter $\overline{EF}$. Thus the segment AECDFB is greater than the semi circle region.

$\angle ADB$ is any angle in the segment.

To Prove:

$\angle ADB$ is less than a right angle i.e $m\angle ADB < 90^\circ$

Construction:

Join point D with E and F such that $\angle EDF$ is in the semi circle.

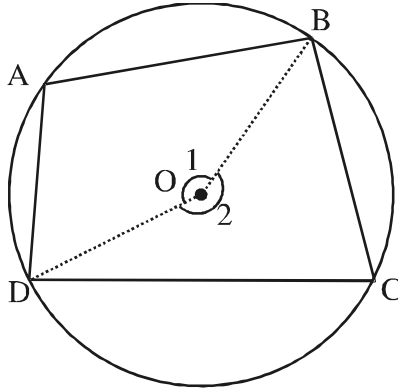
Proof:

Statements	Reasons
$m\angle EDE = 90^\circ$	The measurement of an angle in a semi circle is a right angle.
$\therefore m\angle EDG + m\angle GDH + m\angle HDF = 90^\circ$	$\therefore$
$\therefore m\angle GDH = 90^\circ - [m\angle EDG + m\angle HDF]$	$m\angle BCD + m\angle BAD = 180^\circ$ and $m\angle ABC + m\angle ADC = 180^\circ$
$\Rightarrow m\angle GDH < 90^\circ$	
or $m\angle ADB < 90^\circ$	
Or $\therefore m\angle GDH = 90^\circ$	
$\therefore m\angle AGD + m\angle GDH + m\angle HDB =$	$\therefore \angle GDH$ and $\angle ADB$ are same
OR $m\angle ADG + 90^\circ + m\angle HDB$	
$\Rightarrow m\angle ADB = 90^\circ + m\angle ADG + m\angle HDB$	Addition of equal quantities on both sides of an equation
$\Rightarrow m\angle ADB > 90^\circ$	Addition of angle postulates
	So, The required is proved

Theorem: 12.4

The angles lying in opposite direction of any quadrilateral inscribed in a circle are supplementary.

Figure:



Given:

A circle with centre O. A quadrilateral of vertices ABCD inscribed in the given circle having centre "O".

To Prove:

$$m\angle ADC + m\angle BAD = 180^\circ$$

$$m\angle BCD + m\angle BAD = 180^\circ$$

and $m\angle ABC + m\angle ADC = 180^\circ$

Construction:

Join point B and D with centre of circle "O".

Proof:

Statement	Reasons
$m\angle 1 = 2m\angle BCD - A$	The angle which an arc of a circle subtend at the centre is twice that which it subtends at any point on the remaining part of the circumference.

$m\angle 2 = 2m\angle BAD - B$	The angle which an arc of a circle subtend at the centre is twice that which it subtends at any point on the remaining part of the circumference.
$m\angle 1 + m\angle 2 = 2m\angle BCD + 2m\angle BAD \quad (1)$	Adding the equation A and B
But $m\angle 1 + m\angle 2 = 360^\circ$	In circumference of a circle the measurement of angle is 360° OR four right angles at the centre
$\therefore (1)$ becomes $2[m\angle BCD + m\angle BAD] = 360^\circ$ $\therefore m\angle BCD + m\angle BAD = \frac{360^\circ}{2}$	Taking R.H.S of equation (1)
OR $m\angle BCD + m\angle BAD = 180^\circ$	Dividing both sides by "2"
$m\angle BCD + m\angle BAD$ are supplementary angles. Similarly we can prove that $m\angle ABC$ and $m\angle ADC$ are supplementary	Sum of two angles whose measurement is equal to 180°, the angles are supplementary.

Exercise 12.2

MCQ's

I. Write the correct option against the given box 1.

1. The central of a minor arc of circle is 40° the angle subtended by the corresponding major arc is of measure a
a. 20° b. 80°
c. 60° d. 120°
2. The measurement of an angle in a semi-circle is ____ c
a. acute angle b. obtuse angle
c. right angle d. interior angle
3. Any two angles in the same _____ c
a. chord b. sector
c. segment d. area
4. The angle in a major segment of a circle is b
a. less than 45° b. less than 90°
c. greater than 90° but less than 135°
d. greater than 135°
5. Any cyclic parallelogram is a: b
a. square b. rectangle
c. rhombus d. none of these

II. Fill in the blanks.

1. An acute central angle will always intercept a **minor** arc.
2. An inscribed right angle must intercept a **semi-circle**
3. Opposite angles of an inscribed quadrilateral must be **supplementary**
4. Vertical central angle will intercept arcs of the **semi-measure**
5. The angle in a segment less than a semi-circle is **greater** than a right angle.

III. If the statement is correct, write True otherwise write False

1. A central angle in two concentric circles will intercept arcs of the same measure in each circle. ☐ F
2. A parallelogram can be inscribed in a circle. ☐ F
3. An acute central angle will always intercept a minor arc. ☐ T
4. Opposite angles of an inscribed quadrilateral are congruent. ☐ F
5. An obtuse inscribed angle intercepts a major arc. ☐ F