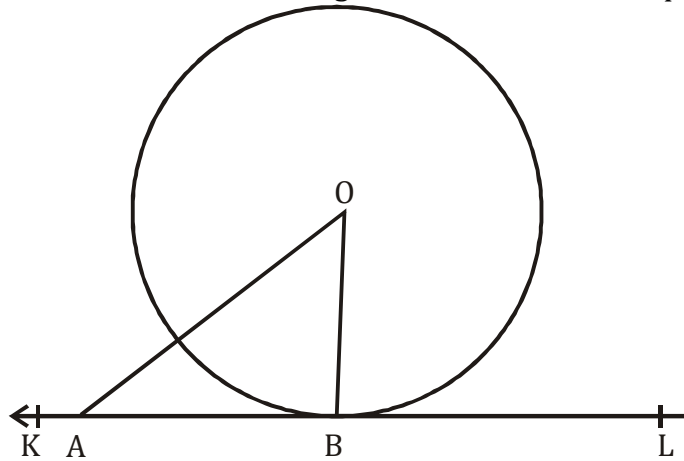


CHAPTER 10

Tangent to a Circle

Theorem: 10.1

If a line is perpendicular to a radial segment of a circle at its outer end and then it is a tangent to the circle at that point.



Given:

A circle with center O. $\overline{OA}$ is radial segment A is its outer end and $\overline{KL} \perp \overline{OA}$.

To Prove:

$\overline{KL}$ is tangent to the circle i.e. $\overline{KL}$ touches the circle only at a point A.

Construction:

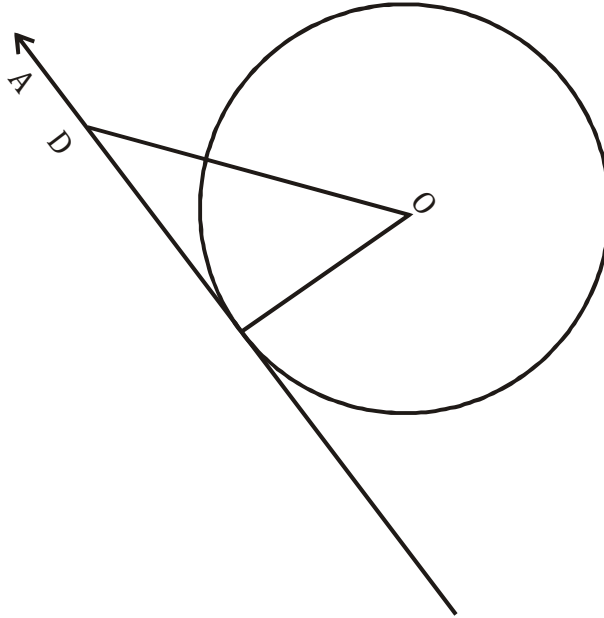
Take a point B other than A at $\overline{KL}$. Join B to O.

Proof:

Statement	Reasons
In triangle AOB $m\angle AOB = 90^\circ$ $\therefore m\overline{OB} > m\overline{OA}$ But $\overline{OA}$ is a radial segment $\therefore$ B is in the exterior of the circle. Similarly it can be proved that any point other than A lying on $\overline{KL}$ lies in the exterior. If the circle i.e $\overline{KL}$ intersects the circle at only one point. OR $\overline{KL}$ is tangent to the circle.	Given Opposite sides of a greater angle By definition of a tangent.

Theorem:10.2

The tangent to the circle and the radial segment joining the point of contact and the center, are perpendicular to each other.

**Given:**

A circle with center O. $\overline{AB}$ is tangent and C is the point of contact.

To Prove:

$$\overline{OC} \perp \overline{AB} \text{ or } m\angle OCA = 90^\circ$$

Construction:

Take a point D on $\overline{AB}$ other than C. join D to O.

Proof:

D is in the exterior of the circle $\therefore \overline{OD} > \overline{OC}$. i.e. $\overline{OC}$ is the shortest distance similarly it can be proved that the line

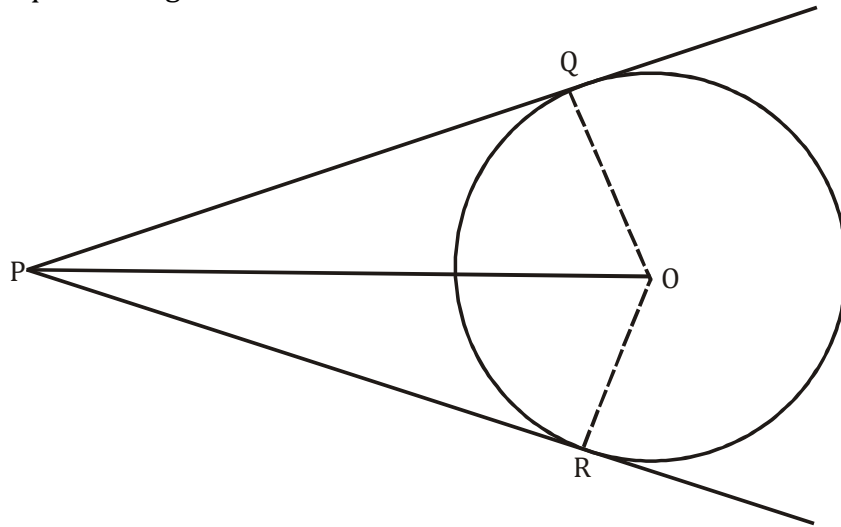
segments joining points on $\overline{AB}$, other than C to O, are greater in measure than $m\overline{OC}$.

Hence $\overline{OC}$ is the shortest line segment or $\overline{OC} \perp \overline{AB}$.

OR $m\angle OCA = 90^\circ$.

Theorem: 10.3

Tangents drawn to a circle from a point outside it are equal in length.



Given:

A circle with center O. P is outside the circle. $\overline{PQ}$ and $\overline{PR}$ are two tangents drawn from P to the circle.

To Prove:

$$\overline{PQ} = \overline{PR}$$

Construction:

Join O to Q and R.

Proof:

Statements	Reasons
Q and R are points of contact $\overline{OQ} \perp \overline{OQ}$ And $\overline{OR} \perp \overline{PR}$ In $\triangle OPQ \leftrightarrow \triangle OPR$ $m\angle OPQ \cong m\angle OPR$ $\overline{OQ} \cong \overline{OR}$ $\overline{OP} \cong \overline{OP}$ $\therefore \triangle OPQ \cong \triangle OPR$ Hence $\overline{PQ} \cong \overline{PR}$	 Tangent is perpendicular to the radial segment at the point of contact. Tangent is perpendicular to the radial segment at the point of contact. Right angles Radial segment of the same circle common SAS postulates Corresponding sides of congruent triangle.

Exercise 10.1

Q1. Find the length of a tangent drawn to a circle of radius 4cm from a point at a distance of 10cm from the center of the circle.

Diagram

Solution: From figure OQ is the hypotenuse of $\triangle OPQ$

$$\begin{aligned}\text{So } (OQ)^2 &= (QP)^2 + (PO)^2 \\ (\text{Hyp})^2 &= (\text{Base})^2 + (\text{Perp})^2 \\ (OQ)^2 &= (PO)^2 + (QP)^2\end{aligned}$$

$$\text{Since } (10)^2 = (4)^2 + (QP)^2$$

$$100 = 16 + (QP)^2$$

$$84 = (QP)^2$$

Taking square root on both sides

$$\sqrt{84} = \sqrt{(QP)^2}$$

$$\sqrt{21 \times 2 \times 2} = QP$$

$$\boxed{QP = 2\sqrt{21}}$$

Q2. In the adjoining figure $\overline{mOP} = a$, $\overline{mPQ} = q$, find $\overline{mOQ}$

Diagram

Solution: From figure OQ is base of triangle $\triangle OQP$

According to Pythagoras theorem

$$(Qr)^2 = (OP)^2 + (OQ)^2$$

$$(Qr)^2 - (OP)^2 = (OQ)^2$$

$$\boxed{(OQ)^2 = \sqrt{a^2 - r^2}}$$

Q3. Prove that the tangents passing through the end points of diameter of a circle are parallel to each other.

Diameter

Solution: Given

Circle O with centre O and diameter AB tangents lines CA and DB tangents at the point A and respectively.

Prove: Lines CA and DB are parallel

Statement	Reason
AB is diameter of circle	Given
AB is perpendicular to CA at A $AB \perp CA$	AC is tangent to the circle
And AB is perpendicular to DB at B $AB \perp DB$	BD is tangent to the circle
$CA \parallel DB$	Lines perpendicular to the same line are parallel to each other

Q4. The four sides of a quadrilateral ABCD are tangents of a circle. Prove that $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{AD}$

Diagram

We have to prove that $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{AD}$

In a quadrilateral circumscribed around a circle the sum of two opposite sides equal to the sum of two remaining opposite

Statement	Reason
ABCD is quadrilateral	Given
AB, BC, CD	Tangent line of a circle are
$\angle B$ and $\angle C$	bisector angles is also tangent
DA	line to the circles

So $m\overline{AB} + m\overline{CD} = m\overline{BC} + m\overline{DA}$ are equal

Q5. A chord $\overline{AC}$ of a circle is produced to P. From a tangent $\overline{PB}$ to a circle a circle is drawn. Prove that

$$m\angle PCB = m\angle ABP$$

Solution: The angle of a tangent and chord is equal to the inscribed angle on the opposite side of the chord

Diagram

Statement	Reason
$m\angle PAC = x$	Given

OA perpendicular to AP $m\angle PAO = 90^\circ$ $m\angle CAO = 90^\circ - x$ $m\angle OCA = 90^\circ - x$ $\angle AOC = \frac{1}{2}(180^\circ - \angle CAO - \angle OCA)$ $\angle AOC = 2x$ Thus $m\angle PCB = m\angle ABP$	Tangent-radius-property Right angle Complementary angles Isosceles triangle T Triangle interior angles sum
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Q6. In the following figure find a
Solution: Diagram make from book

We know that OA is radius of circle
 According to Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$$

$$(a)^2 = (3)^2 + (3)^2$$

$$(a)^2 = 9 + 9 = 18$$

Taking square root on both sides

$$\sqrt{(a)^2} = \sqrt{3 \times 3 \times 2}$$

$$a = 3\sqrt{2}$$

Now to find $a = BO - CO$

$$a = 3\sqrt{2} - 3 \quad \text{Answer}$$

ii. Solution from the figure we see that: To solve triangle of $\triangle OAC$.

According to Pythagoras theorem

$$(AO)^2 = (OC)^2 + (AC)^2$$

$$(AC)^2 = (OC)^2 - (AO)^2$$

$$(AC)^2 = (4)^2 - (2)^2$$

$$(AC)^2 = 16 - 4 = 12$$

$$AC = 2\sqrt{3}$$

Now to find the value of 'a' we have to solve the triangle $\triangle ACB$

$$(AB)^2 = (AC)^2 + (CB)^2$$

$$(a)^2 = (2\sqrt{3})^2 + (6)^2$$

$$(a)^2 = 4 \times 3 + 36$$

$$(a)^2 = 48$$

Taking square root on both sides

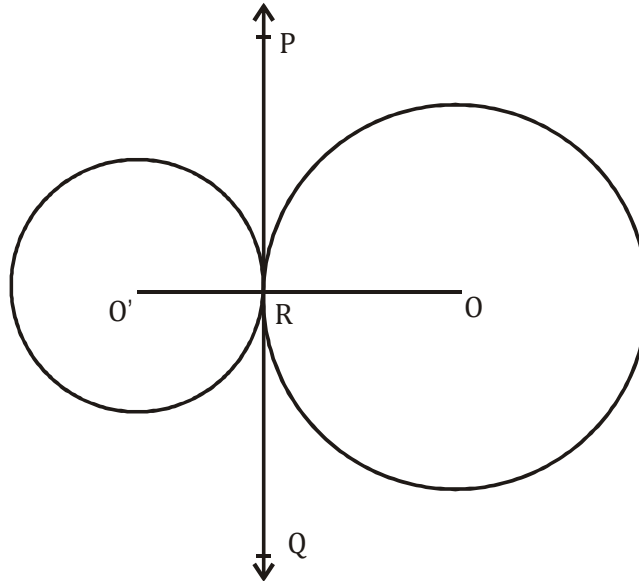
$$\sqrt{(a)^2} = \sqrt{48}$$

$$a = \sqrt{16 \times 3}$$

$$\boxed{a = 4\sqrt{3}}$$

Theorem: 10.4

If two circles touch externally, the distance between their centers is equal to sum of their radii.



Given:

Two circles with center O and O' touch each other. Externally at R. $\overline{OR}$ and $\overline{O'R}$ are the radial segments of these circles.

To Prove:

$$m\overline{OO'} = m\overline{OR} + m\overline{O'R}$$

Construction:

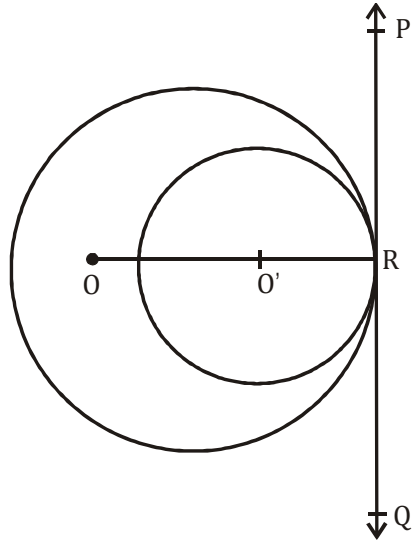
Draw a common tangent $\overline{PQ}$ to both the circles at point R.

Proof:

Statements	Reasons
$m\angle PRO = 90^0$ (1)	$\overline{PQ}$ is tangent at point R.
Similarly $m\angle PRO' = 90^0$ (2)	$\overline{PQ}$ is tangent at point R
OR	
$m\angle PRO + m\angle PRO' = 90^0 + 90^0$ $= 180^0$	Addition of angles postulates.
OR $\overline{ORO'}$ is a line	
OR $m\overline{OO'} = m\overline{OR} + m\overline{O'R}$	Supplementary angles postulates

Theorem: 10.5

If two circles touch internally, the distance between their centers is equal to the difference of their radii.



Given:

Two circles with centers O and O' touch each other internally at point R. $\overline{OR}$ and $\overline{O'R}$ are the radial segments of these circles.

To Prove:

$$m\overline{OO'} = m\overline{OR} - m\overline{O'R}$$

Construction:

Draw a common tangent to the circles at R.

Proof:

Statements	Reasons
$m\angle PRO = 90^0$	$\overline{PQ}$ is tangent to the circle at R.
Similarly $m\angle PRO' = 90^0$	$\overline{PQ}$ is tangent to the circle at R.
$\therefore m\angle PRO = m\angle PRO' = 90^0$	
OR $m\overline{OO'} + m\overline{O'R} = m\overline{OR}$	

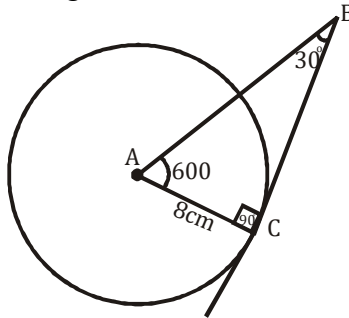
$$\text{OR } m\overline{OO'} = m\overline{OR} - m\overline{O'R}$$

$\overline{OR}$ and $\overline{O'R}$ are the common arms of the same right angle

Exercise 10.2

Q1. If the radius of a circle is 8cm. tangents drawn from the external point P. make an angle of 60° . Find the distance between the center of the circle and the point P.

Solution: Radius of a circle is 8cm and tangent drawn from the external point make angle of 60° .



$$\text{So } \tan 60^\circ = \frac{\text{Perp}}{\text{Base}}$$

$$\tan 60^\circ = \frac{BC}{AC} = \frac{BC}{8}$$

$$\sqrt{3} = \frac{BC}{8} \Rightarrow \boxed{BC = 8\sqrt{3}}$$

According to Pythagoras theorem

$$(\text{Hyp})^2 = (\text{Base})^2 + (\text{Perp})^2$$

$$(\text{Hyp})^2 = (8)^2 + (8\sqrt{3})^2$$

$$(\text{Hyp})^2 = 64 + 192$$

$$(\text{AB})^2 = 256$$

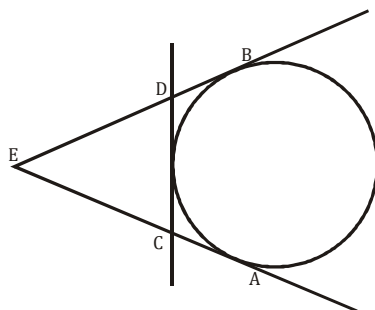
Taking square on both sides

$$(AB)^2 = 16$$

Thus required distance from the centre of circle to external point P.

Q3.

Given: Tangents to a circle are drawn at the ends of a diameter AB another



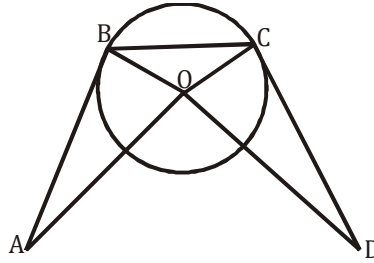
Construction: Draw another tangent which cuts these line at point C and D.

To Prove: $m\overline{CD} = m\overline{AC} + m\overline{BD}$

Statement	Reason
A and B are end points of diameter	Given
$AE \cong DE$	Tangent line is same at diameter
$\triangle ACD$	Is Isosceles triangle
$CE \cong DE$	Distance is same from one tangent to another tangent.
$AC \cong BD$	
Thus	
$m\overline{CD} = m\overline{AC} + m\overline{BD}$	

Q4. Two tangents $\overline{OA}$ and $\overline{OB}$ to a circle are at right angles to one another. $\overline{AC}$ any cord of the circle and $\overline{BD}$ is drawn perpendicular to it. Prove that $m\overline{BD} = m\overline{CD}$.

Solution:



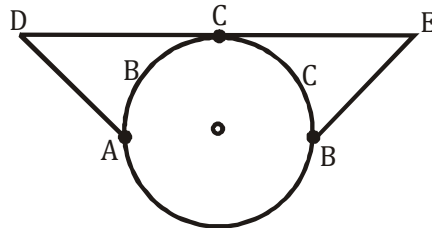
Proof:

Statement	Reason
$\angle BOA = \angle COD$	Vertically opposite angle
$OB = OC$	Radius of the circle
$\angle OBA = \angle OCD = 90^\circ$	Tangent perpendicular to radius
$\triangle ABO \cong \triangle DCO$	A.S.A
$AO = DO$	Corresponding sides of congruent triangles are equal
So	
$m \overline{BD} \cong m \overline{CD}$	

Q6. If $\overline{AD}$, $\overline{DE}$ and $\overline{BE}$ are tangents to a circle as shown.

Prove that $\overline{AD} + \overline{BE} = \overline{DE}$

Solution:



Given: Tangents to a circle are drawn at the ends of a circumference AB.

Construction: Draw an other tangent line at point C, which cuts tangent line at point E and D.

To Prove: Sum of two tangents line is equal to DE tangent line.

Statement	Reason
AD	Tangent line (Given)
BE	Tangent line (Given)
$AD \cong BE$	Tangent line is always same
$DE \cong AD$	from the end point of

$$\begin{aligned} CE &\cong EB \\ DE &= \overline{AD} + \overline{BE} \end{aligned}$$

circumference.

Exercise 10.3

MCQ's

I. Write the correct option against the given box 1.

1. A line which is perpendicular to a radical segment of a circle at its outer end (lying on the circle) is called a

a. secant
b. tangent b

c. chord
d. diameter
2. The line which meets the circle at one point is d

a. secant
b. diameter

c. chord
d. tangent
3. From a point at a distance of 5cm from the centre of a circle of radius 3 cm tangents are drawn to the circle. The length of each tangent will be c

a. 3 cm
b. 5 cm

c. 4 cm
d. 6 cm

II. Fill in the blanks.

1. If a line is a tangent to the circle, then their point of intersection is called the point of **contact**
2. In a circle, a tangent and a diameter that intersect are **perpendicular**
3. A line that intersects a circle at exactly one point is called a **tangent**

4. Two **tangents** can be drawn to a circle from a point outside the circle

III. If statement is correct, write True otherwise write False.

1. Two different lines may be tangent to a circle at the same point. ☐ F
2. A tangent to a circle may contain the centre of the circle ☐ F
3. If two circles are internally tangent, a radius of one may be a diameter of the other. ☐ F