

CHAPTER 01

Quadratic Equations

1. Introduction:

1.1 Linear equations:

An equation of the form $ax + b = 0$ is linear equation in the variable "x" because x has the exponent (Degree) 1.s

For example:

$$\begin{array}{ll} \text{i.} & 2x + 3 = 0 \quad \text{ii.} \quad 7x = 5 \\ \text{iii.} & \sqrt{2}x + 15 = 0 \quad \text{iv.} \quad \frac{3}{2}x + 7 = 0 \end{array}$$

are Linear equations.

Quadratic Equation:

An equation of the form $ax^2 + bx + c = 0$

Where a, b, and c are real numbers and $a \neq 0$ is called a quadratic equation. A quadratic equation is also known as a second degree equation.

For example the following equations are quadratic equations in the variable x.

$$\begin{array}{ll} 1. & 1 \cdot x^2 - 1 \cdot x + 3 = 0 \\ \text{General form} & ax^2 + bx + c = 0 \\ \text{So} & a = 1, \quad b = -1, \quad c = 3 \\ 2. & 1 \cdot x^2 + 1 \cdot x + 1 = 0 \\ \text{So} & a = 1, \quad b = 1, \quad c = 1 \\ 3. & x^2 - 3 = 0 \\ \text{General form} & ax^2 + bx + c = 0 \\ & 1 \cdot x^2 + 0 \cdot x - 3 = 0 \\ & a = 1, \quad b = 0, \quad c = -3 \end{array}$$

4. $x^2 = 0$

General form

$$a \cdot x^2 + bx + c = 0$$

$$1 \cdot x^2 + 0 \cdot x + 0 \cdot c = 0$$

$$a = 1, \quad b = 0, \quad c = 0$$

Note: $ax^2 + bx + c = 0 \quad (1)$

Equation (1) is called the standard (or) general form of a quadratic equation. The coefficient of x^2 must not be zero. Other wise it becomes a linear equation. If $a = 0$ then it is become linear equation.

$$ax^2 + bx + c = 0 \quad \text{if } a = 0$$

$$0 \cdot x^2 + bx + c = 0, \quad 0 \cdot x^2 = 0$$

$$bx + c = 0 \rightarrow (I)$$

So equation is called linear equation.

1.2 ***Solution of Quadratic Equations:***

Recall that all those values of the variable for which the given equation is true are called solutions (OR) roots of the equation and the set of all solutions are called solution set.

For example:

The quadratic equation

$x^2 - 9 = 0$ is true only for $x=3$ and $x = -3$.

$$x^2 - 9 = 0$$

$$x = 3$$

Put

$$(+3)^2 - 9 = 0$$

$$9 - 9 = 0$$

$$0 = 0$$

$$\begin{array}{l}
 \text{Put} \quad \text{L.H.S} = \text{R.H.S} \\
 x = -3, x^2 - 9 = 0 \\
 (-3)^2 - 9 = 0 \\
 +\cancel{9} - \cancel{9} = 0 \\
 0 = 0 \\
 \text{L.H.S} = \text{R.H.S}
 \end{array}$$

Hence $x = 3$ and $x = -3$ are the solutions or roots of the quadratic equation.

$$x^2 - 9 = 0$$

And $\{3, -3\}$ is the solution set.

We may solve a quadratic equation by the following methods.

1. By factorization.
2. By completing square method.
3. By quadratic formula.

1. Solution of a Quadratic Equation by factorization.

To solve the quadratic equation by the method of factorization we proceed as follows.

Step1:

Write the given quadratic equations in its standard form.

$$ax^2 + bx + c = 0$$

Step2:

Factor the left hand side of the equation.

Step3:

Equate each factor to zero separately.

Step4:

Solve the Resultant equations for the values of x.

If $a, b \in \mathbb{R}$ and $a \cdot b = 0$, then either $a = 0$, OR $b = 0$ (OR)
both $a = 0$ and $b = 0$

Example 1:

Solve the equation $x^2 - x - 6 = 0$

Solution:

$$1 \cdot x^2 - x - 6 = 0$$

By Factorization

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow x(x-3) + 2(x-3) = 0$$

$$(x+2)(x-3) = 0$$

$$\Rightarrow x+2=0 \quad \text{OR} \quad x-3=0$$

Subtracting 2 to both sides Adding 3 to both sides

$$x+2-2=0-2$$

$$x-3+3=0+3$$

$$\boxed{x=-2}$$

$$\boxed{x=3}$$

Thus the solutions of the given equation are $-2, 3$ and the solution set is $\{-2, 3\}$.

Example 2:

Solve the equation $2x^2 + 3x - 5 = 0$ by factorization.

Solution: $2x^2 + 3x - 5 = 0$

$$\Rightarrow 2x^2 + 3x - 5 = 0$$

$$\Rightarrow 2x^2 + 5x - 2x - 5 = 0$$

$$\Rightarrow 2x^2 + 5x - 2x - 5 = 0$$

$$\Rightarrow x(2x+5) - 2(x+5) = 0$$

$$\Rightarrow (x-2)(2x+5) = 0$$

$$\Rightarrow x - 2 = 0 \quad \text{or}$$

$$2x + 5 = 0$$

Adding 2 to both
sides

$$x - \cancel{2} + \cancel{2} = 0 + 2 = 2$$

$$x = 2$$

Subtracting 5 from both
sides

$$2x + \cancel{5} - \cancel{5} = 0 - 5$$

Dividing both sides by 2

$$\frac{\cancel{2}x}{\cancel{2}} = -\frac{5}{2}$$

$$x = -\frac{5}{2}$$

Thus the solutions of the given equation are, 1, $-\frac{5}{2}$ and the solution set are $\left\{-\frac{5}{2}, 1\right\}$.

Example 3:

Solve the equation $x^2 - 2x + 1 = 0$ by factorization

Solution: $x^2 - 2x + 1 = 0$

$$\Rightarrow x^2 - 2x + 1 = 0$$

$$\Rightarrow x^2 - x - x + 1 = 0$$

$$\Rightarrow x(x-1) - 1(x-1) = 0$$

$$\Rightarrow (x-1)(x-1) = 0$$

$$x-1 = 0 \quad (OR) \quad x-1 = 0$$

$$\Rightarrow x = 1 \quad (OR) \quad x = 1$$

Thus the solution of the given equation are 1, 1 and solution set is $\{1\}$.

2. Solution of a quadratic equation by completing the square.

Example:

Solve the equation $9x^2 - 12x - 1 = 0$ by completing the square method.

Solution:

Given that quadratic equation

$$9x^2 - 12x - 1 = 0 \dots\dots (I)$$

Dividing both sides by 9, we get

$$\Rightarrow \frac{\cancel{9}x^2}{\cancel{9}} - \frac{12x}{9} - \frac{1}{9} = \frac{0}{9}$$

$$\Rightarrow x^2 - \frac{4}{3}x - \frac{1}{9} = 0$$

Adding $\left(\frac{1}{9}\right)$ to both sides, equation (I)

$$\Rightarrow x^2 - \frac{4}{3}x - \frac{1}{9} + \frac{1}{9} = 0 + \frac{1}{9}$$

$$\Rightarrow x^2 - \frac{4}{3}x = \frac{1}{9}$$

Note: Formula

$$\boxed{a^2 \pm 2ab + b^2 = (a \pm b)^2}$$

$$(x)^2 - 2(x)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 = \left(x - \frac{2}{3}\right)^2$$

Adding $\left(\frac{4}{9}\right)$ to both sides in equation (1), we get

$$\Rightarrow (x)^2 - \frac{4}{3}x^2 + \frac{4}{9} = \frac{1}{9} + \frac{4}{9} L \cdot C \cdot M$$

$$\Rightarrow (x)^2 - 2\left(\frac{2}{3}\right)(x) + \left(\frac{2}{3}\right)^2 = \frac{5}{9}$$

So left hand side

$$\left(x - \frac{2}{3}\right)^2 = \frac{5}{9}$$

Taking square root on both sides

$$\Rightarrow \sqrt{\left(x - \frac{2}{3}\right)^2} = \sqrt{\frac{5}{9}}$$

$$\Rightarrow \left(x - \frac{2}{3}\right) = \pm \frac{\sqrt{5}}{\sqrt{9}}$$

$$\left(x - \frac{2}{3}\right) = \pm \frac{\sqrt{5}}{3}$$

Note:

$$\boxed{\sqrt{9} = 3}$$

$$\sqrt{3 \times 3} = \sqrt{(3)^2} = 3$$

So we can write to this

$$x - \frac{2}{3} = \sqrt{\frac{5}{9}}$$

$$x - \frac{2}{3} = -\frac{\sqrt{5}}{3}$$

Adding $\frac{2}{3}$ to both

Adding $\frac{2}{3}$ to both sides,

sides, we get

we get

$$x - \frac{2}{3} + \frac{2}{3} = \frac{\sqrt{5}}{3} + \frac{2}{3}$$

$$x - \frac{2}{3} + \frac{2}{3} = -\frac{\sqrt{5}}{3} + \frac{2}{3}$$

$$x = \frac{\sqrt{5}}{3} + \frac{2}{3}$$

L.C.M

$$x = \frac{-\sqrt{5} + 2}{3}$$

L.C.M

$$x = \frac{\sqrt{5} + 2}{3}$$

$$x = \frac{2 - \sqrt{5}}{3}$$

$$x = \frac{\sqrt{5} + 2}{3}$$

Thus the solutions of the given equation are

$$x = \frac{2 + \sqrt{5}}{3}, \frac{2 - \sqrt{5}}{3}$$

And the solution set is $\left\{x = \frac{2 + \sqrt{5}}{3}, \frac{2 - \sqrt{5}}{3}\right\}.$

1.3 Quadratic Formula:

Prove that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Proof: As we know that general form of quadratic equation.

$$ax^2 + bx + c = 0$$

Dividing both sides by “a”, we get

$$\begin{aligned} \Rightarrow \quad \frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} &= \frac{0}{a} \\ \Rightarrow \quad x^2 + \frac{b}{a}(x) + \frac{c}{a} &= 0 \end{aligned}$$

Subtracting $\frac{c}{a}$ from both sides, we get

$$\begin{aligned} \Rightarrow \quad x^2 + \left(\frac{b}{a}\right)(x) + \frac{c}{a} - \frac{c}{a} &= 0 - \frac{c}{a} \\ \Rightarrow \quad x^2 + \left(\frac{b}{a}\right)(x) &= -\frac{c}{a} \end{aligned}$$

Now we are using completing square formula.

Adding $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$ to both sides of the equation.

$$\begin{aligned} \Rightarrow \quad x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= \frac{b^2}{4a^2} - \frac{c}{a} \text{ L.C.M} \\ \Rightarrow \quad (x)^2 + 2(x)\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ \text{Complete Square} \\ \Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \end{aligned}$$

Taking square on both sides

$$\Rightarrow \sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \text{Note: } \sqrt{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} \quad \sqrt{4} \cdot \sqrt{a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad 2.a$$

Subtracting $\frac{b}{2a}$ from both sides, we get

$$\Rightarrow x + \frac{b}{2a} - \frac{b}{2a} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \text{ L.C.M}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Is known as quadratic formula

Exercise 1.1

Write the following quadratic equation in the standard form.

Q1. $3x^2 + 4x = 5$

Solution:

$$3x^2 + 4x = 5 \rightarrow (I)$$

As we know that standard form of quadratic equation is

$$ax^2 + bx + c = 0$$

Subtracting both side by 5 in (I), we get

$$3x^2 + 4x - 5 = \cancel{5} - \cancel{5}$$

$$\Rightarrow 3x^2 + 4x - 5 = 0 \rightarrow \text{Standard form}$$

$$\Rightarrow ax^2 + bx + c = 0$$

$$a = 3, b = 4, c = -5$$

Q2. $4x^2 + 5 = 6x$

Solution: $4x^2 + 5 = 6x$

Subtracting $6x$ from both sides, we get

$$\Rightarrow 4x^2 + 5 - 6x = 6x - 6x$$

$$\Rightarrow 4x^2 + 5 - 6x = 0$$

$$4x^2 - 6x + 5 = 0 \text{ Required standard form}$$

$$ax^2 + bx + c = 0$$

$$a = 4, b = -6, c = 5$$

Q3. $2x(x-3) = 3(4x+1)$

Solution: $2x(x-3) = 3(4x+1)$

So $\Rightarrow 2x^2 - 6x = 12x + 3$

Subtracting $12x$ from both sides, we get

$$\Rightarrow 2x^2 - 6x - 12x = \cancel{12x} - \cancel{12x} + 3$$

Subtracting 3 from both sides, we get

$$\Rightarrow 2x^2 - 6x - 12x - 3 = \cancel{3} - \cancel{3}$$

$$2x^2 - 18x - 3 = 0 \text{ (Standard form)}$$

$$ax^2 + bx + c = 0$$

$$a = 2, b = -18, c = 3$$

Q4. $x^2 - 5x = 5x + 6$

Solution: $x^2 - 5x = 5x + 6$

Subtracting $5x$ from both sides, we get

$$\Rightarrow x^2 - 5x - 5x = \cancel{5x} - \cancel{5x} + 6$$

$$x^2 - 10x = 6$$

Subtracting 6 from both sides, we get

$$x^2 - 10x - 6 = \cancel{6} - \cancel{6}$$

$$x^2 - 10x - 6 = 0$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ ax^2 + & bx + & c = 0 \end{matrix}$$

$$x^2 - 10x - 6 = 0$$

Required standard form

$$a = 1, b = -10, c = -6$$

Q5. $4(3x-1) = 2x^2$

Solution:- $4(3x-1) = 2x^2$

Multiplying each term by 4 from left

$$\Rightarrow 4 \times 3x - 4 \times 1 = 2x^2$$

$$12x - 4 = 2x^2$$

Subtracting both sides by $2x^2$, we get

$$12x - 4 - 2x^2 = 2x^2 - 2x^2$$

Rearrange

$$-2x^2 - 4 + 12x = 0$$

$$-2x^2 + 12x - 4 = 0$$

Dividing both sides by -2, we get

$$\frac{-2x^2}{-2} + \frac{12x}{-2} - \frac{-4}{-2} = 0$$

$$x^2 - 6x + 2 = 0$$

$$x^2 - 6x + 2 = 0 \text{ Standard form result}$$

$$ax^2 + bx + c = 0$$

$$a = 1, b = -6, c = 2$$

Q6. $5(x^2+1) = 4+x$

Solution: $5(x^2+1) = 4+x$

$$5x^2 + 5 = 4 + x$$

Subtracting x from both sides, we get

$$\Rightarrow 5x^2 + 5 - x = 4 + x - x$$

$$5x^2 + 5 - x = 4$$

Subtracting 4 from both sides

$$5x^2 - x + 5 - 4 = 4 - 4$$

$$5x^2 - x + 1 = 0 \text{ Standard form}$$

$$ax^2 + bx + c = 0, a = 5, b = -1, c = 1$$

Solve each of the following equations by factorization:

Q7. $x^2 + 5x + 4 = 0$

Solution: $x^2+5x+4=0$

$$\Rightarrow x^2 + 4x + x + 4 = 0$$

$$\Rightarrow x(x+4) + 1(x+4) = 0$$

$$\Rightarrow (x+1)(x+4) = 0$$

$$\Rightarrow x+1=0 \quad \text{OR} \quad x+4=0$$

$$x+1=0$$

Subtracting 1 from both sides

$$x + \cancel{1} - \cancel{1} = 0 - 1$$

$$\boxed{x = -1}$$

$$x+4=0$$

Subtracting 4 from both sides, we get

$$x + \cancel{4} - \cancel{4} = 0 - 4$$

$$\boxed{x = -4}$$

Thus the solution of given equation is $-1, -4$ and the solution set is $\{-1, -4\}$.

Q8. $x^2 - 7x + 12 = 0$

Solution: $x^2 - 7x + 12 = 0$

By factorization method

$$\Rightarrow x^2 - 4x - 3x + 12 = 0$$

$$\Rightarrow x(x-4) - 3(x-4) = 0$$

$$\Rightarrow (x-3)(x-4) = 0$$

$$x-3=0$$

Adding 3 to both sides, we get

$$x - \cancel{3} + \cancel{3} = 0 + 3$$

$$\boxed{x = 3}$$

$$x-4=0$$

Adding 4 to both sides, we get

$$x - \cancel{4} + \cancel{4} = 0 + 4$$

$$\boxed{x = 4}$$

Thus the solution of the given equation is 3, 4 and the solution set is $\{3, 4\}$.

Q9. $x^2 + 3x - 10 = 0$

Solve the given quadratic equation by factorization method

Solution: $x^2 + 3x - 10 = 0$

By factorization method

$$\Rightarrow x^2 + 5x - 2x - 10 = 0$$

$$\Rightarrow x(x + 5) - 2(x - 5) = 0$$

$$\Rightarrow x - 2 = 0 \quad \text{OR} \quad x - 5 = 0$$

$$x - 2 = 0$$

Adding 2 to both sides, we get

$$x - \cancel{2} + \cancel{2} = 0 + 2$$

$$\boxed{x = 2}$$

$$x - 5 = 0$$

Adding 5 to both sides, we get

$$x - \cancel{5} + \cancel{5} = 0 + 5$$

$$\boxed{x = 5}$$

Thus the solution of the given quadratic equation is 2, 5 and the solution set is $\{2, 5\}$.

Q.10 $6x^2 - 13x + 5 = 0$

Solution: $6x^2 - 13x + 5 = 0$

Solve this quadratic equation we use factorization method.

$$6x^2 - 13x + 5 = 0$$

By factorization method

$$\Rightarrow 6x^2 - 10x - 3x + 5 = 0$$

$$\begin{aligned}
&\Rightarrow 2x(3x-5)-1(3x-5)=0 \\
&\Rightarrow (2x-1)(3x-5)=0 \\
&\Rightarrow (2x-1)=0 \quad (3x-5)=0 \\
&\quad 2x-1=0 \quad 3x-5=0 \\
&\text{Adding both 1 to both sides, we get} \quad \text{Adding 5 to both sides, we get} \\
&2x-\cancel{1}+\cancel{1}=0+1 \quad 3x-\cancel{5}+\cancel{5}=0+5 \\
&2x=1 \quad 3x=5 \\
&\text{Dividing both sides by 2} \quad \text{Dividing both sides by 3} \\
&\frac{\cancel{2}x}{\cancel{2}}=\frac{1}{2} \quad \frac{\cancel{3}x}{\cancel{3}}=\frac{5}{3} \\
&\boxed{x=\frac{1}{2}} \quad \boxed{x=\frac{5}{3}}
\end{aligned}$$

Thus the solution of the given equation is $\frac{1}{2}, \frac{5}{3}$ and the solution set is $\left\{\frac{1}{2}, \frac{5}{3}\right\}$.

Q.11 $3(x^2-1)=4(x+1)$

Solution:

By Using factorization method 1st of all write the given quadratic equation in standard form.

$$3(x^2-1)=4(x+1)$$

$$3x^2-3=4x+4$$

Subtracting "4x" and 4 from both sides

$$\Rightarrow 3x^2-3-4x-4=0$$

$$\Rightarrow 3x^2-4x-7=0$$

$$3x^2-4x-7=0$$

By factorization method

$$\begin{aligned}
&\Rightarrow 3x^2 - 4x - 7 = 0 \\
&\Rightarrow 3x^2 - 7x + 3x - 7 = 0 \\
&\Rightarrow x(3x - 7) + 1(3x - 7) = 0 \\
&\Rightarrow (x + 1)(3x - 7) = 0 \\
&\Rightarrow x + 1 = 0 \quad \text{OR} \quad (3x - 7) = 0 \\
&x + 1 = 0 \quad \text{Adding 7 to both sides} \\
&\text{Subtracting 1 from both sides} \quad 3x - \cancel{7} + \cancel{7} = 0 + 7 \\
&x + \cancel{7} - \cancel{7} = 0 - 1 \quad 3x = 7 \\
&x = -1 \quad \text{Dividing both sides by 3} \\
&\quad \quad \quad \frac{\cancel{3}x}{\cancel{3}} = \frac{7}{3}
\end{aligned}$$

Thus the solution of given equation is $-1, \frac{7}{3}$ and

solution set is $\left\{-1, \frac{7}{3}\right\}$.

Q.12 $x(3x - 5) = (x - 6)(x - 7)$

Solution: 1st we can write the given quadratic equation by standard form.

$$\begin{aligned}
&\Rightarrow x(3x - 5) = (x - 6)(x - 7) \\
&\Rightarrow 3x^2 - 5x = x^2 - 13x + 42
\end{aligned}$$

Subtracting x^2 from both sides

$$\Rightarrow 3x^2 - 5x - x^2 = x^2 - x^2 - 13x + 42$$

Subtracting 42 from both sides

$$\begin{aligned}
&\Rightarrow 3x^2 - x^2 - 5x - 42 = -13x + 42 - 42 \\
&\Rightarrow 2x^2 - 5x - 42 = -13x
\end{aligned}$$

Adding 13x to both sides

$$\Rightarrow 2x^2 - 5x + 13x - 42 = -1\cancel{x} + 1\cancel{x}$$

$$\Rightarrow 2x^2 + 8x - 42 = 0$$

Dividing both sides by 2

$$\Rightarrow \frac{2x^2}{2} + \frac{8x}{2} - \frac{42}{2} = \frac{0}{2}$$

$$\Rightarrow x^2 + 4x - 21 = 0$$

$$\Rightarrow x^2 + 7x - 3x - 21 = 0$$

$$\Rightarrow x(x+7) - 3(x+7) = 0$$

$$\Rightarrow (x-3)(x+7) = 0$$

$$\Rightarrow (x-3) = 0 \quad \text{OR} \quad (x+7) = 0$$

adding both side 3 subtracting both side 7

$$x - \cancel{3} + \cancel{3} = 0 + 3$$

$$x = 3$$

$$x + \cancel{7} - \cancel{7} = 0 - 7$$

$$x = -7$$

Thus the solution of the given quadratic equation is 3, -7 and the solution set $\{+3, -7\}$.

Q.13 $(x-3)^2 = 4$

Solution: $(x-3)^2 = 4$

Expand left hand side by $(a-b)^2$ formula.

$$\Rightarrow (x)^2 + (3)^2 - 2(x)(3) = 4$$

$$\Rightarrow x^2 + 9 - 6x = 4$$

$$\Rightarrow x^2 - 6x + 9 = 4$$

$$\Rightarrow x^2 - 6x + 9 - 4 = \cancel{4} - \cancel{4}$$

$$\Rightarrow x^2 - 6x + 5 = 0$$

Now using factorization method

$$\Rightarrow x^2 - 6x + 5 = 0$$

$$\Rightarrow x^2 - 5x - x + 5 = 0$$

$$\Rightarrow x(x-5) - 1(x-5) = 0$$

$$\Rightarrow (x-1)(x-5) = 0$$

$(x-1) = 0$	$(x-5) = 0$
$x - 1 = 0$	$x - 5 = 0$
adding both sides 1	adding both sides 5
$x - \cancel{1} + \cancel{1} = 0 + 1$	$x - \cancel{5} + \cancel{5} = 0 + 5$
$x = 1$	$x = 5$

Thus the solution of given quadratic equation is 1, 5 and the solution set is $\{1, 5\}$.

Solve each of the following equation by completing square:

Q14. $x^2 + 6x - 40 = 0$

Solution: $x^2 + 6x - 40 = 0$

Adding both sides by 40, we get

$$\Rightarrow x^2 + 6x - \cancel{40} + \cancel{40} = 0 + 40$$

$$\Rightarrow x^2 + 6x = 40$$

In completing formula

$$\begin{array}{l} (a)^2 + 2(a)(b) + (b)^2 \\ (x)^2 + 2(x)(3) + (3)^2 \\ x^2 + 6x = 40 \end{array}$$

Adding 9 to both sides

$$\Rightarrow x^2 + 6x + 9 = 40 + 9$$

$$\Rightarrow x^2 + 6x + 9 = 49$$

$$\Rightarrow (x)^2 + 2(x)(3) + (3)^2 = 49$$

$$\Rightarrow (x+3)^2 = 49$$

Taking square root on both sides

$$\sqrt{(x+3)^2} = \pm\sqrt{49}$$

$$x + 3 = \pm 7$$

$$\text{So } x + 3 = 7$$

$$\text{So } x + 3 = -7$$

$$x + 3 = 7$$

$$x + 3 = -7$$

subtracting 3 from both sides

subtracting 3 from both sides

$$x + 3 - 3 = 7 - 3$$

$$x + 3 - 3 = -7 - 3$$

$$\boxed{x = 4}$$

$$\boxed{x = -10}$$

Thus the solution of the given equation is 4, -10 and the solution set is {4, -10}.

Q.15 $x^2 + 10x + 11 = 0$

Solution $x^2 + 10x + 11 = 0$

By completing square method

$$x^2 + 10x + 11 = 0$$

Subtracting 11 from both sides, we get

$$\Rightarrow x^2 + 10x + 11 - 11 = 0 - 11$$

$$\Rightarrow x^2 + 10x = -11$$

$$\boxed{\begin{aligned} (a)^2 + 2(a)(b) + (b)^2 &= (a+b)^2 \\ (x)^2 + 2(x)(-5) + (-5)^2 &= (x-5)^2 \end{aligned}}$$

$$\Rightarrow x^2 + 10x = -11$$

Adding 25 or $(5)^2$ to both sides

$$\Rightarrow x^2 + 10x + 25 = -11 + 25$$

$$\Rightarrow \left[(x)^2 + 2(x)(5) + (5)^2 \right] = 14$$

Completing square formula

Taking square root on both side

$$\Rightarrow \sqrt{(x+5)^2} = \pm\sqrt{14}$$

$$\Rightarrow (x+5) = \pm\sqrt{14}$$

$$\Rightarrow (x - 5) = \sqrt{14} \quad (x - 5) = -\sqrt{14}$$

adding 5 to both sides, adding 5 to both sides

$$\Rightarrow x - \cancel{5} + \cancel{5} = \sqrt{14} + 5 \quad x - \cancel{5} + \cancel{5} = -\sqrt{14} + 5$$

$$\boxed{x = -\sqrt{14} + 5}$$

$$\boxed{x = \sqrt{14} + 5}$$

The solution set of the given quadratic equation is

$5 + \sqrt{14}$, $5 - \sqrt{14}$ and solution set is

$$\{5 + \sqrt{14}, 5 - \sqrt{14}\}.$$

Q.16 $4x^2 + 12x = 0$

Solution: $4x^2 + 12x = 0$

By completing square method dividing both side by 4

$$\Rightarrow \frac{\cancel{4}x^2}{\cancel{4}} + \frac{12x}{4} = \frac{0}{4}$$

$$\Rightarrow x^2 + 3x = 0$$

$$\begin{array}{l} (x)^2 + 3(x) + (\quad)^2 \\ (x)^2 + 2(x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 \end{array}$$

Using formula

$$(a + b)^2 = a^2 + b^2 + 2ab$$

So $x^2 + 3x = 0$

Adding $\left(\frac{9}{4}\right)$ or $\left(\frac{3}{2}\right)^2$ to both sides

$$\Rightarrow (x)^2 + 3(x) + \left(\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2$$

$$\Rightarrow \left(x + \frac{3}{2}\right)^2 = \frac{9}{4}$$

Taking square root on both sides

$$\Rightarrow \sqrt{\left(x + \frac{3}{2}\right)^2} = \sqrt{\frac{9}{4}}$$

$$\Rightarrow \left(x + \frac{3}{2}\right) = \sqrt{\left(\frac{3}{2}\right)^2}$$

$$\Rightarrow \left(x + \frac{3}{2}\right) = \pm \left(\frac{3}{2}\right)$$

$$\Rightarrow x + \frac{3}{2} = \frac{3}{2}$$

Subtracting $\left(\frac{3}{2}\right)$ to both sides

$$\Rightarrow x + \frac{\cancel{3}}{\cancel{2}} - \frac{\cancel{3}}{\cancel{2}} = \frac{3}{2} - \frac{3}{2}$$

$$\Rightarrow x = \frac{3}{2} - \frac{3}{2}$$

Taking L.C.M

$$\Rightarrow x = \frac{3 - 3}{2}$$

$$\Rightarrow x = \frac{0}{2} = 0$$

$$\boxed{x = 0}$$

$$x + \frac{3}{2} = -\frac{3}{2}$$

Subtracting $\left(\frac{3}{2}\right)$ from both sides

$$x + \frac{\cancel{3}}{\cancel{2}} - \frac{\cancel{3}}{\cancel{2}} = -\frac{3}{2} - \frac{3}{2}$$

$$x = -\frac{3}{2} - \frac{3}{2} \qquad x = -\frac{\cancel{6}}{\cancel{2}}$$

$$x = \frac{-3 - 3}{2} \qquad \boxed{x = -3}$$

Thus the solution of given equation is 0, 3 and solution set is $\{0, -3\}$.

Q. 17 $5x^2 - 10x - 840 = 0$

Solution: $5x^2 - 10x - 840 = 0$

Adding both sides by 840, we get

$$\Rightarrow 5x^2 - 10x + \cancel{840} + \cancel{840} = 0 + 840$$

$$\Rightarrow 5x^2 - 10x = 840$$

Dividing both sides by 5

$$\Rightarrow \frac{\cancel{5}x^2}{\cancel{5}} - \frac{10x}{5} = \frac{840}{5}$$

$$\Rightarrow x^2 - 2x = 168$$

$$\boxed{\begin{aligned} (a)^2 + 2(a)(b) + (b)^2 &= (a+b)^2 \\ (x)^2 + 2(x)(1) + (1)^2 &= (x+1)^2 \end{aligned}}$$

$$\Rightarrow x^2 - 2x = 168$$

Adding 1 to both sides

$$\Rightarrow x^2 - 2x + 1 = 168 + 1$$

$$\Rightarrow (x - 1)^2 = 169$$

Taking square root on both sides

$$\sqrt{(x - 1)^2} = \sqrt{169}$$

$$\Rightarrow (x - 1) = \sqrt{13}$$

$$\Rightarrow x - 1 = \pm 13$$

$$\Rightarrow \begin{array}{ll} x - 1 = 13 & x - 1 = -13 \end{array}$$

Adding 1 to both sides Adding 1 to both sides

$$\Rightarrow x - \cancel{1} + \cancel{1} = 13 + 1 \quad \Rightarrow \quad x - \cancel{1} + \cancel{1} = -13 + 1$$

$$\boxed{x = 14}$$

$$\boxed{x = -12}$$

Thus the solution of given quadratic equation is 14, -12 and solution set is $\{14, -12\}$.

Q.18 $9x^2 - 6x + \frac{5}{9} = 0$

Solution: $9x^2 - 6x + \frac{5}{9} = 0$

By completing square method

$$\Rightarrow 9x^2 - 6x + \frac{5}{9} - \frac{5}{9} = 0 - \frac{5}{9}$$

Subtracting $\frac{5}{9}$ from both sides

$$\Rightarrow 9x^2 - 6x = -\frac{5}{9}$$

Dividing both sides by 9, we get

$$\Rightarrow \frac{9x^2}{9} - \frac{6x}{9} = -\frac{5}{9 \times 9}$$

$$\Rightarrow x^2 - \left(\frac{2}{3}\right)(x) = -\frac{5}{81} \text{ ----(I)}$$

By completing square method

$$\boxed{\begin{aligned} (a)^2 - 2(a)(b) + (b)^2 &= (a-b)^2 \\ (x)^2 - 2(x)\left(\frac{1}{3}\right) + (1)^2 &= \left(x - \frac{1}{3}\right)^2 \end{aligned}}$$

So adding $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ on both sides of equ (I)

$$\Rightarrow x^2 - \left(\frac{2}{3}\right)(x) + \left(\frac{1}{3}\right)^2 = -\frac{5}{81} + \left(\frac{1}{3}\right)^2$$

$$\Rightarrow \left(x - \frac{1}{3}\right)^2 = -\frac{5}{81} + \frac{1}{9}$$

Taking L.C.M

$$\Rightarrow \left(x - \frac{1}{3}\right)^2 = \frac{-5+9}{81} = \frac{4}{81}$$

$$\Rightarrow \sqrt{\left(x - \frac{1}{3}\right)^2} = \sqrt{\frac{4}{81}}$$

$$\Rightarrow x - \frac{1}{3} = \sqrt{\frac{(2)^2}{(9)^2}}$$

$$\Rightarrow x - \frac{1}{3} = \sqrt{\left(\frac{2}{9}\right)^2}$$

$$\Rightarrow x - \frac{1}{3} = \pm \frac{2}{9}$$

$$\Rightarrow x - \frac{1}{3} = \frac{2}{9}$$

$$\Rightarrow x - \frac{1}{3} = -\frac{2}{9}$$

Adding both sides $\frac{1}{3}$

Adding both sides $\frac{1}{3}$

$$\Rightarrow x - \frac{1}{3} + \frac{1}{3} = \frac{2}{9} + \frac{1}{3}$$

$$\Rightarrow x - \frac{1}{3} + \frac{1}{3} = -\frac{2}{9} + \frac{1}{3}$$

$$\Rightarrow x = \frac{2+3}{9}$$

$$\Rightarrow x = \frac{-2+3}{9}$$

$$\boxed{x = \frac{5}{9}}$$

$$\boxed{x = \frac{1}{9}}$$

Thus the solution set of given quadratic equation is $\left\{\frac{5}{9}, \frac{1}{9}\right\}$

Solve each of the following equation by Quadratic Formula

Q.19 $x^2 - 8x + 15 = 0$

Solution: $x^2 - 8x + 15 = 0$

As we know that standard form $ax^2 + bx + c = 0$

$$a = 1, b = -8, c = 15$$

So Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \dots \dots (I)$$

So, putting the values in equation I

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64-60}}{2}$$

$$x = \frac{8 \pm \sqrt{4}}{2} \text{ or } \frac{8 \pm 2}{2}$$

$$\Rightarrow x = \frac{8+2}{2} \Rightarrow x = \frac{8-2}{2}$$

$$\Rightarrow x = \frac{10}{2} \Rightarrow x = \frac{6}{2}$$

$$\boxed{x = 5}$$

$$\boxed{x = 3}$$

Thus the solution of the given quadratic equation is 5, 3 and solution set is {5, 3}.

Q. 20 $x^2 - 2x - 4 = 0$

Solution: $x^2 - 2x - 4 = 0$

As we know that General or standard form $ax^2 + bx + c = 0$

$$\Rightarrow ax^2 + bx + c = 0$$

$$\Rightarrow x^2 - 2x - 4 = 0$$

so $a = 1$, $b = -2$, $c = -4$

Now using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ so putting the values of a, b, c}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+16}}{2}$$

$$x = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm \sqrt{4 \times 5}}{2}$$

$$x = \frac{2 \pm \sqrt{4} \times \sqrt{5}}{2} = \frac{2 \pm 2\sqrt{5}}{2}$$

$$x = \frac{2 + 2\sqrt{5}}{2}$$

$$x = \frac{2 - 2\sqrt{5}}{2}$$

$$x = \frac{2(1 + \sqrt{5})}{2}$$

$$x = \frac{2(1 - \sqrt{5})}{2}$$

$$\boxed{x = 1 + \sqrt{5}}$$

$$\boxed{x = 1 - \sqrt{5}}$$

$$\boxed{x = \{1 + \sqrt{5}, 1 - \sqrt{5}\}} \text{ Answer}$$

Q.21

$$4x^2 + 3x = 0$$

Solution:

$$4x^2 + 3x = 0$$

As we know that General or standard form $ax^2 + bx + c = 0$

$$ax^2 + bx + c = 0$$

$$4x^2 + 3x - 0.c = 0$$

$$\boxed{a=4}$$

$$\boxed{b=3}$$

$$\boxed{c=0}$$

Using Quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ ----- (I)}$$

Putting the values in equation I

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(0)(4)}}{2(4)}$$

$$x = \frac{-3 \pm \sqrt{9-0}}{8} = \frac{-3 \pm \sqrt{9}}{8} = \frac{-3 \pm 3}{8}$$

$$\begin{array}{ll}
 x = \frac{-\cancel{3} + \cancel{3}}{8} & x = \frac{-3 - 3}{8} \\
 x = \frac{0}{8} & x = \frac{-\cancel{6}}{\cancel{8}} \\
 \boxed{x=0} & \boxed{x = -\frac{3}{4}}
 \end{array}$$

Hence the solution of given equation is $0, -\frac{3}{4}$ and solution set is $\{0, -\frac{3}{4}\}$.

Q.22 $3x(x - 2) + 1 = 0$

Solution: $\Rightarrow 3x(x - 2) + 1 = 0$

$\Rightarrow 3x^2 - 6x + 1 = 0$

As we know the general form of

$\Rightarrow ax^2 + bx + c = 0$

$\Rightarrow 3x^2 - 6x + 1 = 0$

$a = 3, b = -6, c = 1$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ ----- (I)}$$

Putting the values of a, b, c in equation (I)

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{6 \pm \sqrt{36 - 12}}{6} = \frac{6 \pm \sqrt{24}}{6}$$

$$x = \frac{6 \pm \sqrt{6 \times 4}}{6} = \frac{6 \pm 2\sqrt{6}}{6}$$

$$x = \frac{2(3 \pm \sqrt{6})}{6} = \frac{3 \pm \sqrt{6}}{3}$$

$$\boxed{x = \frac{3 + \sqrt{6}}{3}} \quad \text{OR} \quad \boxed{x = \frac{3 - \sqrt{6}}{3}}$$

So, hence the solution of given quadratic equation are $\frac{3 + \sqrt{6}}{3}$, $\frac{3 - \sqrt{6}}{3}$ and the solution set is $\left\{ \frac{3 + \sqrt{6}}{3}, \frac{3 - \sqrt{6}}{3} \right\}$.

Q.23 $6x^2 - 17x + 12 = 0$

Solution: $6x^2 - 17x + 12 = 0$

$$\Rightarrow ax^2 + bx + c = 0 \quad \text{Standard Form}$$

$$a = 6, b = -17, c = 0$$

Using quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Putting the values of a, b, and c.

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(12)(6)}}{2(6)}$$

$$x = \frac{17 \pm \sqrt{289 - 288}}{12}$$

$$x = \frac{17 \pm \sqrt{1}}{12} = \frac{17 \pm 1}{12}$$

$$x = \frac{17 + 1}{12} \quad x = \frac{17 - 1}{12}$$

$$x = \frac{18}{12} \quad x = \frac{16}{12}$$

$$\boxed{x = \frac{3}{2}} \quad \boxed{x = \frac{4}{3}}$$

So the solution set of given equation is $\left\{ \frac{3}{2}, \frac{4}{3} \right\}$.

Q. 24 $(x - 1)(x + 3) = 5(x + 2) - 3$

Solution: $(x - 1)(x + 3) = 5(x + 2) - 3$
 $(x^2 + 3x - x - 3) = (5x + 10) - 3$
 $x^2 + 2x - 3 = 5x + 10 - 3$
 $\Rightarrow x^2 + 2x - 3 = 5x + 7$

Subtracting $5x$ from both sides, we get

$$\Rightarrow x^2 + 2x - 5x - 3 = \cancel{5x} - \cancel{5x} + 7$$

$$\Rightarrow x^2 - 3x - 3 = 7$$

Subtracting 7 from both sides, we get

$$\Rightarrow x^2 - 3x - 3 - 7 = \cancel{7} - \cancel{7}$$

$$\Rightarrow x^2 - 3x - 10 = 0$$

As we now that general or standard form of quadratic equation is

$$ax^2 + bx + c = 0$$

$$a = 1, b = -3, c = -10$$

So using Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ putting the values}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+40}}{2} = \frac{3 \pm \sqrt{49}}{2}$$

$$x = \frac{3 \pm 7}{2}$$

$$x = \frac{3 + 7}{2} \qquad x = \frac{3 - 7}{2}$$

$$x = \frac{10}{2} \qquad x = \frac{-4}{2}$$

$$\boxed{x=5} \qquad \boxed{x=-2}$$

Q. 25 $\frac{x^2}{3} - \frac{x}{2} = \frac{1}{24}$

Solution:

We can write the given Quadratic equation in standard form

$$\frac{x^2}{3} - \frac{x}{12} = \frac{1}{24}$$

Multiplying both side by 24

$$\Rightarrow \frac{x^2}{3} \times 24 - \frac{x}{12} \times 24 = \frac{1}{24} \times 24$$

$$\Rightarrow \frac{24x^2}{3} - \frac{24x}{12} = \frac{24}{24}$$

$$\Rightarrow 8x^2 - 2x - 1 = 0$$

so $a = 8, b = -2, c = -1$

Using Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ putting the values}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(8)(-1)}}{2(8)}$$

$$x = \frac{2 \pm \sqrt{4 + 32}}{16} = \frac{2 \pm \sqrt{36}}{16}$$

$$x = \frac{2 \pm 6}{16}$$

$$x = \frac{2 + 6}{16}$$

$$x = \frac{2 - 6}{16}$$

$$x = \frac{8}{16}$$

$$x = \frac{-4}{16}$$

$$\boxed{x = \frac{1}{2}}$$

$$\boxed{x = -\frac{1}{4}}$$

Solution set of the given Quadratic Equation is $\left\{\frac{1}{2}, -\frac{1}{4}\right\}$.

.....

1.4 Solution of Equations Reducible to Quadratic Form

Type -1

The equation $ax^4 + bx^2 + c = 0$ is not a quadratic equation. However it can be reduced to a quadratic equation. By making substitutions $y = x^2$ and $y^2 = x^4$

Which change the equation in to

$$ay^2 + by + c = 0$$

An equation such as $ax^4 + bx^2 + c = 0$ is said to be quadratic in form if it can be written as

$$ay^2 + by + c = 0, \text{ where } a \neq 0 \text{ and } y = x^2$$

Example

Solve $12x^4 - 11x^2 + 2 = 0$

Solution: $12x^4 - 11x^2 + 2 = 0 \rightarrow (I)$

Now using type I we can reduce the given equation in quadratic equation.

Put $x^2 = y$

$$\Rightarrow 12x^4 - 11x^2 + 2 = 0$$

$$\Rightarrow 12(x^2)^2 - 11(x^2) + 2 = 0$$

Put $x^2 = y$

$$\Rightarrow 12(y)^2 - 11(y) + 2 = 0$$

$$\Rightarrow 12y^2 - 11y + 2 = 0$$

Now by factorization method

$$12y^2 - 11y + 2 = 0$$

$$\Rightarrow 12y^2 - 8y - 3y + 2 = 0$$

$$\Rightarrow 4y(3y - 2) - 1(3y - 2) = 0$$

$\Rightarrow (4y-1)(3y-2)=0$	
$\Rightarrow 4y-1=0$	$\Rightarrow 3y-2=0$
$4y-1=0$	$3y-2=0$
adding 1 to both sides	adding 2 to both sides
$4y - \cancel{1} + \cancel{1} = 1$	$3y - \cancel{2} + \cancel{2} = 0 + 2$
$4y = 1$	$3y = 2$
Dividing both sides by 4	dividing both sides by 3
$y = \frac{1}{4}$	$y = \frac{2}{3}$
Now Replace $y = x^2$	Now Replace $y = x^2$
$x^2 = \frac{1}{4}$	$x^2 = \frac{2}{3}$
Taking square root on both sides	Taking square root on both side
$\sqrt{x^2} = \sqrt{\frac{1}{4}}$	$\sqrt{x^2} = \sqrt{\frac{2}{3}}$
$x = \pm \frac{1}{2}$	$x = \pm \sqrt{\frac{2}{3}}$

Hence the solution set is $\left\{ \sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}, \frac{1}{2}, -\frac{1}{2} \right\}$. Answer

Type-2 Equation of the form

$$ap(x) + \frac{b}{p(x)} = c \dots \dots (I),$$

Example 1:

Solve: $2x + \frac{4}{x} = 9$

Solution: $2x + \frac{4}{x} = 9$

So multiplying both sides by x, we get

$$\Rightarrow x \left(2x + \frac{4}{x} \right) = 9x$$

$$\Rightarrow 2x^2 + \frac{4 \times \cancel{x}}{\cancel{x}} = 9x$$

$$\Rightarrow 2x^2 + 4 = 9x$$

Subtracting $9x$ from both sides, we get

$$\Rightarrow 2x^2 + 4 - 9x = 9\cancel{x} - 9\cancel{x}$$

$$\Rightarrow 2x^2 - 9x + 4 = 0$$

By factorization

$$\Rightarrow 2x^2 - 8x - x + 4 = 0$$

$$\Rightarrow 2x(x - 4) - 1(x - 4) = 0$$

$$\Rightarrow (2x - 1)(x - 4) = 0$$

$$(2x - 1) = 0$$

$$x - 4 = 0$$

Adding 1 to both sides

Adding 4 to both sides

$$2x - \cancel{1} + \cancel{1} = 0 + 1$$

$$x - \cancel{4} + \cancel{4} = 0 + 4$$

$$2x = 1$$

$$\boxed{x = 4}$$

Dividing both sides by 2

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{1}{2}$$

$$\boxed{x = \frac{1}{2}}$$

Hence given equations are $\frac{1}{2}, 4$ and solution set is

$$\left\{ \frac{1}{2}, 4 \right\}$$

Type-3 Reciprocal equations of the form

$$a \left(x^2 + \frac{1}{x^2} \right) + b \left(x + \frac{1}{x} \right) + c = 0$$

And
$$a\left(x^2 + \frac{1}{x^2}\right) + b\left(x - \frac{1}{x}\right) + c = 0$$

Solve the following equations.

$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0 \dots\dots(I)$$

Solution:
$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

Put
$$x + \frac{1}{x} = y \rightarrow (I)$$

Squaring both sides

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = (y)^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + \left(\cancel{x} - \cancel{x}\right)\left(\frac{1}{\cancel{x}}\right) = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 = y^2$$

Subtracting 2 from both sides, we get

$$\Rightarrow x^2 + \frac{1}{x^2} + \cancel{2} - \cancel{2} = y^2 - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

$$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0 \dots\dots(1)$$

$$2y^2 - 9y + 10 = 0$$

Putting value of $x^2 + \frac{1}{x^2}$, $x + \frac{1}{x}$ in equation-I

$$\Rightarrow 2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$$

$$\Rightarrow 2(y^2 - 2) - 9(y) + 14 = 0$$

$$\Rightarrow 2y^2 - 4 - 9y + 14 = 0$$

$$\Rightarrow 2y^2 - 9y + 10 = 0$$

So by factorization method

$$\Rightarrow 2y^2 - 9y + 10 = 0$$

$$\Rightarrow 2y^2 - 5y - 4y + 10 = 0$$

$$\Rightarrow y(2y - 5) - 2(2y - 5) = 0$$

$$\Rightarrow (y - 2)(2y - 5) = 0$$

$$2y - 5 = 0$$

$$\Rightarrow y - 2 = 0$$

Adding 5 to both sides,

$$2y - \cancel{5} + \cancel{5} = 0 + 5$$

$$2y = 5$$

Dividing both sides by 2,

$$\frac{\cancel{2}y}{\cancel{2}} = \frac{5}{2}$$

$$y = \frac{5}{2}$$

Putting value of y,

$$x + \frac{1}{x} = \frac{5}{2}$$

Multiply both sides by 2,

$$2 \times x + \frac{1}{x} \times 2 = \frac{5}{\cancel{2}} \times \cancel{2}$$

$$2x + \frac{2}{x} = 5$$

Multiply both sides by x

$$x(2x) + \frac{2}{\cancel{x}}(x) = 5(x)$$

$$2x^2 + 2 = 5x$$

Subtract 5x from both sides

$$\Rightarrow 2x^2 + 2 - 5x = 5x - 5x$$

$$\Rightarrow 2x^2 - 5x + 2 = 0$$

Adding 2 to both sides

$$y - \cancel{2} + \cancel{2} = 0 + 2$$

$$y = 2$$

As we know that

$$y = x + \frac{1}{x}$$

put the value of y

$$x + \frac{1}{x} = 2$$

$$(x)x + \frac{1}{\cancel{x}} \cdot \cancel{x} = 2(x)$$

$$x^2 + 1 = 2x$$

$$\Rightarrow x^2 + 1 - 2x = 2x - 2x$$

$$x^2 + 1 - 2x = 0$$

$$(x - 1)^2 = 0$$

$$x = 1, 1$$

By factorization we get

$$\Rightarrow 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow 2x(x-2) - 1(x-2) = 0$$

$$\Rightarrow (2x-1)(x-2) = 0$$

$$\Rightarrow 2x-1=0, x-2=0$$

$$\Rightarrow 2x-1=0$$

$$\Rightarrow 2x - \cancel{x} + \cancel{x} = 0 + 1$$

$$2x = 1$$

Dividing both sides by 2,

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{1}{2}$$

$$\boxed{x = \frac{1}{2}}$$

$$x-2=0$$

$$x - \cancel{x} + \cancel{x} = 0 + 2$$

$$\boxed{x = 2}$$

$$x = 2, \frac{1}{2}$$

Hence the solutions are $\left(2, \frac{1}{2}, 1, 1\right)$ and the solution set is $\left\{2, \frac{1}{2}, 1\right\}$.

Type-4 **Equation that involve of the form a^x where $a > 0$ and $a \neq 0$, are called exponential equations.**

Example1.

Solve: $4 \cdot 2^{2x} - 10 \cdot 2^x + 4 = 0$

Solution: $4 \cdot 2^{2x} - 10 \cdot 2^x + 4 = 0$

We may write the given equation as

$$4(2^x)^2 - 10(2^x) + 4 = 0 \dots (I)$$

Put $2^x = y$

Replace these values in equation-I

$$4(y)^2 - 10(y) + 4 = 0$$

So it becomes quadratic equation

$$4y^2 - 10y + 4 = 0$$

Dividing both sides by 2,

$$\Rightarrow 2y^2 - 5y + 2 = 0$$

By factorization method

$$\Rightarrow 2y^2 - 4y - y + 2 = 0$$

$$\Rightarrow 2y(y - 2) - 1(y - 2) = 0$$

$$\Rightarrow (2y - 1)(y - 2) = 0$$

$$\Rightarrow 2y - 1 = 0$$

$$y - 2 = 0$$

Adding 2 to both sides

Add 2 to both sides

$$2y - \cancel{1} + \cancel{1} = 0 + 1$$

$$y - \cancel{2} + \cancel{2} = 0 + 2$$

$$2y = 1$$

$$\boxed{y = 2}$$

Dividing both sides by 2

Replace $2^x = y$

$$\frac{\cancel{2}y}{\cancel{2}} = \frac{1}{2}$$

$$2^x = 2^1$$

$$\cancel{2}^x = \cancel{2}^1$$

$$\boxed{y = \frac{1}{2}}$$

$$\boxed{x = 1}$$

Now replace

$$y = 2^x$$

$$2^x = \frac{1}{2}$$

$$\cancel{2}^x = \cancel{2}^{-1}$$

$$\boxed{x = -1}$$

Thus the solution of the given quadratic equation are 1, -1,
and solution set is $\{1, -1\}$.

Type-5 Equation of the form

$$(x+a)(x+b)(x+c)(x+d) = k$$

Where

$$a+b=c+d$$

Exercise 1.2

Solve the following equations:

Q1. $x^4 - 5x^2 + 4 = 0$

Solution: $x^4 - 5x^2 + 4 = 0$

So according to type I we can write the given equation

$$\Rightarrow (x^2)^2 - 5(x)^2 + 4 = 0 \dots (1)$$

So put $x^2 = y$

$$\Rightarrow y^2 - 5y + 4 = 0$$

So it become quadratic equation

$$y^2 - 5y + 4 = 0$$

By factorization we get

$$\Rightarrow y^2 - 4y - y + 4 = 0$$

$$\Rightarrow y(y-4) - 1(y-4) = 0$$

$$(y-1)(y-4) = 0$$

$$y-1=0$$

$$y - \cancel{1} + \cancel{1} = 0 + 1$$

Adding both side 1

$$y = 1$$

Replacing $y = x^2$

$$x^2 = 1$$

$$y-4=0$$

$$y - \cancel{4} + \cancel{4} = 0 + 4$$

Adding both side 4

$$y = 4$$

Replacing $y = x^2$

$$x^2 = 4$$

Taking Square root

$$\sqrt{x^2} = \sqrt{1}$$

$$\boxed{x = \pm 1}$$

taking square root

$$\sqrt{x^2} = \sqrt{4}$$

$$\boxed{x = \pm 2}$$

Thus the solution set is $\{1, -1, 2, -2\}$

Q2. $x^4 - 7x^2 + 12$

Solution: $x^4 - 7x^2 + 12 \dots (i)$

So according to type i. we can write eq (i)

$$\Rightarrow (x^2)^2 - 7(x^2) + 12 = 0 \dots (ii)$$

Put $x^2 = y$ in eq (ii)

$$\Rightarrow y^2 - 7y + 12 = 0 \quad -7y = -4y - 3y$$

So it became quadratic equation

$$\Rightarrow y^2 - 7y + 12 = 0$$

By factorization

$$y^2 - 4y - 3y + 12 = 0$$

$$y(y-4) - 3(y-4) = 0$$

$$(y-3)(y-4) = 0$$

$$y-3=0$$

$$y-4=0$$

Add 3 to both sides

add 4 to both sides

$$y - \cancel{3} + \cancel{3} = 0 + 3$$

$$y - \cancel{4} + \cancel{4} = 0 + 4$$

$$\boxed{y=3}$$

$$\boxed{y=4}$$

Thus the solution set is $\{3, 4\}$

Now replace $y = x^2$

Now replace $y = x^2$

$$x^2 = 4$$

$$x^2 = 3$$

Taking square of both side

Taking square of both side

$$\sqrt{x^2} = \sqrt{4}$$

$$\sqrt{x^2} = \sqrt{3}$$

$$\boxed{x = \pm 2}$$

$$\boxed{x = \pm \sqrt{3}}$$

Hence the solution set of the given equation

$$x = \{\sqrt{3}, -\sqrt{3}, 2, -2\}$$

Q3. $6x^4 - 13x^2 + 5 = 0$

Solution:

According to type-I.

$$6x^4 - 13x^2 + 5 = 0$$

$$6(x^2)^2 - 13(x^2) + 5 = 0 \dots (1)$$

So put $x^2 = y$

$$6(y)^2 - 13(y) + 5 = 0$$

So it become quadratic equation

$$6y^2 - 13y + 5 = 0$$

By factorization method

$$\Rightarrow 6y^2 - 10y - 3y + 5 = 0 \quad -13y = -10y - 3y$$

$$\Rightarrow 2y(3y - 5) - 1(3y - 5) = 0$$

$$\Rightarrow (2y - 1)(3y - 5) = 0$$

$$\Rightarrow 2y - 1 = 0$$

$$\Rightarrow 3y - 5 = 0$$

$$\Rightarrow 2y - 1 = 0$$

$$\Rightarrow 3y - 5 = 0$$

Add 1 to both side

Add 5 to both side

$$\Rightarrow 2y - \cancel{1} + \cancel{1} = 0 + 1$$

$$\Rightarrow 3y - \cancel{5} + \cancel{5} = 0 + 5$$

$$\Rightarrow 2y = 1$$

$$\Rightarrow 3y = 5$$

Divide both side by 2

Divide both side by 3

$$\Rightarrow \boxed{y = \frac{1}{2}}$$

$$\Rightarrow \boxed{y = \frac{5}{3}}$$

Replace y by x^2

Replace y by x^2

$$\Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x^2 = \frac{5}{3}$$

Taking square root

Taking square root

$$\Rightarrow \sqrt{x^2} = \sqrt{\frac{1}{2}}$$

$$\Rightarrow \sqrt{x^2} = \sqrt{\frac{5}{3}}$$

$$\boxed{x = \pm \sqrt{\frac{1}{2}}}$$

$$\boxed{x = \pm \sqrt{\frac{5}{3}}}$$

Hence solution set is $\left\{\pm\sqrt{\frac{1}{2}}, \pm\sqrt{\frac{5}{3}}\right\}$ OR
 $\left\{\sqrt{\frac{1}{2}}, \sqrt{\frac{5}{3}}, -\sqrt{\frac{1}{2}}, -\sqrt{\frac{5}{3}}\right\}$

Q4. $x + \frac{1}{x} = 2$

Solution: $x + \frac{1}{x} = 2$

So according to type ii.

Multiply both side by x.

$$\Rightarrow x\left(x + \frac{1}{x}\right) = 2.x$$

$$\Rightarrow x^2 + \frac{x}{x} = 2x$$

$$\Rightarrow x^2 + 1 = 2x$$

Subtracting 2x from both side

$$\Rightarrow x^2 - 2x + 1 = 2x - 2x$$

$$\Rightarrow x^2 - 2x + 1 = 0 \text{ (Quadratic equation)}$$

$$(a^2 - b^2) = a^2 + b^2 - 2ab$$

By completing square method

$$\Rightarrow (x-1)^2 = 0$$

$$\Rightarrow (x-1)(x-1) = 0$$

$$\Rightarrow x-1=0$$

Adding both side 1

$$\Rightarrow x-1=0$$

$$\Rightarrow x - \cancel{1} + \cancel{1} = 0 + 1$$

$$\boxed{x=1}$$

$$\Rightarrow (x-1)=0$$

Adding both side 1

$$\Rightarrow x-1=0$$

$$\Rightarrow x - \cancel{1} + \cancel{1} = 0 + 1$$

$$\boxed{x=1}$$

Thus the solution of given quadratic equation is 1,1 and solution set is $\{1,1\}$ or $\{1\}$

Q5.
$$(x+2) - \frac{1}{x+2} = \frac{3}{2}$$

Solution:

According to type ii.

$$(x+2) - \frac{1}{(x+2)} = \frac{3}{2} \rightarrow (1)$$

As we know that

$$y = x+2 \text{ and } \frac{1}{y} = \frac{1}{x+2}$$

Put value of y in equation 1.

$$\Rightarrow y - \frac{1}{y} = \frac{3}{2}$$

Multiplying both sides by "y".

$$\Rightarrow y \left(y - \frac{1}{y} \right) = \frac{3}{2} \cdot y$$

$$\Rightarrow y^2 - \cancel{y} = \frac{3}{2} y$$

$$\Rightarrow y^2 - 1 = \frac{3}{2} y$$

Multiplying both sides by 2

$$\Rightarrow 2(y^2 - 1) = \frac{3}{\cancel{2}} y \cdot \cancel{2}$$

$$\Rightarrow 2y^2 - 2 = 3y$$

Subtracting 3y from both sides

$$\Rightarrow 2y^2 - 3y - 2 = 3\cancel{y} - 3\cancel{y}$$

So it is a quadratic equation

$$2y^2 - 3y - 2 = 0$$

$$2y^2 - 4y + y - 2 = 0$$

$$2y(y-2) + 1(y-2) = 0$$

$$(2y+1)(y-2) = 0$$

$$\Rightarrow 2y+1=0$$

$$y-2=0$$

Subtracting 1 from both sides Add "2" to both sides

$$\Rightarrow 2y + \cancel{1} - \cancel{1} = 0 - 1$$

$$\Rightarrow y - \cancel{2} + \cancel{2} = 0 + 2$$

$$\Rightarrow 2y = -1$$

$$\boxed{y = 2}$$

Divide both sides by "2"

$$\frac{\cancel{2}y}{\cancel{2}} = -\frac{1}{2}$$

$$\boxed{y = -\frac{1}{2}}$$

Now replace "y" by $x+2$

We get

$$x+2 = -\frac{1}{2}$$

$$x+2=2$$

Subtract "2" from both sides

Subtract "2" from both sides

$$x = -\frac{1}{2} - \frac{2}{1}$$

$$x + \cancel{2} - \cancel{2} = \cancel{2} - \cancel{2}$$

$$\boxed{x = 0}$$

$$x = \frac{-1-4}{2}$$

$$\boxed{x = -\frac{5}{2}}$$

Hence solution set is $\left\{-\frac{5}{2}, 0\right\}$

Q.6. $x - \frac{4}{x} = 2$

Solution: $x - \frac{4}{x} = 2$

According to type-2

Multiplying both sides by "x"

$$\Rightarrow x\left(x - \frac{4}{x}\right)2x$$

$$\Rightarrow x^2 - 4 = 2x$$

Subtracting “2x” from both side

$$\Rightarrow x^2 - 2x - 4 = 2x - 2x$$

$$\Rightarrow x^2 - 2x - 4 = 0$$

Now by quadratic formula

$$a=1, \quad b=-2, \quad c=-4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Putting the value we get

$$\Rightarrow x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$\Rightarrow x = \frac{+2 \pm \sqrt{4+16}}{2}$$

$$\Rightarrow x = \frac{+2 \pm \sqrt{4+16}}{2} = x = \frac{+2 \pm \sqrt{4 \times 5}}{2}$$

$$x = \frac{+2 \pm \sqrt{4} \times \sqrt{5}}{2} = x = \frac{+2 \pm 2\sqrt{5}}{2}$$

Taking 2 common

$$x = \frac{2(1 \pm \sqrt{5})}{2}$$

$$\boxed{x = 1 + \sqrt{5}} \quad \boxed{x = 1 - \sqrt{5}}$$

Thus the solution of the given quadratic equation are

$x = 1 + \sqrt{5}, 1 - \sqrt{5}$ and the solution set is $\{1 + \sqrt{5}, 1 - \sqrt{5}\}$

Q7. $\frac{x}{x+1} + \frac{x+1}{x} = \frac{13}{6}$

Solution: $\frac{x}{x+1} + \frac{x+1}{x} = \frac{13}{6} \dots\dots(1)$

So According to type-2

Suppose $\frac{x}{x+1} = y$ so

$$\Rightarrow \frac{x+1}{x} = \frac{1}{y}$$

Substituting these values in eq (1)

$$\Rightarrow y + \frac{1}{y} = \frac{13}{6}$$

Multiplying both sides by 6y

$$\Rightarrow 6y \left(y + \frac{1}{y} \right) = \frac{13}{6} \times 6y$$

$$\Rightarrow 6y^2 + \frac{6\cancel{y}}{\cancel{y}} = 13y$$

$$\Rightarrow 6y^2 + 6 = 13y$$

Subtracting 13y from both sides

$$\Rightarrow 6y^2 - 13y + 6 = 13\cancel{y} - 13\cancel{y}$$

$$\Rightarrow 6y^2 - 13y + 6 = 0$$

So it become standard form of quadratic equation

By factorization method we get

$$\Rightarrow 6y^2 - 9y - 4y + 6 = 0$$

$$\Rightarrow 3y(2y-3) - 2(2y-3) = 0$$

$$\Rightarrow (3y-2)(2y-3) = 0$$

$$\Rightarrow 3y - 2 = 0$$

Add 2 to both sides

$$3y - \cancel{2} + \cancel{2} = 0 + 2$$

$$\Rightarrow 3y = 2$$

Divide both by 3

$$\Rightarrow \frac{\cancel{3}y}{\cancel{3}} = \frac{2}{3}$$

$$\Rightarrow \boxed{y = \frac{2}{3}}$$

Now replace y by $\frac{x}{x+1}$

$$\Rightarrow \frac{x}{x+1} = \frac{2}{3}$$

By cross multiplication

$$\Rightarrow 3x = 2(x+1)$$

$$\Rightarrow 3x = 2x + 2$$

Subtract 2x from both side

$$\Rightarrow 3x - 2x = \cancel{2x} - \cancel{2x} + 2$$

$$\boxed{x = 2}$$

$$\Rightarrow 2y - 3 = 0$$

Add 3 to both sides

$$\Rightarrow 2y - \cancel{3} + \cancel{3} = 0 + 3$$

$$\Rightarrow 2y = 3$$

Divide both by 2

$$\Rightarrow \frac{\cancel{2}y}{\cancel{2}} = \frac{3}{2}$$

$$\Rightarrow \boxed{y = \frac{3}{2}}$$

Now replace y by $\frac{x}{x+1}$

$$\Rightarrow \frac{x}{x+1} = \frac{3}{2}$$

by cross multiplication

$$\Rightarrow 2x = 3(x+1)$$

$$\Rightarrow 2x = 3x + 3$$

Subtract 3x from both side

$$2x - 3x = \cancel{3x} - \cancel{3x} + 3$$

$$-x = 3$$

$$\boxed{x = -3}$$

Hence the solution set of the quadratic equation is $\{2, -3\}$

Q8. $3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0$

Solution:

So according to type iii.

$$3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0 \dots\dots(1)$$

Let us suppose $x + \frac{1}{x} = y$

Squaring on both sides

$$\left(x + \frac{1}{x}\right)^2 = (y)^2 \text{ Expand by formula}$$

$$x^2 + \frac{1}{x^2} + 2 \cdot x \cdot \frac{1}{x} = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2$$

Subtracting both sides 2

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 - 2 = y^2 - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Now putting these in eq (i)

$$\Rightarrow 3\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0$$

$$\Rightarrow 3(y^2 - 2) - 16(y) + 26 = 0$$

$$\Rightarrow 3y^2 - 6 - 16y + 26 = 0$$

By factorization method

$$\Rightarrow 3y^2 - 6y - 10y + 20 = 0$$

$$\Rightarrow 3y(y - 2) - 10(y - 2) = 0$$

$$\Rightarrow (3y - 10)(y - 2) = 0$$

$$3y - 10 = 0$$

$$y - 2 = 0$$

Adding 10 to both sides

$$3y - 10 + 10 = 0 + 10$$

$$3y = 10$$

Dividing both sides by 3

$$\frac{3y}{3} = \frac{10}{3}$$

$$y = \frac{10}{3}$$

Adding 2 to both sides

$$y - 2 + 2 = 0 + 2$$

$$y = 2$$

$$\text{As } y = x + \frac{1}{x}$$

$$x + \frac{1}{x} = 2$$

Multiply both side by "x"

$$\text{So } y = x + \frac{1}{x}$$

$$\Rightarrow x + \frac{1}{x} = \frac{10}{3}$$

Multiplying both sides by $3x$

$$\Rightarrow 3x^2 + \frac{3x}{x} = \frac{10 \cancel{30}x}{\cancel{3}}$$

$$\Rightarrow 3x^2 + 3 = 10x$$

Subtract $10x$ from both sides

$$\Rightarrow 3x^2 - 10x + 3 = 10x - 10x$$

$$\Rightarrow 3x^2 - 10x + 3 = 0$$

$$\Rightarrow 3x^2 - 9x - x + 3 = 0$$

$$\Rightarrow 3x(x-3) - 1(x-3) = 0$$

$$\Rightarrow 3x(x-3) - 1(x-3) = 0$$

$$\Rightarrow 3x - 1 = 0, x - 3 = 0$$

Add 1 to both sides

$$\Rightarrow 3x - \cancel{1} + \cancel{1} = 0 + 1$$

$$\Rightarrow 3x = 1$$

Divide both sides by 3

$$\boxed{x = \frac{1}{3}}$$

$$x - 3 = 0$$

Add 3 to both sides

$$\boxed{x = 3}$$

Hence the required answer is $\left\{\frac{1}{3}, 3, 1, 1\right\}$

$$\text{Q9.} \quad 6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x - \frac{1}{x}\right) + 12 = 0$$

$$\text{Solution:} \quad 6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x - \frac{1}{x}\right) + 12 = 0 \dots\dots(1)$$

$$x\left(x + \frac{1}{x}\right) = 2x$$

$$\Rightarrow x^2 + \frac{x}{x} = 2x$$

Subtract $2x$ from both sides

$$x^2 - 2x + 1 = 2x - 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x - 1 = 0, x - 1 = 0$$

$$\boxed{x=1} \quad \boxed{x=1}$$

So according to type iii.

Suppose $x - \frac{1}{x} = y \rightarrow (A)$

Squaring both sides

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = (y)^2$$

Expand by formula

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 \cdot x \cdot \frac{1}{x} = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = y^2$$

Add 2 to both sides

$$\Rightarrow x^2 + \frac{1}{x^2} - \cancel{2} + \cancel{2} = y^2 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2 \dots\dots (B)$$

Now putting these values in equation (1)

$$\Rightarrow 6(y^2 + 2) - 25(y) + 12 = 0$$

$$\Rightarrow 6y^2 + 12 - 25y + 12 = 0$$

$$\Rightarrow 6y^2 - 25y + 24 = 0$$

$$\Rightarrow 6y^2 - 16y - 9y + 24 = 0$$

$$\Rightarrow 2y(3y - 8) - 3(3y - 8) = 0$$

$$\Rightarrow (2y - 3) = 0$$

$$\Rightarrow (3y - 8) = 0$$

$$\Rightarrow 2y - 3 = 0$$

$$\Rightarrow 3y - 8 = 0$$

Add 3 to both sides

Add 8 to both sides

$$\Rightarrow 2y - \cancel{3} + \cancel{3} = 0 + 3$$

$$\Rightarrow 3y - \cancel{8} + \cancel{8} = 0 + 8$$

$$\Rightarrow 2y = 3$$

$$\Rightarrow 3y = 8$$

Divide both sides by 2

Divide both sides by

$$\Rightarrow \frac{\cancel{2}y}{\cancel{2}} = \frac{3}{2}$$

$$\Rightarrow \frac{\cancel{3}y}{\cancel{3}} = \frac{8}{3}$$

$$y = \frac{3}{2}$$

$$\Rightarrow y = \frac{8}{3}$$

Put $y = x - \frac{1}{x}$

Put $y = x - \frac{1}{x}$

$$x - \frac{1}{x} = \frac{3}{2}$$

$$x - \frac{1}{x} = \frac{8}{3}$$

Multiplying both sides by $2x$

Multiplying both sides by

$$2x \left(x - \frac{1}{x} \right) = \frac{3}{2} \cdot 2x$$

$$3x \left(x - \frac{1}{x} \right) = \frac{8}{3} \cdot 3x$$

$$2x^2 - \frac{2x}{x} = 2x \times \frac{3}{2}$$

$$3x^2 - \frac{3x}{x} = 3x \times \frac{8}{3}$$

$$2x^2 - 2 = 3x$$

$$3x^2 - 3 = 8x$$

Subtract $3x$ from both sides

Subtract $8x$ from both sides

$$\Rightarrow 2x^2 - 3x - 2 = 3x - 3x$$

$$\Rightarrow 3x^2 - 8x - 3 = 8x - 8x$$

$$\Rightarrow 2x^2 - 3x - 2 = 0$$

$$\Rightarrow 3x^2 - 8x - 3 = 0$$

By factorization

by factorization

$$\Rightarrow 2x^2 - 4x + x - 2 = 0$$

$$\Rightarrow 3x^2 - 9x + x - 3 = 0$$

$$\Rightarrow 2x(x-2) + 1(x-2) = 0$$

$$\Rightarrow 3x(x-3) + 1(x-3) = 0$$

$$\Rightarrow (2x+1) = 0, (x-2) = 0$$

$$\Rightarrow (3x+1) = 0, (x-3) = 0$$

$$\Rightarrow 2x+1=0, x-2=0$$

$$\Rightarrow 3x+1=0, x-3=0$$

$$\Rightarrow x = -\frac{1}{2}, x = 2$$

$$\Rightarrow \boxed{x = -\frac{1}{3}} \quad \boxed{x = 3}$$

$$\Rightarrow \boxed{x = -\frac{1}{2}} \quad \boxed{x = 2}$$

Thus the solution set of the given Quadratic equation are

$$\left\{ -\frac{1}{2}, 2, -\frac{1}{3}, 3 \right\}$$

Q10. $10 \left(x^2 + \frac{1}{x^2} \right) - 63 \left(x - \frac{1}{x} \right) + 52 = 0$

Solution: $10\left(x^2 + \frac{1}{x^2}\right) - 63\left(x - \frac{1}{x}\right) + 52 = 0 \dots (1)$

Suppose $x - \frac{1}{x} = y$

Squaring on both sides

$$\left(x - \frac{1}{x}\right)^2 = (y)^2$$

Expand by formula

$$x^2 + \frac{1}{x^2} - 2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 - 2 = y^2 + 2$$

$$x^2 + \frac{1}{x^2} = y^2 + 2$$

$$\Rightarrow 10\left(x^2 + \frac{1}{x^2}\right) - 63\left(x - \frac{1}{x}\right) + 52 = 0$$

$$\Rightarrow 10(y^2 + 2) - 63(y) + 52 = 0$$

$$\Rightarrow 10y^2 + 20 - 63y + 52 = 0$$

$$\Rightarrow 10y^2 - 63y + 72 = 0$$

Now a = 10, b = -63, c = 72

Using Quadratic formula

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{-(-63) \pm \sqrt{(-63)^2 - 4(10)(72)}}{2(10)}$$

$$y = \frac{63 \pm \sqrt{3969 - 2880}}{20}$$

$$y = \frac{63 \pm \sqrt{1089}}{20}$$

$$y = \frac{63 \pm 33}{20}$$

$$y = \frac{63+33}{20}$$

$$y = \frac{96}{20}$$

$$y = \frac{24}{5}$$

$$\text{Put } y = x - \frac{1}{x}$$

$$\frac{24}{5} = x - \frac{1}{x}$$

Multiplying both sides by 5x

$$\cancel{24} \times \cancel{5} x = \left(x - \frac{1}{x} \right) 5x$$

$$24x = 5x^2 - 5$$

$$\Rightarrow 5x^2 - 24x - 5 = 0$$

By factorization

$$\Rightarrow 5x^2 - 25x + x - 5 = 0$$

$$\Rightarrow 5x(x-5) + 1(x-5) = 0$$

$$(5x+1)(x-5) = 0$$

$$\boxed{x = -\frac{1}{5}}$$

$$\boxed{x = 5}$$

So the solution set $\left\{ -\frac{1}{5}, 5, -\frac{1}{2}, 2 \right\}$

$$\text{Q11.} \quad \left(x^2 + \frac{1}{x^2} \right) - \left(x - \frac{1}{x} \right) - 4 = 0$$

$$\text{Solution:} \quad \left(x^2 + \frac{1}{x^2} \right) - \left(x - \frac{1}{x} \right) - 4 = 0 \dots\dots(1)$$

$$\text{Put } x - \frac{1}{x} = y \dots\dots(A) \text{ Squaring both sides}$$

$$y = \frac{63-33}{20}$$

$$y = \frac{30}{20}$$

$$y = \frac{3}{2}$$

$$y = x - \frac{1}{x}$$

$$\frac{3}{2} = x - \frac{1}{x}$$

Multiplying both sides by 2x

$$3x = 2x^2 - 2$$

$$\Rightarrow 2x^2 - 3x - 2 = 0$$

$$\Rightarrow 2x^2 - 4x + x - 2 = 0$$

$$\Rightarrow 2x(x-2) + 1(x-2) = 0$$

$$\Rightarrow (2x+1)(x-2) = 0$$

$$\boxed{x = -\frac{1}{2}}$$

$$\boxed{x = 2}$$

$$\left(x - \frac{1}{x}\right)^2 = (y)^2$$

Expand by formula

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 \cdot x \cdot \frac{1}{x} = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = y^2$$

Add 2 to both sides

$$\Rightarrow x^2 + \frac{1}{x^2} - \cancel{2} + \cancel{2} = y^2 + 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2 \dots\dots (B)$$

Putting the value of (A) and (B) in equation 1.

$$\Rightarrow (y^2 + 2) - (y) - 4 = 0$$

$$\Rightarrow (y^2 + 2) - (y) - 4 = 0$$

$$\Rightarrow y^2 + 2 - 4 - y = 0$$

$$\Rightarrow y^2 - y - 2 = 0$$

By factorization

$$\Rightarrow y^2 - 2y + y - 2 = 0$$

$$\Rightarrow y(y - 2) + 1(y - 2) = 0$$

$$\Rightarrow y + 1 = 0, \quad y - 2 = 0$$

$$y + 1 = 0, \quad \text{OR} \quad y - 2 = 0$$

Subtract 1 from both sides

$$y + \cancel{1} - \cancel{1} = 0 - 1$$

$$y = -1$$

Put value of y in eq (A)

$$x - \frac{1}{x} = 1$$

Multiply both sides by x.

Add 2 to both sides

$$y - \cancel{2} + \cancel{2} = 0 + 2$$

$$y = 2$$

Put value of y in eq (A)

$$x - \frac{1}{x} = 2$$

Multiply both sides by x

$$x\left(x - \frac{1}{x}\right) = 1 \cdot x$$

$$x^2 - \frac{x}{x} = x$$

$$x^2 - 1 = x$$

Subtract "x" from both sides

$$x^2 - x - 1 = x - x$$

$$x^2 - x - 1 = 0$$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, b = -1, c = -1$$

$$x = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\boxed{x = \frac{1 \pm \sqrt{5}}{2}}$$

$$x\left(x - \frac{1}{x}\right) = 2 \cdot x$$

$$x^2 - \frac{x}{x} = 2x$$

$$x^2 - 1 = 2x$$

subtract "2x" from both sides

$$x^2 - 2x - 1 = 2x - 2x$$

$$x^2 - 2x - 1 = 0$$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{2 \pm 2\sqrt{2}}{2}$$

$$\boxed{x = 1 \pm \sqrt{2}}$$

$$\text{So the Solution set} = \left\{ \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}, 1+\sqrt{2}, 1-\sqrt{2} \right\}$$

Q12. $3x^2 - 10 \cdot 3^x + 9 = 0$

Solution: $3x^2 - 10 \cdot 3^x + 9 = 0$

According to type iv.

$$\Rightarrow (3^x)^2 - 10(3)^x + 9 = 0$$

Put $3^x = y$

$$\Rightarrow y^2 - 10y + 9 = 0$$

By factorization method

$$\Rightarrow y^2 - 9y - y + 9 = 0$$

$$\begin{aligned}
&\Rightarrow y(y-9)-1(y-9)=0 \\
&\Rightarrow (y-1)=0, (y-9)=0 \\
&\Rightarrow y-1=0 \qquad \qquad \Rightarrow y-9=0 \\
&\text{Adding both sides 1} \qquad \text{Adding both sides 9} \\
&\Rightarrow y-\cancel{1}+\cancel{1}=0+1 \Rightarrow y-\cancel{9}+\cancel{9}=0+9 \\
&y=1 \qquad \qquad \qquad y=9 \\
&\text{Put } y=3^x \qquad \qquad \text{Put } 3^x=9 \\
&\text{When we put } x=0 \qquad (3)^x=3 \times 3 \\
&\text{Then L.H.S=R.H.S} \qquad \cancel{3}^x=\cancel{3}^2 \\
&(3)^0=1 \qquad \qquad \boxed{x=2} \\
&1=1 \\
&\text{So } \boxed{x=0}
\end{aligned}$$

Answer $\{0, 2\}$

Q13. $3 \cdot 3^{2x+1} - 10 \cdot 3^x + 1 = 0$

Solution: $3 \cdot 3^{2x+1} - 10 \cdot 3^x + 1 = 0$

$$\begin{aligned}
&\Rightarrow 3 \cdot 3^{2x} \cdot 3^1 - 10 \cdot 3^x + 1 = 0 \\
&\Rightarrow 9 \cdot (3)^{2x} - 10(3)^x + 1 = 0 \\
&\Rightarrow 9 \cdot (3^x)^2 - 10(3)^x + 1 = 0 \dots (1)
\end{aligned}$$

Put in equation (1)

$$\begin{aligned}
&\Rightarrow 9(y)^2 - 10(y) + 1 = 0 \\
&\Rightarrow 9y^2 - 10y + 1 = 0
\end{aligned}$$

So by factorization method

$$\begin{aligned}
&\Rightarrow 9y^2 - 9y - y + 1 = 0 \\
&\Rightarrow 9y(y-1) - 1(y-1) = 0 \\
&\Rightarrow (9y-1)(y-1) = 0 \\
&\Rightarrow 9y-1=0 \qquad \qquad \Rightarrow y-1=0 \\
&\text{Add 1 to both sides} \qquad \text{Add 1 to both sides} \\
&\Rightarrow 9y-\cancel{1}+\cancel{1}=0+1 \Rightarrow y-\cancel{1}+\cancel{1}=0+1
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow 9y = 1 && y = 1 \\
&\Rightarrow \frac{\cancel{9}y}{\cancel{9}} = \frac{1}{9} && \text{Put } 3^x = 1 \\
&\text{Put } 3^x = y && \text{When } x = 0 \text{ so } (3)^0 = 1 \\
&3^x = \frac{1}{9} && \boxed{x = 0} \\
&(3)^x = (9)^{-1} \\
&(3)^x = ((3)^2)^{-1} \\
&(3)^x = (9)^{-1} \\
&(\cancel{3})^x = (\cancel{3})^{-2} \\
&\boxed{x = -2}
\end{aligned}$$

So solution of given equation $\{0, -2\}$

Q14. $5^{x+1} + 5^{2-x} = 126$

Solution: $5^{x+1} + 5^{2-x} = 126$

$$\Rightarrow 5^x \cdot 5^1 + 5^2 \cdot 5^{-x} = 126$$

$$\Rightarrow 5(5)^x + 25(5)^{-x} = 126$$

$$\Rightarrow 5(5)^x + 25(5^x)^{-1} = 126$$

Put $5^x = y$

$$\Rightarrow 5(y) + 25(y)^{-1} = 126$$

$$\Rightarrow 5(y) + \frac{25}{y} = 126$$

Multiplying both sides by "y"

$$\Rightarrow y \left(5y + \frac{25}{y} \right) = 126 \cdot y$$

$$\Rightarrow 5y^2 + \frac{25\cancel{y}}{\cancel{y}} = 126y$$

$$\Rightarrow 5y^2 + 25 = 126y$$

Subtract 126y from both sides

$$\Rightarrow 5y^2 - 126y + 25 = \cancel{126y} - \cancel{126y}$$

$$\Rightarrow 5y^2 - 126y + 25 = 0$$

By factorization method

$$\Rightarrow 5y^2 - 125y - y + 25 = 0$$

$$\Rightarrow 5y(y - 25) - 1(y - 25) = 0$$

$$\Rightarrow 5y - 1 = 0$$

$$\Rightarrow y - 25 = 0$$

$$5y - \cancel{1} + \cancel{1} = 0 + 1$$

$$\Rightarrow y - \cancel{25} + \cancel{25} = 0 + 25$$

$$\Rightarrow 5y = 1$$

$$y = 25$$

$$\Rightarrow \frac{\cancel{5}y}{\cancel{5}} = \frac{1}{5}$$

$$\text{Put } y = 5^x$$

$$(5)^x = (25)$$

$$y = \frac{1}{5}$$

$$(\cancel{5})^x = (\cancel{5})^2$$

$$\boxed{x = 2}$$

$$\text{Put } y = 5^x$$

$$(\cancel{5})^x = (\cancel{5})^{-1}$$

$$\boxed{x = -1}$$

Thus the solution set of given quadratic equation $\{-1, 2\}$

Q15. $2^{x+1} + 2^{-x+5} - 20 = 0$

Solution: $2^{x+1} + 2^{-x+5} - 20 = 0$

$$\Rightarrow 2^x 2^1 + 2^{-x} 2^5 - 20 = 0$$

$$\Rightarrow (2)^x 2^1 + (2)^{-x} 32 - 20 = 0$$

So put $(2)^x = y$

$$\Rightarrow 2y + 32(y)^{-1} - 20 = 0$$

$$\Rightarrow 2y + 32\left(\frac{1}{y}\right) - 20 = 0$$

Multiplying both sides by "y"

$$\Rightarrow 2y^2 + 32 - 20y = 0$$

$$\Rightarrow 2y^2 - 20y + 32 = 0$$

Dividing both sides by "2"

$$\Rightarrow y^2 - 10y + 16 = 0$$

By factorization method.

$$\Rightarrow y^2 - 8y - 2y + 16 = 0$$

$$\Rightarrow y(y - 8) - 2(y - 8) = 0$$

$$\Rightarrow (y - 2)(y - 8) = 0$$

$$\Rightarrow y - 2 = 0$$

$$\Rightarrow y - 8 = 0$$

$$y - 2 = 0$$

$$y - 8 = 0$$

Adding both sides 2

Adding both sides 8

$$y - \cancel{2} + \cancel{2} = 0 + 2$$

$$y - \cancel{8} + \cancel{8} = 0 + 8$$

$$\boxed{y = 2}$$

$$\boxed{y = 8}$$

Put $y = 2^x$

$$2^x = 8$$

$$(\cancel{2})^x = (\cancel{2})^1$$

$$\cancel{2}^x = (\cancel{2})^3$$

$$\boxed{x = 1}$$

$$\boxed{x = 3}$$

The solution set is $\{1, 3\}$

Q. 16 $(x - 3)(x + 9)(x + 5)(x - 7) = 385$

Solution: $(x - 3)(x + 9)(x + 5)(x - 7) = 385$

According to type-5

$$\Rightarrow (x + a)(x + b)(x + c)(x + d) = c$$

So here

$$a = -3, b = 9, c = 5, d = -7$$

thus

$$a + c = b + d$$

$$-3 + 5 = 9 - 7$$

$$2 = 2$$

$$\Rightarrow [(x - 3)(x + 9)(x + 5)(x - 7)] = 385$$

$$\Rightarrow [(x - 3)(x + 9)][(x + 5)(x - 7)] = 385$$

$$\Rightarrow [(x^2 + 5x - 3x - 15)][(x^2 - 7x + 9x - 63)] = 385$$

$$\Rightarrow (x^2 + 2x - 15)(x^2 + 2x - 63) = 385$$

$$\text{Put } x^2 + 2x = y$$

$$\Rightarrow (y - 15)(y - 63) = 385$$

$$\Rightarrow y^2 - 63y - 15y + 945 - 385 = 0$$

$$\Rightarrow y^2 - 78y + 560 = 0$$

By factorization

$$\Rightarrow y^2 - 8y - 70y + 560 = 0$$

$$\Rightarrow y(y - 8) - 70(y - 8) = 0$$

$$\Rightarrow (y - 70)(y - 8) = 0$$

$$\Rightarrow y - 70 = 0$$

adding both sides 70

$$y - \cancel{70} + \cancel{70} = 0 + 70$$

$$y = 70$$

thus put

$$x^2 - 2x = y$$

$$x^2 - 2x = 70$$

$$x^2 - 2x - 70 = 0$$

Now using Quadratic formula

$$a = 1, b = -2, c = -70$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y - 8 = 0$$

adding both sides 8

$$y - \cancel{8} + \cancel{8} = 0 + 8$$

$$y = 8$$

$$x^2 - 2x = 8$$

$$x^2 - 2x - 8 = 0$$

by factorization method

$$x^2 - 4x + 2x - 8 = 0$$

$$x(x - 4) + 2(x - 4) = 0$$

$$(x + 2)(x - 4) = 0$$

$$x + 2 = 0 \quad x - 4 = 0$$

$$x = -2 \quad x = 4$$

Putting the values of a, b, c

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-70)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 280}}{2} \Rightarrow x = \frac{2 \pm \sqrt{284}}{2}$$

$$x = \frac{2 \pm \sqrt{71 \times 4}}{2} \Rightarrow x = \frac{-2 \pm 2\sqrt{71}}{2}$$

$$x = \frac{-2 \pm \sqrt{71}}{2}$$

$$x = -1 + \sqrt{71}, x = -1 - \sqrt{71}$$

Thus the solution set of given Quadratic Equation is

$$x = \{-2, 4, -1 + \sqrt{71}, -1 - \sqrt{71}\}$$

Q. 17 $(x+1)(x+2)(x+3)(x+4)-1=0$

Solution: According to type-V

$$(x+a)(x+b)(x+c)(x+d)=c$$

So here $a=1, b=2, c=3, d=4$

thus $a+c=b+d$

$$1+4=2+3$$

$$5=5$$

Thus multiplying a and d, multiplying b and c

$$\Rightarrow [(x+1)(x+4)][(x+2)(x+3)] + 1 = 0$$

$$\Rightarrow [(x^2+4x+x+4)][(x^2+2x+3x+6)] = 385$$

$$\Rightarrow (x^2+5x+4)(x^2+5x+6)+1=0 \text{----(1)}$$

So put $x^2 + 5x = y$

$$\Rightarrow (y+4)(y+6) + 1 = 0$$

$$\Rightarrow [y^2 + 6x + 4y + 24] + 1 = 0$$

$$\Rightarrow y^2 + 10x + 25 = 0$$

By factorization method

$$\Rightarrow y^2 + 5x + 5x + 25 = 0$$

$$\Rightarrow y(y+5) + 5(y+5) = 0$$

$$\Rightarrow (y+5)(y+5) = 0$$

$$\Rightarrow (x+5) = 0$$

$$\Rightarrow x+5=0$$

Subtracting both sides 5

$$\Rightarrow (x+5) = 0$$

$$\Rightarrow x+5=0$$

Subtracting both sides 5

$\Rightarrow x + \cancel{5} - \cancel{5} = 0 - 5$ $\Rightarrow x = -5$	$\Rightarrow x + \cancel{5} - \cancel{5} = 0 - 5$ $\Rightarrow x = -5$
--	--

Put $x^2 + 5x = -5$
 $x^2 + 5x + 5 = 0$

So it become a quadratic equation

$$a = 1, b = 5, c = 5$$

$$\Rightarrow y = x^2 + 5x$$

$$\Rightarrow x^2 + 5x = -5$$

$$\Rightarrow x^2 + 5x + 5 = 0$$

So it becomes Quadratic equation

$$a = 1, b = 5, c = 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Putting the values

$$x = \frac{-(5) \pm \sqrt{(-5)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 - 20}}{2} = \frac{-5 \pm \sqrt{5}}{2}$$

$$x = \frac{-5 + \sqrt{5}}{2}, \frac{-5 - \sqrt{5}}{2}$$

Thus the solution set of the given Quadratic equation

$$\text{is } x = \left\{ \frac{-5 + \sqrt{5}}{2}, \frac{-5 - \sqrt{5}}{2} \right\}$$

1.5 Radical Equation:

And equation in which the variable appears in one or more radicands is called Radical Equation.

For example:

- i. $\sqrt{x-2} = 3$
- ii. $\sqrt{2x+3} = 2x+5$
- iii. $\sqrt{x+5} = \sqrt{2x-1}$

Types of Radical Equation

Type-1

Equation of the form $\sqrt{ax+b} = cx+d$

Example:

Solve: $\sqrt{27-3x} = x-3$

$$\begin{aligned} \text{Squaring both sides } \Rightarrow & (\sqrt{27-3x})^2 = (x-3)^2 \\ \Rightarrow & 27-3x = (x)^2 + (3)^2 - 2(x)(3) \\ \Rightarrow & 27-3x = x^2+9-6x \end{aligned}$$

Subtracting both sides 27

$$\begin{aligned} \Rightarrow & \cancel{27} - \cancel{27} - 3x = x^2+9-6x-27 \\ \Rightarrow & -3x = x^2+9-6x-27 \end{aligned}$$

Adding both sides 3x

$$\begin{aligned} \Rightarrow & -\cancel{3x} + \cancel{3x} = x^2+9-6x-27+3x \\ \Rightarrow & x^2-3x-18=0 \end{aligned}$$

So by factorization

$$\begin{aligned} \Rightarrow & x^2-6x+3x-18=0 \\ & x(x-6)+3(x-6)=0 \\ \Rightarrow & (x+3)(x-6)=0 \\ x+3=0 & \qquad \qquad x-6=0 \\ \text{subtracting both sides "3"} & \quad \text{adding both sides "6"} \\ x+\cancel{3}-\cancel{3}=0-3 & \qquad x-\cancel{6}+\cancel{6}=0+6 \\ \boxed{x=-3} & \qquad \qquad \boxed{x=6} \end{aligned}$$

Now is necessary to check the solution in the original equation

$$\sqrt{27 - 3x} = x - 3$$

so put $x = 6$

$$\sqrt{27 - 3(6)} = 6 - 3$$

$$\sqrt{27 - 18} = 3$$

$$\sqrt{9} = 3$$

$$\boxed{3 = 3 \text{ true}}$$

if $x = -3$

$$\sqrt{27 - 3(-3)} = -3 - 3$$

$$\sqrt{27 + 9} = -6$$

$$\sqrt{36} = -6$$

$$\sqrt{(6)^2} = -6$$

$$\boxed{6 = -6 \text{ false}}$$

On checking, we find that $x = -3$ is an extraneous root, thus the solution set is $\{6\}$

Type-2

Equation of the form

$$\sqrt{x + a} + \sqrt{x + b} = \sqrt{x + c}$$

Type-3

Equation of the form

$$\sqrt{x^2 + px + m} + \sqrt{x^2 + px + n} = q$$

Exercise 1.3

Solve the following equation

Q. 1.

$$\sqrt{5x + 21} = x + 3$$

Solution: $\sqrt{5x + 21} = x + 3$ ----- (Type-I)

Taking square on both sides

$$(\sqrt{5x + 21})^2 = (x + 3)^2$$

$$\Rightarrow 5x + 21 = x^2 + 9 + 6x$$

Subtracting both sides $5x$

$$\Rightarrow \cancel{5x} - \cancel{5x} + 21 = x^2 + 9 + 6x - 5x$$

Subtracting 21 from both sides

$$\Rightarrow \cancel{21} - \cancel{21} = x^2 + x + 9 - 21$$

$$0 = x^2 + x - 12$$

$$\Rightarrow x^2 + x - 12 = 0$$

Now by factorization

$$\Rightarrow x^2 + 4x - 3x - 12 = 0$$

$$\Rightarrow x(x + 4) - 3(x + 4) = 0$$

$$(x - 3)(x + 4) = 0$$

$$x - 3 = 0 \quad \text{OR} \quad x + 4 = 0$$

Adding 3 to both sides Adding 4 to both sides

$$x - \cancel{3} + \cancel{3} = 0 + 3 \quad x + \cancel{4} - \cancel{4} = 0 - 4$$

$$\boxed{x = 3}$$

$$\boxed{x = -4}$$

Now it is necessary to check the solution in the original equation

$$\sqrt{5x + 21} = x + 3 \quad \sqrt{5x + 21} = x + 3$$

If $x = 3$ then If $x = -4$ then

$$\sqrt{5(3) + 21} = 3 + 3 \quad \sqrt{5(-4) + 21} = -4 + 3$$

$$\sqrt{15 + 21} = 6 \quad \sqrt{-20 + 21} = -1$$

$$\sqrt{36} = 6 \quad \sqrt{1} = -1$$

$$\boxed{6 = 6 \quad \text{true}}$$

$$\boxed{1 = -1 \quad \text{False}}$$

On checking we find that $x = -4$ is an extraneous root. Thus the solution set is $\{3\}$

Q. 2 $\sqrt{2x - 1} = x - 2$

Solution: $\sqrt{2x - 1} = x - 2$

According to type-I. Squaring on both sides

By factorization method

$$x^2 - 5x - x + 5 = 0$$

$$x(x - 5) - 1(x - 5) = 0$$

$$(x - 5)(x - 1) = 0$$

$$\Rightarrow x - 1 = 0 \quad \text{or} \quad x - 5 = 0$$

Adding 1 to both Adding 5 to both

sides

$$x - \sqrt{x} + \sqrt{x} = 0 + 1$$

$$\boxed{x = 1}$$

sides

$$x - \sqrt{x} + \sqrt{x} = 0 + 5$$

$$\boxed{x = 5}$$

Gives $x = 1$ or $x = 5$

Now it is necessary to check the solution in the original equation

$$\sqrt{2x - 1} = x - 1 \text{ -----(1)}$$

if $x = 1$ then $\sqrt{2(1) - 1} = \sqrt{1} - 1$

$$\sqrt{1} = 0$$

$$\boxed{1 \neq 0 \quad \text{False}}$$

$$\sqrt{2x - 1} = x - 2$$

if $x = 5$ then $\sqrt{2(5) - 1} = 5 - 2$

$$\sqrt{10 - 1} = 3$$

$$\sqrt{9} = 3$$

$$\boxed{3 = 3 \quad \text{True}}$$

On checking, we find that $x = 1$ is an extraneous root. Thus the solution set is $x = \{5\}$

Q. 3 $\sqrt{3x + 7} = 3x + 5$

Solution: $\sqrt{3x + 7} = 3x + 5 \text{ -----(A)}$

According to type-I

Squaring both sides of eq (A)

$$(\sqrt{3x + 7})^2 = (3x + 5)^2$$

$$\Rightarrow 3x + 7 = (3x)^2 + (5)^2 + 2(3x)(5)$$

$$\Rightarrow 3x + 7 = 9x^2 + 25 + 30x$$

Subtracting $3x + 7$ from both sides

$$\begin{aligned}
&\Rightarrow (\cancel{3}x + \cancel{7}) - (3\cancel{x} + \cancel{7}) \\
&= 9x^2 + 25 + 30x - (3x + 7) \\
&0 = 9x^2 + 25 + 30x - 3x - 7 \\
&0 = 9x^2 + 27x + 18 \\
&\Rightarrow 9x^2 + 27x + 18 = 0
\end{aligned}$$

Dividing both sides by 3

$$\begin{aligned}
&\Rightarrow \frac{\cancel{9}x^2}{\cancel{3}} + \frac{\cancel{27}x}{\cancel{3}} + \frac{\cancel{18}}{\cancel{3}} = \frac{0}{\cancel{3}} \\
&3x^2 + 9x + 6 = 0
\end{aligned}$$

So by factorization method

$$\begin{aligned}
&\Rightarrow 9x^2 + 6x + 3x + 6 = 0 \\
&\Rightarrow 3x(x + 2) + 3(x + 2) = 0 \\
&\Rightarrow (3x + 3)(x + 2) = 0
\end{aligned}$$

$$3x + 3 = 0$$

$$x + 2 = 0$$

So finding the value of "x"

Subtracting 2 from

$$3x + \cancel{3} - \cancel{3} = 0 - 3$$

both sides

$$3x = -3$$

$$x + \cancel{2} - \cancel{2} = 0 - 2$$

Dividing both sides by 3

$$x = -2$$

$$\cancel{3}x / \cancel{3} = \frac{-\cancel{3}}{\cancel{3}}$$

$$x = -1$$

Gives $\boxed{x = -1}$ and $\boxed{x = -2}$

Now it is necessary to check the solution in the original equation

$$\sqrt{3x + 7} = 3x + 5$$

$$\sqrt{3x + 7} = 3x + 5$$

If $x = -1$ then

If $x = -2$ then

$$\begin{array}{ll} \sqrt{3(-1) + 7} = 3(-1) + 5 & \sqrt{3(-2) + 7} = 3(-2) + 5 \\ \sqrt{-3 + 7} = -3 + 5 & \sqrt{-6 + 7} = -6 + 5 \\ \sqrt{4} = 2 & \sqrt{1} = -1 \\ \boxed{2 = 2} \quad \text{True} & \boxed{1 = -1} \quad \text{False} \end{array}$$

Hence our solution set is $\{-1\}$

Answer is $\boxed{\mathbf{x} = \{-1\}}$

Q. 4 $\sqrt{4x + 5} = 2x - 5$

Solution: $\sqrt{4x + 5} = 2x - 5$ -----(1)

According to type-I

Squaring both sides eq (1)

$$\begin{aligned} \Rightarrow (\sqrt{4x + 5})^2 &= (2x - 5)^2 \\ \Rightarrow 4x + 5 &= (2x)^2 + (5)^2 - 2(2x)(5) \\ \Rightarrow 4x + 5 &= 4x^2 + 25 - 20x \end{aligned}$$

Subtracting 4x from both sides

$$\Rightarrow \cancel{4x} \cancel{+ 5} = 4x^2 + 25 - 20x \cancel{- 4x}$$

Dividing both sides by 5

$$\Rightarrow 4x^2 - 24x + 20 = 0$$

Dividing both sides by 4

$$x^2 - 6x + 5 = 0$$

By factorization

$$\begin{aligned} \Rightarrow x^2 - 5x - x + 5 &= 0 \\ \Rightarrow x(x - 5) - 1(x - 5) &= 0 \\ (x - 1)(x - 5) &= 0 \\ \Rightarrow x - 1 = 0 &\Rightarrow x - 5 = 0 \\ x - \cancel{1} + \cancel{1} = 0 + 1 &\quad x - \cancel{5} + \cancel{5} = 0 + 5 \\ \boxed{x = 1} &\quad \boxed{x = 5} \end{aligned}$$

Gives $x = 6$ and $x = -3$

Now it is necessary to check the solution in the original equation

$$\begin{array}{ll} \sqrt{4x+5} = 2x-5 & \text{if } x = 5, \text{ then} \\ \text{If } x = 1, \text{ then} & \sqrt{4(5)+5} = 2(5) - 5 \\ \sqrt{4(1)+5} = 2(1) - 5 & \sqrt{25} = 10 - 5 \\ \boxed{\sqrt{10} = -3 \text{ False}} & \boxed{5 = 5 \text{ True}} \end{array}$$

So on checking we find that $x = 1$ is an extraneous root. Thus the solution set is $\{5\}$ answer.

Q. 5 $\sqrt{6x-5} = 3x-10$

Solution: $= \sqrt{6x-5} = 3x-10$

Taking square on both sides

$$\Rightarrow (\sqrt{6x-5})^2 = (3x-10)^2$$

$$\Rightarrow 6x-5 = (3x-10)^2$$

Using formula $(a-b)^2 = (a)^2 + (b)^2 - 2(a)(b)$

So $\Rightarrow 6x-5 = (3x)^2 + (10)^2 - 2(3x)(10)$

$$\Rightarrow 6x-5 = 9x^2 + 100 - 60x$$

Subtracting '6x' on both sides

$$\Rightarrow \cancel{6x} - \cancel{6x} - 5 = 9x^2 + 100 - 60x - 6x$$

$$\Rightarrow -5 = 9x^2 + 100 - 66x$$

Adding '5' to both sides

$$\Rightarrow -\cancel{5} + \cancel{5} = 9x^2 + 100 - 66x + 5$$

$$0 = 9x^2 + 105 - 66x + 5$$

Arrange the values

$$\Rightarrow 9x^2 - 66x + 105 = 0$$

Dividing both sides by '3'

$$\Rightarrow \frac{\cancel{9}x^2}{\cancel{3}} - \frac{\cancel{66}x}{\cancel{3}} + \frac{\cancel{105}}{\cancel{3}} = \frac{0}{3}$$

$$3x^2 - 22x + 35 = 0$$

Using quadratic formula

$$x = \frac{-b \pm \sqrt{(b)^2 - 4ac}}{2a}$$

Here

$$a = 3, \quad b = -22, \quad c = 35$$

$$x = \frac{-(-22) \pm \sqrt{(-22)^2 - 4(3)(35)}}{2(3)}$$

$$x = \frac{22 \pm \sqrt{480 - 420}}{6}$$

As

$$\sqrt{64} = \pm 8$$

So

$$x = \frac{22 \pm 8}{6} \Rightarrow x = \frac{22 + 8}{6}$$

$$x = \frac{30}{6} \Rightarrow x = \frac{\cancel{30}^5}{\cancel{6}^1}$$

$$x = 5$$

$$x = \frac{22 - 8}{6} \Rightarrow x = \frac{14}{6}$$

$$\boxed{x = \frac{7}{3}}$$

Now it is necessary to check solution in the original equation.

Which

$$\sqrt{6x - 5} = 3x - 10$$

Put

$$x = 5$$

$$\sqrt{6(5) - 5} = 3(5) - 10$$

$$\sqrt{30 - 5} = 15 - 10$$

$$\sqrt{25} = 5$$

$$\boxed{5 = 5} \text{ True}$$

Thus the solution set is $\{5\}$

If

$$x = \frac{7}{3}$$

$$\sqrt{6\left(\frac{7}{3}\right) - 5} = 3\left(\frac{7}{3}\right) - 10$$

$$\sqrt{\frac{42}{3} - 5} = \frac{21}{3} - 10$$

$$\sqrt{\frac{42-15}{3}} = \frac{21-30}{3}$$

$$\boxed{\sqrt{\frac{27}{3}} = \frac{-7}{3}} \text{ False}$$

Q. 6

$$\sqrt{29 - 4x} = 2x + 3$$

Solution:

According to type-I

$$\sqrt{29 - 4x} = 2x + 3 \text{----- (A)}$$

Squaring both sides eq (A)

$$\Rightarrow (\sqrt{29 - 4x})^2 = (2x + 3)^2$$

Expand by formula

$$\Rightarrow 29 - 4x = 4x^2 + 9 + 12x$$

Adding 4x to both sides

$$\Rightarrow 29 - \cancel{4x} + \cancel{4x} = 4x^2 + 9 + 12x + 4x$$

$$\Rightarrow 29 = 4x^2 + 16x + 9$$

Subtracting 29 to both sides

$$\Rightarrow \cancel{29} - \cancel{29} = 4x^2 + 16x + 9 - 29$$

$$\Rightarrow 0 = 4x^2 + 16x - 20$$

$$\Rightarrow 4x^2 + 16x - 20 = 0$$

Dividing both side by 2

$$\Rightarrow \frac{\cancel{4}x^2}{\cancel{2}} + \frac{\cancel{16}x}{\cancel{2}} - \frac{\cancel{20}}{\cancel{2}} = \frac{0}{\cancel{2}}$$

$$\Rightarrow 2x^2 + 8x - 10 = 0$$

Dividing both side 2

$$\Rightarrow \frac{\cancel{2}x^2}{\cancel{2}} + \frac{\cancel{8}x}{\cancel{2}} - \frac{\cancel{10}}{\cancel{2}} = \frac{\cancel{0}}{\cancel{2}}$$

$$\Rightarrow x^2 + 4x - 5 = 0$$

By factorization

$$\Rightarrow x^2 + 5x - x - 5 = 0$$

$$\Rightarrow x(x + 5) - 1(x + 5) = 0$$

$$\Rightarrow (x + 5)(x - 1) = 0$$

$$\Rightarrow x - 1 = 0 \quad \Rightarrow x + 5 = 0$$

Adding both sides 1 Subtracting both sides

$$x - \cancel{1} + \cancel{1} = 0 + 1$$

$$\boxed{x = 1}$$

$$5$$

$$x + \cancel{5} - \cancel{5} = 0 - 5$$

$$\boxed{x = -5}$$

Now it is necessary to check the solution in the original equation

$$\sqrt{29 - 4x} = 2x + 3$$

If $x = 1$ then we get

$$\sqrt{29 - 4(1)} = 2(1) + 3 \quad \sqrt{29 - 4(5)} = 2(5) + 3$$

$$\sqrt{25} = 5$$

$$\sqrt{9} = 10 + 3$$

$$\boxed{5 = 5} \text{ True}$$

$$\boxed{3 = 13} \text{ False}$$

On checking we find that $x = 5$ is an extraneous root. Thus the solution set is $\{6\}$

Q7. $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Solution: $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Taking square on both sides

$$\Rightarrow (\sqrt{x+7} + \sqrt{x+2})^2 = (\sqrt{6x+13})^2$$

Using formula $(a+b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow (\sqrt{x+7})^2 + (\sqrt{x+2})^2 + 2(\sqrt{x+7})(\sqrt{x+2}) = 6x + 13$$

$$\Rightarrow x + 7 + x + 2 + 2\sqrt{x^2 + 2x + 7x + 14} = 6x + 13$$

$$\Rightarrow 2x + 9 + 2\sqrt{x^2 + 9x + 14} = 6x + 13$$

Subtract $2x+9$ from both sides

$$\Rightarrow \cancel{2x} + \cancel{9} - \cancel{2x} - \cancel{9} + 2\sqrt{x^2 + 9x + 14} = 6x + 13 - 2x - 9$$

$$\Rightarrow 2\sqrt{x^2 + 9x + 14} = 4x + 4$$

Taking '4' common from R.H.S

$$\Rightarrow 2\sqrt{x^2 + 9x + 14} = 4(x + 1)$$

Dividing both sides by '2'

$$\Rightarrow \frac{2\sqrt{x^2 + 9x + 14}}{2} = \frac{4(x + 1)}{2}$$

$$\Rightarrow \sqrt{x^2 + 9x + 14} = 2(x + 1)$$

Again taking square on both sides

$$\Rightarrow (\sqrt{x^2 + 9x + 14})^2 = (2(x + 1))^2$$

$$\Rightarrow x^2 + 9x + 14 = 4(x^2 + 2x + 1)$$

$$\Rightarrow x^2 + 9x + 14 = 4x^2 + 8x + 4$$

Subtract $x^2 + 9x + 14$ to both sides

$$\Rightarrow \cancel{x^2} + \cancel{9x} + \cancel{14} - \cancel{x^2} - \cancel{9x} - \cancel{14} = 4x^2 + 8x + 4 - x^2 - 9x - 14$$

$$\Rightarrow 4x^2 - x^2 + 8x - 9x + 4x - 14 = 0$$

$$\Rightarrow 3x^2 - x - 10 = 0$$

By factorization

$$\Rightarrow 3x^2 - 6x + 5x - 10 = 0$$

$$\Rightarrow 3x(x - 2) + 5(x - 2) = 0$$

$$(3x + 5)(x - 2) = 0$$

$$x - 2 = 0$$

$$3x + 5 = 0$$

Adding '2' to both

Subtracting '5' from both

sides

$$x - \cancel{2} + \cancel{2} = 0 + 2$$

$$\boxed{x = 2}$$

sides

$$3x + \cancel{3} - \cancel{3} = 0 - 5$$

$$3x = -5$$

Dividing both sides by 3

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{-5}{3}$$

$$\boxed{x = \frac{-5}{3}}$$

Solution set is $\left\{2, -\frac{5}{3}\right\}$

Now to check the answer

$$\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$$

Taking L.H.S

$$\sqrt{x+7} + \sqrt{x+2}$$

When $x=2$

$$\begin{aligned} \Rightarrow & \sqrt{9} + \sqrt{4} \\ \Rightarrow & \sqrt{3^2} + \sqrt{2^2} \\ & = 3 + 2 \end{aligned}$$

$$\text{L.H.S} = 5$$

Now taking R.H.S

$$\Rightarrow \sqrt{6x+13}$$

When

$$\begin{aligned} x &= 2 \\ &= \sqrt{6(2)+13} \\ \Rightarrow & \sqrt{12+13} = \sqrt{25} \\ & \sqrt{5^2} = 5 \end{aligned}$$

L.H.S=R.H.S, hence the value of $x=2$ is true

Q8. $5x + \sqrt{3x+1} = \sqrt{5x+1}$

Solution: $5x + \sqrt{3x+1} = \sqrt{5x+1}$

Taking square on both sides

$$\Rightarrow (\sqrt{x} + \sqrt{3x+1})^2 = (\sqrt{5x+1})^2$$

Using formula $(a+b)^2 = a^2 + b^2 + 2ab$

$$\Rightarrow (\sqrt{x})^2 + (\sqrt{3x+1})^2 + 2(\sqrt{x})(\sqrt{3x+1}) = 5x + 1$$

$$\Rightarrow x + 3x + 1 + 2(\sqrt{3x^2 + x}) = 5x + 1$$

$$\Rightarrow 4x + 1 + 2(\sqrt{3x^2 + x}) = 5x + 1$$

Subtracting $4x+1$ to both sides

$$\Rightarrow \cancel{4x} + \cancel{1} - \cancel{4x} - \cancel{1} + 2(\sqrt{3x^2 + x}) = 5x + 1 - 4x - 1$$

$$\Rightarrow 2\sqrt{3x^2 + x} = 5x - 4x$$

$$\Rightarrow 2\sqrt{3x^2 + x} = x$$

Taking square on both sides

$$\Rightarrow (2\sqrt{3x^2 + x})^2 = (x)^2$$

$$\Rightarrow 4(3x^2 + x) = x^2$$

$$\Rightarrow 12x^2 + 4x = x^2$$

Subtracting x^2 from both sides

$$\Rightarrow 12x^2 - x^2 + 4x = \cancel{x^2} - \cancel{x^2}$$

$$11x^2 + 4x = 0$$

Taking 'x' common

$$x(11x + 4) = 0$$

Either $x=0$ OR $11x+4=0$

$$11x + 4 = 0$$

Subtract '4' from both sides

$$11x + \cancel{4} - \cancel{4} = 0 - 4$$

$$11x = -4$$

$$\frac{\cancel{11}x}{\cancel{11}} = -\frac{4}{11}$$

$$\boxed{x = -\frac{4}{11}}$$

Solution set $= \left\{0, \frac{-4}{11}\right\}$

Now to check the answer

$$\sqrt{x} + \sqrt{3x+1} = \sqrt{5x+1}$$

Taking L.H.S

$$\sqrt{x} + \sqrt{3x+1}$$

When

$$x=0$$

So,

$$\sqrt{0} + \sqrt{3(0)+1}$$

$$0 + \sqrt{0+1}$$

$$0+1=1$$

$$\text{L.H.S}=1$$

Now taking R.H.S

$$\sqrt{5x+1}$$

When

$$x = 0$$

$$\sqrt{5(0)+1}$$

$$\sqrt{0+1}=1$$

$$\text{R.H.S}=1$$

L.H.S=R.H.S, the value of $x=0$ is true

Q9. $\sqrt{6x+40} - \sqrt{x+21} = \sqrt{x+5}$

Solution: $\sqrt{6x+40} - \sqrt{x+21} = \sqrt{x+5}$

Taking square on both side

$$\Rightarrow (\sqrt{6x+40} - \sqrt{x+21})^2 = (\sqrt{x+5})^2$$

$$\Rightarrow (\sqrt{6x+40})^2 + (\sqrt{x+21})^2 - 2(\sqrt{6x+40})(\sqrt{x+21}) = (\sqrt{x+5})^2$$

$$\Rightarrow (\sqrt{6x+40})^2 + (\sqrt{x+21})^2 - 2(\sqrt{6x+40})(\sqrt{x+21}) = \sqrt{x+5}$$

$$\Rightarrow 6x + 40 + x + 21 - 2\left(\sqrt{(6x + 40)(x + 21)}\right) = \sqrt{x + 5}$$

$$\Rightarrow 7x + 61 - 2\left(\sqrt{6x^2 + 166x + 840}\right) = x + 5$$

Subtracting $x + 5$ from both sides

$$\Rightarrow 7x - x + 61 + 5 - 2\left(\sqrt{6x^2 + 166x + 840}\right) = x - x + 5 - 5$$

Adding $2\sqrt{6x^2 + 166x + 840}$ to both sides

$$\Rightarrow 7x - x + 61 + 5 - \cancel{2\sqrt{6x^2 + 166x + 840}} + \cancel{2\sqrt{6x^2 + 166x + 840}} = 2\sqrt{6x^2 + 166x + 840}$$

$$\Rightarrow 6x + 56 = 2\sqrt{6x^2 + 166x + 840}$$

Divide both sides by 2

$$\Rightarrow \frac{\cancel{6}x}{\cancel{2}} + \frac{\cancel{56}}{\cancel{2}} = \frac{\cancel{2}\sqrt{6x^2 + 166x + 840}}{\cancel{2}}$$

$$\Rightarrow 3x + 28 = \sqrt{6x^2 + 166x + 840}$$

Squaring both sides

$$\Rightarrow (3x + 28)^2 = \left(\sqrt{6x^2 + 166x + 840}\right)^2$$

$$\Rightarrow 9x^2 + 168x + 784 = 6x^2 + 166x + 840$$

Subtracting $6x^2$, $166x$ and 840 from both sides

$$\Rightarrow 9x^2 + 168x + 784 - 6x^2 - 166x - 840 = 9x^2 - 6x^2 + 168x - 166x + 784 - 840$$

$$\Rightarrow 3x^2 + 2x - 56 = 0 \text{ Here } a = 3, b = 2, c = -56$$

Now according to quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Putting these values in the above formula

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-56)}}{2(3)}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{4+672}}{6}$$

$$\Rightarrow x = \frac{-2 + \sqrt{676}}{6}$$

$$\Rightarrow x = \frac{-2 \pm 26}{6}$$

$$+x = \frac{-2+26}{6} = \frac{24}{6} = 4 \dots\dots (i)$$

$$-x = \frac{-2-26}{6} = \frac{-28}{6} = -\frac{14}{3} \dots\dots (ii)$$

$$\boxed{S.Set = \{4\}} \text{ Answer}$$

Verification:

$$\begin{aligned} & \sqrt{6x+40} - \sqrt{x+21} = \sqrt{x+5} \\ \text{Put } x = 4 & \quad \sqrt{6 \times 4 + 40} - \sqrt{4+21} = \sqrt{4+5} \\ & \sqrt{64} - \sqrt{25} = \sqrt{9} \\ & 8 - 5 = 3 \\ & 3 = 3 \end{aligned}$$

So, solution set = {4}

$\therefore x = -\frac{14}{3}$ Do not satisfy the given equation

Q10. $\sqrt{2x-3} + \sqrt{2x+4} = \sqrt{6x+13}$

Solution: $\sqrt{2x-3} + \sqrt{2x+4} = \sqrt{6x+13}$

Taking square on both sides

$$\left(\sqrt{2x-3} + \sqrt{2x+4}\right)^2 = \left(\sqrt{6x+13}\right)^2$$

As we know formula

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\Rightarrow \left(\sqrt{2x-3}\right)^2 + \left(\sqrt{2x+4}\right)^2 + 2\left(\sqrt{2x-3}\right)\left(\sqrt{2x+4}\right) = 6x+13$$

$$\Rightarrow \sqrt{2x-3} + \sqrt{2x+4} + 2\left(\sqrt{2x-3}\right)\left(\sqrt{2x+4}\right) = 6x+13$$

$$\Rightarrow 2x - 3 + 2x + 4 + \sqrt{4x^2 + 2x - 12} = 6x + 13$$

$$\Rightarrow 4x + 1 + 2\sqrt{4x^2 + 2x - 12} = 6x + 13$$

Subtracting $4x + 1$ from both sides

$$\Rightarrow 2\sqrt{4x^2 + 2x - 12} = 2x + 12$$

Dividing both sides by 2

$$\Rightarrow \frac{2\sqrt{4x^2 + 2x - 12}}{2} = \frac{2(x + 6)}{2}$$

$$\Rightarrow \sqrt{4x^2 + 2x - 12} = x + 6$$

Squaring both sides again, we get

$$\Rightarrow \left(\sqrt{4x^2 + 2x - 12}\right)^2 = (x + 6)^2$$

$$\Rightarrow 4x^2 + 2x - 12 = x^2 + 36 + 12x$$

$$\Rightarrow 4x^2 - x^2 + 2x - 12x - 12 - 36 = 0$$

$$\Rightarrow 3x^2 - 10x - 48 = 0$$

By factorization the equation

$$\Rightarrow 3x^2 - 18x + 10x - 48 = 0$$

$$\Rightarrow 3x(x - 6) + 8(x - 6) = 0$$

$$\Rightarrow (3x + 8)(x - 6) = 0$$

Either

$$1. \quad \Rightarrow 3x + 8 = 0$$

$$\Rightarrow \frac{3x}{3} = -\frac{8}{3}$$

$$x = -\frac{8}{3}$$

$$2. \quad x - 6 = 0$$

$$x - 6 + 6 = 0 + 6$$

$$x = 6$$

$x = -\frac{8}{3} \text{ OR } x = 6$

But on checking we find that $-\frac{8}{3}$ is an extraneous root.

Thus the solution set = {6}

Q11. $\sqrt{x^2 + 2x + 4} + \sqrt{x^2 + 2x + 9} = 5$

Solution: $\sqrt{x^2 + 2x + 4} + \sqrt{x^2 + 2x + 9} = 5$

Let $\sqrt{x^2 + 2x + 4} = a \rightarrow A$ and $\sqrt{x^2 + 2x + 9} = b = B$

So, equation becomes

$$a + b = 5 \rightarrow 1$$

$$\text{So, } a^2 = x^2 + 2x + 4 \text{ and } b^2 = x^2 + 2x + 9$$

$$a^2 - b^2 = (x^2 + 2x + 4) - (x^2 + 2x + 9)$$

$$a^2 - b^2 = \cancel{x^2} + \cancel{2x} + 4 - \cancel{x^2} - \cancel{2x} - 9$$

$$a^2 - b^2 = -5 \quad a^2 - b^2 = (a + b)(a - b)$$

$$\frac{\cancel{5}(a - b)}{\cancel{5}} = -\frac{\cancel{5}}{\cancel{5}} \Rightarrow a - b = -1 \rightarrow 2$$

Adding equation-1 and equation-2

$$a + b = 5$$

$$a - b = -1$$

$$2a = 4$$

$$\Rightarrow \frac{\cancel{2}a}{\cancel{2}} = \frac{\cancel{4}}{\cancel{2}}$$

$$a = 2$$

Taking equation-1 i.e. $a + b = 5$

Put the value of "a"

$2 + b = 5$ subtracting 2 from both sides

$$\cancel{2} - \cancel{2} + b = 5 - 2$$

$$\boxed{b = 3} \dots\dots (II)$$

Taking equation-A

$$a = \sqrt{x^2 + 2x + 4} \text{ put the value of "a"}$$

$$\sqrt{x^2 + 2x + 4} = 2$$

Taking square

$$= \left(\sqrt{x^2 + 2x + 4} \right)^2 = (2)^2$$

$$\Rightarrow x^2 + 2x + 4 = 4$$

Subtracting 4 from both sides

$$\Rightarrow x^2 + 2x + \cancel{4} - \cancel{4} = \cancel{4} - \cancel{4}$$

$$\Rightarrow x^2 + 2x = 0$$

$$x(x + 2) = 0$$

Either $x = 0$ OR $x + 2 = 0$

$$\boxed{x = -2} \text{ Answer}$$

Also taking equation B

$$b = \sqrt{x^2 + 2x + 9}$$

Put the value of "b"

$$3 = \sqrt{x^2 + 2x + 9}$$

Taking square on both sides

$$(3)^2 = \left(\sqrt{x^2 + 2x + 9} \right)^2$$

$$9 = x^2 + 2x + 9$$

Subtracting 9 from both sides

$$\cancel{9} - \cancel{9} = x^2 + 2x + \cancel{9} - \cancel{9}$$

$$\Rightarrow x^2 + 2x = 0$$

$$\Rightarrow x(x + 2) = 0$$

Either $x = 0$ OR $x + 2 = 0 \Rightarrow x = -2$

Now it is necessary to check the solution in the original equation

Verification:

Taking original equation

$$\sqrt{x^2 + 2x + 4} + \sqrt{x^2 + 2x + 9} = 5$$

First put $x = 0$

$$\sqrt{(0)^2 + 2(0) + 4} + \sqrt{(0)^2 + 2(0) + 9} = 5$$

$$\sqrt{4} + \sqrt{9} = 5$$

$$2 + 3 = 5$$

$$5 = 5$$

Hence Required Solution set is $\{0\}$

Q12. $\sqrt{2x^2 + 3x + 5} + \sqrt{2x^2 + 3x + 1} = 2$

Solution: $\sqrt{2x^2 + 3x + 5} + \sqrt{2x^2 + 3x + 1} = 2$

Suppose $\sqrt{2x^2 + 3x + 5} = a \dots\dots (I)$

$$\sqrt{2x^2 + 3x + 1} = b \dots\dots (II)$$

Squaring equation-I and equation-II

$$\Rightarrow \left(\sqrt{2x^2 + 3x + 5} \right)^2 = a^2$$

$$\Rightarrow \left(\sqrt{2x^2 + 3x + 1} \right)^2 = b^2$$

$$\Rightarrow 2x^2 + 3x + 5 = a^2 \dots\dots (III)$$

$$\Rightarrow 2x^2 + 3x + 1 = b^2 \dots\dots (IV)$$

Subtracting equation-IV from equation-III

$$\Rightarrow (2x^2 + 3x + 5) - (2x^2 + 3x + 1) = a^2 - b^2$$

$$\Rightarrow \cancel{2x^2} + \cancel{3x} + 5 - \cancel{2x^2} - \cancel{3x} - 1 = a^2 - b^2$$

$$4 = a^2 - b^2$$

$$(a + b)(a - b) = 4$$

$$(2)(a - b) = 4$$

$$a - b = 2 \dots\dots (A)$$

$$a + b = 2 \dots\dots (B)$$

Adding both equations

$$a - b = 2$$

$$\frac{a + b = 2}{2a = 4}$$

Put $a + 2$ in equation-A or equation-B

$$a - b = 2$$

$$2 - b = 2$$

$$\cancel{2} - \cancel{2} - b = \cancel{2} - \cancel{2}$$

$$b = 0$$

So

$$a = \sqrt{2x^2 + 3x + 5}$$

$$2 = \sqrt{2x^2 + 3x + 5}$$

Squaring both sides

$$(2)^2 = (\sqrt{2x^2 + 3x + 5})^2$$

$$\Rightarrow 4 = 2x^2 + 3x + 5$$

Subtracting 4 from both sides

$$\Rightarrow \cancel{4} - \cancel{4} = 2x^2 + 3x + 5 - 4$$

$$\Rightarrow 2x^2 + 3x + 1 = 0$$

By factorization

$$2x^2 + 2x + x + 1 = 0$$

$$2x(x+1) + 1(x+1) = 0$$

$$(2x+1)(x+1) = 0$$

$$\begin{array}{ll} 2x+1=0 & x+1=0 \\ \boxed{x = -\frac{1}{2}} & \boxed{x = -1} \end{array}$$

And

$$b = \sqrt{2x^2 + 3x + 1}$$

Put $b = 0$

$$0 = \sqrt{2x^2 + 3x + 1}$$

Taking square root on both sides

$$2x^2 + 3x + 1 = 0$$

By factorization

$$2x^2 + 2x + x + 1 = 0$$

$$2x(x+1)+1(x+1)=0$$

$$(2x+1)(x+1)=0$$

$$2x+1=0 \quad x+1=0$$

$$x = -\frac{1}{2}$$

$$x = -1$$

Hence our Required Answer is $\{-1\}$

Exercise No 1.4

Solve each of the following equation by factorization:

Q1. $3x^2+8x+5=0$

Solution: $3x^2+8x+5=0$

By sum and product Rule

$$\Rightarrow 3x^2+8x+5=0$$

$$\Rightarrow 3x^2+5x+3x+5=0$$

$$\Rightarrow x(3x+5)+1(3x+5)=0$$

$$\Rightarrow (x+1)(3x+5)=0$$

$$x+1=0$$

$$\Rightarrow 3x+5=0$$

Subtracting both sides

Subtracting both sides 5

$$1$$

$$3x + \cancel{5} - \cancel{5} = 0 - 5$$

$$x + \cancel{1} - \cancel{1} = 0 - 1$$

$$3x = -5 \text{ dividing both side } 3$$

$$x = -1$$

$$\cancel{3}x / \cancel{3} = -5 / 3$$

$$x = -\frac{5}{3}$$

Thus the solution of given Quadratic equation is -1, -5/3 and the solution set is $\{-1, -\frac{5}{3}\}$

Q2. $10x^2+19x-15=0$

Solution: $10x^2+19x-15=0$

By sum and product rule:

$$\begin{aligned}\Rightarrow 10x^2 + 25x - 6x - 15 &= 0 \\ \Rightarrow 5x(2x + 5) - 3(2x + 5) &= 0 \\ \Rightarrow (5x - 3)(2x + 5) &= 0\end{aligned}$$

$$\Rightarrow 5x - 3 = 0 \quad 2x + 5 = 0$$

Adding both sides 3 Subtracting both sides 5

$$5x - \cancel{3} + \cancel{3} = 0 + 3 \quad 2x + \cancel{5} - \cancel{5} = 0 - 5$$

$$5x = 3 \quad 2x = -5$$

Dividing both sides 5 Dividing both sides 2

$$\cancel{5}x / \cancel{5} = 3 / 5 \quad \cancel{2}x / \cancel{2} = -5 / 2$$

$$\boxed{x = 3/5}$$

$$\boxed{x = -5/2}$$

Thus the solution of given Quadratic equation is $3/5, -5/2$ and solution set is $\{3/5, -5/2\}$

Solve each of the following equation by completing square:

Q3. $9x^2 - 24x - 65 = 0$

Solution:

$$9x^2 - 24x - 65 = 0$$

Adding both sides 65

$$\Rightarrow 9x^2 - 24x - \cancel{65} + \cancel{65} = 0 + 65$$

$$\Rightarrow 9x^2 - 24x = 65$$

$$\boxed{\text{Note: } (a - b)^2 = a^2 + b^2 - 2ab}$$

$$(3x)^2 - 2(3x)(4) + (4)^2$$

Adding both sides 16

$$\Rightarrow 9x^2 - 24x + 16 = 65 + 16$$

$$\Rightarrow (3x)^2 - 2(3x)(4) + (4)^2 = 81$$

$$\Rightarrow (3x - 4)^2 = 81$$

Taking square root on both sides

$$\Rightarrow \sqrt{(3x - 4)^2} = \sqrt{81}$$

$$\Rightarrow 3x - 4 = \pm 9$$

$$3x - 4 = 9$$

$$3x - 4 = -9$$

Adding both sides

Adding both sides 4

$$4$$

$$3x - \cancel{4} + \cancel{4} = -9 + 4$$

$$3x - \cancel{4} + \cancel{4} = 9 + 4$$

$$3x = -5$$

$$3x = 13$$

Dividing both sides by 3

Dividing both

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{-5}{3}$$

sides 3

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{13}{3}$$

$$\boxed{x = -\frac{5}{3}}$$

$$\boxed{x = \frac{13}{3}}$$

Thus the solution set is $\left\{\frac{13}{3}, -\frac{5}{3}\right\}$

Q4. $4x^2 - 6x - 5 = 0$

Solution: $4x^2 - 6x - 5 = 0$

Adding 5 to both sides

$$\Rightarrow 4x^2 - 6x - \cancel{5} + \cancel{5} = 0 + 5$$

$$\Rightarrow 4x^2 - 6x = 5$$

$$\text{Note: } (a - b)^2 = a^2 + b^2 - 2ab$$

$$(2x)^2 - 2(2x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2$$

Adding $\left(\frac{3}{2}\right)^2$ or $\left(\frac{9}{4}\right)$ to both sides

$$\Rightarrow 4x^2 - 6x + \left(\frac{3}{2}\right)^2 = 5 + \frac{9}{4}$$

$$\Rightarrow (2x)^2 - 2(2x)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 = \frac{20+9}{4}$$

$$\Rightarrow \left(2x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

Taking square root on both sides

$$\Rightarrow \sqrt{\left(2x - \frac{3}{2}\right)^2} = \sqrt{\frac{29}{4}}$$

$$\Rightarrow 2x - \frac{3}{2} = \pm \frac{\sqrt{29}}{2}$$

$$\Rightarrow 2x = \pm \frac{\sqrt{29}}{2} + \frac{3}{2}$$

$$\Rightarrow 2x = \frac{3 \pm \sqrt{29}}{2} \Rightarrow x = \frac{3 \pm \sqrt{29}}{4}$$

Thus the solution set is

$$\left\{ \frac{3 + \sqrt{29}}{4}, \frac{3 - \sqrt{29}}{4} \right\}$$

Solve each of the following equation by Quadratic formula

Q5. $3x^2 + 8x + 4 = 0$

Solution: $3x^2 + 8x + 4 = 0$

comparing $3x^2 + 8x + 4 = 0$ with $ax^2 + bx + c = 0$

So $a = 3, b = 8, c = 4$

Using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ Putting the values}$$

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{64 - 48}}{6}$$

$$x = \frac{-8 \pm \sqrt{16}}{6} = \frac{-8 \pm 4}{6}$$

$$x = \frac{-8 + 4}{6}, \frac{-8 - 4}{6}$$

$$x = -\frac{4}{6}, -\frac{12}{6} \text{ or}$$

$$x = -\frac{2}{3}, -2$$

Thus the solution set is

$$\left\{ -\frac{2}{3}, -2 \right\}$$

Q6. $4x^2 + x - 3 = 0$

Solution: $4x^2 + x - 3 = 0$

comparing $4x^2 + x - 3 = 0$ with $ax^2 + bx - c = 0$

So $a = 4, b = 1, c = -3$

Now using the quadratic formula

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{8} = \frac{-1 \pm \sqrt{49}}{8}$$

$$x = \frac{-1 \pm 7}{8}$$

$$x = \frac{-1 + 7}{8}, \frac{-1 - 7}{8}$$

$$x = \frac{6}{8}, -\frac{8}{8} \quad \text{or}$$

$$x = \frac{3}{4}, -1$$

Thus the solution set of the given quadratic equation is $\left\{\frac{3}{4}, -1\right\}$

Solve the following equations:

Q7. $x^4 - 9x^2 + 14 = 0$

Solution: $x^4 - 9x^2 + 14 = 0$ -----(1)

Now according to type-II

Suppose $x^2 = y$

$$\Rightarrow x^4 - 9x^2 + 14 = 0$$

$$\Rightarrow (x^2)^2 - 9(x^2) + 14 = 0$$

$$\Rightarrow (y)^2 - 9(y) + 14 = 0$$

Now by factorization

$$\Rightarrow y^2 - 7y - 2y + 14 = 0$$

$$\Rightarrow y(y - 7) - 2(y - 7) = 0$$

$$\Rightarrow (y - 2)(y - 7) = 0$$

$$y - 2 = 0$$

$$y - 7 = 0$$

Adding both sides 2

Adding both sides 7

$$y - \cancel{2} + \cancel{2} = 0 + 2$$

$$y - \cancel{7} + \cancel{7} = 0 + 7$$

$$y = 2$$

$$y = 7$$

Replace y by x^2

Replace y by x^2

$$x^2 = 2$$

$$x^2 = 7$$

Taking square root
on both sides

Taking square root
on both sides

$$\sqrt{x^2} = \sqrt{2}$$

$$\sqrt{x^2} = \sqrt{7}$$

$$\boxed{x = \pm\sqrt{2}}$$

$$\boxed{x = \pm\sqrt{7}}$$

So the solution set is $\{\pm\sqrt{2}, \pm\sqrt{7}\}$

Q8. $3x^4 - 13x^2 + 10 = 0$

Solution: $3x^4 - 13x^2 + 10 = 0$

According to type-II

Suppose

$$x^2 = y$$

$$\Rightarrow 3x^4 - 13x^2 + 10 = 0$$

$$\Rightarrow 3(x^2)^2 - 13(x^2) + 10 = 0$$

$$\Rightarrow 3(y)^2 - 13(y) + 10 = 0$$

$$\Rightarrow 3y^2 - 13y + 10 = 0$$

By factorization

$$\Rightarrow 3y^2 - 10y - 3y + 10 = 0$$

$$\Rightarrow y(3y - 10) - 1(3y - 10) = 0$$

$$\Rightarrow (y - 1)(3y - 10) = 0$$

$$y - 1 = 0$$

OR

$$3y - 10 = 0$$

Adding both sides 1

$$y - 1 + 1 = 0 + 1$$

$$y = 1$$

Replace y by x^2

$$x^2 = 1$$

Taking square root

both sides

$$\sqrt{x^2} = \sqrt{1}$$

$$\boxed{x = \pm 1}$$

Adding both sides 10

$$3y - 10 + 10 = 0 + 10$$

$$\frac{3y}{3} = \frac{10}{3} \text{ dividing both sides by } 3$$

$$y = \frac{10}{3}$$

Replace y by x^2

$$x^2 = \frac{10}{3}$$

Taking square root both sides

$$\sqrt{x^2} = \sqrt{\frac{10}{3}}$$

$$\boxed{x = \pm \sqrt{10/3}}$$

Thus the solution set is $\left\{\pm 1, \pm \sqrt{10/3}\right\}$

Q9.
$$\frac{x+1}{x} - \frac{x}{x+1} = \frac{5}{6}$$

Solution:
$$\frac{x+1}{x} - \frac{x}{x+1} = \frac{5}{6} \text{-----(1)}$$

Suppose
$$\frac{x+1}{x} = y \text{ then } \frac{x}{x+1} = \frac{1}{y}$$

So
$$y - \frac{1}{y} = \frac{5}{6}$$

Put in equation-(1) y and $\frac{1}{y}$

Multiplying both sides by 6y

$$\Rightarrow 6y \left(y - \frac{1}{y} \right) = \frac{5}{6} \times 6y$$

$$\Rightarrow 6y^2 - \frac{6\cancel{y}}{\cancel{y}} = 5y$$

$$6y^2 - 6 = 5y$$

Subtracting 5y from both sides

$$\Rightarrow 6y^2 - 6 - 5y = \cancel{5y} - \cancel{5y}$$

$$\Rightarrow 6y^2 - 5y - 6 = 0$$

By factorization method

$$\Rightarrow 6y^2 - 9y + 4y - 6 = 0$$

$$\Rightarrow 3y(2y - 3) + 2(2y - 3) = 0$$

$$\Rightarrow (3y + 2)(2y - 3) = 0$$

$$\Rightarrow 3y + 2 = 0$$

$$\Rightarrow 3y + \cancel{2} - \cancel{2} = 0 - 2$$

$$y = -\frac{2}{3}$$

$$\text{put } y = \frac{x+1}{x}$$

$$\frac{x+1}{x} = -\frac{2}{3}$$

By cross multiplication

$$\Rightarrow (x+1) 3 = -2 \times x$$

$$\Rightarrow 3x + 3 = -2x$$

$$\Rightarrow 3x + 3 + 2x = -\cancel{2x} + \cancel{2x}$$

$$\Rightarrow x + 3 = 0$$

$$\Rightarrow x + \cancel{3} - \cancel{3} = 0 - 3$$

$$\boxed{x = -\frac{3}{5}}$$

$$2y + 3 = 0$$

$$2y + \cancel{3} - \cancel{3} = 0 - 3$$

$$y = -\frac{3}{2}$$

$$\text{put } y = \frac{x+1}{x}$$

$$\frac{x+1}{x} = -\frac{3}{2}$$

By cross multiplication

$$\Rightarrow (x+1) 2 = -3 \times x$$

$$\Rightarrow 2x + 2 = -3x$$

$$\Rightarrow 2x + 2 - 3x = -\cancel{3x} + \cancel{3x}$$

$$\Rightarrow -x + 2 = 0$$

$$\Rightarrow -x + \cancel{2} - \cancel{2} = 0 - 2$$

$$\boxed{x = 2}$$

Thus the solution set of given Quadratic equation is $\{-\frac{3}{5}, 2\}$

Q10. $\frac{x+2}{x-2} - \frac{x-2}{x+2} = \frac{5}{6}$

Solution: $\frac{x+2}{x-2} - \frac{x-2}{x+2} = \frac{5}{6}$ -----(1)

Suppose $\frac{x+2}{x-2} = y$ then $\frac{x-2}{x+2} = \frac{1}{y}$

$$\frac{x+2}{x-2} - \frac{x-2}{x+2} = \frac{5}{6}$$

$$y - \left(\frac{1}{y}\right) = \frac{5}{6}$$

Multiplying both sides by

$$y \left(y - \frac{1}{y} \right) = \frac{\cancel{6}y \times 5}{\cancel{6}}$$

$$\Rightarrow 6y^2 - \frac{6\cancel{y}}{\cancel{y}} = 5y$$

$$\Rightarrow 6y^2 - 5y - 6 = 0$$

By factorization

$$\Rightarrow 6y^2 - 9y + 4y - 6 = 0$$

$$\Rightarrow 3y(2y - 3) + 2(2y - 3) = 0$$

$$\Rightarrow (3y + 2)(2y - 3) = 0$$

$$3y + 2 = 0$$

$$2y - 3 = 0$$

subtracting both sides 2

adding both sides

$$3y + \cancel{2} - \cancel{2} = 0 - 2$$

$$2y - \cancel{3} + \cancel{3} = 0 + 3$$

$3y = -2$ divide both sides by 3

$2y = 3$ divide both sides by 2

$$\frac{3y}{3} = \frac{-2}{3}$$

$$\frac{2y}{2} = \frac{3}{2}$$

$$y = -\frac{2}{3}$$

$$y = \frac{3}{2}$$

$$\text{put } y = -\frac{2}{3}$$

$$\text{put } y = \frac{3}{2}$$

$$\frac{x+2}{x-2} = -\frac{2}{3}$$

$$\frac{x+2}{x-2} = \frac{3}{2}$$

by cross multiplication

by cross multiplication

$$\Rightarrow 3(x+2) = -2(x-2)$$

$$\Rightarrow 2(x+2) = 3(x-2)$$

$$\Rightarrow 3x + 6 = -2x + 4$$

$$\Rightarrow 2x + 6 = 3x - 6$$

$$\Rightarrow 3x + 2x = 4 - 6$$

$$\Rightarrow 2x - 3x = -6 - 6$$

$$5x = -2$$

$$-x = -12$$

$$\boxed{x = \frac{2}{5}}$$

$$\boxed{x = 12}$$

Hence the required answer $\left\{ \frac{2}{5}, 12 \right\}$

Q11. $\left(x^2 - \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$

Solution: $\left(x^2 - \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0 \text{-----(A)}$

$$\Rightarrow \left(x^2 - \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

Taking square on both sides

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = y^2$$

$$\Rightarrow (x)^2 + \left(\frac{1}{x}\right)^2 - 2(x)\left(\frac{1}{x}\right) = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} - 2 = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 + 2 \text{-----(II)}$$

Adding "2" to both sides

putting these values in equation (A)

$$\Rightarrow \left(x^2 - \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

$$\Rightarrow (y^2 + 2) - 3(y) - 2 = 0$$

$$\Rightarrow y^2 + 2 - 3(y) - 2 = 0$$

$$\Rightarrow y^2 + 3y = 0$$

$$\Rightarrow y(y + 3) = 0$$

$$\Rightarrow y = 0$$

$$y = 0$$

$$\Rightarrow y + 3 = 0$$

$$y = -3$$

Putting $y = x - \frac{1}{x}$

$$x - \frac{1}{x} = 0$$

Putting $y = x - \frac{1}{x}$

$$x - \frac{1}{x} = -3$$

Multiplying both sides by x Multiplying both sides by x

$$\Rightarrow x \left(x - \frac{1}{x} \right) = 0 \times x \quad \Rightarrow x \left(x - \frac{1}{x} \right) = -3 \times x$$

$$\Rightarrow x^2 - 1 = 0 \quad \Rightarrow x^2 - 1 = -3x$$

$$\Rightarrow x^2 - \cancel{x} + \cancel{x} = 0 + 1 \quad \Rightarrow x^2 - \cancel{x} + \cancel{x} = -3x + 1$$

$$x^2 = 1 \quad x^2 + 3x - 1 = 0$$

Taking square root both sides (a=1, b=3, c=-1)

$$\sqrt{x^2} = \sqrt{1}$$

$$\boxed{x = \pm 1}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-1)}}{2(1)}$$

$$\boxed{x = \frac{-3 \pm \sqrt{13}}{2}}$$

$$\text{Solution set} = \left\{ 1, -1, \frac{-3 + \sqrt{13}}{2}, \frac{-3 - \sqrt{13}}{2} \right\}$$

Q12. $\left(x^2 - \frac{1}{x^2} \right) - 10 \left(x + \frac{1}{x} \right) + 18 = 0$ -----(A)

Solution: $\left(x^2 - \frac{1}{x^2} \right) - 10 \left(x + \frac{1}{x} \right) + 18 = 0$

Suppose $x + \frac{1}{x} = y$

Taking square both sides

$$\left(x + \frac{1}{x} \right)^2 = y^2$$

$$\Rightarrow (x)^2 + \left(\frac{1}{x} \right)^2 + 2(x) \left(\frac{1}{x} \right) = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} + \cancel{2} - \cancel{2} = y^2 - 2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Putting these values in (A)

$$\Rightarrow (y^2 - 2) - 10(y) + 18 = 0$$

$$\Rightarrow y^2 - 2 - 10y + 18 = 0$$

$$\Rightarrow y^2 - 10y + 16 = 0$$

By factorization method

$$\Rightarrow y^2 - 8y - 2y + 16 = 0$$

$$\Rightarrow y(y - 8) - 2(y - 8) = 0$$

$$\Rightarrow (y - 8)(y - 2) = 0$$

$$y - 2 = 0 \quad \text{OR} \quad y - 8 = 0$$

$$y - \cancel{2} + \cancel{2} = 0 + 2 \quad y - \cancel{8} + \cancel{8} = 0 + 8$$

$$\boxed{y = 8}$$

$$\boxed{y = 2}$$

$$\text{Put } y = x + \frac{1}{x}$$

$$\text{Put } y = x + \frac{1}{x}$$

$$\Rightarrow x + \frac{1}{x} = 2$$

$$\Rightarrow x + \frac{1}{x} = 8$$

$$\Rightarrow x^2 + 1 = 2x$$

$$\Rightarrow x^2 + 1 = 8x$$

$$\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x^2 - 8x + 1 = 0$$

$$\Rightarrow (x - 1)^2 = 0$$

$$a = 1, b = -8, c = 1$$

$$\Rightarrow (x - 1)(x - 1) = 0$$

Using quadratic formula

$$\Rightarrow x - 1 = 0, \\ x = 1,$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(1)}}{2(1)}$$

$$\Rightarrow x - 1 = 0$$

$$x = \frac{8 \pm \sqrt{60}}{2} = \frac{8 \pm \sqrt{15 \times 4}}{2}$$

$$\boxed{x = 1}$$

$$x = \frac{8 \pm 2\sqrt{15}}{2} = \frac{2(4 \pm \sqrt{15})}{2}$$

$$\boxed{x = 4 \pm \sqrt{15}}$$

Hence the solution set of given Quadratic equation is

$$S.S = \{1, 1, 4 + \sqrt{15}, 4 - \sqrt{15}\}$$

Q13. $2^{2x+3} - 9 \cdot 2^x + 1 = 0$

Solution: $2^{2x+3} - 9 \cdot 2^x + 1 = 0$

$$\Rightarrow 2^{2x+3} - 9 \cdot 2^x + 1 = 0$$

$$\Rightarrow 2^{2x} \cdot 2^3 - 9(2)^x + 1 = 0$$

$$\Rightarrow (2^x)^2 \cdot (2 \times 2 \times 2) - 9(2)^x + 1 = 0$$

$$\Rightarrow 8(2^x)^2 - 9(2^x) + 1 = 0$$

$$\Rightarrow 8(y)^2 - 9(y) + 1 = 0 \quad \text{Suppose } 2^x = y$$

$$\Rightarrow 8y^2 - 9y + 1 = 0$$

By factorization

$$\Rightarrow 8y^2 - 8y - y + 1 = 0$$

$$\Rightarrow 8y(y - 1) - 1(y - 1) = 0$$

$$\Rightarrow (8y - 1)(y - 1) = 0$$

$$8y - 1 = 0 \quad y - 1 = 0$$

$$8y - 1 + 1 = 0 + 1 \quad y - 1 + 1 = 0 + 1$$

$$\boxed{y = \frac{1}{8}}$$

$$\boxed{y = 1}$$

$$\text{put } y = (2)^x$$

$$(2)^x = 1$$

$$(2)^x = \frac{1}{8}$$

$$\text{put } x = 0$$

$$(2)^x = 8^{-1}$$

$$(2)^0 = 1 \quad 1 = 1$$

$$\boxed{x = 0}$$

$$(2)^x = (2)^{-3}$$

$$\boxed{x = -3}$$

Thus solution set S.S = $\{0, -3\}$

Q14. $4^{1+x} + 4^{1-x} = 10$

Solution:

$$4^{1+x} + 4^{1-x} = 10$$

$$\Rightarrow (4)^1 \cdot (4)^x + (4)^1 \cdot (4)^{-x} = 10$$

$$\Rightarrow (4) \cdot (4^x) + (4)(4^x)^{-1} = 10$$

put $4^x = y$

$$\Rightarrow (4) \cdot (y) + (4)(y)^{-1} = 10$$

$$\Rightarrow 4y + \frac{4}{y} = 10$$

Multiplying both sides by y

$$y \left(4y + \frac{4}{y} \right) = 10y$$

$$4y^2 + 4 = 10y$$

$$\Rightarrow 4y^2 - 10y + 4 = 0 \text{ (By factorization)}$$

$$\Rightarrow 4y^2 - 8y - 2y + 4 = 0$$

$$\Rightarrow 4y(y - 2) - 2(y - 2) = 0$$

$$\Rightarrow (4y - 2)(y - 2) = 0$$

$$4y - 2 = 0$$

$$y - 2 = 0$$

$$4y - 2 + 2 = 0 + 2$$

$$y - 2 + 2 = 0 + 2$$

$$4y = 2$$

$$y = 2$$

$$y = \frac{2}{4} = \frac{1}{2}$$

$$y = 2$$

$$(4)^x = 2$$

Now put $y = (4)^x$

$$(2 \times 2)^x = (2)^1$$

$$\begin{aligned}
 y &= \frac{1}{2} & (2)^{2x} &= (2)^{-1} \\
 (4)^x &= (2)^{-1} & 2x &= 1 \\
 (2 \times 2)^x &= (2)^{-1} & \text{Dividing both sides by} & \\
 (2)^{2x} &= (2)^{-1} & 2 & \\
 2x &= -1 & \frac{2x}{2} &= \frac{1}{2} \\
 \text{Divide both sides by} & & \boxed{x = -\frac{1}{2}} & \\
 2 & & & \\
 \frac{2x}{2} &= -\frac{1}{2} & & \\
 \boxed{x = -\frac{1}{2}} & & &
 \end{aligned}$$

Thus the solution set $\left\{-\frac{1}{2}, \frac{1}{2}\right\}$

Q15. $(x+1)(x+3)(x+5)(x+7) + 16 = 0$

Solution: $(x+1)(x+3)(x+5)(x+7) + 16 = 0$

General form $(x+a)(x+b)(x+c)(x+d) = E$

$$a + b = c + d$$

Then we make pair to these

Hence, $a = 1, b = 3, c = 5, d = 7$

$$a + d = b + c$$

$$1 + 7 = 3 + 5$$

$$8 = 8$$

So, $[(x+1)(x+7)][(x+3)(x+5)] + 16 = 0$

$$\Rightarrow (x^2 + 7x + x + 7)(x^2 + 3x + 5x + 15) + 16 = 0$$

$$\Rightarrow (x^2 + 8x + 7)(x^2 + 8x + 15) + 16 = 0$$

Suppose $x^2 + 8x = y$

$$\begin{aligned}
&\Rightarrow (y+7)(y+15) + 16 = 0 \\
&\Rightarrow y^2 + 15y + 7y + 105 + 16 = 0 \\
&\Rightarrow y^2 + 22y + 121 = 0 \text{ By factorization} \\
&\Rightarrow y^2 + 11y + 11y + 121 = 0 \\
&\Rightarrow y(y+11) + 11(y+11) = 0 \\
&\Rightarrow (y+11)(y+11) = 0 \\
&\quad y+11 = 0 \qquad y+11 = 0 \\
&\quad y+11 - 11 = 0 - 11 \quad y+11 - 11 = 0 - 11 \\
&\quad \boxed{y = -11} \qquad \boxed{y = -11}
\end{aligned}$$

$$\begin{aligned}
\text{Put } y &= x^2 + 8x \\
&\Rightarrow x^2 + 8x = -11 \\
&\Rightarrow x^2 + 8x + 11 = 0 \\
&\quad a = 1, b = 8, c = 11
\end{aligned}$$

Using Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Putting these values

$$\begin{aligned}
x &= \frac{-8 \pm \sqrt{64 - 4(1)(11)}}{2(1)} \\
x &= \frac{-8 \pm \sqrt{64 - 44}}{2} \\
x &= \frac{-8 \pm \sqrt{20}}{2} \\
x &= \frac{-8 \pm \sqrt{5 \times 4}}{2} = \frac{-8 \pm \sqrt{4} \times \sqrt{5}}{2} \\
&\quad \boxed{x = -4 \pm \sqrt{5} \quad S.S}
\end{aligned}$$

Solve the following Radical Equation

16. $\sqrt{x+1} + \sqrt{x+9} = \sqrt{x+16}$

Solution: $\sqrt{x+1} + \sqrt{x+9} = \sqrt{x+16}$

Squaring on both sides

$$\Rightarrow (\sqrt{x+1} + \sqrt{x+9})^2 = (\sqrt{x+16})^2$$

Expand by formula

$$\begin{aligned} (a+b)^2 &= a^2 + b^2 + 2ab \\ \Rightarrow (\sqrt{x+1})^2 + (\sqrt{x+9})^2 + 2(\sqrt{x+1})(\sqrt{x+9}) &= x+16 \\ \Rightarrow x+1+x+9+2(\sqrt{x^2+9x+x+9}) &= x+16 \\ \Rightarrow 10+x+2(\sqrt{x^2+10x+9}) &= 16 \\ \Rightarrow 2(\sqrt{x^2+10x+9}) &= 16-10-x \\ \Rightarrow 2(\sqrt{x^2+10x+9}) &= (6-x) \end{aligned}$$

Squaring again we should get

$$\begin{aligned} \Rightarrow (2)^2 (\sqrt{x^2+10x+9})^2 &= (6-x)^2 \\ \Rightarrow 4(x^2+10x+9) &= (36+x^2-12x) \\ \Rightarrow 4x^2+40x+36 &= 36+x^2-12x \\ \Rightarrow 3x^2+56x &= 0 \\ 3x+56 &= 0 \\ \boxed{x=0} \quad \boxed{x = -56/3} \end{aligned}$$

Verification:

Given that:

Original equation $\sqrt{x+1} + \sqrt{x+9} = \sqrt{x+16}$

if $x = 0$ then

$$\sqrt{0+1} + \sqrt{0+9} = \sqrt{0+16}$$

$$\sqrt{1} + \sqrt{9} = \sqrt{16}$$

$$1 + 3 = 4$$

$$\boxed{4 = 4 \quad \text{TRUE}}$$

if $x = -\frac{56}{3}$ then

$$\sqrt{-\frac{56}{3} + 1} + \sqrt{-\frac{56}{3} + 9} = \sqrt{-\frac{56}{3} + 16}$$

False

Answer is $\boxed{\{0\}}$

Q17. $\sqrt{x-1} + \sqrt{x-6} = \sqrt{x+15}$

Solution: $\sqrt{x-1} + \sqrt{x-6} = \sqrt{x+15}$

Squaring on both sides

$$\Rightarrow (\sqrt{x-1} + \sqrt{x-6})^2 = (\sqrt{x+15})^2$$

$$\Rightarrow (\sqrt{x-1})^2 + (\sqrt{x-6})^2 + 2(\sqrt{x-1})(\sqrt{x-6}) = x+15$$

$$\Rightarrow x-1 + x-6 + 2\sqrt{(x-1)(x-6)} = x+15$$

$$\Rightarrow x-7 + 2\sqrt{(x^2-7x+6)} = 15$$

$$\Rightarrow 2\sqrt{(x^2-7x+6)} = 22-x$$

Again square on both sides

$$\Rightarrow \left(2\sqrt{(x^2-7x+6)}\right)^2 = (22-x)^2$$

$$\Rightarrow 4(x^2-7x+6) = (22)^2 + (x)^2 - 2(22)(x)$$

$$\Rightarrow 4x^2 - 28x + 24 = 484 + x^2 - 44x$$

$$\Rightarrow 4x^2 - 28x + 24 - 484 - x^2 + 44x = 0$$

$$\Rightarrow 3x^2 - 16x + 460 = 0$$

By Quadratic Formula

$$a = 3, b = 16, c = -460$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(16) \pm \sqrt{(16)^2 - 4(3)(-460)}}{2(3)}$$

$$x = \frac{-16 \pm \sqrt{256 + 5520}}{6}$$

$$x = \frac{-16 \pm \sqrt{5776}}{6} = \frac{-16 \pm 76}{6}$$

$$x = \frac{-16 + 76}{6}, \quad x = \frac{-16 - 76}{6}$$

$$x = \frac{\cancel{60}}{\cancel{6}}, \quad x = -\frac{\cancel{92}}{\cancel{6}}$$

$$\boxed{x = 10} \quad \boxed{x = -\frac{46}{3}}$$

Verification:

Original equation $\sqrt{x-1} + \sqrt{x-6} = \sqrt{x+15}$

if $x = 10$ then $\sqrt{10-1} + \sqrt{10-6} = \sqrt{10+15}$

Thus it is true that $x = 10$

$$\sqrt{9} + \sqrt{4} = \sqrt{25}$$

$$3 + 2 = 5$$

$$\boxed{5 = 5}$$

$$\boxed{S.S = \{10\}} \quad \textbf{Answer}$$

Q18.

$$\sqrt{x^2 + 5x + 1} + \sqrt{x^2 + 5x + 2} = 1$$

Solution:

$$\sqrt{x^2 + 5x + 1} + \sqrt{x^2 + 5x + 2} = 1$$

Suppose

$$x^2 + 5x = y \text{ so}$$

$$\Rightarrow \sqrt{y+1} + \sqrt{y+2} = 1$$

Squaring on both sides

$$\begin{aligned}\Rightarrow & \left(\sqrt{y+1} + \sqrt{y+2} \right)^2 = (1)^2 \\ \Rightarrow & \left(\sqrt{y+1} \right)^2 + \left(\sqrt{y+2} \right)^2 + 2\left(\sqrt{y+1} \right)\left(\sqrt{y+2} \right) = 1 \\ \Rightarrow & y+1+y+2+2\sqrt{(y+1)(y+2)} = 1 \\ \Rightarrow & 2y+3+2\sqrt{y^2+3y+2} = 1 \\ \Rightarrow & 2\sqrt{y^2+3y+2} = -2y\end{aligned}$$

Again square on both sides

$$\begin{aligned}\Rightarrow & \left(\sqrt{y^2+3y+2} \right)^2 = (y)^2 \\ \Rightarrow & y^2+3y+2 = y^2 \\ \Rightarrow & \cancel{y^2} + 3y + 2 - \cancel{y^2} = 0 \\ \Rightarrow & 3y + 2 = 0\end{aligned}$$

Put

$$\begin{aligned}y &= x^2 + 5x \\ 3y + 2 &= 0 \\ 3(x^2 + 5x) + 2 &= 0 \\ 3x^2 + 15x + 2 &= 0\end{aligned}$$

let

$$a = 3, b = 15, c = 2$$

Using Quadratic formula

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(15) \pm \sqrt{(15)^2 - 4(3)(2)}}{2(3)} \\ x &= \frac{-15 \pm \sqrt{225 - 24}}{6} \\ x &= \frac{-15 \pm \sqrt{201}}{6}\end{aligned}$$

Hence Required Answer

$$x = \frac{-15 \pm \sqrt{201}}{6}$$

Exercise 1.5

MCQ's

I. Write the correct option in the given box:

1. The factors $x^2 - 8x - 20$ are b
 - a. $(x+10)(x-2)$
 - b. $(x-10)(x+2)$
 - c. $(x-5)(x+4)$
 - d. $(x+5)(x-4)$
2. Which expression is a perfect square? d
 - a. $x^2 + 4$
 - b. $y^2 - 9$
 - c. $x^2 + 4x + 1$
 - d. $x^2 + \frac{1}{x^2} - 2$
3. $\sqrt{2x+3} = 4$ is called c
 - a. Quadratic equation
 - b. linear equation
 - c. Radical equation
 - d. none of these
4. The sum of $x + 1$ and the reciprocal $x - 1$ is c
 - a. $\frac{x}{x-1}$
 - b. $\frac{x}{x+1}$
 - c. $\frac{x^2}{x-1}$
 - d. $\frac{x^2}{x+1}$
5. The common solution of the quadratic equations $x^2 - 7x + 10 = 0$ and $x^2 - 10x + 16 = 0$ is d
 - a. - 2
 - b. 3
 - c. 5
 - d. 2
6. Which equation has exactly two solutions? d
 - a. $2x + 3 = 0$
 - b. $2^x = 0$
 - c. $\frac{1}{x} - 1 = 0$
 - d. $x^2 + 2x - 1 = 0$

III. If the statement is correct, write True otherwise write False

1. Every quadratic equation can be solved by factorization ☐ F
2. Every quadratic equation can be solved by completing the square. ☐ T
3. Every quadratic equation can be solved using quadratic formula. ☐ T
4. $ax^2 + bx + c = 0$ is called the quadratic equation in x if $x \neq 0$ and a, b, c are real numbers. ☐ T
5. The equation $x - \frac{3}{x-1} = 1 - \frac{3}{x-1}$ has on solution. ☐ F