Unit 9

# DIFFERENTIAL EQUATIONS

Differential equation is an equation containing the derivative of an unknown function (dependent variable) of one or more independent variables for example.

$$\frac{dy}{dx} = xy \qquad y(x) = ?$$

$$x\frac{dy}{dx} + y = 0 \qquad y(x) = ?$$

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + xy = 0 \qquad y(x) = ?$$

- The order of a differential equation is the order of the highest-derivative in the equation.

$$\frac{dy}{dx} + xy = 0 \qquad \text{order} = 1$$

$$\frac{d^2y}{dx^2} + x^2 = 0 \qquad \text{order} = 2$$

- Degree of a differential equation is the power of the highest-order derivative in the equation.

$$\frac{dy}{dx} = \frac{y+x}{y-x} \qquad \text{degree} = 1$$

$$\left(\frac{dy}{dx}\right)^2 + x^2\frac{dy}{dx} = 1 \qquad \text{degree} = 2$$

Perfect24u.com

**Solution of a differential equitation:**

A solution of a differential equation is a functions which satisfy the differential equation.

For example, the solutions of differential equation

$$\frac{dy}{dx} - y = 0 \qquad \rightarrow \text{(i) is}$$



www.perfect24u.com

$y = ce^x \qquad \rightarrow (ii)$

As $\frac{dy}{dx} = ce^x$ and

$L.H.S = \frac{dy}{dx} - y = ce^x - ce^x = 0$ R.H.S

**General solution:**

Solution of the form given in equation (ii) above is called the general solution of the differential equation in (i) as it contains the **arbitrary constant c**.

The arbitrary constant can be determined by using an initial condition, such as $y(0)=2$.

**Particular solution:**

Using the initial condition $y(0)=2$, we can find the particular solution of differential equation given in (i) above.

Substituting $x=0$ and $y=1$ in (ii), we get

$2=ce^0 \rightarrow c=2$

Thus particular solution of (i) is, $y=2e^x \rightarrow (iii)$

## EXERCISE 9.1

**Q.1 Find the order and degree of each of the following ordinary differential equations. Also indicate the linear and non-linear in these differential equations.**

Perfect24u.com

**a)** $\frac{dy}{dx} = x^2 + y$

**Solution:** order =1, degree =1

**This is the first order linear differential equation.**

**b)** $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 11y = 3x$

**Solution:** order =2, degree =1



www.perfect24u.com

**This is the second order linear differential equation.**

**c)** $\frac{d^3y}{dx^3} + 2\left(\frac{dy}{dx}\right)^3 - y = 0$

**Solution:** order =3, degree =1

**It is the non-linear as the first order derivative has a power of three.**

**Q.2 In each case, show that the indicated function is a solution of the differential equation.**

**a)** $y = e^x + e^{2x}, \quad \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

**Solution:** Given differential equation is

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0 \rightarrow \quad \text{(i)}$$

and $y = e^x + e^{2x} \quad \rightarrow \quad$ (ii)

So, $\frac{dy}{dx} = e^x + 2e^{2x} \quad \rightarrow \quad$ (iii)

and, $\frac{d^2y}{dx^2} = e^x + 2e^{2x}\,(2) = e^x + 4e^{2x} \rightarrow \quad$ (iv)

Perfect24u.com

Putting (ii), (iii) and (iv) in L.H.S of (i)

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y \rightarrow$ (i)

L.H.S $= e^x + 4e^{2x} - 3\,(e^x + 2e^{2x}) + 2\,(e^x + e^{2x})$

$= e^x + 4e^{2x} - 3e^x - 6e^{2x} + 2e^x + 2e^{2x}$

$= 0$ R.H.S

**As the given function satisfies the differential equation, so it is the solution of the differential equation.**



www.perfect24u.com

**b)** $y = x - x\ln x, \quad x\frac{dy}{dx} + x - y = 0$

**Solution:** Given differential equation is

$$x\frac{dy}{dx} + x - y = 0 \quad \rightarrow \quad (i)$$

and $y = x - x\ln x \quad \rightarrow \quad (ii)$

$$\text{So, } \frac{dy}{dx} = \frac{d}{dx}(x - x\ln x) = 1 - \left[x.\frac{d}{dx}(\ln x) + \ln x\frac{d}{dx}(x)\right]$$

$$\frac{dy}{dx} = 1 - x\left(\frac{1}{x}\right) - \ln x.1 = 1 - 1 - \ln x$$

$$\frac{dy}{dx} = -\ln x \quad \rightarrow \quad (iii)$$

Putting (ii) and (iii) in L.H.S of (i)

$$\text{L.H.S} = x\frac{dy}{dx} + x - y = x(-\ln x) + x - (x - x\ln x)$$

$$= -x\ln x + x - x + x\ln x$$

$$= 0 \text{ R.H.S}$$

**As the given function satisfies the differential equation, so it is the solution of the differential equation.**

Perfect24u.com

**c)** $y = (x + c)e^{-x}, \quad \frac{dy}{dx} + y = e^{-x}$

**Solution:** Given differential equation is

$$\frac{dy}{dx} + y = e^{x} \quad \rightarrow (i)$$

and $y = (x+c)e^{-x} \quad \rightarrow (ii)$

$$\text{So, } \frac{dy}{dx} = \frac{d}{dx}\left[(x + c)e^{-x}\right] = (x + c)\frac{d}{dx}\left(e^{-x}\right) + e^{-x}\frac{d}{dx}(x + c)$$

$$\frac{dy}{dx} = (x + c)e^{-x}(-1) + e^{-x}(1) = e^{-x}\left[-x - c + 1\right]$$



www.perfect24u.com

$$\frac{dy}{dx} = e^{-x}[1-x-c] \quad \rightarrow (iii)$$

Putting (ii) and (iii) in L.H.S of (i)

$$\text{L.H.S} = \frac{dy}{dx} + y = e^{-x}(1-x-c) + (x+c)e^{-x}$$

$$= e^{-x}[1-x-c+x+c]$$

$$= e^{-x}(1)$$

$$= e^{-x}\,\text{R.H.S}$$

**As the given function satisfies the differential equation, so it is the solution of the differential equation.**

**Q.3 For each case of the following equations, determine whether or not it becomes linear when divided by dx or dy?**

**a) $(x+y)dy = (x-y)dx$**

**Solution:** Given differential equation $(x+y)dy=(x-y)dx$

Divide by dx

$$(x+y)\frac{dy}{dx} = (x-y)$$

Perfect24u.com

**It is non-linear differential equation because y is in product with $\frac{dy}{dx}$**

**b) $ady + by\sin x\,dx = 0$**

**Solution:** Given that, $ady + by\sin x\,dx=0$

Divide by dx

$$a\frac{dy}{dx} + by\sin x = 0$$

**Hence, it is the linear, no non-linear term in y or its derivatives**



www.perfect24u.com

**c) $3ydx + 2xdy = 0$**

**Solution:** Given that, $3ydx + 2xdy = 0$

Divide by dx

$$3y + 2x\frac{dy}{dx} = 0$$

**Hence, it is the linear, no non-linear term in y or its derivatives**

**d) $e^x dy + xy^{\frac{1}{3}} dx = 0$**

**Solution:** Given that, $e^x dy + xy^{\frac{1}{3}} dx = 0$

Divide by dx

$$e^x \frac{dy}{dx} + xy^{\frac{1}{3}} = 0$$

**Non-linear, not linear in y**

**Q.4 In each case, use the initial condition and the general solution of the differential equation to determine a particular solution.**

**a) $xy = c, y(2) = 1$**

**Solution:** Given general solution is $xy = c \rightarrow (i)$

and initial condition is; $y(2) = 1$

Substituting $x = 2$ and $y = 1$ in (i), $xy = c$

$(2)(1) = c$

$c = 2$

So, the particular solution is,

**$xy = 2$**

**b) $y = x - x\ln x + c, y(1) = 2$**

**Solution:** Given general solution is $y = x - x\ln x + c \rightarrow (i)$

Perfect24u.com



www.perfect24u.com

and initial condition is; $y(1)=2$

Substituting $x=1$ and $y=2$ in (i), $y=x-x\ln x+c$

$2=1-1\cdot\ln(1)+c$

$c=2-1\ln 1=1+0 \qquad \therefore \ln 1=0$

$c=1$

So, the particular solution is,

**$y=x-x\ln x+1$**

**c)** **$\sin(xy)+y=c,\ y\left(\frac{\pi}{4}\right)=1$**

**Solution:** Given general solution is $\sin(xy)+y=c \rightarrow$ (i)

and initial condition is; $y\left(\frac{\pi}{4}\right)=1$

Substituting $x=\frac{\pi}{4}$ and $y=1$ in (i)

$$\sin\left(\frac{\pi}{4}\cdot 1\right)+1=c \quad \Rightarrow \quad c=1+\sin\left(\frac{\pi}{4}\right)$$

$$c=1+\frac{1}{\sqrt{2}}=\frac{\sqrt{2}+1}{\sqrt{2}}$$

Perfect24u.com

So, the particular solution is,

**$\sin(xy)+y=\frac{\sqrt{2}+1}{\sqrt{2}}$**

**d)** **$\frac{y^2}{x}=\frac{x^2}{2}+c,\ y(1)=1$**

**Solution:** Given general solution is $\frac{y^2}{x}=\frac{x^2}{2}+c \rightarrow$ (i)

and initial condition is; $y(1)=1$



www.perfect24u.com

Substituting x=1 and y=1 in (i)

$$\frac{(1)^2}{1}=\frac{(1)^2}{2}+c \qquad \Rightarrow \qquad c=1-\frac{1}{2}$$

$$c=\frac{1}{2}$$

So, the particular solution is,

$$\frac{y^2}{2}=\frac{x^2}{2}+\frac{1}{2}$$

$$\frac{y^2}{2}=\frac{x^2}{2}+\frac{1}{2} \qquad \Rightarrow \qquad \mathbf{y^2=x^2+1}$$

**Q.5 Solve the following initial value problems;**

**a)** $\mathbf{\frac{dy}{dx}=\cos x, \quad y(0)=1}$

**Solution:** Given that; $\frac{dy}{dx}=\cos x$

By separating variables, we get

$dy=\cos dx$

Perfect24u.com

Taking integration on both sides

$$\int dy=\int \cos dx$$

$\mathbf{y=\sin x+c} \rightarrow$ (i) (General solution)

Now given that y(0)=1 (Using initial condition)

Substituting x=0 and y=1 in (i)

$1=\sin 0+c \qquad \Rightarrow \qquad c=1 \qquad \therefore \sin 0=0$



www.perfect24u.com

Putting in (i)

**y=sinx+1** Particular solution

**b)** $\frac{dy}{dx} = x^2, \quad y(0)=1$

**Solution:** Given that; $\frac{dy}{dx} = x^2$

By separating variables, we get

$dy = x^2\,dx$

Taking integral on both sides

$\int dy = \int x^2 dx$

$y = \frac{x^3}{3} + c \rightarrow (i)$ (General solution)

Now given that $y(0)=1$

Substituting $x=0$ and $y=1$ in (i)

$1 = \frac{(0)^3}{3} + c \quad \Rightarrow \quad c=1$ Putting in (i)

$y = \frac{x^3}{3} + 1$ (Particular solution)

Perfect24u.com

**c)** $\frac{dy}{dx} = 2xy^2, \quad y(3)=-1$

**Solution:** Given that; $\frac{dy}{dx} = 2xy^2$

By separating variables, we get

$\frac{dy}{y^2} = 2xdx$

Taking integral on both sides

$\int y^{-2} dy = \int 2xdx$



www.perfect24u.com

$$\frac{y^{-1}}{-1} = 2\bullet\frac{x^2}{2} + c$$

$$\frac{-1}{y} = x^2 + c \quad \rightarrow (i) \quad \text{(General solution)}$$

Now given that $y(3) = -1$ (Using initial condition)

Substituting $x=3$ and $y=-1$ in (i)

$$\frac{-1}{-1} = (3)^2 + c \qquad \Rightarrow \qquad c+9=1$$

$c=-8$ Putting in (i)

$$\frac{-1}{y} = x^2 - 8$$

$$-y = \frac{1}{x^2 - 8}$$

$$y = \frac{1}{8 - x^2}$$

**d)** $\dfrac{dy}{dx} + y = y^2, \; y(0) = \dfrac{1}{2}$

**Solution:** Given that; $\dfrac{dy}{dx} + y = y^2$

$$\frac{dy}{dx} = y^2 - y$$

Perfect24u.com

By separating variables, we get

$$\frac{dy}{y^2 - y} = 1\bullet dx$$

Taking integration on both sides

$$\int \frac{1}{y^2 - y} dy = \int 1 dx$$

$$\int \frac{1}{y^2 - y} dy = x + c \qquad \rightarrow \qquad (i)$$



www.perfect24u.com

Find the partial fractions of $\frac{1}{y^2-y}$

Let $\frac{1}{y^2-y}=\frac{1}{y(y-1)}=\frac{A}{y}+\frac{B}{y-1}$

Multiplying by y(y–1)

$1=A(y-1)+B(y) \quad \rightarrow \quad$ (ii)

To find A, put y=0; $1=A(0-1)+B(0)$

$A=-1$

To find B put y=1, $1=A(1-1)+B(1)$

$B=1$

So, (i) becomes

$$\int\left(\frac{-1}{y}+\frac{1}{y-1}\right)dy=x+c$$

$$-\ln y+\ln(y-1)=x+c$$

$$\ln\left(\frac{y-1}{y}\right)=x+c \qquad \Rightarrow \qquad \frac{y-1}{y}=e^{x+c}$$

$$\frac{y-1}{y}=e^c e^x \qquad \Rightarrow \qquad \frac{y-1}{y}=ce^x \rightarrow \text{(ii) General solution}$$

Perfect24u.com

Now as given that $y(0)=\frac{1}{2}$

Substituting x=0 and $y=\frac{1}{2}$ in (ii)

$$\frac{1/2-1}{1/2}=ce^0$$

$c=-1$

Putting in (ii)



www.perfect24u.com

$$\frac{y-1}{y} = -e^x$$

OR

$$y-1 = -ye^x$$

$$y + ye^x = 1$$

$$y\left(1+e^x\right) = 1$$

$$\mathbf{y = \frac{1}{1+e^x}} \quad \text{(Particular solution)}$$

**e)** $\mathbf{y\frac{dy}{dx} + xy^2 - x = 0, \quad y(0) = -1}$

**Solution:** Given that; $y\frac{dy}{dx} + xy^2 - x = 0$

$$y\frac{dy}{dx} + x\left(y^2-1\right) = 0$$

$$y\frac{dy}{dx} = -x\left(y^2-1\right)$$

By separating variables, we get

$$\frac{y}{y^2-1}dy = -x\,dx$$

Perfect24u.com

Taking integration on both sides

$$\int \frac{y}{y^2-1}dy = \int -x\,dx$$

Multiplying and Dividing by 2

$$\frac{1}{2}\int \frac{2y}{y^2-1}dy = \frac{-x^2}{2} + c$$

$$\frac{1}{2}\ln\left(y^2-1\right) = \frac{-x^2}{2} + c \qquad \text{Multiplying by 2}$$



www.perfect24u.com

$$\ln|y^2-1| = -x^2+2c$$

Taking e on both sides

$$e^{\ln|y^2-1|} = e^{-x^2+2c}$$

$$y^2-1 = e^{-x^2}\cdot e^{2c}$$

$$y^2-1 = c_1 e^{-x^2} \because c_1 = e^{2c}$$

**$y^2 = c_1 e^{-x^2} + 1 \rightarrow$ (i)** (General solution)

Now, given that $y(0) = -1$

Substituting $x=0$ and $y=-1$ in (i)

$$(-1)^2 = c_1 e^{-(0)^2} + 1$$

$$1 = c_1 \cdot 1 + 1 \quad \Rightarrow \quad c_1 = 0$$

Putting in (i)

$$y^2 = 0e^{-x^2} + 1 \quad \Rightarrow \quad y^2 = 1$$

**$y=1$** (Particular solution)

Perfect24u.com

**f)** **$2\frac{dy}{dx} = 4xe^{-x}, \quad y(0) = 42$**

**Solution:** Given that; $2\frac{dy}{dx} = 4xe^{-x}$

By separating variables, we get

$$dy = \frac{4xe^{-x}}{2}dx$$

Taking integration on both sides



www.perfect24u.com

$$\int dy = \int 2x \cdot e^{-x}\, dx$$

$$y = 2\int xe^{-x}dx \qquad \rightarrow \qquad (i)$$

Integrated by parts

$$y = 2\left[ x \cdot \frac{e^{-x}}{-1} - \int 1 \cdot \frac{e^{-x}}{-1} dx \right]$$

$$y = -2xe^{-x} + 2 \cdot \frac{e^{-x}}{-1} + c$$

**$y = -2xe^{-x} - 2e^{-x} + c$** $\rightarrow$ (ii) (General solution)

Now, given that $y(0) = 42$

Substituting $x = 0$ and $y = 42$ in (i)

$42 = -2(0)e^{-(0)} - 2e^{-(0)} + c$

$42 = -2(1) + c \qquad \Rightarrow \qquad c = 44$

Putting in (i)

**$y = -2x\,e^{-x} - 2\,e^{-x} + 44$** (Particular solution)

Perfect24u.com



www.perfect24u.com

## EXERCISE 9.2

**Q.1 Find general solution of the following differential equations.**

**a)** $\dfrac{2x\dfrac{dy}{dx}-2y}{x^2}=0$

**Solution:** Given that; $\dfrac{2x\dfrac{dy}{dx}-2y}{x^2}=0$

$$2x\frac{dy}{dx}-2y=0$$

$$2x\frac{dy}{dx}=2y \qquad \Rightarrow \qquad x\frac{dy}{dx}=y$$

Separating variables

$$\frac{dy}{y}=\frac{1}{x}dx$$

Taking integral on both sides

$$\int\frac{dy}{y}=\int\frac{dx}{x}$$

$$\ln y=\ln x+\ln c$$

$$\ln y=\ln(xc) \Rightarrow e^{\ln y}=e^{\ln(xc)}$$

**y=xc** (General solution)

Perfect24u.com

**b)** $\dfrac{dy}{x}+ydx=2dx$

**Solution:** Given that; $\dfrac{dy}{x}+ydx=2dx$

$$\frac{dy}{x}+(y-2)dx=0$$

$$\frac{dy}{x}=-(y-2)dx$$



www.perfect24u.com

Separating variables

$$\frac{dy}{y-2} = -x\,dx,$$

Taking integral on both sides

$$\int \frac{dy}{y-2} = \int -x dx$$

$$\ln|y-2| = -\frac{x^2}{2} + c$$

$$e^{\ln|y-2|} = e^{\left(-\frac{x^2}{2}+c\right)}$$

$$y-2 = e^{-\frac{x^2}{2}} \cdot e^c \qquad \text{let} \qquad e^c = c_1$$

$$y-2 = c_1 e^{-\frac{x^2}{2}}$$

$$\mathbf{y = c_1\, e^{-\frac{x^2}{2}} + 2} \quad \text{(General solution)}$$

**c)** $\mathbf{\left(\frac{dy}{dx}\right)^2 = 1 - y^2}$

**Solution:** Given that; $\left(\frac{dy}{dx}\right)^2 = 1 - y^2$

Perfect24u.com

Taking square root on both sides

$$\frac{dy}{dx} = \pm\sqrt{1-y^2}$$

Separating variables

$$\frac{dy}{\sqrt{1-y^2}} = \pm dx$$

Taking integral on both sides

$$\int \frac{dy}{\sqrt{1-y^2}} = \pm \int dx$$



www.perfect24u.com

$\sin^{-1}(y) = \pm(x + c)$

$y = \sin[\pm(x + c)]$

**$y = \pm \sin(x + c)$** (General solution)

**d)** $\mathbf{e^x \frac{dy}{dx} + y^2 = 0}$

**Solution:** Given that; $e^x \frac{dy}{dx} + y^2 = 0$

$e^x \frac{dy}{dx} = -y^2$

Separating variables

$\frac{dy}{y^2} = \frac{-1}{e^x} dx$

Taking integral on both sides

$\int \frac{dy}{y^2} = \int -\frac{1}{e^x} dx$

$\int y^{-2} dy = -\int e^{-x} dx$

$\frac{y^{-1}}{-1} = -\frac{e^{-x}}{-1} + c$

$\frac{-1}{y} = e^{-x} + c$

$\frac{1}{y} = -(e^{-x} + c)$

$\mathbf{y = \frac{-1}{(e^{-x} + c)}}$ (General solution)

**e)** $\mathbf{\sqrt{1 - x^2}\, dy = \sqrt{1 - y^2}\, dx}$

**Solution:** Given that; $\sqrt{1 - x^2}\, dy = \sqrt{1 - y^2}\, dx$

Perfect24u.com



www.perfect24u.com

Separating variables

$$\frac{1}{\sqrt{1-y^2}}dy = \frac{1}{\sqrt{1-x^2}}dx$$

Taking integral on both sides

$$\int \frac{1}{\sqrt{1-y^2}}dy = \int \frac{1}{\sqrt{1-x^2}}dx$$

$\sin^{-1}y = \sin^{-1}x + c$

**$y = \sin(\sin^{-1}x + c)$** (General solution)

**f)** **$\text{cosec}^2 x\,dy + \sec y\,dx = 0$**

**Solution:** Given that; $\text{cosec}^2 x\,dy + \sec y\,dx = 0$

$\text{cosec}^2 x\,dy = -\sec y\,dx$

Separating variables

$$\frac{1}{\sec y}dy = \frac{-1}{\text{cose}^2 x}dx$$

$\cos y\,dy = -\sin^2 x\,dx$

Taking integral on both sides

$$\int \cos y\,dy = \int -\sin^2 dx$$

Perfect24u.com

$$\sin y = -\int\left(\frac{1-\cos 2x}{2}\right)dx \quad \Rightarrow \sin y = \frac{1}{2}\int 1\,dx - \int \cos 2x\,dx$$

$$\sin y = -\frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$$

$$\sin y = \frac{\sin 2x}{4} - \frac{x}{2} + c$$

**$y = \sin^{-1}\left(\frac{\sin 2x}{4} - \frac{x}{2} + c\right)$** (General solution)



www.perfect24u.com

**Q.2 Reduce the following differential equations in separable form and then solve.**

**a)** $y' = (x+y)^2$

**Solution:** Given that; $y' = (x+y)^2 \quad \rightarrow \quad (i)$

Let $u=x+y \quad \rightarrow \quad (ii)$

$$\frac{du}{dx} = \frac{d}{dx}(x+y) \quad \Rightarrow \quad \frac{du}{dx} = \frac{d}{dx}(x) + \frac{d}{dx}(y)$$

$$\frac{du}{dx} = 1 + \frac{dy}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} - 1 \quad \rightarrow \quad (iii)$$

Substituting (ii) and (iii) into (i)

$$\frac{du}{dx} - 1 = u^2$$

$$\frac{du}{dx} = 1 + u^2$$

Separating variables

$$\frac{du}{1+u^2} = 1.dx$$

Taking integral on both sides

$$\int \frac{du}{1+u^2} = \int dx$$

Perfect24u.com

$\tan^{-1}(u) = x+c$

$u = \tan(x+c)$

Now substituting u from (ii)

$x+y = \tan(x+c)$

**$y = \tan(x+c) - x$**

**b)** $y' = \tan(x+y) - 1$

**Solution:** Given that; $y' = \tan(x+y) - 1 \quad \rightarrow \quad (i)$



www.perfect24u.com

Let $u=x+y$ $\rightarrow$ (ii)

$$\frac{du}{dx}=\frac{d}{dx}(x+y) \Rightarrow \frac{du}{dx}=1+\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{du}{dx}-1 \rightarrow \text{(iii)}$$

Substituting (ii) and (iii) into (i)

$$\frac{du}{dx}-1=\tan u-1$$

$$\frac{du}{dx}=\tan u$$

Separating variables $\frac{du}{\tan u}=dx$,

Taking integral on both sides

$$\int\frac{du}{\tan u}=\int 1dx$$

$$\int\frac{\cos u}{\sin u}du=\int 1dx \rightarrow \text{(iv)}$$

Let $t=\sin u$ $\rightarrow$ (v)

$$\frac{dt}{du}=\cos u$$

$dt=\cos u du$ putting in (iv)

$$\int\frac{dt}{t}=\int 1dx$$

$\ln|t|=x+c$ put value of t from (v)

$\ln|\sin u|=x+c$

$e^{\ln|\sin u|}=e^{x+c}$

$\sin u=e^x\cdot e^c$ $e^c=c_1$

$\sin u=c_1e^x$

Perfect24u.com



www.perfect24u.com

$u=\sin^{-1}(c_1e^x)$

Putting value of u from (ii)

$x+y=\sin^{-1}(c_1e^x)$

$\mathbf{y=\sin^{-1}(c_1e^x)-x}$

**c)** $\mathbf{y' = \left(x+e^y-1\right)e^{-y}}$

**Solution:** Given that; $y' = \left(x+e^y-1\right)e^{-y}$

$y'e^y = \left(x+e^y-1\right) \quad \rightarrow \quad$ (i)

Let $u=x+e^y-1 \quad \rightarrow \quad$ (ii)

$$\frac{du}{dx} = 1+e^y\frac{dy}{dx}$$

$$e^y\frac{dy}{dx} = \frac{du}{dx}-1 \quad \rightarrow \quad \text{(iii)}$$

Substituting (ii) and (iii) into (i)

$$\frac{du}{dx}-1 = u$$

$$\frac{du}{dx} = u+1,$$

Separating variables

$$\frac{du}{u+1} = 1\cdot dx,$$

Perfect24u.com

Taking integral on both sides

$$\int\frac{du}{u+1} = \int dx$$

$\ln|u+1| = x+c \quad \Rightarrow \quad e^{\ln|u+1|} = e^{x+c}$

$u+1=e^x\cdot e^c \quad \therefore \quad e^c=c_1$

$u+1=c_1\cdot e^x$



www.perfect24u.com

Substituting u from (ii)

$x+e^y-1+1=c_1\cdot e^x$

$e^y=c_1e^x-x$

$\ln e^y=\ln(c_1e^x-x)$

$y=\ln(c_1e^x-x)$

$e^y=e^{\ln}(c_1e^x-x)$

$\mathbf{e^y=c_1e^x-x}$

**d)** $\mathbf{y\frac{dy}{dx}+xy^2-x=0}$

**Solution:** Given that; $y\frac{dy}{dx}+xy^2-x=0$

$y\frac{dy}{dx}=-xy^2+x$

$y\frac{dy}{dx}=-x\left(y^2-1\right)$

Separating variables

$\frac{y}{y^2-1}dy=-xdx$

Taking integral on both sides

$\int\frac{y}{y^2-1}dy=\int-xdx$

$\frac{1}{2}\int\frac{2y}{y^2-1}dy=\int-xdx$

Multiplying and dividing by 2

$\frac{1}{2}\ln\left|y^2-1\right|=\frac{-x^2}{2}+c \quad \Rightarrow \quad \ln\left|y^2-1\right|=-x^2+2c$

$e^{\ln\left|y^2-1\right|}=e^{-x^2+2c}$

Perfect24u.com



www.perfect24u.com

$y^2-1=e^{-x^2}\cdot e^{2c}$ $\quad\therefore\quad$ $e^{2c}=c_1$

$y^2-1=c_1\,e^{-x^2}$

$y^2=c_1\,e^{-x^2}+1$

$\mathbf{y=\sqrt{c_1\,e^{-x^2}+1}}$

**e)** $\mathbf{\dfrac{dy}{dx}=\dfrac{y}{x}+\dfrac{x^2}{y^2}}$

**Solution:** Given that; $\dfrac{dy}{dx}=\dfrac{y}{x}+\dfrac{x^2}{y^2}\ \rightarrow$ (i)

Let $u=\dfrac{y}{x}\rightarrow$ (ii)

$\dfrac{1}{u}=\dfrac{x}{y}$

By cross multiplication

$y=ux$

Diff w.r.t x

$\dfrac{dy}{dx}=u\dfrac{dx}{dx}+x\dfrac{du}{dx}$

$\dfrac{dy}{dx}=u\cdot 1+x\dfrac{du}{dx}\quad\rightarrow\quad$ (iii)

Perfect24u.com

Substituting (ii) and (iii) in (i)

$x\dfrac{du}{dx}+u=u+\dfrac{1}{u^2}$

$x\dfrac{du}{dx}=\dfrac{1}{u^2}$

Separating variables

$u^2du=\dfrac{1}{x}dx$

Taking integral on both sides



www.perfect24u.com

$$\int u^2 du = \int \frac{1}{x} dx$$

$$\frac{u^3}{3} = \ln x + \ln c$$

$$\frac{u^3}{3} = \ln|xc|$$

Substituting u from (ii)

$$\frac{1}{3}\left(\frac{y}{x}\right)^3 = \ln|xc|$$

$$\frac{y^3}{3x^3} = \ln|xc|$$

$$\frac{y^3}{x^3} = 3\ln|xc|$$

$$\frac{y^3}{x^3} = \ln|xc|^3$$

$$e^{\frac{y^3}{x^3}} = |xc|^3$$

$$\mathbf{e^{\left(\frac{y}{x}\right)^3} = x^3 c}$$

**f)** $\mathbf{(y-3)\,dy = (x^3+1)\,dx}$

Perfect24u.com

**Solution:** Given that; $(y-3)\,dy = (x^3+1)\,dx$

Taking integral on both sides

$$\int (y-3)\,dy = \int (x^3+1)\,dx$$

$$\frac{y^2}{2} - 3y = \frac{x^4}{4} + x + c$$

Multiplying both sides by 2

$$\mathbf{y^2 - 6y = \frac{x^4}{2} + 2x + c}$$



www.perfect24u.com

**Q.3 Solve the following homogeneous differential equations.**

**a)** $\dfrac{dy}{dx}=\dfrac{x+y}{x-y}$

**Solution:** Given that; $\dfrac{dy}{dx}=\dfrac{x+y}{x-y}\rightarrow$ (i)

Let $u=\dfrac{y}{x} \Rightarrow y=ux \rightarrow$ (ii)

Diff w.r.t x

$$\frac{dy}{dx}=u\frac{dx}{dx}+x\frac{du}{dx}$$

$$\frac{dy}{dx}=u+x\frac{du}{dx} \rightarrow \text{(iii)}$$

Substituting (ii) and (iii) in (i)

$$\frac{dy}{dx}=\frac{x+y}{x-y}\rightarrow \text{(i)}$$

$$u+x\frac{du}{dx}=\frac{x+ux}{x-ux}$$

$$u+x\frac{du}{dx}=\frac{x(1+u)}{x(1-u)}$$

$$u+x\frac{du}{dx}=\frac{1+u}{1-u}$$

$$x\frac{du}{dx}=\frac{1+u}{1-u}-\frac{u}{1}=\frac{1+u-u+u^2}{1-u}$$

$$x\frac{du}{dx}=\frac{1+u^2}{1-u}$$

Separating variables and taking integral on both sides

$$\int\frac{1-u}{1+u^2}du=\int\frac{1}{x}dx$$

Perfect24u.com



www.perfect24u.com

$$\int\left(\frac{1}{1+u^2}-\frac{u}{1+u^2}\right)du=\int\frac{1}{x}dx$$

$$\int\frac{1}{1+u^2}du-\frac{1}{2}\int\frac{2u}{1+u^2}du=\ln x+\ln c$$

$$\tan^{-1}u-\frac{1}{2}\ln\left|1+u^2\right|=\ln|xc|$$

Substituting $u=\frac{y}{x}$

$$\tan^{-1}\left(\frac{y}{x}\right)-\frac{1}{2}\ln\left|1+\frac{y^2}{x^2}\right|=\ln|xc|$$

$$\tan^{-1}\left(\frac{y}{x}\right)=\ln|xc|+\frac{1}{2}\ln\left|1+\frac{y^2}{x^2}\right|$$

$$\tan^{-1}\left(\frac{y}{x}\right)=\ln|xc|+\ln\left|1+\frac{y^2}{x^2}\right|^{\frac{1}{2}}$$

$$\boldsymbol{\tan^{-1}\left(\frac{y}{x}\right)=\ln\left(xc\left|1+\frac{y^2}{x^2}\right|^{\frac{1}{2}}\right)}$$

**b)** $\boldsymbol{\frac{dy}{dx}=\frac{xy-y^2}{x^2}}$

Perfect24u.com

**Solution:** Given that; $\frac{dy}{dx}=\frac{xy-y^2}{x^2}\rightarrow$ (i)

Let $u=\frac{y}{x}$ $\Rightarrow$ $y=ux\rightarrow$ (ii)

Diff w.r.t x

and $\frac{dy}{dx}=u\frac{dx}{dx}+x\frac{du}{dx}$



www.perfect24u.com

$$\frac{dy}{dx} = u + x\frac{du}{dx} \qquad \rightarrow \qquad \text{(iii)}$$

Substituting (ii) and (iii) in (i)

$$u + x\frac{du}{dx} = \frac{x(ux) - (ux)^2}{x^2}$$

$$u + x\frac{du}{dx} = \frac{ux^2 - u^2x^2}{x^2}$$

$$u + x\frac{du}{dx} = \frac{x^2(u - u^2)}{x^2}$$

$$u + x\frac{du}{dx} = u - u^2$$

$$x\frac{du}{dx} = -u^2,$$

Separating variables

$$\frac{du}{u^2} = -\frac{1}{x}dx,$$

Taking integral on both sides

$$\int u^{-2}du = \int -\frac{1}{x}dx$$

Perfect24u.com

$$\frac{u^{-1}}{-1} = -\ln x - \ln c$$

$$\frac{-1}{u} = -\ln x - \ln c$$

$$\frac{1}{u} = \ln x + \ln c = \ln|xc|$$

Putting $u = \frac{y}{x}$



www.perfect24u.com

$\frac{x}{y} = \ln|xc|$ Taking "e" on both sides

$e^{\frac{x}{y}} = e^{\ln|xc|}$

$xc = e^{\frac{x}{y}}$

$x = \frac{1}{c}e^{\frac{x}{y}}$

$\mathbf{x = ce^{\frac{x}{y}}}$

**c)** $\mathbf{\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}}$

**Solution:** Given that; $\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$ → (i)

Diff w.r.t x

Let $u = \frac{y}{x}$ ⇒ $y = xu$ → (ii)

and $\frac{dy}{dx} = u + x\frac{du}{dx}$ → (iii)

Substituting (ii) and (iii) in (i)

Perfect24u.com

$u + x\frac{du}{dx} = \frac{x^2 + 3(ux)^2}{2x(ux)}$

$u + x\frac{du}{dx} = \frac{x^2 + 3u^2x^2}{2x^2u} = \frac{x^2(1 + 3u^2)}{x^2(2u)}$

$u + x\frac{du}{dx} = \frac{1 + 3u^2}{2u}$

$x\frac{du}{dx} = \frac{1 + 3u^2}{2u} - \frac{u}{1} = \frac{1 + 3u^2 - 2u^2}{2u}$



www.perfect24u.com

$$x\frac{du}{dx}=\frac{1+u^2}{2u},$$

Separating variables

$$\frac{2udu}{1+u^2}=\frac{1}{x}dx,$$

Taking integral on both sides

$$\int\frac{2u}{1+u^2}du=\int\frac{1}{x}dx$$

$$\ln\left|1+u^2\right|=\ln x+\ln c$$

$$\ln\left|1+u^2\right|=\ln|xc|$$

$$e^{\ln\left|1+u^2\right|}=e^{\ln|xc|}$$

$$1+u^2=xc$$

$$u^2=xc-1$$

Putting $u=\frac{y}{x}$ $\qquad\qquad$ $\because \frac{1}{c}=c$

$$\frac{y^2}{x^2}=xc-1 \quad \Rightarrow y^2=cx^3-x^2$$

$$cx^3=x^2+y^2$$

$$x^3=\frac{1}{c}\left(x^2+y^2\right)$$

$$\mathbf{x^3=c\left(x^2+y^2\right)}$$

**d)** $\mathbf{\frac{dy}{dx}=\frac{\sqrt{x^2-y^2}+y}{x}}$

**Solution:** Given that; $\frac{dy}{dx}=\frac{\sqrt{x^2-y^2}+y}{x}\rightarrow$ (i)

Perfect24u.com



www.perfect24u.com

Let $u=\frac{y}{x} \quad \Rightarrow \quad y=xu \quad \rightarrow$ (ii)

and $\frac{dy}{dx}=u+x\frac{du}{dx} \quad \rightarrow$ (iii)

Substituting (ii) and (iii) in (i)

$$u+x\frac{du}{dx}=\frac{\sqrt{x^2-(ux)^2}+(ux)}{x}=\frac{\sqrt{x^2-x^2u^2}+ux}{x}$$

$$u+x\frac{du}{dx}=\frac{\sqrt{x^2(1-u^2)}+ux}{x}=\frac{x\left(\sqrt{1-u^2}+u\right)}{x}$$

$$u+x\frac{du}{dx}=u+\sqrt{1-u^2}$$

$$x\frac{du}{dx}=\sqrt{1-u^2}$$

Separating variables

$$\frac{du}{\sqrt{1-u^2}}=\frac{1}{x}dx,$$

Taking integral on both sides

$$\int\frac{du}{\sqrt{1-u^2}}=\int\frac{1}{x}dx$$

Perfect24u.com

$$\sin^{-1}(u)=\ln x+\ln c$$

$$\sin^{-1}(u)=\ln(xc)$$

Taking e on both sides

$$e^{\sin^{-1}(u)}=xc$$

$$\mathbf{e^{\sin^{-1}\left(\frac{y}{x}\right)}=xc}$$



www.perfect24u.com

**e)** $\dfrac{dy}{dx} = \dfrac{xy + y^2}{x^2 + xy + y^2}$

**Solution:** Given that; $\dfrac{dy}{dx} = \dfrac{xy + y^2}{x^2 + xy + y^2} \rightarrow$ (i)

Let $u = \dfrac{y}{x} \Rightarrow y = ux \rightarrow$ (ii)

and $\dfrac{dy}{dx} = u + x\dfrac{du}{dx} \rightarrow$ (iii)

Substituting (ii) and (iii) in (i)

$$u + x\frac{du}{dx} = \frac{x(ux) + (ux)^2}{x^2 + x(ux) + (ux)^2} = \frac{ux^2 + u^2x^2}{x^2 + ux^2 + u^2x^2}$$

$$u + x\frac{du}{dx} = \frac{x^2(u + u^2)}{x^2(1 + u + u^2)}$$

$$u + x\frac{du}{dx} = \frac{u + u^2}{1 + u + u^2}$$

$$x\frac{du}{dx} = \frac{u + u^2}{1 + u + u^2} - \frac{u}{1}$$

$$x\frac{du}{dx} = \frac{u + u^2 - u - u^2 - u^3}{1 + u + u^2} = \frac{-u^3}{1 + u + u^2}$$

Perfect24u.com

Separating variables

$$\left(\frac{1 + u + u^2}{u^3}\right)du = \frac{-1}{x}dx,$$

Taking integral on both sides

$$\int\left(\frac{1 + u + u^2}{u^3}\right)du = \int\frac{-1}{x}dx \qquad \because\ 2\ln y = \ln y^2$$

$$\int\left(\frac{1}{u^3} + \frac{u}{u^3} + \frac{u^2}{u^3}\right)du = -\int\frac{1}{x}dx \qquad \frac{1}{c} = -2\ln c$$



www.perfect24u.com

$$\int\left(u^{-3}+u^{-2}+\frac{1}{u}\right)du=-\ln x+\ln c$$

$$-\frac{1}{2u^2}-\frac{1}{u}+\ln\left(\frac{y}{x}\right)=-\ln|x|-\ln|c|$$

$$-\frac{x^2}{2y^2}-\frac{x}{y}+\ln y-\cancel{\ln x}=-\cancel{\ln x}-\ln c$$

Multiplying both side by $-2y^2$

$x^2 + 2xy - 2y^2\ln y^2 = y^2(-2\ln c)$

$$x^2 + 2xy - 2y^2\ln y^2 = (y^2)\left(\frac{1}{c}\right)$$

**$c(x^2 + 2xy - y^2\ln y^2) = y^2$**

**Q.4 Reduce the following differential equation in the standard form of homogeneous form and then solve.**

**a)** $$x\frac{dy}{dx}=y+\sqrt{x^2+y^2},\ y(4)=3$$

**Solution:** Given that; $x\frac{dy}{dx}=y+\sqrt{x^2+y^2}$ divide by x

$$\frac{dy}{dx}=\frac{y}{x}+\frac{\sqrt{x^2+y^2}}{x}=\frac{y}{x}+\sqrt{\frac{x^2}{x^2}+\frac{y^2}{x^2}}$$

Perfect24u.com

$$\frac{dy}{dx}=\frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2} \quad \rightarrow \quad \text{(i)}$$

Let $u=\frac{y}{x} \quad \Rightarrow \quad y=ux \quad \rightarrow \quad$ (ii)

$$\frac{dy}{dx}=u+x\frac{du}{dx} \quad \rightarrow \quad \text{(iii)}$$

Substituting (ii) and (iii) in (i)



www.perfect24u.com

$$u + x\frac{du}{dx} = u + \sqrt{1+u^2}$$

$$x\frac{du}{dx} = \sqrt{1+u^2}$$

Separating variables

$$\frac{du}{\sqrt{1+u^2}} = \frac{1}{x}dx$$

Taking integral on both sides

$$\int \frac{du}{\sqrt{1+u^2}} = \int \frac{1}{x}dx$$

$$\int \frac{1}{\sqrt{1+u^2}}du = \ln x + \ln c \qquad \rightarrow \qquad \text{(iv)}$$

$$\int \frac{1}{\sqrt{1+u^2}}du$$

Let $u=\tan\theta$

$$\frac{du}{d\theta} = \sec^2\theta \qquad \Rightarrow \qquad du = \sec^2\theta d\theta$$

$$\int \frac{\sec^2\theta d\theta}{\sqrt{1+\tan^2\theta}} = \int \frac{\sec^2\theta}{\sqrt{\sec^2\theta}}d\theta = \int \frac{\sec^2\theta}{\sec\theta}d\theta = \int \sec\theta d\theta$$

Perfect24u.com

Multiplying and dividing by $(\sec\theta + \tan\theta)$

$$= \int \frac{\sec\theta(\sec\theta+\tan\theta)d\theta}{(\sec\theta+\tan\theta)}$$

$$= \int \frac{(\sec^2\theta+\tan\theta\sec\theta)}{\sec\theta+\tan\theta}d\theta = \ln|\sec\theta+\tan\theta|$$

$$= \ln\left|\sqrt{1+\tan^2\theta}+\tan\theta\right|$$

As $\tan\theta = u$



www.perfect24u.com

$$\int \frac{1}{\sqrt{1+u^2}}du = \ln\left|\sqrt{1+u^2}+u\right|$$

Substituting in (iv)

$$\ln\left|\sqrt{1+u^2}+u\right| = \ln x + \ln c$$

$$\ln\left|\sqrt{1+u^2}+u\right| = \ln|xc|$$

$$e^{\ln\left|\sqrt{1+u^2}+u\right|} = e^{\ln|xc|}$$

$$\sqrt{1+u^2}+u = xc$$

Substituting u from (ii) $\left(u = \frac{y}{x}\right)$

$$\sqrt{1+\frac{y^2}{x^2}}+\frac{y}{x} = xc \qquad \rightarrow \qquad (v)$$

Putting x=4 and y=3

$$\sqrt{1+\frac{(3)^2}{(4)^2}}+\frac{3}{4} = 4c$$

$$\sqrt{\frac{16+9}{16}}+\frac{3}{4} = 4c$$

$$\frac{5}{4}+\frac{3}{4} = 4c \quad \Rightarrow \quad 4c = \frac{8}{4} = 2$$

$$c = \frac{2}{4} = \frac{1}{2}$$

Putting in (v)

$$\sqrt{1+\frac{y^2}{x^2}}+\frac{y}{x} = \frac{1}{2}x$$

$$\sqrt{\frac{x^2+y^2}{x^2}}+\frac{y}{x} = \frac{1}{2}x$$

Perfect24u.com



www.perfect24u.com

$$\frac{1}{x}\sqrt{x^2+y^2}+\frac{y}{x}=\frac{1}{2}x$$

Multiplying by x

$$\sqrt{x^2+y^2}+y=\frac{1}{2}x^2$$

OR

$$\sqrt{x^2+y^2}=\frac{1}{2}x^2-y$$

Squaring both sides

$$\left(\sqrt{x^2+y^2}\right)^2=\left(\frac{1}{2}x^2-y\right)^2$$

$$x^2+y^2=\frac{1}{4}x^4-2\left(\frac{1}{2}x^2\right)(y)+y^2$$

$$x^2+y^2-\frac{1}{4}x^4+x^2y-y^2=0$$

$$x^2-\frac{1}{4}x^4+x^2y=0$$

$$x^2y=\frac{1}{4}x^4-x^2$$

Divide by $x^2$

$$\mathbf{y=\frac{1}{4}x^2-1}$$

Perfect24u.com

**b)** $\mathbf{\left(x^4+y^4\right)dx=2x^3ydy, y(1)=0}$

**Solution:** Given that; $\left(x^4+y^4\right)dx=2x^3ydy$ divide by $x^4$

$$\left(\frac{x^4}{x^4}+\frac{y^4}{x^4}\right)dx=\frac{2x^3y}{x^4}dy$$



www.perfect24u.com

$$\left(1+\left(\frac{y}{x}\right)^4\right)dx=\frac{2y}{x}dy$$

$$1+\frac{y^4}{x^4}=2\frac{y}{x}\frac{dy}{dx} \qquad \rightarrow \quad \text{(i)}$$

Let $u=\frac{y}{x} \quad \Rightarrow \quad y=ux \rightarrow \quad \text{(ii)}$

Diff w.r.t x

and $\frac{dy}{dx}=u+x\frac{du}{dx} \qquad \rightarrow \quad \text{(iii)}$

Substituting (ii) and (iii) in (i)

$$1+u^4=2u\left(u+x\frac{du}{dx}\right)$$

$$1+u^4=2u^2+2ux\frac{du}{dx}$$

$$2ux\frac{du}{dx}=1+u^4-2u^2$$

Separating variables

$$\frac{2udu}{1-2u^2+u^4}=\frac{1}{x}dx$$

Perfect24u.com

$$\int\frac{2udu}{(1-u^2)^2}du=\int\frac{1}{x}dx$$

$$\int\frac{2u}{(1-u^2)^2}du=\int\frac{1}{x}dx$$

$$\int\frac{2u}{(1-u^2)^2}du=\ln x+\ln c$$

$$\int\frac{+2u}{(1-u^2)^2}du=\ln|xc| \qquad \rightarrow \quad \text{(iv)}$$



www.perfect24u.com

Now, $\int \frac{2u}{(1-u^2)^2} du$ $\rightarrow$ (v)

Let $t=1-u^2$

$\frac{dt}{du} = -2u$

$-dt = +2udu$ Putting in (v)

$\int \frac{-dt}{t^2} = -\int t^{-2} dt = \frac{-t^{-1}}{-1} = \frac{1}{t}$

Putting $t=1-u^2$

$\int \frac{2u}{(1-u^2)^2} = \frac{+1}{1-u^2}$ Putting in (iv)

$\frac{1}{1-u^2} = \ln|xc|$

Substituting (ii) $(u=\frac{y}{x})$

$\frac{1}{1-\frac{y^2}{x^2}} = \ln|xc|$

$\frac{1}{\left(\frac{x^2-y^2}{x^2}\right)} = \ln|xc| \Rightarrow \frac{x^2}{x^2-y^2} = \ln|xc|$

Perfect24u.com

$x^2 - y^2 = \frac{x^2}{\ln|xc|} \Rightarrow y^2 = x^2 - \frac{x^2}{\ln|xc|}$ $\rightarrow$ (vi)

Putting $x=1$ and $y=0$ in (vi)

$(0)^2 = (1)^2 - \frac{(1)^2}{\ln|1c|}$

$1 - \frac{1}{\ln c} = 0 \Rightarrow \frac{1}{\ln c} = 1$



www.perfect24u.com

$\ln c = 1$

$e^{\ln c} = e^1$

$c = e$

Putting in (vi)

$$y^2 = x^2 - \frac{x^2}{\ln(xe)} = x^2 - \frac{x^2}{\ln x + \ln e} = x^2 - \frac{x^2}{\ln x + 1}$$

$$y^2 = \frac{x^2 \ln x + x^2 - x^2}{\ln x + 1}$$

$$\mathbf{y^2 = \frac{x^2 \ln x}{\ln x + 1}}$$

**Q.5 The slope of a family of curves at a point p(x,y) is $\frac{y-1}{1-x}$, determine the equation of the curve that passes through the point p(4,–3).**

**Solution:** Let the equation of the curve is $y = f(x)$, then

$$\frac{dy}{dx} = \frac{y-1}{1-x}$$

Separating variables

$$\frac{dy}{y-1} = \frac{dx}{1-x}$$

Perfect24u.com

Taking integral on both sides

$$\int \frac{dy}{y-1} = \int \frac{dx}{1-x}$$

$$\ln|y-1| = -\ln|1-x| + \ln c$$

$$\ln|y-1| + \ln|1-x| = \ln c$$

$$\ln|y-1| + \ln|1-x| = \ln c$$



www.perfect24u.com

$$\ln\left|(y-1)(1-x)\right| = \ln c$$

$$e^{\ln|(y-1)(1-x)|} = e^{\ln c}$$

$$\mathbf{(y-1)(1-x) = c} \rightarrow \text{(i)}$$

Putting x=4 and y=–3

$$(-3-1)(1-4)=c$$

$$(-4)(-3)=c \quad \Rightarrow \quad c=12$$ Putting in (i)

$$(y-1)(1-x)=12$$

OR

$$y-xy-1+x=12$$

$$y(1-x)=13-x$$

$$\mathbf{y = \frac{13-x}{1-x} = \frac{x-13}{x-1}}$$

**Q.6 Find the solution curve of the differential equation $xyy'=3y^2+x^2$ which passes through the point p(–1,2)**

**Solution:** Given differential equation

$xyy'=3y^2+x^2$ divide by $x^2$

Perfect24u.com

$$\frac{xyy'}{x^2} = \frac{3y^2}{x^2} + \frac{x^2}{x^2}$$

$$\frac{y}{x}y' = 3\frac{y^2}{x^2} + 1 \quad \rightarrow \quad \text{(i)}$$

Let $u = \frac{y}{x}$ $\Rightarrow$ $y=ux$ $\rightarrow$ (ii)

Substituting (ii) and (iii) in (i)

$$u\left(u + x\frac{du}{dx}\right) = 3u^2 + 1$$



www.perfect24u.com

$$u^2 + ux\frac{du}{dx} = 3u^2 + 1$$

$$ux\frac{du}{dx} = 2u^2 + 1$$

Separating variables and taking integral on both sides

$$\int \frac{udu}{2u^2+1} = \int \frac{1}{x}dx$$

$$\frac{1}{4}\int \frac{4udu}{2u^2+1} = \ln x + \ln c$$

$$\frac{1}{4}\ln\left|2u^2+1\right| = \ln|xc| \qquad \Rightarrow \qquad \ln\left|2u^2+1\right|^{\frac{1}{4}} = \ln|xc|$$

$$e^{\ln\left|2u^2+1\right|^{\frac{1}{4}}} = e^{\ln|xc|}$$

$$(2u^2+1)^{\frac{1}{4}} = xc$$

Substituting u from (ii) i.e $\left(u = \frac{y}{x}\right)$

$$\left(2\frac{y^2}{x^2}+1\right)^{\frac{1}{4}} = xc \qquad \rightarrow \qquad \text{(iii)}$$

Putting x=–1 and y=2

$$\left(2\frac{(2)^2}{(-1)^2}+1\right)^{\frac{1}{4}} = -1\cdot c$$

$$(9)^{\frac{1}{4}} = -c$$

Putting in (iii)

$$\left(\frac{2y^2}{x^2}+1\right)^{\frac{1}{4}} = (9)^{\frac{1}{4}}(x)$$

Perfect24u.com



www.perfect24u.com

**Q.7 Determine the particular solution y=f(t) of the homogenous differential equation $t^2y'=y^2+2ty$ with initial condition y(1) =2**

**Solution:** Given differential equation is

$t^2y'=y^2+2ty$ divide by $t^2$

$$y' = \frac{y^2}{t^2} + \frac{2y}{t} \quad \rightarrow \quad (i)$$

Let $u = \frac{y}{t} \Rightarrow y=ut \quad \rightarrow \quad (ii)$

Diff w.r.t t

$$\frac{dy}{dt} = u\frac{dt}{dt} + t\frac{du}{dt}$$

$$\frac{dy}{dt} = u + t\frac{du}{dt} \quad \rightarrow \quad (iii)$$

Substituting (ii) and (iii) in (i)

$$u + t\frac{du}{dt} = u^2 + 2u$$

$$t\frac{du}{dt} = u^2 + u$$

Separating variables

$$\frac{du}{u^2+u} = \frac{1}{t}dt,$$

Perfect24u.com

Taking integral on both sides

$$\int \frac{du}{u+u^2} = \int \frac{1}{t}dt \quad \Rightarrow \int \frac{1}{u(1+u)}du = \ln t + \ln c$$

$$\int \frac{1}{u(1+u)}du = \ln|tc| \quad \rightarrow \quad (ii)$$

Now, let $\frac{1}{u(1+u)} = \frac{A}{u} + \frac{B}{1+u}$



www.perfect24u.com

Multiplying by u (1+u)

$1=A(1+u)+B(u) \quad \rightarrow \quad$ (iii)

To find A, putting u=0 in (iii)

$1=A(1+0)+B(0) \quad \Rightarrow \quad \mathbf{A=1}$

To find B, putting 1+u=0 $\Rightarrow$ u=–1 in (iii)

$1=A(1+(-1))+B(-1) \quad \Rightarrow \quad \mathbf{B=-1}$

Thus

$$\int \frac{1}{u(1+u)}du = \int\left(\frac{1}{u}+\frac{-1}{1+u}\right)du = \ln u - \ln|1+u|$$

Putting in (ii)

$$\ln u - \ln|1+u| = \ln|tc|$$

$$\ln\left|\frac{u}{1+u}\right| = \ln|tc|$$

$$e^{\ln\left|\frac{u}{1+u}\right|} = e^{\ln|tc|}$$

$$\frac{u}{1+u} = tc,$$

Substituting (ii), i.e. $\left(u=\frac{y}{t}\right)$

Perfect24u.com

$$\frac{\frac{y}{t}}{1+\frac{y}{t}} = ct \quad \Rightarrow \quad \frac{\frac{y}{t}}{\frac{t+y}{t}} = ct$$

$$\frac{y}{t+y} = ct \quad \Rightarrow \quad y=ct^2+cty$$

$$y-cty=ct^2 \quad \Rightarrow \quad y(1-ct)=ct^2$$

$$\mathbf{y=\frac{ct^2}{1-ct}} \quad \rightarrow \quad \text{(iii)}$$



www.perfect24u.com

Putting t=1 and y=2

$$2=\frac{c(1)^2}{1-c(1)} \quad \Rightarrow \quad 2=\frac{c}{1-c}$$

$$2-2c=c \quad \Rightarrow \quad 2=3c$$

$$c=\frac{2}{3}$$

Putting in (iii)

$$y=\frac{\frac{2}{3}t^2}{\left(1-\frac{2}{3}t\right)}=\frac{\frac{2}{3}t^2}{\frac{3-2t}{3}}$$

$$\mathbf{y=\frac{2t^2}{3-2t}}$$

**Q.8 A particle move along x-axis so that its velocity at any point in equal to half its abscissa minus three time the time. At a time t=2, x=–4, determine the motion of a particle along the x-axis.**

**Solution:** From given information the differential equation describing the particle motion is given

Perfect24u.com

$$v=\frac{dx}{dt}=+\frac{x}{2}-3t \text{ and } x(2)=-4$$

$$\frac{dx}{dt}=\frac{x-6t}{2} \quad \rightarrow \quad \text{(i)}$$

$$\text{Let } u=x-6t \quad \rightarrow \quad \text{(ii)}$$

$$\frac{du}{dt}=\frac{dx}{dt}-6$$

$$\frac{dx}{dt}=\frac{du}{dt}+6 \quad \rightarrow \quad \text{(iii)}$$



www.perfect24u.com

$$\frac{du}{dt}+6=\frac{u}{2} \quad \Rightarrow \quad \frac{du}{dt}=\frac{u}{2}-6=\frac{u-12}{2}$$

Separating variable of integrating

$$\int\frac{du}{u-12}=\int\frac{1}{2}dt$$

$$\ln|u-12|=\frac{1}{2}t+c$$

Substituting u from (ii)

$$\ln|x-6t-12|=\frac{1}{2}t+c$$

$$e^{\ln|x-6t-12|}=e^{\frac{1}{2}t+c}$$

$$x-6t-12=e^{\frac{1}{2}t}e^{c} \qquad \text{let } e^{c}=c_1$$

$$x-6t-12=c_1e^{\frac{1}{2}t} \quad \rightarrow \quad \text{(iv)}$$

Putting t=2 and x=–4

$$-4-6(2)-12=c_1e^{\frac{1}{2}(2)}$$

$$-28=c_1e \quad \Rightarrow \quad c_1=\frac{-28}{e} \text{ putting in (iv)}$$

Perfect24u.com

$$x-6t-12=\frac{-28}{e}e^{\frac{1}{2}t}$$

$$\mathbf{x(t)=6t+12-28e^{\left(\frac{1}{2}t-1\right)}}$$

**Q.9 The rate of consumption of oil (billions of barrel) is given $\frac{dx}{dt}=1.2e^{0.04t}$, t=0 corresponds to 1990. Find the total amount of oil**



www.perfect24u.com

**consumed from year 1990 to 1995. At this rate, how much oil will be used in 8 (t=8) years?**

**Solution:** Given that the rate of consumption is

$$\frac{dx}{dt} = 1.2\,e^{0.04t}$$

$$dx = 1.2\,e^{0.04t}dt$$

$$\int dx = \int 1.2\,e^{0.04t}dt$$

$$x = \frac{1.2\,e^{0.04t}}{0.04} + c$$

$$\mathbf{x = 30\,e^{0.04t} + c} \quad \rightarrow \quad \text{(i)}$$

Putting t=0 and x=0 (initial consumption is zero)

$$0 = 30\,e^{0.04(0)} + c \quad \Rightarrow \quad c+30=0$$

$c=-30$ putting in (i)

$$x=30e^{0.4t}-30 \quad \rightarrow \quad \text{(ii)}$$

Now at 1995, t=5

$$x(t)=30(e^{0.4t}-1)$$

$$x(0)=30(e^{0}-1)=0$$

$$x(1)=30(e^{0.04}-1)=1.23$$

$$x(2)=30(e^{0.08}-1)=2.4$$

$$x(3)=30(e^{0.12}-1)=3.81$$

$$x(4)=30(e^{0.16}-1)=5.22$$

$$x(5)=30(e^{0.20}-1)=6.63$$

Then the total use

$$x(0)+x(1)\longrightarrow x(5)=19.29$$

At t=8

$$x(8)=30e^{0.4(8)}-30=30\,e^{0.32}-30$$

$$\mathbf{x(8)=30(e^{0.32}-1)}$$



Perfect24u.com

www.perfect24u.com

**Q.10 The rate of infection of a disease (in people per month) is given by $\frac{dI}{dt}=\frac{100}{t^2+1}$. Where t is the time in months since the disease broke out. Find the total number of infected people over the first four months of the disease.**

**Solution:** Given that the rate of infection is

$$\frac{dI}{dt}=\frac{100t}{t^2+1}$$

To find I(4)=?

$$\frac{dI}{dt}=\frac{100t}{t^2+1}$$

$$\int dI=\int\left(\frac{100t}{t^2+1}\right)dt$$

$$I=50\int\frac{2t}{t^2+1}dt$$

**$I=50\ln(t^2+1)+c$** $\rightarrow$ (i)

Now, at t=0, I=0, putting in (i)

$0=50\ln(0^2+1)+c \quad \Rightarrow \quad 50\ln(1)+c=0$

$50(0)+c=0 \Rightarrow c=0$ putting in (i)

$I=50\ln(t^2+1)+0$

**$I=50\ln(t^2+1)$**

Now, $I(1)=50\ln(1^2+1)=50\ln(2)$

**I(1)=34.46**

$I(2)=50\ln(2^2+1)=50\ln(5)=$ **80.17 people**

$I(3)=50\ln(3^2+1)=50\ln(10)=$ **115.13 people**

And

$I(4)=50\ln(4^2+1)=50\ln(17)=$ **141.66 people**

Perfect24u.com



www.perfect24u.com

**Q.11 The rate of reaction to drug is given by $\frac{dR}{dx} = 2x^2e^{-x}$. Where x is the number of hours since the drug was administered. Find the total reaction to the drug from x=1 to x=6.**

**Solution:** Given that the rate or reaction is

$$\frac{dR}{dx} = 2x^2e^{-x}$$

$$\int dR = \int 2x^2e^{-x}\,dx$$

Using integration by part

$$R=2\left[x^2\left(\frac{e^{-x}}{-1}\right)-\int(2x)\left(\frac{e^{-x}}{-1}\right)dx\right]$$

$$R=-2x^2e^{-x}+4\int xe^{-x}dx$$

Integration by part again

$$R=-2x^2e^{-x}+4\left[x\left(\frac{e^{-x}}{-1}\right)-\int(1)\left(\frac{e^{-x}}{-1}\right)dx\right]$$

$$R=-2x^2e^{-x}-4xe^{-x}+4\int e^{-x}dx$$

$$R=-2x^2e^{-x}-4xe^{-x}+\frac{4e^{-x}}{(-1)}+c$$

Perfect24u.com

$$R=-2x^2e^{-x}-4xe^{-x}-4e^{-x}+c \quad \rightarrow \quad \text{(i)}$$

At x=0, R=0, putting in (i)

$$0=-2(0)^2e^{-(0)}-4(0)e^{-0}-4e^{-0}+c$$

$$-4+c=0$$

$$c=4$$

Putting in (i)

$$R=-2x^2e^{-x}-4xe^{-x}-4e^{-x}+4$$

Now,



www.perfect24u.com

$R(1) = -2(1)^2e^{-1} - 4(1)e^{-1} - 4e^{-1} + 4$

$R(1) = 32.1 = 0.321\%$

$R(2) = -2(2)^2e^{-1} - 4(2)e^{-1} - 4e^{-1} + 4$

$R(2) = 129.3 = 1.293\%$

$R(3) = -2(3)^2e^{-1} - 4(3)e^{-1} - 4e^{-1} + 4$

$R(3) = 230.7 = 2.307\%$

$R(4) = -2(4)^2e^{-1} - 4(4)e^{-1} - 4e^{-1} + 4$

$R(4) = 304.8 = 3.04\%$

$R(5) = -2(5)^2e^{-1} - 4(5)e^{-1} - 4e^{-1} + 4$

$R(5) = 350.2 = 3.50\%$

$R(6) = -2(6)^2e^{-1} - 4(6)e^{-1} - 4e^{-1} + 4$

$R(6) = 375.3 = 3.573\%$

**Total reaction:** $R(1) + R(2) + \ldots\ldots\ldots + R(6)$

$$0.32 + 1.29 + 3.05 + 3.50 + 3.75 = 14.72$$

**Q.12 The rate of increase of the number cellular phones subscribers (in millions) Since service began, is given by $\frac{ds}{dx} = 0.38x + 0.04$. Where x is the number of year since 1998, when the service started. There were 0.25 million subscribers in year 1998 (x=0). Find a function that gives the number of subscribers for the year 2004.**

Perfect24u.com

**Solution:** Given that the rate of increase of subscribed

$$\frac{dS}{dx} = 0.38x + 0.04 \text{ and } x(0) = 0.25$$

$$\int dS = \int (0.38x + 0.04)\,dx$$

$$S = \frac{0.38x^2}{2} + 0.04x + c$$



www.perfect24u.com

$\mathbf{S = 0.19x^2 + 0.04x + c} \rightarrow$ (i)

Putting x=0 and s=0.25 in (i)

$0.25 = 0.19\ (0)^2 + 0.04(0) + c \Rightarrow$ **c=0.25**

Putting (i)

**$S=0.19x^2 + 0.04x + 0.25$**

Now for year 2004, x = 2004 – 1998 = 6

$S(6)=0.19(6)^2 + 0.04(6) + 0.25$

**S(6)=7.33 million**

**Q.13 Determine the equations of the orthogonal trajectories of the following families.**

**a) $y=cx^3$**

**Solution:** for given family F, $y=cx^3 \rightarrow$ (i)

Slope of F, $\left(\frac{dy}{dx}\right)_F = 3cx^2 \rightarrow$ (ii)

But from (i) $c = \frac{y}{x^3}$, putting in (ii)

$$\left(\frac{dy}{dx}\right)_F = 3 \cdot \frac{y}{x^3} \cdot x^2$$

$$\mathbf{\left(\frac{dy}{dx}\right)_F = \frac{3y}{x} \rightarrow (iii)}$$

Let the orthogonal family is G, then the slope of orthogonal family G is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F}$$

Perfect24u.com



www.perfect24u.com

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{3y}{x}\right)}$$

$$\left(\frac{dy}{dx}\right)_G = \frac{-x}{3y}$$

Separating variables and integrating

$$\int ydy = \int -\frac{1}{3}xdx$$

$$\frac{y^2}{2} = -\frac{1}{3}\frac{x^2}{2} + c$$

$$y^2 = -\frac{1}{3}x^2 + 2c$$

Let $2c = c_1$

$$y^2 = -\frac{x^2}{3} + c_1 \qquad \text{or} \qquad 3y^2 + x^2 = 3c_1$$

Let, $3\,c_1 = c_2$

**$3y^2 + x^2 = c_2$**

**b)** **$xy = c$**

**Solution:** For given family F, $xy = c \rightarrow$ (i)

Perfect24u.com

To find the slope of given curves, taking derivative of (i)

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c)$$

$$x\frac{dy}{dx} + y\cdot\frac{dx}{dx} = 0$$

$$x\frac{dy}{dx} + y = 0 \qquad \Rightarrow \qquad x\frac{dy}{dx} = -y$$



www.perfect24u.com

$$\left(\frac{dy}{dx}\right)_F = -\frac{y}{x} \rightarrow (ii)$$

Let the orthogonal family in G, then the slope of orthogonal family G is $\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F}$

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(-y/x\right)} = \frac{x}{y}$$

Separating variables and integrating

$$\int y dy = \int x dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c \quad \text{Multiplying by 2}$$

$$y^2 = x^2 + 2c$$

Let $2c = c_1$

$$\mathbf{y^2 - x^2 = c_1}$$

**c)** **$y=cxe^x$**

Perfect24u.com

**Solution:** For given family F, $y=cxe^x \rightarrow$ (i) $c = \frac{y}{xe^x}$

To find the slope of given family, taking derivative of (i)

$$\left(\frac{dy}{dx}\right)_F = \frac{d}{dx}\left(cxe^x\right)$$

$$\left(\frac{dy}{dx}\right)_F = c\left[x\frac{d}{dx}\left(e^x\right) + e^x\frac{d}{dx}(x)\right]$$

$$\left(\frac{dy}{dx}\right)_F = c\left(xe^x + e^x\right) \rightarrow \quad (ii)$$



www.perfect24u.com

But from (i), $c=\frac{y}{xe^x}$, putting in (ii)

$$\left(\frac{dy}{dx}\right)_F = \frac{y}{xe^x}\left[xe^x + e^x\right]$$

$$\left(\frac{dy}{dx}\right)_F = y\left[1+\frac{1}{x}\right] \rightarrow \text{(iii)}$$

Let the orthogonal family in G, then the slope of orthogonal family G is given by $\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{y\left(1+\frac{1}{x}\right)}$

Separating variables and integrating

$$\int ydy = \int \frac{-1}{\left(1+\frac{1}{x}\right)}dx$$

$$\frac{y^2}{2} = \int \frac{-1}{\frac{(x+1)}{x}}dx = \int \frac{-x}{x+1}dx$$

$$\frac{y^2}{2} = -\int \frac{x+1-1}{x+1}dx = -\int\left(\frac{x+1}{x+1}-\frac{1}{x+1}\right)dx$$

$$\frac{y^2}{2} = -\int 1dx + \int \frac{1}{x+1}dx = -x+\ln(x+1)+c$$

Perfect24u.com

Multiplying by 2

$y^2 = -2x+2\ln(x+1)+2c$ let $2c=c_1$

**$y_2=-2x+2\ln(x+1)+c_1$**

**d)** **$y^2=x^2+c$**

**Solution:** For given family F, $y^2=x^2+c \rightarrow$ (i)

To find the slope of given family F, derivate (i)



www.perfect24u.com

$$\frac{d}{dx}(y^2)=\frac{d}{dx}(x^2+c)$$

$$2y\frac{dy}{dx}=2x+0 \qquad \Rightarrow \qquad \frac{dy}{dx}=\frac{2x}{2y}$$

$$\left(\frac{dy}{dx}\right)_F=\frac{x}{y} \quad \rightarrow (ii)$$

Let the orthogonal trajectories are G, then the slope of G is given by

$$\left(\frac{dy}{dx}\right)_G=\frac{-1}{\left(\frac{dy}{dx}\right)_F}=\frac{-1}{\left(\frac{x}{y}\right)}$$

$$\left(\frac{dy}{dx}\right)_G=\frac{-y}{x}$$

Separating variable and integrating

$$\int\frac{dy}{y}=\int-\frac{1}{x}dx \qquad \rightarrow \qquad (iii)$$

$$\ln y=-\ln x+\ln c=\ln\left(\frac{c}{x}\right)$$

$$e^{\ln y}=e^{\ln\left(\frac{c}{x}\right)}$$

**$y=\frac{c}{x}$ or $xy=c$**

Perfect24u.com

**e) y=csin2x**

**Solution:** For given family F, y=csin2x → (i)

To find the slope of given family F, taking the derivative of (i)

$$\frac{dy}{dx}=\frac{d}{dx}(c\sin 2x)$$

$$\frac{dy}{dx}=c\cos 2x\cdot 2=2c\cos 2x$$



www.perfect24u.com

But from (i) $c=\frac{y}{\sin 2x}$

So, $\left(\frac{dy}{dx}\right)_F = 2y\frac{\cos 2x}{\sin 2x} \rightarrow$ (ii)

Let the orthogonal trajectory is G, then the slope of orthogonal trajectory is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{\left(2y\frac{\cos 2x}{\sin 2x}\right)}$$

Separating variables and integrating

$$\int y dy = \int -\frac{\sin 2x}{2\cos 2x} dx$$

$$\frac{y^2}{2} = \frac{1}{4}\int \frac{-2\sin 2x}{\cos 2x} dx \Rightarrow \qquad \frac{y^2}{2} = \frac{1}{4}\ln|c\cos 2x| + \ln c$$

$$\frac{y^2}{2} = \ln|\cos 2x|^{\frac{1}{4}} + \ln c = \ln|c(\cos 2x)|^{\frac{1}{4}}$$

$$\mathbf{y^2 = 2\ln|c(\cos 2x)|^{\frac{1}{4}}}$$

**f)** $\mathbf{e^x \cos y = c}$

**Solution:** For given family F, $e^x \cos y = c \rightarrow$ (i)

Perfect24u.com

To find the slope of given family, taking the derivative of (i)

$$\frac{d}{dx}\left(e^x \cos y\right) = \frac{d}{dx}(c)$$

$$e^x \frac{d}{dx}(\cos y) + \cos y \frac{d}{dx}\left(e^x\right) = 0$$

$$e^x(-\sin y)\frac{dy}{dx} + \cos y e^x = 0$$

$$e^x \sin y \frac{dy}{dx} = e^x \cos y$$



www.perfect24u.com

$$\left(\frac{dy}{dx}\right)_F = \frac{\cos y}{\sin y} \rightarrow \quad \text{(ii)}$$

Let the orthogonal trajectory G, the slope of G is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{\left(\frac{\cos y}{\sin y}\right)}$$

$$\left(\frac{dy}{dx}\right)_G = -\frac{\sin y}{\cos y}$$

Separating variables, and integrating

$$\int \frac{\cos y}{\sin y} dy = \int -dx$$

$$\ln|\sin y| = -x + c$$

$$e^{\ln|\sin y|} = e^{-x+c} \qquad \Rightarrow \qquad \sin y = e^{-x} \cdot e^{c}$$

Let $e^c = c_1$

$$\sin y = c_1 e^{-x}$$

**g)** $y = \sqrt{x + c}$

**Solution:** For given family F, $y = \sqrt{x + c}$

$$y^2 = x + c \rightarrow \quad \text{(i)}$$

Perfect24u.com

To find the slope of given family F, taking the derivative of (i)

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x + c)$$

$$2y\frac{dy}{dx} = 1 + 0$$

$$\left(\frac{dy}{dx}\right)_F = \frac{1}{2y} \qquad \rightarrow \qquad \text{(ii)}$$



www.perfect24u.com

Let the orthogonal trajectory is G, then the slope of G is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{\left(\frac{1}{2y}\right)} = -2y$$

$$\boldsymbol{\left(\frac{dy}{dx}\right)_G = -2y}$$

Separating variables, and integrating

$$\int \frac{1}{y} dy = \int -2dx$$

$$\ln y = -2x + c$$

$$e^{\ln y} = e^{-2x+c} = e^{-2x} \cdot e^{c} \quad \Rightarrow \quad \text{Let } e^{c} = c_1$$

$$\mathbf{y = c_1 e^{-2x}} \quad \text{or} \quad \mathbf{c_1 = ye^{2x}}$$

**h)** $\mathbf{y=x^2+c}$

**Solution:** For given family F, $y=x^2+c \quad \rightarrow$ (i)

To find the slope of given family F, taking the derivative of (i)

$$\left(\frac{dy}{dx}\right)_F = \frac{d}{dx}\left(x^2 + c\right)$$

$$\boldsymbol{\left(\frac{dy}{dx}\right)_F = 2x} \quad \rightarrow \text{(ii)}$$

Perfect24u.com

Let the orthogonal trajectory is G, then the slope of G is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{2x}$$

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{2x}$$

Separating variables, and integrating

$$\int dy = \int \frac{-1}{2x} dx$$



www.perfect24u.com

$$y = \frac{-1}{2}\ln x + c$$

OR

$$y = \frac{-1}{2}\ln x + \ln c = \ln x^{\frac{-1}{2}} + \ln c = \ln\left(cx^{-\frac{1}{2}}\right)$$

$$\mathbf{y = \ln\left(\frac{c}{\sqrt{x}}\right)}$$

**i)** $\mathbf{e^x \sin y = c}$

**Solution:** For given family F, $e^x \sin y = c \quad \rightarrow$ (i)

To find the slope of given family F, taking the derivative of (i)

$$\frac{d}{dx}\left(e^x \sin y\right) = \frac{d}{dx}(c)$$

$$e^x \frac{d}{dx}(\sin y) + \sin y \frac{d}{dx}\left(e^x\right) = 0$$

$$e^x \cos y \cdot \frac{dy}{dx} + \sin y\, e^x = 0 \quad \Rightarrow \quad e^x \cos y \frac{dy}{dx} = -\sin y\, e^x$$

$$\mathbf{\left(\frac{dy}{dx}\right)_F = \frac{-\sin y}{\cos y}} \rightarrow \text{(ii)}$$

Let the orthogonal trajectory is G, then the slope of G is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{\left(\frac{-\sin y}{\cos y}\right)}$$

Perfect24u.com

Separating variables, and integrating

$$\int \frac{\sin y}{\cos y} dy = \int 1 dx$$

$$-\ln|\cos y| = x + c$$

$$\ln|\cos y| = -x - c$$

$$e^{\ln|\cos y|} = e^{-x-c}$$



www.perfect24u.com

$\cos y = e^{-x} \cdot e^{-c}$

Let $e^{-c} = c_1$

$\cos y = c_1 e^{-x}$

$\mathbf{e^x \cos y = c_1}$

**j)** $\mathbf{\cos x \cosh y = c}$

**Solution:** For given family F, $\cos x \cosh y = c \rightarrow$ (i)

To find the slope of given family F, taking the derivative of (i)

$$\frac{d}{dx}(\cos x \cosh y) = \frac{d}{dx}(c)$$

$$\cos x \frac{d}{dx}(\cosh y) + \cosh y \frac{d}{dx}(\cos x) = 0$$

$$\cos x \sinh y \frac{dy}{dx} + \cosh y(-\sin x) = 0$$

$$\cos x \sinh y \frac{dy}{dx} = \sin x \cosh y$$

$$\left(\frac{dy}{dx}\right)_F = \frac{\sin x \cosh y}{\cos x \sinh y} \rightarrow \quad \text{(ii)}$$

Let the orthogonal trajectory is G, then the slope of G is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{\left(\frac{\sin x \cosh y}{\cos x \sinh y}\right)}$$

Perfect24u.com

Separating the variables and integrating

$$\int \frac{\cosh y}{\sinh y} dy = -\int \frac{\cos x}{\sin x} dx$$

$$\ln|\sinh y| = -\ln|\sin x| + \ln c = \ln\left|\frac{c}{\sin x}\right|$$

$$e^{\ln|\sinh y|} = e^{\ln\left|\frac{c}{\sin x}\right|}$$

$$\sinh y = \frac{c}{\sin x}$$

$\mathbf{\sinh y \sin x = c}$



www.perfect24u.com

**k)** $x\cos y = c$ **(Book question is wrong)**

**Solution:** For given family F, $x\cos y = c \rightarrow$ (i)

To find the slope of given family F, taking the derivative of (i)

$$\frac{d}{dx}(x\cos y) = \frac{d}{dx}(c)$$

$$x\frac{d}{dx}(\cos y) + \cos y\frac{d}{dx}(x) = 0$$

$$x(-\sin y)\frac{dy}{dx} + \cos y \cdot 1 = 0$$

$$-x\sin y\frac{dy}{dx} = -\cos y$$

$$\left(\frac{dy}{dx}\right)_F = \frac{+\cos y}{x\sin y} \rightarrow \text{(ii)}$$

Let the orthogonal trajectory is G, then the slope of G is given by

$$\left(\frac{dy}{dx}\right)_G = \frac{-1}{\left(\frac{dy}{dx}\right)_F} = \frac{-1}{\left(\frac{\cos y}{x\sin y}\right)}$$

$$\left(\frac{dy}{dx}\right)_G = \frac{-x\sin y}{\cos y}$$

Separating the variables and integrating

$$\int \frac{\cos y}{\sin y}dy = \int -x dx$$

$$\ln|\sin y| = -\frac{x^2}{2} + c$$

$$e^{\ln|\sin y|} = e^{-\frac{x^2}{2}} + c$$

$$\sin y = e^{-\frac{x^2}{2}} \cdot e^c$$

Let $e^c = c_1$

$$\sin y = c_1 e^{-\frac{x^2}{2}}$$

Perfect24u.com



www.perfect24u.com