Unit 06

PLANE ANALYTIC GEOMETRY STRAIGHT LINE

Distance between two points:

The distance between two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by:

$$|P_1P_2| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \quad \text{or} \quad \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \qquad \text{(i)}$$

Equation (i) is called distance formula.

Dividing a line segment in a given ratio:

Let a point P(x,y) divides $A(x_1,y_1)$ and $B(x_2,y_2)$, in the ratio $m_1: m_2$ then the coordinates of points P(x,y) are given by:

$$P(x,y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

If $m_1=m_2=m$, then P(x,y) is the midpoint of segment AB and is given by

$$P(x,y) = M(x,y) = \left(\frac{mx_2+mx_1}{m+m}, \frac{my_2+my_1}{m+m}\right)$$

$$M(x,y) = \left(\frac{m(x_1+x_2)}{2m}, \frac{m(y_1+y_2)}{2m}\right)$$

$$M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Perfect24u.com

Midpoint formula

EXERCISE 6.1

Q.1 In each case, find the length of the line segment PQ joining the two points.

a) P(–3,–5), Q(–3,–7)

Solution: According to distance formula, the length of line joining to points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by;

www.perfect24u.com

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow (i)$$

Here $x_1 = -3$, $x_2 = -3$, $y_1 = -5$, $y_2 = 7$

Putting the values in (i)

$$|PQ| = \sqrt{((-3)-(-3))^2 + ((-7)-(-5))^2} = \sqrt{(-3+3)^2 + (-7+5)^2}$$

$$|PQ| = \sqrt{(0)^2 + (2)^2}$$

$$|PQ| = \sqrt{4}$$

$$\mathbf{|PQ| = 2}$$

b) P(–4,3), Q(7,2)

Solution: According to distance formula, the length of line joining to points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by;

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow (i)$$

Here $x_1 = -4$, $x_2 = 7$, $y_1 = 3$, $y_2 = 2$

Putting the values in (i)

$$|PQ| = \sqrt{((7)-(-4))^2 + ((2)-(3))^2} = \sqrt{(7+4)^2 + (2-3)^2}$$

$$|PQ| = \sqrt{(11)^2 + (-1)^2}$$

$$|PQ| = \sqrt{121+1}$$

$$\mathbf{|PQ| = \sqrt{122}}$$ **Book answer is wrong**

c) P(5,5) Q(5,–6)

Solution: According to distance formula, the length of line joining to points P(x,y) and $Q(x_2, y_2)$ is given by;

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow (i)$$

Here $x_1 = 5$, $x_2 = 5$, $y_1 = 5$, $y_2 = -6$

Perfect24u.com

www.perfect24u.com

Putting the values in (i)

$$|PQ| = \sqrt{((5)-(5))^2 + ((-6)-(5))^2} = \sqrt{(0)^2 + (-11)^2}$$

$$|PQ| = \sqrt{121}$$

$$\mathbf{|PQ| = 11}$$

d) P(0,0) Q(0,3).

Solution: According to distance formula, the length of line joining to points P(x,y) and Q(x_2, y_2) is given by;

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \rightarrow (i)$$

Here $x_1=0$, $x_2=0$, $y_1=0$, $y_2=3$

Putting the values in (i)

$$|PQ| = \sqrt{((0)-(0))^2 + ((3)-(0))^2} = \sqrt{(0)^2 + (3)^2} = \sqrt{9}$$

$$\mathbf{|PQ| = 3}$$

Q.2 The three points are A(–1,3) B(2,1) and C(5,–1) Show that

a) $\mathbf{|AB| + |BC| = |AC|}$

Solution: To show $|AB| + |BC| = |AC|$

Perfect24u.com

According to distance formula, the distance between two points $P_1(x_1,y_1)$ of $P_2(x_2,y_2)$ is given by:

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Now for the given points, A(–1,3), B(2,1) and C(5,–1)

$$|AB| = \sqrt{((2)-(-1))^2 + ((1)-(3))^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4}$$

$$\mathbf{|AB| = \sqrt{13}}$$

$$|BC| = \sqrt{(5-2)^2 + (-1-1)^3} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4}$$

www.perfect24u.com

$|\mathbf{BC}| = \sqrt{13}$

and $|AC| = \sqrt{((5)-(-1))^2 + ((-1)-(3))^2} = \sqrt{(6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52}$

$|\mathbf{AC}| = 2\sqrt{13}$

$|AB| + |BC| = \sqrt{13} + \sqrt{13} = 2\sqrt{13} = |AC|$

Hence $|\mathbf{AB}| + |\mathbf{BC}| = |\mathbf{AC}|$

Q.3 Show that the points A(0,–2), B(3,1), C(0,4) and D(–3,1) are the vertices of a square ABCD.

Note: For a square all the sides are of same length and the two diagonals are of same length.

Here A(0,–2), B(3,1), C(0,4) and D(–3,1)

According to the distance formula, the distance between two points, $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by:

Here A(0,–2), $x_1=0$, $y_1=-2$

B(3,1), $x_2=3$, $y_2=-1$

C(0,4), $x_3=0$, $y_3=4$

D(–3,1), $x_4=-3$, $y_4=1$

$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Perfect24u.com

Now for the given points A,B,C and D, sides

$|AB| = \sqrt{(3-0)^2 + (1-(-2))^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$|BC| = \sqrt{(0-3)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$|CD| = \sqrt{(-3-0)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

$|DA| = \sqrt{(-3-0)^2 + (1-(-2))^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

www.perfect24u.com

And the diagonals

$$|AC| = \sqrt{(0-0)^2 + (4-(-2))^2} = \sqrt{(0)^2 + (6)^2} = \sqrt{0+36} = 6$$

$$|BD| = \sqrt{(-3-3)^2 + (1-1)^2} = \sqrt{(-6)^2 + (0)^2} = \sqrt{36+0} = 6$$

$$|AB| = |CD|$$

$$\mathbf{3\sqrt{2} = 3\sqrt{2}}$$

$$|AD| = |BC|$$

$$\mathbf{3\sqrt{2} = 3\sqrt{2}}$$

As all the sides are of same length and also the diagonals are of same length, so the points A,B, C and D are vertices of a square.

Q.4 Show that the points A(13, –1), B(–9,3), and C(–3,–9) are the vertices of a right angle triangle ABC.

Solution:

According to Pythagoras theorem in a right angle triangle the sum of squares of two sides is equal to the square of third side. To apply this rule to the given points, we have to find the length of sides of triangle ΔABC.

According to distance formula, the distance between two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by;

$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Now for the given points A, B and C

A(13, –1), B(–9,3), and C(–3,–9)

$$|AB| = \sqrt{(13+9)^2 + (3+1)^2} = \sqrt{576+16} = \sqrt{592}$$

$$\mathbf{|AB| = \sqrt{529}} \text{ …….. (i)}$$

B(–9, 3) and (–3, –9)

Perfect24u.com
www.perfect24u.com

$|BC| = \sqrt{(9+3)^2 + (3+9)^2} = \sqrt{36+144} = \sqrt{180}$

$\mathbf{|BC| = \sqrt{180}}$ …….. (ii)

Now A(13, –1), C(–3, –9)

$|AC| = \sqrt{(13+3)^2 + (-1+9)^2} = \sqrt{256+64} = \sqrt{320}$

$\mathbf{|AC| = \sqrt{320}}$ …….. (iii)

Now $|AB|^2 = 592$, $|BC|^2 = 180$, $|AC|^2 = 320$

$|AB|^2 \neq |BC|^2 + |AC|^2$

$592 \neq 180 + 320 = 500$

$592 \neq 500$

Hence it is not right angle triangle.

Q.5 Show that the points A(21,–2), B(15,10), and C(–5,0) and D(1,–12) are the vertices of a rectangle ABCD.

Note: For a rectangle the opposite sides and the two diagonals are of same length. According to distance formula, the distance between two points, $P_1(x_1,y_1)$ & $P_2(x_2,y_2)$ is given as,

$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Now for the given points A,B,C and D sides are Hence

$|AB| = \sqrt{(15-21)^2 + (10+2)^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}$ $|AB| = |CD|$

$|BC| = \sqrt{(-5-15)^2 + (0-10)^2} = \sqrt{400+100} = \sqrt{500} = 10\sqrt{5}$ $|BC| = |AD|$

$|CD| = \sqrt{(1+5)^2 + (-12-0)^2} = \sqrt{36+144} = \sqrt{180} = 6\sqrt{5}$

$|AD| = \sqrt{(1-21)^2 + (-12+2)^2} = \sqrt{400+100} = \sqrt{500} = 10\sqrt{5}$

and the diagonals

$|AC| = \sqrt{(-5-21)^2 + (0+2)^2} = \sqrt{676+4} = \sqrt{680} = 2\sqrt{170}$

Perfect24u.com

www.perfect24u.com

$$|BD| = \sqrt{(1-15)^2 + (-12-10)^2} = \sqrt{(-14)^2 + (-22)^2} = \sqrt{196+484} = \sqrt{680} = 2\sqrt{170}$$

As all the opposite sides and the two diagonals are of same length, so points A, B, C and D are the vertices of a rectangle.

Q.6 Show that the points A(4,3), B(3,1) and C(1,2) are the vertices of an isosceles triangle, ABC.

Solution:

For isosceles triangle two sides are of same length According to distance formula, the distance between two points, $P_1(x_1,y_1)$ & $P_2(x_2,y_2)$ is given as,

$$|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

For the given points A,B, and C sides are

$$|AB| = \sqrt{(3-4)^2 + (1-3)^2} = \sqrt{1+4} = \sqrt{5}$$

$$|BC| = \sqrt{(1-3)^2 + (2-1)^2} = \sqrt{4+1} = \sqrt{5}$$

$$|AC| = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$$ Hence $|\mathbf{AB}| = |\mathbf{BC}|$

As the two sides are of same length, so the points A,B and C are the vertices of an isosceles triangles ABC.

Perfect24u.com

Q.7 Find the point which is equidistant from the points A(2,1), B(–4,3) and C(–6,5)

Solution:

Let the required points be P(x,y) then, for the given points A, B and C

$|PA| = |PB|$ using distance formula.

$$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x+4)^2 + (y-3)^2}$$

Taking square of both sides

$$x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 + 8x + 16 + y^2 - 6y + 9$$

www.perfect24u.com

$-12x+4y-20=0$ divide by -4

$3x-y+5=0$ $\rightarrow$ (i)

Also $|PA|=|PC|$

$$\sqrt{(x-2)^2+(y-1)^2}=\sqrt{(x+6)^2+(y-5)^2}$$

Taking square of both sides

$$x^2-4x+4+y^2-2y+1=x^2+12x+36+y^2-10y+25$$

$-16x+8y-56=0$ divide by -8

$2x-y+7=0$ $\rightarrow$ (ii)

Subtracting (ii) from (i)

$3x-y+5=0$

$\pm 2x \mp y \pm 7=0$

$x-2=0 \quad \Rightarrow \quad \mathbf{x=2}$

putting in (i)

$3(2)-y+5=0$

$11-y=0$

$y=11$

Hence the point is $P(x, y)=P(2,11)$

Perfect24u.com

Q.8 In each case, find the midpoint of the line segment PQ jointing the two points $P(x_1,y_1)$ and $Q(x_2,y_2)$

a. $P(10,20)$ $Q(-12,-8)$

Solution: The midpoint of two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by

$$M(x, y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right) \rightarrow (i)$$

Here $x_1=10$, $x_2=-12$, $y_1=20$, $y_2=-8$

Putting in (i)

www.perfect24u.com

$$M(x, y) = \left(\frac{(10)+(-12)}{2}, \frac{(20)+(-8)}{2}\right)$$

$$M(x, y) = \left(\frac{-2}{2}, \frac{12}{2}\right)$$

$$\mathbf{M(x, y) = (-1, 6)}$$

b. **P(a, –b) Q (–a, b)**

Solution: The midpoint of two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow (i)$$

Here $x_1 = a$, $x_2 = -a$, $y_1 = -b$, $y_2 = b$

Putting in (i)

$$M(x, y) = \left(\frac{(a)+(-a)}{2}, \frac{(-b)+(b)}{2}\right)$$

$$M(x, y) = \left(\frac{a-a}{2}, \frac{-b+b}{2}\right)$$

$$M(x, y) = \left(\frac{0}{2}, \frac{0}{2}\right)$$

$$\mathbf{M(x, y) = (0, 0)}$$

Perfect24u.com

c. **P(a – b, a + b) Q (a + b, a – b)**

Solution: The midpoint of two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow (i)$$

Here $x_1 = a - b$, $x_2 = a + b$ $y_1 = a + b$, $y_2 = a - b$

Putting in (i)

$$M(x, y) = \left(\frac{a-b+a+b}{2}, \frac{a+b+a-b}{2}\right)$$

www.perfect24u.com

$$M(x, y) = \left(\frac{2a}{2}, \frac{2a}{2}\right)$$

$$\mathbf{M(x, y) = (a, a)}$$

d. $\mathbf{P\left(\frac{-1}{2}, \frac{-2}{4}\right), Q\left(\frac{3}{5}, \frac{4}{7}\right)}$

Solution: The midpoint of two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow (i)$$

Here $x_1 = \frac{-1}{2}$, $x_2 = \frac{3}{5}$, $y_1 = \frac{-2}{4}$, $y_2 = \frac{4}{7}$

Putting in (i)

$$M(x, y) = \left(\frac{-1/2 + 3/5}{2}, \frac{-2/4 + 4/7}{2}\right)$$

$$M(x, y) = \left(\frac{-5 + 6/10}{2}, \frac{-14 + 16/28}{2}\right)$$

$$M(x, y) = \left(\frac{1}{20}, \frac{2}{56}\right)$$

$$\mathbf{M(x, y) = \left(\frac{1}{20}, \frac{1}{28}\right)}$$

Perfect24u.com

Q.9 In each case, find the coordinates of the point P(x, y) which divides the segment PQ joining the two points

a. P(1,2) Q (3,4) in the ratio 5:7

Solution: The points R(x, y) that divides the line between points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is ratio $m_1:m_2$ is given by

$$R(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right) \rightarrow (i)$$

www.perfect24u.com

Here $x_1=1$, $x_2=3$, $y_1=2$, $y_2=4$, $m_1=5$, $m_2=7$

Putting these value in (i)

$$R(x, y)=\left(\frac{(5)(3)+(7)(1)}{5+7},\frac{(5)(4)+(7)(2)}{5+7}\right)$$

$$M(x, y)=\left(\frac{15+7}{12},\frac{20+14}{12}\right)=\left(\frac{22}{12},\frac{34}{12}\right)$$

$$\mathbf{M(x, y)=\left(\frac{11}{6},\frac{17}{6}\right)}$$

b. P(3,4), Q (–6,2) in the ratio 3:–2

Solution: The points R(x, y) that divides the line between points $P(x_1,y_1)$ and $Q(x_2,y_2)$ in ratio $m_1:m_2$ is given by;

$$R(x, y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)\rightarrow(i)$$

Here $x_1=3$, $x_2=-6$, $y_1=4$, $y_2=2$, $m_1=3$, $m_2=-2$

Putting these value in (i)

$$R(x, y)=\left(\frac{(3)(-6)+(-2)(3)}{3-2},\frac{(3)(2)+(-2)(4)}{3-2}\right)$$

$$R(x, y)=\left(\frac{-18-6}{1},\frac{6-8}{1}\right)$$

R(x, y) = (–24,–2)

Perfect24u.com

c. P(1,–2), Q (4,7) in the ratio –2:3

Solution: The points R(x, y) that divides the line between points $P(x_1,y_1)$ and $Q(x_2,y_2)$ in ratio $m_1:m_2$ is given by

$$R(x, y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)\rightarrow(i)$$

Here $x_1=1$, $x_2=4$, $y_1=-2$, $y_2=7$, $m_1=-2$, $m_2=3$

Putting these value in (i)

www.perfect24u.com

$$R(x, y)=\left(\frac{(-2)(4)+(3)(1)}{-2+3}, \frac{(-2)(7)+(3)(-2)}{-2+3}\right)$$

$$R(x, y)=\left(\frac{-8+3}{1}, \frac{-14-6}{1}\right)$$

R(x, y) = (–5,–20)

d. P(–6,7), Q (5,–4) is the ratio $\frac{2}{7}:1$

Solution: The points R(x, y) that divides the line between points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is ratio $m_1:m_2$ is given by

$$R(x , y)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right) \rightarrow (i)$$

Here $x_1=-6$, $x_2=5$, $y_1=7$, $y_2=-4$, $m_1=\frac{2}{7}$, $m_2=1$

Putting these value in (i)

$$R(x, y)=\left(\frac{(2/7)(5)+(1)(-6)}{2/7+1}, \frac{(2/7)(-4)+(1)(7)}{2/7+1}\right)$$

$$R(x, y)=\left(\frac{10/7-6/1}{2+7/7}, \frac{-8/7+7/1}{2+7/7}\right)=\left(\frac{10-42/7}{9/7}, \frac{-8+49/7}{9/7}\right)$$

R(x, y) = $\left(\frac{-32}{9}, \frac{41}{9}\right)$

Perfect24u.com

Q.10 In each case, find the coordinates of the end point (P or Q) of the line segment PQ, when the midpoint of the segment PQ are the following

a. D(2,–1) P (–1,–3) Q=?

Solution: Let the unknown point Q(x, y). As D(2,–1) is the midpoint of PQ, So

$$D(2,-1)=\left(\frac{-1+x}{2}, \frac{-3+y}{2}\right)$$

www.perfect24u.com

Equating the coordinates on both sides

$$\frac{-1+x}{2}=2 \quad \Rightarrow -1+x=4$$

$x=5$

and $\frac{-3+y}{2}=-1 \quad \Rightarrow -3+y=-2$

$y=1$

Thus **Q(x, y) = Q(5,1)**

b. **D(–3,–4)** **Q(2,4)** **P=?**

Solution: Let the unknown point be P(x, y). As D(–3,–4) is the midpoint of PQ, So

$$D(-3,-4)=\left(\frac{x+2}{2},\frac{y+4}{2}\right)$$

Equating the coordinates on both sides

$$\frac{x+2}{2}=-3 \quad \Rightarrow x+2=-6$$

$x=-8$

and $\frac{y+4}{2}=-4 \quad \Rightarrow y+4=-8$

$y=-12$

Thus **P(x, y) = P(–8,–12)**

Perfect24u.com

c. **D(1,2),** **P(4,5)** **Q=?**

Solution: Let the unknown point be Q(x, y). As D(1,2) is the midpoint of segment PQ, So

$$D(1,2)=\left(\frac{4+x}{2},\frac{5+y}{2}\right)$$

Equating the coordinates on both sides

$$\frac{4+x}{2}=1 \quad \Rightarrow 4+x=2$$

www.perfect24u.com

$x=-2$

and $\frac{5+y}{2}=2 \quad \Rightarrow 5+y=4$

$y=-1$

Thus **Q(x, y) = Q(–2,–1)**

d. D(–3,5), Q(3,2) P=?

Solution: Let the unknown point be P(x, y). As D(–3,5) is the midpoint of segment PQ, So

$$D(-3,5)=\left(\frac{x+3}{2},\frac{y+2}{2}\right)$$

Equating the coordinates on both sides

$\frac{x+3}{2}=-3 \quad \Rightarrow x+3=-6$

$x=-9$

and $\frac{y+2}{2}=5 \quad \Rightarrow y+2=10$

$y=8$

Thus **P(x, y) = P(–9,8)**

Perfect24u.com

Q.11 In each case, in what ratio is the line segment PQ joining the two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ divided by the point R(x, y):

a. P(8,10) Q (–12,6) $R\left(\frac{-4}{7},\frac{58}{7}\right)$

Solution: Let the ratio is $m_1:m_2$ then by ratio formula

$$R(x_1y)=\left(\frac{x_2m_1+m_1x_2}{m_1+m_2},\frac{y_2m_1+m_2y_1}{m_1+m_2}\right)$$

$$R\left(\frac{-4}{7},\frac{58}{7}\right)=\left(\frac{(m_1)(-12)+(m_2)(8)}{m_1+m_2},\frac{(m_1)(6)+(m_2)(10)}{m_1+m_2}\right)$$

www.perfect24u.com

Equating the coordinates on both sides

$$\frac{-12m_1+8m_2}{m_1+m_2}=\frac{-4}{7} \quad \Rightarrow \quad -84m_1+56m_2=-4m_1-4m_2$$

$-80m_1+60m_2=0$, dividing by -20

$4m_1-3m_2=0 \rightarrow$(i)

$4m_1=3m_2$

$$\frac{m_1}{m_2}=\frac{3}{4}$$

So the ratio is $m_1:m_2 = 3:4$

b. P(–1,4), Q(5,–2), R(–1,2)

Solution: Let the ratio is $m_1:m_2$ then by ratio formula

$x_1 = -1, y_1 = 4, x_2 = 5, y_2 = -2$

$$R(-1,2)=\left(\frac{5(m_1)+(-1)(m_2)}{m_1+m_2},\frac{(-2)(m_1)+(m_2)(4)}{m_1+m_2}\right)$$

Equating the coordinates on both sides

$$-1=\frac{5m_1-m_2}{m_1+m_2}$$

$(m_1+m_2)(-1) = 5m_1-m_2$

$-m_1-m_2 = 5m_1-m_2$

$-m_1-5m_1 = -m_2+m_2$

$-6m_1 = 0$

$m_1 = 0, m_2 = 0, m_1:m_2 = 0:0$

c. P(4,4), Q(10,16), R(6,8)

Solution: $x_1 = 4, y_1 = 4, x_2 = 10, y_2 = 16$ and $x = 6, y = 8$

Let the ratio is $m_1:m_2$ then by ratio formula

$$R(6,8)=\left(\frac{10m_1+4m_2}{m_1+m_2},\frac{16m_1+4m_2}{m_1+m_2}\right)$$

Comparing coordinates on both sides

Perfect24u.com

www.perfect24u.com

$$6 = \frac{10m_1 + 4m_2}{m_1 + m_2}$$

$6(m_1+m_2) = 10m_1 + 4m_2$

$6m_1 + 6m_2 = 10m_1 + 4m_2$

$6m_1 - 10m_1 = 4m_2 - 6m_2$

$-4m_1 = -2m_2$

$2m_1 = m_2$

$$\frac{m_1}{m_2} = \frac{1}{2}$$

So the ratio is; $m_1 : m_2 = 1 : 2$

d. P(–1,–2), Q (3,2), R(2,1)

Solution:

$x_1 = -1, y_1 = -2, x_2 = 3, y_2 = 2$ and $x = 2, y = 1$

Let the ratio be $m_1:m_2$, then by ratio formula

$$R(2,1) = \left(\frac{3m_1 - m_2}{m_1 + m_2}, \frac{2m_1 - 2m_2}{m_1 + m_2}\right)$$

Equating the coordinates on both sides

$$2 = \frac{3m_1 - m_2}{m_1 + m_2}$$

Perfect24u.com

$2(m_1 + m_2) = 3m_1 - m_2$

$2m_1 + 2m_2 = 3m_1 - m_2$

$2m_1 - 3m_1 = -m_2 - 2m_2$

$\not{-}m_1 = \not{-}3m_2$

$$\frac{m_1}{m_2} = \frac{3}{1}$$

So, the ratio is $m_1:m_2 = 3 : 1$

www.perfect24u.com

Q.12 Find the centroid of the triangle ABC, whose vertices are the following

a. A(4,–2) B (–2,4) C(5,5)

Solution: The centroid G(x, y) of triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and (x_3,y_3) is given by

$$G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)\rightarrow(i)$$

Here $x_1=4$, $x_2=-2$, $x_3=5$

$y_1=-2$, $y_2=4$, $y_3=5$

Putting these values in (i)

$$G(x,y)=\left(\frac{4-2+5}{3},\frac{-2+4+5}{3}\right)=\left(\frac{7}{3},\frac{7}{3}\right)$$

$$\mathbf{G(x,y)=\left(\frac{7}{3},\frac{7}{3}\right)}$$

b. A(3,5) B(4,6) C(3,–1) (book question is wrong)

Solution: The centroid G(x, y) of triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and (x_3,y_3) is given by

$$G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)\rightarrow(i)$$

Perfect24u.com

Here $x_1=3$, $x_2=4$, $x_3=3$

$y_1=5$, $y_2=6$, $y_3=-1$

Putting these values in (i)

$$G(x,y)=\left(\frac{3+4+3}{3},\frac{5+6-1}{3}\right)=\left(\frac{10}{3},\frac{10}{3}\right)$$

$$\mathbf{G(x,y)=\left(\frac{10}{3},\frac{10}{3}\right)}$$

www.perfect24u.com

c. **A(1,1)** **B (–2,–2)** **C(4,5)**

Solution: The centroid G(x,y) of a triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and (x_3,y_3) is given by

$$G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)\rightarrow(i)$$

Here $x_1=1, x_2=-2, x_3=4$

$y_1=1, y_2=-2, y_3=5$

Putting these values in (i)

$$G(x,y)=\left(\frac{1-2+4}{3},\frac{1-2+5}{3}\right)=\left(\frac{3}{3},\frac{4}{3}\right)$$

$$\mathbf{G(x,y)=\left(1,\frac{4}{3}\right)}$$

d. **A(1,4)** **B (2,6)** **C(3,–1)**

Solution: The centroid G(x, y) of a triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and (x_3,y_3) is given by

$$G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)\rightarrow(i)$$

Here $x_1=1, x_2=2, x_3=3$

$y_1=4, y_2=6, y_3=-1$

Perfect24u.com

Putting these values in (i)

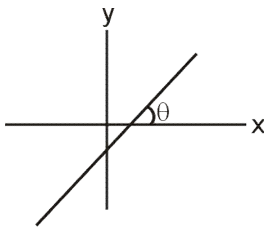
$$G(x,y)=\left(\frac{1+2+3}{3},\frac{4+6+(-1)}{3}\right)=\left(\frac{6}{3},\frac{9}{3}\right)$$

$$\mathbf{G(x,y)=(2,3)}$$

www.perfect24u.com

Slope of a straight line:

The slope of a line is the tangent of the angle it makes with the positive x-axis and shows the steepness of the straight line denoted as $m=\tan\theta$.



Slope formula:

If two points on a straight line has coordinates $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$, then the slope m is given by:

$$m=\frac{y_2-y_1}{x_2-x_1}$$

Two straight lines with slopes m_1 and m_2 are;

1. Parallel if $m_1=m_2$
2. Perpendicular if $m_1m_2=-1$

- The slope of line parallel to x-axis is zero.
- The slope of line perpendicular to x-axis is undefined.

The equation of a straight line:

The equation of the straight line can be obtained by a number of ways

a) **Slope-intercept form:** The equation of straight line with slope m and y-intercept c is given by:

$y=mx+c$ (slope-intercept form)

b) **Point-slope form:** The equation of straight line with slope m and passing through point $p(x_1,y_1)$ is given by:

$y-y_1=m(x-x_1)$ (point-slope form)

c) **Two-point form:** The equation of straight line passing through two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1} \text{ or } \frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

Perfect24u.com

www.perfect24u.com

d) **Double-intercepts form:** The equation of straight line with intercepts a and b on the x-axis and y-axis respectively is given by:

$$\frac{x}{a}+\frac{y}{b}=1$$

e) **Normal form:** The equation of straight line, where the length of perpendicular from origin to line is P and the perpendicular makes an angle θ with x-axis is given by:

$x\cos\theta+y\sin\theta=P$

The more general form of the straight line equation is

$ax+by+c=0$

Which can be reduced to other forms as;

Slope intercept form: $ax+by+c=0$

$by=-ax-c$ divide by b

$$y=\frac{-a}{b}x-\frac{c}{b}$$

Slope $=\frac{-a}{b}$, y–intercept$=\frac{-c}{b}$

Perfect24u.com

Intercepts from: $ax+by+c=0$

$ax+by=-c$ divide by $-c$

$$\frac{ax}{-c}+\frac{by}{-c}=1 \quad \Rightarrow \quad \frac{x}{\left(\frac{-c}{a}\right)}+\frac{y}{\left(\frac{-c}{b}\right)}=1$$

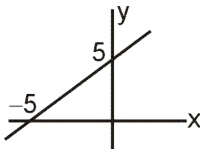
x-intercept$=\frac{-c}{a}$, y-intercept $=\frac{-c}{b}$

www.perfect24u.com

EXERCISE 6.2

Q.1 Find the equation of lines that are represented on the coordinate planes.

a)



Solution:

Here, from the given figure, both the x-intercept and intercept can be seen i.e.

x-intercept, $a=-5$

y-intercept, $b=5$

Using the double intercept form of straight line equation

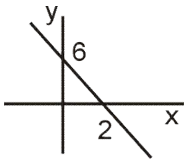
$\frac{x}{a}+\frac{y}{b}=1$ putting values of a and b

$\frac{x}{-5}+\frac{y}{5}=1$ Multiplying by –5

$x-y=-5$

$x-y+5=0$

b)



Solution:

Here, from the given figure, both the x-intercept and intercept can be seen i.e.

Perfect24u.com

x-intercept, $a=2$

y-intercept, $b=6$

Using the two intercepts form of straight line equation

$\frac{x}{a}+\frac{y}{b}=1$ Putting values of a and b

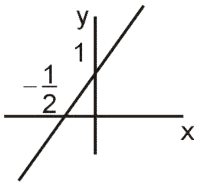
$\frac{x}{2}+\frac{y}{6}=1$ Multiplying by 6

$3x+y=6$

www.perfect24u.com

3x + y –6=0

c)



Solution:

Here, from the given figure, both the x-intercept and intercept can be seen i.e.

x-intercept, $a=\frac{-1}{2}$,

y-intercept, b=1

Using the two intercepts form of straight line equation

$\frac{x}{a}+\frac{y}{b}=1$ Putting values of a and b

$\frac{x}{\left(-1/2\right)}+\frac{y}{1}=1$ Multiplying by $\frac{-1}{2}$

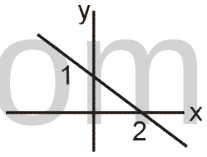
$\frac{x}{\left(-1/2\right)}\cdot\left(-1/2\right)+\frac{y}{1}\left(-1/2\right)=1\left(-1/2\right)$

$x-\frac{1}{2}y=\frac{-1}{2}$ multiplying by 2

$2x-y=-1$

2x – y + 1 = 0

d)



Solution:

Here, from the given figure, both the x-intercept and intercept can be seen i.e.

x-intercept, a=2

y-intercept, b=1

Using the two intercepts form of straight line equation

$\frac{x}{a}+\frac{y}{b}=1$ Putting values of a and b

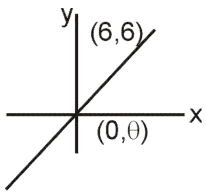
Perfect24u.com

www.perfect24u.com

$\frac{x}{2}+\frac{y}{1}=1$ Multiplying by 2

$x + 2y = 2$

$x + 2y - 2 = 0$



e)

Solution:

The two points on the straight line are A(0,0) and $B_2(6,6)$.

Using the two points form of straight line equation

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}\rightarrow (i)$$

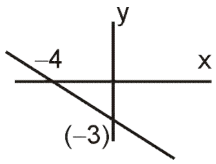
Here $x_1=0$, $y_1=0$, $x_2=6$, $y_2=6$

Putting values in (i)

$$\frac{y-0}{x-0}=\frac{6-0}{6-0}$$

$$\frac{y}{x}=\frac{6}{6} \quad \Rightarrow \quad \frac{y}{x}=1$$

$y = x$ $\Rightarrow$ $x - y=0$



f)

Solution:

Perfect24u.com

Here, from the given figure, both the x-intercept and intercept can be seen i.e.

x-intercept, a=–4

y-intercept, b=–3

Using the two intercepts form of straight line equation

$\frac{x}{a}+\frac{y}{b}=1$ putting values of a and b

$\frac{x}{-4}+\frac{y}{-3}=1$ multiplying by –12

www.perfect24u.com

$$\frac{x}{-4}(-12)+\frac{y}{-3}(-12)=1(-12)$$

$3x + 4y = -12$

$3x + 4y + 12 = 0$

Q.2 Arrange these lines in order of steepness (shallowest to steepest).

a) $y=\frac{2}{5}x+4$ **b)** $y=\frac{1}{3}x-5$ **c)** $y=1x-3$

d) $y=0.3x+6$ **e)** $y=0.01x-200$

Solution: As the measure of steepness of a line is the slope of the line, so first we find the slopes of all the fine lines.

Using the slope intercept form of straight line

$y=mx + c$, comparing with lines from (a) to (e)

Slope of line given in (a), $m_1, =\frac{2}{5}=0.4$

Slope of line given in (b), $m_2, =\frac{1}{3}=0.33$

Slope of line given in (c), $m_3, =1$

Slope of line given in (d), $m_4, =0.3$

Slope of line given in (e), $m_5, =0.01$

Putting these slopes in ascending order

$m_5<m_4<m_2<m_1<m_3$

So, the order of steepness is

(e), (d), (b), (a), (c)

Perfect24u.com

www.perfect24u.com

Q.3 In each case, find the slope if it is defined.

a) $P_1(-5,4)$ and $P_2(3,6)$

Solution: The, slope of a line passing through $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1=-5$, $x_2=3$, $y_1=4$, $y_2=6$

Thus, slope $m = \frac{6-4}{3-(-5)} = \frac{2}{8} = \frac{1}{4}$

b) $P_1(2,-1)$ and $P_2(-5,6)$

Solution: The, slope of a line passing through points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1=2$, $x_2=-5$, $y_1=-1$, $y_2=6$

Thus, slope $m = \frac{6-(-1)}{-5-2} = \frac{6+1}{-7} = \frac{7}{-7} = -1$

c) Through the origin and (–4,6)

Solution: Here $P_1(0,0)$ and $P_2(-4,6)$

Perfect24u.com

The slope of line through points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1=0$, $x_2=-4$, $y_1=0$, $y_2=6$

Thus, slope $m = \frac{6-0}{-4-0} = \frac{6}{-4} = \frac{-3}{2}$

$$m = \frac{-3}{2}$$

www.perfect24u.com

d) 4x+7y=1

Solution: Re arranging the given line to the slope intercept form, $y=mx+c \rightarrow (i)$

Given $4x + 7y = 1$

$7y = -4x + 1$ divide by 7

$y = \frac{-4x}{7} + \frac{1}{7} \rightarrow (ii)$

Comparing (i) and (ii)

Slope, $m = \frac{-4}{7}$

e) Parallel to 2y–4x=7

Solution: For parallel lines the slope in same, so the slope of required line is same as the slope of given line i.e. $m_1=m_2$

Re-arranging the given line to the slope-intercept form

$y=mx + c \rightarrow (i)$

$2y - 4x = 7$

$2y = 4x + 7$ divide by 2

$y = 2x + \frac{7}{2} \rightarrow (ii)$

Comparing (i) and (ii)

Slope, $m_1=2$

So, the slope of required line is,

$m_2=2$

f) Perpendicular to 6x=y–3

Solution: For two perpendicular lines with slopes m_1 and m_2

$m_1 \cdot m_2=-1 \rightarrow (i)$

Let the slope of given line is m_1 and m_2 for required line. To find m_1, re-arrange the gives line to slope-intercept form,

Perfect24u.com

www.perfect24u.com

$y = mx + c \rightarrow$ (ii)

$6x = y - 3$

$y = 6x + 3 \rightarrow$ (iii)

Comparing (ii) and (iii)

$m_1 = 6$

Putting in (i)

$6(m_2) = -1$

$m_2 = \frac{-1}{6}$

The slope of required line.

Q.4 What are the x and y intercepts for each of the following lines?

a) $y = 2x + 6$

Solution: Re-arranging the given equation to the intercepts form of straight line i.e.

$\frac{x}{a} + \frac{y}{b} = 1 \rightarrow$ (i)

$y=2x+6 \quad \Rightarrow \quad -2x+y=6$ divide by 6

$\frac{-2x}{6} + \frac{y}{6} = 1$

Perfect24u.com

$\frac{x}{(-3)} + \frac{y}{(6)} = 1 \rightarrow$ (ii) comparing (i) and (ii)

x-intercept, a=–3 and y-intercept, b=6

b) $y = -3x + 9$

Solution: Re-arranging the given equation to the intercepts form of straight line i.e.

$\frac{x}{a} + \frac{y}{b} = 1 \rightarrow$ (i)

www.perfect24u.com

$y=-3x+9$

$3x+y=9$ divide by 9

$\frac{3x}{9}+\frac{y}{9}=1$

$\frac{x}{3}+\frac{y}{9}=1 \rightarrow$ (ii) comparing (i) and (ii)

x-intercept, a=3 and y-intercept, b=9

c) y = x + 2

Solution: Re-arranging the given equation to the intercepts form of straight line i.e.

$\frac{x}{a}+\frac{y}{b}=1 \rightarrow$ (i)

$y=x+2$

$-x+y=2$ divide by 2

$\frac{-x}{2}+\frac{y}{2}=1$

$\frac{x}{-2}+\frac{y}{2}=1 \rightarrow$ (ii) comparing (i) and (ii)

x-intercept, a=–2 and y-intercept, b=2

Perfect24u.com

d) y = 2x – 8

Solution: Re-arranging the given equation to the intercepts form of straight line i.e.

$\frac{x}{a}+\frac{y}{b}=1 \rightarrow$ (i)

$y = 2x - 8$ divide by 8

$\frac{-2x}{-8}+\frac{y}{-8}=1$

www.perfect24u.com

$\frac{x}{4}-\frac{y}{8}=1 \rightarrow$ (ii) comparing (i) and (ii)

x-intercept, a=4 and y-intercept, b= –8

e) y = 5x + 10

Solution: Re-arranging the given equation to the intercepts form of straight line i.e.

$\frac{x}{a}+\frac{y}{b}=1 \rightarrow$ (i)

$y = 5x + 10$

$-5x + y = 10$ divide by 10

$\frac{-5x}{10}+\frac{y}{10}=1$

$\frac{x}{(-2)}+\frac{y}{(10)}=1 \rightarrow$ (ii) comparing (i) and (ii)

x-intercept, a=–2 and y-intercept, b=10

f) y = 5x – 10

Solution: Re-arranging the given equation to the intercepts form of straight line i.e.

Perfect24u.com

$\frac{x}{a}+\frac{y}{b}=1 \rightarrow$ (i)

$y = 5x – 10$

$5x – y = 10$ divide by 10

$\frac{5x}{10}-\frac{y}{10}=1$

$\frac{x}{2}+\frac{y}{-10}=1 \rightarrow$ (ii) comparing (i) and (ii)

x-intercept, a=2 and y-intercept, b=–10

www.perfect24u.com

Q.5 In each case, use the normal form to find out the equation of a line for the following assumptions?

a) $P = 4, \theta = 60^o$

Solution: The normal form of the equation of straight line is given by

$x\cos\theta + y\sin\theta = P \rightarrow (i)$

Here $P=4$ and $\theta=60^o$

Putting these values in (i)

$x\cos 60^o + y\sin 60^o = 4$

$$x\left(\frac{1}{2}\right) + y\left(\frac{\sqrt{3}}{2}\right) = 4 \qquad \text{multiplying by 2}$$

$$x + \sqrt{3}y = 8$$

$$x + \sqrt{3}y - 8 = 0$$

b) $P=8, \theta=90^o$

Solution: The normal form of the equation of straight line is given by

$x\cos\theta + y\sin\theta = P \rightarrow (i)$

Here $P=8$ and $\theta=90^o$

Putting these values in (i)

$x\cos 90^o + y\sin 90^o = 8$

$$x(0) + y(1) = 8$$

$y=8$

c) $P=5, \theta=30^o$

Solution: The normal form of the equation of straight line is given by

$x\cos\theta + y\sin\theta = P \rightarrow (i)$

Here $P=5$ and $\theta=30^o$

Putting these values in (i)

$x\cos 30^o + y\sin 30^o = 5$

Perfect24u.com

www.perfect24u.com

$$x\left(\frac{\sqrt{3}}{2}\right)+y\left(\frac{1}{2}\right)=5$$

Multiplying by 2

$\sqrt{3}x+y=10 \quad \Rightarrow \sqrt{3}x+y-10=0$

d) P=2, θ=45°

Solution: The normal form of the equation of straight line is given by

$x\cos\theta + y\sin\theta = P \quad \rightarrow (i)$

Here P=2 and θ=45°

Putting these values in (i)

$x\cos45^\circ + y\sin45^\circ = 2$

$$x\left(\frac{1}{\sqrt{2}}\right)+y\left(\frac{1}{\sqrt{2}}\right)=2$$

Multiplying by $\sqrt{2}$

$x+y=2\sqrt{2}$

$x+y-2\sqrt{2}=0$

Q.6 In each case, find the equation of a line, if the x-intercepts and the y-intercept of the line are the following:

a) x:(4,0), y:(0,6)

Solution: The equation of straight line with x-intercept a and y-intercept b is given by

$$\frac{x}{a}+\frac{y}{b}=1 \rightarrow (i)$$

Here a=4 and b=6

Putting these values in (i)

Perfect24u.com

www.perfect24u.com

$\frac{x}{4}+\frac{y}{6}=1$ multiply by 12

$\frac{x}{4}(12)+\frac{y}{6}(12)=1(12)$

$3x + 2y = 12 \Rightarrow \quad 3x + 2y - 12 = 0$

b) x:(–4,0), y:(0,–8)

Solution: The equation of line with x-intercept a and y-intercept b is given by:

$\frac{x}{a}+\frac{y}{b}=1 \rightarrow (i)$

Here $a=-4, b=-8$

Putting these values in (i)

$\frac{x}{-4}+\frac{y}{-8}=1$ Multiplying by–8

$\frac{x}{-4}(-8)+\frac{y}{8}(-8)=1(-8)$

$2x + y = -8$

$2x+y+8=0$

c) x:(5,0), y:(0,10)

Solution: The equation of line with x-intercept a and y-intercept b is given by:

Perfect24u.com

$\frac{x}{a}+\frac{y}{b}=1 \rightarrow (i)$

Here $a = 5, b = 10$

Putting these values in (i)

$\frac{x}{5}+\frac{y}{10}=1$ Multiplying by 10

$\frac{x}{5}(10)+\frac{y}{10}(10)=1(10)$

www.perfect24u.com

$2x + y = 10$

$2x + y - 10 = 0$

d) **x:(–1,0), y:(0,5)**

Solution: The equation of line with x-intercept a and y-intercept b is given by:

$$\frac{x}{a}+\frac{y}{b}=1 \rightarrow (i)$$

Here $a = -1$, $b = 5$

Putting these values in (i)

$\frac{x}{1}+\frac{y}{5}=1$ Multiplying by 5

$$\frac{x}{-1}(5)+\frac{y}{5}(5)=1(5)$$

$-5x + y = 5$

$5x - y + 5 = 0$

Q.7 **In each case, find the equation of a line, that passes through the pair of points**

a) **O(0,0), A(2,6)**

Perfect24u.com

Solution: The two point's form of straight line equation is given by

$$y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1) \text{ OR } \frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \rightarrow (i)$$

Here $x_1=0$, $x_2=2$, $y_1=0$, $y_2=6$, putting these value in (i)

$$\frac{y-0}{x-0}=\frac{6-0}{2-0} \quad \Rightarrow \quad \frac{y}{x}=3$$

$y = 3x$

$3x - y = 0$

www.perfect24u.com

b) E(1,0), F(2,5)

Solution: The two point's form of straight line equation is given by

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \rightarrow \text{(i)}$$

Here $x_1=1$, $y_1=0$, $x_2=2$, $y_2=5$

Putting these values in (i)

$$\frac{y-0}{x-1}=\frac{5-0}{2-1} \quad \Rightarrow \quad \frac{y}{x-1}=\frac{5}{1}$$

$$y = 5x - 5$$

$$\mathbf{5x - y - 5 = 0}$$

c) I(1,1), J(3,3)

Solution: The two point's form of straight line equation is given by

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \rightarrow \text{(i)}$$

Here $x_1=1$, $y_1=1$, $x_2=3$, $y_2=3$

Putting these values in (i)

$$\frac{y-1}{x-1}=\frac{3-1}{3-1} \quad \Rightarrow \quad \frac{y-1}{x-1}=1$$

$y-1=x-1$

$$\mathbf{x-y=0}$$

Perfect24u.com

d) M(3,1), N(–1,3)

Solution: The two point's form of straight line equation is given by

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1} \rightarrow \text{(i)}$$

Here $x_1=3$, $x_2=-1$, $y_1=1$, $y_2=3$ Putting these values in (i)

$$\frac{y-1}{x-3}=\frac{3-1}{-1-3} \quad \Rightarrow \quad \frac{y-1}{x-3}=\frac{2}{-4}$$

www.perfect24u.com

$$\frac{y-1}{x-3}=\frac{-1}{2}$$

$2y - 2 = -x + 3$

$x + 2y - 5 = 0$

Q.8 In each case, find the equation of a line, that passes through the point $A(x_1,y_1)$ having slope m:

a) $A(1,2)$, $m=4$

Solution: By point-slope form, the equation of straight line passing through point $P(x_1, y_1)$ with slope m is gives by;

$y - y_1 = m(x - x_1) \rightarrow$ (i)

Here $m=4$, $x_1=1$, $y_1=2$ Putting these values in (i)

$y - 2 = 4(x - 1)$

$y - 2 = 4x - 4 \quad \Rightarrow \quad 4x - y - 2 = 0$

b) $A(-1,-2)$, $m=\frac{-1}{2}$

Solution: By point-slope form, the equation of straight line with slope m and passing through point $p(x_1,y_1)$ is gives by;

$y-y_1=m(x-x_1) \rightarrow$ (i)

Here $m=\frac{-1}{2}$, $x_1=-1$, $y_1=-2$ Putting these values in (i)

Perfect24u.com

$y-(-2)=\frac{-1}{2}(x-(-1))$

$y+2=\frac{-1}{2}(x+1) \quad \Rightarrow \quad 2y+4=-x-1$

$x + 2y + 5 = 0$

c) $A(-3,5)$, $m = -3$

Solution: By point-slope form, the equation of straight line with slope m and passing through point $p(x_1,y_1)$ is gives by;

www.perfect24u.com

$y - y_1 = m(x - x_1) \rightarrow$ (i)

Here $m = -3$, $x_1 = -3$, $y_1 = 5$ Putting these values in (i)

$y - 5 = -3(x-(-3)) \Rightarrow y-5 = -3(x+3)$

$y - 5 = -3x - 9$

$3x + y + 4 = 0$

d) A(7, –8), m = 5

Solution: By point-slope form, the equation of straight line with slope m and passing through point $p(x_1,y_1)$ is gives by;

$y - y_1 = m(x - x_1) \rightarrow$ (i)

Here $m = 5$, $x_1 = 7$, $y_1 = -8$ Putting these values in (i)

$y-(-8) = 5(x - 7) \Rightarrow y + 8 = 5x - 35$

$5x - y - 43 = 0$

Q.9 In each case, find the equation of a line, that exists the intercept c and slope m:

a) c=2, m=2

Solution: By the slope-intercept form of straight line the equation of straight line with slope m and y-intercept c is given by;

Perfect24u.com

$y = mx + c \rightarrow$ (i)

Here $m = 2$, $c = 2$ Putting these values in (i)

$y = 2x + 2$

$2x - y + 2 = 0$

b) c=4, m=8

Solution: By the slope-intercept form of straight line the equation of straight line with slope m and y-intercept C is given by:

$y = mx + C \rightarrow$ (i)

www.perfect24u.com

Here m = 8, C = 4 Putting these values in (i)

$y = 8x + 4$

$8x - y + 4 = 0$

c) **$c = -4, m = \frac{1}{2}$**

Solution: By the slope-intercept form of straight line the equation of straight line with slope m and y-intercept c is given by:

$y = mx + c \rightarrow$ (i)

Here $m = \frac{1}{2}$, $c = -4$ Putting these values in (i)

$y = \frac{1}{2}x + (-4) \quad \Rightarrow \quad y = \frac{1}{2}x - 4$ Multiplying by 2

$2y = x - 8$

$x - 2y - 8 = 0$

d) **$c = \frac{1}{2}, m = -3$**

Solution: By the slope-intercept form of straight line the equation of straight line with slope m and y-intercept c is given by:

$y = mx + c \rightarrow$ (i)

Here $m = -3$, $c = \frac{1}{2}$ Putting these values in (i)

$y = -3x + \frac{1}{2}$ Multiplying by 2

$2y = -6x + 1$

$6x + 2y - 1 = 0$

Perfect24u.com

www.perfect24u.com

Note: For a point $P(x_1,y_1)$ and straight line

$ax + by + c = 0$

The point $p(x_1,y_1)$ lies on the line if $ax_1+by_1+c=0$

The point $p(x_1,y_1)$ lies below the line if $ax_1+by_1+c<0$

The point $p(x_1,y_1)$ lies above the line if $ax_1+by_1+c>0$

Distance of a point from a line:

Position of a point $P(x_1,y_1)$ w.r.t a line $ax+by+c=0$;

For $b>0$, if

$ax_1+by_1+c>0$, $P(x_1, y_1)$ lies above the line

$ax_1+by_1+c=0$, $P(x_1, y_1)$ lies on the line

$ax_1+by_1+c<0$, $P(x_1, y_1)$ lies below the line

Perpendicular distance from a point $P(x_1,y_1)$ to a line l:

$ax+by+c=0$ is given by

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

Angle between two lines:

Perfect24u.com

The angle between two co-planar lines l_1 and l_2 with slopes m_1 and m_2 is given by;

$$\tan\theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

The line through the point of intersection of two line:

$l_1: a_1x+b_1y+c_1=0$ and $l_2: a_2x+b_2y+c_2=0$ is given by;

$l:(a_1x+b_1y+c_1)+\lambda(a_2x+b_2y+c_2)=0$

www.perfect24u.com

EXERCISE 6.3

Q.1 In each case, show that the point $P(x_1,y_1)$ lies above or below the line $ax + by + c = 0$. Also show that the point $P(x_1, y_1)$ and the origin lies on the same side or on the opposite side of the line $ax + by + c = 0$.

a) P(4,–5), 4x–3y–17=0

Solution:

Here the line is $4x - 3y - 17 = 0$

As $b = -3 < 0$,

Multiplying the equation by –1

$-4x + 3y + 17 = 0 \rightarrow$ (i)

Now putting x=4, y=–5 in (i)

$-4(4)+3(-5)+17=-16-15+17=-14<0$

So, the point P(4,–5) lies below the line.

Now for the origin 0(0,0) put x=0, y=0 in (i)

$-4(0)+3(0)+17=17>0$

So, Origin lies above the given line

As P(4,–5) lies below in the line and origin lies above the line, so the point and origin are on the opposite side of line 4x–3y–17=0

Perfect24u.com

b) P(–3,8), 5x+7y+9=0

Solution: Here, the line is $5x+7y+9=0 \rightarrow$(i) $(b=7>0)$

Putting x=–3 and y=8 in (i)

$5(-3)+7(8)+9=-15+56+9=50>0$

The point P(–3,8) lies above the line

Now, for the origin O(0,0) put x=0, y=0 in (i)

$5(0)+7(0)+9=9>0$

So, origin lies above the line.

www.perfect24u.com

As, both P(–3,8) and origin O(0,0) lies above the line, So the point and origin are on the same side of line.

c) P(20,–15), 6x+9y+13=0

Solution: Here, the line is 6x+9y+13=0 →(i) (b=9>0)

Putting x=20 and y=–15 in (i)

6(20)+9(–15)+13=120–135+13=–2<0

The point P(20,–15) lies below the line

Now, for the origin O(0,0) put x=0, y=0 in (i)

6(0)+9(0)+13=13>0

So, origin lies above the line.

As, P(20,–15) lies, below the line and origin O(0,0) lies above the line, So the point and origin are on opposite sides of line 6x+9y+13=0.

Q.2 In each case, show that the point $P(x_1,y_1)$ and $Q(x_2,y_2)$ are on the same sides or on the opposite side of the line ax + by + c = 0.

a) P(–4,2), Q(11,–3), 5x + 14y – 11 = 0

Solution: Here the line is 5x+14y–11=0 (b=14>0) →(i)

For point P(–4,2), put x=–4, y=2 in (i)

5(–4)+14(2)–11=–20+28–11=–3 <0

Point P(–4,2) lies below the line

Now, for Q(11,–3) put x=11, y=–3 in (i)

5(11)+14(–3)–11=55–42–11=2>0

Point Q (11,–3) lies above the line.

As P(–4,2) lies below the line and Q(11,–3) lies above the line, so the two points lies on opposite sides of line 5x+14y–11=0

b) P(–3,5), Q(1,–2) 2x–3y–10=0

Solution: Here the line is 2x+3y–10=0. As b=–3<0, multiplying by–1

Perfect24u.com

www.perfect24u.com

$-2x+3y+10=0 \quad \rightarrow$(i)

For P(–3,5), put x=–3, y=5 in (i)

$-2(-3)+3(5)+10=6+15+10=31>0$

Point P(–3,5) lies above the line

Now, for Q(1,–2), put x=1, y=–2 in (i)

$-2(1)+3(-2)+10=-2-6+10=2>0$

Point Q (1,–2) lies above the line.

As both P(–3,5) and Q(1,–2) lies above the line, so the two points are on same side of line 2x–3y–10=0

c) P(–3,2), Q(4,5); 3x+7y–15=0

Solution: Here the line is $3x+7y-15=0 \rightarrow$ (i)

As (b=7>0)

For P(–3,2), put x=–3 and y=2 in (i)

$3(-3)+7(2)-15=-9+14-15$

$=-10<0$

P(–3,2) lies below the line

Now, for Q(4,5), put x=3, y=5 in (i)

$3(4)+7(5)-15=12+35-15$

$=32>0$

Point Q (4,5) lies above the line.

As points P(–3,2) lies below the line and Q(4,5) lies above the line, so the two points lies on opposite sides of line, 3x+7y–15=0

Q.3 In each case, find the perpendicular distance from the line ax+by+c=0 to the point (x_1,y_1)

a) P(3,–4), 4x–3y+6=0

Solution: The perpendicular distance of point $P(x,y_1)$ to the line

Perfect24u.com

www.perfect24u.com

ax + by + c = 0 is given by

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \rightarrow (i)$$

Here $x_1=3$, $y_1=-4$, $a=4$, $b=-3$, $c=6$

Putting these values in (i)

$$d=\frac{|4(3)-3(-4)+6|}{\sqrt{4^2+(-3)^2}}=\frac{|12+12+6|}{\sqrt{16+9}}=\frac{30}{5}$$

$d=6$

b) P(5,8), 3x–2y+7=0

Solution: The perpendicular distance of point $P(x,y_1)$ to the line ax + by + c = 0 is given by

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \rightarrow (i)$$

Here $a=3$, $b=-2$, $c=7$, $x_1=5$, $y_1=8$

Putting these values in (i)

$$d=\frac{|3(5)-2(8)+7|}{\sqrt{3^2+(-2)^2}}=\frac{|15-16+7|}{\sqrt{13}}=\frac{6}{\sqrt{13}}$$

$$d=\frac{6}{\sqrt{13}}$$

Perfect24u.com

c) P(3,–1), 5x+12y–16=0

Solution: The perpendicular distance of point $P(x,y_1)$ to the line ax + by + c=0 is given by

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \rightarrow (i)$$

Here $a=5$, $b=12$, $c=-16$, $x_1=3$, $y_1=-1$

Putting these values in (i)

www.perfect24u.com

$$d=\frac{|5(3)+12(-1)-16|}{\sqrt{5^2+12^2}}=\frac{|15-12-16|}{\sqrt{25+144}}=\frac{|-13|}{\sqrt{169}}$$

$$d=\frac{13}{13} \qquad \Rightarrow \qquad d=1$$

d) P(–54,71), 4x+3y–17=0

Solution: The perpendicular distance of point $P(x,y_1)$ to the line $ax + by + c = 0$ is given by

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \rightarrow (i)$$

Here $a=4$, $b=3$, $c=-17$, $x_1=-54$, $y_1=71$

Putting all these values in (i)

$$d=\frac{|4(-54)+3(71)-17|}{\sqrt{4^2+3^2}}=\frac{|-216+213-17|}{\sqrt{16+9}}=\frac{|-20|}{\sqrt{25}}$$

$$d=\frac{20}{5} \qquad \Rightarrow \qquad d=4$$

e) P(–60,25), 3x+7y+5=0

Solution: The perpendicular distance of point $P(x,y_1)$ to the line $ax + by + c = 0$ is given by

Perfect24u.com

$$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \rightarrow (i)$$

Here $a=3$, $b=7$, $c=5$, $x_1=-60$, $y_1=25$

Putting all these values in (i)

$$d=\frac{|3(-60)+7(25)+5|}{\sqrt{3^2+7^2}}=\frac{|-180+175+5|}{\sqrt{9+49}}=\frac{|0|}{\sqrt{58}}$$

$d=0$

So, P(–60,25) lies on the line $3x+7y+5=0$

www.perfect24u.com

Q.4 In each case, find the angle θ from the line L_1 to line L_2, if the slope of the lines L_1 and L_2 are the following.

a) $\mathbf{L_1: m_1 = \frac{1}{2}}$, $\mathbf{L_2: m_2 = 3}$

Solution: If θ is the angle between the lines L_1 and L_2 with slopes m_1 and m_2 respectively, then

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow (i)$$

Here $m_1 = \frac{1}{2}$, $m_2 = 3$

Putting these values in (i)

$$\tan\theta = \frac{3 - 1/2}{1 + (3)1/2} = \frac{\frac{6-1}{2}}{\frac{2+3}{2}} = \frac{5}{5}$$

$\tan\theta = 1 \quad \Rightarrow \quad \theta = \tan^{-1}(1)$

$\theta = 45^0$ angle between L_1 and L_2

b) $\mathbf{L_1: m_1 = 2}$, $\mathbf{L_2: m_2 = 3}$

Solution: If θ is the angle between the lines L_1 and L_2 with slopes m_1 and m_2 respectively, then

Perfect24u.com

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow (i)$$

Here $m_1 = 2$, $m_2 = 3$

Putting these values in (i)

$$\tan\theta = \frac{3-2}{1+(3)(2)} = \frac{1}{7}$$

$$\theta = \tan^{-1}\left(\frac{1}{7}\right), 180 - \tan^{-1}\left(\frac{1}{7}\right)$$

www.perfect24u.com

c) $L_1: m_1 = -3$, $L_2: m_2 = 8$

Solution: If θ is the angle between the lines L_1 and L_2 with slopes m_1 and m_2 respectively, then

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow (i)$$

Here $m_1 = -3$, $m_2 = 8$

Putting these values in (i)

$$\tan\theta = \frac{8 - (-3)}{1 + (-3)(8)} = \frac{8 + 3}{1 - 24} = \frac{11}{-23}$$

$$\theta = \tan^{-1}\left(\frac{-11}{23}\right) = 180 - \tan^{-1}\left(\frac{11}{23}\right), \tan^{-1}\left(\frac{11}{23}\right)$$

d) $L_1: m_1 = 0$, $L_2: m_2 = 2$

Solution: If θ is the angle between the lines L_1 and L_2 with slopes m_1 and m_2 respectively, then

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow (i)$$

Here $m_1 = 0$, $m_2 = 2$

Putting these values in (i)

$$\tan\theta = \frac{2 - 0}{1 + (2)(0)} = \frac{2}{1} = 2$$

$$\theta = \tan^{-1}(2)$$

Perfect24u.com

Q.5 In each case, find the angle θ, from the line L_1 to line L_2

a) L_1: joins (1,2) and (7,–1), L_2 : Joins (3,2) and (5,6) try to obtain acute angle.

Solution: For the angle θ between lines L_1 and L_2 with slopes m_1 and m_2 respectively;

www.perfect24u.com

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow (i)$$

Now for L_1: joining (1,2) and (7,1)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-2}{7-1} = \frac{-3}{6} = \frac{-1}{2}$$

and for L_2: joining (3,2) and (5,6)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{5-3} = \frac{4}{2} = 2$$

Putting m_1 and m_2 in (i)

$$\tan\theta = \frac{2 - \left(-1/2\right)}{1 + (2)\left(-1/2\right)} = \frac{2 + 1/2}{1-1} = \frac{5/2}{0} = \infty$$

$$\theta = \tan^{-1}(\infty) = 90^\circ$$

$$\theta = 90^\circ$$

b) **L_1: joins (2,7) and (7,10),**

L_2 : Joins (1,1) and (–5,3)

Solution: For the angle θ between lines L_1 and L_2 with slopes m_1 and m_2 respectively;

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow (i)$$

Perfect24u.com

Now for L_1: joining (2,7) and (7,10)

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10-7}{7-2} = \frac{3}{5}$$

and L_2: joining (1,1) and (–5,3)

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-1}{-5-1} = \frac{2}{-6} = \frac{-1}{3}$$

Putting m_1 and m_2 in (i)

www.perfect24u.com

$$\tan\theta = \frac{-1/3 - 3/5}{1 + \left(-1/3\right)\left(3/5\right)} = \frac{\frac{-5-9}{15}}{1 - 1/5} = \frac{-14/15}{5 - 1/5}$$

$$\tan\theta = \frac{\frac{-14}{15}}{\frac{4}{5}} = \frac{-14}{15} \times \frac{5}{4} = \frac{-7}{6}$$

$$\theta = \tan^{-1}\left(\frac{-7}{6}\right)$$

Q.6 In each case, find the angle θ, from the line L_1 to line L_2

a) L_1: x–2y+3=0, L_2 : 3x–y+7=0

Solution: First we find the slopes of L_1 and L_2. The slope of line ax+by+c=0 is given by $m = \frac{-a}{b}$

$x - 2y + 3 = 0$

For L_1: a=1, b=–2, thus

$$m_1 = \frac{-1}{-2} = \frac{1}{2}$$

$3x - y + 7 = 0$

For L_2: a=3, b=–1, and

$$m_2 = \frac{-a}{b} = \frac{-3}{-1} = 3$$

Perfect24u.com

If θ is the angle between lines L_1 and L_2 with slopes m_1 and m_2 then

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow \text{(i)}$$

Putting values of m_1 and m_2

$$\tan\theta = \frac{3 - 1/2}{1 + (3)\left(1/2\right)} = \frac{\frac{6-1}{2}}{\frac{2+3}{2}} = \frac{5}{5} = 1$$

www.perfect24u.com

$\theta = \tan^{-1}(1)$

$\theta = 45^0$

b) **L_1: 2x+4y–10=0, L_2 : 5x–3y+1=0**

Solution: First we find the slopes of L_1 and L_2. The slope of line ax+by+c=0 is given by $m = \frac{-a}{b}$

For L_1: 2x+4y–10=0

a=2, b=4, thus

$$m_1 = \frac{-a}{b} = \frac{-2}{4} = \frac{-1}{2}$$

For L_2: 5x–3y+1=0, a=5, b=–3

$$m_2 = \frac{-a}{b} = \frac{-5}{-3} = \frac{5}{3}$$

If θ is the angle between lines L_1 and L_2 with slopes m_1 and m_2 then

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow \text{(i)}$$

Putting values of m_1 and m_2

$$\tan\theta = \frac{5/3 - (-1/2)}{1 + (-1/2)(5/3)} = \frac{\frac{5}{3} + \frac{1}{2}}{\frac{1}{1} - \frac{5}{6}} = \frac{\frac{10+3}{6}}{\frac{6-5}{6}}$$

Perfect24u.com

$$\tan\theta = \frac{13}{1}$$

$\theta = \tan^{-1}(13)$

c) **L_1: 2x+y–8=0, L_2 : 3x+2y–2=0**

Solution: First we find the slopes of L_1 and L_2. The slope of line ax+by+c=0 is given by $m = \frac{-a}{b}$

For L_1: 2x+y–8=0

www.perfect24u.com

$a=2, b=1$, thus

$$m_1 = \frac{-a}{b} = \frac{-2}{1} = -2$$

For L_2: $3x+2y-2=0$

$a=3, b=2$

$$m_2 = \frac{-a}{b} = \frac{-3}{2}$$

If θ is the angle between lines L_1 and L_2 with slopes m_1 and m_2 then

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow \text{(i)}$$

Putting values of m_1 and m_2

$$\tan\theta = \frac{-3/2 + 2/1}{1 + (-2)\left(-3/2\right)} = \frac{\frac{-3+4}{2}}{1+3} = \frac{\left(\frac{1}{2}\right)}{4} = \frac{1}{8}$$

$$\theta = \tan^{-1}\left(\frac{1}{8}\right)$$

d) **L_1: $3x - 5y + 22 = 0$, L_2: $6x + y - 22 = 0$**

Solution: First we find the slopes of L_1 and L_2. The slope of line $3x-5y+22=0$ is given by $m = \frac{-a}{b}$

Perfect24u.com

For L_1: $3x - 5y + 22 = 0$

$a = 3, b = -5$, thus

$$m_1 = \frac{-a}{b} = \frac{-3}{-5} = \frac{3}{5}$$

For L_2: $6x + y - 22 = 0$

$a = 6, b = 1$

$$m_2 = \frac{-a}{b} = \frac{-6}{1} = -6$$

www.perfect24u.com

If θ is the angle between lines L_1 and L_2 with slopes m_1 and m_2 then

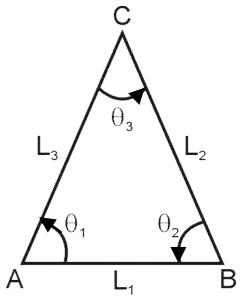
$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} \rightarrow \text{(i)}$$

Putting values of m_1 and m_2

$$\tan\theta = \frac{-6 - 3/5}{1 + (-6)(3/5)} = \frac{\frac{-30-3}{5}}{1 - \frac{18}{5}} = \frac{\frac{-33}{5}}{\frac{5-18}{5}}$$

$$\tan\theta = \frac{-33}{-13} = \frac{33}{13}$$

$$\theta = \tan^{-1}\left(\frac{33}{13}\right)$$



Q.7 In each case, find the angles of triangle ABC whose vertices are the following.

a) A(1,2) B(4,2) and C(–2,3)

Solution: The angle of the triangle ABC, i.e. θ_1, θ_2 and θ_3 (shown in figure) are the angles between lines AB, BC and AC.

The slopes of AB, BC and AC are;

Slope of AB, $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-2}{4-1} = \frac{0}{3} = 0$

Perfect24u.com

Slope of BC, $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{-2-4} = \frac{1}{-6} = \frac{-1}{6}$

Slope of AC, $m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-2}{-2-1} = \frac{1}{-3} = \frac{-1}{3}$

For angle θ_1

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{-1/3 - 0}{1 + (0)(-1/3)} = \frac{-1/3}{1} = \frac{-1}{3}$$

www.perfect24u.com

$$\theta_1 = \tan^{-1}\left(\frac{-1}{3}\right)$$

For angle θ_2

$$\tan\theta_2 = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{0 - \left(-1/6\right)}{1 + (0)\left(-1/6\right)} = \frac{1}{6}$$

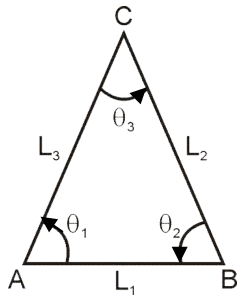
$$\theta_2 = \tan^{-1}\left(\frac{1}{6}\right)$$

For angle θ_3

$$\tan\theta_3 = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-1/6 - \left(-1/3\right)}{1 + \left(-1/6\right)\left(-1/3\right)} = \frac{-1/6 + 1/3}{1 + 1/18}$$

$$\tan\theta_3 = \frac{-1 + 2/6}{18 + 1/18} = \frac{1/6}{19/18} = \frac{1}{6} \times \frac{18}{19} = \frac{3}{19}$$

$$\theta_3 = \tan^{-1}\left(\frac{3}{19}\right)$$



b) A(3,–4) B(1,5) and C(2, –4)

Solution: The angle of the triangle ABC, i.e. θ_1, θ_2 and θ_3 (shown in figure) are the angles between lines AB, BC and AC.

Perfect24u.com

The slopes of AB, BC and AC are;

Slope of AB, $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 + 4}{1 - 3} = \frac{9}{-2} = \frac{-9}{2}$

Slope of BC, $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 5}{2 - 1} = \frac{-9}{1} = -9$

Slope of AC, $m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 + 4}{2 - 3} = \frac{0}{-1} = 0$

For angle θ_1

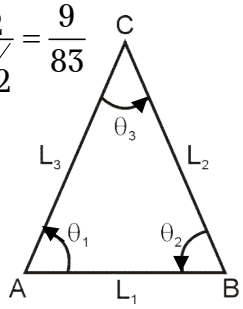
www.perfect24u.com

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{0 - \left(-9/2\right)}{1 + (0)\left(-9/2\right)} = \frac{9}{2}$$

$$\theta_1 = \tan^{-1}\left(9/2\right)$$

For angle θ_2

$$\tan\theta_2 = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{-9/2 - (-9)}{1 + \left(-9/2\right)(-9)} = \frac{-9/2 + 9}{1 + 81/2} = \frac{9/2}{83/2} = \frac{9}{83}$$



$$\theta_2 = \tan^{-1}\left(\frac{9}{83}\right)$$

For angle θ_3

$$\tan\theta_3 = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-9 - (0)}{1 + (-9)(0)} = \frac{-9}{1} = -9$$

$$\theta_3 = \tan^{-1}(-9) = 180^\circ - \tan^{-1}(9)$$

c) A(–4,0) B(2,0) and C(2, 5)

Solution: The angle of the triangle ABC, i.e. θ_1, θ_2 and θ_3 (shown in figure) are the angles between lines AB, BC and AC.

The slopes of AB, BC and AC are;

Perfect24u.com

Slope of AB, $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 0}{2 + 4} = \frac{0}{6} = 0$

Slope of BC, $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{2 - 2} = \frac{5}{0} = \infty$

Slope of AC, $m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{2 + 4} = \frac{5}{6}$

For angle θ_1

www.perfect24u.com

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_1 m_3} = \frac{\frac{5}{6} - 0}{1 + (0)\left(\frac{5}{6}\right)} = \frac{\frac{5}{6}}{1} = \frac{5}{6}$$

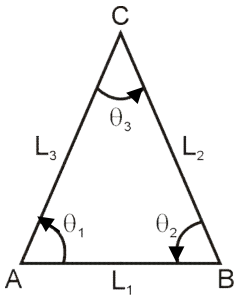
$$\theta_1 = \tan^{-1}\left(\frac{5}{6}\right)$$

For angle θ_2

$$\tan\theta_2 = \frac{m_1 - m_2}{1 + m_1 m_2} \quad \Rightarrow \quad \text{Divide by } m_2$$

$$\tan\theta_2 = \frac{\frac{m_1}{m_2} - 1}{\frac{1}{m_2} + m_1} = \frac{\frac{0}{\infty} - 1}{\frac{1}{\infty} + 0} = \frac{-1}{0}$$

$$\theta_2 = \tan^{-1}(\infty) = 90^\circ$$



For angle θ_3

$$\tan\theta_3 = \frac{m_2 - m_3}{1 + m_2 m_3} \quad \Rightarrow \quad \text{Divide by } m_2$$

$$\tan\theta_3 = \frac{1 - \frac{m_3}{m_2}}{\frac{1}{m_2} + m_3} = \frac{1 - \frac{\frac{5}{6}}{\infty}}{\frac{1}{\infty} + \frac{5}{6}} = \frac{1 - 0}{0 + \frac{5}{6}} = \frac{6}{5}$$

$$\theta_3 = \tan^{-1}\left(\frac{6}{5}\right)$$

Perfect24u.com

d) A(–1,–1) B(–1,4) and C(2, 0)

Solution: The angle of the triangle ABC, i.e. θ_1, θ_2 and θ_3 (shown in figure) are the angles between lines AB, BC and AC.

The slopes of AB, BC and AC are;

Slope of AB, $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 + 1}{-1 + 1} = \frac{5}{0} = \infty$

www.perfect24u.com

Slope of BC, $m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{2+1} = \frac{-4}{3}$

Slope of AC, $m_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0+1}{2+1} = \frac{1}{3}$

For angle θ_1

$$\tan\theta_1 = \frac{m_3 - m_1}{1 + m_1 m_3} \quad \Rightarrow \quad \text{divide by } m_1$$

$$\tan\theta_1 = \frac{\frac{m_3}{m_1} - 1}{\frac{1}{m_1} + m_3} = \frac{\frac{1/3}{\infty} - 1}{\frac{1}{\infty} + \frac{1}{3}} = \frac{0-1}{0+\frac{1}{3}} = -3$$

$$\theta_1 = \tan^{-1}(-3) = \tan^{-1}(3)$$

For angle θ_2

$$\tan\theta_2 = \frac{m_1 - m_2}{1 + m_1 m_2} \quad \Rightarrow \quad \text{divide by } m_1$$

$$\tan\theta_2 = \frac{1 - \frac{m_2}{m_1}}{\frac{1}{m_1} + m_2} = \frac{1 - \frac{(-4/3)}{\infty}}{\frac{1}{\infty} + (-4/3)} = \frac{1-0}{0-4/3} = \frac{-3}{4}$$

Perfect24u.com

$$\theta_2 = \tan^{-1}(3/4)$$

For angle θ_3

$$\tan\theta_3 = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-4/3 - 1/3}{1 + (-4/3)(1/3)} = \frac{-5/3}{1 - \frac{4}{9}}$$

$$\tan\theta_3 = \frac{\frac{-5}{3}}{\frac{9-4}{9}} = \frac{-5/3}{\frac{5}{9}} = \frac{-5}{3} \times \frac{9}{5} = -3$$

www.perfect24u.com

$$\theta_3 = \tan^{-1}(-3) = \tan^{-1}(3)$$

Q.8 Find the equation of the straight line from the family of straight of lines through the point of intersection of straight lines.

a) 2x–3y+4=0 and 3x+4y–5=0 and is perpendicular to the line 6x–7y–18=0

Solution: First we find the point of intersection of the two given straight lines.

$2x-3y+4=0 \rightarrow$ (i)

$3x+4y-5=0 \rightarrow$ (ii)

Multiplying (i) by 4 and (ii) by 3 and then adding

$$8x - 12y + 16 = 0$$
$$9x + 12y - 15 = 0$$

$17x+1=0 \quad \Rightarrow \quad 17x=-1$

$x = \frac{-1}{17}$ putting in (i)

$$2\left(-1/17\right) - 3y + 4 = 0 \quad \Rightarrow \quad -3y = -4 + \frac{2}{17}$$

$$-3y = \frac{-68+2}{17} = \frac{-66}{17}$$

$$y = \frac{-66}{17} \times \frac{1}{-3} = \frac{22}{17}$$

So, the point of intersection is $P = \left(\frac{-1}{17}, \frac{22}{17}\right)$

Now, the slope of third line $m_1 = \frac{-a}{b} = \frac{-6}{-7} = \frac{6}{7}$

Let the slope of required line in m_2, as the required line is perpendicular to 3rd line

Perfect24u.com

www.perfect24u.com

$m_1m_2=-1$

$$m_2=\frac{-1}{m_1}=\frac{-1}{\left(6/7\right)}=\frac{-7}{6}$$

So, we have to find the equation of line passing through $P\left(\frac{-1}{17},\frac{22}{17}\right)$ and having slope, $m_2=\frac{-7}{6}$

Using point-slope formula, $y-y_1=m(x-x_1)$

Putting $x_1=\frac{-1}{17}$, $y_1=\frac{22}{17}$, $m=\frac{-7}{6}$

$$y-\frac{22}{17}=\frac{-7}{6}\left(x-\left(-1/17\right)\right)$$

$$y-\frac{22}{17}=\frac{-7}{6}\left(x+\frac{1}{17}\right)$$

$$y-\frac{22}{17}=\frac{-7}{6}x-\frac{7}{102}$$ multiplying by 102

$102y-132=-119x-7$

$119x+102y-125=0$

b) $l_1=3x-4y+1=0$, $l_2=5x+y-1=0$ and cuts up equal intercepts from the axes

Perfect24u.com

Solution: The equation of line through the intersection of l_1 and l_2 is given by $l_1+\lambda l_2=0$

$(3x-4y+1)+\lambda(5x+y-1)=0 \rightarrow$ (i)

For x-intercept put $y=0$ in (i)

$3x-4(0)+1+\lambda(5x+0-1)=0$

$3x+1+5\lambda x-\lambda=0$

$(3+5\lambda)x=\lambda-1 \quad \Rightarrow \quad x=\frac{\lambda-1}{3+5\lambda} \rightarrow$(ii)

For y– intercept, $x=0$ in (ii)

www.perfect24u.com

$3(0) - 4y + 1 + \lambda(5(0) + y - 1) = 0$

$-4y + 1 + \lambda y - \lambda = 0$

$y(\lambda - 4) = \lambda - 1 \quad \Rightarrow \quad y = \frac{\lambda - 1}{\lambda - 4} \rightarrow (iii)$

The intercepts are of equal length, then equating (ii) and (iii)

$$\frac{\lambda - 1}{3 + 5\lambda} = \frac{\lambda - 1}{\lambda - 4}$$

$$\frac{1}{3 + 5\lambda} = \frac{1}{\lambda - 4}$$

$3 + 5\lambda = \lambda - 4$

$4\lambda = -7$

$$\lambda = \frac{-7}{4}$$

Putting in (i)

$$(3x - 4y + 1) + \frac{-7}{4}(5x + y - 1) = 0$$

Multiplying by 4

$12x - 16y + 4 - 35x - 7y + 7 = 0$

$-23x - 23y + 11 = 0$

$23x + 23y - 11 = 0$

Perfect24u.com

c) l_1: x–2y=a, l_2: x+3y=2a and is parallel to the line l_3: 3x+4y=0

Solution: The equation of line through the point of intersection of l_1 and l_2 is given by:

$l_1 + \lambda l_2 = 0$ putting l_1 and l_2

$(x - 2y - a) + \lambda(x + 3y - 2a) = 0 \rightarrow (i)$

$x - 2y - a + \lambda x + 3\lambda y - 2a\lambda = 0$

$(1 + \lambda)x - (2 - 3\lambda)y - (a + 2a\lambda) = 0 \rightarrow (ii)$

www.perfect24u.com

The slope of line is thus

$$m_1 = \frac{-a}{b} = \frac{-(1+\lambda)}{-(2-3\lambda)} = \frac{1+\lambda}{2-3\lambda} \rightarrow \text{(iii)}$$

The slope of third line $l_3: 3x + 4y = 0$ is

$$m_2 = \frac{-a}{b} = \frac{-3}{4} \rightarrow \text{(iv)}$$

As the required line is parallel to l_3

$$m_1 = m_2$$

$$\frac{1+\lambda}{2-3\lambda} = \frac{-3}{4}$$

$$4 + 4\lambda = -6 + 9\lambda \quad \Rightarrow \quad 5\lambda = 10 \quad \Rightarrow \quad \lambda = 2$$

Put in (i)

$$(x - 2y - a) + 2(x + 3y - 2a) = 0$$

$$x - 2y - a + 2x + 6y - 4a = 0$$

$$3x + 4y - 5a = 0$$

d) $l_1: 2x-y=0$, $l_2: 3x+2y=0$ and is perpendicular to line $l_3: 3x+y-6=0$

Solution: The equation of line passing through the intersection of l_1 and l_2 is given by:

$l_1 + \lambda l_2 = 0$ putting l_1 and l_2

$$(2x - y) + \lambda(3x + 2y) = 0$$

$$2x - y + 3\lambda x + 2\lambda y = 0$$

$$(2+3\lambda)x - (1 - 2\lambda)y = 0 \rightarrow \text{(i)}$$

The slope of line in (i)

$$m_1 = \frac{-a}{b} = \frac{-(2+3\lambda)}{-(1-2\lambda)} = \frac{2+3\lambda}{1-2\lambda} \rightarrow \text{(ii)}$$

The slope of line l_3: $m_2 = \frac{-a}{b} = \frac{-3}{1} = -3$

Perfect24u.com

www.perfect24u.com

As the required line is perpendicular to l_3,

$m_1m_2=-1 \quad \Rightarrow \quad \left(\frac{2+3\lambda}{1-2\lambda}\right).(-3)=-1$

$\frac{6+9\lambda}{1-2\lambda}=1 \quad \Rightarrow \quad 6+9\lambda=1-2\lambda$

$11\lambda=-5$

$\lambda=\frac{-5}{11}$ putting in (i)

$(2x-y)+\left(\frac{-5}{11}\right)(3x+2y)=0$ Multiplying by 11

$22x-11y-15x-10y=0$

$7x-21y=0$ divide by 7

$x-3y=0$

Q.9 Find the equation of the straight line from the family of straight lines through the point of intersection of the lines.

a) l_1: 2x+y+1=0, l_2: 2x+3y+5=0, that touches the point of intersection of l_3: x–y=0 and l_4: x+y=0

Solution: The equation of line passing through the intersection of l_1 and l_2 is given by:

$l_1+\lambda l_2=0$ putting l_1 and l_2

$(2x+y+1)+\lambda(2x+3y+5)=0 \rightarrow$ (i)

To find the point of intersection of l_3 and l_4

$l_3: x-y=0$

$l_4: x+y=0$ Adding l_3 and l_4

$2x=0$

Perfect24u.com

www.perfect24u.com

$x=0$ putting l_3

$0-y=0 \Rightarrow y=0$

So, the point of intersection of l_3 and l_4 is origin, 0(0,0).

As this point lies on required line, so will satisfy equation (i). i.e.

$(2(0)+0+1)+\lambda(2(0)+3(0)+5)=0$

$1+5\lambda=0$

$5\lambda=-1 \Rightarrow \lambda=-1/5$

Putting in (i)

$$(2x+y+1)+\left(\frac{-1}{5}\right)(2x+3y+5)=0$$ Multiplying by 5

$10x+5y+5-2x-3y-5=0$

$8x+2y=0 \Rightarrow 4x+y=0$

b) l_1: 2x–3y–4=0, l_2: 3x+y+5=0, that touches the point of intersection of l_3: x+y–3=0 and l_4: x–5y+1=0

Solution: The equation of line passing through the point of intersection of l_1 and l_2 is given by:

$l_1+\lambda l_2=0$ putting l_1 and l_2

$(2x-3y-4)+\lambda(2x+y+5)=0 \rightarrow$ (i)

To find the point of intersection of l_3 and l_4

Subtracting l_4 from l_3

$x+y-3=0$

$\pm x \mp 5y \pm 1=0$

$6y-4=0 \Rightarrow 6y=4$

Perfect24u.com

www.perfect24u.com

$y=\frac{2}{3}$ putting l_3

$x+\frac{2}{3}-3=0 \quad \Rightarrow \quad x-\frac{7}{3}=0 \quad \Rightarrow \quad x=\frac{7}{3}$

So, the point of intersection of l_3 and l_4 is $P=\left(\frac{7}{3},\frac{2}{3}\right)$

As this point lies on required line, so will satisfy equation (i) i.e.

$\left(2\left(\frac{7}{3}\right)-3\left(\frac{2}{3}\right)-4\right)+\lambda\left(3\left(\frac{7}{3}\right)+\frac{2}{3}+5\right)=0$

$\frac{14}{3}-\frac{6}{3}-4+7\lambda+\frac{2}{3}\lambda+5\lambda=0$ Multiplying by 3

$14-6-12+21\lambda+2\lambda+15\lambda=0$

$38\lambda=4$

$\lambda=\frac{4}{38}=\frac{2}{19}$

Putting in (i)

$(2x-3y-4)+\frac{2}{19}(3x+y+5)=0$ Multiplying by 19

$38x-57y-76+6x+2y+10=0$

$44x-55y-66=0$ Divide by 11

$4x-5y-6=0$

Perfect24u.com

www.perfect24u.com

Concurrency of straight line:

For two straight line; $l_1: a_1x+b_1y+c_1=0$ and

$l_2: a_2x+b_2y+c_2=0$

The point of intersection is obtained by the joint solution of the two equation i.e.

$a_1x+b_1y=c_1$

$a_2x+b_2y=-c_2$

$AX=B$

$X=A^{-1}B \quad \rightarrow \quad (i)$

For the straight lines; $l_1: a_1x+b_1y+c_1=0$

$l_2: a_2x+b_2y+c_2=0$ and

$l_3: a_3x+b_3y+c_3=0$

The lines l_1, l_2 and l_3 are concurrent if the determinant

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Area of a triangular region: the area of a triangular region bounded by points $P_1(x_1,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ is given by;

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Perfect24u.com

EXERCISE 6.4

Q.1 In each case, find the point of intersection P(x, y) of the pair of lines

a) $l_1: 2x+4y-10=0$, $l_2: 5x-3y+1=0$

Solution:

$2x+4y = 10 \rightarrow (i)$

www.perfect24u.com

$5x-3y=-1 \quad \rightarrow$ (ii)

Use augmented matrix form

$$A_b=\left[\begin{array}{cc|c} 2 & 4 & 10 \\ 5 & -3 & -1 \end{array}\right]$$

By row operation

$$A_b=\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 5 & -3 & -1 \end{array}\right] \sim \frac{1}{2}R_1, R_2-5R_1$$

$$A_b=\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -3-10 & -1-25 \end{array}\right]$$

$$A_b=\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -13 & -26 \end{array}\right]$$

Backward substitution

$-13y=-26$

y = 2

$x+2y=5$

$x+2(2)=5$

$x+4=5$

$x=5-4=1$

x = 1

Hence the point of intersection is (1,2)

b) **l_1: 2x+y–8=0, l_2: 3x+2y–2=0**

Solution:

$2x+y=8 \rightarrow$ (i)

$3x+2y=2 \rightarrow$ (ii)

Use augmented matrix form

Perfect24u.com

www.perfect24u.com

$$A_b = \begin{bmatrix} 2 & 1 & 8 \\ 3 & 2 & 2 \end{bmatrix} \sim \frac{R_1}{2}$$

By row operation

$$A_b = \begin{bmatrix} 1 & 1/2 & 4 \\ 3 & 2 & 2 \end{bmatrix} \sim R_2 - 3R_1$$

$$A_b = \begin{bmatrix} 1 & 1/2 & 4 \\ 0 & 2 - 1/2 & 4 - 12 \end{bmatrix}$$

$$A_b = \begin{bmatrix} 1 & 1/2 & 4 \\ 0 & 1/2 & -8 \end{bmatrix}$$

Backward substitution

$\frac{1}{2}y = -8$

y = –16

$x + \frac{y}{2} = 4$

$x - \frac{16}{2} = 4$

$x - 8 = 4$

x = 12

Hence the point of intersection is (–16,12)

c) **l_1: 3x–5y+22=0,** **l_2: 6x+y–22=0**

Solution:

3x–5y = –22 → (i)

6x+y = 22 → (ii)

Use augmented matrix form

Perfect24u.com

www.perfect24u.com

$$A_b = \begin{bmatrix} 3 & -5 & -22 \\ 6 & 1 & 22 \end{bmatrix} \sim R_2 - 2R_1$$

By row operation

$$A_b = \begin{bmatrix} 3 & -5 & -22 \\ 0 & 1+10 & 22+44 \end{bmatrix}$$

$$A_b = \begin{bmatrix} 3 & -5 & -22 \\ 0 & 11 & 66 \end{bmatrix}$$

Backward substitution

$0x + 11y = 66$

$11y = 66$

÷ing both sides by 66

$$\frac{11y}{11} = \frac{66}{11}$$

y = 6

$3x - 5y = -22 \rightarrow$ (ii)

putting $y = 6$ in equation (iii)

$3(x) - 5(6) = -22$

$3x - 30 = -22$

$3x = -22 + 30$

$3x = 8$

x = $8/3$

Hence the point of intersection is $(8/3, 6)$

d) l_1: 8x–7y+1=0, l_2: 10x–11y+35=0

Solution:

$8x - 7y = -1 \rightarrow$ (i)

Perfect24u.com

www.perfect24u.com

$10x - 11y = -35 \rightarrow$ (ii)

Use augmented matrix form

$$A_b = \left[\begin{array}{cc|c} 8 & -7 & -1 \\ 10 & -11 & -35 \end{array}\right] \sim \frac{R_1}{8}$$

By row operation

$$A_b = \left[\begin{array}{cc|c} 1 & -\frac{7}{8} & -\frac{1}{8} \\ 10 & -11 & -35 \end{array}\right] R_2 - 10R_1$$

$$A_b = \left[\begin{array}{cc|c} 1 & -\frac{7}{8} & -\frac{1}{8} \\ 0 & -11 + \frac{70}{8} & -35 + \frac{\overset{5}{\cancel{10}}}{\cancel{8}_{4}} \end{array}\right]$$

$$A_b = \left[\begin{array}{cc|c} 1 & -\frac{7}{8} & -\frac{1}{8} \\ 0 & -88 + \frac{70}{8} & -35 + \frac{5}{4} \end{array}\right]$$

$$A_b = \left[\begin{array}{cc|c} 1 & -\frac{7}{8} & -\frac{1}{8} \\ 0 & -\frac{18}{8} & -140 + \frac{5}{4} \end{array}\right]$$

$$A_b = \left[\begin{array}{cc|c} 1 & -\frac{7}{8} & -\frac{1}{8} \\ 0 & -\frac{9}{4} & -\frac{135}{4} \end{array}\right]$$

Perfect24u.com

Backward substitution

$$0 - \frac{9}{\cancel{4}}y = \frac{-135}{\cancel{4}}$$

$-9y = -135$

÷ing both sides by -9

$$\frac{\cancel{9}y}{\cancel{9}} = \frac{\overset{15}{\cancel{-135}}}{\cancel{-9}}$$

www.perfect24u.com

y = 15

$8x - 7y = -1 \rightarrow$ (ii)

putting y = 15 in equation (iii)

$8(x) - 7(15) = -1$

$8x - 105 = -1$

$8x = -1 + 105$

$3x = 8$

$$\frac{\cancel{8}x}{\cancel{8}} = \frac{\overset{13}{\cancel{104}}}{\cancel{8}}$$

x = 13

Hence the point of intersection is (13,15)

Q.2 Show that the following lines are concurrent. If the lines are concurrent, then find out the point at which the given lines can make the concurrency.

a) l_1: x–y–2=0, l_2: 2x–y–5=0, l_3: 11x–5y–28=0

Solution:

Three lines $l_1: a_1x+b_1y+c_1=0$

$l_2: a_2x+b_2y+c_2=0$

$l_3: a_3x+b_3y+c_3=0$

Perfect24u.com

are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$ ---- (i)

Here $a_1=1$, $a_2=2$ $a_3=11$ $b_1=-1$, $b_2=-1$ $b_3=-5$, $c_1=-2$, $c_2=-5$, $c_3=-28$

Putting these values in L.H.S of (i)

$$\begin{vmatrix} 1 & -1 & -2 \\ 2 & -1 & -5 \\ 11 & -5 & -28 \end{vmatrix} = 1\begin{vmatrix} -1 & -5 \\ -5 & -28 \end{vmatrix} + 1\begin{vmatrix} 2 & -5 \\ 11 & -28 \end{vmatrix} - 2\begin{vmatrix} 2 & -1 \\ 11 & -5 \end{vmatrix}$$

www.perfect24u.com

$= 1(28-25)+1(-56+55)-2(-10+11)$

$= 3-1-2=0$

So, the lines are concurrent

Now, to find the point of concurrency, we find the point of intersection of l_1 and l_2

l_1: $x-y-2=0 \rightarrow$ (i)

l_2: $2x-y-5=0 \rightarrow$ (ii)

Subtracting (ii) from (i)

$x-y-2=0$

$\pm 2x \mp y \mp 5=0$

$-x+3=0 \quad \Rightarrow \quad x=3$ putting in (i)

$3-y-2=0$

$1-y=0 \quad \Rightarrow \quad y=1$

So, The point of concurrency is P (3,1)

b) l_1: $x+2y-3=0$, l_2: $2x-y+4=0$, l_3: $x+4y-7=0$

Solution:

Three line, l_1: $a_1x+b_1y+c_1=0$

l_2: $a_2x+b_2y+c_2=0$

l_3: $a_3x+b_3y+c_3=0$

are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \rightarrow$ (i)

Perfect24u.com

Here $a_1=1$, $b_1=2$, $c_1=-3$, $a_2=2$, $b_2=-1$, $c_2=4$ $a_3=1$, $b_3=4$, $c_3=-7$

Putting these values in L.H.S of (i)

$$\begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 1 & 4 & -7 \end{vmatrix} = 1\begin{vmatrix} -1 & 4 \\ 4 & -7 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 1 & -7 \end{vmatrix} - 3\begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix}$$

www.perfect24u.com

$$= 1(7-16) - 2(-14-4) - 3(8+1)$$

$$= -9 + 36 - 27$$

$$= 0$$

So, the lines are concurrent

Now, to find the point of concurrency, we find the point of intersection of l_1 and l_2

l_1: $x+2y-3=0$ $\rightarrow$ (i)

l_2: $2x-y+4=0$ $\rightarrow$ (ii)

Multiplying (ii) by 2 and adding with (i)

$x + 2y - 3 = 0$ $\rightarrow$ (i)

$\underline{4x - 2y + 8 = 0}$ $\rightarrow 2$(ii)

$5x+5=0 \quad \Rightarrow \quad 5x=-5 \quad \Rightarrow \quad x=-1$

Putting in (i)

$-1+2y-3=0$

$2y-4=0 \quad \Rightarrow \quad 2y=4 \quad \Rightarrow \quad y=2$

So, The point of concurrency of l_1, l_2 and l_3 is P(–1,2)

c) l_1:3 x+2y–1=0, l_2: 2x–3y+4=0, l_3: x+y–2=0

Solution:

Perfect24u.com

Three line, $l_1: a_1x+b_1y+c_1=0$

$l_2: a_2x+b_2y+c_2=0$

$l_3: a_3x+b_3y+c_3=0$

are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \rightarrow$(i)

Here $a_1=3$, $b_1=2$, $c_1=-1$, $a_2=2$, $b_2=-3$, $c_2=4$, $a_3=1$, $b_3=1$, $c_3=-2$

Putting these values in L.H.S of (i)

www.perfect24u.com

$$\begin{vmatrix} 3 & 2 & -1 \\ 2 & -3 & 4 \\ 1 & 1 & -2 \end{vmatrix} = 3\begin{vmatrix} -3 & 4 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 1 & -2 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$$

$$= 3(6-4) - 2(-4-4) - 1(2+3)$$

$$= 6 + 16 - 5 = 17 \neq 0$$

So, the lines are not concurrent

d) l_1: x+2y+5=0, l_2: 3x—+5y+1=0, l_3: 4x+7y+6=0

Solution:

Three line, $l_1: a_1x+b_1y+c_1=0$

$l_2: a_2x+b_2y+c_2=0$

$l_3: a_3x+b_3y+c_3=0$

are concurrent if, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \rightarrow (i)$

Here $a_1=1$, $b_1=2$, $c_1=5$, $a_2=3$, $b_2=5$, $c_2=1$, $a_3=4$, $b_3=7$, $c_3=6$

Putting these values in L.H.S of (i)

$$\begin{vmatrix} 1 & 2 & 5 \\ 3 & 5 & 1 \\ 4 & 7 & 6 \end{vmatrix} = 1\begin{vmatrix} 5 & 1 \\ 7 & 6 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 4 & 6 \end{vmatrix} + 5\begin{vmatrix} 3 & 5 \\ 4 & 7 \end{vmatrix}$$

Perfect24u.com

$$= 1(30-7) - 2(18-4) + 5(21-20) = 23 - 28 + 5$$

$$= 0$$

So, the lines are concurrent

Now, to find the point of concurrency

$l_1: x+2y+5=0 \quad \rightarrow (i)$

$l_2: 3x+5y+1=0 \quad \rightarrow (ii)$

Multiplying (i) by 3 and subtracting from (ii)

www.perfect24u.com

$3x+5y+1=0 \rightarrow$ (ii)

$\pm 3x \pm 6y \pm 15 = 0 \rightarrow$ 3(i)

$-y-14=0 \quad \Rightarrow \quad y+14=0 \quad \Rightarrow \quad y=-14$

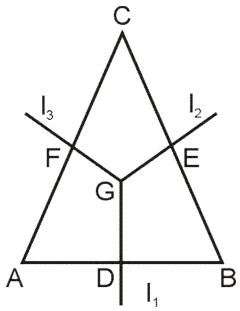
Putting in (i)

$x+2(-14)+5=0 \quad \Rightarrow \quad x-28+5=0 \quad \Rightarrow \quad x=+23$

Thus the point of concurrency is

P (23,–14)

Q.3 If ABC is a triangle with vertices A(0, 0) B(8, 6) and C(12, 0), the show that:



a) **Right Bisector:** right bisectors of a triangle are the lines perpendicular to each side and passing through the midpoint of the side.

For line l_1: l_1 passes through the midpoint D of AB and is perpendicular to AB. The midpoint of AB, by midpoint formula.

$$D:(x,y)=\left(\frac{0+8}{2},\frac{0+6}{2}\right)=D(x,y)=(4,3)$$

Slope of AB, $m_1=\frac{y_2-y_1}{x_2-x_1}=\frac{6-0}{8-0}=\frac{3}{4}$

Perfect24u.com

Slope of perpendicular to AB, $m_2=\frac{-4}{3}$

The equation of right-bisector l_1 using point-slope formula $m_2=\frac{-4}{3}$, D(4,3)

$y-y_1=m(x-x_1)$

$y-3=\frac{-4}{3}(x-4)$

$3y-9=-4x+16$

www.perfect24u.com

$4x+3y-25=0 \rightarrow$ (i)

For line l_2: l_2 passes through the midpoint E of BC and is perpendicular to BC. The midpoint E of BC, by midpoint formula.

$$E(x,y)=\left(\frac{8+12}{2},\frac{6+0}{2}\right)\Rightarrow E(x,y)=(10,3)$$

Slope of BC, $m_3=\frac{y_2-y_1}{x_2-x_1}=\frac{0-6}{12-8}=\frac{-6}{4}=\frac{-3}{2}$

Slope of perpendicular to BC, $m_4=\frac{2}{3}$

The equation of right-bisector l_2 using point-slope formula $m_4=\frac{2}{3}$, E(10,3)

$y-y_1=m(x-x_1)$

$y-3=\frac{2}{3}(x-10)$

$3y-9=2x-20$

$2x-3y-11=0 \rightarrow$ (ii)

For line l_3: l_3 passes through the midpoint F of AC and is perpendicular to AC. The midpoint F of AC, by midpoint formula.

$$F(x,y)=\left(\frac{0+12}{2},\frac{0+0}{2}\right)=F(x,y)=(6,0)$$

Perfect24u.com

Slope of AC, $m_5=\frac{y_2-y_1}{x_2-x_1}=\frac{0-0}{12-0}=0$

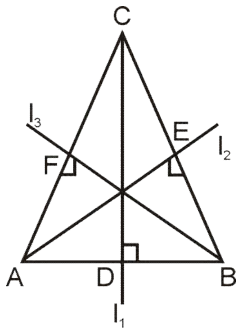
Slope of perpendicular to AC, $m_6=\frac{-1}{0}=-\infty$

The equation of right-bisector l_3 using point-slope formula $m_6=-\infty$, F(6,0)

$y-y_1=m(x-x_1) \quad \Rightarrow \quad \frac{y-y_1}{m}=x-x_1$

www.perfect24u.com

$$\frac{y-0}{-\infty} = x-6 \qquad \Rightarrow \qquad x-6=0 \rightarrow (iii)$$



From (i), (ii) and (iii)

$l_1 : 4x+3y-25=0 \rightarrow (i)$

$l_2 : 2x-3y-11=0 \rightarrow (ii)$

$l_3 : x-6=0 \quad \rightarrow (iii)$

$$\text{Matrix form } A = \begin{bmatrix} 4 & 3 & -25 \\ 2 & -3 & -11 \\ 1 & 0 & -6 \end{bmatrix}$$

Expand by R_3

$$|A| = \begin{vmatrix} 4 & 3 & -25 \\ 2 & -3 & -11 \\ 1 & 0 & -6 \end{vmatrix} = 1\begin{vmatrix} 3 & -25 \\ -3 & -11 \end{vmatrix} - 0\begin{vmatrix} 4 & -25 \\ 2 & -11 \end{vmatrix} - 6\begin{vmatrix} 4 & 3 \\ 2 & -3 \end{vmatrix}$$

$= (-33-75)-6(-12-6)$

$= -108+108$

$= 0$

So, the right bisectors are concurrent

b) The altitude of the triangle ABC is concurrent.

Solution: Altitude

Perfect24u.com

The altitude is the line passing through a vertex and perpendicular to opposite side.

To show that altitudes are concurrent, first we have to find the equations of altitudes. Thus

For line l_1: l_1 is perpendicular to AB, and passes through C.

$$\text{Slope of AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{6-0}{8-0} = \frac{6}{8} = \frac{3}{4}$$

$$m_1 = \frac{3}{4}$$

www.perfect24u.com

Slope of perpendicular to AB, $m_2 = \frac{-4}{3}$

The equation of altitude l_1 using point slope formula.

$m = \frac{-4}{3}$, C (12,0)

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-4}{3}(x - 12) \quad \Rightarrow \quad 3y = -4x + 48$

$4x + 3y - 48 = 0 \rightarrow$ (i)

For line l_2: l_2 is perpendicular to BC, and passes through vertex A(0,0).

Slope of BC $= \frac{y_3 - y_2}{x_3 - x_2} = \frac{0-6}{12-8} = \frac{-6}{4} = \frac{-3}{2}$

$m_3 = \frac{-3}{2}$

Slope of perpendicular to BC, $m_4 = \frac{2}{3}$

The equation of altitude l_2 using point-slope formula.

$m = \frac{2}{3}$, A (0,0)

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{2}{3}(x - 0) \Rightarrow \quad 3y = 2x$

$2x - 3y = 0 \rightarrow$ (ii)

For line l_3: l_3 is perpendicular to AC, and passes through vertex B(8,6).

Slope of AC, $m_5 = 0$

Slope of perpendicular to AC, $m_6 = -\infty$

The equation of altitude l_3 using point-slope formula.

$m_6 = \infty$, B (8,6)

Perfect24u.com

www.perfect24u.com

$y-y_1=m(x-x_1)$

$$\frac{y-y_1}{m}=x-x_1$$

$$\frac{y-6}{-\infty}=x-8$$

$x-8=0 \rightarrow$(iii)

$4x+3y-48=0$

$2x-3y=0$

$x+0y-8=0$

$$A=\begin{bmatrix}4 & 3 & -48\\ 2 & -3 & 0\\ 1 & 0 & -8\end{bmatrix}$$

From (i), (ii) and (iii)

$$A=\begin{vmatrix}4 & 3 & -48\\ 2 & -3 & 0\\ 1 & 0 & -8\end{vmatrix}=1\begin{vmatrix}3 & -48\\ -3 & 0\end{vmatrix}+0\begin{vmatrix}4 & -48\\ 2 & 0\end{vmatrix}-8\begin{vmatrix}4 & 3\\ 2 & -3\end{vmatrix}$$

$=1(0-144)-8(-12-6)$

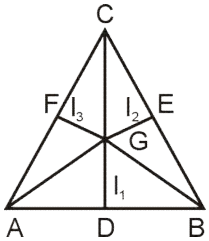
$=-144+144$

$=0$

So, the altitudes are concurrent

Perfect24u.com

c) The medians of the triangle ABC are concurrent.



Solution: Medians

The medians of a triangle are lines that joint the midpoint of each side with the opposite vertex.

To show that medians of a triangle are concurrent, first we have to find the equations of median lines.

For line l_1: l_1 passes through midpoint D of AB and vertex C.

www.perfect24u.com

The midpoint D of AB is D (4,3)

Using two point formula to find the equation of CD; C(12,0) D(4,3)

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-0}{x-12}=\frac{3-0}{4-12} \quad \Rightarrow \quad \frac{y}{x-12}=\frac{3}{-8}$$

$$-8y = 3x - 36$$

$$3x + 8y - 36 = 0 \quad \rightarrow (i)$$

For line l_2: l_2 is passes through midpoint E of BC, and vertex A

Using two points formula to find the equation of AE;

A(0,0) E (10,3)

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-0}{x-0}=\frac{3-0}{10-0} \Rightarrow \frac{y}{x}=\frac{3}{10} \quad \Rightarrow 10y = 3x$$

$$3x-10y=0 \quad \rightarrow (ii)$$

For line l_3: l_3 passes through midpoint F of AC and vertex B.

Using two points formula to find the equation of BF, B(8,6), F(6,0)

$$\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$$

Perfect24u.com

$$\frac{y-6}{x-8}=\frac{0-6}{6-8} \Rightarrow \frac{y-6}{x-8}=\frac{-6}{-2} \Rightarrow \frac{y-6}{x-8}=\frac{3}{1}$$

$$y - 6 = 3x - 24$$

$$3x - y - 18 = 0 \rightarrow (iii)$$

$$A = \begin{bmatrix} 3 & 8 & -36 \\ 3 & -10 & 0 \\ 3 & -1 & -18 \end{bmatrix}$$

www.perfect24u.com

From (i) (ii) and (iii)

$$A=\begin{vmatrix}3 & 8 & -36\\ 3 & -10 & 0\\ 3 & -1 & -18\end{vmatrix}=-36\begin{vmatrix}3 & -10\\ 3 & -1\end{vmatrix}-0\begin{vmatrix}3 & 8\\ 3 & -1\end{vmatrix}-18\begin{vmatrix}3 & 8\\ 3 & -10\end{vmatrix}$$

$$=-36(-3+30)-18(-30-24)$$

$$=-972+972=0$$

$$=0$$

So, the medians are concurrent.

Q.4 Find the area of the triangle region whose vertices are the following

a) $P_1(0,0), P_2(2,4), P_3(-2,2)$.

Solution: The area if triangle with vertices $P(x,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ is given by:

$$A=\frac{1}{2}\begin{vmatrix}x_1 & y_1 & 1\\ x_2 & y_2 & 1\\ x_3 & y_3 & 1\end{vmatrix}$$

Here $x_1=0$, $y_1=0$, $x_2=2$, $y_2=4$, $x_3=-2$, $y_3=2$

So, $A=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 2 & 4 & 1\\ -2 & 2 & 1\end{vmatrix}$ expanding by first row

Perfect24u.com

$$A=\frac{1}{2}\left[0\begin{vmatrix}4 & 1\\ 2 & 1\end{vmatrix}-0\begin{vmatrix}2 & 1\\ -2 & 1\end{vmatrix}+1\begin{vmatrix}2 & 4\\ -2 & 2\end{vmatrix}\right]$$

$$A=\frac{1}{2}[0-0+1(4+8)]=\frac{1}{2}(12)$$

$$A=6$$

b) $P_1(-1,-2), P_2(2,5), P_3(5,2)$.

Solution: The area if triangle with vertices $P(x,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ is given by:

www.perfect24u.com

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Here $x_1=-1$, $y_1=-2$, $x_2=2$, $y_2=5$, $x_3=5$, $y_3=2$

$$\text{So, } A = \frac{1}{2}\begin{vmatrix} -1 & -2 & 1 \\ 2 & 5 & 1 \\ 5 & 2 & 1 \end{vmatrix} = \frac{1}{2}\left[-1\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 5 & 2 \end{vmatrix}\right]$$

$$A = \frac{1}{2}\left[-1(5-2) + 2(2-5) + 1(4-25)\right] = \frac{1}{2}[-3-6-21]$$

$A = 15$ (As area can't be negative)

c) **$P_1(4,-5)$, $P_2(5,-6)$, $P_3(3,1)$.**

Solution: The area if triangle with vertices $P(x,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ is given by:

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Here $x_1=4$, $y_1=-5$, $x_2=5$, $y_2=-6$, $x_3=3$, $y_3=1$

Perfect24u.com

$$\text{So, } A = \frac{1}{2}\begin{vmatrix} 4 & -5 & 1 \\ 5 & -6 & 1 \\ 3 & 1 & 1 \end{vmatrix} = \frac{1}{2}\left[4\begin{vmatrix} -6 & 1 \\ 1 & 1 \end{vmatrix} + 5\begin{vmatrix} 5 & 1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 5 & -6 \\ 3 & 1 \end{vmatrix}\right]$$

$$A = \frac{1}{2}\left[4(-6-1) + 5(5-3) + 1(5+18)\right]$$

$$A = \frac{1}{2}[-28+10+23] = \frac{1}{2}(5)$$

$$A = \frac{5}{2}$$

www.perfect24u.com

Q.5 Find the area of region bounded by the triangle ABC whose vertices are A(a,b+c), B(a,b–c) and C(–a,c).

a) A(a, b + b), B(a, b – c), C(–a, c).

Solution: The area of triangle with vertices $P_1(x_1,y_1)$, $P_2(x_2,y_2)$ and $P_3(x_3,y_3)$ is given by:

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Here $x_1 = a$, $y_1 = b + c$, $x_2 = a$, $y_2 = b–c$, $x_3 = –a$, $y_3 = c$

So, area A is

$$A = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ a & b-c & 1 \\ -a & c & 1 \end{vmatrix}$$

$$A = \frac{1}{2}\left[a\begin{vmatrix} b-c & 1 \\ c & 1 \end{vmatrix} - a\begin{vmatrix} b+c & 1 \\ c & 1 \end{vmatrix} - a\begin{vmatrix} b+c & 1 \\ b-c & 1 \end{vmatrix}\right]$$

$$A = \frac{1}{2}\left[a(b-c-c) - a(b+c-c) - a(b+c-b+c)\right]$$

$$A = \frac{1}{2}\left[ab - 2ac - ab - 2ac\right]$$

Perfect24u.com

$$A = \frac{1}{2}\left[-4ac\right] \Rightarrow \quad A = 2ac \text{ (Area can't be negative)}$$

Q.6 Show that the area bounded by the triangle ABC whose vertices are the following

a) A(–3,6), B(3,2), C(6,0).

Solution: The area of triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ is given by:

www.perfect24u.com

$$A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Here $x_1=-3$, $y_1=6$, $x_2=3$, $y_2=2$, $x_3=6$, $y_3=0$

So, area A is

$$A=\frac{1}{2}\begin{vmatrix} -3 & 6 & 1 \\ 3 & 2 & 1 \\ 6 & 0 & 1 \end{vmatrix}=\frac{1}{2}\left[6\begin{vmatrix} 6 & 1 \\ 2 & 1 \end{vmatrix}-0\begin{vmatrix} -3 & 1 \\ 3 & 1 \end{vmatrix}+1\begin{vmatrix} -3 & 6 \\ 3 & 2 \end{vmatrix}\right]$$

$$A=\frac{1}{2}\left[6(6-2)+1(-6-18)\right]==\frac{1}{2}[24-24]=0$$

b) **A(–2,4), B(3,–6), C(1,–2).**

Solution: The area of triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ is given by:

$$A=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Here $x_1=-2$, $y_1=4$, $x_2=3$, $y_2=-6$, $x_3=1$, $y_3=-2$

So, area A is

Perfect24u.com

$$A=\frac{1}{2}\begin{vmatrix} -2 & 4 & 1 \\ 3 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}=\frac{1}{2}\left[-2\begin{vmatrix} -6 & 1 \\ -2 & 1 \end{vmatrix}-3\begin{vmatrix} 4 & 1 \\ -2 & 1 \end{vmatrix}+1\begin{vmatrix} 4 & 1 \\ -6 & 1 \end{vmatrix}\right]$$

$$A=\frac{1}{2}\left[-2(-6+2)-3(4+2)+1(4+6)\right]$$

$$A=\frac{1}{2}[8-18+10]=0$$

c) **Are the vertices of triangle ABC in parts a and b above, collinear?**

Solution: Yes, the verities of triangles ABC in part a and b above are collinear as the area of triangle in both parts is zero.

www.perfect24u.com

Q.7 Find two first degree straight lines in x and y, when the second degree homogeneous equations are following.

a) $3x^2-2xy-5y^2=0$

Solution:

Method-I

The second degree homogenous equation $ax^2 + 2hxy + by^2 = 0$

Represent two straight line.

$$ax+(h\pm\sqrt{h^2-ab})y=0$$

Here the given equation of the form

$a = 3, b = -5, h = -1$

$$3x+\left(-1\pm\sqrt{(-1)^2-(3)(-5)}\right)y=0$$

$\Rightarrow$ $$3x+\left(-1\pm\sqrt{1+15}\right)y=0$$

$\Rightarrow$ $3x + (-1 \pm 4)y = 0$

$\Rightarrow$ $3x + (-1+4)y = 0$ and $3x + (-1-4)y = 0$

$\Rightarrow$ $3x + (3y) = 0$ and $3x - 5y = 0$

$3x + 3y = 0$

$3(x + y) = 0$

$3x - 5y = 0$

$(x + y)(3x - 5y = 0)$

Perfect24u.com

Method-II

$\Rightarrow$ $3x^2 - 2xy - 5y^2=0$ (By factorization method)

$\Rightarrow$ $3x^2 - 5xy + 3xy - 5y^2=0$

$\Rightarrow$ $x(3x - 5y) + y(3x - 5y)=0$

$\Rightarrow$ $(3x - 5y)(x + y)=0$

www.perfect24u.com

So, the two straight lines are

l_1: $3x - 5y = 0$ and l_2: $x + y = 0$

b) $\mathbf{4x^2 - 9xy + 5y^2 = 0}$

Solution:

Method-I

Given second degree homogeneous equation

$4x^2 - 9xy + 5y^2 = 0$

General form of second degree homogenous equation

$ax^2 + 2hxy + by^2 = 0$

$a = 4, h = -9/2, b = 5$

So required lines are

$$ax + (h \pm \sqrt{h^2 - ab})y = 0$$

Putting the values

$$\Rightarrow \quad 4x + \left[\left(\frac{-9}{2}\right) \pm \sqrt{\left(\frac{-9}{2}\right)^2 - (4)(5)}\right] y = 0$$

$$\Rightarrow \quad 4x + \left[\frac{-9}{2} \pm \sqrt{\frac{81}{4} - 20}\right] y = 0$$

$$\Rightarrow \quad 4x + \left[\frac{-9}{2} \pm \sqrt{\frac{1}{4}}\right] y = 0$$

$$\Rightarrow \quad 4x + \left[\frac{-9}{2} \pm \frac{1}{2}\right] y = 0$$

$$\Rightarrow \quad 4x + \left[\frac{-9}{2} + \frac{1}{2}\right] y = 0 \text{ and } 4x + \left[\frac{-9}{2} - \frac{1}{2}\right] y = 0$$

$4x - 4y = 0$ and $4x - 5y = 0$

Thus the required equation of the form $x - y = 0$ and $4x - 5y = 0$

Perfect24u.com

www.perfect24u.com

Method-II

$\Rightarrow \quad 4x^2-9xy+5y^2=0$

$\Rightarrow \quad 4x^2-4xy-5xy+5y^2=0$

$\Rightarrow \quad 4x(x-y)-5y(x-y)=0$

$\Rightarrow \quad (x-y)(4x-5y)=0$

So, the two straight lines are

l_1: $x-y=0$ and l_2: $4x-5y=0$

c) $\mathbf{x^2+xy+y^2=0}$

Solution:

Method-I

$x^2+xy+y^2=0$

General form of second degree homogeneous equation of the form

$ax^2 + 2hxy + by^2 = 0$

compare with

$x^2 + xy + y^2 = 0$

$a = 1, h = \frac{1}{2}, b = 1$

Equation of the form

$ax+(h\pm\sqrt{h^2-ab})y=0$

Putting the values

$\Rightarrow \quad 1.x+\left[\frac{1}{2}\pm\sqrt{\left(\frac{1}{2}\right)^2-(1)(1)}\right]y=0$

$\Rightarrow \quad x+\left[\frac{1}{2}\pm\sqrt{\frac{1}{4}-1}\right]y=0$

$\Rightarrow \quad x+\left[\frac{1}{2}\pm\sqrt{\frac{1-4}{4}}\right]y=0$

Perfect24u.com

www.perfect24u.com

$$\Rightarrow \quad x + \left[\frac{1}{2} \pm \sqrt{\frac{-3}{4}}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{1}{2} \pm \frac{\sqrt{-3}}{2}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{1}{2} \pm \frac{\sqrt{3}i}{2}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{1 \pm \sqrt{3}i}{2}\right] y = 0$$

$$x + \left[\frac{1+\sqrt{3}i}{2}\right] y = 0 \text{ and } x + \left[\frac{1-\sqrt{3}i}{2}\right] y = 0$$

Method-II

$x^2+xy+y^2=0$

$x^2+xy+y^2=1$ divide by x^2

$$\frac{y^2}{x^2} + \frac{y}{x} + 1 = 0 \quad \Rightarrow \quad \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right) + 1 = 0$$

Which is quadratic in $\frac{y}{x}$ with a=1, b=1 and c=1

Perfect24u.com

$$\text{Thus, } \frac{y}{x} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-1 \pm \sqrt{1^2-4(1)(1)}}{2(1)} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$l_1 : y = \left(\frac{-1+\sqrt{-3}}{2}\right) x \text{ and } l_2 : y = \left(\frac{-1-\sqrt{-3}}{2}\right) x$$

d) $\mathbf{x^2-7xy+6y^2=0}$

Solution:

Method-I

Given second degree homogeneous equation

www.perfect24u.com

$4x^2 - 7xy + 6y^2 = 0$

General form of second degree homogenous equation

$\Rightarrow \quad ax^2 + 2hxy + by^2 = 0$

$\Rightarrow \quad 4x^2 - 7xy + 6y^2 = 0$

$a = 1, h = {}^{-7}\!/_{2}, b = 6$

So required lines are

$$ax + (h \pm \sqrt{h^2 - ab})y = 0$$

Putting the values a, h and b

$$\Rightarrow \quad 1.x + \left[\left(\frac{-7}{2}\right) \pm \sqrt{\left(\frac{-7}{2}\right)^2 - (1)(6)}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{-7}{2} \pm \sqrt{\frac{49}{4} - 6}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{-7}{2} \pm \sqrt{\frac{49 - 24}{4}}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{-7}{2} \pm \sqrt{\frac{25}{4}}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{-7}{2} \pm \frac{5}{2}\right] y = 0$$

Perfect24u.com

$$\Rightarrow \quad x + \left[\frac{-7 \pm 5}{2}\right] y = 0$$

$$\Rightarrow \quad x + \left[\frac{-7 + 5}{2}\right] y = 0 \text{ and } x + \left[\frac{-7 - 5}{2}\right] y = 0$$

$x - y = 0$ and $x - 6y = 0$

Method-II

$\Rightarrow \quad x^2 - 7xy + 6y^2 = 0$

www.perfect24u.com

$\Rightarrow \quad x^2-6xy-xy+6y^2=0$

$\Rightarrow \quad x(x-6y)-y(x-6y)=0$

$\Rightarrow \quad (x-6y)(x-y)=0$

The two straight lines are then

l_1: $x-6y=0$ and l_2: $x-y=0$

Q.8 Find the angle between the lines represented by the following second degree homogeneous equations.

a) $3x^2-2xy-5y^2=0$

Solution: The angle between the two straight lines represented by the homogenous equation

$ax^2+2hxy+by^2=0$ is given by

$$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b} \quad \rightarrow(i)$$

For $3x^2-2xy-5y^2=0$

$a=3, 2h=-2 \Rightarrow \quad h=-1$, and $b=-5$ Putting these values in (i)

$$\tan\theta=\frac{2\sqrt{(-1)^2-(3)(-5)}}{3-5}=\frac{2\sqrt{1+15}}{-2}=\frac{2(4)}{-2}$$

$\tan\theta=-4$

$\theta=\tan^{-1}=(-4)$

Perfect24u.com

b) $4x^2-9xy+5y^2=0$

Solution: The angle between the two straight lines represented by the homogenous equation

$ax^2+2hxy+by^2=0$ is given by

$$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b} \quad \rightarrow(i)$$

For $4x^2-9xy+5y^2=0$

www.perfect24u.com

$4x^2-9xy+5y^2=0$

$a=4,\ b=5,\ 2h=-9 \quad \Rightarrow \quad h=\frac{-9}{2}$ Putting these values in (i)

$$\tan\theta=\frac{2\sqrt{\left(-9/2\right)^2-(4)(5)}}{4+5}=\frac{2\sqrt{\frac{81}{4}-20}}{9}$$

$$\tan\theta=\frac{2\sqrt{\frac{81-80}{4}}}{9}=\frac{2\sqrt{\frac{1}{4}}}{9}=\frac{2\left(1/2\right)}{9}=\frac{1}{9}$$

$$\theta=\tan^{-1}\left(\frac{1}{9}\right)$$

c) $\mathbf{x^2+xy+y^2=0}$

Solution: The angle between the two straight lines represented by the homogenous equation

$$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b} \quad \rightarrow (i)$$

For $x^2+xy+y^2=0$

$a=1,\ b=1,\ 2h=1 \quad \Rightarrow \quad h=\frac{1}{2}$ Putting these values in (i)

$$\tan\theta=\frac{2\sqrt{\left(1/2\right)^2-(1)(1)}}{1+1}=\frac{2\sqrt{\frac{1}{4}-1}}{2}=\sqrt{\frac{-3}{4}}$$

$$\tan\theta=\frac{\sqrt{-3}}{2}$$

$$\theta=\tan^{-1}\left(\frac{\sqrt{-3}}{2}\right)$$

d) $\mathbf{x^2-7xy+6y^2=0}$

Solution: The angle between the two straight lines represented by the homogenous equation

Perfect24u.com

www.perfect24u.com

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} \quad \rightarrow (i)$$

For $x^2-7xy+6y^2=0$

$a=1, b=6, 2h=-7 \quad \Rightarrow \quad h=\frac{-7}{2}$ Putting these values in (i)

$$\tan\theta = \frac{2\sqrt{\left(-7/2\right)^2 - (1)(6)}}{1+6} = \frac{2\sqrt{\frac{49}{4}-6}}{7} = \frac{2\sqrt{\frac{49-24}{4}}}{7}$$

$$\tan\theta = \frac{2\sqrt{\frac{25}{4}}}{7} = \frac{2\left(5/2\right)}{7} = \frac{5}{7}$$

$$\theta = \tan^{-1}\left(\frac{5}{7}\right)$$

Q.9 Show that the first degree straight lines is x and y represented by the following second degree homogeneous equations are; coincident, perpendicular, or neither.

a) $x^2+5xy-y^2=0$

Solution:

Perfect24u.com

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{\left(\frac{5}{2}\right)^2 - (1)(-1)}}{1+(-1)}$$

$$\tan\theta = \frac{2\sqrt{\frac{25}{4}+1}}{0}$$

$$\tan\theta = \infty$$

www.perfect24u.com

$\theta = \tan^{-1}(\infty) = 90°$

b) $2x^2 - xy - y^2 = 0$

Solution:

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$\tan\theta = \frac{2\sqrt{\left(\frac{-1}{2}\right)^2 - (2)(-1)}}{2 + (-1)}$$

$$\tan\theta = \frac{2\sqrt{\frac{1}{4} + 2}}{2 - 1}$$

$$\tan\theta = \frac{2\sqrt{\frac{9}{4}}}{1}$$

$$\tan\theta = \cancel{2} \times \frac{3}{\cancel{2}}$$

$\theta = \tan^{-1}(3)$

This is neither 90° nor 0° or 180°

c) $x^2 + 3xy + 9y^2 = 0$

Solution:

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$\tan\theta = \frac{2\sqrt{\left(\frac{3}{2}\right)^2 - (1)(9)}}{1 + 9}$$

$$\tan\theta = \frac{2\sqrt{\frac{9}{4} - 9}}{10}$$

Perfect24u.com

www.perfect24u.com

$$\tan\theta = \frac{2\sqrt{\frac{-27}{4}}}{10}$$

$$\tan\theta = \frac{\sqrt{-27}}{10}$$

$$\theta = \tan^{-1}\left(\frac{3\sqrt{-3}}{10}\right)$$

This is neither 90° nor 0° or 180°

d) $\mathbf{x^2+4xy+8y^2=0}$

Solution:

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{(2)^2-(1)(8)}}{1+8}$$

$$\tan\theta = \frac{2\sqrt{4-8}}{9}$$

$$\tan\theta = \frac{2\sqrt{-4}}{9}$$

$$\tan\theta = \frac{2(2i)}{9}$$

$$\theta = \tan^{-1}\left(\frac{4i}{9}\right)$$

This is neither 90° nor 0° or 180°

Perfect24u.com

Q.10 Find a joint equation of straight line that passes through origin and

a) Perpendicular to the lines represented by $3x^2-7xy+2y^2=0$

Solution: First we will find the lines represented by the Homogeneous equation.

www.perfect24u.com

$\Rightarrow \quad 3x^2-7xy+2y^2=0$

$\Rightarrow \quad 3x^2-6xy-xy+2y^2=0$

$\Rightarrow \quad 3x(x-2y)-y(x-2y)=0$

$\Rightarrow \quad (x-2y)(3x-y)=0$

The lines are

$l_1: x-2y=0 \rightarrow (i)$ and $l_2: 3x-y=0 \rightarrow (ii)$

Slope of l_1: $m_1=\frac{-a}{b}=\frac{-1}{-2}=\frac{1}{2}$

Slope of line perpendicular to l_1: $m_2=-2$

Equation of line passing through origin and slope $m_2=-2$, using point-slope formula.

$y-y_1=m(x-x_1)$

$y-0=-2(x-0) \Rightarrow \quad y=-2x$

$l_3: 2x+y=0 \quad \Rightarrow (iii)$

Slope of l_2: $m_3=\frac{-a}{b}=\frac{-3}{-1}=3$

Slope of line perpendicular to l_2, $m_4=\frac{-1}{3}$ equation of line perpendicular to l_2 and passing through the origin and having slope $m_4=\frac{-1}{3}$

Perfect24u.com

$y-0=\frac{-1}{3}(x-0)$

$y=\frac{-1}{3}x \quad \Rightarrow \quad 3y=-x$

$l_4: x+3y=0 \quad \rightarrow (iv)$

Now, the joint equation of l_3 and l_4

$\Rightarrow \quad (2x+y)(x+3y)=0$

$\Rightarrow \quad 2x^2+6xy+xy+3y^2=0$

www.perfect24u.com

$\Rightarrow \quad 2x^2+7xy+3y^2=0$

b) Perpendicular to the lines represented by $x^2-2\tan\theta xy-y^2=0$

Solution: First we will find the lines represented by the Homogeneous equation.

$x^2-2\tan\theta xy-y^2=0$

$y^2+2\tan\theta xy-x^2=0$ divide by x^2

$$\frac{y^2}{x^2}+2\tan\theta\frac{y}{x}-1=0$$

Which is a quadratic in $\frac{y}{x}$, with

$a=1, b=2\tan\theta, c=-1$

$$\frac{y}{x}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\tan\theta\pm\sqrt{(2\tan\theta)^2-4(1)(-1)}}{2(1)}$$

$$\frac{y}{x}=\frac{-2\tan\theta\pm\sqrt{4\tan^2\theta+4}}{2}=\frac{-2\tan\theta\pm\sqrt{4\left(\tan^2\theta+1\right)}}{2}$$

$$\frac{y}{x}=\frac{-2\tan\theta\pm\sqrt{4\sec^2\theta}}{2}=\frac{-2\tan\theta\pm 2\sec\theta}{2}$$

$$\frac{y}{x}=-\tan\theta\pm\sec\theta$$

Perfect24u.com

When $\frac{y}{x}=-\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}=\frac{1-\sin\theta}{\cos\theta}$

$$l_1: y=\left(\frac{1-\sin\theta}{\cos\theta}\right)x \quad \rightarrow (i)$$

and $\frac{y}{x}=-\tan\theta-\sec\theta=-\frac{\sin\theta}{\cos\theta}-\frac{1}{\cos\theta}$

$$\frac{y}{x}=\frac{-\sin\theta-1}{\cos\theta}=-\frac{(1+\sin\theta)}{\cos\theta}$$

www.perfect24u.com

$l_2: y=\left(\frac{1+\sin\theta}{\cos\theta}\right)x \quad \rightarrow (ii)$

Slope of l_1: $m_1=\frac{1-\sin\theta}{\cos\theta}$

Slope of line perpendicular to l_1, $m_2=\frac{-\cos\theta}{1-\sin\theta}$

Slope of l_2, $m_3=\frac{-(1+\sin\theta)}{\cos\theta}$

Slope of perpendicular to l_2, $m_4=\frac{\cos\theta}{1+\sin\theta}$

Equation of line perpendicular to l_1 and passing through the origin

$y-0=\frac{-\cos\theta}{1-\sin\theta}(x-0)$

$y=\frac{-\cos\theta}{1-\sin\theta}x$

$l_3: \frac{\cos\theta}{1-\sin\theta}x+y=0 \rightarrow (iii)$

Equation of line perpendicular to l_2 and passing through the origin

$y-0=\frac{\cos\theta}{1+\sin\theta}(x-0) \quad \Rightarrow \quad y=\frac{\cos\theta}{1+\sin\theta}x$

$l_4: \frac{\cos\theta}{1+\sin\theta}x-y=0 \rightarrow (iv)$

Perfect24u.com

Now, joint equation of (iii) and (iv)

$\left(\frac{\cos\theta}{1-\sin\theta}x+y\right)\left(\frac{\cos\theta}{1+\sin\theta}x-y\right)=0$

$\frac{\cos^2\theta}{1-\sin^2\theta}x^2-\frac{\cos\theta}{1-\sin\theta}xy+\frac{\cos\theta}{1+\sin\theta}xy-y^2=0$

$\frac{\cos^2\theta}{\cos^2\theta}x^2-\left(\frac{\cos\theta}{1-\sin\theta}-\frac{\cos\theta}{1+\sin\theta}\right)xy-y^2=0$

www.perfect24u.com

$$\Rightarrow \quad x^2 - \left(\frac{\cos\theta + \cos\theta\sin\theta - \cos\theta + \cos\theta\sin\theta}{(1-\sin\theta)(1+\sin\theta)}\right)xy - y^2 = 0$$

$$\Rightarrow \quad x^2 - \frac{2\cos\theta\sin\theta}{1-\sin^2\theta}xy - y^2 = 0$$

$$\Rightarrow \quad x^2 - \frac{2\cos\theta\sin\theta}{\cos^2\theta}xy - y^2 = 0$$

$$\Rightarrow \quad x^2 - \frac{2\sin\theta}{\cos\theta}xy - y^2 = 0$$

$$x^2 - 2\tan\theta xy - y^2 = 0$$

c) Perpendicular to the lines represented by $ax^2+2hxy+by^2=0$

Solution: First we find the lines represented by the homogeneous equation.

$ax^2-2hxy+by^2=0$, divide by x^2

$$b\frac{y^2}{x^2} + 2h\frac{y}{x} + a = 0$$

$$b\left(\frac{y}{x}\right) + 2h\left(\frac{y}{x}\right) + a = 0 \qquad \text{Using quadratic equation}$$

$$\frac{y}{x} = \frac{-2h \pm \sqrt{(2h)^2 - 4(b)(a)}}{2(b)}$$

$$\frac{y}{x} = \frac{-2h \pm \sqrt{4h^2 - 4ab}}{2b}$$

$$\frac{y}{x} = \frac{-2\left(-h \pm \sqrt{h^2 - ab}\right)}{2b}$$

Slope of l_1: $m_1 = \dfrac{-h + \sqrt{b^2 - ab}}{2b}$

Slope of line perpendicular to l_1; $m_2 = \dfrac{-b}{-h + \sqrt{h^2 - ab}}$

Slope of l_2, $m_3 = \dfrac{-h + \sqrt{h^2 - ab}}{-b}$

Perfect24u.com

www.perfect24u.com

Slope of perpendicular to l_2, $m_4 = \dfrac{-b}{-h-\sqrt{h^2-ab}}$

Equation of straight line with slope m_2 is $y = m_2x$

$$y = \left(\frac{-b}{-h+\sqrt{h^2-ab}}\right)x$$

$$y = \left(\frac{-b}{-h+\sqrt{h^2-ab}}\right)x = 0 \rightarrow (i)$$

Equation of straight line with slope m_4 is $y = m_4x$

$$y = \left(\frac{-b}{-h-\sqrt{h^2-ab}}\right)x$$

$$y = \left(\frac{-b}{-h-\sqrt{h^2-ab}}\right)x = 0 \rightarrow (ii)$$

Now, joint equation of (i) and (ii)

$$\left(y-\left(\frac{-b}{-h+\sqrt{h^2-ab}}\right)x\right)\left(y-\left(\frac{-b}{-h-\sqrt{h^2-ab}}\right)x\right)=0$$

$$\Rightarrow \quad y^2+\frac{b^2x^2}{\left(-h+\sqrt{h^2-ab}\right)\left(-h-\sqrt{h^2-ab}\right)}+\frac{bxy}{\left(-h+\sqrt{h^2-ab}\right)}+\frac{bxy}{\left(-h-\sqrt{h^2-ab}\right)}=0$$

$$\Rightarrow \quad y^2+\frac{b^2x^2}{-h^2-\left(h^2-ab\right)}+bxy\left[\frac{1}{-h+\sqrt{h^2-ab}}+\frac{1}{-h-\sqrt{h^2-ab}}\right]=0$$

Perfect24u.com

$$\Rightarrow \quad y^2+\frac{b^2x^2}{ab}+bxy\left[\frac{-h-\sqrt{h^2-ab}-h+\sqrt{h^2-ab}}{h^2-\left(h^2-ab\right)}\right]=0$$

$$\Rightarrow \quad y^2+\frac{bx^2}{a}+\frac{bxy(-2h)}{ab}=0$$

$$\Rightarrow \quad \frac{bx^2}{a}+y^2-\frac{-2hxy}{a}=0 \qquad \text{Multiplying by a}$$

$$\Rightarrow \quad bx^2+ay^2-2hxy=0$$

$$ay^2-2hxy+bx^2=0$$

www.perfect24u.com