Unit 01

FUNCTIONS AND LIMITS

Introduction:

To say that a quantity y is a function of some other quantity x means, in ordinary language, that the value of y depends on the value of x. For example, the volume V of a cube is a function of the length s of a side. Infact, the dependence of V on s can be made the formula $V = s^3$ where s is independent variable and V is dependent variable.

Function:

A function $y = f(x)$ is a rule that assigns a unique value to y for each value of x. the variable x is known as independent variable while the variable y is known as dependent variable. The typical example of functions is the relation between the price (y) and the number of items purchased (x) of a certain product. If the price of single item is p the relation above can be expressed as

$y = px$

Definition-1

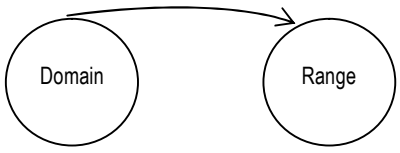
A binary relation f from X and Y with $\text{dom} f = X$ such that for each element $x \in X$ (= domf), there corresponds exactly one (unique) element $y \in$ range f such that $(x, y) \in f$ is called a function from X into Y.

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In this case we can write

$f : X \longrightarrow Y$

(Read f as a function from X into Y)



X Y

This implies $f(x) = y$ for some $x \in X$ and $y \in Y$.

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Definition-2

A function y = f(x) is a rule that assigns for each value of the independent variable x a unique value of the dependent variable y.

Domain and Range:

For a function y=f(x), the set of values of the independent variable x for which the output y is a real number, is known as the domain of f(x). The set of values for the dependent variable y is called as the range of f(x).

Example 1: For the function, y=f(x)=2x

As all real values of x assigns a real value to y, so the domain.

Domain: D = {x|x∈IR} and Range: R = {y|y∈IR}

Example 2: For the function $y=g(x)=x^2$

The domain is the set of real numbers, while the range is the set of non–negative real numbers as x^2 cannot be negative for any real value x.

Domain: D = {x|x∈IR} and Range: R = {y|y ≥ 0}

Compound function:

A function that defined by more than one equation is called a compound function. Compound functions arises in situations where the input–output relation changes for different values of inputs such as the case where price of some product changes with number of items.

For example; let the price of a certain pen is 5 for the first 100 pens and 4 for the number in excess of 100, the function describing the relation between number of pens(x) and price(y) can be expressed as

$$y = \begin{cases} 5x & x \le 100 \\ 4x + 100 & x > 100 \end{cases}$$

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Modulus Function or Absolute value function

A function f: x ⟶ y defined by f(x) = |x| for all x ∈ X is called an absolute value function and |x| is the absolute value of x.

$$\text{Where } |x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$

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Domain of absolute function is the set of all real numbers and range is the set of non-negative real numbers.

Example:

a) $f(x) = |x|$

Find the domain and range of above functions and draw their graphs.

Solution:

For $f(x)=|x|$, the function f(x) can be written as

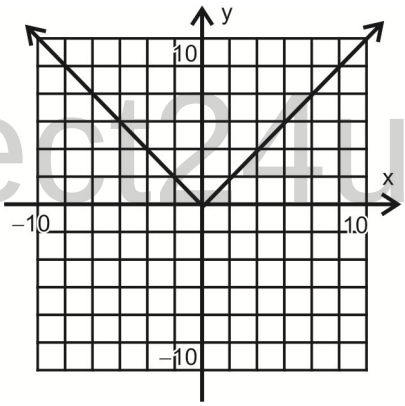
$$f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$y = f(x) = |x| \rightarrow \begin{matrix} \text{Domain} : D = \{x \mid x \in R\} \\ \text{Range} : R = \{y \mid y \geq 0\} \end{matrix}$$

To draw the graph of f(x) we take some x-y pairs.

x	–3	–2	–1	0	1	2	3
f(x)=y=|x|	3	2	1	0	1	2	3

We plot the points (–3,3), (–2,2), (–1,1), (0,0), (1,1), (2,2), (3,3) and draw a continuous line through these points to get the required group.



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One-to-One Function:

A function f: x→y is said to be one-to-one function, if distinct element of x have distinct image is y.

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For example; if x_1 and x_2 are distinct elements of x then $f(x_1) \neq f(x_2)$ in y. Equivalently if $f(x_1)=f(x_2)$, then $x_1 = x_2$.

Example:

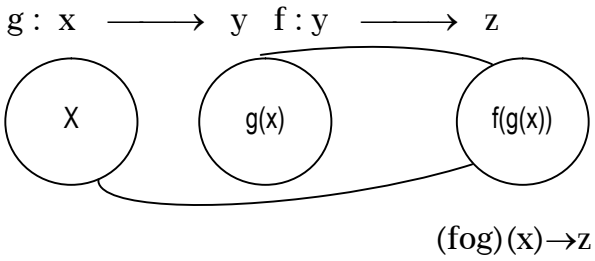
Function $y = f(x) = 2x$ is one-to-one function as every value y corresponds to a unique value x.

Function $y = f(x) = x^2$ is not one-to-one function as $y = 1$ corresponds to two values of x i.e. $x = \pm 1$

Composite Function:

If g is a function with domain x and range y and f is a function with domain y and range z then the composition of f and g is defined as

$(fog)(x) = f(g(x))$



We have $x \to y \to z$

Inverse of a function:

The inverse of a function $y=f(x)$ is a function $x=f^{-1}(y)$. The domain of a function f(x) is the range of its inverse function while the range of f(x) is the domain of the inverse function.

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Domain: f^{-1} = Range f

Range: f^{-1} = Domain f

Example:

For function $y=f(x) = 2x+2$.

Its inverse function is $x=2y+2$, i.e. $f^{-1}(x) = y = \frac{x-2}{2}$

Rule1: For a function $y=f(x)$, the inverse function is the function $x=f^{-1}(y)$.

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Rule2: To find the inverse function of a function y=f(x), inter-change the variables x and y and write resultant expression in terms of y.

e.g. $f(x)=2x+3$

this can be written as $y=2x+3$, interchanging x and y, we get $x=2y+3$, after rearranging in terms of y.

$2y= x-3$

and $y=\frac{x-3}{2}$ which is the required inverse function, thus

$f^{-1}(x)=\frac{x-3}{2}$

EXERCISE 1.1

Q1. Identify the independent and dependent variables for the following problems.

a) P = 64d

b) F(c) = $\frac{9}{5}$c +32

c) C(F) = $\frac{5}{9}$(F – 32)

d) s =f(r, θ) = r.θ

e) F=θ(m, a) = ma

f) SA = $\theta(\ell, w, h) = 2\ell w + 2\ell h + 2wh$

Solution:

No	Question	Independent Variables	Dependent Variables
a	P = 64d	d	P
b	F(c) = $\frac{9}{5}$c + 32	c	F
c	C(F)= $\frac{5}{9}$(F – 32)	F	C
d	S = f(r, θ) = r.θ	r, θ	S
e	F = θ(m_1a) = ma	m, a	F
f	SA = (l, w, h) = 2lw+2lh+2wh	l, w, h	SA

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Q2. Evaluate the following functions for the indicated independent variable.

a) $\mathbf{f(x)=3x^2+7x-5;}$

$\mathbf{f(3), f(-4), f(a+h)}$

Solution:

Putting x = 3

$f(3)=3(3)^2+7(3)-5=0$

$f(3)= 27 + 21 - 5$

$f(3)= 43$

Putting x = –4

$f(-4)=3(-4)^2+7(-4)-5$

$f(-4)=48-28-5$

$f(-4)=15$

Putting x = (a + h)

$f(a+h)=3(a+h)^2+7(a+h)-5$

$f(a+h)=3a^2+6ah+3h^2+7a+7h-5$

b) $\mathbf{f(t)=\frac{t+5}{t-3}, f(2), f(7.4), f(-3.7)}$

Solution:

Putting t = 2

$f(2) = \frac{2+5}{2-3} = \frac{7}{-1}$

$f(2) = -7$

Putting t = 7.4

$f(7.4) = \frac{7.4+5}{7.4-3}$

$f(7.4) = \frac{12.4}{4.4}$

$f(7.4) = 2.8$

Putting t = –3.7

$f(-3.7) = \frac{-3.7+5}{-3.7-3}$

$f(-3.7) = \frac{1.3}{-6.7}$

$f(-3.7) = -0.19$

c) $\mathbf{g(R)=\frac{R^2-R+6}{R-3};}$

$\mathbf{g(2), g(3), g\left(\frac{3}{8}\right)}$

Solution:

Putting R = 2

$g(2) = \frac{2^2-2+6}{2-3}$

$g(2) = \frac{8}{-1}$

$g(2) = -8$

Putting R = 3

$g(3) = \frac{3^2-3+6}{3-3}$

$g(3) = \frac{12}{0}$

$g(3) = \infty$

Putting R = $\frac{3}{8}$

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$$g\left(\frac{3}{8}\right)=\frac{\left(\frac{3}{8}\right)^2-\frac{3}{8}+6}{\frac{3}{8}-\frac{3}{1}}$$

$$g\left(\frac{3}{8}\right)=\frac{\frac{9}{64}-\frac{3}{8}+6}{\frac{3-24}{8}}$$

$$g\left(\frac{3}{8}\right)=\frac{\frac{9-24+384}{64}}{\frac{-21}{8}}$$

$$g\left(\frac{3}{8}\right)=\frac{369}{64}\times\frac{8}{-21}$$

$$g\left(\frac{3}{8}\right)=-2.1964$$

d) **$f(t)=3t^2+2t-\sqrt{t}$;**

$f(3.217), f(5.613), f(\pi)$

Solution: Putting t = 3.217

$f(3.217)=3(3.217)^2+2(3.217)-\sqrt{3.217}$

$f(3.217) = 31.0473+6.434-1.7936$

$f(3.217) = 35.6877$

Putting t = 5.613

$f(5.613)=3(5.613)^2+2(5.613)-\sqrt{5.613}$

$f(5.613)= 94.5173+11.226-2.3692$

$f(5.613)= 103.3741$

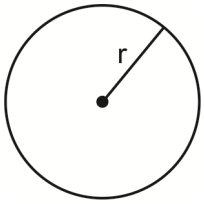
Putting $t = \pi$

$f(\pi) = 3\pi^2+2\pi-\sqrt{\pi}$

$f(\pi) = 29.60+6.2832-1.7724$

$f(\pi) = 34.111$

Q3. **The circumference of a circle is given by $C(r)=2\pi r$, where r is the length of the radius (see the graph). Find**



a) **Solution:**

Putting r = 2.34 in

$C(2.34 \text{ in.}) = 2\pi(2.34 \text{ in})$

$C(2.34 \text{ in.}) = 14.70 \text{ in}$

b) **Solution:**

Putting r = 6.41

$C(6.41\text{m})=2\pi(6.41 \text{ m})$

$C(6.41\text{m})= 40.27 \text{ in}$

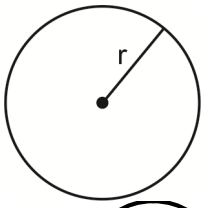
c) **Solution:**

Putting r = $\frac{5}{11}$

$C\left(\frac{5}{11}\text{ in.}\right)=2\pi\left(\frac{5}{11}\text{ in.}\right)$

$C\left(\frac{5}{11}\text{ in.}\right)= 2.856 \text{ in.}$

Q4. **The area of a circle is given by $A(r)=\pi r^2$, where r is the length of the radius (see the graph). Find;**



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a) A(2.34 in.)

Solution:

Putting t = 2.34

$A(2.34\text{ in.}) = \pi(2.34\text{ in.})^2$

$= 17.20\text{ in}^2$

b) A(6.41 in.)

Solution: $A(6.41\text{ m}) = \pi(6.41\text{ m})^2$

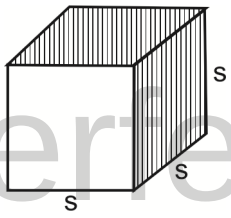
$= 129.08\text{m}^2$

c) $A\left(\frac{5}{11}\text{ in.}\right)$

Solution: $A\left(\frac{5}{11}\text{ in.}\right) = \pi\left(\frac{5}{11}\text{ in.}\right)^2$

$= 0.65\text{ in}^2$

Q5. The total surface area of a cube is given by the function $f(s) = 6s^2$, where s is the length of the side of the cube (see the graph). Find



a) f(3.75m)

Solution: $f(3.75\text{ m}) = 6(3.75\text{ m})^2$

$= 84.375\text{ m}^2$

b) f(6.05 in.)

Solution: $f(6.05\text{ in.}) = 6(6.05\text{ in.})^2$

$= 219.615\text{ in}^2$

$= 220\text{ in}^2$

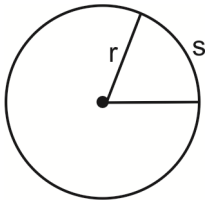
c) f(13.42 mm)

Solution: f(13.42mm)

$= 6(13.42\text{mm})^2$

$= 1080.5781\text{mm}^2$

Q6. The measure of the angle θ in radius is given by $\theta = f(s, r) = \frac{s}{r}$, where s is the length of arc determined by ∠θ and r is the length of the radius of the circle (see the graph). Find



a) f(4.71, 3)

Solution: $\theta = f(4.71, 3) = \frac{4.71}{3}$

$= 1.57\text{ rad}$

b) f(15.71, 5)

Solution: $\theta = f(15.71, 5) = \frac{15.71}{5}$

$= 3.142\text{rad}$

Q7. If $f(x) = \begin{cases} x - 3 & \text{for } x < 0 \\ 2x + 5 & \text{for } x \geq 0 \end{cases}$ then find out

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a) f(–1) = ?

Solution:

$f(x) = x - 3$

Put $x = -1$

$f(-1) = -1-3$

$f(-1) = -4$

b) f(0) = ?

Solution:

$f(x) = 2x + 5$

Put $x = 0$

$f(0) = 2(0) + 5$

$f(0) = 5$

c) f(1) = ?

Solution:

$f(x) = 2x + 5$

Put $x = 1$

$f(1) = 2(1) + 5$

$f(1) = 7$

Q8. If $f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{for } x \neq 2 \\ 4 & \text{for } x = 2 \end{cases}$ **then find out**

a) f(0) = ?

Solution: $f(x) = \frac{x^2-4}{(x-2)}$

Put $x = 0$

$f(0) = \frac{(0)^2-4}{0-2} = \frac{-4}{-2}$

$f(0) = 2$

b) f(2)

Solution: $f(2) = 4$

c) f(4)

Solution: $f(x) = \frac{x^2-4}{(x-2)}$

Put $x = 4$

$f(4) = \frac{4^2-4}{4-2}$

$f(4) = \frac{16-4}{2}$

$f(4) = \frac{12}{2}$

$f(4) = 6$

Q9. Indicate whether each table specifies a function y=f(x)

i) **Domain Range**

$\begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

Solution: This is a function as there is no repetition in domain. Every independent variable is mapped to a unique value of dependent variable.

ii) **Domain Range**

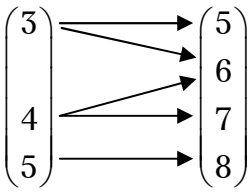
$\begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} \longrightarrow \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix}$

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Solution: This is a function as every independent variable is mapped to a unique value of dependent variable.

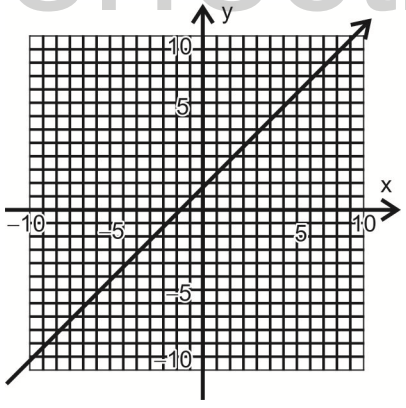
c) **Domain** **Range**



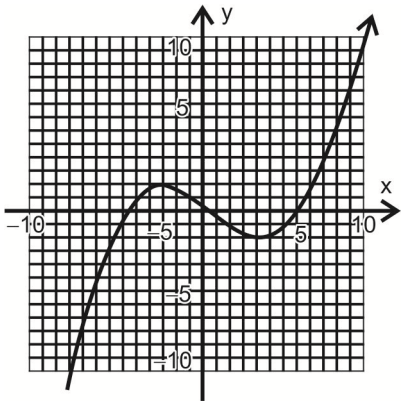
Solution: This is "not a function" as independent variable 3 is mapped to two values of dependent variable 5 and 6, similarly independent variable 4 is mapped to 6 and 7.

Q10. Indicate whether each graph specifies a function y=f(x)

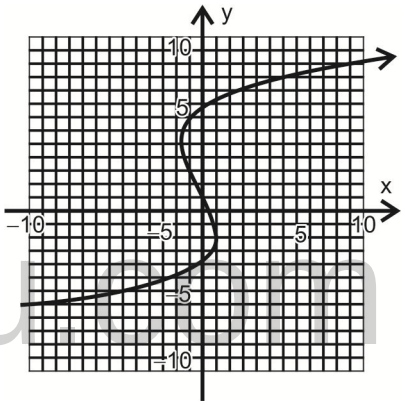
Rule: The graph a function satisfies vertical line test which means that a vertical line will always pass through the graph at only one point, at the most.



a) The given graph represents a "function" as any vertical line passes through the graph only once.



b) The graph represents a "function" as any vertical line crosses it at only one point.



c) The graph shown doesn't represent a function as the vertical line at x=0 (y –axis) crosses the graph at three point which mean that one value of independent variable (x=0) is mapped to

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three values of dependent variable.

Q11. Determine the Domain and Range of the following function.

a) $y = 3x + 4$

Solution:

Domain=R(set of real numbers)

Every real value x is assigned a real value y.

Domain: D = {x|x∈IR}

Range = (set of real numbers)

Every real number can be obtained from the given function.

Range: R = {y|y∈IR}

b) $f(t) = t^2 + 5$

Solution:

Domain=R(set of real numbers)

Every real number value of x is assigned a real value y.

Domain: D = {x|x∈IR}

Range=[5, ∞), real numbers between 5 and ∞ including 5.

The t^2 always results in a non–negative number with a minimum value of 0 at t=0, resulting in the minimum value of function f(0)=5

Range: R = {y|y ≥ 0}

c) $SA=f(r) = 4\pi r^2$

Solution:

Domain: D = {r|r∈IR}

Range = [0, ∞), real numbers between 0 and ∞, including 0.

Range: R = {f(r), f(r) ≥ 0}

Q12. Find the composite function f[g(x)] and g[f(x)] of the following function.

a) $f(x)=x^2+1$, $g(x) =2x$

Solution:

$f(x)=x^2+1$

$f[g(x)]=[g(x)]^2 + 1$

i) $f[g(x)]= f(2x)$

$f[g(x)]= (2x)^2+1$

$f[g(x)]= 4x^2+1$

$g[f(x)]=2[f(x)]^2$

ii) $g[f(x)]= g[x^2+1]$

$g[f(x)]= 2(x^2+1)$

$g[f(x)]= 2x^2+2$

b) $f(x) = \sin x$, $g(x) =1- x^2$

Solution:

i) $f[g(x)]= f(1 - x^2)$

$f[g(x)]= \sin(1 - x^2)$

ii) $g[f(x)] = g[\sin x]$

$g[f(x)] = 1-(\sin x)^2$

$g[f(x)] = 1-\sin^2 x$

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$g[f(x)] = \cos^2 x$

c) $\mathbf{f(t) = \sqrt{t}, \quad g(t) = t^2}$

Solution:

i) $f[g(t)] = f(t^2)$

$f[g(t)] = \sqrt{t^2}$

$f[g(t)] = t$

ii) $g[f(t)] = g\left(\sqrt{t}\right)$

$g[f(t)] = \left(\sqrt{t}\right)$

$g[f(t)] = t$

d) $\mathbf{f(u) = \frac{u-1}{u+1}, g(x) = \frac{u+1}{1-u}}$

Solution:

i) $f[g(u)] = f\left[\frac{u+1}{1-u}\right]$

$f[g(u)] = \left[\frac{\frac{u+1}{1-u} - 1}{\frac{u+1}{1-u} + 1}\right]$

$f[g(u)] = \left[\frac{\frac{u+1-1+u}{(1-u)}}{\frac{u+1+1-u}{(1-u)}}\right]$

$f[g(u)] = \frac{2u}{2}$

$f[g(u)] = u$

ii) $g[f(u)] = g\left[\frac{u-1}{u+1}\right]$

$g[f(u)] = \left[\frac{\frac{u-1}{u+1} + \frac{1}{1}}{\frac{1}{1} - \frac{u-1}{u+1}}\right]$

$g[f(u)] = \left[\frac{\frac{u-1+u+1}{u+1}}{\frac{u+1-u+1}{u+1}}\right]$

$g[f(u)] = \frac{2u}{2}$

$g[f(u)] = u$

e) $\mathbf{f(x) = \sin x, \ g(x) = 2x+3}$

Solution:

i) $f[g(x)] = f(2x+3)$

$f[g(x)] = \sin(2x+3)$

ii) $g[f(x)] = f(\sin x)$

$g[f(x)] = 2\sin x + 3$

f) $\mathbf{f(x) = \frac{1}{x}, \quad g(x) = \tan x}$

Solution:

i) $f[g(x)] = f(\tan x)$

$f[g(x)] = \frac{1}{\tan x}$

$f[g(x)] = \cot x$

ii) $g[f(x)] = g\left(\frac{1}{x}\right)$

$g[f(x)] = \tan\frac{1}{x}$

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Q13. Determine the inverse function of each of the following functions.

a) $y=f(x) = x+5$

Method-I

Solution:

Given that

$y = f(x) = x+5$

$y = x+5$

Interchanging x and y, we get

$x = y+5$

arranging in terms of y,

$y = x - 5$

Required inverse function

i.e. $f^{-1}(x) = x - 5$

Method-II

a) $y=f(x) = x+5$

Solution:

$y=f(x) \Rightarrow x=f^{-1}(y)$

Step-I

$y = x + 5$

then $x = y - 5$

Step-II

Replace x by $f^{-1}(y)$

$f^{-1}(y) = y - 5$

Step-III

Replace y by x

$f^{-1}(x) = x - 5$

b) $y = f(x) = 2x + 7$

Method-I

Solution:

Given that

$y = f(x) = 2x + 7$

$y = 2x + 7$

Interchanging x and y, we get

$x = 2y + 7$

Rearranging in terms of y

$2y = x - 7$

$y = \frac{x-7}{2}$

Required inverse function

i.e. $f^{-1}(x) = \frac{x-7}{2}$

Method-II

$y = f(x) = 2x + 7$

Solution:

$y = f(x) \Rightarrow x = f^{-1}(y)$

Step-I

$y = 2x + 7$

$\Rightarrow \quad y - 7 = 2x$

$\Rightarrow \quad x = \frac{y-7}{2}$

Step-II

Replace x by $f^{-1}(y)$

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$$f^{-1}(y) = \frac{y-7}{2}$$

Step-III

Replace y by x

$$f^{-1}(x) = \frac{x-7}{2}$$

c) $y = f(x) = 2(x-4)$

Method-I

Solution:

Given that,

$$y = f(x) = 2(x-4)$$

$$y = 2(x-4)$$

Interchanging x and y, we get

$$x = 2(y-4)$$

Rearranging in terms of y,

$$x = 2y - 8$$

$$2y = x + 8$$

$$y = \frac{x+8}{2}$$

Which is the Required inverse function

i.e. $f^{-1}(x) = \frac{x+8}{2}$

Method-II

$y = f(x) = 2x + 7$

Solution:

$$y = f(x) \Rightarrow x = f^{-1}(y)$$

Step-I

$$y = 2(x-4)$$

$$\Rightarrow \quad y = 2x - 8$$

$$\Rightarrow \quad y + 8 = 2x$$

$$\Rightarrow \quad x = \frac{y+8}{2}$$

Step-II

Replace x by $f^{-1}(y)$

$$f^{-1}(y) = \frac{x+8}{2}$$

Step-III

Replace x by y

$$f^{-1}(x) = \frac{y+8}{2}$$

d) $y = f(x) = \frac{x+4}{2}$

Method-I

Solution:

Given that

$$y = f(x) = \frac{x+4}{2}$$

$$y = \frac{x+4}{2}$$

Interchange x and y, we get

$$x = \frac{y+4}{2}$$

Rearrange in terms of y

$$y + 4 = 2x$$

$$y = 2x - 4$$

Which is the required inverse function

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i.e. $f^{-1}(x) = 2x - 4$

Method-II

$$y = f(x) = \frac{x+4}{2}$$

Solution:

$y = f(x) \Rightarrow x = f^{-1}(y)$

Step-I

$$y = \frac{x+4}{2}$$

$\Rightarrow \quad 2y = x + 4$

$\Rightarrow \quad x = 2y - 4$

Step-II

Replace x by $f^{-1}(y)$

$f^{-1}(y) = 2y - 4$

Step-III

Replace y by x

$f^{-1}(x) = 2x - 4$

Q14. Graph each of the following absolute functions.

a) $y = |x - 4|$

Solution:

By the definition of mod. function can be written as

$y = +(x - 4)$, for $x - 4 \geq 0$ i.e. $x \geq 4$

and $y = -(x-4) = 4-x$ for $4-x < 0$, i.e. $x < 4$

Hence the given function reads

$$y = \begin{cases} x - 4 & \text{for } x \geq 4 \\ 4 - x & \text{for } x < 4 \end{cases}$$

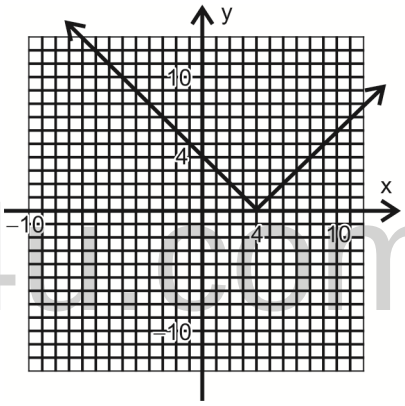
Thus the function has two branches namely

(a) When $x \geq 4$, the function is $y = x - 4$. To draw the graph of this function, we tabulate some of its value as follows.

x	4	5	6	7	8	9	10
$y=x-4$	0	1	2	3	4	5	6

(b) When $x < 4$, the function is $y = 4 - x$. To draw the graph of this function, we tabulate some of its value as follows.

x	3	2	1	0	–1	–2
$y=4-x$	1	2	3	4	5	6



b) $y = |-4 - x|$

Solution:

By the definition of mod. function can be written as

$y = +(-x - 4)$, for $-4 - x \geq 0$ i.e. $x \leq -4$

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and $y=-(-x-4)$ for $-4-x<0$, i.e. $x>-4$

Hence the given function reads

$$y=\begin{cases}-x-4 & \text{for } x\le -4\\ x+4 & \text{for } x<-4\end{cases}$$

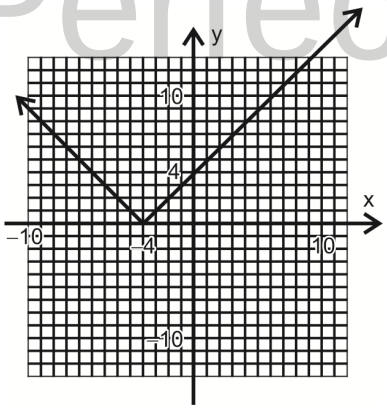
Thus the function has two branches namely

(a) When $x \le -4$, the function is $y = -4-x$. To draw the graph of this function, we tabulate some of its value as follows.

x	–4	–5	–6	–7	–8	–9
y=–4–x	0	1	2	3	4	5

(b) When $x > -4$, the function is $y = 4+x$. To draw the graph of this function, we tabulate some of its values as follows.

x	–3	–2	–1	0	1	2	3
y=x+4	1	2	3	4	5	6	7



c) $y=|2x+5|$

Solution:

By the definition of mod. function can be written as

$y=+(2x+5)$, for $2x+5\ge 0$ i.e. $x \ge -5/2$

and $y=-(2x+5)$ for $2x+5<0$, i.e. $x<-5/2$

Hence the given function reads

$$y=\begin{cases}2x+5 & \text{for } x\ge -5/2\\ -(2x+5) & \text{for } x<-5/2\end{cases}$$

Thus the function has two branches namely

(a) When $x \ge -5/2$, the function is $y = 2x+5$. To draw the graph of this function, we tabulate some of its value as follows.

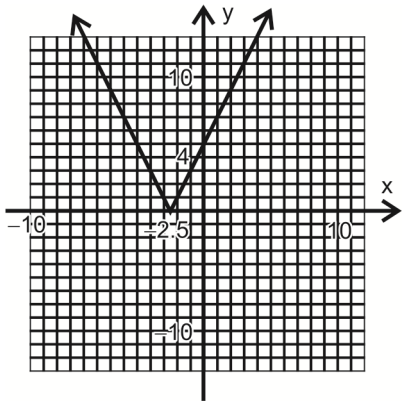
x	$-5/2$	–2	–1	0	1
y=2x+5	0	1	3	5	7

(b) When $x < -5/2$, the function is $y = -(2x+5)$. To draw the graph of this function, we tabulate some of its values as follows.

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x	–3	–4	–5	–6	–7	–8
y=–(2x+5)	1	3	5	7	9	11



d) $y = -|x|$

Solution:

By the definition of mod. function can be written as

$y=-(x)$ when $x \geq 0$

and $y=-(-x)=x$ when $x < 0$

Hence the given function reads

$$y = \begin{cases} x, & x \geq 0 \\ -x & x < 0 \end{cases}$$

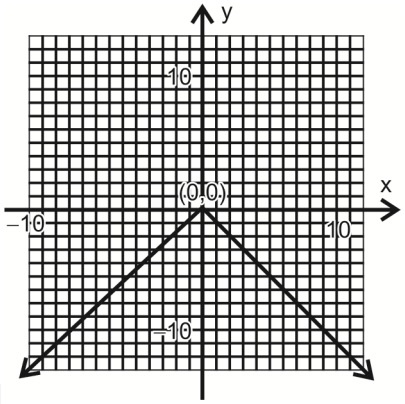
Thus the function has two branches namely

(a) When $x \geq 0$, the function is $y=-x$. To draw the graph of this function, we tabulate some of its value as follows.

x	0	1	+2	3	4	+5
y=–x	0	–1	–2	–3	–4	–5

(b) When $x < 0$, the function is $y = x$. To draw the graph of this function, we tabulate some of its values as follows.

x	–1	–2	–3	–4	–5
y=x	–1	–2	–3	–4	–5



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Polynomial function:

A function of the form

$f(x)=P_n(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots\ldots+a_1x^1+a_0$

With non-negative n is called a polynomial function, of degree n.

If $n=0$, $f(x) = a_0$ is called a constant function

If $n=1$, $f(x) = a_1x+a_0$ is called linear function

If $n=2$, $f(x) = a_2x^2+a_1x+a_0$ is called quadratic function

If $n=3$, $f(x) = a_3x^3+a_2x^2+a_1x+a_0$ is called cubic function

Algebraic function:

A function made of the algebraic operation such as addition, subtraction, multiplication division or roots of polynomial is called an algebraic function.

Example:

$$f(x) = \frac{3x+2}{x+1}$$

Trigonometric function:

Function containing sine, cosine, tangent, cotangent, secant and cosecant of variables is called trigonometric function.

Example is $y=\sin x + \cos x$

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Transcendental function:

Function which are not algebraic are called transcendental functions for example, $f(x)=x + \tan x$.

Exponential function:

An equation of the form $f(x) = b^x$, $b > 0$, $b \neq 1$, b is a positive constant define an exponential function for each different constant b, called the base.

Example, $y=f(x)=10^x$

Exponents' laws:

For real numbers a and b,

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1) $a^x a^y = a^{x+y}$

2) $\frac{a^x}{a^y} = a^{x-y}$

3) $\left(a^x\right)^y = a^{xy}$

4) $(ab)^x = a^x b^x$

5) $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

6) $a^x = a^y$ **if and only if** $x=y$

7) **For** $x\neq 0$, $a^x = b^x$ **if and only if** $a=b$

Logarithmic function:

The inverse of an exponential function is called a logarithmic function. For $b > 0$ and $b \neq 1$ the logarithmic function is:

$y = \log_b x$ which is equivalent to $x = b^y$

Common logarithms:

Common logarithms are logarithms with base 10.

Logarithm to the base 10

$y = \log_{10} x$ means $10^y = x$

Natural logarithms:

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Natural logarithms are logarithms with base e.

Logarithm of the natural base e.

$y = \ln x$ means $e^y = x$

Properties of logarithm:

If b, M and N are positive real number, $b\neq 1$. Also for positive real numbers p and x.

1. $\log_b 1 = 0$
2. $\log_b b = 1$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x,\ x>0$

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5. $\log_b MN = \log_b M + \log_b N$ 6. $\log_b \frac{M}{N} = \log_b M - \log_b N$

7. $\log_b M^p = p\log_b M$ 8. $\log_b M = \log_b N, M = N$

Hyperbolic functions:

The following function are called the hyperbolic functions.

$\sinh x = \frac{e^x - e^{-x}}{2}$, read as sine hyperbolic x

$\cosh x = \frac{e^x + e^{-x}}{2}$, $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

Like the trigonometric identities, these identities hold for hyperbolic functions.

$\cosh^2 x - \sinh^2 x = 1$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

EXERCISE 1.2

Q1.Simplify the following functions.

a) $\left(4^{3x}\right)^{2y}$

Solution: $\left(4^{3x}\right)^{2y} = 4^{3x \times 2y}$

$\left(4^{3x}\right)^{2y} = 4^{6xy}$

b) $10^{3x-1} . 10^{4-x}$

Solution: $10^{3x-1} . 10^{4-x} = 10^{3x-1+4-x}$

$= 10^{2x+3}$

c) $\frac{e^{x-3}}{e^{x-4}}$

Solution: $\frac{e^{x-3}}{e^{x-4}} = e^{(x-3)-(x-4)}$

$= e^{x-3-x+4}$

$= e^1$

$= e$

d) $\frac{e^x}{e^{3-x}}$

Solution: $\frac{e^x}{e^{3-x}} = e^{x-(3-x)}$

$= e^{x-3+x}$

$= e^{2x-3}$

e) $\left(2e^{1.2t}\right)^3$

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Solution: $\left(2e^{1.2t}\right)^3 = (2)^3.\left(e^{1.2t}\right)^3$

$= 8.e^{1.2t\times 3}$

$= 8e^{3.6t}$

f) $\left(3e^{-1.4x}\right)^2$

Solution: $\left(3e^{-1.4x}\right)^2 = (3)^2\left(e^{-1.4x}\right)^2$

$= 9.e^{-1.4x\times 2}$

$= 9e^{-2.8x}$

Q.2 Solve the following equations

a) $10^{2-3x} = 10^{5x-6}$

Solution: Given $10^{2-3x} = 10^{5x-6}$

To find; $x = ?$

$10^{2-3x} = 10^{5x-6}$

$\Rightarrow$ $2 - 3x = 5x - 6$

$\Rightarrow$ $2 + 6 = 5x + 3x$

$\Rightarrow$ $8x = 8$

$x = 1$

b) $5^{3x} = 5^{4x-2}$

Solution: Given $5^{3x} = 5^{4x-2}$

To find $x = ?$

$\Rightarrow$ $5^{3x} = 5^{4x-2}$

$\Rightarrow$ $5^{3x} = 5^{4x-2}$

$\Rightarrow$ $3x = 4x - 2$

$x = 2$

c) $4^{5x-x^2} = 4^{-6}$

Solution: Given $4^{5x-x^2} = 4^{-6}$

To find $x = ?$

$4^{5x-x^2} = 4^{-6}$

$4^{5x-x^2} = 4^{-6}$

$(5x - x^2) = -6$

$\Rightarrow$ $5x - x^2 = -6$

$\Rightarrow$ $x^2 - 5x - 6 = 0$

$\Rightarrow$ $x^2 - 6x + x - 6 = 0$

$\Rightarrow$ $(x - 6)(x + 1) = 0$

$\Rightarrow$ $x = 6, x = -1$

$x = \{6, -1\}$

d) $7^{x^2} = 7^{2x+3}$

Solution: $7^{x^2} = 7^{2x+3}$

To find $x = ?$

$7^{x^2} = 7^{2x+3}$

$7^{x^2} = 7^{2x+3}$

$x^2 = (2x + 3)$

$\Rightarrow$ $x^2 = 2x + 3$

$\Rightarrow$ $x^2 - 2x - 3 = 0$

$\Rightarrow$ $x^2 - 3x + x - 3 = 0$

$\Rightarrow$ $(x - 3)(x + 1) = 0$

$x = \{3, -1\}$

e) $5^3 = (x + 2)^3$

Solution: Given $5^3 = (x + 2)^3$

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To find x = ?

$5^3 = (x+2)^3$

Taking cube on both sides

$(5^3)^{\frac{1}{3}} = [(x+2)^3]^{\frac{1}{3}}$

$5 = x + 2$

$5 - 2 = x$

$x = 3$

f) $\mathbf{(1-x)^5=(2x-1)^5}$

Solution: Given that

$(1-x)^5=(2x-1)^5$

To find x = ?

As components are same, so the basis will be equal i.e.

$1 - x = 2x - 1$

$-x - 2x = -1 - 1$

$-3x = -2$

$x = \frac{2}{3}$

g) $\mathbf{(x-3)e^x = 0}$

Solution: Given; $(x-3)e^x = 0$

To find x = ?

As $(x-3)e^x = 0$

Then $x-3=0$ or $e^x \neq 0$

For, $x-3=0 \Rightarrow x = 3$

Thus, $x = 3$

h) $\mathbf{2xe^{-x}=0}$

Solution: Given $2xe^{-x}=0$

To find x = ?

As $2xe^{-x}=0$

Thus $2x = 0$ or $e^{-x} \neq 0$

For $2x = 0 \Rightarrow x = 0$

Thus $x = 0$

i) $\mathbf{3xe^{-x}+x^2e^{-x}=0}$

Solution: To find x = ?

$\Rightarrow 3xe^{-x}+x^2e^{-x}=0$

$\Rightarrow (3x+x^2)e^{-x}=0$

$\Rightarrow 3x+x^2=0$ or $e^{-x} \neq 0$

For $x(3+x)=0$

$x = \{0, -3\}=0$

j) $\mathbf{x^2e^x - 5xe^x = 0}$

Solution: To find x = ?

$\Rightarrow x^2e^x - 5xe^x = 0$

$\Rightarrow e^x(x^2 - 5x) = 0$

$\Rightarrow (x^2 - 5x)=0$ or $e^{-x} \neq 0$

For, $x^2 - 5x = 0$

$x(x-5)=0$

$x = 0$ and $x = 5$

thus $x = \{0, 5\}$

Q3. Rewrite in the equivalent exponential form, the following logarithmic functions.

a) $\mathbf{\log_3 27 = 3}$

b) $\mathbf{\log_2 32 = 5}$

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c) $\log_{10}1 = 0$

d) $\log_e 1 = 0$

e) $\log_4 8 = \frac{3}{2}$

f) $\log_9 27 = \frac{3}{2}$

Solution:

S.No.	Logarithmic form	Exponential form
a	$\log_3 27 = 3$	$3^3 = 27$
b	$\log_2 32 = 5$	$2^5 = 32$
c	$\log_{10} 1 = 0$	$10^o = 1$
d	$\log_e 1 = 0$	$e^o = 1$
e	$\log_4 8 = \frac{3}{2}$	$4^{\frac{3}{2}} = 8$
f	$\log_9 27 = \frac{3}{2}$	$9^{\frac{3}{2}} = 27$

Q4. Rewrite in the equivalent logarithmic form, the following exponential functions.

a) $49 = 7^2$

b) $36 = 6^2$

c) $8 = 4^{\frac{3}{2}}$

d) $9 = 27^{\frac{2}{3}}$

e) $A = b^u$

f) $M = b^x$

Solution:

S.No.	Exponential form	Logarithmic form
a	$49 = 7^2$	$\log_7 49 = 2$
b	$36 = 6^2$	$\log_6 36 = 2$
c	$8 = 4^{\frac{3}{2}}$	$\log_4 8 = \frac{3}{2}$
d	$9 = 27^{\frac{2}{3}}$	$\log_{27} 9 = \frac{2}{3}$
e	$A = b^u$	$\log_b A = u$
f	$M = b^x$	$\log_b M = x$

Q5. Find x, y and b without a scientific calculator use.

a) $\log_3 x = 2$ x = ?

Solution: Given that $\log_3 x = 2$

Writing in the exponential form

$3^2 = x$

$x = 9$

b) $\log_2 x = 2$, x = ?

Solution: Given that $\log_2 x = 2$

Writing in exponential form

$2^2 = x$

$x = 4$

c) $\log_7 49 = y$, y = ?

Solution: Given that $\log_7 49 = y$

Writing in exponential form

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$7^y = 49$

$7^y = 7^2$ equating basis

$y = 2$

d) $\log_b 10^{-4} = -4$

Solution: Given $\log_b 10^{-4} = -4$

Rewriting in exponential form

$10^{-4} = b^{-4}$

For same exponent, bases will be the same.

$b = 10$

e) $\log_{1/3} 9 = y$

Solution:

Re-writing in exponential form,

$\left(\frac{1}{3}\right)^y = 9$

$\left(\frac{1}{3}\right)^y = \left(\frac{1}{3}\right)^{-2}$

For equal bases, exponents are equal i.e. $y = -2$

f) $\log_b 1000 = \frac{3}{2}$

Solution:

Re-writing in exponential form,

$b^{\frac{3}{2}} = 1000$

$b^{\frac{3}{2}} = (10)^3$

$b^{\frac{3}{2}} = \left((100)^{\frac{1}{2}}\right)^3$

$b^{\frac{3}{2}} = (100)^{\frac{3}{2}}$

As exponents are same, so by equating basis

$b = 100$

Q6. Solve the following equations for the unknown x.

a) $\log_b x = \frac{2}{3}\log_b 8 + \frac{1}{2}\log_b 9 - \log_b 6$

Solution:

$\log_b x = \frac{2}{3}\log_b 8 + \frac{1}{2}\log_b 9 - \log_b 6$

$\log_b x = \log_b (8)^{\frac{2}{3}} + \log_b (9)^{\frac{1}{2}} - \log_b 6$

$\log_b x = \log_b 4 + \log_b 3 - \log_b 6$

$\log_b x = \log_b \frac{(4)(3)}{6}$

$x = 2$

b) $\log_b x = \frac{2}{3}\log_c 27 + 2\log_b 2 - \log_b 3$

Solution:

$\log_b x = \frac{2}{3}\log_c 27 + 2\log_b 2 - \log_b 3$

$\log_b x = \log_b 27^{\frac{2}{3}} + \log_b 2^2 - \log_b 3$

$\log_b x = \log_b 9 + \log_b 4 - \log_b 3$

$\log_b x = \log_b \frac{9 \times 4}{3}$

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$\cancel{\log_b} x = \cancel{\log_b} 12$

$x = 12$

c) $\log_b x = \frac{3}{2}\log_b 4 - \frac{2}{3}\log_b 8 + 2\log_b 2$

Solution:

$\log_b x = \frac{3}{2}\log_b 4 - \frac{2}{3}\log_b 8 + 2\log_b 2$

$\log_b x = \log_b 4^{\frac{3}{2}} - \log_b 8^{\frac{2}{3}} + \log_b 2^2$

$\log_b x = \log_b 8 - \log_b 4 + \log_b 4$

$\log_b x = \log_b \frac{8 \times 4}{4}$

$\log_b x = \log_b 8$

$x = 8$

d) $\log_b x + \log_b (x-4) = \log_b 21$

Solution:

$\log_b x + \log_b (x-4) = \log_b 21$

$\log_b (x)(x-4) = \log_b 21$

$\log_b (x^2 - 4x) = \log_b 21$

$\Rightarrow \quad x^2 - 4x = 21$

$\Rightarrow \quad x^2 - 4x - 21 = 0$

By factorization

$\Rightarrow \quad x^2 - 7x + 3x - 21 = 0$

$\Rightarrow \quad (x-7)(x+3) = 0$

$\Rightarrow \quad x = 7$, As $\log(-3)$ = undefined

e) $\log_{10}(x-1) - \log_{10}(x+1) = 1$

Solution:

$\log_{10}(x-1) - \log_{10}(x+1) = 1$

$\log_{10}\frac{(x-1)}{(x+1)} = 1$

$\cancel{\log_{10}}\frac{(x-1)}{(x+1)} = \cancel{\log_{10}} 10$

$\Rightarrow \quad \frac{x-1}{x+1} = \frac{10}{1}$

$\Rightarrow \quad x - 1 = 10x + 10$

$\Rightarrow \quad -11 = 9x$

$\Rightarrow \quad x = -\frac{11}{9}$, no solution as

$\log_{10}\left(-\frac{11}{9}\right)$ = undefined

f) $\log_{10}(x+6) - \log_{10}(x-3) = 1$

Solution:

$\log_{10}(x+6) - \log_{10}(x-3) = 1$

$\log_{10}\left(\frac{x+6}{x-3}\right) = 1$

$\cancel{\log_{10}}\left(\frac{x+6}{x-3}\right) = \cancel{\log_{10}} 10$

$\Rightarrow \quad \frac{x+6}{x-3} = \frac{10}{1}$

$\Rightarrow \quad x+6 = 10x - 30$

$\Rightarrow \quad 36 = 9x$

$x = 4$

Q7. Suppose the sale of a certain product are approximately by

$S(t) = 125 + 83\log(5t+1)$

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Where S(t) is sales in thousand of dollar t years after the product was introduced on the market. Find

a) S(0)=?

Solution:

As $S(t)=125+83\log(5t+1)$

For $S(0)$ put $t = 0$, thus

$S(0)=125+83\log(5(0)+1)$

$S(0)=125+83\log(1)$

$\therefore \quad \log 1 = 0$

$S(0)=125+83(0)$

$S(0)=125$ thousand

b) S(2) = ?

Solution:

As $S(t)=125+83\log(5t+1)$

For $S(2)$ put $t = 2$, this

$S(2)=125+83\log(5(2)+1)$

$S(2)=125+83\log(11)$

$S(2)=125+83(1.04)$

$S(2)=211.44$ thousand

c) S(4) = ?

Solution:

As $S(t)=125+83\log(5t+1)$

For $S(4)$ put $t= 4$, thus

$S(4)=125+83\log(5(4)+1)$

$S(4)=125+83\log(21)$

$S(4)=125+83(1.32)$

$S(4) = 234.74$ thousand

d) S(31) = ?

Solution:

As $S(t)=125+83\log(5t+1)$

For $S(31)$ put $t= 31$, thus

$S(31)=125+83\log(5(31)+1)$

$S(31)=125+83\log(156)$

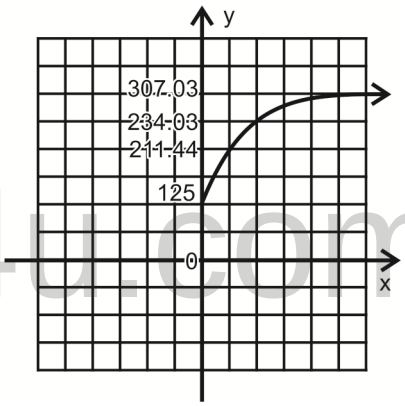
$S(31)=125+83(2.19)$

$S(31)= 307.03$ thousand

e) Draw and interpret the graph.

Solution: From parts (a) to (d) above or from equation.

t	0	2	4	31
S(t)	125	211.44	234.74	307.03



Interpretation of Graph:

As the function is a logarithm function, so the value of function increases as x increases but its rate of increase, decreases as x increases.

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EXERCISE 1.3

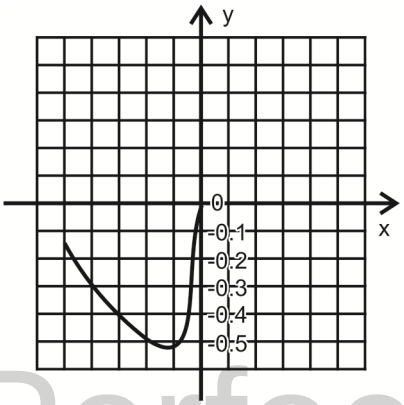
Q1. Using a calculator and point–by–point to plot the following exponential functions:

a) $h(x) = x(2^x)$; [–5, 0]

Solution:

A few pairs of x and y for the given function are as follows

x	–5	–4	–3	–2	–1	0
y	$-\frac{5}{32}$	$-\frac{1}{4}$	$-\frac{3}{8}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0

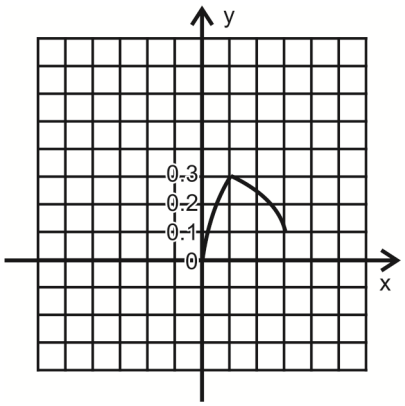


b) $m(x) = x(3^{-x})$; $(0, 3)$

Solution:

Taking few pairs of x and y from given function.

x	0	1	2	3
y	0	$\frac{1}{3} = 0.33$	0.22	0.111

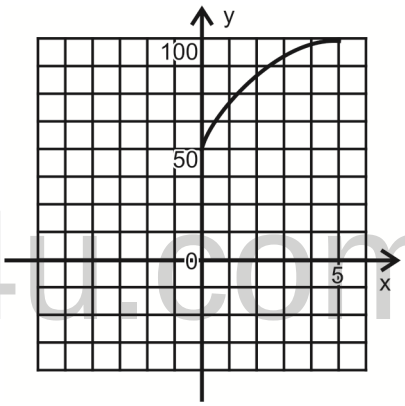


c) $N = \frac{100}{1 + e^{-t}}$; $[0, 5]$

Solution:

Pairs of t and N from the given function are

t	0	1	2	3	4	5
N	50	73.11	88.08	95.26	98.2	99.33



d) $N = \frac{200}{1 + 3e^{-t}}$; $[0, 5]$

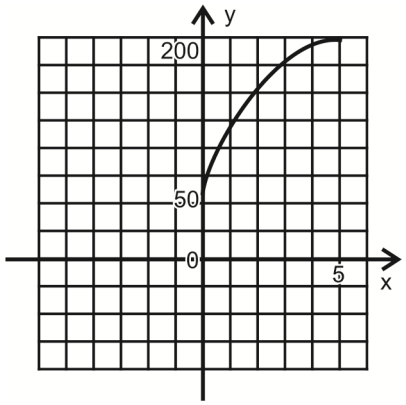
Solution:

Pairs of t and N from the given functions are

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T	0	1	2	3	4	5
N	50	95.08	142.24	174	189.60	196.04

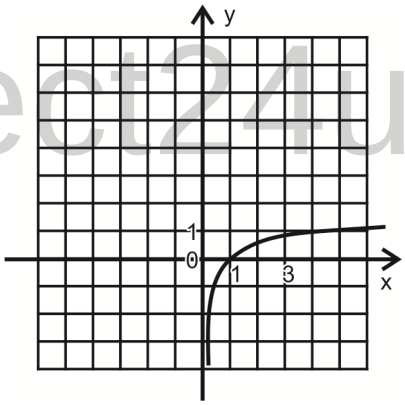


Q2. Using a calculator and point–by–point to plot the following Logarithmic functions:

a) y = lnx

Solution: Taking some x, y pairs for the given function.

x : negative	0	1	e = 2.71	$e^2 = 7.39$	$e^3 = 20.08$
y : Undefined	$-\infty$	0	1	2	3



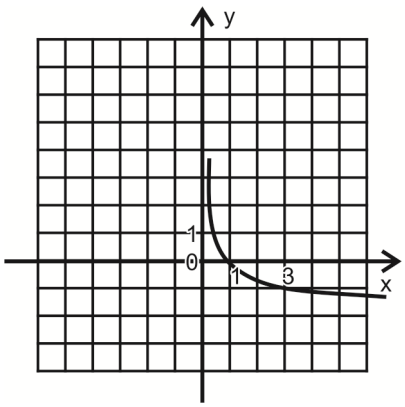
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b) u = –*l*n x

Solution: Taking some x, y pairs for the given function.

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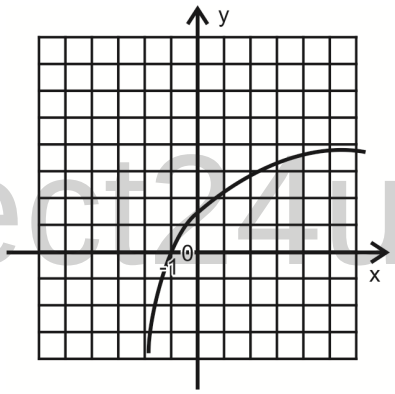
x : negative	0	1	e = 2.71	e^2 = 7.39	e^3 = 20.08
u : Undefined	+ ∞	0	–1	–2	–3



c) $y = 2\,ln(x + 2)$

Solution: Pairs of x and corresponding values of y are given below

x	–2	–1	0	1	2	6
y	– ∞	0	1.39	2.20	2.77	4.16



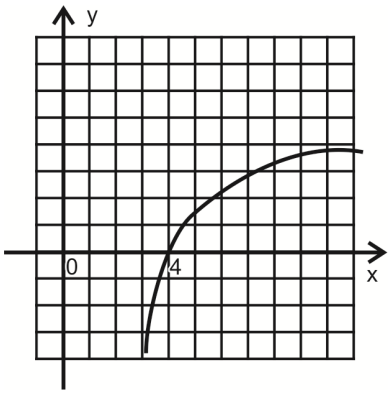
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d) $y = 4\,ln(x - 3)$

Solution: Few pairs of x and y values are

x	3	4	5	6	7	11
y	– ∞	0	2.78	4.40	5.54	8.32

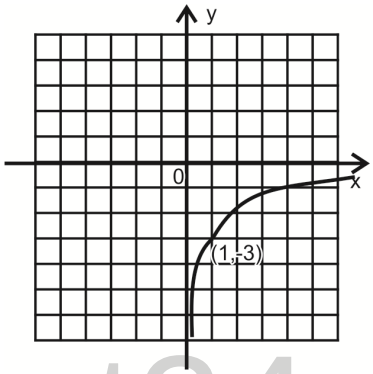
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e) $y = 4\ln x - 3$

Solution: Pairs of x and y values are as follows

x	0	1	2	3	4	5
y	$-\infty$	–3	–0.23	1.39	2.54	3.44



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Q3. Sketch the following parametric curves;

a) $(x(t), y(t)) = (3 - t, 2t)$

Solution: First we eliminate the parameter t. To do so we use the following relation from given parametric form.

$x = 3 - t$ .....(i)

$y = 2t$ .....(ii)

From (i) $t = 3 - x$; put in (ii)

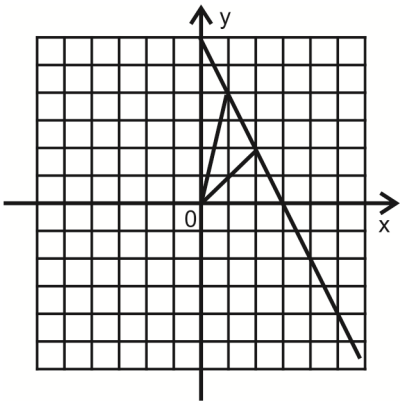
$y = 2(3 - x)$

$y = 6 - 2x$

Now taking some pairs of x and y to sketch the graph

x	0	1	2	3	4
y	6	4	2	0	–2

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b) $(x(t), y(t)) = (4\cos t, -3\sin t)$

Solution: First of all we eliminate the parameter t to find the ratio between x and y.

Here, $x(t) = 4\cos t$ .....(i)

$y(t) = -3\sin t$ .....(ii)

From (i) $\cos t = \frac{x}{4}$

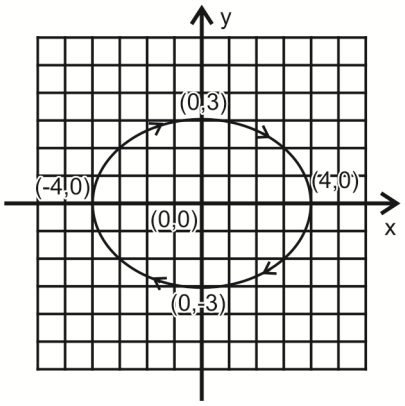
From (ii) $\sin t = -\frac{y}{3}$

Squaring and adding (i) and (ii)

$$\cos^2 t + \sin^2 t = \left(\frac{x}{4}\right)^2 + \left(-\frac{y}{3}\right)^2$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

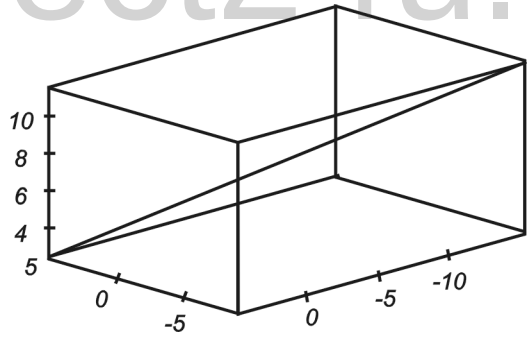
Which is the equation of ellipse with following graph.



c) $(x(t), y(t), z(t)) = (3+2t, 5-3t, 2-4t)$

t	0	1	2	3	4
(x, y, z)	(3, 5, 2)	(5, 2, –2)	(7, –1, –6)	(9, –4, –10)	(11, –7, –14)

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Limit of a function:

A function f(x) of variable x is said to have a limiting value L at x=c if f(x) approaches L as x approaches c, denoted by

$$\lim_{x\to c} f(x) = L$$

The limit exists only if

1. f(x) becomes arbitrarily close to L as x approach to c.
2. The limiting value of f(x) is same from both the left & right side of c.

Theorems on limits of functions:

1. **Constant rule:**

   If $f(x)=k$, where k is a constant, then $\lim_{x\to c} f(x) = k$.

2. **Limit rule:**

   $$\lim_{x\to c} x = c$$

3. **Multiple rule:**

   $$\lim_{x\to c} kf(x) = k\lim_{x\to c} f(x)$$

4. **Sum/Difference rule:**

   $$\lim_{x\to c}\left[f(x)\pm g(x)\right] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$$

5. **Product rule:**

   $$\lim_{x\to c}\left[f(x)\cdot g(x)\right] = \lim_{x\to c} f(x).\lim_{x\to c} g(x)$$

6. **Polynomial rule:**

   If p(x) is a polynomial, then $\lim_{x\to c} p(x) = p(c)$

7. **Quotient rule:**

   $$\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}; \lim_{x\to c} g(x) \neq 0$$

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8. **Power rule:**

$$\lim_{x\to c}[f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$$

9. **Equal functions rule:**

If $f(x) = g(x)$ for all x, then $\lim_{x\to c} f(x) = \lim_{x\to c} g(x)$

Theorems on limits of sequences:

For the sequences a_n and b_n, if $\lim_{n\to\infty} a_n = L$ and $\lim_{n\to\infty} b_n = M$ then;

1. **Linearity rule:**

$$\lim_{n\to\infty}[ra_n + sb_n] = rL + sM.$$

2. **Product rule:**

$$\lim_{n\to\infty}(a_n b_n) = LM$$

3. **Quotient rule:**

$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{L}{M};\ M \neq 0$$

4. **Root rule:**

$$\lim_{n\to\infty}\sqrt[m]{a_n} = \sqrt[m]{L}$$

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Some important limits:

1. $$\lim_{x\to a}\left(\frac{x^n - a^n}{x - a}\right) = na^{n-1}$$

2. $$\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x = e$$

3. $$\lim_{x\to 0}(1+x)^{\frac{1}{x}} = e$$

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4. $\lim_{x\to 0} \frac{a^x - 1}{x} = \ln a$

5. $\lim_{x\to 0} \frac{(1+x)^n - 1}{x} = n$

6. $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Continuous and discontinuous functions:

The left-hand limit of a function f(x) is L if the value of function approaches L as x approaches to c from the left i.e. x<c, denoted as

$$\lim_{x\to c^-} f(x) = L$$

Right-hand limit of a function f(x) at x=c is L if the value of function approaches L as x approaches c from right i.e. x>c, denoted as

$$\lim_{x\to c^+} f(x) = L$$

A function has a limiting value L at x=c if both the left and right-hand limits at c exist and are equal.

$$\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = L$$

$$\lim_{x\to c} f(x) = L$$

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A function is said to be continuous at a point x=c if

1. The function is defined at c; i.e. f(c) is define.
2. The function has a definite limiting value at c i.e. $\lim_{x\to c} f(x)$ exists
3. The limiting value of function at c is same as the value of function at c. i.e. $\lim_{x\to c} f(x) = f(c)$

A function is said to be continuous in an interval, if the function is continuous at every point of that interval.

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EXERCISE 1.4

Q1. Evaluate the following limits.

a) $\lim\limits_{x\to -2}\left(x^2+3x-7\right)$

Solution: $\lim\limits_{x\to -2}\left(x^2+3x-7\right)$

$=\lim\limits_{x\to -2}\left[x^2\right]+\lim\limits_{x\to -2}[3x]-\lim\limits_{x\to -2}[7]$

Sum rule

$=(-2)^2+3(-2)-7$

$=4-6-7$

$=-9$

b) $\lim\limits_{x\to 3}(x+5)(2x-7)$

Solution: $\lim\limits_{x\to 3}(x+5)(2x-7)$

$=\lim\limits_{x\to 3}(x+5).\lim\limits_{x\to 3}(2x-7)$

Product rule

$=(3+5)(2(3)-7)$

$=(8)(-1)$

$=-8$

c) $\lim\limits_{z\to 1}\dfrac{z^2+z-3}{z+1}$

Solution: $\lim\limits_{z\to 1}\dfrac{z^2+z-3}{z+1}$

$=\dfrac{\lim\limits_{z\to 1}\left(z^2+z-3\right)}{\lim\limits_{z\to 1}(z+1)}$

Quotient rule

$=\dfrac{(1)^2+(1)-3}{1+1}$

$=\dfrac{-1}{2}$

d) $\lim\limits_{x\to 4}\left(\dfrac{1}{x}+\dfrac{3}{x-5}\right)$

Solution: $\lim\limits_{x\to 4}\left(\dfrac{1}{x}+\dfrac{3}{x-5}\right)$

Sum rule

$=\lim\limits_{x\to 4}\left(\dfrac{1}{x}\right)+\lim\limits_{x\to 4}\left(\dfrac{3}{x-5}\right)$

Quotient Rule

$=\dfrac{\lim\limits_{x\to 4}(1)}{\lim\limits_{x\to 4}(x)}+\dfrac{\lim\limits_{x\to 4}(3)}{\lim\limits_{x\to 4}(x-5)}$

$=\dfrac{1}{4}+\dfrac{3}{4-5}$

$=\dfrac{1}{4}+\dfrac{3}{-1}=\dfrac{1}{4}-\dfrac{3}{1}$

$=\dfrac{1-12}{4}$

$=-\dfrac{11}{4}$

e) $\lim\limits_{x\to 1}\left[\dfrac{x^2+3x+2}{x^2+x+2}\right]^2$

Solution: $\lim\limits_{x\to 1}\left[\dfrac{x^2+3x+2}{x^2+x+2}\right]^2$

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Power rule

$$= \left[\frac{\lim\limits_{x\to 1}\left(x^2+3x+2\right)}{\lim\limits_{x\to 1}\left(x^2+x+2\right)}\right]^2$$

Quotient Rule

$$= \left(\frac{(1)^2+3(1)+2}{(1)^2+1+2}\right)^2$$

$$= \left(\frac{6}{4}\right)^2 = \frac{\cancel{36}}{\cancel{16}}$$

$$= \frac{9}{4}$$

f) $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}$

Solution: Given $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}$

$$\frac{\sqrt{1}-1}{1-1} = \frac{0}{0}$$

Direct substitution of x=1 results is an indeterminate form $\left(\frac{0}{0}\right)$

Multiplying and dividing by $\sqrt{x}+1$

Thus, $\lim\limits_{x\to 1}\dfrac{\sqrt{x}-1}{x-1}\times\dfrac{\sqrt{x}+1}{\sqrt{x}+1}$

$$= \lim_{x\to 1}\frac{\left(\sqrt{x}\right)^2-(1)^2}{(x-1)\left(\sqrt{x}+1\right)}$$

$$= \lim_{x\to 1}\frac{\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt{x}+1\right)}$$

$$= \lim_{x\to 1}\frac{1}{\left(\sqrt{x}+1\right)}$$

$$= \frac{1}{\lim\limits_{x\to 1}\left(\sqrt{x}+1\right)} = \frac{1}{\sqrt{1}+1}$$

$$= \frac{1}{2}$$

g) $\lim\limits_{x\to 0}\dfrac{\frac{1}{x+3}-\frac{1}{3}}{x}$

Solution: Given $\lim\limits_{x\to 0}\dfrac{\frac{1}{x+3}-\frac{1}{3}}{x}$

$$= \frac{\frac{1}{0+3}-\frac{1}{3}}{0} = \frac{\frac{1}{3}-\frac{1}{3}}{0} = \frac{0}{0}$$

Direct substitution of x=0 results in an indeterminate from $\left(\frac{0}{0}\right)$

Thus, $\lim\limits_{x\to 0}\dfrac{\frac{1}{x+3}-\frac{1}{3}}{x}$ (LCM)

$$= \lim_{x\to 0}\frac{\left(\frac{\cancel{3}-x-\cancel{3}}{3(x+3)}\right)}{x}$$

$$= \lim_{x\to 0}\frac{-\cancel{x}}{3(x+3)\cancel{(x)}}$$

$$= \lim_{x\to 0}\frac{-1}{3(x+3)}$$

Using limit

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$$= \frac{-1}{3(0+3)}$$

$$= -\frac{1}{9}$$

h) $\lim\limits_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1}$

Solution: $\lim\limits_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1}$

$$= \frac{\frac{1}{1}-1}{1-1} = \frac{1-1}{0} = \frac{0}{0}$$

Direct substitution of x=1 result is indeterminate form $\left(\frac{0}{0}\right)$

Thus, $\lim\limits_{x\to 1} \dfrac{\frac{1}{x}-\frac{1}{1}}{x-1}$ can be re-written as

$$= \lim_{x\to 1} \frac{\left(\frac{1-x}{x}\right)}{\left(\frac{x-1}{1}\right)}$$

$$= \lim_{x\to 1} \frac{1-x}{x} \times \frac{1}{x-1}$$

$$= \lim_{x\to 1} \frac{-(x-1)}{x(x-1)}$$

$$= \lim_{x\to 1} \frac{-1}{x}$$

$$= \frac{-1}{1}$$

$$= -1$$

i) $\lim\limits_{x\to 0} \dfrac{1-\sin x}{\cos^2 x}$

Solution: $\lim\limits_{x\to 0} \dfrac{1-\sin x}{\cos^2 x}$

$$= \frac{\lim\limits_{x\to 0}(1-\sin x)}{\lim\limits_{x\to 0}(\cos^2 x)}$$

Quotient Rule

$$= \frac{1-\sin 0}{\cos^2 0}$$

$$= \frac{1-0}{(1)^2} = \frac{1}{1}$$

$$= 1$$

j) $\lim\limits_{x\to 0} \dfrac{\tan x}{x}$

Solution: $\lim\limits_{x\to 0} \dfrac{\tan x}{x} = \dfrac{\tan(0)}{0} = \dfrac{0}{0}$

Direct substitution of x=0, results in indeterminate for $\left(\frac{0}{0}\right)$

So, $\lim\limits_{x\to 0} \dfrac{\tan x}{x}$

Can be re-written as

$$= \lim_{x\to 0} \frac{\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{x}{1}\right)}$$

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$$= \lim_{x\to 0} \frac{\sin x}{\cos} . \frac{1}{x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} . \frac{1}{\cos x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} . \lim_{x\to 0} \frac{1}{\cos x}$$

$$= (1)\frac{1}{(1)}$$

$$= 1$$

k) $\lim\limits_{x\to 0} \dfrac{\mathbf{secx-1}}{\mathbf{xsecx}}$

Method-I

Solution: $\lim\limits_{x\to 0} \dfrac{\sec x - 1}{x \sec x}$

$$= \frac{\sec(0)-1}{0.\sec 0} = \frac{1-1}{0} = \frac{0}{0}$$

Direct substitution of x=0 results in indeterminate form is $\left(\frac{0}{0}\right)$

Multiplying numerator and denominator by secx+1

$$= \lim_{x\to 0} \frac{\sec x - 1}{x \sec x} \times \frac{(\sec x + 1)}{(\sec x + 1)}$$

$$= \lim_{x\to 0} \frac{\sec^2 x - 1}{x \sec x (\sec x + 1)}$$

$$= \lim_{x\to 0} \frac{\tan^2 x}{x \sec x (\sec x + 1)}$$

$$= \lim_{x\to 0} \frac{\left(\sin^2 x\right)}{\left(\cos^2 x\right)} \Big/ x\left(\frac{1}{\cos x}\right)(\sec x + 1)$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x \cos x (\sec x + 1)}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} . \frac{\sin x}{\cos x (\sec x + 1)}$$

$$= 1 . \frac{\sin 0}{\cos 0 (\sec 0 + 1)} = \frac{0}{1(1+1)}$$

$$= 0$$

Method-II

Solution: $\lim\limits_{x\to 0} \dfrac{\sec x - 1}{x \sec x}$

As we know that

$$\because \sec x = \frac{1}{\cos x}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\cos x} - 1}{x . \frac{1}{\cos x}}$$

$$= \lim_{x\to 0} \frac{\frac{1-\cos x}{\cos x}}{\frac{x}{\cos x}}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{\cancel{\cos x}} \times \frac{\cancel{\cos x}}{x}$$

$$= \lim_{x\to 0} \frac{1-\cos x}{x}$$

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Hence again it $\left(\frac{0}{0}\right)$ form

Multiplying and dividing by $1+\cos x$

$$= \lim_{x\to 0} \frac{1-\cos x}{x} \times \frac{1+\cos x}{1+\cos x}$$

$$= \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{1-\cos^2 x}{x.(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x.(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} . \frac{\sin x}{(1+\cos x)}$$

$$= \lim_{x\to 0} \left(\frac{\sin x}{x}\right) . \lim_{x\to 0} \frac{\sin x}{(1+\cos x)}$$

$$= 1.\frac{\sin 0}{1\cos(0)} = \frac{1.0}{2} = \frac{0}{2} = 0$$

l) $\lim\limits_{x\to 0} \frac{\tan^2 x}{x}$

Solution: $= \frac{\tan^2(0)}{0} = \frac{0}{0}$

Direct substitution of x=0 results in indeterminate form is $\left(\frac{0}{0}\right)$

$$= \lim_{x\to 0} \frac{\tan^2 x}{x}$$

$$= \lim_{x\to 0} \frac{\left(\frac{\sin^2 x}{\cos^2 x}\right)}{x}$$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x\cos^2 x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} . \frac{\sin x}{\cos^2 x}$$

$$= 1.\frac{\sin 0}{\cos^2 0}$$

$$= \frac{1\times 0}{(1)^2}$$

$$= 0$$

Q2. Use Algebra and the rules of limits to evaluate the following limits.

a) $\lim\limits_{x\to 4} \frac{-6}{(x-4)^2}$

Solution: $\lim\limits_{x\to 4} \frac{-6}{(x-4)^2}$

$$= \frac{-6}{(4-4)^2} = \frac{-6}{0} = \text{undefined}$$

b) $\lim\limits_{x\to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$

Solution: Direct substitution of x=0 gives $\lim\limits_{x\to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$

$$= \lim_{x\to 0} \frac{\frac{1}{0+3} - \frac{1}{3}}{0}$$

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$$= \frac{\frac{1}{3} - \frac{1}{3}}{0} = \frac{0}{0}$$

Which is indeterminate form, so we rewrite the given limit as

$$\lim_{x\to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$$

$$= \lim_{x\to 0} \frac{\frac{3-x-3}{3(x+3)}}{\frac{x}{1}}$$

$$= \lim_{x\to 0} \frac{-x}{3x(x+3)}$$

$$= \lim_{x\to 0} \frac{-1}{3(x+3)}$$

$$= \frac{-1}{3(0+3)} = \frac{-1}{9}$$

c) $\lim_{x\to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5}$

Solution:

Direct substitution of x=5 gives

$$= \lim_{x\to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5}$$

$$= \frac{\lim_{x\to 5}\left(\sqrt{x} - \sqrt{5}\right)}{\lim_{x\to 5}(x-5)}$$

$$= \frac{\sqrt{5} - \sqrt{5}}{5-5} = \frac{0}{0}$$

Which is indeterminate form, so we rewrite the given limit as

$$= \lim_{x\to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5}$$

Multiplying and dividing by $\sqrt{x} + \sqrt{5}$

$$= \lim_{x\to 5} \frac{\sqrt{x} - \sqrt{5}}{x-5} \times \frac{\sqrt{x} + \sqrt{5}}{\sqrt{x} + \sqrt{5}}$$

$$= \lim_{x\to 5} \frac{\left(\sqrt{x}\right)^2 - \left(\sqrt{5}\right)^2}{(x-5)\left(\sqrt{x} + \sqrt{5}\right)}$$

$$= \lim_{x\to 5} \frac{\cancel{(x-5)}}{\cancel{(x-5)}\left(\sqrt{x} + \sqrt{5}\right)}$$

$$= \lim_{x\to 5} \frac{1}{\sqrt{x} + \sqrt{5}}$$

$$= \frac{1}{\lim_{x\to 5} \sqrt{x} + \sqrt{5}}$$

$$= \frac{1}{\sqrt{5} + \sqrt{5}}$$

$$= \frac{1}{2\sqrt{5}}$$

d) $\lim_{x\to 25} \frac{\sqrt{x} - 5}{x - 25}$

Solution:

Direct substitution of x=25 gives

$$\lim_{x\to 25} \frac{\sqrt{x} - 5}{x - 25}$$

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$$= \frac{\lim_{x\to 25}\left(\sqrt{x}-5\right)}{\lim_{x\to 25}(x-25)}$$

$$= \frac{\sqrt{25}-5}{25-25} = \frac{0}{0}$$

Which is indeterminate form

$$\lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}$$

Multiplying and dividing by $\sqrt{x}+5$

$$= \lim_{x\to 25}\frac{\sqrt{x}-5}{x-25}\times\frac{\sqrt{x}+5}{\sqrt{x}+5}$$

$$= \lim_{x\to 25}\frac{\left(\sqrt{x}\right)^2-(5)^2}{(x-25)\left(\sqrt{x}+5\right)}$$

$$= \lim_{x\to 25}\frac{\cancel{(x-25)}}{\cancel{(x-25)}\left(\sqrt{x}+5\right)}$$

$$= \lim_{x\to 25}\frac{1}{\sqrt{x}+5}$$

$$= \frac{1}{\sqrt{25}+5}$$

$$\frac{1}{5+5} = \frac{1}{10}$$

Q3. Find the limit of the convergent of the following sequences.

a) $\left\{\frac{5n}{n+7}\right\}$

Solution: We have to find

$$= \lim_{n\to\infty}\left(\frac{5n}{n+7}\right)$$

Direction substitution of $n=\infty$ gives $\left(\frac{\infty}{\infty}\right)$ which is an indeterminate form, so we divide the numerator and denominator by n, thus

$$= \lim_{n\to\infty}\left(\frac{5n}{n+7}\right)$$

$$= \lim_{n\to\infty}\left(\frac{\frac{5n}{n}}{\frac{n+7}{n}}\right) = \lim_{n\to\infty}\frac{5}{1+\frac{7}{n}}$$

Applying limit

$$= \frac{5}{1+\frac{7}{\infty}} = \frac{5}{1+0}$$

$$= 5$$

b) $\left\{\frac{4-7n}{8+n}\right\}$

Solution: We have to find

$$\lim_{n\to\infty}\left\{\frac{4-7n}{8+n}\right\}$$

Direct substitution of $n=\infty$ gives $\left(\frac{-\infty}{\infty}\right)$, which is an indeterminate form, so we divide numerator and denominator by n, thus.

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$$\lim_{n\to\infty}\frac{4-7n}{8+n}$$

Dividing by n

$$=\lim_{n\to\infty}\frac{\left(\frac{4-7n}{n}\right)}{\left(\frac{8+n}{n}\right)}=\lim_{n\to\infty}\frac{\frac{4}{n}-7}{\frac{8}{n}+1}$$

$$=\frac{\frac{4}{\infty}-7}{\frac{8}{\infty}+1}=\frac{0-7}{0+1}$$

$$=-7$$

c) $\left\{\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}\right\}$

Solution: We have to evaluate

$$\lim_{n\to\infty}\left\{\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}\right\}$$

Direct substitution of $n=\infty$ gives $\frac{\infty}{\infty}$, which is an indeterminate form, so we divided numerator and denominator by n, thus

$$\lim_{n\to\infty}\left\{\frac{8n-500\sqrt{n}}{2n+800\sqrt{n}}\right\}$$

$$=\lim_{n\to\infty}\frac{\left\{\frac{8n-500\sqrt{n}}{n}\right\}}{\left\{\frac{2n+800\sqrt{n}}{n}\right\}}$$

$$=\lim_{n\to\infty}\left\{\frac{8-\frac{500}{\sqrt{n}}}{2+\frac{800}{\sqrt{n}}}\right\}$$

$$=\frac{\lim_{n\to\infty}\left(8-\frac{500}{\sqrt{n}}\right)}{\lim_{n\to\infty}\left(2+\frac{800}{\sqrt{n}}\right)}$$

$$=\frac{8-\frac{500}{\sqrt{\infty}}}{2+\frac{800}{\sqrt{\infty}}}=\frac{8-0}{2+0}$$

$$=4$$

Q4. Weekly sales (in rupees) at big store x weeks after the end of an advertising campaign are given by.

$$S(x)=5000+\frac{3600}{x+2}$$

Find the sales for the indicated weeks limits

a) S(2)

Solution: As $S(x)=5000+\frac{3600}{x+2}$

$$S(2)=5000+\frac{3600}{2+2}$$

$$S(2)=5000+\frac{\cancel{3600}}{\cancel{4}}$$

$$S(2)=5000+900$$

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$S(2) = 5900$

b) $\lim_{x\to 5} S(x)$

Solution:

$$\lim_{x\to 5} S(x) = \lim_{x\to 5}\left[5000 + \frac{3600}{x+2}\right]$$

$$= 5000 + \lim_{x\to 5}\left[\frac{3600}{x+2}\right]$$

$$= 5000 + \frac{3600}{\lim_{x\to 5}(x+2)}$$

$$= 5000 + \frac{3600}{5+2}$$

$$= 5000 + \frac{3600}{7}$$

$$= 5000 + 514.28$$

$$= 5514.28$$

c) $\lim_{x\to 16} S(x)$

Solution:

$$\lim_{x\to 16} S(x) = \lim_{x\to 16}\left(5000 + \frac{3600}{x+2}\right)$$

$$= 5000 + \frac{3600}{\lim_{x\to 16}(x+2)}$$

$$= 5000 + \frac{3600}{16+2}$$

$$= 5000 + \frac{3600}{18}$$

$$= 5000 + 200$$

$$= 5200$$

Q5. Use the properties of continuous function to test the continuity and discontinuity of the following functions.

a) $f(x) = 2x - 3$

Solution:

Given function is, $f(x) = 2x - 3$

Since f(x) is a polynomial function, therefore defined for all real values of x and the limiting value of function as every x=c is the same at the value of f(x) at x=c, i.e.

$$\lim_{x\to c} f(x) = f(c)$$

Therefore, the given function is a continuous function for all real values of x.

b) $f(x) = 3 - 5x$

Solution:

Given function is $3 - 5x$

Since g(x) is a polynomial function, therefore is defined for all real values x and the limiting value of function at every x = c is the same as the value of g(x) at x=c, i.e.

$$\lim_{x\to c} g(x) = g(c)$$

c) $h(x) = \frac{2}{x-5}$

Solution: The function h(x) is defined for all values of x except x=5, where

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$$h(5)=\frac{2}{5-5}=\frac{2}{0}=\text{undefined}$$

Thus the given function h(x) is continuous function for all values of x accept x = 5.

d) $\mathbf{k(x)=\frac{x}{x+3}}$

Solution: The function k(x) is defined for all values of k except x= – 3, where

$$f(-3)=\frac{-3}{-3+3}=\frac{-3}{0}=\text{undefined}$$

Thus the given function k(x) is not a continuous function due to discontinuity at x =–3.

e) $\mathbf{g(x)=\frac{x-5}{(x-3)(x+2)}}$

Solution: The given function f(x) is continuous for all values of x except x=3 and x =–2

Where,

$$f(-2)=\frac{-2-5}{(-2-3)(-2+2)}=\frac{-7}{0}$$

$$=\text{Undefined}$$

And $$f(3)=\frac{3-5}{(3-3)(3+2)}=\frac{-2}{0}$$

$$=\text{Undefined}$$

So the given function is continuous for all values of x, except x =–2 and x =3.

f) $\mathbf{F(x)=\frac{1}{x(x+7)}}$

Solution: The given function f(x) is continuous for all values of x except x=0 and x =–7

Where,

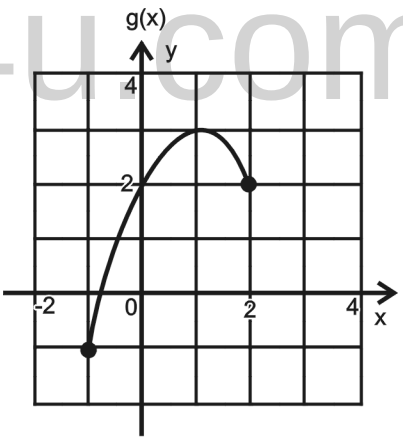
$$F(-7)=\frac{1}{-7(-7+7)}$$

$$F(-7)=\frac{1}{-7(0)}=\text{Undefined}$$

And $$F(0)=\frac{1}{0(0+7)}=\text{Undefined}$$

Thus the given function is continuous for all values of x, except x =0 and x =–7.

Q6. Use the graph of the function g(x) to answer the following questions.



g(x)= $-x^2+2x+2$

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a) Is g(x) continuous on the open interval (–1, 2)?

Solution:

The function g(x) satisfies all the conditions of continuity on the open interval (–1, 2) i.e. for all values of x=c in (–1, 2).

i) g(c) is defined

ii) $\lim_{x\to c} g(x)$ exist and

iii) $\lim_{x\to c} g(x) = g(c)$

So the function g(x) is continuous as the open interval (–1, 2).

b) Is g(x) continuous form the right at x=–1? Is $\lim_{x\to -1^+} g(x) = g(-1)$?

Solution: From the given figures

$$\lim_{x\to -1^+} g(-1) = -1$$

And g(–1) = –1

Thus $\lim_{x\to -1^+} g(x) = g(-1)$

And so g(x) is continuous from the right at x =– 1.

c) Is g(x) continuous form the left at x =2?

Is $\lim_{x\to 2^-} g(x) = g(2) = ?$

Solution: From the given figure,

$$\lim_{x\to 2^-} g(x) = 2 \text{ and } g(2)=2$$

Thus $\lim_{x\to 2^-} g(x) = g(2)$

And so g(x) is continuous from the left at x =2.

d) Is g(x) continuous on the closed interval [–1, 2]?

Solution: From the given figure,

$$\lim_{x\to -1^-} g(x) = \text{undefined}$$

So, the function is not continuous at x=–1 and therefore not continuous on closed interval [–1,2].

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