PROBLEMS

4.1 Find the resultant of the following forces:

(i) 10 N along x-axis

(ii) 6 N along y-axis and

(iii)4 N along negative x-axis. (8.5 N making 45° with x-axis)

Solution: F_x = Net force along x-axis = 10.4 = 6 N

F_y = Force along y-axis = 5 N

Magnitude of the resultant force = F = ?

$F = \sqrt{F_x^2 + F_y^2}$

$F = \sqrt{(6)^2 + (6)^2}$

$F = \sqrt{36 + 36}$

$= \sqrt{72} = 8.5$ N

Now, $\theta = \tan^{-1} = F_y / F_x$

$\theta = \tan^{-1} = 6 / 6$

$\theta = \tan^{-1} (1)$

$\theta = 45$ with x-axis

4.2 Find the perpendicular components of a force of 50 N making an angle of 30° with x-axis. (43.3 N, 25 N)

Solution: Force F = 50 N

Angle $\theta = 30$

$F_x = ?$ and $F_y = ?$

$F_x = F \cos \theta$

$F_x = 50 \times \cos 30$

$= 50\ N \times 0.866$ ($\because \cos 30 = 0.866$)

$F_x = 43.3\ N$

Similarly, $F_y = F \sin \theta$

$F_y = 50 \times 0.5$ ($\because \sin 30 = 0.5$)

$F_y = 25\ N$

4.3 Find the magnitude and direction of a force, if its x-component is 12 N and y-component is 5 N. (13 N making 22.6° with x-axis)

Solution: $F_x = 12\ N$

$F_y = 5\ N$

(i) Magnitude of the force = F = ?
(ii) Direction of the force = θ = ?

$F = \sqrt{F_x^2 + F_y^2}$
$F = \sqrt{(12)^2 + (5)^2}$
$F = \sqrt{144+25}$
$F = 13\ N$

(ii) $\theta = \tan^{-1} = F_y / F_x$

$\theta = \tan^{-1} = 12 / 5$

$\theta = \tan^{-1} (2.4)$

$\theta = 22.6$ with x-axis

Angle $\theta = 30$

$F_x = ?$ and $F_y = ?$

$F_x = F \cos \theta$

$F_x = 50 \times \cos 30$

$= 50\ N \times 0.866$ ($\because \cos 30 = 0.866$)

$F_x = 43.3\ N$

Similarly, $F_y = F \sin \theta$

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Solution: $F_x = 12\ N$

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(i) Magnitude of the force = F = ?
(ii) Direction of the force = θ = ?

$F = \sqrt{F_x^2 + F_y^2}$
$F = \sqrt{(12)^2 + (5)^2}$
$F = \sqrt{144+25}$
$F = 13\ N$

(ii) $\theta = \tan^{-1} = F_y / F_x$

$\theta = \tan^{-1} = 12 / 5$

$\theta = \tan^{-1} (2.4)$

$\theta = 22.6$ with x-axis

4.4 A force of 100 N is applied perpendicularly on a spanner at a distance of 10 cm from a nut. Find the torque produced by the force. (10 Nm)

Solution: Force = F = 100 N

Distance = L = 10 cm = 0.1 m

Torque T = ?

Torque T = F × L

= 100 N × 0.1 m

= 10 Nm

4.5 A force is acting on a body making an angle of 30° with the horizontal. The horizontal component of the force is 20 N. Find the force. (23.1 N)

Solution: Angle θ = 30 (with x-axis)

Horizontal component of force F_x = 20 N

Force F = ?

$F_x = F \cos \theta$

20 N = F cos 30

20 N = F × 0.866 (∵ cos 30 = 0.866)

F = 20 N / 0.866 = 23.09

F = 23.1 N

4.6 The steering of a car has a radius 16 cm. Find the torque produced by a couple of 50 N. (16 Nm)

Solution: Radius = r = L = 16 cm = 16/100 m = 0.16 m

Couple arm = L = 16 cm = 16/100 m = 0.16 m

Force = F = 50 N

Torque T = ?

Torque T = F × L

= 50 N × (2 × 0.16)

= 16 Nm

4.7 A picture frame is hanging by two vertical strings. The tensions in the strings are 3.8 N and 4.4 N. Find the weight of the picture frame. (8.2 N)

Solution: Tension T_1 = 3.8 N

Tension T_2 = 4.4 N

Weight of the picture frame = w = ?

When the picture is in equilibrium, then

$\sum F_x = 0$ and $\sum F_y = 0$

Therefore T – w = 0

Or $(T_1 + T_2) - w = 0$

$T_1 + T_2 = w$

3.8 + 4.4 = w

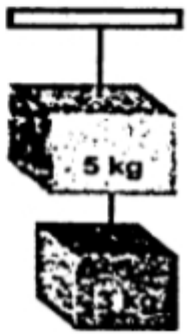
W = 8.2 N

4.8 Two blocks of mass 5 kg and 3 kg are suspended by the two strings as shown. Find the tension in each string. (80 N, 30 N)

Solution: Mass of large block = M = 5 kg

Mass of large block = m = 3 kg

Tension produced in each string = T_1 = ? and T_2 = ?



$T_1 = w_1 + w_2$

$T_1 = Mg + mg$

$T_1 = (M + m)g$

$T_1 = (3+5) \times 10$

$= 8 \times 10$

$= 80$ N

Also, $T_2 = mg$

$T_2 = 3 \times 10 = 30$ N

4.9 A nut has been tightened by a force of 200 N using 10 cm long spanner. What length of a spanner is required to loosen the same nut with 150 N force? (13.3 cm)

Solution: Force = $F_1 = 200$ N

Length = $L_1 = 10$ cm = 10 / 100 = 0.1 m

Length of the spanner to tighten the same nut:

Force = $F_2 = 150$ N

Length = $L_2 = ?$

Since $T_1 = T_2$

$F_1 \times L_1 = F_2 \times L_2$

$200 \times 0.1 = 150 \times L_2$

$20 = 150 \times L_2$

$L_2 = 20 / 150 = 0.133$ m $= 0.133 \times 100 = 13.3$ cm

4.10 A block of mass 10 kg is suspended at a distance of 20 cm from the center of a uniform bar 1 m long. What force is required to balance it at its center of gravity by applying the force at the other end of the bar? (40 N)

$T_1 = w_1 + w_2$

$T_1 = Mg + mg$

$T_1 = (M + m)g$

$T_1 = (3+5) \times 10$

$= 8 \times 10$

$= 80$ N

Also, $T_2 = mg$

$T_2 = 3 \times 10 = 30$ N

4.9 A nut has been tightened by a force of 200 N using 10 cm long spanner. What length of a spanner is required to loosen the same nut with 150 N force? (13.3 cm)

Solution: Force = $F_1 = 200$ N

Length = $L_1 = 10$ cm = 10 / 100 = 0.1 m

Length of the spanner to tighten the same nut:

Force = $F_2 = 150$ N

Length = $L_2 = ?$

Since $T_1 = T_2$

$F_1 \times L_1 = F_2 \times L_2$

$200 \times 0.1 = 150 \times L_2$

$20 = 150 \times L_2$

$L_2 = 20 / 150 = 0.133$ m $= 0.133 \times 100 = 13.3$ cm

4.10 A block of mass 10 kg is suspended at a distance of 20 cm from the center of a uniform bar 1 m long. What force is required to balance it at its center of gravity by applying the force at the other end of the bar? (40 N)

Solution: Mass of the block = m = 10 kg

Length of the bar = l = 1 m

Moment arm of w_1 = L_1 = 20 cm = 0.2 m

Moment arm of w_2 = L_2 = 50 cm = 0.5 m

Force required to balance the bar F_2 = ?

By applying principle of moments:

Clockwise moments = anticlockwise moments

$F_1 \times L_1 = F_2 \times L_2$

$mg \times L_1 = F_2 \times L_2$

$(10 \times 10) \times 0.2 = F_2 \times 0.5$

$20 = F_2 \times 0.5$

$F_2 = 20 / 0.5 = 200 / 5 = 40$ N

ClassNotes